Effective field theories for general extensions of the Standard Model with new particles

Juan Carlos Criado Álamo

Supervisor: Manuel Pérez-Victoria Moreno de Barreda

UNIVERSIDAD DE GRANADA

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Autor: Juan Carlos Criado Álamo
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Contents

CHAPTER 1

Introduction

The current situation in particle physics involves a large number of both theoretical proposals and experimental measurements. The relation between the two is often intricate, as every new physics model comes with its own set of motivations and predictions, and each measurement that is performed has consequences for many theoretical models. The purpose of this thesis is to set the basis for a general, organized and efficient way of dealing with these issues.

At first sight, a naive systematic approach to the problem may be devised: pick a representative set of models together with a sufficiently extensive set of observables, and compute every observable for each model. This procedure suffers from several drawbacks. Firstly, it is not easy to decide which models and observables to include: if there are too many, the task becomes impossible in practice, but one risks not being general enough otherwise. Secondly, it is inefficient: similar calculations will be performed many times. Lastly, it does not scale well: if a new kind of model becomes interesting, one has to recompute the value of all observables; and if a new experiment is designed, then one has to go back to every model to compute the observables that are going to be measured. Roughly speaking, the number of calculations that need to be performed grows as the product of the number of models and the number of observables of interest.

The use of an Effective Field Theory (EFT) solves these problems by splitting the calculations in two parts: matching high-energy models to the EFT and computing observables using the EFT only. Because an EFT is a general parametrization of the physics involving the degrees of freedom it contains (within some range of energies), it is guaranteed that no model or observable is discarded. Moreover, part of the repetitive work that had to be done for each model is included in the calculation of observables in the EFT, which only needs to be done once. In addition, the scaling with the number of models and the number of observables is improved: the number of calculations grows roughly like the sum of the two.[1]

Two EFTs are commonly used to describe the interactions of the Standard Model (SM) particles: the Higgs EFT (HEFT) and the Standard Model EFT (SMEFT) (see

[1] It should be noted that this works when working at a fixed precision. To improve the precision, it is necessary to extend the effective Lagrangian with extra terms, which means that both observables and matching must be recomputed.

section 3.7). They differ in how the SM symmetries are implemented. In the HEFT, the electroweak gauge group is realized non linearly. For this reason, perturbative unitarity is broken for energies around 4π times higher than the electroweak scale. In the SMEFT, all fields belong to linear representations of the electroweak group. It can be viewed as a particular case of the HEFT in which some relations between parameters are imposed. The advantage of using it (apart from its simplicity in comparison with the HEFT) is that it does not break perturbativity just above the electroweak scale. This means that its cutoff scale is arbitrary in principle.

At any rate, neither the SMEFT nor the HEFT can describe the resonant production of new particles that are not present in the SM. Their purpose is to describe the low-energy effects of such extra degrees of freedom, when there is some separation between their masses and the probed energies. They would not be of use in a hypothetical discovery of a new particle through its direct production in an experiment. That is, in the program introduced above of splitting the calculations that relate new physics models to experimental observables, these EFTs only cover indirect effects.

To describe resonances, it is necessary to introduce new fields. If one wishes to proceed in a fully general way, without theoretical prejudices or any further experimental knowledge about the high-energy physics, every possible new field should be included. In general, the extensions of the SM with new fields can be classified in two groups: those that contain unstable particles that decay into the SM ones and those that do not. Many of the concrete models for physics beyond the SM belong to the first class. For a field to create a particle that decays into the SM, it must have the same Lorentz and gauge quantum numbers as some composite SM operator. In this thesis, we construct an EFT for these fields together with the SM ones, which we call the Beyond the SM EFT (BSMEFT).

More precisely, the BSMEFT is an EFT with symmetry given by the linearly-realized SM gauge group, and whose field content consists of the SM fields together with those extra fields for which at least one linear coupling to the SM is allowed by Lorentz and gauge invariance (see chapter 7). The linear realization of the symmetries is required for perturbative unitarity to hold not much above the electroweak scale. The requirement of linear couplings to the SM is only made to obtain the quantum numbers of the new fields, and then all their relevant interactions are considered, including those that are non linear. Having these quantum numbers is a necessary condition for the new particles to have decays into the SM ones, but it is not sufficient: although their decay is allowed by the SM symmetries, it may be forbidden by new ones. In this way, the BSMEFT also includes many models with stable particles. On the other hand, the presence of linear couplings is necessary for leading-order effects in loop expansions: at tree level, only those fields with linear interactions can have single production, decay or indirect effects. The BSMEFT has a cutoff scale above the masses of the extra particles. At each order in the expansion in inverse powers of this cutoff, only a finite number of possibilities for the quantum numbers of the new fields are allowed by the linear coupling condition. This makes the theory manageable: the representations of all the fields and their Lagrangian can be explicitly written and studied.

The BSMEFT further splits the calculations connecting concrete models and experimental data, bringing its own advantages. Two tasks must be performed to relate its parameters to experimental observables in an efficient and systematic way: computing observables in which resonant production of new degrees of freedom may be important, and matching to the SMEFT. Then, for each model in the large class of those that are particular cases of the BSMEFT, one does not need to do any calculations. Instead, one just has to identify how it fits inside the BSMEFT. The relation with observables and with the SMEFT is automatically given by particularizing the general calculations, which can be done once and for all. If a new physics model is not a particular case of the BSMEFT, but its lightest particles are contained in the BSMEFT, the heavier ones can always be integrated out, and the result treated using the BSMEFT.

An example of the usefulness of the BSMEFT is given by matching it at tree level to the SMEFT with operators of dimension 6 or less. This is done in this work (in chapter 8). The result is a complete tree-level dictionary between extensions of the SM with new particles and the dimension-6 SMEFT. This dictionary can be used to translate constraints over the SMEFT coming from experimental data into bounds over the parameters of models with new particles. If a deviation from the SM is detected and parametrized with the SMEFT, one can use the dictionary to find out which possible new particles can generate it. For example, one may obtain all the representations and interactions of the new fields that can generate the observed anomalies in LHCb (as done in section 8.6), or enumerate all high-energy models with tree-level indirect effects in Higgs physics (as in section 8.7).

Another application of the BSMEFT that we consider in this thesis (in chapter 9) is the model-independent study of vector-like quarks. They appear in many well-motivated scenarios beyond the SM. A model-independent analysis can be performed using the adequate sector of the BSMEFT. This allows for the study of both direct and indirect effects. General properties of vector-like quarks can be extracted, which apply to any model that contains them.

In the context of EFTs for physics beyond the SM, one has to deal with large numbers of operators and fields. For this reason, it is convenient and even necessary in practice to develop computer tools that make faster and less error-prone calculations. In this work, we present two such tools, whose aim is to automatize some of the most common tasks one has to perform in this setting (see part II).

In particular, the use of bases of operators is of great practical importance. They drastically reduce the number of operators that must be included in the effective Lagrangian. To rewrite a Lagrangian in terms of a basis, field redefinitions must be performed. At leading order in the EFT expansion, this is equivalent to the use of equations of motion. Higher order terms may be important in the EFTs we deal with for a number of reasons. For example, they could give the leading contribution to some observables, if the symmetries forbid contributions from lower order terms. In this case, it becomes crucial to understand the effects of field redefinitions at higher orders, which we also study in this thesis (in chapter 4).

The thesis is organized as follows:

- In part I, several topics related to EFTs as used in particle physics are discussed. Chapter 3 is a review of the subject. It introduces the EFT construction and some related ideas, such as power counting, renormalization and matching. It also gives a presentation of non-abelian gauge theories and the particular case of the SMEFT. In chapter 4, the effects of field redefinitions at higher orders are analyzed. It is shown that they cannot be reproduced using just the equations of motion. The interplay between redefinitions, renormalization and matching is also studied.

- In part II, two computer tools are presented. `MatchingTools`, which is introduced in chapter 5, is a Python package that does two kinds of EFT calculations: tree-level matching and reduction of an effective Lagrangian to a basis of operators. `BasisGen`, another Python package, presented in chapter 6, computes operators bases for EFTs. Both tools work in a very general setting: any Lorentz-invariant non-abelian gauge theory can be treated with them.

- Part III is dedicated to general extensions of the SM with new particles. We study them using the BSMEFT, which is introduced in chapter 7. The representations under the SM symmetry group of all new fields of the BSMEFT are presented, together with a general effective Lagrangian for them. In chapter 8, the full tree-level matching of the BSMEFT to the SMEFT is performed and the dictionary obtained from it is presented. Two examples of use of this dictionary are given. In chapter 9, the sector of the BSMEFT that only contains new quarks is studied. We focus on the case in which their couplings are not necessarily renormalizable. As explained there, vector-like quarks with non-renormalizable interactions have new features that are not present when only renormalizable interactions are allowed, including new production and decay channels, and, in some cases, long lifetimes.

Most of the results presented in this thesis have been published in the following articles: part of the language and notation used in chapter 3 and the results in chapter 4 can be found in ref. [3]; `MatchingTools` and `BasisGen` were presented in ref. [4] and ref. [5], respectively; the representations and Lagrangian that appear in chapter 7 were constructed in refs. [6–10]; the dictionary and first example given in chapter 8 were presented in ref. [10]; the second example in chapter 8 was presented in ref. [11]; and the results in chapter 9 appeared in ref. [12].

CHAPTER 2

Introducción

La situación actual en física de partículas involucra un gran número de propuestas teóricas y medidas experimentales. La relación entre ambas es muchas veces compleja, ya que cada modelo de nueva física tiene su propio conjunto de motivaciones y predicciones, y cada medida que se realiza tiene consecuencias para muchos modelos teóricos. El propósito de esta tesis es sentar las bases de una estrategia general, organizada y eficiente para lidiar con estos problemas.

A primera vista, se puede pensar en una aproximación sistemática sencilla a este problema: elegir un conjunto representativo de modelos, junto con un conjunto de observables suficientemente completo, y calcular todos los observables en cada modelo. Este procedimiento tiene varias desventajas. Primero, no es fácil decidir qué modelos y observables incluir: si hay demasiados, la tarea será imposible en la práctica, pero se está en riesgo de no ser suficientemente general en caso contrario. Segundo, es ineficiente: muchas veces se realizarán cálculos similares. Por último, no tiene buena escalabilidad: si un nuevo tipo de modelo resulta interesante, hay que recalcular el valor de todos los observables; y si se diseña un nuevo experimento, entonces hay que volver a cada modelo para calcular los observables que se van a medir. *Grosso modo*, el número de cálculos a realizar crece como el producto del número de modelos y el número de observables de interés.

El uso de una Teoría de Campos Efectiva (EFT, por sus siglas en inglés) resuelve estos problemas dividiendo los cálculos en dos partes: *matching* de modelos de altas energías con la EFT y cálculo de observables usando únicamente la EFT. Como una EFT es una parametrización general de la física que involucra los grados de libertad que contiene (dentro de cierto rango de energías), está garantizado que ningún modelo u observable se va a descartar. Además, parte del trabajo repetitivo que tenía que hacerse para cada modelo está incluido en el cálculo de observables de la EFT, que sólo ha de realizarse una vez. La escalabilidad con el número de modelos y el número de observables también mejora: el número de cálculos crece aproximadamente como la suma de los dos.[1]

[1]Nótese que esto funciona cuando se trabaja a una precisión fija. Para mejorar la precisión, es necesario extender el Lagrangiano efectivo con términos extra, lo cual significa que tanto los observables como el *matching* tiene que ser recalculados.

Hay dos EFTs que se usan habitualmente para describir las interacciones de las partículas del Modelo Estándar (SM): la EFT del Higgs (HEFT) y la EFT del Modelo Estándar (SMEFT) (ver sección 3.7). Estas difieren en la implementación de las simetrías del SM. En la HEFT, el grupo *gauge* electrodébil tiene una realización no lineal. Por esta razón, unitariedad está rota a nivel perturbativo para energías alrededor de 4π veces más altas que la escala electrodébil. En la SMEFT, todos los campos pertenecen a representaciones lineales del grupo electrodébil. Se la puede ver como un caso particular de la HEFT en el que se imponen algunas relaciones entre parámetros. La ventaja de usarla (aparte de su simplicidad en comparación con la HEFT) es que no rompe perturbatividad justo debajo de la escala electrodébil. Esto significa que su escala de *cutoff* es arbitraria en principio.

En cualquier caso, ni la SMEFT ni la HEFT pueden describir la producción resonante de nuevos grados de libertad que no estén presentes en el SM. Su propósito es describir los efectos a bajas energías de estos grados de libertad extra, cuando hay cierta separación entre sus masas y las energías exploradas. No serían de utilidad en el hipotético descubrimiento de una nueva partícula a través de su producción directa en un experimento. Esto es, en el programa que hemos introducido anteriormente consistente en dividir los cálculos que relacionan modelos de nueva física con observables experimentales, estas EFTs solo incluyen efectos indirectos.

Para describir resonancias, es necesario introducir nuevos campos. Si se quiere proceder de manera completamente general, sin prejuicios teóricos ni ningún conocimiento experimental sobre la física de altas energías, todo nuevo campo posible debe ser incluido. En general, las extensiones del SM con nuevos campos pueden clasificarse en dos grupos: aquellas que contienen partículas inestables y aquellas que no. Muchos de los modelos concretos para física más allá del SM pertenecen a la primera clase. Para que un campo cree partículas que decaen al SM, tiene que tener los mismos números cuánticos que algún operador compuesto del SM. En esta tesis, construiremos una EFT para estos campos junto con los del SM, la cual llamaremos la EFT Más Allá del SM (BSMEFT, del inglés Beyond SM EFT).

Más concretamente, la BSMEFT es una EFT con el grupo *gauge* del SM realizado linealmente, y cuyo contenido de campos consiste en los campos del SM junto con aquellos campos extra que tienen al menos un acoplamiento lineal con el SM que esté permitido por invariancia Lorentz y *gauge* (ver capítulo 7). La realización lineal de las simetrías se requiere para que exista unitariedad perturbativa no mucho más arriba de las escala electrodébil. La condición de acoplamiento lineal se utiliza solamente para obtener los números cuánticos de los nuevos campos, y después se tienen en cuenta todas sus interacciones relevantes, incluyendo aquellas que son no lineales. Tener estos números cuánticos es condición necesaria para que las nuevas partículas tengan desintegraciones que dan lugar a partículas del SM, pero no es suficiente: aunque su desintegración esté permitida por las simetrías del SM, puede estar prohibida por nuevas simetrías. De esta manera, la BSMEFT también incluye muchos modelos con partículas estables. Por otra parte, la presencia de acoplamientos lineales es necesaria para tener efectos a orden dominante en expansiones en *loops*: a nivel árbol, sólo aquellos campos con interacciones lineales pueden tener producción simple, desintegración o efectos indirectos. La BSMEFT tiene una escala de *cutoff* por encima de las masas de las partículas extra. A cada orden en la expansión en potencias inversas del *cutoff*, solo un número finito de posibilidades para los números cuánticos de los nuevos campos está permitido por la restricción de acoplamiento lineal. Esto hace que la teoría sea tratable: las representaciones de todos los nuevos campos y su Lagrangiano pueden escribirse y estudiarse explícitamente.

La BSMEFT divide de nuevo en dos partes los cálculos necesarios para conectar modelos y datos experimentales, lo cual conlleva sus propias ventajas. Hay dos tareas a realizar para relacionar sus parámetros con observables experimentales de una manera eficiente y sistemática: calcular observables en los que la producción resonante de nuevos grados de libertad pueda ser importante, y realizar *matching* con la SMEFT. Para cada modelo entre todos aquellos que son casos particulares de la BSMEFT, no es necesario realizar ningún cálculo. En lugar de ello, sólo hay que identificar cómo encaja en dentro de la BSMEFT. La relación con observables y con la SMEFT puede encontrarse automáticamente particularizando los cálculos generales, lo cuales solo es necesario realizar una vez. Si un modelo de nueva física no es un caso particular de la BSMEFT, pero sus partículas más ligeras están contenidas en la BSMEFT, las más pesadas siempre pueden eliminarse mediante su integración, y el resultado tratado usando la BSMEFT.

Un ejemplo de la utilidad de la BSMEFT aparece cuando se considera su *matching* a nivel árbol con la SMEFT con operadores de dimensión 6 o inferior. Esto se realizará en este trabajo (en el capítulo 8). El resultado es un diccionario completo a nivel árbol entre extensiones del SM con nuevas partículas y la SMEFT de dimensión 6. Este diccionario puede usarse para traducir restricciones experimentales sobre la SMEFT a límites sobre los parámetros de modelos con nuevas partículas. Si se detecta una desviación del SM y se parametriza usando la SMEFT, se puede usar el diccionario para encontrar qué posibles nuevas partículas pueden generarla. Por ejemplo, se pueden obtener todas las representaciones y interacciones de los nuevos campos que pueden generar las anomalías de LHCb (como se hace en la sección 8.6), o enumerar todos los modelos de altas energías con efectos indirectos a nivel árbol en física del Higgs (como en la sección 8.7).

Otra aplicación de la BSMEFT que consideramos en esta tesis (en el capítulo 9) es el estudio independiente del modelo de *quarks vector-like*. Estos aparecen en muchos escenarios bien motivados de física más allá del SM. Un estudio independiente del modelo puede realizarse utilizando el sector adecuado de la BSMEFT. Esto permite estudiar efectos tanto directos como indirectos. Así se pueden extraer propiedades generales de *quarks vector-like*, que aplican a cualquier modelo que los contenga.

En el marco de EFTs para física más allá del SM, se trabaja con grandes cantidades de operadores y campos. Por esta razón, es conveniente e incluso necesario en la práctica el desarrollo de herramientas computacionales que realicen cálculos más rápidamente y con una menor propensión a errores. En este trabajo, presentamos dos de estas herramientas, cuyo objetivo es automatizar algunas de las tareas más comunes que se han de realizar en este contexto (ver parte II).

En particular, el uso de bases de operadores es de gran importancia práctica. Estas reducen drásticamente el número de operadores que se deben incluir en el Lagrangiano efectivo. Para reescribir un Lagrangiano en términos de una base, hay que realizar redefiniciones de campos. A orden dominante en la expansión de la EFT, esto es equivalente a usar las ecuaciones de movimiento. Los términos de orden superior pueden ser importantes en las EFTs con las que trabajamos por varias razones. Por ejemplo, podrían dar la contribución dominante a ciertos observables, si las simetrías prohíben contribuciones de términos de orden más bajo. En este caso, resulta crucial entender los efectos de redefiniciones a órdenes superiores, que también estudiaremos en esta tesis (en el capítulo 4).

La tesis está organizada de la siguiente manera:

- En la parte I, se discuten varios temas relacionados con EFTs tal y cómo se usan en física de partículas. El capítulo 3 es una introducción a la construcción de EFTs y algunas ideas que las rodean, como contaje de potencias, renormalización y *matching*. También se da una presentación de teorías *gauge* no abelianas y el caso particular de la SMEFT. En el capítulo 4 se analizan los efectos de redefiniciones a órdenes superiores. Se muestra que estos no se pueden reproducir usando solamente las ecuaciones de movimiento. También se estudia la relación de redefinciones con renormalización y *matching*.

- En la parte II, se presentan dos herramientas computacionales. `MatchingTools`, que se introduce en el capítulo 5, es un paquete de Python que hace dos tipos de cálculos en EFTs: *matching* a nivel árbol y reducción de un Lagrangiano efectivo a una base de operadores. `BasisGen`, otro paquete de Python, presentado en el capítulo 6, calcula bases de operadores para EFTs. Ambas herramientas trabajan en un marco muy general: pueden tratar con cualquier teoría *gauge* no abeliana invariante Lorentz.

- La parte III está dedicada extensiones general del SM con nuevas partículas. Estas se estudian usando la BSMEFT, que se introduce en el capítulo 7. Allí se presentan las representaciones bajo el grupo de simetría del SM de todos los nuevos campos de la BSMEFT, junto con un Lagrangiano efectivo general para estos. En el capítulo 8, se realiza el *matching* completo a nivel árbol entre la BSMEFT y la SMEFT, y se presenta el diccionario que se obtiene de este. Se dan dos ejemplos de uso de este diccionario. En el capítulo 9, se estudia el sector de la BSMEFT que solo contiene nuevos quarks, centrándose en el caso en el que sus acoplamientos no son necesariamente renormalizables. Como se explica allí, los *quarks* vector-like con interacciones no renormalizables tienen propiedades que no están presentes cuando solo se permiten interacciones renormalizables. Estas nuevas propiedades incluyen nuevos canales de producción y desintegración, y, en algunos casos, vidas medias largas.

La mayoría de resultados presentados en esta tesis se han publicado en los siguientes artículos: parte del lenguaje y notación utilizados en el capítulo 3 y los resultados en el capítulo 4 se pueden encontrar en la ref. [3]; `MatchingTools` y `BasisGen` se han presentado en la ref. [4] y la ref. [5], respectivamente; las representaciones y el Lagrangiano que aparecen en el capítulo 7 se han construido en las refs. [6–10]; el diccionario y el primer ejemplo dados en el capítulo 8 se han presentado en la ref. [10]; el segundo ejemplo en el capítulo 8 se ha presentado en la ref. [11]; y los resultados en el capítulo 9 han aparecido en la ref. [12].

Part I

Effective field theory and field redefinitions

CHAPTER 3

Effective field theories for particle physics

3.1 Introduction

The core idea underlying the Effective Field Theory (EFT) framework is the observation that the low-energy behavior of a physical system can be described without detailed knowledge about its high-energy physics [13–16]. More concretely, to parametrize the local quantum relativistic dynamics of a system below some energy Λ, it is sufficient to use a local quantum field theory that only includes particles with masses below Λ. The propagation of heavier degrees of freedom induces, in principle, non-local couplings of the light particles. However, these non-local interactions can be expanded as infinite towers of local ones, classified according to the strength of their effects. Once a finite precision is set, only a finite number of such interactions is needed to compute observables. The precision of the calculation can always be improved, at the price of introducing more interactions, which increases the number of free parameters.

This is the quantum field theory implementation of a procedure used across all areas of physics. Whenever there is a small parameter in the theoretical description of some physical phenomenon, a perturbative treatment of the problem can be performed. One expands the quantities of interest as power series in the small parameter. Setting it to zero gives the lowest order approximation. Then, corrections can be computed by including the first terms in the series. Better approximations are produced by taking into account more terms. In the EFTs used in particle physics, the small parameter is usually E/Λ, where E is the typical energy of the process being studied. Taking the limit $\Lambda \to \infty$ amounts to neglecting the new physics that may appear at (or above) the finite scales.

The applications of EFTs can be classified in two main categories: bottom-up and top-down. In the bottom-up approach, the high-energy physics are unknown, and the purpose of the EFT is to parametrize the low-energy physics. In the top-down case, one wants to study the low-energy regime of some theory. A simpler, more adequate description in terms of the relevant degrees of freedom in this regime can be constructed. In this context, the two theories are called "fundamental" and "effective",

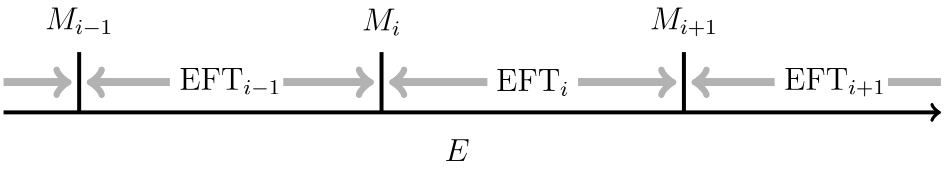

Figure 3.1: Tower of EFTs EFT_i. E is the typical energy at which each EFT is a good description. The M_i are the masses of the particles. EFT_{i-1} contains all the particles in EFT_i except for those with mass M_i, and is matched to EFT_i at the scale M_i.

respectively. The parameters of the effective theory are adjusted so that the value of an observable computed using the it matches the one obtained from the fundamental one. This procedure is known as matching.

The ideas of EFT are deeply related to renormalization. In the Wilsonian approach to this topic, a hard momentum cutoff is imposed to regulate divergent quantities. The Wilsonian renormalization group evolution is given by the variation of the parameters of the theory as the cutoff is changed. In this context, it is natural to think of regularized theories as EFTs for energies below the cutoff scale. This point of view had great influence, historically, in the acceptance of effective field theory as a well-defined framework for describing physical phenomena. However, we will not adopt it in this work, as it does not represent how EFTs are used in practical applications to particle physics today [13].

Instead, in modern particle physics phenomenology, mass-independent renormalization schemes are used, such as dimensional regularization with $\overline{\mathrm{MS}}$. EFTs become here a necessary technical tool. Although mass-independent schemes have many advantages, they present the problem that, for energies E much below the mass M of some particle in the theory, perturbative expansions break down, because of the appearance of large logarithms, as $\log(M/E)$. This happens because the renormalization group evolution towards the infrared for the parameters of the fundamental theory is not adequate when the scale M is crossed. A solution to this problem is achieved by matching the fundamental theory to an effective theory not including the particle with mass M. The renormalization group evolution of the effective theory eliminates the large logarithms. This leads to the standard practice for dealing with situations with several relevant scales: they can be described by a tower of EFTs (see figure 3.1). Each one of them parametrizes the physics between two consecutive mass thresholds. The renormalization group evolution can be performed in each theory, using matching to go from one to the next.

One important feature of EFTs is that they provide a systematic classification of interactions according to the relative size of their effects. Each term in an effective Lagrangian has a coefficient of the form c/Λ^n, where c is an adimensional parameter. For the perturbative expansion to work, c must be $\lesssim 4\pi$. The corresponding exponent n can be derived using dimensional analysis.[1] At tree level, the contribution of any diagram to some amplitude can be estimated as a product with one factor of $c(E/\Lambda)^n$ for each insertion of the corresponding operator. In principle, loops can break this direct relation, but it is recovered if a mass-independent renormalization scheme is

[1] Dimensional analysis arguments can be extended to include other parameters besides $1/\Lambda$. See section 3.3.

used. In this way, EFTs provide a framework in which the contribution to physical observables of each term in the Lagrangian is directly estimated.

Several EFTs are used to describe the different systems studied in particle physics. In the low-energy regime some of the relevant EFTs are: heavy quark effective theory [17, 18], non-relativistic QCD [19, 20], chiral perturbation theory [21, 22], and soft-collinear effective theory [23, 24]. At higher energies, just below the electroweak scale, the Weak Effective Theory (WET) is used [25–27]. Above the electroweak scale, one uses HEFT or the SMEFT, to study the physics of the SM degrees of freedom. We will the present in chapter 7 an EFT (which we call the BSMEFT) that is valid at even higher energies, above the masses of extra particles not present in the SM.

This chapter is organized as follows: section 3.2 is a brief review of the usual EFT construction in particle physics. In section 3.3, a general presentation of power-counting methods is given. Section 3.4 outlines the relation between EFTs and renormalization. In section 3.5 a general presentation of matching is given, together with a summary of one of the available methods for tree-level matching, which is used later in the thesis. Section 3.6 focuses on one of the most common types of EFTs in particle physics: gauge theories. One of the most important examples, the SMEFT, is presented in section 3.7. We conclude in section 3.8.

3.2 The effective field theory construction

This section is a review of the construction of an EFT to describe the scattering of a collection of quantum relativistic particles at energies below some scale Λ. It is assumed that their dynamics is weakly coupled. The main piece of information that is needed as an input for the construction is the set of all relevant particles with mass below Λ, which is the so-called cutoff scale.

There must be a unitary representation U of the (universal cover of the) Poincaré group $P = \mathbb{R}^4 \rtimes SL(2, \mathbb{C})$ over the Hilbert space of states. The one-particle states are eigenvectors of momentum and spin. Therefore, they can be labeled as $|p\sigma\rangle$, where p is the eigenvalue of momentum and σ denotes collectively the eigenvalue of spin and possibly the particle type. There is a one-to-one correspondence between particles and orbits of P in the space of one-particle states. Apart from Poincaré invariance, other symmetries may be present. The Hilbert space should also be equipped with unitary representations of them.

It is well known that quantum field theory is the suitable framework to describe the local interactions of these particles. For each value of the label σ, one introduces a quantum field φ such that $\langle 0| \varphi(0) |p\sigma\rangle \neq 0$. We will use the symbol ϕ to denote the collection of all fields, and index them using Latin letter indices. The implementation of Poincaré symmetry in the space of fields requires that there is a representation ρ of the Lorentz group over the target space. Then, the action of a Poincaré transformation (L, a) over the fields ϕ is given by

$$U(L, a)\phi^i(x)U(L, a)^\dagger = \rho^i_j(L^{-1})\phi^j(Lx + a). \tag{3.1}$$

All observed elementary particles have spin ≤ 1. The field-theoretical description of particles with spin 0 or 1/2 is straightforward: they correspond to scalar and spinor fields, respectively. Massive spin-1 particles are described by vector fields. For the massless case, gauge invariance must be introduced. Massive spin-1 particles can be embedded in a gauge theory through the Higgs mechanism.[2] Higher-spin particles can also be incorporated in the formalism described here. In general, those that are massless will also require gauge invariance.

A local operator at some spacetime point x is defined as a polynomial in the fields and their derivatives evaluated at x. The name *local operator* (or simply *operator*) is also used to refer to functions that assign, to every spacetime point x, a local operator at x. We make use of this last definition in this text. The set of all local operators naturally has the structure of an algebra. An important subalgebra is that of invariant operators, which are those that are invariant under the symmetry group, including the gauge group, if present. A local action is the spacetime integral of a local invariant operator, whereas a quasi-local action is an infinite sum of local ones. The invariant operator that appears inside the integral of the action is known as the Lagrangian density, or simply the Lagrangian.

Each operator that is a monomial in the fields represents a local interaction of the corresponding particles. Because there is an infinite number of them, an organizing principle is needed to determine the relative importance of each interaction. Any EFT should be equipped with a splitting of the algebra $\mathcal{A}$ of local operators into a direct sum

$$\mathcal{A} = \bigoplus_{n=n_{\min}}^{\infty} \mathcal{A}_n, \tag{3.2}$$

of finite-dimensional vector spaces $\mathcal{A}_n$. The subspace of invariant operators inherits this structure: $\mathcal{A}^{\text{inv}} = \bigoplus_{n=n_{\min}}^{\infty} \mathcal{A}_n^{\text{inv}}$, where the $\mathcal{A}_n^{\text{inv}}$ are finite dimensional. Then, it is prescribed that the action of the EFT is of the form

$$S[\phi] = \sum_{n=n_{\min}}^{n_{\max}} \sum_{\mathcal{O}\in\mathcal{B}_n} \lambda^n c_{\mathcal{O}} \int d^4x\, \mathcal{O}(\phi), \tag{3.3}$$

where $\mathcal{B}_n$ is a basis of $\mathcal{A}_n^{\text{inv}}$, $\lambda = 1/\Lambda$ is the inverse of the cutoff scale and the $c_{\mathcal{O}}$ are adimensional coefficients, known as Wilson coefficients. The splitting of the algebra of operators together with the prescription shown in eq. (3.3) is known as a power-counting rule, because it assigns a power of λ to certain operators. An insertion in some Feynman diagram of a term of the action containing a factor of λ^n will give a factor in the diagram of order $(E/\Lambda)^n$ or less, with E representing here the low-energy scales involved in the process.[3] Therefore, if one wishes to produce predictions up to a finite precision ε, it is sufficient to choose $n_{\max} \sim \log(\varepsilon)/\log(E/\Lambda)$ for the action to parametrize with full generality the physics of interest. To summarize, an EFT is defined by the following elements:

1. The particle content, including the spin of each particle. Equivalently, the field content and representation of the target space under the Lorentz group can be given.

2. The gauge group and global symmetry group, if they exist, together with their representation over the space of one-particle states, or, equivalently, over the target space of the fields.

[2] See section 3.6 for a review of gauge theories and the Higgs mechanism.
[3] This is preserved at the quantum level only if a mass-independent scheme is used. See section 3.4.

3. The power-counting rule.

Once these elements are provided, one can construct a general effective action as in eq. (3.3). For any finite precision ε one can find the correct $n_{\max}$. The number of free parameters of the theory, the Wilson coefficients $c_{\mathcal{O}}$, is then finite. One can use the experimentally measured values of any complete set of observables to fix the values of the $c_{\mathcal{O}}$. Then, predictions for other observables can be computed.

The purpose of an EFT as defined here is to parametrize the scattering amplitudes of the particles being considered. Once the general form of the effective action is set, scattering amplitudes can be computed in terms of the Wilson coefficients. We give below a brief presentation of how this is done. It is convenient to define the generating function, which, for a given action S, local operator F and source J is:

$$Z(S,F)[J] := \int \mathcal{D}\phi \, \exp\left\{iS[\phi] + J_\alpha F^\alpha(\phi)\right\}, \tag{3.4}$$

with the normalization $Z(S,F)[0] = 1$. We use here the unconventional notation $Z(S,F)$ with explicit parameters S and F because of its convenience for the discussion in this chapter and the next one (chapter 4). If $F(\phi) = \phi$, we will just write $Z(S)$. We ignore renormalization for the moment. We follow the convention of indicating the adjoints of complex fields with distinct labels i, in such a way that a sum over i includes both a field and its adjoint, if not real. Furthermore, we use the compact DeWitt notation $\phi^\alpha = \phi^i(x)$, with repeated collective indices indicating also integration over the space-time variables.

The functional derivatives of $Z(S,F)$ with respect to J_α at $J = 0$ are the Green functions for F in the theory defined by S. The momentum-space Green functions G are given by

$$G(S,F)^{i_1 \dots i_n}(p_1, \dots, p_n) := a^{i_1}_{\alpha_1}(p_1) \dots a^{i_n}_{\alpha_n}(p_n) \frac{\delta^n Z(S,F)}{\delta J_{\alpha_1} \dots \delta J_{\alpha_n}}, \tag{3.5}$$

where $a^i_{jx}(p) = \delta^i_j e^{ipx}$. Similarly, connected Green functions are derived from the function $W(S,F)[J] := -i \log\{Z(S,F)[J]\}$.

An operator F^i such that $\langle 0| F^i(0) |p\sigma\rangle \neq 0$ is called an interpolating field for particle σ. Interpolating fields are important because scattering amplitudes can be obtained from their Green functions. It is a fundamental property of such functions that they present poles when the sum of some of the momenta goes on-shell; that is, when the sum approaches the mass of the corresponding stable particle. Unstable particles also correspond to poles of Green functions, but the points in momentum space at which they are located have a non-vanishing imaginary part. New particles are often discovered through this property: in a collider experiment, one can study the dependence of the cross section on the invariant mass of the final state. If a bump appears in this distribution, it is interpreted as the effect of a pole in the complex plane, corresponding to some new unstable particle. The poles of stable particles are of course experimentally inaccessible, but they usually present measurable tails.

Scattering amplitudes are obtained from Green functions using the LSZ reduction formula. The former are, up to some constant factors, the residue of the later when all external momenta go on shell. Specifically, the LSZ formula is the following asymptotic relation:

$$
\begin{aligned}
G(S, F)^{i_1 \dots i_n}(p_1, \dots, p_r, -p_{r+1}, \dots, -p_n) \\
\sim \sum_{\sigma_1, \dots, \sigma_n} \left(\prod_{k=1}^{r} \frac{\langle p_k \sigma_k | F^{i_k}(0) | 0 \rangle}{p_k^2 - m_k^2} \right) \left(\prod_{k=r+1}^{n} \frac{\langle 0 | F^{i_k}(0) | p_k \sigma_k \rangle}{p_k^2 - m_k^2} \right) \\
\times \langle p_{r+1} \sigma_{r+1}, \dots, p_n \sigma_n | \mathbf{S}(S) | p_1 \sigma_1, \dots, p_r \sigma_r \rangle , \quad (3.6)
\end{aligned}
$$

as all momenta p_k^2 approach the physical mass m_k^2 of some particle for which F^{i_k} is an interpolating field. Here, $\mathbf{S}(S)$ is the S matrix (the infinite time evolution operator) for action S. Notice that this formula allows for the computation of S-matrix elements (scattering amplitudes) $\langle p_{r+1}\sigma_{r+1}, \dots | \mathbf{S}(S) | p_1 \sigma_1, \dots \rangle$ independently of which specific interpolation field F is used. We will return to this topic in section 4.2, as this is one of the key results that make EFTs invariant under field redefinitions.

3.3 Power counting

The EFTs of interest often depend on several parameters, which can be taken to be the cutoff scale Λ and additional dimensionless quantities, such as coupling constants, ratios of masses and 4π factors associated to loops. The EFT is organized as a multiple power series in $\lambda = 1/\Lambda$ and certain combinations of the parameters, which are assumed to be small (compared to the probed energies, if dimensionful). In the following we use η to refer to λ and any of these combinations. For example, chiral perturbation theory is arranged as a power series in λ with $\lambda = 1/(4\pi f)$ and f the pion decay constant. One could consider a simultaneous expansion in $1/f$ at each order in λ, but this expansion is conveniently resummed using the underlying structure of an spontaneously broken theory. To organize systematically these expansions, it is important to have a power-counting rule that assigns a number $N_\eta(\mathcal{O})$ to each operator $\mathcal{O}$ and each parameter η. Then, the "natural" coefficient of an operator $\mathcal{O}$ is given by

$$
C_{\mathcal{O}} \simeq \prod_{\eta} \eta^{N_\eta(\mathcal{O})}. \quad (3.7)
$$

For instance, in chiral perturbation theory, chiral counting dictates that $N_\lambda(\mathcal{O})$ is equal to the number of derivatives in $\mathcal{O}$. In some cases it is convenient to include in the specification of the operator not only fields and derivatives but also powers of particular coupling constants or masses, which are treated as spurions and taken into account in the counting. To guarantee the stability of the loop expansion, the power-counting rule should be such that all the diagrams that can generate an operator give a contribution that is similar to or smaller than its natural coefficient. In particular, this requires

$$
\Delta_\eta(\mathcal{O}_1 \mathcal{O}_2) = \Delta_\eta(\mathcal{O}_1) + \Delta_\eta(\mathcal{O}_2), \quad (3.8)
$$

where $\Delta_\eta(\mathcal{O}) = N_\eta(\mathcal{O}) + c_\eta$ for some c_η independent of the operator.

A simple power-counting rule for λ is derived from the canonical dimensions $\Delta(\mathcal{O})$ of the operators $\mathcal{O}$. One chooses $\Delta_\lambda(\mathcal{O}) = \Delta(\mathcal{O})$ and $c_\lambda = 4$, so that $N_\lambda(\mathcal{O}) = \Delta(\mathcal{O}) - 4$. This is just dimensional analysis: the energy dimensions of each operator are balanced by powers of energy scale Λ, making the action adimensional. The canonical dimension of an operator that is a product of fields and derivatives is the sum of the dimensions of the factors. Derivatives have $\Delta(\partial_\mu) = 1$. The dimension of any field can be derived from its kinetic term.

Naive dimensional analysis (NDA) [28–30] is a power-counting rule that extends dimensional analysis. It is appropriate in many circumstances and enjoys nice properties. In this case the actual numerical coefficients are expected to be approximately equal to their natural values when the UV completion is strongly coupled, and smaller than them when it is weakly coupled. Approximate symmetries or tunings in the fundamental theory can also give rise to smaller coefficients. Certain assumptions on the UV theory allow to incorporate these suppressions systematically in the power-counting rules [31].

3.4 Renormalization

The path integral in eq. (3.4) is a formal object without a well-defined meaning. As is well known, naive calculations at the quantum level give divergent results and a renormalization procedure is needed to obtain finite quantities. A renormalization scheme $\mathcal{R}$ is a regularization for the divergent integrals that appear, together with a substraction scheme: a prescription that changes the original action in a certain way, usually by adding new terms called counterterms. Schematically, eq. (3.4), gets modified into:

$$Z(S,F)[J] := \lim_{\epsilon\to 0}\int_{\mathcal{R}_\epsilon} \mathcal{D}\phi \, \exp\left\{ i\mathcal{R}_\epsilon(S)[\phi] + J_\alpha F^\alpha(\phi)\right\}, \tag{3.9}$$

where ϵ is a parameter known as the regulator, $\int_{\mathcal{R}_\epsilon}\mathcal{D}\phi$ is the regularized path integral and $\mathcal{R}_\epsilon(S)$ is the action including the counterterms prescribed by the renormalization scheme $\mathcal{R}$. The renormalization scheme should ensure that the limit in eq. (3.9) gives a finite result.

Renormalization introduces a new dimensionful parameter: the renormalization scale μ. One is free, in principle, to choose any value of μ and then proceed by fitting the $c_\mathcal{O}$ to experimental data. However, perturbative expansions are typically improved when μ is of the order the energy scales E involved in the calculation, because the expansion parameter usually contains factors of $\log(E/\mu)$. Therefore, it is useful to have a way of translating the values of $c_\mathcal{O}$ from one renormalization scale to another. This is achieved by solving the renormalization group equation, which is the requirement that physical quantities do not depend on μ. This can be implemented in several ways. A possibility is to impose

$$\frac{d}{d\mu} Z(S,F)[J] = 0. \tag{3.10}$$

This is to be understood as a differential equation over the $c_\mathcal{O}$, which are seen as functions of μ. Because $Z(S,F)$ is not a physical object, eq. (3.10) is only a sufficient condition for physical quantities to be independent of μ. As we will see in chapter 4, different actions can give rise to the same scattering amplitudes. Thus, another possibility for the implementation of the renormalization group is given by the equation $d\mathbf{S}(S)/d\mu = 0$, together with some condition that fixes a specific action among those that satisfy it.

The beta functions are defined as $\beta_{c_{\mathcal{O}}} := dc_{\mathcal{O}}/d\log\mu$. When they do not depend on μ, the renormalization scheme is said to be a mass-independent scheme. Such a scheme has the advantage of preserving the power counting of the EFT, so that the operators with a coefficient of order λ^n will always produce effects of order $(E/\Lambda)^n$. An example of a mass independent scheme is dimensional regularization with minimal substraction (MS) (or the more convenient $\overline{\text{MS}}$), which is widely used in particle physics.

The usage of a mass independent scheme comes with some disadvanges. The main problem is that, for a renormalization scale μ much below the mass of some particle in the theory, pertubative expansions are broken, because factors of large logarithms $\log(M/\mu)$ appear in the expansion parameters. The solution is to find an EFT which does not contain the particle of mass M, but reproduces the physical predictions of the original theory for energy scales $E < M$. We will see how this is done in the next section. The renormalization group evolution of the low-energy effective theory eliminates the large logarithms.

3.5 Matching

3.5.1 General considerations

In a matching calculation, one relates two theories by requiring that they produce the same predictions in some range of energies. An energy scale is fixed, separating what we call the high and low-energy regimes. The two theories that are matched are called the "fundamental" theory and "effective" theory. The fundamental theory is supposed to be a valid description of both the high-energy and the low-energy regimes, at least in principle. The effective theory is only valid for low energies, and only contains the relevant light degrees of freedom that are present in this regime. The matching condition is that the effective theory should give the same physical results as the fundamental one, but only in its range of validity. We make this requirement and its variations more precise later in this section.

One could wonder why is it that matching is a useful tool, as it produces an EFT (the effective theory) that merely reproduces the low-energy behavior of the fundamental one, while the information about its high-energy regime is lost. Despite this fact, performing matching calculations may provide many practical advantages. First, if one is going to concentrate only on the low-energy physics, it is often the case that the effective description is simpler and generally more convenient, as it contains a smaller number of degrees of freedom and parameters. Second, at the quantum level, it becomes necessary to employ an effective theory to remove large logarithms in mass-independent renormalization schemes, as explained in section 3.4. Finally, it is possible that the experimentally accessible physics belong to the low-energy regime and the correct fundamental theory is unknown. The effective theory then becomes a convenient way to parametrize experimental results. Different fundamental theories can be matched to it, and as a result their parameters become related to experimental data.

Let S_{UV} be the action of the fundamental theory, which contains a set of light fields ϕ and heavy fields Φ. That is, there is some scale Λ separating the masses of the fields

ϕ and Φ. An effective action $\bar{S}$ for the light fields can be defined by requiring that

$$Z(S_{\rm UV})[J] = Z(\bar{S})[J], \tag{3.11}$$

where the $Z(S_{\rm UV})[J]$ should be understood as having source terms $J_\alpha \phi^\alpha$ only for the light fields ϕ. If sources were added for the heavy fields, the effective action would also depend on them. Then,

$$\exp\left(i\bar{S}[\phi]\right) = \int \mathcal{D}\Phi \exp\left(iS_{\rm UV}[\Phi,\phi]\right). \tag{3.12}$$

The effective action $\bar{S}$ can be found by two equivalent methods: A) requiring that eq. (3.11) is satisfied, which amounts to *matching* the off-shell 1PI functions of the effective theory to the off-shell one-light-particle-irreducible functions of the fundamental theory; B) *integrating out* the heavy degrees of freedom explicitly, i.e. computing directly eq. (3.12), for instance using functional methods. In section 3.5.2, an algebraic method for tree-level matching belonging to class B is presented. This is the method that will be used later in this thesis, in chapters 5 and 8.

The action $\bar{S}$, obtained by any of these methods is non-local. However, a local effective action can be constructed to approximately reproduce the function $Z(\bar{S})$. The approximation is controlled by the dimensionful parameter $\lambda = 1/\Lambda$. Given a (non-local) effective action $\bar{S}$, we define $\lceil \bar{S} \rceil_n$ as the local action containing terms of order λ^n or less and such that

$$Z(\bar{S}) = Z(\lceil \bar{S} \rceil_n) + O(\lambda^{n+1}). \tag{3.13}$$

The exact action $\bar{S}$ and generating function $Z_{\bar{S}}$ can be viewed formally as infinite series in λ:

$$\bar{S}[\phi] = \sum_{k=0}^{\infty} \lambda^k \bar{S}_k[\phi], \qquad Z(\bar{S})[J] = \sum_{m=0}^{\infty} \lambda^m Z(\bar{S})_m[J]. \tag{3.14}$$

Note that each $\bar{S}_k$ is local, as adding derivatives to an operator increases its order in λ. But because $\bar{S}$ is non-local, it turns out that knowing $\bar{S}_k$ for $k \leq n$ for any given n is not enough, in general, to compute $Z(\bar{S})_n$. Even if any sum of a finite number of terms with $k > n$ gives a vanishing contribution to $Z(\bar{S})_n$, the tail $\sum_{k\geq N} \lambda^k \bar{S}_k$ of the series may contribute to it for arbitrarily large N. Therefore, the naive truncation $\lfloor \bar{S} \rfloor_n$ of $\bar{S}$ to order n does not coincide, in general, with the local effective action $S = \lceil \bar{S} \rceil_n$, which gives the correct approximation of $Z(\bar{S})$ to order n.

In the saddle-point expansion, this can be understood in the following way [32]. The saddle-point configuration, which gives the effective action at the tree level, is in practice approximated by a truncated expansion in λ, say to order N. Then, besides the usual quadratic and higher-order terms, the heavy-field expansion of the action about this non-exact saddle point includes linear terms suppressed by λ^{N+1} and higher powers of λ. Despite this suppression, such terms must be taken into account in the integral of the heavy fields Φ. Indeed, the quantum corrections may give contributions to orders $k < N$, independently of how large N is. The essential reason is that loop integrals regularized with dimensionless regulators probe all energy scales, including those higher than Λ. A way of finding these contributions within approach B, based on the method of regions [33], has been proposed in [34]. In the matching approach

A, correcting $\lfloor \bar{S} \rfloor_n$ to find $\lceil \bar{S} \rceil_n$ is not really an issue: in practice, the matching is performed directly between S_{UV} and $S = \lceil \bar{S} \rceil_n$ at some given n. The necessary contributions then appear automatically from diagrams in the fundamental theory involving loops of both light and heavy fields [32, 35, 36].

A renormalization procedure is required to make sense of all these equations. The matching can be performed at the regularized level (with the same dimensionless regulator). This leads to a regularized effective action that can be perturbatively renormalized. But it makes more physical sense to match the renormalized theories, as at the end of the day the aim of matching is to express the renormalized parameters of the local effective action S as functions of the renormalized parameters of S_{UV}. In method B, this can be achieved by adding counterterms to the UV action but refraining from removing the regulator; then the necessary counterterms in the effective theory will be generated (in the same regularization and renormalization scheme) during the matching procedure. The UV behaviours of the fundamental and effective theories are different, and so will be the counterterms. In the standard approach to matching within method A, the renormalized Green functions of the fundamental and effective theories are compared (with removed regulators). This allows great flexibility, as neither the regularization method nor the renormalization scheme need to be the same in both theories. The relation between renormalized parameters depends on these schemes. To preserve this relation, the effective theory should be used in the same scheme used for the matching. In this regard, observe that, because the effective theory is local, all its renormalized couplings and masses can be modified by finite counterterms. Hence, by adapting the scheme to each UV theory, all the UV information in the renormalized parameters of the effective theory can be erased. Scheme independence, however, ensures that the calculations done in such a scheme (which will depend on the UV parameters) will reproduce to the required order the low-energy predictions of the corresponding fundamental theory. In practice, it is preferable to see this information explicitly in the renormalized parameters, so a universal renormalization scheme, such as MS, should be used in the effective theory.

3.5.2 An algebraic method for tree-level matching

At the tree-level, eq. (3.12) becomes $\bar{S}[\phi] = S_{\mathrm{UV}}[\phi, \Phi_c(\phi)]$, where Φ_c is the solution to the (classical) equation of motion

$$\frac{\delta S_{\mathrm{UV}}}{\delta \Phi}[\phi, \Phi_c(\phi)] = 0. \tag{3.15}$$

The UV action splits as

$$S_{\mathrm{UV}}[\phi, \Phi] = -\frac{1}{2} Q_{\alpha\beta} \Phi^\alpha \Phi^\beta + S^\phi_{\mathrm{UV}}[\phi] + S^{\mathrm{int}}_{\mathrm{UV}}[\phi, \Phi], \tag{3.16}$$

where Q is some differential operator not containing any fields, S^ϕ_{UV} is the part of the action that only depends on ϕ and $S^{\mathrm{int}}_{\mathrm{UV}}$ contains only interaction terms. Then, eq. (3.15) can be solved iteratively as

$$\Phi_{c,0}(\phi) = 0, \qquad \Phi^\alpha_{c,k+1}(\phi) = (Q^{-1})^{\alpha\beta} \frac{\delta S^{\mathrm{int}}_{\mathrm{UV}}}{\delta \Phi^\beta}[\phi, \Phi_{c,k}(\phi)]. \tag{3.17}$$

The operator Q contains terms corresponding the masses M (or masses squared M^2) of the heavy fields. Its inverse can then be expanded in powers of ∂_μ/M. Each iteration of the procedure outlined in eq. (3.17) induces a correction $\Phi_{c,k+1} - \Phi_{c,k}$ containing only terms of canonical dimension higher than those in $\Phi_{c,k}$. In an EFT with power counting based on the canonical dimension of the operators, this means that the order of the corrections in the expansion in powers of λ increases with k. The expansion can be truncated at any desired order by reaching a sufficiently high value of k. The expression for Φ_c obtained in this way can be plugged into S_{UV}, giving a truncated expansion in λ for $\bar{S}$. Since we are working at tree level, we do not need to worry about the subtleties of truncated effective actions explained in section 3.5.1.

The procedure described here can be applied in a purely gauge-covariant manner. In eq. 3.16, terms in the action containing covariant derivatives may be split into the quadratic and interaction parts. If the full covariant derivatives are kept inside Q, all the objects in eq. (3.17) become covariant. We collect here an explicitly covariant form of the equations of motion (ready for the application of eq. 3.17) for the following types of fields:

- Scalars:
$$\Phi = \sum_{n=0}^{\infty} (-1)^n \frac{D^{2n}}{M^{2n+2}} \frac{\delta S_{\text{UV}}^{\text{int}}}{\delta \Phi^\dagger}. \tag{3.18}$$

- Fermions:
$$F = \frac{1}{M} \left(i\not{D} F + \frac{\delta S_{\text{UV}}^{\text{int}}}{\delta \bar{F}} \right). \tag{3.19}$$

- Vectors:
$$V = -\frac{1}{M^2} \sum_{n=0}^{\infty} R^n \frac{\delta S_{\text{UV}}^{\text{int}}}{\delta V^\dagger}, \qquad \text{where} \quad (RW)_\mu := \frac{D_\nu D_\mu - \eta_{\mu\nu} D^2}{M^2} W^\nu. \tag{3.20}$$

These equations are used in the implementation of `MatchingTools`, the computer tool for tree-level matching introduced in chapter 5. We will also use them in the construction of the tree-level UV/IR dictionary presented in chapter 8, which is computed both by hand and using `MatchingTools`.

3.6 Gauge theories and the Higgs mechanism

Most of the EFTs used today in particle physics phenomenology are gauge theories. They are used to incorporate spin-1 particles in a manifestly Lorentz-invariant quantum field theory. Vector fields have the correct transformation properties under the Lorentz group to describe them. However, the naive introduction of a vector field for each such particle presents some problems. First, vector fields seem to contain longitudinal polarizations that massless particles do not have. Second, the longitudinal polarizations of massive vector fields have interactions that break perturbative unitarity at energies not much higher than their masses, unless some method is used to restore it, such as the Higgs mechanism. A convenient description of both massless and massive vectors can be given in terms of gauge invariance.

We briefly introduce now the gauge theory construction and return latter to the issue of describing spin-1 particles. Let G be a reductive Lie group. For each of its simple or abelian factors G_k, consider a collection of vectors A^{ka}_μ, one for each generator t^{ka} of G_k. It is convenient to define the Lie algebra-valued field $A^k_\mu = t^{ka} A^{ka}_\mu$.[4] The corresponding field strength tensors are defined as $F^k_{\mu\nu} := \partial_\mu A^k_\nu - \partial_\nu A^k_\mu + i g_k [A^k_\mu, A^k_\nu]$, where the g_k are free parameters known as the gauge coupling constants. Other fields ψ can be present. A representation ρ_ψ of G_k over their target space should be specified. A gauge transformation is defined as a function Ω assigning, to each spacetime point x, an element $\Omega_k(x)$ of every G_k. Its action on the objects defined so far is given by

$$A^k_\mu \mapsto \Omega_k A^k_\mu \Omega_k^{-1} - \frac{i}{g_k} \Omega_k \partial_\mu \Omega_k^{-1}, \tag{3.21}$$

$$F^{ka}_{\mu\nu} \mapsto \mathrm{ad}(\Omega)^a_b F^{kb}_{\mu\nu}, \tag{3.22}$$

$$\psi^i \mapsto \rho_\psi(\Omega)^i_j \psi^j, \tag{3.23}$$

where ad is the adjoint representation and the components $F^{ka}_{\mu\nu}$ of the field strength tensor are defined by $F^k_{\mu\nu} = F^{ka}_{\mu\nu} t^{ka}$. A theory that is invariant under this set of spacetime point-dependent transformations is said to be gauge invariant. The group G is called the gauge group, the fields A^{ka}_μ are the gauge fields, etc. A gauge-covariant operator is a multiplet of operators $\mathcal{O}^i$ that, under a gauge transformation Ω, transforms as $\mathcal{O}^i \mapsto \rho_{\mathcal{O}}(\Omega)^i_j \mathcal{O}^j$, for some representation $\rho_{\mathcal{O}}$ of G. Both field strength tensors and matter fields are examples of covariant operators. Let $\rho(A^k_\mu)$ be the image of A^k_μ under the Lie algebra representation corresponding to ρ. The covariant derivative of a covariant operator $\mathcal{O}$ is defined as

$$D_\mu \mathcal{O}^i := \left(\partial_\mu \delta^i_j + i \sum_k g_k \, \rho_{\mathcal{O}} \left(A^k_\mu \right)^i_j \right) \mathcal{O}^j. \tag{3.24}$$

Covariant derivatives satisfy the basic properties of derivatives: linearity and the Leibniz rule. They have the advantage that, for any covariant operator $\mathcal{O}$, the derivative $D_\mu \mathcal{O}$ is also a covariant operator with the same representation $\rho_{\mathcal{O}}$. The Lagrangian of any gauge-invariant theory with matter fields can be written in terms of the field strength tensors, matter fields and covariant derivatives only.

Gauge theories can be used to describe both massless and massive spin-1 fields. In principle, a gauge transformation can be chosen so that the longitudinal polarizations of the gauge fields are eliminated. Thus, they contain the right number of degrees of freedom needed to describe massless particles. Moreover, a naive mass term $A_\mu A^\mu$ is forbidden by gauge invariance. It is therefore clear that massless spin-1 particles can be directly incorporated through gauge fields.

On the other hand, it seems at first sight that the gauge theory construction cannot accommodate the massive case. This impression turns out not to be true. Consider a gauge invariant Lagrangian and suppose that we want to give non-zero masses to some subset of the gauge fields, denoted by $A^{\hat{a}}_\mu$, that correspond to the generators $t^{\hat{a}}$ of a coset G/H for some subgroup H of the gauge group G. One possibility is to introduce extra terms in the Lagrangian, including the mass term $A^{\hat{a}}_\mu A^{\hat{a}\mu}$ but possibly others, that explicitly break gauge invariance because of the non-covariant appearance of $A^{\hat{a}}_\mu$.

[4] Summation over k is not implicit in this section

An equivalent gauge invariant description is obtained by adding a new scalar field $\xi^{\hat{a}}$ for each $t^{\hat{a}}$. They are collected in a G/H-valued covariant operator $U = e^{i\xi^{\hat{a}}t^{\hat{a}}/f}$ (with f a constant with dimensions of mass) that transforms under gauge transformations as $U \mapsto \Omega U$. All occurrences of $A^{\hat{a}}_\mu$ can then be rewritten in terms of $u_\mu = U^\dagger D_\mu U$. For example, a gauge-invariant mass term is then given by

$$\mathcal{L}_{\text{mass}} = f^2 \operatorname{tr}(u_\mu u^\mu). \tag{3.25}$$

In this way, the gauge bosons acquire a mass of the order of f. A gauge transformation can then be chosen to set the scalar fields $\xi^{\hat{a}}$ to zero, and then the vector fields have a non-vanishing longitudinal component. This is known as the unitary gauge. In this context, the scalar fields are said to be "eaten" by the gauge bosons. Alternatively, one can go to the gauge in which the longitudinal components of the vector are zero, but then they absorbed in non-zero scalar fields.

In this gauge-invariant version of the naive implementation of massive vector fields, it becomes clear that we are dealing with an EFT with cutoff scale not far away from the masses of the vectors. The interactions of the longitudinal components, which we can identify with the $\xi^{\hat{a}}$, always contain derivatives. They appear together with inverse powers of f, which balance the energy dimensions of the corresponding term in the Lagrangian. For this reason, perturbation theory breaks down at energies around $4\pi f$.

The breaking of the perturbativity in this setting can be traced back to the non-linearity of the realization of G in the space of scalar fields. The Higgs mechanism provides an extension of this model that realizes G linearly, restoring perturbativity at the scale $4\pi f$, and allowing for an arbitrary cutoff scale. The scalar sector must contain a field multiplet ϕ in a linear representation of G. Let H be the subgroup of G that leaves invariant any non-zero ϕ. The slices of the target space of ϕ for constant radial component $|\phi|$ will form (at least locally) the manifold G/H. The scalar potential $V(\phi)$ must have a degenerate set of minima given by the equation $|\phi| = v$, for some constant v with dimensions of energy. A vacuum expectation value (vev) $\langle|\phi|^2\rangle = v^2$ is generated. This is why this mechanism is sometimes referred to as "spontaneous symmetry breaking". With these elements, one can construct a mass term for the gauge bosons corresponding to the generators of G/H as

$$\mathcal{L}^{\text{kin}}_{\text{Higgs}} = D_\mu\phi^\dagger D^\mu\phi. \tag{3.26}$$

Then, their masses are of the order of the vev v of ϕ. The relation with the gauge-invariant approach not implementing the Higgs mechanism can be seen by decomposing $\phi = (1 + h/v)U\phi_0$, where U an element of G/H, h is a scalar with zero vev and ϕ_0 is a constant fixed at a minimum of V. Defining, as before $u_\mu = U^\dagger D_\mu U$, we have that

$$\mathcal{L}^{\text{kin}}_{\text{Higgs}} \supset v^2 \operatorname{tr}(u_\mu u^\mu). \tag{3.27}$$

If we were dealing with a global symmetry, the Goldstone bosons would be inside U. In this setting, they correspond to the scalar fields $\xi^{\hat{a}}$ that are eaten by the gauge bosons to get mass. Thus, the $\xi^{\hat{a}}$ are called "would-be Goldstone bosons". It can be proven that, because they are collected together with the extra degree of freedom h in a linear realization of G, the theory is regulated so that the cutoff is no longer restricted to be near the masses of the vectors.

Name	Flavors	Symbol(s)	Poincaré irrep	Mult.
Photon	–	γ/A	massless spin 1	1
Gluon	–	g	massless, spin 1	8
$W^{\pm}$ boson	–	$W^{\pm}$	massive, spin 1	2
Z boson	–	Z	massive, spin 1	1
up-type quarks	up, charm, top	u, c, t	massive, spin 1/2	3
down-type quark	down, strange, bottom	d, s, b	massive, spin 1/2	3
charged leptons	electron, muon, tau	e, μ, τ	massive, spin 1/2	1
neutrinos	e, μ, τ neut.	ν_e, ν_μ, ν_τ	massive, spin 1/2	1
Higgs boson	–	H	massive, spin 0	1

Table 3.1: Known elementary particles together with their irreducible representations under the Poincaré group and multiplity of their possibly degenerate one-particle states.

3.7 The effective theory approach to the Standard Model

The Standard Model (SM) of particle physics is the theory that describes the interactions of elementary particles. Since its proposal in the decade of 1960 [37, 38], all the new particles it predicted have been discovered: the tau lepton [39], the bottom quark [40], the electroweak gauge bosons [41, 42], the top quark [43, 44] and the Higgs boson [45, 46]. The SM explains most of their observed interactions.

A list of all elementary particles known today is presented in table 3.1. Some of them appear in degenerate mass eigenstates, signaling the presence of the SM symmetries. All the fermions come in three copies, known as flavors or generations, which differ only in their masses. The measured interactions of these particles fit well within a gauge theory with gauge group $G_{\mathrm{SM}} := SU(3)\times SU(2)\times U(1)$. The sector related to the $SU(3)$ subgroup is called quantum chromodynamics (QCD), while the $SU(2)\times U(1)$ part corresponds to the electroweak interactions. The electroweak group is broken to $U(1)_Q$. The only unbroken generator is $Q = T + Y$, where T is the $SU(2)$ isospin and Y is the original $U(1)$ charge, the hypercharge.

The fields that create the elementary particles in table 3.1 can be grouped in irreducible representations (irreps) labeled as $(C, T)_Y$, where C is the $SU(3)$ irrep, T is the $SU(2)$ irrep and Y is the hypercharge. The 8 degenerate gluon states are the $SU(3)$ gauge bosons. The $SU(2)$ and $U(1)$ gauge bosons W^a_μ and B_μ are identified in the following way:

$$W^1_\mu = \frac{1}{\sqrt{2}}(W^+_\mu + W^-_\mu), \qquad W^3_\mu = \frac{gZ_\mu + g'A_\mu}{\sqrt{g^2+(g')^2}}, \tag{3.28}$$

$$W^2_\mu = \frac{i}{\sqrt{2}}(W^+_\mu - W^-_\mu), \qquad B_\mu = \frac{-g'Z_\mu + gA_\mu}{\sqrt{g^2+(g')^2}}, \tag{3.29}$$

where g and g' are the gauge coupling constants of $SU(2)$ and $U(1)$, respectively. The 3 degenerate quark states of each kind are collected into $SU(3)$ triplets. The left-handed components of all the fermions belong to $SU(2)$ doublet, whereas their right-handed parts are just singlets. Explicitly:

- The 3 generations of left-handed quark form $(3,2)_{1/6}$ multiplets as

$$q_{L1} = \begin{pmatrix} u_L \\ d_L \end{pmatrix}, \qquad q_{L2} = \begin{pmatrix} c_L \\ s_L \end{pmatrix}, \qquad q_{L3} = \begin{pmatrix} t_L \\ b_L \end{pmatrix}. \tag{3.30}$$

- The 3 generations of left-handed lepton form $(1,2)_{-1/2}$ multiplets as

$$l_{L1} = \begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix}, \qquad l_{L2} = \begin{pmatrix} \nu_{\mu L} \\ \mu_L \end{pmatrix}, \qquad l_{L3} = \begin{pmatrix} \nu_{\tau L} \\ \tau_L \end{pmatrix}. \tag{3.31}$$

- The 3 generations of right-handed up-type quark form $(3,1)_{2/3}$ multiplets, corresponding to right-handed part of the up-type quark fields: u_R, c_R and t_R.
- The 3 generations of right-handed up-type quark form $(3,1)_{-1/3}$ multiplets, corresponding to right-handed part of the up-type quark fields: d_R, s_R and b_R.
- The 3 generations of right-handed leptons form $(1,1)_{-1}$ singlets, corresponding to right-handed part of the lepton fields: e_R, μ_R and τ_R.

Finally, the Higgs boson is collected in a $(1,2)_{1/2}$ multiplet together with the would-be Goldstone bosons of electroweak symmetry breaking, corresponding to the longitudinal components of the massive electroweak bosons.[5] The conventional parametrizations of this Higgs doublet are

$$\phi = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi^1 + i\phi^2 \\ v + H + i\phi^3 \end{pmatrix} = \frac{v+H}{\sqrt{2}} U \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \tag{3.32}$$

where v is the vacuum expectation value of ϕ, given by $\langle |\phi|^2 \rangle = v^2/2$ and U is a G/H-valued field. The representations of all the field strengths and matter fields of this gauge theory under the Lorentz group and G_{SM} are shown in table 3.2.

The SM can be thought of as an EFT for the degrees of freedom presented here, with some unknown cutoff Λ. A simple power counting based on the canonical dimension of the operators can be implemented. As explained in section 3.3, each operator $\mathcal{O}$ with a well-defined canonical dimension $\Delta(\mathcal{O})$ is assigned a power $N_\lambda(\mathcal{O}) = \Delta(\mathcal{O}) - 4$ of $\lambda = 1/\Lambda$. The complete Lagrangian is therefore given by

$$\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{SM}} + \frac{1}{\Lambda}\mathcal{L}^{(5)}_{\text{SMEFT}} + \frac{1}{\Lambda^2}\mathcal{L}^{(6)}_{\text{SMEFT}} + \dots \tag{3.33}$$

where $\mathcal{L}_{\text{SM}}$ contains operators of dimension 4 or less, while $\mathcal{L}^{(d)}_{\text{SMEFT}}$ contains only operators of dimension d. The limit $\Lambda \to \infty$ gives the leading order approximation of this EFT. It turns out that this approximation is enough to fit most of the current

[5] An alternative, more general effective theory for the SM particles is the Higgs EFT (HEFT), in which the Goldstones and the Higgs are treated independently. Thus, G_{SM} is non-linearly realized in its Goldstone sector. The SMEFT is a particular case of the HEFT in which relations between the HEFT parameters are induced by the fact that the Higgs and Goldstone bosons belong to the same multiplet. A basis for the HEFT with up to four derivatives has been developed in [47, 48].

Name	Lorentz irrep	G_{SM} irrep
$G_{\mu\nu}$	$(1,1)$	$(8,1)_0$
$W_{\mu\nu}$	$(1,1)$	$(1,2)_0$
$B_{\mu\nu}$	$(1,1)$	$(1,1)_0$
q_{Li}	$(1/2,0)$	$(3,2)_{1/6}$
l_{Li}	$(1/2,0)$	$(1,2)_{-1/2}$
u_{Ri}	$(0,1/2)$	$(3,1)_{2/3}$
d_{Ri}	$(0,1/2)$	$(3,1)_{-1/3}$
e_{Ri}	$(0,1/2)$	$(1,1)_{-1}$
ϕ	$(0,0)$	$(1,2)_{1/2}$

Table 3.2: Representations of field strengths and matter fields in the SM under the Lorentz group and the gauge group G_{SM}.

experimental data in particle physics. It is customary to use the name "Standard Model" for the theory obtained from this limit, whereas the corresponding EFT for finite cutoff is known as Standard Model EFT (SMEFT). The SM Lagrangian $\mathcal{L}_{\text{SM}}$ is then

$$\begin{aligned}\mathcal{L}_{\text{SM}} = &-\frac{1}{4}G^A_{\mu\nu}G^{A\ \mu\nu} - \frac{1}{4}W^a_{\mu\nu}W^{a\ \mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu}+\\ &+\bar{l}_{Li}\, i\not{D}\, l_{Li} + \bar{q}_{Li}\, i\not{D}\, q_{Li} + \bar{e}_{Ri}\, i\not{D}\, e_{Ri} + \bar{u}_{Ri}\, i\not{D}\, u_{Ri} + \bar{d}_{Ri}\, i\not{D}\, d_{Ri}+\\ &+ (D_\mu\phi)^\dagger D^\mu\phi - V(\phi) - \left(y^e_{ij}\ \bar{e}_{Ri}\phi l_{Lj} + y^d_{ij}\ \bar{d}_{Ri}\phi q_{Lj} + y^u_{ij}\ \bar{u}_{Ri}\tilde{\phi}^\dagger q_{Lj} + \text{h.c.}\right).\end{aligned} \tag{3.34}$$

As usual, $\tilde{\phi} = i\sigma_2\phi^*$ denotes the $SU(2)$ doublet of hypercharge $-1/2$. The Higgs scalar potential is

$$V(\phi) = -\mu^2_\phi\,|\phi|^2 + \lambda_\phi\,|\phi|^4\,. \tag{3.35}$$

There are 18 free parameters in this Lagrangian: 3 gauge coupling constants, the 2 parameters μ and λ of the potential, and 13 parameters in the Yukawa sector. The counting of parameters in the Yukawa sector should be done after fixing a basis in the space of flavors of the different fermion multiplets. *A priori*, there are three 3×3 matrices of Yukawa couplings. A change of basis can always performed so that they take the form

$$y^e = \text{diag}(y_e, y_\mu, y_\tau), \tag{3.36}$$

$$y^d = \text{diag}(y_d, y_s, y_b), \tag{3.37}$$

$$y^u = V^\dagger_{\text{CKM}}\,\text{diag}(y_u, y_c, y_t), \tag{3.38}$$

where the y_i are real parameters and V_{CKM} is a unitary matrix, known as the Cabibbo-Kobayashi-Maskawa (CKM) matrix, which can be parametrized by 3 mixing angles θ_{12}, θ_{13}, θ_{23}, and a CP-violating phase δ.

The vev of the Higgs is related to the Higgs potential parameters as $v = \mu_\phi/\sqrt{\lambda_\phi}$. The masses of all the fields can be obtained by plugging eq. (3.32) into eq. (3.34), keeping only quadratic terms, and diagonalizing them. The masses of the neutrinos are $m_\nu = 0$. The mass of any other fermion ψ is given by $m_\psi = y_\psi v/\sqrt{2}$ where y_ψ is the corresponding Yukawa coupling. The mass of the Higgs is $v/\sqrt{2}$. The masses of the vector bosons are

$$m_W = \frac{1}{2} g v, \qquad m_Z = \frac{1}{2}\sqrt{g^2 + (g')^2} v, \qquad m_A = m_G = 0. \tag{3.39}$$

The SM is a very successful theory, which is able to explain most of the experimental observations in elementary particle physics. However, it is known that it leaves some important fundamental phenomena unexplained. The most obvious problem is that neutrinos are predicted to be massless, while they are known experimentally to be massive. The lack of a description of dark matter and gravity are other issues with the SM, if it were to be the fundamental theory of nature. All this evidence strongly suggests the existence of new degrees of freedom that have not been discovered yet. The scale at which they can resonantly produced sets a finite cutoff for the SMEFT (unless the new particles are light, that is, at or below the electroweak scale, in which case the SMEFT is clearly not a complete description of all elementary particles in this regime).

A wide variety of UV completions of the SM have been proposed. Among then, there are Grand Unification Theories (GUTs) [49], low-energy supersymmetry [50], composite Higgs models [51], extra dimensions [52, 53], and many others, each with different sets of motivations and predictions. Thus, an experimental or phenomenological analysis that considers every individual UV completion in a case by case basis in impractical. Also, the real UV model could be very different from any of the proposed ones. Thus, it is convenient to use the SMEFT, which parametrizes the low-energy regime of any these new physics models through higher-dimensional operators.

As described in chapter 4 and specially in section 4.5.1, it is useful to define a complete set of independent operators, known as a basis, in terms of which the Lagrangian must be written. In tables 3.3, 3.4 and 3.5, a basis operators of dimension 6 or less is presented (excluding those that are quadratic in the fields). This is the basis that will be used in this work. It was defined in ref. [54], refining the proposals of refs. [55–58]. We use the notation specified in appendix A.

The presence of higher-dimensional operators modifies the SM interactions and introduce new ones. The first correction to SM physics comes from dimension 5 operators. Up to flavor indices, there is only one operator of dimension 5: the Weinberg operator, listed in table 3.3. Its most important effect is, remarkably, the introduction of neutrino masses. Operators of dimension 6 are next in importance. There is large number of them, with a wide variety of effects. They have become nowadays a standard tool for parametrizing new physics effects in a model-independent way. Currently, many of the coefficients of dimension-6 operators in the SMEFT have been constrained using experimental data [59–81].

3.8 Conclusions

In this chapter, we have reviewed the theoretical framework that serves as a basis for the work presented in the rest of the thesis: the EFT construction. We have discussed some of the ideas that surround it: power counting, renormalization and matching. We have also introduced the SMEFT, the EFT that parametrizes the interactions of

	Operator	Notation
Dim. 4	$\left(\phi^\dagger\phi\right)^2$	$\mathcal{O}_{\phi 4}$
	$\bar{e}_R\phi^\dagger l_L$	$\mathcal{O}_{y^e}$
	$\bar{d}_R\phi^\dagger q_L$	$\mathcal{O}_{y^d}$
	$\bar{u}_R\tilde{\phi}^\dagger q_L$	$\mathcal{O}_{y^u}$
Dim. 5	$\overline{l_L^c}\tilde{\phi}^*\tilde{\phi}^\dagger l_L$	$\mathcal{O}_5$

Table 3.3: Operators of dimension four and five. in the SMEFT.

	Operator	Notation	Operator	Notation
$(\bar{L}L)(\bar{L}L)$	$\left(\bar{l}_L\gamma_\mu l_L\right)\left(\bar{l}_L\gamma^\mu l_L\right)$	$\mathcal{O}_{ll}$		
	$\left(\bar{q}_L\gamma_\mu q_L\right)\left(\bar{q}_L\gamma^\mu q_L\right)$	$\mathcal{O}_{qq}^{(1)}$	$\left(\bar{q}_L\gamma_\mu\sigma_a q_L\right)\left(\bar{q}_L\gamma^\mu\sigma_a q_L\right)$	$\mathcal{O}_{qq}^{(3)}$
	$\left(\bar{l}_L\gamma_\mu l_L\right)\left(\bar{q}_L\gamma^\mu q_L\right)$	$\mathcal{O}_{lq}^{(1)}$	$\left(\bar{l}_L\gamma_\mu\sigma_a l_L\right)\left(\bar{q}_L\gamma^\mu\sigma_a q_L\right)$	$\mathcal{O}_{lq}^{(3)}$
$(\bar{R}R)(\bar{R}R)$	$\left(\bar{e}_R\gamma_\mu e_R\right)\left(\bar{e}_R\gamma^\mu e_R\right)$	$\mathcal{O}_{ee}$		
	$\left(\bar{u}_R\gamma_\mu u_R\right)\left(\bar{u}_R\gamma^\mu u_R\right)$	$\mathcal{O}_{uu}$	$\left(\bar{d}_R\gamma_\mu d_R\right)\left(\bar{d}_R\gamma^\mu d_R\right)$	$\mathcal{O}_{dd}$
	$\left(\bar{u}_R\gamma_\mu u_R\right)\left(\bar{d}_R\gamma^\mu d_R\right)$	$\mathcal{O}_{ud}^{(1)}$	$\left(\bar{u}_R\gamma_\mu T_A u_R\right)\left(\bar{d}_R\gamma^\mu T_A d_R\right)$	$\mathcal{O}_{ud}^{(8)}$
	$\left(\bar{e}_R\gamma_\mu e_R\right)\left(\bar{u}_R\gamma^\mu u_R\right)$	$\mathcal{O}_{eu}$	$\left(\bar{e}_R\gamma_\mu e_R\right)\left(\bar{d}_R\gamma^\mu d_R\right)$	$\mathcal{O}_{ed}$
$(\bar{L}L)(\bar{R}R)$	$\left(\bar{l}_L\gamma_\mu l_L\right)\left(\bar{e}_R\gamma^\mu e_R\right)$	$\mathcal{O}_{le}$	$\left(\bar{q}_L\gamma_\mu q_L\right)\left(\bar{e}_R\gamma^\mu e_R\right)$	$\mathcal{O}_{qe}$
	$\left(\bar{l}_L\gamma_\mu l_L\right)\left(\bar{u}_R\gamma^\mu u_R\right)$	$\mathcal{O}_{lu}$	$\left(\bar{l}_L\gamma_\mu l_L\right)\left(\bar{d}_R\gamma^\mu d_R\right)$	$\mathcal{O}_{ld}$
	$\left(\bar{q}_L\gamma_\mu q_L\right)\left(\bar{u}_R\gamma^\mu u_R\right)$	$\mathcal{O}_{qu}^{(1)}$	$\left(\bar{q}_L\gamma_\mu T_A q_L\right)\left(\bar{u}_R\gamma^\mu T_A u_R\right)$	$\mathcal{O}_{qu}^{(8)}$
	$\left(\bar{q}_L\gamma_\mu q_L\right)\left(\bar{d}_R\gamma^\mu d_R\right)$	$\mathcal{O}_{qd}^{(1)}$	$\left(\bar{q}_L\gamma_\mu T_A q_L\right)\left(\bar{d}_R\gamma^\mu T_A d_R\right)$	$\mathcal{O}_{qd}^{(8)}$
$(\bar{L}R)(\bar{R}L)$	$\left(\bar{l}_L e_R\right)\left(\bar{d}_R q_L\right)$	$\mathcal{O}_{ledq}$		
$(\bar{L}R)(\bar{L}R)$	$\left(\bar{q}_L u_R\right)i\sigma_2\left(\bar{q}_L d_R\right)^{\mathrm{T}}$	$\mathcal{O}_{quqd}^{(1)}$	$\left(\bar{q}_L T_A u_R\right)i\sigma_2\left(\bar{q}_L T_A d_R\right)^{\mathrm{T}}$	$\mathcal{O}_{quqd}^{(8)}$
	$\left(\bar{l}_L e_R\right)i\sigma_2\left(\bar{q}_L u_R\right)^{\mathrm{T}}$	$\mathcal{O}_{lequ}^{(1)}$	$\left(\bar{l}_L\sigma_{\mu\nu}e_R\right)i\sigma_2\left(\bar{q}_L\sigma^{\mu\nu}u_R\right)^{\mathrm{T}}$	$\mathcal{O}_{lequ}^{(3)}$
B-violating			$\epsilon_{ABC}\left(\bar{d}_R^{cA}u_R^B\right)\left(\bar{q}_L^{cC}i\sigma_2 l_L\right)$	$\mathcal{O}_{duq}$
			$\epsilon_{ABC}\left(\bar{q}_L^{cA}i\sigma_2 q_L^B\right)\left(\bar{u}_R^{cC}e_R\right)$	$\mathcal{O}_{qqu}$
			$\epsilon_{ABC}\left(\bar{d}_R^{cA}u_R^B\right)\left(\bar{u}_R^{cC}e_R\right)$	$\mathcal{O}_{duu}$
			$\epsilon_{ABC}(i\sigma_2)_{\alpha\delta}(i\sigma_2)_{\beta\gamma}\left(\bar{q}_L^{cA\alpha}q_L^{B\beta}\right)\left(\bar{q}_L^{cC\gamma}l_L^\delta\right)$	$\mathcal{O}_{qqq}$

Table 3.4: Basis of dimension-six operators: four-fermion interactions. Flavor indices are omitted.

	Operator	Notation	Operator	Notation
X^3	$\varepsilon_{abc} W_\mu^{a\,\nu} W_\nu^{b\,\rho} W_\rho^{c\,\mu}$	$\mathcal{O}_W$	$\varepsilon_{abc} \tilde{W}_\mu^{a\,\nu} W_\nu^{b\,\rho} W_\rho^{c\,\mu}$	$\mathcal{O}_{\tilde{W}}$
	$f_{ABC} G_\mu^{A\,\nu} G_\nu^{B\,\rho} G_\rho^{C\,\mu}$	$\mathcal{O}_G$	$f_{ABC} \tilde{G}_\mu^{A\,\nu} G_\nu^{B\,\rho} G_\rho^{C\,\mu}$	$\mathcal{O}_{\tilde{G}}$
ϕ^6	$\left(\phi^\dagger \phi\right)^3$	$\mathcal{O}_\phi$		
$\phi^4 D^2$	$\left(\phi^\dagger \phi\right) \Box \left(\phi^\dagger \phi\right)$	$\mathcal{O}_{\phi\Box}$	$\left(\phi^\dagger D_\mu \phi\right) \left(\left(D^\mu \phi\right)^\dagger \phi\right)$	$\mathcal{O}_{\phi D}$
$\psi^2 \phi^2$	$\left(\phi^\dagger \phi\right) \left(\bar{l}_L \phi e_R\right)$	$\mathcal{O}_{e\phi}$		
	$\left(\phi^\dagger \phi\right) \left(\bar{q}_L \phi d_R\right)$	$\mathcal{O}_{d\phi}$	$\left(\phi^\dagger \phi\right) \left(\bar{q}_L \tilde{\phi} u_R\right)$	$\mathcal{O}_{u\phi}$
$X^2 \phi^2$	$\phi^\dagger \phi B_{\mu\nu} B^{\mu\nu}$	$\mathcal{O}_{\phi B}$	$\phi^\dagger \phi \tilde{B}_{\mu\nu} B^{\mu\nu}$	$\mathcal{O}_{\phi \tilde{B}}$
	$\phi^\dagger \phi W_{\mu\nu}^a W^{a\,\mu\nu}$	$\mathcal{O}_{\phi W}$	$\phi^\dagger \phi \tilde{W}_{\mu\nu}^a W^{a\,\mu\nu}$	$\mathcal{O}_{\phi \tilde{W}}$
	$\phi^\dagger \sigma_a \phi W_{\mu\nu}^a B^{\mu\nu}$	$\mathcal{O}_{\phi WB}$	$\phi^\dagger \sigma_a \phi \tilde{W}_{\mu\nu}^a B^{\mu\nu}$	$\mathcal{O}_{\phi \tilde{W} B}$
	$\phi^\dagger \phi G_{\mu\nu}^A G^{A\,\mu\nu}$	$\mathcal{O}_{\phi G}$	$\phi^\dagger \phi \tilde{G}_{\mu\nu}^A G^{A\,\mu\nu}$	$\mathcal{O}_{\phi \tilde{G}}$
$\psi^2 X \phi$	$\left(\bar{l}_L \sigma^{\mu\nu} e_R\right) \phi B_{\mu\nu}$	$\mathcal{O}_{eB}$	$\left(\bar{l}_L \sigma^{\mu\nu} e_R\right) \sigma^a \phi W_{\mu\nu}^a$	$\mathcal{O}_{eW}$
	$\left(\bar{q}_L \sigma^{\mu\nu} u_R\right) \tilde{\phi} B_{\mu\nu}$	$\mathcal{O}_{uB}$	$\left(\bar{q}_L \sigma^{\mu\nu} u_R\right) \sigma^a \tilde{\phi} W_{\mu\nu}^a$	$\mathcal{O}_{uW}$
	$\left(\bar{q}_L \sigma^{\mu\nu} d_R\right) \phi B_{\mu\nu}$	$\mathcal{O}_{dB}$	$\left(\bar{q}_L \sigma^{\mu\nu} d_R\right) \sigma^a \phi W_{\mu\nu}^a$	$\mathcal{O}_{dW}$
	$\left(\bar{q}_L \sigma^{\mu\nu} T_A u_R\right) \tilde{\phi} G_{\mu\nu}^A$	$\mathcal{O}_{uG}$	$\left(\bar{q}_L \sigma^{\mu\nu} T_A d_R\right) \phi G_{\mu\nu}^A$	$\mathcal{O}_{dG}$
$\psi^2 \phi^2 D$	$\left(\phi^\dagger i \overleftrightarrow{D}_\mu \phi\right) \left(\bar{l}_L \gamma^\mu l_L\right)$	$\mathcal{O}_{\phi l}^{(1)}$	$\left(\phi^\dagger i \overleftrightarrow{D}_\mu^a \phi\right) \left(\bar{l}_L \gamma^\mu \sigma_a l_L\right)$	$\mathcal{O}_{\phi l}^{(3)}$
	$\left(\phi^\dagger i \overleftrightarrow{D}_\mu \phi\right) \left(\bar{e}_R \gamma^\mu e_R\right)$	$\mathcal{O}_{\phi e}$		
	$\left(\phi^\dagger i \overleftrightarrow{D}_\mu \phi\right) \left(\bar{q}_L \gamma^\mu q_L\right)$	$\mathcal{O}_{\phi q}^{(1)}$	$\left(\phi^\dagger i \overleftrightarrow{D}_\mu^a \phi\right) \left(\bar{q}_L \gamma^\mu \sigma_a q_L\right)$	$\mathcal{O}_{\phi q}^{(3)}$
	$\left(\phi^\dagger i \overleftrightarrow{D}_\mu \phi\right) \left(\bar{u}_R \gamma^\mu u_R\right)$	$\mathcal{O}_{\phi u}$	$\left(\phi^\dagger i \overleftrightarrow{D}_\mu \phi\right) \left(\bar{d}_R \gamma^\mu d_R\right)$	$\mathcal{O}_{\phi d}$
	$\left(\tilde{\phi}^\dagger i D_\mu \phi\right) \left(\bar{u}_R \gamma^\mu d_R\right)$	$\mathcal{O}_{\phi ud}$		

Table 3.5: Basis of dimension-six operators: operators other than four-fermion interactions. Flavor indices are omitted.

the known elementary particles. It does so in a general way, independently of the new physics that might appear at high energies.

The low-energy effects of unknown high-energy degrees of freedom is taken into account in the SMEFT through the introduction of higher-dimensional operators, whose contribution to observables is suppressed by inverse powers of the cutoff. At energies around the cutoff scale, these new degrees of freedom may be produced and the SMEFT stops being a valid description of the physics. As we will see in part III, one can go beyond the SMEFT while keeping most of its advantages. Under very general conditions, all possible new particles can be enumerated and collected together with the SM ones in an EFT that extends the SMEFT without loosing model independence. Then, both direct and indirect effects of the new particles can be studied, taking advantage of the EFT approach.

CHAPTER 4

Field redefinitions

4.1 Introduction

The description of a given quantum field theory in terms of an action and a renormalization scale (or a cutoff, in the Wilsonian approach) is highly redundant. Firstly, the renormalization group invariance represents a one-parameter redundancy: a change in the renormalization scale (or in the cutoff) can be compensated by a change in the action in such a way that the predictions of the theory are preserved.[1] Secondly, physical observables are invariant under redefinitions of the quantum fields.[2] This property of quantum field theory is sometimes known as the equivalence theorem (not to be confused with the equivalence theorem in the Higgs mechanism). Different versions of this theorem, with different assumptions and in different contexts, have been proved and discussed in the literature [83–89]. Here we have in mind the application of the EFT to the scattering of particles. In this context, the redundancy is given by the freedom in choosing interpolating fields that can create the relevant particles from the vacuum and be used to compute scattering amplitudes.

In this chapter, we explore some aspects of local perturbative field redefinitions in EFTs. By perturbative, we mean that the variation in the fields is treated as a perturbation. These redefinitions have the virtue of being automatically invertible with a local inverse, in a perturbative sense. Moreover, as shown, for instance, in [90] and reviewed below, their effect is particularly simple, as the Jacobian of the transformation can be ignored in methods such as dimensional regularization. Most of the time, the change of the fields will be taken to be suppressed by some positive integer power of the inverse of the cutoff scale $1/\Lambda$. Then, treating it as a perturbation is actually implied by the perturbative expansion of the EFT in powers of $1/\Lambda$. This kind of redefinition mixes different orders in the $1/\Lambda$ expansion of the effective action in a triangular fashion: the n-th order of the redefined action depends only on terms of order $m \leq n$ in the original one.

[1] More generally, any change of renormalization scheme can compensated by a change in the action.

[2] Actually, the renormalization group invariance can be understood as the invariance under a particular type of field redefinition [82].

Perturbative redefinitions are performed customarily by EFT practitioners in order to write general or particular effective actions, consistent with certain symmetries, in reduced forms [54,56–58,89,91,92]. The idea is to eliminate part of the reparametrization redundancy by imposing a condition on the action. This is completely analogous to fixing a gauge in a gauge-invariant theory, and we will borrow this terminology.[3] A standard gauge-fixing condition is to require the vanishing of the coefficients of certain operators. As we review in section 4.5, this can be achieved order by order in $1/\Lambda$ by perturbative redefinitions. If no linear combination of the remaining operators can be redefined away without violating the gauge-fixing condition, then these operators are said to form a non-redundant basis. When eliminating a certain term of order n, the change in the action at orders $m > n$ (and in the other terms at order n) can be absorbed into the operator coefficients of the original action, if it is completely general.[4] From the purely effective point of view, there is often no need to track this redefinition of the coefficients, as they are free parameters to be determined experimentally. For this reason, among others, the "higher-order effects" of the field redefinition are usually ignored. Then, it turns out that the order-by-order algorithm to remove operators with perturbative redefinitions is equivalent to a very simple recipe: using the equations of motion of the action at lowest order ($n = 0$) in any of the terms of order $n \geq 1$ [91].

However, in many situations it is crucial to know the dependence of the coefficients in the redefined action on the coefficients of the original one. This is the case, for instance, when one wants to translate the experimental limits on the coefficients in one basis of operators into limits on the coefficients of the operators in another (reduced) basis. Another common scenario is the one in which the coefficients in a certain effective action are known functions of the parameters of some ultraviolet (UV) completion of interest, and one wants to know the corresponding functional dependence of the operator coefficients in a particular non-redundant basis. In these situations, all the effects of the field redefinitions up to a certain order must be considered if the aimed precision requires a calculation to that order [94]. Analyzing the perturbative structure of these effects—including the impact of quantum corrections and dimensionless couplings—is the main purpose of this chapter. In particular, we clarify the relation between perturbative field redefinitions and the classical equations of motion, which still is, apparently, the source of some confusion. For example, it is well known that many of the corrections of order $n \geq 2$ generated by the perturbative redefinitions are missed by the recipe based on the lowest-order equations of motion. One could try to improve this situation by including higher-order terms in the equation of motion, as done in [27, 95]. We show, however, that the higher-order corrections induced by the redefinitions are not correctly recovered by this extended recipe. The essential reason is that the classical equations of motion only capture the first-order response of the action to variations of the fields. Therefore, using naively the equations of motion, with or without higher-order corrections, gives in general an action that is not equivalent to the original one at the second and higher orders. Imposing equations of motion is not the same as performing field redefinitions.

[3] In fact, this is more than a mere analogy: any quantum field theory has a BRST symmetry associated with field redefinitions [93].

[4] Note that the necessary redefinitions will always preserve the symmetries of the action, see section 4.5.

