Thirty years of stochastics education research: Reflections and challenges

Treinta años de investigación en educación estocástica: Reflexiones y desafíos

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Abstract

In this paper I reflect on the circumstances and reasons that led to the current situation of research in stochastics education and its evolution in the past 30 years, the period in which I was involved in part of these developments. I first argue that many stochastic concepts can be taught at both basic and advanced level; however a good understanding is only acquired when starting from elementary levels and supporting students in upgrading their progressive understanding since elementary school to university. The emergence of research in advanced stochastic is summarised in the next section and the different actions that collaborated to the evolution and development of this research throughout the past 30 years are analysed. I finish with some reflections and suggestions to guide future research on the subject.

Keywords: Stochastics education, summary of research, evolution.

Resumen

En este trabajo reflexiono sobre las circunstancias y razones que han llevado a la actual investigación sobre educación estocástica y su evolución en los últimos 30 años, en los que he colaborado a una parte de estos desarrollos. En primer lugar, argumento que muchos conceptos estadísticos pueden ser enseñados tanto a nivel básico como avanzado, pero el logro de una comprensión adecuada se facilita si se comienza en nivel elemental, apoyando al estudiante a progresar desde la escuela primaria a la universidad. En las siguientes secciones resumo la emergencia de la investigación en estocástica avanzada y las diferentes acciones que llevaron al desarrollo de la misma en los últimos 30 años. Finalizo con algunas reflexiones y sugerencias para guiar la investigación futura en el tema y algunas sugerencias para evitar transmitir a los estudiantes concepciones erróneas y sesgos.

Palabras clave: Educación estocástica, resumen de la investigación, evolución, propuestas de investigación.

1. Introduction

When selecting a topic for this lecture, I decided to revise some previous papers (Artigue, Batanero, & Kent, 2007; Batanero, 2004a and b; Batanero & Godino, 2005) and present a personal view of the evolution experienced by stochastics education research along the past thirty years, the period in which I was involved in some developments in this field. Approaching retirement is a good moment to examine the path that one has travelled, as well as trying to identify the main obstacles found along the way, as well as providing some advice to other scientists about how to overcome these dilemmas. At the same time, this reflection is an opportunity to identify and recognise the assistance and contributions received from many people and institutions along this journey.

Research in stochastic education is now well established into the mathematics education community, due to the ample attention given to statistics and probability in the school and university curricula, in order to respond to the need of empowering all citizens and professionals in statistical and probability literacy (Gal, 2002; 2005). Since including a topic in the curriculum does not automatically assure its correct teaching and learning, a parallel research in the education of teachers to teach stochastics is expanding in

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mathematics education (Batanero, Burrill, & Reading, 2011; Groth & Meletiou-Mavrotheris, 2017). The situation was far different 30 years ago when few mathematics educators were interested in research in this area. Even if the ISI organised a roundtable conference to discuss this topic in 1990 (Hawkins, 1990), the result was a study about the education of teachers around the world, more than an effort to foster research in this area.

Stochastics education faces its own dilemmas, when compared with mathematics education. A paradigmatic example is statistical inference, whose recent research I examined in a related paper (Batanero, 2018). As with other advanced stochastics topics, inference is widely taught at university level and even in some countries in high school, but still is generally misunderstood and misapplied. At university level is usually responsibility of lecturers with a variety of backgrounds, usually statisticians, but also economists, engineers, doctors or psychologists, and very rarely mathematicians or mathematics educators. Few of these lecturers are engaged in stochastics education research, since they are forced to do research in either theoretical or applied stochastics or in their own fields of knowledge. We attend, however, to an exponential increase of publications describing teaching innovations, which mostly make intensive use of simulation, real data or visualizations[1]; but few of these proposals are based on a deep study of the students' learning when using these tools.

In the next sections I first will argue that the frontiers between advanced and elementary stochastics are fuzzy; then, I will describe my personal view of the contributions from different disciplines to stochastics education research, as well as of the main events and actions that lead to the current state of this research. I will finish with a summary of the main current tendencies and offer some ideas to continue research in stochastics education.

2. Advanced versus elementary levels in the teaching of stochastics

Following the European tradition, and, as discussed in Batanero and Borovcnik (2016), I rather prefer to speak of stochastics, because statistics and probability are indissoluble linked, in my view, since we do not collect statistical data from deterministic phenomena, where data are generated through mathematical formulas. There is then always some uncertainty in statistical data, and them randomness is underlying, even if we only are interested in a descriptive study of the data at hand. Moreover, apart games of chance, it is difficult to estimate the probability of events or update their probability with no statistical data available, and then, sampling and estimation ideas are subsumed in the frequentist or Bayesian views of probability, which have a much larger field of application than the classical view. Consequently, the term stochastics is used here to highlight the mutual dependence between probabilistic and statistical knowledge and reasoning, which are tightly interconnected and should be taught together.

Then I will describe my view of the differences between elementary and advanced stochastics. By the time the book by Tall (1991) was published, the main concerns of researchers in advanced mathematical thinking dealt with exploring the cognitive processes underlying the learning of a few mathematical topics (mainly calculus and algebra) with no attention payed to statistics or probability. Artigue, Batanero and Kent (2007) considered later a larger variety of research problems, theoretical frameworks, and

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1. See for example, Gapmider (https://www.gapminder.org/) or the web page Understanding uncertainty (https://understandinguncertainty.org/).

mathematical topics in advanced mathematics, and for the first time, statistics and probability were taken into account. In spite of this fact, the authors recognised the existence of a specific stochastics education research community with few links with mathematics education research.

