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# Time-Domain Numerical Modeling of THz Photoconductive Antennas

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Abstract—This paper presents a computational procedure to simulate the time-domain behavior of photoconductive antennas made of semiconductor and metal materials. Physical modeling of semiconductor devices at terahertz regime can be achieved by applying joint electronic and electromagnetic procedures, e.g., solving a coupled system of equations inferred from Poisson's drift-diffusion and Maxwell's equations. A set of discrete equations are derived by applying a combined finite-difference methodology for the previous steady-state and the finite-difference time-domain procedure for the transient regime. The results for the radiated electric field at broadside direction show good agreement with the experimental results previously reported in the literature.

*Index Terms*—Finite-difference time-domain (FDTD), photoconductive antennas (PACs), semiconductor device modeling, terahertz sources.

#### I. INTRODUCTION

ESPITE being introduced for more than 30 years ago, the design and manufacture of efficient sources operating in the terahertz regime currently remain an active topic of research [1]-[3]. In fact, the development of sources that can provide adequate power will define the future of THz systems, which are usually limited by the loss propagation factor [4]. Today, the choice of THz sources for systems operating in THz regime is made among: 1) laser-based sources (quantum-cascade lasers and optically pumped molecular lasers), which provide maximum power but present drawbacks in terms of size, weight, and cost [2]; 2) the solid-state sources (e.g., the microvacuum traveling-wave tube) [3], as an intermediate solution with advantages in terms of mechanical properties and price but with the disadvantage of reduced power; and 3) the photonics-based sources (e.g., optical rectifiers, photomixers, and photoconductive antennas), which present serious problems in terms of delivered power yet remain unbeatable as low-profile devices [1]. In this context, this paper is intended to be a computational assessment of the radiation parameters in photoconductive antennas,

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as an initial step toward developing photoconductive sources able to increase the delivered power in THz systems. Photoconductive antennas (PCAs) are widely used in applications such as THz spectroscopy [5]. Their ability to produce time-domain waveforms ranging from several tenths of GHz up to 5-10 THz is appropriate for testing signatures of materials in THz regime with a single pulse [6]. The first device designed for this purpose was the Auston switch [7], which confirmed the ability of photoconductive materials to propagate and radiate transient electromagnetic waves within the range of the subpicoseconds, as previously reported [8]. Laser pulses were later used to excite photoconductive antennas and to generate terahertz pulses [5], [9], and since then experimental measurements have been made from different semiconductors (e.g., LT-GaAs [10], [11], InGaAs [12], [13] or SiC [14]), emphasizing the advantages and disadvantages of each photoconductive material. Furthermore, CAD techniques used to design innovative PCAs have also been described. A simple numerical approach was made by formulating an equivalent current injected into the gap, modeled as a function of the incident laser power, which acts as a source term in Maxwell's curl equations [9]. However, this assumption neglects the mutual coupling between the equations governing the current generation at the gap and Maxwell's equations. Other semi-analytical approaches proposed one-dimensional models of the source currents in order to derive the radiated electric field [15], [16], but the assumptions of those techniques are only accomplished for some geometries (e.g., large apertures or symmetric electrodes). Thus, full-wave numerical models were then introduced, and different implementations of finite-difference time-domain (FDTD) techniques were proposed for solving a coupled drift-diffusion model and Maxwell's equation model [17]-[22]. Further improvements to match numerical and experimental data sought a better description of the steady state [23], and a high-accuracy model developed for electron devices [24] was included for photomixers [25].

The main objective of this paper is to present computational procedures and explicit equations leading to reproduce the experimental behavior of PCAs. Eventually, this formulation will enable the exploration of CAD geometries of PCA which may outperform the performance of the actual ones. Also, procedures described in this paper incorporate all of the abovementioned numerical contributions (e.g., solution of Poisson and continuity equations or the employment of the Shockley–Read–Hall recombination model), and add the following improvements in the calculations to assess the performance of PCAs: 1) the accurate nonequilibrium distribution of carriers in the steady state; 2) the consideration of nonhomogeneous mobilities in

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Fig. 1. Typical photoconductive antennas: the (a) face-to-face and (b) stripline dipoles. Metallic parts are depicted in light gray, semiconductor materials in blue (dark gray). Dimensions correspond to validation examples of Section IV.

the semiconductor; 3) the calculation of the divergence of electron and hole currents through Bernoulli's functions, which is required for higher numerical accuracy in nonlinear spatial distribution of carriers; 4) a stable marching-on-in-time procedure, derived through a double update of carrier distributions at each timestep considered for electric- or magnetic-field calculation; and 5) a set of standalone, explicit numerical equations which can be implemented without any external commercial software.

For this purpose, Section II describes the physical phenomena involved in the radiation of THz waves, through the drift-diffusion model coupled to a time-domain version of Maxwell's equations. Section III shows a stable computational algorithm produced by a discretization of the resulting system of partial differential equations. For the sake of brevity, some of the numerical equations have been moved to the Appendix. Finally, some results are depicted not only for the validation of the procedure but also as evidence of the potential of the method to provide physical insight in PCAs.

### II. PHYSICAL DESCRIPTION OF PCAS AT THZ REGIME

As pointed out in the Introduction, several configurations of semiconductor and metallic materials to form the so-called photoconductive switch have been reported [5], [7], [10], [11]. For historical reasons and without loss of generality, the numerical description of PCAs presented here is based on the Auston switch configuration, in which a semiconductor is biased by external electrodes located on the top of the bulk structure in different geometries, such as the face-to-face (FF) or stripline (SL) dipoles (Fig. 1) [26]. When considering these kinds of devices of total dimensions in the micrometer range, the electromagnetic radiation phenomena at THz frequencies can be described by a set of coupled partial differential equations, which have to take into account the following physical processes: 1) the initial redistribution of carriers by a biasing DC electric-field; 2) the generation of electrons and holes (e-h) inside the semiconductor, as a consequence of an impinging laser pulse; and 3) a directive radiation of electromagnetic waves, as a result of the acceleration of these carriers generated under the biased field.

As is well known, the electromagnetic radiation is related to the dynamics of charge and current distributions [27]. However, the relevance of an accurate solution of the steady state should not be neglected because the physical description of the semiconductor depends strongly on the biasing electric field [26] [28]. Thus, the initial distribution of carriers and currents as well as the nonhomogeneous carrier mobilities in the semiconductor has to be inferred from the steady-state solution, as a step prior to calculating radiation parameters of the PCA [29].