Whether the higher-order terms, in particular those induced by field redefinitions, are significant or not in practice, depends on many factors: the experimental precision, the process to be calculated, the theory at hand, the value of E/Λ and the value of the remaining parameters that appear in the action. For instance, it may happen that the first-order contributions are vanishing or very suppressed, due to some symmetry or to some dynamical reason. Then the second-order terms would give the leading correction [96, 97].

Taking into account the higher-order terms generated by field redefinitions is relevant, in particular, for the consistent perturbative matching of a local EFT to a more fundamental UV theory with the same light degrees of freedom. In this respect, we also study the impact on the EFT of field redefinitions performed in the UV theory. We find that, non-trivially, the redefinitions of the light fields do not commute at the quantum level with the matching procedure. This is related to the contribution of heavy-light loops in the UV theory.

The chapter is organized as follows. In section 4.2, we review the effect of local redefinitions on quantum field theories for off-shell and on-shell quantities, paying special attention to the case of perturbative redefinitions.[5] We discuss in particular renormalization and subtleties related to tadpoles. We also present an a toy model illustrating the role of the Jacobian and the sources in the field transformation. In section 4.3, we discuss the relation between field redefinitions and the classical equations of motion. In particular, we give a counterexample to the exact validity of eliminating operators proportional to the equations of motion. In section 4.4, we examine how field redefinitions affect the matching of an EFT to a more fundamental one. We also perform a sample calculation that proves the appearance of non-trivial effects of field redefinitions when a quasi-local action (such as the one obtained from matching) is truncated at a finite order in the $1/\Lambda$ expansion. In section 4.5, we analyze perturbative field redefinitions in which the perturbation is controlled by the same small parameters as the perturbative expansion of the EFT. This refers mainly to the length scale $1/\Lambda$, but also to other dimensionless parameters that may enter in the EFT, such as coupling constants or $1/4\pi$ factors related to loops. We also point out a few effects at higher orders in $1/\Lambda$ or in the loop expansion. Inside this section, we also show that explicit gauge covariance is preserved by covariant field redefinitions and is manifest in the exact equations of motion of a gauge theory. In particular, this implies that the corrections to the SMEFT equations of motion given in ref. [95] in terms of ordinary derivatives and gauge fields can be written in terms of field-strength tensors and covariant derivatives.

4.2 Reparametrization invariance

4.2.1 Reparametrization invariance in general

Consider a quantum field theory constructed as in chapter 3 from an action S. Let $Z(S)[J]$ be the corresponding generating function. Now, let us perform a change of integration variables $\phi \to F(\phi)$, where F is an invertible function. Ignoring regular-

[5]Much of the content of this section can be found in ref. [89]. We also clarify a couple of important details and summarize latter work on renormalization.

ization and renormalization for the moment, we get

$$Z(S)[J] := \int \mathcal{D}\phi \exp\left(iS[\phi] + J_\alpha \phi^\alpha\right), \tag{4.1}$$

$$= \int \mathcal{D}\phi \det\left(\frac{\delta F}{\delta \phi}\right) \exp\left(iS[F(\phi)] + J_\alpha F^\alpha(\phi)\right). \tag{4.2}$$

So, the generating function is invariant under a field redefinition in the action, $S'[\phi] = S[F(\phi)]$, if the redefinition is accompanied by the corresponding Jacobian factor and the corresponding change in the source terms, as specified by eq. (4.2). Usually, the transformation F is taken to respect the symmetry and hermiticity properties of the original action, although this is not strictly necessary: as long as the transformation is invertible, the change of variables is valid and the generating function will remain invariant (see nonetheless comments in [98]).

Both the Jacobian and the modified source terms are required for Z to remain invariant. In particular, they are necessary to cancel possible higher-order poles, as illustrated in section 4.2.2. Fortunately, they can be neglected under certain circumstances, as we now review. This is the usual statement of the equivalence theorem.

The Jacobian of the transformation can be written in terms of ghost fields c, $\bar{c}$:

$$\det \frac{\delta F}{\delta \phi} = \int \mathcal{D}\bar{c}\mathcal{D}c \exp\left(-i\bar{c}_\alpha \frac{\delta F^\alpha}{\delta \phi^\beta} c^\beta\right). \tag{4.3}$$

In the following we consider only local transformations, with $F^{ax}(\phi)$ depending analytically on the value of the fields and their derivatives, up to a finite order, at the point x. Then the Jacobian in terms of ghosts can be simply added to the action, which remains (quasi) local. In general, this contribution to the action has a non-trivial effect (see section 4.2.2). However, most applications involve perturbative field redefinitions

$$F(\phi) = \phi + \lambda G(\phi), \tag{4.4}$$

where G is analytic in λ and all terms proportional to positive powers of λ are to be treated as interactions in perturbation theory. Then, the inverse of the transformation is also local. Moreover, the ghost propagator is equal to the identity and the ghost loops only contain insertions of $\delta G(\phi)/\delta\phi^\alpha$, which by the locality assumption are polynomials of the internal momenta. Therefore the ghost loops will integrate to zero in dimensional regularization [90]. The same will happen to the contributions that were cancelled by these loops. So, in dimensional regularization (and in any regularization with this property), the Jacobian of a local, perturbative transformation is equal to the identity and the ghosts can be ignored. We then have the identity $Z(S) = Z(S', F)$. Let us stress that, for consistency, the quadratic terms in S' that vanish as $\lambda \to 0$ should neither be resummed into the propagators of that theory.

The change in the source terms is important for the invariance of off-shell quantities, but thanks to the LSZ reduction formula[6] it has no impact on the S matrix, at least for local perturbative redefinitions. To understand this, note first that the poles of the momentum-space two-point function of any operator $\mathcal{O}$ are equal to the physical masses m_a of the particles a that this operator can create from the vacuum. The probability

[6] See section 3.2 and ref. [99].

amplitude of creating particle a with momentum p, $\sqrt{\mathcal{Z}^a_{\mathcal{O}}} := \langle ap|\, \mathcal{O}(0)\, |0\rangle \neq 0$, is given by the residues at the poles. The operator $\mathcal{O}$ is then a valid interpolating field that can be used in the reduction formula to find S-matrix elements involving any of the particles a, with wave-function renormalization given by $\sqrt{\mathcal{Z}^a_{\mathcal{O}}}$. For a perturbative field redefinition eq. (4.4),

$$\langle ap|\, F^{i0}(\phi)\, |0\rangle = \langle ap|\, \phi^{i0}\, |0\rangle + O(\lambda). \tag{4.5}$$

Therefore, if $\mathcal{Z}^a_{\phi^i} \neq 0$ when $\lambda \to 0$, then $\mathcal{Z}^a_{F^i(\phi)} \neq 0$. Hence, $F^i(\phi)$ is also a valid interpolating field for the particle a. Moreover, because the physical masses of the particles do not know about the field representation, the poles in the two-point function will remain the same at any order. In terms of generating functionals, all this means that $Z(S)$ and $Z(S')$ give rise to the same S matrix. We will say that they are equivalent on-shell and write

$$Z(S) = Z(S', F) \sim Z(S'). \tag{4.6}$$

Let us emphasize that this results holds for a general perturbative redefinition [89]. The function G in eq. (4.4) can be non-linear, it can contain terms proportional to the field or to the field derivatives and it can contain a non-vanishing constant. The latter might raise some concerns, as the proof of the LSZ formula assumes a vanishing vacuum expectation value (vev) of the operator $\mathcal{O}$. Let us examine this issue. Suppose $\delta Z[J]/\delta J^i(x)|_0 = v^i$. If $v^i \neq 0$, it is customary to write $\phi^i(x) = v^i + h^i(x)$ and work with the shifted fields h^i, which have vanishing vev in the original theory S. Let $\delta Z'[J]/\delta J^i(x)|_0 = \tilde{v}^i$. The corresponding shift is $\phi^i(x) = \tilde{v}^i + \tilde{h}^i(x)$, such that $\tilde{h}^i$ has vanishing vev in the theory S'. The transformation F induces another transformation $\bar{F}$ on the shifted fields: $h^i = \bar{F}^i(\tilde{h}) = F^i(\tilde{v}+\tilde{h}) - v^i = \tilde{h}^i + \lambda \bar{G}^i(\tilde{h})$. At the classical level, it can be easily checked that $F^i(\tilde{v}) = v^i$. This also holds at the quantum level when F is linear. In this case, $\bar{F}$ and $\bar{G}$ have no constant term.[7] Conversely, in this case the transformation $\bar{F}^i(\tilde{h}) = F^i(\tilde{v}+\tilde{h}) - F^i(\tilde{v})$ leads to fields h^i with no vev. For generic non-linear transformations, on the other hand, $F^i(\tilde{v}) \neq v^i$ at the quantum level. This can be seen as a particular consequence of the fact that the quantum action (unlike the classical one) is not a scalar under non-linear field redefinitions. This is due to the lack of covariance of the source terms $J_\alpha \phi^\alpha$: a non-linear field redefinition in this term cannot be absorbed into a redefinition of the sources. Covariant extensions of the effective action have been proposed in [100, 101]. At any rate, in general $\bar{F}$ and $\bar{G}$ will have a constant at $O(\hbar)$, and it is this constant that guarantees vanishing vevs. In practice, this amounts to performing a field transformation, calculating the vevs with the new action and then performing the corresponding shift (in perturbation theory this can be achieved by imposing tadpole cancellation as a renormalization condition, see the corresponding comments in [98]).[8]

It should be remembered that the simplified result eq. (4.6) is not valid for off-shell quantities. We have already mentioned the fact that the vevs of the fields are not

[7] This property is implicit in the discussion of spontaneously broken theories in [89].

[8] Alternatively, it is possible to work with $h' = F^{-1}(v+h) - F^{-1}(v)$, which in general will have a vev at $O(\hbar\lambda)$. The field h' is perturbatively close to $\tilde{h}$ so the results will be the same in perturbation theory, although the presence of tadpoles is an unwanted complication. It can also be used in the LSZ formula, since the contribution of the (constant) difference with $\tilde{h}$ lacks the corresponding pole.

covariant under field redefinitions. As pointed out in [98], care is also needed with unstable particles. Of course, as long as they are rigorously treated as resonances in processes with stable asymptotic states, the LSZ formula holds and eq. (4.6) can be used. The problem with eq. (4.6) arises when one insists in treating the unstable particles as external states, which is extremely useful since most of the particles in the SM (SM) are unstable. For this, different treatments have been proposed (see ref. [102] and references therein). It would be interesting to assess to what extent eq. (4.6) is a good approximation in each of these treatments.

To finish this section, let us discuss in what sense these results survive renormalization. Following the notation in section 3.4, let S be a classical action and $\mathcal{R}(S)$ the corresponding renormalized action according to a renormalization scheme $\mathcal{R}$. The action $S'[\phi] = S[F(\phi)]$ can also be renormalized to give $\mathcal{R}(S')$, which cannot be recovered by just making the same field redefinition in the original renormalized action. That is, $\mathcal{R}(S') \neq \mathcal{R}(S)'$. One nice way of relating the renormalization in both theories has been proposed in [103]. The essential idea is to add sources L_a for all the possible operators. Then, it is shown that to connect both renormalized theories not only the fields but also the sources must be transformed: $\phi \to F(\phi)$, $L \to L'(L)$. This is quite natural in the framework of the renormalization of composite operators [104], which is required here because ϕ in the theory S' is composite from the point of view of the original theory S. Interestingly, in this picture renormalization itself can be seen as a regulator-dependent change of variables [103, 105]. The most important implication of these relations between renormalized theories is that predictivity is preserved: if the observables depend on a finite number of physical parameters, to a given order, in the theory defined in the original variables, the same holds in the theory defined with the new variables (see ref. [106] for an explicit example in a renormalizable theory).

4.2.2 A simple example

To explicitly show how the original Green functions of a theory are reproduced after a redefinition of the fields, we describe here an example of a field redefinition in a simple quantum field theory. We start with a free massless real scalar field ϕ. Its generating function is

$$Z(S_{\text{free}})[J] = \int D\phi \exp\left(-\frac{i}{2}\phi_x(\Box\phi)^x + J_x\phi^x\right). \tag{4.7}$$

A change of variables $\phi \to F(\phi) = \phi + (1/m^2)(\Box\phi + g\phi^3)$ in the path integral gives the following expression, where we have used eq. (4.3):

$$Z(S_{\text{free}})[J] = Z(S, F)[J], \tag{4.8}$$

where $S[\phi, c, \bar{c}] = S_\phi[\phi] + S_c[\phi, c, \bar{c}]$ is given by

$$S_\phi = -\int d^dx \left[\frac{1}{2}\phi\Box\left(1 + \frac{\Box}{m^2}\right)^2\phi + \frac{g}{m^2}\phi^3\Box\left(1 + \frac{\Box}{m^2}\right)\phi + \frac{g^2}{2m^4}\phi^3\Box\phi^3\right], \tag{4.9}$$

$$S_c = -\int d^dx \left[\bar{c}(\Box + m^2)c + 3g\phi^2\bar{c}c\right]. \tag{4.10}$$

We have normalized c to have a canonical kinetic term.

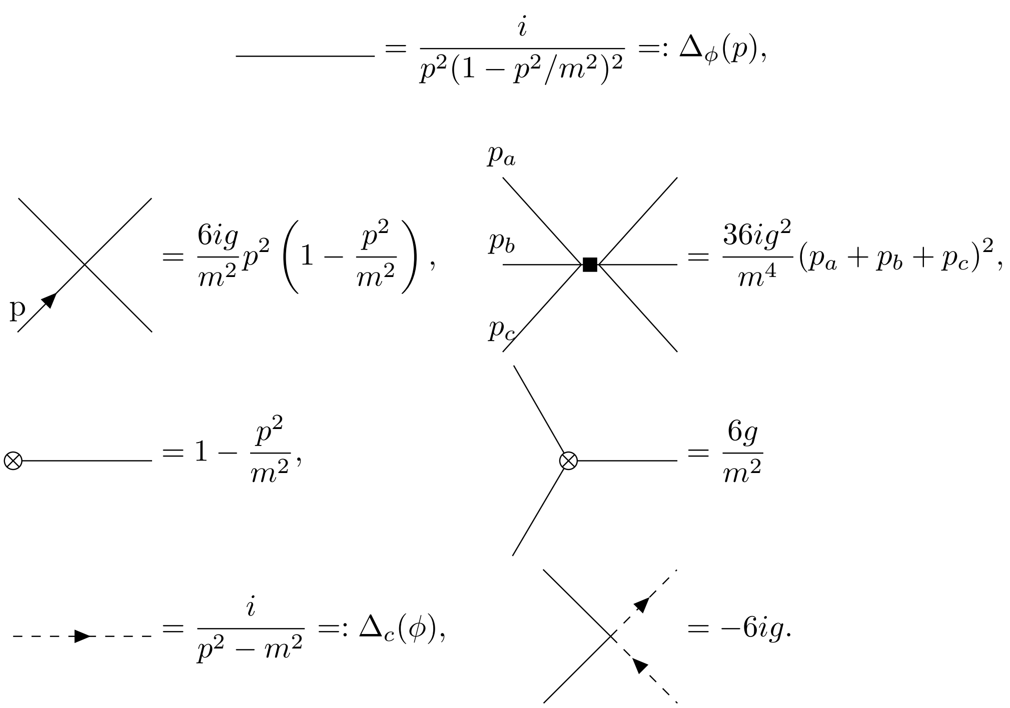

Figure 4.1: Feynman rules from eqs. (4.8), (4.9), (4.10). Crossed dots represent sources. Solid and dotted lines correspond to ϕ and ghosts, respectively. An arrow over a ϕ line is used to specify that the corresponding momentum enters in the factor associated with the vertex it points to. The square that splits the 6-line vertex specifies the three momenta that appear in its associated factor.

The momentum space Green function $G^{(n)}(S,F)$ is the sum over all connected diagrams with n sources constructed using the Feynman rules collected in Figure 4.1. The propagator $\Delta_\phi(p)$ for ϕ contains the physical pole at $p^2 = 0$ but also a new (double) pole at $p^2 = m^2$ that was not present originally. This problematic behavior will be canceled by the momentum dependent vertices and the pole at the same point of the ghost propagator $\Delta_c(p)$.

There are several cancellations between subgraphs of the diagrams we are considering. This is just an example of the more general case nicely discussed in [90]. Three of these cancelllations are shown in Figure 4.2. From the first two equations in this figure, it follows that that we can obtain the full result by summing over a subset of all diagrams: those that do not contain 3-line sources, 6-line vertices, any arrows in external lines or two arrows in the same internal line. In other words, we only need to consider diagrams with 1-line sources, 4-line vertices, no arrows in external lines and at most one arrow in each internal line.

For any diagram, let V be the number of vertices, I the number of internal lines and L the number of loops. We have the relation

$$V - I + L = 1. \tag{4.11}$$

The number of arrows over ϕ lines equals the number of ϕ^4 vertices, so at tree level ($L = 0$) there are no diagrams with less than two arrows in all internal lines. The only exception is the case $V = 0$, which gives the only diagram contributing to $G^{(2)}(p,-p) = i/p^2$. All the other Green functions vanish at tree level.

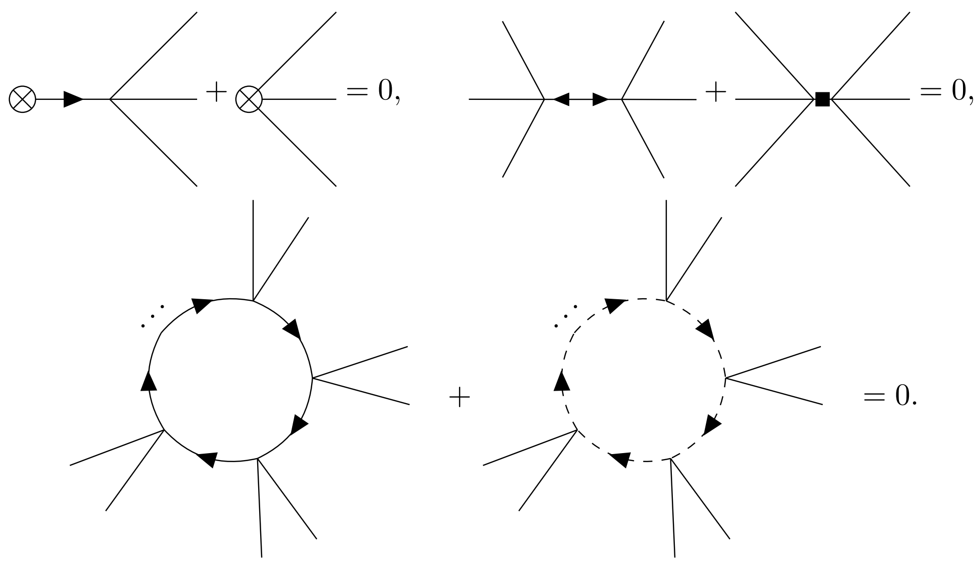

Figure 4.2: Cancellations between subdiagrams.

For $L > 0$, we can reduce the problem by cutting all internal ϕ lines without arrows. The result might be disconnected. For each connected component C the number of ϕ^4 vertices equals the number of internal ϕ lines and the number of $\phi^2\bar{c}c$ vertices equals the number of ghost lines. Therefore, using eq. (4.11), C has exactly one loop ($L = 1$). A 1-loop diagram has as a subgraph one of the two 1-loop diagrams in Figure 4.2, so it must cancel with the diagram obtained by replacing the subgraph with the other 1-loop diagram in the same figure. The cancellation of the connected components after the cut implies the cancellation of the diagrams resulting from joining them back. The conclusion is that the L-loop correction (with $L > 0$) to any Green function is zero.

We have computed all the Green functions to all orders in the loop expansion:

$$G^{(2)}(S,F)(p,-p) = \frac{i}{p^2}, \qquad G(S,F)^{(n>2)} \equiv 0, \tag{4.12}$$

they agree exactly with the $G^{(n)}(S_{\text{free}})$, which are obtained in a more straightforward way from $Z(S_{\text{free}})$. Therefore, they must also be equal order by order in p^2/m^2. This means that if we had worked perturbatively in p^2/m^2 we would have obtained the same results. However the calculations would have had an important difference: the ghost momentum would never appear in the denominator, so their loop integrals would vanish in dimensional regularization. As stated in general in section 4.2, we can ignore the ghosts when the redefinition is perturbative and dimensional regularization is used.

The Green functions $G(S)^{(n)}$ generated with the function $Z(S)$, obtained from eq. (4.8) by replacing $J_\alpha F^\alpha(\phi) \to J_\alpha \phi^\alpha$ in $Z(S,F)$, are equal to the ones computed from $Z(S,F)$ except for the source factors. Now, there is nothing to cancel the first diagram in Figure 4.1, but the corresponding factor has a pole at $p^2 = m^2$ and not at $p^2 = 0$, so its contribution is eliminated by the LSZ formula. The other difference, the p^2/m^2 term in the factor corresponding to the 1-line source, also vanishes on shell. Thus, $Z(S,F) \sim Z(S)$.

In section 4.3, it is proven that some parameter of the action is redundant if and only if the derivative of the action with respect to it is proportional to the equation of motion. This condition is satisfied in this case:

$$\frac{\partial S_\phi}{\partial(1/m^2)} = \frac{\delta S_\phi}{\delta\phi^x}\left(\Box\phi + g\phi^3 - \frac{1}{m^2}\left[\Box^2\phi + 3g\phi^2\Box\phi + g\Box\phi^3 + 3g^2\phi^5\right]\right)^x + O\left(\frac{1}{m^4}\right), \tag{4.13}$$

which means that the parameter $1/m^2$ of the action S_ϕ is redundant. For the parameter g, a similar equation (of the form $\partial S_\phi/\partial g \propto \delta S_\phi/\delta\phi$) can be obtained. However, this is not necessary to eliminate g from S_ϕ because $1/m^2$ can be taken to be zero (as it is redundant) and then S_ϕ becomes independent of g.

4.3 Equations of motion

4.3.1 Equations of motion and redundant operators

The Schwinger-Dyson equations follow from the invariance of the path integral under infinitesimal field redefinitions[9] and can be written succinctly as

$$\int \mathcal{D}\phi \left[i\frac{\delta S}{\delta\phi^\beta} + J^\beta\right] \exp(iS + J_\alpha\phi^\alpha) = 0. \tag{4.14}$$

Differentiation with respect to J gives an infinite set of relations among the Green functions, which can be considered the quantum equations of motion of the theory. In this section, we discuss instead relations between field redefinitions and the *classical* equations of motion, $\delta S/\delta\phi^\alpha = 0$.

For the perturbative redefinition in eq. (4.4), we can Taylor expand the resulting action,

$$S'[\phi] = S[F(\phi)] \tag{4.15}$$

$$= \sum_{m=0}^{\infty} \frac{1}{m!}\lambda^m G^{\alpha_1}(\phi)\cdots G^{\alpha_m}(\phi)\frac{\delta^m S[\phi]}{\delta\phi^{\alpha_1}\ldots\delta\phi^{\alpha_m}} \tag{4.16}$$

$$= S[\phi] + \lambda G^{\alpha_1}(\phi)\frac{\delta S[\phi]}{\delta\phi^{\alpha_1}} + O(\lambda^2) \tag{4.17}$$

$$=: S'_{\text{linear}}[\phi] + O(\lambda^2). \tag{4.18}$$

The term linear in G, of order λ is proportional to $\delta S/\delta\phi$, and thus vanishes if the classical equations of motion of S are used. However, due the higher-order terms, we see that S' is not equal to S'_{linear}, that is, S and S'_{linear} are not related by this field redefinition for any G and λ. As we show below, for a generic G they are actually not related by any local field redefinition. Thus, adding to S a perturbation proportional to its equations of motion does not result in general in an action equivalent to S. Equally, *eliminating terms in the action by imposing the classical equations of motion of the rest of the action does not produce an equivalent theory.* The equivalence only holds at linear order in the perturbation. Note that the perturbation $\lambda G\delta S/\delta\phi$ is neither redundant in the classical limit. Indeed, the relevant equations of motion for a

[9]Conversely, the path integral (4.1) can be understood as a formal solution to the Schwinger-Dyson equations.

tree-level calculation of Green functions include the variation of the perturbation itself and the variation of the source terms.

All this looks pretty straightforward, but apparently there is still some confusion about the limitations of the classical equations of motion, even among experts in EFTs. For example, statements such as "the operators that vanish by the equations of motion are redundant" or "the operators that vanish by the equations of motion give no contribution to on-shell matrix elements", without further qualification, are found every now and then in the specialized literature. To make this point completely clear, we stress that the proofs in [107–110] of the redundancy of equation-of-motion operators are only valid at the linear level, as indicated in these references. Let us briefly review the argument in [107], which is reproduced in the discussion about field redefinitions and equations of motion in the lecture notes [16]. Given an action S and an operator of the form $\mathcal{O}(z) = (f^i \delta S/\delta\phi^i)(z)$, field redefinitions in the path integral are used to show that the correlators $\langle 0|T\phi^{i_1x_1}\dots\phi^{i_nx_n}\mathcal{O}(z)|0\rangle$ in the theory described by S can be written as a sum of terms proportional to delta functions involving the points $i_1\dots i_n$.[10] Then, it follows from the LSZ formula that $\langle p_1\dots p_r|\mathcal{O}(z)|p_{r+1}\dots p_n\rangle$ vanishes, since the number of poles is smaller than n. From this, it is concluded in [16] that the operator $\mathcal{O}$ "can be dropped because it does not contribute to the S matrix". But this conclusion is an unjustified extrapolation of the particular result for S-matrix elements with only one insertion of $\mathcal{O}$.[11] Indeed, the perturbative calculations with the complete action $S+\lambda\mathcal{O}$ involve in general arbitrary powers of the interaction $\lambda\mathcal{O}$, so one needs to also take into account the correlators $\langle 0|T\phi^{i_1x_1}\dots\phi^{i_nx_n}\mathcal{O}(z_1)\dots\mathcal{O}(z_m)|0\rangle$ with $m>1$. It can be checked that these correlators contain terms that are not proportional to any delta function involving the points $x_1,\dots,x_n$. These terms do not need to vanish when the elementary fields are reduced into on-shell particles. Therefore, diagrams with a single insertion of $\mathcal{O}$ do not contribute when the external legs are on shell, but diagrams with two or more insertions do, in general. In section 4.3.2, we check explicitly in a simple example that, already at the tree level, $\langle p_1p_2|T\mathcal{O}(z_1)\mathcal{O}(z_2)|p_3p_4\rangle \neq 0$. All this agrees with eq. (4.18): $\lambda\mathcal{O}$ can be eliminated at the linear order in λ by a perturbative field redefinition, but in doing so other operators proportional to the second and higher powers of λ are generated. The single (and multiple) insertions of these new operators reproduce the effect of the multiple insertions of $\mathcal{O}$.

4.3.2 An example

In ref. [107] (see also ref. [16]) it is proven that the S matrix with one insertion of an operator proportional to the equation of motion vanishes. This is not true, however, for two or more insertions. We check here both statements in the case proposed in exercise 6.1 of [16]. We will compute connected momentum-space Green functions $G^{(m,n)}$ in the theory

$$Z(S,(\phi,\theta))[J^\phi,J^\theta] = \int \mathcal{D}\phi \exp\left(iS[\phi] + J^\phi_x\phi^x + J^\theta_x\theta^x\right) \tag{4.19}$$

[10] This is a simple generalization of the Schwinger-Dyson equations. Dimensional regularization is assumed in order to neglect the Jacobian of the transformation.

[11] The author of ref. [16] warns latter that "working to second order in the equations of motion is tricky" (see ref. [27] for more details). However, as shown in section 4.5 and in the example of section 8.6, using the equations of motion at second order is in general wrong, rather than tricky, while at first order it involves no complications.

$$S[\phi] = -\int d^4x \left(\frac{1}{2}\phi(\Box + m^2)\phi + \frac{\lambda}{4!}\phi^4\right), \tag{4.20}$$

$$\theta = \phi\frac{\delta S}{\delta\phi} = -\phi(\Box + m^2)\phi - \frac{\lambda}{3!}\phi^4. \tag{4.21}$$

They are defined in eq. (3.5). The corresponding Feynman rules are presented in Figure 4.3. We will calculate $G^{(4,1)}$ and $G^{(4,2)}$. The relevant diagrams are shown in Figure 4.4. In terms of them, the Green functions are

$$G^{(4,1)} = A + \sum_{r=1}^{4} B_r, \tag{4.22}$$

$$G^{(4,2)} = \sum_{r=1}^{4}\sum_{k=1}^{2} C_{rk} + \sum_{r=1}^{4}\sum_{\substack{k,l=1\\k\neq l}}^{2} D_{rkl} + \sum_{\substack{r,s=1\\s>r}}^{4}\sum_{\substack{k,l=1\\k\neq l}}^{2} E_{rskl}. \tag{4.23}$$

The S matrix is obtained by taking the residue when all p_i go on-shell. Let Res be the operation

$$\mathrm{Res}(G) = \lim_{p_1^2\to m^2}\lim_{p_2^2\to m^2}\lim_{p_3^2\to m^2}\lim_{p_4^2\to m^2}\left[\left(\prod_{i=1}^{4}(p_i^2 - m^2)\right)G\right]. \tag{4.24}$$

Applying it to each diagram gives

$$\mathrm{Res}(A) = -4\lambda, \qquad \mathrm{Res}(B_r) = \lambda, \tag{4.25}$$

$$\mathrm{Res}(C_{rk}) = -4i\lambda, \qquad \mathrm{Res}(E_{rskl}) = i\lambda, \tag{4.26}$$

$$\mathrm{Res}(D_{rkl}) = i\lambda\left(1 + \frac{(q_l + p_r)^2 - m^2}{(q_l + q_k + p_r)^2 - m^2}\right), \tag{4.27}$$

where all momenta are taken as ingoing. Using eqs. (4.22)-(4.27) we get

$$\mathrm{Res}(G^{(4,1)}) = 0, \tag{4.28}$$

$$\mathrm{Res}(G^{(4,2)}) = i\lambda\left(-12 + \sum_{r=1}^{4}\sum_{\substack{k,l=1\\k\neq l}}^{2}\frac{(q_l + p_r)^2 - m^2}{(q_l + q_k + p_r)^2 - m^2}\right). \tag{4.29}$$

So, indeed, the S-matrix element with one insertion of θ vanishes. However, when two insertions of θ are included, it does not.

4.3.3 Equations of motion and redundant parameters

Another approach to the analysis of redundancies in the action is focusing on *redundant parameters* instead of redundant operators. In this case, there is an exact relation with the classical equations of motion. A parameter ξ in an action S_ξ will be redundant if it can be eliminated by a local field redefinition, i.e. if an invertible F_ξ exists such that $S' = S_\xi \circ F_\xi$ does not depend on ξ. Then, using that $\partial S'/\partial\xi = 0$,

$$\frac{\partial S_\xi}{\partial\xi} = \frac{\partial(F_\xi^{-1})^\alpha}{\partial\xi}\frac{\delta F_\xi^\beta}{\delta\phi^\alpha}\frac{\delta S_\xi}{\delta\phi^\beta}. \tag{4.30}$$

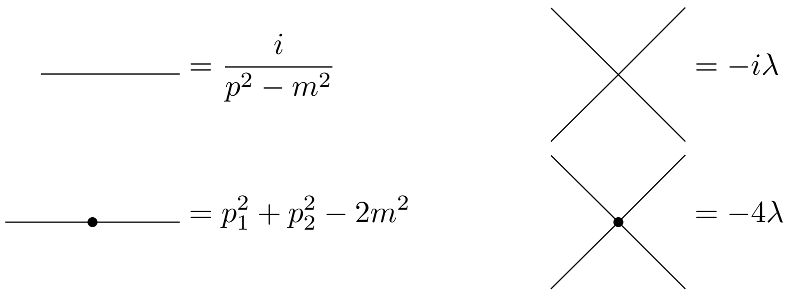

Figure 4.3: Feynman rules for ϕ^4 theory and insertions of the operator θ, represented by a solid dot.

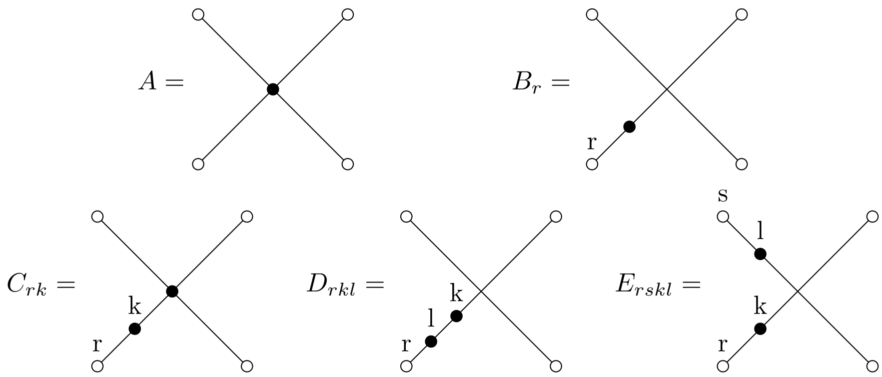

Figure 4.4: Relevant diagrams for the computation of $G^{(4,1)}$ and $G^{(4,2)}$ in the ϕ^4 theory with θ insertions at tree level. Empty (solid) dots denote the sources for ϕ (θ).

We conclude that if ξ is redundant, then $\partial S_\xi/\partial\xi$ vanishes when the classical equations of motion are enforced. The converse implication is also true: if $\partial S_\xi/\partial\xi$ vanishes by the classical equations of motion, then ξ is redundant [111]. Indeed, the variation of S_ξ under an infinitesimal change $\delta\xi$ of the parameter ξ is $\delta S_\xi = (\partial S_\xi/\partial\xi)\delta\xi$. If $(\partial S_\xi/\partial\xi) = f^\alpha \delta S/\delta\phi^\alpha$, then the change δS_ξ can be compensated by the infinitesimal transformation given by eq. (4.4) with $\lambda = \delta\xi$ and $G = -f$, as can be seen in eq. (4.18). That is, $\partial(S_\xi \circ F_\xi)/\partial\xi = 0$. Since this holds for any value of ξ, it follows that $S' = S_\xi \circ F_\xi$ is constant in ξ.

Let us use this last approach to study under which circumstances may $S + \lambda f$ be equivalent to S. Here, f is a local functional of ϕ and neither S nor f depend on λ. As we have just seen, λ is a redundant parameter if and only if

$$f = g^\alpha \frac{\delta}{\delta\phi^\alpha}(S + \lambda f). \tag{4.31}$$

for some local λ-dependent functionals g^α of ϕ. We want to solve this equation for g^α. As we are interested in perturbative redefinitions, we require that g^α has a power expansion $g^\alpha = g^{(0)\alpha} + \lambda g^{(1)\alpha} + \dots$. Comparing the terms of order 0 in λ, we see that, for a solution to exist, it must be possible to write f in the form $f = f^\alpha S_{,\alpha}$, and then $g^{(0)\alpha} = f^\alpha$. Incidentally, this shows once more that the equations of motion can be employed to eliminate terms at first order; the necessary perturbative redefinition with $G^\alpha = -f^\alpha$ follows from eq. (4.30). Writing $g^\alpha = f^\alpha + \lambda \bar{g}^\alpha$, (4.31) reduces to

$$0 = \bar{g}^\alpha S_{,\alpha} + (f^\alpha + \lambda \bar{g}^\alpha)(f^\beta_{,\alpha} S_{,\beta} + f^\beta S_{,\beta\alpha}) \tag{4.32}$$

Looking at the leading order of this equation, we see that, for a solution to exist, we need

$$f^\alpha f^\beta S_{,\beta\alpha} = h^\alpha S_{,\alpha}, \tag{4.33}$$

for some h^α. For a non-trivial action S and a generic f^α, there is no solution to this equation, since the first and second derivatives give a non-homogeneous result when acting on terms in S with different number of fields. A solution exists, however, if $f^\alpha = f^{\alpha\beta} S_{,\beta}$. Actually, in this case there is a solution of eq. (4.31) to all orders in λ, since eq. (4.32) is then of the form

$$0 = \left(\bar{g}^\gamma + f^{\alpha\beta} S_{,\beta} W^\gamma_\alpha + \lambda \bar{g}^\alpha W^\gamma_\alpha\right) S_{,\gamma}, \tag{4.34}$$

where W^γ_α is constructed with $f^{\alpha\beta}$, $S_{,\alpha}$ and their functional derivatives. Thanks to its factorized form, this equation can always be solved recursively, to obtain a local solution $\bar{g}^\alpha$, and thus a local solution g^α, as a power series in λ. From this, the local perturbative redefinition that eliminates λf to all orders can also be obtained recursively, using eq. (4.30). Therefore, we conclude that a perturbation λf is redundant in perturbation theory if f is at least *quadratically* proportional to the equation-of-motion operator $\delta S/\delta\phi$. This result has actually been obtained before in [101, 112]. Here, we have seen that for a general action this condition on f is not only sufficient, but also necessary. A more direct way of checking that $\lambda S_{,\alpha} f^{\alpha\beta} S_{,\beta}$ is redundant is to perform a field redefinition to eliminate it at first order. Then, it is easy to check that the higher order terms have the same form. Therefore, successive field redefinitions will move the effects of the perturbation to higher and higher orders, while preserving the property that the generated terms are quadratic in the equation-of-motion operator. In this way, the effects of the perturbation can be completely eliminated up to an arbitrary power of λ.

4.4 Matching

In this section, we work in the setting and notation introduced in section 3.5.1. In particular, $S_{\rm UV}$ is the action of the fundamental theory, $\bar{S}$ is the non-local action for the light fields that exactly reproduces the effects of $S_{\rm UV}$ and $\lceil \bar{S} \rceil_n$ is the local action approximately reproduces them to order n. Let us perform a general local change of variables involving both the heavy and the light fields, $(\Phi, \phi) \to F(\Phi, \phi) = (F_h(\Phi, \phi), F_l(\Phi, \phi))$. We find

$$Z(S_{\rm UV})[J] = \int \mathcal{D}\phi \mathcal{D}\Phi \det\left(\frac{\delta F}{\delta(\phi, \Phi)}\right) \exp\left\{i S_{\rm UV}[F(\Phi, \phi)] + J_\alpha F^\alpha_l(\Phi, \phi)\right\} \tag{4.35}$$

$$=: Z(S'_{\rm UV}, F_l)[J], \tag{4.36}$$

where $S'_{\rm UV}[\Phi, \phi] = S_{\rm UV}[F(\Phi, \phi)]$. Consider first the particular case with $F_l(\Phi, \phi) = F_l(\phi)$, that is to say, the case in which the new light fields depend only on the original light fields.[12] Then,

$$Z(S_{\rm UV})[J] = Z(\overline{S'}, F_l)[J], \tag{4.37}$$

where

$$\exp\left(i\overline{S'}[\phi]\right) = \int \mathcal{D}\Phi \det\left(\frac{\delta F_h}{\delta\Phi}\right) \exp\left\{iS_{\rm UV}[F_h(\Phi,\phi), F_l(\phi)]\right\} \tag{4.38}$$

$$= \int \mathcal{D}\Phi \exp\left\{iS_{\rm UV}[\Phi, F_l(\phi)]\right\}. \tag{4.39}$$

In the last line we have redefined back the heavy variables for fixed light fields. This change of variables is given by $\Phi = F_h^{-1}(\Phi', F_l(\phi))$, with F_h^{-1} defined by $F^{-1}(\Phi,\phi) = (F_h^{-1}(\Phi,\phi), F_l^{-1}(\Phi,\phi))$.

The last equation shows that

$$\overline{S'}[\phi] = \bar{S}[F_l(\phi)], \tag{4.40}$$

which is also consistent with a change of variables in eq. (3.12). So, for the transformations we are considering now, the heavy field redefinition does not modify $\bar{S}$, while the light field redefinition commutes with the integration of the heavy field.

However, the local version of eq. (4.40),

$$\left\lceil \overline{S'} \right\rceil_n [\phi] \stackrel{?}{=} \left\lceil \bar{S} \right\rceil_n [F_l(\phi)], \tag{4.41}$$

does not hold, in general. Here, both $\lceil \bar{S} \rceil_n$ and $\lceil \overline{S'} \rceil_n$ are defined by eq. (3.13) (with sources coupling linearly to ϕ). Eq. (4.41) is equivalent to

$$Z(\bar{S}, F_l^{-1}) \stackrel{?}{=} Z(\lceil S \rceil_n, F_l^{-1}) + O(\lambda^{n+1}), \tag{4.42}$$

as can be seen by performing a redefinition $\phi \to F_l(\phi)$, using the definition of $\lceil \overline{S'} \rceil_n$, its assumed equality with $\lceil \bar{S} \rceil_n \circ F$ and performing another redefinition $\phi \to F_l^{-1}(\phi)$. But requiring agreement to a given order of the Green functions of ϕ is not the same as requiring agreement to that order of the Green functions of $F_l^{-1}(\phi)$. This means that doing redefinitions does not commute with matching to a local action. To prove this, we give here a counterexample to eq. (4.42). Instead of considering redefinitions of the fields in the action, one can equivalently deal with redefinitions in the source terms, because changes of variables in the path integral relate one case to the other. We will use this fact to simplify the following discussion, in which we consider changes of the source terms only.

Consider the (non-local) action $\bar{S}$ coming from integrating out the field Φ, using eq. (3.12), from the theory defined by the UV action

$$S_{\rm UV}[\Phi,\phi] = -\int d^4x \left\{\frac{1}{2}\phi\Box\phi + \frac{1}{2}\Phi(\Box + M^2)\Phi + g\Phi\phi^2\right\}. \tag{4.43}$$

Let $\bar{S}_{\rm tree}$ be the action obtained by integrating out Φ at tree-level. We take $1/M^2$ as the small parameter that controls the approximation of the EFT. The truncation of $\bar{S}_{\rm tree}$ is

$$\left\lfloor \bar{S}_{\rm tree} \right\rfloor_n [\phi] = -\int d^4x \left\{\frac{1}{2}\phi\Box\phi - \frac{g^2}{2M^2}\phi^2 \left(\sum_{k=0}^{n-1} \frac{(-1)^k \Box^k}{M^{2k}}\right)\phi^2\right\}, \tag{4.44}$$

[12]This kind of redefinition is implicitly performed in the method proposed in ref. [113] to account for the heavy-light loop contributions.

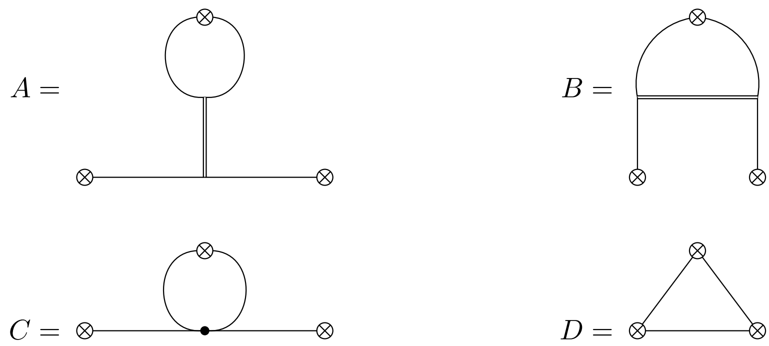

Figure 4.5: Relevant diagrams for the 1-loop 3-point function generated by Z' and Z'_n. A and B are diagrams of $G^{(3)}$, C is a diagram of $G_n^{(3)}$ and D appears in both.

At tree-level, $\lfloor \bar{S}_{\text{tree}} \rfloor_n$ gives the same results as $\bar{S}_{\text{tree}}$ up to order M^{-2n}. The local effective action $\lceil \bar{S} \rceil_n$ is obtained by including both the heavy loop corrections $\bar{S} - \bar{S}_{\text{tree}}$ and the corrections $\lceil \bar{S} \rceil_n - \lfloor \bar{S} \rfloor_n$ due to heavy-light loops. Notice that $\lceil \bar{S} \rceil_n$ will not contain monomials that are odd powers of ϕ because of the $\phi \to -\phi$ symmetry of the original action S_{UV}, that is preserved in the EFT. We will show that the functions

$$Z(\bar{S}, \phi + \lambda\phi^2)[J] = \int \mathcal{D}\phi \exp\left(i\bar{S}[\phi] + J_\alpha(\phi + \lambda\phi^2)^\alpha\right), \tag{4.45}$$

$$Z(\lceil S \rceil_n, \phi + \lambda\phi^2)[J] = \int \mathcal{D}\phi \exp\left(i\lceil \bar{S} \rceil_n[\phi] + J_\alpha(\phi + \lambda\phi^2)^\alpha\right), \tag{4.46}$$

do not satisfy the identity $Z(\bar{S}, \phi + \lambda\phi^2)[J] \stackrel{?}{=} Z(\lceil S \rceil_n, \phi + \lambda\phi^2)[J] + O(1/M^{2n})$ for any $n > 0$. It is enough to see that the 3-point functions $G^{(3)}$ and $G_n^{(3)}$ generated by them are different. The relevant diagrams are presented in Figure 4.5. Because computing Green functions for ϕ with the non-local action $\bar{S}$ is exactly equivalent to computing them with the local action S_{UV}, we present the corresponding diagrams in terms of the Feynman rules for S_{UV}, with double lines representing the propagator for the heavy field Φ. The 4-line dot in diagram C represents the ϕ^4 local interaction in $\lceil \bar{S} \rceil_n$ generated at tree level. We have

$$G^{(3)} = A + B + D + (\text{permutations}), \qquad G_n^{(3)} = C + D + (\text{permutations}). \tag{4.47}$$

Diagram C can be obtained by expanding in powers of $1/M^2$ the heavy propagator inside $A + B$. Thus, for $G_n^{(3)}$ to be equal to $G^{(3)}$ to order n, we should have

$$A + B + (\text{permutations}) \stackrel{?}{=} C + (\text{permutations}) + O\left(\frac{1}{M^{2n+2}}\right). \tag{4.48}$$

This is not true in general. Denoting by p_1, p_2 and p_3 the momenta in each diagram entering in the top, left and right vertex, respectively, we have that, in dimensional regularization,

$$A|_{p_1^2=0} = C|_{p_1^2=0} = 0, \tag{4.49}$$

because when $p_1^2 = 0$ both A and C are scaleless integrals. On the other hand,

$$B|_{p_1^2=0} = -\frac{8ig^2\lambda\mu^{2\epsilon}}{p_2^2 p_3^2} \int \frac{d^{4-2\epsilon}k}{(2\pi)^{4-2\epsilon}} \frac{1}{k^2(k+p_1)^2[(k+p_1+p_2)^2 - M^2]}\bigg|_{p_1^2=0} \tag{4.50}$$

$$= \frac{g^2\lambda}{2\pi^2 p_2^2 p_3^2 (p_2^2 - p_3^2)} \left\{ \log\left(\frac{M^2 - p_3^2}{M^2 - p_2^2}\right) \left[\frac{1}{\epsilon} - \gamma + \log\frac{4\pi\mu^2}{M^2}\right] + \mathrm{Li}_2 \frac{p_2^2}{M^2} - \mathrm{Li}_2 \frac{p_3^2}{M^2} \right.$$
$$\left. + \log^2\left(1 - \frac{p_2^2}{M^2}\right) - \log^2\left(1 - \frac{p_3^2}{M^2}\right) \right\} + O(\epsilon). \qquad (4.51)$$

where we have used the results for 1-loop integrals presented in [114]. The conclusion is that $Z(\bar{S}, \phi + \lambda\phi^2)$ is not approximated by $Z(\lceil S \rceil_n, \phi + \lambda\phi^2)$ to order n, which completes the counterexample to eq. (4.42). Nevertheless, the approximation should be recovered on-shell, as $Z(\bar{S}, \phi + \lambda\phi^2)$ and $Z(\lceil S \rceil_n, \phi + \lambda\phi^2)$ differ only from the original generating functions $Z(\bar{S})$ and $Z(\lceil S \rceil_n)$ by the source terms. This can be checked directly: diagram B does not have a pole at $p_1^2 = 0$ and therefore it does not contribute to the S matrix.

All this discussion applies irrespectively of whether method A or B is employed for the matching. Let us add a few remarks on method A. In this method, the matching is standardly performed for Green functions of the fields ϕ that appear in the action, be it the original or the transformed one. If the comparison with the Green functions for action S_{UV} or S'_{UV} is performed with a general local effective action that includes all the symmetric operators to a given order, then S or S' will be automatically found, respectively. As we have shown, they will be equivalent, but not directly related by the transformation F. A problem may arise if a non-redundant basis is employed. Then it is not possible, in general, to adjust the coefficients in such a way that the off-shell Green functions reproduce those of the fundamental theory with an arbitrary S_{UV}. Indeed, proceeding in this way would be like trying to match Green functions of different fields, ϕ and $\phi' = F(\phi)$. Therefore, any conversion into a reduced basis should be performed after the (off-shell) matching, also in method A. The alternative is to require only agreement for on-shell quantities, as proposed in [91].

In eq. (4.37) and (4.39) we have used in several places (determinant, action and source terms) the fact that F_l is independent of Φ. Therefore, the simple relation eq. (4.40) cannot be extended to the general case in which F_l depends on the heavy fields.[13] Nevertheless, as long as the redefined light field is a valid interpolating field for the light particles, we have

$$Z(S_{\mathrm{UV}}) \sim Z(\bar{S}'') \qquad (4.52)$$

with

$$\exp\left(i\bar{S}''[\phi]\right) = \int \mathcal{D}\Phi \det\left(\frac{\delta F}{\delta(\phi, \Phi)}\right) \exp\left\{iS_{\mathrm{UV}}[F(\Phi, \phi)]\right\}. \qquad (4.53)$$

$\bar{S}''$ (and the corresponding $\lceil \bar{S}'' \rceil_n$) can be used to compute on-shell amplitudes of light particles, even if it has no general simple connection with $\bar{S}$ ($\lceil \bar{S} \rceil_n$).

In addition to these remarks, note that the discussion in the previous section about renormalization before and after the field redefinition also applies to the fundamental and effective renormalized theories that enter the matching.

[13] The redefinition used in the method of ref. [34] to account for heavy-light loops belongs to this more general case.

4.5 Perturbative expansions

4.5.1 Removing reparametrization redundancy

The theory space of possible actions with a given field content can be divided into equivalence classes, with actions in the same class related by field redefinitions (possibly with some restrictions, as discussed in section 4.2). All the actions in the same class give rise to the same S matrix. An elegant way of working with these equivalent classes, which has been mostly employed in non-linear sigma models, is to use a geometric approach, in which the fields are coordinates of a differentiable manifold with a connection [100, 115–122]. This allows to maintain explicit covariance under changes of coordinates (that is, field redefinitions). Here we will study the more mundane (but also useful) approach of choosing a representative for each equivalence class and systematically reducing every action to the corresponding representative [123]. This is what we called "fixing a gauge" in the introduction. In this subsection, we first review how this gauge fixing can be performed order by order in perturbation theory and then examine the consequences of this procedure.

The EFT is organized as a power series in $\lambda = 1/\Lambda$:

$$S[\phi] = \sum_{n=0}^{\infty} \lambda^n S_n[\phi]. \tag{4.54}$$

Let us study the effect of local perturbative redefinitions of order k, of the form $F(\phi) = \phi + \lambda^k G(\phi)$, with $k \geq 1$ and G analytic in λ. Under this redefinition, the action changes into

$$S'[\phi] = S[F(\phi)] \tag{4.55}$$

$$= \sum_{n,m=0}^{\infty} \frac{1}{m!} \lambda^{n+km} G^{\alpha_1}(\phi) \cdots G^{\alpha_m}(\phi) \frac{\delta^m S_n}{\delta\phi^{\alpha_1} \ldots \delta\phi^{\alpha_m}} \tag{4.56}$$

$$= S[\phi] + \lambda^k G^\alpha(\phi) \frac{\delta S_0}{\delta \phi^\alpha} + O\left(\lambda^{k+1}\right). \tag{4.57}$$

In particular, the last line of this equation shows that all the actions that differ by order-k terms proportional to the lowest-order equation of motion belong to the same class to order k. Suppose S_k contains a term of the form $f_k^\alpha(\phi)\delta\mathcal{K}/\delta\phi^\alpha$, with $\mathcal{K}$ any term in S_0. Then, this term can be eliminated by the following field redefinition of order k:

$$F_k^\alpha(\phi) = \phi^\alpha - \lambda^k f_k^\alpha(\phi). \tag{4.58}$$

Obviously, this redefinition has no effect to order $k-1$. At order k, its only effect is to add $-f_k^\alpha \delta S_0/\delta\phi^\alpha$ to the action, which is the same as using the lowest-order equation of motion to change $\delta\mathcal{K}/\delta\phi^\alpha$ by $\delta(\mathcal{K} - S_0)/\delta\phi^\alpha$. The redefinition eq. (4.58) also changes the action at order $k+1$ and higher, as indicated in eq. (4.55).

Therefore, once the lowest order action S_0 is fixed, a representative of each equivalence class can be chosen, order by order, by picking at each order $k \geq 1$ a specific term $\mathcal{K}_k$ (which could be a linear combination of other terms) of S_0 and imposing (besides the hermiticity of the action and invariance under the relevant symmetries) that the coefficients of operators in S_k proportional to $\delta\mathcal{K}_k/\delta\phi^\alpha$ be equal to zero. Identifying these operators may require algebraic manipulations and integration by parts. Note that, for a given $\mathcal{K}_k$, the maximal number of different factors $\delta\mathcal{K}_k/\delta\phi^{ix}$ is equal to the number of different fields ϕ^i. Therefore, to eliminate all the ambiguities at each order k, $\mathcal{K}_k$ should be chosen such that $\delta\mathcal{K}_k/\delta\phi^{ix} \neq 0$ for all i. A standard choice that works for any k is to take $\mathcal{K}_k$ as the sum of all the kinetic terms. Then, any subsequent redefinition of order k would move the action into a different gauge, so the remaining linearly-independent operators that can appear in S_k will form a non-redundant basis of operators at that order. To reach this basis from an arbitrary effective action, one proceeds order by order. Let $S^{(k-1)}$ be the transformed action after consecutive field redefinitions $F_1, \ldots, F_{k-1}$ that put it in the prescribed form to $O(\lambda^{k-1})$ and let $f_k(\phi)$ be the coefficient of $\delta\mathcal{K}_k/\delta\phi^\alpha$ in $S^{(k-1)}$. Then, the field redefinition eq. (4.58) transforms $S^{(k-1)}$ into $S^{(k)}$, which is in the prescribed form to $O(\lambda^k)$. The actions $S^{(k)}$ and S are connected by the field redefinition $F = F_k \circ F_{k-1} \circ \cdots \circ F_1$.

We see that, in order to define a non-redundant basis of operators, it is enough to use the lowest-order equations of motion in the operators to be eliminated [91]. Indeed, for this purpose, and as long as all the algebraically-linearly-independent operators are included from the very beginning, the higher order corrections at each step k are absorbed into coefficients that were arbitrary anyway, so there is no need to worry about them. In fact, the same holds for the coefficients of the non-vanishing operators at order k. So, as described in the last paragraph, it actually suffices to identify a set of appropriate $\mathcal{K}_k$ and put to zero all the terms proportional to $\delta\mathcal{K}_k/\delta\phi^\alpha$. However, we have already stressed that it is often important to know the dependence of the coefficients in the transformed action on the original ones. Then, the redefinition must be performed explicitly. When working to next-to-leading order, $n = 1$, the algorithm has only one step ($k = 1$) and it is sufficient to apply the equations of motion of S_0 to the operators to be eliminated. But when working at orders $n \geq 2$, it is mandatory to include the higher-order corrections in the redefinition. This is the case when one wants to rewrite a known action S in a particular basis. To second order, for instance, this can always be achieved as explained above by a field redefinition $F = F_2 \circ F_1$, where $F_k^\alpha(\phi) = \phi^\alpha + \lambda^k G_k^\alpha(\phi)$, with G_k a λ-independent function of the parameters of S_m , $m \leq k$. The redefined action is

$$S'[\phi] = S[F(\phi)] \tag{4.59}$$

$$= S_0[\phi] + \lambda\left[S_1 + G_1^\alpha(\phi)\frac{\delta S_0}{\delta\phi^\alpha}\right] \tag{4.60}$$

$$+ \lambda^2\left[S_2 + G_1^\alpha(\phi)\frac{\delta S_1}{\delta\phi^\alpha} + \frac{1}{2}G_1^\alpha(\phi)G_1^\beta(\phi)\frac{\delta^2 S_0}{\delta\phi^\alpha\delta\phi^\beta} + G_2(\phi)\frac{\delta S_0}{\delta\phi^\alpha}\right] + O(\lambda^3) \tag{4.61}$$

$$= S_0'[\phi] + \lambda S_1'[\phi] + \lambda^2 S_2'[\phi] + O(\lambda^3). \tag{4.62}$$

We see explicitly that S_k' depends in general on the parameters of all S_n with $n \leq k$, and also that the higher-order effect of F_1 must be taken into account in order to get the correct dependence of the parameters of S_2' on the parameters of S_0, S_1 and S_2. In particular, (4.62) is relevant when comparing, to second order in λ, the constraints on the operator coefficients in one basis with the ones in another basis. The same considerations apply to perturbative matching: *field redefinitions performed to eliminate terms of order k in the effective action have an impact on the matching not only at order k but also at higher orders.* This is readily seen in eq. (4.58) and eq. (4.62), taking S to be the local effective action obtained from matching to a more fundamental theory. For instance, even if we put $G_2 = 0$ in eq. (4.62), we cannot say that to order 2 this S is equivalent to $S_0 + S_1' + S_2$. Changing S_1 by S_1' requires in general a change $S_2 \to S_2'$. Observe also that knowledge of the field redefinition F (in particular of F_1) is needed to find the correct S_2'.

We stressed in section 4.3 that using the exact classical equations of motion is not equivalent to a non-infinitesimal field redefinition, and that it does not lead to an equivalent action. Perturbatively, the effect of an order-k field redefinition can be written as

$$S'[\phi] = S[\phi] + \lambda^k\, G(\phi)^\alpha \frac{\delta S}{\delta\phi^\alpha} + O\left(\lambda^{2k}\right). \tag{4.63}$$

The exact equations of motion only give the linear contribution, starting at λ^k, but miss the remaining $O(\lambda^{2k})$ terms, which are necessary for S' to be equivalent to S. Hence, the equations of motion at higher orders, as used for instance in [27,95], are not sufficient to find the higher-order corrections induced by a field redefinition. In particular, using in S_1 (and S_2) the equation of motion to second order in λ does not give, in general, an action that is equivalent to S to second order. The same conclusions apply to the case in which the equations of motion of S' are used in S_1 (and S_2), as can be seen by exchanging the roles of S and S' and considering the inverse transformation. To obtain the correct S', it is necessary to perform the actual field redefinition in every term of the original action. This can be done either directly or using the functional-derivative expansion in the second line of eq. (4.55).

The redefinitions needed to reduce some gauge-invariant action to a basis do not break its invariance. The whole reducing procedure can be performed in a gauge-covariant way. The $f_k^\alpha(\phi)$ defined above are gauge-covariant operators with the same quantum numbers as $\delta\mathcal{K}/\delta\phi^\alpha$ or, equivalently, of ϕ^α. Therefore, when the redefinition in eq. (4.58) is performed, all the factors that multiply the functional derivative in the redefined action in eq. (4.55) are covariant, as $G^\alpha(\phi) = -f_k^\alpha(\phi)$. We show here that these functional derivatives are also covariant. We denote the action by $S[\phi, A]$, where A are the gauge fields. Consider a redefinition

$$\phi \to \phi' = \phi + \zeta G, \qquad A \to A' = A + \eta H, \tag{4.64}$$

where G and H are covariant operators with G in the same representation as ϕ and H in the adjoint representation. The gauge fields A only appear in the action S through the field strength $F_{\mu\nu}^{(A)} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\mu, A_\nu]$ and the covariant derivative $D_\mu^{(A)} = \partial_\mu - igA_\mu$. For these objects, we have

$$F_{\mu\nu}^{(A')} = F_{\mu\nu}^{(A)} + \eta\left(D_\mu^{(A)} H_\nu - D_\nu^{(A)} H_\mu\right) - i\eta^2 g[H_\mu, H_\nu], \tag{4.65}$$

$$D_\mu^{(A')}\mathcal{O} = D_\mu^{(A)}\mathcal{O} - i\eta g H_\mu \mathcal{O}. \tag{4.66}$$

All the terms in this expressions are covariant, with the same representation under the gauge group. It follows that the transformed action $S[\phi'(\phi, A), A'(\phi, A)]$ is still gauge invariant. Its expansion in ζ, η is

$$S[\phi{+}\zeta G, A{+}\eta H] = \sum_{m,n=0}^{\infty} \frac{\zeta^m\, \eta^n}{(m+n)!}\, G^{\alpha_1}\cdots G^{\alpha_m}\, H^{\beta_1}\cdots H^{\beta_n}\, \frac{\delta^{m+n} S}{\delta\phi^{\alpha_1}\cdots\delta\phi^{\alpha_m}\delta A^{\beta_1}\cdots\delta A^{\beta_n}}. \tag{4.67}$$

Because this is invariant for any ζ and η, it must be invariant order by order in each of them. Now, the covariance of the functional derivatives follows from the covariance of the product of the operators G and H. In particular, the equation of motion operators $\delta S/\delta\phi$ and $\delta S/\delta A$ must be covariant and therefore it is possible to write them in terms of field strengths and covariant derivatives, with no independent occurrences of the gauge fields and partial derivatives.

4.5.2 Power counting

In this section, we follow the notation presented in section 3.3: $N_\eta(\mathcal{O})$ denotes the power of η that corresponds to the operator $\mathcal{O}$. The "natural" coefficient of $\mathcal{O}$ can be found as a product of $\eta^{N_\eta(\mathcal{O})}$ for all η.

Consider a field redefinition given by $F^\alpha(\phi) = \phi^\alpha + f^\alpha(\phi)$, with f local. We can write each f^j as a linear combination of local operators $f^j_1, \ldots, f^j_{n_j}$. The redefinition will be perturbative when $\min\{N_\eta(f^j_1), \ldots, N_\eta(f^j_{n_j})\} > N_\eta(\phi^j)$ for some η. Thanks to the factorization property eq. (3.8), the redefinition preserves the counting rule: an action with natural operator coefficients is transformed into an action with natural operator coefficients whenever the coefficients of the operators in f are natural. The latter means that the coefficient α^j_i of each operator f^j_i is

$$\alpha^j_i \simeq \prod_\eta \eta^{\Delta_\eta(f^j_i)-\Delta_\eta(\phi^j)} = \prod_\eta \eta^{N_\eta(f^j_i)-N_\eta(\phi^j)}. \tag{4.68}$$

This condition will always be satisfied if the redefinition is performed to eliminate any term in an action with natural coefficients.[14] Explicitly, if the redefinition removes a term $\mathcal{Q} = f^\alpha(\delta\mathcal{K}/\delta\phi^\alpha)$, with $\mathcal{K}$ any term in the original action S, an operator $\mathcal{O}$ in the original action will give rise to a sum of terms of the form

$$\mathcal{O}_{[m]} = f^{\alpha_1}\ldots f^{\alpha_m}\frac{\delta^m\mathcal{O}}{\delta\phi^{\alpha_1}\ldots\delta\phi^{\alpha_m}}, \tag{4.69}$$

with power counting given by

$$N_\eta(\mathcal{O}_{[m]}) = N_\eta(\mathcal{O}) + m\cdot(N_\eta(\mathcal{Q}) - N_\eta(\mathcal{K})). \tag{4.70}$$

We have used the factorization property $\Delta_\eta(\mathcal{Q}) = \Delta_\eta(f) + \Delta_\eta(\mathcal{K}) - \Delta_\eta(\phi)$. If the coefficient in $\mathcal{Q}$ happens to be suppressed by a factor ξ, relative to its natural value, while $\mathcal{O}$ has a coefficient suppressed by a factor κ and $\mathcal{K}$ is natural, then the contribution $\mathcal{O}_{[m]}$ in S' will be be suppressed by $\xi^m\kappa$.

In the rest of the section we point out a few implications of this counting when working with the SMEFT [124]. This EFT is usually described as having a power counting determined by the canonical dimension Δ of the operators: $N_\lambda(\mathcal{O}) = \Delta(\mathcal{O}) -$

[14] If the definition has any other purpose, coefficients α^j_i smaller or larger than (4.68) (that is, "under-natural" and "super-natural", respectively) are possible that still preserve the perturbativity of the transformation. Super-natural coefficients will give rise to perturbative corrections that destabilize the hierarchical structure of the original effective action. This will not be reflected in on-shell quantities if all the new terms are included, since the new action is equivalent to the original one. But the perturbative orders will be mixed, which must be taken into account in truncations of the new action.

4. In this case, $\Delta_{1/\Lambda} = \Delta$ and $c_{1/\Lambda} = 4$. We ignore in the following the few operators of dimension 5 and 7. In order to reach some standard basis at dimension 6, one may need to redefine the Higgs doublet ϕ in such a way that the dimension-6 terms proportional to $\Box\phi$ are removed. The necessary cancellation arises from the kinetic term, while the remaining terms of dimension 4 generate other terms (of the form $\mathcal{O}_{[1]}$ in eq. (4.69)) at dimension-6. This is the same as using the dimension-4 Higgs equation of motion in the terms to be eliminated. As discussed in the previous section, there will be corrections at dimension 8, from substituting one ϕ at dimension 6 or two ϕ at dimension 4. Note, however, that an important detail is missing in this discussion: the SMEFT does not start at dimension 4. The gauge-invariant operator $\mathcal{O}_\mu = \phi^\dagger\phi$ has canonical dimension 2. Under the same field redefinition, this super-renormalizable operator gives contributions of the form $(\mathcal{O}_\mu)_{[1]}$ of dimension 4. Even if one can absorb the corrections into a renormalization of the SM couplings, this renormalization modifies the coefficients at dimension 6 (see chapter 8). These linear contributions can also be found using the equations of motion. But on top of this, $\mathcal{O}_\mu$ contributes at dimension 6 with terms of the form $(\mathcal{O}_\mu)_{[2]}$. Indeed, using eq. (4.70) in this particular case, we find $N_\lambda((\mathcal{O}_\mu)_{[2]}) = -2 + 2 \cdot 2 = 2$. Because $(\mathcal{O}_\mu)_{[2]}$ is proportional to $\delta^2\mathcal{O}_\mu/\delta\phi^2$, these dimension-6 contributions will be missed if one only uses the equations of motion. Note that this does not contradict the standard procedure to reach a basis by using the equations of motion, reviewed in section 4.5, because the action at leading order is not given by the dimension-4 terms but by the integral of $\tilde{\mathcal{O}}_\mu = -\mu^2\mathcal{O}_\mu$. Thus, the field redefinition we are considering has nothing to do with the equations of motion of the action at leading order.[15]

Of course, the coefficient μ^2 of $\mathcal{O}_\mu$ is not natural with the counting based on dimensions. Experimentally, we know that there is a hierarchy $\mu \ll \Lambda$. Hence, the new terms $(\mathcal{O}_\mu)_{[1]}$ and $(\mathcal{O}_\mu)_{[2]}$ arising from $\mathcal{O}_\mu$ will carry an extra suppression $(\mu/\Lambda)^2$ and will typically be less important, numerically, than the corresponding dimension-4 and dimension-6 terms. This can be rephrased in a more systematic way by incorporating μ in the power counting: $\Delta_\lambda(\mu^2) = 2$. This modified counting is nothing but dimensional analysis. It follows that $N_\lambda(\tilde{\mathcal{O}}_\mu) = 0$. So, with the new counting $\tilde{\mathcal{O}}_\mu$ is of the same order as the dimension-4 terms, and the SM is the leading order approximation of the SMEFT.

Consider next (differential) cross sections calculated in the SMEFT to order n in $1/\Lambda^2$. They are schematically of the form

$$\sigma \propto \left| A^{(0)} + \frac{1}{\Lambda^2} A^{(1)} + \frac{1}{\Lambda^4} A^{(2)} + \cdots \right|^2, \tag{4.71}$$

where $A^{(n)}$ is the coefficient of Λ^{-2n} in the $1/\Lambda^2$ expansion of the on-shell amplitude. We denote by $A^{(n)}_{i_1 i_2 \ldots i_k}$ the part of $A^{(n)}$ given by diagrams with k insertions of operators, one from S_{i_1}, another one from S_{i_2}, etc. Then, we have:

$$A^{(n)} = \sum_{i_1 + i_2 + \cdots + i_k = n} A^{(n)}_{i_1 i_2 \ldots i_k}. \tag{4.72}$$

[15] The equation of motion at leading order is just $\phi = 0$. This could be used to eliminate recursively all the terms containing the Higgs doublet at dimension 4 and above. This looks strange, but it is consistent with the natural value of μ^2 being of order Λ^2, according to the dimensional counting. Actually, a field with a mass of the order of the cutoff will decouple from the other fields. More precisely, it should be integrated out.

We ignore here phase-space factors, as we are only going to discuss the relative importance of the quadratic and interference terms in the evaluation of the right-hand side of eq. (4.71), which are schematically of the form $A^{(i)}A^{(j)}$, with $i, j \leq n$. Let us nevertheless refer to [30] for an interesting result for the scaling of total cross sections in NDA. Expanding eq. (4.71),

$$\sigma \propto \left|A^{(0)}\right|^2+\frac{2}{\Lambda^2}\,\mathrm{Re}\left(A^{(0)*}A_1^{(1)}\right)+\frac{1}{\Lambda^4}\left[\left|A^{(1)}\right|^2+2\,\mathrm{Re}\left(A^{(0)*}A_{11}^{(2)}+A^{(0)*}A_2^{(2)}\right)\right]+O\left(\frac{1}{\Lambda^6}\right),$$

where we have grouped contributions of the same order. In many applications, only the first two terms need to be taken into account. However, there are processes in which the interference terms $\mathrm{Re}(A^{(0)*}A^{(1)})$ vanish (or are very suppressed) [125]. Then the terms in brackets give the leading correction and must be included in the analysis [126, 127]. Furthermore, it may occur that $\mathrm{Re}(A^{(0)*}A_2^{(2)})$ vanishes at well. This happens often when the process is mediated by one heavy particle in the UV theory [128], since its propagator generates effective operators with the same symmetry properties at all orders. In this scenario, the quadratic term $|A^{(1)}|^2$ and the interference term $\mathrm{Re}(A^{(0)*}A_{11}^{(2)})$ give the only corrections to order $1/\Lambda^4$ and the terms in S_2 are not necessary to compute the leading-order correction to the cross section.

Is this situation preserved by field redefinitions in the EFT? The equivalence theorem tells us that the amplitudes are invariant and comparing order by order we see that the same will hold for each $A^{(i)}$. However, the individual contributions $A_2^{(2)}$ and $A_{11}^{(2)}$ need not be invariant separately. Hence, it is possible that $\mathrm{Re}(A^{(0)*}A_2^{\prime(2)})$ does not vanish any longer, and then the new operators in S_2' cannot be neglected, unless they do not interfere with $A^{(1)}$.

The quadratic terms may also be very relevant if the coefficient of an involved operator $\mathcal{O}^{(1)}$ in S_1 is for some reason $\alpha > 1$. Then, $|A^{(1)}|^2$ and $A^{(0)*}A_{11}^{(2)}$ are enhanced by α with respect to $A^{(0)*}A^{(1)}$. All these terms could then be comparable at sufficiently high energies. In this case, it is mandatory to include them. Furthermore, at second order the effect of operators in S_2 can be neglected if it is known that their coefficients are significantly smaller than α^2. This is the case in certain SM extensions (such as the example in section 8.6). But again, these statements depend on the field coordinates. A field redefinition that removes $\mathcal{O}^{(1)}$ introduces in S_2 operators with an enhancement α^2, so their contributions $\mathrm{Re}(A^{(0)*}A_2^{\prime(2)})$ can no longer be neglected.

4.5.3 The loop expansion

Our previous discussion of power counting also applies to the loop expansion of the EFT and of the fundamental theory. Let us start with the former, which makes no reference to loops in the fundamental theory and is valid also for strongly coupled UV theories.[16] Reintroducing explicitly $\hbar$, we can formally expand the generating functional of the renormalized EFT as

$$Z(S)[J] = \sum_{k=0}^{\infty} \hbar^k Z_{\text{eff}}(S)^{(k)}[J], \tag{4.73}$$

[16] In some interesting cases, the former are related with some other parameter in the fundamental theory. For instance, loops in chiral perturbation theory are related to $1/N_c$ corrections in low-energy QCD. This type of relation has been made precise in gauge-gravity dualities [129].

This actually corresponds to an expansion in the EFT couplings divided by $1/(4\pi)^2$. We have already mentioned that the power counting of the (renormalized) effective action should be consistent with this expansion.

When working in a reduced basis at order λ^n, it is often found that counterterms made out of operators that were removed to reach that basis are necessary to obtain renormalized Green functions. These counterterms (including their arbitrary finite part) can then be written in the reduced basis, to order n, by a perturbative field redefinition in which the perturbation parameter is proportional to $\hbar^m \lambda^n$, with m the loop order of the counterterm. In this way, one finds a reduced renormalized action $(S^R)'$ (instead of the initial renormalized reduced action). As stressed in section 4.2, this action does not give finite Green functions of the elementary field when the regulator is removed. But it does give finite S-matrix elements. So, we can say that the theory described by this action has been renormalized on shell (this concept is not to be confused with an on-shell renormalization scheme). To illustrate this, consider one of the simplest examples of a reduced action: requiring canonical normalization of the kinetic terms in order to remove the exact ambiguity of field rescalings. To obtain finite Green functions, wave function renormalization is required. Then, the renormalized action is no longer in the reduced form. By a regulator-dependent field rescaling, we can, however, transform the renormalized action into a reduced renormalized action, which has canonical kinetic terms; the wave function counterterms are moved into a redefinition of the remaining counterterms. But the Green functions associated to this action are just the Green functions of the bare field (written in terms of renormalized masses and couplings), which are divergent [130]. Nevertheless, these Green functions can be used to calculate finite scattering amplitudes, with the regulator removed after the on-shell reduction. Coming back to the general case, note that, at higher orders in λ, the reduced renormalized action will contain also corrections of order $\hbar^m$ and higher, as indicated in the power-counting formula eq. (4.70). These higher-order counterterms are also required for finiteness of the S matrix.

Importantly, the finite parts of all the redefined counterterms can be fixed in terms of renormalization conditions for each operator in the reduced action (see ref. [88] for a detailed argument in the context of the exact renormalization group). Thus, no independent renormalized couplings associated to redundant operators need to be introduced. This implies that one can describe the renormalization-group evolution of the reduced renormalized couplings in terms of reduced renormalized couplings only, which has led to the definition in [131] of effective beta functions along the reduced directions, depending only on reduced renormalized couplings. The renormalization group equation of on-shell quantities can be written in terms of these effective beta functions. Depending on the aimed precision, the higher-order corrections introduced by the field redefinition may be relevant for the running of reduced couplings. Once again, we stress that using the equations of motion may lead to incorrect results.

The linearized renormalization-group evolution can be described in terms of operator mixing; in this case, the beta functions are just anomalous dimensions. It has been observed in theories of interest that, at one-loop, the anomalous-dimension matrix has many vanishing entries, not explained by power counting [132–138]. This pattern has been explained in terms of the Lorentz structure of the involved operators, which forbids certain mixings at one loop [139].

Let us next consider the loop expansion of the fundamental theory S_{UV}, which we assume to be weakly coupled:

$$Z(S_{\rm UV})[J] = \sum_{k=0}^{\infty} \hbar^k Z(S_{\rm UV})^{(k)}[J]. \tag{4.74}$$

This corresponds to an expansion in the UV couplings divided by $1/(4\pi)^2$. In order to match this expansion, the bare effective action $\bar{S}$ in eq. (3.11) and its local versions $S = \lceil \bar{S} \rceil_n$ must depend explicitly on $\hbar$. We write

$$S[\phi] = \sum_{k=0}^{\infty} \hbar^k S^{(k)}[\phi]. \tag{4.75}$$

Then, each coefficient $Z^{(k)}$ is recovered by combining the powers of $\hbar$ in S with the ones associated to loops (and counterterms) in the EFT. The terms of order $\hbar^k$ in $\bar{S}$ must be corrected as explained in section 3.5 to find the coefficients $S^{(k)}$ in the expansion of the local action S. In approach A to matching, $S^{(k)}$ is given by k-loop diagrams in the UV theory and k-loop diagrams in the effective theory. Consider now a double expansion of S in $\hbar$ and λ:

$$S[\phi] = \sum_{m=0}^{n} \sum_{k=0}^{\infty} \hbar^k \lambda^m S_m^{(k)}[\phi]. \tag{4.76}$$

Note that if all the possible operators are included, then all the $S_m^{(k)}$ with a fixed m will contain the same operators. That is, the quantum corrections can be absorbed into a renormalization of the coefficients. But as discussed above, the point of matching is to compare the renormalized parameters of the EFT with the UV parameters in a renormalization scheme that is independent of the fundamental theory. Let us perform a perturbative field redefinition to eliminate an operator in $S_n^{(j)}$. This will rearrange all $S_m^{(k)}$ with $k \geq j$ and $m \geq n$, in a way consistent with eq. (4.70). Once again, there are practical consequences for the matching workflow. Suppose, for example, that $S_0^{(1)}$ is non vanishing and that we want to eliminate a first order term at the classical level, that is, a term in $S_1^{(0)}$. Then, there will be corrections not only to $S_1^{(0)}$ but also to $S_1^{(1)}$. This means that to calculate the matching at one-loop one must not only integrate out at that level, but also keep track of possible rearrangements of the effective action at the classical level. For this, it is not sufficient to know the final form at the classical level, $(S')_1^{(0)}$. So, the necessary corrections would be missed if one simply added the one-loop result to the results of tree-level matching given in the literature in particular basis. In other words and with more generality, the same light fields should be used in calculating the contributions at each order in the loop expansion.

A related issue is the fact that the classification in [96] of tree-level and loop operators, as those that can be induced or not at the tree-level, respectively, is not stable under field redefinitions. Therefore, this classification is only meaningful in one the following two interpretations: either for classes of operators that can be connected by field redefinitions, as proposed in [133], or for individual operators in the context of a given non-redundant basis of operators. This latter classification is basis-dependent. It turns out that the former is closely related to the pattern of operator mixing [139].

4.6 Conclusions

It is clear that a perturbative transformation, controlled by a small parameter λ, of any function depending analytically on λ will rearrange at all orders its perturbative expansion in λ, with the new coefficients depending on the original ones of the same or lower order. It is also clear that this rearrangement cannot be reproduced by a linear approximation in the perturbation. These simple facts may have non-trivial practical implications for EFTs.

EFTs are treated perturbatively in $1/\Lambda$ and in a loop expansion. When putting together different orders, it is crucial that they are all given in the same field coordinates. Otherwise, inconsistencies will be present, not only off-shell but also in on-shell observables. Preserving the consistency of field redefinitions requires some care when the different orders are calculated independently. Consider, for example, the SMEFT. We compute complete matching of this effective theory to arbitrary UV completions in chapter 8 at the tree-level and to order $1/\Lambda^2$, with Λ the lightest mass of the heavy particles. The results of the matching are given in the Warsaw basis [54]. They are very useful when working to order $1/\Lambda^2$ and at the tree level, but, unfortunately, they cannot be combined with future direct results of tree-level matching at order $1/\Lambda^4$. For this, knowledge of the higher-order terms generated by the lower-order field redefinitions is required. But this information is usually not provided in the literature, including our results in chapter 8, nor can it be recovered without repeating the whole calculation. Similarly, the Warsaw-basis results of tree-level matching cannot be combined with one-loop corrections, even if the latter are transformed into the Warsaw basis. Moreover, in some methods it may be convenient to also perform field redefinitions in the UV action in order to find one-loop corrections to the matching. For consistency, the tree-level contributions must be calculated for the same light fields. Note that an identical situation will arise again and again at higher and higher orders. This is not a fundamental problem, but it conflicts with the idea of building on previous results. The very same issues are relevant for conversions from one basis into another one. In particular, the generalization to higher orders of codes that automatically reduce actions (as in `MatchingTools`, introduced in chapter 5 and in the code presented in ref. [140]) or translate operator coefficients in different bases [141, 142] should implement field redefinitions rather than use equations of motion.

Field redefinitions not only change the action, but they also introduce a determinant (which can be added to the action or ignored in dimensional regularization, for local perturbative redefinitions) and modify the coupling to the sources. The latter effect is crucial in the derivation of Schwinger-Dyson equations and Ward identities. Ignoring it amounts to the bold replacement of a coupling of the source to a sum of composite operators by a linear coupling to the new elementary field. The LSZ formula implies that this replacement has no effect on on-shell quantities. But, as we have discussed, it does have a non-trivial impact on the form of the local effective action after matching and also on renormalization. All these subtle effects are relevant for the standard approach to matching and renormalization in terms of Green functions. However, we should stress that they go away when computing on-shell amplitudes, and might be avoided from the beginning in on-shell matching/renormalization.

Working with non-redundant bases of operators in EFTs has become a standard practice. Besides having a reduced number of operators, these bases have the clear advantage of attaching an unambiguous physical meaning to the set of coefficients that describe the theory to a given order. In particular, flat directions are avoided in comparing with the experimental data. Notwithstanding this, the conversion into non-redundant or reduced bases also has a few drawbacks. The first one is apparent in the example of section 8.6 and, more dramatically, in the example of section 4.2.2: the necessary field redefinitions typically give rise to a more complicated Lagrangian. Of course, this is not so in a truly model-independent approach, in which the starting point is a completely general EFT. But even in this case, the connection to particular UV completions is more intricate. More importantly, the physical predictions are typically more obscure, as the redefinition introduces correlations between operator coefficients that must be precisely preserved.[17] For instance, at first sight it is far from obvious that eq. (4.9) represents a free theory in disguise. Another issue that we have discussed is that reduced actions are not stable under renormalization and renormalization-group evolution, although the departures can be absorbed on-shell into reduced counterterms and effective beta functions. Finally, we have seen that field redefinitions may modify the power counting inherited from (classes of) UV theories, when it cannot be formulated in terms of the effective theory alone. So, such a power counting needs not be apparent in non-redundant bases.

The basis proposed in [133, 134] is optimal in dealing with all the issues just mentioned, but only for particular processes (Higgs physics) and rather specific UV scenarios (universal theories). Let us put forward another possibility: working with a standard over-complete, i.e. non-reduced, basis. In principle, this minimizes the problems pointed out above. Indeed, the connection with UV theories is more transparent, at least at the tree level, and there is flexibility in reproducing the field coordinates used in the matching. Also, if no redefinitions are made after matching, the physical predictions will typically be more obvious, and for simple models will not contain flat directions. The tree-level or loop origin of operators is directly given by the classification in [96]. And finally, from the point of view of the EFT itself, a general action in the over-complete basis is stable under renormalization and gives rise to finite off-shell Green functions that obey standard renormalization group equations. The package `BasisGen`, presented in chapter 6, can generate both reduced and over-complete basis for EFTs, because it performs field redefinitions only optionally.

Working in this approach would first involve selecting a basis at each order, obtained only with algebraic manipulations of the operators (the convenience of the latter should also be assessed in each case). Then, the results of matching and the beta functions would be provided in this basis (with information about possible field redefinitions in the process). And finally, to profit from the advantages of reduced bases, it would be useful to know the conversions of the over-complete basis into non-redundant bases, including higher-order operators generated in the process, or to have the tools to perform automatically this task.

[17] This is also a consequence of some algebraic manipulations performed to reach a given basis, such as Fierz reorderings.

Part II

Computer tools

Introduction: computer tools for effective field theories

To form a general idea of the size of the calculations that are encountered when dealing with EFTs for physics beyond the SM, one can look at the number of fields and terms in the effective Lagrangians. A basis of dimension-6 operators for the SMEFT has 3045 different operators (84 if flavor is neglected). Roughly speaking, the number of operators grows exponentially with their dimension. There are 48 different multiplets of new fields with linear dimension-4 couplings to the SM. Their effective Lagrangian with terms of dimension 5 or less involves thousands of operators.

In view of this situation, it becomes clear that it is convenient to have computer tools to help performing the calculations. Using a computer reduces the time that it takes to perform them and the possibility of introducing human errors. Computers are also useful to exchange information, given the large sets of data available. A complete and coherent set of tools is still under development. There are already several tools with different purposes, overlapping at different places and with different degrees of generality:

- `Rosetta` [141] translates between bases of dimension-6 operators in the SMEFT.
- `DsixTools` [143] and `Wilson` [144] do renormalization group evolution in the dimension-6 SMEFT and the Weak Effective Theory (WET, the EFT just below the electroweak scale), as well as one-loop matching between them.
- `SMEFTsim` [145] and `flavio` [146] compute predictions for observables in the dimension-6 SMEFT (and the WET, in the case of `flavio`).
- `DEFT` [142] generates bases of operators and translates between them, for gauge theories based on the $SU(N)$ groups.
- `smelli` [147] provides a global likelihood for the dimension-6 SMEFT.
- `SmeftFR` [148] generates the Feynman rules for the dimension-6 SMEFT.

In this part of the thesis, we present two Python packages: `MatchingTools` (in chapter 5) and `BasisGen` (in chapter 6). They cover calculations that cannot be performed with previously available tools, with emphasis on being general and efficient. `MatchingTools` does tree-level matching in any Lorentz-invariant EFT with linearly-realized gauge symmetry and operators of arbitrary dimension. It can also reduce the effective Lagrangian that is generated in this way to a basis of operators. `BasisGen` generates operator bases for EFTs under the same setting, for any reductive gauge group. As the number of fields and dimension of the operators is increased, this task easily becomes very computationally expensive. For this reason, `BasisGen` is designed to be as fast as possible without sacrificing generality. It improves the time taken to obtain a basis by two orders of magnitude with respect to previous tools. The use of field redefinitions is optional, allowing for the computation of over-complete basis, which may be convenient for some purposes, as explained in section 4.6.

We will make use of both tools in part III. In chapter 7, `BasisGen` will be used to generate the representations of the new fields in the BSMEFT with linear interactions of dimension 4 or less. It will also be used to compute a basis for the corresponding effective Lagrangian. In chapter 8, `MatchingTools` will perform the matching calculation and the procedure of reducing the Warsaw basis. The correctness of this basis can also be checked using `BasisGen`. In chapter 9, `BasisGen` will be used to produce the representations of the vector-like quarks with linear couplings of dimension 5 or less and their effective Lagrangian.

CHAPTER 5

`MatchingTools`: tree-level matching and reducing

5.1 Introduction

`MatchingTools` is a Python library for doing symbolic calculations in EFTs. It provides the tools to construct general models by defining their field content and their interaction Lagrangian. Once a model is given, the heavy particles can be integrated out at the tree level to obtain an effective Lagrangian in which only the light particles appear. After this matching procedure, some of the terms of the resulting Lagrangian might not be independent. `MatchingTools` contains functions for transforming these terms to rewrite them in terms of any chosen set of operators.

The procedure of matching can be described algebraically in terms of tensor calculus manipulations involving the computation of functional derivatives and the substitution of heavy fields by other previously obtained expressions (see section 3.5.2 and refs. [32, 34, 149–158]). The complexity of the process quickly grows with the number of heavy fields and their interactions. It is in this context where the development of a computer tool to automatize the process becomes necessary.

`MatchingTools` can perform tree-level integration of heavy fields in any given Lagrangian. It has been developed with the application to the SMEFT in mind, but it is able to work with any situation describable by a Lorentz invariant field theory in which the high energy degrees of freedom to be removed are scalars, vector-like or Majorana fermions, or vectors. By introducing the generic solution to their equations of motion, other types of fields can be treated as well. The validity of `MatchingTools` extends to any level in the expansion in inverse powers of the cut-off energy of the EFT.

The Lagrangian resulting from integration usually contains redundancies, as explained in chapter 4: there are operators that can be written in terms of others using identities of the symmetry group, integration by parts and field redefinitions. A complete set of operators that are independent under this set of transformations is called a basis. Several such bases have been described [54, 159, 160].

The transformation of the results of matching to a chosen basis can also be done using `MatchingTools`. One should introduce the identities between tensor expressions needed to transform some operators into others, as well as the desired basis.

There are other tools for the manipulation of bases of operators, such as `Rosetta` [141]. The portion of `MatchingTools` that deals with this calculations differs from it in two main points: first, it allows not only for the transformations between sets of already independent operators, but for the transformation of any set of operators into a basis. Moreover, `MatchingTools` has the ability of doing transformations not with the operators themselves, but with parts of them, allowing for general transformations between parts of tensor expressions into others. Actually, `MatchingTools` can be used as system for tensor calculus manipulations, not necessarily in the context of an EFT. It provides a fast way of doing complex symbolic calculations with many fields and terms involved, which is safe against algebraic errors.

A package that implements a similar way of dealing with the specification of models is `FeynRules` [161,162], thought its objectives are completely distinct. One possible direction for future work with `MatchingTools` is making the connection with `FeynRules`.

Among other computer tools for calculations in the context of the SMEFT we have `DsixTools` [143] (which allows for several calculations including a case of tree level matching) and `SMEFTsim` [145] (which is able to produce theoretical predictions and constraints for the Wilson coefficients of the dimension 6 SMEFT).

`MatchingTools` is available in GitHub (https://github.com/jccriado/matchingtools) and in the PyPI repository (https://pypi.python.org/pypi/matchingtools/), so it can be installed using pip [163] as

```
> pip install matchingtools
```

This chapter is organized as follows: sections 5.2.1, 5.2.2, 5.2.3 and 5.2.4 explain the features of `MatchingTools` and how to use it. Section 8.6 proposes a simple example that serves to see the library in action and as a test case. Some extra features for the applications in physics beyond the SM are introduced in section 5.4. Section 5.5 is an explanation of how to integrate out new types of fields that are not included in `MatchingTools`.

5.2 Interface

5.2.1 Creation of models

In this section we will describe how to create a model using the module `matchingtools.core`. It assumes that the classes and functions that are used are in the namespace. To import all the classes and functions that appear here do

```
from matchingtools.core import (
    Tensor, Operator, OperatorSum
    TensorBuilder, FieldBuilder,
    D, Op, OpSum,
    number_op, power_op
)
```

The `from ... import ...` style is recommended, as the expressions that appear when using this library tend to be long, so having the short names directly accessible is preferable.

Creation of tensors and fields

In MatchingTools, the basic building blocks for everything are the objects of the class Tensor, which we simply call tensors here. Examples of tensors are fields (light and heavy), symmetry group related tensors (such as Pauli matrices) or coupling constants (including gauge couplings, Yukawa couplings and masses).

Tensors have an attribute is_field that is True if and only if they are spacetime dependent (i.e., they are fields). Fields can have derivatives applied to them. The attribute num_of_der counts the number of derivatives that apply to a field. Derivatives are understood here to be covariant derivatives D_μ corresponding to the gauge group of the low energy EFT. Each derivative applies only to one field. The Leibniz rule is used whenever a derivative of a product is encountered. Tensors can be either commuting of anti-commuting, which is distinguished by the attribute statistics. It can be set equal to either boson or fermion, both being variables defined in this module. Finally, all tensors have an attribute indices, a list of integer numbers representing their tensor indices; and an attribute name, an identifier. Other attributes, content and exponent, are for internal use. Names starting with the character '$' are also reserved for internal calculations.

