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ω-Interpolative Ćirić-Reich-Rus-Type Contractions

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Abstract: In this paper, using the concept of ω-admissibility, we prove some fixed point results for interpolate Ćirić-Reich-Rus-type contraction mappings. We also present some consequences and a useful example.

Keywords: metric space; ω-interpolative Ćirić-Reich-Rus-type contraction; fixed point

MSC: 46T99; 47H10; 54H25

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1. Introduction and Preliminaries

In [1], the notion of an interpolative Kannan-type contraction was introduced and the following fixed point theorem was stated:

A self-mapping T on a complete metric space (X,d) such that:

$$d\left(T\xi,T\eta\right)\leq\lambda\left[d\left(\xi,T\xi\right)\right]^{\alpha}\cdot\left[d\left(\eta,T\eta\right)\right]^{1-\alpha},\tag{1}$$

where $\lambda\in[0,1)$ and $\alpha\in(0,1)$, and $\xi,\eta\in X$ with $\xi\neq T\xi$, has a unique fixed point in X.

Very recently, the authors in [2] (see also [3]) pointed out a gap in [1], that is the guaranteed fixed point in the theorem above need not be unique.

The next theorem and its invariants were considered and proven independently by L.B. Ćirić (Serbia), S. Reich (Israel), and I.A. Rus (Romania); see, e.g., [4–11]. Regarding the contributions of these authors, we shall call the following result the Ćirić-Reich-Rus theorem, by which our main result is inspired.

Ćirić-Reich-Rus theorem: A self-mapping T on a complete metric space (X,d) such that:

$$d\left(T\xi,T\eta\right)\leq\lambda\left[d(\xi,\eta)+d(\xi,T\xi)+d(\eta,T\eta)\right],\tag{2}$$

for all $\xi,\eta\in X$, where $\lambda\in\left[0,\frac{1}{3}\right)$, possesses a unique fixed point.

Denote by Ψ the set of all nondecreasing self-mappings ψ on $[0,\infty)$ such that:

$$\sum_{n=1}^{\infty}\psi^{n}(t)<\infty\text{ for each }t>0.$$

Note that for $\psi\in\Psi$, we have $\psi(0)=0$ and $\psi(t)<t$ for each $t>0$; see, e.g., [10,12].

The notion of ω-orbital admissible maps was introduced by Popescu as a refinement of the concept of α-admissible maps of Samet et al. [13].

Definition 1 ([14]). *Let $\omega : X \times X \to [0, \infty)$ be a mapping and $X \neq \emptyset$. A self-mapping $T : X \to X$ is said to be an ω-orbital admissible if for all $s \in X$, we have:*

$$\omega(s, Ts) \geq 1 \Rightarrow \omega(Ts, T^2 s) \geq 1. \tag{3}$$

Many papers used and generalized this above concept in order to prove variant (common) fixed point results (see, for instance, [15–24]). In this setting, the following condition has often been considered in order to avoid the continuity of the involved contractive mappings.

(H) If $\{\eta_n\}$ is a sequence in X such that $\omega(\eta_n, \eta_{n+1}) \geq 1$ for each n and $\eta_n \to \eta \in X$ as $n \to \infty$, then there exists $\{\eta_{n(k)}\}$ from $\{\eta_n\}$ such that $\omega(\eta_{n(k)}, \eta) \geq 1$ for each k.

In this paper, using the notion of ω-admissibility, we initiate the idea of ω-interpolative Ćirić-Reich-Rus-type contraction. We also present some consequences and an example in support of our obtained result.

2. Main Results

First, we initiate the concept of ω-interpolative Ćirić-Reich-Rus-type contractions.