The steady state at any point  $\vec{r}$  of the semiconductor is described by the Poisson's equation for electrostatic fields, as well as by the steady formulation of the continuity equations for holes and electrons as follows:

$$\nabla \cdot (\nabla \phi\left(\vec{r}\right)) = \frac{q}{\varepsilon} \left(n0\left(\vec{r}\right) - p0\left(\vec{r}\right) - C\left(\vec{r}\right)\right) \tag{1}$$

$$\nabla \cdot \vec{J}_{n0}\left(\vec{r}\right) = qR0\left(\vec{r}\right) \tag{2}$$

$$\nabla \cdot \vec{J}_{p0}\left(\vec{r}\right) = -qR0\left(\vec{r}\right) \tag{3}$$

where  $n0(\vec{r})$  and  $p0(\vec{r})$  correspond to the steady-state electric-charge concentration per volume unit for electrons and holes, respectively,  $\phi(\vec{r})$  accounts for the electrostatic potential,  $C(\vec{r}) = N_D^+(\vec{r}) - N_A^-(\vec{r})$  is the net doping concentration distribution, signifying the balance of donor and acceptor ions, and q and  $\varepsilon$  signify the electron charge and the electric permittivity, respectively. Electrostatic field is derived as  $\vec{E}_0(\vec{r}) = -\nabla \phi(\vec{r})$ . In the continuity equations (2) and (3), while  $\vec{J}_{n0}(\vec{r})$  and  $\vec{J}_{p0}(\vec{r})$  are the steady-state electric currents for electron and holes, which can be added to form the total steady-state electric current  $\vec{J}_0(\vec{r})$ , we have

$$\begin{aligned} \vec{J_0} \left( \vec{r} \right) &= \vec{J_{n0}} \left( \vec{r} \right) + \vec{J_{p0}} \left( \vec{r} \right) \\ &= q(\mu_n \left( \vec{r} \right) n0 \left( \vec{r} \right) + \mu_p \left( \vec{r} \right) p0 \left( \vec{r} \right)) \vec{E_0} \left( \vec{r} \right) \\ &+ q V_T \left( \mu_n \left( \vec{r} \right) \nabla n0 \left( \vec{r} \right) - \mu_p \left( \vec{r} \right) \nabla p0 \left( \vec{r} \right) \right) \end{aligned}$$
(4)

and  $R0(\vec{r})$  is the recombination rate of carriers in the semiconductor at steady-state, which can be expressed by the SRH model [30] including Auger recombination [31]

$$R0(\vec{r}) = \left(n0(\vec{r}) p0(\vec{r}) - n_i^2\right) \left(C_{An}n0(\vec{r}) + C_{Ap}p0(\vec{r})\right) + \frac{n0(\vec{r}) p0(\vec{r}) - n_i^2}{\tau_n \left(p_1 + p0(\vec{r})\right) + \tau_p \left(n_1 + n0(\vec{r})\right)}$$
(5)

where  $\tau_n$  and  $\tau_p$  are the electron and hole lifetime,  $C_{An}$  and  $C_{Ap}$  are the Auger recombination coefficients for electron and hole concentrations,  $n_i$  accounts for the intrinsic concentration of carriers, and  $n_1$  and  $p_1$  are auxiliary quantities defined as the electrons and holes concentrations, respectively, which would exist if the Fermi energy level was at the trap energy level.

Transient electromagnetic fields  $\vec{E}(\vec{r},t)$  and  $\vec{H}(\vec{r},t)$  are described by classical Maxwell equations

$$\varepsilon \partial_t \vec{E}\left(\vec{r},t\right) = \nabla \wedge \vec{H}\left(\vec{r},t\right) - \vec{J}\left(\vec{r},t\right) \tag{6}$$

$$\mu \partial_t \vec{H} \left( \vec{r}, t \right) = -\nabla \wedge \vec{E} \left( \vec{r}, t \right) \tag{7}$$

where  $\mu$  is the magnetic permeability, and  $\vec{J}(\vec{r},t)$  is the total current in the semiconductor. In contrast to the usual electromagnetic devices, electromagnetic fields in the semiconductorbased PCAs are produced by adding electron and hole currents  $\vec{J}(\vec{r},t) = \vec{J}_n(\vec{r},t) + \vec{J}_p(\vec{r},t)$ , which are predicted by the driftdiffusion model in the form

$$\vec{J}_{n}(\vec{r},t) = q\mu_{n}(\vec{r})\left(n\left(\vec{r},t\right) + n0\left(\vec{r}\right)\right)\left(\vec{E}\left(\vec{r},t\right) + \vec{E}_{0}\left(\vec{r}\right)\right) + q\mu_{n}(\vec{r})V_{T}\nabla\left(n\left(\vec{r},t\right) + n0\left(\vec{r}\right)\right)$$
(8)

$$\vec{J}_{p}(\vec{r},t) = q\mu_{p}(\vec{r})\left(p\left(\vec{r},t\right) + p0\left(\vec{r}\right)\right)\left(\vec{E}\left(\vec{r},t\right) + \vec{E}_{0}\left(\vec{r}\right)\right) - q\mu_{p}(\vec{r})V_{T}\nabla\left(p\left(\vec{r},t\right) + p0\left(\vec{r}\right)\right)$$
(9)

where  $n(\vec{r},t)$  and  $p(\vec{r},t)$  correspond to the transient electric-charge concentration for electrons and holes, respectively,  $\mu_n(\vec{r}), \mu_p(\vec{r})$  are the non-homogenous field-dependent mobilities for electrons and holes [32], respectively, and  $V_T$  is the thermal voltage of the semiconductor. It is noticeable that (8) and (9) present separately the transient and steady-state contributions of fields and densities of charges, which has been done to allow the numerical discretization explained in Section III. Also, it worth noting that, for these devices, the electrostatic field  $\vec{E}_0(\vec{r})$  is much higher than transient field  $\vec{E}(\vec{r},t)$ , and approximations  $\mu_n(\vec{r},t) \approx \mu_n(\vec{r})$  and  $\mu_p(\vec{r},t) \approx \mu_p(\vec{r})$  can be applied to save computational time without a significant penalty in accuracy. Once known  $\vec{E}_0(\vec{r})$ , several models of field-dependent mobilities can be applied to calculate  $\mu_n(\vec{r})$ [33], being the parallel-field dependent models appropriate for PCAs [28].