To create the tensors and fields of a model, the classes TensorBuilder and FieldBuilder should be used. For example, the Pauli matrices σ^a_{ij} could be defined as

```
sigma = TensorBuilder("sigma")
```

and then used when needed as sigma(i1, i2, i3) where i1, i2 and i3 are the indices. Similarly, a boson field ϕ (with its conjugate ϕ^*) and a fermion f (with its separate chiralities and their conjugates) are defined as

```
phi = FieldBuilder("phi", 1, boson)
phic = FieldBuilder("phic", 1, boson)

fL = FieldBuidler("fL", 1.5, fermion)
fR = FieldBuidler("fR", 1.5, fermion)
fLc = FieldBuidler("fLc", 1.5, fermion)
fRc = FieldBuidler("fRc", 1.5, fermion)
```

The second argument of FieldBuilder is the dimension of the field.

Definition of the interaction Lagrangian

Once all the tensors are created, we are ready to define the interaction Lagrangian. It should be a sum of operators, which in turn are just products of fields. It is defined using the functions OpSum and Op:

```
int_lag = -OpSum(Op(...), Op(...), ...)
```

The minus sign is defined for operator sums and individual operators. The function OpSum creates an object of the class OperatorSum, a container for a list of operators representing their sum. The function Op creates an Operator that contains a list of tensors and represents their product:

```
Op(tensor1(i1, i2, ...), tensor2(i3, i4, ...), ...)
```

Tensors	`t_name = TensorBuilder("t_name")`
Fields	`f_name = FieldBuilder("f_name", dim, statistics)`
Lagrangian	`lag = -OpSum(Op(...), Op(...), ...)`
Operators	`Op(tensor1(i1, i2, ...), ...)`
Derivatives	`Op(..., D(i1, tensor(...)), ...)`
Numeric coefficient	`number_op(number) * Op(...)`
Symbolyc power	`inv_mass_sq = power_op("M", -2)`

Table 5.1: Summary of the tools for the creation of a model.

Positive indices are used to express contraction. During the creation of the model, any index should be contracted with another, so we will only use here positive ones. When indices are repeated inside the same operator, the corresponding contraction is understood. For example, the product of tensors $r_{ij}s_{limnm}t_{njl}$ would be written as

```
Op(r(0, 1), s(3, 0, 4, 5, 4), t(5, 1, 3))
```

To introduce a covariant derivative inside an operator, the appropriate function is `D`, whose first argument is the Lorentz index of the derivative and whose second one is the tensor to which it is to be applied:

```
D(i1, tensor(i2, ...))
```

For numeric coefficients, the function `number_op` creates an operator with only one special tensor representing a number (its name is `"$number"` and has an attribute `content` with the actual number). Multiplication is defined for operators, so the operator $iV_\mu S_a^* D_\mu S_a$ can be expressed as

```
number_op(1j) * Op(V(0), Sc(1), D(0, S(1)))
```

Tensors representing a symbolic constant exponentiated to some power can be created using the function `power_op`, that takes the base (a string) and the exponent (a number) (represented by an extra internal attribute of tensors: `exponent`) and optionally some indices and returns an operator containing only the corresponding tensor. This is useful specially for the masses of the heavy particles, which tend to appear several times with different powers in all calculations.

A summary of the tools presented in this section is shown in table 5.1.

Dealing with spinors

`MatchingTools` uses the two-component spinor formalism to treat spinor fields following the conventions in [164]. The module `matchingtools.core` defines the following tensors to work with them:

- `epsUp` and `epsDown`: the totally anti-symmetric tensors $\epsilon^{\alpha\beta}$ and $\epsilon_{\alpha\beta}$ with two undotted two-component spinor indices defined by $\epsilon^{12} = -\epsilon^{21} = -\epsilon_{12} = \epsilon_{21} = 1$.
- `epsUpDot` and `epsDownDot`: the totally anti-symmetric tensors $\epsilon^{\dot\alpha\dot\beta}$ and $\epsilon_{\dot\alpha\dot\beta}$ with two dotted two-component spinor indices given by $\epsilon_{\dot\alpha\dot\beta} = (\epsilon_{\alpha\beta})^*$ and $\epsilon^{\dot\alpha\dot\beta} = (\epsilon^{\alpha\beta})^*$.

- `sigma4` and `sigma4bar`: the tensors $\sigma^{\mu}_{\alpha\dot{\alpha}}$ and $\bar{\sigma}^{\dot{\alpha}\alpha}_{\mu}$ given by $\sigma^{\mu} = (I_{2\times 2}, \vec{\sigma})$ and $\bar{\sigma}_{\mu} = (I_{2\times 2}, -\vec{\sigma})$, where $\vec{\sigma}$ is the three-vector of the Pauli matrices. The first index of `sigma4` and `sigma4bar` corresponds to the Lorentz index.

5.2.2 Integration

This section explains how to use the classes that represent the heavy fields as well as the function `integrate`, to integrate them out. They belong to the module `matchingtools.integration`. To import them, one can do:

```
from matchingtools.integration import (
    RealScalar, ComplexScalar,
    RealVector, ComplexVector,
    VectorLikeFermion, MajoranaFermion,
    integrate
)
```

To integrate out the heavy fields from a previously defined Lagrangian we should specify which of the fields are heavy. This is done using the classes:

- `RealScalar`. Its constructor receives as arguments the name of the field and the number of indices it has.

- `ComplexScalar`. Requires a field–conjugate field pair. The arguments of the constructor are the name of the field, the name of its conjugate and its number of indices.

- `RealVector`. The arguments are the name of the field and the number of indices. The first index of the field is understood to be the Lorentz vector index.

- `ComplexVector`. The arguments are the name of the field, the name of its conjugate and the number of indices. The first index of both fields should be their corresponding Lorentz vector index.

- `VectorLikeFermion`. The first argument of the constructor is the name of the field. The second and third are the names of the left-handed and right-handed parts. The fourth and fifth are their conjugates. The last is the number of indices. The first index of the each of the four fields is taken to be their two-component spinor index.

- `MajoranaFermion`. The arguments are the name of the field and the name of its conjugate. The first index of both fields should be their two-component spinor index.

The constructors for the bosons have the optional arguments: `order` (default 2), specifying the order in $(D/M)^2$ to which the solution to the equation of motion is to be expanded, and `max_dim` (default 4), representing the maximum allowed dimension for the operators appearing in this expansion. Both bosons and fermions receive the optional argument `has_flavor` (default `True`) stating whether the heavy field has a flavor index. In case it is true, the flavor index is taken to be the last one.

The heavy field classes include the quadratic terms for the kind of particle they represent, as well as the solutions to the equations of motion presented in section 3.5.2. The mass of a field `f` is represented by a tensor whose name is of the form `mass = "M" + f.name`. This tensor has one index if the heavy field has flavor and none otherwise. The first step for integration is defining the heavy fields:

```
heavy_f = HeavyFieldClass("field_name", ...)
```

Given an interaction Lagrangian `int_lag`, the integration is done using the function `integrate`, which takes as arguments a list of the heavy fields, the interaction Lagrangian and a maximum dimension `max_dim` for the operators of the EFT. It returns the corresponding effective Lagrangian:

```
heavy_fields = [heavy_f_1, heavy_f_2, ...]
eff_lag = integrate(
    heavy_fields, int_lag, max_dim
)
```

5.2.3 Transformations of the effective Lagrangian

After integration, the effective Lagrangian contains in general operators that are not independent. To rewrite it in terms of a set of independent operators some manipulations are needed, such as using identities for combinations of tensors related to the symmetry groups, integrating by parts to move derivatives from some fields to others, or using the equations of motion of the light fields.

The `matchingtools.transformations` module introduces the functions for doing this kind of manipulations and for the simplification of the Lagrangian. We will describe here the functions that are imported with

```
from matchingtools.transformations import (
    simplify, apply_rules
)
```

First, the function `simplify` returns a simplified version of the Lagrangian it gets as an argument. Tensors representing a number that appear inside an operator are collected and multiplied. Tensors representing a symbolic constant exponentiated to some power are also collected to give only one tensor with the correct exponent. `simplify` also looks for Kronecker deltas (tensors with the name `"kdelta"` and two indices) removes them by contracting the corresponding indices.

The transformations of a Lagrangian are done using what we call here `rules`. A rule is a pair (a tuple with two elements) whose first element is an operator representing a `pattern` and whose second element is an operator sum representing a `replacement`. They are used by the function `apply_rules` to find occurrences of the pattern and replace them by the replacement. A rule is written as

```
rule = (Op(...), OpSum(Op(...), Op(...), ...))
```

The indices that appear in tensors inside the rule can be general integer numbers. Non-negative integers represent contracted indices, as explained in section 5.2.1. Negative indices are used for free indices and those in the replacement should match the corresponding ones in the pattern. For example the substitution of $\sigma^a_{ij}\sigma^b_{kl}$ by $2\delta_{il}\delta_{kj} - \delta_{ij}\delta_{kl}$ can be done using the rule

```
rule_fierz_SU2 = (
    Op(sigma(0, -1, -2), sigma(0, -3, -4)),
    OpSum(
        number_op(2) * Op(delta(-1, -4),  delta(-3, -2)),
        -Op(delta(-1, -2),  delta(-3, -4))
    )
)
```

To transform the Lagrangian using integration by parts or equations of motion of the light fields the user should also specify the corresponding rules following this procedure.

The function apply_rules repeatedly tries to apply every rule of a list to each operator in an operator sum. If the pattern matches some part of an operator, the rule is applied and the operator sum updated. The first argument to apply_rules is the operator sum, the second is the list of rules and the last one is the number of iterations. It returns the resulting operator sum.

To rewrite the Lagrangian in terms of a chosen set of independent operators the procedure is: define the rules to get to the desired basis, add some rules to identify the operators and apply the function apply_rules. The basis operators should be defined using tensor_op, a function that creates an operator with one tensor inside whose name is the argument of the function. Then write a rule to identify it. For example, for the operator $\mathcal{O}_{\phi D} = (\phi^\dagger D_\mu \phi)(D^\mu \phi)^\dagger \phi$ we would write

```
OphiD = tensor_op("OphiD")
rule_def_OphiD = (
    Op(phic(0), D(1, phi(0)),
       D(1, phic(0)), phi(0)),
    OpSum(OphiD)
)
```

If the basis operator in question has some flavor indices, flavor_tensor_op is to be used instead of tensor_op. It creates a callable object that takes the corresponding free indices as arguments. As an example, for the operator $(O_{e\phi})_{ij} = \bar{l}_{Li} \phi e_{Rj} \phi^\dagger \phi$ we would have:

```
Oephi = flavor_tensor_op("Oephi")
rule_def_Oephi = (
    Op(lLc(0, 1, -1), phi(1), eR(0, -2), phic(2), phi(2)),
    OpSum(Oephi(-1, -2))
)
```

5.2.4 Output

The class matchingtools.output.Writer serves to nicely represent an effective Lagrangian. It is convenient that the final result is represented as a list of the coefficients of the operators in the basis. That is, if each of the terms of the Lagrangian contains a tensor that represents an operator of the basis, we would like to see what are the tensors that multiply each of them. This is what Writer does. If eff_lag is our final effective Lagrangian and op_names is a list of the names of the tensors representing the operators in the basis, one should do

```
eff_lag_writer = Writer(eff_lag, op_names)
```

The constructor admits an optional argument `conjugates`, a dictionary whose keys are the names of all the tensors involved in the final output and whose values are the names of their conjugates. This helps `Writer` collect pairs of conjugate products of tensors returning their real or imaginary part. The string representation can be obtained just by using the `str` method of the class `Writer`. To write it to a text file the user should use

```
eff_lag_writer.write_text_file(filename).
```

The method `write_latex_file` writes a LaTeX file with the representation. It receives four arguments: the name of the output file, the LaTeX representation of the tensors, the LaTeX representation of the coefficients of the basis operators and a list of the strings to be used to represent the indices. The LaTeX representations are given by dictionaries whose keys are the names of the tensors to be represented (or whose coefficient is to be represented) and whose values are the corresponding code. This code should contain placeholders for the necessary indices written as `"{}"` (Python's `format` style). To produce the characters `"{"`, `"}"` in the final code they should appear duplicated in the dictionary values. For a better LaTeX output for the numerical coefficients, the parameter passed to `number_op` in the definitions should be either an `int` or a `fractions.Fraction`. In this context, the imaginary unit can be introduced by multiplying by the operator `core.i_op`.

5.3 An example

In this section we will be creating a simple model to show some of the features of `MatchingTools`. The model is described as follows: it has $SU(2) \times U(1)$ gauge symmetry and contains a complex scalar doublet ϕ (the Higgs) with hypercharge 1/2 and a real scalar triplet Ξ with zero hypercharge that couple as

$$\mathcal{L}_{int} = -\kappa \Xi^a \phi^\dagger \sigma^a \phi - \lambda \Xi^a \Xi^a \phi^\dagger \phi, \tag{5.1}$$

where κ and λ are a coupling constants and σ^a are the Pauli matrices. We will then integrate out the heavy scalar Ξ to obtain an effective Lagrangian which we will finally write in terms of the operators:

$$\begin{array}{ll} \mathcal{O}_{\phi 6} = (\phi^\dagger \phi)^3, & \mathcal{O}_{\phi 4} = (\phi^\dagger \phi)^2, \\ \mathcal{O}_{\phi}^{(1)} = \phi^\dagger \phi (D_\mu \phi)^\dagger D^\mu \phi, & \mathcal{O}_{\phi}^{(3)} = (\phi^\dagger D_\mu \phi)(D^\mu \phi)^\dagger \phi, \\ \mathcal{O}_{D\phi} = \phi^\dagger (D_\mu \phi) \phi^\dagger D^\mu \phi, & \mathcal{O}_{D\phi}^* = (D_\mu \phi)^\dagger \phi (D^\mu \phi)^\dagger \phi. \end{array} \tag{5.2}$$

Notice that this is not an independent set of operators, as some linear combinations of them are total derivatives. Because the purpose of this section is to present a very simple model, we will not be doing integration by parts and therefore we will not simplify the results any further.

5.3.1 Creation of the model

The required imports are

```
from matchingtools.operators import (
    TensorBuilder, FieldBuilder, Op, OpSum,
    number_op, tensor_op, boson, fermion, kdelta
)

from matchingtools.integration import RealScalar, integrate

from matchingtools.transformations import apply_rules

from matchingtools.output import Writer
```

Three tensors will be needed, the Pauli matrices and the coupling constants:

```
sigma = TensorBuilder("sigma")
kappa = TensorBuilder("kappa")
lamb = TensorBuilder("lamb")
```

We will also use three fields: the Higgs doublet, its conjugate and the new scalar:

```
phi = FieldBuilder("phi", 1, boson)
phic = FieldBuilder("phic", 1, boson)
Xi = FieldBuilder("Xi", 1, boson)
```

Now we are ready to write the interaction Lagrangian:

```
interaction_Lagrangian = -OpSum(
    Op(kappa(), Xi(0), phic(1),
       sigma(0, 1, 2), phi(2)),
    Op(lamb(), Xi(0), Xi(0),
       phic(1), phi(1))
)
```

5.3.2 Integration

To integrate out the heavy Ξ we write

```
heavy_Xi = RealScalar("Xi", 1, has_flavor=False)
effective_Lagrangian = integrate(
    [heavy_Xi], interaction_Lagrangian, 6
)
```

5.3.3 Transformations of the effective Lagrangian

After the integration we get operators that contain $(\phi^\dagger\sigma^a\phi)(\phi^\dagger\sigma^a\phi)$. This product can be rewritten in terms of the operator $(\phi^\dagger\phi)^2$. To do this, we can use the $SU(2)$ Fierz identity:

$$\sigma^a_{ij}\sigma^a_{kl} = 2\delta_{il}\delta_{kj} - \delta_{ij}\delta_{kl}. \tag{5.3}$$

We now know that we can define a rule to transform everything that matches the left-hand side of the equality into the expression in the right-hand side with the code

```
fierz_rule = (
    Op(sigma(0, -1, -2), sigma(0, -3, -4)),
    OpSum(
        number_op(2) * Op(kdelta(-1, -4), kdelta(-3, -2)),
        -Op(kdelta(-1, -2), kdelta(-3, -4))
    )
)
```

We should now define the operators in terms of which we want to express the effective Lagrangian

```
Ophi6 = tensor_op("Ophi6")
Ophi4 = tensor_op("Ophi4")
O1phi = tensor_op("O1phi")
O3phi = tensor_op("O3phi")
ODphi = tensor_op("ODphi")
ODphic = tensor_op("ODphic")
```

and then use some rules to express them in terms of the fields and tensors that appear in the effective Lagrangian

```
definition_rules = [
  (Op(phic(0), phi(0), phic(1), phi(1),
      phic(2), phi(2)),
   OpSum(Ophi6)),
  (Op(phic(0), phi(0), phic(1), phi(1)),
   OpSum(Ophi4)),
  (Op(D(2, phic(0)), D(2, phi(0)),
      phic(1), phi(1)),
   OpSum(O1phi)),
  (Op(phic(0), D(2, phi(0)),
      D(2, phic(1)), phi(1)),
   OpSum(O3phi)),
  (Op(phic(0), D(2, phi(0)),
      phic(1), D(2, phi(1))),
   OpSum(ODphi)),
  (Op(D(2, phic(0)), phi(0),
      D(2, phic(1)), phi(1)),
   OpSum(ODphic))
]
```

To apply the $SU(2)$ Fierz identity to every operator until we get to the chosen operators, we do

```
rules = [fierz_rule] + definition_rules
max_iterations = 2
transf_eff_lag = apply_rules(
    effective_Lagrangian, rules,
    max_iterations
)
```

5.3.4 Output

The class `Writer` can be used to represent the coefficients of the operators of a Lagrangian as plain text and write them to a file

```
final_coef_names = [
    "Ophi6", "Ophi4", "O1phi",
    "O3phi", "ODphi", "ODphic"
]
eff_lag_writer = Writer(
    transf_eff_lag, final_coef_names
)
eff_lag_writer.write_text_file(
    "simple_example_results.txt"
)
```

It can also write a LaTeX file with the representation of these coefficients and export it to pdf to show it directly. For this to be done, we should define how the objects that we are using are represented in LaTeX code and the symbols we want to be used as indices

```
latex_tensor_reps = {
    "kappa": r"\kappa",
    "lamb": r"\lambda",
    "MXi": r"M_{{\Xi}}",
    "phi": r"\phi_{}",
    "phic": r"\phi^*_{}"
}

latex_op_reps = {
    "Ophi":
    r"\frac{{\alpha_{{\phi}}}}{{\Lambda^2}}",
    "Ophi4":
    r"\alpha_{{\phi 4}}"
}

latex_indices = ["i", "j", "k", "l"]

eff_lag_writer.write_latex(
    "simple_example", latex_tensor_reps,
    latex_op_reps, latex_indices
)
```

The expected result is a `.tex` file (ready to be compiled) with the coefficients of the operators we defined.

5.4 Extras for beyond the Standard Model applications

`MatchingTools` includes a subpackage called `extras`, with some modules defining tensors and rules that are useful for the applications to physics beyond the SM. These modules are `SU2`, `SU3`, `Lorentz`, `SM` and `SM_dim_6_basis`. Other modules will be added in the future and will be available in the GitHub repository of the program, as well as in its updates in the pypi repository [163].

5.4.1 The `SU2` module

This module defines the following tensors related to $SU(2)$:

- `epsSU2`: The totally antisymmetric tensor ϵ_{ij} with two doublet indices and $\epsilon_{12} = 1$.
- `sigmaSU2`: The Pauli matrices σ^a_{ij}. The first index is the triplet index, whereas the second and third are the doublet ones.
- `CSU2` and `CSU2c`: the Clebsh-Gordan coefficients $C^I_{a\beta}$ with the first index I being a quadruplet index, the second a a triplet index, and the third β a doublet index. The tensor C contracted with the corresponding three objects produces a singlet.
- `epsSU2triplets`: Totally antisymmetric tensor ϵ_{abc} with three $SU(2)$ triplet indices such that $\epsilon_{123} = 1$.
- `fSU2`: Totally antisymmetric tensor with three $SU(2)$ triplet indices given by $f_{abc} = \frac{i}{\sqrt{2}}\epsilon_{abc}$.

It also implements the rules for taking expressions with $\epsilon_{ij}\epsilon_{kl}$, $\sigma^a_{ij}\sigma^a_{kl}$, $C^I_{ap}\epsilon_{pm}\sigma^a_{ij}C^{I*}_{bq}\epsilon_{qn}\sigma^b_{kl}$ or contractions of anti-symmetric tensors, and rewriting them in terms of Kronecker deltas. All the rules are collected in the list `rules_SU2`. The LaTeX representation of the tensors defined is given by the dictionary `latex_SU2`.

5.4.2 The `SU3` module

The $SU(3)$ tensors defined in this module are:

- `epsSU3`: Totally antisymmetric tensor ϵ_{ABC} with three $SU(3)$ triplet indices such that $\epsilon_{123} = 1$.
- `TSU3`: $SU(3)$ generators $(T_A)_{BC} = \frac{1}{2}(\lambda_A)_{BC}$, where λ_A are the Gell-Mann matrices. The first index is the octet index. The second and third are the anti-triplet and triplet ones.
- `fSU3`: $SU(3)$ structure constants f_{ABC}.

The rule for transforming $\epsilon_{ijk}\epsilon_{ilm}$ into a combination of Kronecker deltas is implemented. It is included in the one-element list `rules_SU3`. The LaTeX representation of the tensors defined is in `latex_SU3`.

5.4.3 The `Lorentz` module

This module includes the tensors `epsUp`, `epsUpDot`, `epsDown`, `epsDownDot`, `sigma4`, `sigma4bar` from `matchingtools.operators` and defines:

- `eps4`: Totally antisymmetric tensor $\epsilon_{\mu\nu\rho\sigma}$ with four Lorentz vector indices where $\epsilon_{0123} = 1$.

- `sigmaTensor`: Lorentz tensor

$$\sigma^{\mu\nu} = \frac{i}{4} \left(\sigma^{\mu}_{\alpha\dot{\gamma}} \bar{\sigma}^{\nu\dot{\gamma}\beta} - \sigma^{\nu}_{\alpha\dot{\gamma}} \bar{\sigma}^{\mu\dot{\gamma}\beta} \right). \tag{5.4}$$

The list `rules_Lorentz` contains the rules for substituting $\epsilon^{\alpha\beta}\epsilon^{\dot{\alpha}\dot{\beta}}$ by $\frac{1}{2}\bar{\sigma}^{\mu,\dot{\alpha}\alpha}\bar{\sigma}^{\dot{\beta}\beta}_{\mu}$, $\epsilon_{\alpha\beta}\epsilon_{\dot{\alpha}\dot{\beta}}$ by $\frac{1}{2}\bar{\sigma}^{\mu}_{\alpha\dot{\alpha}}\bar{\sigma}_{\mu,\beta\dot{\beta}}$ and contracted ϵ tensors by combinations of Kronecker deltas.

5.4.4 The `SM` module

Here, the tensors corresponding to the SM fields and its gauge coupling constants, Yukawa couplings and CKM matrix are defined. The SM fields are:

- `phi` and `phic`: The Higgs boson and its conjugate. One $SU(2)$ doublet index.

- `lL` and `lLc`: The left-handed lepton doublet. Its indices are, in order: the two-component spinor index, the $SU(2)$ doublet index and the flavor index.

- `qL` and `qLc`: The left-handed quark doublet. Its indices are: the two-component spinor index, the $SU(3)$ triplet (or anti-triplet) index, the $SU(2)$ doublet index and the flavor index.

- `eR` and `eRc`: The right-handed electron. Indices: two-component spinor and flavor.

- `uR` and `uRc`: The right-handed up quark. Indices: two-component spinor, $SU(3)$ triplet (or antitriplet) and flavor.

- `dR` and `dRc`: The right-handed down quark. Indices: two-component spinor, $SU(3)$ triplet (or antitriplet) and flavor.

- `bFS`: $U(1)$ field strength tensor. Two Lorentz vector indices.

- `wFS`: $SU(2)$ field strength tensor. Two Lorentz vector indices and one $SU(2)$ triplet index.

- `gFS`: $SU(3)$ field strength tensor. Two Lorentz vector indices and one $SU(3)$ octet index.

The constant tensors are:

- `gb` and `gw`: The $U(1)$ and $SU(2)$ gauge coupling constants.

- `ye`, `yec`, `yd`, `ydc`, `yu` and `yuc`: The diagonalized Yukawa matrices for the leptons, the down quarks, the up quarks and their conjugates. They have two indices: the first one corresponds to the flavor of the doublets and the second to the flavor of the singlets.
- `V` and `Vc`: CKM matrix.

The module also includes a list of rules `eoms_SM`, defined to substitute the equations of motion, replacing derivatives of the Standard Model fields by a combination of the other fields. There is a dictionary `latex_SM` containing the LaTeX representation of the tensors that are defined.

5.4.5 The `SM_dim_6_basis` module

In this module, the basis for the SM effective Lagrangian up to dimension six that appears in [9] is defined. The rules to identify them are given in the list `rules_basis_definition`. The LaTeX representation of their coefficients is in `latex_basis_coefs`. Modules containing other bases, such as the one in [54], will be added in the future.

5.5 Using `MatchingTools` with other types of fields

As explained above, `MatchingTools` can integrate scalars, vector-like or Majorana fermions, and vectors in Lorentz-invariant theories. For this purpose, several classes representing the heavy fields are supplied. Other kinds of fields (for instance, with non canonical kinetic terms, spin > 1, or non relativistic) can be treated as well, once the corresponding class for it is provided. Specifically, to treat a new type of field one should define a Python class implementing the following methods:

- `equations_of_motion`. Receives an `OperatorSum` object representing an interaction Lagrangian. Returns a dictionary whose keys are strings with the names of the heavy fields involved (for example, a field and its conjugate, if it is a complex boson) and whose values are `OperatorSum` objects representing the corresponding solution to their equation of motion. These solutions can be written in terms of other heavy fields, but they should be such that iterative substitutions of their respective equations motion reaches a point where no heavy fields appear to the desired order in the dimension of the operators.
- `quadratic_terms`. Does not have any parameters. Returns the kinetic and mass terms of the corresponding heavy field.

For the definition of these methods, it is recommended to use the tools provided by the `core` module. Once such a class is defined, its objects can be included in the list of heavy fields to be passed to `integration.integrate` and they will be dealt with in the same way as the others.

5.6 Conclusions

In this chapter, we have presented `MatchingTools`, a Python library implementing symbolic tree-level matching for any given model. It is also able to transform the resulting Lagrangian using rules specified by the user to remove redundant operators. With this program one can safely automatize these kind of calculations, which practically eliminates the possibility of algebraic errors and drastically reduces the calculation times. Even calculations with complex Lagrangians involving ~ 100 independent terms (thousands of terms in some intermediate steps) can be performed in about thirty seconds (using a 2.6 GHz Intel Core i5 processor).

A direct application of `MatchingTools`, which has also served as an extensive check of its validity, is the integration of all possible new fields that have linear gauge-invariant renormalizable couplings to the Standard Model, keeping terms up to dimension 6 in the results. We present these results in chapter 8.

CHAPTER 6

BasisGen: bases of operators

6.1 Introduction

`BasisGen` is a `Python` package for the automatic generation of bases of operators in EFTs. It accepts any semisimple symmetry group and fields in any of its finite dimensional irreducible representations. It takes into account integration by parts redundancy and, optionally, the use of equations of motion. The implementation is based on well-known methods to generate and decompose representations using roots and weights, which allow for fast calculations, even with large numbers of fields and high-dimensional operators. `BasisGen` can also be used to do some representation-theoretic operations, such as finding the weight system of an irreducible representation from its highest weight or decomposing a tensor product of representations.

The input data needed for this calculation of a basis of operators are the symmetry group G of the theory and the representation of G corresponding to each field. Once they are specified, one can obtain, for every monomial in the fields, the number of independent ways of forming an invariant under the action of G out of it. It must also be taken into account that total derivative terms can be added to the Lagrangian without changing the physics (except for effects of surface terms in the action). This means that some operators with derivatives can be rewritten in terms of others. Moreover, at each order in the effective Lagrangian, the addition of an operator proportional to the equations of motion does not change the S matrix up to higher order effects, as explained in chapter 4. It follows that the equations of motion can be used, for example, to obtain a basis in which all the operators proportional to the functional derivative of the kinetic term have been removed [56–58, 91, 92]. For the SMEFT (see ref. [124] for a review), several bases and (incomplete) sets of independent operators have been computed taking all these facts into account [31, 54, 137, 165]. Computer tools can be used to translate from one basis to another [4, 141, 142, 144].

In the last few years, many developments have been made in the automatization of the generation of operator bases. Hilbert series methods provide an elegant way to compute invariants [166–170]. They can be directly implemented in a computer system with symbolic capabilities, as done for the SMEFT case in the auxiliary `Mathematica` notebook of ref. [169]. One possible drawback of this approach, when used in computer code, is its performance, as an overhead due to the symbolic nature of the calculations might be introduced. The program `DEFT` [142], written in `Python`, uses a different approach to check and generate bases of operators for the SMEFT. The operators are not only counted but they are given explicitly, including their index contraction structure and the fields to which the derivatives are applied (see ref. [97] for a non-automatic calculation of the explicit operators in a basis). Additionally, it can perform changes of bases. The method it implements can be generalized to theories with a symmetry group given by a product of unitary groups.

`BasisGen` uses yet another approach, which is valid for any semisimple symmetry group and avoids the need for symbolic calculations. The algorithms that it uses to deal with representations of semisimple Lie algebras are the classical ones, based on weight vectors. They are reviewed, for example, in ref. [171], and implemented in several computer packages with different purposes [172–177]. To remove integration by parts redundancy, an adaptation of the method in ref. [170] is used. `BasisGen` is $\sim$ 150 times faster than the implementation in the auxiliary notebook of ref. [169]. For example, `BasisGen` takes 3 seconds to compute the 84 dimension-6 operators of the 1-generation SMEFT (in a laptop with a 2,6 GHz Intel Core i5 processor), while the notebook of ref. [169] takes 7 minutes. `DEFT` also takes minutes for the calculation of a dimension-6 basis of the 1-generation SMEFT (according to ref. [142]), although it must be taken into account that it does more work, as the concrete operators are given instead of just being counted.

For computations with EFTs, `BasisGen` assumes 4-dimensional Lorentz invariance. In addition, an internal symmetry group must be specified. This is, in general, the product of the global symmetry group and the gauge group. Derivatives are assumed to be gauge-covariant derivatives, so that the derivative of any field has the same representation under the internal symmetry group as the field itself. The gauge field strengths to be included in a calculation should be provided by the user. The fields must belong to linear irreducible representations of both the Lorentz group and the internal symmetry group. Finally, it is required that a power counting based on canonical dimensions can be used.

In this context, `BasisGen` generates bases of invariant operators. It gives the number of independent invariants that can be formed with each possible field content for an operator. Sets of all covariant operators, with their corresponding irreducible representations (irreps), can also be computed. The basic representation-theoretic functionalities needed for these calculations are: obtaining weight systems of irreps and decomposing their tensor products. An interface for their direct use is provided.

Although `BasisGen` does not provide the explicit index contraction structure of the operators in the basis, the functionality of decomposing tensor products can be used to help in their construction. For a particular field content, one can take the tensor product of the first two fields. Then, for each irrep in the decomposition, take the tensor product with the next field. This process can be iterated, keeping track of the intermediate irreps. In the end, one can obtain all the possible ways of doing the products of the fields that give an invariant. Nevertheless, some extra information (the corresponding Clebsch-Gordan coefficients) is needed to completely determine the operator.

`BasisGen` can be installed using `pip` by doing: `pip install basisgen`. It requires `Python` version 3.5 or higher. Its code can be downloaded from the GitHub repository `https://github.com/jccriado/basisgen`, where some examples of usage

Listing 6.1: Simple EFT example script

```
from basisgen import algebra, irrep, scalar, Field, EFT

phi = Field(
    name='phi',
    lorentz_irrep=scalar,
    internal_irrep=irrep('SU2', '1'),
    charges=[1/2]
)

my_eft = EFT(algebra('SU2'), [phi, phi.conjugate])

invariants = my_eft.invariants(max_dimension=8)

print(invariants)
print("Total:", invariants.count())
```

Listing 6.2: Simple EFT example script's output

```
phi phi*: 1
(phi)^2 (phi*)^2: 1
(phi)^2 (phi*)^2 D^2: 2
(phi)^2 (phi*)^2 D^4: 3
(phi)^3 (phi*)^3: 1
(phi)^3 (phi*)^3 D^2: 2
(phi)^4 (phi*)^4: 1
Total: 11
```

can be found. A simple script using `BasisGen` is presented in listing 6.1. It defines an EFT with internal symmetry group $SU(2) \times U(1)$ for a complex scalar $SU(2)$-doublet field with charge 1/2. It computes a basis of operators of dimension 8 or less. The output is presented in listing 6.2. Each line gives the number of independent invariant operators that can be constructed with each field content.

The rest of this chapter is divided in two sections (apart from the conclusions). They describe `BasisGen`'s implementation (section 6.2) and interface (section 6.3).

6.2 Implementation

6.2.1 Basic operations with representations

In this section, the methods implemented in `BasisGen` to deal representations of semisimple Lie algebras are presented. A representation of a semisimple algebra is just a tensor product of representations of the algebra's simple ideals. Using this fact, `BasisGen` decomposes calculations with semisimple algebras into smaller ones with simple algebras. The basic operations with representations of simple algebras are: the generation of the weight system of an irrep from its highest weight and the decomposition of a reducible representation into a direct sum of irreps. They are both implemented using well-known methods (see refs. [171–177]), which are summarized here, for completeness.

In the Dynkin basis, which we use in what follows, all weights are tuples of integers. Thus, the operations done here involve only addition and multiplication of integer numbers. Each irrep of a simple algebra is uniquely characterized by its highest weight Λ, which is a tuple $(a_1 \dots a_n)$ of non-negative integers. Every such tuple is the highest weight of one irrep. The complete weight system of an irrep may be obtained from its highest weight by the following procedure:

1. Set $W = \{\}$ and $W_{\text{new}} = \{\Lambda\}$.
2. Choose some $\lambda \in W_{\text{new}}$.
3. For each positive component $\lambda_i > 0$, select the ith row α of the Cartan matrix. Append to W_{new} all weights of the form $\lambda - k\alpha$, with $0 < k \leq \lambda_i$.
4. Remove λ from W_{new}. Append it to W.
5. If W_{new} is empty, terminate. Otherwise, go to step 2.

This produces the set W of all weights. The multiplicity n_λ of each weight λ can then be obtained recursively using the Freudenthal formula:

$$n_\lambda = \frac{2\sum_\alpha \sum_{k>0} n_{\lambda+k\alpha}\,(\lambda + k\alpha, \alpha)}{(\Lambda+\delta, \Lambda+\delta) - (\lambda+\delta, \lambda+\delta)}, \tag{6.1}$$

where $\delta = (11\dots1)$ and the summation for α runs over all positive roots.

The algorithm for the decomposition of a reducible representation as a direct sum of irreps is straightforward: from the collection of weights of the representation in question, find the highest and remove from the collection all the weights in the corresponding irrep. Repeat until the collection is empty. Then, the successive highest weights that were found in the process are the highest weights of the irreps in the decomposition. A direct application of this functionality is to decompose the tensor product of irreps. Let W_1 and W_2 be the weight systems of two representations R_1 and R_2. The weight system W of $R_1 \otimes R_2$ is the collection of all $\lambda_1 + \lambda_2$ for $(\lambda_1, \lambda_2) \in W_1 \times W_2$. Once W is constructed, it can be decomposed using the general decomposition algorithm.

In some cases, the symmetric or anti-symmetric tensor power of some representation is needed. If $W = \{\lambda_i\}_{i\in\{1,\dots,n\}}$ is the weight system of some representation R, the weight system of the symmetric tensor power $\mathrm{Sym}^k(R)$ is the collection of weights computed as $\lambda_1 + \cdots + \lambda_k$ for every k-tuple $(\lambda_{i_1}, \dots, \lambda_{i_k})$ where $i_1 \leq \cdots \leq i_k$. The weight system of the anti-symmetric power $\Lambda^k(R)$ is constructed in a similar way, but using all k-tuples $(\lambda_{i_1}, \dots, \lambda_{i_k})$ with $i_1 < \cdots < i_k$ instead.

6.2.2 Constructing invariants in effective theories

`BasisGen` can do calculations for 4-dimensional Lorentz-invariant effective field theories whose internal symmetry group is of the form $G \times U(1)^n$, where G is semisimple. An EFT is specified when the following data are provided:

- The semisimple Lie algebra $\mathfrak{g}$ of G.
- A collection of fields $\phi_1, \dots, \phi_m$. Each ϕ_i must be equipped with:
  - An irrep $R^{(i)}_{\text{Lorentz}}$ of the Lorentz algebra $\mathfrak{su}_2 \oplus \mathfrak{su}_2$.
  - An irrep $R^{(i)}_{\text{internal}}$ of $\mathfrak{g}$.
  - A tuple $\left(c^{(i)}_1, \dots, c^{(i)}_n\right)$ of charges under the $U(1)$ factors.
  - The statistics S_i. Either boson or fermion.
  - A positive real number d_i, specifying the canonical dimension of the field.

It is assumed that a power counting based on canonical dimensions of the fields, with derivatives having dimension 1, can be applied. This is used to reduce the number of possible operators to a finite one.

The main functionality of `BasisGen` is to compute the number of independent invariant operators, constructed with the fields ϕ_i and their (covariant) derivatives, and having dimension less than or equal to some fixed d_{max}. To do this, first, all the possible operator field contents are found. The field content for some operator is identified by a tuple $\mathcal{C} = (e_1, \dots, e_m)$, representing the exponents of each field in the operator: $\mathcal{O} \sim (\phi_1)^{e_1} \cdots (\phi_m)^{e_m}$. For each $\mathcal{C}$, the following (possible reducible) representation is computed:

$$\text{Rep}(\mathcal{C}) = T_1^{e_1}(R^{(1)}) \otimes \cdots \otimes T_m^{e_m}(R^{(m)}), \tag{6.2}$$

where $T_i^k(V)$ is the symmetric power $\text{Sym}^k(V)$ if the statistics S_i are bosonic, and the anti-symmetric power $\Lambda^k(V)$ if they are fermionic. Once $\text{Rep}(\mathcal{C})$ is obtained, it is decomposed into a direct sum of irreps. The number of independent invariant combinations of the fields in $\mathcal{C}$ is then easily obtained as the number of singlet irreps in the decomposition.

To take into account (covariant) derivatives, the same procedure is used, but now including the fields $D_\mu \phi_i$, $\{D_\mu, D_\nu\}\phi_i$, etc. Anti-symmetric combinations of derivatives are automatically discarded, as they are equivalent to field strength tensors. Optionally, the equations of motion of the fields can be applied. This means that, for each $D_{\mu_1} \dots D_{\mu_m} \phi_i$, only the totally symmetric representation is retained (see ref. [168]).

Let I be the set of all operators constructed with the fields and their derivatives (using equations of motion if necessary) that are invariant under the internal symmetry group (but are not necessarily scalars). To eliminate integration by parts redundancy from I, it is first split into the set of operators with zero derivatives I_0, the set of operators with one derivative I_1, etc. Then, the following procedure is applied:

1. Set $R = \{\}$.
2. Take one operator $\mathcal{O}$ from the non-empty I_n with lowest n.

3. Remove $\mathcal{O}$ from I_n and append it to R.

4. Compute the decomposition into irreps of $D_\mu \mathcal{O}$ and eliminate the corresponding operators from I_{n+1}. Compute the decomposition of $\{D_\mu, D_\nu\}\mathcal{O}$ and remove it from I_{n+2}. Continue until the maximum dimension is reached.

5. If all I_k are empty, terminate. Otherwise, go to step 2.

After this is done, a basis (in which integration by parts has been taken into account) is obtained by selecting those operators in R that are scalars. Notice that the irreps in the decomposition of the derivatives of operators are computed and removed. In particular, if no (non-zero) scalar appears in the decomposition, then the corresponding scalar operator will not be eliminated. This avoids the over-counting of integration by parts redundancy in ref. [168] that was pointed out in ref. [169].

6.3 Interface

6.3.1 Basic objects

The basic objects for the usage of `BasisGen` are presented here. All of them can be imported with:

```
from basisgen import (
    algebra, irrep, Field, EFT, boson, fermion,
    scalar, L_spinor, R_spinor, vector, L_tensor, R_tensor
)
```

Functions

`algebra` Creates a (semi)simple Lie algebra from one string argument. The returned object is of the class `SimpleAlgebra` or `SemisimpleAlgebra` from the module `algebra`.

Examples of arguments: `'A3'`, `'C12'`, `'F4'`, `'SU3'`, `'B2+E7'`, `'SU5 x SO6 x Sp10'`.

`irrep` Creates an irreducible representation from 2 string arguments: the first represents the algebra and the second the highest weight[1]. The returned object is of the class `representations.Irrep`.

Example: `irrep('SU4 x Sp7', '1 0 1 0 2 1')`.

The weight system of a `representations.Irrep` object can be obtained by calling its `weights_view` method. Irreps with the same algebra can be multiplied to get the decomposition of their tensor product. Any two irreps can be added to give an irrep of the direct sum of their algebras.

Examples, showing the weights of the octet irrep of $SU(3)$ *(which has highest weight* (11)*) and the decomposition of the product of a triplet* (10) *and an anti-triplet* (01) *as an octet plus a singlet:*

[1] The highest weights for many irreps of several groups can be found, for example in ref. [171]. In particular, notice that the highest weight of an $SU(2)$ irrep is its dimension minus one.

Name	Description	Default
`name`	String identifier	
`lorentz_irrep`	Lorentz group irrep	
`internal_irrep`	Irrep of the internal (semisimple) symmetry group	
`charges`	Charges under an arbitrary number of $U(1)$ factors	`[]`
`statistics`	Either `boson` or `fermion`	`boson`
`dimension`	Canonical dimension of the field	1
`number_of_flavors`	Number of different copies of the same field	1

Table 6.1: Arguments of the `Field` constructor

```
>>> irrep('SU3', '1 1').weights_view()
    (1 1)
(2 -1) (-1 2)
 (0 0) (0 0)
(1 -2) (-2 1)
   (-1 -1)
>>> irrep('SU3', '1 0') * irrep('SU3', '0 1')
[1 1] + [0 0]
```

Classes

`Field` Has an attribute `conjugate`, the conjugate field. The constructor arguments are presented in table 6.1.

`EFT` Constructor arguments:

`internal_algebra` The semisimple Lie algebra of the internal symmetry group.

`fields` A list of `Field` objects representing the field content of the theory.

Methods:

`invariants` Returns a basis of operators, encapsulated in an `EFT.Invariants` object. These can be directly printed (implement `__str__`). They have a method `count` to calculate the total number of operators in the basis, and a method `show_by_classes`, which returns a simplified string representation of the basis, provided a dictionary whose keys are the fields and values are strings representing classes of fields.

`covariants` Returns a collection of all operators with all possible irreps, in the form of a `EFT.Covariants` instance. Its only purpose is to hold the information until it is printed (implements `__str__`).

Both receive the same arguments: `max_dimension`, the maximum dimension of the operators computed; `use_eom` (default: `True`) a boolean to specify whether the equations of motion should be used; `ignore_lower_dimension` (default: `False`), a boolean to specify whether operators with dimension less than `max_dimension` should be included in the results; and `verbose` (default: `False`), a boolean enabling/disabling messages about the progress of the calculations.

Other

The following irreps of the Lorentz group have been defined, for ease of use: `scalar`, `L_spinor`, `R_spinor`, `vector`, `L_tensor`, `R_tensor`. `L_spinor` and `R_spinor` correspond to left and right Weyl spinors, respectively. `L_tensor` and `R_tensor` correspond to the left and right parts of an antisymmetric tensor with two indices.

The statistics of a field can be specified by using the variables `boson` and `fermion`, which are set to the values `BOSON` and `FERMION` of the enum class `Statistics` from the module `statistics`.

6.3.2 The `smeft` module

The `smeft` module contains the definitions of all the SM fields:

- The Higgs doublet `phi` and its conjugate `phic`.
- The left and right parts `GL` and `GR` of the $SU(3)$ field strength.
- The left and right parts `WL` and `WR` of the $SU(2)$ field strength
- The left and right parts `BL` and `BR` of the $U(1)$ field strength.
- The quark doublet `Q` and its conjugate `Qc`.
- The lepton doublet `L` and its conjugate `Lc`.
- The up-type quark singlet `u` and its conjugate `uc`.
- The down-type quark singlet `d` and its conjugate `dc`.
- The electron singlet `e` and its conjugate `ec`.

The bosons are objects of the `Field` class. The fermions are functions that take the number of generations and return a `Field`. Similarly, the function `smeft` takes the number of fermion flavors and returns an `EFT` object representing the SMEFT. The algebra $\mathfrak{su}_3 \oplus \mathfrak{su}_2$ is named `sm_internal_algebra`. A dictionary named `sm_field_classes` is included, to simplify the presentation of the results by passing it as an argument to the method `show_by_classes` of an `EFT.Invariants` object.

Listing 6.3 contains an example script for the computation of bases of arbitrary dimension (passed as an argument to the script) for the 1-generation SMEFT. It gives 84 operators for dimension 6 (in about 3 seconds in a personal computer with a 2,6 GHz Intel Core i5 processor) and 993 operators for dimension 8 (in around 40 seconds in the same computer).

6.4 Conclusions

`BasisGen` computes bases of operators for EFTs in a general setting: the internal symmetry group can be any product of a semisimple group and an arbitrary number of $U(1)$ factors. 4-dimensional Lorentz invariance is assumed to provide support for concrete applications, although adaptations to other spacetime dimensions can be easily made, due to the generality of the core functionalities.

Listing 6.3: SMEFT example

```
from basisgen.smeft import smeft, sm_field_classes
import sys

invariants = smeft(number_of_flavors=1).invariants(
    max_dimension=int(sys.argv[1]),
    verbose=True,
    ignore_lower_dimension=True
)

print(invariants.show_by_classes(sm_field_classes(1)))
print("Number of invariants: {}".format(invariants.count()))
```

We will use `BasisGen` in chapters 7 and 9, to obtain a basis for EFTs with new fields beyond the SM ones.

The decision of using the equations of motion is left to the user, as it may be convenient to work with redundant bases in some cases, as explained in section 4.6. It is also possible not only to compute invariants but to generate all covariant operators, classified by their irreps. This can be useful, for example, to find the representation of fields that couple linearly to an already known theory, which are often the most relevant ones for phenomenology [6–10]. We will also make use of this feature in chapters 7 and 9. An interface for doing basic operations with representations of semisimple groups is also provided.

`BasisGen`'s speed for large numbers of fields and high-dimensional operators makes it possible to calculate bases for the SMEFT or for other EFTs for physics beyond the SM, in times ranging from seconds (for the dimension-8 operators in the SMEFT) to minutes (for higher-dimensional operators or larger number of fields) in personal computers.

Part III

General extensions of the Standard Model

CHAPTER 7

General extensions of the Standard Model

7.1 Introduction

In section 3.7, we introduced the SMEFT: an EFT for the SM particles. Deviations from the SM physics are parametrized in the SMEFT through the introduction of higher-dimensional operators. In this way, it provides a model-independent framework to study new physics. Its range of validity is limited to energies below the threshold of production of any extra degrees of freedom.

To study the direct production of new particles, it is mandatory to incorporate into the EFT the extra fields associated to them. Of course, the problem is that we do not know *a priori* which are the particles and fields that are relevant at the energies that can be accessed in the near future. So, in order to preserve model independence, we need to consider EFTs with arbitrary field content and arbitrary interactions. This also helps in connecting to particular models and hence in providing a rationale for the values of the low-energy parameters.

To explicitly write such an EFT, which we call the BSMEFT, it is necessary to make some assumptions about the high-energy physics. Our aim here is to keep these assumptions minimal, so that we work in a setting as general as possible. For the symmetries, we take 4-dimensional Poincaré invariance together with the SM gauge group $G_{\mathrm{SM}} := SU(3)\times SU(2)\times U(1)$. This choice does not represent a loss of generality: while it is possible that new symmetry groups are relevant at high energy, G_{SM} must be a subgroup of them. Invariance under a larger group fits in the BSMEFT through a particular choice of relations between its free parameters. For the action of the symmetries over the fields, we have to commit to some particular choices in order to have a manageable theory. An important assumption in our construction is that G_{SM} is linearly realized. This is a requisite for the perturbative unitarity of a theory that contains the SM gauge bosons (see section 3.6).[1] We will introduce some extra conditions over the representations of the fields, so that we have a finite number of possibilities for them.

[1] A related effort for the case of the electroweak chiral Lagrangian, in which the Higgs boson is a scalar singlet of the non-linearly realized electroweak symmetry is currently underway [178, 179].

This chapter is organized as follows. In section 7.2, the field content of the BSMEFT is presented. In section 7.3, we collect the sector of their effective Lagrangian that is relevant for leading order effects; that is, those terms that give tree-level contributions to the dimension-6 SMEFT after matching (see chapter 8). We conclude in section 7.4, with a summary of the applications of the BSMEFT, some of which are presented in chapters 8 and 9.

7.2 Field content

In this section, we tackle the task of enumerating the fields that should be included in the BSMEFT. They can be conveniently classified into irreducible representations of the Lorentz and gauge symmetry groups. Our theory should contain all the SM field multiplets (in particular, there should be a scalar in the $(1, 2)_{1/2}$ representation). In addition, we assume that the only chiral fermions are the SM ones. These assumptions are partially justified by the experimental success of the SM, including the discovery of the Higgs boson, precision electroweak data and Higgs data.

For the fields that are not in the SM, we should consider every representation of G_{SM}, according to our general approach of including all possible new fields. To reduce the number of possibilities, we impose the condition that the quantum numbers of each extra field are same as those of some operator made out of the SM fields only. This choice is justified by several reasons. First, it must be satisfied if the particles associated with these fields are unstable and decay into SM ones. Second, these are the fields whose linear interactions with the SM are allowed by the symmetries. The existence of linear interactions is a requirement for many tree-level effects. For example, single production and decay are governed by them at tree level. Moreover, integrating out a new field at tree level only gives non-vanishing contributions if the field has linear couplings.

The full list of assumptions that define the field content of the BSMEFT is:

1. The gauge group $G_{\mathrm{SM}} := SU(3) \times SU(2) \times U(1)$ is linearly realized.
2. The Lorentz and gauge quantum numbers of every field are equal to those of some SM covariant operator. This includes, in particular, all the SM fields.
3. The only fermion fields with chiral transformations under G_{SM} are the ones in the SM. In other words, all the extra fermions are vector-like with respect to G_{SM} or Majorana. This ensures that G_{SM} is non-anomalous.
4. The fields create particles of spin ≤ 1.

The fourth assumption is made to avoid subtle consistency issues with interacting particles of spin > 1 [180].[2] Importantly for the purposes of chapter 8, the first three assumptions ensure that, at energies much smaller than all the (gauge-invariant) masses of the extra particles, the theory is well described by the SMEFT.

Let $\mathcal{L}_{\mathrm{BSM}}$ be the effective Lagrangian of the BSMEFT. The operators of canonical dimension $d > 4$ in $\mathcal{L}_{\mathrm{BSM}}$ have dimensionful coefficients, which can be written as

[2] Local EFTs involving higher-spin particles are possible, with a restricted region of validity determined by their mass, spin and couplings [181].

$\alpha_i f^{4-d}$, with f some mass scale and α_i dimensionless couplings, which can be related with the cutoff Λ by power-counting arguments [28–31, 182–184]. As we have seen in section 3.6, if all the vector bosons in the theory are the additional gauge bosons of an extended gauge symmetry $G \supset G_{\rm SM}$ (spontaneously broken to $G_{\rm SM}$) and $\mathcal{L}_{\rm BSM}$ is invariant under G, with no anomalies, then the BSMEFT is a unitary EFT that can be used to perform perturbative calculations to arbitrary precision at energies below the cutoff Λ. However, in agreement with our model-independent spirit, we will consider here general theories with Proca vector bosons without enforcing any gauge invariance beyond $G_{\rm SM}$.[3] This class of theories contains the ones with extended gauge invariance. All the covariant derivatives we write are thus understood to be covariant with respect to $G_{\rm SM}$ only.

The conditions that we have imposed strongly restrict the quantum numbers of the extra fields. We will prove here a necessary condition over their representation under $G_{\rm SM}$. As in section 3.7, we use the label C for the representation under $SU(3)$, T for the $SU(2)$ isospin and Y for the hypercharge. We define

$$N(C,T,Y) = A(C) + B(T) + Y, \tag{7.1}$$

with A and B defined by the condition $0 \leq A(C), B(T) < 1$ and the equations

$$C(e^{2i\pi/3}I) = e^{2i\pi A(C)}I, \qquad T(-I) = e^{2i\pi B(T)}I. \tag{7.2}$$

where $C(X)$ ($T(X)$) denotes the representation of an element X of the $SU(3)$ ($SU(2)$) algebra defined by C (T). The values A and B may take are limited: $A(C) \in \{0, 1/3, 2/3\}$ and $B(T) \in \{0, 1/2\}$. The set representations of $SU(3)$ is split into three classes by the corresponding value A:

$$0 = A(1) = A(8) = A(10) = A(\overline{10}) = A(27) = \ldots \tag{7.3}$$
$$1/3 = A(3) = A(\overline{6}) = A(15) = A(15') = A(24) = \ldots \tag{7.4}$$
$$2/3 = A(\overline{3}) = A(6) = A(\overline{15}) = A(\overline{15'}) = A(\overline{24}) = \ldots \tag{7.5}$$

In a similar way, B splits the set of $SU(2)$ representations in two subsets: those with integer isospin and the others. Both A and B are additive under the operation of taking tensor products of representations:

$$A(C_1 \otimes C_2) = A(C_1) + A(C_2) \pmod 1, \tag{7.6}$$
$$B(T_1 \otimes T_2) = B(T_1) + B(T_2) \pmod 1. \tag{7.7}$$

We will prove now that if $(C_\mathcal{O}, T_\mathcal{O}, Y_\mathcal{O})$ is the representation of some operator $\mathcal{O}$ constructed with the Standard Model fields, then $N(C_\mathcal{O}, T_\mathcal{O}, Y_\mathcal{O})$ is an integer.[4] First, it can be directly checked that $N(C_\phi, T_\phi, Y_\phi)$ is an integer for any SM field ϕ. Now, from the additivity of A, B and Y it follows that the value of N corresponding to the product $\mathcal{O}\mathcal{Q}$ of two operators $\mathcal{O}$ and $\mathcal{Q}$ is

$$N(C_{\mathcal{O}\mathcal{Q}}, T_{\mathcal{O}\mathcal{Q}}, Y_{\mathcal{O}\mathcal{Q}}) = [A(C_\mathcal{O}) + A(C_\mathcal{Q})] + [B(T_\mathcal{O}) + B(T_\mathcal{Q})] + [Y_\mathcal{O} + Y_\mathcal{Q}] \pmod 1$$

[3]Spin-1 particles could alternatively be described by rank-2 antisymmetric tensor fields, which can be related to our vector formulation by a field redefinition, see [178, 185].

[4]This condition has been given before, in a different form, in ref. [186].

$$= [A(C_{\mathcal{O}}) + B(T_{\mathcal{O}}) + Y_{\mathcal{O}}] + [A(C_{\mathcal{Q}}) + B(T_{\mathcal{Q}}) + Y_{\mathcal{Q}}] \pmod 1$$
$$= N(C_{\mathcal{O}}, T_{\mathcal{O}}, Y_{\mathcal{O}}) + N(C_{\mathcal{Q}}, T_{\mathcal{Q}}, Y_{\mathcal{Q}}) \pmod 1,$$

Therefore, if $N(C_{\mathcal{O}}, T_{\mathcal{O}}, Y_{\mathcal{O}})$ and $N(C_{\mathcal{Q}}, T_{\mathcal{Q}}, Y_{\mathcal{Q}})$ are integers, then $N(C_{\mathcal{OQ}}, T_{\mathcal{OQ}}, Y_{\mathcal{OQ}})$ must also be integer. This completes the proof. Particularizing for specific representations under $SU(3)$, we obtain the relations:

$$T + Y \in \begin{cases} \mathbb{Z} & \text{for } C = 1, 8, 10, 27, \ldots \\ \mathbb{Z} + 2/3 & \text{for } C = 3, \bar{6}, 15, 15', \ldots \end{cases} \tag{7.8}$$

We emphasize that the quantum numbers of all the fields in the BSMEFT must satisfy this condition. Clearly, the number of different representations for new fields is infinite. However, their tree-level effects will be suppressed at least by the inverse power of Λ that corresponds to their linear interactions. Therefore, when working at some fixed order in the expansion in inverse powers of Λ, only those with linear interactions up to that order are relevant at tree level. It turns out that only a finite number of possibilities remain at each order.

In this chapter, we concentrate on those extra fields that can have gauge-invariant linear interactions with the SM fields of dimension $d \leq 4$. They are the relevant fields for chapter 8, as they are the only ones with tree-level contributions to operators of dimension 6 or less in the SMEFT, as shown in section 8.2. This means that they provide the leading contributions to indirect tests. This condition restricts the quantum numbers of the extra fields to be those of operators of dimension 2, 3 and 5/2 that can be built with SM fields. The allowed irreducible representations can be found using `BasisGen` (see chapter 6). All of them, together with the notation we use for each of the corresponding fields, are collected in tables 7.1, 7.2 and 7.3. These new fields have been singled out and studied before, in [6–9].[5] Several subsets of the complete collection have appeared in the literature in different contexts (see for instance [128, 188–192]).

7.3 Explicit BSMEFT Lagrangian

In this section, we present the part of $\mathcal{L}_{\text{BSM}}$ that contributes classically to the SMEFT with operators of dimension 6 or less. This is the relevant Lagrangian for the matching calculation we perform in chapter 8. It is therefore the sector of the BSMEFT with leading-order indirect effects, both in the expansion in loops and in inverse powers of f or the masses of the extra particles. Apart from the SM ones, the only fields

[5] There is actually one exception: the vector field $\mathcal{L}_1$ was not included in [8]. There exists only one gauge-invariant operator of dimension $d \leq 4$ that is linear in this vector and has no any other extra field: the super-renormalizable operator $\mathcal{L}_{1\mu}^{\dagger} D^{\mu}\phi$, which mixes the longitudinal part of $\mathcal{L}_1$ with the Higgs doublet. Such an operator will not appear, in the unitary gauge, if $\mathcal{L}_1$ is the gauge boson of an extended, spontaneously broken gauge invariance. Therefore, in a complete unitary theory, it will not contribute to the SMEFT operators at the leading order. However, it could appear in other gauges and also in phenomenological models, much as pion-vector resonance mixing is included in certain descriptions of low-energy QCD [178, 185]. In these cases it can be eliminated by a field redefinition, which in general generates local operators of dimension 4, 5 and 6 weighted by the vector mass and the dimensional coefficient of the super-renormalizable operator [187]. At the end of the day, as far as low-energy physics is concerned, this is equivalent to integrating the field out, which will be our approach in chapter 8.

Name	$\mathcal{S}$	$\mathcal{S}_1$	$\mathcal{S}_2$	φ	Ξ	Ξ_1	Θ_1	Θ_3
Irrep	$(1,1)_0$	$(1,1)_1$	$(1,1)_2$	$(1,2)_{\frac{1}{2}}$	$(1,3)_0$	$(1,3)_1$	$(1,4)_{\frac{1}{2}}$	$(1,4)_{\frac{3}{2}}$
Name	ω_1	ω_2	ω_4	Π_1	Π_7	ζ		
Irrep	$(3,1)_{-\frac{1}{3}}$	$(3,1)_{\frac{2}{3}}$	$(3,1)_{-\frac{4}{3}}$	$(3,2)_{\frac{1}{6}}$	$(3,2)_{\frac{7}{6}}$	$(3,3)_{-\frac{1}{3}}$		
Name	Ω_1	Ω_2	Ω_4	Υ	Φ			
Irrep	$(6,1)_{\frac{1}{3}}$	$(6,1)_{-\frac{2}{3}}$	$(6,1)_{\frac{4}{3}}$	$(6,3)_{\frac{1}{3}}$	$(8,2)_{\frac{1}{2}}$			

Table 7.1: New scalar bosons contributing to the dimension-six SMEFT at tree level.

Name	N	E	Δ_1	Δ_3	Σ	Σ_1	
Irrep	$(1,1)_0$	$(1,1)_{-1}$	$(1,2)_{-\frac{1}{2}}$	$(1,2)_{-\frac{3}{2}}$	$(1,3)_0$	$(1,3)_{-1}$	
Name	U	D	Q_1	Q_5	Q_7	T_1	T_2
Irrep	$(3,1)_{\frac{2}{3}}$	$(3,1)_{-\frac{1}{3}}$	$(3,2)_{\frac{1}{6}}$	$(3,2)_{-\frac{5}{6}}$	$(3,2)_{\frac{7}{6}}$	$(3,3)_{-\frac{1}{3}}$	$(3,3)_{\frac{2}{3}}$

Table 7.2: New vector-like fermions contributing to the dimension-six SMEFT at tree level.

Name	$\mathcal{B}$	$\mathcal{B}_1$	$\mathcal{W}$	$\mathcal{W}_1$	$\mathcal{G}$	$\mathcal{G}_1$	$\mathcal{H}$	$\mathcal{L}_1$
Irrep	$(1,1)_0$	$(1,1)_1$	$(1,3)_0$	$(1,3)_1$	$(8,1)_0$	$(8,1)_1$	$(8,3)_0$	$(1,2)_{\frac{1}{2}}$
Name	$\mathcal{L}_3$	$\mathcal{U}_2$	$\mathcal{U}_5$	$\mathcal{Q}_1$	$\mathcal{Q}_5$	$\mathcal{X}$	$\mathcal{Y}_1$	$\mathcal{Y}_5$
Irrep	$(1,2)_{-\frac{3}{2}}$	$(3,1)_{\frac{2}{3}}$	$(3,1)_{\frac{5}{3}}$	$(3,2)_{\frac{1}{6}}$	$(3,2)_{-\frac{5}{6}}$	$(3,3)_{\frac{2}{3}}$	$(\bar{6},2)_{\frac{1}{6}}$	$(\bar{6},2)_{-\frac{5}{6}}$

Table 7.3: New vector bosons contributing to the dimension-six SMEFT at tree level.

that appear in this Lagrangian are the ones in tables 7.1, 7.2 and 7.3. The complete BSMEFT be split in the following way:

$$\mathcal{L}_{\mathrm{BSM}} = \mathcal{L}_0 + \mathcal{L}_{\mathrm{S}} + \mathcal{L}_{\mathrm{F}} + \mathcal{L}_{\mathrm{V}} + \mathcal{L}_{\mathrm{mixed}} + \ldots, \tag{7.9}$$

where $\mathcal{L}_0$ contains terms of dimension $d \leq 6$ with only SM fields (and, therefore, it is of the same form as the SMEFT Lagrangian, defined in section 3.7), $\mathcal{L}_{\mathrm{S,F,V}}$ contains terms of dimension $d \leq 5$ with extra scalars, fermions and vectors, respectively, but no products of new fields of different spin, and $\mathcal{L}_{\mathrm{mixed}}$ contains terms of dimension $d \leq 4$ involving products of extra fields of different spin. In writing the dimension-five interactions with the heavy particles we remove redundant operators by using the SM equations of motion. The dots indicate terms that do not contribute in our approximation.

The extra fields can have kinetic or mass mixing with the *a priori* SM ones if they share the same quantum numbers. However, field rotations and rescalings can always be performed in such a way that all the kinetic terms in $\mathcal{L}_{\mathrm{BSM}}$ are diagonal and canonical and all the mass terms are diagonal in the electroweak symmetric phase. All our equations are written with this choice of fields (except for the mixing of ϕ and possible scalars φ with $\mathcal{L}_1$, see footnote 5). Furthermore, we assume that no fields get a non-trivial gauge-invariant vacuum expectation value in the symmetric phase.

This can always be achieved by convenient shifts of the scalar singlets. To match models written in a different "field basis", the shift, diagonalization and canonical normalization must be performed prior to using our formulas.

Working in this "field basis" not only fixes the precise meaning of the couplings in $\mathcal{L}_{\rm BSM}$, but also allows to identify the SM fields that enter in $\mathcal{L}_0$. The SM fermions and gauge fields are the massless fermion and vector eigenstates, respectively, whereas we identify the Higgs doublet ϕ with the $(1,2)_{1/2}$ scalar eigenstate associated to a negative eigenvalue of the squared mass matrix. We assume that this eigenvalue is non-degenerate and that all the other eigenvalues are positive. This is required if we want $\mathcal{L}_{\rm BSM}$ to be described by the SMEFT at low energies.

We proceed now to explicitly write the desired sector of $\mathcal{L}_{\rm BSM}$, with the notation specified in appendix A.

7.3.1 New scalars

The Lagrangian $\mathcal{L}_{\rm S}$ can be written as the sum of two pieces:

$$\mathcal{L}_{\rm S} = \mathcal{L}_{\rm S}^{\rm quad} + \mathcal{L}_{\rm S}^{\rm int}. \tag{7.10}$$

The first one contains the kinetic terms (with covariant derivatives) and mass terms of the new scalars:

$$\mathcal{L}_{\rm S}^{\rm quad} = \sum_\sigma \eta_\sigma \left[(D_\mu \sigma)^\dagger D^\mu \sigma - M_\sigma^2 \sigma^\dagger \sigma \right]. \tag{7.11}$$

Here, σ are the different scalar fields in table 7.1. More than one scalar field in each representation is allowed. The prefactor η_σ takes the value 1 ($\frac{1}{2}$) when σ is in a complex (real) representation of the gauge group. The second piece in (7.10) contains the general interactions of the new scalars with the SM fields and among themselves. We distinguish the terms of dimension $d \leq 4$ and the ones of dimension $d = 5$:

$$\mathcal{L}_{\rm S}^{\rm int} = \mathcal{L}_{\rm S}^{(\leq 4)} + \mathcal{L}_{\rm S}^{(5)}, \tag{7.12}$$

where

$$\begin{aligned}
-\mathcal{L}_{\rm S}^{(\leq 4)} = {} & (\kappa_{\mathcal{S}})_r \mathcal{S}_r \phi^\dagger \phi + (\lambda_{\mathcal{S}})_{rs} \mathcal{S}_r \mathcal{S}_s \phi^\dagger \phi + (\kappa_{\mathcal{S}^3})_{rst} \mathcal{S}_r \mathcal{S}_s \mathcal{S}_t \\
& + \left\{ (y_{\mathcal{S}_1})_{rij} \mathcal{S}_{1r}^\dagger \bar{l}_{Li} i\sigma_2 l_{Lj}^c + \text{h.c.} \right\} \\
& + \left\{ (y_{\mathcal{S}_2})_{rij} \mathcal{S}_{2k}^\dagger \bar{e}_{Ri} e_{Rj}^c + \text{h.c.} \right\} \\
& + \{ (y_\varphi^e)_{rij} \varphi_r^\dagger \bar{e}_{Ri} l_{Lj} + (y_\varphi^d)_{rij} \varphi_r^\dagger \bar{d}_{Ri} q_{Lj} + (y_\varphi^u)_{rij} \varphi_r^\dagger i\sigma_2 \bar{q}_{Li}^T u_{Rj} \\
& \quad + (\lambda_\varphi)_r \left(\varphi_r^\dagger \phi \right) \left(\phi^\dagger \phi \right) + \text{h.c.} \} \\
& + (\kappa_\Xi)_r \phi^\dagger \Xi_r^a \sigma^a \phi + (\lambda_\Xi)_{rs} \left(\Xi_r^a \Xi_s^a \right) \left(\phi^\dagger \phi \right) \\
& + \frac{1}{2} (\lambda_{\Xi_1})_{rs} \left(\Xi_{1r}^{a\dagger} \Xi_{1s}^a \right) \left(\phi^\dagger \phi \right) + \frac{1}{2} (\lambda'_{\Xi_1})_{rs} f_{abc} \left(\Xi_{1r}^{a\dagger} \Xi_{1s}^b \right) \left(\phi^\dagger \sigma^c \phi \right) \\
& + \left\{ (y_{\Xi_1})_{rij} \Xi_{1r}^{a\dagger} \bar{l}_{Li} \sigma^a i\sigma_2 l_{Lj}^c + (\kappa_{\Xi_1})_r \Xi_{1r}^{a\dagger} \left(\tilde{\phi}^\dagger \sigma^a \phi \right) + \text{h.c.} \right\} \\
& + \left\{ (\lambda_{\Theta_1})_r \left(\phi^\dagger \sigma^a \phi \right) C_{a\beta}^I \tilde{\phi}_\beta \epsilon_{IJ} \Theta_{1r}^J + \text{h.c.} \right\}
\end{aligned}$$

$$
\begin{aligned}
&+ \left\{(\lambda_{\Theta_3})_r \left(\phi^\dagger \sigma^a \tilde{\phi}\right) C^I_{\alpha\beta}\tilde{\phi}_\beta \epsilon_{IJ}\Theta^J_{3r} + \text{h.c.}\right\} \\
&+ \left\{(y^{ql}_{\omega_1})_{rij}\omega^\dagger_{1r}\bar{q}^c_{Li} i\sigma_2 l_{Lj} + (y^{qq}_{\omega_1})_{rij}\omega^{A\dagger}_{1r}\epsilon_{ABC}\bar{q}^B_{Li} i\sigma_2 q^{cC}_{Lj}\right. \\
&\quad \left. +(y^{eu}_{\omega_1})_{rij}\omega^\dagger_{1r}\bar{e}^c_{Ri}u_{Rj} + (y^{du}_{\omega_1})_{rij}\omega^{A\dagger}_{1r}\epsilon_{ABC}\bar{d}^B_{Ri}u^{cC}_{Rj} + \text{h.c.}\right\} \\
&+ \left\{(y_{\omega_2})_{rij}\omega^{A\dagger}_{2r}\epsilon_{ABC}\bar{d}^B_{Ri}d^{cC}_{Rj} + \text{h.c.}\right\} \\
&+ \left\{(y^{ed}_{\omega_4})_{rij}\omega^{A\dagger}_{4r}\bar{e}^c_{Ri}d_{Rj} + (y^{uu}_{\omega_4})_{rij}\omega^{A\dagger}_{4r}\epsilon_{ABC}\bar{u}^B_{Ri}u^{cC}_{Rj} + \text{h.c.}\right\} \\
&+ \left\{(y_{\Pi_1})_{rij}\Pi^\dagger_{1r} i\sigma_2 \bar{l}^T_{Li} d_{Rj} + \text{h.c.}\right\} \\
&+ \left\{(y^{lu}_{\Pi_7})_{rij}\Pi^\dagger_{7r} i\sigma_2 \bar{l}^T_{Li} u_{Rj} + (y^{eq}_{\Pi_7})_{rij}\Pi^\dagger_{7r}\bar{e}_{Ri}q_{Lj} + \text{h.c.}\right\} \\
&+ \left\{(y^{ql}_\zeta)_{rij}\zeta^{a\dagger}_r \bar{q}^c_{Li} i\sigma_2\sigma^a l_{Lj} + (y^{qq}_\zeta)_{rij}\zeta^{a\dagger}_r\epsilon_{ABC}\bar{q}^B_{Li}\sigma^a i\sigma_2 q^{cC}_{Lj} + \text{h.c.}\right\} \\
&+ \left\{(y^{ud}_{\Omega_1})_{rij}\Omega^{AB\dagger}_{1r}\bar{u}^{c(A|}_{Ri}d^{|B)}_{Rj} + (y^{qq}_{\Omega_1})_{rij}\Omega^{AB\dagger}_{1r}\bar{q}^{c(A|}_{Li} i\sigma_2 q^{|B)}_{Lj} + \text{h.c.}\right\} \\
&+ \left\{(y_{\Omega_2})_{rij}\Omega^{AB\dagger}_{2r}\bar{d}^{c(A|}_{Ri}d^{|B)}_{Rj} + \text{h.c.}\right\} \\
&+ \left\{(y_{\Omega_4})_{rij}\Omega^{AB\dagger}_{4r}\bar{u}^{c(A|}_{Ri}u^{|B)}_{Rj} + \text{h.c.}\right\} \\
&+ \left\{(y_\Upsilon)_{rij}\Upsilon^{AB\dagger}_r\bar{q}^{c(A|}_{Li} i\sigma_2\sigma^a q^{|B)}_{Lj} + \text{h.c.}\right\} \\
&+ \left\{(y^{qu}_\Phi)_{rij}\Phi^{A\dagger}_r i\sigma_2\bar{q}^T_{Li}T_A u_{Rj} + (y^{dq}_\Phi)_{rij}\Phi^{A\dagger}_r\bar{d}_{Ri}T_A q_{Lj} + \text{h.c.}\right\} \\
&+ (\lambda_{\mathcal{S}\Xi})_{rs}\mathcal{S}_r\Xi^a_s\left(\phi^\dagger\sigma^a\phi\right) + (\kappa_{\mathcal{S}\Xi})_{rst}\mathcal{S}_r\Xi^a_s\Xi^a_t \\
&+ (\kappa_{\mathcal{S}\Xi_1})_{rst}\mathcal{S}_r\Xi^{a\dagger}_{1s}\Xi^a_{1t} + \left\{(\lambda_{\mathcal{S}\Xi_1})_{rs}\mathcal{S}_r\Xi^{a\dagger}_{1s}\left(\tilde{\phi}^\dagger\sigma^a\phi\right) + \text{h.c.}\right\} \\
&+ \left\{(\kappa_{\mathcal{S}\varphi})_{rs}\mathcal{S}_r\varphi^\dagger_s\phi + (\kappa_{\Xi\varphi})_{rs}\Xi^a_r(\varphi^\dagger_s\sigma^a\phi) + (\kappa_{\Xi_1\varphi})_{rs}\Xi^{a\dagger}_{1r}\left(\tilde{\varphi}^\dagger_s\sigma^a\phi\right) + \text{h.c.}\right\} \\
&+ (\kappa_{\Xi\Xi_1})_{rst}f_{abc}\Xi^a_r\Xi^{b\dagger}_{1s}\Xi^b_{1t} + \left\{(\lambda_{\Xi_1\Xi})_{rs}f_{abc}\Xi^{a\dagger}_{1r}\Xi^b_s\left(\tilde{\phi}^\dagger\sigma^c\phi\right) + \text{h.c.}\right\} \\
&+ \left\{(\kappa_{\Xi\Theta_1})_{rs}\Xi^a_r C^I_{\alpha\beta}\tilde{\phi}_\beta\epsilon_{IJ}\Theta^J_{1s} + (\kappa_{\Xi_1\Theta_1})_{rs}\Xi^{a\dagger}_{1r}C^I_{\alpha\beta}\phi_\beta\epsilon_{IJ}\Theta^J_{1s}\right. \\
&\quad \left. + (\kappa_{\Xi_1\Theta_3})_{rs}\Xi^{a\dagger}_{1r}C^I_{\alpha\beta}\tilde{\phi}_\beta\epsilon_{IJ}\Theta^J_{3s} + \text{h.c.}\right\},
\end{aligned}
\tag{7.13}
$$

and

$$
\begin{aligned}
-\mathcal{L}^{(5)}_{\text{S}} = \frac{1}{f}\Bigg[&(\tilde{k}^\phi_{\mathcal{S}})_r\mathcal{S}_r D_\mu\phi^\dagger D^\mu\phi + (\tilde{\lambda}_{\mathcal{S}})_r\mathcal{S}_r|\phi|^4 \\
&+ (\tilde{k}^B_{\mathcal{S}})_r\mathcal{S}_r B_{\mu\nu}B^{\mu\nu} + (\tilde{k}^W_{\mathcal{S}})_r\mathcal{S}_r W^a_{\mu\nu}W^{a\,\mu\nu} + (\tilde{k}^G_{\mathcal{S}})_r\mathcal{S}_r G^A_{\mu\nu}G^{A\,\mu\nu} \\
&+ (\tilde{k}^{\tilde{B}}_{\mathcal{S}})_r\mathcal{S}_r B_{\mu\nu}\tilde{B}^{\mu\nu} + (\tilde{k}^{\tilde{W}}_{\mathcal{S}})_r\mathcal{S}_r W^a_{\mu\nu}\tilde{W}^{a\,\mu\nu} + (\tilde{k}^{\tilde{G}}_{\mathcal{S}})_r\mathcal{S}_r G^A_{\mu\nu}\tilde{G}^{A\,\mu\nu} \\
&+ \left\{(\tilde{y}^e_{\mathcal{S}})_{rij}\mathcal{S}_r\bar{e}_{Ri}\phi^\dagger l_{Lj} + (\tilde{y}^d_{\mathcal{S}})_{rij}\mathcal{S}_r\bar{d}_{Ri}\phi^\dagger q_{Lj} + (\tilde{y}^u_{\mathcal{S}})_{rij}\mathcal{S}_r\bar{u}_{Ri}\tilde{\phi}^\dagger q_{Lj} + \text{h.c.}\right\} \\
&+ (\tilde{k}^\phi_\Xi)_r\Xi^a_r D_\mu\phi^\dagger\sigma^a D^\mu\phi + (\tilde{\lambda}_\Xi)_r\Xi^a_r|\phi|^2\phi^\dagger\sigma^a\phi \\
&+ (\tilde{k}^{WB}_\Xi)_r\Xi^a_r W^a_{\mu\nu}B^{\mu\nu} + (\tilde{k}^{W\tilde{B}}_\Xi)_r\Xi^a_r W^a_{\mu\nu}\tilde{B}^{\mu\nu} \\
&+ \left\{(\tilde{y}^e_\Xi)_{rij}\Xi^a_r\bar{e}_{Ri}\phi^\dagger\sigma^a l_{Lj} + (\tilde{y}^d_\Xi)_{rij}\Xi^a_r\bar{d}_{Ri}\phi^\dagger\sigma^a q_{Lj} + (\tilde{y}^u_\Xi)_{rij}\Xi^a_r\bar{u}_{Ri}\tilde{\phi}^\dagger\sigma^a q_{Lj} + \text{h.c.}\right\} \\
&+ \left\{(\tilde{k}_{\Xi_1})_r\Xi^{a\dagger}_{1r}D_\mu\tilde{\phi}^\dagger\sigma^a D^\mu\phi + (\tilde{\lambda}_{\Xi_1})_r\Xi^{a\dagger}_{1r}|\phi|^2\tilde{\phi}^\dagger\sigma^a\phi + (\tilde{y}^e_{\Xi_1})_{rij}\Xi^{a\dagger}_{1r}\bar{e}_{Ri}\tilde{\phi}^\dagger\sigma^a l_{Lj}\right.
\end{aligned}
$$

$$+(\tilde{y}^{d}_{\Xi_1})_{rij}\Xi^{a\dagger}_{1r}\bar{d}_{Ri}\tilde{\phi}^{\dagger}\sigma^a q_{Lj} + (\tilde{y}^{u}_{\Xi_1})_{rij}\Xi^{a\dagger}_{1r}\bar{q}_{Li}\sigma^a\phi u_{Rj} + \text{h.c.}\Big\}\Bigg]. \tag{7.14}$$

7.3.2 New fermions

As indicated in section 7.1, we exclude the possibility of extra fermions with chiral transformations under the gauge group G_{SM}. Then, in the massive fermion sector, the complex irreducible representations of G_{SM} are carried by vector-like Dirac spinors, while the real irreducible representations are carried by Majorana spinors ψ, with $\psi_L = (\psi_R)^c \equiv \psi_R^c$. The only instances of the latter possibility are the extra leptons N and Σ in table 7.2. In our "field basis", the diagonal mass matrices are given by sums of Dirac mass terms (for the complex representations) and Majorana mass terms (for the real representations).[6]

The general Lagrangian $\mathcal{L}_{\text{F}}$ is given by

$$\mathcal{L}_{\text{F}} = \mathcal{L}_{\text{F}}^{\text{quad}} + \mathcal{L}_{\text{F}}^{\text{int}}, \tag{7.15}$$

where

$$\mathcal{L}_{\text{F}}^{\text{quad}} = \sum_{\psi} \eta_\psi \left[\bar{\psi}\, i\not{D}\, \psi - M_\psi \bar{\psi}\psi\right], \tag{7.16}$$

with ψ labelling the different fields in table 7.2, with an arbitrary number of fields in each irreducible representation, and $\eta_\psi = 1$ ($\eta_\psi = 1/2$) when ψ is Dirac (Majorana), and

$$\mathcal{L}_{\text{F}}^{\text{int}} = \mathcal{L}^{(4)}_{\text{leptons}} + \mathcal{L}^{(4)}_{\text{quarks}} + \mathcal{L}^{(5)}_{\text{leptons}} + \mathcal{L}^{(5)}_{\text{quarks}}, \tag{7.17}$$

where

$$\begin{aligned}
-\mathcal{L}^{(4)}_{\text{leptons}} = {} & (\lambda_N)_{ri}\bar{N}_{Rr}\tilde{\phi}^\dagger l_{Li} + (\lambda_E)_{ri}\bar{E}_{Rr}\phi^\dagger l_{Li} \\
& + (\lambda_{\Delta_1})_{ri}\bar{\Delta}_{1Lr}\phi e_{Ri} + (\lambda_{\Delta_3})_{ri}\bar{\Delta}_{3Lr}\tilde{\phi} e_{Ri} \\
& + \frac{1}{2}(\lambda_\Sigma)_{ri}\bar{\Sigma}^a_{Rr}\tilde{\phi}^\dagger\sigma^a l_{Li} + \frac{1}{2}(\lambda_{\Sigma_1})_{ri}\bar{\Sigma}^a_{1Rr}\phi^\dagger\sigma^a l_{Li} \\
& + (\lambda_{N\Delta_1})_{rs}\bar{N}^c_{Rr}\phi^\dagger\Delta_{1Rs} + (\lambda_{E\Delta_1})_{rs}\bar{E}_{Lr}\phi^\dagger\Delta_{1Rs} \\
& + (\lambda_{E\Delta_3})_{rs}\bar{E}_{Lr}\tilde{\phi}^\dagger\Delta_{3Rs} + \frac{1}{2}(\lambda_{\Sigma\Delta_1})_{rs}\bar{\Sigma}^{c\,a}_{Rr}\tilde{\phi}^\dagger\sigma^a\Delta_{1Rs} \\
& + \frac{1}{2}(\lambda_{\Sigma_1\Delta_1})_{rs}\bar{\Sigma}^a_{1Lr}\phi^\dagger\sigma^a\Delta_{1Rs} + \frac{1}{2}(\lambda_{\Sigma_1\Delta_3})_{rs}\bar{\Sigma}^a_{1Lr}\tilde{\phi}^\dagger\sigma^a\Delta_{3Rs} + \text{h.c.},
\end{aligned} \tag{7.18}$$

$$\begin{aligned}
-\mathcal{L}^{(4)}_{\text{quarks}} = {} & (\lambda_U)_{ri}\bar{U}_{Rr}\tilde{\phi}^\dagger q_{Li} + (\lambda_D)_{ri}\bar{D}_{Rr}\phi^\dagger q_{Li} \\
& + (\lambda^u_{Q_1})_{ri}\bar{Q}_{1Lr}\tilde{\phi} u_{Ri} + (\lambda^d_{Q_1})_{ri}\bar{Q}_{1Lr}\phi d_{Ri} \\
& + (\lambda_{Q_5})_{ri}\bar{Q}_{5Lr}\tilde{\phi} d_{Ri} + (\lambda_{Q_7})_{ri}\bar{Q}_{7Lr}\phi u_{Rj} \\
& + \frac{1}{2}(\lambda_{T_1})_{ri}\bar{T}^a_{1Rr}\phi^\dagger\sigma^a q_{Li} + \frac{1}{2}(\lambda_{T_2})_{ri}\bar{T}^a_{2Rr}\tilde{\phi}^\dagger\sigma^a q_{Li} \\
& + (\lambda_{UQ_1})_{rs}\bar{U}_{Lr}\tilde{\phi}^\dagger Q_{1Rs} + (\lambda_{UQ_7})_{rs}\bar{U}_{Lr}\phi^\dagger Q_{7Rs}
\end{aligned}$$

[6]Note that the particular case of a Dirac fermion Ψ of mass M_Ψ in a real representation of H is equivalent to our description with two degenerate Majorana fields ψ_1 and ψ_2 of mass M_Ψ, with $\Psi_R = 1/\sqrt{2}\,(\psi_{1R} + i\psi_{2R})$ and $\Psi_L = 1/\sqrt{2}\,(\psi^c_{1R} + i\psi^c_{2R})$.

$$
\begin{aligned}
&+ (\lambda_{DQ_1})_{rs}\bar{D}_{Lr}\phi^\dagger Q_{1Rs} + (\lambda_{DQ_5})_{rs}\bar{D}_{Lr}\tilde{\phi}^\dagger Q_{5Rs} \\
&+ \frac{1}{2}(\lambda_{T_1Q_1})_{rs}\bar{T}^a_{1Lr}\phi^\dagger\sigma^a Q_{1Rs} + \frac{1}{2}(\lambda_{T_1Q_5})_{rs}\bar{T}^a_{1Lr}\tilde{\phi}^\dagger\sigma^a Q_{5Rs} \\
&+ \frac{1}{2}(\lambda_{T_2Q_1})_{rs}\bar{T}^a_{2Lr}\tilde{\phi}^\dagger\sigma^a Q_{1Rs} + \frac{1}{2}(\lambda_{T_2Q_7})_{rs}\bar{T}^a_{2Lr}\phi^\dagger\sigma^a Q_{7Rs} + \text{h.c.},
\end{aligned}
\tag{7.19}
$$

$$
\begin{aligned}
-\mathcal{L}^{(5)}_{\text{leptons}} = \frac{1}{f}\Bigg[&(\tilde{\lambda}_N)_{ri}\bar{N}^c_{Rr}\gamma^\mu\left(D_\mu\tilde{\phi}\right)^\dagger l_{Li} \\
&+ (\tilde{\lambda}^l_E)_{ri}\bar{E}_{Lr}\gamma^\mu\left(D_\mu\phi\right)^\dagger l_{Li} + (\tilde{\lambda}^B_E)_{ri}\bar{E}_{Lr}\sigma^{\mu\nu}e_{Ri}B_{\mu\nu} + (\tilde{\lambda}^e_E)_{ri}\bar{E}_{Lr}\phi^\dagger\phi e_{Ri} \\
&+ (\tilde{\lambda}^e_{\Delta_1})_{ri}\bar{\Delta}_{1Rr}\not{D}\phi e_{Ri} + (\tilde{\lambda}^l_{\Delta_1})_{ri}\left(\bar{\Delta}_{1Rr}\phi\right)\left(\phi^\dagger l_{Li}\right) + (\tilde{\lambda}^{l\prime}_{\Delta_1})_{ri}\left(\bar{\Delta}_{1Rr}l_{Li}\right)\left(\phi^\dagger\phi\right) \\
&+ (\tilde{\lambda}^B_{\Delta_1})_{ri}\bar{\Delta}_{1Rr}\sigma^{\mu\nu}l_{Li}B_{\mu\nu} + (\tilde{\lambda}^W_{\Delta_1})_{ri}\bar{\Delta}_{1Rr}\sigma^{\mu\nu}\sigma^a l_{Li}W^a_{\mu\nu} \\
&+ (\tilde{\lambda}^e_{\Delta_3})_{ri}\bar{\Delta}_{3Rr}\not{D}\tilde{\phi}e_{Ri} + (\tilde{\lambda}^l_{\Delta_3})_{ri}\left(\bar{\Delta}_{3Rr}\tilde{\phi}\right)\left(\phi^\dagger l_{Li}\right) \\
&+ (\tilde{\lambda}^l_\Sigma)_{ri}\bar{\Sigma}^{c\,a}_{Rr}\gamma^\mu\left(D_\mu\tilde{\phi}\right)^\dagger\sigma^a l_{Li} + (\tilde{\lambda}^e_\Sigma)_{ri}\bar{\Sigma}^{c\,a}_{Rr}\tilde{\phi}^\dagger\sigma^a\phi e_{Ri} \\
&+ (\tilde{\lambda}^l_{\Sigma_1})_{ri}\bar{\Sigma}^a_{1Lr}\gamma^\mu\left(D_\mu\phi\right)^\dagger\sigma^a l_{Li} + (\tilde{\lambda}^e_{\Sigma_1})_{ri}\bar{\Sigma}^a_{1Lr}\phi^\dagger\sigma^a\phi e_{Ri} \\
&+ (\tilde{\lambda}^W_{\Sigma_1})_{ri}\bar{\Sigma}^a_{1Lr}\sigma^{\mu\nu}e_{Ri}W^a_{\mu\nu}\Bigg] + \text{h.c.},
\end{aligned}
\tag{7.20}
$$

$$
\begin{aligned}
-\mathcal{L}^{(5)}_{\text{quarks}} = \frac{1}{f}\Bigg[&(\tilde{\lambda}^q_U)_{ri}\bar{U}_{Lr}\gamma^\mu\left(D_\mu\tilde{\phi}\right)^\dagger q_{Li} + (\tilde{\lambda}^u_U)_{ri}\bar{U}_{Lr}\phi^\dagger\phi u_{Ri} \\
&+ (\tilde{\lambda}^B_U)_{ri}\bar{U}_{Lr}\sigma^{\mu\nu}u_{Ri}B_{\mu\nu} + (\tilde{\lambda}^G_U)_{ri}\bar{U}_{Lr}T_A\sigma^{\mu\nu}u_{Ri}G^A_{\mu\nu} \\
&+ (\tilde{\lambda}^q_D)_{ri}\bar{D}_{Lr}\gamma^\mu\left(D_\mu\phi\right)^\dagger q_{Li} + (\tilde{\lambda}^d_D)_{ri}\bar{D}_{Lr}\phi^\dagger\phi d_{Ri} \\
&+ (\tilde{\lambda}^B_D)_{ri}\bar{D}_{Lr}\sigma^{\mu\nu}d_{Ri}B_{\mu\nu} + (\tilde{\lambda}^G_D)_{ri}\bar{D}_{Lr}T_A\sigma^{\mu\nu}d_{Ri}G^A_{\mu\nu} \\
&+ (\tilde{\lambda}^u_{Q_1})_{ri}\bar{Q}_{1Rr}\not{D}\tilde{\phi}u_{Ri} + (\tilde{\lambda}^d_{Q_1})_{ri}\bar{Q}_{1Rr}\not{D}\phi d_{Ri} \\
&+ (\tilde{\lambda}^q_{Q_1})_{ri}\left(\bar{Q}_{1Rr}\phi\right)\left(\phi^\dagger q_{Li}\right) + (\tilde{\lambda}^{q\prime}_{Q_1})_{ri}\left(\bar{Q}_{1Rr}q_{Li}\right)\left(\phi^\dagger\phi\right) \\
&+ (\tilde{\lambda}^B_{Q_1})_{ri}\bar{Q}_{1Rr}\sigma^{\mu\nu}q_{Li}B_{\mu\nu} + (\tilde{\lambda}^W_{Q_1})_{ri}\bar{Q}_{1Rr}\sigma^{\mu\nu}\sigma^a q_{Li}W^a_{\mu\nu} \\
&+ (\tilde{\lambda}^G_{Q_1})_{ri}\bar{Q}_{1Rr}\sigma^{\mu\nu}T^A q_{Li}G^A_{\mu\nu} \\
&+ (\tilde{\lambda}^d_{Q_5})_{ri}\bar{Q}_{5Rr}\not{D}\tilde{\phi}d_{Ri} + (\tilde{\lambda}^q_{Q_5})_{ri}\left(\bar{Q}_{5Rr}\tilde{\phi}\right)\left(\phi^\dagger q_{Li}\right) \\
&+ (\tilde{\lambda}^u_{Q_7})_{ri}\bar{Q}_{7Rr}\not{D}\phi u_{Ri} + (\tilde{\lambda}^q_{Q_7})_{ri}\left(\bar{Q}_{7Rr}\phi\right)\left(\tilde{\phi}^\dagger q_{Li}\right) \\
&+ (\tilde{\lambda}^q_{T_1})_{ri}\bar{T}^a_{1Lr}\gamma^\mu\left(D_\mu\phi\right)^\dagger\sigma^a q_{Li} + (\tilde{\lambda}^u_{T_1})_{ri}\bar{T}^a_{1Lr}\phi^\dagger\sigma^a\tilde{\phi}u_{Ri} \\
&+ (\tilde{\lambda}^d_{T_1})_{ri}\bar{T}^a_{1Lr}\phi^\dagger\sigma^a\phi d_{Ri} + (\tilde{\lambda}^W_{T_1})_{ri}\bar{T}^a_{1Lr}\sigma^{\mu\nu}d_{Ri}W^a_{\mu\nu} \\
&+ (\tilde{\lambda}^q_{T_2})_{ri}\bar{T}^a_{2Lr}\gamma^\mu\left(D_\mu\tilde{\phi}\right)^\dagger\sigma^a q_{Li} + (\tilde{\lambda}^u_{T_2})_{ri}\bar{T}^a_{2Lr}\phi^\dagger\sigma^a\phi u_{Ri} \\
&+ (\tilde{\lambda}^d_{T_2})_{ri}\bar{T}^a_{2Lr}\tilde{\phi}^\dagger\sigma^a\phi d_{Ri} + (\tilde{\lambda}^W_{T_2})_{ri}\bar{T}^a_{2Lr}\sigma^{\mu\nu}u_{Ri}W^a_{\mu\nu}\Bigg] + \text{h.c.}\,.
\end{aligned}
\tag{7.21}
$$

