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Expanding Grey Relational Analysis With the Comparable Degree for Dual Probabilistic Multiplicative Linguistic Term Sets and Its Application on the Cloud Enterprise

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ABSTRACT Under the cloud trend of enterprises, how do traditional businesses get on the cloud becomes a worth pondering question. To help those traditional businesses that have no experience to dispel the clouds and see the sun as soon as possible, we are planning to choose one corporation with rich experience to take them into the cloud market. The quintessence of dual probabilistic linguistic term sets (DPLTSs) is that it uses the combination of several linguistic terms and their proportions to reveal decision information by opposite angles. This paper proposes the dual probabilistic multiplicative linguistic preference relations (DPMLPRs) based upon the dual probabilistic multiplicative linguistic term sets (DPMLTSs). Then, it defines the comparable degree between the DPMLPRs and studies the consensus of the group DPMLPR. Moreover, it probes the expanding grey relational analysis (EGRA) under the proposed comparable degree between the DPMLTSs. After that, one example of choosing the experienced cloud cooperative partner is simulated under the dual probabilistic linguistic circumstance. Besides, the comparative analysis is performed by considering the similarity among the EGRA, TODIM, and VIKOR.

INDEX TERMS Dual probabilistic multiplicative linguistic preference relations, comparable degree, consensus, expanding grey relational analysis, multi-criteria decision making.

I. INTRODUCTION

Just like domino effect, since cloud computing [1] was first proposed by Eric Schmidt in 2006, the market for cloud computing is booming. Its research has been gotten a lot of attention from experts in different fields, such as internet of things [2]–[4], cloud storage [5], [6], cloud security [7], [8], cloud education [9], [10] and so on. The essence of cloud computing is to provide services through the network, so its architecture is centered on services, and its objective is to offer customers with faster and more convenient information services.

The currently acknowledged traits of cloud computing can be summarized as follows: (1) Supersize dimension, such as Amazon, IBM, Microsoft and Yahoo, each has hundreds of thousands of servers, "Cloud" is able to offer consumers unheard-of calculating strength; (2) Virtualization, cloud computing permits consumers to make use of application services from facultative situation utilizing all kinds of terminals. The desired resource is derived from the "Cloud" rather than an established concrete existence. The app operates someplace in the "Cloud". However, as a matter of fact, the consumers are not necessary to learn about or concern

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about where the app is operating. With just one laptop or one mobile phone, you are able to do everything we need through web services, even tasks like supercomputing. (3) Dynamic extendibility, the dimension of the cloud can be vibrantly scaled to fulfill the demands of adhibition and consumers scale growth. (4) High reliability, ''Cloud'' uses measures such as the fault tolerance for multiple copies of data and computational node isomorphism to ensure high reliability of services. Cloud computing is more responsible than utilizing local computers. (5) Commonality, cloud computing is not targeted at particular applications. With the help of ''Cloud'', it can structure protean applications. The identical ''Cloud'' can encourage diverse application operations in the mean time. (6) Service on demand, ''Cloud'' is a large resource pool that you are able to purchase according to the requirement, and clouds are able to be charged like water, electricity or gas. (7) Low cost and green energy saving. Because the particular fault-tolerant measures of ''Cloud'' can utilize rare cheap nodes to constitute a cloud, the cloud's automated centralized management eliminates the need for big business to afford cumulatively advanced data center management costs, and the versatility of ''Cloud'' enables the exploitation rate of resources much higher than the conventional system. Moreover, consumers are able to thoroughly enjoy the low-cost benefit of ''Cloud''.

Therefore, many traditional businesses begin to transform the cloud computing industry. However, majority of them do not have the relative experience, it is full of hazard for them to join in the cloud market. So it is a good choice for them to look for a good partner that with the rich experience to get twofold results with half the effort. As far as it goes, the world's four largest cloud computing companies are Amazon Web Services (AWS), Microsoft, Google and Alibaba Cloud. According to their own features, choosing one to collaborate with the four companies is the short cut for those traditional businesses that want to transform in the demand explosion period of cloud industry.

How to determine the selected company becomes the question that we will solve in this paper. The DPLTSs [11] enlarges probabilistic linguistic term sets (PLTSs)' [12] quintessence that uses the combination of several linguistic terms and their proportions to reveal decision information into the membership sentiment and non-membership sentiment. We extend it into the multiplicative linguistic scale [13] and define the dual probabilistic multiplicative linguistic term sets (DPMLTSs). Then we propose the notion of dual probabilistic multiplicative linguistic preference relations (DPMLPRs), and use the DPMLPRs as the implement to do the decision.

As most of the studies on the preference relations (PRs) [14]–[20], the consistency [21]–[26] is the common and essential condition for applying the PRs into the material decision. Different from the majority of researchers [27], [28], this paper defines the comparable degree between the DPMLPRs and utilizes it as the measure to judge the consistency of the DPMLPRs. The reason why we use the comparable degree is that the intrinsic quality between the comparable degree [29]–[31] and the distance measure [32], [33] is same. Moreover, because of the structure of the operator itself, the computation of the comparable degree is also separated into two angles: the membership viewpoint and the non-membership viewpoint.

After acquiring the consistent DPMLPRs, on account of the defined dual probabilistic linguistic weighted geometric aggregation operator (DPLWGA), we can obtain the group DPMLPR. Then on the foundation of the established comparable degree between the individual DPMLPRs and the group DPMLPR, the group consensus [34]–[38] can be checked directly. Moreover, if the consensus cannot be satisfied in the decision-making procedure, then the decision makers (DMs) need to adjust their PRs, until the consensus is satisfied in the end, and the checking is over.

The crucial intention of decision-making is to judge the sort of the alternatives. For the multi-criteria decision-making, the research for weights has been done a lot [39]–[42]. Most of them are divided into the following types: partially known [43]–[45], fully known [46], [47], total unknown [48]–[51]. The weights of criteria in this paper is belong to the third type that is total unknown. On the foundation of classic arithmetic averaging method [52], this paper considers the structural characteristics of DPMLTSs and designs the modified arithmetic averaging method to calculate the weights for criteria. After that, the grey relational analysis (GRA) [53] as one of the more common multi-criteria decision-making method, its superiority lies in that it does not require much of the quantity involved in the decision-making. Moreover, it does not require that the quantities to be determined conform to a typical distribution. The amount of calculation is relatively small, and the results agree well with the qualitative analysis. So the GRA has been expanded in this paper by merging with the proposed comparable degree to calculate the relational coefficient. The GRA based upon the comparable degree is named as expanding GRA (EGRA). Together with the weights of the criteria, the final priority of the alternatives is able to be procured at length.

Furthermore, we apply the proposed procedure to the case mentioned above and to help to determine the selected cooperative partner. Besides, given that the similar principle among the GRA, TODIM [54] and VIKOR [55] that studies the comparable degree between the alternative and ideal alternative, we also expand the TODIM, VIKOR into the expanding TODIM (ETODIM), expanding VIKOR (EVIKOR). Then we compare the EGRA, ETODIM and EVIKOR in the comparative analysis section, and show their several advantages and disadvantages.

In a word, the innovation points of the whole paper can be listed as follows: (1) Define the DPMLPRs; (2) Denote the comparable degree between the individual DPMLPRs; (3) Study the consistency of the individual DPMLPRs; (4) Research the consensus of group DPMLPR; (5) Propose the EGRA method based on the defined comparable degree between the DPMLTSs; (6) Expand the TODIM and VIKOR methods.

The remaining of this paper is structured as follows: Section II lists some necessary notions. Section III defines the DPMLTSs, the basic operations among the DPMLTSs, the comparable degree between the individual DPMLPRs, and study the consistency, consensus of the DPMLPRs. Section IV computes the weights of criteria, introduces the EGRA method, and the integrated multi-criteria decision-making procedure. Section V utilizes a simulation case relevant to the cloud computing industry to clarify the potential and reality of the dual probabilistic multiplicative linguistic multi-criteria group decision-making procedure. Section VI ends with some conclusions.

II. PRELIMINARIES

In this section, we will briefly recall some essential concepts, such as the linguistic terms, the dual probabilistic linguistic term set (DPLTS) and the normalized dual probabilistic linguistic term element (NDPLTE).

A. THE LINGUISTIC TERM SETS

Let $S = \left\{s_\alpha \left| \alpha \in \left[1/q, q\right]\right.\right\}$ be a continuous multiplicative linguistic label set, and q is a adequately large positive integer [13]. Moreover, if $\alpha > \beta$, then $s_\alpha > s_\beta$; if $rec(s_\alpha) = s_\beta$, then $\alpha\beta = 1$; peculiarly, $rec(s_1) = s_1$. Based on the multiplicative linguistic label set S, Xu [13] introduced some basic operational laws for them as follows:

$$\begin{aligned}(s_\alpha)^\mu &= s_{\alpha^\mu}, \mu \in [0,1];\\ s_\alpha \otimes s_\beta &= \max\left\{s_{1/q}, \min\left\{s_{\alpha\beta}, s_q\right\}\right\};\\ s_\alpha \oplus s_\beta &= \max\left\{s_{1/q}, \min\left\{s_{\alpha+\beta}, s_q\right\}\right\}.\end{aligned}$$

B. THE DPLTS

Let X be a fixed set, a DPLTS on X can be signified into the coming type [11]:

$$D = \{\langle x, \wp(p), \Upsilon(p)\rangle, x \in X\} \tag{1}$$

where

$$\wp(p) = \left\{\wp^{(i)}\left(p^{(i)}\right)\left|\wp^{(i)} \in S_1, p^{(i)} \geq 0, \sum_{i=1}^{\#\wp(p)} p^{(i)} \leq 1\right.\right\},$$
$$\Upsilon(p) = \left\{\Upsilon^{(j)}\left(p^{(j)}\right)\left|\Upsilon^{(j)} \in S_1, p^{(j)} \geq 0, \sum_{j=1}^{\#\Upsilon(p)} p^{(j)} \leq 1\right.\right\}.$$

$\wp(p)$ and $\Upsilon(p)$ stand for the conceivable membership and non-membership degrees to the element $x \in X$ for the set D with the conditions that $s_{-q} \leq \wp^+ \oplus \Upsilon^+ \leq s_q$, $s_{-q} \leq \wp^- \oplus \Upsilon^- \leq s_q$, $S_1 = \{s_\alpha |\alpha \in [-q, q]\}$. In addition to that, we call the pair $D = \langle \wp(p), \Upsilon(p)\rangle$ the dual probabilistic linguistic element (DPLTE).

Moreover, in the cause of reducing the trouble of the computation, Xie *et al.* [11] further designed the coming procedure to normalize the DPLTEs (NDPLTEs) as follows:

Assume that $D_1 = \langle \wp_1(p), \Upsilon_1(p)\rangle$ and $D_2 = \langle \wp_2(p), \Upsilon_2(p)\rangle$ are two unlike DPLTEs. For the first step, similar to earn the NPLTSs, there is a need to avoid the deviations in the cardinalities of the two PLTSs $\wp_1(p)$ and $\wp_2(p)$, and to score the PLTSs $\wp_1(p)$ and $\wp_2(p)$ with the identical cardinal numbers: $\#\wp_1(p) = \#\wp_2(p)$. For the second step, we need to replume the PLTSs $\wp_1(p)$ and $\wp_2(p)$ separately in the downward sort. Likewise, the PLTSs $\Upsilon_1(p)$ and $\Upsilon_2(p)$ also need to be treated with the same way. Then we can obtain two new DPLTEs $D_1' = \left\langle\wp_1'(p), \Upsilon_1'(p)\right\rangle$, $D_2' = \left\langle\wp_2'(p), \Upsilon_2'(p)\right\rangle$, where

$$\wp'_l(p) = \left\{\wp'^{(i)}_l\left(p'_{l^{(i)}}\right)\left|\wp'^{(i)}_l \in S, p'^{(i)}_l \geq 0, \sum_{i=1}^{\#\wp'_l(p)} p'^{(i)}_l \leq 1\right.\right\}$$

and

$$\Upsilon'_l(p) = \left\{\Upsilon'^{(j)\left(p'^{(j)}_l\right)}_l\left|\Upsilon'^{(j)}_l \in S, p'^{(j)}_l \geq 0, \sum_{j=1}^{\#\Upsilon'_l(p)} p'^{(j)}_l \leq 1\right.\right\}$$

are revealed in falling sort, $\#\wp_1'(p) = \#\wp_2'(p)$, $\#\Upsilon_1'(p) = \#\Upsilon_2'(p)$, $l = 1, 2$.

Moreover, we offer the definition of score function and accuracy function [11] to compare the different DPLTEs as follows:

For a DPLTE $D = \langle\wp(p), \Upsilon(p)\rangle$, it's score function is:

$$S(D) = s_{\bar{\alpha}-\bar{\beta}} \tag{2}$$

where $\bar{\alpha} = \sum_{i=1}^{\#\wp(p)} I\left(\wp^{(i)}\right)p^{(i)} / \sum_{i=1}^{\#\wp(p)} p^{(i)}$, $\bar{\beta} = \sum_{j=1}^{\#\Upsilon(p)} I\left(\Upsilon^{(j)}\right)r^{(j)}p^{(j)} / \sum_{j=1}^{\#\Upsilon(p)} p^{(j)}$ and $I(\cdot)$ is the function that can obtain the subscript of the corresponding linguistic term.

With regard to two DPLTEs $D_l\,(l = 1, 2)$, if $S(D_1) > S(D_2)$, then D_1 is superior to D_2, denoted by $D_1 \succ D_2$; if $S(D_1) < S(D_2)$, then D_1 is inferior to D_2, denoted by $D_1 \prec D_2$. If $S(D_1) = S(D_2)$, it is tight to tell from two DPLTEs. Thus, we state the accuracy function for the DPLTE as follows:

For a DPLTE $D = \langle\wp(p), \Upsilon(p)\rangle$, it's accuracy function can be ruled as:

$$\begin{aligned}A(D) &= \left(\sum_{i=1}^{\#\wp(p)}\left(p^{(i)}\left(I\left(\wp^{(i)}\right) - \bar{\alpha}\right)\right)^2\right)^{1/2} / \sum_{i=1}^{\#\wp(p)} p^{(i)}\\ &\quad + \left(\sum_{j=1}^{\#\Upsilon(p)}\left(p^{(j)}\left(I\left(\Upsilon^{(j)}\right) - \bar{\beta}\right)\right)^2\right)^{1/2} / \sum_{j=1}^{\#\Upsilon(p)} p^{(j)}\end{aligned} \tag{3}$$

Hence, with regard to two DPLTEs $D_l\,(l = 1, 2)$ with $S(D_1) = S(D_2)$, if $A(D_1) < A(D_2)$, then $D_1 \succ D_2$; if $A(D_1) > A(D_2)$, then $D_1 \prec D_2$; if $A(D_1) = A(D_2)$, then $D_1 \sim D_2$.

