q^2-Index: Quantitative and Qualitative Evaluation Based on the Number and Impact of Papers in the Hirsch Core

F.J. Cabrerizo[a,*], S. Alonso[b], E. Herrera-Viedma[c], F. Herrera[c]

[a] *Department of Software Engineering and Computer Systems, Distance Learning University of Spain, Spain*
[b] *Software Engineering Department, University of Granada, Spain*
[c] *Department of Computer Science and Artificial Intelligence, University of Granada, Spain*

Abstract

Bibliometric studies at the micro level are increasingly requested by science managers and policy makers to support research decisions. Different measures and indices have been developed at this level of analysis. One type of indices, such as the h-index and g-index, describe the most productive core of the output of a researcher and inform about the number of papers in the core. Other indices, such as the a-index and m-index, depict the impact of the papers in the core. In this paper, we present a new index which relates two different dimensions in a researcher's productive core: a quantitative one (number of papers) and a qualitative one (impact of papers). In such a way, we could obtain a more balanced and global view of the scientific production of researchers. This new index, called q^2-index, is based on the geometric mean of h-index and the median number of citations received by papers in the h-core, i.e., the m-index, which allows us to combine the advantages of both kind of indices.

Key words: h-index, m-index, bibliometric measures, aggregation, geometric mean

1. Introduction

The evaluation of the scientific output of researchers by means of the computation of bibliometric measures has attracted significant interest, due to the benefits of obtaining an unbiased and fair criterion (Sidiropoulos et al. (2007)). In fact, nowadays, almost every research assessment decision (accepting research projects, contracting researchers, awarding scientific prices, conceding grants and so on) depends to a great extent upon the scientific merits of the involved researchers. To do so, several different indicators have been used.

*Corresponding author

One of the most recent indicators which, in a short period of time, has became extremely popular is the h-index, introduced by Hirsch (2005). It comprises in a single indicator a measure of quantity and impact of the scientific output of a researcher. The original definition was:

> *"A scientist has index h if h of his or her N_p papers have at least h citations each, and the other $(N_p - h)$ papers have $\leq h$ citations each."*

It has attracted a lot of attention among scientometricians and information scientists, and it has been extended by many authors who have proposed new variations of the h-index (van Eck and Waltman (2008); Egghe (2006); Jin et al. (2007); Schreiber (2008b)), it has been applied to a variety of areas (Csajbók et al. (2007); Oppenheim (2007); Rodriguez et al. (2008); Vanclay (2008)) and it has been analyzed in some studies (Egghe (2008); Rousseau (2008); Ye and Rousseau (2008)). Furthermore, Egghe (2009) and Alonso et al. (2009a) have developed two review papers about the h-index, and a comprehensive list of h-index related publications and additional descriptions on the topic can be found at the web page: `http://sci2s.ugr.es/hindex`.

Burrell (2007) points out that the h-index identifies the most productive core of an author's output in terms of the most cited papers. For this core, consisting of the first h papers, Rousseau (2006) introduced the term *Hirsch core* (h-core), which can be considered as a group of high-performance publications with respect to the scientist's career (Jin et al. (2007)). Taking into account the h-core and according to the results of the analysis developed by Bornmann et al. (2008), two types of indices can be assumed:

- The first type of indices describe the most productive core of the output of a researcher and inform about the number of papers in the core. For example, the h-index (Hirsch (2005)), g-index (Egghe (2006)), hg-index (Alonso et al. (2009b)) and $h^{(2)}$-index (Kosmulski (2006)).
- The second type of indices depict the impact of the papers in the core. For example, the a-index (Jin (2006)), m-index (Bornmann et al. (2008)), ar-index (Jin et al. (2007)) and h_w-index (Egghe and Rousseau (2008)).

Bornmann et al. (2008) indicate that the two index types stand for very different dimensions of the scientist's research output, but they can complement each other very well, and they state:

> *"... we propose the use of any pair of indices as a meaningful indicator for comparing scientists, where one index relates to the number of papers in a researcher's productive core (namely, the h-index or g-index – that is, one of the indices with the highest loadings on this factor in the factor analysis) and the other index relates to the impact of the papers in a researcher's productive core (namely, the a-index or m-index – that is, one of the indices with the highest loadings on this factor in the factor analysis)."*

Following this idea, we propose combining both types of indices using the geometric mean as aggregation operator and, in such a way, to combine the advantages of both types of indices while minimizing their drawbacks.