Definition 2. *Let (X, d) be a metric space. The map $T : X \to X$ is said to be an ω-interpolative Ćirić-Reich-Rus-type contraction if there exist $\psi \in \Psi$, $\omega : X \times X \to [0, \infty)$ and positive reals $\gamma, \beta > 0$, verifying $\gamma + \beta < 1$, such that:*

$$\omega(\xi, \eta)\, d\,(T\xi, T\eta) \leq \psi\Big([d\,(\xi, \eta)]^{\beta} \cdot [d\,(\xi, T\xi)]^{\gamma} \cdot [d\,(\eta, T\eta)]^{1-\gamma-\beta} \Big) \tag{4}$$

for all $\xi, \eta \in X \setminus \operatorname{Fix}(T)$, where $\operatorname{Fix}(T)$ denotes the set of all fixed points of T (that is, points $a \in X$ such that $Ta = a$).

The essential main result is given as follows.

Theorem 1. *Suppose a continuous self-mapping $T : X \to X$ is ω-orbital admissible and forms an ω-interpolative Ćirić-Reich-Rus-type contraction on a complete metric space (X, d). If there exists $\xi_0 \in X$ such that $\omega(\xi_0, T\xi_0) \geq 1$, then T possesses a fixed point in X.*

Proof. Let $\xi_0 \in X$ be a point such that $\omega(\xi_0, T\xi_0) \geq 1$. Let $\{\xi_n\}$ be the sequence defined by $\xi_n = T^n(\xi_0)$, $n \geq 0$. If for some n_0, we have $\xi_{n_0} = \xi_{n_0+1}$, then ξ_{n_0} is a fixed point of T, which ends the proof. Otherwise, $\xi_n \neq \xi_{n+1}$ for each $n \geq 0$. We have $\omega(\xi_0, \xi_1) \geq 1$. Since T is ω-orbital admissible,

$$\omega(\xi_1, \xi_2) = \omega(T\xi_0, T\xi_1) \geq 1.$$

Continuing as above, we obtain that:

$$\omega(\xi_n, \xi_{n+1}) \geq 1 \qquad \text{for all } n \geq 0. \tag{5}$$

Taking $\xi = \xi_n$ and $\eta = \xi_{n-1}$ in (4), we find that:

$$\begin{aligned} d\,(\xi_{n+1}, \xi_n) &\leq \omega\,(\xi_n, \xi_{n-1})\, d\,(T\xi_n, T\xi_{n-1}) \\ &\leq \psi\left([d\,(\xi_n, \xi_{n-1})]^{\beta} [d\,(\xi_n, T\xi_n)]^{\gamma} \cdot [d\,(\xi_{n-1}, T\xi_{n-1})]^{1-\gamma-\beta} \right) \\ &= \psi\left([d\,(\xi_n, \xi_{n-1})]^{\beta} \cdot [d\,(\xi_n, \xi_{n+1})]^{\gamma} \cdot [d\,(\xi_{n-1}, \xi_n)]^{1-\gamma-\beta} \right) \\ &= \psi\left([d\,(\xi_{n-1}, \xi_n)]^{1-\gamma} \cdot [d\,(\xi_n, \xi_{n+1})]^{\gamma} \right). \end{aligned} \tag{6}$$

In particular, as $\psi(t) < t$ for each $t > 0$,

$$d(\xi_{n+1}, \xi_n) \leq \psi\left([d(\xi_{n-1}, \xi_n)]^{1-\gamma} \cdot [d(\xi_n, \xi_{n+1})]^{\gamma}\right) < [d(\xi_{n-1}, \xi_n)]^{1-\gamma} \cdot [d(\xi_n, \xi_{n+1})]^{\gamma}. \tag{7}$$

We derive:

$$[d(\xi_n, \xi_{n+1})]^{1-\gamma} < [d(\xi_{n-1}, \xi_n)]^{1-\gamma}. \tag{8}$$

Therefore:

$$d(\xi_n, \xi_{n+1}) < d(\xi_{n-1}, \xi_n) \quad \text{for all } n \geq 1. \tag{9}$$

