Perspective of the statistical education research based on the onto-semiotic approach

Perspectiva de las investigaciones sobre educación estadística realizadas en el marco del enfoque ontosemiótico

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Abstract

I present a review of the research carried out on the teaching of statistics, probability and combinatorics by analyzing the use made of the Onto-semiotic Approach (OSA) of mathematical knowledge and instruction theoretical tools. I also analyze the role played by research on statistical education in the construction of the OSA, as well as the possibilities that the progressive refinements of this theoretical framework may offer to extend and deepen research in this field.

Keywords: statistical education, theoretical foundations, onto-semiotic approach

Resumen

Se presenta una revisión de los trabajos de investigación realizados sobre la enseñanza de la estadística, probabilidad y combinatoria analizando el uso que se hace en los mismos de las herramientas teóricas del Enfoque Ontosemiótico (EOS) del conocimiento y la instrucción matemáticos. Se analiza, además, el papel desempeñado por las investigaciones sobre educación estadística en la construcción del EOS, así como las posibilidades que pueden ofrecer los progresivos refinamientos de este marco teórico para extender y profundizar en la investigación en dicho campo.

Palabras claves: educación estadística, fundamentos teóricos, enfoque ontosemiótico,

1. Introduction

The development of the Ontosemiotic Approach in Mathematics Education (OSA) (Godino, 2017; Godino & Batanero, 1994; Godino, 2002; Godino, Batanero, & Font, 2007) has been closely linked to its applications to Statistics Education. Consequently, in the article "Institutional and personal meaning of mathematics objects" (Godino & Batanero, 1994), which is considered as the OSA "seed", the concept of arithmetic average is used to contextualise the reflexion about the key notions of mathematics problems, practices, object and meaning.

Furthermore, in the synthesis of the OSA "An ontological and semiotic approach of mathematics cognition" (Godino, 2002) the concept of median is applied to present an technique to analyse the teaching and learning processes that enabled us to determine the institutional and personal meanings used and to identify possible semiotic conflicts in didactic interaction. To be more precise, the process of instruction proposed for the median in a textbook, and one student´s answers to a test, which was applied after carrying out the aforementioned study process, are analysed.

The different research projects and doctoral thesis directed by Carmen Batanero at the University of Granada have been focused on the field of statistics education and, as she is coprotagonist of the elaboration of the OSA, these theses have used the OSA as theoretical framework. Therefore, in this presentation for the CIVEEST conference I have decided to make a retrospective analysis of the role carried out by the different tools of

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the OSA in the works of research on statistics education, many of which have been carried out in University of Granada Statistics Education Research Group. I will also try to suggest possible advances in research on teacher training in statistics education, based on the OSA recent developments, in particular, on the "Didactic-mathematical knowledge and competences" (DMKC) model (Godino, Giacomone, Batanero, & Font, 2017).

The presentation is organized in the following sections. In section 2 the publications that will be analysed and classified into five sections corresponding to the tools of the OSA used and to the statistics topics dealt with, are included. In section 3 the notion of pragmatic meaning of a mathematics object and some publications where this tool is applied are summarised, and in the following sections the publications that use the tools, ontosemiotic configuration, instructional design (configurations and didactic trajectory), didactic suitability and teacher training model, are described.

2. Methodological approach

The OSA tools, whose use we are going to analyse in the statistics education publications are the following:

1. Pragmatic meaning of the mathematics objects, in their institutional and personal version.
2. Ontosemiotic configuration of practices, objects and processes.
3. Didactic configuration. Instructional design
4. Didactic suitability
5. Knowledge and Competences Didactic-mathematical Model (KCDM)

These tools have been developed to date to analyse the epistemological, ontological, semiotic-cognitive, educational-instructional and ecological tools implied in mathematics teaching and learning processes, to approach the problem of mathematics teacher training. In Table 1 I include a selection of papers published, classified according to the OSA tools that they use and the statistics topics they dealt with, which will be analysed in the following sections.

