PhD dissertation

MODELLING HEAT TRANSFER FOR ENERGY EFFICIENCY ASSESSMENT OF BUILDINGS: IDENTIFICATION OF PHYSICAL PARAMETERS

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ABSTRACT

Energy efficiency is one of the two pillars to decrease the use of non-renewable energy besides the use of renewables energies. In fact, buildings are central to the EU's energy efficiency policy, as nearly 40% of the final energy consumption and 36% of greenhouse gas emissions take place in houses, offices, shops and other buildings. The 2030 Energy Efficiency Communication published by the European Commission in July 2014 underpins the key role of the building sector, stating that "the majority of the energy-saving potential is in the building sector". Improving the energy performance of Europe's building stock is crucial, not only to achieve the EU's 2020 targets, but also to meet the longer term objectives of our climate strategy as laid down in the low carbon economy roadmap 2050. In order to profit from this energy-saving potential, a methodology is needed to estimate the savings of new and refurbished buildings. The methodology internationally recognized for the energy efficiency estimation is the International Performance Measurement and Verification Protocol (IPMVP) issued by Efficiency Valuation Organization (EVO). The key point of IPMVP for estimating the savings is to develop a verifiable and reproducible method to estimate energy efficiency. This method should suppress the main technological barrier for energy efficiency estimation, which is to separate, by objective measurements, the intrinsic performance of the building from weather conditions and user behavior, since it has been observed that the energy consumption of similar buildings may vary from 30% to 300%, and about 70% of this variability may be explained by the user behavior. Therefore, the goal is to build a mathematical model with physical meaning considering only the building's intrinsic behavior. For this purpose, buildings may be considered as dynamic systems and heat transfer in buildings may be represented using dynamic models. In this way, heat transfer in buildings may be described by thermal networks which may be stated considering graph theory and thermodynamics, and may be deduced from the classical heat equation. Thermal networks may be expressed as a system of linear differential algebraic equations (DAE) and the system of linear DAE may be transformed into a state-space representation from which an autoregressive model with exogenous (ARX) can be obtained. These different model structures may be used for identifying the physical parameters of thermal networks which implies that this methodology may be useful for identifying the intrinsic performance of buildings and tackling the reduction of non-renewable energy consumption in buildings. This may facilitate the assessment of energy efficiency of buildings within a reproducible framework which allows the comparison between different constructive solutions.

The main original contributions of this dissertation are: 1) thermal networks are stated from graph theory and thermodynamics, leaving back the thermal-electrical analogy; 2) classical heat equation is connected explicitly to a system of DAE (thermal network) by using the finite elements; 3) the transformations for deducing heat transfer models with physical meaning from the classical heat equation are put altogether; 4) transformations between models may are done from thermal networks to autoregressive models with exogenous (ARX) and back; and 5) a criterion for selecting the order of the model by frequency analysis of measurements is proposed.

KEYWORDS: Heat equation, Thermal networks, Differential Algebraic Equations (DAE), State-space, Transfer function, Frequency domain, Time domain, Model order selection, Parameter identification, Heat transfer modelling.

RESUME

La performance énergétique est un pilier pour réduire l'utilisation d'énergie non renouvelable, en plus de l'utilisation des énergies renouvelables. En fait, les bâtiments sont au cœur de la politique des performances énergétiques de l'UE puisque 40% de la consommation finale d'énergie et 36% des émissions de gaz à effet de serre provient des bureaux, magasins et autres bâtiments. La « 2030 Energy Efficiency Communication » publié par la Commission européenne en juillet 2014 sous-tend le rôle clé du secteur de la construction, affirmant que «la majorité du potentiel d'économie d'énergie est dans le secteur de la construction ». L'amélioration de la performance énergétique du parc immobilier de l'Europe est cruciale, non seulement pour atteindre les objectifs 2020 de l'UE, mais aussi pour répondre aux objectifs à long terme de la stratégie climatique européenne pour réduire les émissions de gaz à effet de serre pour le 2050. Pour atteindre cet objectif, il est nécessaire une méthodologie pour estimer la performance énergétique des bâtiments neufs et rénovés. La méthodologie internationalement reconnue pour l'estimation de la performance énergétique est le *Protocole international de mesure et de vérification* (IPMVP) émis par la « Efficiency Valuation Organization (EVO) ». Le point clé de l'IPMVP est le développement d'une méthode vérifiable et reproductible pour estimer la performance énergétique. Cette méthode devrait supprimer la principale barrière technologique pour l'estimation de la performance énergétique, qui consiste à séparer, par des mesures objectives, la performance intrinsèque du bâtiment des conditions météorologiques et du comportement de l'utilisateur, car il a été observé que la consommation énergétique des bâtiments similaires peut varier de 30% à 300%, et environ 70% de cette variabilité peut être expliquée par le comportement de l'utilisateur. Par conséquent, un premier objectif est de construire un modèle mathématique avec une signification physique en considérant seulement le comportement intrinsèque du bâtiment. A cet effet, les bâtiments peuvent être considérés comme des systèmes dynamiques et le transfert de la chaleur dans les bâtiments peut être représenté en utilisant des modèles dynamiques. De cette façon, le transfert de la chaleur dans les bâtiments peut être décrit par des réseaux thermiques obtenus en utilisant la théorie des graphes et de la thermodynamique, et peuvent être déduits de l'équation de la chaleur classique. Les réseaux thermiques peuvent être exprimés comme un système d'équations différentielles et algébriques (DAE) qui peut être transformé en représentation d'état et obtenir un fonction de transfert à partir de laquelle un modèle autorégressif avec des variables exogènes (ARX) peut être obtenu. Ces différentes structures de modèle peuvent être utilisées pour identifier les paramètres physiques des réseaux thermiques, ce qui implique que la méthode peut être utilisée pour identifier la performance intrinsèque des bâtiments et aider à la réduction de la consommation d'énergie dans les bâtiments. Cela peut faciliter l'évaluation de la performance énergétique des bâtiments dans un cadre reproductible qui permet la comparaison entre différentes solutions constructives.

Les principales contributions originales de cette thèse sont: 1) les réseaux thermiques sont présentées à partir de la théorie des graphes et de la thermodynamique, sans considérer l'analogie thermique-électrique; 2) l'équation classique de la chaleur est reliée explicitement avec un système de DAE (réseau thermique) par les éléments finis; 3) différentes transformations pour déduire des modèles du transfert de la chaleur avec signification physique, à partir de l'équation de la chaleur classique, sont présentées toutes ensemble; 4) les transformations entre les modèles sont effectuées à partir des réseaux thermiques jusqu'aux modèles autorégressifs avec des variables exogènes (ARX) et vice-versa; et 5) un critère de sélection de l'ordre du modèle par une analyse de fréquence des mesures est proposé.

MOT CLES : équation de la chaleur, réseaux thermiques, équations différentielles et algébriques (DAE), espace d'état, fonction de transfert, domaine de la fréquence, domaine du temps, identification des paramètres, sélection de l'ordre du modèle, modélisation du transfert de chaleur.

RESUMEN

La eficiencia energética es uno de los pilares para disminuir el uso de energía no renovable, junto a la utilización de energías renovables. De hecho, los edificios son fundamentales para la política de eficiencia energética de la UE, puesto que casi el 40% del consumo final de energía y el 36% de las emisiones de gases de efecto invernadero tienen lugar en casas, oficinas, tiendas y otros edificios. La "2030 Energy Efficiency Communication" publicada por la Comisión Europea en julio 2014 considera el papel clave del sector de la construcción e indica que la mayor parte del potencial de ahorro de energía se encuentra en dicho sector. La mejora de la eficiencia energética del parque inmobiliario europeo es crucial no sólo para lograr los objetivos de la UE en 2020, sino también para cumplir con los objetivos a largo plazo de la estrategia climática tal como se establece el itinerario marcado hasta 2050 para fomentar una economía que reduzca la emisión de gases de efecto invernadero. Para beneficiarse de ese potencial de ahorro de energía se necesita una metodología que estime la eficiencia energética de edificios nuevos y rehabilitados. La metodología internacionalmente reconocida para la estimación de la eficiencia energética viene dada por el protocolo internacional para la medición y verificación del rendimiento energético (IPMVP) emitido por la "Efficiency Valuation Organization" (EVO). El punto clave del IPMVP para la estimación del ahorro energético es desarrollar un método medible y reproducible para estimar la eficiencia energética. Este método debe suprimir la principal barrera tecnológica para la estimación de la eficiencia energética, la cual es separar, por mediciones objetivas, el rendimiento intrínseco del edificio de las condiciones meteorológicas y del comportamiento del usuario, ya que se conoce que el consumo de energía en edificios similares puede variar entre un 30% y un 300%, y aproximadamente el 70% de esta variación puede explicarse por el comportamiento del usuario. Por lo tanto, un primer objetivo es construir un modelo matemático con significado físico considerando sólo el comportamiento intrínseco del edificio. Para este propósito, los edificios pueden ser considerados como sistemas dinámicos y la transferencia de calor entre ellos y su entorno puede representarse utilizando modelos dinámicos. De esta manera, la transferencia de calor en edificios puede ser descrita por redes térmicas obtenidas utilizando la teoría de grafos y la termodinámica, y que pueden ser deducidas de la ecuación clásica del calor. Las redes térmicas se pueden expresar como un sistema de ecuaciones diferenciales y algebraicas (DAE), este sistema de DAE puede ser transformado al espacio de estados para obtener después una función de transferencia a partir de la cual se puede deducir un modelo autorregresivo con variables exógenas (ARX). Estas diferentes estructuras de modelos pueden ser utilizadas para la identificación de los parámetros físicos de la red térmica por lo que esta metodología puede ser útil para identificar el rendimiento intrínseco de los edificios y hacer frente a la reducción del consumo de energía no renovable en ellos. Esto puede facilitar además la evaluación de la eficiencia energética de los edificios dentro de un marco reproducible que permita la comparación entre diferentes soluciones constructivas.

Las principales contribuciones originales de esta tesis son: 1) la red térmica se expresa a partir de la teoría de grafos y de la termodinámica sin utilizar la analogía termo-eléctrica; 2) la ecuación del calor clásica se transforma en un sistema de DAE (red térmica) utilizando elementos finitos; 3) las transformaciones para deducir diferentes modelos de transferencia de calor con significado físico son presentadas conjuntamente en una cadena iniciada en la ecuación del calor clásica; 4) las transformaciones entre modelos son mostradas desde las redes térmicas hasta modelos autorregresivos con variables exógenas (ARX) y viceversa; y 5) un criterio para la selección del orden del modelo utilizando un análisis de frecuencia de las mediciones es presentado.

PALABRAS CLAVE : Ecuación del calor, redes térmicas, ecuaciones diferenciales y algebraicas (DAE), espacio de estados, función de transferencia, dominio de la frecuencia, dominio del

tiempo, selección orden de modelo, identificación de parámetros, modelado de la transferencia de calor.

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NOMENCLATURE

a_{ij}, b_{ij}	parameters of discrete transfer function or ARX model
b_i	temperature source on branch i, °C or K
C_i	thermal or heat capacity in node i, J K^{-1}
c	specific heat capacity, J kg^{-1} K^{-1}
$D(z)$	denominator of a discrete transfer function element
$\det(\mathbf{E})$	determinant of matrix $\mathbf{E}$
e_i	temperature difference over the thermal resistance R_i, °C or K
f_i	heat rate source in node i, W
m_i, n_{ij}	parameters of normalized discrete transfer function or ARX model
N	number of nodes of the thermal network
$N(z)$	numerator of a discrete transfer function element
p	heat rate sources, W m^{-3}
$p(\mathbf{E})$	characteristic polynomial of a matrix $\mathbf{E}$
q	heat transfer rate through a isothermal surface, Wm^{-2}
q_i	heat transfer rate on the branch i, W
R_i	thermal resistance on the branch of the thermal network i, K W^{-1}
s	complex variable
t_k	discrete time
$\mathrm{Tr}(\mathbf{E})$	trace of a matrix $\mathbf{E}$

Greek letters

Δt	sampling time
κ	thermal conductivity, W $m^{-1}K^{-1}$
θ	spatial temperature distribution, °C or K
θ_i	temperature of node, i, °C or K
ρ	density, kg m^{-3}
$\partial/\partial t$ *or* $\dot{\square}$	differential operator in time
$\nabla\cdot$	divergence operator
∇	gradient operator

Vectors and matrices

$\mathbf{A}$	incidence matrix of the thermal network
$\mathbf{A}^T$	transpose of the incidence matrix
$\mathbf{A}_d$	state matrix in the state-space model (discrete time)
$\mathbf{A}_S$	state matrix in the state-space model (continuous time)
$\mathbf{B}_d$	input matrix in the state-space model (discrete time)
$\mathbf{B}_S$	input matrix in the state-space model (continuous time)
$\mathbf{b}$	vector of temperature sources on the branches
$\mathbf{C}$	diagonal matrix of thermal capacities
$\mathbf{C}_S$	output matrix in the state-space model
$\mathbf{D}_S$	feed through matrix in the state-space model
$\mathbf{E}$	generic matrix
$\mathbf{f}$	vector of heat rate sources
$\mathbf{f}_C$	vector of heat rate sources connected to nodes with a thermal capacity
$\mathbf{f}_0$	vector of heat rate sources connected to nodes without a thermal capacity
$\mathbf{G}$	diagonal matrix of thermal conductances
$\mathbf{H}_S$	transfer matrix
H_{ij}	output i regarding to input j component of transfer matrix
$\mathbf{H}_d$	discrete transfer matrix
H_{dij}	output i regarding to input j component of discrete transfer matrix
$\mathbf{I}$	identity matrix
$\mathbf{q}$	vector of heat rates in the branches
$\mathbf{u}$	input vector in the state-space model

Vector in Greek letters

$\boldsymbol{\theta}$	vector of temperatures
$\boldsymbol{\theta}_C$	vector of temperatures in nodes with a thermal capacity
$\boldsymbol{\theta}_{it}$	output vector of ARX model
$\boldsymbol{\theta}_0$	vector of temperatures in nodes without a thermal capacity

SYNTHESES EN FRANÇAIS

Performance énergétique des bâtiments

Cette étude est faite dans le contexte plus large de l'utilisation de l'énergie par la société. Tout au long de l'histoire humaine, les sociétés et les civilisations ont été énergivores alors que les ressources énergétiques disponibles ont été toujours limités (Smil, 2008). Chaque société doit faire face au défi de la gestion de ses ressources énergétiques et à la recherche d'un développement social durable pour l'amélioration du bien-être. Le développement durable est un état d'équilibre entre les activités humaines et l'environnement dans lequel elles sont effectuées. Cela signifie que les actions actuelles de la société pour satisfaire ses besoins ne devraient pas compromettre la possibilité des générations futures de satisfaire leurs propres besoins. En tout cas, l'idée de besoins humains est liée au bien-être qui n'est pas une idée fixe d'un point de vue social. Satisfaire le besoin en énergie implique de résoudre un problème complexe lié aux besoins énergétiques minimaux de la société.

La connaissance des besoins énergétiques minimaux de la société impliquerait quantifier les besoins énergétiques réelles, qui est une énorme tâche dépendante de perspectives et d'objectifs culturels et sociaux variables. Toutefois, indépendamment de la nécessité de la quantifier, l'utilisation efficace de l'énergie impliquerait une minimisation de l'énergie totale utilisée pour couvrir les mêmes besoins, d'un point de vue strictement thermodynamique. Ce fait est le point clé pour renforcer l'efficacité énergétique au niveau mondial et un des piliers pour garantir un développement durable de la société.

En ce sens-là, au niveau européen, la directive sur l'efficacité énergétique (EED) 2012/27/UE a créé un cadre commun de mesures pour la promotion de l'efficacité énergétique au sein de l'Union Européenne ayant comme but pour l'an 2020, d'améliorer de 20% l'efficacité énergétique et d'ouvrir la voie aux nouvelles améliorations de l'efficacité énergétique au-delà de cette date. En particulier, l'efficacité énergétique dans le bâtiment est cruciale pour atteindre cet objectif puisque, dans l'Union Européenne, près de 40% de la consommation finale d'énergie est attribuable aux logements, bureaux et autres bâtiments du secteur privé ou public. En conséquence, un investissement majeur dans les bâtiments est crucial pour l'Union Européenne (UE) pour répondre à ses objectifs énergétiques et climatiques pour 2020. Pour cette raison, dans la période du programme Horizon 2020 (2014-2020), les fonds européens pour les politiques cohésions structurelles et les fonds d'investissement (fonds ESI) jouent un rôle majeur dans le cadre de la rénovation et de la construction de bâtiments.

La directive de performance énergétique des bâtiments (EPBD) 2010/31/UE fixe les exigences minimales des performances énergétiques pour les bâtiments neufs, la rénovation des bâtiments existants et pour les éléments de construction, qui deviennent une référence pour les autorités de gestion. Sur cette ligne, une question très importante est de quantifier les économies d'énergie obtenues par des actions liées à l'amélioration de la performance énergétique des bâtiments. Ces audits énergétiques sont nécessaires pour identifier et certifier les possibilités d'économie d'énergie.

La portée d'un audit énergétique, en ligne avec l'EED, ne comprend pas seulement l'évaluation des caractéristiques techniques du bâtiment, mais aussi l'analyse de la quantité d'énergie consommée par l'utilisation finale et l'impact des changements de comportement. Dans le cas des contrats de performance énergétique, les audits énergétiques fournissent un mécanisme pour évaluer les économies d'énergie qui sont liés au comportement des consommateurs. En outre, pour les projets de

rénovation profonde, qui impliquent des subventions plus élevées, les audits énergétiques détaillés permettent la vérification des améliorations de l'efficacité énergétique et la maîtrise des coûts et des économies d'énergie à long terme. La combinaison entre les mesures standards et les recommandations contenues dans le certificat de performance énergétique peuvent être utilisée pour des projets moins complexes pour identifier les mesures énergétiques à mettre en œuvre dans le cadre de la rénovation du bâtiment. Néanmoins, un audit énergétique est obligatoire pour surveiller et vérifier les économies d'énergie du projet et de comprendre les écarts possibles entre la consommation prévue celle réelle du bâtiment.

Tout d'abord, la vérification des économies d'énergie est nécessaire en raison de l'écart entre la consommation prédite et celle mesurée, qui a des implications importantes pour l'exactitude des évaluations des coûts de cycle de vie du bâtiment, Figure f.1 (Turner & Frankel, 2008).

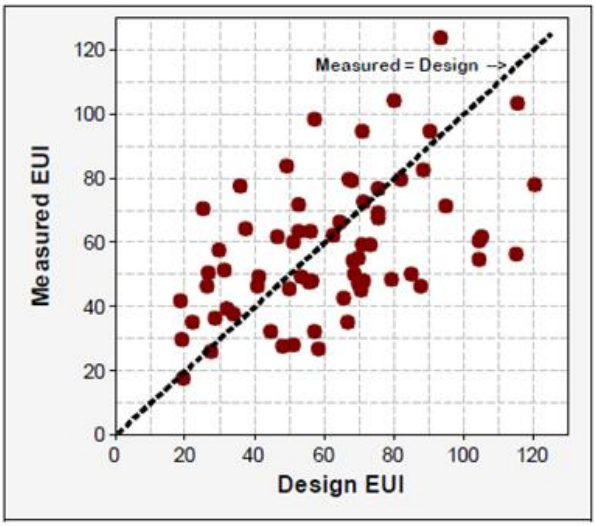


Figure f.1. Energie mesurée versus prévue, EUI (kBtu/sf) (Turner & Frankel, 2008).

Trouver la base de comparaison pour effectuer des audits de l'énergie est nécessaire même si l'écart entre la consommation d'énergie prévue et celle mesurée dans les bâtiments est un problème complexe, impliquant plusieurs aspects (de Wilde, 2014). Cette étude examinerait uniquement une méthodologie basée sur des principes physiques de transfert d'énergie pour étudier les caractéristiques intrinsèques des bâtiments pour garantir sa reproductibilité. Par conséquent, la méthodologie serait soutenue par des principes de transfert de chaleur classique, qui sont effectivement utilisés dans la modélisation du transfert de chaleur dans les bâtiments (EnergyPlus (Seem, 1987), TRNSYS (Stephenson & Mitalas, 1971), ESP-r (Clarke, 2001)). La validation de la méthodologie a été réalisée en utilisant des données expérimentales, en supposant que, théoriquement, le modèle utilisé dans une phase de conception est valide et optimal pour la modélisation du transfert de chaleur.

La mesure de l'efficacité énergétique des bâtiments est faite pour relever les économies d'énergie. Mais, les économies d'énergie représentent l'absence de la consommation d'énergie et ne peuvent pas être mesurées directement, ce qui implique que, pour déterminer les économies d'énergie, il est nécessaire de comparer la consommation d'énergie avant et après la rénovation d'un bâtiment, Figure f.2. L'identification des caractéristiques thermiques intrinsèques des bâtiments devrait nous aider à trouver une base de référence utilisable pour les comparaisons des consommation entre les bâtiments avant et après leur rénovation et entre des différents bâtiments aussi, et donc, pour la mesure de leur efficacité énergétique

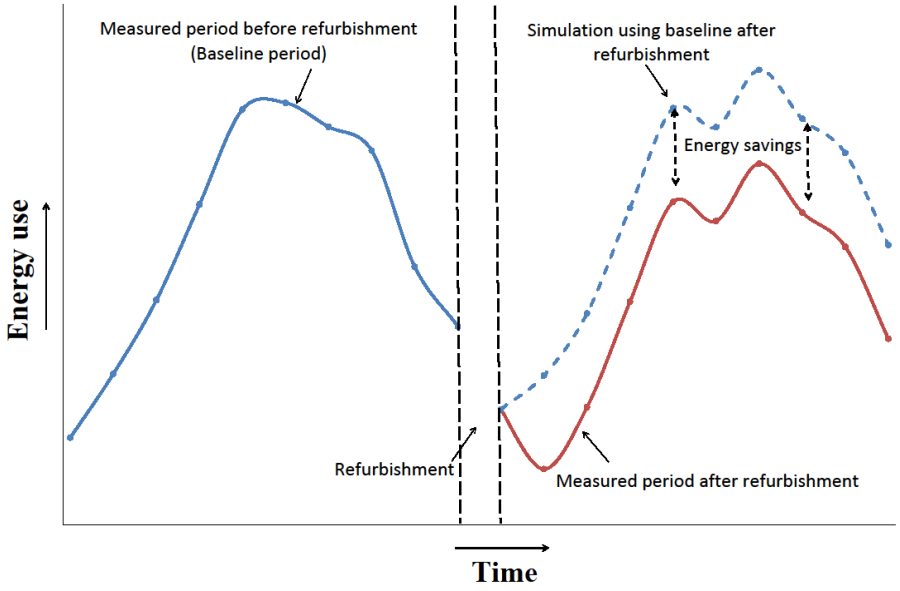


Figure f.2. Estimation des économies d'énergie en utilisant une base de référence obtenue à partir des mesurées prises avant la rénovation d'un bâtiment

Conservation de l'énergie ; chaleur et température

La conservation de l'énergie est l'un des premiers principes de la physique. Il a commencé dans la mécanique et il a été étendu à d'autres domaines de la physique, du classique aux champs modernes. Le principe classique de conservation de l'énergie indique qu'il est possible de définir une fonction d'énergie conservative; à savoir, le changement de l'énergie, ΔE, est égale à l'énergie qui entre dans le système, E_{in}, moins l'énergie qui sort du système, E_{out}:

$$\Delta E = E_{in} - E_{out} \tag{f.1}$$

En théorie l'idée est simple, mais dans la pratique les difficultés apparaissent lorsqu'on veut mesurer l'énergie qui entre et qui sort du système et les transformations entre les différentes formes d'énergie. En revanche, il n'y a pas de mesure absolue de l'énergie puisque la mesure est toujours un changement de l'énergie à partir d'une énergie de référence, choisie par consensus. Ce travail porte sur les principes classiques et il est placé dans le domaine de la thermodynamique classique, qui étudie les systèmes macroscopiques. L'énergie dans l'équation (f.1) peut être considérée comme l'énergie interne du système, U, qui sera égale à l'énergie qui entre/sorte du système (sous la forme de chaleur Q) et par le travail, W, fait par/sur le système. Ce point de vue est considéré par le premier principe de la thermodynamique et il est utilisé pour introduire la chaleur comme une forme d'énergie en transit (Callen, 1985):

$$\Delta U = \pm Q \pm W \tag{f.2}$$

Par conséquent, la chaleur est l'énergie qui entre/sorte du système et, dans l'absence de travail (mécanique ou chimique) est le seul mécanisme pour transférer l'énergie à partir d'un système à l'autre, depuis le point de vue de la thermodynamique. Une question qui ne peut pas être traitée avec le

premier principe de la thermodynamique est quand la chaleur devrait entrer ou sortir du système. Pour répondre à cette question, l'expérience doit être utilisée et les concepts de l'équilibre thermique et de la température apparaissent. Deux systèmes sont en équilibre thermique s'il n'y a pas de transfert d'énergie, sous forme de la chaleur entre eux, une fois qu'ils sont mis en contact thermique. Le principe zéro de la thermodynamique définit la température comme une grandeur physique liée au concept de l'équilibre thermique : "Si deux systèmes sont en équilibre thermique avec un troisième système, ils sont également en équilibre thermique un avec l'autre". Le principe zéro indique une relation d'équivalence et tout état en équilibre thermique peut être caractérisée par une température thermodynamique ou absolue telle que définie par Kelvin (Callen, 1985). Cela signifie que deux systèmes en équilibre thermique entre eux auront, par définition, la même valeur de la température. De cette façon, la température devient une unité de mesuré, comme la longueur, la masse, le temps ou la charge électrique, qui caractérise un état d'équilibre thermique et la chaleur sera transférée entre deux systèmes en contact thermique à des températures différentes produisant un changement de leurs énergies internes.

Ensuite, il est nécessaire de déterminer le sens du flux de la chaleur entre deux systèmes à différentes températures car il n'est pas donné par le principe zéro. A cet effet, le deuxième principe de la thermodynamique affirme que le sens de transfert de chaleur sera à partir du système à température plus élevée vers le système à température plus basse, ce qui équivaut au point de départ habituellement considéré dans le contexte de l'ingénierie sur le transfert de la chaleur (Incropera, 2006), qui dit : quand deux systèmes aux températures différentes sont en contact thermique, de l'énergie est transmise sous forme de la chaleur à partir du système à température plus élevée vers le système à température plus basse.

En plus des concepts d'énergie, chaleur et température, un autre concept qui apparaît dans le contexte de l'ingénierie est l'énergie thermique. L'énergie thermique est généralement définie comme la partie de l'énergie interne d'un système qui dépend seulement de sa température (Incropera, 2006). Dans les cas où l'énergie interne peut être modélisée comme étant seulement dépendante de la température, cette énergie interne sera égale à l'énergie thermique. Enfin, il est nécessaire de remarquer que la chaleur est l'énergie en transit et, que d'un point de vue thermodynamique, elle ne peut pas être stockée tandis que l'énergie thermique est une partie de l'énergie interne du système qui peut être « stockée ». Néanmoins, les deux concepts sont souvent utilisés indistinctement dans des contextes d'ingénierie où on utilise le concept de la "chaleur stockée" lorsque la température d'un système est incrémentée. Le stockage de l'énergie thermique ou « de la chaleur » exige limites adiabatiques parfaits entre le système et son environnement ; dans la pratique, il est très important d'être conscient que « parfait » est une abstraction théorique.

Chaleur (transfert de la chaleur)

Cette étude traite des problèmes dynamiques de transfert d'énergie sous forme de chaleur (transfert de chaleur), tandis que la thermodynamique est une science de l'équilibre ; il est important de noter qu'aucun changement de phase ou réaction chimique ne sera considérés dans cette thèse. Dans ce contexte, le système thermodynamique (système macroscopique) sera synonyme d'un nœud (élément), par définition. Un nœud sera caractérisé par les propriétés physiques suivantes : température, densité, volume et capacité thermique massique. Un nœud sera indivisible et il peut être un solide, un liquide ou un gaz. Un système solide, liquide ou gazeux macroscopique peut être représenté par un nœud unique, ou par plusieurs nœuds lorsque les sous-systèmes macroscopiques de l'ensemble du système ne sont pas en équilibre thermique. En bref, un nœud sera un système thermodynamique

macroscopique en équilibre thermique et plusieurs nœuds mis en contact thermique, qui arrivent à être en équilibre thermique, pourraient être considérés comme un seul nœud.

En plus de cela, le transfert d'énergie sous forme de chaleur (transfert de chaleur) entre les nœuds sera modélisé comme proportionnelle à la différence de températures, c'est à dire, comme un processus linéaire. La constante de proportionnalité sera la conductance thermique entre nœuds, qui peut être interprété comme une propriété physique entre les nœuds. Elle ne doit pas être associée qu'à un seul nœud, car il y a une résistance au transfert de la chaleur et, par définition, au moins deux nœuds doivent être mis en contact thermique.

La littérature présente habituellement trois modes de transfert de la chaleur : la conduction, la convection et le rayonnement (Incropera, 2006). La définition de ces trois modes est au-delà des limites de la thermodynamique, car elle nécessite l'utilisation de la notion de particules microscopiques qui constituent le système. Il est généralement clair dans la littérature ce qui est la définition de la conduction et du rayonnement, mais il est plus difficile à comprendre ce qui est la convection puisque le mot est utilisé d'une manière différente selon le contexte ((Incropera, 2006), (von Böckh, 2012) , (Sidebotham, 2015), (Pert, 2013)). Tout d'abord, la conduction de la chaleur dans les solides est généralement expliquée dans la littérature en utilisant la température comme un champ conservative (Carslaw, 1986), et il peut être étendu aux fluides et aux gazes en supposant aucun mouvement macroscopique ((Incropera, 2006), (Sidebotham, 2015)). La conduction de la chaleur peut être considérée, dans le contexte de la dynamique des fluides (bien que la chaleur n'est pas un fluide), en tant que la diffusion de la température. Le malentendu arrive lorsque les fluides sont considérés car il semble à un mouvement de masse macroscopique à l'intérieur du système qui produit un mouvement d'énergie et il peut être interprété comme un mouvement de la température per se. En mécanique des fluides, ceci est appelé la convection ou l'advection indistinctement et il est indépendant de la diffusion au sein de l'équation de conservation. Néanmoins, dans le contexte du transfert de chaleur, « la convection » désigne le transfert de chaleur par convection entre un solide et un fluide en mouvement ; par conséquent, il est défini dans une couche limite et il est supposé que le transfert d'énergie sous forme de la chaleur se fait par la couche limite vers le solide ((von Böckh, 2012), (Sidebotham, 2015)).

Quoi qu'il en soit, l'idée principale qui est considérée dans la présente étude est que le transfert de la chaleur peut être considéré comme un processus linéaire. La conduction de la chaleur dans les solides peut être décrite par une loi linéaire, la loi de Fourier, qui est introduite dans l'équation de la chaleur pour décrire le transfert de la chaleur par conduction, tandis que le transfert de chaleur par convection et par rayonnement est ajouté comme des conditions aux limites. Le point clé sera d'ajouter le transfert de chaleur par convection et par rayonnement en utilisant des lois linéaires comme cela est fait pour le transfert de la chaleur par conduction. La loi de refroidissement de Newton peut être prise en compte pour le transfert de chaleur par convection uniquement et en définissant une température moyenne de rayonnement, une loi linéaire peut être obtenue pour le rayonnement. L'advection (ou la convection tel que défini dans le contexte de la mécanique des fluides) ne sera pas considérée.

Par conséquent, la linéarisation des processus du transfert de la chaleur permet l'utilisation d'une expression simple, l'équation de la chaleur, qui inclura tous les modes de transfert de la chaleur (à l'échelle macroscopique), en utilisant des conductivités et des capacités thermiques. Cette méthodologie est une simplification du problème du transfert de chaleur pour faciliter sa résolution ; chaque processus de transfert de chaleur peut être étudié de manière plus détaillée. Dans ce travail, la linéarisation des processus de transfert de la chaleur est choisie puisque les logiciels du transfert de chaleur utilisés pour analyser le transfert de chaleur dans les bâtiments admettent habituellement ces

hypothèses (EnergyPlus (Seem, 1987), TRNSYS (Stephenson & Mitalas, 1971), ESP-r (Clarke, 2001)). L'équation de la chaleur est une équation de conservation de l'énergie qui doit être comprise comme une expression unifiée dedans la validité des hypothèses, c'est à dire, elle ne prétend pas être une expression universelle car elle est clairement limitée aux hypothèses classiques macroscopiques et la validité de toutes les approches linéaires. L'objectif est de faciliter l'étude du transfert de chaleur et, en ce cas-là, le transfert de la chaleur dans les bâtiments.

De cette manière, le problème de transfert de chaleur est réduit à l'existence d'un gradient des températures entre les nœuds qui sont reliés. Ce travail donne une méthodologie à appliquer à la caractérisation du transfert de chaleur dans les bâtiments pour être utilisée dans les applications courantes, comme l'identification des paramètres des modèles thermiques. Par conséquent, il ne constitue pas une description détaillée à l'échelle du laboratoire. Le plus important sera de donner des estimations des paramètres avec des incertitudes connues, à savoir de donner la probabilité d'obtenir une valeur particulière d'un paramètre physique et des critères différents pour indiquer si la valeur du paramètre physique est statistiquement significative. Une incertitude relativement élevée n'est pas un problème en soi-même. Les modèles et les méthodes utilisent des approches mais la précision n'est pas seulement une question des méthodes théoriques ; elle est une question de ressources et des limitations expérimentales. Il est important de faire un compromis entre la théorie et l'incertitude expérimentale. La modélisation de tous les processus de transfert de chaleur comme des processus linéaires, qui considèrent la température comme un champ de potentiel conservatif, c'est une approche et même avec cette approche le problème d'étudier le transfert de la chaleur se complique pour les systèmes complexes, tels que les bâtiments. Différentes approches peuvent être envisagées et il existe une vaste littérature à consulter sur cette question ((Carslaw & Jaeger, 1986), (Duffie et Beckman, 2006), (Incropera, 2006), (von Böckh, 2012), (Sidebotham, 2015)). Cette thèse porte sur un problème particulier : l'identification des paramètres physiques intrinsèques, bien que les applications à la simulation et le contrôle sont étroitement liés.

Modélisation du transfert de la chaleur dans les bâtiments

L'idée est de construire un modèle physique qui estime de manière appropriée le bilan énergétique dans les bâtiments. Le problème est essentiellement de rapprocher le comportement thermique d'un bâtiment en utilisant un modèle mathématique. Cette thèse obtienne des modèles mathématiques à partir des premiers principes physiques en essayant de garder autant que possible des informations physiques. L'hypothèse d'état d'équilibre peut être considéré ((Naveros, et al., 2012), (Castillo et al., 2014)), mais les modèles dynamiques sont plus indiqués pour une meilleure compréhension du problème puisque les bâtiments sont des systèmes dynamiques ((Ogata , 2010), (Crassidis & Jenkins, 2011)).

Les problèmes direct et inverse

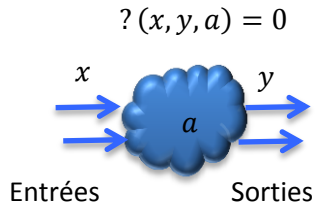
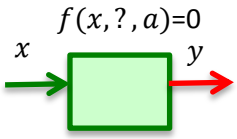
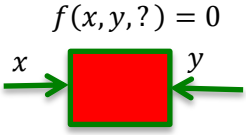
Traditionnellement, les études présentées dans la littérature ont proposé différentes représentations du modèle dynamique pour l'analyse de la consommation d'énergie dans les bâtiments, et une distinction est faite entre la résolution du problème direct (de simulation) ou la résolution du problème inverse (d'identification) (Rabl, 1988). Chaque modèle respecte le principe de conservation de l'énergie, qui exprime le bilan énergétique entre les bâtiments et leur environnement. Chaque représentation de modèle peut être utilisée pour résoudre tous les deux problèmes : direct et inverse. D'une part, le problème direct, où les propriétés des matériaux de construction sont connues, est surtout résolu à partir des principes de la théorie du transfert de chaleur en utilisant l'analogie thermique-électrique ((Fraisse, et al., 2002), (Ramallo-González, et al., 2013)). D'autre part, le problème inverse en général est résolu en connaissant les difficultés pour obtenir des résultats ayant

une signification physique ((Jiménez et Heras, 2005), (Bacher & Madsen, 2011), (Mejri, et al., 2011)). L'objectif est de relier les différentes représentations de modèle en utilisant comme point de départ l'équation de la chaleur, qui est une équation aux dérivées partielles (PDE), et une loi linéaire pour décrire tous les modes de transfert de chaleur.

D'un autre point de vue, les modèles mathématiques utilisés pour résoudre le problème direct sont considérés comme des modèles de type boîte blanche et ceux utilisés pour résoudre le problème inverse sont considérés comme de boîte noire ou de boîte grise, Tableau f.1 (Ghiaus, 2014). Les modèles boîte blanche, qui sont fondés sur des considérations théoriques pour un processus physique, sont utilisés pour la simulation de la performance du système. Les modèles de type boîte noire, qui représentent les relations d'entrée-sortie qui correspondent à des mesures du système sans aucune connaissance de son fonctionnement interne, sont utilisés dans des applications pratiques de contrôle. Ils sont obtenus par des techniques d'identification expérimentales. Pour la modélisation de type boîte grise, la structure du modèle et les paramètres du modèle sont obtenus par une combinaison de premiers principes et mesures. Cette approche est particulièrement utile pour les applications de control lorsque le modèle est exprimé sous une forme appropriée telle que la fonction de transfert ou la représentation dans l'espace d'état (Ghiaus, et al., 2007).

Si les modèles de type boîte noire sont proposés pour résoudre le problème inverse dans l'identification du système (Ljung, 1999), les paramètres identifiés ne peuvent pas être connectés avec des paramètres physiques (Ogata, 2010). Néanmoins, si le problème commence à partir de premiers principes, il est possible d'obtenir un modèle de boîte grise, et la signification physique des paramètres peut être conservée. De cette façon, cette thèse propose des modèles d'identification construits à partir de premiers principes gardant un sens physique, à savoir, ils sont des modèles de type boîte blanche ou boîte grise, dépendant de son application pour résoudre les problèmes directs ou inverses, respectivement. Les hypothèses d'une loi linéaire pour décrire le transfert de la chaleur et le principe de conservation de l'énergie peut être comprises comme premiers principes car ils sont des hypothèses établies en physique qui sont considérées « vérités » indépendamment du fait qu'ils peuvent être vérifiées par des expériences.

Tableau f.1. Classification des problèmes direct et inverse dans cette dissertation

Système Physique		Classe de problème		
		Direct	**Inverse**	
				
		$\{x,a\} \rightarrow y$	$\{x,y\} \rightarrow a$	
		Simulation	**Identification**	
		Boîte Blanche	*Boîte Grise*	*Boîte Noire*
$f(\cdot)$	Structure du modèle depuis :	Premiers principes	Premiers principes / Heuristique	Heuristique
a	Paramètres du modèle	Connu	**Estimé**	**Estimé**
x	Entrées du système physique	Mesuré	Mesuré	Mesuré
y	Sorties du système physique	**Estimé**	Mesuré	Mesuré

Réseaux thermiques en considérant la théorie de graphes et l'algèbre linéaire

L'algèbre linéaire étudie des espaces vectoriels et des transformations linéaires (Strang, 2009). Un espace vectoriel sur un champ F est un ensemble $\mathbf{V}$ avec deux opérations binaires. Les éléments de $\mathbf{V}$ sont des vecteurs formés par des éléments de F qui présente des éléments scalaires. Une des opérations est l'addition de vecteurs et l'autre est la multiplication scalaire. En l'algèbre linéaire, les matrices simplifient l'expression de la combinaison linéaire en utilisant l'addition de vecteurs et la multiplication scalaire. Les opérations d'addition et de multiplication dans un espace vectoriel doivent satisfaire les axiomes suivants :

- l'addition est associative, $(\mathbf{u}+\mathbf{v})+\mathbf{w}=\mathbf{u}+(\mathbf{w}+\mathbf{v})$;
- l'addition est commutative, $\mathbf{u}+\mathbf{v}=\mathbf{v}+\mathbf{u}$;
- élément identité de l'addition: il existe un élément $\mathbf{0}\in\mathbf{V}$, appelé vecteur nul, tel que $\mathbf{u}+\mathbf{0}=\mathbf{u}$ pour tous $\mathbf{u}\in\mathbf{V}$;
- éléments d'addition inverse: pour chaque $\mathbf{v}\in\mathbf{V}$, il existe un élément $-\mathbf{v}\in\mathbf{V}$, dite l'inverse additif de $\mathbf{v}$, tel que $\mathbf{v}+(-\mathbf{v})=\mathbf{0}$;
- la multiplication scalaire est distributive par rapport à l'addition de vecteurs, $a(\mathbf{u}+\mathbf{v})=a\mathbf{u}+a\mathbf{v}$;
- la multiplication scalaire est distributive par rapport à l'addition de champ, $(a+b)\mathbf{v}=a\mathbf{v}+b\mathbf{v}$;
- compatibilité de la multiplication scalaire avec la multiplication de champ, $a(b\mathbf{v})=(ab)\mathbf{v}$;
- élément identité de la multiplication scalaire, $1\mathbf{v}=\mathbf{v}$, où 1 représente l'identité multiplicatif dans le champ F;

où $\mathbf{u},\mathbf{v}$ et $\mathbf{w}$ sont des vecteurs arbitraires de $\mathbf{V}$ et a,b sont des scalaires en F.

D'autre part, les graphes sont représentés par un ensemble de nœuds et un ensemble d'arêtes qui relient deux nœuds ou un nœud à lui-même. Les graphes sont des modèles très polyvalents pour analyser les problèmes dans lesquels les nœuds et les arêtes peuvent avoir une interprétation physique (Gross & Yellen, 2005). Les réseaux thermiques sont des graphes où les nœuds représentent des températures qui sont reliés par des arêtes qui sont pondérés avec des résistances thermiques. En particulier, les réseaux thermiques seront graphes orientés où les arêtes connectent les nœuds dans le sens des températures descendantes. L'étude des réseaux thermiques peut être placée dans la théorie algébrique des graphes. Un réseau thermique peut être représenté par une matrice d'incidence avec dimensions M (arêtes) par N (nœuds) (Ghiaus, 2013) ; par conséquent, un ensemble de réseaux thermiques, qui peuvent être représentés par un ensemble de matrices m × n, est un espace vectoriel qui peut être étudié en utilisant l'algèbre linéaire (Strang, 1987).

Le présent travail propose l'utilisation de l'algèbre linéaire comme base pour l'étude des réseaux thermiques au lieu de l'analogie thermoélectrique, qui est habituellement utilisée. Les réseaux électriques sont également étudiés en utilisant l'algèbre linéaire, car ils sont essentiellement des graphes qui peuvent être représentés par des matrices, c. à d. ils sont aussi un espace vectoriel (Strang, 2007). Néanmoins, ce travail considère important de ne pas faire appel à l'analogie thermique / électrique puisque, traité par la théorie des graphes le cadre théorique des réseaux thermiques apparaît de façon indépendante comme pour les réseaux électriques.

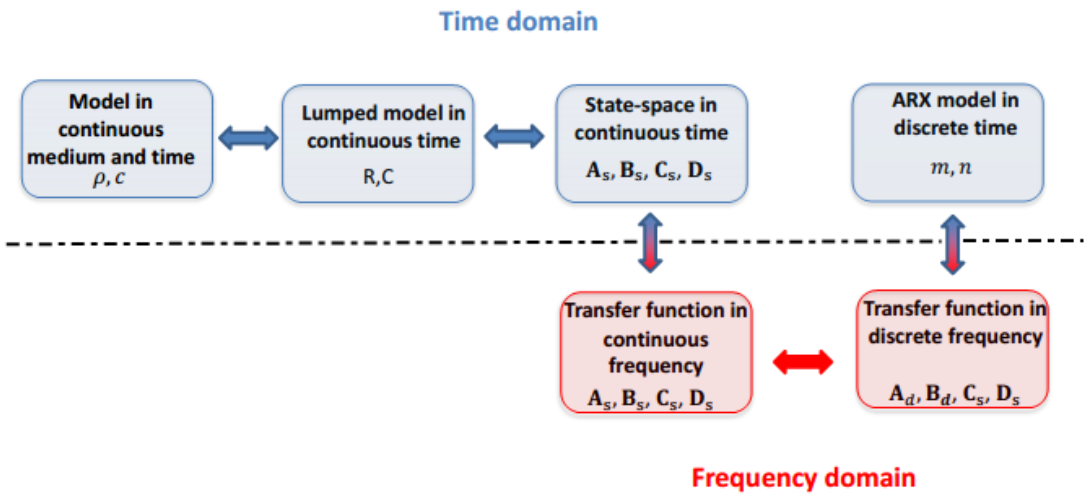


Figure f.3. Schéma de la connexion entre l'équation de la chaleur et un modèle autorégressive avec des variables exogènes

La méthodologie proposée cherche à estimer la performance énergétique des bâtiments par la résolution des problèmes d'identification des paramètres à l'aide des mesures in situ et considère une séparation des caractéristiques thermiques intrinsèques des bâtiments, leur utilisation et le comportement de l'utilisateur. Ceci est une question importante pour simplifier la solution du problème d'estimation de l'efficacité énergétique du bâtiment. Ce travail est axé sur l'identification des caractéristiques thermiques intrinsèques des bâtiments pour fixer une référence pour être utilisé pour les comparaisons de l'efficacité énergétique entre différents bâtiments ou entre les mêmes bâtiments avant et après sa rénovation. Dans la méthodologie proposée, le problème classique de transfert de chaleur dans les bâtiments est résolu en considérant d'une part l'intégration de l'espace de l'équation de la chaleur et d'autre part son intégration dans le temps, comme il est fait habituellement dans ce domaine ((Fraisse, et al, 2002), (Carslaw & Jaeger, 1986)). Par conséquent, le modèle est obtenu à partir du principe de conservation de l'énergie et en supposant que l'espace et le temps sont continus. Après, le modèle est exprimé par discrétisation de l'espace et linéarisation comme un réseau thermique, à savoir comme un ensemble des équations différentielles et algébriques linéaires et invariant dans le temps (DAE). En utilisant la méthode des éléments finis de Galerkin ((Strang, 2007), (Naveros & Ghiaus, 2015)), la représentation dans l'espace d'état est obtenue. La transformée de Laplace est utilisée pour obtenir la matrice de transfert donnant une relation entre les variables d'entrée et de sortie dans le domaine fréquentiel et, par discrétisation, on obtient la fonction de transfert discrète dans le domaine des fréquences discrètes (transformée Z). Ce modèle est transformé en un modèle autorégressif avec des variables exogènes (ARX) en temps discret. Le modèle ARX est une expression intégrée du système représenté dans l'espace d'état (Figure 3). Enfin, les paramètres peuvent être estimés en ajustant le modèle ARX sur les mesures expérimentales et ils peuvent être utilisés pour obtenir une mesure de la performance énergétique intrinsèque d'un bâtiment.

Caractéristiques physiques intrinsèques

L'hypothèse que les caractéristiques physiques intrinsèques d'un bâtiment sont invariantes dans le temps et constantes dans l'espace est considérée. En général, ces caractéristiques physiques peuvent varier dans le temps et dans l'espace, mais ces deux approches peuvent être utilisées étant conscients de leurs limites car ils simplifient la solution de l'équation aux dérivées partielles (PDE), qui donne le bilan énergétique. La double intégration de la PDE, dans l'espace et dans le temps, est obligatoire pour

résoudre à la fois les problèmes direct et inverse. Tout d'abord, par l'intégration dans l'espace on obtient le système dans l'espace discret, qui est paradoxalement l'origine de la PDE avant d'appliquer la limite $\Delta x \to 0$. Ce système discret dans l'espace est le modèle à paramètres localisés, à savoir le réseau thermique finie qui décrit le comportement thermique du bâtiment supposant les paramètres (résistances et capacités thermiques) invariants dans le temps.

Dans ce travail, les résistances et les capacités thermiques, qui caractérisent le transfert de chaleur dans les bâtiments, sont considérés comme les résistances et capacités de matériaux de construction une fois qu'ils sont mis en œuvre, en plus des résistances externes de convection et du rayonnement. L'utilisation des valeurs invariantes dans le temps ne signifie pas qu'ils sont vraiment invariables dans le temps, mais c'est une approche qui nous permet d'obtenir une valeur de résistances et capacités pour vérifier si la valeur est statistiquement significative, à savoir s'il est possible d'indiquer la probabilité d'obtenir cette valeur selon la méthodologie proposée. De cette façon, on obtient une description qui est indépendante de la consommation énergétique des bâtiments et pourrait faciliter une estimation de leur performance énergétique. En outre, ce modèle pourrait être utilisé pour l'analyse de la consommation d'énergie en considérant d'autres facteurs, comme les conditions climatiques, des systèmes de chauffage, ventilation, climatisation, et le comportement des utilisateurs, tout en donnant une description du comportement thermique une fois que le bâtiment est en fonctionnement. En tenant compte de tous ces facteurs, l'évaluation de la performance énergétique dévient plus complexe. Dans chaque application, ces facteurs peuvent être couplés, mais cette étude est axée sur l'identification des paramètres et non sur la consommation d'énergie, donc seulement le comportement thermique intrinsèque est considéré.

Condition aux limites : météo et sources internes

Les conditions météorologiques sont les forces qui excitent les bâtiments, c'est à dire, ce sont les principales entrées du système dynamique en plus des sources d'énergie internes. Pour cette raison, l'étude du transfert de chaleur dans les bâtiments est, tout d'abord, un problème de conditions aux limites puisque les sources d'énergie internes peuvent être éteintes, mais les conditions météorologiques restent toujours.

La réponse des bâtiments, considéré comme des systèmes dynamiques, a deux composantes : 1) la réponse naturelle et 2) la réponse forcée. Après un intervalle de temps, la réponse qui reste est que la réponse forcée parce que les conditions météorologiques ne cessent pas. Ensuite, les caractéristiques intrinsèques des bâtiments peuvent être estimées si on étudie que la réponse des bâtiments à ces conditions aux limites. La réponse forcée des bâtiments à ces conditions peut être utilisé comme une première considération pour donner une valeur de référence de leur performance énergétique malgré d'autres facteurs, comme le comportement de l'utilisateur ou du système de CVC. Une meilleure réponse pour les mêmes conditions météorologiques impliquerait une meilleure performance énergétique en considérant tous les autres facteurs constants. Mathématiquement, cela veut dire que la performance énergétique sera considérée comme une fonction de la réponse forcée d'un bâtiment à des conditions météorologiques en supposant tous les autres facteurs constants. L'obtention d'une valeur de la performance énergétique de cette manière est seulement une première étape ; une fois qu'elle est obtenue, l'addition d'autres facteurs, comme le comportement de l'utilisateur, doit être étudiée.