Hence, the positive sequence $\{d(\xi_{n-1}, \xi_n)$ is decreasing. Eventually, there is a real $\ell \geq 0$ in order that $\lim_{n\to\infty} d(\xi_{n-1}, \xi_n) = \ell$. Taking into account (9),

$$[d(\xi_{n-1}, \xi_n)]^{1-\gamma} \cdot [d(\xi_n, \xi_{n+1})]^{\gamma} \leq [d(\xi_{n-1}, \xi_n)]^{1-\gamma} \cdot [d(\xi_{n-1}, \xi_n)]^{\gamma} = d(\xi_{n-1}, \xi_n),$$

so (6) together with the nondecreasing character of ψ lead to:

$$d(\xi_{n+1}, \xi_n) \leq \psi\left([d(\xi_{n-1}, \xi_n)]^{1-\gamma} \cdot [d(\xi_n, \xi_{n+1})]^{\gamma}\right) \leq \psi\Big(d(\xi_{n-1}, \xi_n)\Big).$$

By repeating this argument, we get:

$$d(\xi_n, \xi_{n+1}) \leq \psi\Big(d(\xi_{n-1}, \xi_n)\Big) \leq \psi^2\Big(d(\xi_{n-2}, \xi_{n-1})\Big) \leq \cdots \leq \psi^n\Big(d(\xi_0, \xi_1)\Big). \tag{10}$$

Taking $n \to \infty$ in (10) and using the fact $\lim_{n\to\infty} \psi^n(t) = 0$ for each $t > 0$, we deduce that $\ell = 0$, that is,

$$\lim_{n\to\infty} d(\xi_n, \xi_{n+1}) = 0.$$

We assert that $\{\xi_n\}$ is a Cauchy sequence, that is $\lim_{n\to\infty} d(\xi_n, \xi_{n+p}) = 0$ for all $p \in \mathbb{N}$. On account of the triangle inequality together with (10), we find:

$$d(\xi_n, \xi_{n+p}) \leq \psi^n\Big(d(\xi_0, \xi_1)\Big) + \cdots + \psi^{n+r-1}\Big(d(\xi_0, \xi_1)\Big) \leq \sum_{i=n}^{\infty} \psi^i\Big(d(\xi_0, \xi_1)\Big).$$

Letting $n \to \infty$ in the inequality above, we conclude that the right-hand side tends to zero. Thus, $\{\xi_n\}$ is a Cauchy sequence. Regarding the completeness of the metric space (X, d), we deduce that there is some $\xi \in X$ so that:

$$\lim_{n\to\infty} d(\xi_n, \xi) = 0. \tag{11}$$

Since T is continuous, we have $\xi = \lim_{n\to\infty} \xi_{n+1} = \lim_{n\to\infty} T\xi_n = T\Big(\lim_{n\to\infty} \xi_n\Big) = T\xi$. □

In what follows, we replace the continuity criteria by a weakened condition (H).

Theorem 2. *Suppose a self-mapping $T : X \to X$ is ω-orbital admissible and forms an ω-interpolative Ćirić-Reich-Rus-type contraction on a complete metric space (X, d). Suppose also that the condition (H) is fulfilled. If there exists $\xi_0 \in X$ such that $\omega(\xi_0, T\xi_0) \geq 1$, then T possesses a fixed point in X.*

Proof. By the proof of Theorem 1 verbatim, we conclude that the constructed sequence $\{\xi_n\}$ is Cauchy and (11) holds. Suppose the condition (H) holds. We argue by contradiction by assuming that $\xi \neq T\xi$.

Recall that $\xi_{n(k)} \neq T\xi_{n(k)}$ for each $k \geq 0$. Due to (H), there is a partial subsequence $\{\xi_{n(k)}\}$ of $\{\xi_n\}$ such that $\omega\left(\xi_{n(k)}, \xi\right) \geq 1$ for all k. Since $\{d\left(\xi_{n(k)}, \xi\right)\} \to 0$, $\{d\left(\xi_{n(k)}, T\xi_{n(k)}\right)\} \to 0$ and $d(\xi, T\xi) > 0$, there is $N \in \mathbb{N}$ such that, for each $k \geq N$,

$$d\left(\xi_{n(k)}, \xi\right) \leq d(\xi, T\xi) \quad \text{and} \quad d\left(\xi_{n(k)}, T\xi_{n(k)}\right) \leq d(\xi, T\xi).$$