III. THE DUAL PROBABILISTIC MULTIPLICATIVE LINGUISTIC TERM SETS

Considering the multiplicative linguistic label set [13] and the defined DPLTS together, next we extend the DPLTS into the environment of multiplicative linguistic label set, and study the basic operations in the following section.

A. THE DPMLTS

Let X be a fixed set, a DPMLTS on X can be shown as the following style:

$$D = \{\langle x, \wp(p), \Upsilon(p)\rangle, x \in X\} \tag{4}$$

where

$$\wp(p) = \left\{ \wp^{(i)}\left(p^{(i)}\right) \middle| \wp^{(i)} \in S, p^{(i)} \geq 0, \sum_{i=1}^{\#\wp(p)} p^{(i)} \leq 1 \right\},$$

$$\Upsilon(p) = \left\{ \Upsilon^{(j)}\left(p^{(j)}\right) \middle| \Upsilon^{(j)} \in S, p^{(j)} \geq 0, \sum_{j=1}^{\#\Upsilon(p)} p^{(j)} \leq 1 \right\}.$$

$\wp(p)$ and $\Upsilon(p)$ stand for the conceivable membership and non-membership degrees to the element $x \in X$ for the set D with the situations that $s_{1/q} \leq \wp^{+} \otimes \Upsilon^{+} \leq s_q$, $s_{1/q} \leq \wp^{-} \otimes \Upsilon^{-} \leq s_q$. Additionally, we call the pair $D = \langle \wp(p), \Upsilon(p)\rangle$ the dual multiplicative probabilistic linguistic element (DPMLTE).

Then on behalf of better applying the DPMLTEs in to the practical case, we regulate the essential operation for the DPMLTEs as follows:

For two DPMLTEs $D_1 = \langle \wp_1(p), \Upsilon_1(p)\rangle$ and $D_2 = \langle \wp_2(p), \Upsilon_2(p)\rangle$, then the multiplicative operation is

$$\begin{aligned} D_1 \otimes D_2 &= \langle \wp_1(p), \Upsilon_1(p)\rangle \otimes \langle \wp_2(p), \Upsilon_2(p)\rangle \\ &= \langle \wp_1(p) \otimes \wp_2(p), \Upsilon_1(p) \otimes \Upsilon_2(p)\rangle \end{aligned} \tag{5}$$

Based on the Ref. [3], where

$$\wp_1(p) \otimes \wp_2(p) = \bigcup\nolimits_{\wp_1^{(i_1)} \in \wp_1(p), \wp_2^{(i_2)} \in \wp_2(p)} \left\{ \left(\wp_1^{(i_1)} \otimes \wp_2^{(i_2)}\right) \left(p_1^{(i_1)} p_2^{(i_2)}\right) | i_1 = 1, 2, \ldots, \#\wp_1(p), i_2 = 1, 2, \ldots, \#\wp_2(p) \right\}$$

$$\Upsilon_1(p) \otimes \Upsilon_2(p) = \bigcup\nolimits_{\Upsilon_1^{(j_1)} \in \Upsilon_1(p), \Upsilon_2^{(j_2)} \in \Upsilon_2(p)} \left\{ \left(\Upsilon_1^{(j_1)} \otimes \Upsilon_2^{(j_2)}\right) \left(p_1^{(j_1)} p_2^{(j_2)}\right) | j_1 = 1, 2, \ldots, \#\Upsilon_1(p), j_2 = 1, 2, \ldots, \#\Upsilon_2(p) \right\}$$

The power operation is

$$(D_1)^{\lambda} = \langle \wp_1(p), \Upsilon_1(p)\rangle^{\lambda} = \left\langle (\wp_1(p))^{\lambda}, (\Upsilon_1(p))^{\lambda}\right\rangle \tag{6}$$

where

$$(\wp_1(p))^{\lambda} = \cup_{\wp_1^{(i_1)} \in \wp_1(p)} \times \left\{ \left(\wp_1^{(i_1)}\right)^{\lambda} \left(p_1^{(i_1)}\right)^{\lambda} | i_1 = 1, 2, \ldots, \#\wp_1(p) \right\}.$$

Then let $D_1, D_2, \ldots, D_n$ be a set of DPMLTEs, then the dual probabilistic multiplicative linguistic weighted geometric aggregated (DPMLWGA) operator can be expressed as:

$$DPMLWGA(D_1, D_2, \ldots, D_n) = \left\langle \underset{i=1}{\overset{n}{\otimes}} (\wp_i(p))^{\omega_i}, \underset{i=1}{\overset{n}{\otimes}} \left(\Upsilon_j(p)\right)^{\omega_i} \right\rangle \tag{7}$$

where $\omega = (\omega_1, \omega_2, \ldots, \omega_n)^T$ is the weight vector with respect to the DPMLTEs, and fulfills $\omega_i \in [0, 1]$ and $\sum_{i=1}^{n} \omega_i = 1$.

B. THE DPMLPR

In the cause of applying the DPMLTSs to the decision-making procedure, in the following, we define the dual probabilistic multiplicative linguistic preference relation (DPMLPR) as follows:

A DPMLPR on the mentioned set $S = \left\{ s_\alpha \middle| \alpha \in \left[1/q, q\right] \right\}$ is defined as the matrix $D = \left(d_{ij}\right)_{n \times n}$, $d_{ij} = \left\langle \wp_{ij}(p), \Upsilon_{ij}(p)\right\rangle$, which meets

$$\wp_{ij}(p) = \Upsilon_{ji}(p), \quad \Upsilon_{ij}(p) = \wp_{ji}(p) \tag{8}$$

$i \neq j$, for $i, j = 1, 2, \ldots, n$.
Moreover, if $i = j$, then

$$\wp_{ii}(p) = \Upsilon_{ii}(p) = \langle \{s_1(1)\}, \{s_1(1)\}\rangle.$$

It is common knowledge that the consistency of PRs is the essential requirement for logical decision-making. So it is no exception to study the consistency of the defined DPMLPRs.

For a DPMLPR $D = \left\langle \wp_{ij}(p), \Upsilon_{ij}(p)\right\rangle$, if D is consistent, then it should satisfy the following conditions: for $\forall i, k, j = 1, 2, \ldots, n$,

$$\begin{cases} \wp_{ij}(p) = \wp_{ik}(p) \otimes \wp_{kj}(p) \\ \Upsilon_{ij}(p) = \Upsilon_{ik}(p) \otimes \Upsilon_{kj}(p) \end{cases} \tag{9}$$

which means that the DPMLPR D is consistent if and only if the membership PR $\wp = \wp_{ij}(p)$ and the non-membership $\Upsilon = \Upsilon_{ij}(p)$ are consistent at the same time.

For the single membership PR $\wp = \wp_{ij}(p)$, its consistent PR $C\wp = C\wp_{ij}(p)$, where

$$C\wp_{ij}(p) = \sqrt[n]{\underset{k=1}{\overset{n}{\otimes}} \left[\wp_{ik}(p) \otimes \wp_{kj}(p)\right]}.$$

By learning from the Ref. [56], its consistency index can be calculated as follows:

$$CI_{\wp} = 1 - \sum_{i,j=1, i<j}^{n} \frac{2}{(n-1)(n-2)} \left| \log e_{\wp_{ij}} - \log e_{C\wp_{ij}} \right| \tag{10}$$

where for a DPMLTE $D = \langle \wp(p), \Upsilon(p)\rangle$, the expected value of the DPMLTE is

$$E_D = \left\langle \sum_{i=1}^{\#\wp(p)} p^{(i)} I\left(\wp^{(i)}\right), \sum_{j=1}^{\#\Upsilon(p)} p^{(j)} I\left(\Upsilon^{(j)}\right) \right\rangle = \left\langle e_{\wp(p)}, e_{\Upsilon(p)}\right\rangle \tag{11}$$

Example 1: For one DPMLTE $D = \langle \{s_{1/2}(0.4), s_3(0.6)\}, \{s_2(0.3), s_1(0.5)\} \rangle$ on the certain linguistic term set $S = \{s_\alpha | \alpha \in [1/9, 9]\}$, then expected value of the DPMLTE is

$$E_D = \left\langle \sum_{i=1}^{\#\wp(p)} p^{(i)} I\left(\wp^{(i)}\right), \sum_{j=1}^{\#\Upsilon(p)} p^{(j)} I\left(\Upsilon^{(j)}\right) \right\rangle = \langle 2.0, 1.1 \rangle .$$

Then for the DPMLPR D, its consistency index can be computed as below:

$$CI = 1 - \sum_{i,j=1, i<j}^{n} \frac{1}{(n-1)(n-2)} \times \left(\left|\log e_{\wp_{ij}} - \log e_{C\wp_{ij}}\right| + \left|\log e_{\Upsilon_{ij}} - \log e_{C\Upsilon_{ij}}\right| \right) \quad (12)$$

Moreover, the consistency procedure can be expressed as the Algorithm 1:

Algorithm 1 The Procedure to Adjust the Consistency

Step 1. Set the threshold value for the consistency index Ξ, and calculate the respective consistency index CI_i $(i = 1, 2, \ldots, n)$ for the DPMLPRs;

Step 2. Judge the consistency of the DPMLPRs, if $CI_i > \Xi$, then go to **Step 4**; Otherwise, go to the next step.

Step 3. Modify the elements of the DPMLPRs according to the following method:

$$\begin{cases} \wp'_{ij}(p) = (\wp_{ij}(p))^{\theta} \otimes (C\wp_{ij}(p))^{1-\theta} \\ \Upsilon'_{ij}(p) = (\Upsilon_{ij}(p))^{\theta} \otimes (C\Upsilon_{ij}(p))^{1-\theta} \end{cases} \quad (13)$$

where $\theta \in [0, 1]$ is a regulation parameter.

Step 4. Let $D' = D$, then go back to **Step 1**.

C. THE COMPATIBILITY DEGREE FOR DPMLPRS

For the obtained consistent DPMLPRs, we are devote to study the consensus of the group DPMLPRs. Usually, people like to choose the distance measure [32], [33] or the similarity measure [57]–[59] as the foundation to analyze the consensus of the group PR. In this paper, with an eye to the similar practical meaning among the distance measure, similarity measure and comparable degree, we utilize the comparable degree between the DPMLPRs as the foundation to research the consensus of group PR in the following subsection:

Before introducing the comparable degree between the DPMLPRs, we first give the notion of comparable degree for two DPMLTEs. For any two DPMLTEs $D_1 = \langle \wp_1(p), \Upsilon_1(p) \rangle$ and $D_2 = \langle \wp_2(p), \Upsilon_2(p) \rangle$, the comparable degree between two DPMLTEs can be calculated as follows:

$$C(D_1, D_2) = \frac{1}{2} \left(\left|\log e_1^{\wp} - \log e_2^{\wp}\right| + \left|\log e_1^{\Upsilon} - \log e_2^{\Upsilon}\right| \right) \quad (14)$$

Furthermore, for two different DPMLPRs $D_1 = \left(d_{ij}^1\right)_{n\times n} = \left(\left\langle \wp_{ij}^1(p), \Upsilon_{ij}^1(p) \right\rangle\right)_{n\times n}$ and $D_2 = \left(d_{ij}^2\right)_{n\times n} = \left(\left\langle \wp_{ij}^2(p), \Upsilon_{ij}^2(p) \right\rangle\right)_{n\times n}$, the comparable degree of D_1 and D_2 can be defined as:

$$C(D_1, D_2) = \frac{1}{n(n-1)} \left[\frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} \left(\begin{array}{l} \left|\log e_{ij1}^{\wp} - \log e_{ij2}^{\wp}\right| + \\ \left|\log e_{ij1}^{\Upsilon} - \log e_{ij2}^{\Upsilon}\right| \end{array} \right) \right] \quad (15)$$

where $E_{D_1} = \left\langle e_{ij1}^{\wp}, e_{ij1}^{\Upsilon} \right\rangle$ and $E_{D_2} = \left\langle e_{ij2}^{\wp}, e_{ij2}^{\Upsilon} \right\rangle$ are the homologous expected value of the different DPMLPRs D_1 and D_2, respectively.

Moreover, if $e_{ij1}^{\wp} = e_{ij2}^{\wp}, e_{ij1}^{\Upsilon} = e_{ij2}^{\Upsilon}, i, j = 1, 2, \ldots, n$, than we call DPMLPRs D_1 and D_2 are perfectly compatible.

Theorem 1: For two DPMLPRs D_1 and D_2, then

(a) $C(D_1, D_2) \geq 0$;

(b) $C(D_1, D_2) = C(D_2, D_1)$;

(c) $C(D_1, D_2) = 0$, if D_1 and D_2 are perfectly compatible.

It is easy to see that Eqs. (a-c) are apparent. Therefore, the proof is omitted.

Theorem 2: For three different DPMLPRs D_1, D_2 and D_3, we have

$$C(D_1, D_3) \leq C(D_1, D_2) + C(D_2, D_3)$$

Proof: $C(D_1, D_3)$, as shown at the top of the next page.

Definition 1: For two DPMLPRs D_1 and D_2, if

$$C(D_1, D_2) \leq \delta \quad (16)$$

then we call that D_1 and D_2 are of acceptable compatibility, where δ is the threshold value of acceptable compatibility.

For a set of DPMLPRs $D_1, D_2, \ldots, D_n$, the group DPMLPR D can be expressed as the following form, D, as shown at the top of the next page.

Theorem 3: For a set of DPMLPRs $D_i = \left(d_{st}^i\right)_{n\times n} = \left(\left\langle \wp_{st}^i(p), \Upsilon_{st}^i(p) \right\rangle\right)_{n\times n}$, one DPMLPR $D^* = \left(d_{st}^*\right)_{n\times n} = \left(\left\langle \wp_{st}^*(p), \Upsilon_{st}^*(p) \right\rangle\right)_{n\times n}$, $s = 1, 2, \ldots, n$, $t = 1, 2, \ldots, n$ and $D = (d_{st})_{n\times n} = (\langle \wp_{st}(p), \Upsilon_{st}(p) \rangle)_{n\times n}$ is the group DPMLPR of the set of DPMLPRs D_i $(i = 1, 2, \ldots, n)$ by utilizing the weight vector $\omega = (\omega_1, \omega_2, \ldots, \omega_n)^T$, then if $C(D_i, D^*) \leq \delta, i = 1, 2, \ldots, n$, then $C(D, D^*) \leq \delta$, where δ is the threshold value of acceptable compatibility.