Ce travail ne considère pas la réponse naturelle mais la réponse forcée. Cela signifie que ce travail permet de résoudre un système dynamique forcé, qui sera décrit par des équations différentielles et algébriques (DAE), en considérant des conditions aux limites et non des conditions initiales. Les conditions initiales peuvent être importantes pour décrire le comportement du bâtiment en fonction des

sources d'énergies internes utilisées pour la maîtrise de l'ambiance intérieure (systèmes de CVC), tandis que les conditions aux limites seront plus importantes pour la caractérisation intrinsèque des bâtiments. Les études sur les problèmes aux valeurs initiales sont nombreuses dans la littérature (Incropera, 2006). D'une part, le problème de valeur initiale considère le changement des conditions intérieures pour atteindre les conditions de confort ; ce changement ne peut pas être instantané car le système (le bâtiment) a une inertie en raison de sa capacité de stockage de l'énergie. D'autre part, le problème des conditions aux limites considère les conditions météorologiques.

Cette étude ne traite pas la réponse naturelle et il considère qu'une réponse forcée et une réponse à l'état stable ne sont pas toujours pareilles. Seulement une force constante donnera une réponse stable (invariante dans le temps). Les conditions météorologiques ne sont pas forces en état d'équilibre (invariants dans le temps), car ils dépendent du temps, mais elles peuvent être considérées à l'état stationnaire "périodique" et cela va produire une réponse forcée stable et « périodique » du bâtiment ((Kumarm, 2013), (Thirumaleshwar, 2009)). Les trois exigences suivantes définissent une réponse stable et « périodique »:

1) La réponse naturelle (sorties) aux entrées est amortie et se rapproche de zéro lorsque le temps tend vers l'infini.

2) Chaque entrée (les conditions météorologiques ou les sources d'énergie internes) est à l'état d'équilibre périodique, puis, les entrées donnent le sens de l'état d'équilibre périodique» de la sortie.

3) Les entrées restent indéfiniment.

La première proposition est accomplie car l'amortissement d'un bâtiment n'est pas infini. La seconde proposition est accomplie parce que les conditions météorologiques suivent des cycles temporels (quotidien, annuels, …) et les sources d'énergie internes peuvent être contrôlés pour l'accomplir. Il est intéressant de noter que dans le domaine fréquentiel, la météo correspond à la fréquence plus élevée (période de quelques heures ou jours), tandis que le climat correspond aux fréquences plus basses (période de plusieurs mois ou années). La troisième proposition est clairement accomplie par les conditions météorologiques, car ils ne cessent jamais à varier et elle peut être accomplie par des sources d'énergie internes pendant la durée de l'expérimentation.

Une fois l'utilisation des conditions aux limites sont présentés, le problème à résoudre peut être considéré comme : 1) un problème fixe-fixe (conditions aux limites de Dirichlet); 2) un problème fixe-libre (conditions aux limites mêlées). Dans le premier, les données météorologiques et les sources de chaleur internes sont les conditions aux limites qui forcent le système dynamique ; ils ne doivent pas être des constantes pour atteindre un état stable, mais ils ont besoin d'être périodiques pour donner un état d'équilibre périodique. Dans la seconde, seulement les conditions météorologiques forcent le système ; elles ne sont pas constantes, mais elles peuvent être considérées à l'état stationnaire périodique.

Le comportement des utilisateurs

En plus des caractéristiques thermiques intrinsèques des bâtiments, le comportement de l'utilisateur est un facteur clé pour déterminer la performance énergétique globale (Fabi, et al., 2013). Les occupants sont la principale cause des sources d'énergie internes ; ils ont une influence très importante sur la consommation d'énergie et ils doivent être pris en considération pour une étude globale. Néanmoins, l'idée de cette thèse est d'analyser le problème en séparant les différentes variables qui influent sur la consommation d'énergie. De cette manière, le modèle obtenu pourrait

considérer le comportement des utilisateurs en plus des caractéristiques physiques dans des futures études. Bien que le comportement de l'utilisateur puisse être le facteur plus déterminant, l'idée, qui devrait rester, est qu'un bâtiment plus efficace d'un point de vue intrinsèque sera plus efficace en considérant un comportement de l'utilisateur standard. Ceci est l'avantage de considérer la performance énergétique des bâtiments indépendamment de la consommation d'énergie mesurée. L'écart entre la consommation d'énergie mesurée et prévue est non seulement liée aux caractéristiques intrinsèques thermiques d'un bâtiment mais aussi à d'autres facteurs, comme l'influence de l'utilisateur. Néanmoins, la complexité du comportement de l'utilisateur rend nécessaire la séparation de facteurs pour simplifier l'étude du problème. Ainsi, le comportement des utilisateurs est un point clé qui reste hors de la présente thèse ; les lecteurs peuvent avoir un aperçu du problème en consultant d'autres sources qui devraient être utiles pour développer des études indépendantes qui tiennent compte aussi des aspects sociaux pour analyser la consommation d'énergie du bâtiment.

Aperçu de la thèse

La linéarisation du transfert de chaleur, le principe de la conservation de l'énergie et de l'hypothèse de paramètres invariants en temps sont habituellement acceptés et utilisés pour étudier le transfert de chaleur dans les bâtiments ((Seem, 1987), (Stephenson & Mitalas, 1971), (Clarke , 2001)). Ce travail propose la modélisation du transfert de chaleur dans les bâtiments en utilisant les réseaux thermiques à partir d'un nouveau point de vue sur la base de l'algèbre linéaire et la théorie des graphes au lieu de l'utilisation actuelle des réseaux thermiques comme analogues aux réseaux électriques. Le point de départ ne change pas le sens des réseaux thermiques, ce qui peut être interprété comme la représentation graphique d'un système d'équations différentielles algébriques (DAE) ou vice-versa. Ceci est important car les réseaux thermiques ont été révélées utiles pour la modélisation de transfert de chaleur pour résoudre le problème direct et le problème inverse. Habituellement, le problème direct utilise les réseaux thermiques, à savoir un système déterministe de DAE, pour obtenir l'évolution temporelle de la température dans un nœud particulier du réseau thermique. Théoriquement, les réseaux thermiques peuvent donner une solution stable et unique du problème direct, qui est considéré comme bien posé, en utilisant des conditions initiales et / ou aux limites et en supposant connu les paramètres du réseau thermique. D'autre part, le problème inverse (identification de paramètre), essaie d'obtenir les paramètres du réseau thermique en utilisant les entrées et les sorties du système. En théorie, ce problème ne devrait pas être considéré comme mal posé, car il peut offrir une solution unique et stable d'un point de vue déterministe sous les hypothèses énoncées comme le problème direct. Dans la pratique, les difficultés surgissent et affectent à la fois la résolution du problème direct et inverse, donc tous les deux problèmes peuvent devenir mal posés. Ces difficultés sont dues à : 1) erreurs de modélisation, à savoir les réseaux thermiques sont une approche aux observations physiques; et 2) des erreurs de mesure, le problème de transfert de chaleur n'est pas résolu en utilisant des fonctions continues mais il est résolu en utilisant des données mesurées discrètes avec des erreurs. Des erreurs de modélisation et de mesure sont toujours mélangées et elles ne peuvent pas être séparées dans la pratique. En outre, il est important de noter que l'existence d'un modèle parfait qui caractérise un phénomène physique en supposant des hypothèses qui soient « vérité » est seulement une approximation de la réalité ; le transfert de chaleur ne fait pas exception. Ces hypothèses doivent être vérifiées par des observations afin d'être acceptées et de faire connaître leurs limites de validité.

Le chapitre 2 introduit les réseaux thermiques à partir de la théorie des graphes et des principes fondamentaux de la physique liés au transfert de la chaleur et les principes de la thermodynamique. C'est la base de ce travail : les réseaux thermiques sont des graphes qui peuvent être étudié avec l'algèbre linéaire. Il est également montré la connexion des réseaux thermiques avec l'équation aux

dérivées partielles, l'équation de la chaleur. L'utilisation des réseaux thermiques considère l'espace discret en lieu du ce continu en utilisant une linéarisation. Cela peut être considéré comme une perte d'information à partir d'un point de vue classique. Néanmoins, d'un point de vue pragmatique, la théorie classique de la conduction de la chaleur suppose l'espace et le temps continus par hypothèse. Il peut être considéré que la réalité ne peut être modélisée par parfaitement les mathématiques; en fait, le point de départ est toujours discret dans l'espace et le temps, et les équations aux dérivées partielles sont obtenues en considérant que les intervalles de temps et de l'espace tendent vers zéro. La vision classique donne plus d'importance à l'analyse mathématique qu'à l'algèbre, mais il n'y a pas de raisons pratiques pour accepter cette déclaration au-delà de la croyance que l'espace et le temps continus décrivent parfaitement la réalité, et les observations physiques ne sont que des mesures discrètes imparfaites de la réalité. Aujourd'hui, nous sommes dans l'ère de l'informatique et des données où les mathématiques discrètes et algèbre linéaire peuvent être considérées plus utiles que les mathématiques continues et l'analyse mathématique, ou, selon les mots de Gilbert Strang: « de nombreuses applications des mathématiques sont discrète plutôt que continue » et « les vecteurs et les matrices sont devenus la langue de savoir »(Strang, 2009).

Le chapitre 3 présente ensemble toutes les transformations depuis l'équation de la chaleur au modèle autorégressive avec des variables exogènes (ARX). Ces transformations sont bien connues séparément même si elles ne sont pas présentées dans la littérature comme la chaîne des transformations proposées dans cette thèse. L'équation de la chaleur est exprimée par un système d'équations différentielles et algébriques (DAE) en utilisant la méthode des éléments finis ; on montre comment un résultat similaire peut être obtenu en utilisant la méthode des volumes finis et la méthode des différences finies. On montre comment le système de DAE peut être mis en représentation d'état (Ghiaus, 2013). Ensuite, la représentation dans l'espace d'état est transformée en la fonction de transfert, qui est liée à un modèle ARX. Le point clé est que le modèle ARX est obtenu par déduction à partir du modèle physique et non ad hoc.

Dans le chapitre 4 il est montré comment les transformations de paramètres, qui supposent des changements de variables pour les paramètres des modèles, peuvent être utilisées pour obtenir les paramètres des réseaux thermiques à partir du modèle représenté dans l'espace d'état, comme une fonction de transfert ou comme un modèle ARX. Cela constituera une preuve pour montrer comment le problème de les transformations des paramètres, une fois les paramètres du modèle ARX sont obtenus, peut être considéré un problème bien posé sous les hypothèses énoncées et d'un point de vue déterministe. La structure des modèles n'est pas récupérée ; elle est connue car elle est obtenue par déduction comme il est exprimé dans le chapitre 3. Les transformations entre les différentes structures des modèles sont utilisées pour connecter ses paramètres.

Le chapitre 5 explique comment l'ordre du modèle peut être réduit en tenant compte des erreurs de mesure. Ce chapitre présente les limites de mesurabilité comme critère pour la réduction de l'ordre de modèle qui est une question complexe généralement traités à partir d'un point de vue seulement mathématique. Le choix de l'ordre du modèle doit permettre la moindre perte d'informations en utilisant la représentation de la fonction de transfert. Théoriquement, un modèle d'ordre élevé serait plus précis qu'un modèle d'ordre réduit, car il tiendrait plus d'informations et il serait mieux pour décrire les observations. En pratique, si la différence entre la réponse d'un modèle d'ordre inférieur et un modèle d'ordre supérieur ne sont pas mesurables, l'utilisation du modèle d'ordre supérieur impliquerait une augmentation de la complexité sans garantir des résultats plus précis. En bref, la précision du modèle ne devrait pas être supérieure à la précision des observations donc le modèle d'ordre inférieur serait un meilleur choix pour simplifier la résolution du problème du transfert de la chaleur.

Le chapitre 6 montre l'application de la méthodologie présentée dans les chapitres précédents et utilise les données mesurées d'une paroi qui est testé dans des conditions météorologiques réelles. Ce chapitre utilise un modèle ARX obtenu par déduction à partir d'un réseau thermique. Les paramètres physiques estimés sont la valeur *U*, la transmittance de l'énergie solaire dynamique et la capacité thermique de la paroi. Les estimations des paramètres physiques sont présentées avec leurs incertitudes estimées avec techniques de Monte Carlo pour la propagation des erreurs.

Enfin, les conclusions les plus pertinentes du point de vue de l'auteur sont brièvement exposées.

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SÍNTESIS EN ESPAÑOL

Eficiencia energética en la edificación

El presente estudio está motivado en un contexto general por la necesidad del uso de la energía por parte de las sociedades. A lo largo de la historia de la humanidad, las sociedades y las civilizaciones han sido grandes consumidoras de energía, mientras que los recursos energéticos disponibles han sido siempre limitados (Smil, 2008). Cada sociedad debe hacer frente, en su tiempo presente, con el reto de la gestión de sus recursos energéticos y buscar un desarrollo social sostenible para mejorar el bienestar humano. El desarrollo sostenible puede ser entendido como un estado de equilibrio entre las actividades humanas y el medio ambiente en el que se llevan a cabo. Esto significa que las acciones actuales de la sociedad para proveer sus necesidades no deben comprometer la posibilidad de las generaciones futuras para proveer sus propias necesidades. En cualquier caso, la idea de las necesidades humanas se relaciona con el bienestar y no es una idea fija desde el punto de vista social a lo largo del tiempo. Dentro de un contexto energético, controlar el consumo de energía implica resolver un problema complejo vinculado a la determinación de las necesidades energéticas mínimas de una sociedad en particular.

La determinación de dichas necesidades energéticas mínimas implicaría su cuantificación para las distintas sociedades actuales, lo cual es una enorme tarea dependiente de objetivos culturales y sociales variables. Sin embargo, independientemente de la cuantificación de las necesidades energéticas, cuestión fuera del alcance del presente estudio, el uso eficiente de la energía implicaría una minimización de la energía total utilizada para cubrir las mismas necesidades de energía desde un punto de vista estrictamente termodinámico. Este hecho es el punto clave para considerar la eficiencia energética como uno de los pilares que pueden garantizar un desarrollo global sostenible junto al uso de energías renovables.

En este sentido, en el ámbito europeo, la Directiva sobre Eficiencia Energética (EED) 2012/27 / UE establece un marco común de medidas para el fomento de la eficiencia energética dentro de la Unión Europea con el fin de garantizar el objetivo de mejorar un 20% la eficiencia energética para 2020, y allanar el camino a nuevas mejoras de la eficiencia energética más allá de esa fecha. En particular, la eficiencia energética de los edificios es fundamental para lograr ese objetivo, ya que en la Unión Europea cerca de un 40% del consumo final de energía se debe a viviendas, oficinas y otros edificios, tanto en el sector público como en el privado. En consecuencia, una inversión importante en los edificios es fundamental para cumplir con los objetivos en materia de energía y cambio climático. Por esta razón, se está impulsando durante el período del programa Horizonte 2020 (2014-2020), entre otras cuestiones, la mejora de la eficiencia energética de los edificios.

La Directiva para el Rendimiento Energético de Edificios (EPBD) 2010/31/UE establece los requisitos mínimos de eficiencia energética para nuevos edificios, edificios ya existentes que sean renovados y para elementos constructivos específicos, y supone un punto de referencia para la gestión de la eficiencia energética por parte de las autoridades públicas. En esta línea, un tema muy importante es cuantificar el ahorro de energía debido a las acciones relacionadas con la mejora de la eficiencia energética de los edificios y la realización de auditorías fiables para certificar su eficiencia energética. Esto permitiría identificar las oportunidades de ahorro de energía y cuantificar el ahorro energético.

El alcance de una auditoría energética, en línea con la Directiva sobre Eficiencia Energética (EED), no sólo incluye la evaluación de las características técnicas del edificio, sino también el análisis de la cantidad de energía consumida en su utilización final, así como el impacto de los cambios de comportamiento en los usuarios. Como tal, los certificados de eficiencia energética pueden hacer aportaciones importantes al ahorro de energía. En el caso de contratos de rendimiento energético, las auditorías energéticas proporcionan un mecanismo para evaluar el ahorro de energía, incluyendo los relacionados con el comportamiento de los consumidores. Por otra parte, para los proyectos de renovación total que implican subvenciones superiores, una metodología para auditar detallada permitiría la supervisión y verificación de las mejoras de eficiencia energética, incluyendo el ahorro de costes y energía a largo plazo. La combinación de medidas estándar individuales y las recomendaciones que figuran en el certificado de eficiencia energética se podrían utilizar para los proyectos de menor complejidad. En resumen, una correcta auditoría energética es necesaria para monitorizar y verificar el ahorro de energía del proyecto y entender las posibles discrepancias entre la calificación de eficiencia energética a priori y el consumo de energía real del edificio una vez está operativo.

En primer lugar, las auditorías energéticas son necesarias debido a la brecha entre el consumo energético estimado y medido de los edificios, brecha que supone importantes desviaciones para la evaluación de los costes del ciclo de vida de un edificio, Figura e.1 (Turner & Frankel, 2008).

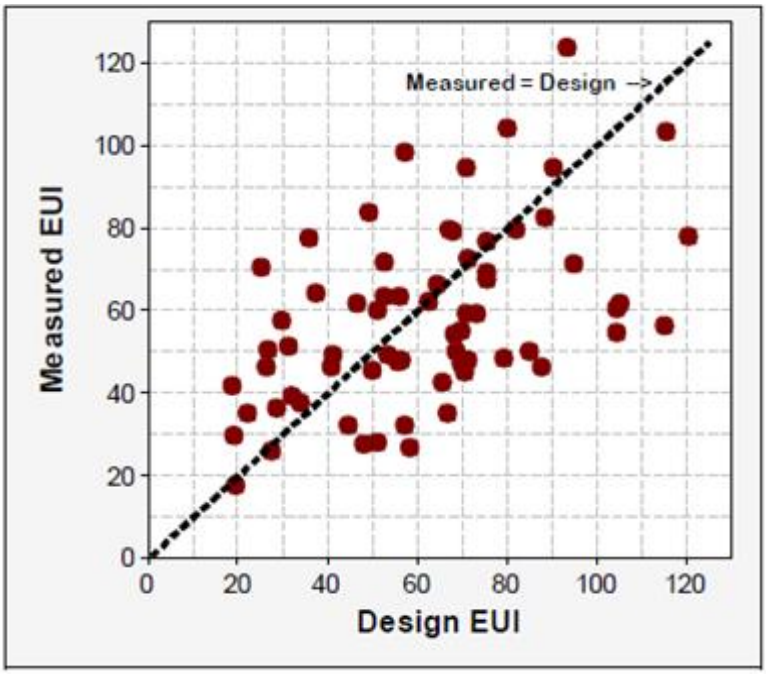


Figure e.1. Uso de energía medida frente a estimada (Turner & Frankel, 2008).

En segundo lugar, las auditorías energéticas necesitan una referencia fija para ser efectivas a la hora de reducir la brecha entre el consumo de energía estimado y medido en los edificios, a pesar de que dicha brecha es un problema complejo que implica varios aspectos (de Wilde, 2014). Este estudio desarrolla y considera solamente una metodología basada en los principios físicos de transferencia de energía para el estudio de las características intrínsecas de los edificios. La metodología persigue garantizar su reproducibilidad dentro del rango de variación determinado por las condiciones de contorno (temperatura ambiente, irradiancia solar en superficie,…) que influyen en las variables de

confort edificio; y se apoya en los principios de transferencia de calor clásicos, que se utilizan actualmente en el modelado de la transferencia de calor en edificios (EnergyPlus (Seem, 1987), TRNSYS (Stephenson y Mitalas, 1971), ESP-r (Clarke, 2001)).

La estimación de la eficiencia energética de los edificios busca aumentar el ahorro de energía. El problema es que el ahorro de energía representa la ausencia de uso de energía y no se pueden medir directamente, lo que implica que para determinar dicho ahorro de energía se necesita comparar el consumo de energía antes y después de, por ejemplo, la rehabilitación de un edificio, Figura e.2. La identificación de las características térmicas intrínsecas de los edificios debe ayudar a fijar una referencia para comparar de forma cuantitativa la eficiencia energética entre diferentes edificios, y entre edificios antes y después de su restauración.

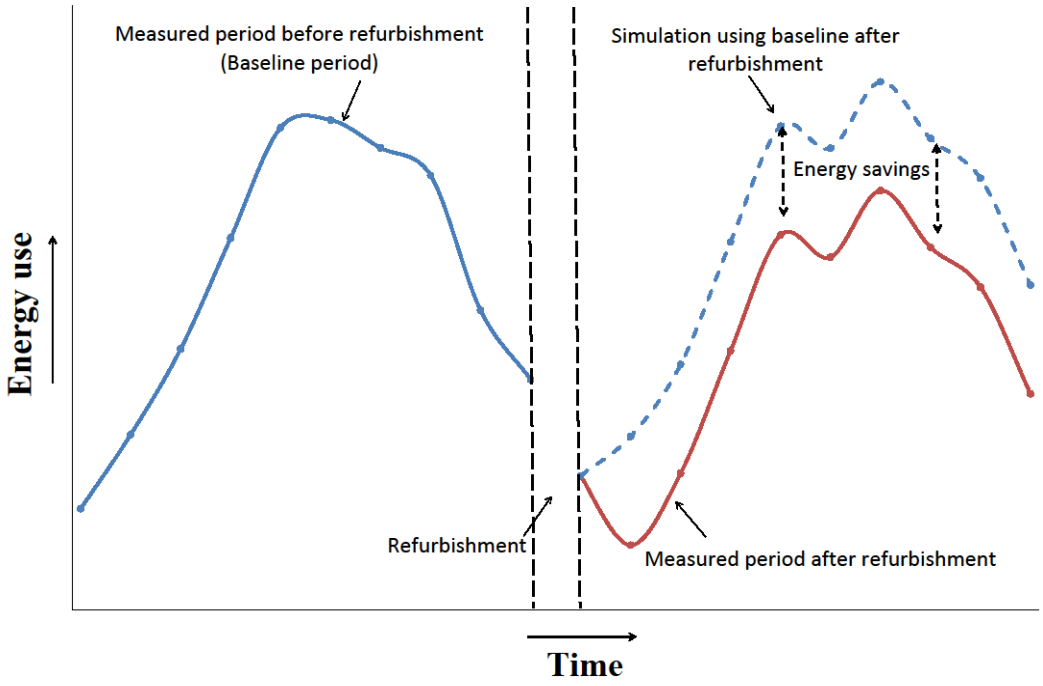


Figure e.2. Energy savings using a baseline obtained from measurements previous to building refurbishment

Conservación de Energía, calor y temperatura

La conservación de energía es un primer principio de la física que se inició en la mecánica antes de extenderse a otros campos, que abarcan desde las teorías clásicas a las modernas. El principio clásico de la conservación de la energía afirma que es posible definir una función conservativa, es decir, el cambio de energía, ΔE, es igual a la energía que entra en el sistema, E_{in}, menos la energía que sale del mismo, E_{out}:

$$\Delta E = E_{in} - E_{out} \tag{e.1}$$

En teoría, la idea es simple; en la práctica, las dificultades aparecen cuando queremos medir la energía que entra y sale del sistema, y las transformaciones entre diferentes formas de energía.

Además no hay una medida absoluta de la energía, sino que la medida es dada siempre por un cambio de energía a partir de una energía cero de referencia fijada por consenso. Este trabajo se ocupa de los principios clásicos y se sitúa dentro del dominio de la termodinámica clásica que estudia los sistemas macroscópicos. La energía considerada en la ecuación (e.1) puede ser vista como el cambio de energía interna del sistema, ΔU, debido a la energía que entra / sale del sistema en forma de calor, Q, y al trabajo, W, realizado sobre / por el sistema. Este punto de vista es considerado por la primera ley de la termodinámica y se utiliza para definir el calor como una forma de energía en tránsito (Callen, 1985):

$$\Delta U = \pm Q \pm W \qquad \text{(e.2)}$$

Por lo tanto, el calor es energía que entra / sale de un sistema y, en ausencia de trabajo (mecánico o químico) es el único mecanismo para transferir energía desde un sistema a otro desde el punto de vista de la termodinámica clásica. Una cuestión que no puede ser abordada mediante la primera ley de la termodinámica es cuando el calor saldrá o entrará al sistema. Para responder a esta pregunta la experiencia es utilizada, y aparecen los conceptos de equilibrio térmico y temperatura. Dos sistemas están en equilibrio térmico si no hay transferencia de energía, en forma de calor, entre ellos cuando se ponen en contacto térmico. El principio cero de la termodinámica define la temperatura como una variable física basada en el concepto de equilibrio térmico: "Si dos sistemas están en equilibrio térmico con un tercer sistema, entonces también se encuentran en equilibrio térmico entre sí". La ley cero establece una relación de equivalencia y supone que cualquier estado de equilibrio térmico puede caracterizarse por una temperatura termodinámica o absoluta según la definición de Kelvin (Callen, 1985). Esto significa que dos sistemas en equilibrio térmico entre ellos tendrán, por definición, la misma temperatura. De esta manera, la temperatura se convierte en una unidad de medida (como la longitud, la masa, la carga eléctrica o el tiempo) que caracteriza un estado de equilibrio térmico, y el calor será transferido entre dos sistemas en contacto térmico que estén a diferentes temperaturas produciéndose un cambio en sus energías internas.

A continuación, es necesario determinar el sentido del flujo de calor entre dos sistemas a diferentes temperaturas, ya que éste no viene fijado por el principio cero. Para este propósito, el segundo principio de la termodinámica afirma que el sentido de la transferencia de calor, entre dos sistemas en contacto térmico, será desde el sistema con temperatura más alta hacia el sistema con temperatura más baja, que es equivalente al punto de inicio que generalmente se considera en ingeniería acerca de la transferencia de calor (Incropera, 2006).

Junto a los conceptos de energía interna, calor y temperatura, otro concepto que aparece en el contexto de la ingeniería es el de energía térmica. La energía térmica se define generalmente como la parte de la energía interna de un sistema que sólo depende de su temperatura (Incropera, 2006). En los casos donde la energía interna puede ser modelada como dependiente sólo de la temperatura, ésta será igual a la energía térmica. Finalmente, es necesario notar que el calor es energía en tránsito y no puede ser almacenado desde un punto de vista termodinámico, mientras que la energía térmica es una parte de la energía interna del sistema por lo que sí se puede "almacenar". Sin embargo, ambos conceptos se utilizan a menudo indistintamente en contextos de ingeniería donde se utiliza el concepto de "calor almacenado", cuando se incrementa la temperatura de un sistema. El almacenamiento de energía térmica o "calor" requeriría paredes adiabáticas perfectas entre el sistema y su entorno; en la práctica es muy importante ser consciente de que "perfectas" es una abstracción teórica.

Calor (transferencia de calor)

Este estudio se ocupa de problemas dinámicos de la transferencia de energía en forma de calor (transferencia de calor), mientras que la termodinámica estudia sistemas en equilibrio; es importante tener en cuenta que no son considerados ni cambios de fase ni reacciones químicas. En este contexto, un sistema simple de la termodinámica (sistema macroscópico) será sinónimo de un nodo (elemento) por definición. Un nodo se caracteriza por las siguientes propiedades físicas: temperatura, densidad, volumen y capacidad de calor específico. Un nodo será, en principio, indivisible y puede ser sólido, líquido o gaseoso. Un sistema macroscópico sólido, líquido o gaseoso puede ser representado por un único nodo o por varios nodos, cuando se consideren subsistemas macroscópicos del sistema que no estén en equilibrio térmico. En resumen, un nodo será un sistema termodinámico macroscópico en equilibrio térmico y varios nodos puesto en contacto térmico, y que estén en equilibrio térmico, podrían considerarse como un único nodo.

Además, la transferencia de energía en forma de calor (transferencia de calor) entre los nodos se modela como directamente proporcional a la diferencia de temperaturas, es decir, como un proceso lineal. La constante de proporcionalidad será la conductancia térmica inter-nodos, que puede interpretarse como una propiedad física entre nodos. No estará asociada a un solo nodo, ya que es una resistencia a la transferencia de calor y, por definición, al menos dos nodos necesitan ser puestos en contacto térmico.

La literatura generalmente presenta tres modos de transferencia de calor: conducción, convección y radiación (Incropera, 2006). La definición de estos tres modos está más allá de los límites de la termodinámica. Por lo general, la definición de conducción y radiación es clara en la literatura, pero la definición de convección se utiliza de una forma diferente dependiendo del contexto ((Incropera, 2006), (von Böckh, 2012), (Sidebotham, 2015), (Pert, 2013)). En primer lugar, la conducción de calor en los sólidos se explica generalmente utilizando la temperatura como un campo conservativo (Carslaw, 1986), y se puede extender a los fluidos, incluidos los gases suponiendo que no haya movimiento macroscópico ((Incropera, 2006), (Sidebotham, 2015)). La conducción de calor se puede ver, en un contexto de dinámica de fluidos (aunque el calor no es un líquido), como una difusión de la temperatura. La problemática surge cuando se consideran fluidos y gases, puesto que aparece un movimiento de masa en el interior del sistema que produce un movimiento de energía que también se podría interpretar como un movimiento del campo de temperatura. En la mecánica de fluidos esto se llama convección o advección indistintamente y se presenta como un término independiente a la difusión dentro de la ecuación de conservación. No obstante, en el contexto de transferencia de calor, "convección" significa la transferencia de calor por convección entre un sólido y un fluido en movimiento (generalmente aire). En consecuencia, se define en una capa límite y se supone que la transferencia de energía en forma de calor se realiza entre la capa límite (inmóvil) y la superficie del sólido ((von Böckh, 2012), (Sidebotham, 2015)).

En cualquier caso, la idea principal que se considera en el presente estudio es que la transferencia de calor, sea cual sea su causa, puede ser vista como un proceso lineal. La conducción de calor en los sólidos considera una ley lineal, la ley de Fourier, que se introduce en la ecuación en derivadas parciales del calor para describir la transferencia de calor por conducción, mientras que las transferencias de calor por convección y radiación se añaden como condiciones de contorno. El punto clave será añadir también la transferencia de calor por convección y radiación mediante el uso de leyes lineales. La ley de Newton de enfriamiento puede ser considerada para la transferencia de calor por convección, y para la transferencia de calor por radiación puede definirse una temperatura radiante media con la que obtener una ley lineal. No será considerada la advección (o convección como se define en el contexto de la mecánica de fluidos).

La linealización de los procesos de transferencia de calor permite la utilización de una sola expresión, la ecuación del calor, que incluirá todos los modos de transferencia de calor (a escala macroscópica) mediante el uso de conductividades térmicas y capacidades. Esta metodología es una simplificación del problema de transferencia de calor para facilitar su resolución; cada proceso de transferencia de calor se puede estudiar de manera más detallada por separado. En este trabajo, se elige esta opción ya que el software de transferencia de calor que se utiliza para el análisis de la transferencia de calor en edificios asume por lo general estas hipótesis (EnergyPlus (Seem, 1987), TRNSYS (Stephenson y Mitalas, 1971), el ESP-r (Clarke, 2001)). La ecuación de calor que incluye todos los modos de transferencia de calor linealizados es una ecuación de conservación de energía que debe ser entendida como una expresión unificada dentro del intervalo de validez de las hipótesis consideradas, es decir, no pretende ser una expresión universal ya que está claramente limitada a los supuestos clásicos macroscópicos y a la validez de linealidad de los procesos. El objetivo es facilitar el estudio de la transferencia de calor en los edificios.

De este modo, el problema de la transferencia de calor se ve reducido a la existencia de un gradiente de temperatura entre nodos. Este trabajo presenta una metodología para ser aplicada a la caracterización de transferencia de calor en edificios en aplicaciones diarias. Por lo tanto, no se considera una descripción detallada a escala de laboratorio, pero sí supondrá la estimación de la incertidumbre de los cálculos, es decir, dará la probabilidad de obtener un determinado valor de un parámetro físico y criterios diferentes para indicar si el valor del parámetro físico es estadísticamente significativo. La incertidumbre en las estimaciones no es un problema en sí misma, modelos y métodos utilizan aproximaciones, y la precisión no es sólo una cuestión de los métodos teóricos, sino también una cuestión de la capacidad tecnológica y experimental. Es importante hacer un compromiso entre teoría, costo experimental e incertidumbre. La modelización de todos los procesos de transferencia de calor como un proceso lineal, considera la temperatura como un campo conservativo e incluso utilizando ese enfoque simplificado el problema de estudiar la transferencia de calor se complica para sistemas complejos, tales como edificios. Existen otras opciones y pueden ser considerados otros enfoques dados en la literatura sobre este tema ((Carslaw y Jaeger, 1986), (Duffie y Beckman, 2006), (Incropera, 2006), (von Böckh, 2012), (Sidebotham, 2015)). Esta tesis se ocupa de un problema particular: la identificación parámetros de elementos constructivos, aunque su aplicación para simulación y control está estrechamente relacionada.

Modelado de la transferencia de calor en edificios

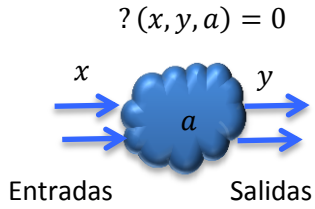
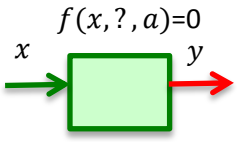
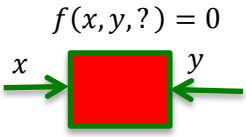
La idea es construir un modelo físico que establezca adecuadamente el balance de energía en los edificios. El problema es esencialmente aproximar el comportamiento térmico de un edificio utilizando un modelo matemático. Esta tesis obtiene modelos matemáticos a partir de primeros principios de la física para conservar la mayor cantidad de conocimiento posible. Pueden suponerse modelos en estado de equilibrio ((Naveros, et al., 2012), (Castillo et al., 2014)), aunque para una mejor comprensión del problema serán considerados modelos dinámicos ((Ogata , 2010), (Crassidis y Jenkins, 2011)).

Problema directo e inverso

Tradicionalmente, estudios anteriores han propuesto diferentes representaciones de modelos dinámicos para el análisis del consumo de energía en edificios, y han hecho una distinción entre la

solución del problema directo (simulación) o del problema inverso (identificación) (Rabl, 1988). Todos los modelos respetan el principio de conservación de la energía, que expresa el equilibrio energético entre un edificio y sus alrededores; y todos los modelos pueden utilizarse tanto en la resolución del problema directo como en la del problema inverso. Por un lado, el problema directo, donde las propiedades de los materiales de construcción se supone conocida, se resuelve considerando la analogía termo-eléctrica como punto de inicio y busca de criterios que permiten determinar la simplificación del problema dentro de ese marco teórico ((Fraisse, et al., 2002), (Ramallo-González, et al., 2013)). Por otro lado, el problema inverso generalmente se ha resuelto utilizando un criterio similar y tratando de resolver las dificultades para obtener resultados con significado físico ((Jiménez y Heras, 2005), (Bacher y Madsen, 2011), (Mejri, et al., 2011)). Un objetivo de este trabajo es mostrar la conexión entre las diferentes representaciones de modelos utilizando como punto de partida la ecuación del calor, que es una ecuación en derivadas parciales (PDE), considerando leyes lineales para describir los diferentes modos de transferencia de calor.

Tabla e.1. Clasificación del modelado del problema directo e inverso en este contexto.

Sistema Físico		Tipo de modelado del problema		
		Directo	**Inverso**	
				
		$\{x,a\} \rightarrow y$	$\{x,y\} \rightarrow a$	
		Simulation	**Identificación**	
		Caja Blanca	*Caja Gris*	*Caja Negra*
$f(\cdot)$	Estructura del modelo desde	Primeros principios	Primeros principios / Heurística	Heurística
a	Parámetros del modelo	Conocidos	**Estimados**	**Estimados**
x	Entradas del sistema físico	Medidas	Medidas	Medidas
y	Salidas del Sistema físico	**Estimadas**	Medidas	Medidas

Desde otro punto de vista (Ghiaus, 2014), los modelos matemáticos utilizados para resolver el problema directo suelen considerarse modelos de caja blanca y se consideran modelos de caja negra o gris aquellos modelos empleados en la resolución del problema inverso (identificación), Tabla e.1. Los modelos de caja blanca, que están basados en la física del sistema, se utilizan para la simulación del comportamiento del sistema. Los modelos de caja negra, que representan las relaciones de entrada-salida que se ajusten a las medidas sin ningún conocimiento de su funcionamiento interno, se utilizan en aplicaciones de control prácticas y se obtienen mediante técnicas experimentales de identificación. En el modelado de caja gris, la estructura del modelo y los parámetros del modelo se obtienen mediante una combinación de primeros principios de la física y medidas. Este último enfoque es especialmente útil para aplicaciones de control cuando el modelo se expresa en una forma adecuada tal como la función de transferencia o el espacio de estados (Ghiaus, et al., 2007).

Cuando se proponen modelos de caja negra para resolver el problema inverso (identificación) (Ljung, 1999), los parámetros identificados no se pueden conectar con los parámetros físicos (Ogata,

2010). Sin embargo, si el problema se plantea a partir de primeros principios para obtener un modelo de caja gris, el significado físico de los parámetros puede mantenerse. Este trabajo propone construir a partir de primeros principios los modelos utilizados para resolver el problema de identificación de parámetros y poder mantener el significado físico de éstos, es decir, utilizar modelos que serán de caja blanca o gris dependiendo sólo de su aplicación para la resolución del problema directo (simulación) o inverso (identificación), respectivamente . El uso de una ley lineal para describir la transferencia de calor, y el principio de conservación de la energía, pueden entenderse como primeros principios de la física puesto que se establecen como axiomas aceptados como "ciertos" con independencia de que puedan ser verificados por la experimentación.

Redes térmicas considerando la teoría de grafos y el álgebra lineal

El álgebra lineal estudia los espacios vectoriales y las aplicaciones lineales (Strang, 2009). Un espacio vectorial sobre un campo F es un conjunto $\mathbf{V}$ junto con dos operaciones binarias. Los elementos de $\mathbf{V}$ son vectores formados por elementos de F que son elementos escalares. Una de las operaciones es la suma de vectores, elemento a elemento, y la otra es la multiplicación escalar. En álgebra lineal, el uso de matrices simplifica las combinaciones lineales que se pueden generar utilizando la suma de los vectores y la multiplicación escalar. Las operaciones de suma y multiplicación en un espacio vectorial deberán cumplir los siguientes axiomas:

- Suma asociativa, $(\mathbf{u}+\mathbf{v})+\mathbf{w}=\mathbf{u}+(\mathbf{w}+\mathbf{v})$;
- Suma conmutativa, $\mathbf{u}+\mathbf{v}=\mathbf{v}+\mathbf{u}$;
- Elemento identidad de la suma: existe un elemento $\mathbf{0}\in\mathbf{V}$, llamado vector cero, tal que $\mathbf{u}+\mathbf{0}=\mathbf{u}\;\forall\;\mathbf{u}\in\mathbf{V}$;
- Elemento inverso de la suma: para cada $\mathbf{v}\in\mathbf{V}$, existe un elemento $-\mathbf{v}\in\mathbf{V}$, llamado el inverso aditivo de $\mathbf{v}$, tal que $\mathbf{v}+(-\mathbf{v})=\mathbf{0}$;
- Multiplicación escalar distributiva respecto a la suma de vectores, $a(\mathbf{u}+\mathbf{v})=a\mathbf{u}+a\mathbf{v}$;
- Multiplicación escalar distributiva respecto a la adición de escalares, $(a+b)\mathbf{v}=a\mathbf{v}+b\mathbf{v}$;
- Multiplicación escalar distributiva con la multiplicación de escalares, $a(b\mathbf{v})=(ab)\mathbf{v}$;
- Elemento identidad de la multiplicación escalar, $1\mathbf{v}=\mathbf{v}$, donde 1 es la identidad multiplicativa en el campo F;

donde $\mathbf{u},\mathbf{v}\;y\;\mathbf{w}$ son vectores arbitrarios en $\mathbf{V}$ y $a\;y\;b$ son escalares en F.

Por otro lado, los grafos están representados por un conjunto de nodos y un conjunto de aristas que conectan dos nodos entre sí, o un nodo a sí mismo. Los grafos son modelos muy versátiles para el análisis de problemas cuando se les asigna significado físico a los nodos y las aristas (Gross & Yellen, 2005). Las redes térmicas pueden ser consideradas grafos, donde los nodos representan temperaturas que están conectadas por aristas que representan resistencias térmicas. En particular, una red térmica será un grafo dirigido (dígrafo) puesto que las aristas conectarán los nodos en el sentido de las temperaturas decrecientes. Además, el estudio de las redes térmicas se puede colocar dentro de la teoría de grafos algebraica, de esta forma una red térmica puede ser representada por una matriz incidente de dimensiones *m*-aristas por *n*-nodos (Ghiaus, 2013). Con lo que la red térmica estará representada por una matriz y podrá ser estudiada como espacio vectorial utilizando el álgebra lineal (Strang, 1987). El presente trabajo propone el uso del álgebra lineal como base para el estudio de las redes térmicas en lugar de la analogía termo-eléctrica ampliamente utilizada actualmente. Es posible dejar la analogía termo-eléctrica al margen porque desde la teoría de grafos aparece el marco teórico de la red térmica sin necesidad de emplearla.

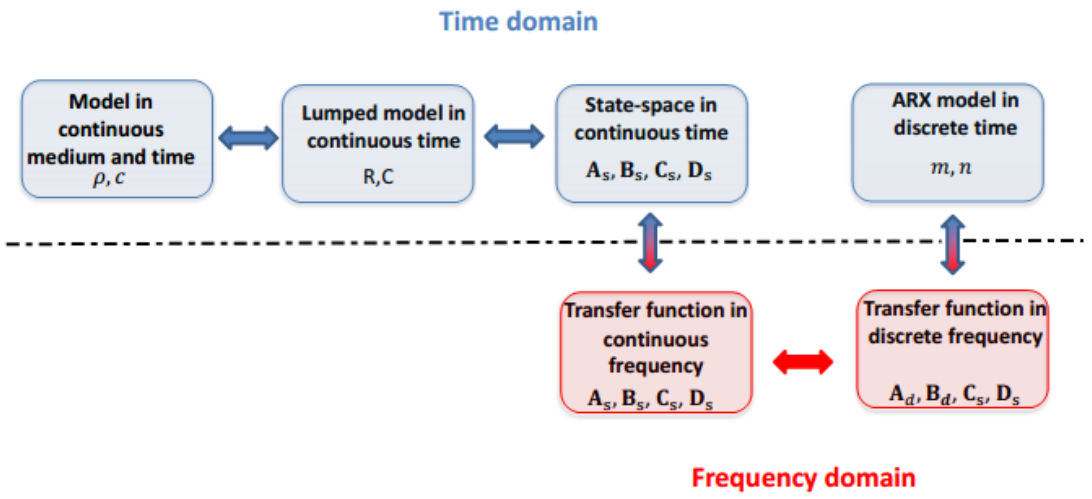


Figure e.3. Esquema conectando la ecuación de calor con un modelo autorregresivo con variables exógenas

Por otra parte, la metodología debe servir para estimar el rendimiento energético de los edificios mediante la resolución de un problema de identificación de parámetros utilizando mediciones in situ, y considerando una separación entre las características térmicas intrínsecas de los edificios, su utilización y la conducta de los usuarios. Esta separación es importante para simplificar la solución del problema de estimar la eficiencia energética de los edificios. Este trabajo se centra en la identificación de las características térmicas intrínsecas de edificios para fijar una referencia que pueda ser empleada para comparar el comportamiento energético entre diferentes edificios o entre un mismo edificio antes y después de ser renovado. La metodología propuesta puede conectarse con el problema clásico de la transferencia de calor en los edificios, que utiliza la ecuación del calor, si ésta se resuelve teniendo en cuenta en primer lugar su integración espacial y en segundo lugar su integración temporal, al contrario de lo que normalmente se realiza en este campo ((Fraisse, et al, 2002), (Carslaw y Jaeger, 1986)). De este modo, se obtiene un modelo considerando el principio de conservación de la energía y suponiendo el espacio y el tiempo continuos. Seguidamente, el modelo se expresa discretizando el espacio y usando una linealización como una red térmica, es decir, como un sistema de ecuaciones diferenciales y algebraicas lineal e invariante en el tiempo respecto a sus parámetros (DAE), utilizando para ello el método Galerkin de elementos finitos ((Strang, 2007), (Naveros y Ghiaus, 2015)), y después se obtiene la representación en el espacio de estados. A partir del espacio de estados, se aplica la transformada de Laplace para obtener la función de transferencia que relaciona las variables de entrada y salida del modelo en el dominio de la frecuencia continua. Tras ésto, se realiza una discretización para obtener la función de transferencia en el dominio de la frecuencia discreta (transformada Z). El modelo en frecuencia discreta puede transformarse de forma inversa en un modelo autorregresivo con variables exógenas (ARX) en el dominio del tiempo discreto. Dicho modelo ARX es una expresión discreta del sistema en el espacio de estados integrado en el tiempo, Figura e.3. Por último, los parámetros pueden estimarse utilizando cualquier modelo para ser empleados en obtener una medida de la eficiencia energética intrínseca experimental de un edificio.

Características físicas intrínsecas

En general, las propiedades térmicas intrínsecas de un edificio pueden ser variables, pero pueden ser consideradas invariantes en el tiempo siendo conscientes de las limitaciones que esta hipótesis implica. Dicha hipótesis simplifica (y hace posible en muchos casos) la solución de la ecuación del

calor en derivadas parciales (PDE). Los lectores deben ser conscientes de que la doble integración, en espacio y tiempo, de la PDE es obligatoria para resolver tanto el problema directo (simulación) como el problema inverso (identificación). En primer lugar, a partir de la integración del espacio se recupera el sistema de espacio discreto, que es en verdad el origen de la ecuación del calor antes de aplicar el límite $\Delta x \to 0$. Este sistema discreto en el espacio es también conocido como un modelo de parámetros "concentrados", es decir, una red térmica finita que describe el comportamiento térmico del edificio suponiendo invariantes en el tiempo los parámetros, i.e. las resistencias y capacitancias térmicas.

En este trabajo, las resistencias y capacitancias, que caracterizan a la transferencia de calor de los edificios, son las resistencias y capacitancias de los materiales de construcción, una vez que se construyen teniendo en cuenta la mano de obra, y las resistencias externas debido a la convección y radiación. El uso de valores invariables en el tiempo no quiere decir que no varíen con el tiempo realmente, pero este enfoque permite obtener un valor de las resistencias y las capacitancias junto a su incertidumbre para comprobar si el valor estimado es estadísticamente significativo, es decir, si es posible indicar la probabilidad de obtener un valor constante representativo siguiendo la metodología propuesta, y si la hipótesis de invariancia temporal es válida. De esta manera, se obtiene una descripción que es independiente del consumo de energía de los edificios y puede facilitar una estimación estándar de la eficiencia energética en la construcción. Además, podría ser utilizada para el análisis del consumo de energía junto a otros factores como las condiciones meteorológicas, sistemas de climatización y de ventilación, o el comportamiento del usuario, dando así toda una descripción del comportamiento térmico de un edificio una vez está operativo. Teniendo en cuenta todos estos factores, las opciones para medir la eficiencia energética de un edificio se torna más compleja. En la práctica, estos factores deben considerarse en su conjunto, pero este estudio se centra en la identificación de parámetros intrínsecos para estimar la eficiencia energética, no en el consumo de energía.

Condiciones de contorno

Las condiciones meteorológicas son las fuerzas que excitan los edificios, es decir, son las entradas que excitan el sistema dinámico junto de las fuentes internas de energía. Por esta razón, el estudio de la transferencia de calor en edificios es, en primer lugar, un problema de condiciones de contorno relacionado con las condiciones meteorológicas que siempre lo excitan. Puesto que las fuentes de energía internas pueden estar o no activas.

La respuesta de los edificios como sistemas dinámicos tendrá dos componentes: 1) la respuesta natural y 2) la respuesta forzada. Después de un cierto intervalo de tiempo, la respuesta que perdura solamente es la respuesta forzada. Las condiciones meteorológicas no cesan, entonces, el estudio de las características intrínsecas de los edificios se puede hacer teniendo en cuenta la respuesta forzada de los edificios a dichas condiciones de contorno. Y las características intrínsecas podrán emplearse para obtener un valor estándar de la eficiencia energética de edificios, antes de sopesar otros factores como el comportamiento de los usuarios o los sistemas de climatización. Una mejor respuesta para las mismas condiciones meteorológicas implicaría un mejor rendimiento energético teniendo en cuenta todos los demás factores constantes. La obtención de un valor de la eficiencia energética de este modo es sólo un primer paso; una vez que se obtiene, la adición de otros factores debe ser estudiada.

Esta tesis no se ocupa de la respuesta natural, que también es importante, sólo de la respuesta forzada. Esto significa que este trabajo resuelve un sistema dinámico de forzado, que se describirá usando ecuaciones diferenciales y algebraicas (DAE), considerando condiciones de contorno y no

condiciones de valor inicial. El problema de condiciones de valor inicial sería importante para describir el comportamiento del edificio como una función de las fuentes de energía internas utilizadas para aclimatar los espacios interiores que no tengan un régimen periódico sino puntual (sistemas de frío-calor). Lo estudios sobre problemas de valor inicial son numerosos en la literatura (Incropera, 2006). Por un lado, el problema de valor inicial se refiere a la modificación de las condiciones interiores para alcanzar las condiciones de confort; este cambio no puede ser instantáneo ya que el sistema (el edificio) tiene inercia al cambio debido a la capacidad de almacenamiento de energía. Por otro lado, el problema de contorno aquí tratado se ocupa de las condiciones interiores que tendrá un edificio debido a las condiciones meteorológicas. Estas condiciones interiores serán determinadas por las características invariables de la envolvente del edificio.

Este estudio no tendrá en cuenta la respuesta natural y considerará que la respuesta forzada no tiene porqué ser una respuesta constante. De hecho, las condiciones meteorológicas dependen del tiempo, aunque se considerará que están en un estado de equilibrio "periódico" y producirán una respuesta forzada "periódica" de las condiciones interiores del edificio ((Kumarm, 2013), (Thirumaleshwar, 2009)). Los siguientes tres requisitos definen un estado de equilibrio "periódico":

1) La respuesta natural (salidas) a las entradas se amortigua y se acerca a cero cuando aumenta el tiempo sin límite.

2) Cada entrada, condiciones meteorológicas o fuentes internas de energía, está en estado de equilibrio periódico, y son las entradas las que dan el significado de "estado de equilibrio periódico" a las salidas (condiciones interiores).

3) Las entradas (condiciones meteorológicas) se aplican indefinidamente.

La primera hipótesis implica que la constante de amortiguamiento de un edificio no es infinita. La segunda puede suponerse cumplida por las condiciones meteorológicas, ya que siguen el ciclo diario (y de las estaciones) y las fuentes internas de energía podrían ser controladas para ser periódicas. La tercera hipótesis se cumple para las condiciones meteorológicas puesto que nunca se detienen, y se podría lograr bajo experimento para las fuentes internas de energía.

Una vez que se presenta el uso de condiciones de contorno, el problema a resolver se puede considerar como: 1) un problema de condiciones de contorno fijo-fijo (condiciones de frontera de Dirichlet) 2) un problema de contorno fijo-libre (condiciones de frontera mixtas). En el primero, las condiciones meteorológicas y las fuentes de frío-calor internas son las condiciones de contorno que fuerzan el sistema dinámico. En el segundo, sólo las condiciones climáticas fuerzan el sistema.