Taking $\xi = \xi_{n(k)}$ and $\eta = \xi$ in (4), we get that:

$$\begin{aligned} d\left(\xi_{n(k)+1}, T\xi\right) &\leq \omega\left(\xi_{n(k)}, \xi\right) d\left(T\xi_{n(k)}, T\xi\right) \\ &\leq \psi\Big(\left[d\left(\xi_{n(k)}, \xi\right)\right]^{\beta} \cdot \left[d\left(\xi_{n(k)}, T\xi_{n(k)}\right)\right]^{\gamma} \cdot \left[d(\xi, T\xi)\right]^{1-\gamma-\beta} \Big). \end{aligned} \tag{12}$$

As ψ is nondecreasing, it follows from (12) that:

$$\begin{aligned} d\left(\xi_{n(k)+1}, T\xi\right) &\leq \psi\Big(\left[d(\xi, T\xi)\right]^{\beta} \cdot \left[d(\xi, T\xi)\right]^{\gamma} \cdot \left[d(\xi, T\xi)\right]^{1-\gamma-\beta} \Big) \\ &= \psi\Big(d(\xi, T\xi)\Big). \end{aligned}$$

Letting $k \to \infty$, we find that:

$$0 < d(\xi, T\xi) \leq \psi\Big(d(\xi, T\xi)\Big) < d(\xi, T\xi),$$

which is a contradiction. Thus, $\xi = T\xi$. $\square$

In what follows, we introduce the notion of ω-interpolative Kannan-type contractions.

Definition 3. *The self-mapping T on the metric space (X, d) is called an ω-interpolative Kannan-type contraction if there exist $\psi \in \Psi$, $\omega : X \times X \to [0, \infty)$ and $\beta \in (0, 1)$ such that:*

$$\omega(\xi, \eta)\, d(T\xi, T\eta) \leq \psi\Big(\left[d(\xi, T\xi)\right]^{\beta} \cdot \left[d(\eta, T\eta)\right]^{1-\beta} \Big), \tag{13}$$

for all $\xi, \eta \in X \setminus \mathrm{Fix}(T)$.

The following one is our second main result.

Theorem 3. *Let $T : X \to X$ be an ω-orbital admissible and ω-interpolative Kannan-type contraction mapping on a complete metric space (X, d). Assume also that either T is continuous on (X, d) or (H) holds. If there exists $\xi_0 \in X$ so that $\omega(\xi_0, T\xi_0) \geq 1$, then there exists a fixed point of T in X.*

We skipped the proof due to the verbatim proof of Theorem 1.

By considering $\omega(x, y) = 1$ in Theorem 1, we state the following.

Corollary 1. *Let T be a self-mapping on a complete metric space (X, d) such that:*

$$d(T\xi, T\eta) \leq \psi\Big(\left[d(\xi, \eta)\right]^{\beta} \cdot \left[d(\xi, T\xi)\right]^{\gamma} \cdot \left[d(\eta, T\eta)\right]^{1-\gamma-\beta} \Big), \tag{14}$$

for all $\xi, \eta \in X \setminus \mathrm{Fix}(T)$, where $\gamma, \beta > 0$ are positive reals satisfying $\gamma + \beta < 1$. Then, T admits a fixed point.

Corollary 2. *Let T be a self-mapping on a complete metric space (X, d) such that:*

$$d(T\xi, T\eta) \leq \psi\Big(\left[d(\xi, T\xi)\right]^{\beta} \cdot \left[d(\eta, T\eta)\right]^{1-\beta} \Big), \tag{15}$$

for all $\xi, \eta \in X \setminus \mathrm{Fix}(T)$, where $0 < \beta < 1$. Then, T admits a fixed point in X.