One reason for this separation was the variety of students' previous knowledge and interest, since statistics is taught, for example, to geographers, scientists, sociologists, psychologists, educators, or health care professionals, who do not always receive a previous calculus or algebra service course. This often means that advanced stochastic is taught to students with little experience in advanced mathematics. Unfortunately, many of these courses present the topic as a list of recipes with little emphasis in promoting sound stochastic reasoning, which lead to the students' dissatisfaction and lack of interest to apply this knowledge in their future work, when needed.

Moreover, as argued by Artigue et al. (2007), the distinction between advanced and elementary stochastics topics is very fuzzy, since current curricula include advanced stochastics ideas, such as correlation or confidence intervals in secondary or high school in many countries. Moreover, it is possible to reduce the difficulty of many of these topics with a teaching approach based on simulation. As a result, many statistics and probability ideas, including inference maybe teach as both elementary and advanced level, depending on the depth of the study of the same and of the kind of students to which we teach these ideas.

Take, for example and apparently simple concept, such as independence. Although the definition of independence of events, given by the product rule is apparently simple, its application to different contexts is quite difficult, as noticed in different research (Díaz, Batanero, & Contreras, 2010). Thus, people relate independence to causality or order of events, even when they are not applicable and, moreover, in applications from real life, statistical data will seldom conform to perfect independence. In these applications we cannot use the product rule and will only accept independence in case the data pass some statistical tests. But, the application of these tests requires an understanding of the logic of hypothesis-testing and sampling distributions, which are both advanced stochastic ideas, in which students present many difficulties. Moreover, sampling distributions are generated under the hypothesis of independence of observations, and consequently all these concepts depend on each other and create a circular situation. I made a similar analysis of the apparently basic idea of randomness, a concept with multiple different meanings (Batanero, 2016), and again linked to independence and statistical tests.

Consider again statistical test; most students and teachers will conceived them as advanced stochastic ideas. In a previous paper (Batanero, Díaz, & López-Martín, 2017) we described the steps needed to solve statistical tests in the Fisher's, Neyman-Pearson's Bayesian and re-sampling methodologies. Using ideas from the onto-semiotic approach (Godino, 2002; Godino & Batanero, 1994; Godino, Batanero, & Font, 2007) we analysed the mathematical practices typical from each of these methods and identified part of the different mathematical objects (problems, concepts, properties and procedures) linked to each of these practices. We concluded that the difficulty of each of these approaches, all of them directed to carry out a statistical test may be different for the students. Therefore, the complexity of statistical tests will also depend on the methodological approach and on the degree of formalization used in teaching.

Moreover, if we consider recent suggestions directed to informal introduction of statistical inference (e.g.. Noll, Gebresenbet y Glover, 2016) the number and depth of concepts involved in a statistical test diminish, partly because the method is based on resampling, instead of using frequentist statistics and partly, because often the teaching is concentrated in only learning to simulate the situation and a basic rule for identifying the results that lead to rejection of the hypothesis with no discussion of the concepts underlying the procedure.

From another point of view, we could argue that the complexity of a mathematical concept will depend on the level of algebraic reasoning required from students to work with and to apply the particular concept. Different authors have studied algebraic thinking and defined levels of complexity for the tasks proposed to students and the solutions to these tasks. In the six levels hierarchy introduced by Godino, Neto, Wilhelmi, Aké, Etchegaray and Lasa (2015), the algebraic levels 4 and 5 are characterised by the presence of parameters (level 4) and the need to operate with parameters (level 5). Parameters do not only appear in stochastics in the study of inference, when students are requested to estimate parameters, such as the population mean or proportion from data collected in a random sample. Parameters also serve to specify the density function or probability function for random variables with different theoretical distributions, such as normal or binomial distributions. In the study of regression, students should estimate the parameters of the fitting curve. Therefore, depending on whether the students only estimate the parameters or operate with the same (for example, when standardizing a normal distribution) the algebraic level required in these activities are level 4 or 5 in Godino et al. model. Moreover the upper level 6 is characterised by the operation with algebraic structures and will appear in the formal definition of compound experiments, their sample spaces and their probability measures.

Of course, it is possible to informally approach all these concepts (what we often do) and then reduce the algebraic level required from the students. For example, when using informal approaches to inference based on simulation we do not explicit the underlying parameters to the students; we just make them consider if a given result is likely or unlikely, given the data obtained in a number of simulations of the situation we test. Then, students only work with variables, and sometimes do not need to solve equations or operate with variables and then we substitute a level 4 or 5 algebraic method to a level 2 or even level 1 procedure.

Consequently, I view the adjective “advanced” as not privative of particular stochastic idea and inapplicable to others. I rather believe that it will depend on the way we approach the concept in teaching and the type of situations where we apply the concept. It is reasonable and good to start the teaching of a topic at the simplest possible formalisation level and for some students, probably this knowledge is all they need. However, when we deal with professionals in different fields, high school students in scientific specialities or even teachers, it is important that they progress to upper levels if we want them acquire a sound stochastic knowledge.

