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A proposal for normalized impedance functions of inclined piles in non-homogeneous media

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Abstract

This work presents impedance functions for inclined pile groups embedded in different halfspaces whose stiffness continuously increases with depth. The results are obtained through a three-dimensional harmonic model where the soil response is modelled through the reciprocity theorem in elastodynamics and the use of Green's functions for the layered halfspace, while the piles are represented by finite elements as Timoshenko's beams. Linear behaviour of soil and piles is assumed. The use of several normalization schemes for the representation of the impedance functions is discussed, highlighting the benefits and drawbacks of each choice and their effects on the interpretation of the obtained results. As a result, expressions for the dimensionless impedance functions and frequency are proposed in order to synthesize the results of the different soil profiles into the same curves. The final objective of the proposed normalization is to transform the well-known impedance functions for the homogeneous halfspace into the corresponding curves for a specific non-homogeneous profile that can be used, e.g., in a substructuring methodology. Despite the fact that the presence of soil non-homogeneity increases the dependence of the impedance functions on the frequency, good agreements between the homogeneous and non-homogeneous profiles can be achieved for a range of frequency of interest.

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Keywords: inclined piles; layered halfspace; impedance functions; normalization schemes; Green's functions

1. Introduction

The computation of pile impedance functions has been an important subject of study in order to better understand the inertial response of pile foundations. These complex-valued frequency-dependent functions represent the stiffness and damping contribution of the soil-foundation system to the base of the supported structure. The impedance functions are usually part of substructuring methodologies directly in frequency-domain [e.g. 1,2] or in time-domain through the use of processes such as lumped parameter models [see e.g. 3].

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The work at hand presents impedance functions for inclined pile groups embedded in non-homogeneous halfspaces, whose properties vary continuously with depth. The effects of pile inclination in the foundation impedance functions have been documented by several authors [e.g. 4–6], especially for homogeneous media. On the other hand, during the last decades the inertial response of vertical piles embedded in non-homogeneous profiles has also been studied through different approaches [7–12].

One obstacle that should be faced when studying impedance functions for non-homogeneous soils is defining the normalization scheme. Generally, in order to present the results as dimensionless quantities, some of the soil properties (e.g. shear wave velocity or Young's modulus) are used. For varying profiles the choice of different expressions can lead to significantly different results. In this work, some of the most used normalization schemes are listed and a proposal is made in order to synthesize the impedance functions of the different soil profiles into the ones of the corresponding homogeneous soil.

2. Normalization Schemes for Pile Impedance Functions

One classical way [9–12] to present the normalized impedance functions $\tilde{K}_j(\omega)$ for pile groups is dividing the frequency-dependent impedance $K_j(\omega)$ by the static stiffness of the single pile embedded in the same soil $K^S_{j_o}$ times the number of piles in the group N:

$$\tilde{K}_j(\omega) = \frac{K_j(\omega)}{K^S_{j_o} N} \qquad j = H(\text{horizontal}), V(\text{vertical}); \qquad \tilde{K}_R(\omega) = \frac{K_R(\omega)}{K^S_{R_o} N + \sum_{i=1}^{N} x_i^2 K^S_{V_o}} \quad (\text{rocking}) \tag{1}$$

where $K_j(\omega)$ can represent the stiffness or damping component of the impedance function. Note that for the case of the rocking impedance of pile groups, the contribution of the static vertical stiffness of the single pile times the square of the distance to the rotation axis x_i is also included in the normalization scheme. The main advantage of this dimensionless expression is the fact that group and dynamic effects can clearly be measured. On the contrary, the major drawback of this scheme is the necessity of knowing the single pile static response for the specific physical problem.

The other generalized normalization scheme consists in dividing the impedance terms by a reference value of the soil Young's E^*_s or shear G^*_s modulus times the pile diameter d to the power of m:

$$\tilde{K}_j(\omega) = \frac{K_j(\omega)}{E^*_s d^m} \qquad m = 1 \text{ for } j = H, V; \quad m = 3 \text{ for } j = R \tag{2}$$

The main benefit of this scheme resides in its facility for computing the impedance functions corresponding to a particular physical problem once the soil properties and pile dimensions are fixed. On the other hand, for non-homogeneous profiles the use of this expression requires a proper definition of the reference value of the soil properties. Different approaches can be found in the literature. The simplest one is to choose a reference point of the pile and take the corresponding properties of soil at this level. Usually a depth equal to the pile diameter [7] or to the pile length [8] is used. When using this approach, one has to carefully compare the results of the different non-homogeneous soils, as significantly softer (especially when using E^d_s) or harder (if E^L_s) homogeneous profiles can present the same reference value. Another option is to define the reference soil elastic constant as its mean value along the pile length [13] in order to incorporate the contribution of the whole profile.