7.3.3 New vectors

For the extra vectors, we write

$$\mathcal{L}_{\mathrm{V}} = \mathcal{L}_{\mathrm{V}}^{\mathrm{quad}} + \mathcal{L}_{\mathrm{V}}^{\mathrm{int}}, \tag{7.22}$$

where[7]

$$\mathcal{L}_{\mathrm{V}}^{\mathrm{quad}} = \sum_V \eta_V \left(D_\mu V_\nu^\dagger D^\nu V^\mu - D_\mu V_\nu^\dagger D^\mu V^\nu + M_V^2 V_\mu^\dagger V^\mu \right), \tag{7.23}$$

with V on the right-hand side labelling the different fields in table 7.3, with an arbitrary number of fields in each irreducible representation, and $\eta_V = 1$ ($\eta_V = 1/2$) when V is in a complex (real) representation of H, and

$$\mathcal{L}_{\mathrm{V}}^{\mathrm{int}} = \mathcal{L}_{\mathrm{V}}^{(\leq 4)} + \mathcal{L}_{\mathrm{V}}^{(5)}, \tag{7.24}$$

where

$$\begin{aligned}
-\mathcal{L}_{\mathrm{V}}^{(\leq 4)} =\; & (g_{\mathcal{B}}^l)_{rij} \mathcal{B}_r^\mu \bar{l}_{Li} \gamma_\mu l_{Lj} + (g_{\mathcal{B}}^q)_{rij} \mathcal{B}_r^\mu \bar{q}_{Li} \gamma_\mu q_{Lj} + (g_{\mathcal{B}}^e)_{rij} \mathcal{B}_r^\mu \bar{e}_{Li} \gamma_\mu e_{Lj} \\
& + (g_{\mathcal{B}}^d)_{rij} \mathcal{B}_r^\mu \bar{d}_{Li} \gamma_\mu d_{Lj} + (g_{\mathcal{B}}^u)_{rij} \mathcal{B}_r^\mu \bar{u}_{Li} \gamma_\mu u_{Lj} + \left\{ (g_{\mathcal{B}}^\phi)_r \mathcal{B}_r^\mu \phi^\dagger i D_\mu \phi + \text{h.c.} \right\} \\
& + \left\{ (g_{\mathcal{B}_1}^{du})_{rij} \mathcal{B}_{1r}^{\mu\dagger} \bar{d}_{Ri} \gamma_\mu u_{Rj} + (g_{\mathcal{B}_1}^\phi)_r \mathcal{B}_{1r}^{\mu\dagger} i D_\mu \phi^T i\sigma_2 \phi + \text{h.c.} \right\} \\
& + \frac{1}{2} (g_{\mathcal{W}}^l)_{rij} \mathcal{W}_r^{\mu a} \bar{l}_{Li} \sigma^a \gamma_\mu l_{Lj} + \frac{1}{2} (g_{\mathcal{W}}^q)_{rij} \mathcal{W}_r^{\mu a} \bar{q}_{Li} \sigma^a \gamma_\mu q_{Lj} \\
& + \left\{ \frac{1}{2} (g_{\mathcal{W}}^\phi)_r \mathcal{W}_r^{\mu a} \phi^\dagger \sigma^a i D_\mu \phi + \text{h.c.} \right\} \\
& + \left\{ \frac{1}{2} (g_{\mathcal{W}_1})_r \mathcal{W}_{1r}^{\mu a\dagger} i D_\mu \phi^T i\sigma_2 \sigma^a \phi + \text{h.c.} \right\} \\
& + (g_{\mathcal{G}}^q)_{rij} \mathcal{G}_r^{\mu A} \bar{q}_{Li} \gamma_\mu T_A q_{Lj} + (g_{\mathcal{G}}^u)_{rij} \mathcal{G}_r^{\mu A} \bar{u}_{Li} \gamma_\mu T_A u_{Rj} + (g_{\mathcal{G}}^d)_{rij} \mathcal{G}_r^{\mu A} \bar{d}_{Ri} \gamma_\mu T_A d_{Rj} \\
& + \left\{ (g_{\mathcal{G}_1})_{rij} \mathcal{G}_{1r}^{A\mu\dagger} \bar{d}_{Ri} T_A \gamma_\mu u_{Rj} + \text{h.c.} \right\} \\
& + \frac{1}{2} (g_{\mathcal{H}})_{rij} \mathcal{H}_r^{\mu a A} \bar{q}_{Li} \gamma_\mu \sigma^a T_A q_{Lj} \\
& + \left\{ (\gamma_{\mathcal{L}_1})_r \mathcal{L}_{1r\mu}^\dagger D^\mu \phi + \text{h.c.} \right\} \\
& + i (g_{\mathcal{L}_1}^B)_{rs} \mathcal{L}_{1r\mu}^\dagger \mathcal{L}_{1s\nu} B^{\mu\nu} + i (g_{\mathcal{L}_1}^W)_{rs} \mathcal{L}_{1i\mu}^\dagger \sigma^a \mathcal{L}_{1j\nu} W^{a\,\mu\nu} \\
& + i (g_{\mathcal{L}_1}^{\tilde{B}})_{rs} \mathcal{L}_{1r\mu}^\dagger \mathcal{L}_{1s\nu} \tilde{B}^{\mu\nu} + i (g_{\mathcal{L}_1}^{\tilde{W}})_{rs} \mathcal{L}_{1r\mu}^\dagger \sigma^a \mathcal{L}_{1s\nu} \tilde{W}^{a\,\mu\nu} \\
& + (h_{\mathcal{L}_1}^{(1)})_{rs} \left(\mathcal{L}_{1r\mu}^\dagger \mathcal{L}_{1s}^\mu \right) \left(\phi^\dagger \phi \right) + (h_{\mathcal{L}_1}^{(2)})_{rs} \left(\mathcal{L}_{1r\mu}^\dagger \phi \right) \left(\phi^\dagger \mathcal{L}_{1s}^\mu \right) \\
& + \left\{ (h_{\mathcal{L}_1}^{(3)})_{rs} \left(\mathcal{L}_{1r\mu}^{1\dagger} \phi \right) \left(\mathcal{L}_{1s}^{\dagger\mu} \phi \right) + \text{h.c.} \right\} \\
& + \left\{ (g_{\mathcal{L}_3})_{rij} \mathcal{L}_{3r}^{\mu\dagger} \bar{e}_{Ri}^c \gamma_\mu l_{Lj} + \text{h.c.} \right\} \\
& + \left\{ (g_{\mathcal{U}_2}^{ed})_{rij} \mathcal{U}_{2r}^{\mu\dagger} \bar{e}_{Ri} \gamma_\mu d_{Rj} + (g_{\mathcal{U}_2}^{lq})_{rij} \mathcal{U}_{2r}^{\mu\dagger} \bar{l}_{Li} \gamma_\mu q_{Lj} + \text{h.c.} \right\} \\
& + \left\{ (g_{\mathcal{U}_5})_{rij} \mathcal{U}_{5r}^{\mu\dagger} \bar{e}_{Ri} \gamma_\mu u_{Rj} + \text{h.c.} \right\}
\end{aligned}$$

[7] For each V, this covariant Proca Lagrangian describes a particle of spin 1 coupled to the SM gauge fields. Other choices of the kinetic term would give rise to ghosts.

$$
\begin{aligned}
&+ \left\{ (g^{ul}_{\mathcal{Q}_1})_{rij}\, \mathcal{Q}^{\mu\dagger}_{1r} \bar{u}^c_{Ri} \gamma_\mu l_{Lj} + (g^{dq}_{\mathcal{Q}_1})_{rij}\, \mathcal{Q}^{A\mu\dagger}_{1r} \epsilon_{ABC} \bar{d}^B_{Ri} \gamma_\mu i\sigma_2 q^{cC}_{Lj} + \text{h.c.} \right\} \\
&+ \Big\{ (g^{dl}_{\mathcal{Q}_5})_{rij}\, \mathcal{Q}^{\mu\dagger}_{5r} \bar{d}^c_{Ri} \gamma_\mu l_{Lj} + (g^{eq}_{\mathcal{Q}_5})_{rij}\, \mathcal{Q}^{\mu\dagger}_{5r} \bar{e}^c_{Ri} \gamma_\mu q_{Lj} \\
&\quad + (g^{uq}_{\mathcal{Q}_5})_{rij}\, \mathcal{Q}^{A\mu\dagger}_{5r} \epsilon_{ABC} \bar{u}^B_{Ri} \gamma_\mu q^{cC}_{Lj} + \text{h.c.} \Big\} \\
&+ \left\{ \frac{1}{2} (g_{\mathcal{X}})_{rij}\, \mathcal{X}^{a\mu\dagger}_r \bar{l}_{Li} \gamma_\mu \sigma^a q_{Lj} + \text{h.c.} \right\} \\
&+ \left\{ \frac{1}{2} (g_{\mathcal{Y}_1})_{rij}\, \mathcal{Y}^{AB\mu\dagger}_{1r} \bar{d}^{(A|}_{Ri} \gamma_\mu i\sigma_2 q^{c|B)}_{Lj} + \text{h.c.} \right\} \\
&+ \left\{ \frac{1}{2} (g_{\mathcal{Y}_5})_{rij}\, \mathcal{Y}^{AB\mu\dagger}_{5r} \bar{u}^{(A|}_{Ri} \gamma_\mu i\sigma_2 q^{c|B)}_{Lj} + \text{h.c.} \right\} \\
&+ \Big\{ (\zeta_{\mathcal{L}_1 \mathcal{B}})_{rs} \left(\mathcal{L}^\dagger_{1r\mu} \phi \right) \mathcal{B}^\mu_s + (\zeta_{\mathcal{L}_1 \mathcal{B}_1})_{rs} \tilde{\mathcal{L}}^\dagger_{1r\mu} \phi \mathcal{B}^{\mu\dagger}_{1s} \\
&\quad + (\zeta_{\mathcal{L}_1 \mathcal{W}})_{rs} \left(\mathcal{L}^\dagger_{1r\mu} \sigma^a \phi \right) \mathcal{W}^{a\mu}_s + (\zeta_{\mathcal{L}_1 \mathcal{W}_1})_{rs} \tilde{\mathcal{L}}^\dagger_{1r\mu} \sigma^a \phi \mathcal{W}^{a\mu\dagger}_{1s} + \text{h.c.} \Big\} ,
\end{aligned} \tag{7.25}
$$

and

$$
\begin{aligned}
-\mathcal{L}^{(5)}_{\text{V}} = \frac{1}{f} \mathcal{L}^{\mu\dagger}_{1r} \Big[&(\tilde{\gamma}^{(1)}_{\mathcal{L}_1})_r \left(\phi^\dagger D_\mu \phi \right) \phi + (\tilde{\gamma}^{(2)}_{\mathcal{L}_1})_r \left(D_\mu \phi^\dagger \phi \right) \phi + (\tilde{\gamma}^{(3)}_{\mathcal{L}_1})_r \left(\phi^\dagger \phi \right) D_\mu \phi \\
&+ (\tilde{\gamma}^B_{\mathcal{L}_1})_r B_{\mu\nu} D^\nu \phi + (\tilde{\gamma}^{\tilde{B}}_{\mathcal{L}_1})_r \tilde{B}_{\mu\nu} D^\nu \phi \\
&+ (\tilde{\gamma}^W_{\mathcal{L}_1})_r W^a_{\mu\nu} \sigma^a D^\nu \phi + (\tilde{\gamma}^{\tilde{W}}_{\mathcal{L}_1})_r \tilde{W}^a_{\mu\nu} \sigma^a D^\nu \phi \\
&+ (\tilde{g}^{eDl}_{\mathcal{L}_1})_{rij} \bar{e}_{Ri} D_\mu l_{Lj} + (\tilde{g}^{Del}_{\mathcal{L}_1})_{rij} D_\mu \bar{e}_{Ri} l_{Lj} + (\tilde{g}^{dDq}_{\mathcal{L}_1})_{rij} \bar{d}_{Ri} D_\mu q_{Lj} \\
&+ (\tilde{g}^{Ddq}_{\mathcal{L}_1})_{rij} D_\mu \bar{d}_{Ri} q_{Lj} + (\tilde{g}^{qDu}_{\mathcal{L}_1})_{rij} i\sigma_2 \bar{q}^T_{Li} D_\mu u_{Rj} + (\tilde{g}^{Dqu}_{\mathcal{L}_1})_{rij} i\sigma_2 D_\mu \bar{q}^T_{Li} u_{Rj} \\
&+ (\tilde{g}^{du}_{\mathcal{L}_1})_{rij} \tilde{\phi} \bar{d}_{Ri} \gamma_\mu u_{Rj} + (\tilde{g}^e_{\mathcal{L}_1})_{rij} \phi \bar{e}_{Ri} \gamma_\mu e_{Rj} + (\tilde{g}^d_{\mathcal{L}_1})_{rij} \phi \bar{d}_{Ri} \gamma_\mu d_{Rj} \\
&+ (\tilde{g}^u_{\mathcal{L}_1})_{rij} \phi \bar{u}_{Ri} \gamma_\mu u_{Rj} + (\tilde{g}^l_{\mathcal{L}_1})_{rij} \phi \bar{l}_{Ri} \gamma_\mu l_{Lj} + (\tilde{g}^{l\prime}_{\mathcal{L}_1})_{rij} \left(\sigma^a \phi \right) \left(\bar{l}_{Li} \gamma_\mu \sigma^a l_{Lj} \right) \\
&+ (\tilde{g}^q_{\mathcal{L}_1})_{rij} \phi \bar{q}_{Li} \gamma_\mu q_{Lj} + (\tilde{g}^{q\prime}_{\mathcal{L}_1})_{rij} \left(\sigma^a \phi \right) \left(\bar{q}_{Li} \gamma_\mu \sigma^a q_{Lj} \right) \Big] + \text{h.c.} .
\end{aligned} \tag{7.26}
$$

7.3.4 Mixed terms

$\mathcal{L}_{\text{mixed}}$ can be further decomposed as

$$
\mathcal{L}_{\text{mixed}} = \mathcal{L}_{\text{SF}} + \mathcal{L}_{\text{SV}} + \mathcal{L}_{\text{VF}}, \tag{7.27}
$$

where the different pieces are given by

$$
\begin{aligned}
-\mathcal{L}_{\text{SF}} = &(\lambda_{\mathcal{S}E})_{rsi} \mathcal{S}_r \bar{E}_{Ls} e_{Ri} + (\lambda_{\mathcal{S}\Delta_1})_{rsi} \mathcal{S}_r \bar{\Delta}_{1Rs} l_{Li} \\
&+ (\lambda_{\mathcal{S}U})_{rsi} \mathcal{S}_r \bar{U}_{Ls} u_{Ri} + (\lambda_{\mathcal{S}D})_{rsi} \mathcal{S}_r \bar{D}_{Ls} d_{Ri} + (\lambda_{\mathcal{S}Q_1})_{rsi} \mathcal{S}_r \bar{Q}_{1Rs} q_{Li} \\
&+ (\lambda_{\Xi\Delta_1})_{rsi} \Xi^a_r \bar{\Delta}_{1Rs} \sigma^a l_{Li} + (\lambda_{\Xi\Sigma_1})_{rsi} \Xi^a_r \bar{\Sigma}^a_{1Ls} e_{Ri} \\
&+ (\lambda_{\Xi Q_1})_{rsi} \Xi^a_r \bar{Q}_{1Rs} \sigma^a q_{Li} + (\lambda_{\Xi T_1})_{rsi} \Xi^a_r \bar{T}^a_{1Ls} d_{Ri} + (\lambda_{\Xi T_2})_{rsi} \Xi^a_r \bar{T}^a_{2Ls} u_{Ri} \\
&+ (\lambda_{\Xi_1\Delta_3})_{rsi} \Xi^{a\dagger}_{1r} \bar{\Delta}_{3Rs} \sigma^a l_{Li} + (\lambda_{\Xi_1\Sigma})_{rsi} \Xi^{a\dagger}_{1r} \bar{\Sigma}^{c\ a}_{Rs} e^c_{Ri} \\
&+ (\lambda_{\Xi_1 Q_5})_{rsi} \Xi^{a\dagger}_{1r} \bar{Q}_{5Rs} \sigma^a q_{Li} + (\lambda_{\Xi_1 Q_7})_{rsi} \Xi^a_{1r} \bar{Q}_{7Rs} \sigma^a q_{Li} \\
&+ (\lambda_{\Xi_1 T_1})_{rsi} \Xi^{a\dagger}_{1r} \bar{T}^a_{1Ls} u_{Ri} + (\lambda_{\Xi_1 T_2})_{rsi} \Xi^a_{1r} \bar{T}^a_{2Ls} d_{Ri} + \text{h.c.} ,
\end{aligned} \tag{7.28}
$$

$$\begin{aligned}
-\mathcal{L}_{\rm SV} &= (\delta_{\mathcal{B}\mathcal{S}})_{rs}\mathcal{B}_{r\mu}D^{\mu}\mathcal{S}_s + (\delta_{\mathcal{W}\Xi})_{rs}\mathcal{W}_{r,\mu}D^{\mu}\Xi_s \\
&+ \left\{(\delta_{\mathcal{L}^1\varphi})_{rs}\mathcal{L}^{1\dagger}_{1r\mu}D^{\mu}\varphi_s + (\delta_{\mathcal{W}^1\Xi_1})_{rs}\mathcal{W}^{1\dagger}_{1r\mu}D^{\mu}\Xi_{1s} + \text{h.c.}\right\} \\
&+ (\varepsilon_{\mathcal{S}\mathcal{L}_1})_{rst}\mathcal{S}_r\mathcal{L}^{\dagger}_{1s\mu}\mathcal{L}^{\mu}_{1t} + (\varepsilon_{\Xi\mathcal{L}_1})_{rst}\Xi^a_r\mathcal{L}^{\dagger}_{1s\mu}\sigma^a\mathcal{L}^{\mu}_{1t} \\
&+ \left\{(\varepsilon_{\Xi_1\mathcal{L}_1})_{rst}\Xi^a_{1i}\mathcal{L}^{\dagger}_{1s\mu}\sigma^a\tilde{\mathcal{L}}^{\mu}_{1t} + \text{h.c.}\right\} \\
&+ \left\{(g_{\mathcal{S}\mathcal{L}_1})_{rs}\phi^{\dagger}\left(D_{\mu}\mathcal{S}_r\right)\mathcal{L}^{\mu}_{1s} + (g'_{\mathcal{S}\mathcal{L}_1})_{rs}\left(D_{\mu}\phi\right)^{\dagger}\mathcal{S}_r\mathcal{L}^{\mu}_{1s}\right. \\
&\quad + (g_{\Xi\mathcal{L}_1})_{rs}\phi^{\dagger}\sigma^a\left(D_{\mu}\Xi^a_r\right)\mathcal{L}^{\mu}_{1s} + (g'_{\Xi\mathcal{L}_1})_{rs}\left(D_{\mu}\phi\right)^{\dagger}\sigma^a\Xi^a_r\mathcal{L}^{\mu}_{1s} \\
&\quad \left. + (g_{\Xi_1\mathcal{L}_1})_{rs}\tilde{\phi}^{\dagger}\sigma^a\left(D_{\mu}\Xi^a_{1r}\right)^{\dagger}\mathcal{L}^{\mu}_{1s} + (g'_{\Xi_1\mathcal{L}_1})_{rs}\left(D_{\mu}\tilde{\phi}\right)^{\dagger}\sigma^a\Xi^{a\dagger}_{1r}\mathcal{L}^{\mu}_{1s} + \text{h.c.}\right\}, \qquad (7.29)
\end{aligned}$$

and

$$\begin{aligned}
-\mathcal{L}_{\rm VF} &= (z_{N\mathcal{L}_1})_{rsi}\bar{N}^c_{Rr}\gamma^{\mu}\tilde{\mathcal{L}}^{\dagger}_{1s\mu}l_{Li} + (z_{E\mathcal{L}_1})_{rsi}\bar{E}_{Lr}\gamma^{\mu}\mathcal{L}^{\dagger}_{1s\mu}l_{Li} \\
&+ (z_{\Delta_1\mathcal{L}_1})_{rsi}\bar{\Delta}_{1Rr}\gamma^{\mu}\mathcal{L}_{1s\mu}e_{Ri} + (z_{\Delta_3\mathcal{L}_1})_{rsi}\bar{\Delta}_{3Rr}\gamma^{\mu}\tilde{\mathcal{L}}_{1s\mu}e_{Ri} \\
&+ (z_{\Sigma\mathcal{L}_1})_{rsi}\bar{\Sigma}^{c\ a}_{Rr}\gamma^{\mu}\tilde{\mathcal{L}}^{\dagger}_{1s\mu}\sigma^a l_{Li} + (z_{\Sigma_1\mathcal{L}_1})_{rsi}\bar{\Sigma}^a_{1Lr}\gamma^{\mu}\mathcal{L}^{\dagger}_{1s\mu}\sigma^a l_{Li} \\
&+ (z_{U\mathcal{L}_1})_{rsi}\bar{U}_{Lr}\gamma^{\mu}\tilde{\mathcal{L}}^{\dagger}_{1s\mu}q_{Li} + (z_{D\mathcal{L}_1})_{rsi}\bar{D}_{Lr}\gamma^{\mu}\mathcal{L}^{\dagger}_{1s\mu}q_{Li} \\
&+ (z^u_{Q_1\mathcal{L}_1})_{rsi}\bar{Q}_{1Rr}\gamma^{\mu}\tilde{\mathcal{L}}_{1s\mu}u_{Ri} + (z^d_{Q_1\mathcal{L}_1})_{rsi}\bar{Q}_{1Rr}\gamma^{\mu}\mathcal{L}_{1s\mu}d_{Ri} \\
&+ (z_{Q_5\mathcal{L}_1})_{rsi}\bar{Q}_{5Rr}\gamma^{\mu}\tilde{\mathcal{L}}_{1s\mu}d_{Ri} + (z_{Q_7\mathcal{L}_1})_{rsi}\bar{Q}_{7Rr}\gamma^{\mu}\mathcal{L}_{1s\mu}u_{Ri} \\
&+ (z_{T_1\mathcal{L}_1})_{rsi}\bar{T}^a_{1Lr}\gamma^{\mu}\mathcal{L}^{\dagger}_{1s\mu}\sigma^a q_{Li} + (z_{T_2\mathcal{L}_1})_{rsi}\bar{T}^a_{2Lr}\gamma^{\mu}\tilde{\mathcal{L}}^{\dagger}_{1s\mu}\sigma^a q_{Li} + \text{h.c.}\,. \qquad (7.30)
\end{aligned}$$

No renormalizable operators exist that contain extra scalars, fermions and vectors simultaneously.

Finally, in order to keep track of the dimensionality of the different contributions to the operators in the effective Lagrangian presented in section 8.5 we collect here the mass dimensions of the different types of couplings appearing in the new physics Lagrangians introduced above:

$$[\kappa] = 1, \quad [\lambda] = [\lambda'] = 0, \quad [y] = 0, \qquad (7.31)$$

$$[\tilde{k}] = 0, \quad [\tilde{\lambda}] = 0, \quad [\tilde{y}] = 0, \qquad (7.32)$$

$$[g] = [g'] = 0, \quad [\gamma] = 1, \quad [h] = 0, \quad [\zeta] = 1, \qquad (7.33)$$

$$[\tilde{g}] = 0, \quad [\tilde{\gamma}] = 0, \qquad (7.34)$$

$$[\delta] = 1, \quad [\varepsilon] = 1, \quad [z] = 0. \qquad (7.35)$$

7.4 Conclusions

In this chapter, we have presented the BSMEFT, an EFT for the model-independent description of physics beyond the SM. Unlike the SMEFT, the BSMEFT can describe the resonant production of degrees of freedom that are not present in the SM. This makes it a useful tool to parametrize new physics effects that cannot be taken into account in other ways. Its generality makes it the right tool to avoid dealing with concrete new physics models in case by case basis: the analyses made within the

BSMEFT can be used to study any of them, by choosing particular values of its parameters. In addition, because it is an EFT that includes every operator allowed by the symmetries, it can help discovering types of new physics effects that may be missed because of the theoretical prejudice that unavoidably goes into model building.

Performing tree-level matching of the BSMEFT to the SMEFT amounts to doing tree-level matching between any extension of the SM with new fields and the SMEFT once and for all. The result can be presented in the form of a dictionary, which can be used to explore both the low-energy effects of all possible new particles and every possible new particle that can generate some low energy effect. In chapter 8, this dictionary will be computed, allowing for operators of dimension 6 or less in the SMEFT. The relevant new fields for this calculation have been presented in this chapter, in tables 7.1, 7.2 and 7.3. The corresponding sector of $\mathcal{L}_{\mathrm{BSM}}$ has been written in section 7.3.

In chapter 9, we will use the BSMEFT to study vector-like quarks. We will consider the possibility that their linear interactions are not necessarily renormalizable. Thus, their allowed representations under G_{SM} are not only those in table 7.2. We will find that they have experimental signatures that are not present when only renormalizable interactions are permitted. As with any application of the BSMEFT, the results of this analysis will be independent of the particular model to which the vector-like quarks belong.

CHAPTER 8

Complete tree-level matching to the SMEFT

8.1 Introduction

As explained in section 3.7, the SMEFT provides an essentially model-independent parameterization of experimental data, inside a range of energies where new degrees of freedom that are not contained in the SM cannot be produced. The task of relating the SMEFT parameters to experiment can be done once and for all, independently of any choice of new physics models.[1] Then, these parameters can be connected to the parameters of specific new-physics models through the process of matching. This reintroduces the model dependence in the process of comparing experimental data to new physics. Both calculations can actually be developed simultaneously and almost independently. Put together, they allow us to use experimental data to test theories beyond the SM, even when the new particles they bring about cannot be produced.

In chapter 7, we have introduced the BSMEFT. As the SMEFT, the BSMEFT parametrizes experimental data in a model-independent way. However, the BSMEFT includes new degrees of freedom, allowing for the parametrization of their resonant production. To do so without losing generality, it includes every possible new field under general assumptions. They key requirement for these extra fields is that they can have linear couplings to the SM ones. This is a necessary condition for them to have tree-level effects when integrated out.

In this chapter, we perform tree-level matching between the BSMEFT and the dimension-6 SMEFT. Because the BSMEFT contains every field that can give non-vanishing contributions in this calculation, we obtain as a result every possible such contribution, independently of the specific model to which the extra fields belong. We present our results in the form of a complete tree-level UV/IR dictionary up to dimension 6 for the SMEFT. Parts of this dictionary have already been computed before, for quarks [6], leptons [7], vectors [8] and scalars [9]. Here, we calculate the complete dictionary, including both the already known contributions and the missing

[1] Global fits have to be updated if there is new experimental data or new theoretical calculations within the context of the EFT.

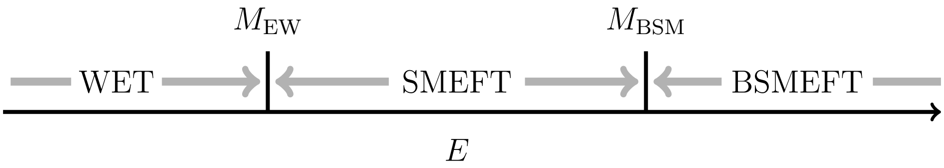

Figure 8.1: Tower of EFTs around and above the electroweak scale M_{EW}. M_{BSM} represents the mass of the new particles in the BSMEFT. Tree-level matching between the SMEFT and the WET has been performed in [25].

pieces. In figure 8.1 we show the hierarchy of EFTs that we are considering around the electroweak scale and above.

We stress that this is a very general result with many practical applications. Consider any weakly-coupled high-energy extension of the SM with new fields. To integrate out these fields at tree level, one can just identify the part of BSMEFT that corresponds to this model and then read the contribution to the SMEFT from the dictionary. Conversely, suppose that one wants to find out which weakly-coupled UV completions of the SM can produce some effect parametrized by the SMEFT. Then, one can use the dictionary in the opposite way, and directly obtain from it the possible extra fields that generate the effect, together with the necessary interactions.

We give all our results in the Warsaw basis [54], following the SM conventions in ref. [135] for the relations between redundant operators.[2] This allows the direct use of our results together with the anomalous dimensions computed in [135, 136, 193, 194] (see also [133, 134]) to have a proper leading order calculation with possible large logarithms resummed.[3]

This chapter is organized as follows. The general contribution to the tree-level matching for effective operators up to dimension 6 is computed in section 8.2. In section 8.3, we provide a guide to use our results both in the bottom-up and in a top-down fashion. The top-down dictionary is given in section 8.4 and the bottom-up one, which collects the expressions of the Wilson coefficients as functions of the UV parameters, is reproduced in section 8.5. Then, we give two specific examples of use of the dictionary: the application to the reported anomalies in certain B-meson observables, in section 8.6; and the study of Higgs physics in simple models with one or two new fields, in section 8.7. We conclude in section 8.8.

8.2 Effective Lagrangian and tree-level matching

In order to study the physics of $\mathcal{L}_{\mathrm{BSM}}$ as defined in section 7.3 at energy scales much smaller than all the masses of the extra particles, the heavy fields can be integrated out to find the corresponding effective Lagrangian, organized as a power series in the

[2] Our results can be easily translated into other popular bases by using publicly available codes [141].

[3] There has been an important progress recently towards the automation of one-loop matching calculations [32, 34, 113, 157, 158, 195–197] which would allow for consistent one-loop calculations in the new models and, eventually, next-to-leading order ones when the two-loop SMEFT anomalous dimensions are available.

inverse masses:

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_0 + \sum_{n=2}^{\infty} \mathcal{L}_{\text{eff}}^{(n)}. \tag{8.1}$$

where $\mathcal{L}_0$, defined in eq. (7.9), is the sector of $\mathcal{L}_{\text{BSM}}$ containing terms of dimension $d \leq 6$ with only SM fields and the $\mathcal{L}_{\text{eff}}^{(n)}$ contain the corrections to $\mathcal{L}_0$ from tree-level matching, containing Lorentz and gauge invariant local operators $\mathcal{O}^{(n)}$ of canonical dimension n (constructed with the SM fields),

$$\mathcal{L}_{\text{eff}}^{(n)} = \sum_j C_j^{(n)} \mathcal{O}_j^{(n)}. \tag{8.2}$$

This effective Lagrangian will be a SMEFT with particular Wilson coefficients $C_j^{(n)}$, of mass dimension $4-n$. The dimensions are provided by the masses and other scales in $\mathcal{L}_{\text{BSM}}$.

Not all the operators $\mathcal{O}^{(n)}$ are independent. Making use of algebraic identities and field redefinitions, certain linear combinations can be eliminated from $\mathcal{L}_{\text{eff}}^{(n)}$, at the price of changing $\mathcal{L}_{\text{eff}}^{(>n)}$ (see chapter 4). Taking this redundance into account, several operator bases have been defined to dimension $n = 6$. Here, we employ the Warsaw basis defined in [54]. The operators in that basis are collected in section 3.7. The main purpose of this chapter is to calculate the corresponding coefficients $C^{(\leq 6)}$ in the classical approximation, as functions of the couplings and masses in $\mathcal{L}_{\text{BSM}}$.

Note that the generated operators have the same form as the ones in $\mathcal{L}_0$. The non-trivial contributions we are interested in can be distinguished when there is sufficient information on $\mathcal{L}_0$. This is the case if the coefficients of the non-renormalizable terms in $\mathcal{L}_0$ are suppressed by a scale larger than the masses of the new particles, and also if they are fixed by symmetries or are known functions of the parameters of a given UV completion of $\mathcal{L}_{\text{BSM}}$. The requirement of a soft UV behaviour also imposes some constraints [178, 185].

The individual contributions of heavy fields and the collective contributions of heavy fields with the same spin (except for the ones involving the vector $\mathcal{L}_1$) have been calculated before in [6–9]. Here, we also incorporate the mixed contributions of heavy particles of different spin, the contributions of $\mathcal{L}_1$ and the contribution of the operators of dimension $d = 5$ in $\mathcal{L}_{\text{BSM}}$.

Let us explain the systematics of the integration procedure. With this aim, we first write the part of $\mathcal{L}_{\text{BSM}}$ involving new fields as

$$\mathcal{L}_{\text{BSM}} - \mathcal{L}_0 = \eta_{(i)} A_i^\dagger \Delta_{(i)}^{-1} A^i + \sum_{m,n} A_{j_1}^\dagger \cdots A_{j_n}^\dagger W_{i_1 \ldots i_m}^{j_1 \ldots j_n} A^{i_1} \cdots A^{i_m}, \tag{8.3}$$

where A^i represent all possible extra fields in $\mathcal{L}_{\text{BSM}}$, $\Delta_{(i)}$ is the covariant propagator for A^i and $W_{i_1 \ldots i_m}^{j_1 \ldots j_n}$ are operators constructed with the SM fields, including the identity operator. The factor $\eta_{(i)} = 1$ $(1/2)$ yields canonical normalization for complex (real) fields (see section 7.3). Lorentz and Dirac indices are implicit. In general, these operators carry a reducible representation of H, but the ones with a single index i belong to the same irreducible representation as A^i or $A_i^\dagger$. The integration at the classical level can be performed by i) using the equations of motion of $\mathcal{L}_{\text{BSM}}$ to eliminate the heavy fields and ii) expanding the propagators of the heavy fields in inverse powers of $D_{(i)}/M_{(i)}$:

$$\Delta_{(i)} = -\frac{1}{M_{(i)}^2}\left(1 - \frac{D_{(i)}^2}{M_{(i)}^2}\right) + O(1/M^6) \quad \text{(scalars)}, \tag{8.4}$$

$$\Delta_{(i)} = -\frac{i\not{D}_{(i)} + M_{(i)}}{M_{(i)}^2}\left(1 - \frac{D_{(i)}^2}{M_{(i)}^2}\right) + O(1/M^5) \quad \text{(fermions)}, \tag{8.5}$$

$$\Delta_{(i)}^{\mu\nu} = \frac{\eta^{\mu\nu}}{M_{(i)}^2} + \frac{D_{(i)}^{\nu} D_{(i)}^{\mu} - \eta^{\mu\nu} D_{(i)}^2}{M_{(i)}^4} + O(1/M^6) \quad \text{(vectors)}. \tag{8.6}$$

The result at any finite order in $D_{(i)}/M_{(i)}$ is a local Lagrangian. We have performed the calculations in this algebraic fashion, keeping only the operators of dimension $n \leq 6$. To deal in an efficient manner with the large number of terms that appear in this process and minimize the possibility of errors, we have employed the symbolic code `MatchingTools`, presented in chapter 5, where we have implemented the algebraic relations and field redefinitions necessary to express our results in terms of the Warsaw-basis operators defined in section 3.7. All the calculations have been double-checked by hand and against previous results in the literature.

We have performed field redefinitions in the linear approximation. This is the same as using the equations of motion of the SM fields. We have shown in chapter 4 that some contributions to Wilson coefficients of dimension-6 operators are missed in this approach. They come from the $\phi^\dagger\phi$ operator, when keeping terms that are quadratic in the perturbation of the field introduced by the redefinition. All these quadratic contributions are suppressed by μ^2/M^2, with respect to the natural coefficient dictated by canonical dimension, so they are numerically of the same size as the natural ones for operators of dimension 8. For this reason, we do not show them in our results. Following section 4.5.2, we can incorporate μ^2 in the power counting by defining $N_{1/M}(\mu^2) = -2$, where M the cutoff of the SMEFT, which corresponds to the masses of the extra fields. Then, what we are doing in our results is keeping only those terms $\mathcal{O}$ such that $N_{1/M}(\mathcal{O}) \leq 2$.

Step i) above can be performed in terms of Feynman diagrams. In figure 8.2, we show the tree-level Feynman diagrams with heavy field propagators that contribute to $\mathcal{L}_{\text{eff}}$ to order $n = 6$. The blobs in this figure represent the SM operators $W^{i_1 \cdots i_m}_{j_1 \cdots j_n}$ with m incoming and n outgoing lines, and the arrowed lines represent the covariant propagators $\Delta_{(i)}$. The arrows have no significance for real representations. In order to see that these are the only non-trivial tree-level diagrams contributing to $\mathcal{L}_{\text{eff}}$, note first that the canonical dimension of each term in the expansion of the propagators is non-negative, while the canonical dimension of each blob is equal to the canonical dimension of its corresponding interaction in eq. (8.3) minus the one carried away by the bosonic or fermionic heavy fields. Consider a particular connected tree-level diagram. Let B_f^d be the number of blobs in the diagram with at least one fermionic index and corresponding to interactions of canonical dimension d, and B_b^d be the number of blobs in the diagram with no fermionic indices and corresponding to interactions of canonical dimension d. Let L_f and L_b be, respectively, the number of fermionic and bosonic propagators in the diagram and let X_f be the number of blocks with uninterrupted heavy-fermion lines. The canonical dimensions n of each term in the diagram, after

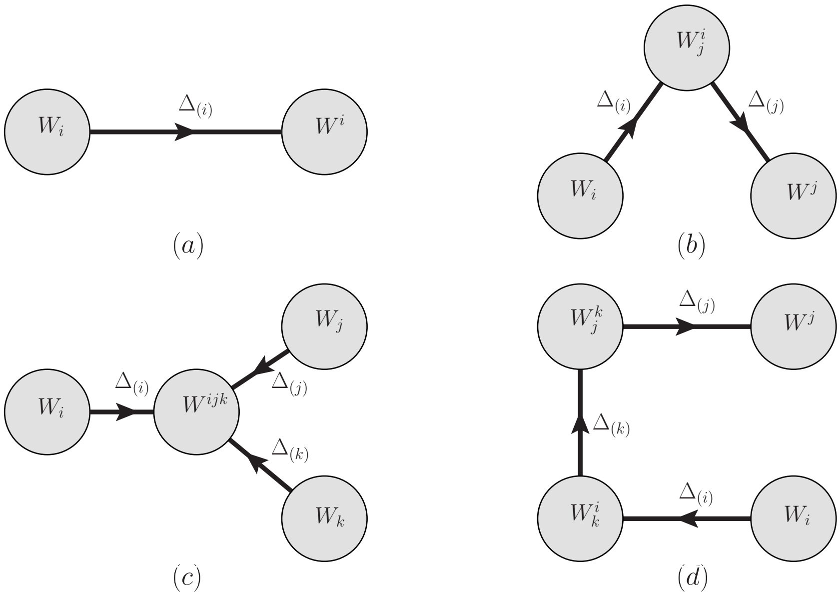

Figure 8.2: Feynman diagrams contributing to $\mathcal{L}_{\text{eff}}$ to dimension $n = 6$. Non-equivalent permutations of the arrow directions shown here should be considered as well.

the propagator expansions, obey

$$n \geq \sum_d d(B_b^d + B_f^d) - 2L_b - 3L_f. \tag{8.7}$$

From the topological relations $L_b + L_f + 1 = \sum_d (B_b^d + B_f^d)$ and $L_f + X_f = \sum_d B_f^d$, the bound

$$n \geq 2 + X_f + \sum_d \left[(d-2)B_b^d + (d-3)B_f^d\right] \tag{8.8}$$

follows. Using the facts that $B_b^d = 0$ if $d < 3$ and $B_f^d = 0$ if $d < 4$, we find in particular that

$$n \geq B + 2, \tag{8.9}$$

with $B = \sum_d (B_b^d + B_f^d)$ the total number of blobs. Therefore, only diagrams with 4 blobs or less can contribute to $n \leq 6$. We also see from (8.8) that only interactions of canonical dimension $d \leq 6$ can contribute to $n \leq 6$. But the operators with $d = 6$ only give the trivial contribution of a diagram with one blob and no propagators, which is nothing but the term already present in $\mathcal{L}_0$. This justifies our restriction to operators with $d \leq 5$ in the explicit expression of $\mathcal{L}_{\text{BSM}}$ written in section 7.3. Finally, we observe that both the operators of dimension $d = 5$ and the ones involving more than one heavy field can only contribute to $n \leq 6$ in the presence of super-renormalizable operators of dimension $d = 3$, and that operators of dimension $d = 5$ with more than one heavy field do not contribute to this order.

Note that in diagrams (a), (b) and (c) of figure 8.2, all the propagators are contracted with one-index operators W_i or W^i, which arise from terms in $\mathcal{L}_{\text{BSM}}$ with only one heavy field (A^i or $A_i^\dagger$). In diagram (d), on the other hand, the propagator $\Delta_{(k)}$ is attached only to operators with two indices, W_j^k and W_k^i. However, upon the covariant-derivative expansion at finite order of the other two propagator, $\Delta_{(i)}$ and $\Delta_{(j)}$, the blobs they connect collapse into one-index local operators $\widetilde{W}^k = W_j^k[\Delta_{(j)}]W^j$ and $\widetilde{W}_k = W_k^i[\Delta_{(i)}]W_i$, with [.] indicating the derivative expansion. The operators $\widetilde{W}^k$ and $\widetilde{W}_k$ are in the same Lorentz and gauge representation as W^k and W_k, respectively. Moreover, to allow for a dimension-six contribution, both of them must have canonical dimension $d = 4$. Hence, the fields A^k ($A_k^\dagger$) associated to $\widetilde{W}_k$ ($\widetilde{W}^k$) must also belong to a representation that can couple linearly to the SM fields to give a scalar gauge-invariant operator of dimension ≤ 4. We conclude that, as promised, only the heavy fields in the irreducible representations of tables 7.1, 7.2 and 7.3 contribute at the tree level to the effective Lagrangian to dimension six.

We can draw another interesting corollary from this discussion. Let us define *tree-level operators* as those for which there exists a renormalizable UV theory that induces them at the tree-level, when the effective Lagrangian is written in the Warsaw basis, and *loop operators* as those for which no such theory exists.[4] As we have just argued, tree-level operators of dimension six can only be generated by the diagrams in figure 8.2 and only by extra fields that allow for linear couplings to SM operators. This is also true if, instead of using the EFT $\mathcal{L}_{\rm BSM}$ as a starting point, we directly integrate out at the classical level all the fields beyond the SM in a renormalizable completion of $\mathcal{L}_{\rm BSM}$. Therefore, our results in section 8.5 explicitly show which operators are tree-level: those that (potentially) receive contributions in the absence of non-renormalizable interactions, that is, when $f \to \infty$ and $\gamma_{\mathcal{L}_1} \to 0$. Conversely, the operators that can *only* have, at most, $1/f$ or $\gamma_{\mathcal{L}_1}$ contributions are loop operators.[5] Even if the latter are connected to $\mathcal{L}_{\rm BSM}$ by tree-level diagrams, they cannot be generated at the tree level in any renormalizable completion of it. That is, the necessary dimension-five interactions are only generated by loop diagrams in any such UV completion. If this completion is weakly coupled, their coefficients will have a loop suppression that carries over to the Wilson coefficients in the SMEFT. Of course, such a suppression will not occur if the UV completion is strongly coupled. This classification agrees with the one in [96], as it should, since we employ the same criteria.

8.3 Results of the matching: user guide

The tree-level integration of the 48 fields of spin 0, 1/2 and 1 that can contribute to the dimension-six SMEFT, via the interactions in eqs. (7.13)-(7.30), generates all the effective operators in the basis of ref. [54], with the exception of the four operators $\mathcal{O}_{G,\tilde{G},W,\tilde{W}}$. The explicit expressions of the contributions to the different Wilson coefficients are collected in section 8.5. In this section we offer a basic guidance so that

[4]The requirement of renormalizability is crucial to make the distinction. Without constraints on the dimension of the interactions, any gauge-invariant operator could be trivially induced at the tree level by directly including it in the UV theory. Considering a complete basis gives definite physical meaning to each operator. Of course, which operators are potentially generated at tree or loop level depends on the particular choice of basis, but the implications for physical observables remain unchanged.

[5]The possibility of generating operators of this type with tree-level diagrams involving higher-dimensional interactions was pointed out and emphasized in [198].

users can quickly find the required entries of the UV/IR dictionary inside our long and numerous equations.

We present our results by writing, for each operator, all the possible contributions of all the multiplets to its Wilson coefficient. The results for the different operators have been organized in the following way:

- Pure four-fermion operators (section 8.5.3), classified according to the structure of chiralities of the fields in the operator, i.e. $(\bar{L}L)(\bar{L}L)$, $(\bar{R}R)(\bar{R}R)$, $(\bar{L}L)(\bar{R}R)$, $(\bar{L}R)(\bar{R}L)$, $(\bar{L}R)(\bar{L}R)$, and, separately, the baryon-number (B) violating interactions.

- Pure bosonic interactions (section 8.5.4). We follow the classification of ref. [54] and include here the operators of the form ϕ^6, $\phi^4 D^2$ and $X^2\phi^2$, where X refers to a field-strength tensor.

- Interactions between bosons and fermions (section 8.5.5). We again follow the classification of ref. [54], and separate the operators of the form $\psi^2\phi^3$, $X\psi^2\phi$ and $\psi^2\phi^2 D$.

Unless otherwise stated, for each Wilson coefficient, the contributions of the different types of fields are ordered in the following way:

$$C_i = C_i^{\text{Scalars}} + C_i^{\text{Fermions}} + C_i^{\text{Vectors}} + C_i^{\text{Mixed}} + \frac{1}{f} C_i^{\text{dim 5}}, \qquad (8.10)$$

where C_i^P, $P =$ Scalars, Fermions, Vectors, contains the information from the integration of only one type of spin, in the same order as presented in tables 7.1, 7.2 and 7.3, respectively. Each of these are further separated, with the contributions from one type of particle appearing first, and mixing between particles of same spin, afterwards:

$$C_i^P = \sum_{m \in P} C_i^m + \sum_{m,n \in P} C_i^{mn} + \sum_{m,n,p \in P} C_i^{mnp}. \qquad (8.11)$$

The contributions coming from Lagrangian interactions between particles of different spin, eqs. (7.28)-(7.30), are contained in C_i^{Mixed}. The coefficient $C_i^{\text{dim 5}}$ includes the dimension-six interactions generated by the non-renomalizable couplings in eqs. (7.14), (7.20), (7.21) and (7.26). These can be easily distinguished noting the prefactor $1/f$. Finally, some of the new particles induce modifications on the kinetic term of the SM Higgs doublet in the EFT. Our results are given in a basis where all fields are canonically normalized, and we include such corrections into a renormalization of the Higgs doublet $\phi \to Z_\phi^{-\frac{1}{2}}\phi$, with $Z_\phi^{-\frac{1}{2}}$ given in eq. (8.18). The corresponding factors of $Z_\phi^{-\frac{n_\phi}{2}}$ renormalizing operators with n_ϕ scalar doublets are shown explicitly in the coefficients.

Finally, for those operators that are non-hermitian we only report the coefficient of the interaction in tables 3.3, 3.4 and 3.5. The corresponding contributions to the coefficients of the hermitian conjugates can be obtained by complex conjugation.

The results of the matching can be employed in both directions:

Top-down

To facilitate the matching of particular models with the SMEFT—for instance to profit from the abundant model-independent constraints phrased in this language (see, e.g. [59–75])—we have collected in tables 8.1, 8.2 and 8.3, in section 8.4, the different operators resulting from the integration of each of the scalar, fermion and vector multiplets, respectively. It turns out that all the operators that receive contributions involving couplings between different types of extra fields (with the same or different spin) can always be generated as well by at least one of the particles entering in the interaction *individually*. Therefore, tables 8.1-8.3 contain all the information necessary to identify which operators can be generated in any scenario.

In this way, these tables show all the operators that can be generated given the field content of the model. One can then look at the corresponding Wilson coefficients in section 8.5 and use eqs. (8.10) and (8.11) to find the explicit contributions in terms of the masses and couplings of the new particles.

Bottom-up

Our results can also be used in a bottom-up fashion, to find the explicit SM extensions that can give rise to a given set of effective interactions. To identify which multiplets contribute to each dimension-six operator in the EFT, one simply needs to look at the labels of the masses in the denominators of each term in the expression of the Wilson coefficient. For operators involving the SM scalar doublet, one must also take into account that $\mathcal{L}_1$ and φ can contribute to the renormalization of the scalar doublet Z_ϕ. Finally, upon integration of the $\mathcal{L}_1$ vector field, the effects of its interactions with the vectors $\mathcal{B}$, $\mathcal{B}_1$, $\mathcal{W}$ and $\mathcal{W}_1$ —parameterized by the $\zeta_{\mathcal{L}_1 V}$ couplings in the Lagrangian (7.25)— can be described in a compact form by using modified couplings of $\mathcal{B}$, $\mathcal{B}_1$, $\mathcal{W}$ and $\mathcal{W}_1$ to the corresponding SM scalar currents. Explicitly, they can be described by replacing

$$(g^\phi_{\mathcal{B}})_r \quad\rightarrow\quad (\hat{g}^\phi_{\mathcal{B}})_r \equiv (g^\phi_{\mathcal{B}})_r - i\frac{(\zeta_{\mathcal{L}_1\mathcal{B}})^*_{sr}(\gamma_{\mathcal{L}_1})_s}{M^2_{\mathcal{L}_{1s}}}, \tag{8.12}$$

$$(g^\phi_{\mathcal{W}})_r \quad\rightarrow\quad (\hat{g}^\phi_{\mathcal{W}})_r \equiv (g^\phi_{\mathcal{W}})_r - 2i\frac{(\zeta_{\mathcal{L}_1\mathcal{W}})^*_{sr}(\gamma_{\mathcal{L}_1})_s}{M^2_{\mathcal{L}_{1s}}}, \tag{8.13}$$

$$(g^\phi_{\mathcal{B}_1})_r \quad\rightarrow\quad (\hat{g}^\phi_{\mathcal{B}_1})_r \equiv (g^\phi_{\mathcal{B}_1})_r + i\frac{(\zeta_{\mathcal{L}_1\mathcal{B}_1})_{sr}(\gamma_{\mathcal{L}_1})_s}{M^2_{\mathcal{L}_{1s}}}, \tag{8.14}$$

$$(g^\phi_{\mathcal{W}_1})_r \quad\rightarrow\quad (\hat{g}^\phi_{\mathcal{W}_1})_r \equiv (g^\phi_{\mathcal{W}_1})_r + 2i\frac{(\zeta_{\mathcal{L}_1\mathcal{W}_1})_{sr}(\gamma_{\mathcal{L}_1})_s}{M^2_{\mathcal{L}_{1s}}}. \tag{8.15}$$

Writing the solution in terms of the $\hat{g}^\phi_V$ couplings has the advantage of simplifying significantly many of the expressions, but obscures a bit the origin of the contribution. So, besides looking at the explicit masses, one should take into account that any $\hat{g}^\phi_V$ coupling implicitly involves a dependence on the couplings and mass of the field(s) $\mathcal{L}_1$.

For instance,

$$
\begin{array}{l}
(\hat{g}^{\phi}_{\mathcal{B}})_r \equiv (g^{\phi}_{\mathcal{B}})_r - i\frac{(\zeta_{\mathcal{L}_1\mathcal{B}})^*_{rs}(\gamma_{\mathcal{L}_1})_s}{M^2_{\mathcal{L}_{1s}}} \\
\downarrow \\
\Delta C = \frac{(\hat{g}^{\phi}_{\mathcal{B}})^2_r}{M^2_{\mathcal{B}_r}} \quad \longrightarrow \quad \Delta C = \frac{(g^{\phi}_{\mathcal{B}})^2_r}{M^2_{\mathcal{B}_r}} - 2i\frac{(g^{\phi}_{\mathcal{B}})_r(\zeta_{\mathcal{L}_1\mathcal{B}})^*_{sr}(\gamma_{\mathcal{L}_1})_s}{M^2_{\mathcal{B}_r}M^2_{\mathcal{L}_{1s}}} - \frac{(\zeta_{\mathcal{L}_1\mathcal{B}})^*_{sr}(\gamma_{\mathcal{L}_1})_s(\zeta_{\mathcal{L}_1\mathcal{B}})^*_{tr}(\gamma_{\mathcal{L}_1})_t}{M^2_{\mathcal{B}_r}M^2_{\mathcal{L}_{1s}}M^2_{\mathcal{L}_{1t}}}.
\end{array}
\tag{8.16}
$$

Remember, nevertheless, that the vector multiplets $\mathcal{L}_1$ will not contribute at all if they are the gauge bosons of an extended gauge invariance.

Similarly, the tree-level matching leads to a redefinition of the coefficients of the SM operators. Then, there are indirect effects in the dimension-six coefficients when the original SM couplings, which wear a *hat*, are written in terms of the redefined ones, without *hat*, as specified in eqs. (8.23), (8.24) and (8.25). Moreover, the covariant kinetic term of the Higgs doublet is modified in the presence of $\gamma_{\mathcal{L}_1}$, which leads to the Higgs-field renormalization in eq. (8.18). Therefore, one should also keep track of the Yukawa couplings $\hat{y}^{e,u,d}$ and the quartic coupling $\hat{\lambda}_\phi$ in order to check which fields can contribute to the Wilson coefficients.

We include reminders of all these implicit dependences, where appropriate, in section 8.5.

8.4 Operators generated by each field multiplet

In this section we provide the representation of each heavy multiplet introduced in chapter 7 in terms of operators of dimension $n \leq 6$ in the low energy effective Lagrangian. The results for the corresponding coefficients are given in section 8.5. See section 8.3 for details.

8.5 Complete contributions to Wilson coefficients

In this section we present the contributions to the dimension-six SMEFT induced by the heavy scalars, fermions and vectors introduced in chapter 7. See section 8.3 for details.

8.5.1 Redefinitions of Standard Model interactions

Upon integrating the heavy fields $\mathcal{L}_1$ and φ out, the kinetic term of the SM Higgs doublet receives extra contributions, yielding a non-canonically normalized field:

$$\mathcal{L}_{\text{kin},\phi} = Z_\phi D_\mu \phi^\dagger D^\mu \phi, \tag{8.17}$$

where

$$Z_\phi \equiv 1 - \frac{(\gamma_{\mathcal{L}_1})^*_r(\gamma_{\mathcal{L}_1})_r}{M^2_{\mathcal{L}_{1r}}} - \frac{\hat{\mu}^2_\phi(\delta_{\mathcal{L}_1\varphi})^*_{ts}(\gamma_{\mathcal{L}_1})_t(\delta_{\mathcal{L}_1\varphi})_{rs}(\gamma_{\mathcal{L}_1})^*_r}{M^2_{\mathcal{L}_{1r}}M^2_{\varphi_s}M^2_{\mathcal{L}_{1t}}}. \tag{8.18}$$

In what follows, we renormalize $\phi \to Z_\phi^{-1/2}\phi$ and present our results in a basis where all fields have canonical kinetic terms (in the electroweak exact phase). The operators with n_ϕ doublets are therefore renormalized with $Z_\phi^{-n_\phi/2}$. (This includes also the operators

Fields	Operators
$\mathcal{S}$	$\mathcal{O}_{\phi 4}$, $\mathcal{O}_{\phi}$, $\mathcal{O}_{\phi\Box}$, $\mathcal{O}_{\phi B}$, $\mathcal{O}_{\phi\tilde{B}}$, $\mathcal{O}_{\phi W}$, $\mathcal{O}_{\phi\tilde{W}}$, $\mathcal{O}_{\phi G}$, $\mathcal{O}_{\phi\tilde{G}}$, $\mathcal{O}_{e\phi}$, $\mathcal{O}_{d\phi}$, $\mathcal{O}_{u\phi}$
$\mathcal{S}_1$	$\mathcal{O}_{ll}$
$\mathcal{S}_2$	$\mathcal{O}_{ee}$
φ	$\mathcal{O}_{le}$, $\mathcal{O}_{qu}^{(1)}$, $\mathcal{O}_{qu}^{(8)}$, $\mathcal{O}_{qd}^{(1)}$, $\mathcal{O}_{qd}^{(8)}$, $\mathcal{O}_{ledq}$, $\mathcal{O}_{quqd}^{(1)}$, $\mathcal{O}_{lequ}^{(1)}$, $\mathcal{O}_{\phi}$, $\mathcal{O}_{e\phi}$, $\mathcal{O}_{d\phi}$, $\mathcal{O}_{u\phi}$
Ξ	$\mathcal{O}_{\phi 4}$, $\mathcal{O}_{\phi}$, $\mathcal{O}_{\phi D}$, $\mathcal{O}_{\phi\Box}$, $\mathcal{O}_{\phi WB}$, $\mathcal{O}_{\phi W\tilde{B}}$, $\mathcal{O}_{e\phi}$, $\mathcal{O}_{d\phi}$, $\mathcal{O}_{u\phi}$
Ξ_1	$\mathcal{O}_{\phi 4}$, $\mathcal{O}_{5}$, $\mathcal{O}_{ll}$, $\mathcal{O}_{\phi}$, $\mathcal{O}_{\phi D}$, $\mathcal{O}_{\phi\Box}$, $\mathcal{O}_{e\phi}$, $\mathcal{O}_{d\phi}$, $\mathcal{O}_{u\phi}$
Θ_1	$\mathcal{O}_{\phi}$
Θ_3	$\mathcal{O}_{\phi}$
ω_1	$\mathcal{O}_{qq}^{(1)}$, $\mathcal{O}_{qq}^{(3)}$, $\mathcal{O}_{lq}^{(1)}$, $\mathcal{O}_{lq}^{(3)}$, $\mathcal{O}_{eu}$, $\mathcal{O}_{ud}^{(1)}$, $\mathcal{O}_{ud}^{(8)}$, $\mathcal{O}_{quqd}^{(1)}$, $\mathcal{O}_{quqd}^{(8)}$, $\mathcal{O}_{lequ}^{(1)}$ $\mathcal{O}_{lequ}^{(3)}$, $\mathcal{O}_{duq}$, $\mathcal{O}_{qqu}$, $\mathcal{O}_{qqq}$, $\mathcal{O}_{duu}$
ω_2	$\mathcal{O}_{dd}$
ω_4	$\mathcal{O}_{uu}$, $\mathcal{O}_{ed}$, $\mathcal{O}_{duu}$
Π_1	$\mathcal{O}_{ld}$
Π_7	$\mathcal{O}_{lu}$, $\mathcal{O}_{qe}$, $\mathcal{O}_{lequ}^{(1)}$, $\mathcal{O}_{lequ}^{(3)}$
ζ	$\mathcal{O}_{qq}^{(1)}$, $\mathcal{O}_{qq}^{(3)}$, $\mathcal{O}_{lq}^{(1)}$, $\mathcal{O}_{lq}^{(3)}$, $\mathcal{O}_{qqq}$
Ω_1	$\mathcal{O}_{qq}^{(1)}$, $\mathcal{O}_{qq}^{(3)}$, $\mathcal{O}_{ud}^{(1)}$, $\mathcal{O}_{ud}^{(8)}$, $\mathcal{O}_{quqd}^{(1)}$, $\mathcal{O}_{quqd}^{(8)}$
Ω_2	$\mathcal{O}_{dd}$
Ω_4	$\mathcal{O}_{uu}$
Υ	$\mathcal{O}_{qq}^{(1)}$, $\mathcal{O}_{qq}^{(3)}$
Φ	$\mathcal{O}_{qu}^{(1)}$, $\mathcal{O}_{qu}^{(8)}$, $\mathcal{O}_{qd}^{(1)}$, $\mathcal{O}_{qd}^{(8)}$, $\mathcal{O}_{quqd}^{(8)}$

Table 8.1: Operators generated by the heavy scalar fields introduced in table 7.1.

in $\mathcal{L}_0$.) We will show these factors explicitly wherever they are needed, such that all the Wilson coefficients C_i in this section are defined as the coefficients multiplying the corresponding operators with canonical fields in the effective Lagrangian. Let us make two observations about Z_ϕ. First, the effect of the second term in eq. (8.18) on the Wilson coefficients of dimension-six operators will have have an extra suppression of the form $\hat{\mu}_\phi^2/M^2$, with M a heavy mass scale, comparable to the one of the typical Wilson coefficients of dimension-eight operators with respect to the dimension-six ones. Hence, even if we include it for completeness of the dimension-six results, for most practical purposes this second term can be neglected. The first term, on the other hand, will not be suppressed if the dimensionful coupling $\gamma_{\mathcal{L}_1}$ is of order $M_{\mathcal{L}_1}$. Second, Z_ϕ is non-trivial only when $\gamma_{\mathcal{L}_1} \neq 0$, so it can be ignored in perturbative unitary extensions of the SM.

The contributions to the renormalizable SM interactions in table 3.3 are given by

$$
\begin{aligned}
Z_\phi^{\frac{1}{2}} (C_{y^e})_{ij} = &- \frac{\hat{\mu}_\phi^2 (\delta_{\mathcal{L}_1\varphi})_{sr} (\gamma_{\mathcal{L}_1})_s^* (y_\varphi^e)_{rij}}{M_{\varphi_r}^2 M_{\mathcal{L}_{1s}}^2} - \frac{\hat{\mu}_\phi^2 \hat{y}_{ij}^e (\delta_{\mathcal{L}_1\varphi})_{ts}^* (\gamma_{\mathcal{L}_1})_t (\delta_{\mathcal{L}_1\varphi})_{rs} (\gamma_{\mathcal{L}_1})_r^*}{2M_{\mathcal{L}_{1r}}^2 M_{\varphi_s}^2 M_{\mathcal{L}_{1t}}^2} \\
&+ \frac{1}{f} \left\{ \frac{\hat{\mu}_\phi^2 (\tilde{g}_{\mathcal{L}_1}^{eDl})_{rij} (\gamma_{\mathcal{L}_1})_r^*}{2M_{\mathcal{L}_{1r}}^2} + \frac{\hat{\mu}_\phi^2 (\tilde{g}_{\mathcal{L}_1}^{Del})_{rij} (\gamma_{\mathcal{L}_1})_r^*}{2M_{\mathcal{L}_{1r}}^2} \right\},
\end{aligned}
\tag{8.19}
$$

Fields	Operators
N	$\mathcal{O}_5$, $\mathcal{O}_{\phi l}^{(1)}$, $\mathcal{O}_{\phi l}^{(3)}$
E	$\mathcal{O}_{e\phi}$, $\mathcal{O}_{eB}$, $\mathcal{O}_{\phi l}^{(1)}$, $\mathcal{O}_{\phi l}^{(3)}$
Δ_1	$\mathcal{O}_{e\phi}$, $\mathcal{O}_{eB}$, $\mathcal{O}_{eW}$, $\mathcal{O}_{\phi e}$
Δ_3	$\mathcal{O}_{e\phi}$, $\mathcal{O}_{\phi e}$
Σ	$\mathcal{O}_5$, $\mathcal{O}_{e\phi}$, $\mathcal{O}_{\phi l}^{(1)}$, $\mathcal{O}_{\phi l}^{(3)}$
Σ_1	$\mathcal{O}_{e\phi}$, $\mathcal{O}_{eW}$, $\mathcal{O}_{\phi l}^{(1)}$, $\mathcal{O}_{\phi l}^{(3)}$
U	$\mathcal{O}_{u\phi}$, $\mathcal{O}_{uB}$, $\mathcal{O}_{uG}$, $\mathcal{O}_{\phi q}^{(1)}$, $\mathcal{O}_{\phi q}^{(3)}$
D	$\mathcal{O}_{d\phi}$, $\mathcal{O}_{dB}$, $\mathcal{O}_{dG}$, $\mathcal{O}_{\phi q}^{(1)}$, $\mathcal{O}_{\phi q}^{(3)}$
Q_1	$\mathcal{O}_{d\phi}$, $\mathcal{O}_{u\phi}$, $\mathcal{O}_{dB}$, $\mathcal{O}_{dW}$, $\mathcal{O}_{dG}$, $\mathcal{O}_{uB}$, $\mathcal{O}_{uW}$, $\mathcal{O}_{uG}$, $\mathcal{O}_{\phi d}$, $\mathcal{O}_{\phi u}$, $\mathcal{O}_{\phi ud}$
Q_5	$\mathcal{O}_{d\phi}$, $\mathcal{O}_{\phi d}$
Q_7	$\mathcal{O}_{u\phi}$, $\mathcal{O}_{\phi u}$
T_1	$\mathcal{O}_{d\phi}$, $\mathcal{O}_{u\phi}$, $\mathcal{O}_{dW}$, $\mathcal{O}_{\phi q}^{(1)}$, $\mathcal{O}_{\phi q}^{(3)}$
T_2	$\mathcal{O}_{d\phi}$, $\mathcal{O}_{u\phi}$, $\mathcal{O}_{uW}$, $\mathcal{O}_{\phi q}^{(1)}$, $\mathcal{O}_{\phi q}^{(3)}$

Table 8.2: Operators generated by the heavy vector-like fermions in table 7.2.

Fields	Operators
$\mathcal{B}$	$\mathcal{O}_{ll}$, $\mathcal{O}_{qq}^{(1)}$, $\mathcal{O}_{lq}^{(1)}$, $\mathcal{O}_{ee}$, $\mathcal{O}_{dd}$, $\mathcal{O}_{uu}$, $\mathcal{O}_{ed}$, $\mathcal{O}_{eu}$, $\mathcal{O}_{ud}^{(1)}$, $\mathcal{O}_{le}$, $\mathcal{O}_{ld}$, $\mathcal{O}_{lu}$, $\mathcal{O}_{qe}$, $\mathcal{O}_{qu}^{(1)}$, $\mathcal{O}_{qd}^{(1)}$, $\mathcal{O}_{\phi D}$, $\mathcal{O}_{\phi\Box}$, $\mathcal{O}_{e\phi}$, $\mathcal{O}_{d\phi}$, $\mathcal{O}_{u\phi}$, $\mathcal{O}_{\phi l}^{(1)}$, $\mathcal{O}_{\phi q}^{(1)}$, $\mathcal{O}_{\phi e}$, $\mathcal{O}_{\phi d}$, $\mathcal{O}_{\phi u}$
$\mathcal{B}_1$	$\mathcal{O}_{\phi 4}$, $\mathcal{O}_{ud}^{(1)}$, $\mathcal{O}_{ud}^{(8)}$, $\mathcal{O}_{\phi}$, $\mathcal{O}_{\phi D}$, $\mathcal{O}_{\phi\Box}$, $\mathcal{O}_{e\phi}$, $\mathcal{O}_{d\phi}$, $\mathcal{O}_{u\phi}$, $\mathcal{O}_{\phi ud}$
$\mathcal{W}$	$\mathcal{O}_{\phi 4}$, $\mathcal{O}_{ll}$, $\mathcal{O}_{qq}^{(3)}$, $\mathcal{O}_{lq}^{(3)}$, $\mathcal{O}_{\phi}$, $\mathcal{O}_{\phi D}$, $\mathcal{O}_{\phi\Box}$, $\mathcal{O}_{e\phi}$, $\mathcal{O}_{d\phi}$, $\mathcal{O}_{u\phi}$, $\mathcal{O}_{\phi l}^{(3)}$, $\mathcal{O}_{\phi q}^{(3)}$
$\mathcal{W}_1$	$\mathcal{O}_{\phi 4}$, $\mathcal{O}_{\phi}$, $\mathcal{O}_{\phi D}$, $\mathcal{O}_{\phi\Box}$, $\mathcal{O}_{e\phi}$, $\mathcal{O}_{d\phi}$, $\mathcal{O}_{u\phi}$
$\mathcal{G}$	$\mathcal{O}_{qq}^{(1)}$, $\mathcal{O}_{qq}^{(3)}$, $\mathcal{O}_{dd}$, $\mathcal{O}_{uu}$, $\mathcal{O}_{ud}^{(8)}$, $\mathcal{O}_{qu}^{(8)}$, $\mathcal{O}_{qd}^{(8)}$
$\mathcal{G}_1$	$\mathcal{O}_{ud}^{(1)}$, $\mathcal{O}_{ud}^{(8)}$
$\mathcal{H}$	$\mathcal{O}_{qq}^{(1)}$, $\mathcal{O}_{qq}^{(3)}$
$\mathcal{L}_1$	$\mathcal{O}_{\phi 4}$, $\mathcal{O}_{y^e}$, $\mathcal{O}_{y^d}$, $\mathcal{O}_{y^u}$, $\mathcal{O}_{le}$, $\mathcal{O}_{qu}^{(1)}$, $\mathcal{O}_{qu}^{(8)}$, $\mathcal{O}_{qd}^{(1)}$, $\mathcal{O}_{qd}^{(8)}$, $\mathcal{O}_{ledq}$, $\mathcal{O}_{quqd}^{(1)}$, $\mathcal{O}_{lequ}^{(1)}$, $\mathcal{O}_{\phi}$, $\mathcal{O}_{\phi D}$, $\mathcal{O}_{\phi\Box}$, $\mathcal{O}_{\phi B}$, $\mathcal{O}_{\phi\tilde{B}}$, $\mathcal{O}_{\phi W}$, $\mathcal{O}_{\phi\tilde{W}}$, $\mathcal{O}_{\phi WB}$, $\mathcal{O}_{\phi W\tilde{B}}$, $\mathcal{O}_{e\phi}$, $\mathcal{O}_{d\phi}$, $\mathcal{O}_{u\phi}$, $\mathcal{O}_{eB}$, $\mathcal{O}_{eW}$, $\mathcal{O}_{dB}$, $\mathcal{O}_{dW}$, $\mathcal{O}_{uB}$, $\mathcal{O}_{uW}$, $\mathcal{O}_{\phi l}^{(1)}$, $\mathcal{O}_{\phi l}^{(3)}$, $\mathcal{O}_{\phi q}^{(1)}$, $\mathcal{O}_{\phi q}^{(3)}$, $\mathcal{O}_{\phi e}$, $\mathcal{O}_{\phi d}$, $\mathcal{O}_{\phi u}$
$\mathcal{L}_3$	$\mathcal{O}_{le}$
$\mathcal{U}_2$	$\mathcal{O}_{lq}^{(1)}$, $\mathcal{O}_{lq}^{(3)}$, $\mathcal{O}_{ed}$, $\mathcal{O}_{ledq}$
$\mathcal{U}_5$	$\mathcal{O}_{eu}$
$\mathcal{Q}_1$	$\mathcal{O}_{lu}$, $\mathcal{O}_{qd}^{(1)}$, $\mathcal{O}_{qd}^{(8)}$, $\mathcal{O}_{duq}$
$\mathcal{Q}_5$	$\mathcal{O}_{qe}$, $\mathcal{O}_{qu}^{(1)}$, $\mathcal{O}_{qu}^{(8)}$, $\mathcal{O}_{ledq}$, $\mathcal{O}_{duq}$, $\mathcal{O}_{qqu}$
$\mathcal{X}$	$\mathcal{O}_{lq}^{(1)}$, $\mathcal{O}_{lq}^{(3)}$
$\mathcal{Y}_1$	$\mathcal{O}_{qd}^{(1)}$, $\mathcal{O}_{qd}^{(8)}$
$\mathcal{Y}_5$	$\mathcal{O}_{qu}^{(1)}$, $\mathcal{O}_{qu}^{(8)}$

Table 8.3: Operators generated by the heavy vector bosons presented in table 7.3.

$$Z_\phi^{\frac{1}{2}}\left(C_{y^d}\right)_{ij} = -\frac{\hat{\mu}_\phi^2(\delta_{\mathcal{L}1\varphi})_{sr}(\gamma_{\mathcal{L}_1})_s^*(y_\varphi^d)_{rij}}{M_{\varphi_r}^2 M_{\mathcal{L}_{1s}}^2} - \frac{\hat{\mu}_\phi^2\hat{y}_{ij}^d(\delta_{\mathcal{L}1\varphi})_{ts}^*(\gamma_{\mathcal{L}_1})_t(\delta_{\mathcal{L}1\varphi})_{rs}(\gamma_{\mathcal{L}_1})_r^*}{2M_{\mathcal{L}_{1r}}^2 M_{\varphi_s}^2 M_{\mathcal{L}_{1t}}^2} + \frac{1}{f}\left\{\frac{\hat{\mu}_\phi^2(\tilde{g}_{\mathcal{L}_1}^{dDq})_{rij}(\gamma_{\mathcal{L}_1})_r^*}{2M_{\mathcal{L}_{1r}}^2} + \frac{\hat{\mu}_\phi^2(\tilde{g}_{\mathcal{L}_1}^{Ddq})_{rij}(\gamma_{\mathcal{L}_1})_r^*}{2M_{\mathcal{L}_{1r}}^2}\right\}, \tag{8.20}$$

$$Z_\phi^{\frac{1}{2}}\left(C_{y^u}\right)_{ij} = \frac{\hat{\mu}_\phi^2(\delta_{\mathcal{L}1\varphi})_{sr}^*(\gamma_{\mathcal{L}_1})_s(y_\varphi^u)_{rji}^*}{M_{\varphi_r}^2 M_{\mathcal{L}_{1s}}^2} - \frac{\hat{\mu}_\phi^2\hat{y}_{ij}^u(\delta_{\mathcal{L}1\varphi})_{ts}^*(\gamma_{\mathcal{L}_1})_t(\delta_{\mathcal{L}1\varphi})_{rs}(\gamma_{\mathcal{L}_1})_r^*}{2M_{\mathcal{L}_{1r}}^2 M_{\varphi_s}^2 M_{\mathcal{L}_{1t}}^2} + \frac{1}{f}\left\{-\frac{\hat{\mu}_\phi^2(\tilde{g}_{\mathcal{L}_1}^{qDu})_{rij}^*(\gamma_{\mathcal{L}_1})_r}{2M_{\mathcal{L}_{1r}}^2} - \frac{\hat{\mu}_\phi^2(\tilde{g}_{\mathcal{L}_1}^{Dqu})_{rij}^*(\gamma_{\mathcal{L}_1})_r}{2M_{\mathcal{L}_{1r}}^2}\right\}, \tag{8.21}$$

$$\begin{aligned}
Z_\phi^2 C_{\phi 4} =& \frac{(\kappa_{\mathcal{S}})_r(\kappa_{\mathcal{S}})_r}{2M_{\mathcal{S}_r}^2} + \frac{(\kappa_\Xi)_r(\kappa_\Xi)_r}{2M_{\Xi_r}^2} - \frac{2\hat{\mu}_\phi^2(\kappa_\Xi)_r(\kappa_\Xi)_r}{M_{\Xi_r}^4} + \frac{2(\kappa_{\Xi_1})_r^*(\kappa_{\Xi_1})_r}{M_{\Xi_{1r}}^2} \\
&- \frac{4\hat{\mu}_\phi^2(\kappa_{\Xi_1})_r^*(\kappa_{\Xi_1})_r}{M_{\Xi_{1r}}^4} + \frac{\hat{\mu}_\phi^2(\hat{g}_{\mathcal{B}_1}^\phi)_r^*(\hat{g}_{\mathcal{B}_1}^\phi)_r}{M_{\mathcal{B}_{1r}}^2} + \frac{\hat{\mu}_\phi^2(\hat{g}_{\mathcal{W}}^\phi)_r^*(\hat{g}_{\mathcal{W}}^\phi)_r}{2M_{\mathcal{W}_r}^2} + \frac{\hat{\mu}_\phi^2(\hat{g}_{\mathcal{W}_1}^\phi)_r^*(\hat{g}_{\mathcal{W}_1}^\phi)_r}{4M_{\mathcal{W}_{1r}}^2} \\
&- \frac{\hat{\mu}_\phi^2 g_2(g_{\mathcal{L}_1}^W)_{sr}(\gamma_{\mathcal{L}_1})_s^*(\gamma_{\mathcal{L}_1})_r}{M_{\mathcal{L}_{1r}}^2 M_{\mathcal{L}_{1s}}^2} + \frac{\hat{\mu}_\phi^2(h_{\mathcal{L}_1}^{(1)})_{rs}(\gamma_{\mathcal{L}_1})_r^*(\gamma_{\mathcal{L}_1})_s}{M_{\mathcal{L}_{1r}}^2 M_{\mathcal{L}_{1s}}^2} \\
&+ \frac{2\hat{\mu}_\phi^2\,\mathrm{Im}\left((\hat{g}_{\mathcal{W}}^\phi)_r\right)(\delta_{\mathcal{W}\Xi})_{rs}(\kappa_\Xi)_s}{M_{\mathcal{W}_r}^2 M_{\Xi_s}^2} + \frac{2\hat{\mu}_\phi^2(\delta_{\mathcal{W}\Xi})_{ts}(\delta_{\mathcal{W}\Xi})_{tr}(\kappa_\Xi)_r(\kappa_\Xi)_s}{M_{\Xi_r}^2 M_{\Xi_s}^2 M_{\mathcal{W}_t}^2} \\
&+ \frac{2\hat{\mu}_\phi^2\,\mathrm{Im}\left((\hat{g}_{\mathcal{W}_1}^\phi)_r^*(\delta_{\mathcal{W}_1\Xi_1})_{rs}(\kappa_{\Xi_1})_s\right)}{M_{\Xi_{1s}}^2 M_{\mathcal{W}_{1r}}^2} + \frac{4\hat{\mu}_\phi^2(\delta_{\mathcal{W}_1\Xi_1})_{st}^*(\delta_{\mathcal{W}_1\Xi_1})_{sr}(\kappa_{\Xi_1})_r(\kappa_{\Xi_1})_t^*}{M_{\Xi_{1r}}^2 M_{\mathcal{W}_{1s}}^2 M_{\Xi_{1t}}^2} \\
&- \frac{2\hat{\mu}_\phi^2\,\mathrm{Re}\left((g_{\mathcal{S}\mathcal{L}_1}^{(2)})_{rs}(\gamma_{\mathcal{L}_1})_s\right)(\kappa_{\mathcal{S}})_r}{M_{\mathcal{S}_r}^2 M_{\mathcal{L}_{1s}}^2} - \frac{\hat{\mu}_\phi^2(\varepsilon_{\mathcal{S}\mathcal{L}_1})_{rts}(\kappa_{\mathcal{S}})_r(\gamma_{\mathcal{L}_1})_t^*(\gamma_{\mathcal{L}_1})_s}{M_{\mathcal{S}_r}^2 M_{\mathcal{L}_{1s}}^2 M_{\mathcal{L}_{1t}}^2} \\
&- \frac{2\hat{\mu}_\phi^2\,\mathrm{Re}\left((\delta_{\mathcal{L}_1\varphi})_{rs}(\gamma_{\mathcal{L}_1})_r^*(\lambda_\varphi)_s\right)}{M_{\varphi_s}^2 M_{\mathcal{L}_{1r}}^2} - \frac{2\hat{\mu}_\phi^2\hat{\lambda}_\phi(\delta_{\mathcal{L}_1\varphi})_{ts}^*(\gamma_{\mathcal{L}_1})_t(\delta_{\mathcal{L}_1\varphi})_{rs}(\gamma_{\mathcal{L}_1})_r^*}{M_{\mathcal{L}_{1r}}^2 M_{\varphi_s}^2 M_{\mathcal{L}_{1t}}^2} \\
&+ \frac{2\hat{\mu}_\phi^2\,\mathrm{Re}\left((g_{\Xi\mathcal{L}_1}^{(2)})_{rs}(\gamma_{\mathcal{L}_1})_s\right)(\kappa_\Xi)_r}{M_{\Xi_r}^2 M_{\mathcal{L}_{1s}}^2} + \frac{\hat{\mu}_\phi^2(\varepsilon_{\Xi\mathcal{L}_1})_{srt}(\kappa_\Xi)_s(\gamma_{\mathcal{L}_1})_r^*(\gamma_{\mathcal{L}_1})_t}{M_{\mathcal{L}_{1r}}^2 M_{\Xi_s}^2 M_{\mathcal{L}_{1t}}^2} \\
&- \frac{4\hat{\mu}_\phi^2\,\mathrm{Re}\left((g_{\Xi\mathcal{L}_1}^{(1)})_{rs}^*(\gamma_{\mathcal{L}_1})_s^*\right)(\kappa_\Xi)_r}{M_{\Xi_r}^2 M_{\mathcal{L}_{1s}}^2} - \frac{4\hat{\mu}_\phi^2\,\mathrm{Re}\left((g_{\Xi_1\mathcal{L}_1}^{(1)})_{rs}^*(\gamma_{\mathcal{L}_1})_s^*(\kappa_{\Xi_1})_r\right)}{M_{\mathcal{L}_{1s}}^2 M_{\Xi_{1r}}^2} \\
&+ \frac{2\hat{\mu}_\phi^2\,\mathrm{Re}\left((\delta_{\mathcal{L}_1\varphi})_{rs}(\gamma_{\mathcal{L}_1})_r^*(\kappa_{\mathcal{S}\varphi})_{ts}\right)(\kappa_{\mathcal{S}})_t}{M_{\varphi_s}^2 M_{\mathcal{S}_t}^2 M_{\mathcal{L}_{1r}}^2} + \frac{2\hat{\mu}_\phi^2\,\mathrm{Re}\left((\delta_{\mathcal{L}_1\varphi})_{rs}(\gamma_{\mathcal{L}_1})_r^*(\kappa_{\Xi\varphi})_{ts}\right)(\kappa_\Xi)_t}{M_{\mathcal{L}_{1r}}^2 M_{\Xi_t}^2 M_{\varphi_s}^2} \\
&+ \frac{4\hat{\mu}_\phi^2\,\mathrm{Re}\left((\delta_{\mathcal{L}_1\varphi})_{rs}(\gamma_{\mathcal{L}_1})_r^*(\kappa_{\Xi_1\varphi})_{ts}^*(\kappa_{\Xi_1})_t\right)}{M_{\varphi_s}^2 M_{\mathcal{L}_{1r}}^2 M_{\Xi_{1t}}^2} \\
&+ \frac{1}{f}\Bigg\{-\frac{\hat{\mu}_\phi^2(\tilde{k}_{\mathcal{S}}^\phi)_r(\kappa_{\mathcal{S}})_r}{M_{\mathcal{S}_r}^2} + \frac{\hat{\mu}_\phi^2(\tilde{k}_\Xi^\phi)_r(\kappa_\Xi)_r}{M_{\Xi_r}^2} + \frac{2\hat{\mu}_\phi^2\,\mathrm{Re}\left((\tilde{\gamma}_{\mathcal{L}_1}^{(3)})_r(\gamma_{\mathcal{L}_1})_r^*\right)}{M_{\mathcal{L}_{1r}}^2}
\end{aligned}$$

$$- \frac{2\hat{\mu}_\phi^2 \, \mathrm{Im}\left((\tilde{\gamma}^W_{\mathcal{L}_1})_r (\gamma_{\mathcal{L}_1})_r^*\right) g_2}{M^2_{\mathcal{L}_{1r}}} \Bigg\}. \tag{8.22}$$

These contributions can be absorbed into redefinitions of the SM Yukawa and quartic Higgs couplings:

$$\hat{y}^{e,u,d}_{ij} = Z_\phi^{\frac{1}{2}} (y^{e,u,d}_{ij} - \left(C_{y^{e,u,d}}\right)_{ij}), \tag{8.23}$$

$$\hat{\lambda}_\phi = Z_\phi^2 (\lambda_\phi - C_{\phi 4}). \tag{8.24}$$

Due to the Higgs-field renormalization, the coefficient of the Higgs mass term is also redefined:

$$\hat{\mu}^2_\phi = Z_\phi \, \mu^2_\phi. \tag{8.25}$$

We remind the reader that the hatted couplings on the left-hand side of the last three equations are the coefficients of the corresponding operators—with the original Higgs-field normalization—in the SM part of $\mathcal{L}_{\mathrm{BSM}}$. The corresponding unhatted couplings are the coefficients of these operators—built with canonically-normalized fields—in $\mathcal{L}_{\mathrm{eff}}$. Note that the right-hand sides depend linearly on the explicit hatted couplings on the left-hand sides. Solving this linear system is straightforward.

In terms of the renormalized Higgs field and the redefined couplings μ^2_ϕ, $y^{e,u,d}$ and λ_ϕ, all the heavy-field contributions appear in the Wilson coefficients of higher-dimensional operators. In order to keep our results as compact and clear as possible, we write the dimension-six operators in terms of the original, hatted couplings. They can be readily substituted by the solutions to eqs. (8.23), (8.24) and (8.25) to get the expressions in terms of the redefined couplings. In practice, these expressions can be greatly simplified. Indeed, all the contributions to $C_{y^{e,u,d}}$, except the one inside Z_ϕ, and most of the contributions to $C_{\phi 4}$ are not $O(1)$ but carry an extra suppression μ^2_ϕ / M^2. For calculations to order E^2/M^2, with E a low-energy scale, all these contributions can be neglected. In this approximation, the hatted couplings do not appear on the right-hand sides of eqs. (8.23), (8.24) and (8.25), which thus give explicitly their expressions in terms of the redefined ones.