Comportamiento de los usuarios

Además de las características térmicas intrínsecas de los edificios, el comportamiento del usuario es un factor clave para determinar la eficiencia energética global de un edificio (Fabi, et al., 2013). Los ocupantes son la causa de las fuentes internas de energía y necesitan ser considerados para un estudio integral. No obstante, la idea de este trabajo es analizar el problema mediante la separación de las diferentes variables que afectan el consumo de energía. De esta manera, el modelo obtenido podrá añadir el comportamiento del usuario en futuros estudios. Aunque el comportamiento del usuario puede ser un factor más determinante que otros, la idea que debe permanecer, es que un edificio más eficiente desde el punto de vista intrínseco será más eficiente teniendo en cuenta un comportamiento del usuario estándar. Esta es la ventaja de tener en cuenta la eficiencia energética de los edificios independientemente del consumo de energía real de un edificio. La brecha entre el consumo de energía

predicho y medido no sólo está relacionada con las características térmicas intrínsecas de un edificio sino con el efecto de los usuarios, el cual puede ser más determinante. En cualquier caso, la complejidad del comportamiento del usuario aconseja la separación de este factor para abordar el problema, y el comportamiento del usuario, que es un punto social clave, queda fuera del alcance de la presente disertación.

Síntesis

La linealización de la transferencia de calor, el principio de conservación de la energía y la asunción de parámetros invariantes en el tiempo son hipótesis utilizadas y aceptadas en general para estudiar la transferencia de calor en los edificios ((Seem, 1987), (Stephenson y Mitalas, 1971), (Clarke, 2001)). Este trabajo propone el modelado de la transferencia de calor usando redes térmicas desde un nuevo punto de vista basado en la ecuación del calor, la termodinámica y la teoría de grafos. En cualquier caso, el punto de partida no cambia el significado de las redes térmicas que pueden interpretarse como la representación gráfica de un sistema de ecuaciones diferenciales y algebraicas (DAE) o viceversa. Esto es importante ya que las redes térmicas han demostrado su utilidad para el modelado de la transferencia de calor en la solución del problema directo y del problema inverso. Por lo general, el problema directo utiliza un sistema determinista de DAE para obtener la distribución de temperatura en el tiempo de un nodo de la red térmica. En teoría, las redes térmicas pueden dar una solución estable y única del problema directo, el cual se considera que está bien planteado, utilizando las condiciones iniciales y / o de contorno y suponiendo que se conocen los parámetros de la red térmica. Por otra parte, el problema inverso (identificación de parámetros), trata de obtener los parámetros de la red térmica utilizando las entradas y las salidas del sistema. Este trabajo muestra que el problema de identificación no debe ser visto como un problema mal planteado teóricamente, es decir, puede tener una solución estable y única desde el punto de vista determinista bajo las hipótesis planteadas. En la práctica, surgen dificultades que afectan problema tanto directo como inverso, que pueden llegar a ser mal planteados al no poder obtenerse solución. Estas dificultades se deben a: 1) errores de modelado, es decir, las redes térmicas son una aproximación a las observaciones físicas; y 2) errores de medida, el problema de transferencia de calor no se resuelve mediante funciones continuas sino discretas formadas por datos registrados con un error experimental.

El capítulo 2 muestra cómo pueden construirse redes térmicas utilizando la teoría de grafos y los primeros principios físicos relacionados con el calor y las leyes de la termodinámica. Esa es la base de este trabajo, es decir, las redes térmicas son grafos que pueden ser estudiados utilizando álgebra lineal. También se muestra como históricamente la teoría clásica de la conducción de calor empieza en una expresión finita, para supuesta la continuidad del espacio y el tiempo, llegar a una expresión infinitesimal. El hecho es que las matemáticas no debieran ser tomadas como un fiel reflejo de la realidad, sino que tanto las matemáticas discretas como las continuas son aproximaciones a lo observado, que tampoco es la realidad. De hecho, en la experimentación el punto de partida es siempre discreto en el espacio y el tiempo. Las ecuaciones en derivadas parciales se obtienen considerando cierta la continuidad del espacio y el tiempo y haciendo que sus incrementos tiendan a cero. Esta visión clásica, que facilita la resolución de algunos problemas, da más importancia al cálculo de álgebra, pero no hay razones prácticas para aceptar dicha declaración más allá de la creencia de que el espacio y el tiempo continuo describen perfectamente la realidad, y las observaciones físicas sólo son mediciones discretas imperfectas de esa realidad. Hoy en día estamos en la era de las computadoras y los datos, donde la matemática discreta y álgebra lineal se pueden considerar más útiles que las matemáticas continuas y el cálculo, en palabras de Gilbert Strang: "la mayoría de las aplicaciones de las matemáticas son discretas en lugar de continuas" y "el uso de vectores y matrices se ha convertido en el idioma de saber" (Strang, 2009).

El capítulo 3 introduce las transformaciones necesarias para convertir la ecuación del calor en derivadas parciales en un modelo autorregresivo con variables exógenas (ARX). Estas transformaciones son bien conocidas por separado, pero la cadena de todas ellas juntas no se encuentra en la literatura tal y como se propone en esta tesis doctoral. En primer lugar, la ecuación del calor se expresa como un sistema de ecuaciones diferenciales y algebraicas (DAE) usando el método de elementos finitos, y se muestra cómo puede obtenerse un resultado similar usando el método de volumen finito y el método de diferencias finitas. Después, se muestra cómo el sistema de DAE puede escribirse en el espacio de estados (Ghiaus, 2013). A continuación, desde el espacio de estados se obtiene la función de transferencia, y está se conecta con un modelo ARX. La idea clave es obtener el modelo ARX por deducción y no ad-hoc.

El capítulo 4 muestra cómo las transformaciones de parámetros pueden ser entendidas como cambios de variable, y como pueden deshacerse todos los cambios de variable para obtener algebraicamente los parámetros de una red térmica a partir de los parámetros del modelo ARX obtenido desde ella por deducción. Esto supone una demostración de que el problema de transformación de parámetros, una vez obtenidos los parámetros del modelo ARX, es un problema bien planteado bajo las hipótesis establecidas y desde un punto de vista determinista. La estructura de los modelos no se recupera; sino que es conocida y se obtiene por deducción como se muestra en el capítulo 3. El conocimiento de la estructura del modelo se utiliza para deshacer los cambios de variable entre los parámetros de las diferentes estructuras de los modelos.

En el capítulo 5 se explica cómo el orden del modelo puede ser reducido teniendo en cuenta los errores de medida. Este capítulo presenta los límites de mensurabilidad como criterio para la reducción del orden del modelo, un tema complejo que por lo general es tratado exclusivamente desde un punto de vista matemático. La idea es la elección del orden del modelo que permite la menor pérdida de conocimiento utilizando la función de transferencia. En teoría, un modelo de orden alto sería más preciso que un modelo de orden inferior, ya que describiría mejor las observaciones. En la práctica, si la diferencia entre la respuesta de un modelo de orden bajo y un modelo de orden superior no es mensurable, el uso del modelo de orden superior implicaría un incremento de complejidad sin proporcionar resultados más precisos. En resumen, la precisión del modelo no debería ser superior a la precisión de las observaciones en la resolución de un problema de transferencia de calor, y elegir un modelo de orden inferior sería una mejor opción por simplicidad.

El capítulo 6 muestra la aplicación de la metodología utilizando datos experimentales medidos en condiciones meteorológicas reales de un muro simple homogéneo y opaco. Se utiliza un modelo autorregresivo (ARX), obtenido por deducción desde una red térmica utilizando la metodología presentada en los capítulos anteriores. Los parámetros físicos estimados son: el valor de la transmitancia térmica total, la transmitancia de energía solar dinámica y la capacidad calorífica efectiva. Las estimaciones de los parámetros físicos se presentan junto con sus incertidumbres considerando la técnica de Monte Carlo para la propagación de errores.

Por último, las conclusiones más relevantes desde el punto de vista del autor son expuestas brevemente.

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CHAPTER 1

INTRODUCTION

1.1. Building energy efficiency

The present study is done in the broader context of energy use by human societies. Throughout human history, societies and civilizations have been energy guzzlers while the available energy resources have been always limited (Smil, 2008). Each society must deal with the challenge of managing its energy resources seeking a sustainable social development for improving human well-being. Sustainable development may be understood as an equilibrium state between human activities and the environment in which they are carried out. This means that present actions of the society for providing its necessities should not compromise the possibility of future generations for providing their own needs. In any case, the idea of human necessities is related to well-being which is not a fix idea from a social point of view. In the energy context, energy necessity implies to solve a complex problem linked to the statement of minimum energy requirements by the society.

The statement of society minimum energy requirements would imply quantifying the energy needed by a society which is a huge task since it depends on variable cultural and social perspectives and goals. However, independently of energy necessities quantification, which is an interesting question out of the scope of the present study, the efficient use of energy would imply a minimization of the total energy used for covering the same needs. This fact is the key point for boosting energy efficiency as one of the pillars to guarantee a global society sustainable development, besides the use of renewable energies.

In this sense, at European level, the Energy Efficiency Directive (EED) 2012/27/EU[1] establishes a common framework of measures for the promotion of energy efficiency within the European Union in order to ensure the achievement of the Union's 2020 20 % headline target on energy efficiency and to pave the way for further energy efficiency improvements beyond that date. Particularly, energy efficiency in buildings is crucial to achieve that target since in European Union nearly 40% of the final energy consumption is attributable to housing, offices and other buildings within the private or public sector. Accordingly, a major investment in buildings is crucial for the European Union (EU) to meet its 2020 energy and climates objectives. For this reason, in the Horizon 2020 (2014-2020) programming period, the European Structural and Investment Funds (ESI Funds), and specifically Cohesion Policy Funds play a major role in relation to the refurbishment and construction of buildings.

The Energy Performance of Buildings Directive (EPBD) 2010/31/EU sets the minimum energy performance requirements for new buildings, renovation of existing buildings and specific building elements to be a baseline for authorities. On this line, a very important issue is to quantify the energy savings due to actions related to the improvement of energy performance of buildings and for this purpose energy audits and energy performance certificates are required to identify energy saving opportunities.

[1] Directive 2012/27/EU of the European Parliament and of the Council of 25 October 2012 on energy efficiency

The scope of an energy audit, in line with the EED, does not only include the assessment of the technical characteristics of the building but also the analysis of the amount of energy consumed per end-use and the impact of behavioural changes. As such, energy performance certificates can provide important inputs to an energy audit. In the case of energy performance contracting, energy audits provide a mechanism to evaluate energy savings including those linked to consumer behaviour. Furthermore, for important renovation projects which imply higher subsidies, detailed energy audits allow to the community the monitoring and verification of energy efficiency improvements, including long-term cost and energy savings. The combination of single standard measures and the recommendations in the energy performance certificate can be used, for less complex projects, to identify the sustainable energy measures to be implemented as part of the building renovation. Nonetheless, an energy audit is mandatory to monitor and verify the project's energy savings and to understand possible discrepancies between the energy performance rating and the actual energy consumption of the building.

Firstly, energy audit is needed due to the gap between predicted and measured energy performance of buildings, with significant implications for the accuracy of prospective life-cycle cost evaluations for any given building, Figure 1.1 (Turner & Frankel, 2008).

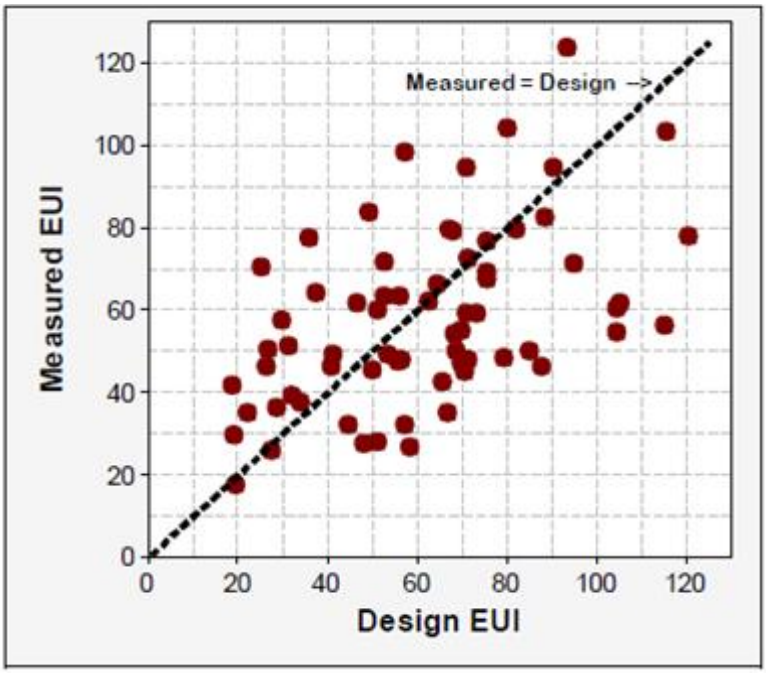


Figure 1.1. Measured versus Design Energy Use Intensities, EUI (kBtu/sf) (Turner & Frankel, 2008).

The use of a baseline in energy audits is needed although the gap between predicted and measured energy consumption in buildings is a complex problem involving several aspects (de Wilde, 2014). This study would consider a division of the problem to simplify its solution and it will only focus on a methodology based on physical principles of energy transfer for studying the intrinsic characteristics of buildings. This guarantees its reproducibility since the methodology would be based on classical heat transfer principles, which are actually used in the modelling of heat transfer in buildings (EnergyPlus (Seem, 1987), TRNSYS (Stephenson & Mitalas, 1971), ESP-r (Clarke, 2001)). The idea is to stablish a methodology which facilitates heat transfer model validation using

experimental data in order to find out if it is suitable for reproducing the observations, supposing that, theoretically, the model is valid and optimal for modelling the heat transfer.

Energy savings represent the reduction of energy use and cannot be measured directly[2], which implies that for determining energy savings it is needed to compare energy use before and after the refurbishment of a building, Figure 1.2. The identification of the intrinsic thermal characteristics of buildings should help us to break the deadlock of fixing a baseline to be used as reference for energy efficiency comparisons between buildings before and after refurbishment or between different buildings.

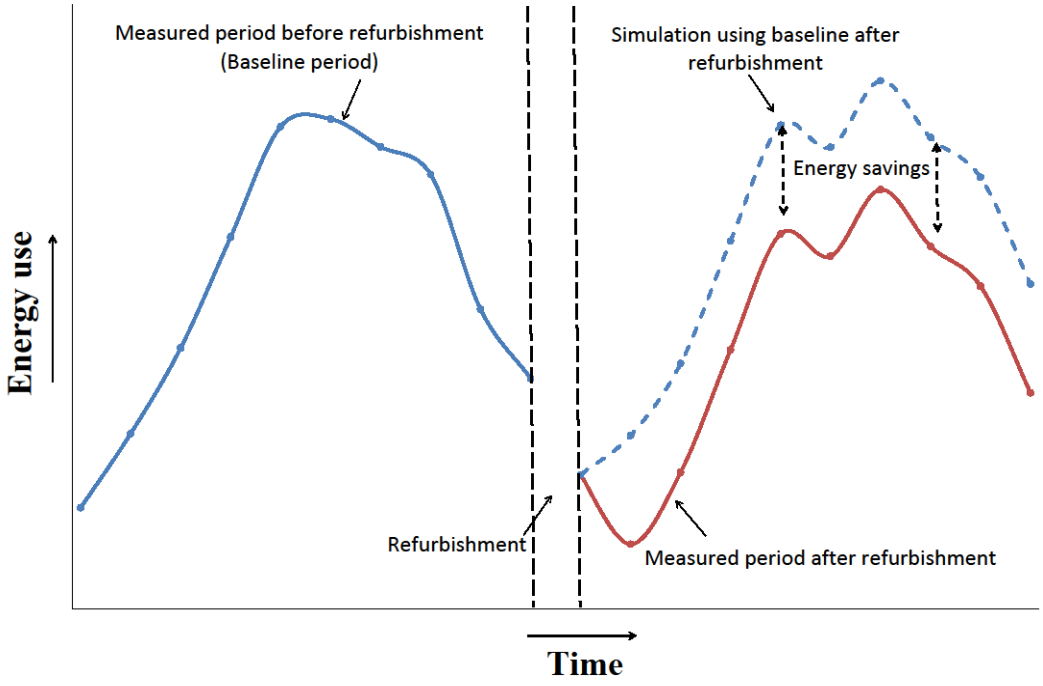


Figure 1.2. Energy savings using a baseline obtained from measurements previous to building refurbishment

1.2. Energy conservation, heat and temperature

Energy conservation is one of the first principles of physics. It started in mechanics and it was extended to other fields of physics, from classical to modern fields. The classical principle of energy conservation states that it is possible to define a conservative energy function, that is the change of energy, ΔE, is equal to the energy that goes into the system, E_{in}, minus the energy that goes out of the system, E_{out}:

$$\Delta E = E_{in} - E_{out} \tag{1.1}$$

In theory, the idea is simple; in practice, the difficulties appear when we want to measure the energy going in and out of the system and the transformations between different forms of energy.

[2] International performance measurement and verification protocol (IPMVP). Concepts and options for determining energy and water savings. Volume 1. Efficiency Valuation Organization, January 2012.

Furthermore, there is no absolute measure of energy since the measure is always a change of energy from a zero chosen by consensus. This work deals with classical principles and it is placed within the domain of classical thermodynamics, which studies macroscopic systems. The energy considered in Eq. (1.1) may be seen as the variation of the internal energy of the system, U, and the energy going in/out the system may be considered as heat, Q, going in/out the system and/or work, W, done on/by the system. This point of view is considered by the first law of thermodynamics and it is used to introduce heat as a form of energy in transit (Callen, 1985):

$$\Delta U = \pm Q \pm W \tag{1.2}$$

Therefore, heat is energy going in/out a system and, in absence of work (mechanical or chemical), it is the only mechanism to transfer energy from a system to another from a thermodynamics point of view. A question which cannot be addressed using the first law of thermodynamics is when heat should go in/out of the system. To answer this question the experience needs to be used and the concepts of thermal equilibrium and temperature appear. Two systems are in thermal equilibrium if there is no energy transfer, in form of heat, between them when they are put in thermal contact. The zeroth law of thermodynamics defines temperature as a physical variable from the concept of thermal equilibrium: "If two systems are in thermal equilibrium with a third system they are also in thermal equilibrium with each other". The zeroth law states an equivalence relation and any state in thermal equilibrium can be characterized by a thermodynamic or absolute temperature as defined by Kelvin (Callen, 1985). This means that two systems in thermal equilibrium between them will have, by definition, the same temperature. In this way, temperature becomes a unit of measurement, as length, mass, electric charge or time, which characterizes a state of thermal equilibrium and heat will be transferred between two systems in thermal contact at different temperatures producing a change of their internal energies.

Next, it is needed to determine the sense of heat flow between two systems at different temperatures since it is not given by the zeroth law. For this purpose, the second law of thermodynamics states that the sense of heat transfer will be from the system at higher temperature towards the system at lower temperature, which is equivalent to the start point usually considered in engineering context about heat transfer (or heat) (Incropera, 2006); that is, when two systems at different temperature are in thermal contact, energy flows in form of heat from the system at higher temperature towards the system at lower temperature.

In addition to the concepts of internal energy, heat and temperature, another concept which appears in engineering context is thermal energy. Thermal energy is usually defined as the part of the internal energy of a system which is only dependent on its temperature (Incropera, 2006). In cases where internal energy may be modelled as only dependent on temperature, it will be equal to the thermal energy. Finally, it is necessary to notice that heat is energy in transit and cannot be stored from a thermodynamic point of view while thermal energy is a part of the internal energy of the system, hence it can be "stored". Nonetheless, both concepts are often used indistinctly in engineering contexts where it is used the concept of "heat stored" when the temperature of a system is incremented. The storage of thermal energy or "heat" requires perfect adiabatic boundaries between the system and its surroundings; in practice it is very important to be aware that "perfect" is a theoretical abstraction.

1.3. Heat (heat transfer)

This study deals with dynamic problems of energy transfer in the form of heat (heat transfer) while thermodynamics is a science of equilibrium; it is important to note that no phase change or

chemical reactions will be considered in this dissertation. In this context, the most basic thermodynamic system (macroscopic system) will be a node (element) by definition. A node will be characterized by the next physical properties: temperature, density, volume and specific heat capacity. A node will be, at beginning, indivisible and it can be solid, liquid or a gaseous. A macroscopic solid, liquid or gaseous system can be represented by a single node or by several nodes, when the macroscopic subsystems of the whole system are not in thermal equilibrium. In short, a node will be a macroscopic thermodynamic system in thermal equilibrium and several nodes put in thermal contact, which are in thermal equilibrium, could be considered as a single node.

In addition to this, energy transfer in form of heat (heat transfer) between nodes will be modelled as proportional to the difference of temperatures, that is to say, as a linear process. The constant of proportionality will be thermal inter-nodes conductance, which may be interpreted as a physical property between nodes. It should not be associated to a single node since it is the reciprocal of the resistance to heat transfer and, by definition, at least two nodes need to be put in thermal contact.

Literature usually presents three modes of heat transfer: conduction, convection and radiation (Incropera, 2006). The definition of these three modes is beyond the limits of the thermodynamics. It is usually clear in the literature what is the definition of conduction and radiation but it is fuzzier to understand what is convection since the word is used in a different way depending on the context ((Incropera, 2006), (von Bockh, 2012), (Sidebotham, 2015), (Pert, 2013)). Firstly, heat conduction in solids is usually explained in the literature using temperature as a conservative field (Carslaw, 1986), and it can be extended to fluids including gases supposing no macroscopic movement ((Incropera, 2006), (Sidebotham, 2015)). Heat conduction can be seen, in a fluid dynamics context (although heat is not a fluid), as a diffusion of temperature. The misunderstanding arrives when fluids are considered since it appears a macroscopic mass movement inside of the system which produces a movement of energy and it can be interpreted as a movement of temperature. In fluid mechanics this is called convection or advection indistinctly and it is independent on diffusion within the energy conservation equation. Nonetheless, in the heat transfer context, "convection" means convective heat transfer between a solid and a moving fluid; consequently, it is defined in a boundary layer and it is supposed that energy transfer in the form of heat is done from the boundary layer towards the solid ((von Bockh, 2012), (Sidebotham, 2015)).

Anyway, the main idea which is considered in the present study is that heat transfer may be modelled as a linear process, at least for small variations of temperatures. Heat conduction in solids considers a linear law, the Fourier law, which is introduced into the partial differential heat equation to describe heat transfer by conduction, while heat transfer due to convective heat transfer and radiation is added as boundary conditions. The key point will be to add convective heat transfer and radiation by using locally linearized laws as it is done for heat transfer by conduction. Newton cooling law may be taken into account for the convective heat transfer and, by defining a mean radiant temperature, a linear law may be obtained for radiation. Advection (or convection as defined in fluid mechanics context) will not be considered.

Therefore, the local linearization of the heat transfer processes allows the use of one single expression, the heat equation, which will include all the heat transfer modes (at macroscopic scale) by using thermal conductivities and capacities. This methodology is a simplification of the problem of heat transfer for facilitating its resolution; each heat transfer process may be studied in a detailed manner. In this work, this option is chosen since heat transfer software used for analysing heat transfer in buildings usually assumes these hypothesis (EnergyPlus (Seem, 1987), TRNSYS (Stephenson & Mitalas, 1971), ESP-r (Clarke, 2001)). The heat equation, including all locally linearized heat transfer

modes, is an energy conservation equation which needs to be understood as a unified expression within the range of validity of the hypothesis, that is to say, it does not pretend to be a universal expression since it is clearly limited to macroscopic classical assumptions and the validity of all the linear approaches. The goal is to facilitate the study of heat transfer in buildings.

In this way, heat transfer problem is reduced to the existence of a gradient of temperatures between nodes. This work develops a methodology to be applied to characterizing heat transfer in buildings to be used in daily applications. Hence, a detailed description at laboratory scale is not considered. The main goal is to give an estimation of the parameters of the model and of their uncertainties, based on in-situ measurements. A relatively high uncertainty is not a problem by itself; the accuracy is not only a question of theoretical methods but also a question of experimental capabilities. It is important to make a compromise between theory, experimental cost and uncertainty. Modelling all heat transfer processes as a linear process, which considers temperature as a conservative potential field, is an approach and even with this approach the problem of studying heat transfer becomes complicated for complex systems, such as buildings. Different approaches may be considered and there is a wide literature about this issue ((Carslaw & Jaeger, 1986), (Duffie & Beckman, 2006), (Incropera, 2006), (von Bockh, 2012), (Sidebotham, 2015)). This dissertation deals with a particular problem: building parameters identification, although simulation and control applications will be closely related.

1.4. Modelling heat transfer in buildings

The idea is to build a physical model that appropriately relates the energy balance in buildings. The problem is essentially to approximate the thermal behavior of a building using a mathematical model. This dissertation will obtain mathematical models from physical first principles trying to keep as much physical information as possible. Steady-state assumptions may be considered (Naveros, et al., 2012), (Castillo, et al., 2014), but dynamic models will be stated for a better understanding of the problem since buildings will be considered as dynamic systems (Ogata, 2010), (Crassidis & Junkins, 2011).

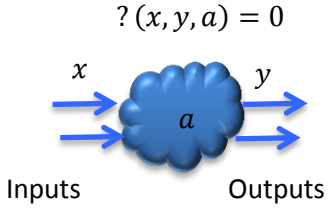
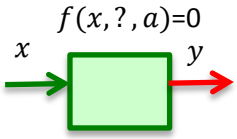
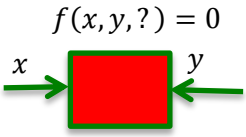
1.4.1. Direct and inverse problem

Traditionally, previous studies have proposed different dynamic model representations for the analysis of energy consumption in buildings; a distinction is done between solving the direct problem (simulation) or solving the inverse problems of model identification (Rabl, 1988). Every model respects the principle of energy conservation, which expresses the energy balance between buildings and their surroundings. Every model representation may be used in both the direct and the inverse problem. On one hand, the direct problem, where properties of building materials are supposed known, is mostly solved from principles of heat transfer theory using the thermal-electrical analogy as a start point and looking for criteria which allow determining the best simplification within this theoretical framework (Fraisse, et al., 2002), (Ramallo-González, et al., 2013). On the other hand, the inverse problem usually has been solved in a similar way finding difficulties to identify results with physical meaning (Jiménez & Heras, 2005), (Bacher & Madsen, 2011), (Mejri, et al., 2011). The aim will be to connect the different model representations using as starting point the heat equation, which is a partial differential equation (PDE), and considering a linear law for describing all the heat transfer modes.

From another point of view, mathematical models used for solving the direct problem are considered white box models and those used for solving the inverse problem are considered black box or grey box models, Table 1.1 (Ghiaus, 2014). The white-box models, which are based on theoretical

considerations for a physical process, are used for system performance simulation. The black-box models, which represent input–output relations that fit the measurements without any knowledge of its internal workings, are used in practical control applications. They are obtained by experimental identification techniques. In grey-box modelling, the structure of the model and the parameters of the model are obtained by a combination of first principles and measurements. This approach is especially useful for control applications when the model is expressed in a suitable form such as transfer function or state space (Ghiaus, et al., 2007).

Table 1.1. Classification of direct and inverse modelling problems in the present dissertation

Physical System		Type of modelling problem		
		Direct	**Inverse**	
				
		$\{x,a\} \rightarrow y$	$\{x,y\} \rightarrow a$	
		Simulation	**Identification**	
		White Box	*Grey Box*	*Black Box*
$f(\cdot)$	Model structure built from	First principles	First principles/ Heuristic	Heuristic
a	Model parameters	Known	**Estimated**	**Estimated**
x	Inputs of the physical system	Measured	Measured	Measured
y	Outputs of the physical system	**Estimated**	Measured	Measured

When black box models are proposed for solving the inverse problem in system identification (Ljung, 1999), the identified parameters cannot be connected with physical parameters (Ogata, 2010). Nonetheless, if the problem is formulated starting from first principles to obtain a grey-box model, the physical meaning of the parameters may be kept. In this way, this work proposes that models for system identification should be built from first principles keeping a physical meaning, i.e., white-box or grey-box models should be used for solving direct or inverse problems, respectively. The assumptions of a linear law for describing heat transfer and the principle of energy conservation may be understood as first principles since they are established on physical assumptions that are supposed "truth" regardless of whether they may be verified by experiments.

1.4.2. Thermal networks from graph theory by using linear algebra

Linear algebra studies vector spaces and linear transformations (Strang, 2009). A vector space over a field F is a set **V** together with two binary operations. The elements of **V** are vectors formed by elements of F which has scalar elements. Vector addition is one of the operations and the other is scalar multiplication. In linear algebra, matrices appear to simplify the expression of linear combination, using addition and scalar multiplication of vectors. The operations of addition and multiplication in a vector space must satisfy the following axioms:

- Associativity of addition, $(\mathbf{u}+\mathbf{v})+\mathbf{w}=\mathbf{u}+(\mathbf{v}+\mathbf{w})$;

- Commutativity of addition, $\mathbf{u} + \mathbf{v} = \mathbf{v} + \mathbf{u}$;
- Identity element of addition: there exists an element $\mathbf{0} \in \mathbf{V}$, called zero vector, such that $\mathbf{u} + \mathbf{0} = \mathbf{u}$ for all $\mathbf{v} \in \mathbf{V}$;
- Inverse elements of addition: for every $\mathbf{v} \in \mathbf{V}$, there exists an element $-\mathbf{v} \in \mathbf{V}$, called the additive inverse of $\mathbf{v}$, such that $\mathbf{v} + (-\mathbf{v}) = \mathbf{0}$;
- Distributivity of scalar multiplication with respect to vector addition, $a(\mathbf{u} + \mathbf{v}) = a\mathbf{u} + a\mathbf{v}$;
- Distributivity of scalar multiplication with respect to field addition, $(a + b)\mathbf{v} = a\mathbf{v} + b\mathbf{v}$;
- Compatibility of scalar multiplication with field multiplication, $a(b\mathbf{v}) = (ab)\mathbf{v}$;
- Identity element of scalar multiplication, $1\mathbf{v} = \mathbf{v}$, where 1 denotes the multiplicative identity in F;

where $\mathbf{u}, \mathbf{v}$ and $\mathbf{w}$ are arbitrary vectors in $\mathbf{V}$ and a, b are scalars in F.

On another hand, graphs are represented by a set of nodes and a set of edges which connects two nodes or one node to itself. Graphs are highly versatile models for analysing problems in which nodes and edges may have a physical interpretation (Gross & Yellen, 2005). Thermal networks are graphs where nodes represent temperatures which are connected by edges which are weighted with thermal resistances. In particular, thermal networks will be directed graphs (digraphs) since the edges will connect the nodes in the sense of decreasing temperatures. In particular, the study of thermal networks may be placed within algebraic graph theory. A thermal network may be represented by an incident matrix with dimensions m edges by n nodes (Ghiaus, 2013); therefore, a set of thermal networks, which may be represented by a set of matrices $n \times m$, is a vector space which can be studied using linear algebra (Strang, 1987).

The present work proposes the use of linear algebra as basis for the study of thermal networks instead of the thermal-electrical analogy, which is usually used. Electrical networks are also studied by using linear algebra since they are basically graphs which can be represented by matrices, i.e. they are also a vector space (Strang, 2007). This work considers important to leave back the thermal-electrical analogy since, by using the graph theory, the theoretical framework for thermal networks appears independently as it also appears for electrical networks (Gross & Yellen, 2005).

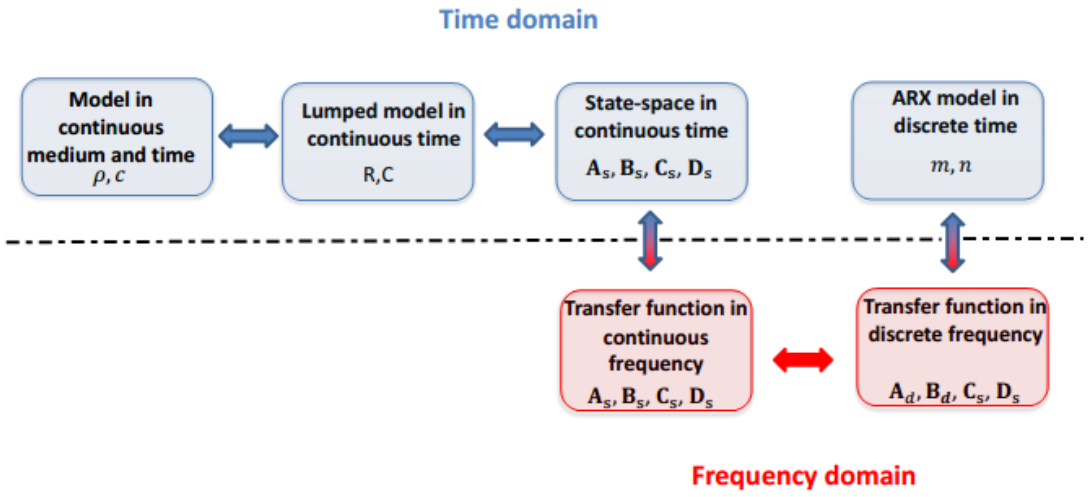


Figure 1.3. Methodology scheme connecting the heat equation with an autoregressive model with exogenous

The methodology seeks to estimate energy performance of buildings by solving parameters identification problem using in-situ measurements and considers a separation of the intrinsic thermal characteristics of buildings, their use and the user behaviour. This is an important question to simplify the solution of the building energy efficiency estimation problem. This work is focused on the identification of the intrinsic thermal characteristics of buildings in order to obtain a baseline to be used as reference for energy efficiency comparisons between different buildings or for the same building before and after refurbishment. In the proposed methodology, the classic problem of heat transfer in buildings is solved considering firstly the space integration of the heat equation and secondly its time integration, as opposed to what is usually done in this field ((Fraisse, et al, 2002), (Carslaw & Jaeger, 1986)). Therefore, the model is obtained considering the energy conservation principle and supposing continuous space and time. The model is obtained from the weak form of the heat equation by space discretization and linearization as a thermal network, i.e. as a set of linear time invariant in parameters differential and algebraic equations (DAE), using Galerkin finite element method (Strang, 2007), (Naveros & Ghiaus, 2015), and it is obtained in state space representation. Laplace transform is used to obtain the transfer matrix giving a relation between output and input variables in continuous frequency domain and a discretization is done to obtain the discrete transfer function in discrete frequency domain (Z transform). The model in discrete frequency is transformed into an autoregressive model with exogenous (ARX) in discrete time. The ARX model is an integrated expression in discrete time of the state space system, Figure 1.3. Finally, the parameters may be estimated by fitting any of the deduced model structures, which have physical meaning, and they may be employed to obtain a measure of the experimental intrinsic energy performance of a building.

1.4.3. Intrinsic physical characteristics

The intrinsic physical characteristics of a building are considered time invariant. In general, building thermal properties may be time invariant but this approach may be used being aware of its limitations since it simplifies the solution of the partial differential equation (PDE), which states the energy balance, in space and time. The double integration of the PDE in space and time is mandatory to solve both the direct and the inverse problem. Firstly, from space integration it is retrieved the discrete space system, which paradoxically is the origin of the PDE before applying the limit $\Delta x \to 0$. This discrete-space system is the lumped parameter model, i.e. the finite thermal network which would describe the thermal behaviour of the building supposing time invariant parameters, resistances and capacitances.

In this work, resistances and capacitances, which characterize heat transfer of buildings, are considered the resistances and capacitances of building materials once they are built taking into account the workmanship and the external resistances due to convection and radiation. The use of lumped time invariant values does not mean that they are really constant and time invariant, but that their variation is negligible for the time scale considered for identification. This hypothesis allows us to obtain the values of the resistances and the capacitances and check if these values are statistically significant, i.e. if it is possible to state the probability of obtaining such values by following the proposed methodology. In this way, it is obtained a description which is independent on energy consumption of buildings and could facilitate a standard estimation of building energy performance. Furthermore, it could be used for analyzing energy consumption in addition to other factors, as weather conditions, energy and ventilation systems or user behavior, by giving a whole description of the thermal behavior once the building is in operation. Taking into account all these factors, the options for considering a building as energy efficient grow and turn more complex. In every virtual application, these factors may be coupled, but this study is focused on parameters identification, not on energy consumption, and the intrinsic thermal behaviour is considered alone.

1.4.4. Boundary conditions: weather data and internal energy sources

Weather data are the forces which excite the buildings, that is to say, they are the inputs exciting the dynamic system besides the internal energy sources; ground temperature may be seen as a function of weather data (geothermal sources are not considered). By this reason, the study of heat transfer in buildings is, firstly, a problem of boundary conditions since internal energy sources may be turned-off, but weather conditions always remain.

The response of buildings as dynamic systems will have two components: 1) the natural response and 2) the forced response. After a time interval, the response which remains is only the forced response. Since the variation of weather conditions never stop, the study of the intrinsic characteristics of buildings may be done considering the response of buildings to these boundary conditions. The forced response of buildings to these conditions may be used as a first consideration for giving a standard value of their energy efficiency. Despite of other factors as user behavior or HVAC system, a better response for same weather conditions would imply a better energy performance considering constant all the others factors. Mathematically, this means that energy efficiency will be only seen as a function of the forced response of a building to weather conditions supposing all the other factors as constants. Obtaining a value of the energy efficiency in this way is only a first step; once it is obtained, the addition of other factors should be studied.

This work does not deal with natural response, which is also important, but with forced response only. This means that this work solves a forced dynamic system, which will be described by differential and algebraic equations (DAE), considering boundary conditions, not initial value conditions. The initial value conditions problem may be important for describing the behavior of the building as a function of internal energy sources, while the boundary conditions are important for the intrinsic characterization of buildings. Studies about initial value problem are numerous in the literature (Incropera, 2006). On one hand, the initial value problem addresses the change in indoor conditions to reach comfort conditions; this change cannot be instantaneous since the system (the building) has inertia due to the capacity of storing energy. On the other hand, the boundary value problem addresses the indoor conditions which will have a particular building due to weather conditions as a function of building characteristics. These conditions will be determined by the invariant characteristics of the building envelope.

This study will not consider the natural response and it uses the fact that a forced response and a steady-state response do not mean always the same thing, since a forced response may be time invariant. Such statement is only accomplished when the forces are constant, but weather conditions are not steady-state forces since they depend on time. Anyway, this work will consider that weather conditions may be considered as inputs which are at “periodic” steady-state and this will produce a “periodic” steady-state forced response of the building (indoor conditions) (Kumarm 2013), (Thirumaleshwar, 2009). The next three requirements define a “periodic” steady-state:

1) The natural response (outputs) to inputs is damped and approaches to zero when time increases without limit.
2) Every input (weather conditions or internal energy sources), is at periodic steady-state, then, the inputs give the meaning of ‘periodic steady-state’ of the output response (indoor conditions).
3) The inputs remain indefinitely forcing the system.

The first statement is accomplished since the damping constant of a building is not infinite. The second statement is accomplished by weather conditions since they follow the daily cycle and internal

energy sources can be controlled. It is interesting to note that outdoor conditions are more than weather; in frequency domain, weather corresponds to the higher frequency (period of hours or days) while climate corresponds to the lower frequencies (period of months or years). The third statement is clearly accomplished by weather conditions since they never stop and it could be also accomplished by internal energy sources.

Once the use of boundary conditions is presented, the problem to solve may be considered as: 1) a fix-fix problem (Dirichlet boundary conditions); 2) a fix-free problem (mixed boundary conditions). In the first, the weather data and internal heat sources are the boundary conditions which force the dynamic system; they do not need to be constants to reach a steady-state but they need to be periodic to reach a periodic steady-state. In the second, only the weather conditions force the system; they are not constants but they may be considered at periodic steady-state.

1.4.5. User behaviour

In addition to the intrinsic thermal characteristics of buildings, user behaviour is a key factor for determining the overall energy performance of a building (Fabi, et al., 2013). Occupants are the cause of internal energy sources; they have a very important influence on energy use and they need to be considered for a holistic study. Nonetheless, the idea of the present study is to analyse the problem by separating the different variables which affect energy consumption. In this way, the model obtained may add user behaviour to physical thermal properties. Although user behaviour may be a factor more determinant than others, the idea, which should remain, is that a more efficient building from an intrinsic point of view will be more efficient considering standard user behaviour. This is the advantage of considering energy efficiency of buildings independently of the real energy consumption of a building. The gap between predicted and measured energy consumption is not only related to the intrinsic thermal characteristics of a building since the effect of users may be predominant, but the complexity of user behaviour makes necessary the variable separation to simplify the problem. Thus, user behaviour is a key social point which remains out of the scope of the present dissertation; readers may have an insight of the problem by consulting other sources which should be useful to develop independent studies which take into account also social aspects for analysing building energy consumption.

1.5. Outline of the thesis

The linearization of the heat transfer, the principle of energy conservation and the assumption of time invariant parameters are hypotheses which are usually utilized and accepted to study heat transfer in buildings ((Seem, 1987), (Stephenson & Mitalas, 1971), (Clarke, 2001)). This work proposes the modelling of heat transfer in buildings using thermal networks from a new point of view based on linear algebra instead of presenting them as networks analogous to electrical networks. Anyway, the starting point does not change the meaning of thermal networks, which may be interpreted as the graphical representation of a system of differential algebraic equations (DAE) or vice-verse. This is important since thermal networks have been proved useful for heat transfer modelling for solving the direct problem and the inverse problem. Usually, the direct problem uses thermal networks, i.e. a system of DAE, for obtaining the variation of temperature in time of a particular node of the thermal network. Theoretically, thermal networks can give a stable and unique solution of the direct problem, which is considered well posed, using initial and/or boundary conditions and supposing known the parameters of the thermal network. On the other hand, the inverse (parameter identification) problem, tries to obtain the parameters of the thermal network using the inputs and the outputs of the system.

This works will show that this problem should not be seen as an ill-posed problem theoretically[3], i.e. it has a stable and unique solution from a deterministic point of view under the stated hypotheses. In practice, difficulties arise which affect both the direct and the inverse problem and they may become ill-posed. These difficulties are due to: 1) modelling errors, i.e. thermal networks are a theoretical description of physical observations; and 2) measurement errors, i.e. the heat transfer problem is not solved using continuous functions but discrete measured data with errors. Model and measurement errors are always mixed and they cannot be separated in practice. Moreover, it is important to notice that the existence of a perfect model that characterizes a phenomenon becomes from assumptions supposed "truth"; heat transfer is not an exception. Such assumptions, although must not be proved by observations, need to be verified from observations in order to be accepted and to state their limits of validity.

Chapter 2 constructs thermal networks from graph theory and physical first principles related to heat and thermodynamics laws. That is the basis of this work, i.e. thermal networks are graphs, and graphs may be described by linear algebra. It is also shown their connection with the partial differential equation, known as the heat equation. This step from continuous to discrete space, using a linearization, may be seen as a loss of information from a classical point of view. Nonetheless, it should not be considered a loss of information from a pragmatic point of view. From a pragmatic point of view, the classic theory of heat conduction, which supposes continuous space and time, could be also considered as based on imperfect "truth" assumptions which are only approximations to the observations. It may be considered that reality cannot be modelled by mathematics perfectly since the starting point is always discrete in space and time, and the partial differential equations are obtained considering the differences in space and time going to zero. This classical vision gives more importance to calculus than to algebra, but there are no practical reasons for accepting such statement beyond the belief that continuous space and time describe perfectly the reality and analytical solutions are perfect solutions, and physical observations are considered to be only imperfect discrete measurements of the reality. Nowadays, we are in the computer and data era when discrete mathematics and linear algebra may be considered more useful than continuous mathematics and calculus; or, in Gilbert Strang's words: "many applications of mathematics are discrete rather than continuous" and "vector and matrices have become the language to know" (Strang, 2009).

Chapter 3 introduces the transformations from heat equation to autoregressive models with exogenous (ARX). These transformations are well-known separately although they are not presented in the literature as the whole chain of transformations proposed in this dissertation. The heat equation is expressed as a system of differential algebraic equations (DAE) using the finite element method, and it is shown how a similar result may be obtained using the finite volume method and the finite differences method. It is also shown how the system of DAE can be put in state-space representation (Ghiaus, 2013). Next, the state-space representation is transformed into transfer function representation, which is connected to an ARX model. The key point is to obtain the ARX model by deduction not ad hoc.

Chapter 4 shows how the parameter transformations, which may be understood as change of variables, can be done back algebraically from ARX model parameters to thermal network parameters. This will constitute a proof for showing how parameter transformations, once ARX parameters are obtained, is a well posed problem under the stated hypotheses and from a deterministic point of view. In this chapter, the structure of the models is not recovered; it is considered known and obtained as

[3] Ill-posed problems. *Encyclopedia of Mathematics.* URL: http://www.encyclopediaofmath.org/index.php?title=Ill-posed_problems&oldid=25322

shown in Chapter 3. The knowledge of model structure is used for undoing the change of variables between the parameters of the different model structures.

Chapter 5 explains how the order of the model may be reduced by considering the measurement errors. This chapter introduces the limits of measurability as a criterion for model order reduction which is a complex issue usually treated from a mathematical point of view only. The idea is to choose the model order which allows the lowest loss of information using the transfer function representation. Theoretically, a high order model would be more accurate than a lower order model since it would keep more information and it would describe better the observation. In practice, if the difference between the answer of a low order model and a higher order model is not measurable, the use of the higher order model would imply an increment of complexity without guaranteeing more accurate results. In short, the accuracy of the model should not be higher than the accuracy of the observations when solving a heat transfer problem and the model of lower order would be a better choice by simplicity.

Chapter 6 shows the application of the methodology using measured data obtained from a wall which is tested under real weather conditions. Therefore, this chapter uses measured data and an ARX model, obtained by deduction from a thermal network using the methodology presented in the previous chapters. The physical parameters are the *U*-value, the dynamic solar energy transmittance and the effective heat capacity. The estimations of the physical parameters are presented together with their uncertainties considering Monte Carlo techniques for propagation of errors.

Finally, the most relevant conclusions from the point of view of the author are briefly exposed.

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CHAPTER 2

THERMAL NETWORKS CONSIDERING GRAPH THEORY AND THERMODYNAMICS

2.1. Introduction

Heat transfer in solids is usually dealt by using calculus. The work by Carslaw is an early textbook reference which shows analytical solutions for the study of heat transfer throughout solids and at their surfaces (Carslaw & Jaeger, 1986)[4]. Carslaw and Jaeger present the heat equation as a partial differential equation for solids and they notice the thermal-electrical analogy. This analogy is mainly based on the similitude between Fourier's and Ohm's laws and it implies that the different variables used in thermal and electrical networks are usually presented as analogues (Weedy, 1988); in fact, Ohm's law was inspired in part by Fourier's law (Narasimhan, 1999). At solid surfaces in contact with a fluid (e. g. air) the analogy with Ohm's law can be extended by considering Newton's cooling law which represents all heat losses between the solid's surface and its surroundings. Heat losses at a solid's surface may be due to convection (fluid-solid) and radiation (surface-surface) phenomena although at Newton's age radiative losses were not known. Nowadays, despite the fact that convective and radiative losses are usually separated, the analogy with Ohm's law can be done by linearizing the radiative losses given by Stefan-Boltzmann's law. The assumption of linear energy flow in form of heat (heat flow) within solids and at their surfaces, independently of the physical linear law considered, enables the use of the heat equation beyond heat conduction within solids.

In chapter 3, this dissertation will state thermal networks independently on thermal-electrical analogy due to the limitations of such analogy (Narasimhan, 1999). Thermal networks should be seen as a particular case within graph theory, which is a wide field of mathematics ((Gross &Yellen, 2005), (Gross, et al., 2013), (Estrada, 2013)). The interpretation of a graph as a thermal network needs to utilize physical principles of heat, i.e. thermodynamics laws. This allows us to propose an alternative to the use of the electrical analogy, since electrical networks are other particular application of graph theory consistent with the electromagnetic laws, which are not analogues to thermodynamics laws. Moreover, thermal networks will have physical meaning and may be studied by using linear algebra.

It will be firstly given an insight about the classical foundations of the heat conduction theory in solids before the information necessary for building a thermal network to be employed in heat transfer modelling of a dynamic system. In addition, it is shown that thermal networks may be seen as the graphical representation of a linear time invariant system of differential algebraic equations (DAE). Thermal networks are formed by nodes (vertices in graph theory) and branches (edges in graph theory) weighted by a thermal resistance. It is important to highlight that thermal networks or the system of DAE will be linear in parameters not in temperature or heat sources, which are the physical variables to be considered. Moreover, linear in parameters does not mean that the thermal network represents a homogeneous system in space (Naveros & Ghiaus, 2015). The density, specific heat capacity or

[4] The first edition was published in 1946 and it was in part the development of a work by Carslaw published in 1906 as it can be read explicitly in the preface of the reference cited.

conductivity may be function of space as well as time in general. An inhomogeneous system would simply require a thermal network of higher order, i.e. a greater number of nodes and branches. Each node of the thermal network has a unique value for each variable at a given time. The parameters of the thermal networks will be supposed time invariant; however, the supplied temperature and heat sources may be time variant. In short, linearity will be related to the parameters of the thermal network or the system of DAE not to the physical variables.

Finally, thermal networks are usually associated to the use of lumped parameter models. As lumped parameter models are historically presented as opposite to distributed parameter systems (Wholers, et al., 1969), it is included a brief discussion about the concepts of lumped and distributed parameter models since it is considered important to notice the subtle difference between them.

2.2. Heat flux (heat flow rate)

Heat flux or heat flow rate is the amount of energy per time unit which goes in form of heat from a system at a higher temperature to another system at a lower temperature.

2.2.1. Linear heat flux

It may be said that the mathematical theory of heat conduction is founded upon a hypothesis suggested by experimental evidences (Carslaw & Jaeger, 1986) which states that the energy which flows, Q, per time and surface unit in one direction across a solid is given by:

$$\frac{Q}{S\Delta t} = \frac{\kappa(\theta_{s1} - \theta_{s2})}{\Delta d} \tag{2.1}$$

where κ is the thermal conductivity which may be considered a constant property dependent on the solid; $(\theta_{s1} - \theta_{s2})$ is the difference of temperatures between both surfaces of the solid; S and Δd are the surface area and the thickness of the solid respectively.

Eq. (2.1) reaches the range of physical law when is carried out to the limits of zero surface, thickness and time. Supposing heat flux density in three dimensions, we obtain:

$$q_S = -\kappa \boldsymbol{\nabla} \theta \tag{2.2}$$

where $\boldsymbol{\nabla}$ is the gradient operator and the density heat flux is noted as q_S. The surface, S, is usually considered constant and it can be put in the right-hand side of Eq. (2.2). Then, it will be obtained the heat flux, q, instead of the density heat flux,

$$q = q_S S \tag{2.3}$$

The importance of Eq. (2.2) resides in its linearity and it is usually known as Fourier's law (Incropera, 2006).