Table 1. Selection of publications according to the OSA tools and the statistics content

OSA Tool	References	Statistics content
Pragmatic meanings	- Godino and Batanero (1994). Institutional and personal meaning of mathematics. - Godino (2002). An ontologic and semiotic approach to mathematics cognition. - Batanero (2005). Meanings of probability in secondary education	Mean Median Probability
Ontosemiotic Configuration	- Batanero (2005). Meanings of probability in secondary education. - Godino (2013a). Design and analysis of tasks for the development of didactic-mathematic knowledge of teachers	Probability
Instructional design	- Tauber, Batanero, and Sánchez (2004). Design, implementation and analysis of a teaching sequence of normal distribution in a university course. - Alvarado and Batanero (2007). Difficulties in understanding the normal approximation to the binomial distribution. - Godino, Rivas, Arteaga, Lasa, and Wilhelmi (2014). Didactical engineering based on the ontologic-semiotic approach of knowledge and mathematical instruction.	Normal distribution Binomial and normal distribution Descriptive statistics
Didactic	- Arteaga, Batanero, Cañadas, and Gea (2012). Evaluating the	Data

suitability	specialised knowledge of statistics in prospective teachers through the analysis of a statistics project. - Arteaga, Contreras, and Cañadas (2014). Knowledge of statistics and students in prospective teachers: an exploratory study. - Arteaga, Batanero, and Gea (2017). The mediational component of the didactic-mathematical knowledge of prospective teachers about statistics: an exploratory evaluation study. - Godino, Rivas, and Arteaga (2012). Inference of didactic suitability indicators from curricular orientations. - Beltrán-Pellicer and Godino (2017). Application of affective suitability indicators in a secondary education probability teaching process. - Beltrán-Pellicer, Godino, and Giacomone (2018). Elaboration of specific suitability indicators for probability: Application for reflecting on teaching practice.	Analysis Statistics graphs Random Projects Elemental statistics and probability Probability
DMKC Model (teacher training)	- Arteaga, P., Batanero, C., Cañadas, G. and Gea. M. (2012). Evaluation of prospective teachers 'specialised knowledge through using the analysis of a statistics project. - Godino (2013a). Design and analysis of tasks to develop teachers' Didactic-mathematical knowledge. - Vásquez, C., and Alsina, A. (2015). Teachers´knowledge of teaching probability: A global analysis from the Didactic-Mathematical Knowledge Model - Vásquez, C. and Alsina, A. (2017). Approximation to common knowledge of content to teach probability from the Didactic-Mathematical Knowledge model. - Arteaga, P., Batanero, C., Contreras, J. M., and Cañadas, G. R. (2015) Statistical graphs complexity and reading levels: a study with prospective teachers. - Batanero, C. Arteaga, P., Serrano, L., and Ruiz. B. (2014). Prospective primary school teachers' perception of randomness. - Arteaga, P., Contreras, J. M., and Cañadas, G. (2014). Knowledge of statistics and students in prospective teachers: an exploratory study. - Godino, J. D., and Batanero, C. (2008). Teacher training based on guided reflexion of the practice. - Godino, J. D., Batanero, C., Roa, R., and Wilhelmi, M. R. (2008). Assessing and developing pedagogical content and statistical knowledge of primary school teachers through project work. - Ortiz, J. J., and Font, V. (2014). Pre-service teachers' common content knowledge regarding the arithmetic mean.	Probability Statistics graphs Data Analysis Randomness Statistics Probability; data anlysis Arithmetics mean

3. Pragmatic meaning

The notion of meaning of a mathematical object interpreted in a pragmatic way is essential in OSA. So, in Godino and Batanero (1994), the following definitions are introduced,

> Meaning of an institutional object IO: It is the system of institutional practices associated with the field of problems from which IO emerges at a given time.
>
> Meaning of personal object PO: It is the system of practices of a person p to solve the field of problems from which the PO object emerges at a given time.

In Godino, Beltrán-Pellicer, Burgos and Giacomone (2017) we suggest that, in this interpretation, you transit from the mathematical object, which initially is a "black box", a label that refers to a mental, ideal or abstract entity, to the practices implied in the use

of this object, which is really a creative interpretation of Pierce´s pragmatic maxim:

> 402. It appears, then, that the rule for attaining the third grade of clearness of apprehension is as follows: Consider what effects, that might conceivably have practical bearings, we conceive the object of our conception to have. Then, our conception of these effects is the whole of our conception of the object. (Peirce, CP 5.402)

This notion of meaning permits us to introduce the study of the social (respectively, personal) practices system structure), from which the mathematical objects emerge, in addition to their temporary evolution and institutional (personal) dependence into the epistemological and didactic problematic.