8.5.2 Dimension five

The only dimension-five operator in the basis receives the following contributions:

$$Z_\phi \, (C_5)_{ij} = -\frac{2(\kappa_{\Xi_1})_r (y_{\Xi_1})^*_{rji}}{M^2_{\Xi_{1r}}} + \frac{(\lambda_N)_{rj}(\lambda_N)x_{ri}}{2M_{N_r}} + \frac{(\lambda_\Sigma)_{rj}(\lambda_\Sigma)_{ri}}{8M_{\Sigma_r}}. \tag{8.26}$$

8.5.3 Four-fermion operators

$(\overline{L}L)\,(\overline{L}L)$

$$\begin{aligned}(C_{ll})_{ijkl} = & \frac{(y_{\mathcal{S}_1})^*_{rjl}(y_{\mathcal{S}_1})_{rik}}{M^2_{\mathcal{S}_{1r}}} + \frac{(y_{\Xi_1})_{rki}(y_{\Xi_1})^*_{rlj}}{M^2_{\Xi_{1r}}} - \frac{(g^l_{\mathcal{B}})_{rkl}(g^l_{\mathcal{B}})_{rij}}{2M^2_{\mathcal{B}_r}} \\ & - \frac{(g^l_{\mathcal{W}})_{rkj}(g^l_{\mathcal{W}})_{ril}}{4M^2_{\mathcal{W}_r}} + \frac{(g^l_{\mathcal{W}})_{rkl}(g^l_{\mathcal{W}})_{rij}}{8M^2_{\mathcal{W}_r}},\end{aligned} \tag{8.27}$$

$$\begin{aligned}\left(C_{qq}^{(1)}\right)_{ijkl} =& \frac{(y_{\omega_1}^{qq})_{rik}(y_{\omega_1}^{qq})_{rlj}^*}{2M_{\omega_{1r}}^2} + \frac{3(y_{\zeta}^{qq})_{rki}(y_{\zeta}^{qq})_{rlj}^*}{2M_{\zeta_r}^2} + \frac{(y_{\Omega_1}^{qq})_{rik}^*(y_{\Omega_1}^{qq})_{rjl}}{4M_{\Omega_{1r}}^2} + \frac{3(y_{\Upsilon})_{rlj}(y_{\Upsilon})_{rki}^*}{4M_{\Upsilon_r}^2} \\ & - \frac{(g_{\mathcal{B}}^{q})_{rkl}(g_{\mathcal{B}}^{q})_{rij}}{2M_{\mathcal{B}_r}^2} - \frac{(g_{\mathcal{G}}^{q})_{rkj}(g_{\mathcal{G}}^{q})_{ril}}{8M_{\mathcal{G}_r}^2} + \frac{(g_{\mathcal{G}}^{q})_{rkl}(g_{\mathcal{G}}^{q})_{rij}}{12M_{\mathcal{G}_r}^2} - \frac{3(g_{\mathcal{H}})_{rkj}(g_{\mathcal{H}})_{ril}}{32M_{\mathcal{H}_r}^2},\end{aligned} \tag{8.28}$$

$$\begin{aligned}\left(C_{qq}^{(3)}\right)_{ijkl} =& - \frac{(y_{\omega_1}^{qq})_{rki}(y_{\omega_1}^{qq})_{rjl}^*}{2M_{\omega_{1r}}^2} - \frac{(y_{\zeta}^{qq})_{rki}(y_{\zeta}^{qq})_{rjl}^*}{2M_{\zeta_r}^2} + \frac{(y_{\Omega_1}^{qq})_{rik}^*(y_{\Omega_1}^{qq})_{rlj}}{4M_{\Omega_{1r}}^2} + \frac{(y_{\Upsilon})_{rki}^*(y_{\Upsilon})_{rjl}}{4M_{\Upsilon_r}^2} \\ & - \frac{(g_{\mathcal{W}}^{q})_{rkl}(g_{\mathcal{W}}^{q})_{rij}}{8M_{\mathcal{W}_r}^2} - \frac{(g_{\mathcal{G}}^{q})_{rkj}(g_{\mathcal{G}}^{q})_{ril}}{8M_{\mathcal{G}_r}^2} + \frac{(g_{\mathcal{H}})_{rkl}(g_{\mathcal{H}})_{rij}}{48M_{\mathcal{H}_r}^2} + \frac{(g_{\mathcal{H}})_{rkj}(g_{\mathcal{H}})_{ril}}{32M_{\mathcal{H}_r}^2},\end{aligned} \tag{8.29}$$

$$\begin{aligned}\left(C_{lq}^{(1)}\right)_{ijkl} =& \frac{(y_{\omega_1}^{ql})_{rki}^*(y_{\omega_1}^{ql})_{rlj}}{4M_{\omega_{1r}}^2} + \frac{3(y_{\zeta}^{ql})_{rki}^*(y_{\zeta}^{ql})_{rlj}}{4M_{\zeta_r}^2} - \frac{(g_{\mathcal{B}}^{q})_{rkl}(g_{\mathcal{B}}^{l})_{rij}}{M_{\mathcal{B}_r}^2} \\ & - \frac{(g_{\mathcal{U}_2}^{lq})_{rjk}^*(g_{\mathcal{U}_2}^{lq})_{ril}}{2M_{\mathcal{U}_{2r}}^2} - \frac{3(g_{\mathcal{X}})_{rjk}^*(g_{\mathcal{X}})_{ril}}{8M_{\mathcal{X}_r}^2},\end{aligned} \tag{8.30}$$

$$\begin{aligned}\left(C_{lq}^{(3)}\right)_{ijkl} =& - \frac{(y_{\omega_1}^{ql})_{rki}^*(y_{\omega_1}^{ql})_{rlj}}{4M_{\omega_{1r}}^2} + \frac{(y_{\zeta}^{ql})_{rki}^*(y_{\zeta}^{ql})_{rlj}}{4M_{\zeta_r}^2} - \frac{(g_{\mathcal{W}}^{q})_{rkl}(g_{\mathcal{W}}^{l})_{rij}}{4M_{\mathcal{W}_r}^2} \\ & - \frac{(g_{\mathcal{U}_2}^{lq})_{rjk}^*(g_{\mathcal{U}_2}^{lq})_{ril}}{2M_{\mathcal{U}_{2r}}^2} + \frac{(g_{\mathcal{X}})_{rjk}^*(g_{\mathcal{X}})_{ril}}{8M_{\mathcal{X}_r}^2}.\end{aligned} \tag{8.31}$$

$(\overline{R}R)\,(\overline{R}R)$

$$(C_{ee})_{ijkl} = \frac{(y_{\mathcal{S}_2})_{rki}(y_{\mathcal{S}_2})^*_{rlj}}{2M^2_{\mathcal{S}_{2r}}} - \frac{(g^e_{\mathcal{B}})_{rkl}(g^e_{\mathcal{B}})_{rij}}{2M^2_{\mathcal{B}_r}}, \tag{8.32}$$

$$(C_{dd})_{ijkl} = \frac{(y_{\omega_2})^*_{rlj}(y_{\omega_2})_{rki}}{M^2_{\omega_{2r}}} + \frac{(y_{\Omega_2})^*_{rik}(y_{\Omega_2})_{rjl}}{2M^2_{\Omega_{2r}}} - \frac{(g^d_{\mathcal{B}})_{rkl}(g^d_{\mathcal{B}})_{rij}}{2M^2_{\mathcal{B}_r}} - \frac{(g^d_{\mathcal{G}})_{rkj}(g^d_{\mathcal{G}})_{ril}}{4M^2_{\mathcal{G}_r}} + \frac{(g^d_{\mathcal{G}})_{rkl}(g^d_{\mathcal{G}})_{rij}}{12M^2_{\mathcal{G}_r}}, \tag{8.33}$$

$$(C_{uu})_{ijkl} = \frac{(y^{uu}_{\omega_4})^*_{rlj}(y^{uu}_{\omega_4})_{rki}}{M^2_{\omega_{4r}}} + \frac{(y_{\Omega_4})^*_{rik}(y_{\Omega_4})_{rjl}}{2M^2_{\Omega_{4r}}} - \frac{(g^u_{\mathcal{B}})_{rkl}(g^u_{\mathcal{B}})_{rij}}{2M^2_{\mathcal{B}_r}} - \frac{(g^u_{\mathcal{G}})_{rkj}(g^u_{\mathcal{G}})_{ril}}{4M^2_{\mathcal{G}_r}} + \frac{(g^u_{\mathcal{G}})_{rkl}(g^u_{\mathcal{G}})_{rij}}{12M^2_{\mathcal{G}_r}}, \tag{8.34}$$

$$(C_{ed})_{ijkl} = \frac{(y^{ed}_{\omega_4})^*_{rik}(y^{ed}_{\omega_4})_{rjl}}{2M^2_{\omega_{4r}}} - \frac{(g^d_{\mathcal{B}})_{rkl}(g^e_{\mathcal{B}})_{rij}}{M^2_{\mathcal{B}_r}} - \frac{(g^{ed}_{\mathcal{U}_2})^*_{rjk}(g^{ed}_{\mathcal{U}_2})_{ril}}{M^2_{\mathcal{U}_{2r}}}, \tag{8.35}$$

$$(C_{eu})_{ijkl} = \frac{(y^{eu}_{\omega_1})^*_{rik}(y^{eu}_{\omega_1})_{rjl}}{2M^2_{\omega_{1r}}} - \frac{(g^u_{\mathcal{B}})_{rkl}(g^e_{\mathcal{B}})_{rij}}{M^2_{\mathcal{B}_r}} - \frac{(g_{\mathcal{U}_5})^*_{rjk}(g_{\mathcal{U}_5})_{ril}}{M^2_{\mathcal{U}_{5r}}}, \tag{8.36}$$

$$\left(C^{(1)}_{ud}\right)_{ijkl} = \frac{(y^{du}_{\omega_1})^*_{rlj}(y^{du}_{\omega_1})_{rki}}{3M^2_{\omega_{1r}}} + \frac{(y^{ud}_{\Omega_1})^*_{rik}(y^{ud}_{\Omega_1})_{rjl}}{3M^2_{\Omega_{1r}}} - \frac{(g^u_{\mathcal{B}})_{rij}(g^d_{\mathcal{B}})_{rkl}}{M^2_{\mathcal{B}_r}} - \frac{(g^{du}_{\mathcal{B}_1})^*_{rli}(g^{du}_{\mathcal{B}_1})_{rkj}}{3M^2_{\mathcal{B}_{1r}}} - \frac{4(g_{\mathcal{G}_1})^*_{rli}(g_{\mathcal{G}_1})_{rkj}}{9M^2_{\mathcal{G}_{1r}}}, \tag{8.37}$$

$$\left(C^{(8)}_{ud}\right)_{ijkl} = -\frac{(y^{du}_{\omega_1})^*_{rlj}(y^{du}_{\omega_1})_{rki}}{M^2_{\omega_{1r}}} + \frac{(y^{ud}_{\Omega_1})^*_{rik}(y^{ud}_{\Omega_1})_{rjl}}{2M^2_{\Omega_{1r}}} - \frac{(g^d_{\mathcal{G}})_{rkl}(g^u_{\mathcal{G}})_{rij}}{M^2_{\mathcal{G}_r}} - \frac{2(g^{du}_{\mathcal{B}_1})^*_{rli}(g^{du}_{\mathcal{B}_1})_{rkj}}{M^2_{\mathcal{B}_{1r}}} + \frac{(g_{\mathcal{G}_1})^*_{rli}(g_{\mathcal{G}_1})_{rkj}}{3M^2_{\mathcal{G}_{1r}}}. \tag{8.38}$$

$(\overline{L}L)\,(\overline{R}R)$

Recall that $\hat{y}^{e,u,d}$ are defined in eq. (8.23).

$$
\begin{aligned}
(C_{le})_{ijkl} =& -\frac{(y^e_\varphi)^*_{rli}(y^e_\varphi)_{rkj}}{2M^2_{\varphi_r}} - \frac{(g^e_{\mathcal{B}})_{rkl}(g^l_{\mathcal{B}})_{rij}}{M^2_{\mathcal{B}_r}} + \frac{(g_{\mathcal{L}_3})^*_{rki}(g_{\mathcal{L}_3})_{rlj}}{M^2_{\mathcal{L}_{3r}}} \\
& - \frac{\hat{y}^{e*}_{li}(\delta_{\mathcal{L}_1\varphi})_{sr}(\gamma_{\mathcal{L}_1})^*_s(y^e_\varphi)_{rkj}}{2M^2_{\varphi_r}M^2_{\mathcal{L}_{1s}}} - \frac{\hat{y}^e_{kj}(\delta_{\mathcal{L}_1\varphi})^*_{sr}(\gamma_{\mathcal{L}_1})_s(y^e_\varphi)^*_{rli}}{2M^2_{\varphi_r}M^2_{\mathcal{L}_{1s}}} \\
& - \frac{\hat{y}^e_{kj}\hat{y}^{e*}_{li}(\delta_{\mathcal{L}_1\varphi})^*_{ts}(\gamma_{\mathcal{L}_1})_t(\delta_{\mathcal{L}_1\varphi})_{rs}(\gamma_{\mathcal{L}_1})^*_r}{2M^2_{\mathcal{L}_{1r}}M^2_{\varphi_s}M^2_{\mathcal{L}_{1t}}} \\
& + \frac{1}{f}\left\{\frac{\hat{y}^{e*}_{li}(\tilde{g}^{eDl}_{\mathcal{L}_1})_{rkj}(\gamma_{\mathcal{L}_1})^*_r}{4M^2_{\mathcal{L}_{1r}}} + \frac{\hat{y}^{e*}_{li}(\tilde{g}^{Del}_{\mathcal{L}_1})_{rkj}(\gamma_{\mathcal{L}_1})^*_r}{4M^2_{\mathcal{L}_{1r}}}\right. \\
& \left. + \frac{\hat{y}^e_{kj}(\tilde{g}^{eDl}_{\mathcal{L}_1})^*_{rli}(\gamma_{\mathcal{L}_1})_r}{4M^2_{\mathcal{L}_{1r}}} + \frac{\hat{y}^e_{kj}(\tilde{g}^{Del}_{\mathcal{L}_1})^*_{rli}(\gamma_{\mathcal{L}_1})_r}{4M^2_{\mathcal{L}_{1r}}}\right\},
\end{aligned}
\tag{8.39}
$$

$$
(C_{ld})_{ijkl} = -\frac{(y_{\Pi_1})^*_{rjk}(y_{\Pi_1})_{ril}}{2M^2_{\Pi_{1r}}} - \frac{(g^d_{\mathcal{B}})_{rkl}(g^l_{\mathcal{B}})_{rij}}{M^2_{\mathcal{B}_r}} + \frac{(g^{dl}_{\mathcal{Q}_5})^*_{rki}(g^{dl}_{\mathcal{Q}_5})_{rlj}}{M^2_{\mathcal{Q}_{5r}}},
\tag{8.40}
$$

$$
(C_{lu})_{ijkl} = -\frac{(y^{lu}_{\Pi_7})^*_{rjk}(y^{lu}_{\Pi_7})_{ril}}{2M^2_{\Pi_{7r}}} - \frac{(g^u_{\mathcal{B}})_{rkl}(g^l_{\mathcal{B}})_{rij}}{M^2_{\mathcal{B}_r}} + \frac{(g^{ul}_{\mathcal{Q}_1})^*_{rki}(g^{ul}_{\mathcal{Q}_1})_{rlj}}{M^2_{\mathcal{Q}_{1r}}},
\tag{8.41}
$$

$$
(C_{qe})_{ijkl} = -\frac{(y^{eq}_{\Pi_7})^*_{rli}(y^{eq}_{\Pi_7})_{rkj}}{2M^2_{\Pi_{7r}}} - \frac{(g^e_{\mathcal{B}})_{rkl}(g^q_{\mathcal{B}})_{rij}}{M^2_{\mathcal{B}_r}} + \frac{(g^{eq}_{\mathcal{Q}_5})^*_{rki}(g^{eq}_{\mathcal{Q}_5})_{rlj}}{M^2_{\mathcal{Q}_{5r}}},
\tag{8.42}
$$

$$
\begin{aligned}
\left(C^{(1)}_{qu}\right)_{ijkl} =& -\frac{(y^u_\varphi)^*_{rjk}(y^u_\varphi)_{ril}}{6M^2_{\varphi_r}} - \frac{2(y^{qu}_\Phi)^*_{rjk}(y^{qu}_\Phi)_{ril}}{9M^2_{\Phi_r}} \\
& - \frac{(g^u_{\mathcal{B}})_{rkl}(g^q_{\mathcal{B}})_{rij}}{M^2_{\mathcal{B}_r}} + \frac{2(g^{uq}_{\mathcal{Q}_5})^*_{rlj}(g^{uq}_{\mathcal{Q}_5})_{rki}}{3M^2_{\mathcal{Q}_{5r}}} + \frac{2(g_{\mathcal{Y}_5})^*_{rlj}(g_{\mathcal{Y}_5})_{rki}}{3M^2_{\mathcal{Y}_{5r}}} \\
& + \frac{\hat{y}^u_{kj}(\delta_{\mathcal{L}_1\varphi})_{rs}(\gamma_{\mathcal{L}_1})^*_r(y^u_\varphi)_{sil}}{6M^2_{\mathcal{L}_{1r}}M^2_{\varphi_s}} + \frac{\hat{y}^{u*}_{li}(\delta_{\mathcal{L}_1\varphi})^*_{rs}(\gamma_{\mathcal{L}_1})_r(y^u_\varphi)^*_{sjk}}{6M^2_{\mathcal{L}_{1r}}M^2_{\varphi_s}} \\
& - \frac{\hat{y}^u_{kj}\hat{y}^{u*}_{li}(\delta_{\mathcal{L}_1\varphi})^*_{ts}(\gamma_{\mathcal{L}_1})_t(\delta_{\mathcal{L}_1\varphi})_{rs}(\gamma_{\mathcal{L}_1})^*_r}{6M^2_{\mathcal{L}_{1r}}M^2_{\varphi_s}M^2_{\mathcal{L}_{1t}}} \\
& + \frac{1}{f}\left\{ - \frac{\hat{y}^u_{kj}(\tilde{g}^{qDu}_{\mathcal{L}_1})_{rli}(\gamma_{\mathcal{L}_1})^*_r}{12M^2_{\mathcal{L}_{1r}}} - \frac{\hat{y}^u_{kj}(\tilde{g}^{Dqu}_{\mathcal{L}_1})_{rli}(\gamma_{\mathcal{L}_1})^*_r}{12M^2_{\mathcal{L}_{1r}}}\right. \\
& \left. - \frac{\hat{y}^{u*}_{li}(\tilde{g}^{qDu}_{\mathcal{L}_1})^*_{rkj}(\gamma_{\mathcal{L}_1})_r}{12M^2_{\mathcal{L}_{1r}}} - \frac{\hat{y}^{u*}_{li}(\tilde{g}^{Dqu}_{\mathcal{L}_1})^*_{rkj}(\gamma_{\mathcal{L}_1})_r}{12M^2_{\mathcal{L}_{1r}}}\right\},
\end{aligned}
\tag{8.43}
$$

$$
\left(C^{(8)}_{qu}\right)_{ijkl} = -\frac{(y^u_\varphi)^*_{rjk}(y^u_\varphi)_{ril}}{M^2_{\varphi_r}} + \frac{(y^{qu}_\Phi)^*_{rjk}(y^{qu}_\Phi)_{ril}}{6M^2_{\Phi_r}}
$$

$$
\begin{aligned}
&- \frac{(g^u_{\mathcal{G}})_{rkl}(g^q_{\mathcal{G}})_{rij}}{M^2_{\mathcal{G}_r}} - \frac{2(g^{uq}_{\mathcal{Q}_5})^*_{rlj}(g^{uq}_{\mathcal{Q}_5})_{rki}}{M^2_{\mathcal{Q}_{5r}}} + \frac{(g_{\mathcal{Y}_5})^*_{rlj}(g_{\mathcal{Y}_5})_{rki}}{M^2_{\mathcal{Y}_{5r}}} \\
&+ \frac{\hat{y}^u_{kj}(\delta_{\mathcal{L}_1\varphi})_{rs}(\gamma_{\mathcal{L}_1})^*_r(y^u_\varphi)_{sil}}{M^2_{\mathcal{L}_{1r}}M^2_{\varphi_s}} + \frac{\hat{y}^{u*}_{li}(\delta_{\mathcal{L}_1\varphi})^*_{rs}(\gamma_{\mathcal{L}_1})_r(y^u_\varphi)^*_{sjk}}{M^2_{\mathcal{L}_{1r}}M^2_{\varphi_s}} \\
&- \frac{\hat{y}^u_{kj}\hat{y}^{u*}_{li}(\delta_{\mathcal{L}_1\varphi})^*_{ts}(\gamma_{\mathcal{L}_1})_t(\delta_{\mathcal{L}_1\varphi})_{rs}(\gamma_{\mathcal{L}_1})^*_r}{M^2_{\mathcal{L}_{1r}}M^2_{\varphi_s}M^2_{\mathcal{L}_{1t}}} \\
&+ \frac{1}{f}\Bigg\{ - \frac{\hat{y}^u_{kj}(\tilde{g}^{qDu}_{\mathcal{L}_1})_{rli}(\gamma_{\mathcal{L}_1})^*_r}{2M^2_{\mathcal{L}_{1r}}} - \frac{\hat{y}^u_{kj}(\tilde{g}^{Dqu}_{\mathcal{L}_1})_{rli}(\gamma_{\mathcal{L}_1})^*_r}{2M^2_{\mathcal{L}_{1r}}} \\
&\qquad - \frac{\hat{y}^{u*}_{li}(\tilde{g}^{qDu}_{\mathcal{L}_1})^*_{rkj}(\gamma_{\mathcal{L}_1})_r}{2M^2_{\mathcal{L}_{1r}}} - \frac{\hat{y}^{u*}_{li}(\tilde{g}^{Dqu}_{\mathcal{L}_1})^*_{rkj}(\gamma_{\mathcal{L}_1})_r}{2M^2_{\mathcal{L}_{1r}}}\Bigg\},
\end{aligned}
\tag{8.44}
$$

$$
\begin{aligned}
\left(C^{(1)}_{qd}\right)_{ijkl} =& - \frac{(y^d_\varphi)^*_{rli}(y^d_\varphi)_{rkj}}{6M^2_{\varphi_r}} - \frac{2(y^{dq}_\Phi)^*_{rli}(y^{dq}_\Phi)_{rkj}}{9M^2_{\Phi_r}} \\
&- \frac{(g^d_{\mathcal{B}})_{rkl}(g^q_{\mathcal{B}})_{rij}}{M^2_{\mathcal{B}_r}} + \frac{2(g^{dq}_{\mathcal{Q}_1})^*_{rlj}(g^{dq}_{\mathcal{Q}_1})_{rki}}{3M^2_{\mathcal{Q}_{1r}}} + \frac{2(g_{\mathcal{Y}_1})^*_{rlj}(g_{\mathcal{Y}_1})_{rki}}{3M^2_{\mathcal{Y}_{1r}}} \\
&- \frac{\hat{y}^{d*}_{li}(\delta_{\mathcal{L}_1\varphi})_{sr}(\gamma_{\mathcal{L}_1})^*_s(y^d_\varphi)_{rkj}}{6M^2_{\varphi_r}M^2_{\mathcal{L}_{1s}}} - \frac{\hat{y}^d_{kj}(\delta_{\mathcal{L}_1\varphi})^*_{sr}(\gamma_{\mathcal{L}_1})_s(y^d_\varphi)^*_{rli}}{6M^2_{\varphi_r}M^2_{\mathcal{L}_{1s}}} \\
&- \frac{\hat{y}^d_{kj}\hat{y}^{d*}_{li}(\delta_{\mathcal{L}_1\varphi})^*_{ts}(\gamma_{\mathcal{L}_1})_t(\delta_{\mathcal{L}_1\varphi})_{rs}(\gamma_{\mathcal{L}_1})^*_r}{6M^2_{\mathcal{L}_{1r}}M^2_{\varphi_s}M^2_{\mathcal{L}_{1t}}} \\
&+ \frac{1}{f}\Bigg\{ \frac{\hat{y}^{d*}_{li}(\tilde{g}^{dDq}_{\mathcal{L}_1})_{rkj}(\gamma_{\mathcal{L}_1})^*_r}{12M^2_{\mathcal{L}_{1r}}} + \frac{\hat{y}^{d*}_{li}(\tilde{g}^{Ddq}_{\mathcal{L}_1})_{rkj}(\gamma_{\mathcal{L}_1})^*_r}{12M^2_{\mathcal{L}_{1r}}} \\
&\qquad + \frac{\hat{y}^d_{kj}(\tilde{g}^{dDq}_{\mathcal{L}_1})^*_{rli}(\gamma_{\mathcal{L}_1})_r}{12M^2_{\mathcal{L}_{1r}}} + \frac{\hat{y}^d_{kj}(\tilde{g}^{Ddq}_{\mathcal{L}_1})^*_{rli}(\gamma_{\mathcal{L}_1})_r}{12M^2_{\mathcal{L}_{1r}}}\Bigg\},
\end{aligned}
\tag{8.45}
$$

$$
\begin{aligned}
\left(C^{(8)}_{qd}\right)_{ijkl} =& - \frac{(y^d_\varphi)^*_{rli}(y^d_\varphi)_{rkj}}{M^2_{\varphi_r}} + \frac{(y^{dq}_\Phi)^*_{rli}(y^{dq}_\Phi)_{rkj}}{6M^2_{\Phi_r}} \\
&- \frac{(g^d_{\mathcal{G}})_{rkl}(g^q_{\mathcal{G}})_{rij}}{M^2_{\mathcal{G}_r}} - \frac{2(g^{dq}_{\mathcal{Q}_1})^*_{rlj}(g^{dq}_{\mathcal{Q}_1})_{rki}}{M^2_{\mathcal{Q}_{1r}}} + \frac{(g_{\mathcal{Y}_1})^*_{rlj}(g_{\mathcal{Y}_1})_{rki}}{M^2_{\mathcal{Y}_{1r}}} \\
&- \frac{\hat{y}^{d*}_{li}(\delta_{\mathcal{L}_1\varphi})_{sr}(\gamma_{\mathcal{L}_1})^*_s(y^d_\varphi)_{rkj}}{M^2_{\varphi_r}M^2_{\mathcal{L}_{1s}}} - \frac{\hat{y}^d_{kj}(\delta_{\mathcal{L}_1\varphi})^*_{sr}(\gamma_{\mathcal{L}_1})_s(y^d_\varphi)^*_{rli}}{M^2_{\varphi_r}M^2_{\mathcal{L}_{1s}}} \\
&- \frac{\hat{y}^d_{kj}\hat{y}^{d*}_{li}(\delta_{\mathcal{L}_1\varphi})^*_{ts}(\gamma_{\mathcal{L}_1})_t(\delta_{\mathcal{L}_1\varphi})_{rs}(\gamma_{\mathcal{L}_1})^*_r}{M^2_{\mathcal{L}_{1r}}M^2_{\varphi_s}M^2_{\mathcal{L}_{1t}}} \\
&+ \frac{1}{f}\Bigg\{ \frac{\hat{y}^{d*}_{li}(\tilde{g}^{dDq}_{\mathcal{L}_1})_{rkj}(\gamma_{\mathcal{L}_1})^*_r}{2M^2_{\mathcal{L}_{1r}}} + \frac{\hat{y}^{d*}_{li}(\tilde{g}^{Ddq}_{\mathcal{L}_1})_{rkj}(\gamma_{\mathcal{L}_1})^*_r}{2M^2_{\mathcal{L}_{1r}}} \\
&\qquad + \frac{\hat{y}^d_{kj}(\tilde{g}^{dDq}_{\mathcal{L}_1})^*_{rli}(\gamma_{\mathcal{L}_1})_r}{2M^2_{\mathcal{L}_{1r}}} + \frac{\hat{y}^d_{kj}(\tilde{g}^{Ddq}_{\mathcal{L}_1})^*_{rli}(\gamma_{\mathcal{L}_1})_r}{2M^2_{\mathcal{L}_{1r}}}\Bigg\}.
\end{aligned}
\tag{8.46}
$$

$(\overline{L}R)\,(\overline{R}L)$ and $(\overline{L}R)\,(\overline{L}R)$

Recall that $\hat{y}^{e,u,d}$ are defined in eq. (8.23).

$$
\begin{aligned}
(C_{ledq})_{ijkl} =& \frac{(y^d_\varphi)_{rkl}(y^e_\varphi)^*_{rji}}{M^2_{\varphi_r}} + \frac{2(g^{lq}_{\mathcal{U}_2})_{ril}(g^{ed}_{\mathcal{U}_2})^*_{rjk}}{M^2_{\mathcal{U}_{2r}}} - \frac{2(g^{eq}_{\mathcal{Q}_5})_{rjl}(g^{dl}_{\mathcal{Q}_5})^*_{rki}}{M^2_{\mathcal{Q}_{5r}}} \\
&+ \frac{\hat{y}^{e*}_{ji}(\delta_{\mathcal{L}_1\varphi})_{sr}(\gamma_{\mathcal{L}_1})^*_s(y^d_\varphi)_{rkl}}{M^2_{\varphi_r}M^2_{\mathcal{L}_{1s}}} + \frac{\hat{y}^d_{kl}(\delta_{\mathcal{L}_1\varphi})^*_{sr}(\gamma_{\mathcal{L}_1})_s(y^e_\varphi)^*_{rji}}{M^2_{\varphi_r}M^2_{\mathcal{L}_{1s}}} \\
&+ \frac{\hat{y}^d_{kl}\hat{y}^{e*}_{ji}(\delta_{\mathcal{L}_1\varphi})^*_{ts}(\gamma_{\mathcal{L}_1})_t(\delta_{\mathcal{L}_1\varphi})_{rs}(\gamma_{\mathcal{L}_1})^*_r}{M^2_{\mathcal{L}_{1r}}M^2_{\varphi_s}M^2_{\mathcal{L}_{1t}}} \\
&+ \frac{1}{f}\Bigg\{ - \frac{\hat{y}^{e*}_{ji}(\tilde{g}^{dDq}_{\mathcal{L}_1})_{rkl}(\gamma_{\mathcal{L}_1})^*_r}{2M^2_{\mathcal{L}_{1r}}} - \frac{\hat{y}^{e*}_{ji}(\tilde{g}^{Ddq}_{\mathcal{L}_1})_{rkl}(\gamma_{\mathcal{L}_1})^*_r}{2M^2_{\mathcal{L}_{1r}}} \\
&\qquad - \frac{\hat{y}^d_{kl}(\tilde{g}^{eDl}_{\mathcal{L}_1})^*_{rji}(\gamma_{\mathcal{L}_1})_r}{2M^2_{\mathcal{L}_{1r}}} - \frac{\hat{y}^d_{kl}(\tilde{g}^{Del}_{\mathcal{L}_1})^*_{rji}(\gamma_{\mathcal{L}_1})_r}{2M^2_{\mathcal{L}_{1r}}}\Bigg\},
\end{aligned} \tag{8.47}
$$

$$
\begin{aligned}
\left(C^{(1)}_{quqd}\right)_{ijkl} =& - \frac{(y^u_\varphi)_{rij}(y^d_\varphi)^*_{rlk}}{M^2_{\varphi_r}} + \frac{4(y^{qq}_{\omega_1})_{rki}(y^{du}_{\omega_1})^*_{rlj}}{3M^2_{\omega_{1r}}} + \frac{4(y^{qq}_{\Omega_1})^*_{rki}(y^{ud}_{\Omega_1})_{rjl}}{3M^2_{\Omega_{1r}}} \\
&- \frac{\hat{y}^{d*}_{lk}(\delta_{\mathcal{L}_1\varphi})_{sr}(\gamma_{\mathcal{L}_1})^*_s(y^u_\varphi)_{rij}}{M^2_{\varphi_r}M^2_{\mathcal{L}_{1s}}} + \frac{\hat{y}^{u*}_{ji}(\delta_{\mathcal{L}_1\varphi})^*_{sr}(\gamma_{\mathcal{L}_1})_s(y^d_\varphi)^*_{rlk}}{M^2_{\varphi_r}M^2_{\mathcal{L}_{1s}}} \\
&+ \frac{\hat{y}^{u*}_{ji}\hat{y}^{d*}_{lk}(\delta_{\mathcal{L}_1\varphi})^*_{ts}(\gamma_{\mathcal{L}_1})_t(\delta_{\mathcal{L}_1\varphi})_{rs}(\gamma_{\mathcal{L}_1})^*_r}{M^2_{\mathcal{L}_{1r}}M^2_{\varphi_s}M^2_{\mathcal{L}_{1t}}} \\
&+ \frac{1}{f}\Bigg\{ \frac{\hat{y}^{d*}_{lk}(\tilde{g}^{qDu}_{\mathcal{L}_1})_{rji}(\gamma_{\mathcal{L}_1})^*_r}{2M^2_{\mathcal{L}_{1r}}} + \frac{\hat{y}^{d*}_{lk}(\tilde{g}^{Dqu}_{\mathcal{L}_1})_{rji}(\gamma_{\mathcal{L}_1})^*_r}{2M^2_{\mathcal{L}_{1r}}} \\
&\qquad - \frac{\hat{y}^{u*}_{ji}(\tilde{g}^{dDq}_{\mathcal{L}_1})^*_{rlk}(\gamma_{\mathcal{L}_1})_r}{2M^2_{\mathcal{L}_{1r}}} - \frac{\hat{y}^{u*}_{ji}(\tilde{g}^{Ddq}_{\mathcal{L}_1})^*_{rlk}(\gamma_{\mathcal{L}_1})_r}{2M^2_{\mathcal{L}_{1r}}}\Bigg\},
\end{aligned} \tag{8.48}
$$

$$
\left(C^{(8)}_{quqd}\right)_{ijkl} = - \frac{4(y^{qq}_{\omega_1})_{rki}(y^{du}_{\omega_1})^*_{rlj}}{M^2_{\omega_{1r}}} + \frac{2(y^{qq}_{\Omega_1})^*_{rki}(y^{ud}_{\Omega_1})_{rjl}}{M^2_{\Omega_{1r}}} - \frac{(y^{dq}_\Phi)^*_{rlk}(y^{qu}_\Phi)_{rij}}{M^2_{\Phi_r}}, \tag{8.49}
$$

$$
\begin{aligned}
\left(C^{(1)}_{lequ}\right)_{ijkl} =& \frac{(y^u_\varphi)_{rkl}(y^e_\varphi)^*_{rji}}{M^2_{\varphi_r}} + \frac{(y^{eu}_{\omega_1})_{rjl}(y^{ql}_{\omega_1})^*_{rki}}{2M^2_{\omega_{1r}}} + \frac{(y^{eq}_{\Pi_7})^*_{rjk}(y^{lu}_{\Pi_7})_{ril}}{2M^2_{\Pi_{7r}}} \\
&+ \frac{\hat{y}^{e*}_{ji}(\delta_{\mathcal{L}_1\varphi})_{sr}(\gamma_{\mathcal{L}_1})^*_s(y^u_\varphi)_{rkl}}{M^2_{\varphi_r}M^2_{\mathcal{L}_{1s}}} - \frac{\hat{y}^{u*}_{lk}(\delta_{\mathcal{L}_1\varphi})^*_{sr}(\gamma_{\mathcal{L}_1})_s(y^e_\varphi)^*_{rji}}{M^2_{\varphi_r}M^2_{\mathcal{L}_{1s}}} \\
&- \frac{\hat{y}^{u*}_{lk}\hat{y}^{e*}_{ji}(\delta_{\mathcal{L}_1\varphi})^*_{ts}(\gamma_{\mathcal{L}_1})_t(\delta_{\mathcal{L}_1\varphi})_{rs}(\gamma_{\mathcal{L}_1})^*_r}{M^2_{\mathcal{L}_{1r}}M^2_{\varphi_s}M^2_{\mathcal{L}_{1t}}} \\
&+ \frac{1}{f}\Bigg\{ - \frac{\hat{y}^{e*}_{ji}(\tilde{g}^{qDu}_{\mathcal{L}_1})_{rlk}(\gamma_{\mathcal{L}_1})^*_r}{2M^2_{\mathcal{L}_{1r}}} - \frac{\hat{y}^{e*}_{ji}(\tilde{g}^{Dqu}_{\mathcal{L}_1})_{rlk}(\gamma_{\mathcal{L}_1})^*_r}{2M^2_{\mathcal{L}_{1r}}} \\
&\qquad + \frac{\hat{y}^{u*}_{lk}(\tilde{g}^{eDl}_{\mathcal{L}_1})^*_{rji}(\gamma_{\mathcal{L}_1})_r}{2M^2_{\mathcal{L}_{1r}}} + \frac{\hat{y}^{u*}_{lk}(\tilde{g}^{Del}_{\mathcal{L}_1})^*_{rji}(\gamma_{\mathcal{L}_1})_r}{2M^2_{\mathcal{L}_{1r}}}\Bigg\},
\end{aligned} \tag{8.50}
$$

$$\left(C_{lequ}^{(3)}\right)_{ijkl} = -\frac{(y_{\omega_1}^{eu})_{rjl}(y_{\omega_1}^{ql})^*_{rki}}{8M_{\omega_{1r}}^2} + \frac{(y_{\Pi_7}^{eq})^*_{rjk}(y_{\Pi_7}^{lu})_{ril}}{8M_{\Pi_{7r}}^2}. \tag{8.51}$$

B-violating

$$(C_{duq})_{ijkl} = \frac{(y_{\omega_1}^{du})^*_{rij}(y_{\omega_1}^{ql})_{rkl}}{M_{\omega_{1r}}^2} + \frac{2(g_{\mathcal{Q}_1}^{dq})^*_{rik}(g_{\mathcal{Q}_1}^{ul})_{rjl}}{M_{\mathcal{Q}_{1r}}^2} - \frac{2(g_{\mathcal{Q}_5}^{uq})^*_{rjk}(g_{\mathcal{Q}_5}^{dl})_{ril}}{M_{\mathcal{Q}_{5r}}^2}, \tag{8.52}$$

$$(C_{qqu})_{ijkl} = \frac{(y_{\omega_1}^{eu})_{rlk}(y_{\omega_1}^{qq})^*_{rij}}{M_{\omega_{1r}}^2} - \frac{2(g_{\mathcal{Q}_5}^{uq})^*_{rki}(g_{\mathcal{Q}_5}^{eq})_{rlj}}{M_{\mathcal{Q}_{5r}}^2}, \tag{8.53}$$

$$(C_{qqq})_{ijkl} = \frac{2(y_{\omega_1}^{qq})^*_{rij}(y_{\omega_1}^{ql})_{rkl}}{M_{\omega_{1r}}^2} - \frac{2(y_{\zeta}^{qq})^*_{rij}(y_{\zeta}^{ql})_{rkl}}{M_{\zeta_r}^2}, \tag{8.54}$$

$$(C_{duu})_{ijkl} = \frac{(y_{\omega_1}^{du})^*_{rij}(y_{\omega_1}^{eu})_{rlk}}{M_{\omega_{1r}}^2} - \frac{2(y_{\omega_4}^{uu})^*_{rjk}(y_{\omega_4}^{ed})_{rli}}{M_{\omega_{4r}}^2}. \tag{8.55}$$

8.5.4 Bosonic operators

ϕ^6 and $\phi^4 D^2$

Recall that $\hat{g}_V^{\phi}$ contains contributions from $\mathcal{L}_1$ (see eqs. (8.12)–(8.15)) and that $\hat{\lambda}_\phi$ is defined in eq. (8.24).

Due to the length of the contributions to the coefficient of the $\mathcal{O}_\phi$ operator we have separated them as follows:

$$Z_\phi^3 C_\phi = C_\phi^{\rm S} + C_\phi^{\rm V} + C_\phi^{\rm SV}, \tag{8.56}$$

where $C_\phi^{\rm S}$, $C_\phi^{\rm V}$, and $C_\phi^{\rm SV}$ are given below.

$$\begin{aligned} C_\phi^{\rm V} =& -\frac{2\hat{\lambda}_\phi(\hat{g}_{\mathcal{B}_1}^{\phi})^*_r(\hat{g}_{\mathcal{B}_1}^{\phi})_r}{M_{\mathcal{B}_{1r}}^2} - \frac{\hat{\lambda}_\phi(\hat{g}_{\mathcal{W}}^{\phi})^*_r(\hat{g}_{\mathcal{W}}^{\phi})_r}{M_{\mathcal{W}_r}^2} - \frac{\hat{\lambda}_\phi(\hat{g}_{\mathcal{W}_1}^{\phi})^*_r(\hat{g}_{\mathcal{W}_1}^{\phi})_r}{2M_{\mathcal{W}_{1r}}^2} \\ &+ \frac{2g_2\hat{\lambda}_\phi(g_{\mathcal{L}_1}^{W})_{sr}(\gamma_{\mathcal{L}_1})^*_s(\gamma_{\mathcal{L}_1})_r}{M_{\mathcal{L}_{1r}}^2 M_{\mathcal{L}_{1s}}^2} - \frac{2\hat{\lambda}_\phi(h_{\mathcal{L}_1}^{(1)})_{rs}(\gamma_{\mathcal{L}_1})^*_r(\gamma_{\mathcal{L}_1})_s}{M_{\mathcal{L}_{1r}}^2 M_{\mathcal{L}_{1s}}^2} \\ &+ \frac{1}{f}\left\{ -\frac{4\hat{\lambda}_\phi \,\mathrm{Re}\left((\tilde{\gamma}_{\mathcal{L}_1}^{(3)})_r(\gamma_{\mathcal{L}_1})^*_r\right)}{M_{\mathcal{L}_{1r}}^2} + \frac{4\hat{\lambda}_\phi \,\mathrm{Im}\left((\tilde{\gamma}_{\mathcal{L}_1}^{W})_r(\gamma_{\mathcal{L}_1})^*_r\right)g_2}{M_{\mathcal{L}_{1r}}^2}\right\}, \end{aligned} \tag{8.57}$$

$$\begin{aligned} C_\phi^{\rm S} =& -\frac{(\lambda_{\mathcal{S}})_{rs}(\kappa_{\mathcal{S}})_r(\kappa_{\mathcal{S}})_s}{M_{\mathcal{S}_r}^2 M_{\mathcal{S}_s}^2} + \frac{(\kappa_{\mathcal{S}_3})_{rts}(\kappa_{\mathcal{S}})_r(\kappa_{\mathcal{S}})_t(\kappa_{\mathcal{S}})_s}{M_{\mathcal{S}_r}^2 M_{\mathcal{S}_s}^2 M_{\mathcal{S}_t}^2} + \frac{(\lambda_\varphi)^*_r(\lambda_\varphi)_r}{M_{\varphi_r}^2} + \frac{4\hat{\lambda}_\phi(\kappa_\Xi)_r(\kappa_\Xi)_r}{M_{\Xi_r}^4} \\ &- \frac{(\lambda_\Xi)_s(\kappa_\Xi)_s(\kappa_\Xi)_r}{M_{\Xi_r}^2 M_{\Xi_s}^2} + \frac{8\hat{\lambda}_\phi(\kappa_{\Xi_1})^*_r(\kappa_{\Xi_1})_r}{M_{\Xi_{1r}}^4} - \frac{2(\lambda_{\Xi_1})_{rs}(\kappa_{\Xi_1})^*_r(\kappa_{\Xi_1})_s}{M_{\Xi_{1r}}^2 M_{\Xi_{1s}}^2} \end{aligned}$$

$$
\begin{aligned}
&+\frac{\sqrt{2}(\lambda'_{\Xi_1})_{rs}(\kappa_{\Xi_1})^*_r(\kappa_{\Xi_1})_s}{M^2_{\Xi_{1r}}M^2_{\Xi_{1s}}}+\frac{(\lambda_{\Theta_1})^*_r(\lambda_{\Theta_1})_r}{6M^2_{\Theta_{1r}}}+\frac{(\lambda_{\Theta_3})^*_r(\lambda_{\Theta_3})_r}{2M^2_{\Theta_{3r}}}\\
&-\frac{2\,\mathrm{Re}\left((\kappa_{\mathcal{S}\varphi})_{rs}(\lambda_\varphi)^*_s\right)(\kappa_{\mathcal{S}})_r}{M^2_{\mathcal{S}_r}M^2_{\varphi_s}}+\frac{(\kappa_{\mathcal{S}\varphi})^*_{rt}(\kappa_{\mathcal{S}})_r(\kappa_{\mathcal{S}\varphi})_{st}(\kappa_{\mathcal{S}})_s}{M^2_{\mathcal{S}_r}M^2_{\mathcal{S}_s}M^2_{\varphi_t}}\\
&-\frac{(\lambda_{\mathcal{S}\Xi})_{sr}(\kappa_{\mathcal{S}})_s(\kappa_\Xi)_r}{M^2_{\Xi_r}M^2_{\mathcal{S}_s}}+\frac{(\kappa_{\mathcal{S}\Xi})_{tsr}(\kappa_{\mathcal{S}})_t(\kappa_\Xi)_s(\kappa_\Xi)_r}{M^2_{\Xi_r}M^2_{\Xi_s}M^2_{\mathcal{S}_t}}\\
&-\frac{2\,\mathrm{Re}\left((\kappa_{\Xi\varphi})_{rs}(\lambda_\varphi)^*_s\right)(\kappa_\Xi)_r}{M^2_{\Xi_r}M^2_{\varphi_s}}+\frac{(\kappa_{\Xi\varphi})^*_{tr}(\kappa_\Xi)_t(\kappa_{\Xi\varphi})_{sr}(\kappa_\Xi)_s}{M^2_{\varphi_r}M^2_{\Xi_s}M^2_{\Xi_t}}\\
&-\frac{4\,\mathrm{Re}\left((\lambda_{\mathcal{S}\Xi_1})_{rs}(\kappa_{\Xi_1})^*_s\right)(\kappa_{\mathcal{S}})_r}{M^2_{\Xi_{1s}}M^2_{\mathcal{S}_r}}+\frac{2(\kappa_{\mathcal{S}\Xi_1})_{trs}(\kappa_{\mathcal{S}})_t(\kappa_{\Xi_1})^*_r(\kappa_{\Xi_1})_s}{M^2_{\Xi_{1r}}M^2_{\Xi_{1s}}M^2_{\mathcal{S}_t}}\\
&-\frac{2\sqrt{2}\,\mathrm{Re}\left((\lambda_{\Xi_1\Xi})_{rs}(\kappa_{\Xi_1})^*_r\right)(\kappa_\Xi)_s}{M^2_{\Xi_s}M^2_{\Xi_{1r}}}-\frac{\sqrt{2}(\kappa_{\Xi\Xi_1})_{srt}(\kappa_\Xi)_s(\kappa_{\Xi_1})^*_r(\kappa_{\Xi_1})_t}{M^2_{\Xi_{1r}}M^2_{\Xi_s}M^2_{\Xi_{1t}}}\\
&-\frac{4\,\mathrm{Re}\left((\kappa_{\Xi_1\varphi})^*_{rs}(\kappa_{\Xi_1})_r(\lambda_\varphi)^*_s\right)}{M^2_{\Xi_{1r}}M^2_{\varphi_s}}+\frac{4(\kappa_{\Xi_1\varphi})^*_{st}(\kappa_{\Xi_1})_s(\kappa_{\Xi_1\varphi})_{rt}(\kappa_{\Xi_1})^*_r}{M^2_{\Xi_{1r}}M^2_{\Xi_{1s}}M^2_{\varphi_t}}\\
&-\frac{\mathrm{Re}\left((\kappa_{\Xi\Theta_1})_{rs}(\lambda_{\Theta_1})^*_s\right)(\kappa_\Xi)_r}{3M^2_{\Theta_{1s}}M^2_{\Xi_r}}+\frac{(\kappa_{\Xi\Theta_1})^*_{rs}(\kappa_\Xi)_r(\kappa_{\Xi\Theta_1})_{ts}(\kappa_\Xi)_t}{6M^2_{\Xi_r}M^2_{\Theta_{1s}}M^2_{\Xi_t}}\\
&-\frac{\mathrm{Re}\left((\kappa_{\Xi_1\Theta_1})_{rs}(\kappa_{\Xi_1})^*_r(\lambda_{\Theta_1})^*_s\right)}{3M^2_{\Xi_{1r}}M^2_{\Theta_{1s}}}+\frac{(\kappa_{\Xi_1\Theta_1})^*_{tr}(\kappa_{\Xi_1})_t(\kappa_{\Xi_1\Theta_1})_{sr}(\kappa_{\Xi_1})^*_s}{6M^2_{\Theta_{1r}}M^2_{\Xi_{1s}}M^2_{\Xi_{1t}}}\\
&-\frac{\mathrm{Re}\left((\kappa_{\Xi_1\Theta_3})_{rs}(\kappa_{\Xi_1})^*_r(\lambda_{\Theta_3})^*_s\right)}{M^2_{\Xi_{1r}}M^2_{\Theta_{3s}}}+\frac{(\kappa_{\Xi_1\Theta_3})^*_{tr}(\kappa_{\Xi_1})_t(\kappa_{\Xi_1\Theta_3})_{sr}(\kappa_{\Xi_1})^*_s}{2M^2_{\Theta_{3r}}M^2_{\Xi_{1s}}M^2_{\Xi_{1t}}}\\
&+\frac{2\,\mathrm{Re}\left((\kappa_{\Xi\varphi})^*_{rs}(\kappa_{\mathcal{S}\varphi})_{ts}\right)(\kappa_\Xi)_r(\kappa_{\mathcal{S}})_t}{M^2_{\Xi_r}M^2_{\mathcal{S}_t}M^2_{\varphi_s}}+\frac{4\,\mathrm{Re}\left((\kappa_{\Xi_1\varphi})_{rs}(\kappa_{\Xi_1})^*_r(\kappa_{\mathcal{S}\varphi})_{ts}\right)(\kappa_{\mathcal{S}})_t}{M^2_{\varphi_s}M^2_{\Xi_{1r}}M^2_{\mathcal{S}_t}}\\
&+\frac{4\,\mathrm{Re}\left((\kappa_{\Xi_1\varphi})^*_{rs}(\kappa_{\Xi_1})_r(\kappa_{\Xi\varphi})^*_{ts}\right)(\kappa_\Xi)_t}{M^2_{\varphi_s}M^2_{\Xi_{1r}}M^2_{\Xi_t}}+\frac{\mathrm{Re}\left((\kappa_{\Xi_1\Theta_1})^*_{rs}(\kappa_{\Xi_1})_r(\kappa_{\Xi\Theta_1})_{ts}\right)(\kappa_\Xi)_t}{3M^2_{\Theta_{1s}}M^2_{\Xi_{1r}}M^2_{\Xi_t}}\\
&+\frac{1}{f}\left\{\frac{2\hat{\lambda}_\phi(\tilde{k}^\phi_{\mathcal{S}})_r(\kappa_{\mathcal{S}})_r}{M^2_{\mathcal{S}_r}}+\frac{(\tilde{\lambda}_{\mathcal{S}})_r(\kappa_{\mathcal{S}})_r}{M^2_{\mathcal{S}_r}}-\frac{2\hat{\lambda}_\phi(\tilde{k}^\phi_\Xi)_r(\kappa_\Xi)_r}{M^2_{\Xi_r}}+\frac{(\tilde{\lambda}_\Xi)_r(\kappa_\Xi)_r}{M^2_{\Xi_r}}\right.\\
&\qquad\left.+\frac{4\,\mathrm{Re}\left((\tilde{\lambda}_{\Xi_1})_r(\kappa_{\Xi_1})^*_r\right)}{M^2_{\Xi_{1r}}}\right\},
\end{aligned}
\tag{8.58}
$$

$$
\begin{aligned}
\frac{C_{\phi}^{\rm SV}}{\hat{\lambda}_{\phi}} = & -\frac{4\,{\rm Im}\left((\hat{g}_{\mathcal{W}}^{\phi})_r\right)(\delta_{\mathcal{W}\Xi})_{rs}(\kappa_{\Xi})_s}{M_{\mathcal{W}_r}^2 M_{\Xi_s}^2} - \frac{4(\delta_{\mathcal{W}\Xi})_{ts}(\delta_{\mathcal{W}\Xi})_{tr}(\kappa_{\Xi})_r(\kappa_{\Xi})_s}{M_{\Xi_r}^2 M_{\Xi_s}^2 M_{\mathcal{W}_t}^2} \\
& - \frac{4\,{\rm Im}\left((\hat{g}_{\mathcal{W}_1}^{\phi})_r^*(\delta_{\mathcal{W}_1\Xi_1})_{rs}(\kappa_{\Xi_1})_s\right)}{M_{\Xi_{1s}}^2 M_{\mathcal{W}_{1r}}^2} - \frac{8(\delta_{\mathcal{W}_1\Xi_1})_{st}^*(\delta_{\mathcal{W}_1\Xi_1})_{sr}(\kappa_{\Xi_1})_r(\kappa_{\Xi_1})_t^*}{M_{\Xi_{1r}}^2 M_{\mathcal{W}_{1s}}^2 M_{\Xi_{1t}}^2} \\
& + \frac{4\,{\rm Re}\left((g_{\mathcal{S}\mathcal{L}_1}^{(2)})_{rs}(\gamma_{\mathcal{L}_1})_s\right)(\kappa_{\mathcal{S}})_r}{M_{\mathcal{S}_r}^2 M_{\mathcal{L}_{1s}}^2} + \frac{2(\varepsilon_{\mathcal{S}\mathcal{L}_1})_{rts}(\kappa_{\mathcal{S}})_r(\gamma_{\mathcal{L}_1})_t^*(\gamma_{\mathcal{L}_1})_s}{M_{\mathcal{S}_r}^2 M_{\mathcal{L}_{1s}}^2 M_{\mathcal{L}_{1t}}^2} \\
& + \frac{4\,{\rm Re}\left((\delta_{\mathcal{L}_1\varphi})_{rs}(\gamma_{\mathcal{L}_1})_r^*(\lambda_{\varphi})_s\right)}{M_{\varphi_s}^2 M_{\mathcal{L}_{1r}}^2} + \frac{4\hat{\lambda}_{\phi}(\delta_{\mathcal{L}_1\varphi})_{ts}^*(\gamma_{\mathcal{L}_1})_t(\delta_{\mathcal{L}_1\varphi})_{rs}(\gamma_{\mathcal{L}_1})_r^*}{M_{\mathcal{L}_{1r}}^2 M_{\varphi_s}^2 M_{\mathcal{L}_{1t}}^2} \\
& - \frac{4\,{\rm Re}\left((g_{\Xi\mathcal{L}_1}^{(2)})_{rs}(\gamma_{\mathcal{L}_1})_s\right)(\kappa_{\Xi})_r}{M_{\Xi_r}^2 M_{\mathcal{L}_{1s}}^2} - \frac{2(\varepsilon_{\Xi\mathcal{L}_1})_{srt}(\kappa_{\Xi})_s(\gamma_{\mathcal{L}_1})_r^*(\gamma_{\mathcal{L}_1})_t}{M_{\mathcal{L}_{1r}}^2 M_{\Xi_s}^2 M_{\mathcal{L}_{1t}}^2} \\
& + \frac{8\,{\rm Re}\left((g_{\Xi\mathcal{L}_1}^{(1)})_{rs}^*(\gamma_{\mathcal{L}_1})_s^*\right)(\kappa_{\Xi})_r}{M_{\Xi_r}^2 M_{\mathcal{L}_{1s}}^2} + \frac{8\,{\rm Re}\left((g_{\Xi_1\mathcal{L}_1}^{(1)})_{rs}^*(\gamma_{\mathcal{L}_1})_s^*(\kappa_{\Xi_1})_r\right)}{M_{\mathcal{L}_{1s}}^2 M_{\Xi_{1r}}^2} \\
& - \frac{4\,{\rm Re}\left((\delta_{\mathcal{L}_1\varphi})_{rs}(\gamma_{\mathcal{L}_1})_r^*(\kappa_{\mathcal{S}\varphi})_{ts}\right)(\kappa_{\mathcal{S}})_t}{M_{\varphi_s}^2 M_{\mathcal{S}_t}^2 M_{\mathcal{L}_{1r}}^2} - \frac{4\,{\rm Re}\left((\delta_{\mathcal{L}_1\varphi})_{rs}(\gamma_{\mathcal{L}_1})_r^*(\kappa_{\Xi\varphi})_{ts}\right)(\kappa_{\Xi})_t}{M_{\mathcal{L}_{1r}}^2 M_{\Xi_t}^2 M_{\varphi_s}^2} \\
& - \frac{8\,{\rm Re}\left((\delta_{\mathcal{L}_1\varphi})_{rs}(\gamma_{\mathcal{L}_1})_r^*(\kappa_{\Xi_1\varphi})_{ts}^*(\kappa_{\Xi_1})_t\right)}{M_{\varphi_s}^2 M_{\mathcal{L}_{1r}}^2 M_{\Xi_{1t}}^2},
\end{aligned}
\tag{8.59}
$$

$$
\begin{aligned}
Z_\phi^2 C_{\phi D} =& -\frac{2(\kappa_\Xi)_r(\kappa_\Xi)_r}{M_{\Xi_r}^4} + \frac{4(\kappa_{\Xi_1})_r^*(\kappa_{\Xi_1})_r}{M_{\Xi_{1r}}^4} - \frac{\mathrm{Re}\left((\hat{g}_{\mathcal{B}}^\phi)_r(\hat{g}_{\mathcal{B}}^\phi)_r\right)}{M_{\mathcal{B}_r}^2} - \frac{(\hat{g}_{\mathcal{B}}^\phi)_r^*(\hat{g}_{\mathcal{B}}^\phi)_r}{M_{\mathcal{B}_r}^2} \\
&+ \frac{(\hat{g}_{\mathcal{B}_1}^\phi)_r^*(\hat{g}_{\mathcal{B}_1}^\phi)_r}{M_{\mathcal{B}_{1r}}^2} - \frac{\mathrm{Re}\left((\hat{g}_{\mathcal{W}}^\phi)_r(\hat{g}_{\mathcal{W}}^\phi)_r\right)}{4M_{\mathcal{W}_r}^2} + \frac{(\hat{g}_{\mathcal{W}}^\phi)_r^*(\hat{g}_{\mathcal{W}}^\phi)_r}{4M_{\mathcal{W}_r}^2} - \frac{(\hat{g}_{\mathcal{W}_1}^\phi)_r^*(\hat{g}_{\mathcal{W}_1}^\phi)_r}{4M_{\mathcal{W}_{1r}}^2} \\
&+ \frac{g_1(g_{\mathcal{L}_1}^B)_{rs}(\gamma_{\mathcal{L}_1})_r^*(\gamma_{\mathcal{L}_1})_s}{M_{\mathcal{L}_{1r}}^2 M_{\mathcal{L}_{1s}}^2} - \frac{(h_{\mathcal{L}_1}^{(2)})_{rs}(\gamma_{\mathcal{L}_1})_r^*(\gamma_{\mathcal{L}_1})_s}{M_{\mathcal{L}_{1r}}^2 M_{\mathcal{L}_{1s}}^2} + \frac{2\,\mathrm{Re}\left((h_{\mathcal{L}_1}^{(3)})_{rs}(\gamma_{\mathcal{L}_1})_r^*(\gamma_{\mathcal{L}_1})_s^*\right)}{M_{\mathcal{L}_{1r}}^2 M_{\mathcal{L}_{1s}}^2} \\
&+ \frac{2\,\mathrm{Im}\left((\hat{g}_{\mathcal{W}}^\phi)_r\right)(\delta_{\mathcal{W}\Xi})_{rs}(\kappa_\Xi)_s}{M_{\mathcal{W}_r}^2 M_{\Xi_s}^2} + \frac{2(\delta_{\mathcal{W}\Xi})_{ts}(\delta_{\mathcal{W}\Xi})_{tr}(\kappa_\Xi)_r(\kappa_\Xi)_s}{M_{\Xi_r}^2 M_{\Xi_s}^2 M_{\mathcal{W}_t}^2} \\
&- \frac{2\,\mathrm{Im}\left((\hat{g}_{\mathcal{W}_1}^\phi)_r^*(\delta_{\mathcal{W}_1\Xi_1})_{rs}(\kappa_{\Xi_1})_s\right)}{M_{\mathcal{W}_{1r}}^2 M_{\Xi_{1s}}^2} - \frac{4(\delta_{\mathcal{W}_1\Xi_1})_{tr}^*(\delta_{\mathcal{W}_1\Xi_1})_{ts}(\kappa_{\Xi_1})_s(\kappa_{\Xi_1})_r^*}{M_{\Xi_{1r}}^2 M_{\Xi_{1s}}^2 M_{\mathcal{W}_{1t}}^2} \\
&- \frac{4\,\mathrm{Re}\left((g_{\Xi\mathcal{L}_1}^{(1)})_{rs}^*(\gamma_{\mathcal{L}_1})_s^*\right)(\kappa_\Xi)_r}{M_{\Xi_r}^2 M_{\mathcal{L}_{1s}}^2} + \frac{4\,\mathrm{Re}\left((g_{\Xi\mathcal{L}_1}^{(2)})_{rs}(\gamma_{\mathcal{L}_1})_s\right)(\kappa_\Xi)_r}{M_{\Xi_r}^2 M_{\mathcal{L}_{1s}}^2} \\
&+ \frac{2(\varepsilon_{\Xi\mathcal{L}_1})_{srt}(\kappa_\Xi)_s(\gamma_{\mathcal{L}_1})_r^*(\gamma_{\mathcal{L}_1})_t}{M_{\mathcal{L}_{1r}}^2 M_{\Xi_s}^2 M_{\mathcal{L}_{1t}}^2} + \frac{4\,\mathrm{Re}\left((g_{\Xi_1\mathcal{L}_1}^{(1)})_{rs}^*(\gamma_{\mathcal{L}_1})_s^*(\kappa_{\Xi_1})_r\right)}{M_{\mathcal{L}_{1s}}^2 M_{\Xi_{1r}}^2} \\
&- \frac{4\,\mathrm{Re}\left((g_{\Xi_1\mathcal{L}_1}^{(2)})_{rs}(\kappa_{\Xi_1})_r^*(\gamma_{\mathcal{L}_1})_s\right)}{M_{\mathcal{L}_{1s}}^2 M_{\Xi_{1r}}^2} - \frac{4\,\mathrm{Re}\left((\varepsilon_{\Xi_1\mathcal{L}_1})_{rst}^*(\kappa_{\Xi_1})_r^*(\gamma_{\mathcal{L}_1})_t(\gamma_{\mathcal{L}_1})_s\right)}{M_{\mathcal{L}_{1s}}^2 M_{\mathcal{L}_{1t}}^2 M_{\Xi_{1r}}^2} \\
&+ \frac{1}{f}\left\{\frac{2(\tilde{k}_\Xi^\phi)_r(\kappa_\Xi)_r}{M_{\Xi_r}^2} - \frac{4\,\mathrm{Re}\left((\tilde{k}_{\Xi_1})_r(\kappa_{\Xi_1})_r^*\right)}{M_{\Xi_{1r}}^2} - \frac{2\,\mathrm{Re}\left((\tilde{\gamma}_{\mathcal{L}_1}^{(1)})_r(\gamma_{\mathcal{L}_1})_r^*\right)}{M_{\mathcal{L}_{1r}}^2}\right. \\
&\quad \left. + \frac{2\,\mathrm{Re}\left((\tilde{\gamma}_{\mathcal{L}_1}^{(2)})_r(\gamma_{\mathcal{L}_1})_r^*\right)}{M_{\mathcal{L}_{1r}}^2} + \frac{2\,\mathrm{Im}\left((\tilde{\gamma}_{\mathcal{L}_1}^{B})_r(\gamma_{\mathcal{L}_1})_r^*\right)g_1}{M_{\mathcal{L}_{1r}}^2}\right\},
\end{aligned}
\tag{8.60}
$$

$$
\begin{aligned}
Z_\phi^2\, C_{\phi\Box} = &-\frac{(\kappa_{\mathcal{S}})_r(\kappa_{\mathcal{S}})_r}{2M_{\mathcal{S}_r}^4}+\frac{(\kappa_{\Xi})_r(\kappa_{\Xi})_r}{2M_{\Xi_r}^4}+\frac{2(\kappa_{\Xi_1})_r^*(\kappa_{\Xi_1})_r}{M_{\Xi_{1r}}^4}\\
&-\frac{\mathrm{Re}\left((\hat{g}_{\mathcal{B}}^{\phi})_r(\hat{g}_{\mathcal{B}}^{\phi})_r\right)}{2M_{\mathcal{B}_r}^2}-\frac{\mathrm{Re}\left((\hat{g}_{\mathcal{W}}^{\phi})_r(\hat{g}_{\mathcal{W}}^{\phi})_r\right)}{8M_{\mathcal{W}_r}^2}-\frac{(\hat{g}_{\mathcal{B}_1}^{\phi})_r^*(\hat{g}_{\mathcal{B}_1}^{\phi})_r}{2M_{\mathcal{B}_{1r}}^2}-\frac{(\hat{g}_{\mathcal{W}}^{\phi})_r^*(\hat{g}_{\mathcal{W}}^{\phi})_r}{4M_{\mathcal{W}_r}^2}\\
&-\frac{(\hat{g}_{\mathcal{W}_1}^{\phi})_r^*(\hat{g}_{\mathcal{W}_1}^{\phi})_r}{8M_{\mathcal{W}_{1r}}^2}+\frac{g_1(g_{\mathcal{L}_1}^{B})_{rs}(\gamma_{\mathcal{L}_1})_r^*(\gamma_{\mathcal{L}_1})_s}{4M_{\mathcal{L}_{1r}}^2M_{\mathcal{L}_{1s}}^2}+\frac{3g_2(g_{\mathcal{L}_1}^{W})_{sr}(\gamma_{\mathcal{L}_1})_s^*(\gamma_{\mathcal{L}_1})_r}{4M_{\mathcal{L}_{1r}}^2M_{\mathcal{L}_{1s}}^2}\\
&-\frac{(h_{\mathcal{L}_1}^{(1)})_{rs}(\gamma_{\mathcal{L}_1})_r^*(\gamma_{\mathcal{L}_1})_s}{2M_{\mathcal{L}_{1r}}^2M_{\mathcal{L}_{1s}}^2}+\frac{\mathrm{Re}\left((h_{\mathcal{L}_1}^{(3)})_{rs}(\gamma_{\mathcal{L}_1})_r^*(\gamma_{\mathcal{L}_1})_s^*\right)}{M_{\mathcal{L}_{1r}}^2M_{\mathcal{L}_{1s}}^2}\\
&+\frac{\mathrm{Im}\left((\hat{g}_{\mathcal{B}}^{\phi})_r\right)(\delta_{\mathcal{BS}})_{rs}(\kappa_{\mathcal{S}})_s}{M_{\mathcal{B}_r}^2M_{\mathcal{S}_s}^2}+\frac{(\delta_{\mathcal{BS}})_{rt}(\delta_{\mathcal{BS}})_{rs}(\kappa_{\mathcal{S}})_s(\kappa_{\mathcal{S}})_t}{2M_{\mathcal{B}_r}^2M_{\mathcal{S}_s}^2M_{\mathcal{S}_t}^2}\\
&-\frac{\mathrm{Im}\left((\hat{g}_{\mathcal{W}}^{\phi})_r\right)(\delta_{\mathcal{W}\Xi})_{rs}(\kappa_{\Xi})_s}{2M_{\mathcal{W}_r}^2M_{\Xi_s}^2}-\frac{(\delta_{\mathcal{W}\Xi})_{ts}(\delta_{\mathcal{W}\Xi})_{tr}(\kappa_{\Xi})_r(\kappa_{\Xi})_s}{2M_{\Xi_r}^2M_{\Xi_s}^2M_{\mathcal{W}_t}^2}\\
&-\frac{\mathrm{Im}\left((\hat{g}_{\mathcal{W}_1}^{\phi})_r^*(\delta_{\mathcal{W}_1\Xi_1})_{rs}(\kappa_{\Xi_1})_s\right)}{M_{\Xi_{1s}}^2M_{\mathcal{W}_{1r}}^2}-\frac{2(\delta_{\mathcal{W}_1\Xi_1})_{st}^*(\delta_{\mathcal{W}_1\Xi_1})_{sr}(\kappa_{\Xi_1})_r(\kappa_{\Xi_1})_t^*}{M_{\Xi_{1r}}^2M_{\mathcal{W}_{1s}}^2M_{\Xi_{1t}}^2}\\
&-\frac{\mathrm{Re}\left((g_{\mathcal{SL}_1}^{(1)})_{rs}^*(\gamma_{\mathcal{L}_1})_s^*\right)(\kappa_{\mathcal{S}})_r}{M_{\mathcal{S}_r}^2M_{\mathcal{L}_{1s}}^2}+\frac{\mathrm{Re}\left((g_{\mathcal{SL}_1}^{(2)})_{rs}(\gamma_{\mathcal{L}_1})_s\right)(\kappa_{\mathcal{S}})_r}{M_{\mathcal{S}_r}^2M_{\mathcal{L}_{1s}}^2}\\
&+\frac{(\varepsilon_{\mathcal{SL}_1})_{rts}(\kappa_{\mathcal{S}})_r(\gamma_{\mathcal{L}_1})_t^*(\gamma_{\mathcal{L}_1})_s}{2M_{\mathcal{S}_r}^2M_{\mathcal{L}_{1s}}^2M_{\mathcal{L}_{1t}}^2}+\frac{\mathrm{Re}\left((g_{\Xi\mathcal{L}_1}^{(1)})_{rs}^*(\gamma_{\mathcal{L}_1})_s^*\right)(\kappa_{\Xi})_r}{M_{\Xi_r}^2M_{\mathcal{L}_{1s}}^2}\\
&-\frac{\mathrm{Re}\left((g_{\Xi\mathcal{L}_1}^{(2)})_{rs}(\gamma_{\mathcal{L}_1})_s\right)(\kappa_{\Xi})_r}{M_{\Xi_r}^2M_{\mathcal{L}_{1s}}^2}-\frac{(\varepsilon_{\Xi\mathcal{L}_1})_{srt}(\kappa_{\Xi})_s(\gamma_{\mathcal{L}_1})_r^*(\gamma_{\mathcal{L}_1})_t}{2M_{\mathcal{L}_{1r}}^2M_{\Xi_s}^2M_{\mathcal{L}_{1t}}^2}\\
&+\frac{2\,\mathrm{Re}\left((g_{\Xi_1\mathcal{L}_1}^{(1)})_{rs}^*(\gamma_{\mathcal{L}_1})_s^*(\kappa_{\Xi_1})_r\right)}{M_{\mathcal{L}_{1s}}^2M_{\Xi_{1r}}^2}-\frac{2\,\mathrm{Re}\left((g_{\Xi_1\mathcal{L}_1}^{(2)})_{rs}(\kappa_{\Xi_1})_r^*(\gamma_{\mathcal{L}_1})_s\right)}{M_{\mathcal{L}_{1s}}^2M_{\Xi_{1r}}^2}\\
&-\frac{2\,\mathrm{Re}\left((\varepsilon_{\Xi_1\mathcal{L}_1})_{rst}^*(\kappa_{\Xi_1})_r^*(\gamma_{\mathcal{L}_1})_t(\gamma_{\mathcal{L}_1})_s\right)}{M_{\mathcal{L}_{1s}}^2M_{\mathcal{L}_{1t}}^2M_{\Xi_{1r}}^2}\\
&+\frac{1}{f}\Bigg\{\frac{(\tilde{k}_{\mathcal{S}}^{\phi})_r(\kappa_{\mathcal{S}})_r}{2M_{\mathcal{S}_r}^2}-\frac{(\tilde{k}_{\Xi}^{\phi})_r(\kappa_{\Xi})_r}{2M_{\Xi_r}^2}-\frac{2\,\mathrm{Re}\left((\tilde{k}_{\Xi_1})_r(\kappa_{\Xi_1})_r^*\right)}{M_{\Xi_{1r}}^2}+\frac{\mathrm{Re}\left((\tilde{\gamma}_{\mathcal{L}_1}^{(2)})_r(\gamma_{\mathcal{L}_1})_r^*\right)}{M_{\mathcal{L}_{1r}}^2}\\
&\quad-\frac{\mathrm{Re}\left((\tilde{\gamma}_{\mathcal{L}_1}^{(3)})_r(\gamma_{\mathcal{L}_1})_r^*\right)}{M_{\mathcal{L}_{1r}}^2}+\frac{\mathrm{Im}\left((\tilde{\gamma}_{\mathcal{L}_1}^{B})_r(\gamma_{\mathcal{L}_1})_r^*\right)g_1}{2M_{\mathcal{L}_{1r}}^2}+\frac{3\,\mathrm{Im}\left((\tilde{\gamma}_{\mathcal{L}_1}^{W})_r(\gamma_{\mathcal{L}_1})_r^*\right)g_2}{2M_{\mathcal{L}_{1r}}^2}\Bigg\}.
\end{aligned}
\tag{8.61}
$$

$X^2\phi^2$

$$Z_\phi C_{\phi B} = -\frac{(g_1)^2(\gamma_{\mathcal{L}_1})_r^*(\gamma_{\mathcal{L}_1})_r}{8M_{\mathcal{L}_{1r}}^4} - \frac{g_1(g_{\mathcal{L}_1}^B)_{rs}(\gamma_{\mathcal{L}_1})_r^*(\gamma_{\mathcal{L}_1})_s}{4M_{\mathcal{L}_{1r}}^2 M_{\mathcal{L}_{1s}}^2} + \frac{1}{f}\left\{\frac{(\tilde{k}_{\mathcal{S}}^B)_r(\kappa_{\mathcal{S}})_r}{M_{\mathcal{S}_r}^2} - \frac{\mathrm{Im}\left((\tilde{\gamma}_{\mathcal{L}_1}^B)_r(\gamma_{\mathcal{L}_1})_r^*\right)g_1}{2M_{\mathcal{L}_{1r}}^2}\right\}, \tag{8.62}$$

$$Z_\phi C_{\phi \tilde{B}} = -\frac{g_1(g_{\mathcal{L}_1}^{\tilde{B}})_{rs}(\gamma_{\mathcal{L}_1})_r^*(\gamma_{\mathcal{L}_1})_s}{4M_{\mathcal{L}_{1r}}^2 M_{\mathcal{L}_{1s}}^2} + \frac{1}{f}\left\{\frac{(\tilde{k}_{\mathcal{S}}^{\tilde{B}})_r(\kappa_{\mathcal{S}})_r}{M_{\mathcal{S}_r}^2} - \frac{\mathrm{Im}\left((\tilde{\gamma}_{\mathcal{L}_1}^{\tilde{B}})_r(\gamma_{\mathcal{L}_1})_r^*\right)g_1}{2M_{\mathcal{L}_{1r}}^2}\right\}, \tag{8.63}$$

$$Z_\phi C_{\phi W} = -\frac{(g_2)^2(\gamma_{\mathcal{L}_1})_r^*(\gamma_{\mathcal{L}_1})_r}{8M_{\mathcal{L}_{1r}}^4} - \frac{g_2(g_{\mathcal{L}_1}^W)_{rs}(\gamma_{\mathcal{L}_1})_r^*(\gamma_{\mathcal{L}_1})_s}{4M_{\mathcal{L}_{1r}}^2 M_{\mathcal{L}_{1s}}^2} + \frac{1}{f}\left\{\frac{(\tilde{k}_{\mathcal{S}}^W)_r(\kappa_{\mathcal{S}})_r}{M_{\mathcal{S}_r}^2} - \frac{\mathrm{Im}\left((\tilde{\gamma}_{\mathcal{L}_1}^W)_r(\gamma_{\mathcal{L}_1})_r^*\right)g_2}{2M_{\mathcal{L}_{1r}}^2}\right\}, \tag{8.64}$$

$$Z_\phi C_{\phi \tilde{W}} = -\frac{g_2(g_{\mathcal{L}_1}^{\tilde{W}})_{rs}(\gamma_{\mathcal{L}_1})_r^*(\gamma_{\mathcal{L}_1})_s}{4M_{\mathcal{L}_{1r}}^2 M_{\mathcal{L}_{1s}}^2} + \frac{1}{f}\left\{\frac{(\tilde{k}_{\mathcal{S}}^{\tilde{W}})_r(\kappa_{\mathcal{S}})_r}{M_{\mathcal{S}_r}^2} - \frac{\mathrm{Im}\left((\tilde{\gamma}_{\mathcal{L}_1}^{\tilde{W}})_r(\gamma_{\mathcal{L}_1})_r^*\right)g_2}{2M_{\mathcal{L}_{1r}}^2}\right\}, \tag{8.65}$$

$$Z_\phi C_{\phi WB} = -\frac{g_1 g_2(\gamma_{\mathcal{L}_1})_r^*(\gamma_{\mathcal{L}_1})_r}{4M_{\mathcal{L}_{1r}}^4} - \frac{g_2(g_{\mathcal{L}_1}^B)_{rs}(\gamma_{\mathcal{L}_1})_r^*(\gamma_{\mathcal{L}_1})_s}{4M_{\mathcal{L}_{1r}}^2 M_{\mathcal{L}_{1s}}^2} - \frac{g_1(g_{\mathcal{L}_1}^W)_{rs}(\gamma_{\mathcal{L}_1})_r^*(\gamma_{\mathcal{L}_1})_s}{4M_{\mathcal{L}_{1r}}^2 M_{\mathcal{L}_{1s}}^2} + \frac{1}{f}\left\{\frac{(\tilde{k}_{\Xi}^{WB})_r(\kappa_{\Xi})_r}{M_{\Xi_r}^2} - \frac{\mathrm{Im}\left((\tilde{\gamma}_{\mathcal{L}_1}^B)_r(\gamma_{\mathcal{L}_1})_r^*\right)g_2}{2M_{\mathcal{L}_{1r}}^2} - \frac{\mathrm{Im}\left((\tilde{\gamma}_{\mathcal{L}_1}^W)_r(\gamma_{\mathcal{L}_1})_r^*\right)g_1}{2M_{\mathcal{L}_{1r}}^2}\right\}, \tag{8.66}$$

$$Z_\phi C_{\phi W\tilde{B}} = -\frac{g_2(g_{\mathcal{L}_1}^{\tilde{B}})_{rs}(\gamma_{\mathcal{L}_1})_r^*(\gamma_{\mathcal{L}_1})_s}{4M_{\mathcal{L}_{1r}}^2 M_{\mathcal{L}_{1s}}^2} - \frac{g_1(g_{\mathcal{L}_1}^{\tilde{W}})_{rs}(\gamma_{\mathcal{L}_1})_r^*(\gamma_{\mathcal{L}_1})_s}{4M_{\mathcal{L}_{1r}}^2 M_{\mathcal{L}_{1s}}^2} + \frac{1}{f}\left\{\frac{(\tilde{k}_{\Xi}^{W\tilde{B}})_r(\kappa_{\Xi})_r}{M_{\Xi_r}^2} - \frac{\mathrm{Im}\left((\tilde{\gamma}_{\mathcal{L}_1}^{\tilde{B}})_r(\gamma_{\mathcal{L}_1})_r^*\right)g_2}{2M_{\mathcal{L}_{1r}}^2} - \frac{\mathrm{Im}\left((\tilde{\gamma}_{\mathcal{L}_1}^{\tilde{W}})_r(\gamma_{\mathcal{L}_1})_r^*\right)g_1}{2M_{\mathcal{L}_{1r}}^2}\right\}, \tag{8.67}$$

$$Z_\phi C_{\phi G} = \frac{1}{f}\frac{(\tilde{k}_{\mathcal{S}}^G)_r(\kappa_{\mathcal{S}})_r}{M_{\mathcal{S}_r}^2}, \tag{8.68}$$

$$Z_\phi C_{\phi \tilde{G}} = \frac{1}{f}\frac{(\tilde{k}_{\mathcal{S}}^{\tilde{G}})_r(\kappa_{\mathcal{S}})_r}{M_{\mathcal{S}_r}^2}. \tag{8.69}$$

8.5.5 Operators with bosons and fermions

There are three types of operators coupling bosonic and fermionic fields: the operators of the form $\psi^2\phi^3$ represent couplings between scalars and fermions only, while those of the form $X\psi^2\phi$ and $\psi^2 D\phi^2$ contain covariant interactions between the SM scalar, fermions and gauge fields.

$\psi^2\phi^3$

Due to the length of the contributions to the coefficients of the different $\psi^2\phi^3$ operators ($\mathcal{O}_{e\phi}$, $\mathcal{O}_{d\phi}$ and $\mathcal{O}_{u\phi}$), we have separated them as follows:

$$Z_\phi^{\frac{3}{2}}\ (C_{e\phi})_{ij} = \hat{y}^{e*}_{ji} a + b^e_{ij} + c^e_{ij}, \tag{8.70}$$

$$Z_\phi^{\frac{3}{2}}\ (C_{d\phi})_{ij} = \hat{y}^{d*}_{ji} a + b^d_{ij} + c^d_{ij}, \tag{8.71}$$

$$Z_\phi^{\frac{3}{2}}\ (C_{u\phi})_{ij} = \hat{y}^{u*}_{ji} a^* + b^u_{ij} + c^u_{ij}, \tag{8.72}$$

where the coefficients a, b^ψ_{ij} and c^ψ_{ij} are defined below (eqs. (8.73)–(8.79)). (The coefficients b^ψ_{ij} and c^ψ_{ij} refer to the contributions from only one type of particle and mixed contributions, respectively.)

Recall also that $\hat{g}^\phi_V$ contains contributions from $\mathcal{L}_1$ (see eqs. (8.12)–(8.15)) and that $\hat{y}^{e,d,u}_{ji}$ and $\hat{\lambda}_\phi$ are defined in eqs. (8.23) and (8.24).

$$
\begin{aligned}
a = {} & \frac{(\kappa_\Xi)_r(\kappa_\Xi)_r}{M_{\Xi_r}^4} + \frac{2(\kappa_{\Xi_1})_r^*(\kappa_{\Xi_1})_r}{M_{\Xi_{1r}}^4} - \frac{i\,\mathrm{Im}\left((\hat{g}_{\mathcal{B}}^\phi)_r(\hat{g}_{\mathcal{B}}^\phi)_r\right)}{2M_{\mathcal{B}_r}^2} - \frac{(\hat{g}_{\mathcal{B}_1}^\phi)_r^*(\hat{g}_{\mathcal{B}_1}^\phi)_r}{2M_{\mathcal{B}_{1r}}^2} \\
& - \frac{i\,\mathrm{Im}\left((\hat{g}_{\mathcal{W}}^\phi)_r(\hat{g}_{\mathcal{W}}^\phi)_r\right)}{8M_{\mathcal{W}_r}^2} - \frac{(\hat{g}_{\mathcal{W}}^\phi)_r^*(\hat{g}_{\mathcal{W}}^\phi)_r}{4M_{\mathcal{W}_r}^2} - \frac{(\hat{g}_{\mathcal{W}_1}^\phi)_r^*(\hat{g}_{\mathcal{W}_1}^\phi)_r}{8M_{\mathcal{W}_{1r}}^2} \\
& + \frac{g_2(g_{\mathcal{L}_1}^W)_{sr}(\gamma_{\mathcal{L}_1})_s^*(\gamma_{\mathcal{L}_1})_r}{2M_{\mathcal{L}_{1r}}^2 M_{\mathcal{L}_{1s}}^2} - \frac{(h_{\mathcal{L}_1}^{(1)})_{rs}(\gamma_{\mathcal{L}_1})_r^*(\gamma_{\mathcal{L}_1})_s}{2M_{\mathcal{L}_{1r}}^2 M_{\mathcal{L}_{1s}}^2} - \frac{i\,\mathrm{Im}\left((h_{\mathcal{L}_1}^{(3)})_{rs}(\gamma_{\mathcal{L}_1})_r^*(\gamma_{\mathcal{L}_1})_s^*\right)}{M_{\mathcal{L}_{1r}}^2 M_{\mathcal{L}_{1s}}^2} \\
& - \frac{i\,\mathrm{Re}\left((\hat{g}_{\mathcal{B}}^\phi)_r\right)(\delta_{\mathcal{B}\mathcal{S}})_{rs}(\kappa_{\mathcal{S}})_s}{M_{\mathcal{B}_r}^2 M_{\mathcal{S}_s}^2} \\
& + \frac{i\left((\hat{g}_{\mathcal{W}}^\phi)_r - 3(\hat{g}_{\mathcal{W}}^\phi)_s^*\right)(\delta_{\mathcal{W}\Xi})_{rs}(\kappa_\Xi)_s}{4M_{\mathcal{W}_r}^2 M_{\Xi_s}^2} - \frac{(\delta_{\mathcal{W}\Xi})_{ts}(\delta_{\mathcal{W}\Xi})_{tr}(\kappa_\Xi)_r(\kappa_\Xi)_s}{M_{\Xi_r}^2 M_{\Xi_s}^2 M_{\mathcal{W}_t}^2} \\
& - \frac{\mathrm{Im}\left((\hat{g}_{\mathcal{W}_1}^\phi)_r^*(\delta_{\mathcal{W}_1\Xi_1})_{rs}(\kappa_{\Xi_1})_s\right)}{M_{\Xi_{1s}}^2 M_{\mathcal{W}_{1r}}^2} - \frac{2(\delta_{\mathcal{W}_1\Xi_1})_{st}^*(\delta_{\mathcal{W}_1\Xi_1})_{sr}(\kappa_{\Xi_1})_r(\kappa_{\Xi_1})_t^*}{M_{\Xi_{1r}}^2 M_{\mathcal{W}_{1s}}^2 M_{\Xi_{1t}}^2} \\
& + \frac{i\,\mathrm{Im}\left((g_{\mathcal{S}\mathcal{L}_1}^{(1)})_{rs}^*(\gamma_{\mathcal{L}_1})_s^*\right)(\kappa_{\mathcal{S}})_r}{M_{\mathcal{S}_r}^2 M_{\mathcal{L}_{1s}}^2} + \frac{\mathrm{Re}\left((g_{\mathcal{S}\mathcal{L}_1}^{(2)})_{rs}(\gamma_{\mathcal{L}_1})_s\right)(\kappa_{\mathcal{S}})_r}{M_{\mathcal{S}_r}^2 M_{\mathcal{L}_{1s}}^2} \\
& + \frac{(\varepsilon_{\mathcal{S}\mathcal{L}_1})_{rts}(\kappa_{\mathcal{S}})_r(\gamma_{\mathcal{L}_1})_t^*(\gamma_{\mathcal{L}_1})_s}{2M_{\mathcal{S}_r}^2 M_{\mathcal{L}_{1s}}^2 M_{\mathcal{L}_{1t}}^2} + \frac{(\delta_{\mathcal{L}_1\varphi})_{sr}(\gamma_{\mathcal{L}_1})_s^*(\lambda_\varphi)_r}{M_{\varphi_r}^2 M_{\mathcal{L}_{1s}}^2} \\
& + \frac{2\hat{\lambda}_\phi(\delta_{\mathcal{L}_1\varphi})_{ts}^*(\gamma_{\mathcal{L}_1})_t(\delta_{\mathcal{L}_1\varphi})_{rs}(\gamma_{\mathcal{L}_1})_r^*}{M_{\mathcal{L}_{1r}}^2 M_{\varphi_s}^2 M_{\mathcal{L}_{1t}}^2} + \frac{\left((g_{\Xi\mathcal{L}_1}^{(1)})_{rs}(\gamma_{\mathcal{L}_1})_s + 3(g_{\Xi\mathcal{L}_1}^{(1)})_{rs}^*(\gamma_{\mathcal{L}_1})_s^*\right)(\kappa_\Xi)_r}{2M_{\Xi_r}^2 M_{\mathcal{L}_{1s}}^2} \\
& - \frac{\mathrm{Re}\left((g_{\Xi\mathcal{L}_1}^{(2)})_{rs}(\gamma_{\mathcal{L}_1})_s\right)(\kappa_\Xi)_r}{M_{\Xi_r}^2 M_{\mathcal{L}_{1s}}^2} - \frac{(\varepsilon_{\Xi\mathcal{L}_1})_{srt}(\kappa_\Xi)_s(\gamma_{\mathcal{L}_1})_r^*(\gamma_{\mathcal{L}_1})_t}{2M_{\mathcal{L}_{1r}}^2 M_{\Xi_s}^2 M_{\mathcal{L}_{1t}}^2} \\
& + \frac{2\,\mathrm{Re}\left((g_{\Xi_1\mathcal{L}_1}^{(1)})_{rs}^*(\gamma_{\mathcal{L}_1})_s^*(\kappa_{\Xi_1})_r\right)}{M_{\mathcal{L}_{1s}}^2 M_{\Xi_{1r}}^2} - \frac{2i\,\mathrm{Im}\left((g_{\Xi_1\mathcal{L}_1}^{(2)})_{rs}(\kappa_{\Xi_1})_r^*(\gamma_{\mathcal{L}_1})_s\right)}{M_{\mathcal{L}_{1s}}^2 M_{\Xi_{1r}}^2} \\
& - \frac{2i\,\mathrm{Im}\left((\varepsilon_{\Xi_1\mathcal{L}_1})_{rst}^*(\kappa_{\Xi_1})_r^*(\gamma_{\mathcal{L}_1})_t(\gamma_{\mathcal{L}_1})_s\right)}{M_{\mathcal{L}_{1s}}^2 M_{\mathcal{L}_{1t}}^2 M_{\Xi_{1r}}^2} \\
& - \frac{(\delta_{\mathcal{L}_1\varphi})_{tr}(\gamma_{\mathcal{L}_1})_t^*(\kappa_{\mathcal{S}\varphi})_{sr}(\kappa_{\mathcal{S}})_s}{M_{\varphi_r}^2 M_{\mathcal{S}_s}^2 M_{\mathcal{L}_{1t}}^2} - \frac{(\delta_{\mathcal{L}_1\varphi})_{rt}(\gamma_{\mathcal{L}_1})_r^*(\kappa_{\Xi\varphi})_{st}(\kappa_\Xi)_s}{M_{\mathcal{L}_{1r}}^2 M_{\Xi_s}^2 M_{\varphi_t}^2} \\
& - \frac{2(\delta_{\mathcal{L}_1\varphi})_{sr}(\gamma_{\mathcal{L}_1})_s^*(\kappa_{\Xi_1\varphi})_{tr}^*(\kappa_{\Xi_1})_t}{M_{\varphi_r}^2 M_{\mathcal{L}_{1s}}^2 M_{\Xi_{1t}}^2} \\
& + \frac{1}{f}\left\{\frac{(\tilde{k}_{\mathcal{S}}^\phi)_r(\kappa_{\mathcal{S}})_r}{2M_{\mathcal{S}_r}^2} - \frac{(\tilde{k}_\Xi^\phi)_r(\kappa_\Xi)_r}{2M_{\Xi_r}^2} - \frac{2i\,\mathrm{Im}\left((\tilde{k}_{\Xi_1})_r(\kappa_{\Xi_1})_r^*\right)}{M_{\Xi_{1r}}^2} - \frac{i\,\mathrm{Im}\left((\tilde{\gamma}_{\mathcal{L}_1}^{(2)})_r(\gamma_{\mathcal{L}_1})_r^*\right)}{M_{\mathcal{L}_{1r}}^2}\right. \\
& \qquad \left. - \frac{\mathrm{Re}\left((\tilde{\gamma}_{\mathcal{L}_1}^{(3)})_r(\gamma_{\mathcal{L}_1})_r^*\right)}{M_{\mathcal{L}_{1r}}^2} + \frac{\mathrm{Im}\left((\tilde{\gamma}_{\mathcal{L}_1}^W)_r(\gamma_{\mathcal{L}_1})_r^*\right)g_2}{M_{\mathcal{L}_{1r}}^2}\right\},
\end{aligned}
\tag{8.73}
$$

$$
\begin{aligned}
b^e_{ij} =& \frac{(\lambda_\varphi)_r (y^e_\varphi)^*_{rji}}{M^2_{\varphi_r}} + \frac{\hat{y}^{e*}_{jk}(\lambda_E)_{rk}(\lambda_E)^*_{ri}}{2M^2_{E_r}} + \frac{\hat{y}^{e*}_{ki}(\lambda_{\Delta_1})_{rj}(\lambda_{\Delta_1})^*_{rk}}{2M^2_{\Delta_{1r}}} \\
&+ \frac{\hat{y}^{e*}_{ki}(\lambda_{\Delta_3})_{rj}(\lambda_{\Delta_3})^*_{rk}}{2M^2_{\Delta_{3r}}} + \frac{\hat{y}^{e*}_{jk}(\lambda_\Sigma)^*_{ri}(\lambda_\Sigma)_{rk}}{4M^2_{\Sigma_r}} + \frac{\hat{y}^{e*}_{jk}(\lambda_{\Sigma_1})_{rk}(\lambda_{\Sigma_1})^*_{ri}}{8M^2_{\Sigma_{1r}}} \\
&+ \frac{i\hat{y}^{e*}_{jk}\,\mathrm{Im}\left((\hat{g}^\phi_{\mathcal{B}})_r\right)(g^l_{\mathcal{B}})_{rik}}{M^2_{\mathcal{B}_r}} - \frac{i\hat{y}^{e*}_{ki}\,\mathrm{Im}\left((\hat{g}^\phi_{\mathcal{B}})_r\right)(g^e_{\mathcal{B}})_{rkj}}{M^2_{\mathcal{B}_r}} + \frac{i\hat{y}^{e*}_{jk}\,\mathrm{Im}\left((\hat{g}^\phi_{\mathcal{W}})_r\right)(g^l_{\mathcal{W}})_{rik}}{4M^2_{\mathcal{W}_r}} \\
&+ \frac{1}{f}\Bigg\{ \frac{(\tilde{y}^e_{\mathcal{S}})^*_{rji}(\kappa_{\mathcal{S}})_r}{M^2_{\mathcal{S}_r}} + \frac{(\tilde{y}^e_\Xi)^*_{rji}(\kappa_\Xi)_r}{M^2_{\Xi_r}} + \frac{2(\tilde{y}^e_{\Xi_1})^*_{rji}(\kappa_{\Xi_1})_r}{M^2_{\Xi_{1r}}} + \frac{i\hat{y}^{e*}_{jk}(\tilde{\lambda}^l_E)_{rk}(\lambda_E)^*_{ri}}{2M_{E_r}} \\
&\quad + \frac{(\tilde{\lambda}^e_E)_{rj}(\lambda_E)^*_{ri}}{M_{E_r}} + \frac{i\hat{y}^{e*}_{jk}(\tilde{\lambda}^l_E)^*_{ri}(\lambda_E)_{rk}}{2M_{E_r}} - \frac{i\hat{y}^{e*}_{ki}(\tilde{\lambda}^e_{\Delta_1})^*_{rk}(\lambda_{\Delta_1})_{rj}}{2M_{\Delta_{1r}}} \\
&\quad + \frac{(\tilde{\lambda}^l_{\Delta_1})^*_{ri}(\lambda_{\Delta_1})_{rj}}{M_{\Delta_{1r}}} + \frac{(\tilde{\lambda}^{l\prime}_{\Delta_1})^*_{ri}(\lambda_{\Delta_1})_{rj}}{M_{\Delta_{1r}}} - \frac{i\hat{y}^{e*}_{ki}(\tilde{\lambda}^e_{\Delta_1})_{rj}(\lambda_{\Delta_1})^*_{rk}}{2M_{\Delta_{1r}}} \\
&\quad - \frac{i\hat{y}^{e*}_{ki}(\tilde{\lambda}^e_{\Delta_3})^*_{rk}(\lambda_{\Delta_3})_{rj}}{2M_{\Delta_{3r}}} + \frac{(\tilde{\lambda}^l_{\Delta_3})^*_{ri}(\lambda_{\Delta_3})_{rj}}{M_{\Delta_{3r}}} - \frac{i\hat{y}^{e*}_{ki}(\tilde{\lambda}^e_{\Delta_3})_{rj}(\lambda_{\Delta_3})^*_{rk}}{2M_{\Delta_{3r}}} \\
&\quad + \frac{i\hat{y}^{e*}_{jk}(\tilde{\lambda}^l_\Sigma)^*_{ri}(\lambda_\Sigma)_{rk}}{2M_{\Sigma_r}} + \frac{i\hat{y}^{e*}_{jk}(\tilde{\lambda}^l_\Sigma)_{rk}(\lambda_\Sigma)^*_{ri}}{2M_{\Sigma_r}} + \frac{(\tilde{\lambda}^e_\Sigma)_{rj}(\lambda_\Sigma)^*_{ri}}{M_{\Sigma_r}} \\
&\quad + \frac{i\hat{y}^{e*}_{jk}(\tilde{\lambda}^l_{\Sigma_1})_{rk}(\lambda_{\Sigma_1})^*_{ri}}{4M_{\Sigma_{1r}}} + \frac{(\tilde{\lambda}^e_{\Sigma_1})_{rj}(\lambda_{\Sigma_1})^*_{ri}}{2M_{\Sigma_{1r}}} + \frac{i\hat{y}^{e*}_{jk}(\tilde{\lambda}^l_{\Sigma_1})^*_{ri}(\lambda_{\Sigma_1})_{rk}}{4M_{\Sigma_{1r}}} \\
&\quad + \frac{\hat{y}^{e*}_{jk}\hat{y}^e_{lk}(\tilde{g}^{eDl}_{\mathcal{L}_1})^*_{rli}(\gamma_{\mathcal{L}_1})_r}{4M^2_{\mathcal{L}_{1r}}} - \frac{\hat{y}^{e*}_{ki}\hat{y}^e_{kl}(\tilde{g}^{eDl}_{\mathcal{L}_1})^*_{rjl}(\gamma_{\mathcal{L}_1})_r}{4M^2_{\mathcal{L}_{1r}}} - \frac{\hat{\lambda}_\phi(\tilde{g}^{eDl}_{\mathcal{L}_1})^*_{rji}(\gamma_{\mathcal{L}_1})_r}{M^2_{\mathcal{L}_{1r}}} \\
&\quad - \frac{\hat{y}^{e*}_{jk}\hat{y}^e_{lk}(\tilde{g}^{Del}_{\mathcal{L}_1})^*_{rli}(\gamma_{\mathcal{L}_1})_r}{4M^2_{\mathcal{L}_{1r}}} + \frac{\hat{y}^{e*}_{ki}\hat{y}^e_{kl}(\tilde{g}^{Del}_{\mathcal{L}_1})^*_{rjl}(\gamma_{\mathcal{L}_1})_r}{4M^2_{\mathcal{L}_{1r}}} - \frac{\hat{\lambda}_\phi(\tilde{g}^{Del}_{\mathcal{L}_1})^*_{rji}(\gamma_{\mathcal{L}_1})_r}{M^2_{\mathcal{L}_{1r}}} \\
&\quad + \frac{i\hat{y}^{e*}_{ki}(\tilde{g}^e_{\mathcal{L}_1})_{rkj}(\gamma_{\mathcal{L}_1})^*_r}{2M^2_{\mathcal{L}_{1r}}} - \frac{i\hat{y}^{e*}_{jk}(\tilde{g}^l_{\mathcal{L}_1})_{rik}(\gamma_{\mathcal{L}_1})^*_r}{2M^2_{\mathcal{L}_{1r}}} - \frac{i\hat{y}^{e*}_{jk}(\tilde{g}^{l\prime}_{\mathcal{L}_1})_{rik}(\gamma_{\mathcal{L}_1})^*_r}{2M^2_{\mathcal{L}_{1r}}} \\
&\quad + \frac{i\hat{y}^{e*}_{ki}(\tilde{g}^e_{\mathcal{L}_1})^*_{rjk}(\gamma_{\mathcal{L}_1})_r}{2M^2_{\mathcal{L}_{1r}}} - \frac{i\hat{y}^{e*}_{jk}(\tilde{g}^l_{\mathcal{L}_1})^*_{rki}(\gamma_{\mathcal{L}_1})_r}{2M^2_{\mathcal{L}_{1r}}} - \frac{i\hat{y}^{e*}_{jk}(\tilde{g}^{l\prime}_{\mathcal{L}_1})^*_{rki}(\gamma_{\mathcal{L}_1})_r}{2M^2_{\mathcal{L}_{1r}}} \Bigg\},
\end{aligned} \tag{8.74}
$$