Proof: $D = (d_{st})_{n\times n} = (\langle L_{st}(p), U_{st}(p) \rangle)_{n\times n}$, where

$$\begin{aligned} \wp_{st}(p) &= \underset{i=1}{\overset{n}{\otimes}} (\wp_{sti}(p))^{\omega_i} \\ &= \underset{i=1}{\overset{n}{\otimes}} \left(\wp_{sti}^{\omega_i}\left(p_{sti}^{\omega_i}\right)\right) = I^{-1}\left(\prod_{i=1}^{n} (I(\wp_{sti}))^{\omega_i}\right) \left(\prod_{i=1}^{n} p_{sti}^{\omega_i}\right), \\ \Upsilon_{st}(p) &= \underset{i=1}{\overset{n}{\otimes}} (\Upsilon_{sti}(p))^{\omega_i} \\ &= \underset{i=1}{\overset{n}{\otimes}} \left(\Upsilon_{sti}^{\omega_i}\left(p_{sti}^{\omega_i}\right)\right) = I^{-1}\left(\prod_{i=1}^{n} (I(\Upsilon_i))^{\omega_i}\right) \left(\prod_{i=1}^{n} p_{sti}^{\omega_i}\right). \end{aligned}$$

Then $E_{D_i} = \left(e_{sti}^{\wp}, e_{sti}^{\Upsilon}\right)$ $(i = 1, 2, \ldots, n)$, $E_D = \left(e_{st}^{\wp}, e_{st}^{\Upsilon}\right)$, $E_{D^*} = \left(e_{st}^{\wp *}, e_{st}^{\Upsilon *}\right)$, $e_{st}^{\wp} = \left(\prod_{i=1}^{n} (I(\wp_{sti}))^{\omega_i}\right) \left(\prod_{i=1}^{n} p_{sti}^{\omega_i}\right)$, $e_{st}^{\Upsilon} = \left(\prod_{i=1}^{n} (I(\Upsilon_i))^{\omega_i}\right) \left(\prod_{i=1}^{n} p_{sti}^{\omega_i}\right)$.

$$
\begin{aligned}
C\left(D_1, D_3\right) &= \frac{1}{n(n-1)}\left[\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\left(\begin{array}{l}\left|\log e_{ij1}^{\wp}-\log e_{ij3}^{\wp}\right|+\\ \left|\log e_{ij1}^{\Upsilon}-\log e_{ij3}^{\Upsilon}\right|\end{array}\right)\right]\\
&= \frac{1}{n(n-1)}\left[\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\left(\begin{array}{l}\left|\log e_{ij1}^{\wp}-\log e_{ij2}^{\wp}+\log e_{ij2}^{\wp}-\log e_{ij3}^{\wp}\right|+\\ \left|\log e_{ij1}^{\Upsilon}-\log e_{ij2}^{\Upsilon}+\log e_{ij2}^{\Upsilon}-\log e_{ij3}^{\Upsilon}\right|\end{array}\right)\right]\\
&\leq \frac{1}{n(n-1)}\left[\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\left(\begin{array}{l}\left|\log e_{ij1}^{\wp}-\log e_{ij2}^{\wp}\right|+\left|\log e_{ij2}^{\wp}-\log e_{ij3}^{\wp}\right|+\\ \left|\log e_{ij1}^{\Upsilon}-\log e_{ij2}^{\Upsilon}\right|+\left|\log e_{ij2}^{\Upsilon}-\log e_{ij3}^{\Upsilon}\right|\end{array}\right)\right]\\
&\leq \frac{1}{n(n-1)}\left[\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\left(\left|\log e_{ij1}^{\wp}-\log e_{ij2}^{\wp}\right|+\left|\log e_{ij1}^{\Upsilon}-\log e_{ij2}^{\Upsilon}\right|\right)\right]\\
&\quad+\frac{1}{n(n-1)}\left[\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\left(\left|\log e_{ij2}^{\wp}-\log e_{ij3}^{\wp}\right|+\left|\log e_{ij2}^{\Upsilon}-\log e_{ij3}^{\Upsilon}\right|\right)\right]\\
&= C\left(D_1, D_2\right)+C\left(D_2, D_3\right)
\end{aligned}
$$

$$
D=\begin{pmatrix} D_{11} & D_{12} & \cdots & D_{1n}\\ D_{21} & D_{22} & \cdots & D_{2n}\\ \vdots & \vdots & \ddots & \vdots\\ D_{m1} & D_{m2} & \cdots & D_{mn}\end{pmatrix}
$$

$$
=\begin{pmatrix}
\left\langle \overset{n}{\underset{i=1}{\otimes}}\left(\wp_{11}^{i}(p)\right)^{\omega_i}, \overset{n}{\underset{i=1}{\otimes}}\left(\Upsilon_{11}^{i}(p)\right)^{\omega_i}\right\rangle & \left\langle \overset{n}{\underset{i=1}{\otimes}}\left(\wp_{12}^{i}(p)\right)^{\omega_i}, \overset{n}{\underset{i=1}{\otimes}}\left(\Upsilon_{12}^{i}(p)\right)^{\omega_i}\right\rangle & \cdots & \left\langle \overset{n}{\underset{i=1}{\otimes}}\left(\wp_{1n}^{i}(p)\right)^{\omega_i}, \overset{n}{\underset{i=1}{\otimes}}\left(\Upsilon_{1n}^{i}(p)\right)^{\omega_i}\right\rangle\\
\left\langle \overset{n}{\underset{i=1}{\otimes}}\left(\wp_{21}^{i}(p)\right)^{\omega_i}, \overset{n}{\underset{i=1}{\otimes}}\left(\Upsilon_{21}^{i}(p)\right)^{\omega_i}\right\rangle & \left\langle \overset{n}{\underset{i=1}{\otimes}}\left(\wp_{22}^{i}(p)\right)^{\omega_i}, \overset{n}{\underset{i=1}{\otimes}}\left(\Upsilon_{22}^{i}(p)\right)^{\omega_i}\right\rangle & \cdots & \left\langle \overset{n}{\underset{i=1}{\otimes}}\left(\wp_{2n}^{i}(p)\right)^{\omega_i}, \overset{n}{\underset{i=1}{\otimes}}\left(\Upsilon_{2n}^{i}(p)\right)^{\omega_i}\right\rangle\\
\vdots & \vdots & \ddots & \vdots\\
\left\langle \overset{n}{\underset{i=1}{\otimes}}\left(\wp_{n1}^{i}(p)\right)^{\omega_i}, \overset{n}{\underset{i=1}{\otimes}}\left(\Upsilon_{n1}^{i}(p)\right)^{\omega_i}\right\rangle & \left\langle \overset{n}{\underset{i=1}{\otimes}}\left(\wp_{n2}^{i}(p)\right)^{\omega_i}, \overset{n}{\underset{i=1}{\otimes}}\left(\Upsilon_{n2}^{i}(p)\right)^{\omega_i}\right\rangle & \cdots & \left\langle \overset{n}{\underset{i=1}{\otimes}}\left(\wp_{nn}^{i}(p)\right)^{\omega_i}, \overset{n}{\underset{i=1}{\otimes}}\left(\Upsilon_{nn}^{i}(p)\right)^{\omega_i}\right\rangle
\end{pmatrix}
$$

Since

$$
\begin{aligned}
&C\left(D_i, D^{*}\right)\\
&\quad=\frac{1}{n(n-1)}\left[\frac{1}{2}\sum_{s=1}^{n}\sum_{t=1}^{n}\left(\begin{array}{l}\left|\log e_{sti}^{\wp}-\log e_{st}^{\wp*}\right|\\ +\left|\log e_{sti}^{\Upsilon}-\log e_{st}^{\Upsilon*}\right|\end{array}\right)\right],\\
&\quad\leq\delta
\end{aligned}
$$

then, $C\left(D, D^{*}\right)$, as shown at the top of the next page. Thus the proof is completed.

Theorem 4: For two sets of DPMLPRs $D_i\ (i=1,2,\ldots,n)$, $D_i\ (i=1,2,\ldots,n)$, D is the group DPMLPR of the set of DPMLPRs $D_i\ (i=1,2,\ldots,n)$ and D is the group DPMLPR of the set of DPMLPRs $D_i\ (i=1,2,\ldots,n)$ by utilizing the same weight vector $\omega=(\omega_1,\omega_2,\ldots,\omega_n)^T$, respectively, then if $C\left(D_i, D_i\right)\leq\delta$, $i=1,2,\ldots,n$, then $C\left(D, D\right)\leq\delta$, where δ is the threshold value of acceptable compatibility.

The proof is similar to that of *Theorem 4*, so the specific proof process is omitted.

Then the group consensus procedure can be listed in Algorithm 2.

IV. METHODOLOGY

In this section, the determination of weights for criteria based on the group DPMLPR and the patulous GRA are presented in detail.

A. THE WEIGHTS FOR CRITERIA

Based on the algorithm in Section III, we can get a group DPMLPR $D=\left(D_{ij}\right)_{n\times n}=\left(\left\langle\wp_{ij}(p),\Upsilon_{ij}(p)\right\rangle\right)_{n\times n}$ with the acceptable consensus degree. Then for the DPMLPR $D=\left(D_{ij}\right)_{n\times n}$, with a view to the construction features of the elements in DPMLPR, the classic arithmetic averaging method [52] cannot be used directly. So we give the following equation to calculate the weights for criteria:

$$
\varpi_i=\sum_{j=1}^{n} I\left(S\left(D_{ij}\right)\right)\Big/\sum_{i=i}^{n}\sum_{j=1}^{n} I\left(S\left(D_{ij}\right)\right) \tag{17}
$$

where $S(\cdot)$ is the score function of D_{ij}.

$$
\begin{aligned}
C\left(D,D^*\right) &= \frac{1}{n(n-1)}\left[\frac{1}{2}\sum_{s=1}^{n}\sum_{t=1}^{n}\left(\begin{array}{l}\left|\log e_{st}^{\wp}-\log e_{st}^{\wp^*}\right| \\ +\left|\log e_{st}^{\Upsilon}-\log e_{st}^{\Upsilon*}\right|\end{array}\right)\right] \\
&= \frac{1}{n(n-1)}\left[\frac{1}{2}\sum_{s=1}^{n}\sum_{t=1}^{n}\left(\begin{array}{l}\left|\log\left\{\left(\prod_{i=1}^{n}\left(I\left(\wp_{sti}\right)\right)^{\omega_i}\right)\left(\prod_{i=1}^{n}\left(p_{sti}\right)^{\omega_i}\right)\right\}-\log e_{st}^{\wp^*}\right| \\ +\left|\log\left\{\left(\prod_{i=1}^{n}\left(I\left(\Upsilon_{sti}\right)\right)^{\omega_i}\right)\left(\prod_{i=1}^{n}\left(p_{sti}\right)^{\omega_i}\right)\right\}-\log e_{st}^{\Upsilon*}\right|\end{array}\right)\right] \\
&= \frac{1}{n(n-1)}\left[\frac{1}{2}\sum_{s=1}^{n}\sum_{t=1}^{n}\left(\begin{array}{l}\left|\log\left\{\left(\prod_{i=1}^{n}I\left(\wp_{sti}\right)\right)^{\omega_i}\left(\prod_{i=1}^{n}p_{sti}\right)^{\omega_i}\right\}-\log e_{st}^{\wp^*}\right| \\ +\left|\log\left\{\left(\prod_{i=1}^{n}I\left(\Upsilon_{sti}\right)\right)^{\omega_i}\left(\prod_{i=1}^{n}p_{sti}\right)^{\omega_i}\right\}-\log e_{st}^{\Upsilon*}\right|\end{array}\right)\right] \\
&= \frac{1}{n(n-1)}\left[\frac{1}{2}\sum_{s=1}^{n}\sum_{t=1}^{n}\left(\begin{array}{l}\left|\sum_{i=1}^{n}\omega_i\left(\log\left\{I\left(\wp_{sti}\right)p_{sti}\right\}-\log e_{st}^{\wp^*}\right)\right| \\ +\left|\sum_{i=1}^{n}\omega_i\left(\log\left\{I\left(\Upsilon_{sti}\right)p_{sti}\right\}-\log e_{st}^{\Upsilon*}\right)\right|\end{array}\right)\right] \\
&\le \frac{1}{n(n-1)}\left[\frac{1}{2}\sum_{s=1}^{n}\sum_{t=1}^{n}\left(\begin{array}{l}\sum_{i=1}^{n}\omega_i\left|\log\left\{I\left(\wp_{sti}\right)p_{sti}\right\}-\log e_{st}^{\wp^*}\right|+ \\ \sum_{i=1}^{n}\omega_i\left|\log\left\{I\left(\Upsilon_{sti}\right)p_{sti}\right\}-\log e_{st}^{\Upsilon*}\right|\end{array}\right)\right] \\
&= \sum_{i=1}^{n}\omega_i\left\{\frac{1}{n(n-1)}\left[\frac{1}{2}\sum_{s=1}^{n}\sum_{t=1}^{n}\left(\begin{array}{l}\left|\log\left\{I\left(\wp_{sti}\right)p_{sti}\right\}-\log e_{st}^{\wp^*}\right|+ \\ \left|\log\left\{I\left(\Upsilon_{sti}\right)p_{sti}\right\}-\log e_{st}^{\Upsilon*}\right|\end{array}\right)\right]\right\} \\
&= \sum_{i=1}^{n}\omega_i\left\{\frac{1}{n(n-1)}\left[\frac{1}{2}\sum_{s=1}^{n}\sum_{t=1}^{n}\left(\begin{array}{l}\left|\log e_{sti}^{\wp}-\log e_{st}^{\wp^*}\right|+ \\ \left|\log e_{sti}^{\Upsilon}-\log e_{st}^{\Upsilon*}\right|\end{array}\right)\right]\right\} \\
&\le \sum_{i=1}^{n}\omega_i\delta=\delta
\end{aligned}
$$

B. THE EXPANDING GREY RELATIVE ANALYSIS METHOD

With regard to the individual dual probabilistic linguistic decision-making matrices given by the DMs for the alternatives $(a_1, a_2, \ldots, a_m)$ with respect to the criteria $(c_1, c_2, \ldots, c_n)$, the group dual linguistic decision-making matrix $M = \left(M_{ij}\right)_{m\times n}$ can be acquired by Eq. (7) as follows:

$$
M = \begin{array}{c} \\ a_1 \\ a_2 \\ \vdots \\ a_m \end{array}
\begin{array}{c} \begin{array}{cccc} c_1 & c_2 & \cdots & c_n \end{array} \\
\begin{bmatrix} M_{11} & M_{12} & \cdots & M_{1n} \\ M_{21} & M_{22} & \cdots & M_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ M_{m1} & M_{m2} & \cdots & M_{mn} \end{bmatrix}\end{array} \tag{18}
$$

Let $M^+ = \left(M_1^+, M_1^+, \ldots, M_n^+\right)^T$ and $M^- = \left(M_1^-, M_1^-, \ldots, M_n^-\right)^T$ be the positive ideal element (PIE) and the negative ideal element (PIE) in M, respectively, where $M_j^+ = \max_i M_{ij}$, $M_j^- = \min_i M_{ij}$, M_j^+ and M_j^- are determined through Eq. (2) or Eq. (3).