An important detail needs to be highlighted. In thermodynamics, time is not a variable of interest since the changes are considered from an equilibrium state to another equilibrium state without taking care about time. For this reason, heat or heat transfer are considered energy in transit with units of energy. In heat transfer, since time is considered, we will talk about heat "rate" transfer or heat flux with units of energy by time and density heat flux with units of energy by time and by space. The differences between heat transfer and heat flux need to be remembered to avoid confusion.

In dynamic conditions, time is relevant and we consider heat flux density as the energy transferred across a surface per unit area and per unit time as it is expressed by Fourier's law. Heat flux occurs only as a consequence of a gradient of temperatures between systems at different temperatures which are connected thermally. Classically, this is interpreted supposing the existence of isothermal surfaces and supposing that temperature is a potential function. In this context, the exchange of energy will be only a function of temperature differences between systems. Hereafter, thermal networks will use nodes (vertices in graph theory) instead of isothermal surfaces. In fact, in the case of isothermal surfaces the energy transfer is supposed to be done between points of different isothermal surfaces. An isothermal surface has a unique temperature by definition; and a node of a thermal network will have a unique temperature too. Nodes may be seen as a generalization of isothermal surfaces since they may be seen as isothermal surfaces or volumes, or even they can be considered points. A node may also represent the mean temperature of a surface of a volume. A volume may be referred to absolute or relative space; a node may represent a fix volume, surface or point, or they may be mobile. In this work, nodes (points, surfaces, volumes) will be fixed but the concept of a node moving through an absolute space could be considered in future studies.

It should be noticed that the origin of the heat conduction theory is not the differential expression, Eq. (2.2), but the algebraic expression, Eq. (2.1), which is verified by the experience (Carslaw & Jaeger, 1986). The origin is very important since the definition of thermal conductivity arises from Eq. (2.1):

$$\kappa = \frac{Qd}{(\theta_{s1} - \theta_{s2})St} \tag{2.4}$$

Therefore, thermal conductivity is not a direct measurable quantity but it needs to be estimated from measurements of other physical variables. From Eq. (2.4), it is usually made the theoretical assumption of a time invariant and constant thermal conductivity between two surfaces, which is referred, upon a classical interpretation, to the resistance of heat for going throughout the volume between the isothermal surfaces (Carslaw & Jaeger, 1986).

In this dissertation, the hypothesis of a constant thermal conductivity will be interpreted in an equivalent way which allows us to use it in a wider context. We will say that the resistance occurs between two nodes instead of between two surfaces. In this way, as it has been explained above, the nodes may be surfaces but they can be also volumes or points and there is a resistance for exchanging energy in form of heat due to their different temperatures between two nodes. The use of nodes, instead of isothermal surfaces allows us to consider the algebraic linear law, Eq. (2.1), in general not only limited to heat conduction within solids.

2.2.2. Energy conservation considering thermodynamics

The next important point is the principle of energy conservation, which is a first principle of physics. From a classical point of view, the energy, E, going out/in of a system is equal to the change of energy within the system. The energy balance equation of a system, as previously presented in Chapter 1, is:

$$\Delta E = E_{out} - E_{in} \tag{2.5}$$

In thermodynamics, it is supposed that energy may be transferred in form of heat from a system in equilibrium to other/s system/s and vice-verse until the equilibrium is reached. The energy balance is done between a system and its surroundings (other/s system/s). In this context, a macroscopic system

in equilibrium, which implies homogeneous temperature, may be a node. It must be noticed that a node may be also a point which represents the mean temperature of a system, or an isothermal surface. The concept of node will be used for building thermal networks based on graph theory, where a node is called a vertex. The aim is to enable the study of the dynamic systems for solving heat transfer problems using linear algebra, since thermodynamics, despite of its name, does not consider the study of dynamic systems.

Physics is a science of measures and measures are comparisons. Therefore, it is always needed to state a baseline for energy measurements, i.e. in the case of energy measurements a zero energy value. In the case of energy transferred in form of heat, the temperature will be the measurement for the quantification of the energy of a system (a node). For this purpose, the increment in enthalpy of a system, ΔH, is defined as (Incropera, 2006):

$$\Delta H = mc\Delta\theta \tag{2.6}$$

where m(kg) is the mass of the system given by its density times its volume, c(J/kgK) is the specific heat capacity of the system and θ(K) is the temperature of the system in degrees Celsius. By convention, zero enthalpy corresponds to zero degrees Celsius. Therefore, the enthalpy, *H*, can be written as:

$$H = mc\theta \tag{2.7}$$

Eq. (2.6) and (2.7) suppose that the change of energy in the system (node) is only due to the change of temperature of the system (node). Such expressions are usually considered from a thermodynamic point of view. In a dynamic context, this expression of the energy of the system may be differentiated in time supposing that only the temperature is time variant:

$$\frac{dH}{dt} = mc\frac{d\theta}{dt} \tag{2.8}$$

This approach will be considered in this dissertation where neither mass transfers nor phase changes are considered. The use of thermal networks will not be limited by these assumptions, but such subjects would need further developments, which are out of the scope of this text.

By definition, the enthalpy, *H*, is the sum of the internal energy of the system, *U*, and the mechanical work done over/by the system (Callen, 1986):

$$\Delta H = \Delta U + \Delta PV + P\Delta V \tag{2.9}$$

One of the main hypotheses assumed in the present dissertation is that the processes to be treated will be at constant pressure and volume. Then, the changes in enthalpy and in internal energy will be equal. That is to say, the exchange of heat will be the only cause of all the energy change in the system and no work is done by/over the system (node). In this way, the energy transfer is due to heat flux between the system and its surroundings, and heat flux is due only to temperature differences between the system and its surroundings. This is the initial hypothesis of textbooks which deal with heat transfer in solids ((Carslaw & Jaeger, 1986), (Incropera, 2006)). In thermal networks, a node (system) may be or not the abstract representation of a solid.

An important point is that the mass of the system (node), given by its density times its volume, and the specific heat capacity will be supposed constant and time invariant. The product of the mass by the specific heat capacity is equal to the heat or thermal capacity. The heat capacity, *C*(J/K):

$$C = mc \tag{2.10}$$

is a measurable physical quantity given by definition as the amount of heat which produces a change of temperature in a system (node) of one degree Celsius:

$$C = \frac{Q}{\Delta\theta} \tag{2.11}$$

From Eq. (2.10) and (2.11) it is obtained:

$$Q = mc\Delta\theta \tag{2.12}$$

In the present study, the heat, Q(J), will be the only cause of the change of temperature of a system and it will correspond exactly with the change of the internal energy that will be given by the change of enthalpy,

$$Q = \Delta H \tag{2.13}$$

2.2.3. The heat equation

The heat equation can be obtained from Eq. (2.5). Since the change of energy of a system (node) is only supposed due to the change of temperature, i.e. heat is the only cause of the change of internal energy without mass transfer or phase change; it may be given by the time variation of the enthalpy, Eq. (2.8):

$$\Delta E = \frac{dH}{dt} \tag{2.14}$$

or:

$$\Delta E = \rho V c \frac{d\theta}{dt} \tag{2.15}$$

where the density, ρ(kg/m^3), the volume, V(m^3), and the specific heat capacity, c(J/kgK), are considered time invariant for the system (node).

Next, the heat flux going into/out of the system (node) is given by:

$$E_{out} - E_{in} = (q_{out} - q_{in})S \tag{2.16}$$

where S is the surface of the system (node).

By using Eq. (2.15) and Eq. (2.16), supposing a cubic volume and considering first heat flux in x-direction, Eq. (2.5) becomes:

$$\rho c S \delta x \frac{\partial\theta}{\partial t} = \delta q_x S \tag{2.17}$$

Canceling the surface and taking the limit of $\delta x \to 0$, it may be obtained:

$$\rho c \frac{\partial\theta}{\partial t} = \frac{\partial q_x}{\partial x} \tag{2.18}$$

It is possible to follow an analogous procedure in y and z directions considering heat flux as a vector, $\mathbf{q} = (q_x, q_y, q_z)$. Then, when Fourier's law is considered and heat sources, p, are added to the system, we obtain the well-known heat equation:

$$\rho c \frac{\partial \theta}{\partial t} = -\boldsymbol{\nabla} \cdot (-\kappa \boldsymbol{\nabla} \theta) + p$$

It is important to notice that we will work with linear algebra not with calculus. Anyway, this simplified way of obtaining the heat equation shows that calculus comes later than linear algebra, i.e. the reasoning begins with discrete space and time before obtaining the partial differential equation.

2.3. Thermal networks considering graph theory

The basis of networks is the graph theory which is considered to be started by Leonard Euler and the problem of the seven bridges of Konigsberg (Estrada, 2013). A formal definition given by Wallis (Wallis, 2013) states that a graph consists of a finite set of objects called vertices together with a set of unordered pairs of vertices called edges. Graphs are usually represented by diagrams in which the vertices are points and edges between two points (x, y) are shown as lines between (the point representing) x and (the point representing) y. Graphs have multiple applications and the general graph theory is wide. In particular, we will be interested in oriented or directed graphs (digraphs). A digraph is like a graph except that each edge is allocated a direction and one vertex is designated a start and the other is an end; a digraph can be restricted to not allow more than one edge to connect two points.

An important application of digraphs is electrical networks which consider algebraic graph theory based on Kirchhoff's voltage and current laws, and Ohm's law (Gross et al., 2013). The fast development of electrical networks explains the use of the thermal-electrical analogy even if Fourier's law precedes Ohm's law (Narasimhan, 1999). But Kirchhoff's voltage law does not apply for temperatures sources since temperatures sources cannot be added algebraically in series nor parallel, and then the sum of the temperatures drops in a close thermal network cannot be zero. This restriction is mandatory for avoiding the violation of thermodynamics laws in the use of thermal networks. That very important and crucial difference, among others, makes not really useful the electric analogy and makes recommendable the statement of thermal networks from graph theory directly despite of the results which could be inferred by the thermal-electrical analogy. Other option usually proposed is the use of thermodynamic bond graphs ((Breedveld, 1986), (Borutzky, 2010), (Borutzky, 2011)). Nonetheless, bond graphs also play with analogies at a level higher than graph theory; the idea is to extend the use of analogies to mechanical or hydraulic systems, further electrical or thermal systems. They propose the use of the same mathematical expressions but changing the meaning of vertices (potentials) and edges (forces) ((Borutzky, 2010), (Borutzky, 2011)).

Nonetheless, thermal networks may be seen as directed graphs and this will be the prime idea in this dissertation instead of the usual analogy with electrical networks ((Carslaw & Jaeger, 1986), (Incropera, 2006), (von Bockh, 2012)), which are directed graphs. Basically, the reasons for not using the electrical analogy reside in the differences between heat and electricity and in its limitation to steady-state problem, as for example, the different rules which apply to the addition of ideal temperature sources, which value will be independent on the system, and ideal electrical voltage sources. The addition of temperature sources must respect the thermodynamic laws; this means that two temperatures sources connected in series cannot be added to obtain an equivalent temperature source which is the algebraic sum of them, while it is well-known that electrical voltage sources in series can be added for giving an equivalent voltage electrical source which is the algebraic sum of

them; in short, Kirchhoff's voltage law has no analogy in thermal systems. Moreover, the difference between the elliptic partial differential equation representing electromagnetic fields and the parabolic partial differential equation representing temperature fields is well-known regarding to the order of the time derivative; as a consequence, there is no analogue term for the inductance, the existence of such term, called inertance, would go against the second principle of thermodynamics (Breedveld, 1982).

Gustav Kirchhoff used the incipient graph theory for his advances in the study of electrical circuits (Gross et al., 2013), as James J. Sylvester used graph theory in his paper Chemistry and Algebra (Estrada, 2013). Electrical and chemical circuits are only applications of the graph theory. Nowadays, the powerful graphs have extended their use over all disciplines as social or information sciences; they do not only remain in chemistry or physics. The advantage of graph theory is that it offers simplicity to the representation of a problem, that allows a new insight which facilitates the analysis of the problem itself. In the case of thermal networks, they will be directed graphs and they will be represented by matrices considering linear algebra in their study. From mathematics to physics, a directed graph will be a thermal network instead of adapting ad-hoc, by an incomplete analogy, an electrical network to be a thermal network.

Then, the problem becomes to put physics on the digraphs to obtain thermal networks. Firstly, the vertices will be the thermal nodes and the edges will be thermal branches. The nodes will be characterized by a unique temperature and will be connected in the sense given by Fourier's law. Thermal resistances will be placed at branches and thermal capacitances at nodes. The absence of mass transfer and phase changes make capacitances time invariant since it is related to the mass of the element and its specific heat. Instead of mass, the coefficient mass per volume is used, i.e. density. This is the approach used in this text. The consideration of mass transfer is an interesting issue and it should be coupled to the solution of the problem in futures works.

2.4. Thermal networks: Differential Algebraic Equations

Basically, a network is a graph where vertices and edges become entities with physical sense. Once a graph is applied to a particular problem, we obtain a network. This gives a non-abstract sense to vertices and edges. In the case of thermal networks, vertices and edges have physical sense, nodes represent temperatures and they are weighted by thermal capacities; the edges represent physical interactions between nodes and they are weighted by thermal resistances from which heat flows from node to node. Thermal networks will suppose by construction the linearity of heat transfer between nodes which is further verified by experience for enough small variations of temperature. This Section is inspired by a previous work by Christian Ghiaus (Ghiaus, 2013). It shows how the thermal network is built by using graphs and the rules given by energy conservation and linear heat transfer between two elements (nodes) at different temperatures.

In the same way in which a set of differential algebraic equations (DAE) can be represented graphically as a thermal network, from a thermal network it is possible to obtain a set of DAE. A thermal network may be arranged in:

- branches, containing thermal resistances, R_k, and temperatures sources, b_k, which are crossed by heat flow rates, q_k, and
- nodes to which thermal capacities and flow sources are connected.

Figure 2.1 shows the basic elements of a thermal network.

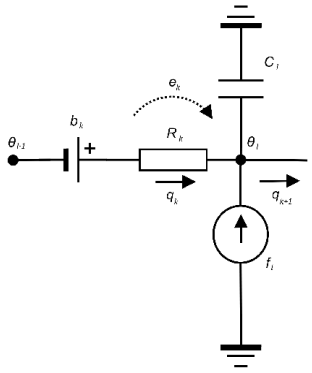

Figure 2.1. Typical branch and nodes in thermal networks (Ghiaus notation)

The temperature difference over a resistance R_k is:

$$e_k = -a_{kl-1}\theta_{l-1} - a_{kl}\theta_l + b_k \tag{2.19}$$

where θ_{l-1} and θ_l are the temperatures in the nodes $l-1$ and l, b_k is the temperature source on the branch k and a_{kl-1}, a_{kl} are elements of the incidence matrix of the thermal network, **A**, with dimension $m \times n$; it is a matrix operator which makes the differences of temperatures. The m rows of the incidence matrix **A** correspond to the branches containing heat rate flows, q_k, crossing the resistances, R_k, and the n columns correspond to the nodes representing the temperatures θ_l.

The elements of the incidence matrix **A** are given by:

$$a_{kl} = \begin{cases} -1 & \text{if flow } q_k \text{ exits from node } \theta_l \\ 0 & \text{if } q_k \text{ is not connected to node } \theta_l \\ +1 & \text{if flow } q_k \text{ enters in node } \theta_l \end{cases}$$

This formalism allows a unique representation for all branches of the model studied. Writing the set of Eq. (2.19) for all branches, we obtain:

$$\mathbf{e} = -\mathbf{A}\boldsymbol{\theta} + \mathbf{b} \tag{2.20}$$

where:

$\mathbf{e} = [e_1, \dots, e_k, \dots e_m]^T$ is the vector of temperature drops over thermal resistances, with m the number of branches in the model, which is equal to the number of resistances in the model;

$\boldsymbol{\theta} = [\theta_1, \dots, \theta_l, \dots, \theta_n]^T$ is the vector of temperature values in the nodes, with n the number of nodes;

$\mathbf{b} = [b_1, \dots, b_k, \dots, b_m]^T$ is the vector of temperature sources on the branches.

The heat flow rate in each branch is:

$$q_k = R_k^{-1} e_k \tag{2.21}$$

Expressed in matrix form for all heat rate flows, Eq. (2.21) gives:

$$\mathbf{q} = \mathbf{G}\mathbf{e} \tag{2.22}$$

where:

$\mathbf{q} = [q_1, \dots, q_k, \dots, q_m]^T$ is the vector of heat rates in the branches

$\mathbf{G} = \begin{bmatrix} R_1^{-1} & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & R_m^{-1} \end{bmatrix}$ is a diagonal matrix of thermal conductances.

The balance of heat rates in a node θ_l states that the variation in time of the energy accumulated in the thermal capacity, $C_l\dot{\theta}_l$, is equal to the algebraic sum of heat rates entering or exiting the node θ_l and the heat rate source f_l connected to the node:

$$C_l\dot{\theta}_l = \sum_k a_{lk}\, q_k + f_l$$

where a_{lk} are the elements of the transpose of the incidence matrix $\mathbf{A}$.

The balance equation for all nodes can be written in matrix form:

$$\mathbf{C}\dot{\boldsymbol{\theta}} = \mathbf{A}^T\mathbf{q} + \mathbf{f} \tag{2.23}$$

where

$\mathbf{C} = \begin{bmatrix} C_1 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & C_n \end{bmatrix}$ is a diagonal matrix of thermal capacities;

$\mathbf{A}^T$**,** is the transpose of the incidence matrix; it is a matrix operator which makes the algebraic sum of heat transfer rates $\mathbf{q}$ in nodes;

$\mathbf{f} = [f_1, \dots, f_l, \dots, f_n]^T$ is the vector of heat rate sources connected to the temperature nodes.

Substituting Eq. (2.20) and Eq. (2.22) in Eq. (2.23), the set of heat balance equations can be written as:

$$\mathbf{C}\dot{\boldsymbol{\theta}} = -\mathbf{A}^T\mathbf{G}\mathbf{A}\boldsymbol{\theta} + \mathbf{A}^T\mathbf{G}\mathbf{b} + \mathbf{f} \tag{2.24}$$

or, by noting $\mathbf{K} \equiv -\mathbf{A}^T\mathbf{G}\mathbf{A}$ and $\mathbf{K}_b \equiv \mathbf{A}^T\mathbf{G}$, as:

$$\mathbf{C}\dot{\boldsymbol{\theta}} = \mathbf{K}\boldsymbol{\theta} + \mathbf{K}_b\mathbf{b} + \mathbf{f} \tag{2.25}$$

Eq. (2.24) gives a set of differential algebraic equations, which are algebraic equations for the nodes with negligible capacity (the elements of matrix $\mathbf{C}$ corresponding to these capacities are supposed zero).

The set of differential algebraic equations, Eq. (2.24), presents in one expression the Eq. (2.20)-(2.23). An alternative way to arrange Eq. (2.20)-(2.23) consists in the use of a matrix of blocks:

$$\begin{bmatrix} \mathbf{G}^{-1} & \mathbf{A} \\ -\mathbf{A}^T & \mathbf{C}\dfrac{d}{dt} \end{bmatrix} \begin{bmatrix} \mathbf{q} \\ \boldsymbol{\theta} \end{bmatrix} = \begin{bmatrix} \mathbf{b} \\ \mathbf{f} \end{bmatrix} \tag{2.26}$$

Eq. (2.26) is an adaptation of the equilibrium equations to dynamic heat transfer problems where the term related to the dynamic of the system, $\mathbf{C}\frac{d}{dt}$, is placed separately of the conductance matrix, $\mathbf{G}$, instead of changing it by an impedance matrix, as it is done for electrical networks (Strang, 2007). In frequency domain, $\mathbf{C}\frac{d}{dt}$ becomes $\mathbf{C}s$**,** where $s = \sigma + j\omega$ is a complex variable. Eq. (2.26) is a generalization for a whole system of the two-port network (or quadripole) representation for an

element of the thermal network ((Athienitis, et al., 1985), (Fraisse et al., 2002), (Chen, 2013)). This is an example about a difference between electrical and thermal networks. The properties of heat transfer allow and need a different formalization of graphs to be implemented as thermal networks instead of as electrical networks.

2.5. Distributed and lumped parameter models

This chapter has shown two ways of dealing with the flow of energy in the form of heat (or heat flow) between systems at different temperatures. Thermal networks may be built without considering the heat equation, as it has been done in this chapter, or may be built from the heat equation as it will be demonstrated in Chapter 3. These two different ways of dealing with heat flow aims to emphasize the difference of considering finite networks or finite systems of differential and algebraic equations (DAE) instead of partial differential equations (PDE) for dealing with the space distribution. In this way, thermal networks are usually described by finite systems of DAE while the heat equation, which is a PDE, may be described by an infinite system of DAE.

It is usually considered that thermal networks are lumped parameter models whose variables are not functions of the space, while the heat equation is a distributed parameter model whose variables are dependent on space. As it has been asseverated previously, a lumped parameter model may be described by a finite system of differential algebraic equations (DAE) and distributed parameter models may be described by partial differential equations (PDE) and a PDE may become an infinite system of DAE (Naveros & Ghiaus, 2015). Therefore, we could consider that lumped and distributed parameter models may be described in general by a system of DAE allowing us to consider the same mathematical formalism for describing both kinds of models. In mathematics, operators, which may be differentials or not, are introduced in order to use the same formalism and defining in the same way both distributed and lumped parameter models (Banks & Kunisch, 1989). Once the same mathematical formalism is used for describing a system in the same formalism whether it is distributed or lumped, the difference remains in the assumption of considering that a lumped parameter model is an approximation of a distributed parameter model, which is supposed to be able of describing the physical system perfectly (Wohlers, et al., 1969).

Such difference should be regarded carefully since in practical applications it is quite subtle and there is no scientific evidence for accepting that a distributed parameter model describes the physical system perfectly. In fact, infinite systems of DAE are mathematical abstractions which are not tractable in practical applications for simulation or identification purposes; hence, finite systems need to be used. It is possible to argue that we do not need to use infinite systems of DAE since we may use PDE and solve them using calculus to find its space distribution of parameters and variables (Banks & Kunisch, 1989), but this is valid for a small number of theoretical solutions supposing known the continuous independent functions which describe the temperature and the heat flux, and supposing that the PDE can be integrated analytically in space and time. Nonetheless, in virtually any application the parameters and the variables of a physical system are only known (measured) at discrete points in space, hence the knowledge of a continuous space function is not possible physically.

The aim of noting the subtle difference between distributed and lumped parameters models is to highlight that for solving distributed parameter problems it is considered that they are usually reduced to lumped parameter problems by space integration. This may be done before or after solving the time integration of the system (van Berkel, et al., 2014). The reduction of the distributed system into a lumped system implies a connection between both representations. Such reduction is usually done by space integration of the distributed system. For this reason, it should be noted that a lumped system

obtained in such way could be considered a distributed system where the spatial dependence is present implicitly. In this way, a lumped model should not be considered the opposite of a distributed model since it may be used for obtaining the variables at every point in space. The question would be to estimate the order of the lumped model needed for describing accurately a thermal system distributed in space (Naveros & Ghiaus, 2015). Nonetheless, literature in different fields usually introduces lumped parameter models without noting the connection to distributed parameter models ((Wohlers, 1969), (Sidebotham, 2012), (Davies & Schmitz, 2015)).

The next chapter will discuss the use of Galerkin method for finite element ((Strang, 2007), (Banks & Kunisch, 1989)), for integrating in space a distributed system for obtaining a lumped system with knowledge of exact values of the variables and the parameters only at selected space locations (nodes), i.e. $\theta(x_1, t) = \theta_1(t) \equiv \theta_1$ and $\theta(x_2, t) = \theta_2(t) \equiv \theta_2$. The approximation is a linearization of the values of temperatures between two adjacent nodes θ_1 and θ_2 using for instance linear piecewise functions. Following, the question about how many lumped elements are needed will be discussed, i.e. what is the order of the system of DAE.

2.6. Conclusions

The heat equation is a partial differential equation (PDE) usually utilized for studying the energy flow in form of heat (or heat flow) within solids by conduction as well as convective and radiative heat flow at solid surfaces. The heat equation is a distributed in parameter model which is obtained using the principle of energy conservation and considering that heat flow, from a system to another system at different temperature, follows a linear (or step-wise linearizable) law. Moreover, the linear law for considering heat flow between systems may be used whatever is the mode of energy exchange: conduction, convection or radiation.

On another hand, the heat flow between systems at different temperatures may be also studied using thermal networks; in this case, systems will be represented by nodes. The most important principle in thermal networks is also the principle of energy conservation. Furthermore, the interactions between the nodes of thermal networks may be also considered governed by a linear law, as it is usually done by classical physics. The use of thermal networks will require working in the range of validity of such assumption since interactions between nodes could be described by a non-linear law. Thermal networks may be introduced from the analogy with electrical networks as usual, but this assumption has important limitations which should remain clear; the main problem observed is that Kirchhoff's voltage law does not apply for thermal networks since temperatures cannot be added algebraically as it is done with electrical voltage. Hence, this dissertation proposes to start with graph theory by using directed graph (digraphs) as basis for constructing and stating thermal networks without considering analogies. Of course, the analogy is useful and gives exact results of the presented theory when its limitations are considered ad-hoc since electrical networks may be also represented by digraphs. Readers may decide the path they prefer to follow for defining thermal networks, the electrical analogy and then, to consider the laws of thermodynamics for imposing restriction to the analogy; or using the graph theory and the laws of thermodynamics directly. This work considers that the construction of thermal networks is simplified by using graph theory directly instead of adapting electrical networks. The use of the analogy requires first a whole knowledge of electrical networks which is not necessary for studying thermal networks.

Finally, it has been explained that the notion of lumped parameter models may be fuzzy since it is usually associated to the use of non-distributed parameter models. Nonetheless, a lumped model considers also the distribution of the variables in space implicitly, the only theoretical difference

regarding to a distributed parameter model is based on the assumption that the distributed parameter model uses the exact value of the variables at every space point, while the lumped parameter model will use only the exact value of the variable at a finite number of space points while for the other values of the variable it will use an approximation, which may be obtained, for instance, by linear interpolation between exact values of the variables. In the case of a thermal network as defined in this text, the distribution of the variables in space is present in the number of nodes of the thermal network, i.e. in the number of differential and algebraic equations (DAE) which will be used for describing a particular thermal system.

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CHAPTER 3

DYNAMIC HEAT TRANSFER MODELLING: FROM HEAT EQUATION TO ARX MODELS

3.1. Introduction

The basis of the study of classical physical system dynamics is the deterministic mathematical representation of time-variation of some physical variables in a control volume. The physical variables of each volume are connected by using the first principles of conservation to form the system. This mathematical model, which constitutes the mathematical model of the system, must guarantee that the system output can be determined for any given set of inputs; this allows us the system analysis ((Ogata, 2010), (Tucsnak & Weiss, 2014)). Mathematical models may be used for optimal estimation of dynamical systems (Crassidis & Junkins, 2011) or control, which are dual problems (Lindquist, 1972).

This chapter presents an insight into the modelling of heat transfer problems, which will be particularly applied for thermal parameter identification of buildings in next chapters. Heat transfer is a physical transport process which is usually described by the classical heat equation, which is a deterministic parabolic partial differential equation (PDE), and boundary and/or initial conditions. Solving the heat equation implies space and time integration and it is possible to change the order in which these integrations are done (van Berkel, et al., 2014). Anyway, in virtually every application space and time discretization needs to be done before solving the integrations. In the study of heat transfer in buildings, the direct (simulation) and the inverse (system identification) problems use different model structures which are presented disconnected from each other ((Rabl, 1988), (Jiménez & Madsen, 2008)). The direct problem usually fixes a white-box model and solves first the time integration of the one-dimensional heat equation using the Laplace transform ((Fraisse, et al., 2002), (Athienitis, et al., 1985)). This is the basis of the conduction transfer function method implemented in several building performance simulation programs such TRANSYS or EnergyPlus ((Stephenson & Mitalas, 1971), (Seem, 1987)). The direct problem can be also treated by employing finite elements to solve first the space integration as it is implemented in ESP-r (Clarke, 2001). On the other hand, the inverse problem gives a black-box or a grey-box model structure and selects the model order and parameters that better represent the dynamical system considering a data-set (Ljung, 1999). For this purpose, different estimation methods may be used ((Verhaegen & Verdult, 2007), (Söderström, 2007), (Diversi, et al., 2007), (Ninness & Henriksen, 2010).

In this sense, it is important to show how it is possible to deduce from first principles and physical laws all the classes of models, or model structures, usually used for analyzing heat transfer processes. The heat equation, which can lead to a system of differential-algebraic equations (DAEs) (Brenan, et al., 1995), is the usual starting point for historical reasons. From this starting point, it will be shown how to deduce different heat transfer model structures which keep their physical meaning. First, the heat equation is expressed as a system of linear differential algebraic equations (DAE) by using the finite element method ((Strang, 1986), (Strang, 2007)), and it is also shown how the use of finite differences or finite volume methods (which may be considered a particular case of finite element) would give the same results mathematically ((Peiró & Sherwin, 2005), (Grossmann, et al., 2007)). This system of linear DAE may be represented graphically as a thermal network as it is defined in Chapter 2 ((Ghiaus, 2013), (Naveros & Ghiaus, 2015)). Next, it is shown how to put the system of

DAE into state-space representation and then transformed into transfer function representation (Ghiaus, 2013). The discrete form of the transfer function is transformed into an autoregressive model with exogenous (ARX), which is a model structure widely used in parameters identification ((Ljung, 1999), (Naveros, et al., 2015)). This methodology, which supposes to solve the heat equation by space and time integration, is schematized in Figure 3.1.

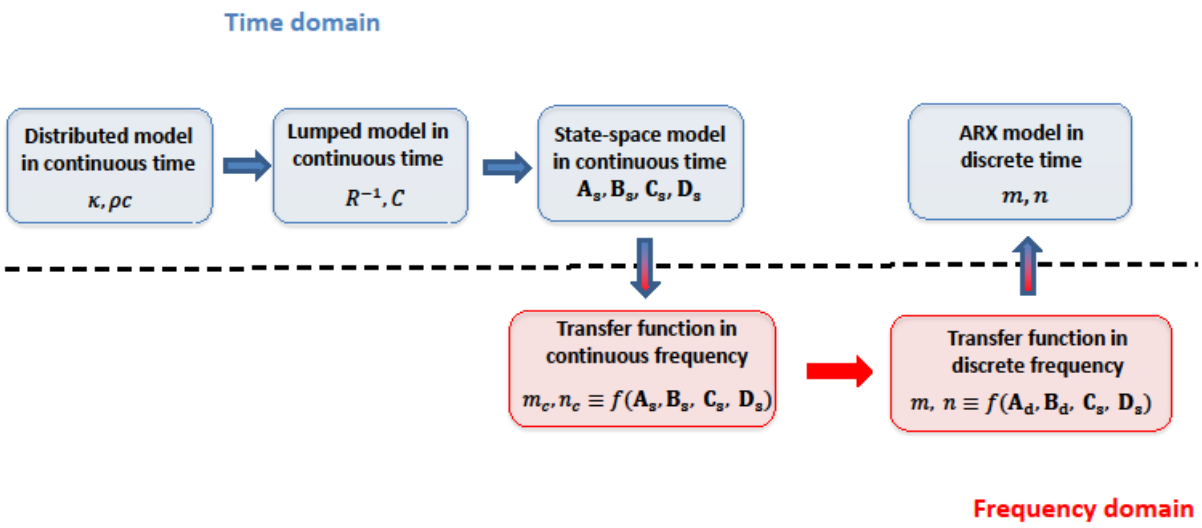

Figure 3.1. Path for deducing an ARX model from the heat equation passing through frequency domain

3.2. From heat equation to a system of differential algebraic equations (DAE)

The heat equation, which is the basis of heat transfer, is deduced using calculus principles (analytical derivatives) and next it is usually solved by using calculus (analytical integrations) (Carslaw & Jaeger, 1986). This manuscript will show how to express the heat equation as a system of differential and algebraic equations (DAE) by using linear algebra (Strang, 2003), in order to be used into the study of heat transfer problems. For this purpose, it is used the finite element method based on Gilbert Strang work ((Strang, 1986), (Strang, 2007)), which may be considered a generalization of finite differences or finite volume method as it can be seen for instance in a book by Grossmann where finite volume is shown as a particular case of finite element using piecewise linear test functions, and finite difference is shown equivalent to finite volume (Grossmann, et al., 2007). The similarities of finite difference, finite element method and finite volume method are also presented by Peiró (Peiró & Sherwin, 2005). Moreover, Gilbert Strang also shows how the use of finite element or finite differences gives similar results by using piecewise linear test functions (Strang, 2007). Following, we will see how the finite element method is applied to solve in space the heat equation and we will show that similar results are obtained by using finite differences or the finite volume method.

Let us consider the heat equation for a continuous isotropic non-homogeneous medium (Carslaw & Jaeger, 1986):

$$\rho c \frac{\partial \theta}{\partial t} = -\boldsymbol{\nabla} \cdot (-\kappa \boldsymbol{\nabla} \theta) + p \tag{3.1}$$

where $\rho = \rho(x, y, z)$ is the volume density, $c = c(x, y, z)$ is the specific heat capacity of the volume, $\kappa = \kappa(x, y, z)$ is the thermal conductivity of the volume, $p = p(x, y, z)$ is a function which includes all heat rate sources supplied to the volume, e.g., by radiation and convection on its surface and by possible internal gains, $\theta = \theta(x, y, z)$ is the function of temperature distribution in the volume.

An important remark is that Eq. (3.1) is a parabolic partial differential equation (PDE) which is classically solved by space and time integration using calculus by supposing a whole knowledge of physical properties and the knowledge of continuous functions describing the boundary and/or initial conditions. Even in the case of accepting the possibility of knowing all the parameters and continuous physical variables, only few cases may be solved analytically. For practical purposes, the heat equation needs to be solved numerically by discretization in space and time for integrating the partial differential expression. In this way, the problem may go from calculus to linear algebra. As it will be shown, the problem may be stated as a system of differential algebraic equations (DAE) instead of using the heat equation. For this aim, it is solved first the space integration, while in the field of heat transfer in buildings some authors usually use the Laplace transform for making first the integral in time ((Stephenson & Mitalas, 1971), (Seem, 1987), (Fraisse, et al., 2002), (Chen, et al., 2013)).

3.2.1. Space discretization of the heat equation: finite elements

The heat equation in its strong form, Eq. (3.1), may be put in a weak form in order to use the finite element method in its resolution. For this, the strong form of the heat equation needs to be integrated:

$$\int_V \rho c \frac{\partial \theta}{\partial t} v \mathrm{d}V = \int_V -\boldsymbol{\nabla} \cdot (-\kappa \boldsymbol{\nabla} \theta) v \mathrm{d}V + \int_V p v \mathrm{d}V \tag{3.2}$$

where the heat equation is integrated over the volume, V, using a test function $v = v(x, y, z)$.

The weak form is usually derived, from Eq. (3.2), in one-dimension since it is possible to extend the result to two or three-dimensions (Strang, 2007). Firstly, by expanding the divergence and gradient operators using Cartesian coordinates, it may be obtained:

$$\int_V \rho c \frac{\partial \theta}{\partial t} v \mathrm{d}V = \int_V \left(\frac{\partial}{\partial x}\left(\kappa \frac{\partial \theta}{\partial \mathrm{x}}\right) + \frac{\partial}{\partial y}\left(\kappa \frac{\partial \theta}{\partial \mathrm{y}}\right) + \frac{\partial}{\partial z}\left(\kappa \frac{\partial \theta}{\partial \mathrm{z}}\right) \right) v \mathrm{d}V + \int_V p v \mathrm{d}V \tag{3.3}$$

This integral formula may be expressed in matrix form as a system of differential algebraic equations (DAE), which will have a graphical correspondence to a thermal network. For obtaining the matrix form, Eq. (3.3) may be simplified by separating the elements on x, y and z coordinates. In this way, three equal parts, independently of each other, are obtained. Next, the part corresponding to the x-coordinate is used for detailing how obtaining the matrix form. The extension to three dimensions may be done considering a test function, $v = v(x) + v(y) + v(z)$, linear in the three coordinates.

For one dimension, considering the total interval $x \in [0,1]$, we obtain:

$$\int_S \mathrm{dydz} \int_0^1 \rho c \frac{\partial \theta}{\partial t} v \mathrm{dx} = \int_S \mathrm{dydz} \int_0^1 \frac{\partial}{\partial x}\left(\kappa \frac{\partial \theta}{\partial \mathrm{x}}\right) v \mathrm{dx} + \int_S \mathrm{dydz} \int_0^1 p v \mathrm{dx} \tag{3.4}$$

or by considering the integration into a cube of volume,

$\mathrm{V}=\int_V \mathrm{dxdydz} = h^3$,

the flux through the surface perpendicular to the *x*-coordinate will be,

$S = \int_S \mathrm{dydz} = h^2$,

and Eq. (3.4) becomes:

$$h^2 \int_0^1 \rho c \frac{\partial \theta}{\partial t} v \mathrm{dx} = h^2 \int_0^1 \frac{\partial}{\partial x}\left(\kappa \frac{\partial \theta}{\partial \mathrm{x}}\right) v \mathrm{dx} + h^2 \int_0^1 p v \mathrm{dx} \tag{3.5}$$

The factors representing the surface (h^2) in Eq. (3.5) may be cancelled:

$$\int_0^1 \rho c \frac{\partial \theta}{\partial t} v \mathrm{dx} = \int_0^1 \frac{\partial}{\partial x}\left(\kappa \frac{\partial \theta}{\partial \mathrm{x}}\right) v \mathrm{dx} + \int_0^1 p v \mathrm{dx} \tag{3.6}$$

Following, Eq. (3.6) will be put in matrix notation by using the finite element Galerkin method (Strang, 2007). For this purpose, it is used a continuous piecewise linear approximation for building the trial functions $\varphi_i(x)$. Let $I_h: 0 = x_1 < x_2 < \cdots < x_N = 1$ be a partition of interval $I = (0,1)$ into subintervals $I_j = (x_j, x_{j-1})$ of length $h_j = x_j - x_{j-1}$; we will suppose by the sake of simplicity $h_j = h$. Let V_h denote the set of continuous piecewise linear functions on I_h that are one at $x = 0$ and $x = 1$. V_h is a finite dimensional vector space, $dim V_h = N$, with a basis consisting of hat functions $\{\varphi_i\}_{i=1}^N$:

$$\varphi_i(x) = \begin{cases} 0, & if\ x \notin [x_{i-1}, x_{i+1}], \\ \dfrac{x - x_{i-1}}{x_i - x_{i-1}}, & if\ x \in [x_{i-1}, x_i], \\ \dfrac{x_{i+1} - x}{x_{i+1} - x_i}, & if\ x \in [x_i, x_{i+1}]; \end{cases} \tag{3.7}$$

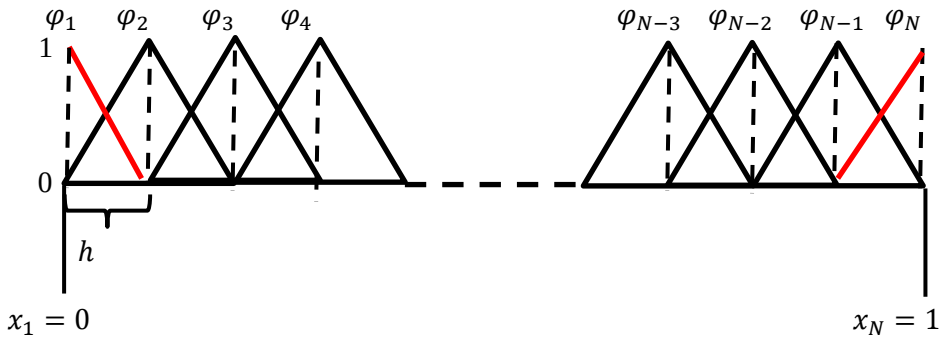

Figure 3.2. Hat functions and semi-hat functions at extremes

Using such trial functions, the temperature function may be defined as:

$$\theta = \sum_{i=1}^{N} \theta_i \, \varphi_i(x) \tag{3.8}$$

and the test function may be defined as a set of *N* functions at each space interval by:

$$v_j = \varphi_j(x) \ \ j = 2, \ldots, N-1; v_1 = 0; v_N = 0 \tag{3.9}$$

These test functions will coincide with the hat functions, except at the extremes ($v_1 \neq \varphi_1, v_N \neq \varphi_N$). Moreover, the function of thermal conductivities may be defined as a constant piecewise function:

$$\kappa(x) = \kappa_i \quad for\ x \in]x_{i-1}, x_i[, \quad i = 1, \ldots, N-1 \tag{3.10}$$

Thermal conductivity does not need to be a continuous function since it is the heat flux and the temperature distribution which need to be continuous. At the junctions of the intervals, the thermal conductivity may change without continuity since the space may be non-homogenous, for instance when there is a change of material ((Carslaw & Jaeger, 1986), (Strang, 2007)).

Next, we consider the integration by parts of the first term in the right-hand side of Eq. (3.5):

$$h^2 \int_0^1 \frac{\mathrm{d}}{\mathrm{d}x}\left(\kappa \frac{\mathrm{d}\theta}{dx}\right) v \mathrm{d}x = -h^2 \int_0^1 \kappa \frac{\mathrm{d}\theta}{\mathrm{d}x}\frac{\mathrm{d}v}{dx}\mathrm{d}x + \; h^2 \kappa \frac{\mathrm{d}\theta}{dx} v\Big|_0^1 \tag{3.11}$$

The second term in the right-hand side becomes zero by choosing the test function $v(0,y,z) = 0$ and $v(1,y,z) = 0$; the first term in the right-hand side of Eq. (3.11) may be written as N expressions ($j = 1, \dots, N$) for each linear piecewise test function (v_j), and each expression can be expanded in N terms ($i = 1, \dots, N$). Developing the addition of the temperature function, Eq. (3.8), which uses the linear piecewise trial functions (φ_i), we obtain:

$$\begin{matrix} N\ trial\ functions\ (\varphi_i) \\ \left.\begin{matrix} -h^2\left(\frac{\mathrm{d}\varphi_1}{\mathrm{d}x}\theta_1 + \cdots + \frac{\mathrm{d}\varphi_N}{\mathrm{d}x}\theta_N\right)\left(\frac{\mathrm{d}v_1}{\mathrm{d}x}\right)\int_0^1 \kappa dx \\ \vdots \\ -h^2\left(\frac{\mathrm{d}\varphi_1}{\mathrm{d}x}\theta_1 + \cdots + \frac{\mathrm{d}\varphi_N}{\mathrm{d}x}\theta_N\right)\left(\frac{\mathrm{d}v_N}{\mathrm{d}x}\right)\int_0^1 \kappa dx \end{matrix}\right\} N\ test\ functions\ (v_j) \end{matrix} \tag{3.12}$$

Since the integral may be considered as an inner product between trial and test functions (Strang, 2007), this allows us to use matrix notation for rewriting expression (3.12) as $\mathbf{K}\boldsymbol{\theta}_{\varphi}$, where $\boldsymbol{\theta}_{\varphi} = (\theta_1 \dots \theta_n)^T$ is the vector of temperatures representing state variables and temperature sources; $\mathbf{K}$ is known as the stiffness matrix and it is a sparse tridiagonal matrix ($N \times N$) with elements different to zero only on the diagonal and on the adjacent diagonals. $\mathbf{K}$ is a symmetric and positive definite matrix, hence invertible.

In practice, for obtaining the stiffness matrix, the test and trial functions, which are defined as linear piecewise functions, may be arranged as vectors and matrices. The total interval [0,1] may be split into $N-1$ subintervals, which may be supposed equal spaced by a length h, and each subinterval will represent a vector component. For instance, the derivatives of a set of trial functions, Eq. (3.7), considering $N = 5$, may be represented as five independent column vectors or a matrix, Figure 3.3-Figure 3.7 and Eq. (3.13)-(3.19). The derivative of the first trial function (φ_1) illustrated in Figure 3.3 may be defined at each interval as 1/h, 0, 0 and 0 respectively, Eq. (3.13):

$$\boldsymbol{\varphi}'_1 = \frac{1}{h}\begin{pmatrix} -1 & 0 & 0 & 0 \end{pmatrix}^T \tag{3.13}$$

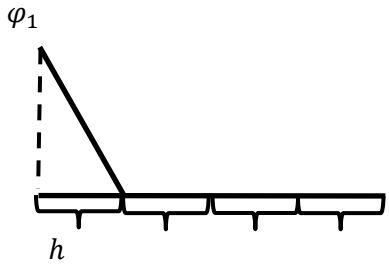

Figure 3.3. First trial function (semi-hat function) represented considering an interval divided in four subintervals of length *h*.

The derivative of the second trial function (φ_2) illustrated in Figure 3.4 may be defined at each interval as 1/h, -1/h, 0 and 0 respectively, Eq. (3.14):

$$\boldsymbol{\varphi}'_2 = \frac{1}{h}\begin{pmatrix}1 & -1 & 0 & 0\end{pmatrix}^T \tag{3.14}$$

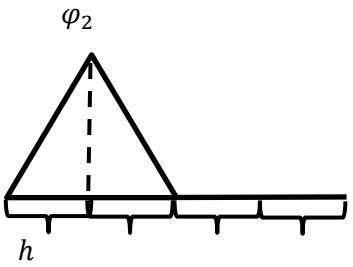

Figure 3.4. Second trial function (hat function) represented considering an interval divided in four subintervals of length *h*.

The derivative of the third trial function (φ_3) considered in Figure 3.5 may be defined at each interval as 1/h, -1/h, 0 and 0 respectively, Eq. (3.15):

$$\boldsymbol{\varphi}'_3 = \frac{1}{h}\begin{pmatrix}0 & 1 & -1 & 0\end{pmatrix}^T \tag{3.15}$$

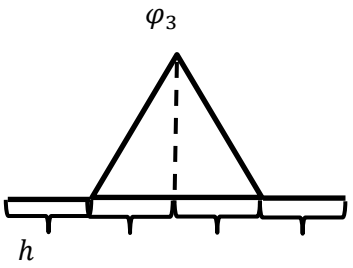

Figure 3.5. Third trial function (hat function) represented considering an interval divided in four subintervals of length *h*

The derivative of the fourth trial function (φ_4) illustrated in Figure 3.6 may be defined at each interval as 0, 0, 1/h and -1/h respectively, Eq. (3.16):

$$\boldsymbol{\varphi}'_4 = \frac{1}{h}\begin{pmatrix}0 & 0 & 1 & -1\end{pmatrix}^T \tag{3.16}$$

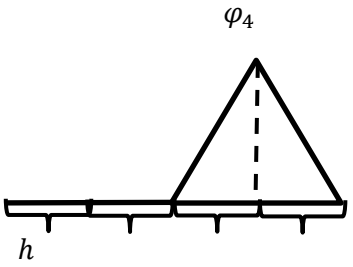

Figure 3.6. Fourth trial function (hat function) represented considering an interval divided in four subintervals of length *h*

The derivative of the fifth trial function (φ_5) illustrated in Figure 3.7 may be obtained for each interval as 0, 0, 0 and 1/h respectively, Eq. (3.17):

$$\boldsymbol{\varphi}'_5 = \frac{1}{h}\begin{pmatrix}0 & 0 & 0 & 1\end{pmatrix}^T \tag{3.17}$$

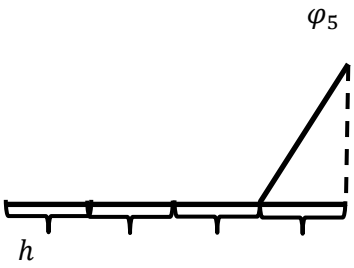

Figure 3.7. Fifth trial function (semi-hat function) represented considering an interval divided in four subintervals of length h

The column vectors, representing the trial functions, may be arranged altogether to form a difference matrix, $\mathbf{A}_\varphi$:

$$\mathbf{A}_\varphi = \frac{1}{h}\begin{pmatrix} \boldsymbol{\varphi}'_1 & \boldsymbol{\varphi}'_2 & \boldsymbol{\varphi}'_3 & \boldsymbol{\varphi}'_4 & \boldsymbol{\varphi}'_5 \\ \hline -1 & 1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & -1 & 1 \end{pmatrix} \tag{3.18}$$

On the other hand, the test functions are equal to the trial functions less at extremes where they may be imposed to be zero for cancelling the second term on the right-hand side of Eq. (3.11) related to boundary conditions (at least one boundary condition) which are required for solving the problem, i.e. for obtaining an invertible stiffness matrix in the same way that after integration the boundary conditions are mandatory for obtaining the integration constants. The next difference matrix, $\mathbf{A}_v$**,** is obtained analogously:

$$\mathbf{A}_v = \frac{1}{h}\begin{pmatrix} v'_1 & v'_2 & v'_3 & v'_4 & v'_5 \\ \hline 0 & 1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & -1 & 0 \end{pmatrix} \tag{3.19}$$

Thermal conductivities may be arranged in a diagonal matrix, $\boldsymbol{\Lambda}$, considering they can be different at each interval $(N-1=4)$ of length h.

$$\boldsymbol{\Lambda} = \begin{pmatrix} \kappa_1 & 0 & 0 & 0 \\ 0 & \kappa_2 & 0 & 0 \\ 0 & 0 & \kappa_3 & 0 \\ 0 & 0 & 0 & \kappa_4 \end{pmatrix} \tag{3.20}$$

In this way, expression (3.12) may be written using matrix notation as:

$$-h^2\mathbf{A}_{\boldsymbol{v}}^{T}h\boldsymbol{\Lambda}\mathbf{A}_{\varphi}\boldsymbol{\theta}_{\varphi} \tag{3.21}$$

where the term $h\boldsymbol{\Lambda}$ comes from the integral given in expression (3.12) made interval by interval:

$$\int_0^1 \kappa \mathrm{dx} = \int_0^h \kappa_1 \mathrm{dx} + \int_h^{2h} \kappa_2 \mathrm{dx} + \int_{2h}^{3h} \kappa_3 \mathrm{dx} + \int_{3h}^1 \kappa_4 \mathrm{dx} \tag{3.22}$$

With this configuration ($N = 5$), we have five trial functions, φ_i, and five test functions, v_j, but only three test functions (v_2, v_3, v_4) are different from zero. Hence the expressions given in expression (3.21) different from zero are only three:

$$-\begin{pmatrix} 0 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{pmatrix} h \begin{pmatrix} \kappa_1 & 0 & 0 & 0 \\ 0 & \kappa_2 & 0 & 0 \\ 0 & 0 & \kappa_3 & 0 \\ 0 & 0 & 0 & \kappa_4 \end{pmatrix} \begin{pmatrix} -1 & 1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & -1 & 1 \end{pmatrix} \begin{pmatrix} \theta_{\varphi 1} \\ \theta_{\varphi 2} \\ \theta_{\varphi 3} \\ \theta_{\varphi 4} \\ \theta_{\varphi 5} \end{pmatrix} = h \begin{pmatrix} 0 \\ \kappa_1(\theta_{\varphi 1} - \theta_{\varphi 2}) + \kappa_2(\theta_{\varphi 3} - \theta_{\varphi 2}) \\ \kappa_2(\theta_{\varphi 2} - \theta_{\varphi 3}) + \kappa_3(\theta_{\varphi 4} - \theta_{\varphi 3}) \\ \kappa_3(\theta_{\varphi 3} - \theta_{\varphi 4}) - \kappa_4(\theta_{\varphi 5} - \theta_{\varphi 4}) \\ 0 \end{pmatrix} \tag{3.23}$$

The result obtained in Eq. (3.23) may be rearranged as:

$$-\mathbf{A}^T\mathbf{G}\mathbf{A}\boldsymbol{\theta} + \mathbf{A}^{\mathbf{T}}\mathbf{G}\mathbf{b} \tag{3.24}$$

where the vector of temperatures, $\boldsymbol{\theta}_{\varphi} = (\theta_{\varphi 1}\ \theta_{\varphi 2}\ \theta_{\varphi 3}\ \theta_{\varphi 4}\ \theta_{\varphi 5})^T$, is split in vector of temperatures representing the state-variables, $\boldsymbol{\theta} = (\theta_1\ \theta_2\ \theta_3\ )^T$ and in a vector of temperature sources (boundary conditions), $\mathbf{b} = (b_1\ 0\ 0\ b_4)^T$. For this purpose, the following changes of variables are considered, $\theta_{\varphi 1} \equiv b_1$, $\theta_{\varphi 2} \equiv \theta_1$, $\theta_{\varphi 3} \equiv \theta_2$ and $\theta_{\varphi 4} \equiv \theta_3$ and $\theta_{\varphi 5} \equiv -b_4$. $\mathbf{A}$ is a difference matrix where all the columns are independent which guarantees that the stiffness matrix ($\mathbf{K} \equiv -\mathbf{A}^T\mathbf{G}\mathbf{A}$) is positive definite (invertible):

$$\mathbf{A} = \begin{pmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & -1 \end{pmatrix} \tag{3.25}$$

and $\mathbf{G}$ is a matrix of thermal conductances, which is obtained by multiplying the length of each interval, h, of the matrix of thermal conductivities, $\boldsymbol{\Lambda}$:

$$\mathbf{G} = h\boldsymbol{\Lambda} = \begin{pmatrix} R_1^{-1} & 0 & 0 & 0 \\ 0 & R_2^{-1} & 0 & 0 \\ 0 & 0 & R_3^{-1} & 0 \\ 0 & 0 & 0 & R_4^{-1} \end{pmatrix} \tag{3.26}$$

Next, the terms which refer the heat rate sources and the energy accumulation in Eq. (3.5) may be put altogether as:

$$h^2 \int_0^1 \left(\rho c \frac{\partial \theta}{\partial t} - p\right) v \mathrm{d}x \tag{3.27}$$

The expression (3.27) may be also put in matrix notation. For instance, by making the integration using the Lagrange interpolation method:

$$\int_0^1 g(x) v_i(x) \mathrm{d}x \approx g(x_i) \int_0^1 v_i(x) \mathrm{d}x \tag{3.28}$$

where in our case $g = h^2(\rho c \dot{\theta} - p)$.

