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A group decision making support system for the Web: how to work in environments with a high number of participants and alternatives

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Abstract

One of the main challenges that the appearance of Web 2.0 and the overall spreading of the Internet have generated is how to tackle with the high number of users and information available. This problem is also inherited by the group decision making problems that can be carried out over the Web. In this article, to solve this issue, a group decision making support system that allows the use of a high number of participants and alternatives is presented. This method allows any number of participants to join the decision making process at any time. Furthermore, they let them provide information only about a certain subset of alternatives. The high participation rate can provide enough information for the decision process to be carried out even if the participants do not provide information about all the high number of available alternatives.

Keywords: fuzzy linguistic modelling; group decision making; computing with words; multi-granular linguistic information.
1. Introduction

Internet has recently experienced a deep change [2, 25, 28]. In its beginnings, it started as a way of consulting information. Nowadays, Internet can be accessed from any part of the world by almost everybody. Traditional Group Decision Making (GDM) algorithms consider a small group of experts and alternatives [4, 7, 20, 22, 26, 29, 31, 34]. This is because they are designed to regulate a group decision making process carried out in a small committee. For example, they are built to be used by the managers of a company in selecting what they should do about a certain inconvenience. Nowadays, because of the new Internet paradigm, GDM methods that allow the participation of a high number of people and can work with a high number of alternatives are demanded. Internet is now designed as a place where everybody has vote and can provide opinions and ideas. Also, there are high amounts of information to discuss about. Therefore, it is extremely important to design GDM methods that can help Internet users to carry out decisions in an organized way.

When the decisions that participants must make are open and everybody can participate, it is important to get the highest possible participation rate. The higher the participation rate is, the more reliable the decisions results are. Therefore, it is important to implement tools that persuade users to participate in the decision making process.

In this article, a new GDM model that allows us to carry out GDM processes that have a high number of participants and alternatives has been designed. Its main purpose is to provide an environment for the Internet actual paradigm. Because of that, it is designed to be open and flexible, that is, to allow participation of a high amount of people and manage a high amount of information. Psychology of persuasian guidelines of Robert B. Cialdini, a well known psychologist about persuasian [10, 15] have been followed to obtain the highest possible participation rate in the process. Furthermore, a multi-granular linguistic approach [26] will be followed in order to ease the way that participants express their preferences to the system.

This paper is organized as follows. In section 2, several concepts needed to understand the designed GDM method are exposed. In section 3, the designed method is described. In section 4, an illustrated application example
is used. In section 5, advantages and drawbacks of the method are discussed. Finally, some conclusions are pointed out.

2. Preliminaries

To make this paper as self-contained as possible, this section is introducing concepts and methods to be referred to thorough this paper. In subsection 2.1, how to use and manage multi-granular fuzzy linguistic information is exposed. In subsection 2.2, psychology of persuasion guidelines used in this article are specified.

2.1. Multi-granular fuzzy linguistic information

It is a fact that human beings feel more comfortable when they can express their preferences using words. In an area such as GDM, where participants must provide their preferences to the system, it is important to provide ways of reducing the user-system communication gap. One way of reducing it consists in using fuzzy linguistic modelling [16, 21]. This linguistic approach uses the concept of linguistic variable defined by Zadeh [32].

Thanks to linguistic variables, participants can express their opinions using words and computational systems can understand what the participants are trying to communicate. It should be noticed that, usually, labels are chosen in the same linguistic term set (LTS). Nonetheless, an LTS has a fixed granularity obligating all the participants to use the same number of labels. This situation can become a disadvantage since it is probable that not all the participants agreed with the number of labels that the used LTS have. Therefore, choosing the granularity of an LTS is a critical matter that affects the participants expression capability. Because the best granularity value for an LTS depends totally on the participant, it is not possible to select a single value that fulfil all the participants requirements.

Multi-granular fuzzy linguistic modelling solves this issue giving to each participant the chance of selecting the LTS that better fits his/her necessities [26]. In such a way, every participant can provide their preferences in an easy way using the LTS that better fit his/her needs. A typical multi-granular GDM process follows the next steps [26]:

- **LTS choosing step**: Every participant selects the LTS that better fits his/her needs.
• **Providing preferences step**: Participants provide their preferences using their chosen LTS.

• **Basic linguistic term set (BLTS) conversion**: All the preferences provided by the participants are expressed using an unique LTS that is called BLTS.

• **Aggregation step**: Information is aggregated into a single collective information piece using the participants individual BLTS expressed information.

• **Exploitation step**: Selection operators are used on the collective information piece in order to create an alternative ranking.

There are several ways of carrying out the transformations among different LTSs. In this article, the method exposed in [18] will be used. It carries out the transformations using linguistic hierarchies and the 2-tuple representation.

2.2. *Psychology of persuasion guidelines*

In cases when the group decision making participants are not forced to cooperate in the group decision making process, it is important to try to persuade them to provide their opinion. This issue was not important in traditional group decision making processes since the set of experts was pre-defined and they all must participate. Nevertheless, users in the Web have free will and they may decide not to participate in the process. Therefore, it is important to design means that improve users’ participation. The more people involved in the process, the more reliable the obtained results will be. Thanks to the application of the psychology of persuasion, it is possible to pinpoint those issues that can affect people participation and try to solve them to increase the participation rate. Thanks to the psychology of persuasion, it is possible to improve issues such as interface presentation, general survey length and, in general, the way that the process is presented and sold to the user.

In [15], Robert M. Groves and Robert B. Cialdini provide a classification of the factors that influence survey participation. We believe that, considering these factors, is possible to persuade Internet users to participate in a GDM open process. According to [15], these factors can be grouped in the following categories:
• Societal-level factors.
• Attributes of the survey design
• Characteristics of the interviewee.
• Attributes of the interviewer.
• Interviewee-Interviewer interaction.