The aim of this paper is to define a new index to characterize the scientific output of researchers, called q^2-index, which is based on the geometric mean of an index describing the number of the papers (quantitative dimension of a researcher's productive core) namely, the h-index and an index depicting the impact of the papers (qualitative dimension of a researcher's productive core) specifically, the median number of citations received by papers in the h-core, i.e., the m-index.

To do so, the paper is set out as follows. In Section 2, we introduce the h- and m- indices, as well as we point out some of their most interesting properties and drawbacks. In Section 3, we present the new index for evaluation purposes, the q^2-index, and we discuss its properties. Section 4 presents a practical example in which the new index is applied and where some of its benefits are shown. Finally, some concluding remarks are pointed out in Section 5.

2. Preliminaries: the *h*- and *m*- indices

The main advantage of the h-index is that it combines a measure of quantity and impact in a single indicator, aspects that traditionally have been measured separately by using different indicators. Another benefit of this indicator is that it is quite simple to compute from the citation data available through the ISI Web of Science (online resource) of the ISI Web of Knowledge. The h-index has been proven to be robust in the sense that it is insensitive to a set of lowly cited papers (Vanclay (2007)). Additionally, increasing the h-index is difficult as each unit increment implies receiving citations in a larger number of papers. Moreover, the h-index is insensitive to one or several outstandingly highly cited papers (which is usually considered as a drawback).

However, the h-index presents other drawbacks that have been pointed out in the literature (Bornmann and Daniel (2007); Bornmann et al. (2008); Costas and Bordons (2007); Jin et al. (2007)). To overcome these issues, several authors have proposed several variants of the h-index, each of them usually centering its attention on a particular aspect of the h-index (van Eck and Waltman (2008); Egghe (2006); Jin et al. (2007); Schreiber (2007a,b, 2008b)). One of the h-related indices is the called m-index. This index, presented by Bornmann et al. (2008), was designed to depict the impact of the papers in the core, while the h-index describes the most productive core of the output of a researcher and informs about the number of papers in the core.

Example 1: Suppose that we want to compare the scientific production of two different researchers. The first researcher has published 20 papers, with the following citation record: 47, 42, 37, 36, 21, 18, 17, 16, 16, 16, 15, 13, 13, 13, 13, 12, 12, 12, 12 and 11. The second researcher has published 20 papers, with the following citation record: 20, 20, 18, 18, 17, 15, 14, 14, 13, 13, 13, 13, 13, 12, 12, 11, 10, 9, 7 and 5. According to the Hirsch definition, both have a h-index of 13, whilst it is obvious that the production of the first researcher has a higher impact. The problem is that the h-index only takes into account the number of papers in the core, but not the impact of each paper in the core.

The m-index (Bornmann et al. (2008)) is defined as the median number of citations received by papers in the h-core. The median is described as the number separating the higher half of a sample from the lower half. Therefore, in this case, the median can be found by arranging all the number of citations received by paper in the h-core in decreasing order and selecting the middle one. The m-index uses the median instead of the arithmetic average because the distribution of citation counts is usually skewed (Bornmann et al. (2008)).

It is easy to prove that $m \geq h$. However, although the m-index is successful in evaluating the production of a researcher incorporating the actual citations of his/her papers in the h-core, it also presents some drawbacks that have to be taken into account.

Example 2: Suppose that we want to compare the scientific production of two different researchers. The first researcher has published 8 papers with the following citation record: 23, 20, 19, 12, 7, 4, 3 and 1. The second researcher has published 15 papers, with the following citation record: 78, 54, 37, 30, 23, 17, 16, 13, 13, 12, 11, 9, 8, 5 and 3. The m-index of first researcher is 19, whereas the m-index of second researcher is 17. However, the scientific production and the number of papers in the core of the second researcher is much greater than the first researcher's one, as it is proven by their h-indices (11 and 5, respectively). Furthermore, if we include a third researcher with a citation record similar to the one of the second researcher but with higher citation rates for his first papers (148, 102, 76, 30, 23, 17, 16, 13, 13, 12, 11, 9, 8, 5 and 3) we can observe that they have the same m- indices. It shows that a few exceptional papers have no influence at all on the m-index.

3. The q^2-index: a new index for evaluation purposes

In this section, we present the q^2-index. It is based on the geometric mean of a quantitative measure (the h-index) and a qualitative measure (the m-index) of h-core. The h-index is used because it is robust and describes the number of the papers (quantitative dimension) in a researcher's productive core, whilst the m-index is used because it depicts the impact of the papers (qualitative dimension) in a researcher's productive core and because it correctly deals with citation distributions which are usually skewed. It can be noticed that the q^2-index is based on two indices which stand for different dimensions of the scientist's research output. Therefore, it obtains a more global view of the scientific production of researchers.