Due to the reality that the comparable degree is similar to the distance measure in physical significance, in the light of the proposed comparable degree between the DPMLTSs, the grey relative coefficient matrices based on the PIE and the NIE are extended as follows:

$$
\mu_{ij}^+ = \frac{\min\limits_{1\le i\le m}\min\limits_{1\le j\le n} C(M_{ij}, M_j^+) + \xi \max\limits_{1\le i\le m}\max\limits_{1\le j\le n} C(M_{ij}, M_j^+)}{C(M_{ij}, M_j^+) + \xi \max\limits_{1\le i\le m}\max\limits_{1\le j\le n} C(M_{ij}, M_j^+)} \tag{19}
$$

$$
\mu_{ij}^- = \frac{\min\limits_{1\le i\le m}\min\limits_{1\le j\le n} C(M_{ij}, M_j^-) + \xi \max\limits_{1\le i\le m}\max\limits_{1\le j\le n} C(M_{ij}, M_j^-)}{C(M_{ij}, M_j^-) + \xi \max\limits_{1\le i\le m}\max\limits_{1\le j\le n} C(M_{ij}, M_j^-)} \tag{20}
$$

Combined with the weights of criteria, the opposite closeness coefficient to the PIE can be determined by the

Algorithm 2 The Procedure to Adjust the Consensus

Step 1: For the set of consistent DPMLPRs $D_i\ (i = 1, 2, \ldots, n)$, with the Eq. (7) and the subjective weight vector of the DMs, it is easy to obtain the group DPMLPR D.

Step 2: Let δ be the threshold value of acceptable compatibility, then calculate the compatibility degree between the individual DPMLPRs $D^{\iota}\ (\iota = 1, 2, \ldots, n)$ and the group DPMLPR $D = \left(D_{ij}\right)_{n\times n} = \left(\left\langle \wp_{ij}(p), \Upsilon_{ij}(p)\right\rangle\right)_{n\times n}$, if $C\left(D^{\iota_o}, D\right) \leq \sigma$, then group DMLPR is of the acceptable consensus, go to **Step 4**; Otherwise, go to the next step.

Step 3: Let $\iota_o = \iota_o + 1$, $D^{\iota_o+1} = D^{\iota_o}$, where

$$\begin{cases} D_{ij}^{\iota_o+1} = \left(\left\langle \wp_{ij}^{\iota_o+1}(p), \Upsilon_{ij}^{\iota_o+1}(p)\right\rangle\right)_{n\times n}, \\ \wp_{ij}^{\iota_o+1}(p) = \eta \wp_{ij}^{\iota_o}(p) \otimes (1-\eta)\, \wp_{ij}(p), \\ \Upsilon_{ij}^{\iota_o+1}(p) = \eta \Upsilon_{ij}^{\iota_o}(p) \otimes (1-\eta)\, \Upsilon_{ij}(p). \end{cases}$$

Then go back to **Step 2** until $C\left(D^{\iota_o}, D\right) \leq \sigma$.

Step 4: Let $\iota_o = \iota_o + 1$, $D^{\iota_o+1} = D^{\iota_o}$, then go back to **Step 1**.

coming equation:

$$OC_i = \frac{\sum_{j=1}^{n} \mu_{ij}^{+} \varpi_j}{\sum_{j=1}^{n} \mu_{ij}^{+} \varpi_j + \sum_{j=1}^{n} \mu_{ij}^{-} \varpi_j} \tag{21}$$

The bigger the opposite closeness coefficient OC_i, the better the alternative.

Then the EGRA method can be illustrated as Algorithm 3:

Step 1: Identify the PIE and the NIE of the group dual probabilistic decision-making matrix;

Step 2: Calculate the respective grey relative coefficients on the foundation of PIE and NIE;

Step 3: Obtain the opposite closeness coefficient for the alternative.

C. THE INTEGRATED PROCESS FOR SOLVING MULTI-CRITERIA GROUP DECISION-MAKING PROBLEM

On the foundation of Section III and the remaining subsection of Section IV, the integrated decision-making procedure can be concluded as follows:

V. SIMULATION EXPERIMENT

So as to make the decision-making procedure more detailed, this section performs a concrete simulation experiment relevant to the assessment for the manifestation of cloud enterprise mentioned above. Moreover, this section has four subsections: the first subsection is the practical experimental procedure to make Section II, III and IV particular; the second and third subsections are the comparative analysis; the four subsection is the sensitivity analysis.

A. EXPERIMENTAL PROCESS

Cloud computing [1] is a type of computing in which vibrantly scalable and always virtualized resources are supplied as a service over the internet. It was first proposed by the CEO Eric Schmidt of Google at the search engine conference in 2006.

According to service types, cloud computing is able to be divided into three types: IaaS (Infrastructure-as-a-Service), consumers can get services from a complete computer infrastructure over the internet; PaaS (Platform-as-a- Service), it uses the software development environment, application environment, etc. as a service to directly provide users with the application platform required by the software; SaaS (Software-as-a- Service), it is a model for providing software over the internet. Instead of purchasing software, users rent web-based software from providers to manage business operations.

The emergence of cloud computing will reshape the IT industry landscape. There will be two clear investment opportunities: one is the new market capacity brought about by the rapid development of the cloud computing industry, and the other is to reshape the emerging industry opportunities brought about by the IT landscape. Considering the broader trend, many corporations are going to the "Cloud". For those IT corporations, they already have own IT costs and IT technology. It is much easier for them to the "Cloud". While for those traditional corporations that lack of network experience want to the "Cloud", they need to bear the cost of trial and error and the risk of failure. It is good choice for those traditional corporations to choose a good partner. Obviously, the so-called good partner shall have rich experience and enough funds to support the traditional industries in need of assistance. Globally, the four giants of the cloud industry are AWS, Microsoft, Google and Alibaba Cloud. As mentioned in Ref. [11], one good partner corporations shall equip with the four features: Corporate value, Independent research and development ability, Corporate size and Product market share. Apparently, the four features are benefit, which means the four features are positively related to the direction of growth.

Considering the future development potential of cloud computing, the enterprise who wants to get twofold results with half the effort chooses to collaborate with one of the four giants of the cloud industries: AWS, Microsoft, Google and Alibaba Cloud. Supposed that the four giants of the cloud industries are four evaluated alternatives $x_i\ (i = 1, 2, 3, 4)$. To evaluate the four enterprises, they entrust one questionnaire enterprise to investigate the impact of four cloud enterprises under the four previously mentioned aspects. The questionnaire enterprise regards the four mentioned-above aspects as four criteria: Corporate value (c_1), Independent research and development ability (c_2), Corporate size (c_3) Product market share (c_4). Obviously, all of the four criteria are benefit. In order to make the evaluation as objective as possible, and consider the DPMLTSs can from the two opposite aspects display the decision-making information, the questionnaire enterprise choose the DPMLTSs as the decision-making tool for evaluation. To some extent, not only reflect the membership degree of the decision-making information, but also the non-membership degree.

Assume that the DPMLPRs that are given by four DMs for the four alternatives with respect to four criteria are as D_1, D_2, D_3, and D_4, as shown at the bottom of this page.

Step 1: Let $\Xi = 0.9$, then we check and improve the consistency of individual DPMLPRs $\tilde{D}_i (i = 1, 2, 3, 4)$ by Algorithm 1 as follows:

TABLE 1. The consistent degree of individual DPMLPRS.

DPMLPRs	D_1	D_2	D_3	D_4
CI	0.6636	0.6161	0.5610	0.6851

Obviously, based on Table 1, all of the four individual DPMLPRs are not consistent. On the foundation of Algorithm 1, they can be adjusted as D_1, D_2, D_3, and D_4, as shown at the next page. The consistent degree of four adjusted individual DPMLPRs are listed as follows:

TABLE 2. The consistent degree of adjusted individual DPMLPRs.

DPMLPRs	$\tilde{D}_1$	$\tilde{D}_2$	$\tilde{D}_3$	$\tilde{D}_4$
CI	0.9210	0.9055	0.9105	0.9247

Step 2: Let the subjective weight of the DMs $\hat{\omega} = (0.3, 0.2, 0.15, 0.35)^T$, then we utilize the aggregation operator (7) to figure out the group DPMLPR D as shown at the bottom of the page 11.

$$D_1 = \begin{pmatrix} \langle\{s_1(1)\},\{s_1(1)\}\rangle & \langle\{s_6(0.1),s_7(0.7),s_8(0.2)\},\{s_{1/4}(0.4),s_{1/3}(0.4)\}\rangle & \langle\{s_4(0.3),s_5(0.4),s_6(0.3)\},\{s_1(0.2),s_2(0.2),s_3(0.2)\}\rangle & \langle\{s_4(0.3),s_5(0.2),s_6(0.2)\},\{s_{1/8}(0.2),s_{1/7}(0.2)\}\rangle \\ \langle\{s_{1/4}(0.4),s_{1/3}(0.4)\},\{s_6(0.1),s_7(0.7),s_8(0.2)\}\rangle & \langle\{s_1(1)\},\{s_1(1)\}\rangle & \langle\{s_{1/3}(0.2),s_{1/2}(0.6),s_1(0.2)\},\{s_{1/4}(0.5),s_{1/3}(0.3),s_{1/2}(0.1)\}\rangle & \langle\{s_{1/5}(0.3),s_{1/4}(0.2),s_{1/3}(0.5)\},\{s_4(0.4),s_6(0.6)\}\rangle \\ \langle\{s_1(0.2),s_2(0.2),s_3(0.2)\},\{s_4(0.3),s_5(0.4),s_6(0.3)\}\rangle & \langle\{s_{1/4}(0.5),s_{1/3}(0.3),s_{1/2}(0.1)\},\{s_{1/3}(0.2),s_{1/2}(0.6),s_1(0.2)\}\rangle & \langle\{s_1(1)\},\{s_1(1)\}\rangle & \langle\{s_3(0.6),s_4(0.4)\},\{s_{1/3}(0.7),s_1(0.3)\}\rangle \\ \langle\{s_{1/8}(0.2),s_{1/7}(0.2)\},\{s_4(0.3),s_5(0.2),s_6(0.2)\}\rangle & \langle\{s_4(0.4),s_6(0.6)\},\{s_{1/5}(0.3),s_{1/4}(0.2),s_{1/3}(0.5)\}\rangle & \langle\{s_{1/3}(0.7),s_1(0.3)\},\{s_3(0.6),s_4(0.4)\}\rangle & \langle\{s_1(1)\},\{s_1(1)\}\rangle \end{pmatrix}$$

$$D_2 = \begin{pmatrix} \langle\{s_1(1)\},\{s_1(1)\}\rangle & \langle\{s_4(0.4),s_5(0.4),s_6(0.2)\},\{s_{1/5}(0.3),s_{1/3}(0.3)\}\rangle & \langle\{s_{1/8}(0.3),s_{1/6}(0.3)\},\{s_3(0.3),s_4(0.2),s_5(0.3)\}\rangle & \langle\{s_3(0.1),s_4(0.7),s_5(0.2)\},\{s_{1/6}(0.8),s_{1/4}(0.1)\}\rangle \\ \langle\{s_{1/5}(0.3),s_{1/3}(0.3)\},\{s_4(0.4),s_5(0.4),s_6(0.2)\}\rangle & \langle\{s_1(1)\},\{s_1(1)\}\rangle & \langle\{s_3(0.2),s_4(0.6)\},\{s_3(0.3),s_4(0.2)\}\rangle & \langle\{s_{1/7}(0.9),s_{1/6}(0.1)\},\{s_{1/2}(0.2),s_1(0.3),s_2(0.5)\}\rangle \\ \langle\{s_3(0.3),s_4(0.2),s_5(0.3)\},\{s_{1/8}(0.3),s_{1/6}(0.3)\}\rangle & \langle\{s_3(0.3),s_4(0.2)\},\{s_3(0.2),s_4(0.6)\}\rangle & \langle\{s_1(1)\},\{s_1(1)\}\rangle & \langle\{s_{1/4}(0.5),s_{1/3}(0.3),s_{1/2}(0.1)\},\{s_6(0.4),s_7(0.1),s_8(0.4)\}\rangle \\ \langle\{s_{1/6}(0.8),s_{1/4}(0.1)\},\{s_3(0.1),s_4(0.7),s_5(0.2)\}\rangle & \langle\{s_{1/2}(0.2),s_1(0.3),s_2(0.5)\},\{s_{1/7}(0.9),s_{1/6}(0.1)\}\rangle & \langle\{s_6(0.4),s_7(0.1),s_8(0.4)\},\{s_{1/4}(0.5),s_{1/3}(0.3),s_{1/2}(0.1)\}\rangle & \langle\{s_1(1)\},\{s_1(1)\}\rangle \end{pmatrix}$$