Taking $\psi(t) = \lambda t$ (where $\lambda \in [0,1)$) in Corollary 1, we state:

Corollary 3. *Let T be a self-mapping on a complete metric space (X, d) such that:*

$$d(T\xi, T\eta) \leq \lambda [d(\xi, \eta)]^{\beta} \cdot [d(\xi, T\xi)]^{\gamma} \cdot [d(\eta, T\eta)]^{1-\gamma-\beta}, \tag{16}$$

for all $\xi, \eta \in X \setminus \mathrm{Fix}(T)$, where γ, β are positive reals verifying $\gamma + \beta < 1$ and $\lambda \in [0,1)$. Then, T has a fixed point in X.

Taking $\psi(t) = \lambda t$ (where $\lambda \in [0,1)$) in Corollary 2, we state:

Corollary 4. *Let T be a self-mapping on a complete metric space (X, d) such that:*

$$d(T\xi, T\eta) \leq \lambda \cdot [d(\xi, T\xi)]^{\beta} \cdot [d(\eta, T\eta)]^{1-\beta}, \tag{17}$$

for all $\xi, \eta \in X \setminus \mathrm{Fix}(T)$, where $0 < \beta < 1$ and $\lambda \in [0,1)$. Then, there exists a fixed point of T.

Remark 1. *Corollary 3 corresponds to Corollary 2.1 in [2].*

Let $(X, d, \preceq)$ be a partially-ordered metric space. Let us consider the following condition.

(G) If $\{\xi_n\}$ is a sequence in X such that $\xi_n \preceq \xi_{n+1}$ for each n and $\xi_n \to \xi \in X$ as $n \to \infty$, then there exists a subsequence $\{\xi_{n(k)}\}$ of $\{\xi_n\}$ such that $\xi_{n(k)} \preceq \xi$ for each k.

Following [23], we may state the following consequences of Theorem 1.

Corollary 5. *Let $(X, d, \preceq)$ be a complete partially-ordered metric space. Let $T : X \to X$ be the mapping such that:*

$$\omega(\xi, \eta)\, d(T\xi, T\eta) \leq \psi\Big([d(\xi, \eta)]^{\beta} \cdot [d(\xi, T\xi)]^{\gamma} \cdot [d(\eta, T\eta)]^{1-\gamma-\beta} \Big),$$

for all $\xi, \eta \in X \setminus \mathrm{Fix}(T)$ with $\xi \preceq \eta$, where $\psi \in \Psi$ and $\gamma, \beta > 0$ are positive reals such that $\gamma + \beta < 1$. Assume that:

(i) T is nondecreasing with respect to $\preceq$;
(ii) there exists $\xi_0 \in X$ such that $\xi_0 \preceq T\xi_0$;
(iii) either T is continuous on (X, d) or (G) holds.

Then, T has a fixed point in X.

Proof. It suffices to take, in Theorem 1,

$$\omega(x, y) = \begin{cases} 1 & \text{if } (x \preceq y) \text{ or } (y \preceq x), \\ 0 & \text{otherwise.} \end{cases}$$

□

Corollary 6. *Let $(X, d, \preceq)$ be a complete partially-ordered metric space and $T : X \to X$ be a given mapping satisfying:*

$$\omega(\xi, \eta)\, d(T\xi, T\eta) \leq \psi\Big([d(\xi, T\xi)]^{\beta} \cdot [d(\eta, T\eta)]^{1-\beta} \Big),$$

for all $\xi, \eta \in X \setminus \mathrm{Fix}(T)$ with $\xi \preceq \eta$, where $\psi \in \Psi$ and $0 < \beta < 1$. Assume that:

(i) T is nondecreasing with respect to $\preceq$;

(ii) *there exists* $\xi_0 \in X$ *such that* $\xi_0 \preceq T\xi_0$*;*
(iii) *either* T *is continuous on* (X,d) *or* (G) *holds.*

Then, T *has a fixed point in* X*.*

Proof. We take in Theorem 3,

$$\omega(x,y) = \begin{cases} 1 & \text{if } (x \preceq y) \text{ or } (y \preceq x), \\ 0 & \text{otherwise.} \end{cases}$$