Godino (2002) propose a first interpretation of the meaning of an object and leaves in second place the notion of practice. The attention is focused on the different types of objects that intervene in the practices, which are considered as the *elements of the meaning*, that is, like the components of the system of operative and discursive practices: lingüistic elements, situations, procedural, conceptual, propositional and argumentative[1]. In the annex of this paper, *the meaning of reference of the median* is described, by using a list of the different elements, which are characteristic of each of the six types of primary objects.

Batanero (2005) follows this interpretation of the meaning of an object to analyse the notion of probability. She affirms, "The meaning of the object would be a composed entity, made up of the set of operative and discursive practices related to this field of problems" (p. 249). For the case of probability, she differentiates between the set of practices linked to the solution of the corresponding field of problems (study, analysis and prediction of random phenomena), which would include all that we do and say to solve this type of problem, and the mathematical object probability, which has historically emerged and continues to evolve as a result of these practices. However, she is aware that one same mathematical object can involve itself different networks of other mathematical objects as well as various levels of complexity and so, its meaning can be different in diverse professional or educational institutions. In this sense, Batanero (2005) describes different meanings of probability related to different moments or historical stages of the development of this mathematical object: intuitive, laplacian, frequentist, subjetive and axiomatic-formal meanings. These different meanings are distinguished from one another by considering that different objects intervene in the practices carried out. The analysis of the different meanings of an object is focused on the identification of the elements que characterize the different meanings.

When solving problems or mathematics tasks it is considered necessary to analyse the mathematics practices carried out with the support of different languages, trying to show the synergic relationships between the same and the different types of non-ostensive objects that necessarily accompany the different representations. To carry out this analysis the OSA, starting from the elementary types of objects primitively defined, widens this typology and introduce the facets and associated processes as well as the notion of ontosemiotic configuration which is described to follow.

4. Ontosemiotic configuration

In the notion of *ontosemiotic configuration* of practices, objects and processes (Godino, Batanero, & Font, 2007), the different types of objects according to their nature and

[1] In Godino (2002), the procedural objects are named as actuative and the argumentative as validative objects.

function are classified in the following categories:

- Languages (terms, expressions, notations, graphs) in their different registers (written, oral, gestual, etc.).
- Problems- situations (intra or extra-mathematic-applications, exercises).
- Concepts-definition (introduced using definitions or descriptions, like straight, point, number, media, function).
- Propositions (statements about concepts).
- Procedures (algorithms, operations, calculus techniques).
- Arguments (statements used to validate or explain the propositions and the deductive or other types of procedures).

These objects, considered as primary, can be contemplated from different points of view or polarities:

1. Ostensive (materials, visible) and non-ostensive objects (abstract, ideal, immaterial).
2. Extensive (particular), intensive objets (general).
3. Personal (relative to individual subjects), institutional (shared in an institution or community of practices).
4. Signifier or signified (meanings) (antecedent or consequent of a semiotic function).
5. Unitary (objets considered globally as one entity) and systemic (considered as systems formed by structured components).

The primary as well as the secondary objects (derived from the application of the dualities) can be considered from the process-product perspective, which provides criteria to distinguish types of primary and secondary mathematical processes. As a result, we have processes of problematization, enunciation, statement, argumentation, particularization-generalization, representation-interpretation (meaning), etc.

Rondero and Font (2015) tackle the problem of mathematics objects complexity, due to their different (pragmatic) partial meanings, which can be identified and to the epistemic configurations associated with each partial meaning, for the case of arithmetic mean. They also deal with the problem of articulating these partial meanings to be able to finally identify the object in a unitary way. The analysis which the authors carried out puts the notion of epistemic configuration into the foreground, by applying it to the identification of the configurations linked to the arithmetic mean.

Rondero and Font (2015) identify the following different intramathematical and extramathematical contexts for the arithmetic mean, to each of which they assign a set of mathematical practices and an epistemic configuration: a) the Babylonic method and Heron method to calculate roots, (CE1); b) the Arquímedes equilibrium of processes method, which is a basic principle of statics (CE2); c) sequences, and arithmetic and geometric progressions (CE3); d) Merton's method to study movement (CE4), and e) calculation of areas, quadrature methods, etc. (CE5). In spite that these configurations are different, they also present articulations, in such a way that they can be related, basing on a highest degree of generality. These partial configurations are articulated in two separate sets: the statistics configuration CE6 and the calculus of defined integrals CE7, from all

of which a global reference *arithmetic mean* object emerges.