One I have clarify these ideas, I first will present a quick summary of the evolution of stochastic education research in the past 30 years, to follow with some landmarks that contributed to change of perspectives or reinforcement of that research. Finally I will try to describe the current tendencies and suggest possible ways to continue research in stochastics education.

3. The development of educational research in stochastic education

A substantial part of the pioneer research related to stochastics thinking and learning has been carried out outside the mathematics education community. Different fields have contributed with various research paradigms and theoretical frameworks, which have been analyzed in different surveys of research, such as Batanero, Chernoff, Engel, Lee and Sánchez (2016); Bakker, Hahn, Kazak and Pratt (2018), Ben-Zvi, Makar and Garfield (2018), Chernoff and Sriraman (2014), Jones (2005), Jones, Lagrall and Mooney (2007), Jones and Thornton (2005), Kapadia and Borovcnik (1991), Shaughnessy (1992; 2007), Shaughnessy, Garfield and Greer (1996), or Zieffler, Garfield, and Fry (2018). Below I include a brief summary of the main contributions to advanced stochastics from psychology, statistics and mathematics education.

Psychological research

Earlier research in probabilistic reasoning started with the pioneer work by Piaget and Inhelder (1951) which investigated the developmental growth of children's probabilistic reasoning and described stages in this development in topics such as randomness, sample space, combinatorics, distribution and convergence. The authors assumed that children's reasoning evolved through different stages of development: preoperational (4 to 7 years); concrete operational (8 to 11 years); and formal operational (beyond 11 years). Although they assumed that the order of the stages was invariant, they also suggested that the age at which any given stage appeared varied considerably and that not all individuals achieve the formal operational stage

Many researchers tried to confirm or complement their description of these stages (see Jones & Thornton, 2005, for a summary), the most influential of which was Fischein (1975) with his theory of intuitions. Fischbein described intuitions as cognitive beliefs that are global and self- evident, and can be influenced by instruction, so that he was in favour of starting the teaching of probability as soon as possible.Although these studies were not directly related to the learning and teaching of probability, their findings have been very influential for including the teaching of probability in the primary school curricula, as well as for promoting new research.

As regards advanced stochastics, the pioneer research also originated in Psychology, and lead to change our confidence in the rationality of human beings. Psychologists convinced us that most educated adults tend to make wrong decisions in different activities linked to their professional work, such as, for example, medicine, army, law or politics.

Different theorerical frameworks tried to explain these apparently irrational behaviours. Among them, the *heuristics and biases* programme (Kahneman, Slovic & Tversky; 1982), was the dominant theoretical framework in the last two decades of the XXth century and is still influencing our research in students' reasoning and learning. The authors following this view assume that people do not follow the normative mathematical rules that guide statistical inference when they make a decision in uncertain situations. They instead use intuitive strategies, which reduce the complex stochastics tasks to simpler problems; although these *heuristics* may be frequently useful, in specific circumstances they produce systematic errors in other problems and are resistant to change. A well known example is the *representativeness heuristics*, by which people tend to estimate the likelihood for an even with only considering how well the event represents some aspects of the parent population. An associated bias is the *belief in the law of small numbers* or belief that even small samples should exactly reflect all the characteristics in the population distribution.

Another theoretical framework was the *adaptive algorithms* (Cosmides & Tooby, 1996; Gigerenzer, 1994). This theory assumes that we use adaptive algorithms, which are acquired by natural selection along a long period of time in a specie; they help solving adaptive problems (such as communicating, or finding food or water). Since adaptive algorithms are shaped by the natural environments, they are more effective when the tasks are presented in a format close to how data are perceived and remembered in ordinary life. According to this theory the difficulty of statistical problems dimminish when data are presented in a natural format of frequencies (absolute frequencies) instead of using rates or percentages (Sedlmeier, 1999). The reason is that frequency representations lead to simpler algorithms that give immediate accessible solution to many statistical problems, while most people would be unable to use the complex algorithm required when information is given by fractions of percentages (Gigerenzer, 1994).

Research in statistics

The stronger contribution to stochastics education originated within statistis itself and started with the creation of the Education Committee by the International Statistical Institute (ISI) at a time where the main concern for the ISI was the need for better statistical information and preparation of statisticians in developing countries (Vere-Jones, 1995; Zieffler et al., 2018). By the end of the XX century, the different applications of statistics began to emphasize increasingly specialized methods and procedures that were reflected in the variey of courses in topics such as bayesian or multivariate statistics, non parametric methods or resampling. At the same time an increasing number of universities started to teach statistis courses to a variety of students. The need to find methods to teach these techniques were reflected in the different invited papers sessions at the ICOTS conferences.

The Education Committee started to pay more attention to statistics education at school level since the mid seventies and organise in 1982 the first *International Conference on Teaching Statistics (ICOTS)* that have continued every four years and intend to reinforce the links between statistics teachers and researchers, as well as users of statistics.These conferences were complemented with a series of Round Table Conferences focussed on specific themes. The journals *Teaching Statistics* first published in 1979 and *Journal of Statistics Education*, started in 1993, soon became main tools to improve statistics education all over the world.

Some statisticians also developed specific frameworks to describe the activity carried out by statisticians, as well as the proceses of statistical modelling and thinking. The most influential example is the theoretical framework developed by Wild and Pfannkuch (1999) to describe statistical thinking, including four components. The first component is termed the statistical investigation cycle PPDAC (Problem, Planning, Data, Analysis, Conclusion); secondly there is an interrogative cycle, the third component in the model describes the types of thinking, present in statistical problem solving and finally the model describes a series of dispositions, such as curiosity, imagination or scepticism.