The joint set of PDEs required for an appropriate numerical model of PCAs in the transient stage is completed by the continuity charge equations for electrons and holes

$$\partial_t n\left(\vec{r},t\right) = -RG\left(\vec{r},t\right) + q^{-1}\nabla \cdot \vec{J}_n\left(\vec{r},t\right)$$
(10)

$$\partial_t p\left(\vec{r},t\right) = -RG\left(\vec{r},t\right) - q^{-1}\nabla \cdot \vec{J}_p\left(\vec{r},t\right)$$
(11)

where  $RG(\vec{r}, t) = R(\vec{r}, t) - G(\vec{r}, t)$  is the net rate of recombination which accounts for the generation-recombination of carriers in the semiconductor. For recombination processes at any point  $(\vec{r}, t)$ , R is

$$R = \frac{np}{\tau_n (p_1 + p) + \tau_p (n_1 + n)} + np(C_{An}n + C_{Ap}p).$$
(12)

For the generation of carriers, function G accounts for the photoelectric phenomena by applying a Gaussian function to describe the increase rate of the (e-h) pairs [34]. Then, generation at any point  $\vec{r} = (x, y, z)$  is

$$G(\vec{r},t) = W_0 \eta_{QE}(z) h(x,y) f(z,t)$$
(13)

where  $W_0 = I_0 \lambda_\gamma \alpha / hc$  is the maximum generation rate, which includes the parameters  $\alpha$  as the photonic absorption coefficient, and  $I_0$  and  $\lambda_\gamma$  as, respectively, the optical intensity and the wavelength of the laser beam. The quantum efficiency  $\eta_{QE}(z)$ is [34]

$$\eta_{QE}(z) = T\xi \left(1 - e^{-\alpha(z-z_0)}\right) \tag{14}$$

where T and  $\xi$  are, respectively, the transmittance in the vacuum-semiconductor interface and the rate of pairs (e-h) which contribute to the total electric current. Finally, the spatial distribution of carriers takes the form

$$h(x,y) = e^{-\left((x - x_0/\sigma_x)^2 + (y - y_0/\sigma_y)^2\right)}$$
(15)

$$f(z,t) = e^{-\left(\alpha(z-z_0) + (t-t_0 - z - z_0/v_m/\sigma_t)^2\right)}$$
(16)

where  $\sigma_x$ ,  $\sigma_y$ , and  $\sigma_t$  are the spatial and temporal distribution of the spot laser,  $(x_0, y_0, z_0)$  is considered the point on the surface where the center of the spot laser impinges, and  $v_m$  corresponds to the velocity of the photons in the semiconductor.

Therefore, once the steady state is solved by applying (1)–(3), (6)–(7), and (10)–(11) accurately describe the carrier distributions and electromagnetic fields in the PCAs in the transient THz regime. However, theoretical solution for practical setups is not available, and computational models can be implemented only through a discretized formulation, which will be developed in the following section.

### III. NUMERICAL SOLUTION OF PCAS AT THZ REGIME

The numerical solution of the PDEs can be achieved by means of finite-difference techniques. However, time-domain procedures are known to have stability issues, and thus an explicit and stable formulation is not straightforward [35]. Moreover, the relevance of an accurate solution of the steady state has been shown [25], not only because of the bias field but also of the non-homogeneous spatial mobility [28]. Therefore, the steady state is solved by combining the usual difference schemes for Poisson's (1) with a local description of the density of carriers based on Bernoulli's functions, as proposed in [24] to improve the numerical models of semiconductors in electron-device simulations.

Thus, for a cuboid mesh of size  $(\Delta x_i, \Delta y_j, \Delta z_k)$ , where  $\{i, j, k\}$  corresponds to the index of the cuboid, the discretized form of Poisson's equation is given by

$$\bar{\delta}_1 \bar{\phi}_1 + \bar{\delta}_2 \bar{\phi}_2 + \delta_\Sigma \phi_{i,j,k} = \frac{q}{\varepsilon} (n 0_{i,j,k} - p 0_{i,j,k} - C_{i,j,k})$$
(17)

where  $\phi_{i,j,k}$ ,  $n0_{i,j,k}$ ,  $p0_{i,j,k}$ , and  $C_{i,j,k}$  are, respectively, the electrostatic potential, electron density, hole density, and net concentration charge at any discrete point of the mesh. Also, auxiliary potential  $\overline{\phi}_1$ ,  $\overline{\phi}_2$  and geometrical  $\delta_{ij}$ , and  $\delta_{\Sigma}$  parameters are defined in Appendix A for the sake of brevity.

As it pointed out above, a finite-difference formulation for the divergence operator in (2) and (3) results in solutions with poor accuracy. The main reason for this is the exponential change in the distributions of carriers between near neighbors in the mesh as a function of the electrostatic potential, which produce significant errors for a numerical calculation of the divergence of the electric currents when a moderate mesh size is used. Instead of reducing the mesh size, which would lead to unaffordable computational resources for PCAs of usual dimensions, we can be use the Bernoulli function  $B(x) = x/e^x - 1$  to describe the local variation of the carrier distribution. In this way, (2) and (3) can be expressed as

$$V_T \sum_{w=1}^{3} \left( \bar{\beta}_{n0w} \left( \bar{n0}_w \right)^T \right) - R0 = 0$$
 (18)

$$V_T \sum_{w=1}^{3} \left( \bar{\beta}_{p0w} \left( \bar{p0}_w \right)^T \right) - R0 = 0$$
 (19)



Fig. 2. Extended Yee's cell to include carrier distributions.

where auxiliary functions  $\bar{\beta}_{n0}$ ,  $\bar{\beta}_{p0}$ ,  $\bar{n}0$ , and  $\bar{p}0$  are detailed in Appendix A. Discrete recombination values  $R0_{i,j,k}$  are calculated by a substitution of  $n0_{i,j,k}$  and  $p0_{i,j,k}$  in (12).