The authors suggest that the unitary-systemic duality, and the articulation of relationships by means of generalization, help to show the complexity of the arithmetic mean, as well as the multiplicity of spaces where the mean intervene within mathematics. The link between the different meanings and their different epistemic configurations is made by taking into account different degrees of generality, as well as using different metaphoric projections (interpret a meaning in terms of others).

In relation to the ontosemiotic configuration, the notion of semiotic function enables us to analyse the processes of interpretation and meaning (semiosis), characteristics of mathematics activity, which is basically a relational activity. The notion of semiotic function is understood as the correspondence between an antecedent object (expression, significant) and another consequent (content, meaning) established by a subject (person or institution) according to a criteria or rule of correspondence.

Thus, Batanero (2005) analyses the reasoning process that a student follows (Figure 1) when solving a probability problem by applying the notion of semiotic function and the types of primary objects; she presents it as the "Semiotic analysis of the student´s solution" (p. 258).

Problem 1. A group of school students sit an English and Mathematics exam respectively. The proportion of students who pass Mathematics is 80% and English 70%. Supposing that the marks of the subjects are independent, what is the probability that one student randomly chosen has passed both subjects?

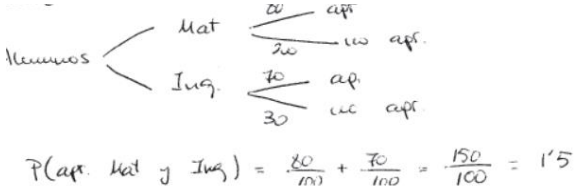

Figure 1. One student´s reply

In Godino (2013a), different OSA tools are applied to analyse the design of tasks directed towards the mathematics and didactic training of teachers. One of these tasks asks to analyse the knowledge used when solving questions related to a fair game based on the results of throwing two dice. In the a priori analysis of the expected result (Figure 2) the determination of knowledge in terms of identification of the mathematics objects involved (first column) and their meanings (second column) is carried out by either referring to the objects or to the role that these objects play in the solution process.

Objects:	Meanings (reference/use):
Situation-problems - Determine whether a game is fair (addition of points when two dice are thrown) - Analyse the data obtained in an empirical experience with the game.	- Development of calculation of probabilities competences - Reflexion on probabilistic intuitions in gaming situations - Development of data analysis competences - Reflexion on the properties of the convergence in small samples
Linguistic elements Terms and verbal expressions	- Words with mathematical meaning; for example, fairness, probability, results.

Figure 2. Recognition of objects and meanings

5. Design and instructional analysis. Didactic configurations and trajectories

The main tool for the microanalysis of the instruction processes is the notion of didactic configuration (Godino, Contreras y Font, 2006), which is defined as any segment of didactic activity (teaching and learning) included between the beginning and the end of a task (situation-problem). So, it includes the teacher´s and the student´s actions as well as

the means planned or used to carry out the task. The sequence of didactic configurations make up a didactic trajectory.

The task that distinguishes a didactic configuration can be formed by different subtasks, each of which can be considered as a sub configuration. In every didactic configuration there is: a) an epistemic configuration (system of practices, objects and institutional mathematics processes required in the task), b) an instructional configuration (system of teaching and learning functions and instructional means used, as well as the interactions between the different components) and c) a cognitive-affective configuration (system of practices, objects and personal mathematics processes that describe the learning and the affective components that accompany it). To follow we analyse three research works in which instructional processes for statistics content are applied, designed, implemented and assessed using theoretical notions of the OSA.

5.1. Teaching the normal distribution

In Tauber, Batanero and Sánchez (2004) a synthesis of the results of Tauber´s doctoral thesis is presented. This thesis was written during the period 1997 – 2000, and was focused on an evaluation of the teaching and learning process in an introductory statistics course, offered as free choice content for students of different university degrees. The instructional process was supported by the use of computers and the advances, difficulties and errors that the students presented along the experience were analysed.

The theoretical tools of the OSA which are applied are basically the notions of institutional and personal meaning of the mathematical objects, used to characterize the meaning of reference of the normal distribution. The meaning of mathematical objects is conceived as the system of practices linked to fields of specific problems; these meanings are characterized by using the distinction of five types of primary mathematics objects, considered as elements of the meaning. The types of objects are named following a first formulation of these primary objects: extensive elements (situations – problems), ostensive elements (languages), actuative elements (procedures), intensive elements (concepts and propositions), validative elements (arguments).