Another important contribution came from collaborations between statisticians and mathematics teachers in the United States,where the American Statistical Association (ASA) together with the National Council of Teachers of Mathematics (NCTM) constituted a committee that produced the *Guidelines for Assessment and Instruction in*

Statistics Education (GAISE, Aliaga et al., 2005; Franklin et al., 2007), a set of recommendations to teach statistics at both school and college levels. These documents inspired further research; such as for example, comparing curricular guidelines in different countries with these recommendations. Other important developments at school levels, such as the Census at school projects (new.censusatschool.org.nz/), as well as the role of other organisations in fostering statistics education are described in Zieffler et al. (2018).

As a consequence of all this interest and the expansion of the teaching of stochastics at universities, research in advanced stochastics also started to progressively grow under the influence of several funded projects and initiatives by the American Statistical Association (e.g. the Undergraduate Statistics Education Initiative (USEI) and the Consortium for the Advancement of Undergraduate Statistics Education (CAUSE)). Most important was the creation and work of the International Association for Statistical Education (IASE) which is summarise in the next sections.

Research in mathematics education

Mathematics educators' research approach is far different from that taken by psychologists. On one hand, the mathematical and epistemological analyses reveal that the complexity of concepts, tasks and students' responses investigated by psychologists is often greater than what psychological research assumes, and suggests the need for re-analysing these objects from a mathematical perspective. On the other hand, centring on isolated types of tasks does not always reveal in depth the students' understanding of a concept, since their responses are sometimes very dependent on the task variables. Theoretical constructs taken from mathematics education also contribute to a different perspective of the same phenomena.

Research in stochastics education, as a subfield of mathematics education tries to respond to the above challenges and it is now well established, as shown by the existence of stochastics working Group at the International Congress of Mathematics Education (ICME), the European Mathematics Education Conferences (CERME), as well as in regional or national mathematics education conferences, such as the Latin-America Mathematics Education Conference (RELME) or the Simposios de la Sociedad Española de Educación Matemática (SEIEM). In these conference stochastics education is considered as another component of mathematics education research, as noticed also in the conferences and invited papers on this topic in these conferences. Moreover, the recent handbooks that summarise mathematics education research have included chapters dealing with stochastic education research (e.g., Borovcnik, & Peard, 1996 Jones, Langrall, & Mooney, 2007; Shaughnessy, 1992; 2007; Shaughnessy, Garfield, & Greer, 1996).

Finally, other signs of the maturity of stochastics education are the different thematic issues that have been recently offered in mathematics education journals and the fact that many mathematics education departments offer the possibility of carrying out a Ph.D or Master thesis in stochastics education.

4. Some landmarks

In the above pagraphs I summarised the different contribution from various disciplines to the current state of research in stochastics education. Today we observe the confluence of there varied research groups and paradigms under the umbrella of the new field of statistics education. Various events have contributed to this integration, which have been fostered from different associations, journals and conferences that helped to difusse research results, theories and methods as they were producing. Zieffler et al. (2018) describe different milestones in the history of statistics education at secondary school and university level from the point of view of changes in educational programs or curricula. Below I instead concentrate in events that have contributed to the current wealth of stochastics education research.

The most important event was the creation in 1991 of the *International Association for Statistics Education (IASE*) as a separate section of the ISI. This association took over the organisation of ICOTS, starting in 1994 and the Round Tables conferences on specific topics every four years: *Introducing data analysis in the schools* (Pereira- Mendoza, 1993), *Role of technology in teaching and learning statistics* (Garfield & Burrill, 1997), *Training researchers in the use of statistics* (Batanero, 2001), *Curricular development in statistics* (Burrill & Camden, 2005), *Joint ICMI/IASE Study* (Batanero, Burrill, & Reading, 2011), *Technology in statistics education* (Gould, 2012) and *Promoting understanding of statistics about society* (Engel, 2016). In addition, the IASE hosts other conferences as satellites to ISI Sessions.

The 2008 IASE Round table, held in Monterrey, Mexico was particular, in the sense that it reinforced formal collaboration between IASE and the International Commission on Mathematical Instruction (ICMI) that have started a few years ago, when the IASE was invited to suggest organisers for the Statistics and the Probability working group at the ICME (International Congress on Mathematical Education) conferences. Since the mid-1980s, ICMI started organising its ICMI studies, aimed to foster research on topics of particular interest in mathematics education (www.mathunion.org/icmi/activities/icmi-studies). These studies share a common struture, which include a specific conference and the preparation and publicationi of a related monograph (www.springer.com/series/6351). The idea arose from the ICMI interest in improving the teaching of statistics at school and the realisation that changing the teaching of this topic in schools will depend on the extent to which teachers could be prepared to teach statistics. Morover, although interest in the education and professional development of mathematics teachers had increased and there was in that date a body of research results on this issue, the specific case of statistics had not been considered.