Another issue in the numerical discretization is to establish proper boundary conditions for handling the finite size of computational mesh. In this case, the boundary condition considered for the electrodes is the Dirichlet boundary condition, i.e., a perfect electric conductor (PEC) material is assumed and thus a fixed electrostatic potential  $\phi = V_a + \phi_b$  where  $V_a$  and  $\phi_b$ are the voltage source and built-in potential, respectively. Furthermore, a constant value of the carrier concentration in equilibrium is assumed for the metal-semiconductor junction in the electrodes

$$n|_{\text{elec}} = \frac{\sqrt{C^2 + 4n_i^2} + C}{2} \tag{20}$$

$$p|_{\text{elec}} = \frac{\sqrt{C^2 + 4n_i^2 - C}}{2}.$$
 (21)

For the interface vacuum-semiconductor, the following Neumann boundary condition is applied:

$$\varepsilon_r \left. \frac{\partial \phi}{\partial \hat{n}} \right|_{\text{sem}} = \left. \frac{\partial \phi}{\partial \hat{n}} \right|_{\text{vac}}$$
(22)

where  $|_{sem}$  and  $|_{vac}$  stand for normal derivative applied to points located at the boundary in the direction of the semiconductor and vacuum, respectively. For carrier distributions, the boundary conditions applied are:

$$\left. \frac{\partial n}{\partial \hat{n}} \right|_{\rm sem} = 0 \tag{23}$$

$$\left. \frac{\partial p}{\partial \hat{n}} \right|_{\text{sem}} = 0.$$
 (24)

Regarding the transient stage, discretization is based of the FDTD [35], which solves Maxwell's equations at discrete

timesteps m, corresponding to  $t = m\Delta t$ , where  $\Delta t$  is the time interval of analysis. For semiconductors, the continuity equations will be considered by modifying the Yee's cell [36]. Fig. 2 shows the discrete unknowns on the new Yee's cell used in our simulator [17], [23].

Numerical formulation of FDTD is achieved by considering the transient current  $\vec{J}_t(\vec{r}, t)$ , which can be calculated by using (8), (9), and (4) to yield

$$\vec{J}_{t}(\vec{r},t) = \vec{J}(\vec{r},t) - \vec{J}_{0}(\vec{r}) = \vec{J}_{s}(\vec{r},t) + (\sigma_{0}(\vec{r}) + \sigma(\vec{r},t)) \vec{E}(\vec{r},t)$$
(25)

where

$$\vec{J}_{s}(\vec{r},t) = \sigma(\vec{r},t) \vec{E}_{0}(\vec{r}) + qV_{T}\mu_{n}(\vec{r})\nabla n(\vec{r},t) - qV_{T}\mu_{p}(\vec{r})\nabla p(\vec{r},t)$$
(26)

$$\sigma_0(\vec{r}) = q\left(\mu_n(\vec{r})n0(\vec{r}) + \mu_p(\vec{r})p0(\vec{r})\right)$$
(27)

$$\sigma(\vec{r},t) = q(\mu_n(\vec{r})n(\vec{r},t) + \mu_p(\vec{r})p(\vec{r},t)).$$
(28)

Source current at *m*th timestep  $\vec{J}_s^m$  can be rewritten in terms of electrostatic field and time-dependent conductivity,  $\vec{E}_0$  and  $\sigma^m$ , respectively, as

$$\vec{J}_s^m = \sigma^m \vec{E}_0 + q V_T \left( \mu_n \nabla n^m - \mu_p \nabla p^m \right).$$
 (29)

Then, transient current is introduced in discrete form of Ampere's equation (7) as

$$\frac{\vec{E}^{m+1/2} - \vec{E}^{m-1/2}}{\varepsilon \Delta t} = \nabla \wedge \vec{H}^m - \vec{J}_s^m - (\sigma_0 + \sigma^m) \frac{\vec{E}^{m+1/2} + \vec{E}^{m-1/2}}{2}$$
(30)

which enables a split formulation of Ampere-Maxwell and Faraday laws, of the form

$$\vec{E}^{m+1/2} = C_a^m \vec{E}^{m-1/2} + C_b^m \left( \nabla \wedge \vec{H}^m - \vec{J}_s^m \right)$$
(31)

$$\vec{H}^{m+1} = \vec{H}^m - \mu_0^{-1} \left( \nabla \wedge \vec{E}^{m+1/2} \right)$$
(32)

where

$$C_a^m = \frac{2\varepsilon - \Delta t(\sigma^m + \sigma_0)}{2\varepsilon + \Delta t(\sigma^m + \sigma_0)}$$
(33)

$$C_b^m = \frac{2\Delta t}{2\varepsilon + \Delta t(\sigma^m + \sigma_0)}.$$
(34)

A noteworthy numerical issue may arise if  $C_a^m < 0$ , requiring a modification of the auxiliary constants [37]

$$C_{\sigma}^{m} = e^{-\Delta t (\sigma^{m} + \sigma_{0})/\varepsilon}$$
(35)

$$C_b^m = \frac{1 - e^{-\Delta t (\sigma^m + \sigma_0)/\varepsilon}}{\sigma^m + \sigma_0}.$$
(36)

It also bears remarking that proposed procedure is prone to instabilities in the numerical solution. To avoid this, the Courant stability condition still holds for the solution of the combined drift-diffusion and FDTD scheme, but other challenging issues appear in this case which differ from the usual electromagnetic



Fig. 3. Scheme of stable marching-on in-time procedure.



Fig. 4. Flowchart diagram of the transient stage. Numbers at the left part are related update procedure of Fig. 3.

case: on one hand, the complete system of equations is nonlinear, because of the recombination function (12) appearing in (10) and (11) and, on the other hand, terms related to distribution of carriers  $n^m$  and  $p^m$  act as diffusive sources of the electromagnetic field. Stable and accurate results are achieved by a double update of the distribution of carriers for each of the electromagnetic fields, as shown in Figs. 3 and 4.