Regardless of the use of Eqs. (1) or (2), results are usually presented as functions of the dimensionless frequency:

$$a_o = \omega d / c^*_s \tag{3}$$

where ω is the angular frequency and c^*_s is the reference value of the soil shear wave velocity. This expression has the same issues that were commented before for Eq. (2). The definition of different equivalent shear wave velocities and their effects on the computation of the soil response are detailed in [14].

Finally, an interesting normalization scheme for single vertical piles in non-homogeneous soils is presented by Rovithis et al. [8]. Their proposed scheme, based on a Winkler formulation, makes use of the average wavenumber along the active length of the pile.

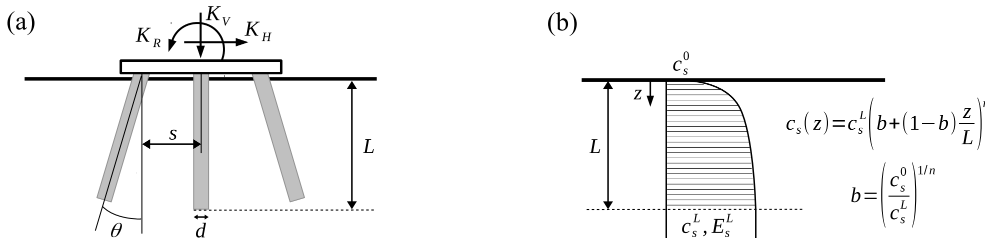

Fig. 1. (a) pile impedance problem; (b) soil profile.

2.1. Proposed normalization scheme

In the present work, the proposed normalization scheme for the pile impedance functions is based on Eqs. (2) and (3) and the use of mean soil properties as reference values. Different from what was done in the authors' previous work [13], the mean soil shear wave velocity $\bar{c}_s$ and Young's modulus $\bar{E}_s$ are defined as:

$$\bar{c}_s = \frac{1}{\alpha_1 L}\int_0^{\alpha_1 L} c_s(z)\,\mathrm{d}z; \qquad \bar{E}_s = \frac{1}{\alpha_2 L}\int_0^{\alpha_2 L} E_s(z)\,\mathrm{d}z; \qquad 0 \le \alpha_i \le 1 \tag{4}$$

where the normalization coefficients α_i indicate the portion of the pile length that is used in order to compute the mean values. Despite the relation that exists between both properties, the values of α_1 and α_2 have to be set independently in order to obtain good results. The first corresponds to the normalization of the frequency and the latter to the normalization of the impedance function.

The proposed scheme is based on the fact that, depending on the impedance term (horizontal, vertical, rocking), the contribution of the superficial layers can be more or less important in relation to the one of the rest of the pile length [9,10].

3. Methodologies

3.1. Computation of impedance functions for inclined piles in non-homogeneous soils

A previously developed [13] three-dimensional time-harmonic numerical model is used for the computation of the impedance functions of inclined pile foundations embedded in non-homogeneous soils. Its formulation is based on the integral equation of the reciprocity theorem and the use of Pak and Guzina's [15] Green's functions for the layered halfspace for representing the soil behaviour. Piles are modelled through finite elements as Timoshenko's beams and are treated as internal load lines acting inside the soil domain. Linear behaviour of soil and pile is assumed.

Fig. 1(a) shows a sketch of the studied problem. For the analysis of the performance of the proposed normalization scheme, some of the results of the 2×2 and 3×3 group configurations studied in [13] are used. The properties of pile and soil are: pile-soil modulus ratio $E_p/E_s^L = 10^3$, density ratio $\rho_s/\rho_p = 0.7$, soil $\nu_s = 0.4$ and pile $\nu_p = 0.25$ Poisson's ratios, soil hysteretic damping coefficient $\beta_s = 5\%$, pile aspect ratio $L/d = 15$ and pile separation $s/d = 5, 10$. Rake angles $\theta = 0^o, 10^o, 30^o$ are considered defining a pile inclination parallel or perpendicular to the horizontal excitation.