Eq. (3.28) may be split in five terms corresponding to five test functions ($N = 5$), but only three will be different from zero as in Eq. (3.23). This allows defining the matrix of thermal capacities, $\mathbf{C}$, as:

$$\mathbf{C} = \begin{pmatrix} C_1 & 0 & 0 \\ 0 & C_2 & 0 \\ 0 & 0 & C_3 \end{pmatrix} \tag{3.29}$$

where $C_i = \rho_i c_i h^3$, and the vector of heat rate sources as:

$$\mathbf{f} = (f_1 \; f_2 \; f_3)^T \tag{3.30}$$

where $f = p_i h^3$

The resulting expression is:

$$\mathbf{C}\dot{\boldsymbol{\theta}} - \mathbf{f} \tag{3.31}$$

where $\dot{\boldsymbol{\theta}} = \left(\frac{\partial \theta_1}{\partial t} \; \frac{\partial \theta_2}{\partial t} \; \frac{\partial \theta_3}{\partial t}\right)^T$ is the time derivative of the vector of temperatures, $\boldsymbol{\theta}$.

Finally, by combining expressions (3.24) and (3.31), it is obtained a system of differential algebraic equations (DAE), which has been deduced from the heat equation in its weak form and is totally equivalent to the system of DAE presented in Chapter 2 which is deduced from thermodynamics and graph theory:

$$\mathbf{C}\dot{\boldsymbol{\theta}} = -\mathbf{A}^T\mathbf{G}\mathbf{A}\boldsymbol{\theta} + \mathbf{A}^T\mathbf{G}\mathbf{b} + \mathbf{f} \tag{3.32}$$

3.2.2. Space discretization of the heat equation: finite differences and finite volume

The same result, given by Eq. (3.32), may be obtained by using finite differences or finite volume method. The reason is that these methods may be understood as particular cases of the finite element method when the space is discretized using piecewise linear functions and a cubic volume mesh. The equivalence between finite differences and finite volume, and how finite volume may be considered a particular case of the finite element method is explained in detail, among others, by Grossmann et al. (Grossmann, et al., 2007).

Following, it is shown how the same solution can be obtained using finite differences (Strang, 2007). In this case, the heat equation becomes a system of DAE from its strong form:

$$\rho c \frac{\partial \theta}{\partial t} = -\boldsymbol{\nabla} \cdot (-\kappa \boldsymbol{\nabla} \theta) + p \tag{3.33}$$

We may use again Cartesian coordinates and write:

$$\rho c \frac{\partial \theta}{\partial t} = \frac{\partial}{\partial x}\left(\kappa \frac{\partial}{\partial \mathrm{x}} \theta_x\right) + \frac{\partial}{\partial y}\left(\kappa \frac{\partial}{\partial \mathrm{y}} \theta_y\right) + \frac{\partial}{\partial z}\left(\kappa \frac{\partial}{\partial \mathrm{z}} \theta_z\right) + p \tag{3.34}$$

Next, it is presented a finite difference scheme for showing the connection to finite element method. As well as for the finite element method, hereafter it is considered only the *x*-coordinate since the extension to three dimensions is straightforward thanks to the linearity on x, y, z coordinates. Eq. (3.35) for one dimension. By omitting the sub-indexes on *x*-variable, equation (3.34) becomes:

$$\rho_i c_i \frac{\partial \theta_i}{\partial t} = \frac{\partial}{\partial x}\left(\kappa_i \frac{\theta_i - \theta_{i-1}}{h}\right) \tag{3.35}$$

and the second difference, centred on θ_i is:

$$\rho_i c_i \frac{\partial \theta_i}{\partial t} = \frac{1}{h}\left(\kappa_{i+1} \frac{\theta_{i+1} - \theta_i}{h} - \kappa_i \frac{\theta_i - \theta_{i-1}}{h}\right) + p_i \tag{3.36}$$

or:

$$\rho_i c_i \frac{\partial \theta_i}{\partial t} = \frac{1}{h^2}(\kappa_i \theta_{i-1} - (\kappa_i + \kappa_{i+1})\theta_i + \kappa_{i+1}\theta_{i+1}) + p_i \tag{3.37}$$

The difference between Eq. (3.37) and Eq. (3.32) vanishes if both sides of Eq. (3.37) are multiplied by the constant volume h^3:

$$\rho_i c_i h^3 \frac{\partial \theta_i}{\partial t} = (h\kappa_i \theta_{i-1} - (h\kappa_i + h\kappa_{i+1})\theta_i + h\kappa_{i+1}\theta_{i+1}) + p_i h^3 \tag{3.38}$$

or:

$$C_i \frac{\partial \theta_i}{\partial t} = \left(R_i^{-1}\theta_{i-1} - \left(R_i^{-1} + R_{i+1}^{-1}\right)\theta_i + R_{i+1}^{-1}\theta_{i+1}\right) + f_i \tag{3.39}$$

This subtle difference (to cancel or not the factor h^3) between the weak and the strong form of the heat equation, once it is discretized, is also noted by Gilbert Strang (Strang, 2007). The generalization of Eq. (3.39), which is obtained from a finite difference scheme, may give a system of DAE identical to that given by the finite element method, Eq. (3.32).

Moreover, as it is previously noted, the finite volume method may give also a similar result. For this purpose, the heat equation may be integrated into a cubic volume ($V = h^3$):

$$\int_V \rho c \frac{\partial \theta}{\partial t} dV = \int_V -\boldsymbol{\nabla} \cdot (-\kappa \boldsymbol{\nabla} \theta) dV + \int_V p dV \tag{3.40}$$

By integrating considering the midpoint or rectangle rule and by applying the Green's theorem, Eq. (3.40) becomes:

$$\rho_i c_i h^3 \frac{\partial \theta_i}{\partial t} = \int_S \kappa \nabla\theta \, dS + p_i h^3 \tag{3.41}$$

Finally, if it is only considered by simplicity the heat flow on *x*-coordinate going in/out of the two surfaces limited by *yz*-coordinates, the last integral expression vanishes in Eq. (3.41) and it is obtained an equivalent result to that obtained by using finite element method, Eq. (3.32):

$$\rho_i c_i h^3 \frac{\partial \theta_i}{\partial t} = h^2 \left(\kappa_{i+1} \frac{\theta_{i+1} - \theta_i}{h} - \kappa_i \frac{\theta_i - \theta_{i-1}}{h} \right) + p_i h^3 \tag{3.42}$$

Once shown how the heat equation (PDE) may be expressed as a system of DAE by discretization using finite element, finite difference or finite volume methods, it will be shown in the next section how transforming this system of DAE into state-space representation.

3.3. From differential algebraic equations to state-space representation

It has been seen that from the heat equation, the system of heat balance equations can be written, by space discretization, as:

$$\mathbf{C}\dot{\boldsymbol{\theta}} = -\mathbf{A}^T\mathbf{G}\mathbf{A}\boldsymbol{\theta} + \mathbf{A}^T\mathbf{G}\mathbf{b} + \mathbf{f} \tag{3.43}$$

or, by noting $\mathbf{K} \equiv -\mathbf{A}^T\mathbf{G}\mathbf{A}$ and $\mathbf{K}_b \equiv \mathbf{A}^T\mathbf{G}$, as:

$$\mathbf{C}\dot{\boldsymbol{\theta}} = \mathbf{K}\boldsymbol{\theta} + \mathbf{K}_b\mathbf{b} + \mathbf{f} \tag{3.44}$$

Eq. (3.43) can be written in state-space form by eliminating the temperatures corresponding to the nodes with negligible thermal capacity, i.e. the nodes for which $C_i = 0$, (Ghiaus, 2013); it is an special case of regular semi-implicit strangeness-free DAE ((Kunkel & Mehrmann, 2006), (Biegler, et al., 2012)). If the Eq. (3.44) is written in blocks, we obtain:

$$\begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{C}_C \end{bmatrix} \begin{bmatrix} \dot{\boldsymbol{\theta}}_0 \\ \dot{\boldsymbol{\theta}}_C \end{bmatrix} = \begin{bmatrix} \mathbf{K}_{11} & \mathbf{K}_{12} \\ \mathbf{K}_{21} & \mathbf{K}_{22} \end{bmatrix} \begin{bmatrix} \boldsymbol{\theta}_0 \\ \boldsymbol{\theta}_C \end{bmatrix} + \begin{bmatrix} \mathbf{K}_{b1} \\ \mathbf{K}_{b2} \end{bmatrix} \mathbf{b} + \begin{bmatrix} \mathbf{I}_{11} & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{f}_0 \\ \mathbf{f}_C \end{bmatrix} \tag{3.45}$$

where

$\dot{\boldsymbol{\theta}}_0, \boldsymbol{\theta}_0$ and $\mathbf{f}_0$ correspond to the nodes without thermal capacity;

$\dot{\boldsymbol{\theta}}_C, \boldsymbol{\theta}_C$ and $\mathbf{f}_C$ correspond to the nodes with thermal capacity;

$\mathbf{C}_C$ is the block of the partitioned matrix $\mathbf{C}$ for which the elements on the diagonal are non-zero;

$\mathbf{K}_{11}, \mathbf{K}_{12}, \mathbf{K}_{21}$ and $\mathbf{K}_{22}$ are blocks of the partitioned matrix $\mathbf{K}$ obtained according to the partitioning of the matrix $\mathbf{C}$;

$\mathbf{K}_{b1}$ and $\mathbf{K}_{b2}$ are blocks of the partitioned matrix $\mathbf{K}_b$ obtained according to the partitioning of the matrix $\mathbf{C}$;

$\mathbf{I}_{11}$ and $\mathbf{I}_{22}$ are identity matrices.

The vector $\boldsymbol{\theta}_0$ can be eliminated from the second row of Eq. (3.45). By multiplying the first row of Eq. (3.45) by $-\mathbf{K}_{21}\mathbf{K}_{11}^{-1}$ we obtain:

$$\begin{bmatrix}\mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{C}_C\end{bmatrix}\begin{bmatrix}\dot{\boldsymbol{\theta}}_0\\ \dot{\boldsymbol{\theta}}_C\end{bmatrix}=\begin{bmatrix}-\mathbf{K}_{21} & -\mathbf{K}_{21}\mathbf{K}_{11}^{-1}\mathbf{K}_{12}\\ \mathbf{K}_{21} & \mathbf{K}_{22}\end{bmatrix}\begin{bmatrix}\boldsymbol{\theta}_0\\ \boldsymbol{\theta}_C\end{bmatrix}+\begin{bmatrix}-\mathbf{K}_{21}\mathbf{K}_{11}^{-1}\mathbf{K}_{b1}\\ \mathbf{K}_{b2}\end{bmatrix}\mathbf{b}+\begin{bmatrix}-\mathbf{K}_{21}\mathbf{K}_{11}^{-1} & \mathbf{0}\\ \mathbf{0} & \mathbf{I}_{22}\end{bmatrix}\begin{bmatrix}\mathbf{f}_0\\ \mathbf{f}_C\end{bmatrix} \tag{3.46}$$

By replacing the second row with the sum of the two rows, Eq. (3.46) becomes:

$$\begin{aligned}[\mathbf{0} \quad \mathbf{C}_C]\begin{bmatrix}\dot{\boldsymbol{\theta}}_0\\ \dot{\boldsymbol{\theta}}_C\end{bmatrix} &= [\mathbf{0} \quad -\mathbf{K}_{21}\mathbf{K}_{11}^{-1}\mathbf{K}_{12}+\mathbf{K}_{22}]\begin{bmatrix}\boldsymbol{\theta}_0\\ \boldsymbol{\theta}_C\end{bmatrix}+(-\mathbf{K}_{21}\mathbf{K}_{11}^{-1}\mathbf{K}_{b1}+\mathbf{K}_{b2})\mathbf{b}\\ &+[-\mathbf{K}_{21}\mathbf{K}_{11}^{-1} \quad \mathbf{I}_{22}]\begin{bmatrix}\mathbf{f}_0\\ \mathbf{f}_C\end{bmatrix}\end{aligned} \tag{3.47}$$

Thus, in Eq. (3.47) all coefficients of $\boldsymbol{\theta}_0$ are zero, which implies that this equation does not depend on the temperatures of the nodes without thermal capacity:

$$\mathbf{C}_C\dot{\boldsymbol{\theta}}_C=(-\mathbf{K}_{21}\mathbf{K}_{11}^{-1}\mathbf{K}_{12}+\mathbf{K}_{22})\boldsymbol{\theta}_C+(-\mathbf{K}_{21}\mathbf{K}_{11}^{-1}\mathbf{K}_{b1}+\mathbf{K}_{b2})\mathbf{b}+[-\mathbf{K}_{21}\mathbf{K}_{11}^{-1} \quad \mathbf{I}_{22}]\begin{bmatrix}\mathbf{f}_0\\ \mathbf{f}_C\end{bmatrix} \tag{3.48}$$

Since the matrix $\mathbf{C}_C$ in Eq. (3.48) is invertible, the state-space representation of the thermal network is:

$$\dot{\boldsymbol{\theta}}_C=\mathbf{C}_C^{-1}(-\mathbf{K}_{21}\mathbf{K}_{11}^{-1}\mathbf{K}_{12}+\mathbf{K}_{22})\boldsymbol{\theta}_C+\mathbf{C}_C^{-1}[-\mathbf{K}_{21}\mathbf{K}_{11}^{-1}\mathbf{K}_{b1}+\mathbf{K}_{b2} \quad -\mathbf{K}_{21}\mathbf{K}_{11}^{-1} \quad \mathbf{I}_{22}]\begin{bmatrix}\mathbf{b}\\ \mathbf{f}_0\\ \mathbf{f}_C\end{bmatrix} \tag{3.49}$$

where the state matrix is

$$\mathbf{A}_S=\mathbf{C}_C^{-1}(-\mathbf{K}_{21}\mathbf{K}_{11}^{-1}\mathbf{K}_{12}+\mathbf{K}_{22}) \tag{3.50}$$

the input matrix is

$$\mathbf{B}_S=\mathbf{C}_C^{-1}[-\mathbf{K}_{21}\mathbf{K}_{11}^{-1}\mathbf{K}_{b1}+\mathbf{K}_{b2} \quad -\mathbf{K}_{21}\mathbf{K}_{11}^{-1} \quad \mathbf{I}_{22}] \tag{3.51}$$

the input vector is

$$\mathbf{u}=[\mathbf{b} \quad \mathbf{f}_0 \quad \mathbf{f}_C]^T \tag{3.52}$$

Considering Eq. (3.49) -(3.52), the differential equations of the state-space model can be written as:

$$\dot{\boldsymbol{\theta}}_C=\mathbf{A}_S\boldsymbol{\theta}_C+\mathbf{B}_S\mathbf{u} \tag{3.53}$$

Algebraic equations of state-space model, related to the vector of temperatures, $\boldsymbol{\theta}$, can be written in a different way depending on the thermal capacity of the particular node.

If the node θ_a has a thermal capacity (i.e. if θ_a is a state variable), the identity matrix $\mathbf{I}$ reproduces θ_a from the vector $\boldsymbol{\theta}_C$:

$$\begin{aligned}\boldsymbol{\theta}_C&=\mathbf{I}\boldsymbol{\theta}_C+\mathbf{0}\mathbf{u}\\ \theta_a&=\theta_{C_a}\end{aligned} \tag{3.54}$$

If the node θ_a does not have a thermal capacity (i.e. if θ_a is not a state variable), then θ_a is an element of the vector $\boldsymbol{\theta}_0$ which can be obtained from the first row of the Eq. (3.46):

$$\begin{aligned}\boldsymbol{\theta}_0&=-\mathbf{K}_{11}^{-1}(\mathbf{K}_{12}\boldsymbol{\theta}_C+\mathbf{K}_{b1}\mathbf{b}+\mathbf{I}_{11}\mathbf{f}_0)\\ &=-\mathbf{K}_{11}^{-1}\left(\mathbf{K}_{12}\boldsymbol{\theta}_C+[\mathbf{K}_{b1} \quad \mathbf{I}_{11} \quad \mathbf{0}]\begin{bmatrix}\mathbf{b}\\ \mathbf{f}_0\\ \mathbf{f}_C\end{bmatrix}\right)\end{aligned} \tag{3.55}$$

or, by simplifying the notation:

$$\boldsymbol{\theta}_0 = \mathbf{C}_S\boldsymbol{\theta}_C + \mathbf{D}_S\mathbf{u} \tag{3.56}$$

where, from Eq. (3.55)-(3.56), $\mathbf{C}_S$ and $\mathbf{D}_S$ are:

$$\mathbf{C}_S = -\mathbf{K}_{11}^{-1}\mathbf{K}_{12} \tag{3.57}$$

$$\mathbf{D}_S = -\mathbf{K}_{11}^{-1}[\mathbf{K}_{b1} \quad \mathbf{I}_{11} \quad \mathbf{0}] \tag{3.58}$$

Thus, if θ_a is an element of $\boldsymbol{\theta}_0$, it can be extracted from the corresponding row a of the Eq. (3.55). In general, the state-space model may be written combining Eq. (3.53), (3.54) and (3.56). Then, the output, θ_a, is a term of the vector of temperatures in the nodes, $\boldsymbol{\theta} = [\boldsymbol{\theta}_C \quad \boldsymbol{\theta}_0]^T$, as:

$$\dot{\boldsymbol{\theta}}_C = \mathbf{A}_S\boldsymbol{\theta}_C + \mathbf{B}_S\mathbf{u} \tag{3.59}$$

$$\begin{bmatrix}\boldsymbol{\theta}_C \\ \boldsymbol{\theta}_0\end{bmatrix} = \begin{bmatrix}\mathbf{I} \\ \mathbf{C}_S\end{bmatrix}\boldsymbol{\theta}_C + \begin{bmatrix}\mathbf{0} \\ \mathbf{D}_S\end{bmatrix}\mathbf{u} \tag{3.60}$$

It must be noted that the output, θ_a, gives the measurement equation of the state-space model; therefore it must be a measurable variable. This does not imply that all the outputs are measurable; in fact, the state-variables will be in general non-measurable and the nodes with negligible heat capacity, usually related to surface temperatures, could be non-measurable in practice.

Finally, the state-space model needs to be integrated in time. For this purpose, a linear state variable transformation can be tried to make the diagonalization of the state-space matrix, $\mathbf{A}_S$, which simplifies the computation of the exponential matrix, $e^{\mathbf{A}_S\Delta t}$. But this implies that there are an infinite number of equivalent state-space representations (Crassidis & Junkins, 2012). Fortunately, there is a particular one state-space representation obtained from the thermal network before the linear state variable transformation. In the next section, this one state-space representation obtained from the thermal network is transformed into transfer function representation using the Laplace transform.

3.4. From state-space to transfer function representation

The transfer function is widely used in engineering for analysing systems in frequency domain, by considering the inputs and the outputs of the system.

3.4.1. Transfer function in continuous time

The relation between the inputs, $\mathbf{u}$, and the particular outputs considering nodes with negligible thermal capacity, $\boldsymbol{\theta}_0$, can be expressed as a set of transfer functions or a transfer matrix.

Considering Eq. (3.53) and Eq. (3.56), by applying the Laplace transform for zero initial conditions, the set of state-space equations can be expressed as:

$$s\boldsymbol{\theta}_C = \mathbf{A}_S\boldsymbol{\theta}_C + \mathbf{B}_S\mathbf{u} \tag{3.61}$$

$$\boldsymbol{\theta}_0 = \mathbf{C}_S\boldsymbol{\theta}_C + \mathbf{D}_S\mathbf{u} \tag{3.62}$$

where $s = \sigma + j\omega$ is the complex variable. From Eq. (3.61) it can be obtained:

$$\boldsymbol{\theta}_C = (s\mathbf{I} - \mathbf{A}_S)^{-1}\mathbf{B}_S\mathbf{u} \tag{3.63}$$

and substituting in Eq. (3.62) it is obtained:

$$\boldsymbol{\theta}_0 = (\mathbf{C}_S(s\mathbf{I} - \mathbf{A}_S)^{-1}\mathbf{B}_S + \mathbf{D}_S)\mathbf{u} \tag{3.64}$$

where:

$$\mathbf{H}_S = \mathbf{C}_S(s\mathbf{I} - \mathbf{A}_S)^{-1}\mathbf{B}_S + \mathbf{D}_S \tag{3.65}$$

is the transfer matrix in which each element is a transfer function relating the output θ_a to a particular input from the input vector, $\mathbf{u}$.

It is important to note that the transfer function is unaffected by a linear state variable transformation (Crassidis & Junkins, 2012). This fact and the possibility of obtaining a canonical state-space representation from the transfer function representation limit the transformation from transfer function representation into a unique state-space representation (Ogata, 2010). This limitation is transcended by applying the methodology presented hereafter. In this way, the transfer function derived from a particular thermal network can be uniquely related to the particular one state-space representation obtained from a particular thermal network.

3.4.2. Transfer function in discrete time

It is possible to go from state-space representation to transfer function representation and to focus the problem in inputs-outputs relation by solving algebraically the differential equations. Eq. (3.61)-(3.62) are obtained by using the Laplace transform supposing variables in continuous time, i.e. the transfer matrix results in continuous frequency domain. Nonetheless, since the measured input and output variables are usually sampled at discrete time steps, the discrete transfer matrix may be obtained in terms of the Z transform, which may be related to the Laplace transform by the starred Laplace transform, $\mathbf{H}_s^*$:

$$\mathbf{H}_S^* = \mathbf{H}_d|_{z=e^{s\Delta t}} \tag{3.66}$$

where Δt is the sample time.

The discrete transfer matrix in discrete frequency domain is:

$$\mathbf{H}_d = \mathbf{C}_S(z\mathbf{I} - \mathbf{A}_d)^{-1}\mathbf{B}_d + \mathbf{D}_S \tag{3.67}$$

In Eq. (3.67), each element is a discrete transfer function relating the output vector $\boldsymbol{\theta}_0$ to a particular input from the input vector $\mathbf{u}$, where the matrices $(\mathbf{A}_d, \mathbf{B}_d)$ may be related to the matrices $(\mathbf{A}_S, \mathbf{B}_S)$, for instance, using the starred Laplace transform and a first order Taylor development: $z \approx 1 + s\Delta t$. Then, for this example, it is obtained that $\mathbf{A}_d \approx \mathbf{I} + \mathbf{A}_S\Delta t$ and $\mathbf{B}_d \approx \mathbf{B}_S\Delta t$, which is equivalent to make a forward Euler integration in time of the state space model. It should be noticed that the matrices are constant in time and input zero order hold is implicit in this approach.

The discrete transfer matrix, Eq. (3.67), may be written as:

$$\mathbf{H_d} = \begin{bmatrix} H_{d11}(z) & \cdots & H_{d1n}(z) \\ \vdots & \ddots & \vdots \\ H_{dm1}(z) & \cdots & H_{dmn}(z) \end{bmatrix} \tag{3.68}$$

where m is the number of measured outputs and n is the number of measured inputs.

3.5. From transfer functions to autoregressive models with exogenous (ARX)

The discrete transfer functions in discrete frequency may be directly transformed into an auto-regressive model with exogenous in discrete time. From Eq. (3.64) it is possible to obtain a relation between measured output at time t_k, and inputs at time t_k and t_{k-l}, and output at previous time t_{k-l}, where l-value is limited by the order of the model, p.

For this purpose, each element of Eq. (3.68) is a discrete transfer function and can be written as:

$$H_{dij}(z) = \frac{\sum_{l=0}^{r_j \leq p} b_{ijl} z^l}{\sum_{l=0}^{p} a_{il} z^l} \tag{3.69}$$

or, by multiplying both denominator and numerator by z^{-p} as:

$$H_{dij}(z^{-1}) = \frac{\sum_{l=0}^{r_j \leq p} b_{ijl} z^{l-p}}{\sum_{l=0}^{p} a_{il} z^{l-p}} \tag{3.70}$$

where p is the maximum delay given by the model order; its upper limit is the length of the measured data time series, L.

If the discrete transfer function operator is applied considering the temperature, θ_i, as output and all the inputs, u_j, then:

$$\left(\sum_{l=0}^{p} a_{il} z^{l-p}\right) \theta_i(\mathrm{z}) = \sum_{j} \sum_{l=0}^{r_j \leq p} b_{ijl} z^{l-p} u_j(\mathrm{z}) \tag{3.71}$$

From Eq. (3.71) dividing both equation sides by a_{ip} it is obtained:

$$\theta_i(z) = \sum_{l=0}^{p-1} m_{ip-l} z^{l-p} \theta_i(\mathrm{z}) + \sum_{j} \sum_{l=0}^{r_j \leq p} n_{ijp-l} z^{l-p} u_j(\mathrm{z}) \tag{3.72}$$

To go from the Z transform in discrete frequency domain to the discrete time domain, the time shifting property of the Z transform, $Z^{-1}\{z^l x(z)\} = x(\mathrm{t}_{k+l})$, can be used. From Eq. (3.72) it is obtained:

$$\theta_i(\mathrm{t}_k) = \sum_{l=0}^{p-1} m_{ip-l} \theta_i\left(\mathrm{t}_{k+l-p}\right) + \sum_{j} \sum_{l=0}^{r_j \leq p} n_{ijp-l} u_j\left(\mathrm{t}_{k-l-p}\right) \tag{3.73}$$

Ljung defines the model given by Eq. (3.73) as an autoregressive model with exogenous (ARX) by entering the equation error as white-noise and as autoregressive moving-average model with exogenous (ARMAX) by entering the equation error as a moving-average of white-noise (Ljung, 1999). It is important to highlight that the parameters of the discrete transfer function are equal to the parameters of the ARX model. Since Eq. (3.73) was obtained from the physical model expressed as a system of DAE, Eq. (3.43), the information on physical parameters of the thermal network (resistances and capacitances) is kept, from a deterministic point of view, in the chain of model transformations and the model can be considered as a white-box or a grey-box model depending on its use (Ghiaus,

2014). Autoregressive models are equivalent to discrete transfer functions; in fact, they can be obtained from discrete state-space models using the delay operator (q) instead of the complex variable (z) since ARX models deduced from state-space representation are the integration in time of such state-space models.

3.6. Conclusions

The present chapter has shown the heat equation (PDE) may be approximated by a system of differential algebraic equations (DAE) by space discretization. This implies that the heat equation may be considered the starting point for the study of heat transfer processes in buildings, although it was shown in Chapter 2 that the system of DAE may be obtained directly considering thermal networks which are built considering thermodynamics principles and the graph theory. The connection is important for showing explicitly the equivalence of both methods, which demonstrates that the choice of the starting discretisation method (i.e. finite element, finite differences or finite volume) does not change the analysis that may be done a posteriori. In fact, the idea remaining is that, in practice, we do not know continuous in space and time functions but only a finite number of function values (discrete in space and time functions). For this reason, even the resolution of the heat equation as the starting point derives for every virtual application in the resolution of a system of DAE.

One of the advantage for utilizing vectors and matrices is that by representing the heat equation in matrix form, the boundary conditions appear explicitly in one expression which is similar for strong (infinite system of DAE) and weak (finite system of DAE) forms. The assumption of space linearity supposes the connection of nodes using straight lines, i.e. it is supposed that the function of temperatures changes linearly between two space points, or more in general nodes, with a known value (measured or estimated). In this way, the theory of thermal networks is given by the work by Gilbert Strang, who shows smartly the connection between calculus and algebra.

Next, the system of DAE is transformed into state-space representation. This step may be seen as a change of variables of the physical parameters, i.e. the parameters of the state-space representation have physical meaning since they depend on the physical parameters. But the physical meaning is shared between the parameters and the knowledge of the change of variables (transformations). The following transformation is from state-space to transfer function representation, which implies going from time domain to frequency domain and may be the first step for the integration in time of the differential equations. This step may be seen as a second change of variables of the physical parameters. For completing the time integration, the transfer function is discretized (for example by using forward Euler's method) and finally, by going back to time domain, it is transformed in an autoregressive with exogenous model (ARX). The parameters of the ARX are the same with those of the discrete transfer function, which have been obtained through a new transformation (change of variables). Therefore, the ARX parameters have physical meaning but their physical meaning is shared with the knowledge of all the transformations (change of variables).

Finally, this chapter presents the transformations in the direct way. When the aim is to solve the heat equation, the whole chain of transformations is not shown explicitly. This chain covers all the possible kinds of model structures which may be used and any of them can be favoured a-priori. The importance of showing all the model structures resides in the fact of doing explicit their connections since such fact is not highlighted in the literature, which usually highlights the part of mathematical methods instead of the part related to modelling heat transfer. Since different model representations may be used indistinctly for simulation, control or system identification, next chapters deal with system identification while simulation and control are out of the scope of this dissertation. Moreover,

the problem has been stated from a deterministic point of view, without considering error propagation in the transformations.

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CHAPTER 4

GENERAL IDENTIFIABILITY OF THERMAL NETWORK PARAMETERS

4.1. Introduction

In the previous chapters, it has been shown how the different models for studying heat transfer can be deduced from the heat equation. This is important because the different models usually employed in the study of heat transfer in buildings for solving both the direct and the inverse (parameter identification) are presented disconnected, and they use often parameters without physical meaning ((Sonderegger, 1978), (Athienitis, et al., 1985), (Rabl, 1988), (Madsen & Holst, 1995), (Bloem & Martin, 2001), (Baker & van Dijk, 2008), (Jimenez & Madsen, 2008), (Jimenez, et al., 2009), (Luo, et al., 2010), (Bacher & Madsen, 2011), (Luo, et al., 2011), (Mejri, et al., 2011), (Lodi, et al., 2012), (Martin, et al., 2012), (Naveros, et al., 2012), (Hazyuk, et al., 2012), (Ramallo-Gonzalez, et al., 2013), (Castillo, et al., 2014), (Biddulph, et al., 2014), (Naveros, et al., 2014)). In particular, the structures of the models are not presented in literature as interconnected altogether but only connections between two adjacent model structures are usually considered: for instance, from state-space to transfer function or vice-verse (Ogata, 2010). In literature, it is usually preferred the use of model structures without a direct connection with physics (black box models) (Ljung, 1999). This point of view is useful in control problems, but for design and a better understanding of a physical system, modelling from first principles (white box or grey box modelling) is more recommendable, when possible, since more information about the system is available for its analysis.

The idea is to show how the path presented in Chapter 1 may be followed in the inverse way too, Figure 4.1, as far as the heat equation and the approximations done in the transformations are supposed valid and optimal for modelling heat transfer. Every model should be validated by experience to certificate the goodness of its use. Mathematical modelling of every physical process may be done following a-priori knowledge or not ((Ljung. 1999), (Ghiaus, 2007), (Ghiaus, 2014)). A mathematical model obtained without using a-priori physical knowledge on the system is a black-box model, while a mathematical model obtained only using a-priori physical knowledge on the system is a white-box model. In between these extremes, a grey box model is obtained from a-priori knowledge and from measurements. This approach is especially useful in system identification for control applications when the model is expressed in a suitable structure such as state-space or transfer function. The concepts of black-box, white-box and grey-box are independent of the model structure by definition. Ljung's definition of the use of a model structure without a direct connection to physics may be misunderstood due to the wide and useful use of transfer functions as black-box models ((Ljung. 1999), (Ogata, 2010)). Nonetheless, no model structure obtained from first principles can be considered a black-box; it is a white-box or grey-box.

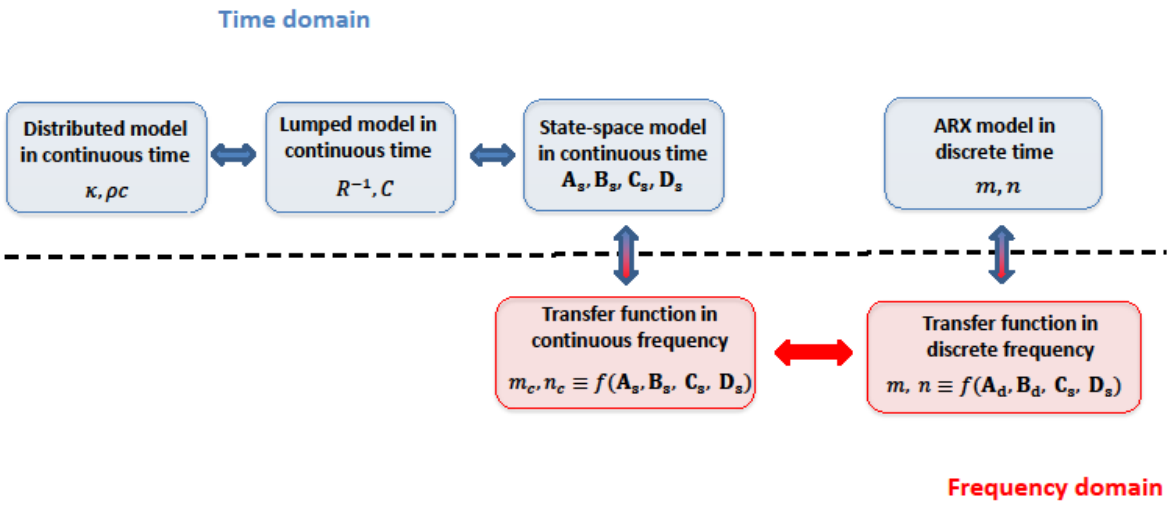

Figure 4.1. Two-way path connecting the heat equation to an ARX model passing through frequency domain

This chapter deals with system identification which consists in giving a class of models, or model structure, for finding within such class the model that fits a given data-set with the minimum loss, according to a given criterion (Ljung, 1999). From this definition, Ljung considers that the construction of a model from data involves three basic entities: the data-set, the model structure and the parameter identification procedure. Among these three entities, Ljung states the choice of the model structure (modelling) as the most important and the most difficult decision to make in the the system identification procedure. This chapter is only focused on heat transfer modelling and it obtains different classes of models independently of the estimation methods (Verhaegen & Verdult, 2007), The starting point may be the heat equation or a system of differential algebraic equations (DAE) directly, from where it has been shown in Chapter 2 that different model structures, from state-space up to an autoregressive model with exogenous (ARX) may be deduced. Next, the inverse path will be followed up to show how to obtain the parameters with a direct physical sense of a system of DAE (or a thermal network).

4.2. Inverse transformation from parameters of a grey-box ARX models to physical parameters of thermal networks

In system identification, discrete transfer functions, or their equivalent autoregressive models with exogenous, are usually built as black box models without taking into account the physical structure of the model beyond the relation between input and output variables.

Nonetheless, since the physical information is kept in the chain of model transformations presented in previous sections, the discrete transfer function obtained from the thermal network may be understood as a grey box model in solving the inverse problem, or it may be understood as a white-box model obtained from first principles, if it is used to solve the direct problem (Ghiaus, 2014).

The discrete transfer function, as it is obtained in Chapter 3 from the heat equation, can be written as:

$$H_{dij}(z^{-1}) = \frac{\sum_{l=0}^{r\leq p} n_{ijp-l} z^{l-p}}{1 + \sum_{l=0}^{p-1} m_{ip-l} z^{l-p}} \tag{4.1}$$

and it is equivalent to the autoregressive model with exogenous:

$$\theta_i(\mathrm{t}_k) = \sum_{l=0}^{p-1} m_{ip-l}\theta_i\left(\mathrm{t}_{k+l-p}\right) + \sum_{j}\sum_{l=0}^{r_j\leq p} n_{ijp-l} u_j\left(\mathrm{t}_{k-l-p}\right) \tag{4.2}$$

The parameters of the discrete transfer function, Eq. (4.1), are identical to the ARX model parameter and they may be obtained by fitting the model using experimental data (Crassidis & Junkins, 2011). Then, the physical parameters of the thermal network, i.e. the values of the resistances and the capacities, may be retrieved from the parameters obtained experimentally for the ARX model or the discrete transfer function. First, it is needed to obtain from the discrete transfer function or ARX model parameters, m_i, n_{ij}, the state-space matrix parameters ($\mathbf{A}_d$, $\mathbf{B}_d$, $\mathbf{C}_S$, $\mathbf{D}_S$). Next, the state-space matrix parameters in continuous time ($\mathbf{A}_S$, $\mathbf{B}_S$) may be derived and finally the physical parameters of the thermal network may be obtained.

4.2.1. Obtaining the discrete state-space matrix parameters from grey-box ARX parameters

A transfer function does not have a unique state-space representation (Ogata, 2010), but this is not a problem when it is also used the knowledge of the transformations from the particular discrete state-space representation ($\mathbf{A}_\mathrm{d}$, $\mathbf{B}_\mathrm{d}$, $\mathbf{C}_S$, $\mathbf{D}_S$) to the particular discrete transfer function. This is an advantage of deducing different models from the physical model.

The parameters of the discrete transfer function may be arranged as it is presented in Eq. (4.1), which may be expressed by simplicity as:

$$H_{dij}(z) = \frac{N(z)}{D(z)} \tag{4.3}$$

where the numerator and the denominator are, respectively:

$$N(z) = \sum_{l=0}^{r\leq p} n_{ijp-l} z^{l-p}$$

$$D(z) = 1 + \sum_{l=0}^{p-1} m_{ip-l} z^{l-p}$$

On the other hand, from the state-space representation it can be obtained the discrete transfer matrix as:

$$\mathbf{H}_d = \mathbf{C}_S(z\mathbf{I} - \mathbf{A}_d)^{-1}\mathbf{B}_d + \mathbf{D}_S \tag{4.4}$$

or by taking into account that:

$$\det\left(\mathbf{E} - \mathbf{x}\mathbf{y}^\mathrm{T}\right) = \det(\mathbf{E})(1 + \mathbf{y}^\mathrm{T}\mathbf{E}^{-1}\mathbf{x}) \tag{4.5}$$

each discrete transfer function, H_{dij}, of the discrete transfer matrix, $\mathbf{H}_d$, with i-outputs and j inputs, can be expressed in a more practical way as:

$$H_{dij} = \frac{\det\left(\mathrm{z}\mathbf{I} - \mathbf{A}_d + \mathbf{b}_{dj}^c \mathbf{c}_i^r\right) - \det(\mathrm{z}\mathbf{I} - \mathbf{A}_d) + \det(\mathrm{z}\mathbf{I} - \mathbf{A}_d) d_{ij}}{\det(\mathrm{z}\mathbf{I} - \mathbf{A}_d)} \tag{4.6}$$

Using that $\det(s\mathbf{I} - \mathbf{E}) = p(\mathbf{E})$, where $p(\mathbf{E})$ is the characteristic polynomial of the matrix $\mathbf{E}$, the previous equation becomes:

$$H_{dij} = \frac{p\left(\mathbf{A}_d - \mathbf{b}_{dj}^c \mathbf{c}_i^r\right) - p(\mathbf{A}_d) + p(\mathbf{A}_d) d_{ij}}{p(\mathbf{A}_d)} \tag{4.7}$$

Eq. (4.7) is used to obtain the transfer function from the state-space representation in software packages as MATLAB®. Hereafter, the inverse procedure is discussed: how to obtain the elements of the matrices of the state-space representation $(\mathbf{A}_\mathrm{d}, \mathbf{B}_\mathrm{d}, \mathbf{C}_S, \mathbf{D}_S)$, from the parameters of the discrete transfer function (m_i, n_{ij}) by comparison of Eq. (4.1) and Eq. (4.7) and the structure of the state-space representation.

Before making the comparison, it is considered that the characteristic polynomial can be expressed, in general, by using exterior algebra (Bourbaki, 1998). For a 2×2 order matrix, using z as eigen-coefficients, the characteristic polynomial is given by $p(\mathbf{E}) = z^2 - \mathrm{Tr}(\mathbf{E})z + \det(\mathbf{E})$ (Crassidis & Junkins, 2011). In general, for a $n \times n$ order matrix, the Faddeev-Leverrier algorithm can be also used to obtain the characteristic polynomial in a recursive way ((Seem, 1987), (Helmberg, et al., 1993), (Braun & Chaturvedi, 2002)). Let the characteristic polynomial be:

$$p(\mathbf{E}) = z^n + \sum_{k=1}^{n} c_k z^{n-k} \tag{4.8}$$

then, the coefficients c_k can be obtained, for $k = 1, 2, \ldots, n$, by using:

$$\begin{aligned} \mathbf{E}_1 &\equiv \mathbf{E} \\ \mathbf{E}_{k+1} &= \mathbf{E}(\mathbf{E}_k - c_k \mathbf{I}) \\ c_k &= -\frac{\mathrm{Tr}(\mathbf{E}_k)}{k} \end{aligned} \tag{4.9}$$

Then, the comparison of Eq. (4.1) and Eq. (4.7) results in a particular system of non-linear algebraic equations for which the number of equations is equal to the number of parameters in Eq. (4.1). In the direct way, the parameters of the discrete transfer function model, Eq. (4.1), are obtained uniquely using the parameters of the state-space matrices obtained from the thermal network in Eq. (4.7), supposing $\det(\mathrm{z}\mathbf{I} - \mathbf{A}_d) \neq 0$. But in the inverse way, although the solution exists when $\det(\mathrm{z}\mathbf{I} - \mathbf{A}_d) \neq 0$, it is not possible to obtain uniquely the particular parameters of the state-space matrices obtained from the thermal network using only the set of non-linear algebraic equations. The solution is not unique since the set of non-linear algebraic equations can be also solved, for instance, using the canonical forms of the state-space representation ((Crassidis & Junkins, 2011), (Ogata, 2010)).

The number of unknowns of the inverse problem is equal to the number of the state-space matrix parameters, $(\mathbf{A}_d, \mathbf{B}_d, \mathbf{C}_S, \mathbf{D}_S)$, and it is given by the sum of all the matrix elements: #States x #States + #States x #Inputs + #Outputs x #States + #Outputs x #Inputs, where #States, #Inputs, #Outputs represents the number of states, inputs and outputs, respectively. In general, this number of unknowns is different than the number of equations that can be obtained which is given by the number of parameters in the discrete transfer function, Eq. (4.1). Table 4.1 presents examples for different states,

outputs and inputs to show that, in general, the number of unknowns is different from the number of equations when only Eq. (4.1) is used. Furthermore, it should be also noted that, as it is presented in Eq. (4.6), the discrete transfer function is obtained by using the determinant of matrices and the determinant of a matrix is not a bijective function.

Nonetheless, since the discrete transfer function is built from a particular state-space representation derived from a particular thermal network, the knowledge about the structure of this particular state-space representation can be used to observe that there are parameters equal to zero and relations between different state-space matrix parameters. This information, about the structure of the model in its state-space representation derived from a thermal network, needs to be used besides the discrete transfer function, or ARX model, parameters, (m_i, n_{ij}), to obtain the discrete state-space matrix parameters ($\mathbf{A}_\mathrm{d}$, $\mathbf{B}_\mathrm{d}$, $\mathbf{C}_S$, $\mathbf{D}_S$).

Table 4.1. Total number of equations relating discrete transfer function coefficients with state-space matrices and maximum number of unknown variables.

States	Outputs	Inputs	Unknowns	Equations
2	1	4	18	14
2	2	4	24	26
3	2	4	35	35

Finally, it is known that different state-space representations give the same transfer function and that the transfer function does not provide information concerning the physical structure of the model (Ogata, 2010). It is very important to notice that this fact is consistent with the present case since the physical information is not provided by the transfer function itself, but by the known transformation of the thermal network expressed in state-space representation from where the transfer function is built.

4.2.1.1. Illustrative example of the procedure to obtain the discrete state-space matrix parameters using the ARX model parameters

The present section shows, without losing generality, the relations between discrete transfer function, or ARX model, parameters and the parameters of the state space matrices in discrete time for the two-state case considering a particular example with one output and four inputs.

For a system having 2 states, 2 outputs and 4 inputs, as it is presented in the second row of Table 4.1, the state-space matrices in discrete time, without taking into account the known physical structure of the thermal network, are:

$$\mathbf{A}_\mathrm{d} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

$$\mathbf{B}_\mathrm{d} = \begin{bmatrix} b_{11} & b_{12} & b_{13} & b_{14} \\ b_{21} & b_{22} & b_{23} & b_{24} \end{bmatrix} = [\mathbf{b}_1^c \quad \mathbf{b}_2^c \quad \mathbf{b}_3^c \quad \mathbf{b}_4^c] \tag{4.10}$$

$$\mathbf{C}_S = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix} = \begin{bmatrix} \mathbf{c}_1^r \\ \mathbf{c}_2^r \end{bmatrix}$$

$$\mathbf{D}_S = \begin{bmatrix} d_{11} & d_{12} & d_{13} & d_{14} \\ d_{21} & d_{22} & d_{23} & d_{24} \end{bmatrix} = \begin{bmatrix} \mathbf{d}_1^r \\ \mathbf{d}_2^r \end{bmatrix}$$

The total of unknowns in the four matrices, $\mathbf{A}_\mathrm{d}$, $\mathbf{B}_\mathrm{d}$, $\mathbf{C_s}$ and $\mathbf{D}_\mathrm{s}$, is equal to their number of elements; in this case, there are 24 unknowns.

The set of non-linear algebraic equations to be used in order to get the state-space matrix parameters are obtained by introducing in Eq. (4.7) the matrices presented in Eq. (4.10). The use of the first output, $i = 1$, implies the use of $\mathbf{c}_1^r$ and $\mathbf{d}_1^r$ and 14 equations are obtained as it is presented in Eq. (4.11). The use of the second output, $i = 2$, implies the use of $\mathbf{c}_2^r$ and $\mathbf{d}_2^r$ and 14 equations are obtained as it is presented in Eq. (4.14). In Eq. (4.14) there are only 12 different equations regarding to the system of Eq. (4.11).

$$\begin{aligned}
m_1 &= -(a_{11} + a_{22}) = -tr(\mathbf{A}_\mathrm{d}) \\
m_2 &= a_{11}a_{22} - a_{12}a_{21} = det(\mathbf{A}_\mathrm{d}) \\
n_{10} &= d_{11} \\
n_{11} &= -d_{11}(a_{11} + a_{22}) + b_{11}c_{11} + b_{21}c_{12} \\
n_{12} &= d_{11}(a_{11}a_{22} - a_{12}a_{21}) - a_{11}b_{21}c_{12} + a_{12}b_{21}c_{11} + a_{21}b_{11}c_{12} - a_{22}b_{11}c_{11} \\
n_{20} &= d_{12} \\
n_{21} &= -d_{12}(a_{11} + a_{22}) + b_{12}c_{11} + b_{22}c_{12} \\
n_{22} &= d_{12}(a_{11}a_{22} - a_{12}a_{21}) - a_{11}b_{22}c_{12} + a_{12}b_{22}c_{11} + a_{21}b_{12}c_{12} - a_{22}b_{12}c_{11} \\
n_{30} &= d_{13} \\
n_{31} &= -d_{13}(a_{11} + a_{22}) + b_{13}c_{11} + b_{23}c_{12} \\
n_{32} &= d_{13}(a_{11}a_{22} - a_{12}a_{21}) - a_{11}b_{23}c_{12} + a_{12}b_{23}c_{11} + a_{21}b_{13}c_{12} - a_{22}b_{13}c_{11} \\
n_{40} &= d_{14} \\
n_{41} &= -d_{14}(a_{11} + a_{22}) + b_{14}c_{11} + b_{24}c_{12} \\
n_{42} &= d_{14}(a_{11}a_{22} - a_{12}a_{21}) - a_{11}b_{24}c_{12} + a_{12}b_{24}c_{11} + a_{21}b_{14}c_{12} - a_{22}b_{14}c_{11}
\end{aligned} \tag{4.11}$$

Therefore, there are 26 equations and 24 unknowns, as it can be observed in Eq. (4.10), (4.11) and (4.14). In general, the number of equations and unknowns is given by the number of states, outputs and inputs and the number of unknowns is not equal to the number of equations, as it is shown in Table 4.1.