Also, the marching-on-in-time procedure for carriers is updated by

$$n^{m+1/2} = n^{m-1/2} - \Delta t \left[ RG^m + V_T^{-1} \sum_{w=1}^3 \left( \bar{\beta}_{nw}^m \left( \bar{n}_w^m \right)^T \right) \right]$$
(37)  
$$p^{m+1/2} = p^{m-1/2} - \Delta t \left[ RG^m + V_T^{-1} \sum_{w=1}^3 \left( \bar{\beta}_{pw}^m \left( \bar{p}_w^m \right)^T \right) \right]$$
(38)

as a solution in the (m + 1/2)th time step, where the net recombination-generation rate is noted as  $RG^m = R^m - G^m$ . It is worth noting that, at the *m*th timestep, the electric field is

TABLE I	
PARAMETERS USED IN THE SIMULATIONS	
LT-GaAs: parameters used in the simulation (T=300k)	
Recombination function parameters	
Electrons	Holes
$ au_n = 0.3 ps \ [38]$	$\tau_p = 0.4 ps$ [38]
$C_{An} = 7 \cdot 10^{-30} cm^6 s^{-1}$ [39]	$\tilde{C}_{Ap} = 7 \cdot 10^{-30} cm^6 s^{-1}$ [39]
$n_1 = 4.5 \cdot 10^6 cm^{-3}$	$p_1 = 4.5 \cdot 10^6 cm^{-3}$
Doping and intrinsic concentration	
$N_D^+ = 1.3 \cdot 10^{16} cm^{-3}$	$n_i = 9 \cdot 10^6 cm^{-3}$ [40]
Mobility model [41] parameters	
$\mu_{n0} = 8000 \frac{cm^2}{Vs} \ [42]$	$\mu_{p0} = 400 \frac{cm^2}{Vs} \ [42]$
Relative permittivity and permeability	
$\varepsilon_r = 13.26$ if $\omega < 6THz$ [43] $\mu_r = 1.0$	
The laser and carriers generating rate function parameters	
$\alpha = 1.0 \mu m^{-1}$ [34]	T=(1-R)=0.99999
$\xi = 0.9999$	$\lambda_{\gamma} = 780 nm$
$I_0 = 5 \frac{W}{\mu m^2}$	$\sigma_x = 1.8 \mu m$
$n_{GaAs}(\lambda_{\gamma}) = 3.83$ [44]	$\sigma_y = 1.8 \mu m$
$v_m = \frac{c_0}{n c_0 A_0}$	$\sigma_t = 80 f s$
$P_{opt,av} = 0.63mW$	$\nu_{\gamma} = 80 M H z$
Applied bias: 30V	

interpolated as  $\vec{E}^m = (\vec{E}^{m+1/2} + \vec{E}^{m-1/2})/(2)$ , while in the (m+1)th timestep this interpolation is no longer needed because the electric field  $\vec{E}^{m+1/2}$  is known. Notably, in (37) and (38), it has been considered that  $n^{-1/2} = p^{-1/2} = n^0 = p^0 = \vec{E}^{-1/2} = \vec{H}^0 = 0$ , because steady-state distributions have been explicitly taking into account through the variables  $\sigma_0$ ,  $\vec{E}_0$  in the coefficients  $C_a^m$ ,  $C_b^m$ ,  $\vec{J}_s^m$  of (31) and (32).

Regarding the boundary conditions of the FDTD method, the convolutional perfectly matched layers (CPMLs) are used [35]. Details of notation and the explicit discretized formulation are given in the Appendix A. For the sake of clarity, a complete flowchart diagram for the calculation is presented in Fig. 4.

## IV. RESULTS

As will be shown, the procedures described above provide satisfactory results when compared with experimental measurements [10]. However, slight differences are detected, mostly due to the inherent complexity of a detailed numerical model of the laboratory setups. A high number of physical values, often omitted in reports of the experiments, are required to form a full computational model. Thus, some of them have been assumed to be typical values reported in the literature. A summary of the used values is presented in Table I, which lists the numerical values used for LT-GaAs, the source laser, and the bias electrodes. At this point, it should be noted that different values of the parameters can be found in previous papers, and Table I includes references from which they were extracted.

For validation purposes, the geometries of the electrodes implemented (Fig. 1) correspond to those described in [10] and [26]: the face-to-face (FF) dipole, where a small region with high electrostatic fields is created, and the strip-line (SL) dipole, where a large region of uniform electrostatic fields exists. Steady-state results are summarized in Fig. 5. The 3-D image of the electrostatic potential of Fig. 5(a) depicts the abrupt change in the inner region of the electrodes, where the



Fig. 5. Steady-state distribution of: (a) electrostatic potential for the FF dipole, (b) n-carriers for the SL dipole, and (c) mobility of n-carriers for the FF dipole.

source laser acts, and it has been validated using a specialized CAD for electronics [33], [29]. The high value of the subsequent electrostatic fields creates a significant drift for photogenerated carriers, thus starting the physical processes to generate THz radiation. Similar results are achieved for the SL dipole. In this case, Fig. 5(b) shows the distribution of n-charges. There is a noticeable accumulation of negative carriers around the electrodes, which needs to be taken into account because the source laser is usually applied near the electrodes in practical application using this PCA. Another noteworthy effect is the presence of strong nonlinear accumulations under the tips of the electrodes, which justifies the attention paid for an adequate computational model in this work. Fig. 5(c) illustrates the nonhomogeneous mobility of n-carriers for the FF dipole, calculated by applying at each point of the semiconductor a parallel-field model of the mobility. Thus, electrostatic field decreases substantially the  $\mu_{n0}$  of Table I (meaning for the mobility of n-carriers in the absence of electric field).



Fig. 6. Photogeneration of electric field at the semiconductor.



Fig. 7. Distribution of the transient current density at the interface of electrodes and LT- GaAs for the FF-dipole: (a) 3 ps and (b) 3.4 ps.

The transient stage begins with the incoming laser pulse. This generation phase is characterized for the generation of pairs (e-h) in the semiconductor. In practice, a thin layer of photosensitive LT-GaAs is placed on the top of a semi-insulating GaAs [45], thereby creating a transient electromagnetic field. Fig. 6 illustrates this, showing also that parallel electric-field predominates in LT-GaAs, justifying the use of parallel mobility models [32], as pointed out in Section II.

Fig. 7 illustrates the induced current pulse on the interface vacuum-semiconductor. Induced currents propagate in time



Fig. 8. Comparison of laser pulse and transient photogenerated current. For the sake of comparison, both lines were normalized to 3.14  $A \cdot \mu m^{-3}$  and 0.02  $A \cdot \mu m^{-2}$ , respectively.

along the geometry of electrodes, located at the left and right of the figure, in similar to the electric currents at RF and microwave dipole antennas. Then, the discontinuities of electrodes act as sources of radiating electromagnetic field [46], and they resonate at frequencies directly related to the length of the dipole [47].

Another remarkable fact is the influence of the recombination process in the bandwidth of the radiated field. Fig. 8 compares in the time and frequency domains the laser pulse and the photogenerated current at the center of the spot. It can be appreciated that the rise time of the induced current is influenced mainly by the waveform of the laser pulse, but, for the case of the decay time, the main factor to be considered is the recombination of photocarriers. Consequently, the bandwidth of the current decreases in comparison to the laser bandwidth, and the electromagnetic field is radiated in the THz regime.