$$D_3 = \begin{pmatrix} \langle\{s_1(1)\},\{s_1(1)\}\rangle & \langle\{s_{1/2}(0.8),s_2(0.1)\},\{s_{1/6}(0.3),s_{1/5}(0.2),s_{1/4}(0.4)\}\rangle & \langle\{s_{1/9}(0.3),s_{1/7}(0.2)\},\{s_{1/5}(0.5),s_{1/4}(0.3),s_{1/3}(0.1)\}\rangle & \langle\{s_{1/4}(0.3),s_{1/3}(0.1),s_{1/2}(0.6)\},\{s_{1/7}(0.5),s_{1/6}(0.2)\}\rangle \\ \langle\{s_{1/6}(0.3),s_{1/5}(0.2),s_{1/4}(0.4)\},\{s_{1/2}(0.8),s_2(0.1)\}\rangle & \langle\{s_1(1)\},\{s_1(1)\}\rangle & \langle\{s_{1/8}(0.3),s_{1/7}(0.2),s_{1/6}(0.4)\},\{s_{1/2}(0.1),s_1(0.3),s_2(0.3)\}\rangle & \langle\{s_{1/8}(0.6),s_{1/7}(0.1),s_{1/6}(0.3)\},\{s_{1/2}(0.2),s_1(0.2),s_2(0.5)\}\rangle \\ \langle\{s_{1/5}(0.5),s_{1/4}(0.3),s_{1/3}(0.1)\},\{s_{1/9}(0.3),s_{1/7}(0.2)\}\rangle & \langle\{s_{1/2}(0.1),s_1(0.3),s_2(0.3)\},\{s_{1/8}(0.3),s_{1/7}(0.2),s_{1/6}(0.4)\}\rangle & \langle\{s_1(1)\},\{s_1(1)\}\rangle & \langle\{s_{1/2}(0.6),s_2(0.1)\},\{s_{1/5}(0.6),s_{1/4}(0.3),s_{1/3}(0.1)\}\rangle \\ \langle\{s_{1/7}(0.5),s_{1/6}(0.2)\},\{s_{1/4}(0.3),s_{1/3}(0.1),s_{1/2}(0.6)\}\rangle & \langle\{s_{1/2}(0.2),s_1(0.2),s_2(0.5)\},\{s_{1/8}(0.6),s_{1/7}(0.1),s_{1/6}(0.3)\}\rangle & \langle\{s_{1/5}(0.6),s_{1/4}(0.3),s_{1/3}(0.1)\},\{s_{1/2}(0.6),s_2(0.1)\}\rangle & \langle\{s_1(1)\},\{s_1(1)\}\rangle \end{pmatrix}$$

$$D_4 = \begin{pmatrix} \langle\{s_1(1)\},\{s_1(1)\}\rangle & \langle\{s_4(0.2),s_5(0.3),s_6(0.2)\},\{s_1(0.5),s_2(0.1),s_3(0.3)\}\rangle & \langle\{s_1(0.1),s_3(0.1)\},\{s_{1/3}(0.2),s_{1/2}(0.4),s_1(0.2)\}\rangle & \langle\{s_{1/5}(0.3),s_{1/3}(0.6)\},\{s_2(0.2),s_3(0.2),s_4(0.2)\}\rangle \\ \langle\{s_1(0.5),s_2(0.1),s_3(0.3)\},\{s_4(0.2),s_5(0.3),s_6(0.2)\}\rangle & \langle\{s_1(1)\},\{s_1(1)\}\rangle & \langle\{s_1(0.2),s_2(0.3),s_3(0.3)\},\{s_{1/7}(0.5),s_{1/5}(0.5)\}\rangle & \langle\{s_{1/8}(0.1),s_{1/7}(0.3),s_{1/6}(0.6)\},\{s_{1/4}(0.3),s_{1/3}(0.3),s_{1/2}(0.3)\}\rangle \\ \langle\{s_{1/3}(0.2),s_{1/2}(0.4),s_1(0.2)\},\{s_1(0.1),s_3(0.1)\}\rangle & \langle\{s_{1/7}(0.5),s_{1/5}(0.5)\},\{s_1(0.2),s_2(0.3),s_3(0.3)\}\rangle & \langle\{s_1(1)\},\{s_1(1)\}\rangle & \langle\{s_6(0.4),s_7(0.1),s_8(0.3)\},\{s_{1/2}(0.3),s_1(0.3),s_2(0.1)\}\rangle \\ \langle\{s_2(0.2),s_3(0.2),s_4(0.2)\},\{s_{1/5}(0.3),s_{1/3}(0.6)\}\rangle & \langle\{s_{1/4}(0.3),s_{1/3}(0.3),s_{1/2}(0.3)\},\{s_{1/8}(0.1),s_{1/7}(0.3),s_{1/6}(0.6)\}\rangle & \langle\{s_{1/2}(0.3),s_1(0.3),s_2(0.1)\},\{s_6(0.4),s_7(0.1),s_8(0.3)\}\rangle & \langle\{s_1(1)\},\{s_1(1)\}\rangle \end{pmatrix}$$

$$D_1 = \left(\begin{array}{c}
\langle\{s_1(1)\},\{s_1(1)\}\rangle\ \langle\{s_{5.1537}(0.1117),\ldots,s_{8.8247}(0.1273)\},\{s_{0.1713}(0.1820),\ldots,s_{0.3494}(0.2003)\}\rangle \\
\langle\{s_{0.1713}(0.1820),\ldots,s_{0.3494}(0.2003)\},\{s_{5.1537}(0.1117),\ldots,s_{8.8247}(0.1273)\}\rangle\ \langle\{s_1(1)\},\{s_1(1)\}\rangle \\
\langle\{s_{0.4947}(0.1817),\ldots,s_{1.2786}(0.1246)\},\{s_{2.8586}(0.1689),\ldots,s_{6.3325}(0.1521)\}\rangle \\
\langle\{s_{0.9375}(0.2383),\ldots,s_{2.1352}(0.0992)\},\{s_{0.3029}(0.1834),\ldots,s_{0.9268}(0.1722)\}\rangle \\
\langle\{s_{0.2219}(0.1794),\ldots,s_{0.4147}(0.1651)\},\{s_{3.9218}(0.1770),\ldots,s_{6.6835}(0.1596)\}\rangle \\
\langle\{s_{1.4142}(0.2225),\ldots,s_{2.7591}(0.2059)\},\{s_{0.3657}(0.2128),\ldots,s_{0.7432}(0.2515)\}\rangle
\end{array} \right.$$

$$\left. \begin{array}{c}
\langle\{s_{2.8586}(0.1689),\ldots,s_{6.3325}(0.1521)\},\{s_{0.4947}(0.1817),\ldots,s_{1.2786}(0.1246)\}\rangle \\
\langle\{s_{3.9218}(0.1770),\ldots,s_{6.6835}(0.1596)\},\{s_{0.2219}(0.1794),\ldots,s_{0.4147}(0.1651)\}\rangle \\
\langle\{s_{0.3029}(0.1834),\ldots,s_{0.9268}(0.1722)\},\{s_{0.9375}(0.2383),\ldots,s_{2.1352}(0.0992)\}\rangle \\
\langle\{s_{0.3657}(0.2128),\ldots,s_{0.7432}(0.2515)\},\{s_{1.4142}(0.2225),\ldots,s_{2.7591}(0.2059)\}\rangle \\
\langle\{s_1(1)\},\{s_1(1)\}\rangle\ \langle\{s_{1.4694}(0.3004),\ldots,s_{2.9225}(0.1759)\},\{s_{0.4664}(0.2940),\ldots,s_{1.3594}(0.1868)\}\rangle \\
\langle\{s_{0.4664}(0.2940),\ldots,s_{1.3594}(0.1868)\},\{s_{1.4694}(0.3004),\ldots,s_{2.9225}(0.1759)\}\rangle\ \langle\{s_1(1)\},\{s_1(1)\}\rangle
\end{array} \right)$$

$$D_2 = \left(\begin{array}{c}
\langle\{s_1(1)\},\{s_1(1)\}\rangle\ \langle\{s_{2.1352}(0.1724),\ldots,s_{4.3735}(0.1401)\},\{s_{0.2740}(0.2614),\ldots,s_{0.4864}(0.1441)\}\rangle \\
\langle\{s_{0.2740}(0.2614),\ldots,s_{0.4864}(0.1441)\},\{s_{2.1352}(0.1724),\ldots,s_{4.3735}(0.1401)\}\rangle\ \langle\{s_1(1)\},\{s_1(1)\}\rangle \\
\langle\{s_{0.9950}(0.2526),\ldots,s_{1.9541}(0.1172)\},\{s_{0.7469}(0.1605),\ldots,s_{1.1826}(0.1972)\}\rangle \\
\langle\{s_{2.1440}(0.2056),\ldots,s_{4.5004}(0.1231)\},\{s_{0.9666}(0.1923),\ldots,s_{1.4600}(0.2530)\}\rangle \\
\langle\{s_{0.3644}(0.3449),\ldots,s_{0.7782}(0.1117)\},\{s_{0.9341}(0.1372),\ldots,s_{1.7160}(0.0910)\}\rangle \\
\langle\{s_{1.0332}(0.1985),\ldots,s_{3.1870}(0.1939)\},\{s_{0.2551}(0.3150),\ldots,s_{0.4090}(0.0826)\}\rangle
\end{array} \right.$$

$$\left. \begin{array}{c}
\langle\{s_{0.7469}(0.1605),\ldots,s_{1.1826}(0.1972)\},\{s_{0.9950}(0.2526),\ldots,s_{1.9541}(0.1172)\}\rangle \\
\langle\{s_{0.9341}(0.1372),\ldots,s_{1.7160}(0.0910)\},\{s_{0.3644}(0.3449),\ldots,s_{0.7782}(0.1117)\}\rangle \\
\langle\{s_{0.9666}(0.1923),\ldots,s_{1.4600}(0.2530)\},\{s_{2.1440}(0.2056),\ldots,s_{4.5004}(0.1231)\}\rangle \\
\langle\{s_{0.2551}(0.3150),\ldots,s_{0.4090}(0.0826)\},\{s_{1.0332}(0.1985),\ldots,s_{3.1870}(0.1939)\}\rangle \\
\langle\{s_1(1)\},\{s_1(1)\}\rangle\ \langle\{s_{0.5415}(0.2628),\ldots,s_{1.0989}(0.0672)\},\{s_{1.6000}(0.2360),\ldots,s_{2.9852}(0.2332)\}\rangle \\
\langle\{s_{1.6000}(0.2360),\ldots,s_{2.9852}(0.2332)\},\{s_{0.5415}(0.2628),\ldots,s_{1.0989}(0.0672)\}\rangle\ \langle\{s_1(1)\},\{s_1(1)\}\rangle
\end{array} \right)$$

$$D_3 = \left(\begin{array}{c}
\langle\{s_1(1)\},\{s_1(1)\}\rangle\ \langle\{s_{0.2554}(0.2659),\ldots,s_{1.2194}(0.1117)\},\{s_{0.0768}(0.2634),\ldots,s_{0.1249}(0.1820)\}\rangle \\
\langle\{s_{0.0768}(0.2634),\ldots,s_{0.1249}(0.1820)\},\{s_{0.2554}(0.2659),\ldots,s_{1.2194}(0.1117)\}\rangle\ \langle\{s_1(1)\},\{s_1(1)\}\rangle \\
\langle\{s_{0.1399}(0.2681),\ldots,s_{0.3597}(0.0765)\},\{s_{0.0859}(0.2614),\ldots,s_{0.1724}(0.1180)\}\rangle \\
\langle\{s_{0.3247}(0.1342),\ldots,s_{1.8536}(0.1133)\},\{s_{0.0646}(0.2477),\ldots,s_{0.1016}(0.1820)\}\rangle \\
\langle\{s_{0.1017}(0.3053),\ldots,s_{0.1898}(0.1140)\},\{s_{0.1454}(0.2977),\ldots,s_{0.4172}(0.1808)\}\rangle \\
\langle\{s_{0.2567}(0.1817),\ldots,s_{1.1632}(0.1613)\},\{s_{0.0893}(0.3355),\ldots,s_{0.1798}(0.1972)\}\rangle
\end{array} \right.$$

$$\left. \begin{array}{c}
\langle\{s_{0.0859}(0.2614),\ldots,s_{0.1724}(0.1180)\},\{s_{0.1399}(0.2681),\ldots,s_{0.3597}(0.0765)\}\rangle \\
\langle\{s_{0.1454}(0.2977),\ldots,s_{0.4172}(0.1808)\},\{s_{0.1017}(0.3053),\ldots,s_{0.1898}(0.1140)\}\rangle \\
\langle\{s_{0.0646}(0.2477),\ldots,s_{0.1016}(0.1820)\},\{s_{0.3247}(0.1342),\ldots,s_{1.8536}(0.1133)\}\rangle \\
\langle\{s_{0.0893}(0.3355),\ldots,s_{0.1798}(0.1972)\},\{s_{0.2567}(0.1817),\ldots,s_{1.1632}(0.1613)\}\rangle \\
\langle\{s_1(1)\},\{s_1(1)\}\rangle\ \langle\{s_{0.2199}(0.3004),\ldots,s_{0.8970}(0.0891)\},\{s_{0.1000}(0.3004),\ldots,s_{0.2032}(0.0959)\}\rangle \\
\langle\{s_{0.1000}(0.3004),\ldots,s_{0.2032}(0.0959)\},\{s_{0.2199}(0.3004),\ldots,s_{0.8970}(0.0891)\}\rangle\ \langle\{s_1(1)\},\{s_1(1)\}\rangle
\end{array} \right)$$

$$D_4 = \left(\begin{array}{c}
\langle\{s_1(1)\},\{s_1(1)\}\rangle\ \langle\{s_{0.9416}(0.1328),\ldots,s_{1.9900}(0.1512)\},\{s_{0.6276}(0.1703),\ldots,s_{2.2628}(0.1868)\}\rangle \\
\langle\{s_{0.6276}(0.1703),\ldots,s_{2.2628}(0.1868)\},\{s_{0.9416}(0.1328),\ldots,s_{1.9900}(0.1512)\}\rangle\ \langle\{s_1(1)\},\{s_1(1)\}\rangle \\
\langle\{s_{0.5568}(0.1756),\ldots,s_{1.7403}(0.1512)\},\{s_{0.8421}(0.0826),\ldots,s_{3.1663}(0.0826)\}\rangle \\
\langle\{s_{0.3375}(0.2383),\ldots,s_{0.6636}(0.2258)\},\{s_{0.5946}(0.1081),\ldots,s_{2.4415}(0.1441)\}\rangle \\
\langle\{s_{0.8499}(0.1512),\ldots,s_{2.9225}(0.1118)\},\{s_{0.4494}(0.1237),\ldots,s_{0.9132}(0.2530)\}\rangle \\
\langle\{s_{0.3786}(0.1806),\ldots,s_{0.9909}(0.1469)\},\{s_{0.2821}(0.1035),\ldots,s_{0.5922}(0.3355)\}\rangle
\end{array} \right.$$

$$\left. \begin{array}{c}
\langle\{s_{0.8421}(0.0826),\ldots,s_{3.1663}(0.0826)\},\{s_{0.5568}(0.1756),\ldots,s_{1.7403}(0.1512)\}\rangle \\
\langle\{s_{0.4494}(0.1237),\ldots,s_{0.9132}(0.2530)\},\{s_{0.8499}(0.1512),\ldots,s_{2.9225}(0.1118)\}\rangle \\
\langle\{s_{0.5946}(0.1081),\ldots,s_{2.4415}(0.1441)\},\{s_{0.3375}(0.2383),\ldots,s_{0.6636}(0.2258)\}\rangle \\
\langle\{s_{0.2821}(0.1035),\ldots,s_{0.5922}(0.3355)\},\{s_{0.3786}(0.1806),\ldots,s_{0.9909}(0.1469)\}\rangle \\
\langle\{s_1(1)\},\{s_1(1)\}\rangle\ \langle\{s_{0.8670}(0.1898),\ldots,s_{1.5777}(0.2526)\},\{s_{0.5694}(0.1335),\ldots,s_{2.6516}(0.0725)\}\rangle \\
\langle\{s_{0.5694}(0.1335),\ldots,s_{2.6516}(0.0725)\},\{s_{0.8670}(0.1898),\ldots,s_{1.5777}(0.2526)\}\rangle\ \langle\{s_1(1)\},\{s_1(1)\}\rangle
\end{array} \right)$$

Step 3: Let $\sigma = 0.7$, then we figure out the comparable degree between the individual DPMLPRs with the group DPMLPR by Eq. (15) as follows:

TABLE 3. The comparable degree between individual dpmlprs and group DPMLPR.