3. A novel GDM method for Web environments

In this section, the novel developed GDM method is presented. It is designed to work in environments where a high number of alternatives and participants are available. Therefore, several tools to ease the way that participants express themselves and to increase participation has been added. From now on, $E = \{e_1, \ldots, e_n\}$ will denote the variable set of participants, $X = \{x_1, \ldots, x_m\}$ will be the fixed set of alternatives and $p^k_{ij}$ denotes the preference relation [20] of participant $k$ for alternative $i$ over alternative $j$. $n$ indicates the number of participants that have participated until the present moment in the GDM process. The designed GDM method follows the next steps:

• **Providing preferences**: Participant $e_k$ provides preferences about a subset $X'_k$ of the set of alternatives $X$. Because the number of available alternatives is high, it is not affordable to make every participant to provide a pairwise comparison of every possible alternative. The solution adopted to solve this is to make them provide only a small part of the required information. This way, they leave the duty of filling the rest of the information to other participants. Because the number of participants, $n$, is expected to become high, there will be enough information to rank all the alternatives in $X$. In this step, a multi-granular fuzzy linguistic modelling approach will be used. Thus, participants will choose the LTS that better fulfil their needs.

• **Collective value calculation**: In this step, the information provided by the participants is aggregated. Because $n$ is high, it is not possible to aggregate all the individual preferences at once. Instead, participant information will be partially aggregated every time that each participant provides it. Two matrices will be generated in this step. In $C$ the
sum of all the provided values is stored while $C^{count}$ stores the number of participants that have assessed each possible pairwise comparison.

- **Selection phase**: Using matrices $C$ and $C^{count}$, operators $GDD$ and $GNDD$ [9] are applied in order to obtain the ranking of alternatives.

- **GDM process participation study**: After providing his/her preferences, every participant must fill a brief questionnaire whose purpose is to provide guidelines of how to increase the survey participation rate. Psychology of persuasion guidelines discussed in 2.2 have been used for the survey design.

In Figure 1, modules interaction scheme is showed. In the following sections, these modules will be explained in more detail. In subsection 3.1, the way of providing preferences is exposed. In subsection 3.2, how the collective matrix is being updated during the GDM process is commented. In subsection 3.3, the selection process carried out to generate the alternative ranking is described. Finally, in subsection 3.4, how the participation survey has been designed is showed.
3.1. Providing preferences module

This module oversees of retrieving preferences from the participants that want to participate in the GDM process. This process is carried out following the next steps:

- **LTS selection**: The participant selects the granularity of the LTS that he/she wants to use. If the participant wants to give an accurate opinion, he/she will choose a LTS with a high granularity value. On the other hand, if the participant is not able to be accurate, he/she is able to choose an LTS with a lower granularity value.

- **Alternatives subset selection**: A subset, $X'$, of the alternatives set $X$ is selected. There are four ways of carrying out this step:
  
  - *Custom selection*: Each participant selects the alternatives that he/she want to opine about. The advantage of this choice is that the participants can provide their preferences about the alternatives that they know more about. This way, the reliability of the information is increased. Another advantage is that participants can select the amount of information that they want to provide. A participant that wants to provide more information will select a high number of alternatives while a participant that does not want to waste much time can contribute with less information. The disadvantage of using this approach is that it does not provide equality. This way, there can be a high amount of information of those alternatives that are more known by the participants and less in the unpopular ones. Consequently, reliability of the whole GDM process could decrease.

  - *Fixed size and random selection*: For each participant $k$, a random subset $X'_k$ of fixed size $l$ is chosen. More probability of being selected is given to those alternatives that have been chosen less times. The main advantage of this method is that it provides equality among the alternatives. Therefore, the same amount of information will be recollected for all the alternatives. Furthermore, using a fixed size $l$ for all the participants guarantees that all the participants provide the same amount of information. This way, their opinion is equally valued. The main disadvantage of using this approach is that everything is preselected and the participant needs are not taken into account.
– **Custom size and random selection:** This option guarantees that all the alternatives will be equally responded but, because participants can select the amount of information to provide, there can be participants that can contribute more than others to the GDM process.

– **Fixed size and custom alternatives selection:** This option allows participants to choose a fixed number of alternatives. It guarantees the same participation in the GDM process by all the participants, but it is possible that the information provided is not equally distributed among all the alternatives.

**Information providing:** Each participant $e_k$ provides his/her preferences using preference relation matrices, $P^k$, over $X'_k$ using the LTS that they have previously selected. For all pair of alternatives $(i, j)$, $i \neq j$, such that, $x_i, x_j \in X'$, a value $p^k_{ij}$ describing how much $i$ is preferred to $j$ must be provided. Depending on the available resources and the expected quantity of participants that will participate in the GDM process, there are two ways of dealing with the provided individual preference matrices $P$:

– **Store $P$ values:** It is desirable that participants can modify their opinions and increase the amount of information provided to the system. This way, if a participant changes his/her mind, he/she can access to his/her preference values and modify them. Also, if he/she want to provide new information about alternatives that were not initially included in his/her $X'$, new alternatives can be added and matrix $P$ updated. Storing the participants alternatives also allows us to implement consensus measures [4, 6, 33] and use proximity values [5] to help participants to bring positions closer together. To include all these features, $P$ values of all the participants that have participate need to be stored.

– **Aggregate $P$ values and discard them:** It it possible that the infrastructure where the GDM process wants to be carried out does not have capability to store all the participants individual preference values. In this case, two solutions can be applied:

  * Store the information locally: Each participant can store locally in his/her access device the information of their own
preferences. The preferences information about a single participant has a low weight. Therefore, it is not costly to store it locally. In such a way, the server will not be overcharged with all the participants individual preferences information. With this approach, the server is only dedicated to store the collective matrices and carry out GDM process computations. The main problem about this configuration is that participants are being obligated to dedicate their own computational resources to the GDM process. Consequently, the connecting device is not totally free of the GDM process computations.