It is assumed that these elements can be varied, according to the institutional and personal contexts in which the solving of a field of problems is faced. The normal distribution for example does not have the same meaning in secondary education as in a statistics courses for researchers or for a statistician who is developing new methods of inference. This leads to differentiating between *institutional meanings and personal meanings*, depending on whether we are dealing with socially shared practices or to the contrary, we are referring to idiosyncratic responses of an individual subject. This suject, when dealing with solving certain types of problems, builds a personal meaning of the mathematics objects involved. When this subject becomes part of an institution (for example, the university) it is possible that the practices acquired individually come into conflict with those admitted for the the said object in the heart of the institution, and so a process of progressive coupling between personal and institutional meanings should be carried out. These processes should lead to a gradual understanding of a mathematics object.

5.2. Approximation of normal distribution to the binomial

In Alvarado and Batanero (2007) the theoretical and practical understanding of the approximation of the binomial distribution to the normal distribution reached by a group of 134 engineering students after an experiment of teaching supported by the use of Excel,

is studied. The use of manipulative material and the computer as reinforcement of the theorem is included in the design. In addition, a previous historical study and of textbooks was carried out to characterize the different meanings of the theorem and which of then should be acquired by the students.

In this paper the notion of didactic configuration, introduced in the OSA by Godino (2003), was used. The authors indicated that the instructional process was organised by taking into account three types of configurations: manipulative, computational and algebraic.

- In manipulative configuration the student works with dice, coins, tokens, paper-pencil or calculator, without using algebraic notation or calculation. The aim is to simulate random experiments, write down the results, study the resulting statistics variables, their distribution and moments and empirically obtain some simple sample distributions. The preferential type of proof is by using the study of examples and counterexamples. The procedures of solution of the activities are empirical and graphical and the language is reduced to verbal and graphical expressions.
- The *computational configuration*, based on computers, notably widens the number and variety of dynamic graphical representations; in addition to the iconic language, it incorporated the simulation and permits to work with the statistics and random variables simultaneously, as well as introducing the ideas of approximation and goodness of fit. By varying the parameter values the effect on the form of simple distribution and goodness of fit, can be studied. The preferred argument is inductive as well as the study of examples, counterexamples and generalization.
- The *algebraic configuration* is characterised by the symbolic language and the deductive demonstration, as well as the algebraic and analytical resource. The procedures are analytical and algebraic, in addition to incorporating the use of distribution tables to calculate probabilities. The most formalized ideas of convergence, moments and typification are included. The demonstrations are deductive.

The evaluation of the learning achieved by the students is done by using the design and application of a questionnaire with five ítems and an open problem.

5.3. Introducing statistics to primary school teachers

In Godino, Rivas, Arteaga, Lasa and Wilhelmi (2014) an instructional process directed to primary school teachers on statistics notions is describedo in order to illustrate the application of the OSA system of theoretical tools and as a version of didactic engineering. The phases of the preliminary study; design, implementation and assessment, analysed according to the epistemic, cognitive and instructional dimensions, are taken into account. The theoretical tools used enable us to reveal *significant didactic facts*, which at the same time determine, on one hand, guidelines to determine the adequate didactic trajectories for the teaching of the statistics topic and on the other hand, basics to evaluate didactic engineering as a methodology of research which can be transferred to different theoretical approaches.

6. Didactic suitability

The didactic suitability of an instruction process is defined as the degree in which the said process (or part of the same) groups together certain features which enable us to call it optimum or adequate to achieve the adaptation between the different personal meanings achieved by the students (learning) and the institutional meanings intended or implemented (teaching), by taking into account the available circumstances and resources (setting). This supposes the coherent and systematic articulation of six facets or dimensions: the epistemic, ecological, cognitive, affective, interactional and mediational facets (Godino, 2013b; Godino et al., 2007). The systems of empirical indicators identified in each facet guide the systematic analysis and reflexion that certain criteria provide for the progressive improvement of the teaching and learning processes.

6.1. Levels of recognition of epistemic, cognitive, affective and mediational suitability

In this section three articles, which inform of the results of primary education teacher training experiences, in which they live a learning experience of statistics and afterwards they are asked to assess different facets of the didactic suitability of the said training process, are described. Previously the teachers have received instruction about the notion of didactic suitability, its components and indicators, in such a way that in this research the focus of attention is assessing the degree of the students´ understanding of the didactic suitability tool. Indirectly, there is some access to partial aspects of their specialised didactic-mathematic knowledge on teaching and learning of statistics.