DPMLPRs	D_1	D_2	D_3	D_4
Comparable degree	0.7652	0.6092	1.1478	0.6028

This table shows that the individual DPMLPRs D_1 and D_3 are not of the acceptable comparable degrees, so we adjust the individual DPMLPRs by Algorithm 2 as D_1' and D_3', as shown at the bottom of this page.

Until all the individual DPMLPRs fulfill with the $C(D_i, D) \leq 0.7$ $(i = 1, 2, 3, 4)$, by using Eq. (7), we can get a group DPMLPR as D, as shown at the top of the next page. Then, we use the Eq. (17) can get the final weight for the criteria as:

$$\varpi = (0.2488, 0.2608, 0.2456, 0.2448)$$

Moreover, through the combination of the subjective weights of DMs and the four individual dual probabilistic linguistic decision-making matrices M_i $(i = 1, 2, 3, 4)$, M_1, M_2, M_3, and M_4, as shown at the top of the next page, the group dual probabilistic multiplicative linguistic decision-making matrix M can be figured up as M, as shown at the top of the page 13.

$$D = \left(\begin{array}{l}
\langle\{s_1(1)\},\{s_1(1)\}\rangle\ \langle\{s_{1.2557}(0.1831),\ldots,s_{3.0679}(0.1272)\},\{s_{0.2334}(0.1994),\ldots,s_{0.3447}(0.1795)\}\rangle \\
\langle\{s_{1.2557}(0.1831),\ldots,s_{3.0679}(0.1272)\},\{s_{0.2334}(0.1994),\ldots,s_{0.3447}(0.1795)\}\rangle\ \langle\{s_1(1)\},\{s_1(1)\}\rangle \\
\langle\{s_{0.4043}(0.2069),\ldots,s_{0.9351}(0.1068)\},\{s_{0.5619}(0.1829),\ldots,s_{1.1558}(0.1337)\}\rangle \\
\langle\{s_{0.7109}(0.2005),\ldots,s_{1.9796}(0.1228)\},\{s_{0.3590}(0.1846),\ldots,s_{0.5414}(0.1846)\}\rangle \\
\langle\{s_{0.3057}(0.1841),\ldots,s_{0.4795}(0.1265)\},\{s_{0.7433}(0.1750),\ldots,s_{1.4310}(0.1597)\}\rangle \\
\langle\{s_{0.8494}(0.2004),\ldots,s_{1.8001}(0.1776)\},\{s_{0.2515}(0.2198),\ldots,s_{0.3879}(0.1930)\}\rangle \\
\langle\{s_{0.5619}(0.1829),\ldots,s_{1.1558}(0.1337)\},\{s_{0.4043}(0.2069),\ldots,s_{0.9351}(0.1068)\}\rangle \\
\langle\{s_{0.7433}(0.1750),\ldots,s_{1.4310}(0.1597)\},\{s_{0.3057}(0.1841),\ldots,s_{0.4795}(0.1265)\}\rangle \\
\langle\{s_{0.3590}(0.1846),\ldots,s_{0.5414}(0.1846)\},\{s_{0.7109}(0.2005),\ldots,s_{1.9796}(0.1228)\}\rangle \\
\langle\{s_{0.2515}(0.2198),\ldots,s_{0.3879}(0.1930)\},\{s_{0.8494}(0.2004),\ldots,s_{1.8001}(0.1776)\}\rangle \\
\langle\{s_1(1)\},\{s_1(1)\}\rangle\ \langle\{s_{0.5836}(0.2527),\ldots,s_{1.4491}(0.1207)\},\{s_{0.4509}(0.1952),\ldots,s_{0.9043}(0.1342)\}\rangle \\
\langle\{s_{0.5836}(0.2527),\ldots,s_{1.4491}(0.1207)\},\{s_{0.4509}(0.1952),\ldots,s_{0.9043}(0.1342)\}\rangle\ \langle\{s_1(1)\},\{s_1(1)\}\rangle
\end{array}\right)$$

$$D_1' = \left(\begin{array}{l}
\langle\{s_1(1)\},\{s_1(1)\}\rangle\ \langle\{s_{2.7168}(0.1681),\ldots,s_{5.2032}(0.1273)\},\{s_{0.2357}(0.1881),\ldots,s_{0.3470}(0.1896)\}\rangle \\
\langle\{s_{0.2357}(0.1881),\ldots,s_{0.3470}(0.1896)\},\{s_{2.7168}(0.1681),\ldots,s_{5.2032}(0.1273)\}\rangle\ \langle\{s_1(1)\},\{s_1(1)\}\rangle \\
\langle\{s_{0.4472}(0.1939),\ldots,s_{1.0934}(0.1154)\},\{s_{1.3816}(0.1892),\ldots,s_{2.7054}(0.1426)\}\rangle \\
\langle\{s_{0.8462}(0.2051),\ldots,s_{2.0559}(0.1104)\},\{s_{0.4596}(0.1785),\ldots,s_{0.7084}(0.1783)\}\rangle \\
\langle\{s_{0.2604}(0.1817),\ldots,s_{0.4459}(0.1445)\},\{s_{1.8506}(0.1682),\ldots,s_{3.0925}(0.1596)\}\rangle \\
\langle\{s_{1.0960}(0.2111),\ldots,s_{2.2286}(0.1912)\},\{s_{0.3454}(0.2219),\ldots,s_{0.5370}(0.2203)\}\rangle \\
\langle\{s_{1.3816}(0.1892),\ldots,s_{2.7054}(0.1426)\},\{s_{0.4472}(0.1939),\ldots,s_{1.0934}(0.1154)\}\rangle \\
\langle\{s_{1.8506}(0.1682),\ldots,s_{3.0925}(0.1596)\},\{s_{0.2604}(0.1817),\ldots,s_{0.4459}(0.1445)\}\rangle \\
\langle\{s_{0.4596}(0.1785),\ldots,s_{0.7084}(0.1783)\},\{s_{0.8462}(0.2051),\ldots,s_{2.0559}(0.1104)\}\rangle \\
\langle\{s_{0.3454}(0.2219),\ldots,s_{0.5370}(0.2203)\},\{s_{1.0960}(0.2111),\ldots,s_{2.2286}(0.1912)\}\rangle \\
\langle\{s_1(1)\},\{s_1(1)\}\rangle\ \langle\{s_{0.9260}(0.2755),\ldots,s_{2.0579}(0.1457)\},\{s_{0.4586}(0.2396),\ldots,s_{1.1088}(0.1583)\}\rangle \\
\langle\{s_{0.4586}(0.2396),\ldots,s_{1.1088}(0.1583)\},\{s_{0.9260}(0.2755),\ldots,s_{2.0579}(0.1457)\}\rangle\ \langle\{s_1(1)\},\{s_1(1)\}\rangle
\end{array}\right)$$

$$D_3' = \left(\begin{array}{l}
\langle\{s_1(1)\},\{s_1(1)\}\rangle\ \langle\{s_{0.5663}(0.2207),\ldots,s_{1.9341}(0.1192)\},\{s_{0.1519}(0.2028),\ldots,s_{0.2075}(0.1807)\}\rangle \\
\langle\{s_{0.1519}(0.2028),\ldots,s_{0.2075}(0.1807)\},\{s_{0.5663}(0.2207),\ldots,s_{1.9341}(0.1192)\}\rangle\ \langle\{s_1(1)\},\{s_1(1)\}\rangle \\
\langle\{s_{0.2485}(0.2101),\ldots,s_{0.5799}(0.0904)\},\{s_{0.2197}(0.2187),\ldots,s_{0.4463}(0.1256)\}\rangle \\
\langle\{s_{0.5239}(0.1882),\ldots,s_{1.9156}(0.1179)\},\{s_{0.1679}(0.1960),\ldots,s_{0.2346}(0.1833)\}\rangle \\
\langle\{s_{0.2127}(0.1999),\ldots,s_{0.3017}(0.1201)\},\{s_{0.3408}(0.1990),\ldots,s_{0.7727}(0.1699)\}\rangle \\
\langle\{s_{0.6153}(0.1896),\ldots,s_{1.4470}(0.1693)\},\{s_{0.1715}(0.2090),\ldots,s_{0.2641}(0.1951)\}\rangle \\
\langle\{s_{0.2197}(0.2187),\ldots,s_{0.4463}(0.1256)\},\{s_{0.2485}(0.2101),\ldots,s_{0.5799}(0.0904)\}\rangle \\
\langle\{s_{0.3408}(0.1990),\ldots,s_{0.7727}(0.1699)\},\{s_{0.2127}(0.1999),\ldots,s_{0.3017}(0.1201)\}\rangle \\
\langle\{s_{0.1679}(0.1960),\ldots,s_{0.2346}(0.1833)\},\{s_{0.5239}(0.1882),\ldots,s_{1.9156}(0.1179)\}\rangle \\
\langle\{s_{0.1715}(0.2090),\ldots,s_{0.2641}(0.1951)\},\{s_{0.6153}(0.1896),\ldots,s_{1.4470}(0.1693)\}\rangle \\
\langle\{s_1(1)\},\{s_1(1)\}\rangle\ \langle\{s_{0.3582}(0.2755),\ldots,s_{1.1401}(0.1037)\},\{s_{0.2344}(0.1746),\ldots,s_{0.4287}(0.1134)\}\rangle \\
\langle\{s_{0.2344}(0.1746),\ldots,s_{0.4287}(0.1134)\},\{s_{0.3582}(0.2755),\ldots,s_{1.1401}(0.1037)\}\rangle\ \langle\{s_1(1)\},\{s_1(1)\}\rangle
\end{array}\right)$$

$$D=\left(\begin{array}{c}
\langle\{s_1(1)\},\{s_1(1)\}\rangle\ \langle\{s_{1.3164}(0.1759),\ldots,s_{3.0771}(0.1301)\},\{s_{0.2705}(0.2017),\ldots,s_{0.4108}(0.1761)\}\rangle\\
\langle\{s_{0.2705}(0.2017),\ldots,s_{0.4108}(0.1761)\},\{s_{1.3164}(0.1759),\ldots,s_{3.0771}(0.1301)\}\rangle\ \langle\{s_1(1)\},\{s_1(1)\}\rangle\\
\langle\{s_{0.4651}(0.2099),\ldots,s_{1.0546}(0.1107)\},\{s_{0.5959}(0.1701),\ldots,s_{1.2494}(0.1341)\}\rangle\\
\langle\{s_{0.7507}(0.2036),\ldots,s_{1.9799}(0.1285)\},\{s_{0.4349}(0.1825),\ldots,s_{0.6694}(0.1870)\}\rangle\\
\langle\{s_{0.3641}(0.1796),\ldots,s_{0.5764}(0.1238)\},\{s_{0.7427}(0.1693),\ldots,s_{1.4089}(0.1562)\}\rangle\\
\langle\{s_{0.8809}(0.2011),\ldots,s_{1.8225}(0.1766)\},\{s_{0.2614}(0.2231),\ldots,s_{0.4025}(0.1848)\}\rangle\\
\langle\{s_{0.5959}(0.1701),\ldots,s_{1.2494}(0.1341)\},\{s_{0.4651}(0.2099),\ldots,s_{1.0546}(0.1107)\}\rangle\\
\langle\{s_{0.7427}(0.1693),\ldots,s_{1.4089}(0.1562)\},\{s_{0.3641}(0.1796),\ldots,s_{0.5764}(0.1238)\}\rangle\\
\langle\{s_{0.4349}(0.1825),\ldots,s_{0.6694}(0.1870)\},\{s_{0.7507}(0.2036),\ldots,s_{1.9799}(0.1285)\}\rangle\\
\langle\{s_{0.2614}(0.2231),\ldots,s_{0.4025}(0.1848)\},\{s_{0.8809}(0.2011),\ldots,s_{1.8225}(0.1766)\}\rangle\\
\langle\{s_1(1)\},\{s_1(1)\}\rangle\ \langle\{s_{0.6027}(0.2388),\ldots,s_{1.4186}(0.1204)\},\{s_{0.5640}(0.1909),\ldots,s_{1.1046}(0.1354)\}\rangle\\
\langle\{s_{0.5640}(0.1909),\ldots,s_{1.1046}(0.1354)\},\{s_{0.6027}(0.2388),\ldots,s_{1.4186}(0.1204)\}\rangle\ \langle\{s_1(1)\},\{s_1(1)\}\rangle
\end{array}\right)$$