As it has been said previously, to obtain the state-space matrix parameters derived from a thermal network, it is also needed to consider the knowledge about its state-space representation structure to obtain extra-relations between its parameters and eliminate unknowns. The state-space representation:

$$\dot{\boldsymbol{\theta}}_C = \mathbf{A}_S\boldsymbol{\theta}_C + \mathbf{B}_S\mathbf{u} \tag{4.12}$$

$$\begin{bmatrix} \boldsymbol{\theta}_C \\ \boldsymbol{\theta}_0 \end{bmatrix} = \begin{bmatrix} \mathbf{I} \\ \mathbf{C}_S \end{bmatrix} \boldsymbol{\theta}_C + \begin{bmatrix} \mathbf{0} \\ \mathbf{D}_S \end{bmatrix} \mathbf{u} \tag{4.13}$$

is obtained by the transformation of the heat equation, following the methodology detailed in Chapter 3 ((Ghiaus, 2013) , (Naveros & Ghiaus, 2015)). The structure of the state-space representation, i.e. the state-space matrices ($\mathbf{A}_S$, $\mathbf{B}_S$, $\mathbf{C}_S$, $\mathbf{D}_S$), is obtained using the known thermal network, i.e. the incidence matrix, $\mathbf{A}$, the conductance matrix, $\mathbf{G}$, and the thermal capacities matrix, $\mathbf{C}$. Then, by time discretization the structure of the discrete matrices ($\mathbf{A}_\mathrm{d}$, $\mathbf{B}_\mathrm{d}$) is inferred.

$$
\begin{aligned}
&m_1 = -(a_{11} + a_{22}) = -tr(\mathbf{A}_\mathrm{d}) \\
&m_2 = a_{11}a_{22} - a_{12}a_{21} = det(\mathbf{A}_\mathrm{d}) \\
&n'_{10} = d_{21} \\
&n'_{11} = -d_{21}(a_{11} + a_{22}) + b_{11}c_{21} + b_{21}c_{22} \\
&n'_{12} = d_{21}(a_{11}a_{22} - a_{12}a_{21}) - a_{11}b_{21}c_{22} + a_{12}b_{21}c_{21} + a_{21}b_{11}c_{22} - a_{22}b_{11}c_{21} \\
&n'_{20} = d_{22} \\
&n'_{21} = -d_{22}(a_{11} + a_{22}) + b_{12}c_{21} + b_{22}c_{22} \\
&n'_{22} = d_{22}(a_{11}a_{22} - a_{12}a_{21}) - a_{11}b_{22}c_{22} + a_{12}b_{22}c_{21} + a_{21}b_{12}c_{22} - a_{22}b_{12}c_{21} \\
&n'_{30} = d_{23} \\
&n'_{31} = -d_{23}(a_{11} + a_{22}) + b_{13}c_{21} + b_{23}c_{22} \\
&n'_{32} = d_{23}(a_{11}a_{22} - a_{12}a_{21}) - a_{11}b_{23}c_{22} + a_{12}b_{23}c_{21} + a_{21}b_{13}c_{22} - a_{22}b_{13}c_{21} \\
&n'_{40} = d_{24} \\
&n'_{41} = -d_{24}(a_{11} + a_{22}) + b_{14}c_{21} + b_{24}c_{22} \\
&n'_{42} = d_{24}(a_{11}a_{22} - a_{12}a_{21}) - a_{11}b_{24}c_{22} + a_{12}b_{24}c_{21} + a_{21}b_{14}c_{22} - a_{22}b_{14}c_{21}
\end{aligned}
\tag{4.14}
$$

The next general procedure, which may be applied independently of the order of the model, uses the structure of the state-space representation and the discrete transfer function, or ARX model, parameters (m_i, n_{ij}) to obtain the values of the state-space matrix parameters. It begins by using the structure of the model, i.e. its graphical representation from which may be built the incidence matrix, $\mathbf{A}$, (composed by 1, 0 and -1 elements), the matrix of thermal conductances, $\mathbf{G}$, the matrix of thermal capacities, $\mathbf{C}$, (diagonal matrices) and the vector of source temperatures, $\mathbf{b}$. This allows us to obtain the state-space matrices ($\mathbf{A}_S$, $\mathbf{B}_S$, $\mathbf{C}_S$, $\mathbf{D}_S$) as a function of the incidence matrix and the thermal conductance and capacity matrices by using:

$$\mathbf{A}_S = \mathbf{C}_C^{-1}(-\mathbf{K}_{21}\mathbf{K}_{11}^{-1}\mathbf{K}_{12} + \mathbf{K}_{22}) \tag{4.15}$$

$$\mathbf{B}_S = \mathbf{C}_C^{-1}[-\mathbf{K}_{21}\mathbf{K}_{11}^{-1}\mathbf{K}_{b1} + \mathbf{K}_{b2} \quad -\mathbf{K}_{21}\mathbf{K}_{11}^{-1} \quad \mathbf{I}_{22}] \tag{4.16}$$

$$\mathbf{C}_S = -\mathbf{K}_{11}^{-1}\mathbf{K}_{12} \tag{4.17}$$

$$\mathbf{D}_S = -\mathbf{K}_{11}^{-1}[\mathbf{K}_{b1} \quad \mathbf{I}_{11} \quad \mathbf{0}] \tag{4.18}$$

where $\mathbf{K}_{11}, \mathbf{K}_{12}, \mathbf{K}_{21}$ and $\mathbf{K}_{22}$ are blocks of the partitioned matrix $\mathbf{K} \equiv \mathbf{A}^T\mathbf{G}\mathbf{A}$ obtained according to the partitioning of the matrix $\mathbf{C}$ (Chapter 3), and $\mathbf{K}_{b1}$ and $\mathbf{K}_{b2}$ are blocks of the partitioned matrix $\mathbf{K}_b \equiv \mathbf{A}^T\mathbf{G}\mathbf{b}$ obtained according to the partitioning of the matrix $\mathbf{C}$ (Chapter 3).

Then, the structure of the matrices ($\mathbf{A}_\mathrm{d}$, $\mathbf{B}_\mathrm{d}$) is inferred. The steps to follow, using the state-space matrices ($\mathbf{A}_\mathrm{d}$, $\mathbf{B}_\mathrm{d}$, $\mathbf{C}_S$, $\mathbf{D}_S$) are:

1) By using the information on state-space matrices, Eq. (4.15)-(4.18), find the matrix parameters which are zero and remove them from Eq. (4.10) reducing the number of unknowns.
2) By using the information on state-space matrices, Eq. (4.15)-(4.18), find the relations between matrix parameters, different from zero, from Eq. (4.10).
3) Update Eq. (4.11) and Eq. (4.14) with the information obtained in step 1 and adding the extra equations obtained in step 2.
4) Use a substitution method to solve the set of equations given by step 3 to obtain all the discrete state-space matrix parameters, Eq. (4.10), which are different from zero. In the first loop, the matrix parameters that can be identified directly from the discrete transfer function, or ARX model, parameters are obtained. The next loops will obtain by substitution new

discrete state-space matrix parameters until all parameters are found or until the number of loops made is equal to the initial number of unknowns.

This automatic procedure does not need to estimate the ARX model parameters to show the identifiability of the thermal network parameters, i.e. to show if there exists a solution for obtaining the thermal network parameters and to show if such solution is unique.

4.2.1.2. Application of the procedure to obtain the discrete state-space matrix parameters using the ARX model parameters

As an example of the procedure, we consider a problem of heat transfer in buildings. The, model consisting in 4 nodes and 5 branches which could represent a building, for which two nodes have non-negligible heat capacity. The structure of the model is given by the network shown in Figure 4.2.

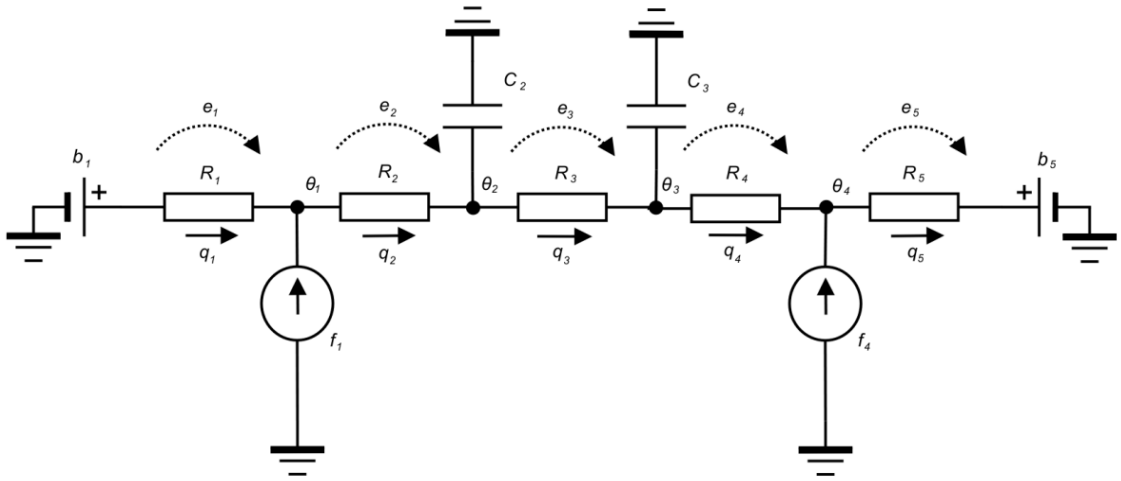

Figure 4.2. Thermal network representing a building using N=4 nodes, 2 nodes with non-negligible thermal capacities, two temperatures sources and two heat rate sources

From Figure 4.2, the incidence matrix, **A**, the diagonal matrix of thermal conductances, **G**, and the diagonal matrix of thermal capacities, **C**, are:

$$\mathbf{A} = \begin{array}{c} \begin{matrix} \theta_1 & \theta_4 & \theta_2 & \theta_3 \end{matrix} \\ \begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & -1 & 0 & 0 \end{bmatrix} \end{array} \begin{matrix} q_1 \\ q_2 \\ q_3 \\ q_4 \\ q_5 \end{matrix}, \quad \mathbf{G} = \begin{bmatrix} R_1^{-1} & 0 & 0 & 0 & 0 \\ 0 & R_2^{-1} & 0 & 0 & 0 \\ 0 & 0 & R_3^{-1} & 0 & 0 \\ 0 & 0 & 0 & R_4^{-1} & 0 \\ 0 & 0 & 0 & 0 & R_5^{-1} \end{bmatrix} \begin{matrix} q_1 \\ q_2 \\ q_3 \\ q_4 \\ q_5 \end{matrix},$$

$$\mathbf{C} = \begin{array}{c} \begin{matrix} \theta_1 & \theta_4 & \theta_2 & \theta_3 \end{matrix} \\ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & C_2 & 0 \\ 0 & 0 & 0 & C_3 \end{bmatrix} \end{array}$$

Next, the general procedure is applied step by step:

1) Evaluating Eq. (4.15)-(4.18); for the present example, the following parameters of the discrete state-space matrices, Eq. (4.10), are zero:
   b_{12}, b_{14}, b_{21} and b_{23} ; c_{12} and c_{21} ; and d_{12}, d_{14}, d_{21} and d_{23}
2) From Eq. (4.15)-(4.18), the following relations between discrete state-space matrix parameters, Eq. (4.10), are found by comparison:

$$d_{11} \to 1 - c_{11}$$
$$a_{11} \to 1 - b_{11} - a_{12}$$
$$a_{22} \to 1 + b_{22} - a_{21}$$

3) Eq. (4.11) is simplified using step 1 and the three new relations shown in step 2 are added to Eq. (4.19), where the known variables of the autoregressive model Eq. (4.2), are in bold.

$$\begin{aligned}
&\boldsymbol{m_1} \to -(a_{11} + a_{22})\\
&\boldsymbol{m_2} \to a_{11}a_{22} - a_{12}a_{21}\\
&\boldsymbol{n_{10}} \to d_{11}\\
&\boldsymbol{n_{11}} \to \boldsymbol{m_1} d_{11} + b_{11}c_{11}\\
&\boldsymbol{n_{12}} \to \boldsymbol{m_2} d_{11} - a_{22}b_{11}c_{11}\\
&\boldsymbol{n_{22}} \to a_{12}b_{22}c_{11}\\
&\boldsymbol{n_{30}} \to d_{13}\\
&\boldsymbol{n_{31}} \to \boldsymbol{m_1} d_{13} + b_{13}c_{11}\\
&\boldsymbol{n_{32}} \to \boldsymbol{m_2} d_{13} - a_{22}b_{13}c_{11}\\
&\boldsymbol{n_{42}} \to a_{12}b_{24}c_{11}\\
&d_{11} \to 1 - c_{11}\\
&a_{11} \to 1 - b_{11} - a_{12}\\
&a_{22} \to 1 + b_{22} - a_{21}
\end{aligned} \tag{4.19}$$

4) Using the information obtained in step 3, the chain of loops may start. At the beginning of each loop, the parameters which are found are in bold (see Appendix A4). They are used to find out new unknown parameters. At each new loop, the parameters that are found are put in bold and they are used by substitution, besides the previously known parameters, up to the point at which all the discrete state-space matrix parameters are found.

The step 4, which uses a substitution method, is shown in the Appendix A4, where it is observed that after seven loops, all the parameters have been obtained. In the present example, which can be extended to every simple chain, all the discrete state-space matrix parameters, derived from the thermal network, are obtained.

Finally, it is needed to use the relations between discrete and continuous state-space matrices, supposing forward Euler approximation: $\mathbf{A}_d = 1 + \mathbf{A}_S \Delta t$ and $\mathbf{B}_d = \mathbf{B}_S \Delta t$, for obtaining the parameters of the continuous state-space matrices $(\mathbf{A}_S, \mathbf{B}_S)$.

4.2.2. Obtaining the thermal network parameters using the state-space matrix parameters

Chapter 3 has detailed how, by using the thermal network characterized by the incidence matrix, **A**, the thermal conductance matrix, **G,** and the thermal capacity matrix, **C**, it can be obtained the state-space matrices $(\mathbf{A}_S, \mathbf{B}_S, \mathbf{C}_S, \mathbf{D}_S)$ by using Eq. (4.15)-(4.18). In the inverse way, the thermal conductance matrix, **G,** and the thermal capacity matrix, **C**, can be obtained in an unique way, since the linear transformations are an isomorphism, by using the state-space matrices $(\mathbf{A}_S, \mathbf{B}_S, \mathbf{C}_S, \mathbf{D}_S)$. The

transformation from the state-space representation to the set of differential algebraic equations (DAE) can be done with the relations shown hereafter.

To this purpose, in agreement with the input vector, as it was defined in Chapter 3:

$$\mathbf{u} = [\mathbf{b} \quad \mathbf{f}_0 \quad \mathbf{f}_C]^T \tag{1}$$

where $\mathbf{b}$ is the vector of temperature sources, $\mathbf{f}_0$ is the vector of heat rate sources connected to the temperature nodes with negligible thermal capacity, and $\mathbf{f}_C$ is the vector of heat rate sources connected to the temperature nodes with non-negligible thermal capacity. Let Eq. (4.16) and (4.18) be expressed in block matrices as:

$$\mathbf{B}_\mathrm{s} = [\mathbf{B}_{\mathrm{s}1} \quad \mathbf{B}_{\mathrm{s}2} \quad \mathbf{B}_{\mathrm{s}3}] \tag{4.20}$$

$$\mathbf{D}_\mathrm{s} = [\mathbf{D}_{\mathrm{s}1} \quad \mathbf{D}_{\mathrm{s}2} \quad \mathbf{D}_{\mathrm{s}3}] \tag{4.21}$$

From Eq. (4.16), (4.18) and Eq. (4.20)-(4.21), the following relations are direct:

$$\mathbf{C}_\mathrm{C} = \mathbf{B}_{\mathrm{s}3}^{-1} \tag{4.22}$$

$$-\mathbf{K}_{11}^{-1} = \mathbf{D}_{\mathrm{s}2} \tag{4.23}$$

From Eq. (4.18), (4.21) and (4.23):

$$\mathbf{K}_{\mathrm{b}1} = \mathbf{D}_{\mathrm{s}2}^{-1}\mathbf{D}_{\mathrm{s}1} \tag{4.24}$$

From Eq. (4.17) and (4.23):

$$\mathbf{K}_{12} = \mathbf{D}_{\mathrm{s}2}^{-1}\mathbf{C}_\mathrm{s} \tag{4.25}$$

From Eq. (4.16), (4.22) and (4.23):

$$\mathbf{K}_{21} = \mathbf{B}_{\mathrm{s}3}^{-1}\mathbf{B}_{\mathrm{s}2}\mathbf{D}_{\mathrm{s}2}^{-1} \tag{4.26}$$

Eq. (4.25) and Eq. (4.26) are related by the fact that $\mathbf{K}_{12} = \mathbf{K}_{21}^T$ since $\mathbf{G}$ is a symmetric matrix (since it is diagonal) which implies that $\mathbf{K} = -\mathbf{A}^T\mathbf{G}\mathbf{A}$ is also a symmetric matrix.

From Eq. (4.16), (4.22), (4.23), (4.24) and (4.26):

$$\mathbf{K}_{\mathrm{b}2} = \mathbf{B}_{\mathrm{s}3}^{-1}(\mathbf{B}_{\mathrm{s}1} - \mathbf{B}_{\mathrm{s}2}\mathbf{D}_{\mathrm{s}2}^{-1}\mathbf{D}_{\mathrm{s}1}) \tag{4.27}$$

From Eq. (4.15), (4.22), (4.23), (4.25) and (4.26):

$$\mathbf{K}_{22} = \mathbf{B}_{\mathrm{s}3}^{-1}(\mathbf{A}_\mathrm{s} - \mathbf{B}_{\mathrm{s}2}\mathbf{D}_{\mathrm{s}2}^{-1}\mathbf{C}_\mathrm{s}) \tag{4.28}$$

It must be noted that for obtaining the matrix $\mathbf{C}_\mathrm{C}$, only $\mathbf{B}_{\mathrm{s}3}^{-1}$ is needed. From $\mathbf{K}_{\mathrm{b}1}$ and $\mathbf{K}_{\mathrm{b}2}$, the matrix $\mathbf{K}_\mathrm{b}$ is obtained by using:

$$\mathbf{C}\dot{\boldsymbol{\theta}} = -\mathbf{A}^T\mathbf{G}\mathbf{A}\boldsymbol{\theta} + \mathbf{A}^T\mathbf{G}\mathbf{b} + \mathbf{f} \tag{4.29}$$

which in blocks is expressed as:

$$\begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{C}_C \end{bmatrix}\begin{bmatrix} \dot{\boldsymbol{\theta}}_0 \\ \dot{\boldsymbol{\theta}}_C \end{bmatrix} = \begin{bmatrix} \mathbf{K}_{11} & \mathbf{K}_{12} \\ \mathbf{K}_{21} & \mathbf{K}_{22} \end{bmatrix}\begin{bmatrix} \boldsymbol{\theta}_0 \\ \boldsymbol{\theta}_C \end{bmatrix} + \begin{bmatrix} \mathbf{K}_{b1} \\ \mathbf{K}_{b2} \end{bmatrix}\mathbf{b} + \begin{bmatrix} \mathbf{I}_{11} & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_{22} \end{bmatrix}\begin{bmatrix} \mathbf{f}_0 \\ \mathbf{f}_C \end{bmatrix} \tag{4.30}$$

Then, the matrix $\mathbf{G}$ can be found by using that $\mathbf{K}_b = \mathbf{A}^T\mathbf{G}$.

Nonetheless, even in the case that $\mathbf{B}_{s3}$ cannot be known because its inputs are not exciting the system, the problem can be solved if it is used the fact that the $\mathbf{K}$ matrix is symmetric, which implies that $\mathbf{K}_{12} = \mathbf{K}_{21}^T$ and $\mathbf{K}_{11}$, $\mathbf{K}_{11}^{-1}$ are symmetric, i.e. $\mathbf{K}_{11}^{-1} = (\mathbf{K}_{11}^{-1})^T$ and $\mathbf{K}_{21}\mathbf{K}_{11}^{-1} = [\mathbf{K}_{11}^{-1}\mathbf{K}_{12}]^T$, obtaining:

$$\mathbf{B}_{s2} = \mathbf{C}_C^{-1}\mathbf{C}_s^T \tag{4.31}$$

Then, it is possible to obtain the non-negligible thermal capacities:

$$\mathbf{C}_C = \mathbf{C}_s^T\mathbf{B}_{s2}^T(\mathbf{B}_{s2}\mathbf{B}_{s2}^T)^{-1} \tag{4.32}$$

where $\mathbf{B}_{s2}^T$ is used to make the pseudoinverse since the matrix $\mathbf{B}_{s2}$ is not always a square matrix. Using Eq. (4.32) it is possible to obtain $\mathbf{B}_{s3}^{-1}$ from Eq. (4.22) to be applied in Eq. (4.26)-(4.28).

By using these transformations, it is possible to transform the state-space representation, characterized by matrices ($\mathbf{A}_s$, $\mathbf{B}_s$, $\mathbf{C}_s$, $\mathbf{D}_s$) into a thermal network, characterized by its parameters $\mathbf{G}$ and $\mathbf{C}$ and its incidence matrix $\mathbf{A}$. These last back transformations can be also used when the state-space matrix parameters are obtained in time domain using the state-space representation without using the Laplace transform (Naveros, et al., 2014).

4.3. Conclusions

Once the different classes of models, or model structures, for the heat transfer processes are deduced from first principles as it is shown in Chapter 3, such modelling procedure connects bi-directionally, i.e. it also connects in the inverse way, thermal networks, state-space and ARX model parameters in a general and automatic way.

The order of the different model structures is finite. For practical applications in system identification, the choice of model order may be done following a-priori physical knowledge on the thermal system, or it may be determined by using estimation methods related to the identification procedure which follows the modelling. Chapter 5 will propose how to deal with the selection of the order of the model following the present methodology.

In practical applications, independently of the estimation of the ARX parameters using experimental data, which will be exemplified in Chapter 6, the bi-directional connection between different model structure parameters gives a new insight on the identifiability of the thermal network parameters from a deterministic point of view. The proposed procedure determines if there exists a solution to identify the thermal network parameters, i.e. resistances and capacities, and if such solution is unique without requiring the estimation of the parameter values.

Any of the model structures, which are deduced from the heat equation (i.e. thermal networks, state-space, transfer function or ARX models), may be employed as:

- white-box models for system simulation, i.e. the output is estimated using a known model with known parameters and measured input data;
- grey-box models for system identification, i.e. the model and its parameters are estimated in part from measured data using any estimation method.

Chapters 3-4 are developed under a deterministic point of view that implies that high order models will be closer to the exact description of the heat equation. From a practical point of view, uncertainty and numerical issues need to be considered after modelling since they will be a limit for system identification as well as for system simulation.

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Appendix A4. Obtaining the state-space matrix elements

The procedure of identification described in Section 4.2 uses four steps. To carry out the two first steps, it is needed to build the state-space matrices using the knowledge about the physical system that is contained in the thermal network, i.e., it is needed to use the incidence matrix, **A**, the conductance matrix, **G**, and the thermal capacity matrix, **C**. For the example used in the present study, representing

a simple chain, the state-space matrices obtained from the thermal network shown in Figure 4.2 are given hereafter.

First, the matrices $\mathbf{K} \equiv -\mathbf{A}^T\mathbf{G}\mathbf{A}$ and $\mathbf{K}_b \equiv \mathbf{A}^T\mathbf{G}$ need to be obtained and expressed in block matrices:

$$\mathbf{K} = \left[\begin{array}{cc:cc} -\frac{1}{R_1}-\frac{1}{R_2} & 0 & \frac{1}{R_2} & 0 \\ 0 & -\frac{1}{R_4}-\frac{1}{R_5} & 0 & \frac{1}{R_4} \\ \hdashline \frac{1}{R_2} & 0 & -\frac{1}{R_2}-\frac{1}{R_3} & 0 \\ 0 & \frac{1}{R_4} & 0 & -\frac{1}{R_3}-\frac{1}{R_4} \end{array}\right] \tag{A4.1}$$

$$\mathbf{K}_b = \left[\begin{array}{ccccc} \frac{1}{R_1} & -\frac{1}{R_2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{R_4} & -\frac{1}{R_5} \\ \hdashline 0 & \frac{1}{R_2} & -\frac{1}{R_3} & 0 & 0 \\ 0 & 0 & \frac{1}{R_3} & -\frac{1}{R_4} & 0 \end{array}\right] \tag{A4.2}$$

Then, from Eq. (4.15), the state matrix, $\mathbf{A}_S$, is:

$$\mathbf{A}_S = \mathbf{C}_C^{-1}(-\mathbf{K}_{21}\mathbf{K}_{11}^{-1}\mathbf{K}_{12} + \mathbf{K}_{22})$$

$$\mathbf{A}_S = \left[\begin{array}{cc} -\frac{1}{(R_1+R_2)C_2}-\frac{1}{R_3C_2} & \frac{1}{R_3C_2} \\ \frac{1}{R_3C_3} & -\frac{1}{(R_4+R_5)C_3}-\frac{1}{R_3C_3} \end{array}\right] \tag{A4.3}$$

From Eq. (4.16), the input matrix, $\mathbf{B}_S$, is:

$$\mathbf{B}_S = \mathbf{C}_C^{-1}[-\mathbf{K}_{21}\mathbf{K}_{11}^{-1}\mathbf{K}_{b1} + \mathbf{K}_{b2} \quad -\mathbf{K}_{21}\mathbf{K}_{11}^{-1} \quad \mathbf{I}_{22}]$$

$$\mathbf{B}_S = \left[\begin{array}{ccccccccc} \frac{1}{(R_1+R_2)C_2} & \frac{1}{(R_1+R_2)C_2} & -\frac{1}{R_3C_2} & 0 & 0 & \frac{R_1}{(R_1+R_2)C_2} & 0 & \frac{1}{C_2} & 0 \\ 0 & 0 & \frac{1}{R_3C_3} & -\frac{1}{(R_4+R_5)C_3} & -\frac{1}{(R_4+R_5)C_3} & 0 & \frac{R_5}{(R_4+R_5)C_3} & 0 & \frac{1}{C_3} \end{array}\right] \tag{A4.4}$$

and using the input vector:

$$\mathbf{u} = [\mathbf{b} \quad \mathbf{f}_0 \quad \mathbf{f}_C]^T$$

$$\mathbf{u} = \begin{bmatrix} b_1 & 0 & 0 & 0 & b_5 & f_{01} & f_{04} & 0 & 0 \end{bmatrix} \tag{A4.5}$$

the input matrix is reduced to:

$$\mathbf{B}_S = \begin{bmatrix} \dfrac{1}{(R_1+R_2)C_2} & 0 & \dfrac{R_1}{(R_1+R_2)C_2} & 0 \\ 0 & -\dfrac{1}{(R_4+R_5)C_3} & 0 & \dfrac{R_5}{(R_4+R_5)C_3} \end{bmatrix} \tag{A4.6}$$

From Eq. (4.17), the output matrix, $\mathbf{C}_S$, is:

$$\mathbf{C}_S = -\mathbf{K}_{11}^{-1}\mathbf{K}_{12}$$

$$\mathbf{C}_S = \begin{bmatrix} \dfrac{R_1}{R_1+R_2} & 0 \\ 0 & \dfrac{R_5}{R_4+R_5} \end{bmatrix} \tag{A4.7}$$

From Eq. (4.18), the feed through matrix, $\mathbf{D}_S$, is:

$$\mathbf{D}_S = -\mathbf{K}_{11}^{-1}[\mathbf{K}_{b1} \quad \mathbf{I}_{11} \quad \mathbf{0}]$$

$$\mathbf{D}_S = \begin{bmatrix} \dfrac{R_2}{R_1+R_2} & -\dfrac{R_1}{R_1+R_2} & 0 & 0 & 0 & \dfrac{R_1R_2}{R_1+R_2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \dfrac{R_5}{R_4+R_5} & -\dfrac{R_4}{R_4+R_5} & 0 & \dfrac{R_4R_5}{R_4+R_5} & 0 & 0 \end{bmatrix} \tag{A4.8}$$

and using the input vector, Eq. (A4.5), the feed through matrix is reduced to:

$$\mathbf{D}_S = \begin{bmatrix} \dfrac{R_2}{R_1+R_2} & 0 & \dfrac{R_1R_2}{R_1+R_2} & 0 \\ 0 & -\dfrac{R_4}{R_4+R_5} & 0 & \dfrac{R_4R_5}{R_4+R_5} \end{bmatrix} \tag{A4.9}$$

By inspection, the elements of Eq. (A4.3), (A4.6), (A4.7) and Eq. (A4.9) which are equal to zero are obtained and extra equations are found to be used in the third step.

A substitution method is proposed in this study to solve the fourth step and to find the values of the elements of the state-space matrices. This method will give the solution for the thermal network representing the physical system after a finite number of steps. In this example, after seven loops the values of all the elements of the state space matrices, which are different to zero, are estimated (Table A4.1).

Once the state-space matrices in discrete time are obtained, the state-space matrices in continuous time can be derived within the limits of the time discretization approach. The thermal network parameters will be always found since the transformation between the set of DAE and the state-space representation in continuous time does not imply any approximation. Finally, the physical parameters of the heat equation can be derived from the thermal network parameters within the limits of the space discretization.

Table A4.1. Third step of the procedure of identification: substitution method

First loop: d_{11}, d_{13} obtained	Second loop: c_{11} obtained	Third loop: b_{11}, b_{13} obtained
$\boldsymbol{m_1} \rightarrow -(a_{11} + a_{22})$	$\boldsymbol{m_1} \rightarrow -(a_{11} + a_{22})$	$\boldsymbol{m_1} \rightarrow -(a_{11} + a_{22})$
$\boldsymbol{m_2} \rightarrow a_{11}a_{22} - a_{12}a_{21}$	$\boldsymbol{m_2} \rightarrow a_{11}a_{22} - a_{12}a_{21}$	$\boldsymbol{m_2} \rightarrow a_{11}a_{22} - a_{12}a_{21}$
$\boldsymbol{n_{10}} \rightarrow \boldsymbol{d_{11}}$	$\boldsymbol{n_{10}} \rightarrow \boldsymbol{d_{11}}$	$\boldsymbol{n_{10}} \rightarrow \boldsymbol{d_{11}}$
$\boldsymbol{n_{11}} \rightarrow \boldsymbol{m_1 d_{11}} + b_{11}c_{11}$	$\boldsymbol{n_{11}} \rightarrow \boldsymbol{m_1 d_{11}} + b_{11}\boldsymbol{c_{11}}$	$\boldsymbol{n_{11}} \rightarrow \boldsymbol{m_1 d_{11}} + \boldsymbol{b_{11} c_{11}}$
$\boldsymbol{n_{12}} \rightarrow \boldsymbol{m_2 d_{11}} - a_{22}b_{11}c_{11}$	$\boldsymbol{n_{12}} \rightarrow \boldsymbol{m_2 d_{11}} - a_{22}b_{11}\boldsymbol{c_{11}}$	$\boldsymbol{n_{12}} \rightarrow \boldsymbol{m_2 d_{11}} - a_{22}\boldsymbol{b_{11} c_{11}}$
$\boldsymbol{n_{22}} \rightarrow a_{12}b_{22}c_{11}$	$\boldsymbol{n_{22}} \rightarrow a_{12}b_{22}\boldsymbol{c_{11}}$	$\boldsymbol{n_{22}} \rightarrow a_{12}b_{22}\boldsymbol{c_{11}}$
$\boldsymbol{n_{30}} \rightarrow \boldsymbol{d_{13}}$	$\boldsymbol{n_{30}} \rightarrow \boldsymbol{d_{13}}$	$\boldsymbol{n_{30}} \rightarrow \boldsymbol{d_{13}}$
$\boldsymbol{n_{31}} \rightarrow \boldsymbol{m_1 d_{13}} + b_{13}c_{11}$	$\boldsymbol{n_{31}} \rightarrow \boldsymbol{m_1 d_{13}} + b_{13}\boldsymbol{c_{11}}$	$\boldsymbol{n_{31}} \rightarrow \boldsymbol{m_1 d_{13}} + \boldsymbol{b_{13} c_{11}}$
$\boldsymbol{n_{32}} \rightarrow \boldsymbol{m_2 d_{13}} - a_{22}b_{13}c_{11}$	$\boldsymbol{n_{32}} \rightarrow \boldsymbol{m_2 d_{13}} - a_{22}b_{13}\boldsymbol{c_{11}}$	$\boldsymbol{n_{32}} \rightarrow \boldsymbol{m_2 d_{13}} - a_{22}\boldsymbol{b_{13} c_{11}}$
$\boldsymbol{n_{42}} \rightarrow a_{12}b_{24}c_{11}$	$\boldsymbol{n_{42}} \rightarrow a_{12}b_{24}\boldsymbol{c_{11}}$	$\boldsymbol{n_{42}} \rightarrow a_{12}b_{24}\boldsymbol{c_{11}}$
$\boldsymbol{d_{11}} \rightarrow 1 - c_{11}$	$\boldsymbol{d_{11}} \rightarrow 1 - \boldsymbol{c_{11}}$	$\boldsymbol{d_{11}} \rightarrow 1 - \boldsymbol{c_{11}}$
$a_{11} \rightarrow 1 - b_{11} - a_{12}$	$a_{11} \rightarrow 1 - b_{11} - a_{12}$	$a_{11} \rightarrow 1 - \boldsymbol{b_{11}} - a_{12}$
$a_{22} \rightarrow 1 + b_{22} - a_{21}$	$a_{22} \rightarrow 1 + b_{22} - a_{21}$	$a_{22} \rightarrow 1 + b_{22} - a_{21}$
Fourth loop: a_{22} obtained	Fifth loop: a_{11} obtained	Sixth loop: a_{12} obtained
$\boldsymbol{m_1} \rightarrow -(a_{11} + \boldsymbol{a_{22}})$	$\boldsymbol{m_1} \rightarrow -(\boldsymbol{a_{11}} + \boldsymbol{a_{22}})$	$\boldsymbol{m_1} \rightarrow -(\boldsymbol{a_{11}} + \boldsymbol{a_{22}})$
$\boldsymbol{m_2} \rightarrow a_{11}\boldsymbol{a_{22}} - a_{12}a_{21}$	$\boldsymbol{m_2} \rightarrow \boldsymbol{a_{11}a_{22}} - a_{12}a_{21}$	$\boldsymbol{m_2} \rightarrow \boldsymbol{a_{11}a_{22}} - \boldsymbol{a_{12}}a_{21}$
$\boldsymbol{n_{10}} \rightarrow \boldsymbol{d_{11}}$	$\boldsymbol{n_{10}} \rightarrow \boldsymbol{d_{11}}$	$\boldsymbol{n_{10}} \rightarrow \boldsymbol{d_{11}}$
$\boldsymbol{n_{11}} \rightarrow \boldsymbol{m_1 d_{11}} + \boldsymbol{b_{11} c_{11}}$	$\boldsymbol{n_{11}} \rightarrow \boldsymbol{m_1 d_{11}} + \boldsymbol{b_{11} c_{11}}$	$\boldsymbol{n_{11}} \rightarrow \boldsymbol{m_1 d_{11}} + \boldsymbol{b_{11} c_{11}}$
$\boldsymbol{n_{12}} \rightarrow \boldsymbol{m_2 d_{11}} - \boldsymbol{a_{22} b_{11} c_{11}}$	$\boldsymbol{n_{12}} \rightarrow \boldsymbol{m_2 d_{11}} - \boldsymbol{a_{22} b_{11} c_{11}}$	$\boldsymbol{n_{12}} \rightarrow \boldsymbol{m_2 d_{11}} - \boldsymbol{a_{22} b_{11} c_{11}}$
$\boldsymbol{n_{22}} \rightarrow a_{12}b_{22}\boldsymbol{c_{11}}$	$\boldsymbol{n_{22}} \rightarrow a_{12}b_{22}\boldsymbol{c_{11}}$	$\boldsymbol{n_{22}} \rightarrow \boldsymbol{a_{12}} b_{22}\boldsymbol{c_{11}}$
$\boldsymbol{n_{30}} \rightarrow \boldsymbol{d_{13}}$	$\boldsymbol{n_{30}} \rightarrow \boldsymbol{d_{13}}$	$\boldsymbol{n_{30}} \rightarrow \boldsymbol{d_{13}}$
$\boldsymbol{n_{31}} \rightarrow \boldsymbol{m_1 d_{13}} + \boldsymbol{b_{13} c_{11}}$	$\boldsymbol{n_{31}} \rightarrow \boldsymbol{m_1 d_{13}} + \boldsymbol{b_{13} c_{11}}$	$\boldsymbol{n_{31}} \rightarrow \boldsymbol{m_1 d_{13}} + \boldsymbol{b_{13} c_{11}}$
$\boldsymbol{n_{32}} \rightarrow \boldsymbol{m_2 d_{13}} - \boldsymbol{a_{22} b_{13} c_{11}}$	$\boldsymbol{n_{32}} \rightarrow \boldsymbol{m_2 d_{13}} - \boldsymbol{a_{22} b_{13} c_{11}}$	$\boldsymbol{n_{32}} \rightarrow \boldsymbol{m_2 d_{13}} - \boldsymbol{a_{22} b_{13} c_{11}}$
$\boldsymbol{n_{42}} \rightarrow a_{12}b_{24}\boldsymbol{c_{11}}$	$\boldsymbol{n_{42}} \rightarrow a_{12}b_{24}\boldsymbol{c_{11}}$	$\boldsymbol{n_{42}} \rightarrow \boldsymbol{a_{12}} b_{24}\boldsymbol{c_{11}}$
$\boldsymbol{d_{11}} \rightarrow 1 - \boldsymbol{c_{11}}$	$\boldsymbol{d_{11}} \rightarrow 1 - \boldsymbol{c_{11}}$	$\boldsymbol{d_{11}} \rightarrow 1 - \boldsymbol{c_{11}}$
$a_{11} \rightarrow 1 - \boldsymbol{b_{11}} - a_{12}$	$\boldsymbol{a_{11}} \rightarrow 1 - \boldsymbol{b_{11}} - a_{12}$	$\boldsymbol{a_{11}} \rightarrow 1 - \boldsymbol{b_{11}} - \boldsymbol{a_{12}}$
$\boldsymbol{a_{22}} \rightarrow 1 + b_{22} - a_{21}$	$\boldsymbol{a_{22}} \rightarrow 1 + b_{22} - a_{21}$	$\boldsymbol{a_{22}} \rightarrow 1 + b_{22} - a_{21}$
Seventh loop: a_{21}, b_{22}, b_{24} obtained		
$\boldsymbol{m_1} \rightarrow -(\boldsymbol{a_{11}} + \boldsymbol{a_{22}})$		
$\boldsymbol{m_2} \rightarrow \boldsymbol{a_{11}a_{22}} - \boldsymbol{a_{12}a_{21}}$		
$\boldsymbol{n_{10}} \rightarrow \boldsymbol{d_{11}}$		
$\boldsymbol{n_{11}} \rightarrow \boldsymbol{m_1 d_{11}} + \boldsymbol{b_{11} c_{11}}$		
$\boldsymbol{n_{12}} \rightarrow \boldsymbol{m_2 d_{11}} - \boldsymbol{a_{22} b_{11} c_{11}}$		
$\boldsymbol{n_{22}} \rightarrow \boldsymbol{a_{12}} \boldsymbol{b_{22}} \boldsymbol{c_{11}}$		
$\boldsymbol{n_{30}} \rightarrow \boldsymbol{d_{13}}$		
$\boldsymbol{n_{31}} \rightarrow \boldsymbol{m_1 d_{13}} + \boldsymbol{b_{13} c_{11}}$		
$\boldsymbol{n_{32}} \rightarrow \boldsymbol{m_2 d_{13}} - \boldsymbol{a_{22} b_{13} c_{11}}$		
$\boldsymbol{n_{42}} \rightarrow \boldsymbol{a_{12}} \boldsymbol{b_{24}} \boldsymbol{c_{11}}$		
$\boldsymbol{d_{11}} \rightarrow 1 - \boldsymbol{c_{11}}$		
$\boldsymbol{a_{11}} \rightarrow 1 - \boldsymbol{b_{11}} - \boldsymbol{a_{12}}$		
$\boldsymbol{a_{22}} \rightarrow 1 + \boldsymbol{b_{22}} - \boldsymbol{a_{21}}$		

CHAPTER 5

ORDER SELECTION OF THERMAL MODELS BY MEASUREMENTS FREQUENCY ANALYSIS

5.1. Introduction

Previous chapters have shown how heat transfer can be modelled by using thermal networks which represent the energy balance in the system formed by the wall and its surroundings, considering that the model is continuous in time and discrete in space. Thermal networks are models in time domain but they can be transformed in a transfer function in frequency domain, which allows using the information about the frequency spectrum of the input time series, which, related to the accuracy of the sensors used for measuring the outputs, gives an appropriate structure of the model, i.e., the order of space discretization that can be used without losing precision (Naveros & Ghiaus, 2015).

In general, mathematical criteria are used to decide on the order of the model. In direct (or simulation) problems, the criterion is mainly based on the comparison of the outputs of the complete (or a very high order) model and the reduced order model. In inverse (or parameter identification) problems, the main idea is to augment the order of the model until the residual (i.e. the difference between the measured output and the output of the model) becomes an uncorrelated white noise ((Ljung, 1999), (Gouda, et al., 2002), (Bacher & Madsen, 2011), (Cole, et al., 2014), (Goyal & Banoach, 2012), (Deng, et al., 2014)).

In practice, the input signals have a limited domain of variation in amplitude and in frequency. The upper limit of the amplitude is given by the nature of the system; its lower limit is given by the accuracy of the measurement devices. Similarly, the lower frequency of the signals depends on the nature of the system and the higher limit is given by the sampling time. Since the large majority of the physical systems are dissipative, the amplitude gain will be constant at low frequencies and reduced at high frequencies. The frequency at which the reduction of the amplitude gain is so high that it cannot be measured due to the limited accuracy of the devices gives the upper limit of the frequency domain on which the frequency response of the complete model and that of the reduced order model need to be similar.

Hereafter this criterion is proposed for the choice of the order of the model and it is shown its application to the experimental parameter identification of a wall under normal weather conditions. The structure of the model is obtained from space and time discretization of a continuous model expressed in state-space form (Ghiaus, 2013). This model will act as a filter for the inputs. The spectral distribution of the input signals reveals the domain on which the two filters, the complete (or the physical process) and the reduced order models, need to give similar results. In this chapter, a homogeneous light wall is used as an example for a detailed frequency study as well as examples of homogeneous building elements with very high and very low time constants. A description of the experimental set-up and of the measured data is presented. Then the methodology proposed in previous chapters is used to study, for homogeneous walls as an example, how an adequate structure

of the model can be chosen based on frequency analysis of the input signals. Low order models are compared with a very high order model (hundreds of states) which could be considered practically continuous in space.

5.2. Experimental set-up and measured data

The measured data used in this chapter stem from a test of a homogeneous lightweight and opaque wall (Guzmán, et al., 2010). It corresponds to the period from 24th April 2010 to 3th May 2010 with a sampling time of 30 minutes, Figure 5.4. This wall was tested under outdoor weather conditions in a test cell (Strachan & Baker, 2008), at "Plataforma Solar de Almeria", of LECE laboratory, in the South East of Spain (*37*° N, *2.4*° W). The measured variables are given in Table 5.1 and a block diagram with inputs and outputs is presented in Figure 5.1. The indoor air temperature was controlled, being maintained at a constant value (Naveros, et al., 2012).

Table 5.1. Measured variables and sensor characteristics

	Measured Variable	Symbol	Units	Accuracy	Range of device	Range of measurements
Inputs						
	Outdoor temperature	T_o	°C	1%	[-200 ÷ 200]	[11 ÷ 31]
	Indoor temperature	T_i	°C	1%	[-200 ÷ 200]	[37 ÷ 41]
	Vertical global solar irradiance	I_{SV}	W/m^2	3 %	[0 ÷ 4000]	[0 ÷ 566]
	Wind speed	w	m/s	2 %	[0 ÷ 50]	[0 ÷ 9]
Outputs						
	Surface outside temperature	T_{so}	°C	1%	[-200 ÷ 200]	[16 ÷ 47]
	Surface inside temperature	T_{si}	°C	1%	[-200 ÷ 200]	[32 ÷ 41]
	Outside heat flux density	Q_o	W/m^2	5 %	[-1000 ÷ 1000]	[-110 ÷ 74]
	Inside heat flux density	Q_i	W/m^2	5 %	[-1000 ÷ 1000]	[-11 ÷ 57]

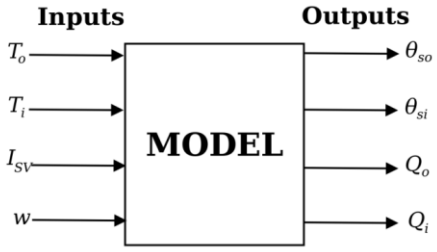


Figure 5.1. Input/output block diagram

5.3. Thermal modelling

As it has been shown in previous chapters, for an isotropic medium (volume), the heat equation is expressed as a partial differential equation (Carslaw & Jaeger, 1986):

$$\rho c \frac{\partial \theta}{\partial t} = -\nabla \cdot (-\kappa \nabla \theta) + p \tag{5.1}$$

where ρ is the medium density, c is the medium specific heat capacity, κ is the thermal conductivity of the medium, p is a function which includes all heat rate sources supplied to the medium, e.g., by

irradiance and convection on its surface and by possible internal gains, θ is the function of temperature distribution in the medium.

It has been demonstrated in Chapter 3 that the heat equation can be written as a set of differential algebraic equations (DAE). The set of DAE may be formed by considering that the medium is constituted by an infinite set of nodes, $N \to \infty$, each one with a temperature θ_l $(l = 1, \dots, N)$, and connected between them by branches, each one with a thermal resistance R_k $(k = 1, \dots, N+1)$ (Ghiaus, 2013):

$$\mathbf{C}_d \dot{\boldsymbol{\theta}}_d = -\mathbf{A}_d^T \mathbf{G} \mathbf{A}_d \boldsymbol{\theta}_d + \mathbf{A}_d^T \mathbf{G} \mathbf{b} + \mathbf{f}_d \tag{5.2}$$

It must be noted that the model given by Eq. (5.1) is based on space continuity assumption, therefore, *N* needs to tend to infinity in Eq. (5.2) to make the distance between nodes to tend to zero, $\boldsymbol{d} \to \mathbf{0}$. In practical application it is considered a finite set of DAE.

The equivalence between both representations is shown in Table 5.2 (Strang, 2007), where for the differential equation,

θ is the function of temperature distribution in the medium,

ρ - the medium density,

c - the medium specific heat capacity,

∇ - the gradient operator,

$\nabla \cdot$ - the divergence operator,

κ - the medium thermal conductivity,

p - the function of heat rate sources supplied to the solid.

for the set of differential algebraic equations (DAE),

$\boldsymbol{\theta}_d$ is the vector of temperatures in the nodes,

$\mathbf{C}_d$ - the diagonal matrix of thermal capacities,

$\mathbf{A}_d$ - the incidence matrix of the thermal network,

$\mathbf{A}_d^T$ - the transpose of the incidence matrix,

$\mathbf{G}$ - the diagonal matrix of thermal conductances,

$\mathbf{b}$ - the vector of temperatures sources on the branches,

$\mathbf{f}_d$ - the vector of heat rate sources.

The system of differential algebraic equations, Eq. (5.2), can be put in state-space representation (Ghiaus, 2013).

Table 5.2. Terminology equivalence between heat equation as a partial differential equation and as a set of differential algebraic equations (DAE)

Heat equation	DAE
θ[K]	$\boldsymbol{\theta}_d$ [K]
ρc [Jm^{-3}K^{-1}]	$\mathbf{C}_d$ [JK^{-1}]
∇ [m^{-1}]	$\mathbf{A}_d$ [-]
$-\nabla\cdot$[m^{-1}]	$\mathbf{A}_d^T$ [-]
κ [Wm^{-1}K^{-1}]	$\mathbf{G}$ [WK^{-1}]
p [Wm^{-3}]	$\mathbf{A}_d^T\mathbf{G}\mathbf{b}+\mathbf{f}_d$ [W]

5.3.1. Advantages of using differential algebraic equations (DAE)

Before moving forward to the study of the choice of the order of the model, it is worth emphasizing the following advantages of using the equivalence between the heat equation and the differential algebraic equation representation.

Heat transfer may be represented by different graphs in order to obtain a particular model to solve the problem. In this dissertation, the choice has been made for thermal networks from a new insight following the work of Gilbert Strang instead of using the thermal-electrical analogy; another example of graph is the balance arrow scheme (Naveros, et al., 2012). Using these techniques, particular cases of Eq. (5.1) may be represented graphically. The differential algebraic equation formulation of the problem shows clearly that all these representations are equivalent. Hereafter, a formal graphical representation of the model is given from which any set of differential algebraic equations (DAE) can be derived.

The system of DAE is a general representation of systems of differential equations for vector valued functions which may represent the classical continuous model described by the heat equation (Eq. (5.1)) as it is shown in Chapter 3. It may be transformed in state-space representation (Ghiaus, 2013), which is suitable for analysis. Different representations can be obtained from DAE ((Rabl, 1988), (Jiménez & Madsen, 2008)):

1. thermal networks;
2. a single differential equation is the simplest particular case included in the set of DAE;
3. differential equations are a particular case of DAE for which there is no algebraic equation;
4. modal analysis is a particular case in which the set of DAE is solved using eigenvalues and eigenvectors (Madsen & Holst, 1995);
5. when the set of DAE is transformed into transfer functions, it can be obtained a particular case of autoregressive models with exogenous, ARX ((Jiménez et al., 2008), (Ljung, 1999)), while keeping the physical interpretation.

Stochastic differential equations and observation equations (Naveros, et al., 2014), may be also a particular case for linear time invariant systems, that can be obtained by adding in Eq. (5.2) the stochastic term, which is an improvement looking for a better performance of the model.

5.4. Differential algebraic equations for a wall

In the case of a wall, the thermal network can be represented as in Figure 5.2 in order to obtain directly a system of DAE arranged in the sense that the differential equations are separated in independent blocks from the algebraic equations.