Finally, the validation of the results is shown in the Fig. 9, where a good agreement between the computational predictions and the experimental data [10] is found for the radiated electromagnetic field in the broadside direction, for FF and SL dipoles. Differences arising for the time-domain responses are again justified by the complexity of the experimental setups. For considering the effect of the dielectric lens (d = 2.7 cm), the semi-analytical approaches of [48] and [49] are applied to the radiated electromagnetic field, derived through a near-to-far-field FDTD algorithm [50]. Also, the receiving antenna effect is taken into account as proposed in [51], and the received current waveform is evaluated by applying

$$i_{\rm rec}(\tau) \propto \int \sigma_{rec}(t-\tau) E_{\rm THz}(t) dt$$
 (39)

where  $E_{\rm THz}(t)$  corresponds to the radiated electromagnetic field calculated by the FDTD procedure, and  $\sigma_{\rm rec}(t)$  is the photoconductance of the receiving PCA, which has been taken as the waveform of transient photogeneration depicted in Fig. 8.

At this point, it also bears remarking that similar results (i.e., radiated electromagnetic field) can be successfully reproduced



Fig. 9. Comparison of detected pulse from the radiated electromagnetic field  $rE_{\theta}$  in the broadside direction ( $\theta = 90^{\circ}, \varphi = 0^{\circ}$ ), showing experimental and numerical responses for the FF (upper) and SL (lower) dipoles. For sake of comparison, simulated responses of time-domain graphs were normalized to (a) 0.186  $\mu$ A and (b) 56.6 nA.

by semi-analytical methods [15], [16], which are computationally faster than the presented procedure. As was stated in the Introduction, full-wave methods are useful for cases where simpler approaches are not applicable and for those aimed to provide a physical analysis of PCAs. On the other hand, a full-wave model of lenses is computationally demanding, and other approaches as asymptotic method-of-moments [52] can be also useful for the analysis and simulation of these devices.

#### V. CONCLUSION

Numerical simulations of PCAs can be implemented through a combined drift-diffusion and Maxwell equations model. For this purpose, the well-known FDTD procedure can be used. A key factor for achieving satisfactory results is to provide an adequate treatment of the steady model and time-update procedure, which is accomplished by applying nonlinear (Bernoulli) functions for the local distribution of carriers. Results not only validate of the code with experimental results but also demonstrate the potential of the method for analyzing the physical phenomena involved through the observation of the inner electromagnetic fields. In summary, the main contribution of this paper is the presentation of specific equations to THz researchers for their implementation as simulation tools in THz experiments.

# APPENDIX

OPERATIONAL EQUATIONS FOR THE SIMULATION

For the sake of compactness, operational expressions arising from the discretization of the theoretical equations have been omitted in the main text. Here, the main results are presented in a matrix and ready-to-implement formulation. Also, it bears mentioning that the symbol  $\odot$  is used to represent an element-by-element matrix product.

For a discrete spatial point  $\{i, j, k\}$  of the mesh at a discrete time m, the discrete form of the carrier distribution surrounding that point in the x, y and z directions can be defined as

$$\widetilde{n}^{m} = \begin{pmatrix} \overline{n}_{1}^{m} \\ \overline{n}_{2}^{m} \\ \overline{n}_{3}^{m} \end{pmatrix} = \begin{pmatrix} n_{i+1,j,k}^{m} & n_{i,j,k}^{m} & n_{i-1,j,k}^{m} \\ n_{i,j+1,k}^{m} & n_{i,j,k}^{m} & n_{i,j-1,k}^{m} \\ n_{i,j,k+1}^{m} & n_{i,j,k}^{m} & n_{i,j,k-1}^{m} \end{pmatrix}$$
(40)

for the electrons. Similarly, for holes, this is

$$\widetilde{p}^{m} = \begin{pmatrix} \overline{p}_{1}^{m} \\ \overline{p}_{2}^{m} \\ \overline{p}_{3}^{m} \end{pmatrix} = \begin{pmatrix} p_{i+1,j,k}^{m} & p_{i,j,k}^{m} & p_{i-1,j,k}^{m} \\ p_{i,j+1,k}^{m} & p_{i,j,k}^{m} & p_{i,j-1,k}^{m} \\ p_{i,j,k+1}^{m} & p_{i,j,k}^{m} & p_{i,j,k-1}^{m} \end{pmatrix}.$$
(41)

The discrete potential matrix surrounding the considered point  $\{i, j, k\}$  is

$$\widetilde{\phi} = \begin{pmatrix} \overline{\phi}_1 \\ \overline{\phi}_2 \end{pmatrix} = \begin{pmatrix} \phi_{i+1,j,k} & \phi_{i,j+1,k} & \phi_{i,j,k+1} \\ \phi_{i-1,j,k} & \phi_{i,j-1,k} & \phi_{i,j,k-1} \end{pmatrix}.$$
 (42)

Also, a matrix of coefficients involving useful coefficients related to the step sizes  $\Delta x_i$ ,  $\Delta y_j$ , and  $\Delta z_k$  of a nonuniform mesh is introduced as follows:

$$\widetilde{\delta} = \begin{pmatrix} \overline{\delta}_1 \\ \overline{\delta}_2 \end{pmatrix} = \begin{pmatrix} \delta_{11} & \delta_{12} & \delta_{13} \\ \delta_{21} & \delta_{22} & \delta_{23} \end{pmatrix}$$
(43)

where

$$\delta_{11} = 2 \left( \Delta x_i \left( \Delta x_i + \Delta x_{i-1} \right) \right)^{-1}$$
  

$$\delta_{12} = 2 \left( \Delta y_j \left( \Delta y_j + \Delta y_{j-1} \right) \right)^{-1}$$
  

$$\delta_{13} = 2 \left( \Delta z_k \left( \Delta z_k + \Delta z_{k-1} \right) \right)^{-1}$$
  

$$\delta_{21} = 2 \left( \Delta x_{i-1} \left( \Delta x_i + \Delta x_{i-1} \right) \right)^{-1}$$
  

$$\delta_{22} = 2 \left( \Delta y_{j-1} \left( \Delta y_j + \Delta y_{j-1} \right) \right)^{-1}$$
  