$$M_1=\left[\begin{array}{c}
\langle\{s_{1/8}(0.5),s_{1/7}(0.3),s_{1/6}(0.1)\},\{s_{1/7}(0.2),s_{1/6}(0.8)\}\rangle\ \langle\{s_5(0.3),s_6(0.2),s_7(0.4)\},\{s_{1/3}(0.1),s_{1/2}(0.3),s_1(0.5)\}\rangle\\
\langle\{s_{1/7}(0.1),s_{1/6}(0.4),s_{1/5}(0.2)\},\{s_{1/2}(0.6),s_2(0.1)\}\rangle\ \langle\{s_2(0.3),s_4(0.6)\},\{s_{1/2}(0.3),s_1(0.5)\}\rangle\\
\langle\{s_7(0.3),s_9(0.3)\},\{s_4(0.7),s_6(0.3)\}\rangle\ \langle\{s_7(0.1),s_8(0.4),s_9(0.3)\},\{s_3(0.2),s_4(0.3)\}\rangle\\
\langle\{s_3(0.4),s_4(0.2),s_5(0.3)\},\{s_{1/5}(0.2),s_{1/4}(0.2)\}\rangle\ \langle\{s_{1/7}(0.1),s_{1/5}(0.8)\},\{s_{1/8}(0.1),s_{1/7}(0.1),s_{1/6}(0.6)\}\rangle\\
\langle\{s_4(0.1),s_6(0.9)\},\{s_{1/9}(0.5),s_{1/8}(0.3),s_{1/7}(0.2)\}\rangle\ \langle\{s_{1/5}(0.3),s_{1/4}(0.4)\},\{s_2(0.2),s_3(0.2),s_4(0.5)\}\rangle\\
\langle\{s_{1/3}(0.1),s_1(0.4)\},\{s_5(0.2),s_6(0.4)\}\rangle\ \langle\{s_{1/8}(0.3),s_{1/6}(0.5)\},\{s_{1/8}(0.3),s_{1/7}(0.2),s_{1/6}(0.2)\}\rangle\\
\langle\{s_{1/5}(0.6),s_{1/3}(0.4)\},\{s_6(0.5),s_7(0.2),s_8(0.2)\}\rangle\ \langle\{s_{1/3}(0.5),s_{1/2}(0.4)\},\{s_{1/6}(0.4),s_{1/5}(0.4),s_{1/4}(0.2)\}\rangle\\
\langle\{s_7(0.2),s_8(0.3),s_9(0.1)\},\{s_3(0.5),s_4(0.1),s_5(0.3)\}\rangle\ \langle\{s_4(0.6),s_5(0.2),s_6(0.2)\},\{s_{1/2}(0.5),s_1(0.1)\}\rangle
\end{array}\right]$$

$$M_2=\left[\begin{array}{c}
\langle\{s_{1/8}(0.5),s_{1/7}(0.3)\},\{s_8(0.2),s_9(0.1)\}\rangle\ \langle\{s_5(0.1),s_7(0.2)\},\{s_8(0.2),s_9(0.2)\}\rangle\\
\langle\{s_{1/2}(0.3),s_2(0.1)\},\{s_4(0.5),s_5(0.3)\}\rangle\ \langle\{s_{1/3}(0.1),s_{1/2}(0.7)\},\{s_{1/5}(0.5),s_{1/4}(0.2),s_{1/3}(0.2)\}\rangle\\
\langle\{s_1(0.1),s_2(0.5),s_3(0.1)\},\{s_1(0.2),s_3(0.3)\}\rangle\ \langle\{s_5(0.8),s_6(0.2)\},\{s_4(0.3),s_6(0.7)\}\rangle\\
\langle\{s_3(0.5),s_4(0.2),s_5(0.2)\},\{s_5(0.4),s_6(0.2),s_7(0.4)\}\rangle\ \langle\{s_7(0.8),s_8(0.2)\},\{s_{1/9}(0.4),s_{1/8}(0.4)\}\rangle\\
\langle\{s_{1/6}(0.6),s_{1/5}(0.2)\},\{s_2(0.4),s_3(0.1),s_4(0.3)\}\rangle\ \langle\{s_3(0.1),s_4(0.1),s_5(0.8)\},\{s_{1/4}(0.1),s_{1/2}(0.8)\}\rangle\\
\langle\{s_6(0.2),s_8(0.4)\},\{s_{1/8}(0.2),s_{1/7}(0.3),s_{1/6}(0.1)\}\rangle\ \langle\{s_3(0.2),s_4(0.4),s_5(0.4)\},\{s_{1/8}(0.4),s_{1/7}(0.1),s_{1/6}(0.1)\}\rangle\\
\langle\{s_{1/2}(0.3),s_1(0.3),s_2(0.3)\},\{s_3(0.3),s_4(0.2),s_5(0.5)\}\rangle\ \langle\{s_{1/5}(0.2),s_{1/4}(0.3),s_{1/3}(0.3)\},\{s_{1/7}(0.2),s_{1/6}(0.8)\}\rangle\\
\langle\{s_{1/5}(0.3),s_{1/3}(0.7)\},\{s_{1/4}(0.5),s_{1/3}(0.1)\}\rangle\ \langle\{s_6(0.1),s_7(0.2),s_8(0.6)\},\{s_{1/3}(0.4),s_{1/2}(0.6)\}\rangle
\end{array}\right]$$

$$M_3=\left[\begin{array}{c}
\langle\{s_6(0.4),s_7(0.1),s_8(0.5)\},\{s_1(0.3),s_2(0.2),s_3(0.5)\}\rangle\ \langle\{s_2(0.3),s_3(0.2)\},\{s_{1/7}(0.3),s_{1/6}(0.2),s_{1/5}(0.3)\}\rangle\\
\langle\{s_4(0.5),s_5(0.3),s_6(0.1)\},\{s_5(0.3),s_6(0.1),s_7(0.3)\}\rangle\ \langle\{s_{1/8}(0.1),s_{1/7}(0.1)\},\{s_7(0.1),s_8(0.2),s_9(0.4)\}\rangle\\
\langle\{s_{1/4}(0.4),s_{1/3}(0.1),s_{1/2}(0.4)\},\{s_{1/3}(0.3),s_1(0.2)\}\rangle\ \langle\{s_5(0.4),s_6(0.1),s_7(0.2)\},\{s_3(0.1),s_4(0.4),s_5(0.2)\}\rangle\\
\langle\{s_1(0.1),s_2(0.5),s_3(0.3)\},\{s_{1/8}(0.2),s_{1/7}(0.6),s_{1/6}(0.2)\}\rangle\ \langle\{s_{1/6}(0.2),s_{1/5}(0.6)\},\{s_2(0.4),s_3(0.3)\}\rangle\\
\langle\{s_3(0.1),s_4(0.6),s_5(0.2)\},\{s_{1/3}(0.2),s_{1/2}(0.4)\}\rangle\ \langle\{s_6(0.2),s_7(0.4),s_8(0.3)\},\{s_{1/5}(0.1),s_{1/4}(0.4),s_{1/3}(0.3)\}\rangle\\
\langle\{s_{1/5}(0.5),s_{1/3}(0.1)\},\{s_5(0.4),s_6(0.5)\}\rangle\ \langle\{s_{1/4}(0.2),s_{1/3}(0.5),s_{1/2}(0.3)\},\{s_1(0.5),s_2(0.1),s_3(0.3)\}\rangle\\
\langle\{s_6(0.4),s_7(0.3),s_8(0.1)\},\{s_4(0.2),s_5(0.1),s_6(0.7)\}\rangle\ \langle\{s_3(0.5),s_4(0.1),s_5(0.3)\},\{s_4(0.1),s_5(0.2),s_6(0.3)\}\rangle\\
\langle\{s_{1/6}(0.5),s_{1/5}(0.2),s_{1/4}(0.3)\},\{s_{1/6}(0.3),s_{1/5}(0.4),s_{1/4}(0.3)\}\rangle\ \langle\{s_3(0.7),s_5(0.2)\},\{s_{1/3}(0.3),s_1(0.4)\}\rangle
\end{array}\right]$$

$$M_4=\left[\begin{array}{c}
\langle\{s_6(0.4),s_7(0.1),s_8(0.3)\},\{s_{1/2}(0.3),s_1(0.3),s_2(0.1)\}\rangle\ \langle\{s_{1/6}(0.1),s_{1/5}(0.2),s_{1/4}(0.3)\},\{s_{1/2}(0.3),s_1(0.2),s_2(0.3)\}\rangle\\
\langle\{s_{1/5}(0.2),s_{1/4}(0.4),s_{1/3}(0.3)\},\{s_{1/7}(0.2),s_{1/6}(0.1),s_{1/5}(0.6)\}\rangle\\
\langle\{s_{1/8}(0.3),s_{1/7}(0.3),s_{1/6}(0.3)\},\{s_{1/8}(0.4),s_{1/7}(0.3),s_{1/6}(0.3)\}\rangle\\
\langle\{s_{1/3}(0.5),s_{1/2}(0.1),s_1(0.4)\},\{s_3(0.3),s_5(0.7)\}\rangle\ \langle\{s_{1/4}(0.5),s_{1/3}(0.3),s_{1/2}(0.2)\},\{s_{1/5}(0.2),s_{1/4}(0.8)\}\rangle\\
\langle\{s_{1/8}(0.5),s_{1/7}(0.2)\},\{s_4(0.4),s_5(0.4),s_6(0.2)\}\rangle\ \langle\{s_8(0.7),s_9(0.2)\},\{s_5(0.2),s_6(0.6),s_7(0.2)\}\rangle\\
\langle\{s_4(0.1),s_5(0.3),s_6(0.2)\},\{s_5(0.2),s_6(0.7),s_7(0.1)\}\rangle\ \langle\{s_{1/7}(0.3),s_{1/6}(0.6),s_{1/5}(0.1)\},\{s_5(0.6),s_6(0.2)\}\rangle\\
\langle\{s_5(0.5),s_6(0.2),s_7(0.2)\},\{s_{1/5}(0.2),s_{1/4}(0.3),s_{1/3}(0.5)\}\rangle\ \langle\{s_{1/2}(0.2),s_1(0.5)\},\{s_{1/2}(0.1),s_2(0.6)\}\rangle\\
\langle\{s_4(0.2),s_5(0.1),s_6(0.3)\},\{s_{1/9}(0.1),s_{1/8}(0.5),s_{1/7}(0.2)\}\rangle\ \langle\{s_4(0.4),s_5(0.1),s_6(0.5)\},\{s_4(0.3),s_6(0.7)\}\rangle\\
\langle\{s_{1/3}(0.2),s_{1/2}(0.4),s_1(0.2)\},\{s_{1/4}(0.6),s_{1/3}(0.1),s_{1/2}(0.2)\}\rangle\ \langle\{s_5(0.6),s_6(0.4)\},\{s_{1/5}(0.3),s_{1/4}(0.3),s_{1/3}(0.2)\}\rangle
\end{array}\right]$$

Step 4: Determine the PIE M_j^+ and the NIE M_j^- for the DPMLPR M and, M_j^+ and M_j^-, as shown at the top of the next page.

Step 5: Let $\xi = 0.5$, by using Eqs. (19) and (20), we figure out the grey relative coefficient matrices μ_{ij}^+ and μ_{ij}^-:

$$\mu^+=\begin{pmatrix} 0.6551 & 0.4437 & 0.4704 & 0.4570\\ 0.6744 & 0.3333 & 0.5966 & 0.3797\\ 1 & 1 & 1 & 0.4482\\ 0.4873 & 0.3685 & 0.3954 & 1\end{pmatrix}$$

and

$$\mu^-=\begin{pmatrix} 1 & 0.4141 & 0.5568 & 0.4982\\ 0.6540 & 1 & 0.5396 & 1\\ 0.6551 & 0.3333 & 0.3954 & 0.4850\\ 0.5660 & 0.5420 & 1 & 0.3797\end{pmatrix}$$

Step 6: Using Eq. (21) to figure out the closeness coefficient:

$$OC = (0.4512, 0.3816, 0.6500, 0.4739)$$

$$\mathrm{M} = \begin{bmatrix} \langle\{s_{0.8660}(0.4472),\ldots,s_{1.1196}(0.2580)\},\{s_{0.7620}(0.2449),\ldots,s_{1.4776}(0.3278)\}\rangle & \langle\{s_{1.9155}(0.1431),\ldots,s_{2.8511}(0.3140)\},\{s_{0.5150}(0.3024),\ldots,s_{0.7732}(0.2491)\}\rangle \\ \langle\{s_{2.1783}(0.2042),\ldots,s_{3.1567}(0.2617)\},\{s_{0.4972}(0.1990),\ldots,s_{0.9803}(0.3224)\}\rangle & \langle\{s_{1.0748}(0.2090),\ldots,s_{1.4805}(0.3375)\},\{s_{0.6762}(0.1611),\ldots,s_{1.1753}(0.4004)\}\rangle \\ \langle\{s_{0.6196}(0.2428),\ldots,s_{1.1254}(0.1452)\},\{s_{1.4060}(0.3849),\ldots,s_{2.6366}(0.2394)\}\rangle & \langle\{s_{0.7459}(0.2569),\ldots,s_{1.3816}(0.2219)\},\{s_{1.4753}(0.2549),\ldots,s_{1.8993}(0.3389)\}\rangle \\ \langle\{s_{0.3494}(0.1639),\ldots,s_{0.5104}(0.2979)\},\{s_{0.8516}(0.2362),\ldots,s_{1.3238}(0.3566)\}\rangle & \langle\{s_{0.3704}(0.2259),\ldots,s_{0.6324}(0.3999)\},\{s_{0.3186}(0.3222),\ldots,s_{0.6654}(0.2366)\}\rangle \\ \langle\{s_{0.9359}(0.2875),\ldots,s_{1.8895}(0.2781)\},\{s_{1.2167}(0.3567),\ldots,s_{2.7147}(0.2956)\}\rangle & \langle\{s_{1.2381}(0.3844),\ldots,s_{2.2381}(0.2226)\},\{s_{2.4915}(0.2573),\ldots,s_{3.6001}(0.3724)\}\rangle \\ \langle\{s_{3.5290}(0.3135),\ldots,s_{4.9264}(0.2259)\},\{s_{2.1169}(0.1702),\ldots,s_{3.0944}(0.3573)\}\rangle & \langle\{s_{0.9427}(0.4026),\ldots,s_{1.4984}(0.3531)\},\{s_{0.7917}(0.2053),\ldots,s_{1.1293}(0.3670)\}\rangle \\ \langle\{s_{1.2679}(0.2662),\ldots,s_{2.4531}(0.2603)\},\{s_{0.5062}(0.2549),\ldots,s_{0.6804}(0.2297)\}\rangle & \langle\{s_{0.5886}(0.2989),\ldots,s_{0.9553}(0.2405)\},\{s_{0.4572}(0.4297),\ldots,s_{0.7218}(0.2266)\}\rangle \\ \langle\{s_{0.6006}(0.2587),\ldots,s_{0.7403}(0.4453)\},\{s_{0.5603}(0.2378),\ldots,s_{0.7580}(0.3681)\}\rangle & \langle\{s_{4.0559}(0.4425),\ldots,s_{5.9625}(0.2764)\},\{s_{0.3487}(0.3704),\ldots,s_{0.7383}(0.2579)\}\rangle \end{bmatrix}$$

$$\begin{aligned} M_j^{+} = (&\langle\{s_{0.9359}(0.2875),\ldots,s_{1.8895}(0.2781)\},\{s_{1.2167}(0.3567),\ldots,s_{2.7147}(0.2956)\}\rangle, \\ &\langle\{s_{3.5290}(0.3135),\ldots,s_{4.9264}(0.2259)\},\{s_{2.1169}(0.1702),\ldots,s_{3.0944}(0.3573)\}\rangle, \\ &\langle\{s_{1.2381}(0.3844),\ldots,s_{2.2381}(0.2226)\},\{s_{2.4915}(0.2573),\ldots,s_{3.6001}(0.3724)\}\rangle, \\ &\langle\{s_{4.0559}(0.4425),\ldots,s_{5.9625}(0.2764)\},\{s_{0.3487}(0.3704),\ldots,s_{0.7383}(0.2579)\}\rangle) \\ M_j^{-} = (&\langle\{s_{0.8660}(0.4472),\ldots,s_{1.1196}(0.2580)\},\{s_{0.7620}(0.2449),\ldots,s_{1.4776}(0.3278)\}\rangle, \\ &\langle\{s_{0.3494}(0.1639),\ldots,s_{0.5104}(0.2979)\},\{s_{0.8516}(0.2362),\ldots,s_{1.3238}(0.3566)\}\rangle, \\ &\langle\{s_{0.5886}(0.2989),\ldots,s_{0.9553}(0.2405)\},\{s_{0.4572}(0.4297),\ldots,s_{0.7218}(0.2266)\}\rangle, \\ &\langle\{s_{0.3704}(0.2259),\ldots,s_{0.6324}(0.3999)\},\{s_{0.3186}(0.3222),\ldots,s_{0.6654}(0.2366)\}\rangle) \end{aligned}$$

Therefore, the priority of the alternatives is $a_3 \succ a_4 \succ a_1 \succ a_2$.