In Arteaga, Batanero, Cañadas and Gea (2012) the specialised knowledge of elementary statistics of 108 prospective primary education teachers, is assessed, using the notion of epistemic suitability applied to a data analysis project. The students are proposed to assess the epistemic suitability, for a statistics study process based on the solution of a data analysis project, taking into account the different components and descriptors proposed in Godino (2009),. The concept of didactic suitability and the use of the guideline had been studied by the participants as part of the subject in a previous session, using examples of application of the different descriptors to other topics. It is required that a prospective teacher should know the mathematics objects proposed to teach the topic and he/she should be able to recognise their presence and adequate use during the development of the teaching. This analysis of the epistemic suitability enables the assessment and at the same time to develop specialised knowledge of statistics.

The authors developed a hierarchy with criteria to define three degrees or levels of recognition and application for the different descriptors of epistemic suitability by the prospective teachers, with level 0 of no knowledge. The system of indicators used in these work is later extended by Gea (2014) to five levels. This procedure enables the authors to assign each one of the students with a level of achievement in the competence of assessing the epistemic suitability of the instruction process experienced, and thus of his/her specialised knowledge of the content. Arteaga et al. (2012) results show that the specialised knowledge in their sample of teachers is scarce, as on average participants do not even reach level 2 (anecdotic application), although a proportion of them that vary between 10% and 35% according to the descriptors reach the highest level of the application.

In Arteaga, Contreras and Cañadas (2014) a study similar to the aforementioned is carried out (Arteaga, et al., 2012) focusing the attention on developing a hierarchy to distinguish levels of competence in the recognition of the indicators of cognitive and affective suitability by the same sample of prospective primary education teachers. The

information from this new study indicates that, although certain descriptors of suitability were easy to apply, overall, the future teachers of the sample had difficulty in assessing the cognitive and affective suitability. Indirectly it can be seen that the participants show scarce knowledge of the cognitive and affective facets of the knowledge of statistics as they are unable to appreciate the students' previous knowledge, their learning or their attitudes towards statistics.

In Arteaga, Batanero and Gea (2017) the experience described in the two works of research we have mentioned are repeated with a new sample of prospective teachers, implementing the same process of instruction based on carrying out a data analysis project. It is intended to put the future teachers in a situation of reflexion about the didactic suitability of the process of study experiences, although in this case, the core is focused on the evaluation of the mediational suitability, that is, of the use of didactic resources in statistics teaching. The assessment of the knowledge of the future teachers of the role of resources in the process of study experienced is also done by developing a hierarchy with three levels of achievement and applying it to the answers given by the students. The data in this new study indicate that, although there are certain descriptors that are easy to apply, overall, the future teachers of the sample also have difficulties in assessing the mediational suitability, which supposes scarce knowledge of statistics teaching as they are unable to evaluate the resources, student organization and the time required for the study.

6.2. Specific suitability indicators for instruction in statistics and probability

Godino, Rivas and Arteaga (2012) describe a methodology directed to the progressive improvement of evaluation instruments of the mathematics instruction processes, based on content analysis of the proposed curricula. The units of analysis are classified depending on the facets and components proposed in the Theory of Didactic Suitability, in order to identify norms and suitability indicators, which are confronted with the system proposed by this theory, so as to identify agreements and complementarities. This methodology is applied, as an example to the Principles and Standards for Mathematics Education (NCTM, 2000), in the area of statistics for the levels K-8.

Using content analysis, in a first phase the text is divided into units of analysis, which are classified according to the facets and components that the Theory of Didactic Suitability proposes. To follow, in a second phase, the said units are compared among themselves and reduced to avoid reiterations. Later, from the analysis of the norms taken from the standards, indicators of didactic suitability for mathematics instruction processes, individualized for specific cases to statistics, are inferred. These indicators are compared with those proposed in Godino (2013b), which gives rise to a revised version, and eventually, extend these indicators. The aim of the described methodology is developing guideline or guide of "refined" suitability indicators, while at the same time limitations and complementarities between both instruments are identified.

Beltrán-Pellicer, Godino and Giacomone (2018) present an Assessment Guide of Didactic Suitability (AGDS) for the study of probability in secondary education. The aim is to produce an instrument that promotes the teacher´s reflexion of specific teaching-learning experiences of probability content. The method of research takes as a starting point, the systematic review of the didactic-mathematic knowledge of each facet in which an educational process is divided: epistemic-ecological, cognitive-affective and instructional (interactional and use of technological resources). To follow, the AGDS is applied to a didactic experience with secondary education students. The results of the said application

reveal this tool potential to enable the reflexion on the practice itself, to establish relationships between the different facets and identify possible improvements in the design of successive teaching cycles.