Conversations started by 2005 between ICMI and the IASE, and they made clear that there was a common interest in organising a Joint Study related to problems in the teaching of statistics within school mathematics.The invitation from ICMI to collaborate on a Joint Study was accepted by the IASE. Subsequently, IASE suggested that this Joint Study would merge with the 2008 IASE Round Table Conference. As a consequence of this agreement, the Joint ICMI/IASE Study Conference with the topic *Teaching statistics in school mathematics. Challenges for teaching and teacher education* was held at the Instituto Tecnológico y de Estudios Superiores, Monterrey (ITESM), Monterrey Campus, Mexico in July 2008 and the resulting monograph was published in 2011 (Batanero, Burrill y Reading, 2011). This study marked a turning point and contributed sustantially to increase research in the education of teachers.

Another milestone for statistics education research was the creation of a specific research journal. Although statisticians have promoted journals like *Teaching Statistics* and *Journal of Statistics Education* to share new teaching methods and ideas from the classroom and eventually inform teachers and lecturers of research related to teaching and learning statistics, *Statistics Education Research Journal*, created in 2001, was the first journal specifically focussed in research in this field. One aim of this journal was promoting research and unifying isolated pieces of research, in order to progressively develop a more general knowledge about statistical education. In 2018, the publication of two issues each year has been ininterrupted (with 17th volumes); various editors, such as for example, Flavia Jolliffe and myself, Iddo Gal, Tom Short, Peter Petocz, Bob delMas, Maxine Pfannkuch, Manfred Borovcnik and Jennifer Kaplan, as well as other prestigious reseachers who edited the different thematic issues helped the journal gain a reputatio in the field and is today indexed in Scopus. Today this journal has been complemented with others, such as *Statistique et Enseignent* or *Technology Innovations in Statistics Education.*

5. Statistics education research in Iberoamerica

Initial language barriers explained the fact that initially there were few statistics educators in the Iberian peninsula and Iberoamerica attending international conferences or publishing in statistics education journals, since English was the main language of communication. Litle by little, however, isolate researchers from different countries in these areas met both at ICOTS and ISI conferences, as well as in statistics or mathematics education conferences that included an statistics education component. These researchers started creating cooperative links, and, as a consecuence, statistics education research has also grotwh quickly in the past decades in the Iberian peninsula and Iberoamerica.

One important event that helped creating these links was the celebration of ICOTS 7 in 2006 in Salvador de Bahia, Brazil. The theme “working cooperatively in statistics education” was reflected in the conference not only in the different topics but in many different ways. Offering reduced fees to researchers in the region served to get over 200 iberoamericans statistics educators in the conference, where at least an Spanish and Portuguese speaking contributed paper session was offered along all the conference calendar. Moreover special interests groups meetings were offered at lunch and evening times in these languages for those interested. Many links were formed through these initiatives that evolved in actions such as the creation of Internet fora, regional conferences and publications.

Today we count on well established research meetings, such as for example, the *Encuentro Colombiano de Educación Estocástica* (ECEE, third edition in 2018) in Colombia, the *Encuentro Internacional de la Enseñanza de la Probabilidad y Estadística* (EIEPE, 8th edition in 2018), in México, Encontro de probabilidades e estatística na escola in Portugal (4th edition in 2017) or the *Encuentro de Didáctica de la Estadística la Probabilidad y el Análisis de Dato*s (EDEPA, 6th edition in 2018) in Costa Rica, the Encuentro Latinoamericano en Educación Estadística held in 2008 in México, as well as the statistics education component in mathematics education or statistics conferences as in other countrie are a sign of the strength that research has acquired in the region.

The bulletin *Hipótesis Alternativa* (http://www.ucv.ve/hipotesis), supported by the IASE as a regional publication for Latin america edited by Audy Salcedo and other bulletins, such as that the ACESEST bulletin or groups, such as RELIEE (Latin american statistics education research network) linked to RELME (Latin american mathematics education conferences) since 2013 help difussing information and mantaining links between researchers. A number of research groups, such as the Grupo de Estudos em Raciocínio

Combinatório do Centro de Educação, Universidade Federal de Pernambuco, leadered by Rute Borba were also created to support this research.

At the University of Granada, the interest towards stochastics education developed with the stablishing of the Doctoral Programme in Mathematics Education in 1988 and the related statistics education research group (http://www.ugr.es/~batanero/). Some statistics lecturers from different universities to start research related to topics they taught to their students and for which they were familiar with their student' difficulties lead us start research on advanced stochastics. As a consequence, different doctoral theses were carried out in advanced statistical topics such as statistical tests (Vallecillos, 1994), the normal distribution (Tauber, 2001), central limit theorem (Alvarado, 2007), confidence intervals (Olivo, 2008), contingency tables (Cañadas, 2012), random variable (Ruiz, 2013) or variance analysis (Vera, 2015). Other theses dealt with textbook analyses, descriptive statistics, probability or the education of teachers, and today part of our doctoral students became university lecturers and are supervising new doctoral dissertations.

Finally, the organisation of the *Jornadas Virtuales sobre Enseñanza de la Estadística, Probabilidad y Combinatoria* in 2013 and 2015, mainly supported by the work of José Miguel Contreras, and which now have changed *to III International Virtual Congress on Statistical Education* (http://civeest.com/) is helping to progressively link iberoamerican colleagues interested in stochastics education research.