Different non-homogeneous profiles are assumed through parameters $c_s^0/c_s^L = 0.25, 0.5, 0.7$ and $n = 0.3, 0.5, 0.9$ (see soil profile definition in Fig. 1(b)). In addition to them, a homogeneous halfspace (E_p^L, c_s^L) is also considered as its results will be used as reference. For all profiles, ρ_s, ν_s and β_s are assumed to have a constant value.

3.2. Computation of optimal values for the normalization coefficients

For each impedance problem and configuration, the pair α_1, α_2 that minimize the difference between the normalized results for non-homogeneous and homogeneous soils is obtained through an exhaustive search ($\Delta\alpha = 0.01$). This method is chosen among other optimization processes (e.g. gradient-based methods, or genetic algorithms) because of the smoothness of the optimization function, the limited search space, and the possible existence of local minima. The difference ε between the normalized impedance function of the non-homogeneous soil $\tilde{K}(a_o)$ and the one

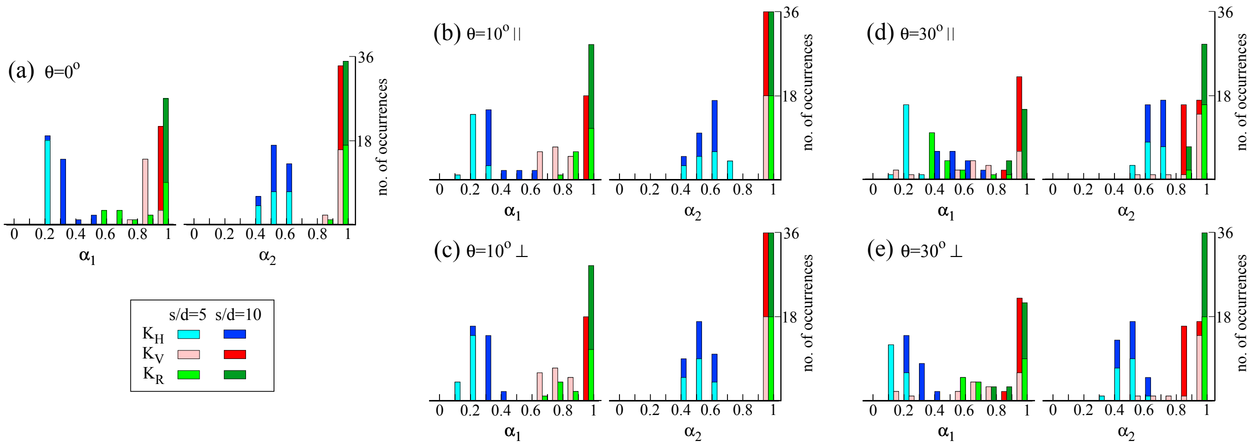

Fig. 2. histograms of coefficients α_1 and α_2 for the pile groups under study: (a) vertical piles; (b) piles inclined $\theta = 10^o$ parallel to horizontal excitation; (c) piles inclined $\theta = 10^o$ perpendicular to horizontal excitation; (d) piles inclined $\theta = 30^o$ parallel to horizontal excitation; (e) piles inclined $\theta = 30^o$ perpendicular to horizontal excitation.

corresponding to the homogeneous profile $\tilde{K}_{hom}(a_o)$ is defined as:

$$\varepsilon = \sqrt{\sum_{m=1}^{N_\omega} w_m |\tilde{K}(a_{o_m}) - \tilde{K}_{hom}(a_{o_m})|^2}; \qquad w_m = \frac{|\tilde{K}_{hom}(a_{o_m})|}{\sum_{n=1}^{N_\omega} |\tilde{K}_{hom}(a_{o_n})|} \tag{5}$$

where N_ω is the number of discrete dimensionless frequency values a_{o_m} used in the analyses; and the weight factor w_m is applied in order to obtain better matches around the peak values of the impedance functions.

4. Results

Fig. 2 shows the distribution of the optimal normalization coefficients α_1, α_2 obtained for the pile group configurations and soil profiles defined before. Each subfigure 2(a-e) corresponds to different pile rake angles (and direction of inclination), and contains a total of 36 pairs per impedance problem. Different colors refer to different impedance terms and the light/dark variants distinguish the results of the two studied separation distances.

Attending to the results, a high problem-dependence is found for the normalization coefficients, as their values strongly changes depending on the group configuration (rake angle, separation distance, etc.). Different optimal values are obtained for α_1 and α_2, being the latter generally higher than the former. Regarding the influence of the separation distance of the pile, smaller coefficients (especially α_1) are obtained for the smallest separation value.