* Discard participants individual preferences: With this approach, after carrying out the aggregation step, the individual preference information is deleted. The main advantage of this approach is that no computational effort is carried out in the participants access device. The main drawback is that, if individual preference information is not stored, it is not possible to allow participants to modify their opinions or giving them feedback about how to reach a consensus.

- **LTS to BLTS conversion**: All the participants opinions that are expressed using an LTS different from the BLTS must be translated into it. In such a way, all the information can be expressed using the same means and, therefore, it is possible for the system to deal with it. Method exposed in [18] will be used to carry out the necessary conversions. In it, a set of LTSs is organized as a hierarchy, each level containing a single LTS. Conversions can be established among different levels of the hierarchy allowing labels from one LTS to be transformed into labels of another one. The used transformation function is showed below:

\[
TF_{t'}^t : l(t, n(t)) \rightarrow l(t', n(t'))
\]

\[
TF_{t'}^t \left( s_i^{n(t)}, \alpha^{n(t)} \right) = \Delta \left( \frac{\Delta^{-1} \left( s_i^{n(t)}, \alpha^{n(t)} \right) \cdot \left( n(t') - 1 \right)}{n(t) - 1} \right)
\]

(1)

where \( l(t, n(t)) \) indicates the hierarchy, \( t \) is the hierarchy level, \( n(t) \) the granularity of the LTS located in hierarchy level \( t \), \( \left( s_i^{n(t)}, \alpha^{n(t)} \right) \) is a 2-tuple value from a LTS with granularity \( n(t) \) as exposed in [17] and
\( \Delta \) and \( \Delta^{-1} \) functions are defined below:

\[
\Delta : [0, g) \rightarrow S \times [-0.5, 0.5)
\]

\[
\Delta(\beta) = (s_i, \alpha) \quad \text{with} \quad \begin{cases} 
  s_i = \text{round}(\beta) \\
  \alpha = \beta - i \\
  i \in [0.5, 0.5)
\end{cases}
\]

(2)

\[
\Delta^{-1} : S \times [-0.5, 0.5) \rightarrow [0, g)
\]

\[
\Delta^{-1}(s_i, \alpha) = i + \alpha = \beta
\]

(3)

where \( S = \{s_0, \ldots, s_i, \ldots, s_{n(t)}\} \) is a linguistic label set, \( \alpha \) is a numerical value located in \([-0.5, 0.5)\) called the symbolic translation. It indicates the distance between the obtained value and the closest label. \( \beta \) is the numerical representation of a concrete linguistic label or an aggregation of them and \( i \) is the closest label index to the \( \beta \) value. Using \( \Delta^{-1} \) and \( \Delta \) functions, it is possible to convert numerical values \( \beta \) into labels in the form \((s_i, \alpha)\) and the other way around.

A conversion example is presented below:

**Example 1.** In a GDM process, participant \( e_1 \) uses the LTS \( S_1 = \{s^{1}_{1}, \ldots, s^{1}_{5}\} \) to express his/her preferences while the rest of the participants use the the LTS \( S_2 = \{s^{1}_{1}, \ldots, s^{1}_{9}\} \). If the participant \( e_1 \) provides the label \( s^{1}_{3} \) for a specific pair of alternatives, that label can be express using \( S_2 \) applying expression (1) as follows:

\[
(s^{1}_{3}, 0) = \Delta \left( \frac{\Delta^{-1}(s^{1}_{3}, 0) \cdot (9 - 1)}{5 - 1} \right) = \Delta \left( \frac{3 \cdot 8}{4} \right) = (s^{2}_{6}, 0)
\]

Some recent applications of 2-tuple linguistic representation can be found in [23, 27, 35].

As it has been previously commented, if participants individual preferences values are stored, the following features can be added to the already defined GDM process:

- **Preferences modification:** It is quite usual that participants have a certain opinion at the beginning of a GDM process and, after taking in other points of view, they change their minds. This way, it is important to allow preferences modification in a GDM process. Nevertheless, it should be noticed that the necessity of this feature is context
dependant. For example, let focus on a GDM process that is about sorting a set of movies. Because liking or disliking a movie is quite a personal choice, the debate can be carried out but it is not really necessary because it is not likely that participants change their opinions. In this case, allowing preferences modification is not a critical matter. Nevertheless, if the GDM process is about sorting a political party preferences, debate is extremely important and it is quite likely that participants change their opinions in the middle of the process. Therefore, in this case, it is indispensable to allow preferences modification.

- **Consensus calculation**: When making a decision in a GDM process, it is important that the participants reach an agreement. Participants should talk among them, carry out a debate, share their point of views and, finally, reach an agreement. If this process is not carried out, decisions are made blindly and bad consequences are more likely to appear. Consensus measures \([3, 12, 13, 14, 24, 30]\), help us to measure and determine if agreement among participants have been reached. This way, if the reached consensus is very low, it will be better to let participants to debate a bit longer. On the other hand, if consensus is high, it means that a high majority the participants support the final decision and, therefore, GDM results are reliable. For carrying out the consensus measures calculations, it is necessary to access and carry out calculations with all the participants individual preferences matrices. Therefore, it is necessary to store all this information in the server. If a enormous amount of participants are participating in the GDM process, due to the necessity of disk space and to the high amount of information needed to work with, this process can take a very high amount of time or even become unmanageable.

- **Participants feedback**: Proximity measures \([9, 30]\) calculate the distance between the collective matrix and each participant individual preferences matrix. If the distance is high, it means that the participant opinion is far from the main stream opinion. If it is low, it means that he/she agrees with the majority of them. With proximity measures, it is also possible to help participants to modify their opinions in order to make them closer to the main stream. To calculate each participant feedback, it is necessary to know the collective preference
matrix and the participant individual preference matrix. Therefore, it is a measure much easier to calculate in a domain with a high number of participants and alternatives than consensus ones. Because participants feedback can work properly if participants individual preferences are stored locally, we recommend to use proximity values in order to approximate consensus. This way, if the majority of participants have low proximity values to the collective matrix, it is easy to determine that consensus is high.