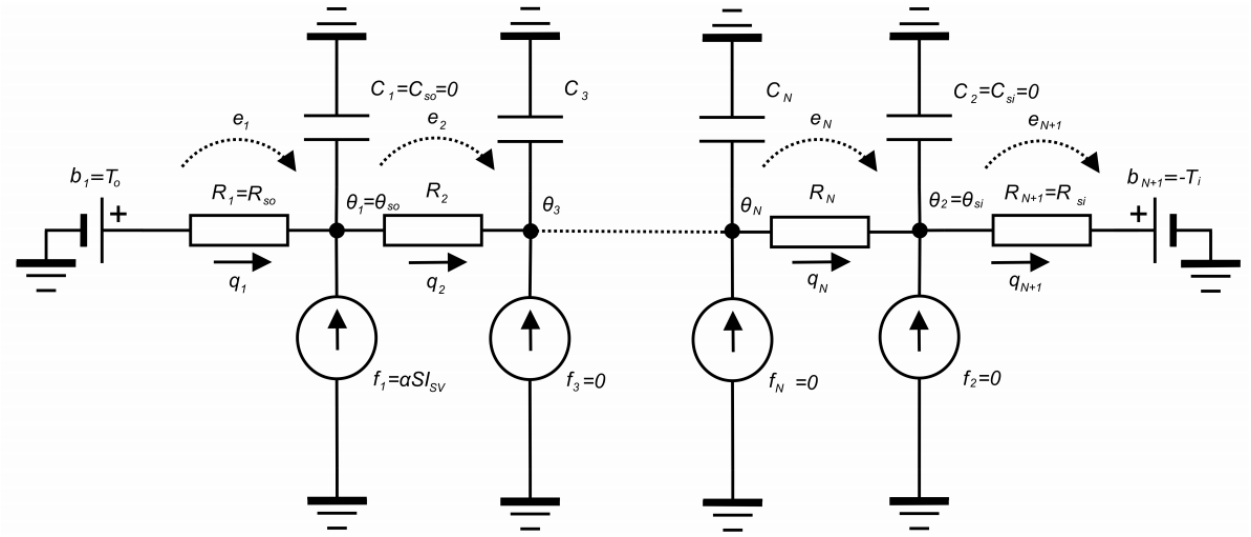


Figure 5.2. Wall nodes scheme having the nodes with negligible heat capacity arranged (*N* nodes)

From Figure 5.2 can be derived the next matrices and vectors for a wall considering *N* nodes:

$$\mathbf{C}=\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & C_3 & 0 & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & C_N \end{bmatrix}$$

where **C** is a re-arranged diagonal matrix of thermal capacities, considering that $C_{so}=0$ and $C_{si}=0$,

$$\boldsymbol{\theta}=\begin{bmatrix}\theta_{so} & \theta_{si} & \cdots & \theta_{N-1} & \theta_N\end{bmatrix}^T$$

is the re-arranged vector of temperatures in nodes,

$$\mathbf{A}=\begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ -1 & 0 & 1 & \ddots & 0 \\ 0 & 0 & -1 & \ddots & 0 \\ \vdots & \ddots & \ddots & \ddots & 0 \\ 0 & 1 & \ddots & \ddots & -1 \\ 0 & -1 & 0 & \cdots & 0 \end{bmatrix}$$

is the re-arranged incidence matrix of the thermal network,

$$\mathbf{G}=\begin{bmatrix} R_{so}^{-1} & 0 & 0 & 0 & 0 \\ 0 & R_2^{-1} & 0 & 0 & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & 0 & R_N^{-1} & 0 \\ 0 & 0 & 0 & 0 & R_{si}^{-1} \end{bmatrix}$$

is the diagonal matrix of the thermal conductances, where R_{so}^{-1} is the outside thermal conductance and R_{si}^{-1} is the inside thermal conductance,

$$\mathbf{b} = \begin{bmatrix} T_o & 0 & \cdots & 0 & -T_i \end{bmatrix}^T$$

is the vector of temperature sources on branches, and

$$\mathbf{f} = \begin{bmatrix} \alpha S I_{SV} & 0 & \cdots & 0 \end{bmatrix}^T$$

is the re-arranged vector of heat rates sources, where α, S are the absorptivity and the surface area of the wall, respectively. In this case, Eq. (5.2) may be written as:

$$\mathbf{C}\dot{\boldsymbol{\theta}} = \mathbf{K}\boldsymbol{\theta} + \mathbf{K}_{\mathrm{b}}\mathbf{b} + \mathbf{f} \tag{5.3}$$

from where we obtain the relations: $\mathbf{K} = -\mathbf{A}^T\mathbf{G}\mathbf{A}$ and $\mathbf{K_b} = \mathbf{A}^T\mathbf{G}$. These relations can be used for a wall once the incidence matrix, **A**, and the diagonal matrix of thermal conductivities, **G**, are known. The partitioned matrix **K** for a wall considering N nodes is:

$$\mathbf{K} = \left[\begin{array}{cc:cccccc} -R_{so}^{-1}-R_2^{-1} & 0 & R_2^{-1} & 0 & \cdots & \cdots & 0 & 0 \\ 0 & -R_N^{-1}-R_{si}^{-1} & 0 & 0 & \cdots & \cdots & 0 & R_N^{-1} \\ \hdashline R_2^{-1} & 0 & -R_2^{-1}-R_3^{-1} & R_3^{-1} & 0 & 0 & \cdots & 0 \\ 0 & 0 & R_3^{-1} & -R_3^{-1}-R_4^{-1} & R_4^{-1} & 0 & \cdots & 0 \\ \vdots & \vdots & 0 & R_4^{-1} & \ddots & \ddots & \ddots & \vdots \\ \vdots & \vdots & 0 & \ddots & \ddots & \ddots & R_{N-2}^{-1} & 0 \\ 0 & 0 & \vdots & \ddots & 0 & R_{N-2}^{-1} & -R_{N-2}^{-1}-R_{N-1}^{-1} & R_{N-1}^{-1} \\ 0 & R_N^{-1} & 0 & \cdots & 0 & 0 & R_{N-1}^{-1} & -R_{N-1}^{-1}-R_N^{-1} \end{array}\right] \tag{5.4}$$

and the partitioned matrix $\mathbf{K_b}$ is:

$$\mathbf{K}_b = \left[\begin{array}{ccccccc} R_{so}^{-1} & -R_2^{-1} & 0 & \cdots & 0 & 0 & 0 \\ 0 & 0 & 0 & \cdots & 0 & R_N^{-1} & -R_{si}^{-1} \\ \hdashline 0 & R_2^{-1} & -R_3^{-1} & 0 & 0 & 0 & 0 \\ 0 & 0 & R_3^{-1} & -R_4^{-1} & 0 & 0 & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & R_{N-1}^{-1} & -R_N^{-1} & 0 \end{array}\right] \tag{5.5}$$

5.4.1. Example for the lowest order dynamical model of a wall

As an illustrative example, let us consider the most simple case, $N = 3$. From Eq. (5.4) and (5.5), the blocks for **K** and $\mathbf{K}_b$ are shown in Eq. (5.6):

$$\left[\begin{array}{cc:c} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \hdashline 0 & 0 & C_3 \end{array}\right] \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{bmatrix} = \left[\begin{array}{cc:c} R_{so}^{-1}+R_2^{-1} & 0 & -R_2^{-1} \\ 0 & R_3^{-1}+R_{si}^{-1} & -R_3^{-1} \\ \hdashline -R_2^{-1} & -R_3^{-1} & R_2^{-1}+R_3^{-1} \end{array}\right] \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix} + \left[\begin{array}{cccc} R_1^{-1} & -R_2^{-1} & 0 & 0 \\ 0 & 0 & R_3^{-1} & -R_4^{-1} \\ \hdashline 0 & R_2^{-1} & -R_3^{-1} & 0 \end{array}\right] \begin{bmatrix} b_1 \\ 0 \\ 0 \\ b_4 \end{bmatrix} + \begin{bmatrix} f_1 \\ 0 \\ 0 \end{bmatrix} \tag{5.6}$$

The blocks from Eq. (5.6) are:

$$\mathbf{K}_{11} = \begin{bmatrix} -R_{so}^{-1} - R_2^{-1} & 0 \\ 0 & -R_3^{-1} - R_{si}^{-1} \end{bmatrix};\ \mathbf{K}_{12} = \begin{bmatrix} R_2^{-1} \\ R_3^{-1} \end{bmatrix}$$
$$\mathbf{K}_{21} = [R_2^{-1} \quad R_3^{-1}];\ \mathbf{K}_{22} = [-R_2^{-1} \ -R_3^{-1}]$$
$$\mathbf{K}_{b1} = \begin{bmatrix} R_{so}^{-1} & -R_2^{-1} & 0 & 0 \\ 0 & 0 & R_3^{-1} & -R_{si}^{-1} \end{bmatrix}$$
$$\mathbf{K}_{b2} = [0 \quad R_2^{-1} \quad -R_3^{-1} \quad 0]$$

Using these blocks, it is possible to obtain the differential and the algebraic equations for a wall from Eq. (5.6). When the number of nodes is $N = 3$, there is only one differential equation and two algebraic equations in Eq. (5.6).

The differential equation for the wall, with $N = 3$, is obtained from Eq. (5.6):

$$C_3\dot{\theta}_3 = \frac{R_{so}^{-1}R_2^{-1}}{R_{so}^{-1} + R_2^{-1}}(T_o - \theta_3) + \frac{R_3^{-1}R_{si}^{-1}}{R_3^{-1} + R_{si}^{-1}}(T_i - \theta_3) + \frac{R_2^{-1}}{R_{so}^{-1} + R_2^{-1}}\,\alpha SI_{SV} \tag{5.7}$$

For a wall, as it is illustrated in Figure 5.2, the two algebraic equations obtained from Eq. (5.6) correspond to the surface temperatures:

$$\boldsymbol{\theta}_0 = \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} \tag{5.8}$$

Eq. (5.8) depends only on resistances placed besides the boundaries of the wall. By notation, $\theta_1 = \theta_{so}$ is the outside surface temperature and $\theta_2 = \theta_{si}$ is the inside surface temperature. The outside surface temperature, θ_{so}, is given by:

$$\theta_{so} = \frac{R_2^{-1}}{R_{so}^{-1} + R_2^{-1}}\theta_3 + \frac{R_{so}^{-1}}{R_{so}^{-1} + R_2^{-1}}T_o + \frac{1}{R_{so}^{-1} + R_2^{-1}}\,\alpha SI_{SV} \tag{5.9}$$

and the inside surface temperature, θ_{si}, is given by:

$$\theta_{si} = \frac{R_3^{-1}}{R_3^{-1} + R_{si}^{-1}}\theta_3 + \frac{R_{si}^{-1}}{R_3^{-1} + R_{si}^{-1}}T_i \tag{5.10}$$

If the heat flux density is wanted in the measurement equation, it is possible to change the observation equation in order to obtain the inside heat flux density, Q_i:

$$Q_i = \frac{1}{S}\frac{R_3^{-1}R_{si}^{-1}}{R_3^{-1} + R_{si}^{-1}}(\theta_3 - T_i) \tag{5.11}$$

and the outside heat flux density, Q_o:

$$Q_o = \frac{1}{S}\frac{R_{so}^{-1}R_2^{-1}}{R_{so}^{-1} + R_2^{-1}}\left(T_o + \frac{1}{R_{so}^{-1}}\,\alpha SI_{SV} - \theta_3\right) \tag{5.12}$$

Eq. (5.7)-(5.12), are equivalent to those presented in a previous work without considering stochastic terms (Naveros, et al., 2014).

5.5. Frequency analysis of the optimal order of the model

Next, it is done the frequency analysis of measurements for estimation of energy efficiency in buildings or building components. In particular, walls are considered.

5.5.1. Transfer function representation

From the differential algebraic equations, it is possible to obtain the transfer function, which is a model in frequency domain (Chapter 3):

$$\mathbf{H}_S = \mathbf{C}_S(s\mathbf{I} - \mathbf{A}_S)^{-1}\mathbf{B}_S + \mathbf{D}_S \tag{5.13}$$

Explicitly, each transfer function H_{ij} from Eq. (5.13), with the number of outputs $i = 1, 2$ and the number of inputs $j = 1, 2, 3$, may be given by:

$$H_{ij} = \frac{\det\left(s\mathbf{I} - \mathbf{A}_S + \mathbf{b}_j^c\mathbf{c}_i^r\right) - \det(s\mathbf{I} - \mathbf{A}_S) + \det(s\mathbf{I} - \mathbf{A}_S)d_{ij}}{\det(s\mathbf{I} - \mathbf{A}_S)} \tag{5.14}$$

In Eq. (5.14) the next notation, where *N* is the number of nodes, is used:

$\mathbf{B}_S = [\mathbf{b}_1^c \quad \mathbf{b}_2^c \quad \mathbf{b}_3^c]$; $\mathbf{b}_j^c = \begin{bmatrix} b_{1j} \\ \vdots \\ b_{N-2j} \end{bmatrix}$ with $j = 1, 2, 3$ corresponding to the inputs;

$\mathbf{C}_S = \begin{bmatrix} \mathbf{c}_1^r \\ \mathbf{c}_2^r \end{bmatrix}$; $\mathbf{c}_i^r = [c_{i1} \quad \cdots \quad c_{iN-2}]$ with $i = 1, 2$, corresponding to the outputs.

Numerator and denominator coefficients in Eq. (5.14) are obtained considering that $\det(\mathbf{E} - s\mathbf{I}) = p(\mathbf{E})$, where $p(\mathbf{E})$ is the characteristic polynomial of matrix $\mathbf{E}$.

For the wall considered as an example in this chapter, the transfer function can be written as:

$$\cdot\mathbf{H}_S = \begin{bmatrix}\mathbf{H}_1 & \mathbf{H}_2\end{bmatrix}^T = \left[\begin{array}{ccc} H_{11} & H_{12} & H_{13} \\ \hline H_{21} & H_{22} & H_{23} \end{array}\right] \tag{5.15}$$

If surface temperatures are used as outputs, then:

$\mathbf{H}_1 = \left[\frac{\theta_{so}}{T_o} \quad \frac{\theta_{so}}{T_i} \quad \frac{\theta_{so}}{I_s}\right]$; $\mathbf{H}_2 = \left[\frac{\theta_{si}}{T_o} \quad \frac{\theta_{si}}{T_i} \quad \frac{\theta_{si}}{I_s}\right]$.

If heat flux densities are used as outputs, then:

$\mathbf{H}_1 = \left[\frac{Q_o}{T_o} \quad \frac{Q_o}{T_i} \quad \frac{Q_o}{I_s}\right]$; $\mathbf{H}_2 = \left[\frac{Q_i}{T_o} \quad \frac{Q_i}{T_i} \quad \frac{Q_i}{I_s}\right]$.

Once the structure of the model is proposed, the transfer function representation for $N = 3$ up to $N \to \infty$ (e.g. $N = 100$), is used in the next section to determine the optimal order of the model which may be chosen to describe adequately the thermal behavior of the wall.

5.5.2. Frequency spectrum

In the study of the time series, Fourier analysis is a useful tool to represent a signal as a sum of harmonic functions. For linear systems, the superposition principle allows to study the problem for each frequency and then to obtain the final result by adding the results corresponding to each frequency.

Since the time series obtained in an experiment are composed of discrete values, the Discrete Fourier Transform is used, in particular, the Fast Fourier Transform algorithm. Before applying this

algorithm to the time series obtained in the experiments presented in this study, an example is introduced in which the time series are obtained as the sum of three discrete harmonic functions. A time series $f(t)$ can be expressed as an infinite sum of harmonic functions:

$$f(t) = \sum_{n=0}^{\infty} A_n sin(2\pi f_n t + \phi_n) \tag{5.16}$$

where A_n are the amplitudes, $f_n = nf$ are the frequencies and ϕ_n are the phases of each harmonic function, respectively. As in practical applications the sum is not infinite, Eq. (5.16) can be approximated by using N_a finite harmonic functions:

$$f(t) \approx f_{N_a}(t) = \sum_{n=0}^{N_a} A_n sin(2\pi f_n t + \phi_n) \tag{5.17}$$

For signals representing variables characterizing the weather (i.e. outdoor temperature, solar irradiation, wind speed), the phases, $\phi_1, \dots, \phi_{N_a}$ are not equal, $\phi_1 \neq \phi_2 \neq \cdots \neq \phi_{N_a}$.

Let us consider, for example, three harmonic functions (by taking $N_a = 3$):

$$f_N(t) = A_0 + A_1 sin(2\pi f_1 t + \phi_1) + A_2 sin(2\pi f_2 t + \phi_2) + A_3 sin(2\pi f_3 t + \phi_3) \tag{5.18}$$

Since the measured time series are discrete, let us note $t = t_k$ with $k = 1, \dots, m$ where m is the number of data in the time series. Then, Eq. (5.18) can be expressed as:

$$f_N(t_k) = A_0 + A_1 sin(2\pi f_1 t_k + \phi_1) + A_2 sin(2\pi f_2 t_k + \phi_2) + A_3 sin(2\pi f_3 t_k + \phi_3) \tag{5.19}$$

A graphical representation of Eq. (5.19) is given in Figure 5.3.

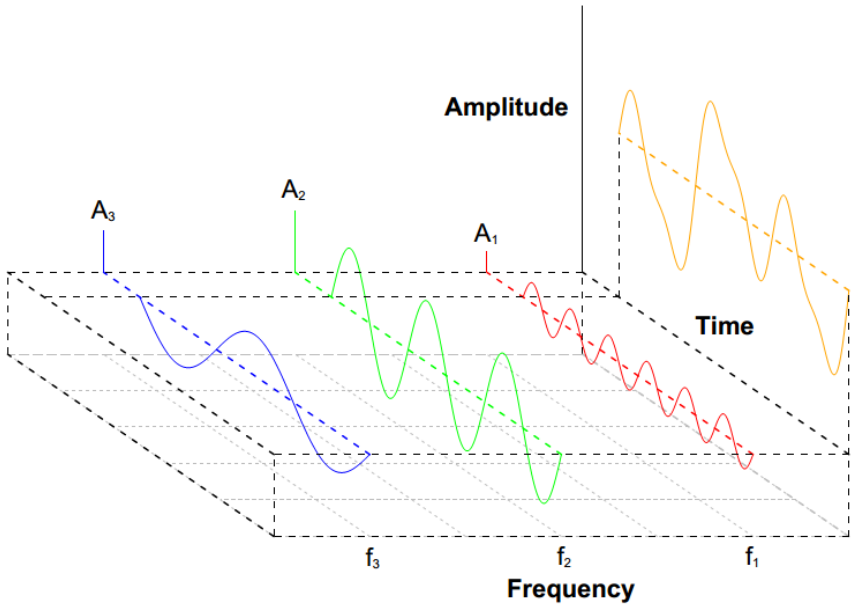


Figure 5.3. Illustration of Fourier decomposition of a time series. The amplitudes A_1, A_2, A_3 correspond to each frequency. The mean (A_0) corresponding to the frequency zero is not drawn.

5.5.3. Frequency spectrum of measured data

The time series of the measured data are represented in Figure 5.4. Their frequency spectrum is represented, after eliminating the mean value, in Figure 5.5. The frequency associated to the daily cycle, corresponding to the *24 hours* period, has the largest amplitude for all inputs; the second large amplitude corresponds to the *12 hours* period (for all measured data, excepting the wind speed). In Figure 5.5, the dashed red lines show the frequency window given the sample-rate and the length of the time series, i.e., the maximum frequency of the time series that can be observed is the inverse of *1 hour* and the minimum frequency is the inverse of *10 days*. The dashed black line is the most relevant frequency present in the time series of the data set, i.e. the inverse of *24 hours*. The dashed green line shows the cut-off frequency, f_c(Hz), of the experimental wall for a first order model, Eq. (5.7) and (5.11), see Appendix A5.

The cut-off frequency is considered as the frequency for which the amplitude gain is about 70% of its value at zero frequency, i.e., the magnitude gain in Bode plot (next section) is reduced about $3dB$, regarding to a strictly proper transfer function. For a first order model the time constant of the wall, $\tau(s/rad)$, is the inverse of 2π times the cut-off frequency ($\tau = 1/2\pi f_c$).

For frequencies higher than the cut-off frequency (Figure 5.5, dashed green line), the signals passing through the wall are damped depending on the model order. This effect can be observed in Figure 5.5f and Figure 5.5h representing the frequency spectrum of the inside surface temperature, θ_{si}, and of the inside heat flux density Q_i, which represent outputs of the system. As expected, Figure 5.5b shows very small amplitudes for the indoor temperature, T_i, which is controlled at (quasi) constant value.

The sensor for outside heat flux density, Figure 5.5g, is placed a few centimeters inside the wall, i.e., it is protected by a thin layer of concrete to avoid the direct incidence of the solar radiation. The wind speed, Figure 5.5d, presents other important amplitudes for frequencies lower than the inverse of *24 hours*.

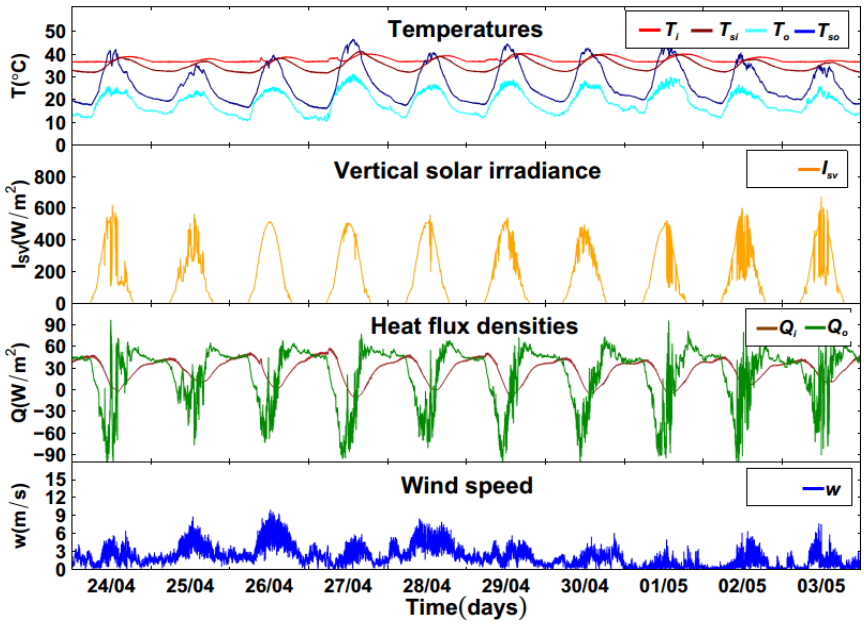
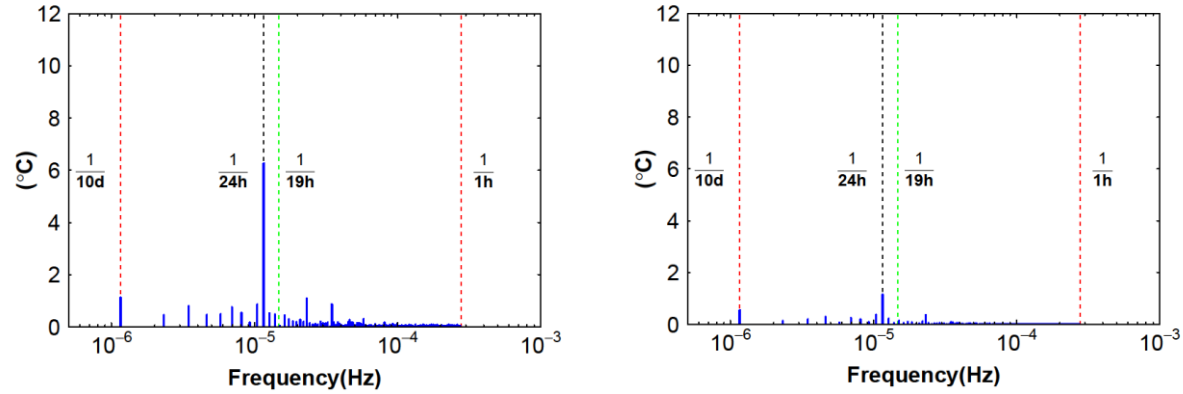
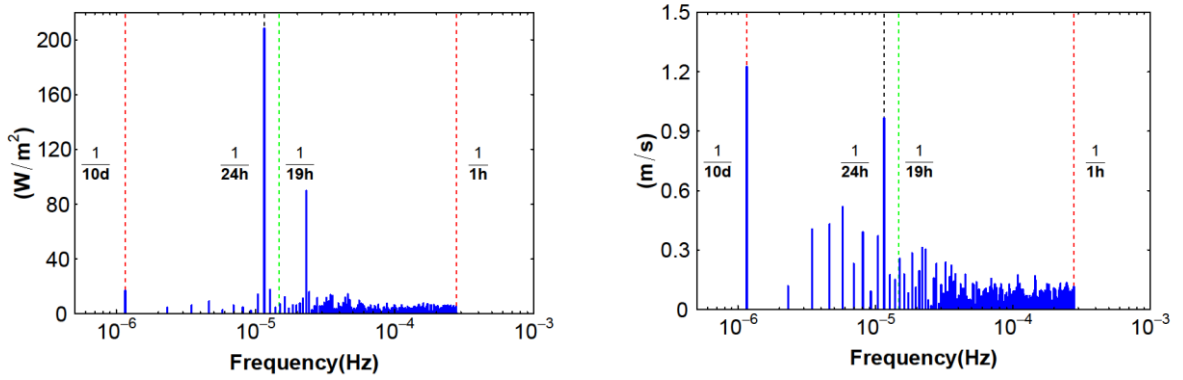


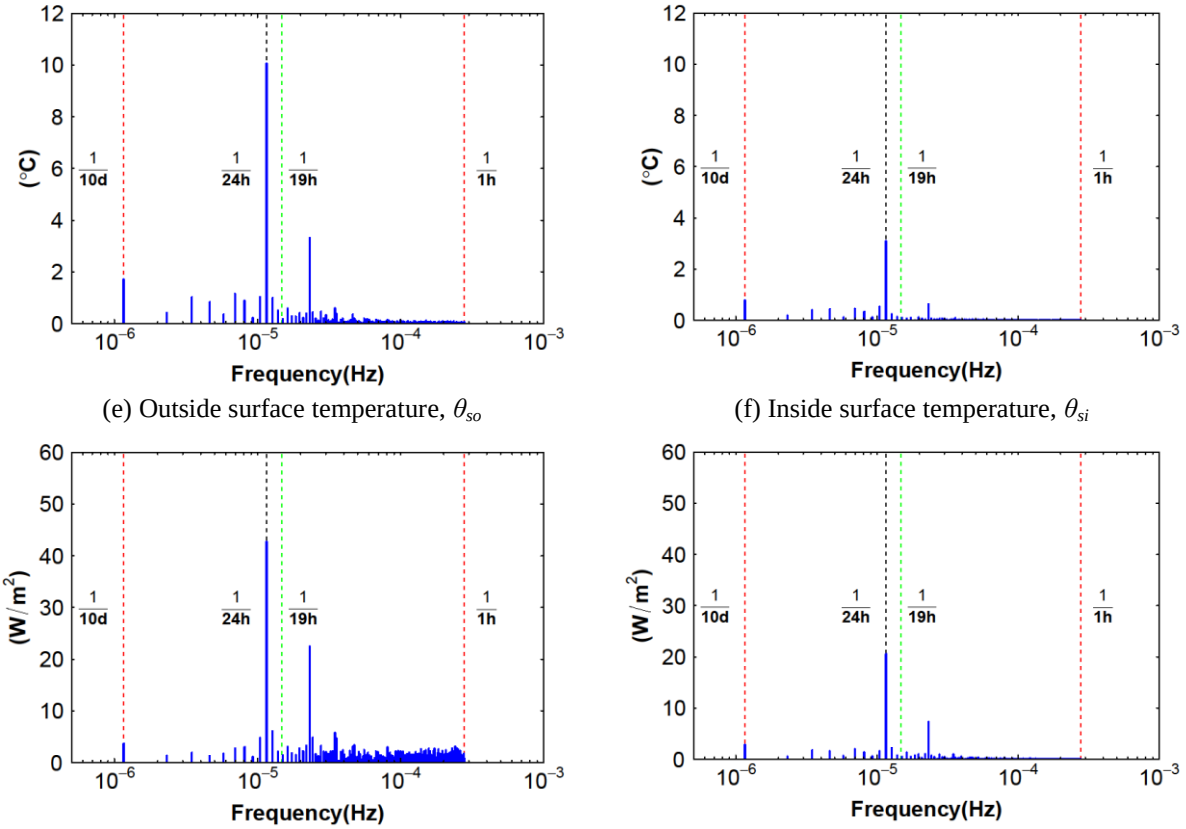
Figure 5.4. Measured data: temperatures, vertical solar irradiance, heat flux densities and wind speed



a) Outside temperature, T_o (b) Inside temperature, T_i



(c) Vertical solar irradiance, I_{SV} (d) Wind speed, w



(e) Outside surface temperature, θ_{so} (f) Inside surface temperature, θ_{si}

(g) Outside heat flux density, Q_o (h) Inside heat flux density, Q_i

Figure 5.5. Frequency spectrum of measured variables: temperatures, vertical solar irradiance, wind speed and heat flux densities

Since the influence on wind speed, inside air speed and long-wave radiation exchange variations are supposed negligible, then the outside and the inside thermal conductances, R_{so}^{-1} and R_{si}^{-1}, respectively, are considered constants. However, these variations may be non-negligible ((Escudero, et

al., 2013), (Naveros, et al., 2014)). In the next section, it is analyzed a theoretical wall with a greater time constant than this experimental wall, and it is compared to other theoretical walls with very high and very low time constants.

5.5.4. Bode diagram and the order of the model

The Bode diagram is a useful tool to be used in frequency domain for the study and the analysis of different output signals regarding the input signals which are the cause of them. This section explains the use of Bode diagram for facilitating the choice of the order of the model by frequency analysis of the measurements.

5.5.4.1. Bode diagram for different outputs

Bode diagrams show the frequency response of models and physical systems. They show the influence of the input on the output as a function of the input frequencies. Since, for linear systems, any input signal can be expressed as a sum of sine or cosine functions with different frequencies and amplitudes, Eq. (5.17), the output is a sum of input signals amplified and shifted in time with values indicated in the Bode magnitude and phase plots.

Figures 5.6-5.9 show the graphical representation of the magnitude plots for the:

- inside surface temperature, θ_{si}.
- outside surface temperature θ_{so}.
- inside heat flux density, Q_i.
- outside heat flux density, Q_o,

as a function of outdoor temperature, T_o, indoor temperature, T_i, and solar irradiance, I_{sv}. Each time, three responses are shown: for the first, second and hundredth order model, i.e. $N = 3,\ 4,\ 100$, respectively. These plots were obtained for a $10m^2$ usual brick wall having the thermal resistance $R = 0.05\ \mathrm{K/W}$ and the thermal capacity, $= 2.1 \cdot 10^6\ \mathrm{J/K}$ (Table 5.7), the values of the outside thermal conductance, $R_{so}^{-1} = 200\ \mathrm{W/K}$, and the inside thermal conductance, $R_{si}^{-1} = 80\ \mathrm{W/K}$, are supposed constant.

In Figures 5.6-5.9, it can be noticed that for all the outputs, the first order has a different behavior than the higher order models within the frequency window (frequencies between dashed red lines, from $1/10\ day$ up to $1/1\ hour$). Nonetheless, as it is shown in Figure 5.5, the main amplitude in the spectra of inputs corresponds to the frequency of $1/24h$. Since this frequency is close to the wall cut-off frequency, $1/45h$, and up to this cut-off frequency there is no difference between different order models (Figures 5.6-5.9), a first order model may be supposed a valid approach. Tables 5.3-5.6 show the ratio between the amplitude of the input and the output signals corresponding to the daily frequency using the first order and the highest order model.

Tables 5.3-5.6 also show the minimum measurable variation of each input signal which could produce a measurable output signal response by taking into account the measurement accuracy of the output signal. Since the input signals can be a sum of signals with different frequencies, the output response depends on frequency, i.e. the gain of the output response varies depending on frequency. The difference between the gains of models having different orders starts to be significant in the output response, depending on the input signal frequencies, for frequencies much higher than the cut-off frequency of the wall, $1/45h$, and the amplitude of the output is damped in comparison to the

amplitude of the input more than for lower frequencies. Therefore, the amplitude of the input needs to be higher in order to have a measurable variation of the output.

Another important consideration is that the transfer function of the inside heat flux density, Q_i, is proper but not strictly proper for the input indoor temperature, T_i, (Figure 5.8b) and that the transfer function of the outside heat flux density, Q_o, is proper but not strictly proper for the inputs outdoor temperature, T_o, (Figure 5.9a) and vertical solar irradiance, I_{SV}, (Figure 5.9c).

Since the transfer function for the heat flux density are not strictly proper, the gain in the output response grows up to the limit given by the wall inside surface conductance, R_{si}^{-1}, for inside heat flux density, Q_i (Figure 5.9b) and by the wall outside surface conductance, R_{so}^{-1}, for outside heat flux density, Q_o (Figure 5.9a). The limits are represented by a horizontal black dashed line. In Figure 5.9c the limit is given by the higher absorption value possible, i.e., $\alpha = 1$, which corresponds to the horizontal black line at zero.

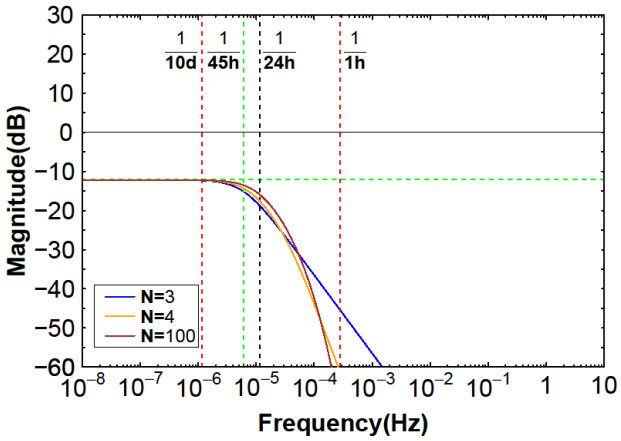
For the not strictly proper transfer functions, the gain of the output response for the highest frequency within the given frequency window, $1/1h$, is also presented to show explicitly its quantitative value. The outputs that have a proper but not strictly proper transfer function will be very sensitive to the high frequencies present in some input signals (as solar irradiance). This sensitivity explains why the heat flux meters need to be protected from direct solar radiation for avoiding excitations of high frequency.

Table 5.3. Minimum measurable variation of inputs, T_o, T_i and I_{SV}, necessary to observe a variation of the output θ_{si} for frequency corresponding to the largest amplitude

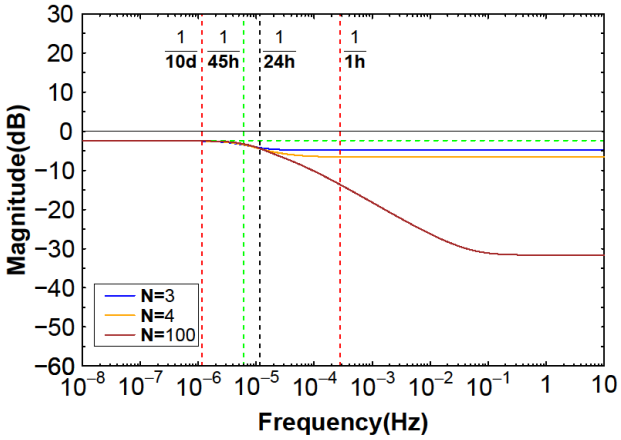
Output (°C)	**Input**	**Figure 7**	**Frequency**	**Magnitude** M(dB)	**Amplitude gain** ($\mathbf{10^{M/20}}$)	**Accuracy Output measurement**	**Minimum measurable variation of Input**
$\boldsymbol{\theta_{si}}$	T_o(°C)	(a) *N*=3	[1/24h]	-19	0.11	0.10	0.89(°C)
$\boldsymbol{\theta_{si}}$	T_o(°C)	(a) *N* =100	[1/24h]	-16	0.16	0.10	0.63(°C)
$\boldsymbol{\theta_{si}}$	T_i(°C)	(b) *N* =3,100	[1/24h]	-04	0.63	0.10	0.16(°C)
$\boldsymbol{\theta_{si}}$	I_{SV}(W/m^2)	(c) *N* =3	[1/24h]	-45	0.01	0.10	18(W/m^2)
$\boldsymbol{\theta_{si}}$	I_{SV}(W/m^2)	(c) *N* =100	[1/24h]	-42	0.01	0.10	13(W/m^2)

Table 5.4. Minimum measurable variation of inputs, T_o, T_i and I_{SV}, necessary to observe a variation of the output θ_{so} for frequency corresponding to the largest amplitude

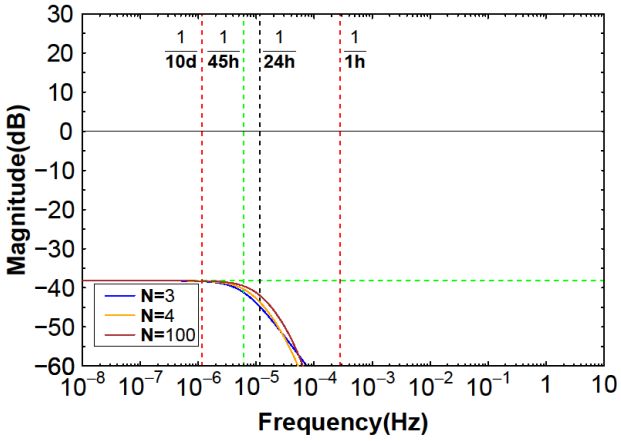
Output (°C)	**Input**	**Figure 8**	**Frequency**	**Magnitude** M(dB)	**Amplitude gain** ($\mathbf{10^{M/20}}$)	**Accuracy Output measurement**	**Minimum measurable variation of Input**
$\boldsymbol{\theta_{so}}$	T_o(°C)	(a) *N*=3,100	[1/24h]	-02	0.79	0.10	0.13(°C)
$\boldsymbol{\theta_{so}}$	T_i(°C)	(b) *N* =3	[1/24h]	-27	0.04	0.10	2.24(°C)
$\boldsymbol{\theta_{so}}$	T_i(°C)	(b) *N* =100	[1/24h]	-24	0.06	0.10	1.60(°C)
$\boldsymbol{\theta_{so}}$	I_{SV}(W/m^2)	(c) *N* =3,100	[1/24h]	-28	0.04	0.10	2.51(W/m^2)



(a) Output θ_{si} / Input T_o

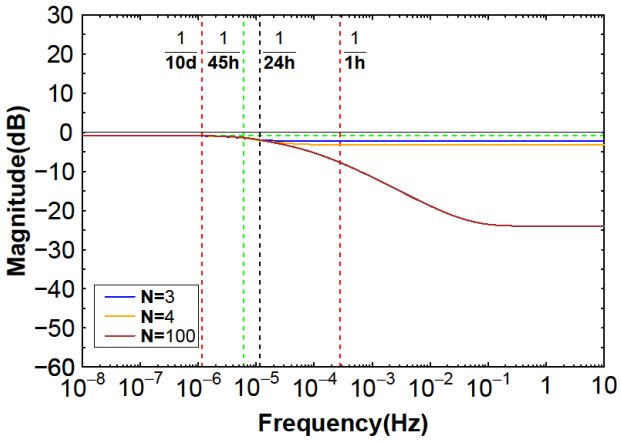


(b) Output θ_{si} / Input T_i

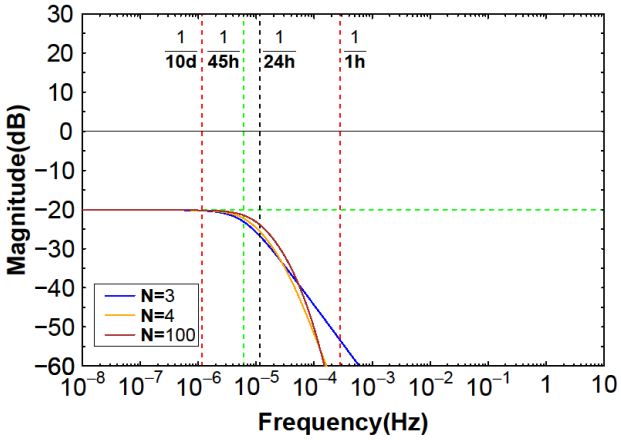


(c) Output θ_{si} / Input I_{SV}

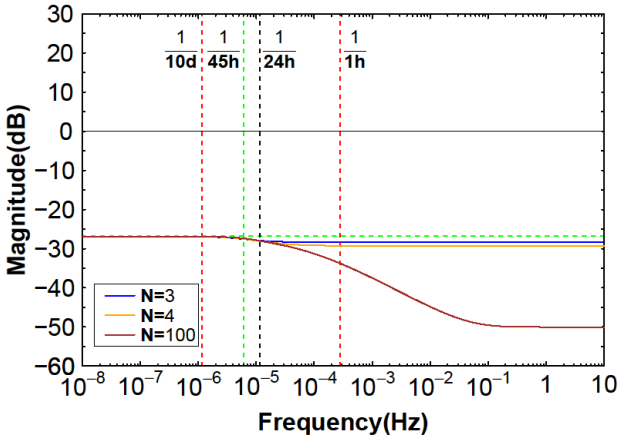
Figure 5.6. Bode plot of output θ_{si} as a function of inputs T_o, T_i and I_{SV}



(a) Output θ_{so} / Input T_o

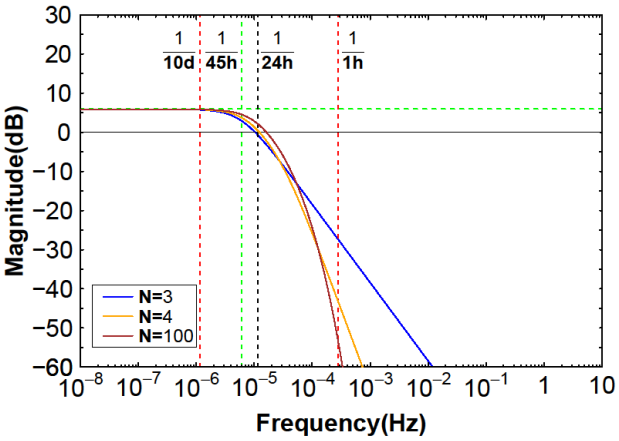


(b) Output θ_{so} / Input T_i

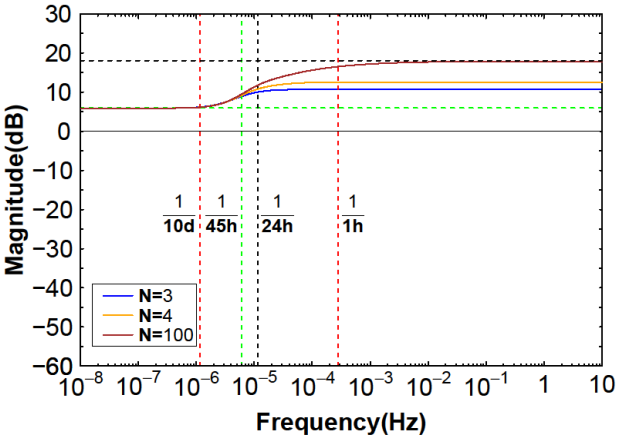


(c) Output θ_{so} / Input I_{SV}

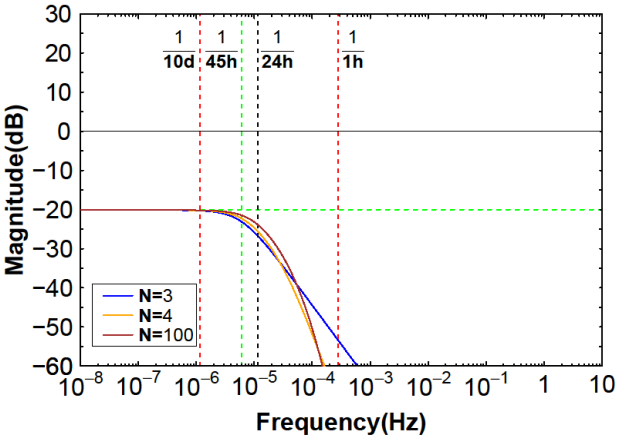
Figure 5.7. Bode plot of output θ_{so} as a function of inputs T_o, T_i and I_{SV}



(a) Output Q_i / Input T_o

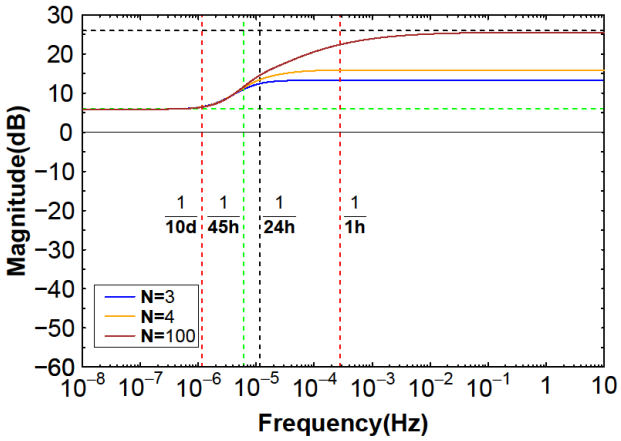


(b) Output Q_i / Input T_i

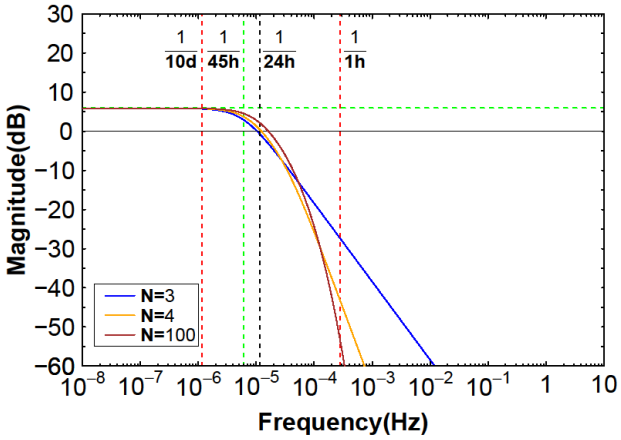


(c) Output Q_i / Input I_{SV}

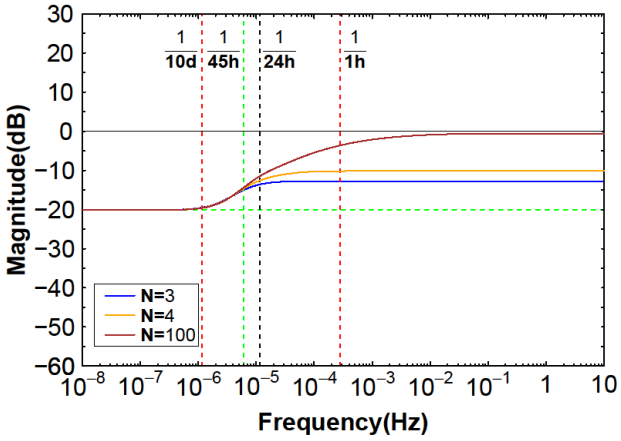
Figure 5.8. Bode plot of output Q_i as a function of inputs T_o, T_i and I_{SV}



(a) Output Q_o / Input T_o



(b) Output Q_o / Input T_i



(c) Output Q_o / Input I_{SV}

Figure 5.9. Bode plot of output Q_o as a function of inputs T_o, T_i and I_{SV}

Table 5.5. Minimum measurable variation of inputs, T_o, T_i and I_{SV}, necessary to observe a variation of the output Q_i for frequency corresponding to the largest amplitude and for the highest frequency (T_i)

Output ($\mathbf{W/m^2}$)	Input	Figure 9	Frequency	Magnitude M(dB)	Amplitude gain ($\mathbf{10^{M/20}}$)	Accuracy Output measurement	Minimum measurable variation of Input
$\boldsymbol{Q_i}$	T_o(°C)	(a) N=3	[1/24h]	-01	0.89	2.50	2.81(°C)
$\boldsymbol{Q_i}$	T_o(°C)	(a) N =100	[1/24h]	02	1.26	2.50	1.99(°C)
$\boldsymbol{Q_i}$	T_i(°C)	(b) N =3	[1/24h]	10	3.16	2.50	0.79(°C)
$\boldsymbol{Q_i}$	T_i(°C)	(b) N =100	[1/24h]	12	3.98	2.50	0.63(°C)
$\boldsymbol{Q_i}$	T_i(°C)	(b) N =3	[1/1h]	11	3.55	2.50	0.70(°C)
$\boldsymbol{Q_i}$	T_i(°C)	(b) N =100	[1/1h]	18	7.94	2.50	0.31(°C)
$\boldsymbol{Q_i}$	I_{SV}($\mathrm{W/m^2}$)	(c) N=3	[1/24h]	-27	0.04	2.50	56($\mathrm{W/m^2}$)
$\boldsymbol{Q_i}$	I_{SV}($\mathrm{W/m^2}$)	(c) N =100	[1/24h]	-24	0.06	2.50	40($\mathrm{W/m^2}$)

Table 5.6. Minimum measurable variation of inputs, T_o, T_i and I_{SV}, necessary to observe a variation of the output Q_o for frequency corresponding to the largest amplitude and for the highest frequency (T_o, I_{sv})

Output ($\mathbf{W/m^2}$)	Input	Figure 10	Frequency	Magnitude M(dB)	Amplitude gain ($\mathbf{10^{M/20}}$)	Accuracy Output measurement	Minimum measurable variation of Input
$\boldsymbol{Q_o}$	T_o(°C)	(a) N=3	[1/24h]	12	3.98	5.00	1.26(°C)
$\boldsymbol{Q_o}$	T_o(°C)	(a) N =100	[1/24h]	15	5.62	5.00	0.89(°C)
$\boldsymbol{Q_o}$	T_o(°C)	(a) N =3	[1/1h]	13	4,47	5.00	1.12(°C)
$\boldsymbol{Q_o}$	T_o(°C)	(a) N =100	[1/1h]	22	12.6	5.00	0.40(°C)
$\boldsymbol{Q_o}$	T_i(°C)	(b) N =3	[1/24h]	-01	0.89	5.00	5.61(°C)
$\boldsymbol{Q_o}$	T_i(°C)	(b) N =100	[1/24h]	02	1.26	5.00	3.97(°C)
$\boldsymbol{Q_o}$	I_{SV}($\mathrm{W/m^2}$)	(c) N=3	[1/24h]	-14	0.20	5.00	25($\mathrm{W/m^2}$)
$\boldsymbol{Q_o}$	I_{SV}($\mathrm{W/m^2}$)	(c) N =100	[1/24h]	-11	0.28	5.00	18($\mathrm{W/m^2}$)
$\boldsymbol{Q_o}$	I_{SV}($\mathrm{W/m^2}$)	(c) N=3	[1/1h]	-13	0.22	5.00	22($\mathrm{W/m^2}$)
$\boldsymbol{Q_o}$	I_{SV}($\mathrm{W/m^2}$)	(c) N =100	[1/1h]	-03	0.71	5.00	7.06($\mathrm{W/m^2}$)

5.5.4.2. Bode diagram for different time constants

The last analysis is done for the inside heat flux density, Q_i, considering three walls with different thermal resistances and thermal capacities: an extremely thin gypsum wall, a usual brick wall and a very thick wall (Table 5.7). The gypsum and the wood walls are considered as extremes. In all cases the values of the outside and the inside thermal conductance are supposed constant, $R_{so}^{-1} = 200\ \mathrm{W/K}$ and $R_{si}^{-1} = 80\ \mathrm{W/K}$., and the wall area is considered $S = 10\ \mathrm{m}^2$.

Figures 5.10a and 5.11a show the gypsum wall with a low time constant, $\tau_1 = 10\ minutes/radians$, Figures 5.10b and 5.11b show the brick wall with a higher time constant, $\tau_2 = 7\ hours/radians$, and Figures 5.10c and 5.11c present the wood wall with a very high time constant, $\tau_3 = 9\ days/radians$.

It is observed in Figures 5.10-5.11 and Table 5.8 that, like for the gypsum wall, there is no significant difference between models of different orders in the frequency domain from $1/10d$ up to $1/1h$ and that there is an amplification of the inputs which allows obtaining a measurable response to the frequency with main amplitude, $1/24h$. On the other hand, for the brick wall, there is a small difference between models of different orders at daily frequency, $1/24h$, which is quantified and may help in the decision about the order selection looking for a trade-off between accuracy and model complexity. Lastly, for the wood wall, the cut-off frequency is out of the frequency window (at $1/58d$). There is a significant difference between models of different orders but the attenuation is so

important for all order models that, in fact, the variation of the input is not measurable. For example, a variation greater than 400°C of the outdoor temperature would be necessary in order to obtain a measurable effect of the indoor heat flux at the $1/24h$ frequency even for high order models, taking into account the accuracy of the measurement devices.