As expected, the optimal coefficients differ depending on the studied impedance problem, being the smallest values the ones corresponding to the horizontal term. As commented before, this agrees with the fact that the superficial layers have more influence on the horizontal impedance of piles in non-homogeneous soils [9,10]. Related to this and to the horizontal-vertical coupling for inclined piles, it is found that, as the rake angle augments, the values of α_i corresponding to the horizontal problem increase, while the ones corresponding to the vertical and rocking impedances decrease. This effect can be seen to a greater extent when piles are inclined parallel to the horizontal excitation.

In order to illustrate the effects of the proposed normalization in the impedance functions, the results of 3×3 pile groups with $s/d = 5$ are presented in Fig. 3 for vertical piles and Fig. 4 for $\theta = 30^o$ inclined piles. Both figures show the horizontal, vertical and rocking normalized impedance functions for the non-homogeneous profiles defined before (grey lines) compared to the ones of the homogeneous profile (black lines). Note that the impedance imaginary component is presented instead of the equivalent dashpot coefficient in order give a better insight of the magnitude of this term at low frequencies. The optimal values of the α_1, α_2 coefficients slightly differ depending on the soil profile. Thus, their mean values are used to obtain the presented results (see Table 1).

Fig. 3 shows the comparison between the proposed optimal normalization (b) and the one using the mean properties along the whole pile length (a). The differences between both schemes are especially significant for the horizontal

Table 1. Mean value and standard deviation for the optimal α_1, α_2 of the non-homogeneous soils under study (3×3 pile groups with $s/d = 5$).

θ	α_1^H	α_2^H	α_1^V	α_2^V	α_1^R	α_2^R
0^o	0.24 ± 0.02	0.58 ± 0.08	0.88 ± 0.05	0.95 ± 0.04	0.68 ± 0.11	0.96 ± 0.03
10^o ∥	0.23 ± 0.02	0.63 ± 0.09	0.81 ± 0.01	1.00 ± 0.01	0.89 ± 0.07	1.00 ± 0.01
10^o ⊥	0.22 ± 0.01	0.57 ± 0.08	0.81 ± 0.01	1.00 ± 0.01	0.82 ± 0.11	1.00 ± 0.00
30^o ∥	0.24 ± 0.03	0.66 ± 0.07	0.68 ± 0.09	0.97 ± 0.04	0.53 ± 0.16	0.93 ± 0.03
30^o ⊥	0.20 ± 0.02	0.54 ± 0.07	0.68 ± 0.09	0.97 ± 0.04	0.60 ± 0.03	0.97 ± 0.02

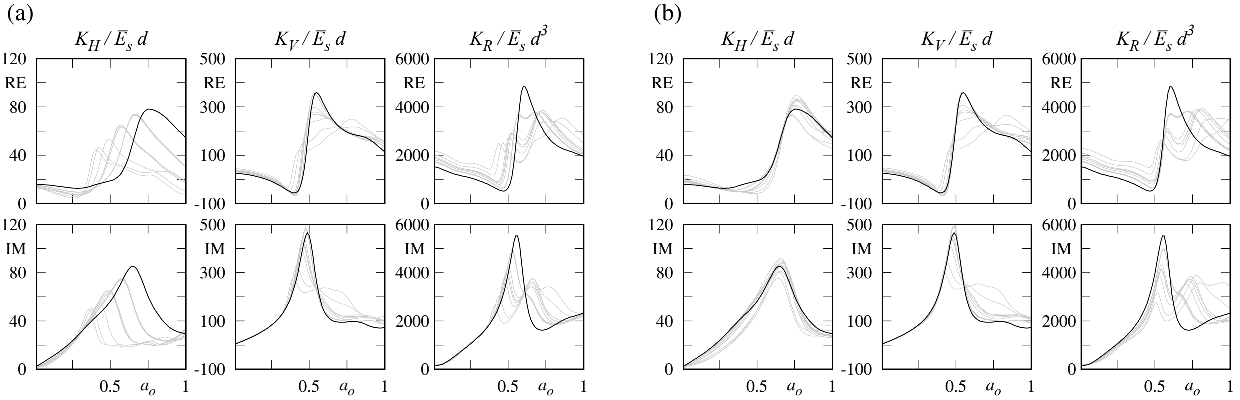

Fig. 3. normalized impedance functions for 3×3 vertical pile groups with $s/d = 5$ embedded in homogeneous (black lines) or non-homogeneous (grey lines) soils; (a) normalization through mean properties along the whole length, $\alpha_1 = \alpha_2 = 1$; (b) normalization through the mean value of optimal α_1, α_2 (Table 1).