6. Tendencies and perspectives

The above summary suggests that stochastic education research pose a number of important challenges to research. First, the existing research has been carried out by different scientific communities, and not just by mathematics educators, and for this reason the sources of information are widespread and not always easily available. At the same time, the diversity of research problems, theoretical frameworks and approaches is very wide, what requires to researcher an additional effort in defining their research questions and selecting an appropriate framework and method to answer these questions. In the Granada stochastic research group a strategy was working in close cooperation with researchers involved in developing the onto-semiotic approach to mathematics education (http://enfoqueontosemiotico.ugr.es/). This cooperation led to more profound epistemological, cognitive and didactic analyses and, consequently, to more innovative research results, at the time that contributed creating new constructs that enriched the theoretical framework.

The mathematics curricula in different countries increase or diminish the relevance given to stochastics education, as a consequence of somewhat random factors and the interest towards stochastics of curricular designers. This fact is analysed for the specific case of probability in the United States primary school curricula by Langrall (2018) and may result in generations of poorly educated citizens. As suggested by Artigue et al. (2007), another explanation for the scarce relevance of stochastics in some curricula is the progressive separation of mathematics from the applications of statistics and the fact that statistics has changed much more rapidly than mathematics, is much more dependent on the context and on information technology, and is usually taught at post-secondary level by non-mathematicians. Mathematics educators responsible of curricular changes may not be familiar with these changes and do not perceive the relevance of stochastics education for students. However, stochastic ideas are informally used today by people with little formal training, since we are exposed in the media and professional work to situations that require stochastic thinking for correct interpretation.

The education of teachers is still a priority research topic. Although much research emerged as a consequence of the research agenda raised at the Joint ICMI/IASE study, still this research is focusing only in the stochastic knowledge of teachers. More attention should be given to teachers' knowledge in the different facets of their didactic-stochastic knowledge, including the epistemic, cognitive, affective, mediational, interactional and ecological facets of their knowledge (Godino, Giacomone, Batanero, & Font, 2017). The analysis of instructional proposals to educate teachers in each of these facets is also needed.

Consequently there is still a long way to advance research in stochastics education, since most states of art, such as those quoted in Section 1 finish with some ideas for a research agenda, part of which are still needed. For example, in Batanero (2015) I suggested that we should try to investigate how different approaches to inference (instead of the prevalent frequentist approach) may help students' difficulties in inference. We find today suggestions to completely change the teaching of inference in favour of diminish the algebraic level of reasoning needed and substitute the theory of inference by intensive simulation with computers. But the success of this informal approach to inference on the students' understanding should still be explored, as preliminary research results are contradictory. A number of specific research questions are listed in Batanero et al. (2016), such as for example, analysing the best way and age to introduce each different meaning of probability in the curriculum or clarifying the way in which probabilistic thinking could contribute to improving mathematical competencies of students. There is also a need for more systematic research about how teachers and students use technology in classrooms and how large-scale assessment may be fostered by technology (Ridgway, 2016)

To sum up, it is difficult to summarise in only a few pages the changes and exponential raising of stochastics education research, and the quick incorporation of Iberoamerican researchers to this tendency, We probably will attend a leadership of Iberoamerican countries in this area in a next future, given the number of researchers involved and the many initiatives described in the paper.

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References

Aliaga, M., Cobb, G., Cuff, C., Garfield, J., Gould, R., Lock, R., ... & Velleman, P. (2005). *Guidelines for assessment and instruction in statistics education (GAISE): College report.* Washington: American Statistical Association. Available from: http://www.amstat.org/asa/files/pdfs/GAISE/GAISEPreK-12_Full.pdf.

Alvarado, H. (2007). *Significados del teorema central del límite en la enseñanza de la estadística en ingeniería.* Ph.D. Universidad de Granada.

Artigue, M., Batanero, C., & Kent, P. (2007). Mathematics thinking and learning at post-secondary level. In F. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 1011-1049). Greenwich, CT: Information Age Publishing & NCTM.

Bakker, A., Hahn, C., Kazak, S. & Pratt, D. (2018). Research on probability and statistics education in ERME: Trends and directions In T. Dreyfus, M. Artigue, D. Potari, S.

Prediger, & K. Ruthven (Eds.), *Developing research in mathematics education - twenty years of communication, cooperation and collaboration in Europe.* Oxon: Routledge.

Batanero, C. (Ed.). (2001). *Training researchers in the use of statistics*. Granada: International Statistics Institute.

Batanero, C. (2004a). Los retos de la cultura estadística. *Yupana*, *1*(1), 27-37.

Batanero, C. (2004b). Statistics education as a field for research and practice. Regular lecture at the *Tenth International Congress on Mathematics Education*. Copenhaguen, July, 2004: International Commission on Mathematics Education.

Batanero, C. (2015). Aproximación informal al contraste de hipótesis. In J.M. Contreras (Ed.), *II Jornadas Virtuales de Didáctica de la Estadística, la Probabilidad y la Combinatoria.* Granada: SEIEM.

Batanero, C. (2016). Understanding randomness: Challenges for research and teaching. In K. Krainer, & N. Vondrová (Eds.). *Proceedings of the Ninth Congress of the European Society for Research in Mathematics Education* (pp. 34-49). Prague: European Society for Research in Mathematics Education.

Batanero, C. (2018). Treinta años de investigación didáctica sobre el análisis inferencial de datos. En A .Avila (Ed.), *Rutas de la educación matemática* (pp. 186-199). México: Sociedad Mexicana de Investigación y Divulgación en Educación Matemática.