$$\delta_{22} = 2 \left( \Delta z_{k-1} \left( \Delta z_k + \Delta z_{k-1} \right) \right)^{-1}$$

and the sum of spatial elements  $\delta_{\Sigma}$  is also useful:

$$\delta_{\Sigma} = -\sum_{r=1}^{2}\sum_{w=1}^{3}\delta_{rw}.$$

Also, discrete mobilities for electrons and holes are reorganized as

$$\widetilde{\mu}_{n} = \begin{pmatrix} \overline{\mu}_{n_{1}} \\ \overline{\mu}_{n_{2}} \end{pmatrix} = \begin{pmatrix} \mu_{n}|_{i+1/2,j,k} & \mu_{n}|_{i,j+1/2,k} & \mu_{n}|_{i,j,k+1/2} \\ \mu_{n}|_{i-1/2,j,k} & \mu_{n}|_{i,j-1/2,k} & \mu_{n}|_{i,j,k-1/2} \end{pmatrix}$$

$$\widetilde{\mu}_{p} = \begin{pmatrix} \overline{\mu}_{p_{1}} \\ \overline{\mu}_{p_{2}} \end{pmatrix} = \begin{pmatrix} \mu_{p}|_{i+1/2,j,k} & \mu_{p}|_{i,j+1/2,k} & \mu_{p}|_{i,j,k+1/2} \\ \mu_{p}|_{i-1/2,j,k} & \mu_{p}|_{i,j-1/2,k} & \mu_{p}|_{i,j,k-1/2} \end{pmatrix}$$

$$(45)$$

and auxiliary matrices involving finite-difference coefficients and mobilities are then defined as

$$\widetilde{\mu}_{n}^{*} = \begin{pmatrix} \overline{\mu}_{n_{1}}^{*} \\ \overline{\mu}_{n_{2}}^{*} \end{pmatrix} = \widetilde{\delta} \odot \widetilde{\mu}_{n} = \begin{pmatrix} \delta_{11}\mu_{n_{11}} & \delta_{12}\mu_{n_{12}} & \delta_{13}\mu_{n_{13}} \\ \delta_{21}\mu_{n_{21}} & \delta_{22}\mu_{n_{22}} & \delta_{23}\mu_{n_{23}} \end{pmatrix}$$

$$\widetilde{\mu}_{p}^{*} = \begin{pmatrix} \overline{\mu}_{p_{1}}^{*} \\ \overline{\mu}_{p_{2}}^{*} \end{pmatrix} = \widetilde{\delta} \odot \widetilde{\mu}_{p} = \begin{pmatrix} \delta_{11}\mu_{p_{11}} & \delta_{12}\mu_{p_{12}} & \delta_{13}\mu_{p_{13}} \\ \delta_{21}\mu_{p_{21}} & \delta_{22}\mu_{p_{22}} & \delta_{23}\mu_{p_{23}} \end{pmatrix}.$$

$$(47)$$

Regarding the electric fields, a generalized form is assumed as follows:

$$\widetilde{E}^{m} = \begin{pmatrix} \overline{E}_{1}^{m} \\ \overline{E}_{2}^{m} \end{pmatrix}$$
$$= \begin{pmatrix} E_{x}|_{i+1/2,j,k}^{m} & E_{y}|_{i,j+1/2,k}^{m} & E_{z}|_{i,j,k+1/2}^{m} \\ E_{x}|_{i-1/2,j,k}^{m} & E_{y}|_{i,j-1/2,k}^{m} & E_{z}|_{i,j,k-1/2}^{m} \end{pmatrix}.$$
(48)

The discrete form of the continuity (10) and (11) is written in terms of  $(\tilde{\beta}_n^m)$  and  $(\tilde{\beta}_p^m)$ . To derive their operational expression, we define an auxiliary matrix

$$\widetilde{\wedge} = \begin{pmatrix} \overline{\wedge}_1 \\ \overline{\wedge}_2 \\ \overline{\wedge}_3 \\ \overline{\wedge}_4 \end{pmatrix} = V_T^{-1} \begin{pmatrix} -\Delta x_i & -\Delta y_j & -\Delta z_k \\ \Delta x_i & \Delta y_j & \Delta z_k \\ -\Delta x_{i-1} & -\Delta y_{j-1} & -\Delta z_{k-1} \\ \Delta x_{i-1} & \Delta y_{j-1} & \Delta z_{k-1} \end{pmatrix}.$$
(49)

Then, we introduce a combination of the modified mobilities and Bernoulli's functions

$$\widetilde{\varsigma}_{n} = \begin{pmatrix} \overline{\varsigma}_{n_{1}}^{m} \\ \overline{\varsigma}_{n_{2}}^{m} \\ \overline{\varsigma}_{n_{3}}^{m} \\ \overline{\varsigma}_{n_{4}}^{m} \end{pmatrix} = \begin{pmatrix} \overline{\mu}_{n_{1}}^{*} \odot B\left(\overline{\wedge}_{1} \odot \overline{E}_{1}^{m}\right) \\ \overline{\mu}_{n_{1}}^{*} \odot B\left(\overline{\wedge}_{2} \odot \overline{E}_{1}^{m}\right) \\ \overline{\mu}_{n_{2}}^{*} \odot B\left(\overline{\wedge}_{3} \odot \overline{E}_{2}^{m}\right) \\ \overline{\mu}_{n_{2}}^{*} \odot B\left(\overline{\wedge}_{4} \odot \overline{E}_{2}^{m}\right) \end{pmatrix}$$
(50)

and

$$\widetilde{\varsigma}_{p} = \begin{pmatrix} \overline{\varsigma}_{p_{1}}^{m} \\ \overline{\varsigma}_{p_{2}}^{m} \\ \overline{\varsigma}_{p_{3}}^{m} \\ \overline{\varsigma}_{p_{4}}^{m} \end{pmatrix} = \begin{pmatrix} \overline{\mu}_{p_{1}}^{*} \odot B \left( \overline{\wedge}_{1} \odot \overline{E}_{1}^{m} \right) \\ \overline{\mu}_{p_{1}}^{*} \odot B \left( \overline{\wedge}_{2} \odot \overline{E}_{1}^{m} \right) \\ \overline{\mu}_{p_{2}}^{*} \odot B \left( \overline{\wedge}_{3} \odot \overline{E}_{2}^{m} \right) \\ \overline{\mu}_{p_{2}}^{*} \odot B \left( \overline{\wedge}_{4} \odot \overline{E}_{2}^{m} \right) \end{pmatrix}$$
(51)