We use the geometric mean because, among its properties, it is easy to compute, it is easily understandable in geometric terms (see Alonso et al. (2009b)), it is not influenced by extremely higher values, and thus, it obtains a value which fuses the information provided by the aggregated values in a more balanced way than other aggregation operators.

The q^2-index of a researcher is computed as the geometric mean of his/her h- and m- indices, that is:

$$q^2 = \sqrt{h \cdot m}$$

It is trivial to demonstrate that $h \leq q^2 \leq m$ and that $q^2 - h \leq m - q^2$, that is, the q^2-index corresponds to a value nearer to h than to m. This property can be seen as a penalization of the m-index in the cases of a very low h-index.

Example 3: In this example, we make use of the data from example 2. The q^2-index of the first researcher is $\sqrt{5 \cdot 19} = 9.75$ ($h = 5$, $m = 19$) and the q^2-index of the second researcher is $\sqrt{11 \cdot 17} = 13.67$ ($h = 11$, $m = 17$). It can be seen how de q^2-index has minimized the effect of a high m-index in the case of a low h-index.

Some advantages of this new index are the following:

- It is very simple to compute once the h- and m- indices have been obtained.
- It provides more granularity than the h-index: The h-index only takes integer values (whilst the q^2-index takes real ones) and, additionally, it is usual to find many cases researchers with very different citation records and production with the same h-index. Moreover, to increase the h-index of a researcher is more difficult than to increase the q^2-index, as any increase in any of the h- or m- indices implies an increase in the q^2 index.
- It takes into account both the quantitative and qualitative dimensions of the researcher's productive core and, therefore, it obtains a more global and balanced view of the scientific production of researchers than if we use the h- and m- indices separately.

4. Case of study based on researchers in Fuzzy Logic and Soft Computing

In this section, we analyze the behavior of the q^2-index and show its benefits in comparison with the h- and m- indices in a real world example where some authors specialized in the fuzzy logic field are compared. Two of the most well recognized journals in the topic are *Fuzzy Sets and Systems* (Dubois and de Baets (online resource)) and *IEEE Transactions on Fuzzy Systems* (Pal (online resource)). Furthermore, we study the correlation among these indices.

In Merigó and Gil-Lafuente (2009), the top authors in the fuzzy logic field, obtained from the ISI Web of Knowledge are presented. We part from the fifteen most cited researchers and we compute the h-, m- and q^2- indices for each one of them. In Table 1, we rank the different researchers according to the h-, m- and q^2- indices. This information has been collected in May 2009 from the Science Citations Index provided by Thomson Scientific in the ISI Web of Science (WoS).

We should point out that the q^2-index provides more granularity than any of the h- and m- indices separately. This is an advantage as it allows to provide a better rank between the researchers. Furthermore, we can see that the q^2-index presents a better characterization of the scientific output of the researchers:

	h		m		q^2
H. Prade	45	L.A. Zadeh	186	L.A. Zadeh	79.52
R.R. Yager	41	M. Sugeno	78	H. Prade	57.31
D. Dubois	41	D. Dubois	75	D. Dubois	55.45
J.C. Bezdek	39	J.C. Bezdek	74	R.R. Yager	54.35
F. Herrera	38	H. Prade	73	J.C. Bezdek	53.72
L.A. Zadeh	34	H. Ishibuchi	73	F. Herrera	49.70
J.M. Mendel	33	R.R. Yager	72	W. Pedrycz	45.96
W. Pedrycz	33	F. Herrera	65	H. Ishibuchi	45.21
S.K. Pal	28	W. Pedrycz	64	J.M. Mendel	44.12
H. Ishibuchi	28	J.M. Mendel	59	M. Sugeno	40.47
J.J. Buckley	27	S.K. Pal	45	S.K. Pal	35.50
N.R. Pal	22	J.J. Buckley	44	J.J. Buckley	34.47
M. Sugeno	21	N.R. Pal	39	N.R. Pal	29.29
D.A. Linkens	21	R. Lowen	38	D.A. Linkens	28.25
R. Lowen	20	D.A. Linkens	38	R. Lowen	27.57

Table 1: Researchers ranked by their h-, m-, and q^2- indices.