Batanero, C., & Borovcnik, M. (2016). *Statistics and probability in high school,* Rotterdamn, The Netherlands: Sense Publishers.

Batanero, B., Burrill, G., & Reading, C. (Eds.), *Teaching statistics in school mathematics. Challenges for teaching and teacher education.* A Joint ICMI/IASE Study. New York: Springer.

Batanero, C., Chernoff, E., Engel, J. Lee, H., & Sánchez, E. (2016). *Research on teaching and learning probabilit*y. New York: Springer. ICME-13. Topical Survey series.

Batanero, C., & Díaz, C. (2007). Meaning and understanding of mathematics. The case of probability. In J.P Van Bendegen y K. François (Eds), *Philosophical dimmensions in mathematics education.* (pp. 107-127). New York: Springer,

Batanero, C., Díaz, C., & López-Martín, M. M. (2017). Significados del contraste de hipótesis, configuraciones epistémicas asociadas y algunos conflictos semióticos. In J. M. Contreras, P. Arteaga, G. R. Cañadas, M.M. Gea, B. Giacomone, & M. M. López-Martín (Eds.), *Actas del Segundo Congreso International Virtual sobre el Enfoque Ontosemiótico del Conocimiento y la Instrucción Matemáticos.* Available from: http://enfoqueontosemiotico.ugr.es/civeos.html.

Batanero, C., & Godino, J. (2005). Perspectivas de la educación estadística como área de investigación. In L. Blanco (Ed.), *Líneas de investigación en Didáctica de las Matemáticas* (pp. 203-226). Badajoz: Universidad de Extremadura.

Ben-Zvi D., Makar K., & Garfield J. (2018). *International handbook of research in statistics education.* New York: Springer,

Borovcnik, M., & Peard, R. (1996). Probability. In A. Bishop, M.A. Clements, C. Keitel, J. Kilpatrick, & C. Laborde (Eds.), *International handbook of mathematics education* (pp. 239-288). Dordrecht: The Netherlands: Kluwer.

Burrill, G., & Camden, M. (2005). *Curricular development in statistics education*: *International Association for Statistical Education 2004 Roundtable.* Voorburg, The Netherlands: International Statistical Institute. Online: https://iase-web.org/Conference_Proceedings.php?p=Curricular_Development_2004.

Cañadas, G. R. (2012). *Comprensión intuitiva y aprendizaje formal de las tablas de contingencia en alumnos de psicología.* Ph.D. thesis. Universidad de Granada

Chernoff, E. J., & Sriraman, B. (Eds.) (2014a), *Probabilistic thinking. Presenting multiple perspectives*. New York: Springer.

Cosmides, L. & Tooby, J. (1996). Are humans good intuitive statisticians after all? Rethinking some conclusions from the literature on judgment under uncertainty. *Cognition, 58,* 1-73.

Díaz, C., Batanero, C., & Contreras, J. M. (2010). Teaching independence and conditional probability. *Boletín de Estadística e Investigación Operativa*, *26*(2), 149-162.

Engel, J. (Ed.) (2016), *Promoting understanding of statistics about society. Proceedings of the Roundtable Conference of the International Association of Statistics Education,* Berlín: IASE. Available from: https://iase-web.org/Conference_Proceedings.php?p=Stat_and_Internet_2003.

Fischbein, E. (1975). *The intuitive source of probability thinking in children.* Dordrecht, The Netherlands: Reidel.

Franklin, C., Kader. G., Mewborn, D., Moreno, J., Peck, R., Perry, M., & Scheaffer, R. (2007). *Guidelines for assessment and instruction in statistics education (GAISE) report: A Pre-K-12 curriculum framework.* Alexandria, VA: American Statistical Association. Available from: www.amstat.org/Education/gaise/.

Gal, I. (2002). Adults' statistical literacy: Meanings, components, responsibilities (with discussion). *International Statistical Review*, *70*(1), 1-51.

Gal, I. (2005). Towards "probability literacy" for all citizens: Building blocks and instructional dilemmas. In G. A. Jones (Ed.), *Exploring probability in school. Challenges for teaching and learning* (pp. 39-63). Dordrecht: Kluwer.

Garfield, J. & Burrill, G. (1997). *Research on the role of technology in teaching and learning statistics, Proceedings of the 1996 IASE Round Table Conference.* Voorburg, The Netherlands: International Statistical Institute. Available from: https://iase-web.org/Conference_Proceedings.php?p=Role_of_Technology_1996.

Gigerenzer, G. (1994). Why the distinction between single-event probabilities and frequencies is important for psychology (and vice-versa). In G. Wright & P. Ayton (Eds.), *Subjective probability* (pp. 129-161). Chichester: Wiley.

Godino, J. D. (2002). Un enfoque ontológico y semiótico de la cognición matemática. *Recherches en Didactiques des Mathematiques*, 22(2-3), 237-284.

Godino, J. D. y Batanero, C. (1994). Significado institucional y personal de los objetos matemáticos. *Recherches en Didactique des Mathématiques*, *14*(3), 325-355.

Godino, J. D., Batanero, C. y Font, V. (2007). The onto-semiotic approach to research in mathematics education. *ZDM*, *39*(1-2), 127-135.

Godino, J. D., Giacomone, B., Batanero, C. y Font, V. (2017). Enfoque ontosemiótico de los conocimientos y competencias del profesor de matemáticas. *Bolema*, *31*(57), 90-113.