In Figure 2, an scheme of a providing preferences module with feedback is showed. On it, the participant provides his/her preferences, information about the LTS used and the alternatives selected. Then, information is transformed from LTS to BLTS and stored in the collective preferences matrix by the collective value calculation module. Finally, participant is given feedback of how to modify their preferences in order to be closer to the main stream opinion. He/She can modify them or leave them as he/she has provided it. Therefore, they are not forced to follow them.
3.2. Collective value calculation module

The collective value calculation module is in charge of adding the preferences that the participants provide to the collective matrix. From now on, it is assumed that participants’ individual preferences are maintained in the system. For carrying out this task, the module follows the next steps:

- **New and modified information identification**: Every time a participant $e_k$ provides information to the GDM process, it must be determined if the information is new or he/she is modifying his/her already provided preferences. This is done by consulting the individual preference matrix of each participant and the participant subset $X'_k$ which indicates the alternatives that the participant has already worked with. According to the results, there are three possible paths to follow:

  1. **New information providing**: In this case, all the information provided by the participant is new. This usually happens when it is the first time that the participant is providing preferences in the GDM process or when he/she is sending data about alternatives that were not previously included in his/her $X'_k$ subset. In this case, all the preferences values are considered new and the adding new information step is carried out.

  2. **Modifying existing information**: The participant has already sent information to the GDM process and all the data provided is about alternatives that were previously included in the alternatives subset $X'_k$. In this case, the participant is just modifying the preferences that he/she has already provided. The modifying the previously provided information step is carried out.

  3. **New information providing and modification of the existing one**: In this case, the participant provides new information and, at the same time, he/she has modified preferences provided before. Information that has already been provided will follow modifying the previously provided information guidelines while new information will be dealt as the Adding the new information step indicates.

- **Adding new information**: When the provided information is new, that is, the participant is not modifying their previous provided preferences, information is aggregated to the collective matrix following the next steps:
1. **Collective preferences matrix update**: Each new preference value $p_{ij}^k$ provided by the participant $k$ is added to $C_{ij}$. Sum operator is used as follows:

$$C_{ij} = C_{ij} + p_{ij}^k$$

(4)

It should be taken into account that $i, j \in X', i \neq j$.

2. **Collective preferences matrix count update**: Each position $(i, j)$ in $C_{\text{count}}$ is increased by 1.

- **Modifying the previously provided information**: When information is being modified, the following expression is used:

$$C_{ij} = C_{ij} - p_{ij}^{k'} + p_{ij}^k$$

(5)

where $p_{ij}^{k'}$ represents the old preference value and $p_{ij}^k$ the new one. $C_{\text{count}}$ does not need to be updated since no new information has been added.

It should be noticed that if participants’ individual preferences values are not stored, it is impossible to distinguish among different individuals. Therefore, all the information must be dealt as new one.

In Figure 3, an scheme of the described process can be observed.

### 3.3. Selection phase module

Using the collective matrix calculated in the previous step, the alternatives ranking is made. For this purpose, GDD and GNDD operators [9] will be used. Ranking is calculated as follows:

1. $C$ matrix contains the sum of all the participants preferences while $C_{\text{count}}$ stores the number of people that have provided each piece of information $C_{ij}$. In order to obtain the collective preference matrix mean values for each pair of alternatives, $C_{\text{mean}}$, the following expression is followed:

$$C_{ij}^{\text{mean}} = C_{ij}/C_{ij}^{\text{count}}, \forall i, j \in [1, m], i \neq j$$

(6)

2. Using $C_{ij}^{\text{mean}}$ GDD operator is calculated. GDD operator calculates the degree in which one alternative dominates the other ones. The operator is calculated as follows:

$$GDD_i = \phi(c_{i1}, c_{i2}, \ldots, c_{i(i-1)}, c_{i(i+1)}, \ldots, c_{in})$$

(7)
3. Similar process is carried out to calculate the GNDD operator. This expression calculates the degree in which one alternative is not dominated by the rest. GNDD operator expression is defined below as follows:

\[
G N D D_i = \phi(c^s_{1i}, c^s_{2i}, \ldots, c^s_{(i-1)i}, c^s_{(i+1)i}, \ldots, c^s_{ni}) \quad (8)
\]

where

\[c^s_{ji} = \max\{c_{ji} - c_{ij}, 1\}\]

4. Final ranking value, \(RV\), is calculated calculating the mean of the GDD and the GNDD degree values:

\[
RV = (GDD_i + GNDD_i)/2, \forall i \in [0, m] \quad (9)
\]

Alternatives are then sorted using the RV values.

A scheme of the described process is showed in Figure 4.

To get a view of the GDM participation in a specific moment, the participation rate measures can be used. For an alternative \(i\), its participation rate is calculated as follows:

\[
PR_i = \phi(c^\text{count}_{1i}, c^\text{count}_{2i}, \ldots, c^\text{count}_{(i-1)i}, c^\text{count}_{(i+1)i}, \ldots, c^\text{count}_{mi}) \quad (10)
\]
The overall participation rate can be calculated aggregating the $PR_i$ values as follows:

$$PR = \phi(PR_1, \ldots PR_m)$$  \hspace{1cm} (11) 

$PR$ value estimates, considering the amount of information stored, the number of hypothetical participants that would have participated in the GDM process if they were all providing pairwise comparison for all the alternatives available to them.