B. RESULT ANALYSIS WITH EXPANDING TODIM

In this subsection, based on the proposed comparable degree, we propose the ETODIM.

As the conventional introduction for the TODIM, it usually concludes the following procedures:(1) Obtain the group decision-making information; (2) Divide the index value into two classifications: the benefit type and the cost type and normalize the group decision-making information; (3) Figure up the relative weight between the fixed indexes; (4) Count the comparative dominance between the selected alternatives; (5) Compute the prospect value on account of the acquired dominance and receive the ranking of the picked alternatives.

In this subsection, different from the traditional TODIM that uses the distance measure to measure the deviation between the alternatives, we use the comparable degree to calculate the comparative dominance between the selected alternatives. Concretely, the ETODIM can be stated below:

Step 1: Acquire the group dual probabilistic linguistic decision-making matrix $M = \left(M_{ij}\right)_{n\times n}$;

Step 2: Normalize the dual probabilistic linguistic decision-making matrix $\bar{M} = \left(\bar{M}_{ij}\right)_{m\times n}$, if

$$\bar{M}_{ij} = \begin{cases} \bar{M}_{ij}, & if\ c_j\ is\ benefit \\ M_{ij}, & if\ c_j\ is\ \cos t; \end{cases}$$

Step 3: Figure up the weights for criteria $\breve{\omega} = (\breve{\omega}_1, \breve{\omega}_2, \ldots, \breve{\omega}_n)$ by the Eq. (17), then we count the comparative weight $\breve{\omega}_{jr} = \breve{\omega}_j/\breve{\omega}_r$, where $\breve{\omega}_r = \max(\breve{\omega}_1, \breve{\omega}_2, \ldots, \breve{\omega}_n)$;

Step 4: Count the comparative dominance $\mathbb{Q}(a_i, a_k) = \sum_{j=1}^{n} \psi_j(a_i, a_k)$ for $\forall (i, k)$, where

$$\psi_j(a_i, a_k) = \begin{cases} \sqrt{\breve{\omega}_{jr} C\left(\bar{M}_{ij}, \bar{M}_{kj}\right) / \sum_{j=1}^{n} \breve{\omega}_{jr}}, & if\ S\left(\bar{M}_{ij}\right) > S\left(\bar{M}_{kj}\right); \\ 0, & if\ S\left(\bar{M}_{ij}\right) = S\left(\bar{M}_{kj}\right); \\ -\frac{1}{\varsigma}\sqrt{C\left(\bar{M}_{kj}, \bar{M}_{ij}\right)\left(\sum_{j=1}^{n} \breve{\omega}_{jr}\right) \Big/ \breve{\omega}_{jr}}, & if\ S\left(\bar{M}_{ij}\right) < S\left(\bar{M}_{kj}\right); \end{cases} \tag{22}$$

Moreover, the parameter ς is the attenuation factor of the losses, here we take $\varsigma = 1$ which manifests that the losses will make contribution to their real value to the global value and (a_i, a_k) is any pair of alternatives.

Step 5: Compute the prospect value of the picked alternatives as:

$$\aleph(a_i) = \frac{\sum_{k=1}^{m} \mathbb{Q}(a_i, a_k) - \min_i\left\{\sum_{k=1}^{m} \mathbb{Q}(a_i, a_k)\right\}}{\max_i\left\{\sum_{k=1}^{m} \mathbb{Q}(a_i, a_k)\right\} - \min_i\left\{\sum_{k=1}^{m} \mathbb{Q}(a_i, a_k)\right\}} \tag{23}$$

Then with the ETODIM method, we can get the comparative weight $\breve{\omega} = (0.9540, 1, 0.9415, 0.9384)$, the comparative dominance matrix

$$\mathbb{Q} = \begin{pmatrix} 0 & 1.8378 & -1.0826 & -2.8501 \\ -7.3405 & 0 & -5.3131 & -7.7640 \\ -3.1812 & -0.0735 & 0 & -5.5764 \\ -2.0592 & 1.9365 & -0.8096 & 0 \end{pmatrix},$$

the prospect values of the picked alternatives are as follows: $\aleph(a_1) = 0.9421$, $\aleph(a_2) = 0$, $\aleph(a_3) = 0.5957$, $\aleph(a_4) = 1$. Then we can get the priority of the alternatives as $a_4 \succ a_1 \succ a_3 \succ a_2$.

C. RESULT ANALYSIS WITH EXPANDING VIKOR

Owing to that the TODIM and the GRA are with the same principle that use the distance as the basis to compute, so we consider to use the other relative classic VIKOR for comparative analysis in this subsection. First, we state simply the classic VIKOR as follows: (1) Seek out the PIE and the NIE for the benefit and cost criteria, respectively; (2) Determine the weight of the criteria; (3) Figure up the ordering value; (4) Count the compromise solution of the chosen alternatives and confirm the priority for the alternatives. Similarly, on the foundation of the suggested comparable degree, we present the following EVIKOR below:

Step 1: For the obtained group dual probabilistic linguistic decision-making matrix $M = (M_{ij})_{n\times n}$, seek out the PIE:

$$M_j^* = \begin{cases} \max_i M_{ij}, & \text{if } c_j \text{ is benefit}; \\ \min_i M_{ij}, & \text{if } c_j \text{ is } \cos t. \end{cases} \quad (24)$$

and the NIE:

$$M_j^- = \begin{cases} \min_i M_{ij}, & \text{if } c_j \text{ is benefit}; \\ \max_i M_{ij}, & \text{if } c_j \text{ is } \cos t. \end{cases} \quad (25)$$

Step 2: Determine the weights for criteria $\vec{\omega} = (\vec{\omega}_1, \vec{\omega}_2, \ldots, \vec{\omega}_n)$ by Eq. (17).

Step 3: Figure up the ordering value Υ_j and Z_j as follows:

$$\begin{cases} \Upsilon_i = \sum_{j=1}^{n} \vec{\omega}_j \frac{C\left(M_j^*, M_{ij}\right)}{C\left(M_j^*, M_j^-\right)} \\ Z_i = \max_i \left(\vec{\omega}_j \frac{C\left(M_j^*, M_{ij}\right)}{C\left(M_j^*, M_j^-\right)} \right) \end{cases}, \quad i = 1, 2, \ldots, m. \quad (26)$$

Step 4: Count the compromise solution of the chosen alternatives as:

$$\begin{cases} \Lambda_i = \rho \frac{\Upsilon_i - \Upsilon^*}{\Upsilon^- - \Upsilon^*} + (1-\rho) \frac{Z_i - Z^*}{Z^- - Z^*} \\ i = 1, 2, \ldots, m \end{cases} \quad (27)$$

where the parameter $\rho \in (0, 1)$ shows the weight of Υ_i and the decision-making tactic of the DMs. The ultimate sort outcome is steady with a decision-making tactic, which accords with the majority rule if $\rho > 0.5$, or the consensus rule if $\rho = 0.5$, or the veto rule if $\rho < 0.5$. $\Upsilon^* = \min(\Upsilon_i)$, $\Upsilon^- = \max(\Upsilon_i)$, $Z^* = \min(Z_i)$, $Z^- = \max(Z_i)$.

Then by using EVIKOR method, the ordering value $\Upsilon = (0.7711, 0.8423, 0.1844, 0.9663)$, $Z = (0.2488, 0.2608, 0.1844, 0.4973)$ and the compromise solution of the chosen alternatives $\Lambda_1 = 0.4781$, $\Lambda_2 = 0.5428$, $\Lambda_3 = 0$, $\Lambda_4 = 1$. Therefore, we can get the priority of the alternatives as $a_3 \succ a_1 \succ a_2 \succ a_4$.

So as to present the results clearly, we give the following table:

TABLE 4. The priority of alternatives with different methods.

Method	The priority of alternatives
EGRA	$a_3 \succ a_4 \succ a_1 \succ a_2$
ETODIM	$a_4 \succ a_1 \succ a_3 \succ a_2$
EVIKOR	$a_3 \succ a_1 \succ a_2 \succ a_4$

Apparently, the obtained optimal decisions by three different methods are different. For the EGRA method, the optimal alternative is a_3. Usually, it is based on the degree of similarity or dissimilarity between the development trends of factors, that is, the "grey correlation degree", as a method to measure the degree of association between factors. It considers the relative comparable degree between the ideal solution and the alternative. It has the advantage of being simple to calculate. For the ETODIM method, the optimal alternative is a_4. It is a typical decision-making method considering the mental behavior of DMs based on the prospect theory. It sorts and optimizes the solution by calculating the dominance of the alternatives over other scenarios. The salient features of it are that it not only accelerates the risk factor in the system, but also enriches the range of decision-making procedure. Moreover, it provides a chance for us to check gains and losses for any two alternatives with regard to any criteria. While for the EVIKOR method, the optimal alternative is a_3. If there is a conflict between the indicators, it sorts the scheme according to a certain method, so as to obtain an optimal solution. Because it maximizes group benefits and minimizes individual losses, it leads to a compromise solution that can be acknowledged by DMs. Moreover, the compromise solution is the optimal solution in the solution space.

D. SENSITIVITY ANALYSIS WITH THE PARAMETER ξ

Let ξ vary from 0 to 1, then we distinguish the variation of three different final priorities by the following figure:

From the figure 2, it is to see when the parameter ξ increases, there are fewer and fewer differences between the schemes of the alternatives. The purpose of decision-making is to choose the preferred alternative among the selected

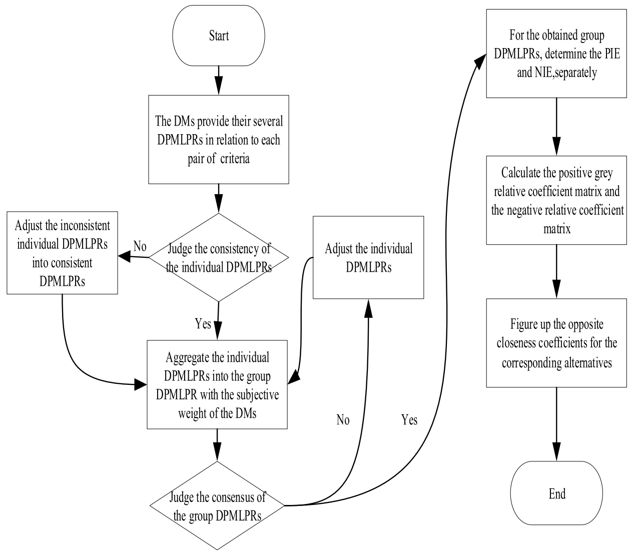

FIGURE 1. The integrated procedure to do the multi-criteria decision-making.

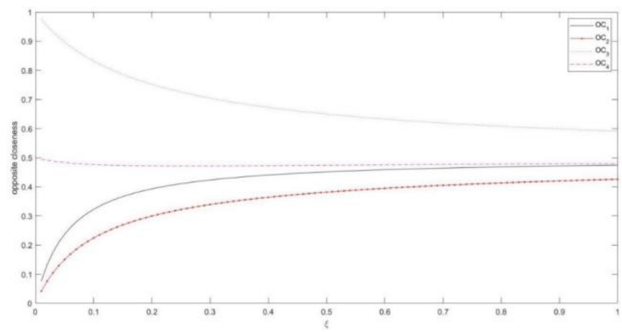

FIGURE 2. The priority of alternatives with the variation of parameter ξ.

alternatives. In this paper, let $\xi = 0.5$, then we not only can obtain the priority of alternatives, but also in the risk neutral status. To some extent, the choice of the parameter is rational.

VI. CONCLUSIONS

In this paper, we have enriched the basic theory of the DPLTSs by putting forward the DPMLTSs and the DPMLPRs, separately. Moreover, we have considered the importance of the consistency of the PRs in the procedure of obtaining the logical decision result, and probed the consistency of the DPMLPRs. Furthermore, on the foundation of the proposed comparable degree between the DPMLPRs, we have researched the consensus of the group DPMLPR. In addition, in order to obtain the final decision result, we have proposed the EGRA method. On the side, we have also developed the ETODIM method and the EVIKOR method based upon the comparable degrees. After that, we have applied the proposed method to settle the problem that mentioned at the beginning of the paper, and helped choose the best cooperative enterprise for cloud enterprise. Finally, the specific execution of the example has demonstrated the effective of the proposed theory. Besides, two comparative analyses have been utilized to highlight the advantages and disadvantages of the proposed method.

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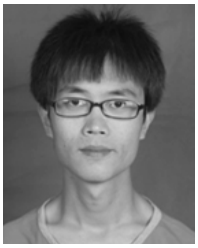

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