$$
\begin{aligned}
c^e_{ij} = &-\frac{(\kappa_{\mathcal{S}\varphi})_{rs}(\kappa_{\mathcal{S}})_r(y^e_\varphi)^*_{sji}}{M^2_{\mathcal{S}_r}M^2_{\varphi_s}}-\frac{(\kappa_{\Xi\varphi})_{sr}(\kappa_\Xi)_s(y^e_\varphi)^*_{rji}}{M^2_{\varphi_r}M^2_{\Xi_s}}\\
&-\frac{2(\kappa_{\Xi_1\varphi})^*_{sr}(\kappa_{\Xi_1})_s(y^e_\varphi)^*_{rji}}{M^2_{\varphi_r}M^2_{\Xi_{1s}}}-\frac{(\lambda_{E\Delta_1})_{rs}(\lambda_E)^*_{ri}(\lambda_{\Delta_1})_{sj}}{M_{E_r}M_{\Delta_{1s}}}\\
&-\frac{(\lambda_{E\Delta_3})_{sr}(\lambda_E)^*_{si}(\lambda_{\Delta_3})_{rj}}{M_{\Delta_{3r}}M_{E_s}}-\frac{(\lambda_\Sigma)^*_{si}(\lambda_{\Sigma\Delta_1})_{sr}(\lambda_{\Delta_1})_{rj}}{2M_{\Delta_{1r}}M_{\Sigma_s}}\\
&-\frac{(\lambda_{\Sigma_1\Delta_1})_{rs}(\lambda_{\Sigma_1})^*_{ri}(\lambda_{\Delta_1})_{sj}}{4M_{\Sigma_{1r}}M_{\Delta_{1s}}}+\frac{(\lambda_{\Sigma_1\Delta_3})_{sr}(\lambda_{\Sigma_1})^*_{si}(\lambda_{\Delta_3})_{rj}}{4M_{\Delta_{3r}}M_{\Sigma_{1s}}}\\
&-\frac{(w_{\mathcal{S}E})_{rsj}(\kappa_{\mathcal{S}})_r(\lambda_E)^*_{si}}{M^2_{\mathcal{S}_r}M_{E_s}}-\frac{(w_{\mathcal{S}\Delta_1})^*_{rsi}(\kappa_{\mathcal{S}})_r(\lambda_{\Delta_1})_{sj}}{M^2_{\mathcal{S}_r}M_{\Delta_{1s}}}\\
&-\frac{(w_{\Xi\Delta_3})^*_{rsi}(\kappa_\Xi)_r(\lambda_{\Delta_1})_{sj}}{M^2_{\Xi_r}M_{\Delta_{1s}}}-\frac{(w_{\Xi\Sigma_1})_{srj}(\kappa_\Xi)_s(\lambda_{\Sigma_1})^*_{ri}}{2M_{\Sigma_{1r}}M^2_{\Xi_s}}\\
&-\frac{2(w_{\Xi_1\Delta_3})^*_{rsi}(\kappa_{\Xi_1})_r(\lambda_{\Delta_3})_{sj}}{M^2_{\Xi_{1r}}M_{\Delta_{3s}}}-\frac{(\lambda_\Sigma)^*_{si}(w_{\Xi_1\Sigma})^*_{rsj}(\kappa_{\Xi_1})_r}{M^2_{\Xi_{1r}}M_{\Sigma_s}}\\
&+\frac{i\hat{y}^{e*}_{jk}(z_{E\mathcal{L}_1})_{rsk}(\lambda_E)^*_{ri}(\gamma_{\mathcal{L}_1})^*_s}{2M_{E_r}M^2_{\mathcal{L}_{1s}}}+\frac{i\hat{y}^{e*}_{jk}(z_{E\mathcal{L}_1})^*_{rsi}(\gamma_{\mathcal{L}_1})_s(\lambda_E)_{rk}}{2M_{E_r}M^2_{\mathcal{L}_{1s}}}\\
&-\frac{i\hat{y}^{e*}_{ki}(z_{\Delta_1\mathcal{L}_1})^*_{rsk}(\gamma_{\mathcal{L}_1})^*_s(\lambda_{\Delta_1})_{rj}}{2M_{\Delta_{1r}}M^2_{\mathcal{L}_{1s}}}-\frac{i\hat{y}^{e*}_{ki}(z_{\Delta_1\mathcal{L}_1})_{rsj}(\lambda_{\Delta_1})^*_{rk}(\gamma_{\mathcal{L}_1})_s}{2M_{\Delta_{1r}}M^2_{\mathcal{L}_{1s}}}\\
&-\frac{i\hat{y}^{e*}_{ki}(z_{\Delta_3\mathcal{L}_1})^*_{srk}(\gamma_{\mathcal{L}_1})_r(\lambda_{\Delta_3})_{sj}}{2M^2_{\mathcal{L}_{1r}}M_{\Delta_{3s}}}-\frac{i\hat{y}^{e*}_{ki}(z_{\Delta_3\mathcal{L}_1})_{srj}(\lambda_{\Delta_3})^*_{sk}(\gamma_{\mathcal{L}_1})^*_r}{2M^2_{\mathcal{L}_{1r}}M_{\Delta_{3s}}}\\
&+\frac{i\hat{y}^{e*}_{jk}(\lambda_\Sigma)^*_{ri}(z_{\Sigma\mathcal{L}_1})_{rsk}(\gamma_{\mathcal{L}_1})_s}{2M_{\Sigma_r}M^2_{\mathcal{L}_{1s}}}+\frac{i\hat{y}^{e*}_{jk}(\lambda_\Sigma)_{sk}(z_{\Sigma\mathcal{L}_1})^*_{sri}(\gamma_{\mathcal{L}_1})^*_r}{2M^2_{\mathcal{L}_{1r}}M_{\Sigma_s}}\\
&+\frac{i\hat{y}^{e*}_{jk}(z_{\Sigma_1\mathcal{L}_1})_{srk}(\lambda_{\Sigma_1})^*_{si}(\gamma_{\mathcal{L}_1})^*_r}{4M^2_{\mathcal{L}_{1r}}M_{\Sigma_{1s}}}+\frac{i\hat{y}^{e*}_{jk}(z_{\Sigma_1\mathcal{L}_1})^*_{rsi}(\gamma_{\mathcal{L}_1})_s(\lambda_{\Sigma_1})_{rk}}{4M_{\Sigma_{1r}}M^2_{\mathcal{L}_{1s}}}\\
&+\frac{i\hat{y}^{e*}_{jk}(\delta_{\mathcal{BS}})_{rs}(g^l_{\mathcal{B}})_{rik}(\kappa_{\mathcal{S}})_s}{M^2_{\mathcal{B}_r}M^2_{\mathcal{S}_s}}-\frac{i\hat{y}^{e*}_{ki}(\delta_{\mathcal{BS}})_{rs}(g^e_{\mathcal{B}})_{rkj}(\kappa_{\mathcal{S}})_s}{M^2_{\mathcal{B}_r}M^2_{\mathcal{S}_s}}\\
&+\frac{i\hat{y}^{e*}_{jk}(\delta_{\mathcal{W}\Xi})_{sr}(g^l_{\mathcal{W}})_{sik}(\kappa_\Xi)_r}{2M^2_{\Xi_r}M^2_{\mathcal{W}_s}}+\frac{2\hat{\lambda}_\phi(\delta_{\mathcal{L}_1\varphi})^*_{sr}(\gamma_{\mathcal{L}_1})_s(y^e_\varphi)^*_{rji}}{M^2_{\varphi_r}M^2_{\mathcal{L}_{1s}}},
\end{aligned}
\tag{8.75}
$$

$$
\begin{aligned}
b^d_{ij} =& \frac{(\lambda_\varphi)_r (y^d_\varphi)^*_{rji}}{M^2_{\varphi_r}} + \frac{\hat{y}^{d*}_{jk}(\lambda_D)_{rk}(\lambda_D)^*_{ri}}{2M^2_{D_r}} + \frac{\hat{y}^{d*}_{ki}(\lambda^d_{Q_1})_{rj}(\lambda^d_{Q_1})^*_{rk}}{2M^2_{Q_{1r}}} \\
&+ \frac{\hat{y}^{d*}_{ki}(\lambda_{Q_5})_{rj}(\lambda_{Q_5})^*_{rk}}{2M^2_{Q_{5r}}} + \frac{\hat{y}^{d*}_{jk}(\lambda_{T_1})_{rk}(\lambda_{T_1})^*_{ri}}{8M^2_{T_{1r}}} + \frac{\hat{y}^{d*}_{jk}(\lambda_{T_2})_{rk}(\lambda_{T_2})^*_{ri}}{4M^2_{T_{2r}}} \\
&+ \frac{i\hat{y}^{d*}_{jk}\,\mathrm{Im}\left((\hat{g}^\phi_{\mathcal{B}})_r\right)(g^q_{\mathcal{B}})_{rik}}{M^2_{\mathcal{B}_r}} - \frac{i\hat{y}^{d*}_{ki}\,\mathrm{Im}\left((\hat{g}^\phi_{\mathcal{B}})_r\right)(g^d_{\mathcal{B}})_{rkj}}{M^2_{\mathcal{B}_r}} + \frac{i\hat{y}^{d*}_{jk}\,\mathrm{Im}\left((\hat{g}^\phi_{\mathcal{W}})_r\right)(g^q_{\mathcal{W}})_{rik}}{4M^2_{\mathcal{W}_r}} \\
&+ \frac{1}{f}\Bigg\{ \frac{(\tilde{y}^d_{\mathcal{S}})^*_{rji}(\kappa_{\mathcal{S}})_r}{M^2_{\mathcal{S}_r}} + \frac{(\tilde{y}^d_{\Xi})^*_{rji}(\kappa_{\Xi})_r}{M^2_{\Xi_r}} + \frac{2(\tilde{y}^d_{\Xi_1})^*_{rji}(\kappa_{\Xi_1})_r}{M^2_{\Xi_{1r}}} + \frac{i\hat{y}^{d*}_{jk}(\tilde{\lambda}^q_D)_{rk}(\lambda_D)^*_{ri}}{2M_{D_r}} \\
&\quad + \frac{(\tilde{\lambda}^d_D)_{rj}(\lambda_D)^*_{ri}}{M_{D_r}} + \frac{i\hat{y}^{d*}_{jk}(\tilde{\lambda}^q_D)^*_{ri}(\lambda_D)_{rk}}{2M_{D_r}} - \frac{i\hat{y}^{d*}_{ki}(\tilde{\lambda}^d_{Q_1})^*_{rk}(\lambda^d_{Q_1})_{rj}}{2M_{Q_{1r}}} \\
&\quad + \frac{(\tilde{\lambda}^q_{Q_1})^*_{ri}(\lambda^d_{Q_1})_{rj}}{M_{Q_{1r}}} + \frac{(\tilde{\lambda}^{q\prime}_{Q_1})^*_{ri}(\lambda^d_{Q_1})_{rj}}{M_{Q_{1r}}} - \frac{i\hat{y}^{d*}_{ki}(\tilde{\lambda}^d_{Q_1})_{rj}(\lambda^d_{Q_1})^*_{rk}}{2M_{Q_{1r}}} \\
&\quad - \frac{i\hat{y}^{d*}_{ki}(\tilde{\lambda}^d_{Q_5})^*_{rk}(\lambda_{Q_5})_{rj}}{2M_{Q_{5r}}} + \frac{(\tilde{\lambda}^q_{Q_5})^*_{ri}(\lambda_{Q_5})_{rj}}{M_{Q_{5r}}} - \frac{i\hat{y}^{d*}_{ki}(\tilde{\lambda}^d_{Q_5})_{rj}(\lambda_{Q_5})^*_{rk}}{2M_{Q_{5r}}} \\
&\quad + \frac{i\hat{y}^{d*}_{jk}(\tilde{\lambda}^q_{T_1})_{rk}(\lambda_{T_1})^*_{ri}}{4M_{T_{1r}}} + \frac{(\tilde{\lambda}^d_{T_1})_{rj}(\lambda_{T_1})^*_{ri}}{2M_{T_{1r}}} + \frac{i\hat{y}^{d*}_{jk}(\tilde{\lambda}^q_{T_1})^*_{ri}(\lambda_{T_1})_{rk}}{4M_{T_{1r}}} \\
&\quad + \frac{i\hat{y}^{d*}_{jk}(\tilde{\lambda}^q_{T_2})_{rk}(\lambda_{T_2})^*_{ri}}{2M_{T_{2r}}} + \frac{(\tilde{\lambda}^d_{T_2})_{rj}(\lambda_{T_2})^*_{ri}}{M_{T_{2r}}} + \frac{i\hat{y}^{d*}_{jk}(\tilde{\lambda}^q_{T_2})^*_{ri}(\lambda_{T_2})_{rk}}{2M_{T_{2r}}} \\
&\quad + \frac{\hat{y}^{d*}_{jk}\hat{y}^d_{lk}(\tilde{g}^{dDq}_{\mathcal{L}_1})^*_{rli}(\gamma_{\mathcal{L}_1})_r}{4M^2_{\mathcal{L}_{1r}}} - \frac{\hat{y}^{d*}_{ki}\hat{y}^d_{kl}(\tilde{g}^{dDq}_{\mathcal{L}_1})^*_{rjl}(\gamma_{\mathcal{L}_1})_r}{4M^2_{\mathcal{L}_{1r}}} - \frac{\hat{\lambda}_\phi(\tilde{g}^{dDq}_{\mathcal{L}_1})^*_{rji}(\gamma_{\mathcal{L}_1})_r}{M^2_{\mathcal{L}_{1r}}} \\
&\quad - \frac{\hat{y}^{d*}_{jk}\hat{y}^d_{lk}(\tilde{g}^{Ddq}_{\mathcal{L}_1})^*_{rli}(\gamma_{\mathcal{L}_1})_r}{4M^2_{\mathcal{L}_{1r}}} + \frac{\hat{y}^{d*}_{ki}\hat{y}^d_{kl}(\tilde{g}^{Ddq}_{\mathcal{L}_1})^*_{rjl}(\gamma_{\mathcal{L}_1})_r}{4M^2_{\mathcal{L}_{1r}}} - \frac{\hat{\lambda}_\phi(\tilde{g}^{Ddq}_{\mathcal{L}_1})^*_{rji}(\gamma_{\mathcal{L}_1})_r}{M^2_{\mathcal{L}_{1r}}} \\
&\quad + \frac{i\hat{y}^{d*}_{ki}(\tilde{g}^d_{\mathcal{L}_1})_{rkj}(\gamma_{\mathcal{L}_1})^*_r}{2M^2_{\mathcal{L}_{1r}}} - \frac{i\hat{y}^{d*}_{jk}(\tilde{g}^q_{\mathcal{L}_1})_{rik}(\gamma_{\mathcal{L}_1})^*_r}{2M^2_{\mathcal{L}_{1r}}} - \frac{i\hat{y}^{d*}_{jk}(\tilde{g}^{q\prime}_{\mathcal{L}_1})_{rik}(\gamma_{\mathcal{L}_1})^*_r}{2M^2_{\mathcal{L}_{1r}}} \\
&\quad + \frac{i\hat{y}^{d*}_{ki}(\tilde{g}^d_{\mathcal{L}_1})^*_{rjk}(\gamma_{\mathcal{L}_1})_r}{2M^2_{\mathcal{L}_{1r}}} - \frac{i\hat{y}^{d*}_{jk}(\tilde{g}^q_{\mathcal{L}_1})^*_{rki}(\gamma_{\mathcal{L}_1})_r}{2M^2_{\mathcal{L}_{1r}}} - \frac{i\hat{y}^{d*}_{jk}(\tilde{g}^{q\prime}_{\mathcal{L}_1})^*_{rki}(\gamma_{\mathcal{L}_1})_r}{2M^2_{\mathcal{L}_{1r}}} \Bigg\},
\end{aligned}
\tag{8.76}
$$

$$
\begin{aligned}
c^d_{ij} = &-\frac{(\kappa_{\mathcal{S}\varphi})_{rs}(\kappa_{\mathcal{S}})_r(y^d_\varphi)^*_{sji}}{M^2_{\mathcal{S}_r}M^2_{\varphi_s}} - \frac{(\kappa_{\Xi\varphi})_{rs}(\kappa_{\Xi})_r(y^d_\varphi)^*_{sji}}{M^2_{\Xi_r}M^2_{\varphi_s}} \\
&- \frac{2(\kappa_{\Xi_1\varphi})^*_{sr}(\kappa_{\Xi_1})_s(y^d_\varphi)^*_{rji}}{M^2_{\varphi_r}M^2_{\Xi_{1s}}} - \frac{(\lambda_{DQ_1})_{sr}(\lambda_D)^*_{si}(\lambda^d_{Q_1})_{rj}}{M_{Q_{1r}}M_{D_s}} \\
&- \frac{(\lambda_{DQ_5})_{rs}(\lambda_D)^*_{ri}(\lambda_{Q_5})_{sj}}{M_{D_r}M_{Q_{5s}}} - \frac{(\lambda_{T_1Q_1})_{rs}(\lambda_{T_1})^*_{ri}(\lambda^d_{Q_1})_{sj}}{4M_{T_{1r}}M_{Q_{1s}}} \\
&- \frac{(\lambda_{T_2Q_1})_{sr}(\lambda_{T_2})^*_{si}(\lambda^d_{Q_1})_{rj}}{2M_{Q_{1r}}M_{T_{2s}}} + \frac{(\lambda_{T_1Q_5})_{rs}(\lambda_{T_1})^*_{ri}(\lambda_{Q_5})_{sj}}{4M_{T_{1r}}M_{Q_{5s}}} \\
&- \frac{(w_{\mathcal{S}D})_{rsj}(\kappa_{\mathcal{S}})_r(\lambda_D)^*_{si}}{M^2_{\mathcal{S}_r}M_{D_s}} - \frac{(w_{\mathcal{S}Q_1})^*_{rsi}(\kappa_{\mathcal{S}})_r(\lambda^d_{Q_1})_{sj}}{M^2_{\mathcal{S}_r}M_{Q_{1s}}} \\
&- \frac{(w_{\Xi Q_7})^*_{sri}(\kappa_{\Xi})_s(\lambda^d_{Q_1})_{rj}}{M_{Q_{1r}}M^2_{\Xi_s}} - \frac{(w_{\Xi T_1})_{rsj}(\kappa_{\Xi})_r(\lambda_{T_1})^*_{si}}{2M^2_{\Xi_r}M_{T_{1s}}} \\
&- \frac{2(w_{\Xi_1 Q_5})^*_{rsi}(\kappa_{\Xi_1})_r(\lambda_{Q_5})_{sj}}{M^2_{\Xi_{1r}}M_{Q_{5s}}} - \frac{(w_{\Xi_1 T_2})_{rsj}(\kappa_{\Xi_1})_r(\lambda_{T_2})^*_{si}}{M^2_{\Xi_{1r}}M_{T_{2s}}} \\
&+ \frac{i\hat{y}^{d*}_{jk}(z_{D\mathcal{L}_1})_{rsk}(\lambda_D)^*_{ri}(\gamma_{\mathcal{L}_1})^*_s}{2M_{D_r}M^2_{\mathcal{L}_{1s}}} + \frac{i\hat{y}^{d*}_{jk}(z_{D\mathcal{L}_1})^*_{rsi}(\gamma_{\mathcal{L}_1})_s(\lambda_D)_{rk}}{2M_{D_r}M^2_{\mathcal{L}_{1s}}} \\
&- \frac{i\hat{y}^{d*}_{ki}(z^d_{Q_1\mathcal{L}_1})^*_{rsk}(\gamma_{\mathcal{L}_1})^*_s(\lambda^d_{Q_1})_{rj}}{2M_{Q_{1r}}M^2_{\mathcal{L}_{1s}}} - \frac{i\hat{y}^{d*}_{ki}(z^d_{Q_1\mathcal{L}_1})_{rsj}(\lambda^d_{Q_1})^*_{rk}(\gamma_{\mathcal{L}_1})_s}{2M_{Q_{1r}}M^2_{\mathcal{L}_{1s}}} \\
&- \frac{i\hat{y}^{d*}_{ki}(z_{Q_5\mathcal{L}_1})^*_{rsk}(\gamma_{\mathcal{L}_1})_s(\lambda_{Q_5})_{rj}}{2M_{Q_{5r}}M^2_{\mathcal{L}_{1s}}} - \frac{i\hat{y}^{d*}_{ki}(z_{Q_5\mathcal{L}_1})_{rsj}(\lambda_{Q_5})^*_{rk}(\gamma_{\mathcal{L}_1})^*_s}{2M_{Q_{5r}}M^2_{\mathcal{L}_{1s}}} \\
&+ \frac{i\hat{y}^{d*}_{jk}(z_{T_2\mathcal{L}_1})_{rsk}(\lambda_{T_2})^*_{ri}(\gamma_{\mathcal{L}_1})_s}{2M_{T_{2r}}M^2_{\mathcal{L}_{1s}}} + \frac{i\hat{y}^{d*}_{jk}(z_{T_2\mathcal{L}_1})^*_{sri}(\gamma_{\mathcal{L}_1})^*_r(\lambda_{T_2})_{sk}}{2M^2_{\mathcal{L}_{1r}}M_{T_{2s}}} \\
&+ \frac{i\hat{y}^{d*}_{jk}(z_{T_1\mathcal{L}_1})_{rsk}(\lambda_{T_1})^*_{ri}(\gamma_{\mathcal{L}_1})^*_s}{4M_{T_{1r}}M^2_{\mathcal{L}_{1s}}} + \frac{i\hat{y}^{d*}_{jk}(z_{T_1\mathcal{L}_1})^*_{sri}(\gamma_{\mathcal{L}_1})_r(\lambda_{T_1})_{sk}}{4M^2_{\mathcal{L}_{1r}}M_{T_{1s}}} \\
&+ \frac{i\hat{y}^{d*}_{jk}(\delta_{\mathcal{B}\mathcal{S}})_{rs}(g^q_{\mathcal{B}})_{rik}(\kappa_{\mathcal{S}})_s}{M^2_{\mathcal{B}_r}M^2_{\mathcal{S}_s}} - \frac{i\hat{y}^{d*}_{ki}(\delta_{\mathcal{B}\mathcal{S}})_{rs}(g^d_{\mathcal{B}})_{rkj}(\kappa_{\mathcal{S}})_s}{M^2_{\mathcal{B}_r}M^2_{\mathcal{S}_s}} \\
&+ \frac{i\hat{y}^{d*}_{jk}(\delta_{\mathcal{W}\Xi})_{sr}(g^q_{\mathcal{W}})_{sik}(\kappa_{\Xi})_r}{2M^2_{\Xi_r}M^2_{\mathcal{W}_s}} + \frac{2\hat{\lambda}_\phi(\delta_{\mathcal{L}_1\varphi})^*_{sr}(\gamma_{\mathcal{L}_1})_s(y^d_\varphi)^*_{rji}}{M^2_{\varphi_r}M^2_{\mathcal{L}_{1s}}},
\end{aligned}
\tag{8.77}
$$

$$
\begin{aligned}
b^u_{ij} = & -\frac{(\lambda_\varphi)^*_r (y^u_\varphi)_{rij}}{M^2_{\varphi_r}} + \frac{\hat{y}^{u*}_{jk}(\lambda_U)_{rk}(\lambda_U)^*_{ri}}{2M^2_{U_r}} + \frac{\hat{y}^{u*}_{ki}(\lambda^u_{Q_1})_{rj}(\lambda^u_{Q_1})^*_{rk}}{2M^2_{Q_{1r}}} \\
& + \frac{\hat{y}^{u*}_{ki}(\lambda_{Q_7})_{rj}(\lambda_{Q_7})^*_{rk}}{2M^2_{Q_{7r}}} + \frac{\hat{y}^{u*}_{jk}(\lambda_{T_1})_{rk}(\lambda_{T_1})^*_{ri}}{4M^2_{T_{1r}}} + \frac{\hat{y}^{u*}_{jk}(\lambda_{T_2})_{rk}(\lambda_{T_2})^*_{ri}}{8M^2_{T_{2r}}} \\
& + \frac{i\hat{y}^{u*}_{jk}\,\mathrm{Im}\left((\hat{g}^\phi_{\mathcal{B}})_r\right)(g^q_{\mathcal{B}})_{rik}}{M^2_{\mathcal{B}_r}} - \frac{i\hat{y}^{u*}_{ki}\,\mathrm{Im}\left((\hat{g}^\phi_{\mathcal{B}})_r\right)(g^u_{\mathcal{B}})_{rkj}}{M^2_{\mathcal{B}_r}} - \frac{i\hat{y}^{u*}_{jk}\,\mathrm{Im}\left((\hat{g}^\phi_{\mathcal{W}})_r\right)(g^q_{\mathcal{W}})_{rik}}{4M^2_{\mathcal{W}_r}} \\
& + \frac{1}{f}\left\{ \frac{(\tilde{y}^u_{\mathcal{S}})^*_{rji}(\kappa_{\mathcal{S}})_r}{M^2_{\mathcal{S}_r}} - \frac{(\tilde{y}^u_{\Xi})^*_{rji}(\kappa_{\Xi})_r}{M^2_{\Xi_r}} + \frac{2(\tilde{y}^u_{\Xi_1})_{rji}(\kappa_{\Xi_1})^*_r}{M^2_{\Xi_{1r}}} \right. \\
& \quad + \frac{i\hat{y}^{u*}_{jk}(\tilde{\lambda}^q_U)_{rk}(\lambda_U)^*_{ri}}{2M_{U_r}} + \frac{(\tilde{\lambda}^u_U)_{rj}(\lambda_U)^*_{ri}}{M_{U_r}} + \frac{i\hat{y}^{u*}_{jk}(\tilde{\lambda}^q_U)^*_{ri}(\lambda_U)_{rk}}{2M_{U_r}} \\
& \quad - \frac{i\hat{y}^{u*}_{ki}(\tilde{\lambda}^u_{Q_1})^*_{rk}(\lambda^u_{Q_1})_{rj}}{2M_{Q_{1r}}} + \frac{(\tilde{\lambda}^{q\prime}_{Q_1})^*_{ri}(\lambda^u_{Q_1})_{rj}}{M_{Q_{1r}}} - \frac{i\hat{y}^{u*}_{ki}(\tilde{\lambda}^u_{Q_1})_{rj}(\lambda^u_{Q_1})^*_{rk}}{2M_{Q_{1r}}} \\
& \quad - \frac{i\hat{y}^{u*}_{ki}(\tilde{\lambda}^u_{Q_7})^*_{rk}(\lambda_{Q_7})_{rj}}{2M_{Q_{7r}}} + \frac{(\tilde{\lambda}^q_{Q_7})^*_{ri}(\lambda_{Q_7})_{rj}}{M_{Q_{7r}}} - \frac{i\hat{y}^{u*}_{ki}(\tilde{\lambda}^u_{Q_7})_{rj}(\lambda_{Q_7})^*_{rk}}{2M_{Q_{7r}}} \\
& \quad + \frac{i\hat{y}^{u*}_{jk}(\tilde{\lambda}^q_{T_1})_{rk}(\lambda_{T_1})^*_{ri}}{2M_{T_{1r}}} + \frac{(\tilde{\lambda}^u_{T_1})_{rj}(\lambda_{T_1})^*_{ri}}{M_{T_{1r}}} + \frac{i\hat{y}^{u*}_{jk}(\tilde{\lambda}^q_{T_1})^*_{ri}(\lambda_{T_1})_{rk}}{2M_{T_{1r}}} \\
& \quad + \frac{i\hat{y}^{u*}_{jk}(\tilde{\lambda}^q_{T_2})_{rk}(\lambda_{T_2})^*_{ri}}{4M_{T_{2r}}} - \frac{(\tilde{\lambda}^u_{T_2})_{rj}(\lambda_{T_2})^*_{ri}}{2M_{T_{2r}}} + \frac{i\hat{y}^{u*}_{jk}(\tilde{\lambda}^q_{T_2})^*_{ri}(\lambda_{T_2})_{rk}}{4M_{T_{2r}}} \\
& \quad - \frac{\hat{y}^{u*}_{jk}\hat{y}^u_{lk}(\tilde{g}^{qDu}_{\mathcal{L}_1})_{rli}(\gamma_{\mathcal{L}_1})^*_r}{4M^2_{\mathcal{L}_{1r}}} + \frac{\hat{y}^{u*}_{ki}\hat{y}^u_{kl}(\tilde{g}^{qDu}_{\mathcal{L}_1})_{rjl}(\gamma_{\mathcal{L}_1})^*_r}{4M^2_{\mathcal{L}_{1r}}} + \frac{\hat{\lambda}_\phi(\tilde{g}^{qDu}_{\mathcal{L}_1})_{rji}(\gamma_{\mathcal{L}_1})^*_r}{M^2_{\mathcal{L}_{1r}}} \\
& \quad + \frac{\hat{y}^{u*}_{jk}\hat{y}^u_{lk}(\tilde{g}^{Dqu}_{\mathcal{L}_1})_{rli}(\gamma_{\mathcal{L}_1})^*_r}{4M^2_{\mathcal{L}_{1r}}} - \frac{\hat{y}^{u*}_{ki}\hat{y}^u_{kl}(\tilde{g}^{Dqu}_{\mathcal{L}_1})_{rjl}(\gamma_{\mathcal{L}_1})^*_r}{4M^2_{\mathcal{L}_{1r}}} + \frac{\hat{\lambda}_\phi(\tilde{g}^{Dqu}_{\mathcal{L}_1})_{rji}(\gamma_{\mathcal{L}_1})^*_r}{M^2_{\mathcal{L}_{1r}}} \\
& \quad + \frac{i\hat{y}^{u*}_{ki}(\tilde{g}^u_{\mathcal{L}_1})_{rkj}(\gamma_{\mathcal{L}_1})^*_r}{2M^2_{\mathcal{L}_{1r}}} - \frac{i\hat{y}^{u*}_{jk}(\tilde{g}^q_{\mathcal{L}_1})_{rik}(\gamma_{\mathcal{L}_1})^*_r}{2M^2_{\mathcal{L}_{1r}}} + \frac{i\hat{y}^{u*}_{jk}(\tilde{g}^{q\prime}_{\mathcal{L}_1})_{rik}(\gamma_{\mathcal{L}_1})^*_r}{2M^2_{\mathcal{L}_{1r}}} \\
& \quad \left. + \frac{i\hat{y}^{u*}_{ki}(\tilde{g}^u_{\mathcal{L}_1})^*_{rjk}(\gamma_{\mathcal{L}_1})_r}{2M^2_{\mathcal{L}_{1r}}} - \frac{i\hat{y}^{u*}_{jk}(\tilde{g}^q_{\mathcal{L}_1})^*_{rki}(\gamma_{\mathcal{L}_1})_r}{2M^2_{\mathcal{L}_{1r}}} + \frac{i\hat{y}^{u*}_{jk}(\tilde{g}^{q\prime}_{\mathcal{L}_1})^*_{rki}(\gamma_{\mathcal{L}_1})_r}{2M^2_{\mathcal{L}_{1r}}} \right\}, \qquad (8.78)
\end{aligned}
$$

$$
\begin{aligned}
c^u_{ij} = {}& \frac{(\kappa_{\mathcal{S}\varphi})^*_{rs}(\kappa_{\mathcal{S}})_r(y^u_\varphi)_{sij}}{M^2_{\mathcal{S}_r}M^2_{\varphi_s}} + \frac{(\kappa_{\Xi\varphi})^*_{sr}(\kappa_{\Xi})_s(y^u_\varphi)_{rij}}{M^2_{\varphi_r}M^2_{\Xi_s}} \\
&+ \frac{2(\kappa_{\Xi_1\varphi})_{rs}(\kappa_{\Xi_1})^*_r(y^u_\varphi)_{sij}}{M^2_{\Xi_{1r}}M^2_{\varphi_s}} - \frac{(\lambda_{UQ_1})_{rs}(\lambda_U)^*_{ri}(\lambda^u_{Q_1})_{sj}}{M_{U_r}M_{Q_{1s}}} \\
&- \frac{(\lambda_{UQ_7})_{rs}(\lambda_U)^*_{ri}(\lambda_{Q_7})_{sj}}{M_{U_r}M_{Q_{7s}}} - \frac{(\lambda_{T_1Q_1})_{sr}(\lambda_{T_1})^*_{si}(\lambda^u_{Q_1})_{rj}}{2M_{Q_{1r}}M_{T_{1s}}} \\
&- \frac{(\lambda_{T_2Q_1})_{sr}(\lambda_{T_2})^*_{si}(\lambda^u_{Q_1})_{rj}}{4M_{Q_{1r}}M_{T_{2s}}} + \frac{(\lambda_{T_2Q_7})_{sr}(\lambda_{T_2})^*_{si}(\lambda_{Q_7})_{rj}}{4M_{Q_{7r}}M_{T_{2s}}} \\
&- \frac{(w_{\mathcal{S}U})_{rsj}(\kappa_{\mathcal{S}})_r(\lambda_U)^*_{si}}{M^2_{\mathcal{S}_r}M_{U_s}} - \frac{(w_{\mathcal{S}Q_1})^*_{rsi}(\kappa_{\mathcal{S}})_r(\lambda^u_{Q_1})_{sj}}{M^2_{\mathcal{S}_r}M_{Q_{1s}}} \\
&+ \frac{(w_{\Xi T_2})_{srj}(\kappa_{\Xi})_s(\lambda_{T_2})^*_{ri}}{2M_{T_{2r}}M^2_{\Xi_s}} + \frac{(w_{\Xi Q_7})^*_{rsi}(\kappa_{\Xi})_r(\lambda^u_{Q_1})_{sj}}{M^2_{\Xi_r}M_{Q_{1s}}} \\
&- \frac{(w_{\Xi_1 T_1})_{srj}(\kappa_{\Xi_1})^*_s(\lambda_{T_1})^*_{ri}}{M_{T_{1r}}M^2_{\Xi_{1s}}} - \frac{2(w_{\Xi_1 Q_7})^*_{rsi}(\kappa_{\Xi_1})^*_r(\lambda_{Q_7})_{sj}}{M^2_{\Xi_{1r}}M_{Q_{7s}}} \\
&+ \frac{i\hat{y}^{u*}_{jk}(z_{U\mathcal{L}_1})_{rsk}(\lambda_U)^*_{ri}(\gamma_{\mathcal{L}_1})_s}{2M_{U_r}M^2_{\mathcal{L}_{1s}}} + \frac{i\hat{y}^{u*}_{jk}(z_{U\mathcal{L}_1})^*_{rsi}(\gamma_{\mathcal{L}_1})^*_s(\lambda_U)_{rk}}{2M_{U_r}M^2_{\mathcal{L}_{1s}}} \\
&- \frac{i\hat{y}^{u*}_{ki}(z^u_{Q_1\mathcal{L}_1})^*_{srk}(\gamma_{\mathcal{L}_1})_r(\lambda^u_{Q_1})_{sj}}{2M^2_{\mathcal{L}_{1r}}M_{Q_{1s}}} - \frac{i\hat{y}^{u*}_{ki}(z^u_{Q_1\mathcal{L}_1})_{srj}(\lambda^u_{Q_1})^*_{sk}(\gamma_{\mathcal{L}_1})^*_r}{2M^2_{\mathcal{L}_{1r}}M_{Q_{1s}}} \\
&- \frac{i\hat{y}^{u*}_{ki}(z_{Q_7\mathcal{L}_1})^*_{rsk}(\gamma_{\mathcal{L}_1})^*_s(\lambda_{Q_7})_{rj}}{2M_{Q_{7r}}M^2_{\mathcal{L}_{1s}}} - \frac{i\hat{y}^{u*}_{ki}(z_{Q_7\mathcal{L}_1})_{rsj}(\lambda_{Q_7})^*_{rk}(\gamma_{\mathcal{L}_1})_s}{2M_{Q_{7r}}M^2_{\mathcal{L}_{1s}}} \\
&+ \frac{i\hat{y}^{u*}_{jk}(z_{T_1\mathcal{L}_1})_{rsk}(\lambda_{T_1})^*_{ri}(\gamma_{\mathcal{L}_1})^*_s}{2M_{T_{1r}}M^2_{\mathcal{L}_{1s}}} + \frac{i\hat{y}^{u*}_{jk}(z_{T_1\mathcal{L}_1})^*_{sri}(\gamma_{\mathcal{L}_1})_r(\lambda_{T_1})_{sk}}{2M^2_{\mathcal{L}_{1r}}M_{T_{1s}}} \\
&+ \frac{i\hat{y}^{u*}_{jk}(z_{T_2\mathcal{L}_1})^*_{sri}(\gamma_{\mathcal{L}_1})^*_r(\lambda_{T_2})_{sk}}{4M^2_{\mathcal{L}_{1r}}M_{T_{2s}}} + \frac{i\hat{y}^{u*}_{jk}(z_{T_2\mathcal{L}_1})_{rsk}(\lambda_{T_2})^*_{ri}(\gamma_{\mathcal{L}_1})_s}{4M_{T_{2r}}M^2_{\mathcal{L}_{1s}}} \\
&+ \frac{i\hat{y}^{u*}_{jk}(\delta_{\mathcal{BS}})_{rs}(g^q_{\mathcal{B}})_{rik}(\kappa_{\mathcal{S}})_s}{M^2_{\mathcal{B}_r}M^2_{\mathcal{S}_s}} - \frac{i\hat{y}^{u*}_{ki}(\delta_{\mathcal{BS}})_{rs}(g^u_{\mathcal{B}})_{rkj}(\kappa_{\mathcal{S}})_s}{M^2_{\mathcal{B}_r}M^2_{\mathcal{S}_s}} \\
&- \frac{i\hat{y}^{u*}_{jk}(\delta_{\mathcal{W}\Xi})_{sr}(g^q_{\mathcal{W}})_{sik}(\kappa_{\Xi})_r}{2M^2_{\Xi_r}M^2_{\mathcal{W}_s}} - \frac{2\hat{\lambda}_\phi(\delta_{\mathcal{L}_1\varphi})_{sr}(\gamma_{\mathcal{L}_1})^*_s(y^u_\varphi)_{rij}}{M^2_{\varphi_r}M^2_{\mathcal{L}_{1s}}}.
\end{aligned}
\tag{8.79}
$$

$X\psi^2\phi$

$$Z_\phi^{\frac{1}{2}}\ (C_{eB})_{ij} = \frac{1}{f}\left\{\frac{(\tilde{\lambda}_E^B)_{rj}(\lambda_E)^*_{ri}}{M_{E_r}} + \frac{(\tilde{\lambda}_{\Delta_1}^B)^*_{ri}(\lambda_{\Delta_1})_{rj}}{M_{\Delta_{1r}}} - \frac{g_1(\tilde{g}_{\mathcal{L}_1}^{eDl})^*_{rji}(\gamma_{\mathcal{L}_1})_r}{8M^2_{\mathcal{L}_{1r}}} + \frac{g_1(\tilde{g}_{\mathcal{L}_1}^{Del})^*_{rji}(\gamma_{\mathcal{L}_1})_r}{8M^2_{\mathcal{L}_{1r}}}\right\}, \tag{8.80}$$

$$Z_\phi^{\frac{1}{2}}\ (C_{eW})_{ij} = \frac{1}{f}\left\{\frac{(\tilde{\lambda}_{\Delta_1}^W)^*_{ri}(\lambda_{\Delta_1})_{rj}}{M_{\Delta_{1r}}} + \frac{(\tilde{\lambda}_{\Sigma_1}^W)_{rj}(\lambda_{\Sigma_1})^*_{ri}}{2M_{\Sigma_{1r}}} - \frac{g_2(\tilde{g}_{\mathcal{L}_1}^{eDl})^*_{rji}(\gamma_{\mathcal{L}_1})_r}{8M^2_{\mathcal{L}_{1r}}} + \frac{g_2(\tilde{g}_{\mathcal{L}_1}^{Del})^*_{rji}(\gamma_{\mathcal{L}_1})_r}{8M^2_{\mathcal{L}_{1r}}}\right\}, \tag{8.81}$$

$$Z_\phi^{\frac{1}{2}}\ (C_{dB})_{ij} = \frac{1}{f}\left\{\frac{(\tilde{\lambda}_D^B)_{rj}(\lambda_D)^*_{ri}}{M_{D_r}} + \frac{(\tilde{\lambda}_{Q_1}^B)^*_{ri}(\lambda_{Q_1}^d)_{rj}}{M_{Q_{1r}}} - \frac{g_1(\tilde{g}_{\mathcal{L}_1}^{dDq})^*_{rji}(\gamma_{\mathcal{L}_1})_r}{8M^2_{\mathcal{L}_{1r}}} + \frac{g_1(\tilde{g}_{\mathcal{L}_1}^{Ddq})^*_{rji}(\gamma_{\mathcal{L}_1})_r}{8M^2_{\mathcal{L}_{1r}}}\right\}, \tag{8.82}$$

$$Z_\phi^{\frac{1}{2}}\ (C_{dW})_{ij} = \frac{1}{f}\left\{\frac{(\tilde{\lambda}_{Q_1}^W)^*_{ri}(\lambda_{Q_1}^d)_{rj}}{M_{Q_{1r}}} + \frac{(\tilde{\lambda}_{T_1}^W)_{rj}(\lambda_{T_1})^*_{ri}}{2M_{T_{1r}}} - \frac{g_2(\tilde{g}_{\mathcal{L}_1}^{dDq})^*_{rji}(\gamma_{\mathcal{L}_1})_r}{8M^2_{\mathcal{L}_{1r}}} + \frac{g_2(\tilde{g}_{\mathcal{L}_1}^{Ddq})^*_{rji}(\gamma_{\mathcal{L}_1})_r}{8M^2_{\mathcal{L}_{1r}}}\right\}, \tag{8.83}$$

$$Z_\phi^{\frac{1}{2}}\ (C_{dG})_{ij} = \frac{1}{f}\left\{\frac{(\tilde{\lambda}_D^G)_{rj}(\lambda_D)^*_{ri}}{M_{D_r}} + \frac{(\tilde{\lambda}_{Q_1}^G)^*_{ri}(\lambda_{Q_1}^d)_{rj}}{M_{Q_{1r}}}\right\}, \tag{8.84}$$

$$Z_\phi^{\frac{1}{2}}\ (C_{uB})_{ij} = \frac{1}{f}\left\{\frac{(\tilde{\lambda}_U^B)_{rj}(\lambda_U)^*_{ri}}{M_{U_r}} + \frac{(\tilde{\lambda}_{Q_1}^B)^*_{ri}(\lambda_{Q_1}^u)_{rj}}{M_{Q_{1r}}} + \frac{g_1(\tilde{g}_{\mathcal{L}_1}^{qDu})_{rji}(\gamma_{\mathcal{L}_1})^*_r}{8M^2_{\mathcal{L}_{1r}}} - \frac{g_1(\tilde{g}_{\mathcal{L}_1}^{Dqu})_{rji}(\gamma_{\mathcal{L}_1})^*_r}{8M^2_{\mathcal{L}_{1r}}}\right\}, \tag{8.85}$$

$$Z_\phi^{\frac{1}{2}}\ (C_{uW})_{ij} = \frac{1}{f}\left\{\frac{(\tilde{\lambda}_{Q_1}^W)^*_{ri}(\lambda_{Q_1}^u)_{rj}}{M_{Q_{1r}}} + \frac{(\tilde{\lambda}_{T_2}^W)_{rj}(\lambda_{T_2})^*_{ri}}{2M_{T_{2r}}} + \frac{g_2(\tilde{g}_{\mathcal{L}_1}^{qDu})_{rji}(\gamma_{\mathcal{L}_1})^*_r}{8M^2_{\mathcal{L}_{1r}}} - \frac{g_2(\tilde{g}_{\mathcal{L}_1}^{Dqu})_{rji}(\gamma_{\mathcal{L}_1})^*_r}{8M^2_{\mathcal{L}_{1r}}}\right\}, \tag{8.86}$$

$$Z_\phi^{\frac{1}{2}}\ (C_{uG})_{ij} = \frac{1}{f}\left\{\frac{(\tilde{\lambda}_U^G)_{rj}(\lambda_U)^*_{ri}}{M_{U_r}} + \frac{(\tilde{\lambda}_{Q_1}^G)^*_{ri}(\lambda_{Q_1}^u)_{rj}}{M_{Q_{1r}}}\right\}. \tag{8.87}$$

$\psi^2\phi^2 D$

Recall that $\hat{g}_V^\phi$ contains contributions from $\mathcal{L}_1$ (see eqs. (8.12)–(8.15)) and that $\hat{y}^{e,u,d}$ are defined in eq. (8.23).

$$
\begin{aligned}
Z_\phi \left(C_{\phi l}^{(1)}\right)_{ij} =& \frac{(\lambda_N)^*_{ri}(\lambda_N)_{rj}}{4M^2_{N_r}} - \frac{(\lambda_E)_{rj}(\lambda_E)^*_{ri}}{4M^2_{E_r}} + \frac{3(\lambda_\Sigma)^*_{ri}(\lambda_\Sigma)_{rj}}{16M^2_{\Sigma_r}} - \frac{3(\lambda_{\Sigma_1})_{rj}(\lambda_{\Sigma_1})^*_{ri}}{16M^2_{\Sigma_{1r}}} \\
& - \frac{\mathrm{Re}\left((\hat{g}^\phi_{\mathcal{B}})_r\right)(g^l_{\mathcal{B}})_{rij}}{M^2_{\mathcal{B}_r}} - \frac{g_1\delta_{ij}(g^B_{\mathcal{L}_1})_{rs}(\gamma_{\mathcal{L}_1})^*_r(\gamma_{\mathcal{L}_1})_s}{4M^2_{\mathcal{L}_{1r}}M^2_{\mathcal{L}_{1s}}} \\
& + \frac{i(\lambda_N)_{rj}(z_{N\mathcal{L}_1})^*_{rsi}(\gamma_{\mathcal{L}_1})^*_s}{4M_{N_r}M^2_{\mathcal{L}_{1s}}} - \frac{i(\lambda_N)^*_{ri}(z_{N\mathcal{L}_1})_{rsj}(\gamma_{\mathcal{L}_1})_s}{4M_{N_r}M^2_{\mathcal{L}_{1s}}} \\
& - \frac{i(z_{E\mathcal{L}_1})^*_{rsi}(\gamma_{\mathcal{L}_1})_s(\lambda_E)_{rj}}{4M_{E_r}M^2_{\mathcal{L}_{1s}}} + \frac{i(z_{E\mathcal{L}_1})_{rsj}(\lambda_E)^*_{ri}(\gamma_{\mathcal{L}_1})^*_s}{4M_{E_r}M^2_{\mathcal{L}_{1s}}} \\
& + \frac{3i(\lambda_\Sigma)_{sj}(z_{\Sigma\mathcal{L}_1})^*_{sri}(\gamma_{\mathcal{L}_1})^*_r}{8M^2_{\mathcal{L}_{1r}}M_{\Sigma_s}} - \frac{3i(\lambda_\Sigma)^*_{ri}(z_{\Sigma\mathcal{L}_1})_{rsj}(\gamma_{\mathcal{L}_1})_s}{8M_{\Sigma_r}M^2_{\mathcal{L}_{1s}}} \\
& + \frac{3i(z_{\Sigma_1\mathcal{L}_1})_{srj}(\lambda_{\Sigma_1})^*_{si}(\gamma_{\mathcal{L}_1})^*_r}{8M^2_{\mathcal{L}_{1r}}M_{\Sigma_{1s}}} - \frac{3i(z_{\Sigma_1\mathcal{L}_1})^*_{rsi}(\gamma_{\mathcal{L}_1})_s(\lambda_{\Sigma_1})_{rj}}{8M_{\Sigma_{1r}}M^2_{\mathcal{L}_{1s}}} \\
& + \frac{1}{f}\left\{ \frac{i(\tilde{\lambda}_N)^*_{ri}(\lambda_N)_{rj}}{4M_{N_r}} - \frac{i(\tilde{\lambda}_N)_{rj}(\lambda_N)^*_{ri}}{4M_{N_r}} \right. \\
& \quad + \frac{i(\tilde{\lambda}^l_E)_{rj}(\lambda_E)^*_{ri}}{4M_{E_r}} - \frac{i(\tilde{\lambda}^l_E)^*_{ri}(\lambda_E)_{rj}}{4M_{E_r}} \\
& \quad + \frac{3i(\tilde{\lambda}^l_\Sigma)^*_{ri}(\lambda_\Sigma)_{rj}}{8M_{\Sigma_r}} - \frac{3i(\tilde{\lambda}^l_\Sigma)_{rj}(\lambda_{\Sigma_0})^*_{ri}}{8M_{\Sigma_r}} \\
& \quad + \frac{3i(\tilde{\lambda}^l_{\Sigma_1})_{rj}(\lambda_{\Sigma_1})^*_{ri}}{8M_{\Sigma_{1r}}} - \frac{3i(\tilde{\lambda}^l_{\Sigma_1})^*_{ri}(\lambda_{\Sigma_1})_{rj}}{8M_{\Sigma_{1r}}} \\
& \quad - \frac{\hat{y}^{e*}_{ki}(\tilde{g}^{eDl}_{\mathcal{L}_1})_{rkj}(\gamma_{\mathcal{L}_1})^*_r}{8M^2_{\mathcal{L}_{1r}}} - \frac{\hat{y}^e_{kj}(\tilde{g}^{eDl}_{\mathcal{L}_1})^*_{rki}(\gamma_{\mathcal{L}_1})_r}{8M^2_{\mathcal{L}_{1r}}} \\
& \quad + \frac{\hat{y}^{e*}_{ki}(\tilde{g}^{Del}_{\mathcal{L}_1})_{rkj}(\gamma_{\mathcal{L}_1})^*_r}{8M^2_{\mathcal{L}_{1r}}} + \frac{\hat{y}^e_{kj}(\tilde{g}^{Del}_{\mathcal{L}_1})^*_{rki}(\gamma_{\mathcal{L}_1})_r}{8M^2_{\mathcal{L}_{1r}}} \\
& \quad - \frac{i(\tilde{g}^l_{\mathcal{L}_1})_{rij}(\gamma_{\mathcal{L}_1})^*_r}{2M^2_{\mathcal{L}_{1r}}} + \frac{i(\tilde{g}^l_{\mathcal{L}_1})^*_{rji}(\gamma_{\mathcal{L}_1})_r}{2M^2_{\mathcal{L}_{1r}}} \\
& \quad \left. - \frac{\mathrm{Im}\left((\tilde{\gamma}^B_{\mathcal{L}_1})_r(\gamma_{\mathcal{L}_1})^*_r\right)g_1\delta_{ij}}{2M^2_{\mathcal{L}_{1r}}} \right\},
\end{aligned}
\tag{8.88}
$$

$$
\begin{aligned}
Z_\phi \left(C_{\phi l}^{(3)}\right)_{ij} =& - \frac{(\lambda_N)_{ri}^*(\lambda_N)_{rj}}{4M_{N_r}^2} - \frac{(\lambda_E)_{rj}(\lambda_E)_{ri}^*}{4M_{E_r}^2} + \frac{(\lambda_\Sigma)_{ri}^*(\lambda_\Sigma)_{rj}}{16M_{\Sigma_r}^2} + \frac{(\lambda_{\Sigma_1})_{rj}(\lambda_{\Sigma_1})_{ri}^*}{16M_{\Sigma_{1r}}^2} \\
& - \frac{\mathrm{Re}\left((\hat{g}_{\mathcal{W}}^\phi)_r\right)(g_{\mathcal{W}}^l)_{rij}}{4M_{\mathcal{W}_r}^2} + \frac{g_2\delta_{ij}(g_{\mathcal{L}_1}^W)_{rs}(\gamma_{\mathcal{L}_1})_r^*(\gamma_{\mathcal{L}_1})_s}{4M_{\mathcal{L}_{1r}}^2 M_{\mathcal{L}_{1s}}^2} \\
& - \frac{i(\lambda_N)_{rj}(z_{N\mathcal{L}_1})_{rsi}^*(\gamma_{\mathcal{L}_1})_s^*}{4M_{N_r}M_{\mathcal{L}_{1s}}^2} + \frac{i(\lambda_N)_{ri}^*(z_{N\mathcal{L}_1})_{rsj}(\gamma_{\mathcal{L}_1})_s}{4M_{N_r}M_{\mathcal{L}_{1s}}^2} \\
& + \frac{i(z_{E\mathcal{L}_1})_{rsj}(\lambda_E)_{ri}^*(\gamma_{\mathcal{L}_1})_s^*}{4M_{E_r}M_{\mathcal{L}_{1s}}^2} - \frac{i(z_{E\mathcal{L}_1})_{rsi}^*(\gamma_{\mathcal{L}_1})_s(\lambda_E)_{rj}}{4M_{E_r}M_{\mathcal{L}_{1s}}^2} \\
& + \frac{i(\lambda_\Sigma)_{sj}(z_{\Sigma\mathcal{L}_1})_{sri}^*(\gamma_{\mathcal{L}_1})_r^*}{8M_{\mathcal{L}_{1r}}^2 M_{\Sigma_s}} - \frac{i(\lambda_\Sigma)_{ri}^*(z_{\Sigma\mathcal{L}_1})_{rsj}(\gamma_{\mathcal{L}_1})_s}{8M_{\Sigma_r}M_{\mathcal{L}_{1s}}^2} \\
& - \frac{i(z_{\Sigma_1\mathcal{L}_1})_{srj}(\lambda_{\Sigma_1})_{si}^*(\gamma_{\mathcal{L}_1})_r^*}{8M_{\mathcal{L}_{1r}}^2 M_{\Sigma_{1s}}} + \frac{i(z_{\Sigma_1\mathcal{L}_1})_{rsi}^*(\gamma_{\mathcal{L}_1})_s(\lambda_{\Sigma_1})_{rj}}{8M_{\Sigma_{1r}}M_{\mathcal{L}_{1s}}^2} \\
& + \frac{1}{f}\Bigg\{ - \frac{i(\tilde{\lambda}_N)_{ri}^*(\lambda_N)_{rj}}{4M_{N_r}} + \frac{i(\tilde{\lambda}_N)_{rj}(\lambda_N)_{ri}^*}{4M_{N_r}} \\
& \quad + \frac{i(\tilde{\lambda}_E^l)_{rj}(\lambda_E)_{ri}^*}{4M_{E_r}} - \frac{i(\tilde{\lambda}_E^l)_{ri}^*(\lambda_E)_{rj}}{4M_{E_r}} \\
& \quad + \frac{i(\tilde{\lambda}_\Sigma^l)_{ri}^*(\lambda_\Sigma)_{rj}}{8M_{\Sigma_r}} - \frac{i(\tilde{\lambda}_\Sigma^l)_{rj}(\lambda_\Sigma)_{ri}^*}{8M_{\Sigma_r}} \\
& \quad - \frac{i(\tilde{\lambda}_{\Sigma_1}^l)_{rj}(\lambda_{\Sigma_1})_{ri}^*}{8M_{\Sigma_{1r}}} + \frac{i(\tilde{\lambda}_{\Sigma_1}^l)_{ri}^*(\lambda_{\Sigma_1})_{rj}}{8M_{\Sigma_{1r}}} \\
& \quad - \frac{\hat{y}_{ki}^{e*}(\tilde{g}_{\mathcal{L}_1}^{eDl})_{rkj}(\gamma_{\mathcal{L}_1})_r^*}{8M_{\mathcal{L}_{1r}}^2} - \frac{\hat{y}_{kj}^{e}(\tilde{g}_{\mathcal{L}_1}^{eDl})_{rki}^*(\gamma_{\mathcal{L}_1})_r}{8M_{\mathcal{L}_{1r}}^2} \\
& \quad + \frac{\hat{y}_{ki}^{e*}(\tilde{g}_{\mathcal{L}_1}^{Del})_{rkj}(\gamma_{\mathcal{L}_1})_r^*}{8M_{\mathcal{L}_{1r}}^2} + \frac{\hat{y}_{kj}^{e}(\tilde{g}_{\mathcal{L}_1}^{Del})_{rki}^*(\gamma_{\mathcal{L}_1})_r}{8M_{\mathcal{L}_{1r}}^2} \\
& \quad - \frac{i(\tilde{g}_{\mathcal{L}_1}^{l\prime})_{rij}(\gamma_{\mathcal{L}_1})_r^*}{2M_{\mathcal{L}_{1r}}^2} + \frac{i(\tilde{g}_{\mathcal{L}_1}^{l\prime})_{rji}^*(\gamma_{\mathcal{L}_1})_r}{2M_{\mathcal{L}_{1r}}^2} \\
& \quad + \frac{\mathrm{Im}\left((\tilde{\gamma}_{\mathcal{L}_1}^W)_r(\gamma_{\mathcal{L}_1})_r^*\right)g_2\delta_{ij}}{2M_{\mathcal{L}_{1r}}^2}\Bigg\},
\end{aligned}
\tag{8.89}
$$

$$
\begin{aligned}
Z_\phi \left(C_{\phi q}^{(1)}\right)_{ij} =& \frac{(\lambda_U)_{rj}(\lambda_U)^*_{ri}}{4M_{U_r}^2} - \frac{(\lambda_D)_{rj}(\lambda_D)^*_{ri}}{4M_{D_r}^2} - \frac{3(\lambda_{T_1})_{rj}(\lambda_{T_1})^*_{ri}}{16M_{T_{1r}}^2} + \frac{3(\lambda_{T_2})_{rj}(\lambda_{T_2})^*_{ri}}{16M_{T_{2r}}^2} \\
&- \frac{\mathrm{Re}\left((\hat{g}_{\mathcal{B}}^{\phi})_r\right)(g_{\mathcal{B}}^q)_{rij}}{M_{\mathcal{B}_r}^2} + \frac{g_1\delta_{ij}(g_{\mathcal{L}_1}^B)_{rs}(\gamma_{\mathcal{L}_1})^*_r(\gamma_{\mathcal{L}_1})_s}{12M_{\mathcal{L}_{1r}}^2 M_{\mathcal{L}_{1s}}^2} \\
&- \frac{i(z_{U\mathcal{L}_1})_{rsj}(\lambda_U)^*_{ri}(\gamma_{\mathcal{L}_1})_s}{4M_{U_r}M_{\mathcal{L}_{1s}}^2} + \frac{i(z_{U\mathcal{L}_1})^*_{rsi}(\gamma_{\mathcal{L}_1})^*_s(\lambda_U)_{rj}}{4M_{U_r}M_{\mathcal{L}_{1s}}^2} \\
&+ \frac{i(z_{D\mathcal{L}_1})_{rsj}(\lambda_D)^*_{ri}(\gamma_{\mathcal{L}_1})^*_s}{4M_{D_r}M_{\mathcal{L}_{1s}}^2} - \frac{i(z_{D\mathcal{L}_1})^*_{rsi}(\gamma_{\mathcal{L}_1})_s(\lambda_D)_{rj}}{4M_{D_r}M_{\mathcal{L}_{1s}}^2} \\
&+ \frac{3i(z_{T_1\mathcal{L}_1})_{rsj}(\lambda_{T_1})^*_{ri}(\gamma_{\mathcal{L}_1})^*_s}{8M_{T_{1r}}M_{\mathcal{L}_{1s}}^2} - \frac{3i(z_{T_1\mathcal{L}_1})^*_{sri}(\gamma_{\mathcal{L}_1})_r(\lambda_{T_1})_{sj}}{8M_{\mathcal{L}_{1r}}^2 M_{T_{1s}}} \\
&- \frac{3i(z_{T_2\mathcal{L}_1})_{rsj}(\lambda_{T_2})^*_{ri}(\gamma_{\mathcal{L}_1})_s}{8M_{T_{2r}}M_{\mathcal{L}_{1s}}^2} + \frac{3i(z_{T_2\mathcal{L}_1})^*_{sri}(\gamma_{\mathcal{L}_1})^*_r(\lambda_{T_2})_{sj}}{8M_{\mathcal{L}_{1r}}^2 M_{T_{2s}}} \\
&+ \frac{1}{f}\Bigg\{ - \frac{i(\tilde{\lambda}_U^q)_{rj}(\lambda_U)^*_{ri}}{4M_{U_r}} + \frac{i(\tilde{\lambda}_U^q)^*_{ri}(\lambda_U)_{rj}}{4M_{U_r}} \\
&\quad + \frac{i(\tilde{\lambda}_D^q)_{rj}(\lambda_D)^*_{ri}}{4M_{D_r}} - \frac{i(\tilde{\lambda}_D^q)^*_{ri}(\lambda_D)_{rj}}{4M_{D_r}} \\
&\quad + \frac{3i(\tilde{\lambda}_{T_1}^q)_{rj}(\lambda_{T_1})^*_{ri}}{8M_{T_{1r}}} - \frac{3i(\tilde{\lambda}_{T_1}^q)^*_{ri}(\lambda_{T_1})_{rj}}{8M_{T_{1r}}} \\
&\quad - \frac{3i(\tilde{\lambda}_{T_2}^q)_{rj}(\lambda_{T_2})^*_{ri}}{8M_{T_{2r}}} + \frac{3i(\tilde{\lambda}_{T_2}^q)^*_{ri}(\lambda_{T_2})_{rj}}{8M_{T_{2r}}} \\
&\quad - \frac{\hat{y}_{ki}^{d*}(\tilde{g}_{\mathcal{L}_1}^{dDq})_{rkj}(\gamma_{\mathcal{L}_1})^*_r}{8M_{\mathcal{L}_{1r}}^2} - \frac{\hat{y}_{kj}^{d}(\tilde{g}_{\mathcal{L}_1}^{dDq})^*_{rki}(\gamma_{\mathcal{L}_1})_r}{8M_{\mathcal{L}_{1r}}^2} \\
&\quad + \frac{\hat{y}_{ki}^{d*}(\tilde{g}_{\mathcal{L}_1}^{Ddq})_{rkj}(\gamma_{\mathcal{L}_1})^*_r}{8M_{\mathcal{L}_{1r}}^2} + \frac{\hat{y}_{kj}^{d}(\tilde{g}_{\mathcal{L}_1}^{Ddq})^*_{rki}(\gamma_{\mathcal{L}_1})_r}{8M_{\mathcal{L}_{1r}}^2} \\
&\quad - \frac{\hat{y}_{kj}^{u}(\tilde{g}_{\mathcal{L}_1}^{qDu})_{rki}(\gamma_{\mathcal{L}_1})^*_r}{8M_{\mathcal{L}_{1r}}^2} - \frac{\hat{y}_{ki}^{u*}(\tilde{g}_{\mathcal{L}_1}^{qDu})^*_{rkj}(\gamma_{\mathcal{L}_1})_r}{8M_{\mathcal{L}_{1r}}^2} \\
&\quad + \frac{\hat{y}_{kj}^{u}(\tilde{g}_{\mathcal{L}_1}^{Dqu})_{rki}(\gamma_{\mathcal{L}_1})^*_r}{8M_{\mathcal{L}_{1r}}^2} + \frac{\hat{y}_{ki}^{u*}(\tilde{g}_{\mathcal{L}_1}^{Dqu})^*_{rkj}(\gamma_{\mathcal{L}_1})_r}{8M_{\mathcal{L}_{1r}}^2} \\
&\quad - \frac{i(\tilde{g}_{\mathcal{L}_1}^{q})_{rij}(\gamma_{\mathcal{L}_1})^*_r}{2M_{\mathcal{L}_{1r}}^2} + \frac{i(\tilde{g}_{\mathcal{L}_1}^{q})^*_{rji}(\gamma_{\mathcal{L}_1})_r}{2M_{\mathcal{L}_{1r}}^2} \\
&\quad + \frac{\mathrm{Im}\left((\tilde{\gamma}_{\mathcal{L}_1}^{B})_r(\gamma_{\mathcal{L}_1})^*_r\right)g_1\delta_{ij}}{6M_{\mathcal{L}_{1r}}^2}\Bigg\},
\end{aligned}
\tag{8.90}
$$

$$
\begin{aligned}
Z_\phi \left(C_{\phi q}^{(3)}\right)_{ij} =& -\frac{(\lambda_U)_{rj}(\lambda_U)^*_{ri}}{4M_{U_r}^2} - \frac{(\lambda_D)_{rj}(\lambda_D)^*_{ri}}{4M_{D_r}^2} + \frac{(\lambda_{T_1})_{rj}(\lambda_{T_1})^*_{ri}}{16M_{T_{1r}}^2} + \frac{(\lambda_{T_2})_{rj}(\lambda_{T_2})^*_{ri}}{16M_{T_{2r}}^2} \\
&- \frac{\mathrm{Re}\left((\hat{g}^\phi_{\mathcal{W}})_r\right)(g^q_{\mathcal{W}})_{rij}}{4M_{\mathcal{W}_r}^2} + \frac{g_2\delta_{ij}(g^W_{\mathcal{L}_1})_{rs}(\gamma_{\mathcal{L}_1})^*_r(\gamma_{\mathcal{L}_1})_s}{4M_{\mathcal{L}_{1r}}^2M_{\mathcal{L}_{1s}}^2} \\
&+ \frac{i(z_{U\mathcal{L}_1})_{rsj}(\lambda_U)^*_{ri}(\gamma_{\mathcal{L}_1})_s}{4M_{U_r}M_{\mathcal{L}_{1s}}^2} - \frac{i(z_{U\mathcal{L}_1})^*_{rsi}(\gamma_{\mathcal{L}_1})^*_s(\lambda_U)_{rj}}{4M_{U_r}M_{\mathcal{L}_{1s}}^2} \\
&+ \frac{i(z_{D\mathcal{L}_1})_{rsj}(\lambda_D)^*_{ri}(\gamma_{\mathcal{L}_1})^*_s}{4M_{D_r}M_{\mathcal{L}_{1s}}^2} - \frac{i(z_{D\mathcal{L}_1})^*_{rsi}(\gamma_{\mathcal{L}_1})_s(\lambda_D)_{rj}}{4M_{D_r}M_{\mathcal{L}_{1s}}^2} \\
&- \frac{i(z_{T_1\mathcal{L}_1})_{rsj}(\lambda_{T_1})^*_{ri}(\gamma_{\mathcal{L}_1})^*_s}{8M_{T_{1r}}M_{\mathcal{L}_{1s}}^2} + \frac{i(z_{T_1\mathcal{L}_1})^*_{sri}(\gamma_{\mathcal{L}_1})_r(\lambda_{T_1})_{sj}}{8M_{\mathcal{L}_{1r}}^2M_{T_{1s}}} \\
&- \frac{i(z_{T_2\mathcal{L}_1})_{rsj}(\lambda_{T_2})^*_{ri}(\gamma_{\mathcal{L}_1})_s}{8M_{T_{2r}}M_{\mathcal{L}_{1s}}^2} + \frac{i(z_{T_2\mathcal{L}_1})^*_{sri}(\gamma_{\mathcal{L}_1})^*_r(\lambda_{T_2})_{sj}}{8M_{\mathcal{L}_{1r}}^2M_{T_{2s}}} \\
&+ \frac{1}{f}\Bigg\{ \frac{i(\tilde{\lambda}^q_U)_{rj}(\lambda_U)^*_{ri}}{4M_{U_r}} - \frac{i(\tilde{\lambda}^q_U)^*_{ri}(\lambda_U)_{rj}}{4M_{U_r}} \\
&\quad + \frac{i(\tilde{\lambda}^q_D)_{rj}(\lambda_D)^*_{ri}}{4M_{D_r}} - \frac{i(\tilde{\lambda}^q_D)^*_{ri}(\lambda_D)_{rj}}{4M_{D_r}} \\
&\quad - \frac{i(\tilde{\lambda}^q_{T_2})_{rj}(\lambda_{T_2})^*_{ri}}{8M_{T_{2r}}} + \frac{i(\tilde{\lambda}^q_{T_2})^*_{ri}(\lambda_{T_2})_{rj}}{8M_{T_{2r}}} \\
&\quad - \frac{i(\tilde{\lambda}^q_{T_1})_{rj}(\lambda_{T_1})^*_{ri}}{8M_{T_{1r}}} + \frac{i(\tilde{\lambda}^q_{T_1})^*_{ri}(\lambda_{T_1})_{rj}}{8M_{T_{1r}}} \\
&\quad - \frac{\hat{y}^{d*}_{ki}(\tilde{g}^{dDq}_{\mathcal{L}_1})_{rkj}(\gamma_{\mathcal{L}_1})^*_r}{8M_{\mathcal{L}_{1r}}^2} - \frac{\hat{y}^{d}_{kj}(\tilde{g}^{dDq}_{\mathcal{L}_1})^*_{rki}(\gamma_{\mathcal{L}_1})_r}{8M_{\mathcal{L}_{1r}}^2} \\
&\quad + \frac{\hat{y}^{d*}_{ki}(\tilde{g}^{Ddq}_{\mathcal{L}_1})_{rkj}(\gamma_{\mathcal{L}_1})^*_r}{8M_{\mathcal{L}_{1r}}^2} + \frac{\hat{y}^{d}_{kj}(\tilde{g}^{Ddq}_{\mathcal{L}_1})^*_{rki}(\gamma_{\mathcal{L}_1})_r}{8M_{\mathcal{L}_{1r}}^2} \\
&\quad + \frac{\hat{y}^{u}_{kj}(\tilde{g}^{qDu}_{\mathcal{L}_1})_{rki}(\gamma_{\mathcal{L}_1})^*_r}{8M_{\mathcal{L}_{1r}}^2} + \frac{\hat{y}^{u*}_{ki}(\tilde{g}^{qDu}_{\mathcal{L}_1})^*_{rkj}(\gamma_{\mathcal{L}_1})_r}{8M_{\mathcal{L}_{1r}}^2} \\
&\quad - \frac{\hat{y}^{u}_{kj}(\tilde{g}^{Dqu}_{\mathcal{L}_1})_{rki}(\gamma_{\mathcal{L}_1})^*_r}{8M_{\mathcal{L}_{1r}}^2} - \frac{\hat{y}^{u*}_{ki}(\tilde{g}^{Dqu}_{\mathcal{L}_1})^*_{rkj}(\gamma_{\mathcal{L}_1})_r}{8M_{\mathcal{L}_{1r}}^2} \\
&\quad - \frac{i(\tilde{g}^{q\prime}_{\mathcal{L}_1})_{rij}(\gamma_{\mathcal{L}_1})^*_r}{2M_{\mathcal{L}_{1r}}^2} + \frac{i(\tilde{g}^{q\prime}_{\mathcal{L}_1})^*_{rji}(\gamma_{\mathcal{L}_1})_r}{2M_{\mathcal{L}_{1r}}^2} \\
&\quad + \frac{\mathrm{Im}\left((\tilde{\gamma}^W_{\mathcal{L}_1})_r(\gamma_{\mathcal{L}_1})^*_r\right)g_2\delta_{ij}}{2M_{\mathcal{L}_{1r}}^2}\Bigg\},
\end{aligned}
\tag{8.91}
$$

$$
\begin{aligned}
Z_\phi\ (C_{\phi e})_{ij} =& \frac{(\lambda_{\Delta_1})_{rj}(\lambda_{\Delta_1})^*_{ri}}{2M^2_{\Delta_{1r}}} - \frac{(\lambda_{\Delta_3})_{rj}(\lambda_{\Delta_3})^*_{ri}}{2M^2_{\Delta_{3r}}} \\
&- \frac{\mathrm{Re}\left((\hat{g}^\phi_{\mathcal{B}})_r\right)(g^e_{\mathcal{B}})_{rij}}{M^2_{\mathcal{B}_r}} - \frac{g_1\delta_{ij}(g^B_{\mathcal{L}_1})_{rs}(\gamma_{\mathcal{L}_1})^*_r(\gamma_{\mathcal{L}_1})_s}{2M^2_{\mathcal{L}_{1r}}M^2_{\mathcal{L}_{1s}}} \\
&+ \frac{i(z_{\Delta_1\mathcal{L}_1})^*_{rsi}(\gamma_{\mathcal{L}_1})^*_s(\lambda_{\Delta_1})_{rj}}{2M_{\Delta_{1r}}M^2_{\mathcal{L}_{1s}}} - \frac{i(z_{\Delta_1\mathcal{L}_1})_{rsj}(\lambda_{\Delta_1})^*_{ri}(\gamma_{\mathcal{L}_1})_s}{2M_{\Delta_{1r}}M^2_{\mathcal{L}_{1s}}} \\
&- \frac{i(z_{\Delta_3\mathcal{L}_1})^*_{sri}(\gamma_{\mathcal{L}_1})_r(\lambda_{\Delta_3})_{sj}}{2M^2_{\mathcal{L}_{1r}}M_{\Delta_{3s}}} + \frac{i(z_{\Delta_3\mathcal{L}_1})_{srj}(\lambda_{\Delta_3})^*_{si}(\gamma_{\mathcal{L}_1})^*_r}{2M^2_{\mathcal{L}_{1r}}M_{\Delta_{3s}}} \\
&+ \frac{1}{f}\Bigg\{ \frac{i(\tilde{\lambda}^e_{\Delta_1})^*_{ri}(\lambda_{\Delta_1})_{rj}}{2M_{\Delta_{1r}}} - \frac{i(\tilde{\lambda}^e_{\Delta_1})_{rj}(\lambda_{\Delta_1})^*_{ri}}{2M_{\Delta_{1r}}} \\
&\quad - \frac{i(\tilde{\lambda}^e_{\Delta_3})^*_{ri}(\lambda_{\Delta_3})_{rj}}{2M_{\Delta_{3r}}} + \frac{i(\tilde{\lambda}^e_{\Delta_3})_{rj}(\lambda_{\Delta_3})^*_{ri}}{2M_{\Delta_{3r}}} \\
&\quad - \frac{\hat{y}^{e*}_{jk}(\tilde{g}^{eDl}_{\mathcal{L}_1})_{rik}(\gamma_{\mathcal{L}_1})^*_r}{4M^2_{\mathcal{L}_{1r}}} - \frac{\hat{y}^{e}_{ik}(\tilde{g}^{eDl}_{\mathcal{L}_1})^*_{rjk}(\gamma_{\mathcal{L}_1})_r}{4M^2_{\mathcal{L}_{1r}}} \\
&\quad + \frac{\hat{y}^{e*}_{jk}(\tilde{g}^{Del}_{\mathcal{L}_1})_{rik}(\gamma_{\mathcal{L}_1})^*_r}{4M^2_{\mathcal{L}_{1r}}} + \frac{\hat{y}^{e}_{ik}(\tilde{g}^{Del}_{\mathcal{L}_1})^*_{rjk}(\gamma_{\mathcal{L}_1})_r}{4M^2_{\mathcal{L}_{1r}}} \\
&\quad - \frac{i(\tilde{g}^e_{\mathcal{L}_1})_{rij}(\gamma_{\mathcal{L}_1})^*_r}{2M^2_{\mathcal{L}_{1r}}} + \frac{i(\tilde{g}^e_{\mathcal{L}_1})^*_{rji}(\gamma_{\mathcal{L}_1})_r}{2M^2_{\mathcal{L}_{1r}}} \\
&\quad - \frac{\mathrm{Im}\left((\tilde{\gamma}^B_{\mathcal{L}_1})_r(\gamma_{\mathcal{L}_1})^*_r\right)g_1\delta_{ij}}{M^2_{\mathcal{L}_{1r}}} \Bigg\},
\end{aligned}
\tag{8.92}
$$

$$
\begin{aligned}
Z_\phi \ (C_{\phi d})_{ij} =& \frac{(\lambda^d_{Q_1})_{rj}(\lambda^d_{Q_1})^*_{ri}}{2M^2_{Q_{1r}}} - \frac{(\lambda_{Q_5})_{rj}(\lambda_{Q_5})^*_{ri}}{2M^2_{Q_{5r}}} \\
& - \frac{\mathrm{Re}\left((\hat{g}^\phi_{\mathcal{B}})_r\right)(g^d_{\mathcal{B}})_{rij}}{M^2_{\mathcal{B}_r}} - \frac{g_1\delta_{ij}(g^B_{\mathcal{L}_1})_{rs}(\gamma_{\mathcal{L}_1})^*_r(\gamma_{\mathcal{L}_1})_s}{6M^2_{\mathcal{L}_{1r}}M^2_{\mathcal{L}_{1s}}} \\
& + \frac{i(z^d_{Q_1\mathcal{L}_1})^*_{rsi}(\gamma_{\mathcal{L}_1})^*_s(\lambda^d_{Q_1})_{rj}}{2M_{Q_{1r}}M^2_{\mathcal{L}_{1s}}} - \frac{i(z^d_{Q_1\mathcal{L}_1})_{rsj}(\lambda^d_{Q_1})^*_{ri}(\gamma_{\mathcal{L}_1})_s}{2M_{Q_{1r}}M^2_{\mathcal{L}_{1s}}} \\
& - \frac{i(z_{Q_5\mathcal{L}_1})^*_{rsi}(\gamma_{\mathcal{L}_1})_s(\lambda_{Q_5})_{rj}}{2M_{Q_{5r}}M^2_{\mathcal{L}_{1s}}} + \frac{i(z_{Q_5\mathcal{L}_1})_{rsj}(\lambda_{Q_5})^*_{ri}(\gamma_{\mathcal{L}_1})^*_s}{2M_{Q_{5r}}M^2_{\mathcal{L}_{1s}}} \\
& + \frac{1}{f}\Bigg\{ \frac{i(\tilde{\lambda}^d_{Q_1})^*_{ri}(\lambda^d_{Q_1})_{rj}}{2M_{Q_{1r}}} - \frac{i(\tilde{\lambda}^d_{Q_1})_{rj}(\lambda^d_{Q_1})^*_{ri}}{2M_{Q_{1r}}} \\
& \quad - \frac{i(\tilde{\lambda}^d_{Q_5})^*_{ri}(\lambda_{Q_5})_{rj}}{2M_{Q_{5r}}} + \frac{i(\tilde{\lambda}^d_{Q_5})_{rj}(\lambda_{Q_5})^*_{ri}}{2M_{Q_{5r}}} \\
& \quad - \frac{\hat{y}^{d*}_{jk}(\tilde{g}^{dDq}_{\mathcal{L}_1})_{rik}(\gamma_{\mathcal{L}_1})^*_r}{4M^2_{\mathcal{L}_{1r}}} - \frac{\hat{y}^d_{ik}(\tilde{g}^{dDq}_{\mathcal{L}_1})^*_{rjk}(\gamma_{\mathcal{L}_1})_r}{4M^2_{\mathcal{L}_{1r}}} \\
& \quad + \frac{\hat{y}^{d*}_{jk}(\tilde{g}^{Ddq}_{\mathcal{L}_1})_{rik}(\gamma_{\mathcal{L}_1})^*_r}{4M^2_{\mathcal{L}_{1r}}} + \frac{\hat{y}^d_{ik}(\tilde{g}^{Ddq}_{\mathcal{L}_1})^*_{rjk}(\gamma_{\mathcal{L}_1})_r}{4M^2_{\mathcal{L}_{1r}}} \\
& \quad - \frac{i(\tilde{g}^d_{\mathcal{L}_1})_{rij}(\gamma_{\mathcal{L}_1})^*_r}{2M^2_{\mathcal{L}_{1r}}} + \frac{i(\tilde{g}^d_{\mathcal{L}_1})^*_{rji}(\gamma_{\mathcal{L}_1})_r}{2M^2_{\mathcal{L}_{1r}}} \\
& \quad - \frac{\mathrm{Im}\left((\tilde{\gamma}^B_{\mathcal{L}_1})_r(\gamma_{\mathcal{L}_1})^*_r\right)g_1\delta_{ij}}{3M^2_{\mathcal{L}_{1r}}} \Bigg\}, \qquad (8.93)
\end{aligned}
$$

$$
\begin{aligned}
Z_\phi\ (C_{\phi u})_{ij} =& -\frac{(\lambda^u_{Q_1})_{rj}(\lambda^u_{Q_1})^*_{ri}}{2M^2_{Q_{1r}}} + \frac{(\lambda_{Q_7})_{rj}(\lambda_{Q_7})^*_{ri}}{2M^2_{Q_{7r}}} \\
& - \frac{\mathrm{Re}\left((\hat{g}^\phi_{\mathcal{B}})_r\right)(g^u_{\mathcal{B}})_{rij}}{M^2_{\mathcal{B}_r}} + \frac{g_1\delta_{ij}(g^B_{\mathcal{L}_1})_{rs}(\gamma_{\mathcal{L}_1})^*_r(\gamma_{\mathcal{L}_1})_s}{3M^2_{\mathcal{L}_{1r}}M^2_{\mathcal{L}_{1s}}} \\
& - \frac{i(z^u_{Q_1\mathcal{L}_1})^*_{sri}(\gamma_{\mathcal{L}_1})_r(\lambda^u_{Q_1})_{sj}}{2M^2_{\mathcal{L}_{1r}}M_{Q_{1s}}} + \frac{i(z^u_{Q_1\mathcal{L}_1})_{srj}(\lambda^u_{Q_1})^*_{si}(\gamma_{\mathcal{L}_1})^*_r}{2M^2_{\mathcal{L}_{1r}}M_{Q_{1s}}} \\
& + \frac{i(z_{Q_7\mathcal{L}_1})^*_{rsi}(\gamma_{\mathcal{L}_1})^*_s(\lambda_{Q_7})_{rj}}{2M_{Q_{7r}}M^2_{\mathcal{L}_{1s}}} - \frac{i(z_{Q_7\mathcal{L}_1})_{rsj}(\lambda_{Q_7})^*_{ri}(\gamma_{\mathcal{L}_1})_s}{2M_{Q_{7r}}M^2_{\mathcal{L}_{1s}}} \\
& + \frac{1}{f}\Bigg\{ -\frac{i(\tilde{\lambda}^u_{Q_1})^*_{ri}(\lambda^u_{Q_1})_{rj}}{2M_{Q_{1r}}} + \frac{i(\tilde{\lambda}^u_{Q_1})_{rj}(\lambda^u_{Q_1})^*_{ri}}{2M_{Q_{1r}}} \\
& \quad + \frac{i(\tilde{\lambda}^u_{Q_7})^*_{ri}(\lambda_{Q_7})_{rj}}{2M_{Q_{7r}}} - \frac{i(\tilde{\lambda}^u_{Q_7})_{rj}(\lambda_{Q_7})^*_{ri}}{2M_{Q_{7r}}} \\
& \quad - \frac{\hat{y}^u_{ik}(\tilde{g}^{qDu}_{\mathcal{L}_1})_{rjk}(\gamma_{\mathcal{L}_1})^*_r}{4M^2_{\mathcal{L}_{1r}}} - \frac{\hat{y}^{u*}_{jk}(\tilde{g}^{qDu}_{\mathcal{L}_1})^*_{rik}(\gamma_{\mathcal{L}_1})_r}{4M^2_{\mathcal{L}_{1r}}} \\
& \quad + \frac{\hat{y}^u_{ik}(\tilde{g}^{Dqu}_{\mathcal{L}_1})_{rjk}(\gamma_{\mathcal{L}_1})^*_r}{4M^2_{\mathcal{L}_{1r}}} + \frac{\hat{y}^{u*}_{jk}(\tilde{g}^{Dqu}_{\mathcal{L}_1})^*_{rik}(\gamma_{\mathcal{L}_1})_r}{4M^2_{\mathcal{L}_{1r}}} \\
& \quad - \frac{i(\tilde{g}^u_{\mathcal{L}_1})_{rij}(\gamma_{\mathcal{L}_1})^*_r}{2M^2_{\mathcal{L}_{1r}}} + \frac{i(\tilde{g}^u_{\mathcal{L}_1})^*_{rji}(\gamma_{\mathcal{L}_1})_r}{2M^2_{\mathcal{L}_{1r}}} \\
& \quad + \frac{2\,\mathrm{Im}\left((\tilde{\gamma}^B_{\mathcal{L}_1})_r(\gamma_{\mathcal{L}_1})^*_r\right)g_1\delta_{ij}}{3M^2_{\mathcal{L}_{1r}}}\Bigg\},
\end{aligned}
\tag{8.94}
$$

$$
\begin{aligned}
Z_\phi\ (C_{\phi ud})_{ij} =& \frac{(\lambda^d_{Q_1})_{rj}(\lambda^u_{Q_1})^*_{ri}}{M^2_{Q_{1r}}} - \frac{(\hat{g}^\phi_{\mathcal{B}_1})_r(g^{du}_{\mathcal{B}_1})^*_{rji}}{M^2_{\mathcal{B}_{1r}}} \\
& + \frac{i(z^u_{Q_1\mathcal{L}_1})^*_{rsi}(\gamma_{\mathcal{L}_1})_s(\lambda^d_{Q_1})_{rj}}{M_{Q_{1r}}M^2_{\mathcal{L}_{1s}}} - \frac{i(z^d_{Q_1\mathcal{L}_1})_{rsj}(\lambda^u_{Q_1})^*_{ri}(\gamma_{\mathcal{L}_1})_s}{M_{Q_{1r}}M^2_{\mathcal{L}_{1s}}} \\
& + \frac{1}{f}\Bigg\{ \frac{i(\tilde{\lambda}^u_{Q_1})^*_{ri}(\lambda^d_{Q_1})_{rj}}{M_{Q_{1r}}} - \frac{i(\tilde{\lambda}^d_{Q_1})_{rj}(\lambda^u_{Q_1})^*_{ri}}{M_{Q_{1r}}} + \frac{i(\tilde{g}^{du}_{\mathcal{L}_1})^*_{rji}(\gamma_{\mathcal{L}_1})_r}{M^2_{\mathcal{L}_{1r}}} \\
& \quad - \frac{\hat{y}^u_{ik}(\tilde{g}^{dDq}_{\mathcal{L}_1})^*_{rjk}(\gamma_{\mathcal{L}_1})_r}{2M^2_{\mathcal{L}_{1r}}} + \frac{\hat{y}^u_{ik}(\tilde{g}^{Ddq}_{\mathcal{L}_1})^*_{rjk}(\gamma_{\mathcal{L}_1})_r}{2M^2_{\mathcal{L}_{1r}}} \\
& \quad + \frac{\hat{y}^{d*}_{jk}(\tilde{g}^{qDu}_{\mathcal{L}_1})^*_{rik}(\gamma_{\mathcal{L}_1})_r}{2M^2_{\mathcal{L}_{1r}}} - \frac{\hat{y}^{d*}_{jk}(\tilde{g}^{Dqu}_{\mathcal{L}_1})^*_{rik}(\gamma_{\mathcal{L}_1})_r}{2M^2_{\mathcal{L}_{1r}}}\Bigg\}.
\end{aligned}
\tag{8.95}
$$

8.6 Example: interpretation of LHCb anomalies

Our UV/IR dictionary is a tool that can be used for different phenomenological purposes, such as finding indirect limits on the parameters of explicit models, constructing BSM models consistent with existing data or analyzing deviations with respect to the SM in terms of new physics. In this section we illustrate the latter application with a particular example: explaining the hints in LHCb data of a violation of lepton flavor universality (LFU) in B-meson decays [199, 200].[6] We will first identify which heavy multiplets can generate the necessary operators and then look at correlated effects that could constrain or test the different possibilities. Our schematic analysis is just intended as an illustration. Most of the results in this section have in fact already appeared in the literature, but our formulation allows for a compact unified discussion of the different explanations.

The measurement of the observables $R_K \equiv \mathrm{Br}(B^+ \to K^+\mu^+\mu^-)/\mathrm{Br}(B^+ \to K^+e^+e^-)$ and $R_{K^*} \equiv \mathrm{Br}(B \to K^*\mu^+\mu^-)/\mathrm{Br}(B \to K^*e^+e^-)$ provides a particularly clean test of LFU of the gauge interactions, since a large component of the SM theory uncertainties cancel in the ratio. The LHCb collaboration has presented measurements of these ratios, both of which deviate from the SM predictions by $\sim 2.6\ \sigma$ [199] and $\sim 2.4\ \sigma$ [200], respectively. These are not the only anomalies in $b \to s\ell^+\ell^-$ processes, with some discrepancies also in the angular distributions of $B \to K^*\mu^+\mu^-$ [202–204], or in the differential branching fractions of $B \to K\mu^+\mu^-$ [202] and $B_s \to \phi\mu^+\mu^-$ [205]. At present, the different deviations follow a pattern that can be consistently explained by the presence of new physics. Altogether, the global fit to all flavour anomalies points to a deviation with respect to the SM hypotheses of $\sim$ 3-5 σ, depending on the estimates assumed for the SM hadronic uncertainties in some of the observables [206–211].