7. Teacher training

A theoretical model of mathematics teacher knowledge (model DMK) has been developed in the OSA framework. This model was initially introduced as a system of analysis categories (Godino, 2009), and was refined and applied in different research works (Pino-Fan & Godino, 2015). One of the developments perspectives for the DMK model is linking the notion of knowledge to that of teacher competence. Additionally, several works of research have been carried out also in the OSA framework, focussing on the mathematics teachers' competences (Font, Breda & Sala, 2015), which have shown the need to count on a teacher knowledge model to be able to evaluate and develop their competences. These two lines of research have come together to generating the model teacher Didactic-Mathematic Knowledge and Competence model (DMKC), recently described in Godino et al. (2017).

The DMKC model is an extension of the DMK model, in which it is proposed that the teachers 'didactic-mathematic knowledge can be organized or developed in accordance with the mathematics, didactic and meta didactic-mathematic dimensions (Figure 3). The mathematics dimension refers to the knowledge that a teacher should have of the school mathematics that he/she teaches; the second dimension refers to the knowledge of aspects involved in the processes of teaching and learning of mathematics (deep knowledge of school mathematics and its interaction with students' cognitive and affective aspects, resources and means, interactions in the classroom and ecological aspects). The meta didactic-mathematical dimension indicates the knowledge that a teacher needs to be able to systematize the reflexion on his/her practice and give valuable judgement about that practice or that of others (Breda, Pino-Fan, & Font, 2017).

The two key competences of mathematics teachers in the DMKC are the mathematics competence and the analysis and didactic intervention competence, whose nucleus (Breda et al., 2017, p. 1897) consists of: "Designing, applying and evaluating their own and other learning sequences, using techniques of didactic analysis and quality criteria, in order to establish planning, implementation and evaluation cycles and to propose improvement". In order to develop this competence the teacher needs, on the one hand, knowledge that enables him/her to describe and explain what has happened in the teaching and learning process, and on the other hand, knowledge to evaluate what has happened and make improvement proposals for future implementations.

To sum up, this mathematics teacher's overall competence of analysis and didactic intervention is made up of five sub-competences (Figure 4) which are associated to five conceptual and methodological tools of the OSA, whose synthetic description can be found in Godino et al. (2017): a) competence for analysing global meanings (based on the identification of situations-problems and the operative, discursive and normative practices implied in their solution); b) competence for ontosemiotic analysis of the practices (identification of the objects and processes implied in the practices); c) competence for managing the configurations and didactic trajectories (identification of the patterns of interaction between the teacher, the student, the content and the resources); d) competence for normative analysis (recognition of the set of norms and metanorms that condition and support the instructional process); and e) competence for analysing the didactic suitability (evaluation of the instructional process and identification of potential

improvement).