- For example, if we compare N.R. Pal with respect to M. Sugeno, we notice that they present a similar h-index (22 and 21, respectively) whilst their m-indices are quite different (39 and 78, respectively). This is detected by the q^2-index, which awards M. Sugeno with respect to N.R. Pal.
- Similarly, if we compare H. Ishibuchi with respect to R.R. Yager, we notice that they present a similar m-index (73 and 72, respectively), whilst their h-indices are very different (28 and 41, respectively). This also is detected by the q^2-index, which awards R.R. Yager with respect to H. Ishibuchi.
- Fuzzy set theory appeared in 1965 with the L.A. Zadeh's seminal paper (Zadeh (1965)). Obviously, the most cited researcher is L.A. Zadeh, as it is shown in Merigó and Gil-Lafuente (2009) and it is proved by his high m-index. However, the h-index does not show the leading position of L.A. Zadeh while his q^2-index ranks him in the first position with a softened distance respect to the following experts in fuzzy research, such as H. Prade, D. Dubois and R.R. Yager, which have a higher h-index.

The h- and m- indices, when are considered separately, do only take into account either the number of the papers in a researcher's productive core or the impact of the papers in a researcher's productive core. However, the q^2-index distinguishes better among researchers because it merges both the quantitative and qualitative dimensions of the papers in the productive core.

The fact that the h- and m- indices measure very different dimensions of the scientist's research output can be proved by the weak correlation between these indices. To quantify it, as it is not clear whether the values of the indices follow a normal distribution, we have computed Spearman's rank-order correlation coefficients ρ (Demsar (2006); García and Herrera (2008); Schreiber (2008a)). In Table 2, we show the Spearman's rank-order correlation coefficients among all the studied indices for this example.

ρ	h	m	q^2
h	1.000	0.576	0.907
m	-	1.000	0.818
q^2	-	-	1.000

Table 2: Spearman's rank-order correlation coefficients ρ

The correlation between the h- and m- indices is 0.576, which is quite low and can be explained because the h-index of some researchers is low while the m-index is high: a few papers in their h-core have received many citations and, therefore, the median number of citations received by papers in the h-core is also high. However, the correlation between the q^2- and h- indices is 0.907 and between the q^2- and m- indices is 0.818, i.e., the correlation between the new index, the q^2-index, and the h- and m- indices is high. The low correlation between the h- and m- indices, and the high correlation among both h- and m- indices and q^2-index, shows how the q^2-index efficiently merges the information provided by h- and m- indices separately. To visualize the correlation among the indices, Figure 1 shows the h- and m- indices in dependence on the q^2-index.

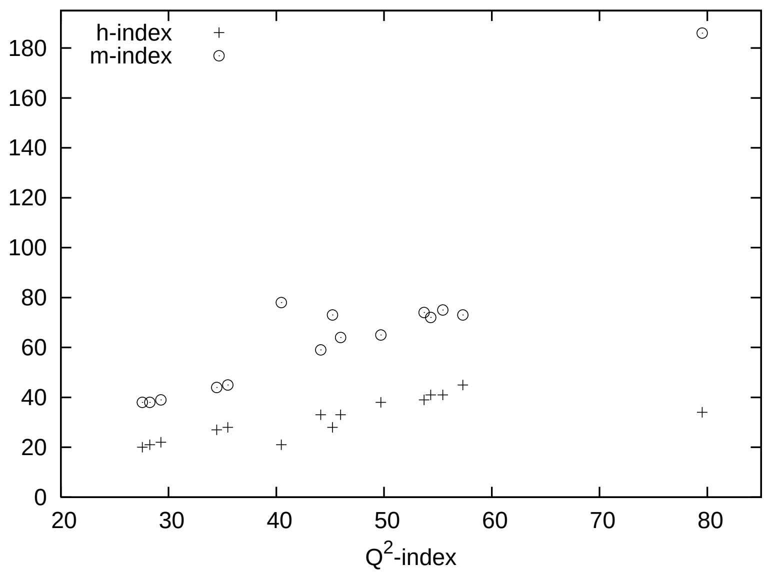

Figure 1: Scatter plot of h- and m- indices versus q^2-index

Hence, the weak correlation between the h- and m- indices justifies the use of the q^2-index, which satisfactorily combines both quantity and quality dimensions of h-core.

5. Concluding Remarks

In this paper, we have presented a new index, the q^2-index, which is based on two indices which stand for very different dimensions of the scientist's research output: the h-index, which describes the quantitative dimension of the papers in the h-core, and the m-index, which depicts the qualitative dimension of the papers in the h-core. Furthermore, we have shown some good properties of this index by means of a real world example where some authors specialized in the fuzzy logic field are compared.

Finally, we point out the good behavior of the q^2-index, which provides a more balanced view of the scientific output of researchers and, consequently, we could evaluate them better.

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