The observed deviations from LFU in B decays are well described by the following four-fermion effective Hamiltonian, valid at energies $E \ll M_W$,

$$\mathcal{H}_{\mathrm{E}ff}^{b\to s\ell\ell} = -V_{tb}V_{ts}^*\frac{\alpha_{\mathrm{em}}}{4\pi}\frac{4G_F}{\sqrt{2}}\sum C_{ij}^{\ell}\mathcal{O}_{ij}^{\ell} + \mathrm{h.c.}, \tag{8.96}$$

where

$$\mathcal{O}_{ij}^{\ell} = (\bar{s}\gamma^\mu P_i b)(\bar{\ell}\gamma_\mu P_j \ell) \tag{8.97}$$

are the different chiral four-fermion operators that can be obtained from the product of two vector currents, with $P_{L,R} = \frac{1}{2}(1 \mp \gamma_5)$. The fit to $R_{K,K*}$ favors an explanation where new physics is present in left-handed leptons and, in particular, points to a sizable deviation from the SM hypotheses in C_{LL}^{ℓ}. For the purpose of this example, we focus the discussion around these interactions. They can be either $C_{LL}^{\mu} < 0$ or $C_{LL}^{e} > 0$, although a global fit to all B anomalies prefers new physics in the muon sector, with $C_{LL}^{\mu} \approx -1.2 \pm 0.3$ [206–211].

Matching $\mathcal{O}_{LL}^{\ell}$ with the dimension-six SMEFT at the tree level results in the following four-fermion contributions to C_{LL}^{ℓ}:

$$C_{LL}^{\ell} = \lambda_t^{-1}\left(C_{lq}^{(1)} + C_{lq}^{(3)}\right)_{\ell\ell 23}, \tag{8.98}$$

where $\lambda_t \equiv V_{tb}V_{ts}^*\frac{\alpha_{\mathrm{em}}}{4\pi}\frac{4G_F}{\sqrt{2}}$, and we are working in a fermion basis with diagonal Yukawa interactions for the down-type quarks. The operators $\mathcal{O}_{\phi q}^{(1,3)}$ and $\mathcal{O}_{\phi l}^{(1,3)}$ also contribute,

[6] Recently, new measurements of the related observables has been reported in ref. [201]

via a modification of the couplings of the Z boson to the relevant quarks and leptons. However, such non-universal anomalous couplings are strongly bounded by LEP data, so we concentrate on the operators $\mathcal{O}_{lq}^{(1)}$ and $\mathcal{O}_{lq}^{(3)}$.

The relevant entries of the UV/IR dictionary are eqs. (8.30) and (8.31). A look at the masses in the denominators of each term allows us to easily identify all the types of multiplets that can contribute to $C_{lq}^{(1)}$ and $C_{lq}^{(3)}$ at the tree level:

$$\{ \overset{(3,3)_{-\frac{1}{3}}}{\zeta}, \quad \overset{(1,1)_0}{\mathcal{B}}, \quad \overset{(1,3)_0}{\mathcal{W}}, \quad \overset{(3,1)_{\frac{2}{3}}}{\mathcal{U}_2}, \quad \overset{(3,3)_{\frac{2}{3}}}{\mathcal{X}} \}. \tag{8.99}$$

Note that for ω_1, $C_{lq}^{(1)} = -C_{lq}^{(3)}$ and therefore $C_{LL}^{\ell} = 0$. This list with one scalar and four vector multiplets agrees with the classification in other studies, see, e.g. [207,212–214]. From eqs. (8.30) and (8.31) we also see that there is no collective contribution with several heavy propagators in the same diagram. Most importantly, we can pinpoint the relevant couplings in $\mathcal{L}_{\text{BSM}}$. This is a simple example of looking at an IR entry of the dictionary to find its UV translation.

For instance, we can readily check in eqs. (8.30) and (8.31) that a product of leptoquark couplings is involved in the case of the scalar ζ and the vector bosons $\mathcal{X}$ and $\mathcal{U}^2$, while the vectors $\mathcal{B}$ and $\mathcal{W}$ contribute through a product of a diagonal lepton coupling and a flavor-changing quark coupling.

With this information, one can proceed to investigate in a systematic way all the different constraints (or signals) arising from other processes that involve the same couplings and particles. Processes involving other couplings will also be of great interest if the anomalies are confirmed. Direct searches with resonant production can be very relevant, but here we focus mostly on indirect searches. They reduce essentially to an analysis of the different operators, besides $\mathcal{O}_{lq}^{(1)}$ and $\mathcal{O}_{lq}^{(3)}$, that are generated when the heavy particles are integrated out. We can distinguish three kinds of contributions to the Wilson coefficients of the other induced operators:

Type I: Contributions that depend *only* on couplings that enter in C_{LL}^{ℓ}. The corresponding observable effects are then correlated with the ones entering in $b \to s\ell^+\ell^-$, and can be used to constrain or probe a given solution to the B-meson anomalies.

Type II: Contributions that depend on these couplings but can be made arbitrarily small by tuning an interaction not entering in C_{LL}^{ℓ}. In this case, the correlations require extra information on that coupling.

Type III: Contributions that do not depend on the couplings that appear in C_{LL}^{ℓ}. These are completely uncorrelated.

In this classification it is of course crucial to take flavor indices into account. Even if contributions of type I are more relevant, an observation of the effects of contributions of type II and III could also be used to support the new physics interpretation and for model discrimination.

Let us examine along these lines the multiplets ζ, $\mathcal{X}$ and $\mathcal{W}$, which have the compelling feature of allowing only for the required left-handed couplings. In this case, we will use the dictionary in the UV to IR direction. Tables 8.1 and 8.3 prove handy for this task, as they list the operators we need to look at for each assumed multiplet.

Scalar leptoquark ζ

The interactions of ζ can be found in eq. (7.13). We see that the scalar ζ has, up to flavor indices, two couplings (besides the gauge couplings, determined by quantum numbers): the lepto-quark coupling y_ζ^{ql} and the coupling to quarks y_ζ^{qq}. A glimpse at table 8.1 tells us that the following operators are induced: $\mathcal{O}_{lq}^{(1,3)}$, $\mathcal{O}_{qq}^{(1,3)}$ and $\mathcal{O}_{qqq}$. Then, we read the precise contributions to their Wilson coefficients from eqs. (8.28)-(8.31) and (8.54). Assuming only one ζ multiplet,

$$(C_{lq}^{(1)})_{ijkl} = 3(C_{lq}^{(3)})_{ijkl} = \frac{3}{4}\frac{(y_\zeta^{ql})_{lj}(y_\zeta^{ql})^*_{ki}}{M_\zeta^2}, \tag{8.100}$$

$$(C_{qq}^{(1)})_{ijkl} = -3(C_{qq}^{(3)})_{ilkj} = \frac{3(y_\zeta^{qq})_{ki}(y_\zeta^{qq})^*_{lj}}{2M_\zeta^2}, \tag{8.101}$$

$$(C_{qqq})_{ijkl} = -\frac{2(y_\zeta^{qq})^*_{ij}(y_\zeta^{ql})_{kl}}{M_\zeta^2}. \tag{8.102}$$

Looking at the flavor structure of (8.100), we see that we need sizable couplings $(y_\zeta^{ql})_{2\ell}$ and $(y_\zeta^{ql})_{3\ell}$ to explain the anomalies. For sufficiently low mass M_ζ, these couplings can be probed by analyses of single and pair production of ζ at the LHC [215]. The very same couplings also contribute to other components of $C_{qq}^{(1,3)}$, and we conclude that

$$C_{LL}^l \neq 0 \longrightarrow \begin{cases} (C_{lq}^{(1)})_{\ell\ell 33} = 3(C_{lq}^{(3)})_{\ell\ell 33} = \frac{3\left|(y_\zeta^{ql})_{3\ell}\right|^2}{4M_\zeta^2} \neq 0, \\ (C_{lq}^{(1)})_{\ell\ell 22} = 3(C_{lq}^{(3)})_{\ell\ell 22} = \frac{3\left|(y_\zeta^{ql})_{2\ell}\right|^2}{4M_\zeta^2} \neq 0. \end{cases} \tag{8.103}$$

These are contributions of type I. The corresponding effects in hadronic-flavor-preserving processes are correlated with the B anomalies. From (8.103) it is also clear that in these processes each of the two couplings can be measured, in principle, independently. Both the flavor-preserving and flavor-violating effects in an electron explanation of the anomalies can be tested in e^+e^- colliders. The observed values of R_{K,K^*} can be reproduced with $C_{LL}^\ell \sim O(1)$, which corresponds to a new physics interaction scale of about 35 TeV, well above the sensitivity of LEP2. Therefore, current e^+e^- data do not provide significant constraints on the relevant couplings. However, they could be tested at future lepton colliders. Any other combination of flavor indices gives contributions of type III, with effects that are uncorrelated with the anomalies. The same holds for the contributions to the operators $\mathcal{O}_{qq}^{(1,3)}$, which involve the quark couplings y_ζ^{qq}. Finally, the baryon-number violating operator $\mathcal{O}_{qqq}$ receives contributions of type II or type III, depending on the flavor indices. Note in particular that the quark couplings for the first family are strongly constrained by the non-observation of proton decay.

Vector leptoquark $\mathcal{X}$

The analysis of the vector multiplet $\mathcal{X}$ is similar, but as we can see in eq. (7.25) in this case there is only one non-gauge coupling (up to flavor indices): the lepto-quark coupling $g_\mathcal{X}$. In table 8.3 we see that only the operators $\mathcal{O}_{lq}^{(1,3)}$ are generated in the

EFT below the mass $M_{\mathcal{X}}$. Assuming only one replica of $\mathcal{X}$, eq. (7.13) gives

$$(C_{lq}^{(1)})_{ijkl} = -3(C_{lq}^{(3)})_{ijkl} = -\frac{3(g_{\mathcal{X}})^*_{jk}(g_{\mathcal{X}})_{il}}{8M_{\mathcal{X}}^2}. \tag{8.104}$$

We see that the contribution of $\mathcal{X}$ to C_{LL}^{ℓ} is proportional to the product of $(g_{\mathcal{X}})^*_{\ell 2}$ and $(g_{\mathcal{X}})_{\ell 3}$. Again, there are correlations with the coefficients of the corresponding hadronic flavor-conserving operators:

$$C_{LL}^{l} \neq 0 \longrightarrow \begin{cases} (C_{lq}^{(1)})_{\ell\ell 33} = -3(C_{lq}^{(3)})_{\ell\ell 33} = -\frac{3|(g_{\mathcal{X}})_{\ell 3}|^2}{8M_{\mathcal{X}}^2} \neq 0, \\ (C_{lq}^{(1)})_{\ell\ell 22} = -3(C_{lq}^{(3)})_{\ell\ell 22} = -\frac{3|g_{\mathcal{X}})_{\ell 2}|^2}{8M_{\mathcal{X}}^2} \neq 0. \end{cases} \tag{8.105}$$

The same discussion in the paragraph below eq. (8.103) applies to this case, except for the fact that now there are no purely-hadronic couplings.

Vector iso-triplet $\mathcal{W}$

As we can check in eq. (7.25), the vector iso-triplet $\mathcal{W}$ has couplings $g_{\mathcal{W}}^{l}$ and $g_{\mathcal{W}}^{q}$ to left-handed fermions and $g_{\mathcal{W}}^{\phi}$ to the Higgs doublet. The latter induces a mixing of the Z' and W' components with the Z and W bosons, respectively. There are also couplings involving a possible vector doublet $\mathcal{L}_1$, which we shall not consider. For masses $M_{\mathcal{W}}$ light enough, the Z' and W' bosons in $\mathcal{W}$ can be produced at hadron colliders if the light-quark couplings are not too small. They then decay into di-leptons (including lepton + MET) [216] and di-bosons [217] through the couplings to leptons and to the Higgs, respectively. Regarding indirect effects, the operators that can be induced are listed in the $\mathcal{W}$ entry of table 8.3. The most relevant ones in the context of the B anomalies are $\mathcal{O}_{lq}^{(3)}$, $\mathcal{O}_{ll}$ and $\mathcal{O}_{qq}^{(3)}$, with Wilson coefficients given by (see eqs. (8.31), (8.27) and (8.29))

$$(C_{lq}^{(3)})_{ijkl} = -\frac{(g_{\mathcal{W}}^{l})_{ij}(g_{\mathcal{W}}^{q})_{kl}}{4M_{\mathcal{W}}^2}, \tag{8.106}$$

$$(C_{ll})_{ijkl} = -\frac{(g_{\mathcal{W}}^{l})_{ij}(g_{\mathcal{W}}^{l})_{kl}}{8M_{\mathcal{W}}^2}, \tag{8.107}$$

$$(C_{qq}^{(3)})_{ijkl} = -\frac{(g_{\mathcal{W}}^{q})_{ij}(g_{\mathcal{W}}^{q})_{kl}}{8M_{\mathcal{W}}^2}. \tag{8.108}$$

We see that to get the necessary C_{LL}^{ℓ} we need sizable couplings $(g_{\mathcal{W}}^{l})_{\ell\ell}$ and $(g_{\mathcal{W}}^{q})_{23}$. The first one must be non-universal, while the second one is explicitly flavor-changing. Schematically, we have the following correlations:

$$C_{LL}^{l} \neq 0 \longrightarrow \begin{cases} (C_{ll})_{\ell\ell\ell\ell} = -\frac{(g_{\mathcal{W}}^{l})^2_{\ell\ell}}{8M_{\mathcal{W}}^2} \neq 0, \\ (C_{qq}^{(3)})_{2323} = -\frac{(g_{\mathcal{W}}^{q})^2_{23}}{8M_{\mathcal{W}}^2} \neq 0, \\ (C_{qq}^{(3)})_{2332} = -\frac{|(g_{\mathcal{W}}^{q})_{23}|^2}{8M_{\mathcal{B}}^2} \neq 0. \end{cases} \tag{8.109}$$

Of particular importance is the contribution to $(C_{qq}^{(3)})_{2323}$, as it generates contributions to $B_s - \bar{B}_s$ mixing amplitudes. Such contributions are tightly constrained, pushing the new physical interaction scale to values of $O(100)$ TeV [218, 219].[7] This case shows that, although $\Delta F = 1$ and $\Delta F = 2$ bounds are uncorrelated in a low-energy operator analysis, correlations may exist and be crucial in specific explanations of the B anomalies. Similar considerations apply often to processes that may not appear to be connected in an effective low-energy description. Of course, the correlations become weaker as more particles (with the same or different quantum numbers) are included, but some of them are unavoidable [224].

Again, other combinations of flavor indices give contributions of type II and III. The contributions of $\mathcal{W}$ to $\psi^2\phi^2 D$ operators,

$$(C_{\phi l}^{(3)})_{ij} = -\frac{\mathrm{Re}\left\{(g_{\mathcal{W}}^{l})_{ij}\; g_{\mathcal{W}}^{\phi}\right\}}{4M_{\mathcal{W}}^2}, \tag{8.110}$$

$$(C_{\phi q}^{(3)})_{ij} = -\frac{\mathrm{Re}\left\{(g_{\mathcal{W}}^{q})_{ij}\; g_{\mathcal{W}}^{\phi}\right\}}{4M_{\mathcal{W}}^2}, \tag{8.111}$$

are of type II for $ij = \ell\ell$ and $ij = 23, 32$, respectively, and of type III otherwise. These operators modify the Z and W couplings to leptons and quarks, so they are constrained by electroweak precision data, by observables sensitive to flavor-changing decays of the Z boson, $B_s - \bar{B}_s$ mixing and by non-resonant processes with di-lepton and di-jet final states at the LHC. But these limits can always be made compatible with the lepton and quark couplings that explain the anomalies by tuning the Higgs coupling $g_{\mathcal{W}}^{\phi}$ to be small. This coupling also induces type-III effects in Higgs physics, via the operators $\mathcal{O}_{\phi D}$, $\mathcal{O}_{\phi}$, $\mathcal{O}_{\phi\Box}$ and $\mathcal{O}_{f\phi}$ $(f = e, d, u)$, with Wilson coefficients

$$C_{\phi D} = -\frac{\mathrm{Im}\left\{(g_{\mathcal{W}}^{\phi})\right\}^2}{2M_{\mathcal{W}}^2}, \qquad C_{\phi} = -\frac{\lambda_\phi (g_{\mathcal{W}}^{\phi})^2}{M_{\mathcal{W}}^2}, \qquad C_{\phi\Box} = -\frac{|g_{\mathcal{W}}^{\phi}|^2}{4M_{\mathcal{W}}^2},$$
$$(C_{e\phi(d\phi)})_{ij} = y_{ji}^{e(d)*} a, \qquad (C_{u\phi})_{ij} = -2y_{ji}^{u*} a^*, \quad a \equiv -\frac{2|g_{\mathcal{W}}^{\phi}|^2 + i\,\mathrm{Im}((g_{\mathcal{W}}^{\phi})^2)}{8M_{\mathcal{W}}^2} \tag{8.112}$$

(Note that we have replaced $\hat{\lambda}_\phi$ and $\hat{y}^{e,d,u}$ by λ_ϕ and $y^{e,d,u}$, respectively, as in the extension we are considering there are no contributions to dimension-four operators.)

Before finishing this section let us point out another possible usage of the UV/IR dictionary for model building. Say we are interested in a given class of models, including one or more of the multiplets that contribute at the tree level to the dimension-six effective Lagrangian. Then we can relax the indirect limits on the corresponding couplings by including other multiplets that (partially) cancel the contributions to the Wilson coefficients of interest. The different possibilities can be easily determined by a scan of our results in section 8.5. For instance, it is easy to see that the contributions of $\mathcal{W}$ to $(C_{ll})_{1111}$, which could be tested at future $e^+e^- \to e^+e^-$ colliders, can be (partially) cancelled, with some tuning, against the ones of a hypercharge 1 scalar singlet $\mathcal{S}_1$ or triplet Ξ_1 [224].

[7] These bounds, together with the ones discussed below, can be relaxed by reducing the $(g_{\mathcal{W}}^{q})_{23}$ and $g_{\mathcal{W}}^{\phi}$ couplings at the expense of increasing the corresponding $(g_{\mathcal{W}}^{l})_{\ell\ell}$ ones [220, 221]. A similar comment applies to the case of $\mathcal{B}$. Such leptophilic vector bosons can be probed at colliders in multi-lepton searches [222, 223].

8.7 Example: Higgs physics in simple models

In this section, we use the UV/IR dictionary to discuss new physics effects in Higgs physics in simple SM extensions with one or two particles. First, we find the part of the SMEFT most important for Higgs physics. Then, we study the correlations found in each model between the effects in Higgs physics and those in other types of observables. The implications in Higgs physics are not always apparent in the dimension-6 Lagrangian, because of possible correlations between the coefficients of operators, and in indirect effects from the modified relations used to obtain the values of the SM input parameters. These implications have been worked out in the literature and can be found, for instance, in ref. [225].

The operators in the Warsaw basis that contain the Higgs field and can be generated at the tree level are listed in the first column of table 8.4. For simplicity, we disregard four-fermion interactions, even if they might influence Higgs physics indirectly. The operators $\mathcal{O}_\phi$, $\mathcal{O}_{\phi\Box}$ and $(\mathcal{O}_{u\phi})_{33}$ are the ones that are currently less constrained by experimental data. The operator $\mathcal{O}_{\phi D}$ and all those of the type $\mathcal{O}_{\phi\psi}$ have been constrained to be small by electroweak precision tests (EWPT), while $\mathcal{O}_{\phi ud}$ is also limited by low-energy data. Experimental data from Higgs physics tell us that the Wilson coefficients of the interactions $(\mathcal{O}_{e\phi})_{33}$ and $(\mathcal{O}_{d\phi})_{33}$ should be well below 1 TeV^{-2}.

Using the UV/IR dictionary, we can readily identify which heavy fields can generate each operator at the tree level. They are listed in the last column of table 8.4, next to each corresponding operator. In the following, we consider a few simplified models that contain one or two of these fields. All these models are particular cases of the BSMEFT. Our selection includes fields that appear frequently in more elaborate setups and illustrates typical features of the latter. Furthermore, all the operators in table 8.4 are generated by this set of models. We first discuss popular extensions of the SM with only one particle, which are severely constrained by EWPT. Then, we study minimal extensions with several particles that preserve custodial symmetry. In this case, the strongest constraints are evaded and strong effects in Higgs physics are allowed. In the explicit results below, it can be observed that many of the contributions to the Wilson coefficients have a definite sign.

8.7.1 Models with one extra particle

Quark singlet: $U \sim (3,1)_{2/3}$

In chapter 9, we study the physics of heavy vector-like quarks in detail. Here, we concentrate on the Higgs physics of a vector-like quark U, with the same quantum numbers as the right-handed top, that only has renormalizable interactions. The relevant sector of $\mathcal{L}_{\text{BSM}}$ is

$$\mathcal{L}_U = \mathcal{L}_{\text{SM}} + i\bar{U}\not{D}U + M_U\bar{U}U - \left((\lambda_U)_i \bar{U}_R\tilde{\phi}^\dagger q_{Li} + \text{h.c.}\right), \tag{8.113}$$

which we have particularized for only one flavor of the field U. To avoid flavour-changing neutral currents, we consider the case in which only one of the three $(\lambda_U)_i$ is non-vanishing. From the results presented in section 8.4, we extract the contributions of U to the SMEFT, which we give in table 8.5. The first two operators contribute to gauge couplings (also in association with one or two Higgs bosons) of the SM quarks

Name	Operator	Constraints	Fields that generate it
$\mathcal{O}_\phi$	$\lvert\phi\rvert^6$	[weak constraints]	$\mathcal{S}$, φ, Ξ, Ξ_1, Θ_1, Θ_3, $\mathcal{B}_1$, $\mathcal{W}$
$\mathcal{O}_{\phi\Box}$	$\lvert\phi\rvert^2\Box\lvert\phi\rvert^2$	[weak constraints]	$\mathcal{S}$, Ξ, Ξ_1, $\mathcal{B}$, $\mathcal{B}_1$, $\mathcal{W}$, $\mathcal{W}_1$
$\mathcal{O}_{\phi D}$	$\lvert\phi^\dagger D_\mu\phi\rvert^2$	EWPT	Ξ, Ξ_1, $\mathcal{B}$, $\mathcal{B}_1$, $\mathcal{W}$, $\mathcal{W}_1$
$\mathcal{O}_{e\phi}$	$\lvert\phi\rvert^2\bar{l}_L\phi e_R$	Higgs data	$\mathcal{S}$, φ, Ξ, Ξ_1, E, Δ_1, Δ_3, Σ, Σ_1, $\mathcal{B}$, $\mathcal{B}_1$, $\mathcal{W}$, $\mathcal{W}_1$
$\mathcal{O}_{d\phi}$	$\lvert\phi\rvert^2\bar{q}_L\phi d_R$	Higgs data	$\mathcal{S}$, φ, Ξ, Ξ_1, D, Q_1, Q_5, T_1, T_2, $\mathcal{B}$, $\mathcal{B}_1$, $\mathcal{W}$, $\mathcal{W}_1$
$\mathcal{O}_{u\phi}$	$\lvert\phi\rvert^2\bar{q}_L\tilde{\phi} u_R$	[weak constraints]	$\mathcal{S}$, φ, Ξ, Ξ_1, U, Q_1, Q_7, T_1, T_2, $\mathcal{B}$, $\mathcal{B}_1$, $\mathcal{W}$, $\mathcal{W}_1$
$\mathcal{O}_{\phi l}^{(1)}$	$(\bar{l}_L\gamma^\mu l_L)(\phi^\dagger i\overleftrightarrow{D}_\mu\phi)$	EWPT	N, E, Σ, Σ_1, $\mathcal{B}$
$\mathcal{O}_{\phi l}^{(3)}$	$(\bar{l}_L\gamma^\mu\sigma^a l_L)(\phi^\dagger i\overleftrightarrow{D}^a_\mu\phi)$	EWPT	N, E, Σ, Σ_1, $\mathcal{W}$
$\mathcal{O}_{\phi q}^{(1)}$	$(\bar{q}_L\gamma^\mu q_L)(\phi^\dagger i\overleftrightarrow{D}_\mu\phi)$	EWPT	U, D, T_1, T_2, $\mathcal{B}$
$\mathcal{O}_{\phi q}^{(3)}$	$(\bar{q}_L\gamma^\mu\sigma^a q_L)(\phi^\dagger i\overleftrightarrow{D}^a_\mu\phi)$	EWPT	U, D, T_1, T_2, $\mathcal{W}$
$\mathcal{O}_{\phi e}$	$(\bar{e}_R\gamma^\mu e_R)(\phi^\dagger i\overleftrightarrow{D}_\mu\phi)$	EWPT	Δ_1, Δ_3, $\mathcal{B}$
$\mathcal{O}_{\phi u}$	$(\bar{u}_R\gamma^\mu u_R)(\phi^\dagger i\overleftrightarrow{D}_\mu\phi)$	EWPT	Q_1, Q_7, $\mathcal{B}$
$\mathcal{O}_{\phi d}$	$(\bar{d}_R\gamma^\mu d_R)(\phi^\dagger i\overleftrightarrow{D}_\mu\phi)$	EWPT	Q_1, Q_5, $\mathcal{B}$
$\mathcal{O}_{\phi ud}$	$(\bar{u}_R\gamma^\mu d_R)(\phi^\dagger iD_\mu\tilde{\phi})$	low-energy data	Q_1, $\mathcal{B}_1$

Table 8.4: Fields that generate each operator containing the Higgs at the tree level, together with the type of experimental data that constrains it the most.

$(C_{\phi q}^{(1)})_{ij}$	$(C_{\phi q}^{(3)})_{ij}$	$(C_{u\phi})_{ij}$
$\frac{(\lambda_U)_i^*(\lambda_U)_j}{4M_U^2}$	$\frac{(\lambda_U)_i^*(\lambda_U)_j}{4M_U^2}$	$y_{jk}^{u*}\frac{(\lambda_U)_i^*(\lambda_U)_k}{4M_U^2}$

Table 8.5: Tree-level contributions to operators with the Higgs from the U heavy vector-like quarks.

whereas the third one contributes to the up-type Yukawa couplings (again plus one or two extra Higgs bosons). Associated WH or ZH production from the first two operators in the case of first and second generation quarks is constrained by EWPT. Other correlations appear once we consider this specific model. Indeed, all three Wilson coefficients are controlled by a single parameter $\lvert(\lambda_U)_i\rvert^2/M_U^2$. Top gauge couplings are not so severely constrained, thus allowing *a priori* for a significant deviation of the top Yukawa coupling. However, in this case EWPT still constrain the parameters of the model through one-loop contributions.

Current direct searches for pair production of this vector-like top put a lower bound of about a TeV for its mass [226,227]. Single production is also sensitive to the Yukawa coupling of the new quarks [228,229].

$C_{\phi D}$	$C_{\phi\Box}$	$(C_{\phi\psi}^{[(1)]})_{ij}$
$-\frac{2(g_{\mathcal{B}}^{\phi})^2}{M_{\mathcal{B}}^2}$	$-\frac{(g_{\mathcal{B}}^{\phi})^2}{2M_{\mathcal{B}}^2}$	$-\frac{(g_{\mathcal{B}}^{\psi})_{ij}(g_{\mathcal{B}}^{\phi})}{M_{\mathcal{B}}^2}$

Table 8.6: Tree-level contributions to operators with the Higgs from the neutral vector singlet $\mathcal{B}$.

Neutral vector singlet: $\mathcal{B} \sim (1,1)_0$

In this model, a vector field $\mathcal{B}$ couples to the SM fermions and the Higgs doublet through renormalizable operators. Therefore, the terms of $\mathcal{L}_{\rm BSM}$ we are interested in are

$$\begin{aligned}\mathcal{L}_{\mathcal{B}} = \mathcal{L}_{\rm SM} + \frac{1}{2}\left((D_\mu\mathcal{B}_\nu)(D^\nu\mathcal{B}^\mu) - (D_\mu\mathcal{B}_\nu)(D^\mu\mathcal{B}^\nu) + M_{\mathcal{B}}^2\mathcal{B}_\mu\mathcal{B}^\mu\right)\\ - \sum_\psi (g_{\mathcal{B}}^{\psi})_{ij}\mathcal{B}^\mu\bar{\psi}_i\gamma_\mu\psi_j - \left[g_{\mathcal{B}}^{\phi}\mathcal{B}^\mu\phi^\dagger iD_\mu\phi + \text{h.c.}\right].\end{aligned} \tag{8.114}$$

The only component of this field is associated to a heavy neutral particle of spin 1, i.e. a Z' boson. We assume that the coupling constant $g_{\mathcal{B}}^{\phi}$ to the Higgs is real. The UV/IR dictionary gives four-fermion operators plus the contributions that appear in table 8.6. Note that the modification of the Higgs kinetic term depends on $C_{\phi\Box} - C_{\phi D}/4$, and therefore vanishes in this model. There is however a modification of the SM-like Higgs coupling to vector bosons via $\mathcal{O}_{\phi D}$. From the EWPT constraints on $\mathcal{O}_{\phi D}$, it follows that $g_{\mathcal{B}}^{\phi}$ should be small. This affects every operator with the Higgs generated by $\mathcal{B}$, which will be suppressed by $g_{\mathcal{B}}^{\phi}$ or $(g_{\mathcal{B}}^{\phi})^2$. Similarly, one has effects in associated ZH production coming from the operators $\mathcal{O}_{\phi\psi}^{[(1)]}$, which are also constrained by EWPT.

Searches for single production of neutral vectors decaying to dileptons, dibosons or dijets exclude additional regions in the parameter space of this model [230–236].

8.7.2 Custodial models

Quark bidoublet: $Q_1 \sim (3,2)_{1/6}$ **and** $Q_7 \sim (3,2)_{7/6}$

In this case, we have a multiplet of heavy vector-like quarks in the $(2,2)_{2/3}$ representation of the $SU(2)_L \times SU(2)_R \times U(1)_X$ symmetry group, where $SU(2)_L$ corresponds to the $SU(2)$ factor in the SM gauge group $G_{\rm SM}$ and the hypercharge of the $U(1)$ factor in $G_{\rm SM}$ is given by $Y = T_3^R + X$.

As we have stressed above, models that extend the SM symmetries, such as this one, are particular cases of the BSMEFT, in which some relations between couplings are imposed. This model corresponds to the sector of the BSMEFT containing the 2 quark doublets Q_1 and Q_7 in the $2_{1/6}$ and $2_{7/6}$ irreps of the electroweak gauge group, respectively, with the associated parameters related by $M_{Q_1} = M_{Q_7} =: M$ and $\lambda_{Q_1} = \lambda_{Q_7} =: \lambda$. We assume that the heavy quarks only couple to the third generation of SM quarks. The relevant part of $\mathcal{L}_{\rm BSM}$ is given by

$$\begin{aligned}\mathcal{L}_{\rm bidoublet} = \mathcal{L}_{\rm SM} + i\bar{Q}_7\not{D}Q_7 + i\bar{Q}_1\not{D}Q_1 + M\left(\bar{Q}_7Q_7 + \bar{Q}_1Q_1\right)\\ - \left[\lambda\left(\bar{Q}_{7L}\phi t_R + \bar{Q}_{1L}\tilde{\phi}t_R\right) + \text{h.c.}\right].\end{aligned} \tag{8.115}$$

$(C_{u\phi})_{33}$
$\frac{y_{33}^{u*}\lvert\lambda\rvert^2}{M^2}$

Table 8.7: Tree-level contributions to operators with the Higgs from the quark doublets Q_7 and Q_1, with the interactions in eq. (8.115).

Because of the extended symmetry, the contributions to the $\mathcal{O}_{\phi\psi}$ operators from both doublets cancel each other. Only the operator $(\mathcal{O}_{u\phi})_{33}$ is generated by a tree-level integration, with a positive Wilson coefficient. The explicit value of $(C_{u\phi})_{33}$ in this SM extension is given in table 8.7. Therefore, this is a model which can give large negative contributions to the top Yukawa coupling without producing any other effects at the tree level. Note that one-loop constraints are under control for this particular model: contributions to the T parameter are protected by custodial symmetry, bounds from the S parameter are mild, and the contributions of the new quarks to Higgs production via gluon fusion compensate the reduction in the top Yukawa coupling.

The mass of the extra quarks is bounded from below by direct pair production limits, similarly to the case of the singlet (subsection 8.7.1).

Hypercharge zero vector triplet: $\mathcal{W} \sim (1,3)_0$

The hypercharge zero vector triplet contains a Z' and a W'. It couples to the SM doublets. The relevant part of the BSMEFT Lagrangian is:

$$\begin{aligned}\mathcal{L}_{\mathcal{W}} = \mathcal{L}_{\mathrm{SM}} &+ \frac{1}{2}\left((D_\mu \mathcal{W}_\nu^a)(D^\nu \mathcal{W}^{\mu a}) - (D_\mu \mathcal{W}_\nu^a)(D^\mu \mathcal{W}^{\nu a})\right) \\ &+ \frac{1}{2} M_{\mathcal{W}}^2 \mathcal{W}_\mu^a \mathcal{W}^{\mu a} - \frac{1}{2}(g_{\mathcal{W}}^l)_{ij} \mathcal{W}^{\mu a} \bar{l}_{Li} \sigma^a \gamma_\mu l_{Lj} \\ &- \frac{1}{2}(g_{\mathcal{W}}^q)_{ij} \mathcal{W}^{\mu a} \bar{q}_{Li} \sigma^a \gamma_\mu q_{Lj} - \left[\frac{1}{2}(g_{\mathcal{W}}^\phi) \mathcal{W}^{\mu a} \phi^\dagger \sigma^a i D_\mu \phi + \text{h.c.}\right]. \end{aligned} \tag{8.116}$$

We assume that $g_{\mathcal{W}}^\phi$ is real. Table 8.8 summarizes the tree-level contributions of $\mathcal{W}$ to operators with the Higgs, which can be read from the dictionary. Unlike the case of the vector singlet $\mathcal{B}$, the coupling $g_{\mathcal{W}}^\phi$ is allowed to be large in this case, because the contribution to the T parameter is zero. Therefore, in this model there can be large contributions controlled by $g_{\mathcal{W}}^\phi$ to $\mathcal{O}_\phi$ (which modifies the Higgs trilinear coupling), to $\mathcal{O}_{\phi\Box}$ (that changes the Higgs kinetic term) and to $\mathcal{O}_{\psi\phi}$ (which in this model modifies Yukawa couplings in an universal way). While the mixing of $\mathcal{W}$ with the SM gauge boson induces a custodial symmetric modification on the SM-like Higgs to vector couplings (this effect is captured by $\mathcal{O}_{\phi\Box}$), the net effect in other couplings follows from a modification of the SM relations used to derive the values of the inputs of the model.

As for the singlet (subsection 8.7.1), direct searches for single production of Z' and W' apply here.

Pair of vector singlets: $\mathcal{B} \sim (1,1)_0$ and $\mathcal{B}_1 \sim (1,1)_1$

Here, as in the quark bidoublet case, we have the extended symmetry $SU(2)_L \times SU(2)_R \times U(1)_X$. A pair of vector singlets $\mathcal{B}$ and $\mathcal{B}_1$ combine to form a $(1,3)_0$ rep-

C_ϕ	$C_{\phi\Box}$	$(C_{\psi\phi})_{ij}$	$(C^{(3)}_{\phi\psi})_{ij}$
$-\frac{\lambda_\phi (g^\phi)^2}{M_{\mathcal{W}}^2}$	$-\frac{3(g^\phi_{\mathcal{W}})^2}{8M_{\mathcal{W}}^2}$	$-\frac{y^{\psi *}_{ji}(g^\phi_{\mathcal{W}})^2}{4M_{\mathcal{W}}^2}$	$-\frac{(g^\psi_{\mathcal{W}})_{ij}(g^\phi_{\mathcal{W}})}{4M_{\mathcal{W}}^2}$

Table 8.8: Tree-level contributions to operators with the Higgs from the hypercharge zero vector triplet $\mathcal{W}$.

C_ϕ	$C_{\phi\Box}$	$(C_{\psi\phi})_{ij}$
$-\frac{4\lambda_\phi (g^\phi)^2}{M^2}$	$-\frac{3(g^\phi)^2}{2M^2}$	$-\frac{y^{\psi *}_{ji}(g^\phi)^2}{M^2}$

Table 8.9: Tree-level contributions to operators with the Higgs from the pair of vector singlets $\mathcal{B}$ and $\mathcal{B}_1$.

resentation of this group (see ref. [237] for a discussion of the effects of the $\mathcal{B}_1$ vector alone). The induced relations between BSMEFT parameters are: $M_{\mathcal{B}} = M_{\mathcal{B}_1} =: M$ and $g^\phi_{\mathcal{B}} = g^\phi_{\mathcal{B}_1}/\sqrt{2} =: g^\phi$. Thus, this model is described by the following terms in $\mathcal{L}_{\text{BSM}}$:

$$\begin{aligned}\mathcal{L}_{\text{vector-pair}} = \mathcal{L}_{\text{SM}} &+ \mathcal{L}^{\mathcal{B}}_{\text{kin}} + \mathcal{L}^{\mathcal{B}_1}_{\text{kin}} \\ &+ M^2\left(\frac{1}{2}\mathcal{B}_\mu\mathcal{B}^\mu + \mathcal{B}^\dagger_{1\mu}\mathcal{B}^\mu_1\right) \\ &- \left[g^\phi\left(\mathcal{B}^\mu\phi^\dagger iD_\mu\phi + \sqrt{2}\mathcal{B}^{\mu\dagger}_1 iD_\mu\phi^T i\sigma_2\phi\right) + \text{h.c.}\right].\end{aligned} \tag{8.117}$$

For simplicity, we have not included in the previous equation the fermionic couplings to the heavy vectors. Their effects are independent from the ones discussed in this subsection.

Using the UV/IR dictionary, we obtain table 8.9. The contribution to $\mathcal{O}_{\phi D}$ from $\mathcal{B}$ cancels the one from $\mathcal{B}_1$. This means that the limits on g^ϕ are milder than in the case with $\mathcal{B}$ alone (subsection 8.7.1). In fact, this coupling is not constrained by EWPT when the fermion couplings vanish. Therefore, large effects in $\mathcal{O}_\phi$, $\mathcal{O}_{\phi\Box}$ and $\mathcal{O}_{\psi\phi}$ are allowed in this model. The discussion of the effects in Higgs physics/couplings coming from g^ϕ in this model is similar to that in the case of $\mathcal{W}$.

Direct searches at the LHC are sensitive to the Higgs couplings of the new vectors via vector boson production of the new fields and decays into dibosons. Other channels are typically more restrictive when the couplings to fermions are not small.

8.8 Conclusions

In this chapter we have shown that the matching between the IR and UV descriptions can be performed once and for all at the leading order, namely for operators of canonical dimension up to 6 and at the classical level. The idea is to map the model-independent low-energy EFT approach to arbitrary models of new physics. With this purpose, we have considered a completely general extension of the SM, the BSMEFT (see chapter 7), subject only to a few mild assumptions. This extension has an arbitrary number of new scalars, fermions and vectors, with no restrictions on their gauge quantum numbers nor on their possible interactions. In particular, we have made no assumption about renormalizability.

We have integrated out the new heavy particles in the BSMEFT at the tree-level and have computed the Wilson coefficients of the corresponding SMEFT operators of dimension up to 6 in the Warsaw basis. This is the main contribution of this chapter. We report our results in the form of a UV/IR dictionary. A top-down approach to the analysis of new physics would first use our section 8.4, where we list all the operators that are generated for specific new particles. In section 8.5, on the other hand, we give our results organized from the bottom-up point of view, by writing the contribution to each Wilson coefficient from an arbitrary number of new particles. This dictionary greatly simplifies the task of analyzing the low-energy implications of explicit models and obtaining the corresponding bounds on their parameters. It also helps disentangle the origin of possible anomalies eventually observed in experiments. We have included a short section to guide the reader through our results and have provided a simple example to illustrate the use of this dictionary.

It is interesting that all operators in the Warsaw basis, except for the ones involving three field strength tensors, are generated in our tree-level integration. This would naively seem to contradict the arguments in ref. [96], which, up to the presence of $\mathcal{L}_1$, share our assumptions. In fact there is no contradiction since, as we have shown, tree-level contributions to operators that are classified as "loop generated" in [96] only arise due to non-renormalizable, dimension-five operators in our SM extension, which can only be generated in turn at the loop level in any weakly-coupled renormalizable UV completion of that theory. (See [198] for a related discussion.) However, we have included these operators in our dictionary because they could be unsuppressed in strongly-coupled completions.

We conclude by emphasizing that we have provided a complete classification of all possible extensions of the SM (with new particles up to spin 1) with low-energy implications at the leading order. These implications are encoded in tree-level contributions to the Wilson coefficients of the dimension-six operators in the SMEFT, which we have computed explicitly in terms of the masses and couplings of the new particles. This result can in principle be extended to operators of higher dimension: as long as the classical approximation is used, the number of extra fields and extra couplings to be considered will be finite (even if huge). On the other hand, at the loop level this endeavor faces an additional problem: there are infinitely-many types of extra fields that can contribute, already at one loop, to dimension-six operators. The reason is that fields without linear couplings to the SM need also be considered in this case. So, a complete matching to general extensions beyond the classical approximation will need to deal with this difficulty.

CHAPTER 9

Vector-like quarks with non-renormalizable interactions

9.1 Introduction

So far, we have considered the sector of the BSMEFT in which the new fields have dimension-4 linear couplings. In this chapter, we study the possibility of relaxing this bound over the dimension. We do so for the case of vector-like quarks. That is, spin-1/2 color triplets whose left-handed and right-handed components transform in the same way the electroweak gauge group. In general, new heavy fermions must be vector-like in order to have an explicit gauge-invariant mass term.

Vector-like quarks appear in many motivated extensions of the SM, for diverse reasons. In models with additional symmetries, they may complete multiplets that include SM fermions [51, 238, 239]. They may also be necessary for the cancellation of the anomalies of an extended gauge group [240]. In models with (partially) composite quarks [241], they emerge effectively as resonances, while in models in extra dimensions, they show up as Kaluza-Klein modes when the quarks propagate through the bulk [242]. Vector-like quarks are also used to relax the bounds from precision observables [243] or to avoid strong fine tuning in the Higgs sector [244, 245]. Here, we will not worry about the origin of the vector-like quarks or the details of the model in which they appear. Instead, we follow a systematic model-independent approach by studying a general effective field theory that describes the new quarks and their interactions with the SM fields. Our conclusions can be easily translated to specific models.

At the renormalizable level, the possible gauge-invariant interactions of the extra quarks in the electroweak symmetric phase are the ones with the gauge bosons, determined by their quantum numbers, and Yukawa interactions involving either two extra quarks or one extra quark and one SM quark (see section 7.3). Upon electroweak breaking, the Yukawa couplings give rise to off-diagonal terms in the quark mass matrix, which translate into the mixing of mass eigenstates in the interaction terms with the Z and W bosons and the Higgs (beyond the mixing in the original Yukawas). Many of the observable effects of the new quarks, such as their decay into SM particles, their single production and the induced modifications of the light-quark couplings, are associated to their mixing with the SM quarks, which is suppressed when their gauge-invariant mass is larger than the Z mass [246]. This suppression is stronger for heavy vector-like quarks that are not directly connected by Yukawa couplings to the SM quarks. Therefore, the effects of mixing are sizable only in the presence of vector-like quarks with gauge quantum numbers that allow for such couplings. Assuming that electroweak breaking is mostly triggered by the vev v of one or more Higgs doublets, in agreement with limits on the ρ parameter, there are seven different multiplets of vector-like quarks that carry the appropriate quantum numbers. They are ones shown in table 7.2. Vector-like quarks with this property will be called "renormalizable" vector-like quarks (RVLQ), even if they can also have non-renormalizable interactions. Their components have electric charges in the set $\{\pm 1/3, \pm 2/3, \pm 4/3, \pm 5/3\}$. The most general renormalizable extension of the SM with arbitrary combinations of the seven types of RVLQ was explicitly written in ref. [6]. In that work, the leading indirect effects beyond the SM, including flavour-changing neutral currents, right-handed charged currents and a non-unitary CKM matrix, were studied by integrating the heavy quarks out and using the results in ref. [59] for the relevant flavourful part of the SMEFT at dimension 6. The loop contributions of these multiplets to oblique parameters have also been calculated in refs. [247–249]. Regarding direct searches, refs. [250, 251] provide a comprehensive and detailed guide to the LHC phenomenology of minimal renormalizable extensions of the SM with vector-like quarks that mix dominantly with the third family. Several other works have been devoted to collider searches of RVLQ, see for instance refs. [252–256].

Allowing non-renormalizable interactions in the BSMEFT allows us to assess the robustness of the standard limits on vector-like quarks and to explore possible new observable signals. For simplicity, we will consider simple extensions with only one quark multiplet at a time. All the possible particles not included in the effective Lagrangian, such as additional extra quarks or extra scalars, are assumed to be heavier than the cutoff Λ; their effects are then encoded into the Wilson coefficients of the EFT. As usual, the effective Lagrangian is to be expanded in inverse powers of Λ. When Λ is much higher than the probed energies E and the Higgs vev v, all the effects of higher-dimensional operators will be suppressed by powers of E/Λ and/or v/Λ with respect to the effects of the renormalizable ones and will typically give rise to small corrections to the known results. However, some processes may require the presence of higher-dimensional interactions, which will then provide the leading contributions. In particular, this is always the case for quark multiplets that can only couple linearly and gauge invariantly to the SM fermions at the non-renormalizable level. As we will see, the phenomenology of these multiplets can indeed be different from the one of RVLQ.

In fact, relaxing the requirement of renormalizability enlarges the list of vector-like quarks that can mix with the SM ones and, more generally, have linear couplings with SM operators.[1] In this chapter, we only study explicitly the leading corrections to renormalizable theories with vector-like quarks. So, we will truncate the effective Lagrangian at order $1/\Lambda$, that is, we will consider only operators of canonical dimension $n \leq 5$. The quark multiplets that can have linear couplings to this order are collected in table 9.1. As can be checked there, there are five new multiplets, in addition to the

[1] Interestingly, non-renormalizable linear interactions of other colour representations, beyond singlets and triplets, are also allowed.

seven RVLQ, which we have presented before in table 7.2. The new ones will be called "non-renormalizable" vector-like quarks (NRVLQ). The only gauge-invariant operator that can be built with the SM fields at dimension 5 is the Weinberg operator, which involves only leptons and is thus irrelevant in our context. Therefore, the relevant dimension 5 operators always contain at least one of the extra quarks in table 9.1. In order to simplify the analysis, we will assume that the extra quarks do not couple to the first two SM families. This assumption can easily be dropped, at the price of introducing more free parameters. We study the mixing with the third family of SM quarks and the associated phenomenology, including indirect effects on electroweak and Higgs observables and the production and decay of the new quarks. We will see that for some multiplets there are new single production mechanisms and new decay channels, which can be sizable in some regions of parameter space. A significant feature of the vector-like quarks without renormalizable interactions is that their widths are suppressed. For dimensionless couplings of order 1 and a cutoff Λ larger than 5 TeV, it turns out that their lifetimes are larger than the typical QCD times and thus non-perturbative effects, including hadronization, will take place before decay. For still larger values of Λ, the NRVLQ, or more precisely the hadrons they form, will be long lived. These quarks would then elude the usual searches, which assume prompt decays, and lead instead to alternative signatures, such as tracks with anomalous ionization, long time of flight or displaced vertices.

Extra quarks with non-renormalizable interactions have been studied before in the context of pseudo-Goldstone composite Higgs models [245, 257–259]. This is a particular subclass of the theories included in our general model-independent framework, with Λ identified with the symmetry breaking scale f. But in the pseudo-Goldstone scenario, the assumed symmetry breaking pattern allows to easily resum the $1/f$ expansion. Then, f can be pretty low without loosing predictive power.[2] The vector-like quarks in those models belong to multiplets of an extended symmetry and, for the popular choices in the literature, decompose under the SM gauge group into a subset of the seven RVLQ representations. Here, we want to follow a model-independent approach, so we do not make any assumptions about the nature of the Higgs, about symmetries beyond the SM ones or about the representations of the quarks (except for the requirement of linear interactions). Another study of non-renormalizable interactions for new quarks, similar in spirit to the one in this paper, was presented in ref. [261]. There, the first three multiplets in table 9.1, coupled via operators involving the Higgs, were considered. We generalize this work by including all the relevant multiplets and operators at dimension 5. In particular, we consider multiplets without dimension-4 interactions, which present the most dramatic changes with respect to the usual phenomenology of vector-like quarks. On the other hand, the flavour structure we assume is more restrictive than the one in ref. [261], which allowed for couplings to the light families of SM quarks.

We have implemented in `FeynRules 2.0` [162] the EFT for each vector-like multiplet in table 9.1. All the simulations have been performed with `MadGraph5_aMC@NLO` [262, 263] with the UFO files generated with `FeynRules`.

The chapter is organized as follows. In section 9.2, we introduce the EFT for vector-

[2] The EFT descriptions of these models are valid up to a cutoff higher than f, associated to additional resonances or strong coupling. In explicit holographic models, these effects are incorporated and the cutoff can be much higher for many purposes [260].

like quarks, find the constraints on quantum numbers for linear interactions and write explicitly the general Lagrangian for an arbitrary multiplet with all the operators of dimension up to 5. We also comment briefly on the possible ultraviolet (UV) origin of the non-renormalizable operators. In section 9.3, we diagonalize the mass matrices that appear in the Higgs phase for the components with the same electric charges as the SM quarks. Section 9.4 is devoted to indirect effects of the new quarks and to the corresponding limits from Higgs, electroweak and top data. Production at hadron colliders is discussed in section 9.5, while the decay of the new quarks is examined in section 9.6. In this last section, we explain why the branching ratios to Higgs and Z bosons are approximately equal for all multiplets but one. We also obtain a simple formula to reinterpret the mass limits provided by the LHC collaborations in the case with additional decay modes. Our method is based on the one in ref. [264]. We present our conclusions in 9.7.

9.2 General extensions of the Standard Model with vector-like quarks

We consider here the sector of the general theory $\mathcal{L}_{\text{BSM}}$ (defined in chapter 7) that contains new quarks. We obtained in section 7.2 a general constraint over the representation of any SM operator, and thus of any field with a gauge-invariant linear coupling. In the case of color triplets, it reads

$$T + Y + 1/3 \in \mathbb{Z}, \tag{9.1}$$

with T the isospin of the $SU(2)$ representation and Y the hypercharge. It is also true that, given a representation of $SU(2) \times U(1)$ satisfying eq. (9.1), there is a product of SM fields that produces this representation. Indeed, consider first the products $\phi^k (\phi^*)^l$ of the Higgs doublet and its conjugate. They generate all representations with $T + Y \in \mathbb{Z}$. Then, the operators of the form $\phi^k (\phi^*)^l q$ give all the possibilities satisfying eq. (9.1). So this formula allows to find easily the quark multiplets with linear couplings.

Higher-dimensional multiplets couple linearly to the SM through higher-dimensional operators. Therefore, the effects of higher-dimensional multiplets tend to be more suppressed than the lower-dimensional ones. As we have just explained, at each order in inverse powers of the cutoff Λ, which is given by the dimension of the operators, there is a finite number of multiplets with linear couplings to SM fields. This number increases with the order in $1/\Lambda$. We focus in the following on the next-to-leading order in this expansion, which is $O(1/\Lambda)$. Equivalently, we impose a maximum dimension of 5 for the operators in the effective Lagrangian. There are twelve possible multiplets with linear couplings at this order, listed in table 9.1. The ones in the first seven rows, called RVLQ in this paper, can have linear interactions of dimension 4. These are the multiplets that have been studied in the past. For natural values of the couplings, the dimension-5 operators will generate small corrections to their properties. The remaining five multiplets, which we call NRVLQ, cannot have dimension-4 linear couplings. Therefore, for these multiplets the dimension-5 interactions will give leading-order effects. Let us stress that RVLQ can have non-renormalizable linear interactions and that NRVLQ have renormalizable quadratic interactions with the gauge fields, besides the kinetic and mass terms. Let us also note in passing that, besides the singlet and triplet representations, other irreducible representations of $SU(3)$ are possible for spin-1/2 particles with dimension-5 linear couplings to the SM. The extra eight possibilities for their representations under $SU(3)\times SU(2)\times U(1)$ are:

$$(6,1)_{-2/3},\ (6,1)_{1/3},\ (6,2)_{-1/6},\ (8,1)_{1},\ (8,2)_{1/2},\ (15,1)_{2/3},\ (15,1)_{-1/3},\ (15,2)_{1/6}. \tag{9.2}$$

Coming back to extra quarks, the dimension-5 operators containing exactly one vector-like quark can have one of the following two schematic forms: $\bar{Q}\phi\phi q$ and $\bar{Q}\sigma^{\mu\nu}qF_{\mu\nu}$, where ϕ is the Higgs doublet, q and Q represent SM and extra quark multiplets, respectively, and $F_{\mu\nu}$ is the field-strength tensor of a SM gauge field. We do not consider operators with the field content $\bar{Q}\phi qD$, with D a covariant derivative, because they can be eliminated using integration by parts and field redefinitions, up to $O(1/\Lambda^2)$ corrections. The interactions allowed for each multiplet are presented in table 9.1. They can be found using `BasisGen` (see chapter 6). It is important to note that the interactions of the form $\bar{Q}\phi\phi q$ will typically give physical effects suppressed by powers of v/Λ, while the effects of interactions of the form $\bar{Q}\sigma^{\mu\nu}qF_{\mu\nu}$ are suppressed by powers of E/Λ, with E the characteristic energy of the process ($E\simeq M$ for on-shell extra quarks). In the rest of this paper, we study sector of the BSMEFT that is relevant for each one of the possible multiplets Q at a time. The corresponding dimension-5 effective Lagrangian $\mathcal{L}=\mathcal{L}_{\text{SM}}+\mathcal{L}_Q^{\text{free}}+(\mathcal{L}_Q^{\text{lin}}+\mathcal{L}_Q^{\text{quad}}+\text{h.c.})$, with

$$\mathcal{L}_Q^{\text{free}} = \bar{Q}(i\not{D}-M)Q, \tag{9.3}$$

$$-\mathcal{L}_U^{\text{lin}} = \lambda_i\,\bar{U}_R\tilde{\phi}^\dagger q_{Li} + y_i\,(\bar{U}_L u_{Ri})(\phi^\dagger\phi) + w_{Bi}\,\bar{U}_L\sigma^{\mu\nu}u_{Ri}B_{\mu\nu} + w_{Gi}\,\bar{U}_L\lambda^A\sigma^{\mu\nu}u_{Ri}G^A_{\mu\nu}, \tag{9.4}$$

$$-\mathcal{L}_D^{\text{lin}} = \lambda_i\,\bar{D}_R\phi^\dagger q_{Li} + y_i\,(\bar{D}_L d_{Ri})(\phi^\dagger\phi) + w_{Bi}\,\bar{D}_L\sigma^{\mu\nu}d_{Ri}B_{\mu\nu} + w_{Gi}\,\bar{D}_L\lambda^A\sigma^{\mu\nu}d_{Ri}G^A_{\mu\nu}, \tag{9.5}$$

$$\begin{aligned}-\mathcal{L}_{Q_1}^{\text{lin}} = {}& \lambda_{ui}\,\bar{Q}_{1L}\tilde{\phi}u_{Ri} + \lambda_{di}\,\bar{Q}_{1L}\phi d_{Ri} + y_{ui}\,(\bar{Q}_{1R}\tilde{\phi})(\tilde{\phi}^\dagger q_{Li}) + y_{di}\,(\bar{Q}_{1R}\phi)(\phi^\dagger q_{Li})\\ &+ w_{Bi}\,\bar{Q}_{1R}\sigma^{\mu\nu}q_{Li}B_{\mu\nu} + w_{Wi}\,\bar{Q}_{1R}\sigma^a\sigma^{\mu\nu}q_{Li}W^a_{\mu\nu} + w_{Bi}\,\bar{Q}_{1R}\lambda^A\sigma^{\mu\nu}q_{Li}G^A_{\mu\nu},\end{aligned} \tag{9.6}$$

$$-\mathcal{L}_{Q_7}^{\text{lin}} = \lambda_i\,\bar{Q}_{7L}\phi u_{Ri} + y_i\,(\bar{Q}_{7R}\phi)(\tilde{\phi}^\dagger q_{Li}), \tag{9.7}$$

$$-\mathcal{L}_{Q_5}^{\text{lin}} = \lambda_i\,\bar{Q}_{5L}\tilde{\phi}d_{Ri} + y_i\,(\bar{Q}_{5R}\tilde{\phi})(\phi^\dagger q_{Li}), \tag{9.8}$$

$$-\mathcal{L}_{T_1}^{\text{lin}} = \lambda_i\,\bar{T}^a_{1R}\phi^\dagger\sigma^a q_{Li} + y_{ui}\,\bar{T}^a_{1L}u_{Ri}\phi^\dagger\sigma^a\tilde{\phi} + y_{di}\,\bar{T}^a_{1L}d_{Ri}\phi^\dagger\sigma^a\phi + w_i\,\bar{T}^a_{1L}\sigma^{\mu\nu}d_{Ri}W^a_{\mu\nu}, \tag{9.9}$$

$$-\mathcal{L}_{T_2}^{\text{lin}} = \lambda_i\,\bar{T}^a_{2R}\tilde{\phi}^\dagger\sigma^a q_{Li} + y_{ui}\,\bar{T}^a_{2L}u_{Ri}\phi^\dagger\sigma^a\phi + y_{di}\,\bar{T}^a_{2L}d_{Ri}\tilde{\phi}^\dagger\sigma^a\phi + w_i\,\bar{T}^a_{2L}\sigma^{\mu\nu}u_{Ri}W^a_{\mu\nu}, \tag{9.10}$$

$$-\mathcal{L}_{T_4}^{\text{lin}} = y_i\,\bar{T}^a_{4L}d_{Ri}\phi^\dagger\sigma^a\tilde{\phi}, \tag{9.11}$$

$$-\mathcal{L}_{T_5}^{\text{lin}} = y_i\,\bar{T}^a_{5L}u_{Ri}\tilde{\phi}^\dagger\sigma^a\phi, \tag{9.12}$$

$$-\mathcal{L}_{F_1}^{\text{lin}} = y_i\,\bar{F}^a_{1R}C^a_{bc}q_{Lic}\phi^\dagger\sigma^b\phi + w_i\,\bar{F}^a_{1R}C^a_{bc}\sigma^{\mu\nu}q_{Lic}W^b_{\mu\nu}, \tag{9.13}$$

$$-\mathcal{L}_{F_5}^{\text{lin}} = y_i\,\bar{F}^a_{5R}C^a_{bc}q_{Lic}\phi^\dagger\sigma^b\tilde{\phi}, \tag{9.14}$$

$$-\mathcal{L}_{F_7}^{\text{lin}} = y_i\,\bar{F}^a_{7R}C^a_{bc}q_{Lic}\tilde{\phi}^\dagger\sigma^b\phi, \tag{9.15}$$

$$\begin{aligned}-\mathcal{L}_Q^{\text{quad}} = {}& W_B\,(\bar{Q}_L\sigma^{\mu\nu}Q_R)B_{\mu\nu} + W_W\,(\bar{Q}_L\sigma^{\mu\nu}\mathbf{T}^a_Q Q_R)W^a_{\mu\nu} + W_G\,(\bar{Q}_L\sigma^{\mu\nu}\mathbf{t}^A_Q Q_R)G^A_{\mu\nu}\\ &+ Y_1\,(\bar{Q}_L Q_R)(\phi^\dagger\phi) + Y_2\,(\bar{Q}_L\,\mathbf{T}^a_Q Q_R)(\phi^\dagger\sigma^a\phi),\end{aligned} \tag{9.16}$$

where $\mathbf{T}_Q^A$ ($\mathbf{t}_Q^a$) are the generators of $SU(2)$ ($SU(3)$) in the representation of Q. The rest the group-theory notation used here is defined in appendix A. The index i indicates the SM fermion family and $W_W = Y_2 = 0$ for singlets. We have used the following notation for coefficients of operators that are linear in Q: λ_i is the coefficient of $\bar{Q}q_i\phi$, y_i is for $\bar{Q}q_i\phi\phi$, and w_i is for $\bar{Q}\sigma^{\mu\nu}q_iF_{\mu\nu}$. When there is more than one possibility, the corresponding coupling constants are differentiated by an additional subindex, which indicates the SM field that unambiguously determines the operator. Observe that we include all the gauge-invariant operators of dimension equal to or smaller than 5 that can be constructed with the field content of the theory. The condition of linear couplings is used to select the representations of the vector-like quarks, but not to restrict their interactions in the EFT. Note also that the λ_i parameters are dimensionless, whereas y_i, w_i, Y and W have dimensions of inverse energy and are expected to be of order Λ^{-1}.

This new notation for the couplings turns out to be more convenient for our purposes here than the one defined in section 7.2. A direct conversion between the two can be done. For the coefficients of renormalizable operators, we have $\lambda_{(u,d)i} = (\lambda_Q^{(u,d)})_{3i}$. For the non-renormalizable Yukawas of singlets and triplets: $y_{(u,d)i} = (\tilde{\lambda}_Q^{(u,d)})_{3i}/f$, while for the Q_5 and Q_7 doublets: $y_i = (\tilde{\lambda}_Q^q)_{3i}/f$. For the dimension-5 Yukawa couplings of the Q_1 singlet, we use a basis here that differs from the one in section 7.2. In this case, we have $y_{ui} = (\tilde{\lambda}_Q^{q'})_{3i}/f$ and $y_{di} = ((\tilde{\lambda}_Q^q)_{3i} + (\tilde{\lambda}_Q^{q'})_{3i})/f$. Finally, the relation between the couplings for field strengths are given by $w_{(B,W)i} = (\tilde{\lambda}_Q^{(B,W)})_{3i}/f$ and $w_{(G)i} = (\tilde{\lambda}_Q^G)_{3i}/f$.

We will consider in this paper only couplings to the third family of SM quarks. This choice is made to reduce the dimensionality of the parameter space and to automatically satisfy the most stringent flavour limits. It is also motivated by theoretical ideas in different models. This means that λ_i, y_i and w_i are taken to be vanishing for $i = 1, 2$. Accordingly, we simplify the name of the non-vanishing couplings in the following way:

$$\begin{aligned} &\lambda = \lambda_3; \;\; \lambda_t = \lambda_{u3}; \;\; \lambda_b = \lambda_{d3}; \\ &y = y_3; \;\;\; y_t = y_{u3}; \;\;\; y_b = y_{d3}; \\ &w = w_3; \;\;\; w_B = w_{B3}; \;\;\; w_W = w_{W3}; \;\;\; w_G = w_{G3}. \end{aligned} \tag{9.17}$$

Let us briefly comment on possible ultraviolet completions that can give rise to the dimension 5 operators at low energies. The Yukawa-like operators $\bar{Q}q\phi\phi$, of dimension 5, can be generated at the tree level in a completion with one additional field: either a colour-neutral scalar $\mathbf{S}$, with interactions $\mathbf{S}\bar{Q}q$ and $\mathbf{S}\phi\phi$ or an additional quark $\mathbf{Q}$, with interactions $\mathbf{Q}\phi Q$ and $\mathbf{Q}\phi q$. The mass of the extra particle, which is assumed to be larger than M, sets the cutoff scale Λ of the EFT $\mathcal{L}$. The Feynman diagrams that contribute to the dimension-5 Yukawas are shown in figure 9.1. The quantum numbers of the extra field must allow for the gauge-invariant vertices in the diagrams. This means that the heavy scalar $\mathbf{S}$ belongs to one of the representations 1_0, 3_0 and 3_1 of $SU(2) \times U(1)$, while the heavy quark $\mathbf{Q}$ belongs to one of the representations in the first seven rows of table 9.1, so it is also a RVLQ (but assumed to be heavier than the ones in the effective Lagrangian). The operators of the form $\bar{Q}\sigma_{\mu\nu}qF^{\mu\nu}$, on the other hand, cannot be generated at tree-level in a renormalizable ultraviolet theory. In figure 9.2 we show a one-loop diagram that contributes to these effective operators in a theory with an extra scalar multiple $\mathbf{S}$, which must be either a singlet or a triplet

Name	Irrep	$\bar{Q}\phi q$	$\bar{Q}\phi\phi q$	$\bar{Q}\sigma^{\mu\nu}qF_{\mu\nu}$
U	$1_{2/3}$	✓	✓	✓
D	$1_{-1/3}$	✓	✓	✓
Q_1	$2_{1/6}$	✓	✓	✓
Q_5	$2_{-5/6}$	✓	✓	✗
Q_7	$2_{7/6}$	✓	✓	✗
T_1	$3_{-1/3}$	✓	✓	✓
T_2	$3_{2/3}$	✓	✓	✓
T_4	$3_{-4/3}$	✗	✓	✗
T_5	$3_{5/3}$	✗	✓	✗
F_1	$4_{1/6}$	✗	✓	✓
F_5	$4_{-5/6}$	✗	✓	✗
F_7	$4_{7/6}$	✗	✓	✗

Table 9.1: Irreps $(2T+1)_Y$ under $SU(2)_L \times U(1)_Y$ and linear interactions of new quarks with dimension-5 linear couplings. The subscript in the name of each multiplet is the absolute value of the numerator of its hypercharge, when written as an irreducible fraction. An explicit formula for this integer number is $|2+4\widetilde{T}+3(Y-2/3)/(1-\widetilde{T})|$ where $\widetilde{T} = T \pmod 1$.

Figure 9.1: Tree-level diagrams that generate the $\bar{Q}q\phi\phi$ operator in UV completions of $\mathcal{L}$ with additional extra quarks (left) and additional scalars (right).

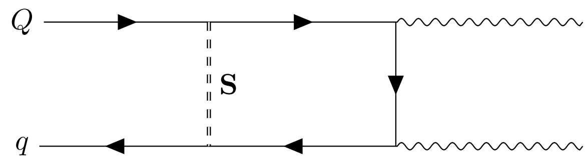

Figure 9.2: A one-loop diagram that generates the $\bar{Q}\sigma_{\mu\nu}qF^{\mu\nu}$ operator in a UV completion of $\mathcal{L}$ with new scalars.

of $SU(2)$, and a singlet or an octet of $SU(3)$. That is, there are 4 possibilities: $(1,1)_0$, $(1,3)_0$, $(8,1)_0$ and $(8,3)_0$. The coefficients w of these "magnetic" operators are thus naturally suppressed by a loop factor in weakly coupled completions. In addition, because a quark mass insertion m_Q is needed for the chiralities of the external lines to match those of the effective operator, the suppression with the UV scale m_S is not $1/m_S$ as expected from the EFT power counting, but m_Q/m_S^2. An explicit model with a U vector-like quark and a scalar singlet has been studied in [265].

9.3 Mixing

The multiplets in table 9.1 can be decomposed into component fields with well-defined electric charge:

$$Q_1 = \begin{pmatrix} T^0 \\ B^0 \end{pmatrix}, \qquad Q_5 = \begin{pmatrix} B^0 \\ Y \end{pmatrix}, \qquad Q_7 = \begin{pmatrix} X \\ T^0 \end{pmatrix}, \tag{9.18}$$

$$T_1 = \begin{pmatrix} T^0 \\ B^0 \\ Y \end{pmatrix}, \qquad T_2 = \begin{pmatrix} X \\ T^0 \\ B^0 \end{pmatrix}, \qquad T_4 = \begin{pmatrix} B^0 \\ Y \\ Y' \end{pmatrix}, \qquad T_5 = \begin{pmatrix} X' \\ X \\ T^0 \end{pmatrix}, \tag{9.19}$$

$$F_1 = \begin{pmatrix} X \\ T^0 \\ B^0 \\ Y \end{pmatrix}, \qquad F_5 = \begin{pmatrix} T^0 \\ B^0 \\ Y \\ Y' \end{pmatrix}, \qquad F_7 = \begin{pmatrix} X' \\ X \\ T^0 \\ B^0 \end{pmatrix}. \tag{9.20}$$

The components are denoted by symbols in the set $\{X', X, T^0, B^0, Y, Y'\}$, with electric charges given by

$$Q(X') = 8/3, \qquad Q(B^0) = -1/3, \tag{9.21}$$
$$Q(X) = 5/3, \qquad Q(Y) = -4/3, \tag{9.22}$$
$$Q(T^0) = 2/3, \qquad Q(Y') = -7/3. \tag{9.23}$$

Upon electroweak breaking, the fields T^0 (B^0) will mix, in general, with all the SM up-type (down-type) quarks. However, with our flavour restriction and neglecting the tiny off-diagonal CKM elements of the third family, the new quarks mix only with the top and bottom quarks. The relevant mass terms have the form

$$\mathcal{L}_{\text{mass}} = -\begin{pmatrix} \bar{t}^0_L & \bar{T}^0_L \end{pmatrix} \begin{pmatrix} m^t_{11} & m^t_{12} \\ m^t_{21} & m^t_{22} \end{pmatrix} \begin{pmatrix} t^0_R \\ T^0_R \end{pmatrix} \tag{9.24}$$
$$- \begin{pmatrix} \bar{b}^0_L & \bar{B}^0_L \end{pmatrix} \begin{pmatrix} m^b_{11} & m^b_{12} \\ m^b_{21} & m^b_{22} \end{pmatrix} \begin{pmatrix} b^0_R \\ B^0_R \end{pmatrix} + \text{h.c.}, \tag{9.25}$$

with the superindex 0 emphasizing that the fields are weak eigenstates, i.e. the components of the gauge-covariant multiplets.[3] The elements of the diagonal of each of the mass matrices are $m_{11} \sim v$, which arises from the SM Yukawa coupling $\bar{q}\phi q$, and $m_{22} \simeq M$. For RVLQ, one of the off-diagonal elements, $m_{ij} \sim v$, comes from the operator $\bar{Q}\phi q$, and the other one, $m_{ji} \sim yv^2$, comes from $\bar{Q}\phi\phi q$. For NRVLQ, only one of the off-diagonal elements, $m_{ij} \sim yv^2$, is non-zero. The precise values of the entries of the mass matrices are given in table 9.2. The mixing angles that relate weak and mass eigenstates are obtained by diagonalizing the corresponding mass matrices:

$$\begin{pmatrix} t_{L,R} \\ T_{L,R} \end{pmatrix} = \begin{pmatrix} c^t_{L,R} & -e^{i\phi_t} s^t_{L,R} \\ e^{-i\phi_t} s^u_{L,R} & c^t_{L,R} \end{pmatrix} \begin{pmatrix} t^0_{L,R} \\ T^0_{L,R} \end{pmatrix}, \tag{9.26}$$

$$\begin{pmatrix} b_{L,R} \\ B_{L,R} \end{pmatrix} = \begin{pmatrix} c^b_{L,R} & -e^{i\phi_b} s^b_{L,R} \\ e^{-i\phi_b} s^b_{L,R} & c^b_{L,R} \end{pmatrix} \begin{pmatrix} b^0_{L,R} \\ B^0_{L,R} \end{pmatrix}, \tag{9.27}$$

[3]Note that we use the symbol t^0_R for the right-handed SM weak eigenstate of electric charge 3/2 (-1/3), which is in fact the unique component of the SM iso-singlet u_{R3} (d_{R3})of hypercharge 3/2 (-1/3). Of course, t^0_L (b^0_L) are the upper and lower components of the SM iso-doublet q_{L3}.

	m^t_{12}	m^t_{21}	m^t_{22}	m^b_{12}	m^b_{21}	m^b_{22}
U	$\frac{\lambda^* v}{\sqrt{2}}$	$\frac{y v^2}{2}$	$\hat{M}$	–	–	–
D	–	–	–	$\frac{\lambda^* v}{\sqrt{2}}$	$\frac{y v^2}{2}$	$\hat{M}$
Q_1	$\frac{(y_u)^* v^2}{2}$	$\frac{\lambda_u v}{\sqrt{2}}$	$\hat{M} - \frac{Y_2 v^2}{4}$	$\frac{(y_d)^* v^2}{2}$	$\frac{\lambda_d v}{\sqrt{2}}$	$\hat{M} + \frac{Y_2 v^2}{4}$
Q_5	–	–	–	$\frac{y^* v^2}{2}$	$\frac{\lambda v}{\sqrt{2}}$	$\hat{M} - \frac{Y_2 v^2}{4}$
Q_7	$\frac{y^* v^2}{2}$	$\frac{\lambda v}{\sqrt{2}}$	$\hat{M} + \frac{Y_2 v^2}{4}$	–	–	–
T_1	$\lambda^* v$	$\frac{y_u v^2}{\sqrt{2}}$	$\hat{M} - \frac{Y_2 v^2}{2}$	$-\frac{\lambda^* v}{\sqrt{2}}$	$-\frac{y_d v^2}{2}$	$\hat{M}$
T_2	$\frac{\lambda^* v}{\sqrt{2}}$	$-\frac{y_u v^2}{2}$	$\hat{M}$	$\lambda^* v$	$\frac{y_d v^2}{\sqrt{2}}$	$\hat{M} + \frac{Y_2 v^2}{2}$
T_4	–	–	–	0	$\frac{y v^2}{\sqrt{2}}$	$\hat{M} - \frac{Y_2 v^2}{2}$
T_5	0	$\frac{y v^2}{\sqrt{2}}$	$\hat{M} + \frac{Y_2 v^2}{2}$	–	–	–
F_1	$-\frac{y^* v^2}{\sqrt{6}}$	0	$\hat{M} - \frac{Y_2 v^2}{4}$	$-\frac{y^* v^2}{\sqrt{6}}$	0	$\hat{M} + \frac{Y_2 v^2}{4}$
F_5	$\frac{y^* v^2}{\sqrt{2}}$	0	$\hat{M} - \frac{3Y_2 v^2}{4}$	$\frac{y^* v^2}{\sqrt{6}}$	0	$\hat{M} - \frac{Y_2 v^2}{4}$
F_7	$-\frac{y^* v^2}{\sqrt{6}}$	0	$\hat{M} + \frac{Y_2 v^2}{4}$	$-\frac{y^* v^2}{\sqrt{2}}$	0	$\hat{M} + \frac{3Y_2 v^2}{4}$

Table 9.2: Mass matrix elements. We use the notation $\hat{M} = M + Y_1 v^2/2$. The 11 component is always just the SM contribution: $m^{t,b}_{11} = \lambda^{t,b}_{\text{SM}} v/\sqrt{2}$.

where t, T, b and B are the mass eigenstates, $c^{t,b}_{L,R} := \cos\theta^{t,b}_{L,R}$ and $s^{t,b}_{L,R} := \sin\theta^{t,b}_{L,R}$, with $\theta^{t,b}_{L,R}$ the mixing angle. In what follows, we take $\phi_t = \phi_b = 0$, since non-trivial phases $\phi_{t,b}$ can be ignored for the observables discussed here. The explicit expressions for the mixing angles in terms of $m^{t,b}_{ij}$ are (see also ref. [266])

$$\tan 2\theta^{t,b}_L = \frac{2\left|m^{t,b}_{11}(m^{t,b}_{21})^* + m^{t,b}_{12}(m^{t,b}_{22})^*\right|}{|m^{t,b}_{11}|^2 - |m^{t,b}_{12}|^2 - |m^{t,b}_{21}|^2 + |m^{t,b}_{22}|^2}, \tag{9.28}$$

$$\tan 2\theta^{t,b}_R = \frac{2\left|(m^{t,b}_{11})^* m^{t,b}_{12} + (m^{t,b}_{21})^* m^{t,b}_{22}\right|}{|m^{t,b}_{11}|^2 - |m^{t,b}_{12}|^2 - |m^{t,b}_{21}|^2 + |m^{t,b}_{22}|^2}. \tag{9.29}$$

From these formulas and the scale dependence of each entry it can then be seen that, for $M \gg v$ (in agreement with experimental limits, see below), the mixing angles are suppressed by v/M, at least. Furthermore, $\theta_L \gg \theta_R$ if $|m_{12}| \gg |m_{21}|$, and viceversa. For natural values of the couplings and $\Lambda > 1$ TeV, one of the off-diagonal couplings is indeed much larger than the other, so the off-diagonal couplings involving heavy and light quark eigenstates will be mostly chiral (especially in the b sector). For RVLQ, the dominant mixing angle is θ_L for even isospin and θ_R for those with odd isospin. For NRVLQ, instead, the dominant mixing angle is θ_R for even isospin and θ_L for odd isospin. Note, however, that for some RVLQ the limits from electroweak precision tests are quite strict [251]. For these multiplets, the off-diagonal entries might be comparable and then the interactions involving both chiralities would be relevant.

9.4 Indirect effects

In this section, we discuss the indirect effects of heavy quarks in low-energy physics, Higgs physics and top physics, which are summarized in table 9.3. NRVLQ typically generate smaller contributions than RVLQ, as any insertion of a dimension-5 operator introduces a suppression of $1/\Lambda$. For the same reason, the effects of the dimension-5 interactions of RVLQ will naturally be small corrections to the ones coming only from dimension-4 interactions, when they are present.

Integrating out the RVLQ at tree level gives contributions to dimension-6 operators in the SMEFT. The low-energy effective Lagrangian, which can be read from the results shown in section 8.5, is presented in table 9.4, with the corresponding effective operators defined in section 3.7 (table 3.5). Observe that the dimension-6 terms without extra quarks in the EFT $\mathcal{L}$, which we are not writing here, will give additional contributions to the corresponding dimension-6 operators in the SMEFT. However, these contributions will be suppressed by M^2/Λ^2 or M/Λ relative to the ones from integrating out the RVLQ. Still, they might be relevant for M/Λ not small, depending on the values of the couplings.[4] Here we assume that even in this case they do not cancel against the ones in table 9.4.

	Observable	Coupling	Loop order
EWPT	S and T parameters	$\lambda_{(t)}$, $y_{(t)}$	one loop
	$Z \to bb$	$\lambda_{(t)}$, $y_{(t)}$	one loop
		$\lambda_{(b)}$, $y_{(b)}$	tree level
Higgs	$H \to bb$	$\lambda_{(b)}$, $\lambda_{(b)} y_{(b)}$	tree level
	ttH production	λ	tree level
	$gg \to H$, $H \to gg$	$\lambda_{(t)}$, Y_1	one loop
	double Higgs production	$\lambda_{(t)}$, Y_1	one loop
Top	top single production	$\lambda_{(t)} w_W$	tree level
	top pair production	$\lambda_{(t)}$, $\lambda_{(t)} w_B$	tree level
	$tt\gamma$ and ttZ production	$\lambda_{(t)}$, $\lambda_{(t)} w_B$, $\lambda_{(t)} w_W$	tree level
low-energy $\not{C}\not{P}$	electron/neutron EDM	$\lambda_{(t)}$, $\lambda_{(t)}\lambda_{(b)}$, $\lambda_{(t)} w_F$	two loops

Table 9.3: Summary of indirect effects of heavy quarks. The subindex (q) means that only the couplings to the SM quark q should be taken. The dependence on products of couplings may involve complex conjugation of some of them.

On the other hand, the NRVLQ do not contribute at tree level to the dimension-6 SMEFT. Therefore, their indirect effects are small. Their leading tree-level contributions of NRVLQ have at least dimension 8 and will not be written explicitly.

Electroweak precision observables

Electroweak precision observables set the strongest limits on the Yukawa couplings of each multiplet. In the mass-eigenstate basis, the mixing between the SM b quark

[4] This is nothing but a more precise formulation of the usual caution one should exert in general with indirect bounds.

	$\mathcal{L}_{\text{nh}}$	$\mathcal{L}_{\text{h}}$
U	$\frac{\lambda(w_B)^*}{M}\mathcal{O}_{tB} + \frac{\lambda(w_G)^*}{M}\mathcal{O}_{tG} + \left(\frac{(\lambda^t_{\text{SM}})^*\lvert\lambda\rvert^2}{2M^2} + \frac{\lambda^* y}{M}\right)\mathcal{O}_{t\phi}$	$\frac{\lvert\lambda\rvert^2}{4M^2}\mathcal{O}^{(1)}_{\phi q} - \frac{\lvert\lambda\rvert^2}{4M^2}\mathcal{O}^{(3)}_{\phi q}$
D	$\frac{\lambda(w_B)^*}{M}\mathcal{O}_{bB} + \frac{\lambda(w_G)^*}{M}\mathcal{O}_{bG} + \left(\frac{(\lambda^b_{\text{SM}})^*\lvert\lambda\rvert^2}{2M^2} + \frac{\lambda^* y}{M}\right)\mathcal{O}_{b\phi}$	$-\frac{\lvert\lambda\rvert^2}{4M^2}\mathcal{O}^{(1)}_{\phi q} - \frac{\lvert\lambda\rvert^2}{4M^2}\mathcal{O}^{(3)}_{\phi q}$
Q_1	$\frac{\lambda_t(w_B)^*}{M}\mathcal{O}_{tB} + \frac{\lambda_t(w_W)^*}{M}\mathcal{O}_{tW} + \frac{\lambda_t(w_G)^*}{M}\mathcal{O}_{tG}$ $+\frac{\lambda_b(w_B)^*}{M}\mathcal{O}_{bB} + \frac{\lambda_b(w_W)^*}{M}\mathcal{O}_{bW} + \frac{\lambda_b(w_G)^*}{M}\mathcal{O}_{bG}$ $+\left(\frac{(\lambda^t_{\text{SM}})^*\lvert\lambda_t\rvert^2}{2M^2} + \frac{\lambda_t(y_t)^*}{M}\right)\mathcal{O}_{t\phi}$ $+\left(\frac{(y^b_{\text{SM}})^*\lvert\lambda_b\rvert^2}{2M^2} + \frac{\lambda_b(y_b)^*}{M}\right)\mathcal{O}_{b\phi}$	$-\frac{\lvert\lambda_t\rvert^2}{2M^2}\mathcal{O}_{\phi t} + \frac{\lvert\lambda_b\rvert^2}{2M^2}\mathcal{O}_{\phi b}$ $+\frac{\lambda_b(\lambda_t)^*}{M^2}\mathcal{O}_{\phi tb}$
Q_5	$\left(\frac{(\lambda^b_{\text{SM}})^*\lvert\lambda\rvert^2}{2M^2} + \frac{\lambda y^*}{M}\right)\mathcal{O}_{b\phi}$	$-\frac{\lvert\lambda\rvert^2}{2M^2}\mathcal{O}_{\phi b}$
Q_7	$\left(\frac{(\lambda^t_{\text{SM}})^*\lvert\lambda\rvert^2}{2M^2} + \frac{\lambda y^*}{M}\right)\mathcal{O}_{t\phi}$	$\frac{\lvert\lambda\rvert^2}{2M^2}\mathcal{O}_{\phi t}$
T_1	$\frac{\lambda(w_W)^*}{M}\mathcal{O}_{bW} + \left(\frac{(y^t_{\text{SM}})^*\lvert\lambda\rvert^2}{4M^2} + \frac{\lambda^* y_t}{M}\right)\mathcal{O}_{t\phi}$ $+\left(\frac{(y^b_{\text{SM}})^*\lvert\lambda\rvert^2}{8M^2} + \frac{\lambda^* y_b}{2M}\right)\mathcal{O}_{b\phi}$	$-\frac{3\lvert\lambda\rvert^2}{16M^2}\mathcal{O}^{(1)}_{\phi q} + \frac{\lvert\lambda\rvert^2}{16M^2}\mathcal{O}^{(3)}_{\phi q}$
T_2	$\frac{\lambda(w_W)^*}{M}\mathcal{O}_{tW} + \left(\frac{(y^t_{\text{SM}})^*\lvert\lambda\rvert^2}{8M^2} - \frac{\lambda^* y_b}{2M}\right)\mathcal{O}_{t\phi}$ $+\left(\frac{(y^b_{\text{SM}})^*\lvert\lambda\rvert^2}{4M^2} + \frac{\lambda^* y_b}{M}\right)\mathcal{O}_{b\phi}$	$\frac{3\lvert\lambda\rvert^2}{16M^2}\mathcal{O}^{(1)}_{\phi q} + \frac{\lvert\lambda\rvert^2}{16M^2}\mathcal{O}^{(3)}_{\phi q}$

Table 9.4: Dimension-6 effective Lagrangian generated by tree-level matching of the EFT with each multiplet to the SMEFT. The contributions to Hermitian and non-Hermitian operators are separated in $\mathcal{L}_{\text{h}}$ and $\mathcal{L}_{\text{nh}}$. The complete effective Lagrangian is $\mathcal{L}_{\text{h}} + (\mathcal{L}_{\text{nh}} + \text{h.c.})$. The definitions of the operators $\mathcal{O}_i$ are given in table 3.5.

and the B component of a given multiplet induces a modification of the Zbb coupling, which affects the R_b, A^b_{FB}, A_b and R_c observables at tree level. t–T mixing changes the Ztt coupling. Insertions of this modified interaction in diagrams with loops of the top quark also generate corrections to these observables, as well as to the S and T parameters.

For the renormalizable multiplets, the origin of these effects can be easily identified in the unbroken phase. They come from tree-level and one-loop diagrams containing the $\mathcal{O}_{\phi q}$-type operators generated by tree level matching. Notice that the non-renormalizable multiplets will also have contributions to these observables, but to obtain them one needs to keep dimension-8 operators, which indicates that their effects will be smaller.

In ref. [251], the limits on the mixing angles from electroweak precision observables were computed, assuming renormalizability. The corrections from dimension-5 interactions can be neglected for RVLQ. However, for NRVLQ, the dimension-5 contribution is the leading one. Following the method in ref. [251], we can use the experimental measurements of R_b, A^b_{FB}, A_b and R_c to obtain the following bounds: $s_L < 0.13$ for the triplet T_4, $s^d_L < 0.02$ for the quadruplet F_7 and $s^d_L < 0.03$ for the quadruplet F_5. These limits are already satisfied by the mixing angles

$$\theta \sim \frac{yv^2}{M} \lesssim 0.02, \tag{9.30}$$

for $y \leq (3\,\mathrm{TeV})^{-1}$, $M \geq 1\,\mathrm{TeV}$. The quadruplet F_1 produces a Zbb coupling with an extra suppression of m_b/M, so it is even less constrained. The limits from S and T are weaker than the ones from $Z \to bb$ when there is a B component in the multiplet. The only multiplet without such component among the non-renormalizable ones is $T_5 \sim 3_{5/3}$. In this case both the limits from $Z \to bb$ and from S and T may be relevant. Anyway, since these effects are loop suppressed, as long as $y/M \leq (1.7\,\mathrm{TeV})^{-2}$, this multiplet satisfies these constraints.

Higgs physics

The $\mathcal{O}_{t\phi}$ operator introduces a modification of the top Yukawa coupling, which can be measured using ttH production. This process has been observed at the LHC [267,268]. The current uncertainty for the top Yukawa coupling is however too large for the effects of $\mathcal{O}_{t\phi}$ to be relevant. The situation could improve in future experiments [269].

The presence of $\mathcal{O}_{t\phi}$ also changes gluon fusion Higgs production, through its appearance in diagrams with loops of the top quark. In addition, there are contributions to $gg \to H$ from the heavy-quark loops. At the renormalizable level, the contribution of the T loops is cancelled quite precisely by the effect of t loops with insertions of $\mathcal{O}_{t\phi}$ (such cancellation does not happen for B loops) [251]. In the presence of $Qq\phi\phi$ operators the cancellation is spoiled by the contributions to $\mathcal{O}_{t\phi}$ proportional to λy. However, this contribution is suppressed not only by M/Λ but also by the small mixing. The dimension-5 interactions with Y_1 give yet another contribution to this process (see also ref. [261]). This can be computed by one-loop matching to the SMEFT. The relevant part of the effective Lagrangian is

$$\mathcal{L}_{\text{1-loop}} \supset \frac{(2T+1)\alpha_s\,\mathrm{Re}(Y_1)}{12\pi M}\mathcal{O}_{\phi G}, \tag{9.31}$$

where $\mathcal{O}_{\phi G} = \phi^\dagger\phi\, G^A_{\mu\nu}G^{A,\mu\nu}$. As we can see, the coefficient of the induced operator is not suppressed by the mixing. Bounds on the coefficient on this operator have been calculated in ref. [77]. They can be translated into limits for the parameters of our theory:

$$\frac{|\,\mathrm{Re}(Y_1)|}{M} < \frac{1}{(2T+1)(1.25\,\mathrm{TeV})^2}, \tag{9.32}$$

where T is the isospin of the corresponding multiplet. Of course, both $\mathcal{O}_{\phi G}$ and $\mathcal{O}_{t\phi}$ contribute to the $H \to gg$ partial width, through tree-level and one-loop diagrams, respectively. This is discussed in detail in ref. [251]. These operators modify also double Higgs production, which has not been observed yet but could be measured at the HL-LHC [270]. Similarly, there are loop contributions to other vector-boson decay modes of the Higgs.

On the other hand, the $H \to bb$ decay channel is modified at the tree level by the operator $\mathcal{O}_{b\phi}$. Because the contribution to this operator from dimension-4 couplings is suppressed by the Yukawa coupling of the bottom quark, while the dimension-5 contribution does not contain this suppression, it is possible that the dimension-5 interaction dominates. Using the limit on the coefficient of $\mathcal{O}_{b\phi}$ from ref. [77] (with milder flavour assumptions), we find the bound $|y_{(b)}|/M \lesssim (0.2\,\mathrm{TeV})^{-2}$.

Top physics

Several of the dimension-6 SMEFT operators generated at tree level are relevant for the production of the top quark. $\mathcal{O}_{tW}$ and $\mathcal{O}_{\phi q}^{(3)}$ contribute to single production, whereas $\mathcal{O}_{tG}$ contributes to pair production [271]. In ref. [272], upper limits on the coefficients of these operators are derived. They range from approximately $(0.5\,\mathrm{TeV})^{-2}$ to $(0.8\,\mathrm{TeV})^{-2}$. Again, the natural values of these coefficients in our case, which are given by $\sim \lambda^2/2M$ and $\sim \lambda w/M$, already satisfy these limits. The same happens for the operators $\mathcal{O}_{tB}$, $\mathcal{O}_{\phi t}$ and $\mathcal{O}_{\phi q}^{(1)}$, which contribute to $tt\gamma$ and ttZ production, and have even weaker limits.

Low-energy CP violation

The imaginary part of the coefficients of the operators $\mathcal{O}_{t\phi}$, $\mathcal{O}_{\phi tb}$, $\mathcal{O}_{tW}$, $\mathcal{O}_{tB}$, $\mathcal{O}_{bW}$ and $\mathcal{O}_{tG}$ affects the electric dipole moment of the electron and the neutron. These low-energy observables must be computed by performing the RG running of the coefficients down to the electroweak scale and integrating out the top quark. In ref. [76, 273, 274], strong limits on the imaginary part of the coefficients have been obtained, ranging from $(2\,\mathrm{TeV})^{-2}$ to $(42\,\mathrm{TeV})^{-2}$. Our UV parameters enter these coefficients with the combination $\lambda w^*/M$, so either their absolute value is very small, or all their phases must be almost equal. A trivial way of satisfying these limits is by imposing that all parameters are real.

9.5 Production at the LHC

All the vector-like quarks can be produced in pairs at hadron colliders by their coupling to gluons, which is determined by the value of α_s at the relevant energy. Given M, the production cross section is fixed and it is the same for all the multiplets. One of the several tree-level diagrams contributing to pair production is represented in figure 9.3a. On the other hand, the T, B states can be singly produced via their mixing with the SM t, b quarks. The corresponding process is represented in figure 9.3b.