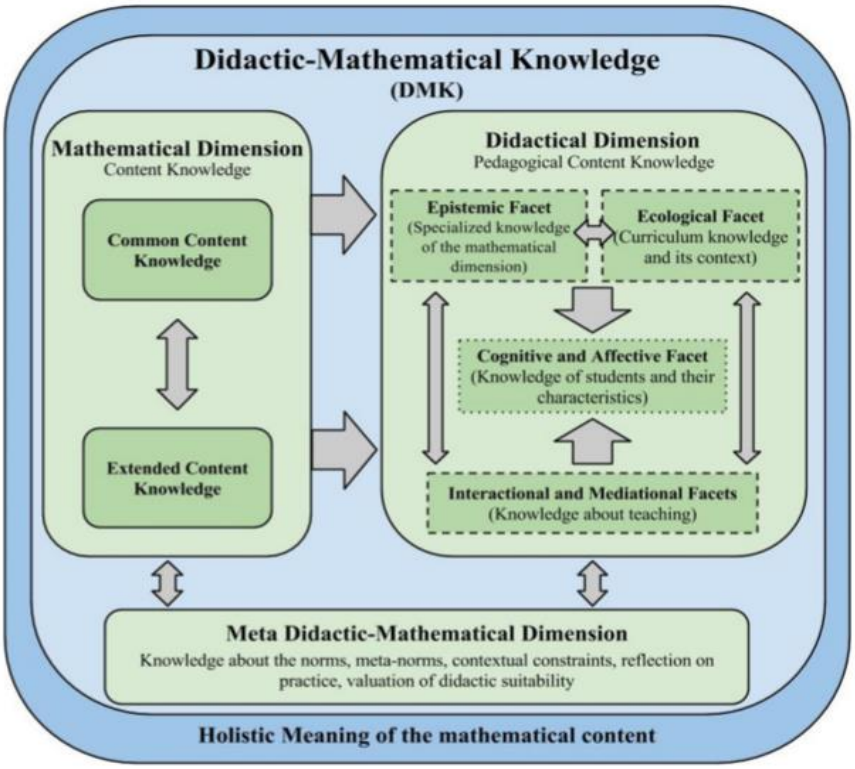

Figure 3. Dimension and Components of Didactic-Mathematic Knowledge (DMK) (Pino-Fan & Godino, 2015, p. 103)

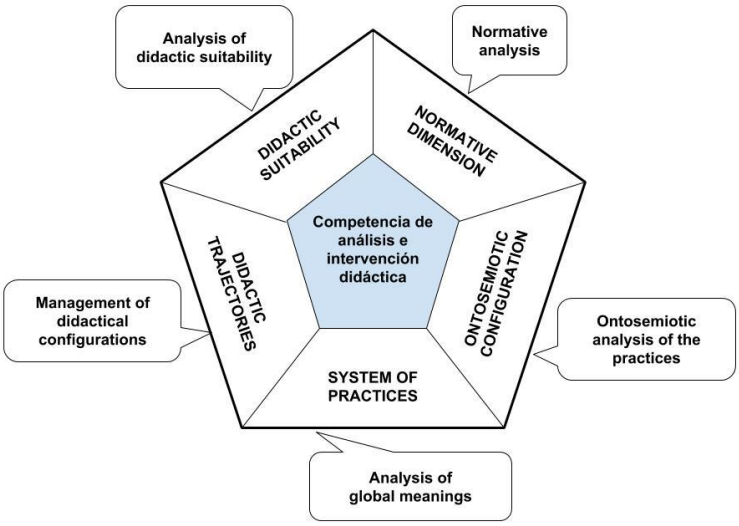

Figure 4. Components of the competence of analysis and didactic intervention (Godino, Batanero, Font, & Giacomone, 2016, p. 295)

8. Final reflexions

The application of the OSA theoretical tools in the analysis and didactic intervention has progressively been enriched, revealing that this theoretic framework is dynamic, since the tools are refined as the different works of research are applied. This enables us to make

an overall macroscopic analysis, like the characterization of the pragmatic meanings of an object, for example, probability (Batanero, 2005), or the evaluation of the didactic suitability of an instruction process in its different facets. However, the analysis can also be made at a microscopic level of the solution of the problem, from the institutional (epistemic) as well as the personal (cognitive) points of view.

The application of the *ontosemiotic configuration of practices, objects and processes* in all its elements is complex, and depends on the level of study established in the research or the teaching, so frequently it has been simplified. In particular, to identify all the objects that intervene in the sequence of resolutive practices and the set of semiotic functions that are established between these objects is very laborious. In Contreras, Molina-Portillo, Godino, Rodríguez-Pérez and Arteaga (2017) the notion *critical semiotic function* has been introduced in order to focus the attention on the knowledge, which is essential to solve a task. Depending on the aims of the research, the attention can be focused on some semiotic functions that are essential for a correct interpretation of the diagram or the solution of a task. If the person who is interpreting the graph or solving the problem does not establish this semiotic function, a semiotic conflict is produced, this being understood to be a disparity of meanings between two subjects (people and institutions) which prevents the communicative interaction.

The panorama presented in this paper about the research carried out on statistics education based on the use of the OSA, reveals the close relationship of the development and evolution of this theoretical framework with research in statistics, probability and combinatorics education, as well as with teacher training. In Figure 4 five sub-competences of the general competence of analysis and didactic intervention, which the teachers should develop, are summarised. While the development of the competence of didactic suitability analysis has been the focus of attention in the statistics field (Arteaga, et al., 2012; 2014; 2017; Gea, 2014) it would be interesting to promote research projects on teacher training that would train them in the development of the remaining competences, in particular, the global analysis of the meanings of the objects in the statistics, probability and combinatorics fields, the ontosemiotic analysis of the practices, the configurations and didactic trajectories and the awareness of the set of norms that support and condition the teaching and learning processes.

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