It is also possible to estimate the minimum possible number of participants that must participate in a GDM process for a specific $PR$ value. Let $m$ be the number of alternatives available in the GDM process, $a$ the average number of alternatives that each participant chooses, $n$ the $PR$ value that is desired and $e$ the number of participants that have to participate in order to get the desired $PR$ value, $e$ can be calculated as follows:

$$e = \frac{a^2}{m^2} \cdot n$$  \hspace{1cm} (12) 

For example, for having a $PR$ of 5, considering that participants will choose
a mean of 4 alternatives to participate and having a total amount of 20 alternatives, at least 125 participants must participate in the GDM process. This way, it is possible to have an idea of the moment in time that the selection process should be applied and how reliable the results will probably be. It should also be pointed out that, using that expression, it is possible to calculate the average number of alternatives chosen by the participants at the end of the selection phase. It is important to notice that expression (12) is also assuming that there not exist any preference among the alternatives, that is, participants will select with the same probability any of the available alternatives.

3.4. GDM process participation study

Using Robert M. Groves and Robert B. Cialdini psychology of persuasion guidelines [15], a brief survey has been designed to improve the participation rate. The survey is designed to be long enough to provide useful information but short enough to not tired participants. It is passed to participants after the providing preferences step. Using their responses, suggestions are built and showed to the GDM designer who can follow them or leave them aside. In Figure 5, a scheme of this process can be seen. The set of the chosen 7 questions are described below:

1. **Did the survey took too long to fill?**: When using a fixed size of $X'$ for all of the participants, the answer to this question indicates if the number of alternatives chosen is correct or should be reduced. To get reliable information, it is important not to exhaust participants with too much information to provide. If most of participants indicates that the survey was too long, the system will suggest the GDM designer to reduce the number of alternatives included in each $X'$.

2. **Is the Web page attractive?**: As commented in section 2.2, the liking principle is critical in order to obtain people cooperation. In this case, because Internet is being used, the liking principle will be fulfilled if the providing preference mean is liked by the participants. If most of participants respond that the web page used for carrying out the GDM process is not attractive, the system will suggest the GDM designer to modify the design.

3. **Do you feel that the GDM topic is important to you or your community?**: This question tries to check if the social validation principle is fulfilled. If most of participants respond negatively, the GDM
designer will be asked to enhance how important is each vote for the community.

4. **Do you trust the company that is carrying out the GDM process?** With this question, authority principle is being checked. If the majority of participants respond “no”, the GDM designer will be asked to provide information about why the company or institution should be trusted. This way, it is expected that participants will be more willing to participate.

5. **Do you think that it has been a great opportunity to participate?** This question checks the scarcity principle. If participants respond negatively, the GDM designer will be asked to enhance the importance of getting a high participation rate in the GDM process and how valuable their opinion are for the GDM process purpose.

6. **Do you feel that your participation is going to help?** This question checks if the GDM designed is applying helping tendencies in order to improve participation rate. If the answer is negative, the GDM designer is asked to clarify that the company/institution carrying out the GDM process really needs people participation.

7. **How would you improve this survey?** Participants opinion can be really valuable in order to make improvements to the survey. GDM designer can study them and carry out changes to increase the value of the provided information.

It should be considered that the GDM process designer is not forced to make any change to the GDM process. The process participation module is only in charge of providing suggestions.

4. **Illustrative Example**

In this section, an illustrative example that shows how our designed GDM process works is shown. Imagine that a company such as the trusted IMDB [11, 19] wants to establish a ranking of the most rated movies in its webpage using the Internet users opinion. The company selects the 20 most rated movies and ask participants from all around the world to rank them using pairwise comparison. The alternatives are shown in Table 1.

Establishing a pairwise comparison among 20 alternatives will imply that every user has to provide \(20 \cdot 20 - 20 = 380\) preference values. Therefore,
Figure 5: Process participation module scheme.

<table>
<thead>
<tr>
<th>$x_1$</th>
<th>The Shawshank Redemption</th>
<th>$x_{11}$</th>
<th>The Lord of the Rings I</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_2$</td>
<td>The Godfather</td>
<td>$x_{12}$</td>
<td>The Empire Strikes Back</td>
</tr>
<tr>
<td>$x_3$</td>
<td>The Godfather II</td>
<td>$x_{13}$</td>
<td>Inception</td>
</tr>
<tr>
<td>$x_4$</td>
<td>The Dark knight</td>
<td>$x_{14}$</td>
<td>Forrest Gump</td>
</tr>
<tr>
<td>$x_5$</td>
<td>Pulp Fiction</td>
<td>$x_{50}$</td>
<td>One Flew Over the Cuckoo's Nest</td>
</tr>
<tr>
<td>$x_6$</td>
<td>Il buono, il brutto, il cattivo</td>
<td>$x_{16}$</td>
<td>The Lord of the Rings II</td>
</tr>
<tr>
<td>$x_7$</td>
<td>Schindler’s List</td>
<td>$x_{17}$</td>
<td>Goodfellas</td>
</tr>
<tr>
<td>$x_8$</td>
<td>12 Angry Men</td>
<td>$x_{18}$</td>
<td>Matrix</td>
</tr>
<tr>
<td>$x_9$</td>
<td>The Lord of the Rings III</td>
<td>$x_{19}$</td>
<td>Star Wars</td>
</tr>
<tr>
<td>$x_{10}$</td>
<td>Fight Club</td>
<td>$x_{20}$</td>
<td>Shichinin no samurai</td>
</tr>
</tbody>
</table>

Table 1: Illustrative example alternatives list.
to make participants to provide preferences values to every pair of alternatives is unaffordable. Furthermore, it is possible that participants have not watched all the movies on the list making them unable to provide reliable values when referring to certain alternatives.

Thanks to the designed GDM process, participants can select the alternatives that they want to provide information about. This way, the number of preferences values that each participant has to provide are reduced and they also are able to provide information about alternatives that they really know about.

For this example, it will be assumed that 30 participants will attend the GDM process. For the sake of simplicity, it will only be shown preferences of three of them: $e_1$, $e_2$ and $e_3$.