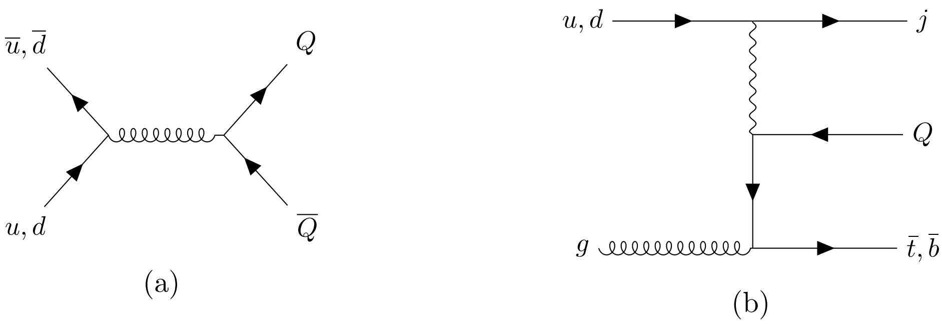

Figure 9.3: Production of heavy quarks in hadron colliders: (a) example diagram for pair production; (b) single production in association with a light jet j and a heavy SM quark $q = t, b$.

When the heavy quarks have low mass, the cross section for pair production is larger than the one for single production. As their mass increases, and for fixed collider energy, the later eventually becomes the main production mechanism. This has been studied for RVLQ mixing with the third family in ref. [251]. For these multiplets, the addition of dimension-5 interactions with natural values of the y couplings and $\Lambda \geq 2$ TeV does not change significantly the results, as they give a small correction to the cross section. Here we are assuming that the dimension-4 couplings saturate the electroweak limits. In the case of NRVLQ, for natural values of the y couplings and $\Lambda \geq 2$ TeV, pair production is larger than single production for the range of masses that can be tested at colliders in the present and near future. Some examples of the dependence of the production cross section on the the mass are shown in figure 9.4.

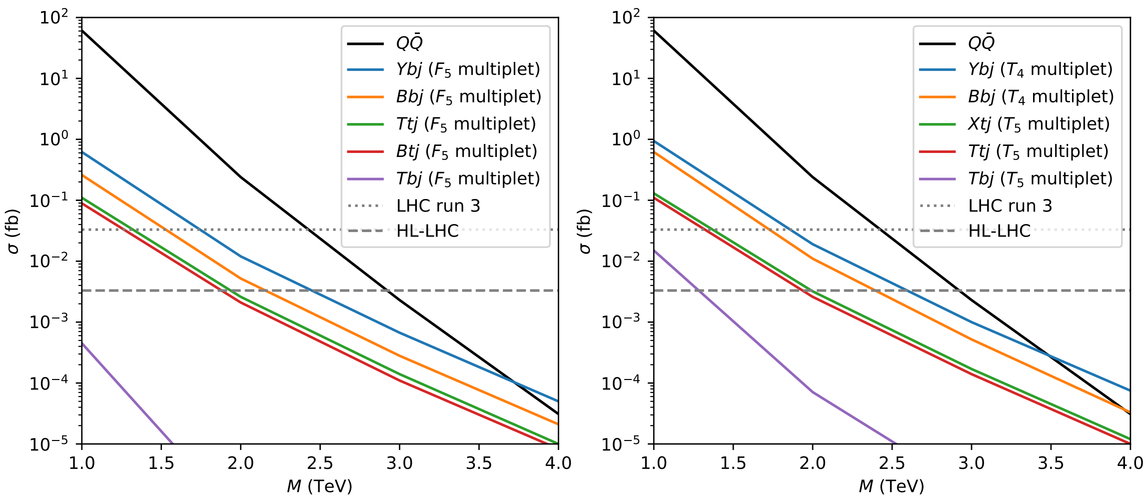

Figure 9.4: Cross section for different processes for production of heavy quarks with $y = (4\,\text{TeV})^{-1}$ and a center-of-mass energy of 14 TeV. The left plot corresponds to the F_5 quadruplet, while the right plot is for the T_5 and T_4 triplets. Pair production dominates for masses below $\simeq 3.5$ TeV. The dotted and dashed gray lines represent the minimum cross section needed to obtain at least 10 events at the corresponding collider, assuming that the expected integrated luminosity is reached [275].

The operators $\overline{Q}\sigma^{\mu\nu}qF_{\mu\nu}$ open new single production channels, which are suppressed by $(M/\Lambda)^2$ instead of $\sin^2\theta$. In figure 9.5, we show the two main mechanisms, which produce a heavy quark in association with a SM third generation quark. Other single production processes are possible with b quarks from the protons in the initial state . In this way, the B component of multiplets with these operators can be generated alone, while the T component can be produced together with a jet or a W boson. As an example, we show in figure 9.6 the cross section of the T production processes involving these operators, for the U multiplet. For $w = (4\,\text{TeV})^{-1}$ these cross sections are large. However, these couplings are generated in renormalizable UV completions only at one loop, so the natural value for w is expected to have a suppression of $1/16\pi^2$ in weakly coupled UV completions. Including this suppression gives cross sections that are smaller than pair production.

A concrete model with $\overline{Q}\sigma^{\mu\nu}qF_{\mu\nu}$ operators has been tested experimentally, as presented in ref. [276], for the case of the multiplet Q_1. This analysis focuses on a particular direction in parameter space, which in our notation corresponds to: $g_s w^G = g w^W = -g' w^B/6$, with the coefficients of all the other operators set to zero. The search is for the decay into γb. Under these conditions, M-dependent limits over the

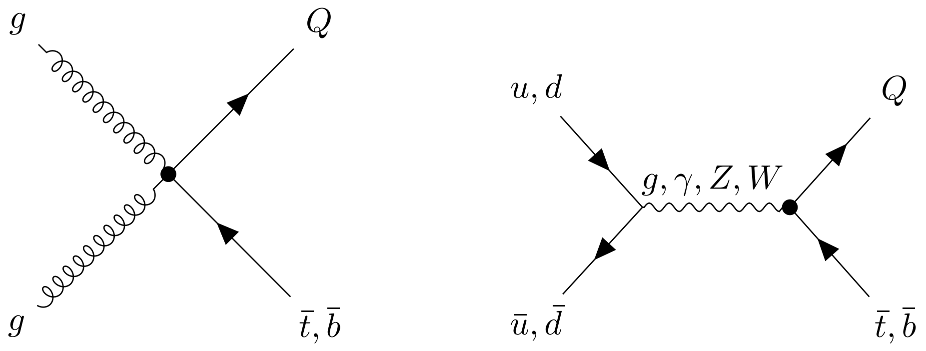

Figure 9.5: Single production with $\overline{Q}\sigma^{\mu\nu}qF_{\mu\nu}$-type operators.

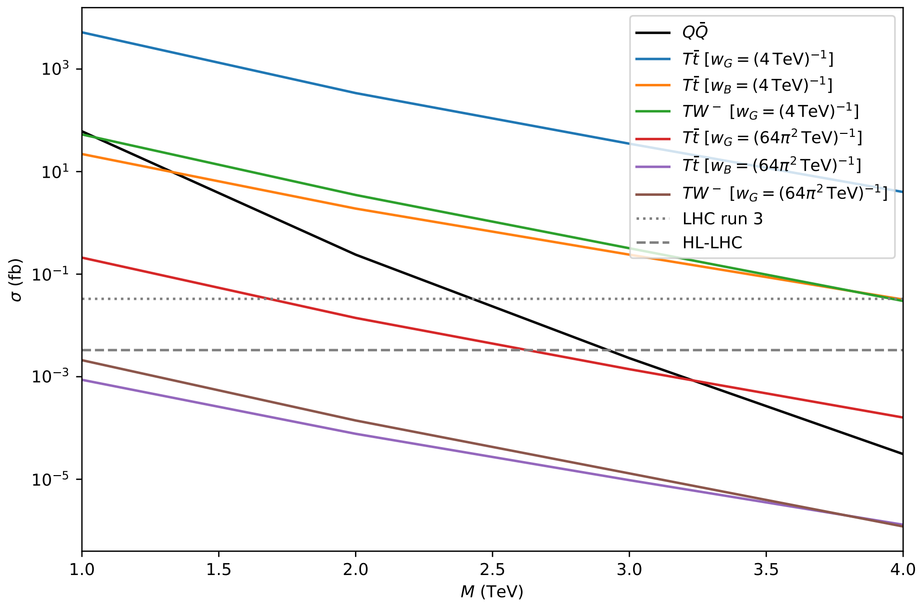

Figure 9.6: Cross section for different processes involving $\overline{Q}\sigma^{\mu\nu}qF_{\mu\nu}$, for production of heavy quarks in the U model, with a center-of-mass energy of 14 TeV.

coefficients of the operators have been obtained, for masses between $M = 1\,\mathrm{TeV}$ and $M = 1.8\,\mathrm{TeV}$. Translated into our notation, the bounds for these two masses are $w^G \lesssim (7\,\mathrm{TeV})^{-1}$ and $w^G \lesssim (5\,\mathrm{TeV})^{-1}$, respectively.

9.6 Decay

9.6.1 Lifetime

In this section, we study the decays of the heavy quarks. Barring cancellations with other heavy physics, electroweak precision tests require small mixings. In this case, the splittings between the different components of the extra quark multiplet are small (of a few GeV at most for masses below 2 TeV). This in turn implies that the decays from one component to another are very suppressed. The T and B states can decay via mixing into Ht, Zt, W^+b and Hb, Zb, W^-b, respectively. They can also decay into $t\gamma$, tg and $b\gamma$, bg, respectively, in the presence of w couplings. The X and Y states decay via mixing mainly into W^+t and W^-b, respectively. Their three-body decays are also sizable. Finally, X' and Y' have no two-body decays, as their charges differ by at least two units from the ones of the SM quarks.

The decay width of RVLQ is typically large enough for them to have prompt decays and small enough for a good narrow width approximation. The NRVLQ, on the other hand, have smaller and smaller widths for larger and larger values of the cutoff Λ. In figure 9.7, we show the dependence of the total width of T and B with the dimension-5 Yukawa coupling y for each type of NRVLQ, for $M = 2$ TeV. For widths below the QCD scale (see the discussion below), we have extrapolated the results calculated for larger couplings.

For small enough widths, i.e. long lifetimes, the phenomenology of the vector-like quarks can be completely different from the one in the standard searches of these particles. First, when the width is smaller than the QCD scale Λ_{QCD}, non-perturbative effects, including hadronization, will be significant before the quarks have time to decay. One possibility is the formation near threshold of $Q\bar{Q}$ quarkonium states. This has been studied in ref. [277] (see also the review in ref. [278]) and generalized in ref. [186] to higher color representations. Possible signatures would have di-photon and di-lepton resonant final states. But the production cross-section is suppressed by the wave function at the origin and the cross sections are small. For instance, for M above at the 0.01 TeV, ref. [186] shows that the cross section into $\gamma\gamma$ for quarks with masses above 1 TeV is below 0.01 fb. In fact, most of the time the heavy quarks will fragment independently forming Qq meson states and also baryons with light quarks from the vacuum. This is completely analogous to the case of b-quarks forming B mesons. For $M \gg \Lambda_{\text{QCD}}$, the mass and partial decay widths of the hadrons will inherit the properties of the heavy quark, up to small QCD corrections. Moreover, most of the energy resides in the hadron containing the heavy quark, leaving only a small fraction to light particles in the accompanying jet, and gluon radiation only softens the spectrum slightly [277]. Hence, the standard type of search for vectorlike quarks will be mostly blind to the fact that the quarks hadronize, as long as they decay promptly (that is, for lifetimes below 10^{-14} s).

For widths smaller than $\sim 10^{-12}$ GeV, the hadrons carrying the heavy quark will be long-lived. In this context, they are called R-hadrons. Their phenomenology at the LHC has been studied in detail, especially for squarks and gluinos in supersymmetric models. R-hadrons interact hadronically as they move through the detector, but in these processes the heavy quark acts mostly as a spectator of the low-energy scattering of light partons. Compared to SM hadrons, their energy loss in the calorimeter is small. Possible signatures include (see ref. [279] for a review of the phenomenology of long-lived particles):

- Tracks with anomalous ionization, from the slower speed of the heavy quarks in comparison to SM particles and/or non-standard charges. Note that Qq mesons formed with X, X', Y or Y' will always be charged, while those with T and B can be charged or neutral. In these searches, one must take into account the fact that the charge of the R-hadrons may change due to the hadronic interactions of the light partons with the detector material.

- Delayed detector signals, due again to the small speed. In the extreme case, it is possible for a quasi-stable R-hadron to loose all its energy and stop at the hadronic calorimeter; its eventual decay would give out-of-time signals.

- Displaced vertices from the delayed decay of the heavy quark. The final states produced by R-hadrons with vector-like quarks are very different from the ones in supersymmetric theories and other scenarios considered thus far. So, a dedicated search for displaced vertices of vector-like quarks would be necessary to probe this scenario.

The relevance of each of the signatures depends crucially on the lifetime of the R-hadron, which is of the order of the lifetime of the heavy quark, as calculated ignoring QCD. In table 9.5, we give the values of $1/y$ above which i) non-perturbative QCD is important (Λ_{QCD}), ii) displaced vertices can be observed (Λ_{disp}) and iii) the heavy quark is stable within detector distances ($\Lambda_{\text{long lived}}$).

	Λ_{QCD}					Λ_{disp}	$\Lambda_{\text{long lived}}$
	T_4	T_5	F_1	F_5	F_7		
X'	–	1.0	–	–	1.0	5×10^5	5×10^7
X	–	4.6	3.1	–	3.9	10^6	10^8
T	–	5.3	3.7	5.7	5.6	10^6	10^8
B	5.3	–	3.7	5.7	5.7	10^6	10^8
Y	4.6	–	3.1	3.9	–	10^6	10^8
Y'	1.1	–	–	1.1	–	5×10^5	5×10^7

Table 9.5: Value of $1/y$ (in TeV) at which the total width reaches the scales $\Lambda_{\text{QCD}} = 0.2\,\text{GeV}$, $\Lambda_{\text{disp}} = 10^{-12}\,\text{GeV}$ and $\Lambda_{\text{long lived}} = 10^{-16}\,\text{GeV}$. For Λ_{disp} and $\Lambda_{\text{long lived}}$ only an estimate of the order of magnitude is provided, obtained by extrapolation of the results above Λ_{QCD}.

As a reference, ATLAS has recently put bounds on the mass of long-lived supersymmetric R-hadrons, using ionization energy loss and time-of-flight information [280]. This search is quite model-independent and can be adapted to the case of vector-like quarks (which are also color-triplets but fermions, rather than scalars). Comparing with the limits on production cross sections for squarks and sgluinos, we estimate a lower bound close to 1500 GeV on the mass of detector stable vector-like quarks.

9.6.2 Branching ratios into Hq, Zq and Wq

In the following, we concentrate on branching ratios, having in mind mostly the case with prompt decays. Consider RVLQ. If the dimension-5 couplings are turned off, T essentially decays only into Ht, Zt or W^+b, while B decays into Hb, Zb or W^-t. Changing the specific values of the parameters in these models has a small effect in the branching ratios. This means that the branching ratios are approximately determined by the choice of multiplet. Because the sum of branching ratios must be one

$$BR(Q \to Hq) + BR(Q \to Zq) + BR(Q \to W^{\pm}q') = 1, \tag{9.33}$$

(with $Q = T, B$ and $q, q' = t, b$) it suffices to know two branching ratios of Q to be able to know the third. Any two branching ratios BR_1 and BR_2 of Q form a point in the triangle $BR_1 + BR_2 \leq 1$, $BR_{1,2} \geq 0$. Thus, each multiplet determines a point in this triangle (or a short segment, taking into account variations of the values of the parameters). This is the usual method for representing graphically the branching ratios of vector-like quarks [281].

The addition of dimension-5 interactions modifies these points, both by changing the corresponding partial widths and by introducing new decay channels. Then, eq. (9.33) no longer holds. For any choice of the values of the parameters, the branching ratios define a point p in the multi-dimensional simplex determined by $BR_i \leq 1$, $\sum_i BR_i = 1$. In particular, the branching ratios into Ht, Zt and W^+b define a point that falls inside the tetrahedron

$$\Sigma := BR(Q \to Hq) + BR(Q \to Zq) + BR(Q \to W^{\pm}q') \leq 1, \tag{9.34}$$

$$BR(Q \to Hq), BR(Q \to Zq), BR(Q \to W^{\pm}q') \geq 0. \tag{9.35}$$

For their graphical representation, we have chosen to plot the projections of p into the $BR(Q \to Zq)$—$BR(Q \to Hq)$ plane and into the $BR(Q \to W^{\pm}q')$—$BR(Q \to Hq)$ plane, as shown in figure 9.8.

The results for RVLQ are presented in figures 9.10, 9.11, 9.12 and 9.13, while the branching ratios of NRVLQ are presented in figures 9.14, 9.15, 9.16 and 9.17. Each segment is obtained by evaluating at $M = 1\,\text{TeV}$ and at $M = 2\,\text{TeV}$ while keeping all the other parameters fixed. The value of the coefficients of dimension-5 operators is chosen to be $(2\,\text{TeV})^{-1}$. This pretty large value has been chosen to visually highlight the directions of the corrections induced on the branching ratios for RVLQ. For lower, probably more realistic values of the coefficients (especially for w), these corrections will be smaller. For the multiplets without dimension-4 interactions, this value of the coefficients ensures that the decay width is much higher than the QCD scale, so that that QCD effects can be neglected. The branching ratios do not change much with the value of the corresponding coefficient in the range from $(2\,\text{TeV})^{-1}$ down to the values in which the total width equals Λ_{QCD}.

As it can be clearly seen in the figures, most branching ratios points lie near or directly over the $BR(Q \to Hq) = BR(Q \to Zq)$ diagonal. This happens in all cases where the coefficients of the $\bar{Q}\sigma^{\mu\nu}qF_{\mu\nu}$-type operators vanish, except for the F_1 multiplet. To show why, we define $X^{L,R}_{Qq}$ and $Y^{L,R}_{Qq}$ as the following coefficients in the Lagrangian:

$$\mathcal{L}_Z = -\frac{g}{2c_W}\bar{q}\not{Z}\left(\pm X^L_{qQ}P_L \pm X^R_{qQ}P_R\right)Q + \text{h.c.},$$
$$\mathcal{L}_H = -\frac{gm_Q}{2m_W}\bar{q}H\left(Y^L_{qQ}P_L + Y^R_{qQ}P_R\right)Q,$$

the equality of the braching ratios follows from the equality in magnitude of the dominant $X^{L,R}_{Qq}$ and the dominant $Y^{L,R}_{Qq}$. The weak eigenstates q^0, Q^0 couple to the Z boson as

$$\mathcal{L}_Z = -\frac{g}{2c_W}\sum_{\chi=L,R}\begin{pmatrix}\bar{q}^0_\chi & \bar{Q}^0_\chi\end{pmatrix}\not{Z}\begin{pmatrix}2T_3(q^0_\chi) - 2Q_e(q^0_\chi)s^2_W & 0\\ 0 & 2T_3(Q^0_\chi) - 2Q_e(Q^0_\chi)s^2_W\end{pmatrix}\begin{pmatrix}q^0_\chi\\ Q^0_\chi\end{pmatrix},$$

where T_3 denotes the third component of isospin and Q_e denotes electric charge. After the unitary transformation in equations (9.26) and (9.27), we get

$$X^{L,R}_{qQ} = 2s_{L,R}\,c_{L,R}\left[T_3(q^0_{L,R}) - T_3(Q^0_{L,R})\right].$$

On the other hand, the quark gauge eigenstates q^0, Q^0 couple to the Higgs as

$$\mathcal{L}_H = -\frac{1}{\sqrt{2}} \begin{pmatrix} \bar{q}^0_L & \bar{Q}^0_L \end{pmatrix} H \begin{pmatrix} y_{11} & y_{12} \\ y_{21} & 0 \end{pmatrix} \begin{pmatrix} q^0_R \\ Q^0_R \end{pmatrix}.$$

Generally, one of the off-diagonal elements is negligible. This happens because the dimension-4 and dimension-5 Yukawas always contribute to different elements of the y_{ij} matrix. Either one of them is zero or, when both are present, the dimension-5 one is smaller. This means that one of the mixing angles $\theta_{L,R}$ dominates. For multiplets with dimension-4 couplings, the chirality with the dominant mixing angle θ_D is $D = L$ for singlets and triplets and $D = R$ for doublets. For multiplets without dimension-4 couplings it is $D = R$ for triplets and $D = L$ for quadruplets.

The dominant off-diagonal element y_D is related to the corresponding mixing angle as $y_D \simeq x\sqrt{2}m_Q s_D/v$, where $x = 1$ in the cases with dimension-4 interactions and $x = 2$ in the ones with only dimension-5 ones. This factor is necessary because of the different the relation between the mass and Yukawa terms in both cases. The dominant HqQ coupling is, then

$$Y^D_{Qq} \simeq x s_D c_D.$$

For $X^D_{Qq} \simeq Y^D_{Qq}$ it is necessary and sufficient that

$$\left| T_3(q^0_D) - T_3(Q^0_D) \right| = x/2. \tag{9.36}$$

It can be checked case by case that this relation is satisfied for all multiplets except for F_1. In this case, we have $|T_3(q^0_L) - T_3(Q^0_L)| = 0$.

9.6.3 Extra decay channels and limits on mass

The experimental analyses of searches of pair-produced vector-like quarks usually combine the information on the different final states to put lower bounds on the heavy quark masses, as a function of the branching ratios to Wq', Zq and Hq [226, 227]. Eq. (9.33) is assumed in these analyses, so the results are not directly valid beyond the renormalizable level. However, they can be adapted to the case where other decay channels are present. This has been discussed previously in refs. [264, 282]. We derive here a simple formula for the corrected mass limit due to the presence of extra decays. Experimental data determines an upper limit L_{exp} on the sum of the cross-sections for the production and decay of a pair of heavy quarks, weighted by the efficiency for each decay channel (see ref. [264]):

$$\sigma_{pp\to Q\bar{Q}}(M) \sum_{ij} \epsilon_{ij} BR_i BR_j < L_{\text{exp}}, \tag{9.37}$$

where M is the mass of the heavy quark, i and j run over all the decay channels, and ϵ_{ij} is the corresponding efficiency. A limit on the mass can be derived from this inequality. In the usual experimental analyses, it is assumed that the sum of the branching ratios into these three channels is $\Sigma = 1$.

We consider now the case $\Sigma < 1$. We will obtain a lower limit on the mass of some heavy quark with branching ratios BR_i. Some assumption has to be made about the efficiency $\epsilon_{ia} = \epsilon_{ai}$ for the channels a that are not Hq, Zq or $W^\pm q'$. We adopt here the conservative choice $\epsilon_{ia} = 0$. Let M_1 be the lower limit on the mass for the branching ratios $BR_i^\Sigma = BR_i/\Sigma$, whose sum is 1, so that M_1 is known from experimental analyses. We define the mass M_Σ by the equation

$$\Sigma^2 \sigma_{pp\to Q\bar{Q}}(M_\Sigma) = \sigma_{pp\to Q\bar{Q}}(M_1). \tag{9.38}$$

Then, we have the identity

$$\sigma_{pp\to Q\bar{Q}}(M_\Sigma) \sum_{ij} \epsilon_{ij} BR_i BR_j = \sigma_{pp\to Q\bar{Q}}(M_1) \sum_{ij} \epsilon_{ij} BR_i^\Sigma BR_j^\Sigma. \tag{9.39}$$

Because M_1 is the limit obtained from eq. (9.37) for branching ratios BR_i^Σ, it follows from this identity that M_Σ is the limit for BR_i. We now proceed to find an analytic solution to eq. (9.38). The production cross-section $\sigma_{pp\to Q\bar{Q}}(M)$ can be approximated, for masses around $\widetilde{M} = 1.1\,\text{TeV}$ by an exponential:

$$\sigma_{pp\to Q\bar{Q}}(M) \simeq \sigma_{pp\to Q\bar{Q}}(\widetilde{M}) \exp\left(-\frac{M^{1/2} - \widetilde{M}^{1/2}}{f^{1/2}/2}\right), \tag{9.40}$$

where $f = 20.5\,\text{GeV}$. In the range $[0.8, 1.4]\,\text{TeV}$, the difference between the cross section produced by this formula and the one obtained using MadGraph increases towards the extremes of the interval and is at most 3%. Plugging eq. (9.40) in eq. (9.38) gives

$$M_\Sigma = (M_1^{1/2} + f^{1/2} \log \Sigma)^2. \tag{9.41}$$

We have thus found a lower bound M_Σ on the mass of any heavy quark as a function of the lower bound M_1 it would have if its branching ratios into Ht, Zq and $W^\pm q'$ were rescaled by the same factor $1/\Sigma$, so that eq. (9.33) would hold. Here, $f = 20.5\,\text{GeV}$ is just a constant. In table 9.6, we present the limits calculated using this formula, for different choices of the values of the parameters for each model, taking the bound M_1 from ref. [226]. In all cases the couplings of dimension-4 operators are chosen to saturate the electroweak limits. In figure 9.9, we show the corrections induced by the use of this formula on the results of ref. [226], for the value $\Sigma = 1/2$.

We have emphasized the presence of alternative decay channels at the non-renormalizable level. In tables 9.7 and 9.8, we give the decay channels with branching ratio > 0.01 other than Zq, $W^\pm q'$ and Hq for T and B, together with the maximum value they get and the interaction that generates them. We choose the values $M = 2\,\text{TeV}$ and $w, y = (2\,\text{TeV})^{-1}$, again quite extreme, in order to maximize these alternative branching ratios (including those of three-body decays). The tables also include three-body channels that survive when $w = y = 0$. For RVLQ, the values of the couplings λ are chosen to approximately saturate the electroweak limits. For smaller values of λ, the alternative channels will have larger branching ratios. Large branching ratios are found for channels involving "magnetic" operators. The reason is that the partial widths are suppressed in this case by $(M/\Lambda)^2$, in comparison with the $(v/M)^2$ suppression of the decay widths of standard channels. Note however that the value of w we use is $\sim 16\pi^2$ times too high in weakly coupled completions. A detailed analysis of the decays of a U vector-like quark into $t\gamma$ and tg at the LHC has been performed in ref. [283].

$U \sim 1_{2/3}$	
Only dim. 4	1300
y	1310
w_B	1010
w_G	< 800

$D \sim 1_{2/3}$	
Only dim. 4	1200
y	1190
w_B	< 800
w_G	< 800

$Q_1 \sim 2_{1/6}$	
Only dim. 4	1340
y_t	1340
y_b	1120
w_B	830
w_W	1250
w_G	< 800

$Q_5 \sim 2_{-5/6}$	
Only dim. 4	1130
y	1130

$Q_7 \sim 2_{7/6}$	
Only dim. 4	1360
y	1350

$T_1 \sim 3_{-1/3}$	
Only dim. 4	1220
y_t	1250
y_b	1200
w	970

$T_2 \sim 3_{2/3}$	
Only dim. 4	1130
y_t	1130
y_b	1130
w	1260

$T_4 \sim 3_{-4/3}$	
y	1130

$T_5 \sim 3_{5/3}$	
y	1360

$F_1 \sim 4_{1/6}$	
y	1030
w	1010

$F_5 \sim 4_{-5/6}$	
y	1200

$F_7 \sim 4_{7/6}$	
y	1130

Table 9.6: Mass limits for each multiplet and different values of the couplings. In the right column, a lower bound on the mass of the heavy quark (in TeV) is displayed, assuming that the corresponding coupling in the left column has a value of $(2\,\text{TeV})^{-1}$ and the other dimensionful couplings vanish. The dimensionless couplings λ are always chosen to saturate the corresponding electroweak precision bounds.

In the case of X and Y, the decays into W^+t and W^-b, respectively, have branching ratios in the range 60–90%. The remaining decays are into three particles, two of which are always W^+t or W^-b. The branching ratios for these channels are collected in tables 9.9 and 9.10. The states X' and Y' have only three-body decays. They always decay into W^+W^+t and W^-W^-b, respectively.

Multiplet	Decay products	Maximum BR	Coupling
U	$b\bar{b}t$	0.02	λ, y
	$tt\bar{t}$	0.01	λ, y
	γt	0.71	w_B
	gt	0.93	w_G
Q_1	tW^+W^-	0.08	λ_t, y_b
	$tt\bar{t}$	0.01	λ_t, y_t, y_b
	bHW^+	0.11	y_b
	bZW^+	0.04	λ_b, y_t, y_b
	γt	0.77	w_B
	gt	0.99	w_G
Q_7	tW^+W^-	0.08	λ
T_1	bHW^+	0.10	y_b
	tW^+W^-	0.07	λ, y_t, y_b
	bZW^+	0.83	w
	$tt\bar{t}$	0.01	λ, y_t, y_b
	$b\gamma W^+$	0.01	w
T_2	bHW^+	0.17	y_b
	tW^+W^-	0.25	w
	bZW^+	0.06	λ, y_t, y_b
	$tb\bar{b}$	0.01	λ, y_t, y_b
	γt	0.21	w
T_5	tW^+W^-	0.08	y
F_1	bHW^+	0.30	y
	bZW^+	0.23	w
	tW^+W^-	0.66	w
	γt	0.09	w
F_5	bHW^+	0.10	y
	tW^+W^-	0.07	y
F_7	bHW^+	0.28	y
	bZW^+	0.10	y
	tW^+W^-	0.09	y

Table 9.7: Extra decay channels of T with branching ratio larger than 0.01 for $M = 2\,\text{TeV}$, when the couplings λ are fixed to the values that saturate electroweak precision limits. The last column displays the coupling constant which, when set to $(2\,\text{TeV})^{-1}$, gives the maximum BR in the corresponding channel. The appearance of λ indicates that the channel in question is present already in the case with dimension-4 interactions only.

Multiplet	Decay products	Maximum BR	Coupling
D	γb	0.77	w_B
	gb	0.99	w_G
Q_1	tHW^-	0.12	y_t
	tZW^-	0.04	λ_t, y_t, y_b
	bW^+W^-	0.08	λ_b, y_t, y_b
	$bt\bar{t}$	0.02	λ_t, y_t, y_b
	γb	0.77	w_B
	gb	0.99	w_G
Q_5	bW^+W^-	0.08	λ, y
	$bt\bar{t}$	0.01	λ, y
T_1	tHW^-	0.19	y_t
	tZW^-	0.06	λ, y_t, y_b
	$bt\bar{t}$	0.01	λ, y_t, y_b
	bW^+W^-	0.90	w
	γb	0.13	w
T_2	tHW^-	0.10	λ
	bW^+W^-	0.07	λ, y_t, y_b
	tZW^-	0.12	w
	$bt\bar{t}$	0.02	λ, y_t, y_b
T_4	bW^+W^-	0.08	y
F_1	tHW^-	0.30	y
	bW^+W^-	0.69	w
	γb	0.09	w
	tZW^-	0.20	w
F_5	tHW^-	0.28	y
	tZW^-	0.10	y
	bW^+W^-	0.09	y
F_7	tHW^-	0.10	y
	bW^+W^-	0.07	y

Table 9.8: Extra decay channels of B with branching ratio larger than 0.01 for $M = 2\,\text{TeV}$, when the couplings λ are fixed to the values that saturate electroweak precision limits. The last column displays the coupling constant which, when set to $(2\,\text{TeV})^{-1}$, gives the maximum BR in the corresponding channel. The appearance of λ indicates that the channel in question is present already in the case with dimension-4 interactions only.

Multiplet	Decay products	Maximum BR	Coupling
Q_7	tHW^+	0.12	y
	tZW^+	0.04	λ, y
T_2	tHW^+	0.10	λ, y_d
	bW^+W^+	0.04	λ, y_u, y_d
	tZW^+	0.12	w
	$tt\bar{b}$	0.02	λ, y_u, y_d
T_5	tHW^+	0.29	y
	tZW^+	0.11	y
F_1	tHW^+	0.32	y
	tZW^+	0.82	w
F_7	tHW^+	0.32	y

Table 9.9: Decay channels of X other than W^+t with branching ratio larger than 0.01 for $M = 2\,\text{TeV}$, when the couplings λ are fixed to the values that saturate electroweak precision limits. The last column displays the coupling constant which, when set to $(2\,\text{TeV})^{-1}$, gives the maximum BR in the corresponding channel. The appearance of λ indicates that the channel in question is present already in the case with dimension-4 interactions only.

Multiplet	Decay products	Maximum BR	Coupling
Q_5	bHW^-	0.10	λ, y
	bZW^-	0.04	λ, y
	$bb\bar{t}$	0.02	λ, y
T_1	bHW^-	0.10	λ, y_u, y_d
	bZW^-	0.83	w
	tW^-W^-	0.04	λ, y_u, y_d
T_4	bHW^-	0.29	y
	bZW^-	0.11	y
F_1	bHW^-	0.32	y
	bZW^-	0.82	w
F_5	bHW^-	0.32	y

Table 9.10: Decay channels of Y other than W^-b with branching ratio larger than 0.01 for $M = 2\,\text{TeV}$, when the couplings λ are fixed to the values that saturate electroweak precision limits. The last column displays the coupling constant which, when set to $(2\,\text{TeV})^{-1}$, gives the maximum BR in the corresponding channel. The appearance of λ indicates that the channel in question is present already in the case with dimension-4 interactions only.

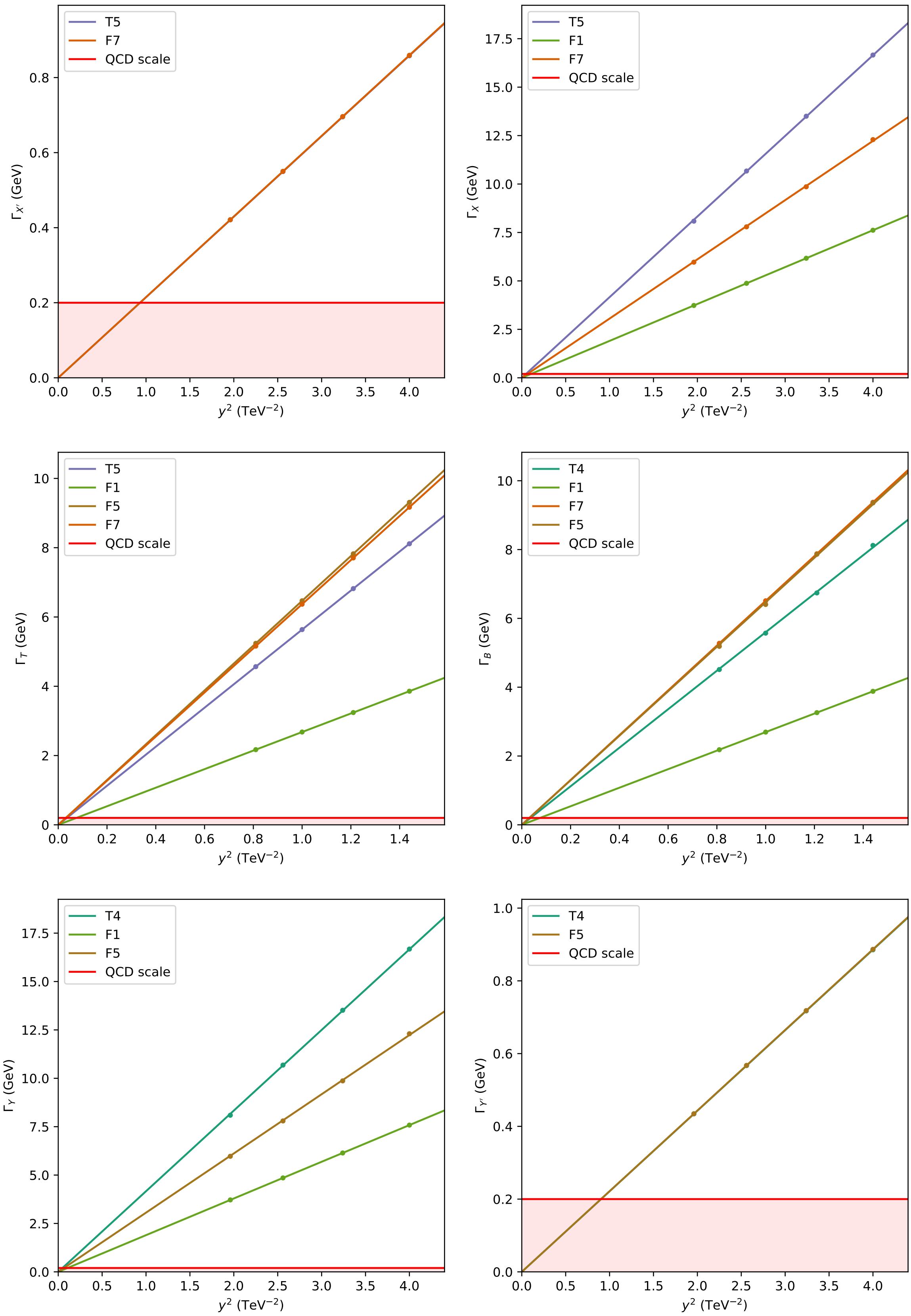

Figure 9.7: Total decay width of T (left) and B (right) vs the dimension-5 Yukawa coupling y for each multiplet without dimension-4 couplings and $M_Q = 2\,\text{TeV}$.

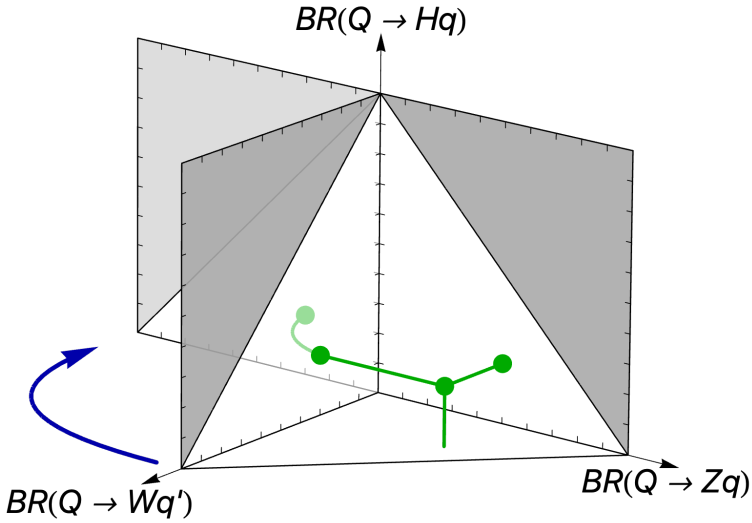

Figure 9.8: Representation of the $(BR(Q \to Zq), BR(Q \to W^{\pm}q'), BR(Q \to Hq))$ point as its projections into the $BR(Q \to Zq)$—$BR(Q \to Hq)$ plane and into the $BR(Q \to W^{\pm}q')$—$BR(Q \to Hq)$ plane.

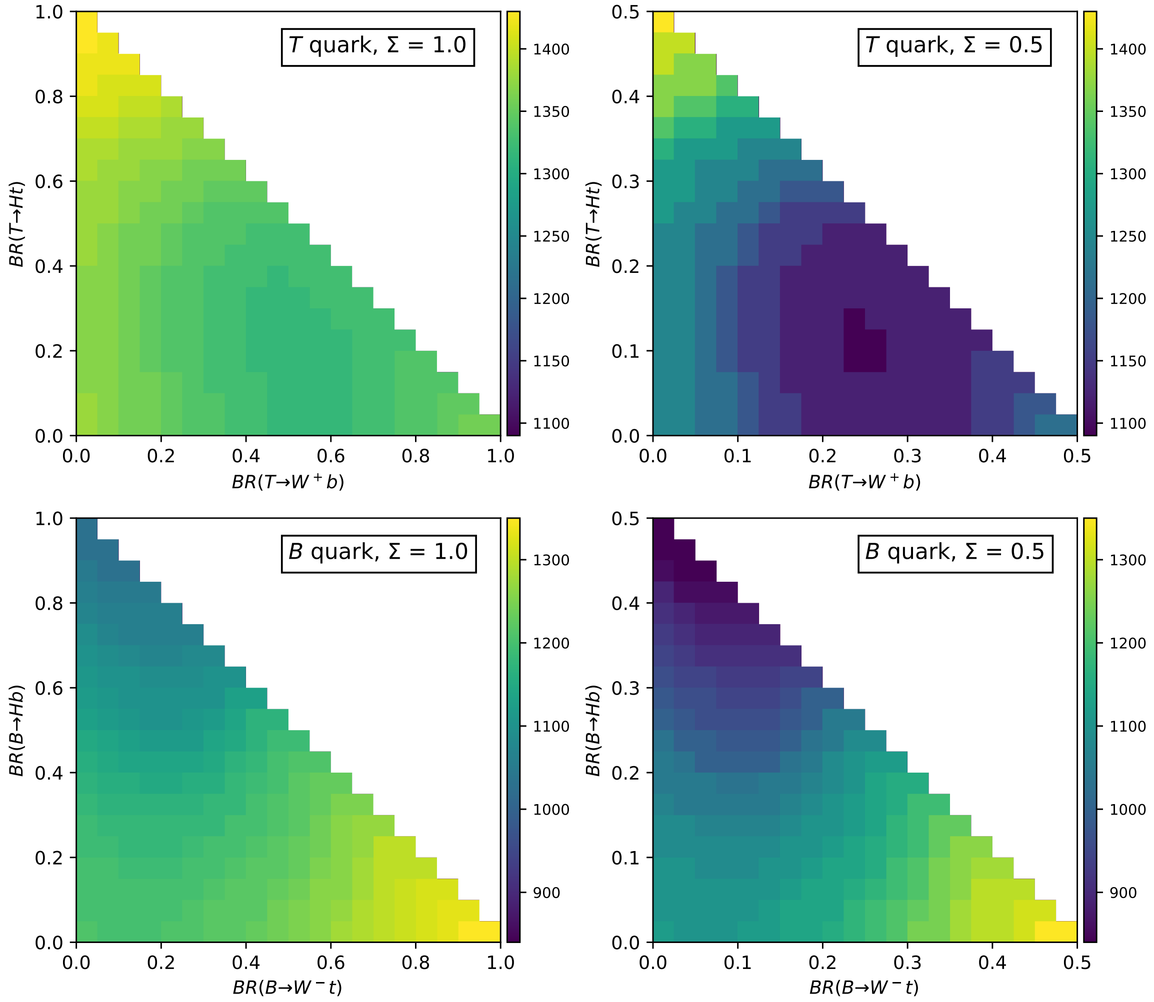

Figure 9.9: Left plots: lower bounds for the masses of heavy quarks presented in ref. [226] assuming that the sum of branching ratios into Hq, Zq and $W^{\pm}q'$ is $\Sigma = 1$. Right plots: corrected lower bounds for the case in which $\Sigma = 0.5$.

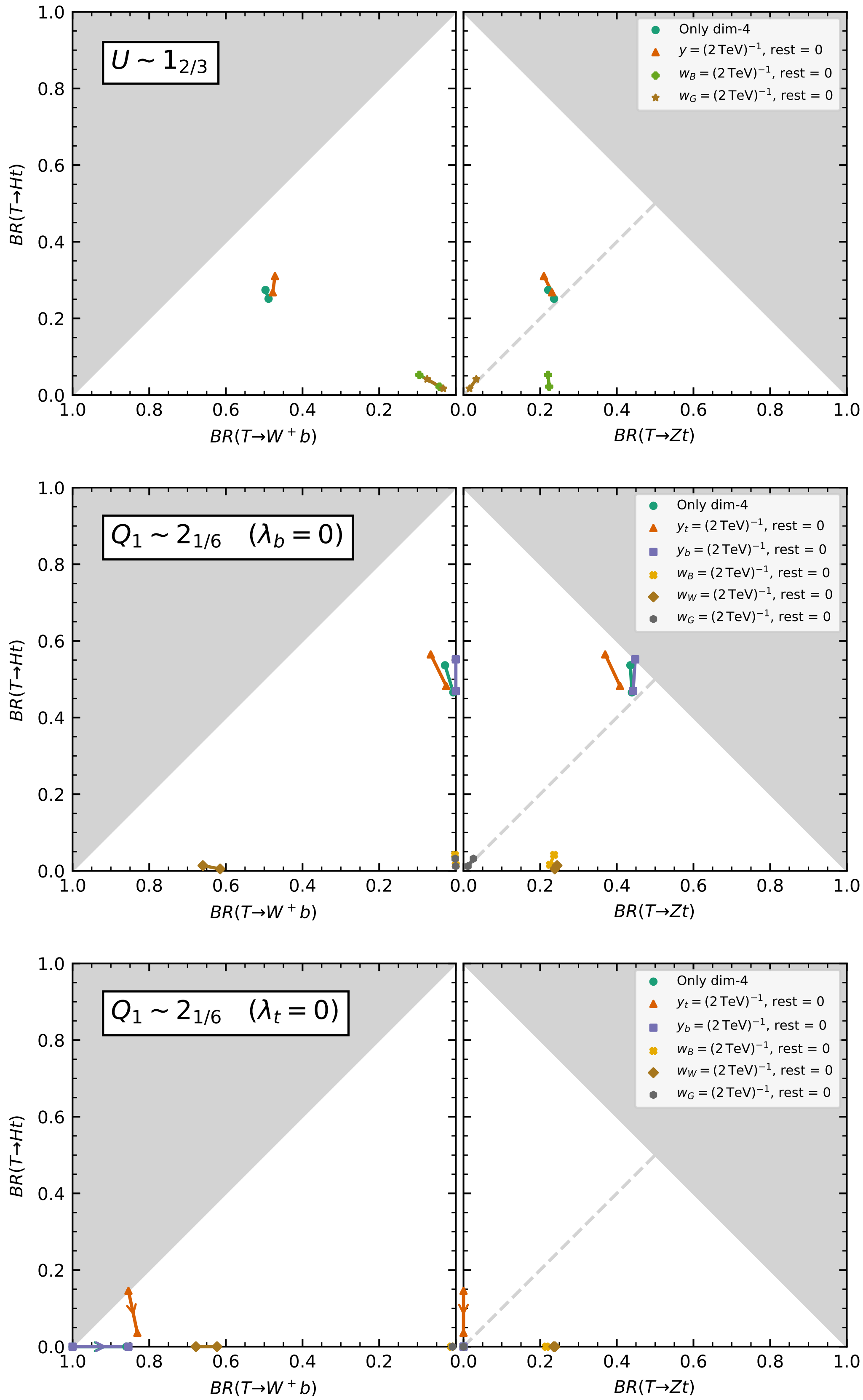

Figure 9.10: Branching ratios of T into Ht, Zt and W^+b for various values of the parameters in the U, Q_7 and Q_1 models. The dimensionless couplings λ are always chosen to saturate the corresponding electroweak precision bounds.

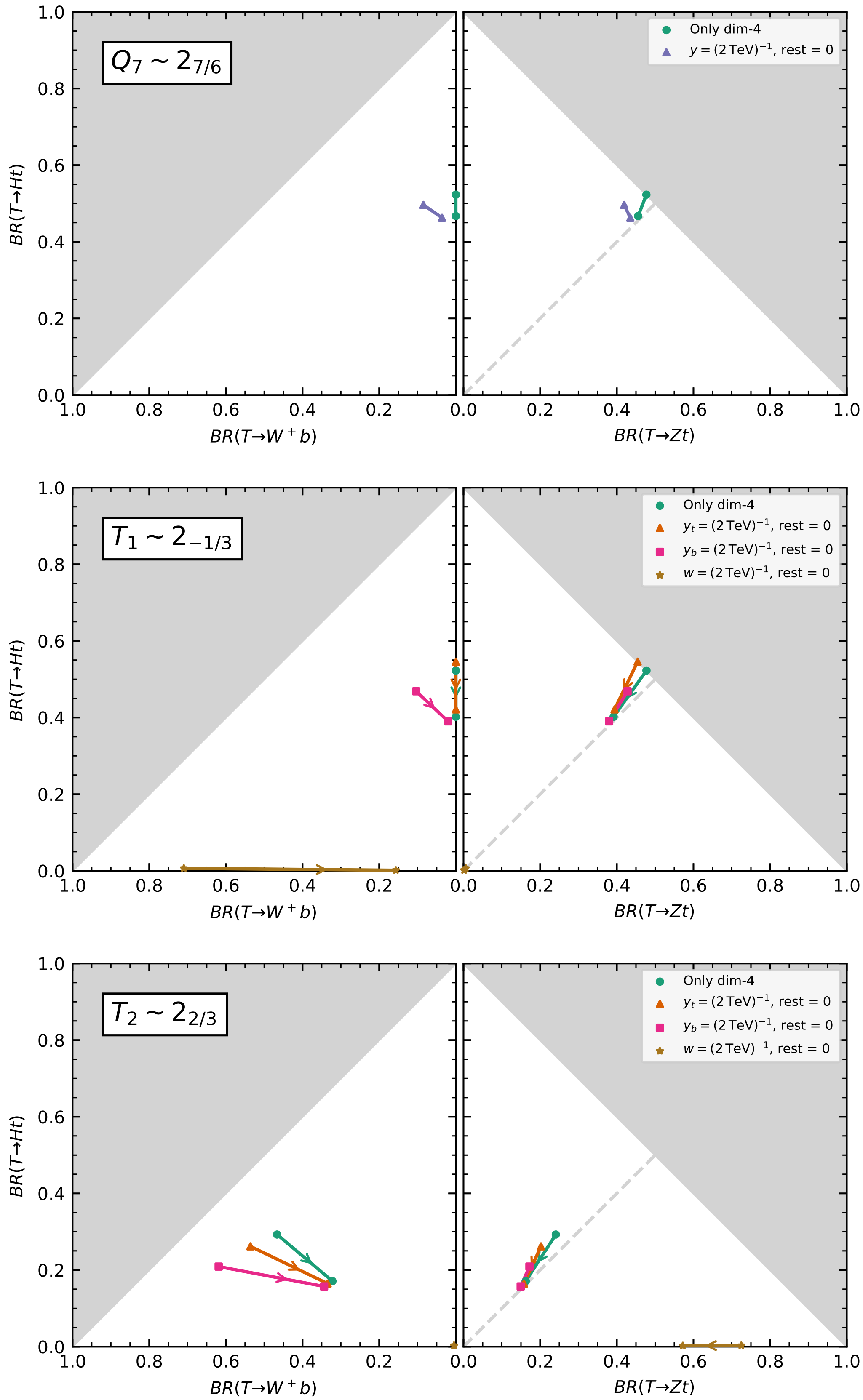

Figure 9.11: Branching ratios of T into Ht, Zt and W^+b for various values of the parameters in the Q_1, T_2 and T_1 models. The dimensionless couplings λ are always chosen to saturate the corresponding electroweak precision bounds.

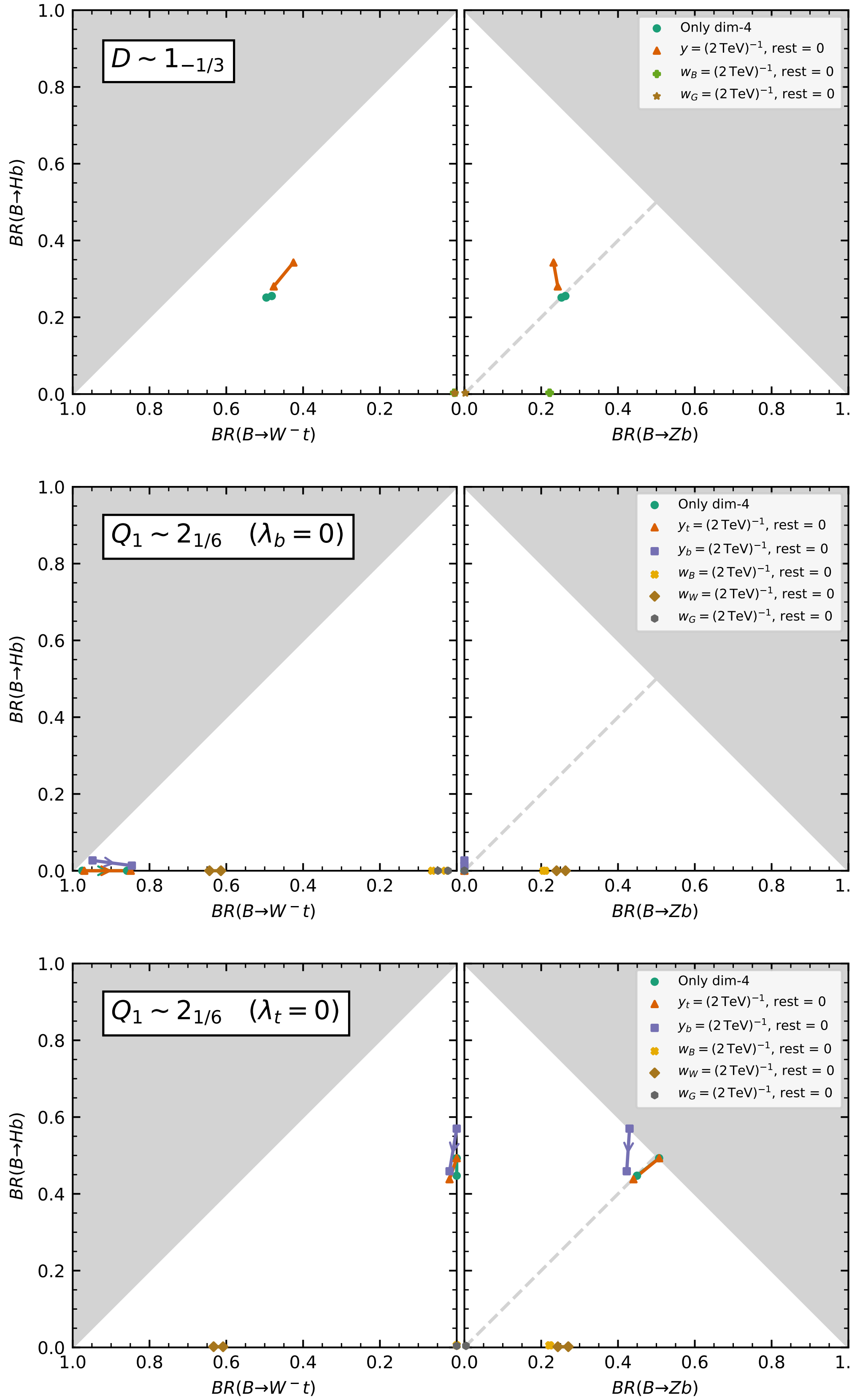

Figure 9.12: Branching ratios of B into Hb, Zb and W^-t for various values of the parameters in the D and Q_1 models. The dimensionless couplings λ are always chosen to saturate the corresponding electroweak precision bounds.

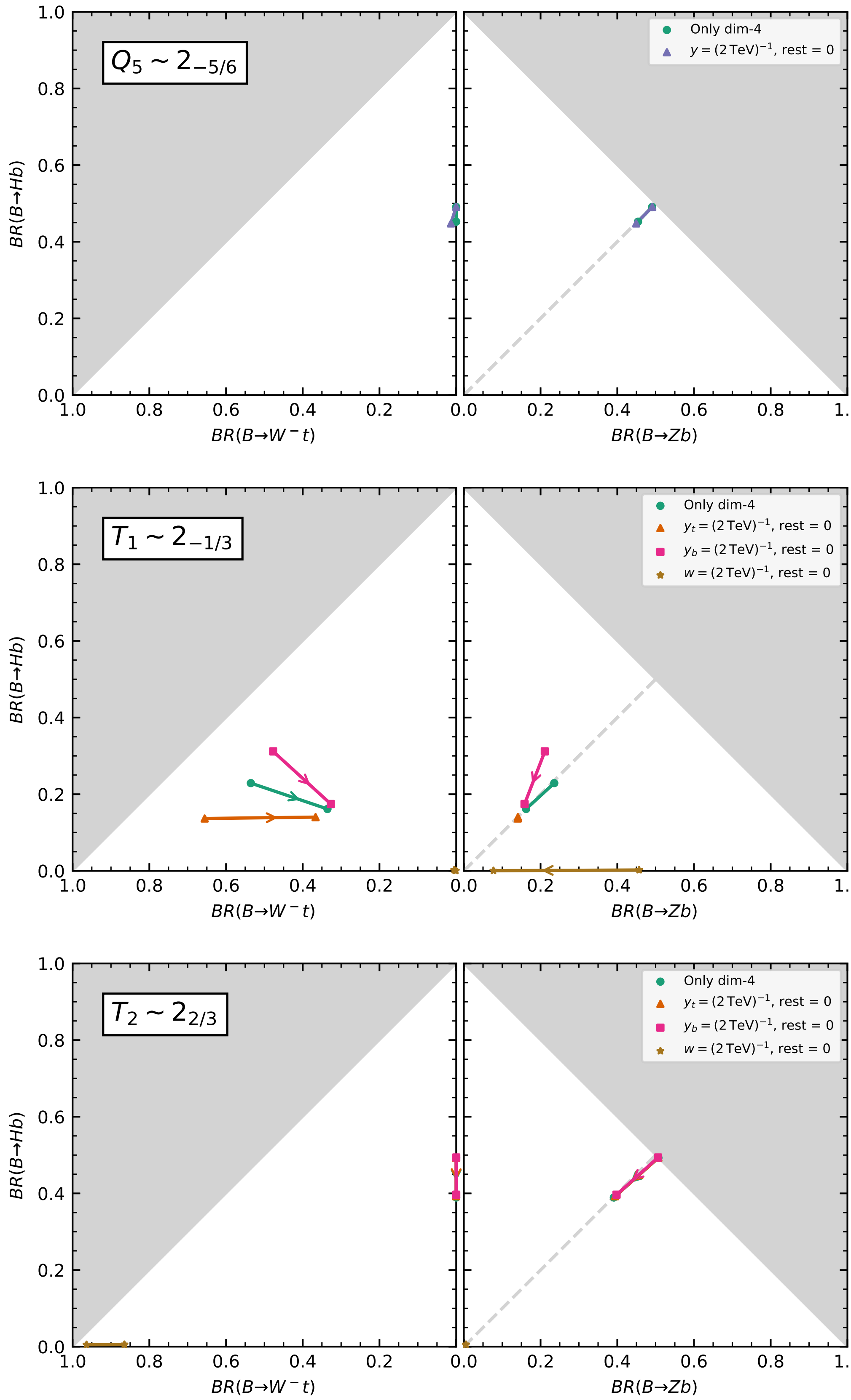

Figure 9.13: Branching ratios of B into Hb, Zb and $W^{-}t$ for various values of the parameters in the Q_5, T_2 and T_1 models. The dimensionless couplings λ are always chosen to saturate the corresponding electroweak precision bounds.

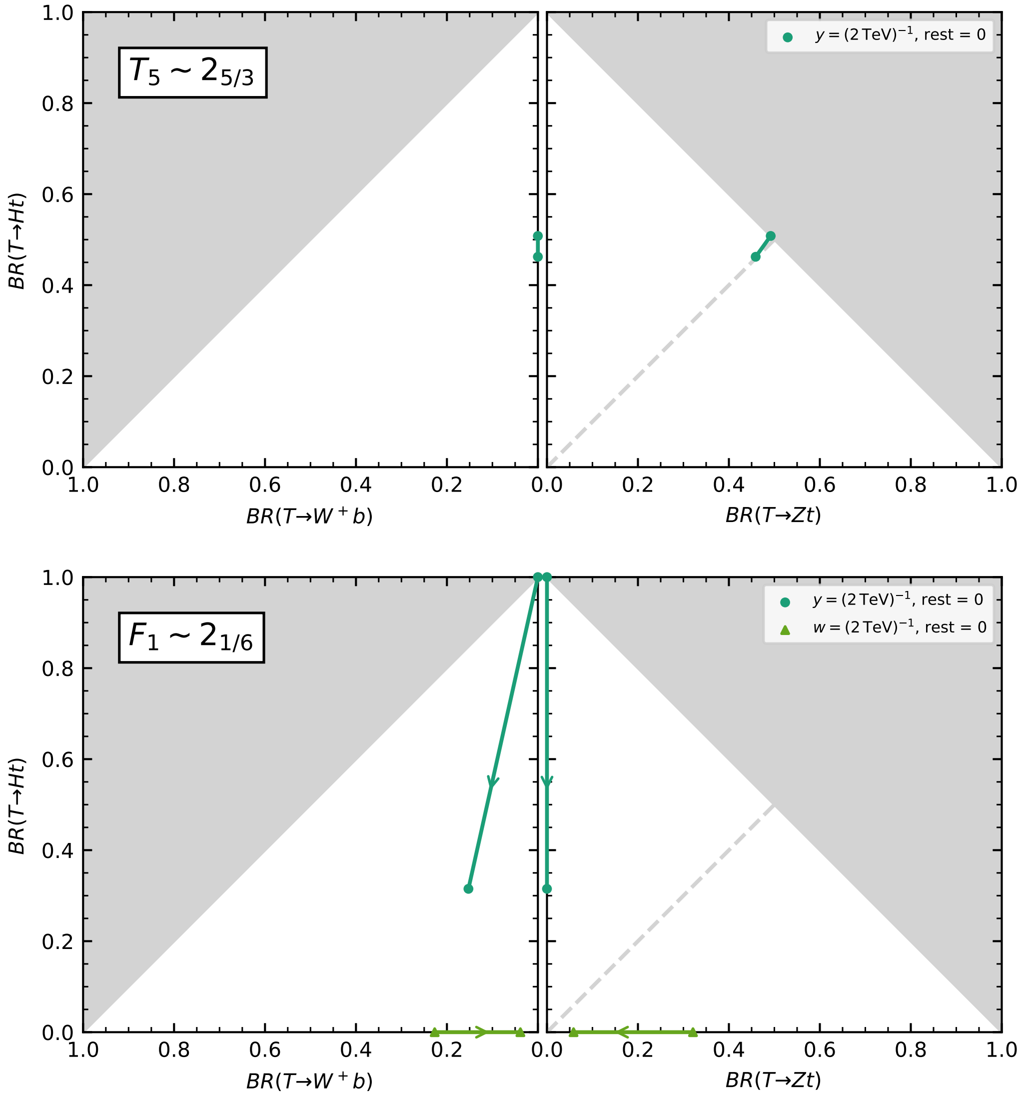

Figure 9.14: Branching ratios of T into Ht, Zt and W^+b for various values of the parameters in the T_5 and F_7 models.

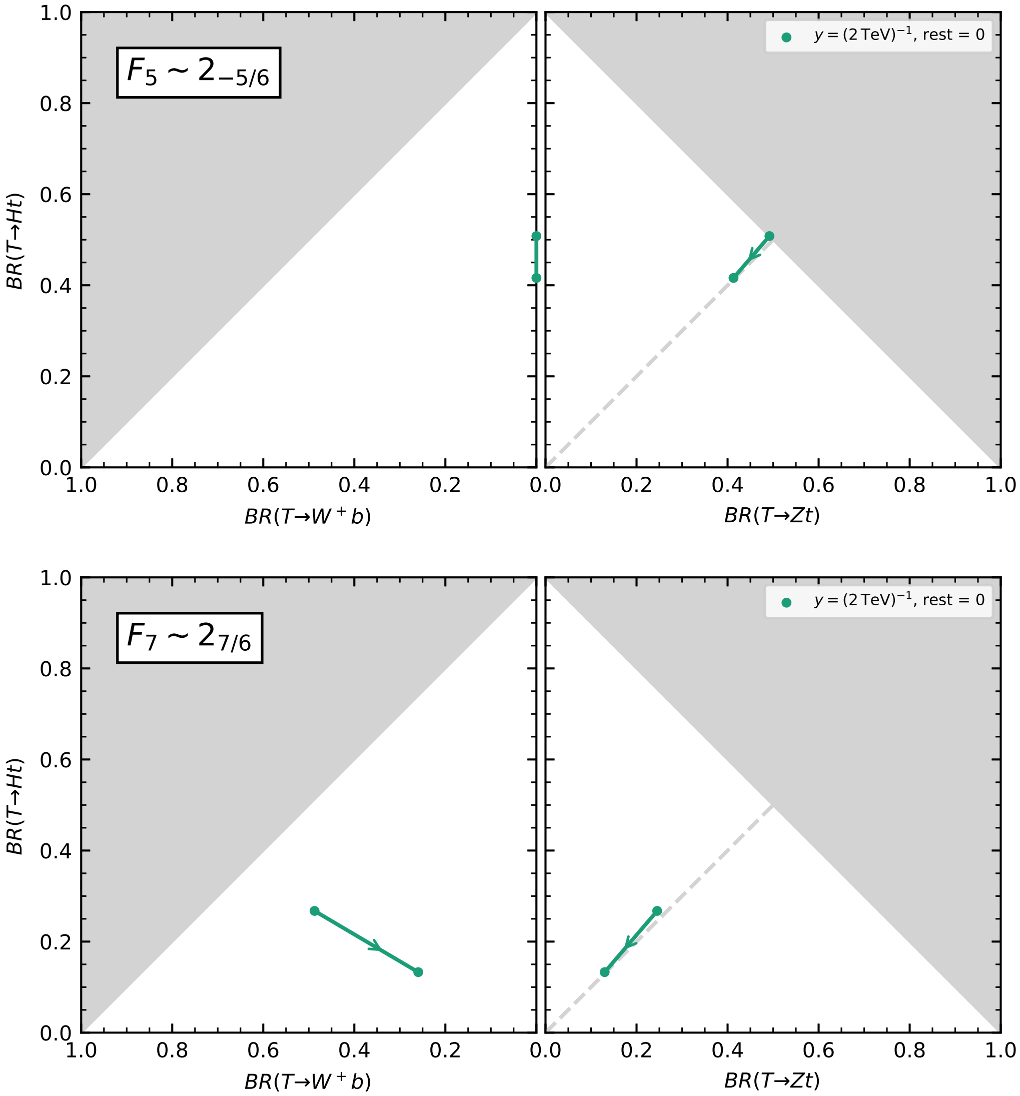

Figure 9.15: Branching ratios of T into Ht, Zt and W^+b for various values of the parameters in the F_1 and F_5 models.

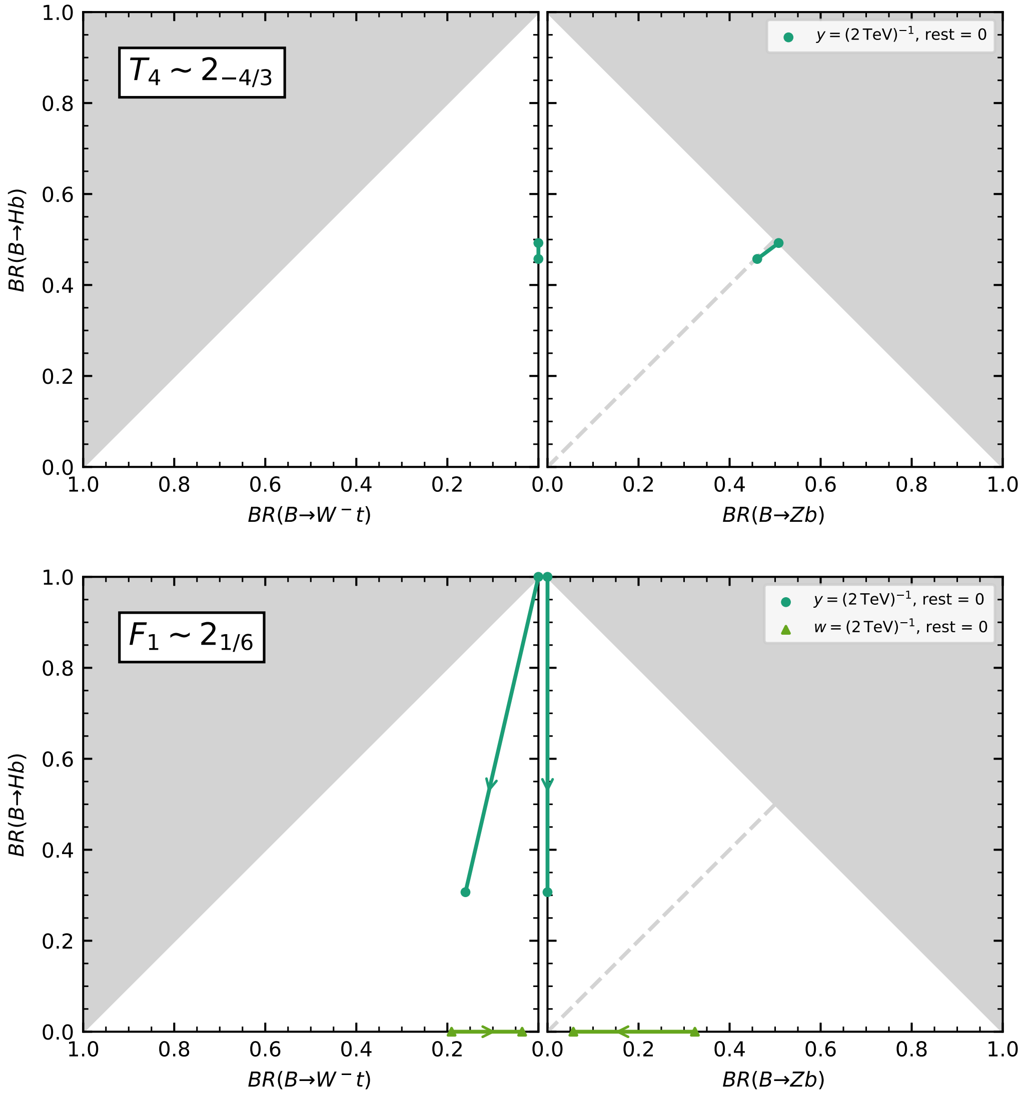

Figure 9.16: Branching ratios of B into Hb, Zb and W^-t for various values of the parameters in the T_4 and F_7 models.

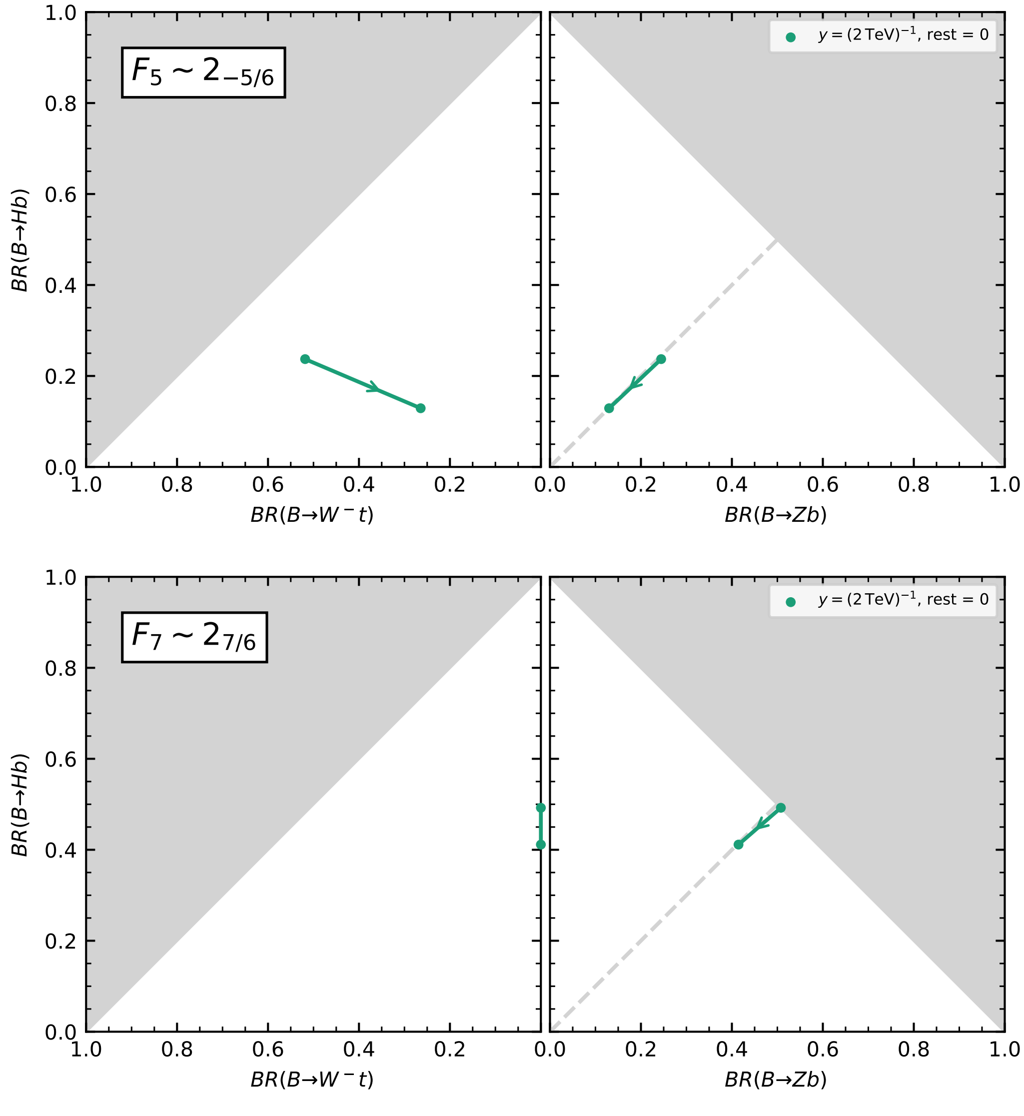

Figure 9.17: Branching ratios of T into Ht, Zt and W^-t for various values of the parameters in the F_1 and F_5 models.

9.7 Conclusions

We have used the BSMEFT to study the phenomenology of vector-like extra quarks near the TeV scale, which is to a large extent governed by gauge invariance and power counting. To start with, extra quarks can always be pair produced at hadron colliders by their gauge coupling to gluons. Once produced, they will decay into SM particles if they have gauge-invariant linear interactions with them.

At the renormalizable level, this is only possible for seven different gauge-covariant multiplets. These are the multiplets that can have Yukawa couplings with the Higgs doublet, which mix the extra quarks among themselves and with the SM ones. The latter mixing gives rise to decays into a SM quark and either Z, W or Higgs bosons. In simple extensions with only one vector-like multiplet, these are the only significant decay modes. Furthermore, in the motivated case of exclusive mixing with the third generation, the branching ratios are fixed by the quantum numbers of the multiplet. The mixing is also responsible for indirect effects, mass splittings and single production.

This simple picture can be modified in three ways (or combinations of them). First, one can consider general couplings to all the three SM generations [6, 284–287]. This typically requires flavour symmetries to evade the strong flavour constraints. Sizable mixing with the valence quarks in the proton would increase the importance of single production [286]. Second, it is possible to consider several vector-like quark multiplets, or other additional particles, like scalars or vector bosons. This may give rise to new production mechanisms [288] or new decay modes [264, 282, 289], in addition to the standard ones described above. Third, one can drop the assumption of renormalizability. This is the path we have explored in this chapter.

We have proposed a model-independent approach based on the BSMEFT, valid up to a cutoff scale Λ and constructed with the SM fields and the fields that represent arbitrary new vector-like quarks. This is a faithful description of any model with new vector-like quarks, as long as the new physics not explicitly included appears at scales higher than Λ. In particular, our EFT describes well the case of additional particles when they are heavier than Λ. As usual, the effective Lagrangian is defined by its expansion in inverse powers of Λ. The lowest order, formed by operators of canonical dimension ≤ 4, corresponds to the usual renormalizable theories with extra vector-like quarks. The interactions of higher dimension give contributions to observables suppressed by powers of μ/Λ, with $\mu = E, M, v$ the characteristic scale of the process. Even if suppressed, these interactions can be very relevant for proceses that do not exist at the renormalizable level.

In our explicit phenomenological analysis we have worked with the sectors of the BSMEFT with only one vector-like quark multiplet and we have truncated it at the next-to-leading order, i.e. at canonical dimension 5. For simplicity we have also assumed couplings to the third generation only. At this order, there are twelve irreducible representations of extra quarks that can decay into SM particles (and be singly produced). Up to field redefinition ambiguities, four new types of interactions appear at dimension 5: the Yukawa-type operators $\bar{Q}q\phi\phi$ and $\bar{Q}Q\phi\phi$ and the "magnetic" operators $\bar{Q}\sigma^{\mu\nu}qF_{\mu\nu}$ and $\bar{Q}\sigma_{\mu\nu}QF_{\mu\nu}$. The latter have effects that increase with the energy of the process.

We have distinguished two types of vector-like quarks. Those in the seven representations that allow for renormalizable linear interactions, and those in the remaining five representations. For the extra quark in the first group, and for natural values of the coupling constants, the dimension-5 interactions typically give only small corrections to the standard phenomenology of vector-like quarks. One exception is the possibility of new indirect effects in Higgs physics. Moreover, in strongly-coupled UV completions avoiding the loop suppression in the "magnetic" couplings, there can be new single production modes with cross sections larger than the one of pair production and also new decay modes (into qg, for instance) with large branching ratios. Of course, all these effects depend on the cutoff and will be negligible if Λ is much larger than the TeV scale.

For the quarks in the five multiplets that do not have renormalizable linear interactions (two triplets and four quadruplets), the dimension-5 operators give the leading contributions. In this case, all the indirect bounds can be easily evaded without explicit tuning of couplings, for moderate values of Λ. Pair production is still possible and the decay (possibly after hadronization) will be prompt if Λ is not too high. Some non-standard decay modes, including three-body decays, can be sizable and the measurements of decays of T and B into Zq, Wq and Hq could easily give rise to new points in the corresponding triangles. In this respect, we have given a simple formula to recast the combination limits given by the ATLAS and CMS collaborations, which assume the absence of other decay channels. For $\Lambda \gtrsim 10^6$ TeV, the decays of the hadrons containing the heavy quarks will be non-prompt. The usual searches will not be sensitive to vector-like quarks in this regime, but one can instead resort to the signatures associated to coloured and charged long-lived particles. Taking advantage of these signatures would require dedicated searches of vector-like quarks, specially in the case of displaced vertices formed by their decay products.

New operators involving the extra quarks appear at yet higher orders in the $1/\Lambda$ expansion. At dimension 6 one should include four fermion operators [290]. In particular, the interactions of the form $qqqQ$ will give rise to new single production mechanisms, which can have observable cross sections at the LHC for Λ of a few TeV when the couplings to the first generation are allowed. Moreover, at each order new types of vector-like quarks will be able to decay into SM particles. Their lifetime will be suppressed by the corresponding power of M/Λ. Finally, in principle it is also possible that new vector-like quarks exist in gauge representations with $T + Y + 1/3 \notin \mathbb{Z}$. They would be stable or else decay into additional stable particles. However, there are very strong constraints on the abundances of stable strongly interacting (and charged) particles, in particular from searches of rare nuclei [291–295]. [5]

[5] See, nevertheless, ref. [296] for comments on the robustness of such bounds and a proposal of coloured dark matter.

CHAPTER 10

Conclusions

In this work, we have reviewed and extended a model-independent framework for the study of physics beyond the SM, based on EFT. We have developed new computer tools that help automatizing the most common types of calculations in this context. We have contributed to setting the basis for the phenomenological analysis of new particles and their interactions through the introduction of the BSMEFT, an EFT that includes every possible new field under general assumptions.

In chapter 4, we have studied one of the most useful mathematical tools in the practical implementation of EFTs: field redefinitions. They allow for the reduction of the number of interactions considered in any EFT. A complete set of independent local operators that is minimal in the sense that it cannot be reduced using redefinitions is what is called a basis. Although the use of bases and redefinitions is convenient, some caution is needed in many of their applications. A common practice when working at leading order in the EFT expansion is to use the equations of motion of the fields, instead of redefinitions. They capture, in fact, the leading order effects of redefinitions. However, we have shown that this procedure cannot be extended to higher orders.

The results of many calculations in EFT, as matching or the computation of the renormalization group evolution, are usually presented in terms of an operator basis. The use of redefinitions (or equations of motion) to arrive to a basis generates a loss of information that cannot be recovered from the final result, unless all the field redefinitions that have been used are explicitly given. This has led us to the proposal of the definition and use of over-complete bases in intermediate steps of calculations. The idea is not to replace the usual bases, but to complement them. For example, given an over-complete basis and a reduced basis, it would be useful to do off-shell matching calculations using the former, and then have a dictionary to directly translate the results to the later.

In chapters 5 and 6, two Python packages have been presented: `MatchingTools` and `BasisGen`. They automatize the generation of bases of operators, tree-level matching and reduction to a basis. In the present situation in phenomenology of physics beyond the SM, in which one deals with EFTs with many fields and operators, it has become crucial to develop a reliable set of computer tools in which lengthy calculations can be implemented. The advantages are manifold: first, the use of computer tools almost always faster than doing calculations by hand; second, the possibility of the introduction of human errors is drastically reduced; in addition, repeated calculations with minor changes are more easily performed. Both `MatchingTools` and `BasisGen` contribute in advancing in this direction. The correctness of their results has been checked by hand and against the previous results in the literature. They have been developed paying attention to performance, specially in the case of `BasisGen`, which improves the speed of previous tools that do similar calculations by a factors of about a hundred.

In chapter 7, the formalism for general extensions of the SM with new fields has been introduced. We have constructed the BSMEFT, which includes all possible new particles that can decay into SM ones. The corresponding fields have gauge-invariant linear couplings to the SM. They are the most relevant ones for phenomenology in many situations: they have single production, decay and indirect effects at tree level. The generality of the BSMEFT makes it useful for parametrizing many new physics effects. It can be used to connect the Wilson coefficients of the SMEFT to the parameters of any high-energy model with tree-level contributions to them. Contrary to the SMEFT, the BSMEFT is able describe resonant production of new degrees of freedom. In the rest of the thesis, we consider some of its applications.

In chapter 8, the complete tree-level dictionary between the dimension-6 SMEFT and its high-energy completions has been provided. It has been computed by tree-level matching of the SMEFT to the BSMEFT. This dictionary is a useful result to explore both the low-energy consequences of high-energy models and the possible high-energy explanations of low-energy effects. For each combination of particles in some theory beyond the SM, one can find to which operators of the SMEFT they contribute at tree-level, as well as their contribution the corresponding Wilson coefficients. Then, constraints on the SMEFT coefficients can be used to put bounds on the parameters of the high-energy model. If some new physics effect is observed, implying that some Wilson coefficient is non-zero, one can find what possible new particles can be responsible, together with the high-energy interactions that are necessary to generate the effect.

In chapter 9, next-to-leading order effects in the BSMEFT have been studied, for the case of vector-like quarks. In general, the only extra fields with leading-order effects are those with dimension-4 linear couplings to the SM. One can relax the condition over the dimension of these operators to analyze the robustness of the approximation and to study new experimental signatures that may not be possible at the lowest order. We have focused on the phenomenology of vector-like quarks with dimension-4 and/or dimension-5 linear couplings. We have found that new production and decay channels appear at dimension 5. Those quarks whose linear interactions start at dimension 5 are less constrained in general by their indirect effects. However, they also generate new experimental signals due to the fact that their effective couplings are naturally small, because they are suppressed by inverse powers of the cutoff scale. In consequence, they may hadronize and produce delayed detector signals and displaced vertices.

CHAPTER 11

Conclusiones

En este trabajo, hemos revisado y extendido el marco independiente del modelo que proporcionan las EFTs para el estudio de física más allá del Modelo Estándar. Hemos desarrollado nuevas herramientas informáticas que ayudan en la automatización de los tipos más habituales de cálculos en este contexto. También hemos contribuido a sentar las bases del análisis fenomenológico de nuevas partículas e interacciones a través de la introducción de la BSMEFT, una EFT que incluye cada posible nuevo campo bajo asunciones generales.

En el capítulo 4, hemos estudiado una de las herramientas matemáticas más útiles en la implementación práctica de EFTs: las redefiniciones de campos. Estas permiten reducir considerablemente el número de interacciones a tener en cuenta en cualquier EFT. Un conjunto completo de operadores locales independientes tal que no se puede reducir usando redefiniciones es lo que se conoce como una base. Aunque el uso de bases y redefiniciones es conveniente, es necesario hacerlo con cierta precaución. Una práctica común cuando se trabaja a orden dominante es el uso de las ecuaciones de movimiento de los campos en lugar de redefiniciones. De hecho, las ecuaciones de movimiento capturan los efectos dominantes de las redefiniciones. Sin embargo, como hemos demostrado, este procedimiento no puede extenderse a órdenes superiores.

Los resultados de muchos cálculos en EFTs, como el *matching* o el cálculo de la evolución con el grupo de renormalización, se presentan normalmente en términos de una base de operadores. El uso de redefiniciones (o de ecuaciones de movimiento) para llegar a una base genera una pérdida de información que no puede recuperarse a partir el resultado final, a menos que todas las redefiniciones utilizadas se proporcionen explícitamente. Esto nos ha conducido a la propuesta de la definición y uso de bases redundantes en pasos intermedios de cálculos. La idea no es reemplazar la bases habituales, sino complementarlas. Por ejemplo, dada una base redundante una reducida, sería útil hacer cálculos de *matching off-shell* usando la primera, y luego tener un diccionario para traducir directamente los resultados a la segunda.

En los capítulos 5 y 6, se han presentado dos paquetes de Python: `MatchingTools` y `BasisGen`. Estos automatizan la generación de bases de operadores, la realización de *matching* a nivel árbol y la reducción de un Lagrangiano hasta escribirlo en términos de una base de operadores. En la situación actual en fenomenología más allá del SM, en la que hay que trabajar con EFTs con muchos campos y operadores, se ha convertido en una necesidad el desarrollo de un conjunto sólido de herramientas informáticas en las que implementar los largos cálculos que aparecen. Las ventajas son múltiples: primero, el uso de estas herramientas es prácticamente siempre más rápido que la realización de los cálculos a mano; segundo, la posibilidad de introducir errores humanos se reduce en gran medida; por último, la repetición de cálculos con cambios menores puede hacerse con mayor facilidad. `MatchingTools` y `BasisGen` contribuyen a avanzar en este dirección. La corrección de sus resultados se ha comprobado a mano y usando los resultados presentados en la literatura. Estas herramientas se han desarrollado prestando atención a la eficiencia, especialmente en el caso de `BasisGen`, que mejora la velocidad de herramientas anteriores que hacen cálculos similares por factores que llegan a alrededor de cien.

En el capítulo 7, se ha introducido el formalismo para extensiones generales del SM con nuevos campos. Hemos construido la BSMEFT, que incluye todas las posible nuevas partículas que pueden decaer a las del SM. Estas son las más relevantes para fenomenología en muchas situaciones: tienen producción simple, desintegración y efectos indirectos a nivel árbol. La generalidad de la BSMEFT la hace útil para parametrizar muchos efectos de nueva física. Se la puede usar para conectar los coeficientes de Wilson de la SMEFT con los parámetros de cualquier modelo de altas energías con contribuciones a estos a nivel árbol. Al contrario que la SMEFT, la BSMEFT puede describir producción resonante de nuevos grados de libertad. En el resto de esta tesis consideramos algunas de sus aplicaciones.

En el capítulo 8, se ha proporcionado el diccionario completo a nivel árbol entre la SMEFT de dimensión 6 y sus posible extensiones a altas energías. Este se ha calculado usando matching a nivel árbol entre la SMEFT y la BSMEFT. Este diccionario es útil para explorar las consecuencias a bajas energías de modelos para más altas energías, así como las posibles explicaciones a altas energías de efectos de bajas energías. Para cada combinación de partículas en cualquier teoría más allá del SM, se puede encontrar qué operadores de la SMEFT generan a nivel árbol, así como su contribución a los coeficientes de Wilson correspondientes. Entonces, las restricciones sobre coeficientes de la SMEFT pueden usarse para poner límites a los parámetros del modelo de altas energías. Si algún efecto de nueva física se observa, implicando que algún coeficiente de Wilson es distinto de cero, se puede encontrar qué nuevas partículas pueden ser responsables, junto con las interacciones de altas energías que son necesarias para generar el efecto.

En el capítulo 9, se estudian algunos efectos a órdenes subdominantes en la BSMEFT. En general, los únicos campos extra con efectos a orden dominante son aquellos con acoplamientos lineales de dimensión-4 al SM. Esta condición sobre la dimensión de los operadores puede relajarse para analizar la robustez de la aproximación y para estudiar nuevas características experimentales distintivas que pueden no ser posibles al orden más bajo. Nos hemos concentrado en la fenomenología de *quarks vector-like* con acoplamientos lineales de dimensión 4 y/o 5. Hemos encontrado que a dimensión 5 aparecen nuevos canales de producción y desintegración. Aquellos nuevos *quarks* cuyas interacciones lineales empiezan en dimensión 5 están menos restringidos a través de sus efectos indirectos. Sin embargo, generan nuevos efectos experimentales debido al hecho de que sus acoplamientos son naturalmente pequeños, ya que están suprimidos por potencias inversas de la escala de *cutoff*. En consecuencia, pueden hadronizar y producir señales retrasadas en el detector y vértices desplazados.

APPENDIX A

Standard Model group-theory notation

In this work we use a notation where color indices are labeled by capital letters, A, B, C, running over the dimensionality of the corresponding $SU(3)_c$ representation. Whenever possible, objects in the fundamental representations of $SU(2)_L$ and $SU(3)_c$ have been written as row or column vectors, with matrix products implied. The superscript symbol "T" indicates transposition of the $SU(2)_L$ indices exclusively. When showing these indices explicitly, we use the following different labels, depending on the $SU(2)_L$ representation: $\alpha, \beta = \frac{1}{2}, -\frac{1}{2}$ for $SU(2)_L$ doublets; $a, b, c = 1, 2, 3$ for the components of $SU(2)_L$ adjoints/triplets in Cartesian coordinates; and $I, J, K = \frac{3}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{3}{2}$ for the components of the $SU(2)_L$ quadruplets.

The symbols $T_A = \frac{1}{2}\lambda_A$ and f_{ABC}, $A, B, C = 1, \ldots, 8$, denote the $SU(3)_c$ generators and structure constants, respectively, with λ_A the Gell-Mann matrices. ϵ_{ABC} (ε_{abc}) , $A, B, C = 1, 2, 3$ $(a, b, c = 1, 2, 3)$ is the totally antisymmetric tensor in color (weak isospin) indices; σ_a or σ^a, $a = 1, 2, 3$ are the Pauli matrices; $\sigma_{\mu\nu} = \frac{i}{2}[\gamma_\mu, \gamma_\nu]$; and $\tilde{A}_{\mu\nu} = \frac{1}{2}\varepsilon_{\mu\nu\rho\sigma}A^{\rho\sigma}$ is the Hodge-dual of the field strength $A_{\mu\nu}$.

In the construction of the different $SU(2)_L$ invariants we also use the following:

- The isospin-1 product of two triplets is obtained through:

$$f_{abc} = \frac{i}{\sqrt{2}}\varepsilon_{abc}.$$

- Quadruplets are obtained from the product of an isospin-1 field and a doublet by means of

$$C^{3/2}_{a\beta} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 0 \\ -i & 0 \\ 0 & 0 \end{pmatrix}, \qquad C^{1/2}_{a\beta} = \frac{1}{\sqrt{6}}\begin{pmatrix} 0 & 1 \\ 0 & -i \\ -2 & 0 \end{pmatrix},$$

$$C^{-1/2}_{a\beta} = -\frac{1}{\sqrt{6}}\begin{pmatrix} 1 & 0 \\ i & 0 \\ 0 & 2 \end{pmatrix}, \quad C^{-3/2}_{a\beta} = -\frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 1 \\ 0 & i \\ 0 & 0 \end{pmatrix}.$$

- The singlet product of two quadruplets is obtained through the $SU(2)$ product

$$\epsilon_{IJ} = \frac{1}{2}\begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}.$$

Finally, for $SU(3)_c$ indices, we use the following notation for the symmetric product of colored fields:

$$\psi_1^{(A|} \ldots \psi_2^{|B)} \equiv \frac{1}{2}\left(\psi_1^A \ldots \psi_2^B + \psi_1^B \ldots \psi_2^A\right).$$

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