which allows the calculation of auxiliary matrices  $\widetilde{\beta}_n^m$  and  $\widetilde{\beta}_p^m$  of the form

$$\widetilde{\beta}_{n}^{m} = \begin{pmatrix} \overline{\beta}_{n_{1}}^{m} \\ \overline{\beta}_{n_{2}}^{m} \\ \overline{\beta}_{n_{3}}^{m} \end{pmatrix} = \begin{pmatrix} \overline{\zeta}_{n_{1}}^{m} \\ -\overline{\zeta}_{n_{2}}^{m} - \overline{\zeta}_{n_{3}}^{m} \\ \overline{\zeta}_{n_{4}}^{m} \end{pmatrix}$$
(52)

and

$$\widetilde{\beta}_{p}^{m} = \begin{pmatrix} \overline{\beta}_{p_{1}}^{m} \\ \overline{\beta}_{p_{2}}^{m} \\ \overline{\beta}_{p_{3}}^{m} \end{pmatrix} = \begin{pmatrix} \overline{\zeta}_{p_{3}}^{m} \\ -\overline{\zeta}_{p_{1}}^{m} - \overline{\zeta}_{p_{4}}^{m} \\ \overline{\zeta}_{p_{2}}^{m} \end{pmatrix}.$$
(53)

For the steady-state solution, the dependence of (31) is expressed in terms of the scalar electric potential  $\phi$  of the form

$$\widetilde{E}_{0} = \begin{pmatrix} \frac{\phi_{i,j,k} - \phi_{i+1,j,k}}{\Delta x_{i}} & \frac{\phi_{i,j,k} - \phi_{i,j+1,k}}{\Delta y_{j}} & \frac{\phi_{i,j,k} - \phi_{i,j,k+1}}{\Delta z_{k}} \\ \frac{\phi_{i-1,j,k} - \phi_{i,j,k}}{\Delta x_{i-1}} & \frac{\phi_{i,j-1,k} - \phi_{i,j,k}}{\Delta y_{j-1}} & \frac{\phi_{i,j,k-1} - \phi_{i,j,k}}{\Delta z_{k-1}} \end{pmatrix}$$

$$= \begin{pmatrix} \bar{E}_{0,1} \\ \bar{E}_{0,2} \end{pmatrix} \tag{54}$$

and steady-state distribution of carriers is

$$\widetilde{n0} = \begin{pmatrix} \overline{n0}_1 \\ \overline{n0}_2 \\ \overline{n0}_3 \end{pmatrix} = \begin{pmatrix} n0_{i+1,j,k} & n0_{i,j,k} & n0_{i-1,j,k} \\ n0_{i,j+1,k} & n0_{i,j,k} & n0_{i,j-1,k} \\ n0_{i,j,k+1} & n0_{i,j,k} & n0_{i,j,k-1} \end{pmatrix}$$
(55)

for the electrons. Also

$$\widetilde{p0} = \begin{pmatrix} \overline{p0}_1 \\ \overline{p0}_2 \\ \overline{p0}_3 \end{pmatrix} = \begin{pmatrix} p0_{i+1,j,k} & p0_{i,j,k} & p0_{i-1,j,k} \\ p0_{i,j+1,k} & p0_{i,j,k} & p0_{i,j-1,k} \\ p0_{i,j,k+1} & p0_{i,j,k} & p0_{i,j,k-1} \end{pmatrix}$$
(56)

for holes. Auxiliary matrices  $\beta_{n0}$  and  $\beta_{p0}$  are defined equivalently to (52) and (53) in the form

$$\widetilde{\beta}_{n0} = \begin{pmatrix} \overline{\beta}_{n0_1} \\ \overline{\beta}_{n0_2} \\ \overline{\beta}_{n0_3} \end{pmatrix} = \begin{pmatrix} \overline{\varsigma}_{n0_1} \\ -\overline{\varsigma}_{n0_2} - \overline{\varsigma}_{n0_3} \\ \overline{\varsigma}_{n0_4} \end{pmatrix}$$
(57)

and

$$\widetilde{\beta}_{p0} = \begin{pmatrix} \overline{\beta}_{p0_1} \\ \overline{\beta}_{p0_2} \\ \overline{\beta}_{p0_3} \end{pmatrix} = \begin{pmatrix} \overline{\varsigma}_{p0_3} \\ -\overline{\varsigma}_{p0_1} - \overline{\varsigma}_{p0_4} \\ \overline{\varsigma}_{p0_2} \end{pmatrix}$$
(58)

where

$$\begin{pmatrix} \bar{\varsigma}_{n0_1} \\ \bar{\varsigma}_{n0_2} \\ \bar{\varsigma}_{n0_3} \\ \bar{\varsigma}_{n0_4} \end{pmatrix} = \begin{pmatrix} \bar{\mu}_{n_1}^* \odot B \left( \bar{\Lambda}_1 \odot \bar{E}_{0,1} \right) \\ \bar{\mu}_{n_1}^* \odot B \left( \bar{\Lambda}_2 \odot \bar{E}_{0,1} \right) \\ \bar{\mu}_{n_2}^* \odot B \left( \bar{\Lambda}_3 \odot \bar{E}_{0,2} \right) \\ \bar{\mu}_{n_2}^* \odot B \left( \bar{\Lambda}_4 \odot \bar{E}_{0,2} \right) \end{pmatrix}$$
(59)

and

$$\begin{pmatrix} \bar{\varsigma}_{p0_1} \\ \bar{\varsigma}_{p0_2} \\ \bar{\varsigma}_{p0_3} \\ \bar{\varsigma}_{p0_4} \end{pmatrix} = \begin{pmatrix} \bar{\mu}_{p_1}^* \odot B \left( \bar{\wedge}_1 \odot \bar{E}_{0,1} \right) \\ \bar{\mu}_{p_1}^* \odot B \left( \bar{\wedge}_2 \odot \bar{E}_{0,1} \right) \\ \bar{\mu}_{p_2}^* \odot B \left( \bar{\wedge}_3 \odot \bar{E}_{0,2} \right) \\ \bar{\mu}_{p_2}^* \odot B \left( \bar{\wedge}_4 \odot \bar{E}_{0,2} \right) \end{pmatrix}.$$
(60)

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