First, participants select the alternatives that they want to discuss, that is, their personal $X'$, and the LTSs that they want to use to provide their preferences. In order to express themselves, participants $e_1$ and $e_2$ select $S^5$ while $e_3$ selects $S^9$. Both LTSs are defined below:

$$S^5 = \{s^5_1, s^5_2, s^5_3, s^5_4, s^5_5\}$$
$$S^9 = \{s^9_1, s^9_2, s^9_3, s^9_4, s^9_5, s^9_6, s^9_7, s^9_8, s^9_9\}$$

It is important to notice that the higher the index, the more the participants prefer one alternative over the other. This way, $s^x_1$ indicates that the experts totally prefer the second option over the first where $x$ indicates the granularity of the LTS. On the contrary, if participants select $s^x_x$, they indicate that they totally prefer the first option over the second. Finally, $s^x_{x/2}$ indicates that they have more or less the same opinion about both alternatives. It is also remarkable that the higher the granularity, the more specific their preferences will be.

Alternatives sets $X'$ selected by the three participants are showed below:

$$X'_1 = \{x_1, x_2, x_{17}\}$$
$$X'_2 = \{x_2, x_{14}, x_{17}, x_{20}\}$$
$$X'_3 = \{x_2, x_4, x_{16}\}$$
Table 2: Collective preference matrix for participants $e_1$ and $e_2$.

<table>
<thead>
<tr>
<th></th>
<th>$x_1$</th>
<th>$x_2$</th>
<th>$x_{14}$</th>
<th>$x_{17}$</th>
<th>$x_{20}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_1$</td>
<td>-2</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>$x_2$</td>
<td>3</td>
<td>-2</td>
<td>5</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>$x_{14}$</td>
<td>0</td>
<td>1</td>
<td>-2</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>$x_{17}$</td>
<td>5</td>
<td>9</td>
<td>4</td>
<td>-5</td>
<td></td>
</tr>
<tr>
<td>$x_{20}$</td>
<td>0</td>
<td>2</td>
<td>3</td>
<td>1</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 3: $C^{count}$ matrix for participants $e_1$ and $e_2$.

<table>
<thead>
<tr>
<th></th>
<th>$x_1$</th>
<th>$x_2$</th>
<th>$x_{14}$</th>
<th>$x_{17}$</th>
<th>$x_{20}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_1$</td>
<td>-1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>$x_2$</td>
<td>1</td>
<td>-1</td>
<td>2</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>$x_{14}$</td>
<td>0</td>
<td>1</td>
<td>-1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>$x_{17}$</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>-1</td>
<td></td>
</tr>
<tr>
<td>$x_{20}$</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>-</td>
</tr>
</tbody>
</table>

After this selection, they proceed to provide their preferences. Values provided for these three participants can be seen below:

$$P_1 = \begin{pmatrix} -s_5^2 & s_1^5 \\ s_3^5 & -s_4^5 \\ s_5^5 & s_4^5 & - \end{pmatrix} P_2 = \begin{pmatrix} -s_2^5 & s_1^5 & s_1^5 \\ s_3^5 & -s_2^5 & s_1^5 \\ s_5^5 & s_4^5 & - \end{pmatrix} P_3 = \begin{pmatrix} -s_9^9 & s_8^9 \\ s_9^9 & -s_3^9 \\ s_2^9 & s_1^9 & - \end{pmatrix}$$

It should be noticed that positions $(i,j)$ in each matrix refers to different alternatives. The BLTS used for storing preferences in the collective matrix is $S^5$. Therefore, preferences from participants $e_1$ and $e_2$ can be aggregated without any previous transformation. In Tables 2 and 3, filled collective values after aggregating preferences from participants $e_1$ and $e_2$ can be seen. The index value of each label has been used.

It can be seen that because $e_1$ and $e_2$ have alternatives $x_2$ and $x_{17}$ in common, values referring to $C(2,17)$ and $C(17,2)$ are the aggregation of preference values from $e_1$ and $e_2$. Consequently, $C^{count}$ sets the $(2,17)$ and $(17,2)$ position values as 2 because there are two participants that have provided preferences for alternatives $x_2$ and $x_{17}$.
Before aggregating $e_3$ preferences, it is necessary to transform the labels and express them using $S^5$. Each label of the matrix is transformed using expression (1). Results are showed below:

$$P_3 = \begin{pmatrix} s_5^5 & (s_4^5, 0.5) \\ s_3^5 & - \\ (s_1^5, 0.5) & s_2^5 \\ & - \end{pmatrix}$$

(13)

Now that $P_3$ is expressed using the BLTS, information can be aggregated. After all the 30 participants have attended the GDM process, GDD and GNDD values are calculated using expressions (7) and (8) respectively. Results can be seen in Table 4. After carrying out the mean as indicated in expression (9) and sort the results, alternatives ranking is as follows:

$$\{x_9, x_{14}, x_{20}, x_{17}, x_{16}, x_{15}, x_{18}, x_{19}, x_7, x_{13}, x_{12}, x_6, x_{10}, x_{11}, x_4, x_5, x_3, x_2, x_1\}.$$

Therefore, $x_9$ is the most popular movie while $x_1$ is the least one.

After carrying out the GDM process, participation rate for each alternative will be calculated. Results can be seen in Table 5. The overall participation rate, $PR$, is 2.255. This way, a mean of only 2.255 participants have provided information for each pair of alternatives. Because the number is too low, this means that results obtained are not reliable, more participants must participate in the GDM process to obtain accurate results.

Using expression (12), it is possible to deduce the mean number of alter-
Table 5: Participation rate value for each alternative.