Godino, J. D., Neto, T., Wilhelmi, M., Aké, L., Etchegaray, S., & Lasa, A. (2015, February). Algebraic reasoning levels in primary and secondary education. In K. Krainer, & N. Vondrová (Eds.). *Proceedings of the Ninth Congress of the European Society for Research in Mathematics Education* (pp. 426-432). Prague: European Society for Research in Mathematics Education.

Gould, R. (2012) (Ed). *Technology in statistics education. Virtualities and realities. Proceedings of the Roundtable Conference of the International Association of Statistics Education*. Cebu, the Philipines. Available from https://iase-web.org/Conference_Proceedings.php?p=Technology_in_Statistics_Education_2012.

Groth, R., & Meletiou-Mavrotheris, M. (2018). Research on statistics teachers' cognitive and affective characteristics. In D. Be-_Zvi, K. Makar, & J.B. Garfield (Eds.), *International handbook of research in statistics education* (pp. 327-355). Springer, Cham.

Hawkins, A. (Ed.). (1990). Training teachers to teach statistics: *Proceedings of the International Statistical Institute Round Table Conference*, Budapest, Hungary, 23-27 July, 1988. International Statistical Institute.

Jones, G. A. (2005). *Exploring probability in schools. Challenges for teaching and learning*. New York: Springer.

Jones, G., Langrall, C., & Mooney, E. (2007). Research in probability: responding to classroom realities. In F. Lester (Ed.), *Second handbook of research on mathematics teaching and learning*. Greenwich, CT: Information Age Publishing and NCTM.

Jones, G. A., & Thornton, C. A. (2005). An overview of research into the teaching and learning of probability. In G. A. Jones (Ed.), *Exploring probability in school: Challenges for teaching and learning* (pp. 65-92). Dordrecht: Kluwer Academic Publishers.

Kahneman, D., Slovic, P., & Tversky, A. (1982). *Judgment under uncertainty: Heuristics and biases.* New York: Cambridge University Press.

Kapadia, R., & Borovcnik M. (Eds.) (1991). *Chance encounters*. Dordrecht: The Netherlands: Kluwer.

Langrall, C. W. (2018). The status of probability in the elementary and lower secondary school mathematics curriculum: The rise and fall of probability in school mathematics in the United States. In C. Batanero (Ed.), *Teaching and Learning Stochastics* (pp. 39-50). Springer, Cham.

Noll, J., Gebresenbet, M., y Glover, E. D. (2016). A modeling and simulation approach to informal inference: successes and challenges. In D. Ben-Zvi (Ed.), *The teaching and learning of statistics* (pp. 139-150). New York: Springer

Olivo, E. (2008). *Significados de los intervalos de confianza para los estudiantes de ingeniería en México.* Ph. D. Thesis. Universidad de Granada.

Pereira-Mendoza, L. (Ed.)) (1993). Introducing data analysis into schools: Who should teach it and how? *Proceedings of the IASE Round Table Conference*. Voorburg, The Netherlands: International Statistical Institute.

Piaget, J., & Inhelder, B. (1975). *The origin of the idea of chance in children.* New York: Norton (Original work published in 1951).

Ridgway, J. (2016). Implications of the data revolution for statistics education. *International Statistical Review, 84*(3), 528-549.

Ruiz, B. (2013). *Un análisis epistemológico y cognitivo de la variable aleatoria.* Ph.D. thesis. Universidad de Granada.

Sedlmeier, P. (1999). *Improving statistical reasoning. Theoretical models and practical implications.* Mahwah, NJ: Erlbaum

Shaughnessy, J.M. (1992). Research in probability and statistics: Reflections and directions. In D.A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 465–494). New York: Macmillan.

Shaughnessy, J. M. (2007). Research on statistics learning and reasoning. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 957-1009). Charlotte, NC: Information Age Publishing.

Shaughnessy, J. M., Garfield, J., & Greer, B. (1996). Data handling. In A. J. Bishop, K. Clements, C. Keitel, J. Kilpatrick, & C. Laborde (Eds.), *International handbook of mathematics education* (v.1, pp. 205-237). Dordrecht, Netherlands: Kluwer.

Tauber, L. (2001). *La construcción del significado de la distribución normal en un curso de análisis de datos.* Ph.D. thesis. Universidad de Sevilla.

Vallecillos A. (1994). *Estudio teórico-experimental de errores y concepciones sobre el contraste estadístico de hipótesis en estudiantes universitarios.* Ph.D. Universidad de Granada.

Vera, O. (2015). *Comprensión del análisis de varianza elemental por estudiantes universitarios.* Ph.D. Universidad de Granada.

Vere-Jones, D. (1997). The coming of age of statistical education. *International Statistical Review*, *63*(1), 3-23.

Wild, C., & Pfannkuch, M. (1999). Statistical thinking in empirical enquiry (with discussion). *International Statistical Review, 67*(3), 223-265.

Tall, D. (Ed.). (1991). *Advanced mathematical thinking.* Dordrecht: Kluwer.

Zieffler, A., Garfield J., & Fry E. (2018) What is statistics education? In D. Ben-Zvi, K. Makar, & J. Garfield (Eds.), *International handbook of research in statistics education* (pp. 37-70). New York: Springer.