□

Corollary 7. *Suppose that the subsets* A_1 *and* A_2 *of a complete metric space* (X,d) *are closed. Suppose also that* $T : A_1 \cup A_2 \to A_1 \cup A_2$ *satisfies:*

$$\omega(\xi,\eta)\, d(T\xi,T\eta) \leq \psi\Big([d(\xi,\eta)]^{\beta} \cdot [d(\xi,T\xi)]^{\gamma} \cdot [d(\eta,T\eta)]^{1-\gamma-\beta} \Big)$$

for all $\xi \in A_1$ *and* $\eta \in A_2$*, such that* $\xi,\eta \notin \mathrm{Fix}(T)$*, where* $\psi \in \Psi$ *and* $\gamma,\beta > 0$ *are positive reals such that* $\gamma+\beta<1$*. If* $T(A_1) \subseteq A_2$ *and* $T(A_2) \subseteq A_1$*, then there exists a fixed point of* T *in* $A_1 \cap A_2$*.*

Proof. It suffices to take, in Theorem 1,

$$\omega(x,y) = \begin{cases} 1 & \text{if } (A_1 \times A_2) \cup (A_2 \times A_1), \\ 0 & \text{otherwise.} \end{cases}$$

□

Corollary 8. *Let* A_1 *and* A_2 *be two nonempty closed subsets of a complete metric space* (X,d)*. Suppose that* $T : A_1 \cup A_2 \to A_1 \cup A_2$ *satisfies:*

$$\omega(\xi,\eta)\, d(T\xi,T\eta) \leq \psi\Big([d(\xi,T\xi)]^{\beta} \cdot [d(\eta,T\eta)]^{1-\gamma-\beta} \Big)$$

for all $\xi \in A_1$ *and* $\eta \in A_2$ *such that* $\xi,\eta \notin \mathrm{Fix}(T)$*, where* $\psi \in \Psi$ *and* $0<\beta<1$*. If* $T(A_1) \subseteq A_2$ *and* $T(A_2) \subseteq A_1$*, then there exists a fixed point of* T *in* $A_1 \cap A_2$*.*

Proof. It suffices to take, in Theorem 3,

$$\omega(x,y) = \begin{cases} 1, & \text{if } (A_1 \times A_2) \cup (A_2 \times A_1), \\ 0, & \text{otherwise.} \end{cases}$$

□

Theorem 1 is supported by the following.

Example 1. *Let us consider the set* $X = [0,2]$ *endowed with* $d(x,y) = |x-y|$*. Let* T *be a self-mapping on* X *defined by:*

$$Tx = \begin{cases} \frac{3}{2}, & \text{if } x \in [1,2] \\ \frac{1}{3}, & \text{if } x \in [0,1). \end{cases}$$

Take:

$$\omega(x,y) = \begin{cases} 1, & \text{if } x,y \in [1,2] \\ 0, & \text{otherwise.} \end{cases}$$

Let $x, y \in X$ be such that $x \neq Tx$, $y \neq Ty$ and $\omega(x,y) \geq 1$. Then, $x, y \in [1,2]$ and $x, y \notin \{\frac{3}{2}\}$. We have $Tx = Ty = \dfrac{3}{2}$. Hence, (4) holds. For $x_0 = 2$, we have:

$$\omega(2, T2) = \omega\left(2, \frac{3}{2}\right) = 1.$$

Now, let $x, y \in X$ be such that $\omega(x,y) \geq 1$. It yields that $x, y \in [1,2]$, so $Tx = Ty \in [1,2]$. Hence, $\omega(Tx, Ty) \geq 1$, that is T is ω-orbital admissible. Notice that T is not continuous. We shall show that (H) holds. Let $\{x_n\}$ be a sequence in X such that $\omega(x_n, x_{n+1}) \geq 1$ for each $n \in \mathbb{N}$. Then, $\{x_n\} \subset [1,2]$. If $\{x_n\} \to u$ as $n \to \infty$, we have $|x_n - u| \to 0$ as $n \to \infty$. Hence, $u \in [1,2]$, and so, $\omega(x_n, u) = 1$. All conditions of Theorem 1 hold. Note that $\frac{1}{3}$ and $\frac{3}{2}$ are two fixed points of T.

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