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_1$</td>
<td>2.25</td>
<td>$x_{11}$</td>
<td>2.1</td>
</tr>
<tr>
<td>$x_2$</td>
<td>1.6</td>
<td>$x_{12}$</td>
<td>2.9</td>
</tr>
<tr>
<td>$x_3$</td>
<td>2.05</td>
<td>$x_{13}$</td>
<td>2.75</td>
</tr>
<tr>
<td>$x_4$</td>
<td>2.5</td>
<td>$x_{14}$</td>
<td>2.45</td>
</tr>
<tr>
<td>$x_5$</td>
<td>2.45</td>
<td>$x_{15}$</td>
<td>2.25</td>
</tr>
<tr>
<td>$x_6$</td>
<td>1.85</td>
<td>$x_{16}$</td>
<td>2.1</td>
</tr>
<tr>
<td>$x_7$</td>
<td>1.45</td>
<td>$x_{17}$</td>
<td>2.3</td>
</tr>
<tr>
<td>$x_8$</td>
<td>2.1</td>
<td>$x_{18}$</td>
<td>2.25</td>
</tr>
<tr>
<td>$x_9$</td>
<td>2.05</td>
<td>$x_{19}$</td>
<td>2.55</td>
</tr>
<tr>
<td>$x_{10}$</td>
<td>2.55</td>
<td>$x_{20}$</td>
<td>2.6</td>
</tr>
</tbody>
</table>

natives selected by each participant:

$$a = \sqrt{\frac{m^2 \cdot n}{e}} = \sqrt{\frac{20^2 \cdot 2.255}{30}} = 5.48$$ (14)

Therefore, it is estimated that each participant selects between 5 and 6 alternatives to participate in the GDM process. Taking that into account, it can be estimated how many numbers of participants will be needed until reaching a desirable $PR$. If a $PR$ value of 10 is desired, the following number of participants cooperation will be needed:

$$e = \frac{20^2}{4^2} \cdot 10 = 250$$ (15)

Thanks to expression (12), it is possible to estimate results without having to carry out the selection phase which is usually costly due to the high number of participants and alternatives that the GDM process deals with.

5. Discussion

The new paradigm of Internet and Web applications have promoted the adaptation of traditional algorithms that were designed for working in personal computers. Implementing algorithms over the Internet requires them to overcome the limitations imposed by the framework that they were firstly designed for. In the case of GDM methods, features of traditional GDM processes that are not supported by the Internet and how they have been solved in this paper are described below:
• **Low number of participants**: Traditional GDM methods were designed for a small group of people to make decisions. Nevertheless, in the Internet framework, information can be provided by any user. Therefore, it is desirable for GDM processes to be open and allow participation of every single person that want to join in. Theoretically, it does not exist a maximum allowed number of participants that can participate but, in practice, computational means have limits due to the high amount of information that the system can manage. These limits have a dramatically impact in the following aspects:

1. **Collective matrix calculation**: Collective matrix is calculated aggregating all the individual preferences provided by the participants. When the set of participants is high, because there are a lot of pieces of information to aggregate, this calculation can become computational intensive. To solve this issue, collective matrix is not calculated aggregating all the information at once. On the contrary, each participant piece of information is aggregated at the same moment it is received. This way, collective matrix is in constant update and the aggregation computational time is expanded over all the GDM providing preferences step time reducing its impact in the final results calculation.

2. **Information storing**: As it has been commented, having a high number of participants implies having a high number of individual preference values. With the computational resources available nowadays, it can be possible to have a server that stores all this information but, what can be done if our resources are limited? In that case, 2 solutions have been given: 1) to make each participant to store locally their individual preference values and, 2) to only keep the collective matrix. The second option implies a big disadvantage, that is, participants cannot modify their preferences once that they have provided them. First option can be carried out, for example, implementing an Android or IOS app.

3. **Consensus calculation**: Consensus measures [9] are traditionally calculated using the individual preferences matrices from the participants, comparing them two by two. When an extremely high number of participant is expected, to carry out two by two computations of each individual preference is completely unaffordable. Especially, it is totally unaffordable if there is not enough space to
store the individual preferences in the server. To solve this issue, it has been proposed the use of proximity measures to help participants to get closer to a consensus. Because proximity measures just take the individual preference information of each participant and the collective one, it is affordable to generate suggestions for the participants to increase consensus. It is important to point out that no solution has been given for the task of calculating a global consensus value. It should be pointed out also that, when dealing with an extremely high number of participants, to make them reach a consensus is, if not difficult, an almost impossible task. In conclusion, because having a high number of participants implies a high number of individual preference values, to be efficient, operations using the individual preference values should be avoided when possible.

4. The GDM round concept: In the traditional GDM methods, the concept of GDM round is conceived. In each round, participants debate, provide their preferences and consensus measures are calculated. When the consensus measures are high, final results are calculated and GDM process is ended. If they are low, another round is carried out. When having an open GDM process with a high number of participants, it is impossible to check if every participant has participated in the debate or if there are new participants that want to join and participate later. Luckily there exists platforms like Twitter or Facebook that can deal with a high amount of people and give them tools to carry out a debate. In the designed GDM method on this paper, the round concept is present in a fuzzy way. GDM rounds are carried out as usual but decision results are calculated in fixed dates, without considering any consensus measures. Also, because the GDM process is designed to be as open as the Internet, participants can to join the GDM process at any time and in any round. Furthermore, there is no debate participation verification since is not affordable to track participants when there is a high amount of them.

5. Participation rate needed: Traditional GDM processes were defined for a predefined set of participants to participate. Participants are supposed to participate because it is assumed that it is their duty. Nevertheless, if an open GDM process is assumed, any Internet user can become a participant and, therefore, they do not
have any duty to participate in the process. Consequently, means to attract participants are needed to convince as many people as possible to participate in the GDM process. The designed GDM process use Cialdini psychology of persuasion guidelines to generate suggestions about how to attract more participants to the process. Participants answer a set of questions that give us an idea of how attractive the designed GDM process is for them and how to improve it.