problem. The proposed normalization makes the curves of the depth-varying profiles to match those of the homogeneous soil. On the contrary, the use of optimal α_1, α_2 has a minor impact on the vertical or rocking impedances for the vertical pile group. This can be understood as the optimal coefficients for this configurations have values close to unity (which corresponds to integrating the properties along the whole length). A limitation of the proposed scheme can be observed attending to the real component of the rocking impedance. As $\alpha_2 \leq 1$, the normalized curves can only be higher than the ones corresponding to the whole-length mean properties. Thus, if the results of the non-homogeneous profiles are greater than the ones of the homogeneous soil; the proposed normalization can not synthesize their results into the same curves. Another limitation that should be considered is the fact that the impedance curves for non-homogeneous soils present a higher frequency-dependency than the ones of the homogeneous profile. This frequency-dependency can not be avoided through the proposed scheme.

Finally, Fig. 4 presents the optimal normalized impedances for a pile inclination of $\theta = 30^o$ either parallel (a) or perpendicular (b) to the horizontal excitation. For the inclined pile group, better agreement between the non-homogeneous and homogeneous profiles is found for the vertical and rocking terms. These curves become smoother as the pile inclination augments, diminishing the discrepancies produced by the frequency-dependency of non-homogeneous impedances. However, this oscillatory behaviour is still present, especially when the pile is inclined parallel to the horizontal excitation. Owing to the horizontal-vertical coupling, for the inclined pile configuration higher differences (compared to the vertical pile group) between the studied profiles are found in the normalized horizontal impedance functions.

5. Conclusions

A normalization scheme for impedance functions of pile groups in non-homogeneous soils is proposed, making use of the mean soil properties along a portion of the pile length defined by two coefficients α_1, α_2. Despite being strongly problem-dependent, optimal values of the normalization coefficients can be used to synthesize the results of all soil profiles into the same curves. This way, the existent impedance functions for homogeneous soils can be

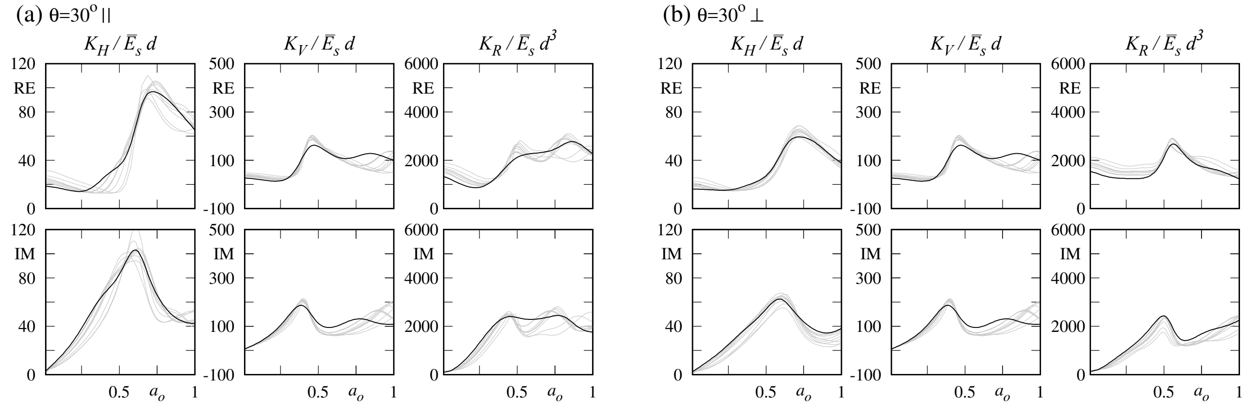

Fig. 4. normalized impedance functions for 3×3 inclined pile groups with $s/d = 5$ embedded in homogeneous (black lines) or non-homogeneous (grey lines) soils through the mean value of optimal α_1, α_2 (Table 1); (a) piles inclined $\theta = 30^o$ parallel to horizontal excitation; (b) piles inclined $\theta = 30^o$ perpendicular to horizontal excitation.

transformed into the ones corresponding to a specific non-homogeneous profile. Good agreements are obtained for the horizontal impedances of the pile groups, as well as for the vertical and rocking curves of inclined pile groups.

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