- **Low number of alternatives**: Traditional GDM processes were designed to work with a low number of alternatives. Nevertheless, since Internet is able to hold a high amount of information, it would be desirable to design GDM processes that can work with a high number of alternatives. To design such a GDM method, it is necessary to surpass the following challenges:

  1. *Pairwise information providing*: When having a high amount of alternatives and, specially, when it is desired that participants express themselves using pairwise comparison, participants must provide a high amount of information. This situation is clearly unaffordable since participants will get tired and abandon the GDM process if they are asked to provide, for example, 300 values. If reliable information wants to be obtained from participants, it is necessary to not push them too hard. To solve this issue, the designed GDM method allows participants (or GDM process designers) to select the number of alternatives that they want to deal with. This way, participants can work with that reduced set of alternatives making the amount of information requested more reasonable. It should be noticed that this approach will only work if it is expected that a high number of participants are going to cooperate. This way, every participant subset concentrates in providing information about a specific set of alternatives and, all together, they can provide enough information to make a reasonable ranking including all the available alternatives. Consequently, in this case, having a high number of participants is an advantage.

  2. *Preference matrix computations*: Having a big alternatives set means dealing with big preference matrices. Therefore, GDD and GNDD calculations can become computationally intensive.
In particular, GDD just sums the rows of the collective preferences matrix. Therefore, it is more efficient that GNDD which must calculate an intermediate matrix $C^s$. No suggestions have been given about how to solve this issue. We suggest avoiding calculating GNDD and make the ranking using only GDD if GNDD computations become too intensive. If GDD and GNDD computations are both too intensive because of the preferences matrices length, we suggest avoiding using preference relations and use utility vectors or preferences orderings. It is possible to carry out conversions among preference relations, preference orderings and utility vector using the expressions contained in [8].

3. **Dealing with missing information:** As it has been pointed out, if participants are allowed to provide information only about a small subset of the entire set of alternatives, a high participation rate of participants is needed. Enough information must be recollected to carry out a reliable GDM ranking calculation. In this paper, it has been assumed that, at the selection step execution moment, enough information has been provided, but, what can be done if we find out that there is alternatives that has never been chosen by any participant? If it is possible to establish relations among the alternatives, one option is to assign the values of the alternative that is more like it. In [1], several ways of carrying out this kind of processes are described. Nevertheless, if no relations can be made among the alternatives, inferring methods cannot be performed. This can happen because it is considered that every choice indicates a different path. In this case, we suggest removing that alternative from the ranking. This decision is based in the fact that if no participant has provided any information about it, they do not appreciate it. In other words, it is considered that not showing interest about a specific alternative is a symptom that participants do not like it. It is important to notice that an alternative have missing information in it if at least one pairwise value including it has not been provided by any participant.

In Table 6, traditional GDM methods issues and how they have been solved in this paper are exposed.

Thanks to all the presented improvements, it is possible to carry out
High number of participants

<table>
<thead>
<tr>
<th></th>
<th>Update it with every preference providing.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Collective matrix</td>
<td></td>
</tr>
<tr>
<td>calculation</td>
<td></td>
</tr>
<tr>
<td>Information storing</td>
<td>In the server, locally or not to store.</td>
</tr>
<tr>
<td>Consensus calculation</td>
<td>Not affordable, use proximity measures.</td>
</tr>
<tr>
<td>GDM rounds</td>
<td>Participants can join at any time.</td>
</tr>
<tr>
<td></td>
<td>Cannot track their movements.</td>
</tr>
<tr>
<td>Participation rate</td>
<td>Use psychology of persuasion guidelines to attract participants.</td>
</tr>
</tbody>
</table>

High number of alternatives

<table>
<thead>
<tr>
<th></th>
<th>Use reduced set of alternatives.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Information providing</td>
<td></td>
</tr>
<tr>
<td>Preference matrix</td>
<td>GDD and GNDD, GDD only</td>
</tr>
<tr>
<td>computations</td>
<td>or not use preference relations.</td>
</tr>
<tr>
<td>Missing information</td>
<td>Omit alternatives with no information or infer missing information.</td>
</tr>
</tbody>
</table>

Table 6: Traditional GDM methods and proposed solutions.

complex group decision making processes over the Internet. For instance, some examples are exposed below:

- **Selection of the most preferred destinations:** Imagine that a travel webpage want to know which are the preferred destinations of its users. This way, it is possible to apply our method using the webpage visitors as participants and the different destinations as alternatives.

- **Selection of the best smartphones:** Imagine that a computer store webpage want to know which are the most popular smartphones among its visitors. For this purpose, they can apply our methodology.

- **Recommending videos:** Imagine that a webpage like youtube want to rate the most visited videos in their web. This way, using our method, a group decision making method can be carried out in order to sort them. Debate among participant can help to decide which one is the best.

6. Conclusions

In this paper, a novel GDM method design that works in environments where a high number of participants and alternatives are available has been
developed. With the new Internet paradigm, the Internet user is becoming more involved in the Internet information providing. This fact has led to the creation of a huge network that stores a really high amount of information that is available to every user in the net. GDM methods were initially designed to work in environments with a low number of participants and alternatives. That is, they were designed for a small group to decide among a small set of alternatives. Nevertheless, the Internet paradigm is open to everybody and there is a lot of information to deal with. Therefore, changes must be made to traditional algorithms in order to fulfil these new requirements. In this paper, a novel GDM method that is capable of working with these types of environments and overcoming their needs, have been presented.

To increase and attract as much participation as possible, Cialdini psychology of persuasion guidelines have been followed. Participants fill a small survey about the GDM process that allow the system to provide suggestions about how to introduce improvements that attracts people attention and enhance their willing to participate on it.

The developed method has only been tested with toy problems. As future research, we are planning on applying the developed method in real world problems to analyse how the method behaves and fix all the issues that could appear.

Acknowledgements

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References


Highlights

- Our novel developed method is able to work in environments that have a high number of experts and alternatives.

- It is a very useful method for open environments like Web 2.0.

- The experts do not need to provide preferences for all the alternatives. They can select the ones that they prefer.

- The proposed method takes into account the amount of information stored in the server.

- Cialdini guidelines are followed in order to make the group decision process easy for the experts to use.