In all cases, the minimum measurable variation of the input is given by the highest order model since it is the one closest to the real model. The use of the first order model may be useful when the differences between both models can be considered negligible.

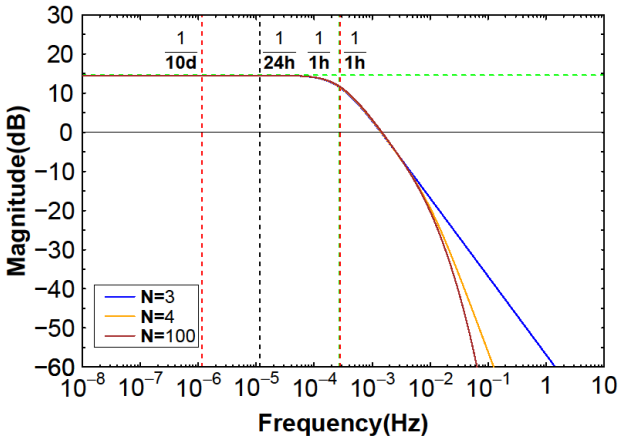
It can be also noticed that the cut-off frequency shows the frequency limit from which it is important to take into consideration the order of the model. For example, for the wood wall (Figures 5.10c and 5.11c), it can be seen that the response of a higher order model is less damped than that of the first order model roughly up to the frequency corresponding to $1/10d$, and much more damped for higher frequencies. A similar behavior is observed for the gypsum and brick walls, but beyond the limit of the daily frequency towards higher frequencies. Furthermore, the cut-off frequency also shows the frequency limit from where it is needed to check if the output response has measurable amplitude, taking into account the amplitude variation of the inputs.

Table 5.7. Thermal characteristics of the three theoretical walls used in comparison (walls area: $S = 10m^2$)

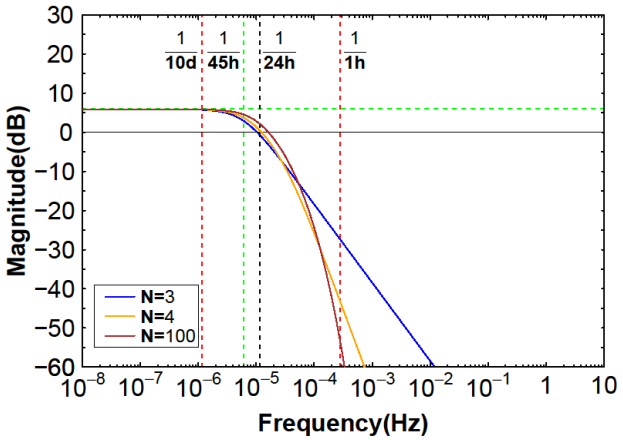
Wall material	Density ($\mathbf{kg/m^3}$)	Specific heat capacity ($\mathbf{J/kgK}$)	Width ($\mathbf{m}$)	Heat capacity, $\boldsymbol{C}$ ($\mathbf{J/K}$)	Thermal conductance, $\boldsymbol{R^{-1}}$ ($\mathbf{W/K}$)	Time constant, $\boldsymbol{\tau}$ ($\mathbf{s/rad}$)
Gypsum ($\boldsymbol{\tau_1}$)	1800	837	0.01	$1.5 \cdot 10^5$	810	$5.9 \cdot 10^2$
Brick ($\boldsymbol{\tau_2}$)	1700	837	0.15	$2.1 \cdot 10^6$	30	$2.6 \cdot 10^4$
Wood ($\boldsymbol{\tau_3}$)	850	2386	0.50	$1.0 \cdot 10^7$	3	$8.0 \cdot 10^5$

Table 5.8. Minimum measurable variation of inputs necessary to observe an output variation for frequencies corresponding to the largest amplitude and for the highest frequency of the three walls used in comparison

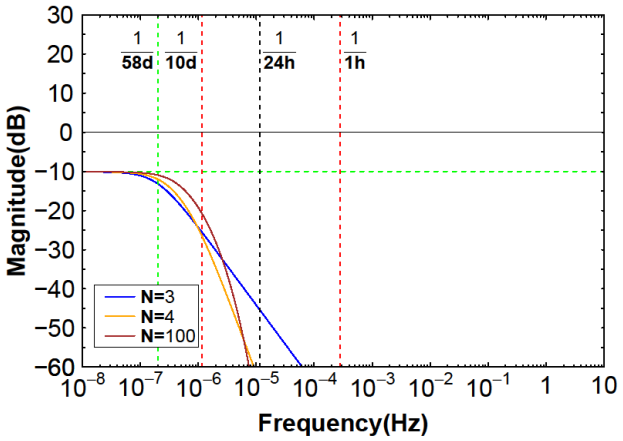
Output ($\mathbf{W/m^2}$)	Input	Wall	Figures 11-12	Frequency	Magnitude M(dB)	Amplitude gain ($\mathbf{10^{M/20}}$)	Accuracy Output measurement	Minimum measurable variation of Input
$\boldsymbol{Q_i}$	T_o(°C)	Gypsum	(11a) *N*=3,100	[1/24h]	15	5.62	2.50	0.44(°C)
$\boldsymbol{Q_i}$	T_o(°C)	Gypsum	(11a) *N* =3,100	[1/1h]	12	3.98	2.50	0.63(°C)
$\boldsymbol{Q_i}$	T_o(°C)	Brick	(11b) *N* =3	[1/24h]	-01	0.89	2.50	2.81(°C)
$\boldsymbol{Q_i}$	T_o(°C)	Brick	(11b) *N* =100	[1/24h]	02	1.26	2.50	1.99(°C)
$\boldsymbol{Q_i}$	T_o(°C)	Brick	(11b) *N* =3	[1/1h]	-27	0.04	2.50	56(°C)
$\boldsymbol{Q_i}$	T_o(°C)	Brick	(11b) *N* =100	[1/1h]	-51	0.00	2.50	890(°C)
$\boldsymbol{Q_i}$	T_o(°C)	Wood	(11c) *N* =3	[1/24h]	-45	0.01	2.50	445(°C)
$\boldsymbol{Q_i}$	T_o(°C)	Wood	(11c) *N* =100	[1/24h]	-75	0.00	2.50	14000(°C)
$\boldsymbol{Q_i}$	T_i(°C)	Gypsum	(12a) *N*=3,100	[1/24h]	15	5.62	2.50	0.44(°C)
$\boldsymbol{Q_i}$	T_i(°C)	Gypsum	(12a) *N* =3,100	[1/1h]	17	7.08	2.50	0.35(°C)
$\boldsymbol{Q_i}$	T_i(°C)	Brick	(12b) *N*=3	[1/24h]	10	3.16	2.50	0.79(°C)
$\boldsymbol{Q_i}$	T_i(°C)	Brick	(12b) *N* =100	[1/24h]	12	3.98	2.50	0.63(°C)
$\boldsymbol{Q_i}$	T_i(°C)	Brick	(12b) *N*=3	[1/1h]	11	3.55	2.50	0.70(°C)
$\boldsymbol{Q_i}$	T_i(°C)	Brick	(12b) *N* =100	[1/1h]	17	7.08	2.50	0.35(°C)
$\boldsymbol{Q_i}$	T_i(°C)	Wood	(12c) *N* =3	[1/24h]	-04	0.63	2.50	3.96(°C)
$\boldsymbol{Q_i}$	T_i(°C)	Wood	(12c) *N* =100	[1/24h]	10	3.16	2.50	0.79(°C)
$\boldsymbol{Q_i}$	T_i(°C)	Wood	(12c) *N* =3	[1/1h]	-04	0.63	2.50	3.96(°C)
$\boldsymbol{Q_i}$	T_i(°C)	Wood	(12c) *N* =100	[1/1h]	15	5.62	2.50	0.44(°C)



(a) Gypsum wall, $\tau_2 < \tau_1$

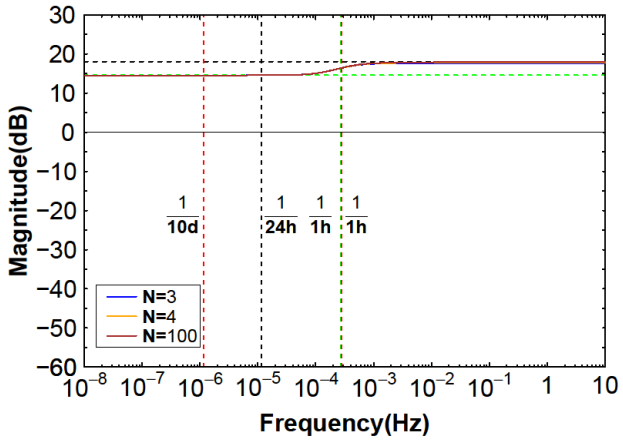


(b) Brick wall, τ_1

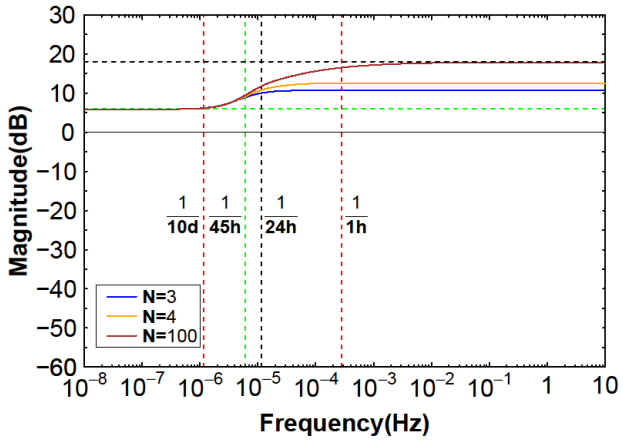


(c) Wood wall, $\tau_3 > \tau_1$

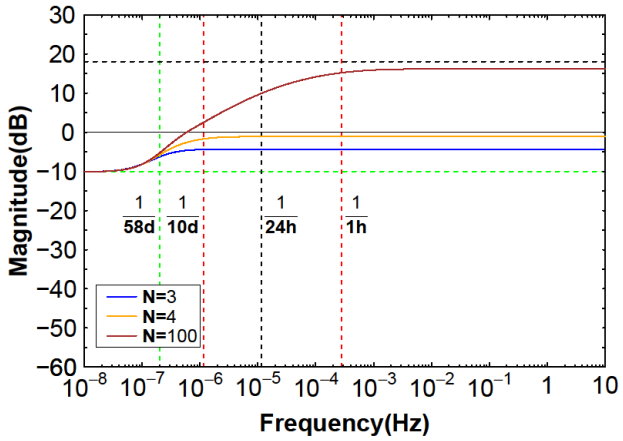
Figure 5.10. Bode plot of output Q_i as a function of input T_o for walls with different time constants



(a) Gypsum wall, $\tau_2 < \tau_1$



(b) Brick wall, τ_1



(c) Wood wall, $\tau_3 > \tau_1$

Figure 5.11. Bode plot of output Q_i as a function of input T_i for walls with different time constants

5.6. Conclusions

Simulation-based estimation of energy efficiency in buildings gives very large errors. Therefore, methods for energy efficiency estimation in buildings by measurements need to be developed. Much advancement was achieved in experimental identification of building thermal parameters. One important problem that still needs clarification is the choice of the order of the model. The heat transfer analysis starts from the heat equation which may be put in an infinite matrix form. In matrix representation, the continuous and the discrete models have the same formalism and the choice of the model structure is based on the discretization of the differential algebraic equations which model the physical phenomena.

The discretization represents a model order reduction (from infinity to a finite value). The aim of the model is to reproduce the measurable input-output relation of the variables related by the physical system. Then, the precision of the model does not need to be higher than the achievable accuracy of the measurements and the model order needs to take into account the domain of variation of the variables.

The transfer of the signal from input to output implies a change of amplitude in function of frequency. Spectrum analysis of the inputs of the system reveals the frequency domain on which the model needs to be valid. The domain of validity of the model is limited by the length of the measured signal and the sampling time step. This domain is reduced by considering the frequency range in which the amplitudes of input signals are large enough to be measured. Furthermore, the frequency domain is reduced by taking into account the measurable variation of output signals as a function of the amplitude variation of the input signals and the attenuation done by the system.

The present chapter gives a criterion, based on the frequency spectrum of the inputs and on the frequency response of the dynamic models models, to be applied for choosing the order of the model used on building thermal parameter identification. This can optimize the characterization of building energy performance.

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APPENDIX A5. Thermal conductance units and time constant estimation

In the present chapter, all expressions consider the thermal conductance, R^{-1}(W/K), to keep the usual physical dimensions of the thermal or heat capacity C(J/K) and to use the heat rate sources, f_l, in watts (W). The thermal conductance divided by square meters, *U*-value (W/m^2K), could be used in all expressions if the thermal or heat capacity are divided by square meters and the heat rate sources are also divided by square meters. Both alternatives are consistent and the first option was chosen.

For a first order model, the total thermal resistance of a wall, R_T(K/W), is:

$$R_T = R_{so} + R_2 + R_3 + R_{si} \tag{A5.1}$$

where R_{so}, R_{si} are the outside and the inside thermal resistances, respectively, and:

$$R = R_2 + R_3 \tag{A5.2}$$

is the wall thermal resistance (K/W); considering a homogeneous wall: $R_2 = R_3$.

The cut-off frequency of a wall, $f_c(s^{-1})$, for a first order model, is obtained by applying Laplace transform to Eq. (5.7) and (5.11).

Before applying the Laplace transform, Eq. (5.7) and (5.11) can be written as:

$$\frac{d\theta}{dt} = \frac{R_o^{-1}}{C}(T_o - \theta) + \frac{R_i^{-1}}{C}(T_i - \theta) + \frac{gS}{C}I_V \tag{A5.3}$$

$$Q_i = \frac{1}{S}R_i^{-1}(\theta - T_i) \tag{A5.4}$$

where θ (K), is the wall temperature, C(J/K) is the thermal or heat capacity of the wall, and the next useful relations are used:

$$\frac{1}{R_o^{-1}} = \frac{1}{R_{so}^{-1}} + \frac{1}{R_2^{-1}} \tag{A5.5}$$

$$\frac{1}{R_i^{-1}} = \frac{1}{R_{si}^{-1}} + \frac{1}{R_3^{-1}} \tag{A5.6}$$

$$g = \frac{R_2^{-1}}{R_{so}^{-1} + R_2^{-1}}\alpha \tag{A5.7}$$

Rearranging Eq. (A5.3) and applying Laplace transform:

$$L\left\{\frac{d\theta(t)}{dt}\right\} = L\left\{-\frac{R_o^{-1}+R_i^{-1}}{C}\theta(t) + \frac{R_o^{-1}}{C}T_o(t) + \frac{R_i^{-1}}{C}T_i(t) + \frac{gS}{C}I_{SV}(t)\right\} \quad \text{(A5.8)}$$

it is obtained:

$$\left(s + \frac{R_o^{-1}+R_i^{-1}}{C}\right)\theta(s) = \frac{R_o^{-1}}{C}T_o(s) + \frac{R_i^{-1}}{C}T_i(s) + \frac{gS}{C}I_{SV}(s) \quad \text{(A5.9)}$$

Solving Eq. (A5.9) results in:

$$\theta(s) = \frac{1}{\left(1 + s\frac{C}{R_o^{-1}+R_i^{-1}}\right)}\left(\frac{R_o^{-1}}{R_o^{-1}+R_i^{-1}}T_o(s) + \frac{R_i^{-1}}{R_o^{-1}+R_i^{-1}}T_i(s) + \frac{gS}{R_o^{-1}+R_i^{-1}}I_{SV}(s)\right) \quad \text{(A5.10)}$$

Then, applying Laplace transform to Eq. (A5.4) and using Eq. (A5.10):

$$Q_i(s) = \frac{1}{S}R_i^{-1}\left(\frac{1}{\left(1 + s\frac{C}{R_o^{-1}+R_i^{-1}}\right)}\left(\frac{R_o^{-1}}{R_o^{-1}+R_i^{-1}}T_o(s) + \frac{R_i^{-1}}{R_o^{-1}+R_i^{-1}}T_i(s) + \frac{gS}{R_o^{-1}+R_i^{-1}}I_{SV}(s)\right) - T_i(s)\right) \quad \text{(A5.11)}$$

or:

$$Q_i(s) = \frac{1}{S}\left(\frac{\frac{R_o^{-1}R_i^{-1}}{R_o^{-1}+R_i^{-1}}}{\left(1 + s\frac{C}{R_o^{-1}+R_i^{-1}}\right)}T_o(s) - \frac{\frac{R_o^{-1}R_i^{-1}}{R_o^{-1}+R_i^{-1}} + s\frac{R_i^{-1}C}{R_o^{-1}+R_i^{-1}}}{\left(1 + s\frac{C}{R_o^{-1}+R_i^{-1}}\right)}T_i(s) + \frac{\frac{gSR_i^{-1}}{R_o^{-1}+R_i^{-1}}}{\left(1 + s\frac{C}{R_o^{-1}+R_i^{-1}}\right)}I_{SV}(s)\right) \quad \text{(A5.12)}$$

It can be noted that in Eq. (A12):

$$R_T^{-1} = \frac{R_o^{-1}R_i^{-1}}{R_o^{-1}+R_i^{-1}} \quad \text{(A5.13)}$$

From Eq. (A12), the transfer functions relating the inside heat flux density, $Q_i(\mathrm{W/m^2})$, which is an output of the model, with the different inputs, it can be obtained:

a) the transfer function for the input: outdoor temperature, $T_o(\mathrm{K})$

$$H_{11} = \frac{1}{S}\frac{R_T^{-1}}{\left(1 + s\frac{C}{R_o^{-1}+R_i^{-1}}\right)} \quad \text{(A5.14)}$$

b) the transfer function for the input: indoor temperature, $T_i(\mathrm{K})$

$$H_{12} = -\frac{1}{S}\frac{R_T^{-1} + s\dfrac{R_i^{-1}C}{R_o^{-1}+R_i^{-1}}}{\left(1 + s\dfrac{C}{R_o^{-1}+R_i^{-1}}\right)} \tag{A5.15}$$

c) the transfer function for the input: solar irradiance, $I_{SV}(\mathrm{W/m^2})$

$$H_{13} = \frac{\dfrac{gR_i^{-1}}{R_o^{-1}+R_i^{-1}}}{\left(1 + s\dfrac{C}{R_o^{-1}+R_i^{-1}}\right)} \tag{A5.16}$$

From Eq. (A5.14-A5.16) it is deduced that the time constant of the wall, including the outside and the inside thermal resistances, is given by:

$$\tau = \frac{C}{R_o^{-1}+R_i^{-1}} \tag{A5.17}$$

and the cut-off frequency by:

$$f_c = \frac{1}{2\pi}\frac{R_o^{-1}+R_i^{-1}}{C} \tag{A5.18}$$

It is important to notice that the units of the time constant are seconds divided by radians, though usually the radians are omitted since they are really a non-dimensional unit. By this reason to obtain the cut-off frequency, which is in hertz, it is necessary to use the factor $2\pi\ radians$.

CHAPTER 6

PARAMETERS IDENTIFICATION: WALL EXAMPLE

6.1. Introduction

This last chapter applies the results of modelling heat transfer for parameters identification of buildings, which is a key point within the actual effort which is being carried out in order to optimize energy use and to improve energy efficiency in buildings (Yang, et al., 2014). This effort implies in part the use of dynamic models for heat transfer modelling of buildings and building components (Martín, et al., 2012) to solve the inverse problem (identification) (Ghiaus, 2014). The inverse problem seeks to identify the parameters of a model that characterizes the thermal behavior of buildings or building components (Rabl, 1988).

The principle of parameter identification is to propose a model and then to find the set of parameters that minimizes the error between the measured output and the output predicted by the model. For this purpose, the different model structures presented in previous chapters are usually proposed for modelling heat transfer in buildings ((Sonderegger, 1978), (Braun & Chaturvedi, 2002), (Jiménez, et al., 2008a), (Mejri, et al., 2011), (Bacher & Madsen, 2011)) and the parameters of the models are identified using different methods from measured data ((Jiménez, et al., 2008b), (Jiménez & Madsen, 2008)). The parameters of the models may have an explicit physical meaning as it is the case of thermal networks (Ghiaus, 2013), or not, as it is the case of autoregressive models with exogenous (ARX) (Ljung, 1999). In this manuscript it has been demonstrated that these different model structures are equivalent and that they can be converted from one into another. The transformations between models make it possible to obtain the physical parameters from models that do not contain them explicitly, such as ARX models.

These transformations, as shown in Chapters 3-4, may be considered from thermal networks which represent graphically a system of differential algebraic equations (DAE) ((Ghiaus, 2013), (Strang, 1986), (Strang, 2003)). The parameters of the thermal network have physical meaning since they may be explicitly related to the physical parameters of the heat equation (Naveros & Ghiaus, 2015). Next, an ARX model is deduced through a chain of transformations from the thermal network. Hence, the parameters of the ARX model, which are obtained from the physical parameters, are unique. This chapter uses the knowledge of model transformations and model structures, as shown in Chapters 3-4. The procedure is exemplified for the identification of the physical parameters (*U*-value, dynamic solar energy transmittance and effective heat capacity) of a simple wall supposed homogeneous by using data measured in-situ ((Strachan, 1993), (Vandaele, et al., 1990), (Strachan & Baker, 2008)). The wall has been already studied previously by using different methodologies ((Naveros, et al., 2012), (Naveros, et al., 2014)).

The experimental setup and the measured data are outlined in the next section. The parameters of the deduced ARX model, and their uncertainties, are estimated by using ordinary

least squares which are sensitive to the measurement errors. Next, it is done the identification of the parameters with physical sense (thermal network parameters) using the ARX model parameters and the chain of transformations presented in previous chapters.

6.2. Experimental set-up and data

The measured data used in this paper were obtained for a homogenous lightweight opaque wall, which was briefly described in Chapter 5. The wall, installed in a full-size test cell, was tested under outdoor weather conditions (Figure 6.1) at LECE laboratory ("Plataforma Solar de Almeria" (Jiménez, 2011)) in the South East of Spain (37.1° N, 2.4° W).

Figure 6.1. Exterior view of the test cell: wall tested with south orientation

6.2.1. Test-cell description

The test cell is divided in two insulated rooms: the test room and the auxiliary room. The test room contains the tested wall which is oriented to the south. The wall is made of ceramic bricks joined using sand and concrete mortar; its exterior side is plastered with mortar and its interior side is plastered with gypsum. The wall surface area, $S(\mathrm{m}^2)$, is $298\mathrm{cm}$ width by $276\mathrm{cm}$ height and the total wall thickness is $15\mathrm{cm}$: $2\mathrm{cm}$ mortar, $11.5\mathrm{cm}$ brick and $1.5\mathrm{cm}$ gypsum (Guzmán, et al., 2010).

The experiments were carried out under outdoor weather conditions. The weather at test location may be split in only two seasons: 1) summer (June-September) which is dry and extremely hot; 2) winter (October-May) which is dry and cold. The outdoor air ambient temperature varies considerably between day and night. The sky is usually very clear and the daily vertical global solar irradiation is significantly higher in winter, $22\mathrm{MJ/m}^2$, than in summer, $12\mathrm{MJ/m}^2$ (Naveros, et al., 2014).

The indoor air temperature in the test room was maintained constant and a ventilator was used to avoid indoor air temperature stratification. The set-point was 45°C for winter conditions and 18°C for summer conditions, chosen to maximize the temperature difference between indoor and outdoor air ambient.

6.2.2. Measurements

The measurement variables taken into account are illustrated in Table 6.1. The following list summarizes the sensors which were used in the experiment:

- Air temperature: 6 platinum thermo-resistance, PT100 1/10 DIN, within a ventilated solar radiation shield. Voltage is directly measured using a 4-wire connection.
- Vertical global solar irradiance: 1 pyranometer model CM11 manufactured by Kipp & Zonen, standard according to ISO 9060:1960. Voltage is directly measured using a differential connection.
- Heat flux density: 7 sensors model HFP01 manufactured by Hukseflux. Voltage is directly measured using a differential connection.

The system of data acquisition has the following characteristics: 1) 16-bit A/D resolution; 2) range of measurements fitted to the output of the sensors; 3) modules distributed to minimize wiring.

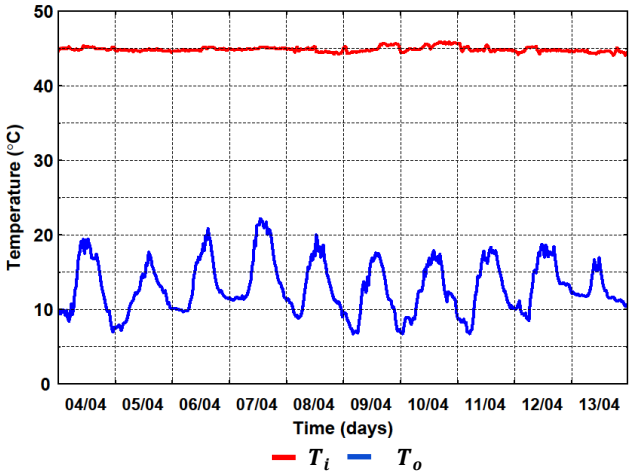
Table 6.1. Measured variables and sensor characteristics

Measured Variable	Symbol	Units	Accuracy	Range of device	Range of measurements
Inputs					
Outdoor temperature	T_o	°C	0.1°C	[-100÷100]	[1÷45]
Indoor temperature	T_i	°C	0.1°C	[-100÷100]	[15÷50]
Vertical global solar Irradiance	I_{sv}	W/m^2	3 %	[0÷4000]	[0÷900]
Output/Input					
Inside heat flux density	Q_i	W/m^2	5 %	[-2000÷2000]	[-75÷85]

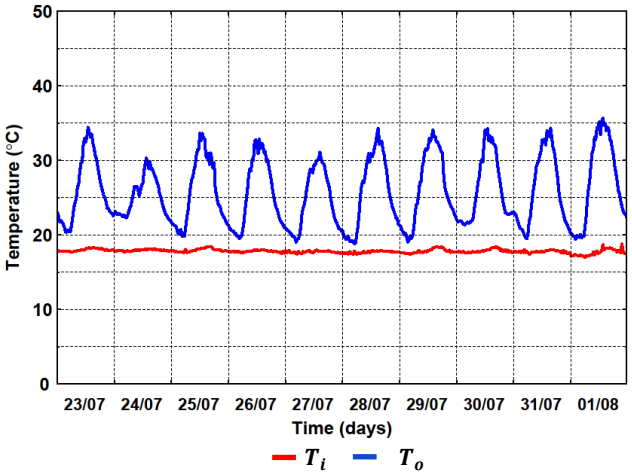
6.2.3. Data

Data were collected from 5th March 2010 to 11th October 2010. Data are sampled every 30 minutes and split into 21 sets of 10 days (Table 6.2). The total length of the experiment seeks to provide statistically significant results. The sample time and the length of data series are chosen to be consistent with a previous work (Naveros, et al., 2014). As an example, two data series for April and July are plotted in Figure 6.2.

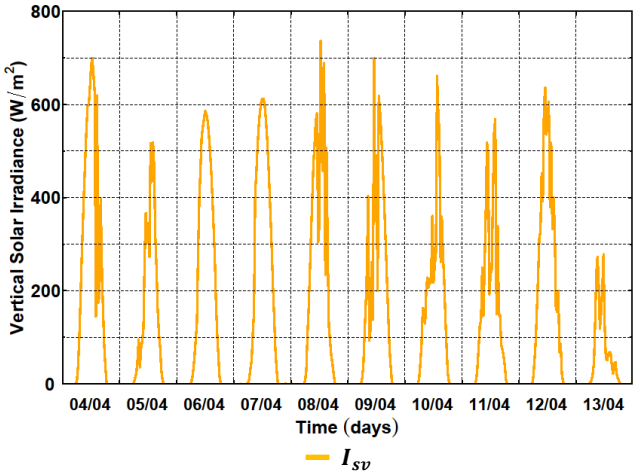
Two different periods can be clearly distinguished. The first one is characterized by a large difference between indoor and outdoor air temperatures and by a high level of vertical global solar irradiance (Figure 6.2a and Figure 6.2c); it runs from March until mid-June (series 01-10 in Table 6.2). The second period is characterized by a small difference between indoor and outdoor air temperatures and by a low level of vertical global solar irradiance measured on the wall surface (Figure 6.2b and Figure 6.2d); it runs from mid-June until October (series 11-21 in Table 6.2).



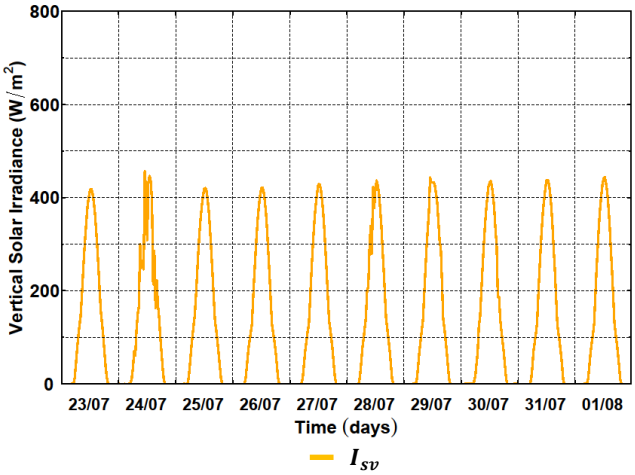
(a) Indoor, T_i, and outdoor, T_o, air temperatures



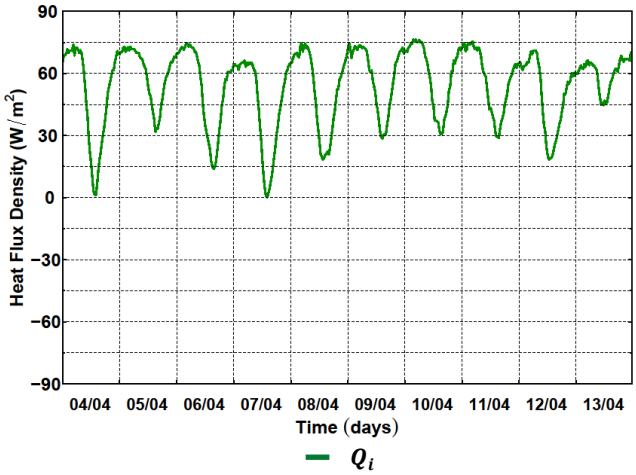
(b) Indoor, T_i, and outdoor, T_o, air temperatures



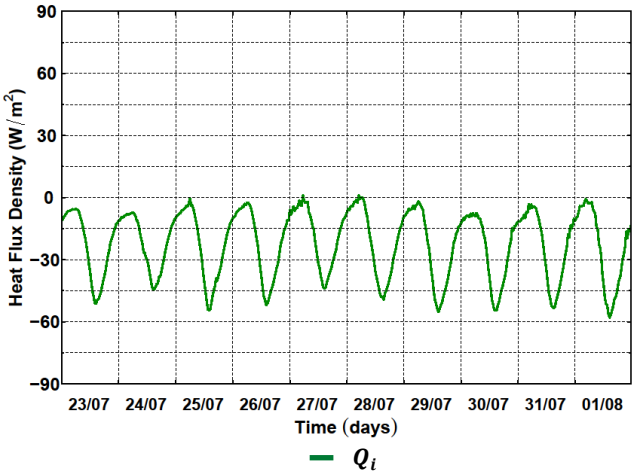
(c) Vertical global solar irradiance



(d) Vertical global solar irradiance



(e) Inside heat flux density



(f) Inside heat flux density

Figure 6.2. Measured data representation for Series 4: from 4th to 13th April 2010 (left) and Series 14: from 23nd July to 1st August 2010 (right)

Heat flux density passing throughout the wall is mainly driven by two effects: air ambient temperatures difference and solar irradiance. Heat flux density ranges in similar absolute values in both periods. In the first period, heat goes out from test room (Figure 6.2e), while in the second period heat goes into the test room (Figure 6.2f).

Table 6.2. Date of data set analyzed

Series	Date
01	05/03/2010–14/03/2010
02	15/03/2010–24/03/2010
03	25/03/2010–03/04/2010
04	04/04/2010–13/04/2010
05	14/04/2010–23/04/2010
06	24/04/2010–03/05/2010
07	04/05/2010–13/05/2010
08	20/05/2010–29/05/2010
09	30/05/2010–08/06/2010
10	09/06/2010–18/06/2010
11	23/06/2010–02/07/2010
12	03/07/2010–12/07/2010
13	13/07/2010–22/07/2010
14	23/07/2010–01/08/2010
15	02/08/2010–11/08/2010
16	12/08/2010–21/08/2010
17	22/08/2010–31/08/2010
18	01/09/2010–10/09/2010
19	11/09/2010–20/09/2010
20	21/09/2010–30/09/2010
21	01/10/2010–10/10/2010

6.3. Parameters identification

From a thermal network, it has been shown in Chapter 3 that it may be deduced an ARX model:

$$\theta_i(\mathrm{t}_k) = \sum_{l=0}^{q-1} m_{iq-l}\theta_i\left(\mathrm{t}_{k+l-q}\right) + \sum_{j}\sum_{l=0}^{r_j \leq q} n_{ijq-l}u_j\left(\mathrm{t}_{k+l-q}\right) \tag{6.1}$$

The measured output, $\theta_i(\mathrm{t}_k)$, and the measured inputs, $u_j(\mathrm{t}_k)$, are time series and they may be arranged as column vectors; the length of the time series is L. Then, Eq. (6.1) may be arranged in matrix form:

- Left side is a single column vector, with a number of rows equals to the length of the time series minus the model order, $L - q$.
- Right side is a matrix, an independent basis set, which multiplies the parameters single column vector. The matrix has a number of rows equals to the length of the time series minus the order of the model, $L - q$, and a number of columns given by the order of the model and the number of inputs, $(q-1) + \sum_j r_j$. The parameters columns vector has a number of rows equals to the number of matrix columns, $(q-1) + \sum_j r_j$.

As an example, considering only one output, θ, and one input, u, Eq. (6.1) becomes:

$$\begin{pmatrix} \theta(t_k) \\ \theta(t_{k+1}) \\ \vdots \\ \theta(t_L) \end{pmatrix} = \begin{pmatrix} \theta(t_{k-q}) & \cdots & \theta(t_{k-1}) & u(t_{k-q}) & \cdots & u(t_k) \\ \theta(t_{k-q+1}) & \cdots & \theta(t_k) & u(t_{k-q+1}) & \cdots & u(t_{k+1}) \\ \vdots & \cdots & \vdots & \vdots & \cdots & \vdots \\ \theta(t_{L-q}) & \cdots & \theta(t_{L-1}) & u(t_{L-q}) & \cdots & u(t_L) \end{pmatrix} \begin{pmatrix} m_q \\ \vdots \\ m_1 \\ n_q \\ \vdots \\ n_0 \end{pmatrix} \tag{6.2}$$

From Eq. (6.2) it is easiest to visualize that, for instance, Gauss' principle of linear least squares may be used to fit the ARX model, Eq. (6.1), and to estimate its parameters: m, n.

Using the ARX model parameters, m, n, which are estimated, it is possible to obtain the coefficients of the matrices in discrete time $\mathbf{A}_d$, $\mathbf{B}_d$, $\mathbf{C}_S$ and $\mathbf{D}_S$. Next, the coefficients of the matrices in continuous time $\mathbf{A}_S$ and $\mathbf{B}_S$, may be derived and finally the identification of physical parameters may be done.

6.3.1. Wall example

The procedure of identification using the whole chain of transformations stated in this dissertation is exemplified for a wall hereafter. The use of ARX models does not pretend to favour this structure regarding to other model structures but to show explicitly how it may be possible to use it ARX models obtained by deduction.

6.3.1.1. Differential algebraic equations

A wall, as any thermal system, may be lumped as it is shown in Chapter 3. Then, a thermal network may describe the conductive, convective and radiative heat balance of the studied thermal system (Figure 6.3).

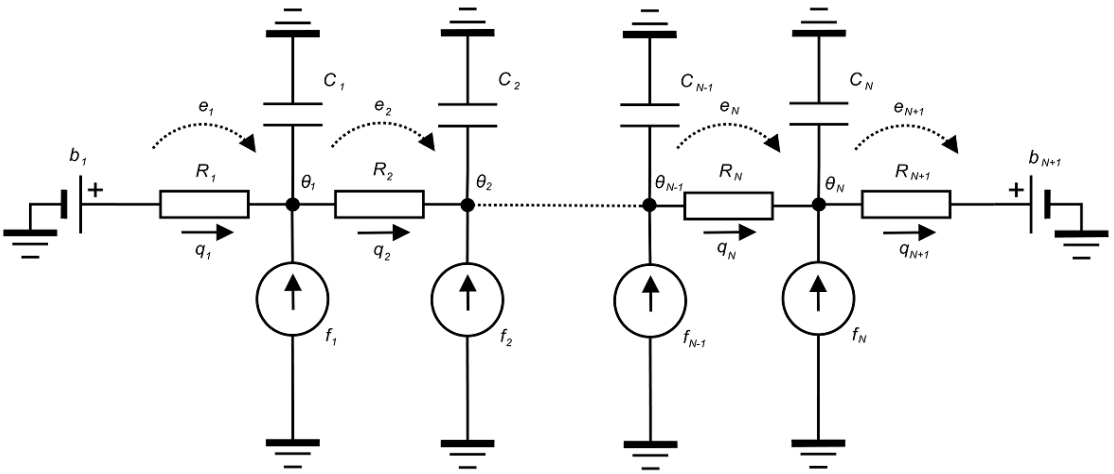


Figure 6.3. Thermal network using Ghiaus notation (Ghiaus, 2013): wall represented by *N* nodes

The use of a first order lumped model, *N*=3, represented by one differential and two algebraic equations, is used and detailed as an example, Figure 6.4.

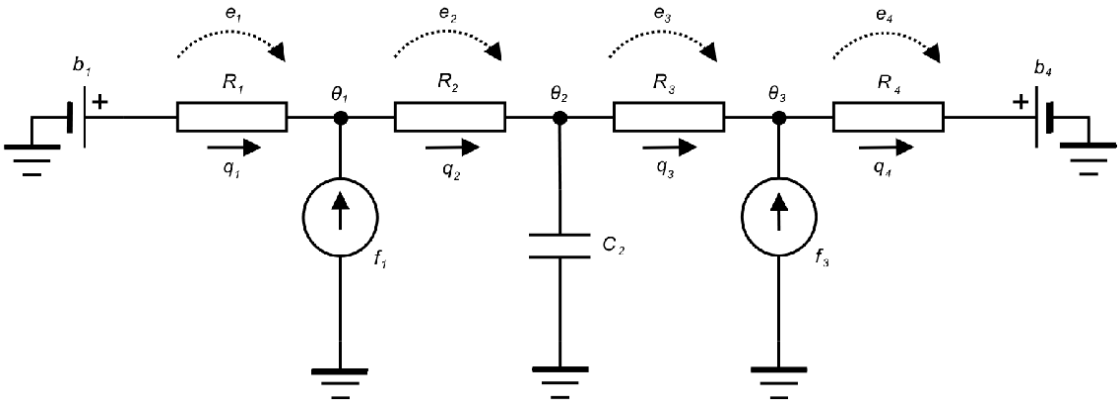


Figure 6.4. Thermal network using Ghiaus notation (Ghiaus, 2013): wall represented by 3 nodes.

Let us note

$\mathbf{e} = [e_1 \;\; e_2 \;\; e_3 \;\; e_4]^T$

$\mathbf{b} = [b_1 \;\; b_2 \;\; b_3 \;\; b_4]^T \equiv [T_o \;\; 0 \;\; 0 \;\; -T_i]^T$

$\boldsymbol{\theta} = [\theta_1 \;\; \theta_2 \;\; \theta_3 \;]^T \equiv [\theta_{so} \; \theta \;\; \theta_{si} \;]^T$

For the thermal network, Figure 6.4, which represents the wall, the temperature differences for each thermal resistance are:

$$\begin{cases} e_1 = b_1 - \theta_1 \\ e_2 = \theta_1 - \theta_2 \\ e_3 = \theta_2 - \theta_3 \\ e_4 = \theta_3 + b_4 \end{cases} \tag{6.3}$$

The incidence matrix can be obtained from Figure 6.4, as it is shown in Chapter 3:

$$\mathbf{A} = . \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & -1 \end{bmatrix} \tag{6.4}$$

Then, the system of Eq. (6.3) can be written in matrix form as:

$$\begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \end{bmatrix} = - \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix} + \begin{bmatrix} b_1 \\ 0 \\ 0 \\ b_4 \end{bmatrix} \tag{6.5}$$

The heat transfer rates in the network branches can be expressed as:

$$\begin{cases} q_1 = R_1^{-1} e_1 \\ q_2 = R_2^{-1} e_2 \\ q_3 = R_3^{-1} e_3 \\ q_4 = R_4^{-1} e_4 \end{cases} \tag{6.6}$$

By putting

$$\mathbf{q} = [q_1 \;\; q_2 \;\; q_3 \;\; q_4]^T \equiv [q_{co} \;\; q_{w1} \;\; q_{w2} \;\; q_{ci}]^T$$

$$\mathbf{G} = \begin{bmatrix} R_1^{-1} & 0 & 0 & 0 \\ 0 & R_2^{-1} & 0 & 0 \\ 0 & 0 & R_3^{-1} & 0 \\ 0 & 0 & 0 & R_4^{-1} \end{bmatrix} \equiv \begin{bmatrix} R_{co}^{-1} & 0 & 0 & 0 \\ 0 & R_{w1}^{-1} & 0 & 0 \\ 0 & 0 & R_{w2}^{-1} & 0 \\ 0 & 0 & 0 & R_{ci}^{-1} \end{bmatrix}$$

the system of Eq. (6.6) can be written in matrix form as:

$$\begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix} = \begin{bmatrix} R_1^{-1} & 0 & 0 & 0 \\ 0 & R_2^{-1} & 0 & 0 \\ 0 & 0 & R_3^{-1} & 0 \\ 0 & 0 & 0 & R_4^{-1} \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \end{bmatrix} \tag{6.7}$$

The heat balance in each temperature node $\boldsymbol{\theta} = [\theta_{so} \;\; \theta \;\; \theta_{si}\,]^T$ gives:

$$\begin{cases} 0 = q_1 - q_2 + f_1 \\ C_2\dot{\theta}_2 = q_2 - q_3 + f_2 \\ 0 = q_3 - q_4 + f_3 \end{cases} \tag{6.8}$$

Noting

$$\mathbf{C} = \begin{bmatrix} C_1 & 0 & 0 \\ 0 & C_2 & 0 \\ 0 & 0 & C_3 \end{bmatrix} \equiv \begin{bmatrix} 0 & 0 & 0 \\ 0 & C & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\mathbf{f} = [f_1 \;\; f_2 \;\; f_3]^T \equiv [\alpha S I_{sv} \;\; 0 \;\; 0]^T$$

$$\mathbf{A}^T = \begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

we obtain in matrix form:

$$\begin{bmatrix} C_1 & 0 & 0 \\ 0 & C_2 & 0 \\ 0 & 0 & C_3 \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix} + \begin{bmatrix} f_1 \\ 0 \\ 0 \end{bmatrix} \tag{6.9}$$

Taking into account Eq. (6.5) and (6.7) we could write Eq. (6.9) as:

$$\begin{bmatrix} C_1 & 0 & 0 \\ 0 & C_2 & 0 \\ 0 & 0 & C_3 \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{bmatrix} = -\begin{bmatrix} R_1^{-1} + R_2^{-1} & -R_2^{-1} & 0 \\ -R_2^{-1} & R_2^{-1} + R_3^{-1} & -R_3^{-1} \\ 0 & -R_3^{-1} & R_3^{-1} + R_4^{-1} \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix} + \begin{bmatrix} R_1^{-1} & -R_2^{-1} & 0 & 0 \\ 0 & R_2^{-1} & -R_3^{-1} & 0 \\ 0 & 0 & R_3^{-1} & -R_4^{-1} \end{bmatrix} \begin{bmatrix} b_1 \\ 0 \\ 0 \\ b_4 \end{bmatrix} + \begin{bmatrix} f_1 \\ 0 \\ 0 \end{bmatrix} \tag{6.10}$$

To obtain a matrix relation where nodes with non-negligible heat capacities are separated, as it was seen in previous section, the system given in Eq. (6.10) is rearranged. First, interchanging rows 2 and 3 in matrix $\mathbf{C}$ implies interchanging rows 2 and 3 in $-\mathbf{A}^T\mathbf{G}\mathbf{A}$, $\mathbf{A}^T\mathbf{G}$, and $\mathbf{f}$:

$$\begin{bmatrix} C_1 & 0 & 0 \\ 0 & 0 & C_3 \\ 0 & C_2 & 0 \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{bmatrix} = -\begin{bmatrix} R_1^{-1} + R_2^{-1} & -R_2^{-1} & 0 \\ 0 & -R_3^{-1} & R_3^{-1} + R_4^{-1} \\ -R_2^{-1} & R_2^{-1} + R_3^{-1} & -R_3^{-1} \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix} + \begin{bmatrix} R_1^{-1} & -R_2^{-1} & 0 & 0 \\ 0 & 0 & R_3^{-1} & -R_4^{-1} \\ 0 & R_2^{-1} & -R_3^{-1} & 0 \end{bmatrix} \begin{bmatrix} b_1 \\ 0 \\ 0 \\ b_4 \end{bmatrix} + \begin{bmatrix} f_1 \\ 0 \\ 0 \end{bmatrix} \tag{6.11}$$

Second, interchanging columns 2 and 3 in matrix $\mathbf{C}$ implies interchanging rows 2 and 3 in vector $\dot{\boldsymbol{\theta}}$ and requires interchanging column 2 and 3 in matrix $-\mathbf{A}^T\mathbf{G}\mathbf{A}$ and rows 2 and 3 in vector $\boldsymbol{\theta}$ to conserve relations between $\dot{\boldsymbol{\theta}}$ and $\boldsymbol{\theta}$:

$$\begin{bmatrix} C_1 & 0 & 0 \\ 0 & C_3 & 0 \\ 0 & 0 & C_2 \end{bmatrix}\begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_3 \\ \dot{\theta}_2 \end{bmatrix} = -\begin{bmatrix} R_1^{-1}+R_2^{-1} & 0 & -R_2^{-1} \\ 0 & R_3^{-1}+R_4^{-1} & -R_3^{-1} \\ -R_2^{-1} & -R_3^{-1} & R_2^{-1}+R_3^{-1} \end{bmatrix}\begin{bmatrix} \theta_1 \\ \theta_3 \\ \theta_2 \end{bmatrix} + \begin{bmatrix} R_1^{-1} & -R_2^{-1} & 0 & 0 \\ 0 & 0 & R_3^{-1} & -R_4^{-1} \\ 0 & R_2^{-1} & -R_3^{-1} & 0 \end{bmatrix}\begin{bmatrix} b_1 \\ 0 \\ 0 \\ b_4 \end{bmatrix} + \begin{bmatrix} f_1 \\ 0 \\ 0 \end{bmatrix} \tag{6.12}$$

Since for nodes 1 and 3 the heat capacity is considered negligible, Eq. (6.12) can be written as

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & C_2 \end{bmatrix}\begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_3 \\ \dot{\theta}_2 \end{bmatrix} = -\begin{bmatrix} R_1^{-1}+R_2^{-1} & 0 & -R_2^{-1} \\ 0 & R_3^{-1}+R_4^{-1} & -R_3^{-1} \\ -R_2^{-1} & -R_3^{-1} & R_2^{-1}+R_3^{-1} \end{bmatrix}\begin{bmatrix} \theta_1 \\ \theta_3 \\ \theta_2 \end{bmatrix} + \begin{bmatrix} R_1^{-1} & -R_2^{-1} & 0 & 0 \\ 0 & 0 & R_3^{-1} & -R_4^{-1} \\ 0 & R_2^{-1} & -R_3^{-1} & 0 \end{bmatrix}\begin{bmatrix} b_1 \\ 0 \\ 0 \\ b_4 \end{bmatrix} + \begin{bmatrix} f_1 \\ 0 \\ 0 \end{bmatrix} \tag{6.13}$$

Once the heat balance matrix expression is rearranged, the partition matrix, $\mathbf{K}$, is:

$$\mathbf{K} = \left[\begin{array}{cc:c} -R_1^{-1}-R_2^{-1} & 0 & R_2^{-1} \\ 0 & -R_3^{-1}-R_4^{-1} & R_3^{-1} \\ \hdashline R_2^{-1} & R_3^{-1} & -R_2^{-1}-R_3^{-1} \end{array}\right] \tag{6.14}$$

where the blocks are:

$$\mathbf{K}_{11} = \begin{bmatrix} -R_1^{-1}-R_2^{-1} & 0 \\ 0 & -R_3^{-1}-R_4^{-1} \end{bmatrix};\ \mathbf{K}_{12} = \begin{bmatrix} R_2^{-1} \\ R_3^{-1} \end{bmatrix}$$

$$\mathbf{K}_{21} = [R_2^{-1} \quad R_3^{-1}];\ \mathbf{K}_{22} = [-R_2^{-1}-R_3^{-1}]$$

and to partition matrix $\mathbf{K}_b$:

$$\mathbf{K}_b = \left[\begin{array}{cc:cc} R_1^{-1} & -R_2^{-1} & 0 & 0 \\ 0 & 0 & R_3^{-1} & -R_4^{-1} \\ \hdashline 0 & R_2^{-1} & -R_3^{-1} & 0 \end{array}\right] \tag{6.15}$$

where the blocks are:

$$\mathbf{K}_{b1} = \begin{bmatrix} R_1^{-1} & -R_2^{-1} & 0 & 0 \\ 0 & 0 & R_3^{-1} & -R_4^{-1} \end{bmatrix}$$

$$\mathbf{K}_{b2} = [0 \quad R_2^{-1} \quad -R_3^{-1} \quad 0]$$

Other relations to consider when the number of nodes is $N = 3$ are:

$$\mathbf{C}_C = [C_2];\ \mathbf{I}_{22} = [1];\ \mathbf{f} = [\mathbf{f}_0 \quad \boldsymbol{f_C}]^T;\ \mathbf{f}_0 = [\alpha SI_s \quad 0];\ \mathbf{f}_C = [0]$$

6.3.1.2. State-space and measurement equations

By introducing the block matrices given by Eq. (6.14), for $N = 3$, into(Chapter 3):

$$\mathbf{A}_S = \mathbf{C}_C^{-1}(-\mathbf{K}_{21}\mathbf{K}_{11}^{-1}\mathbf{K}_{12} + \mathbf{K}_{22}) \tag{6.16}$$

$$\mathbf{B}_S = \mathbf{C}_C^{-1}[-\mathbf{K}_{21}\mathbf{K}_{11}^{-1}\mathbf{K}_{b1} + \mathbf{K}_{b2} \quad -\mathbf{K}_{21}\mathbf{K}_{11}^{-1} \quad \mathbf{I}_{22}] \tag{6.17}$$

the next state space matrices are obtained:

$\mathbf{A}_S = \frac{1}{C_2}\left(-\frac{R_1^{-1}R_2^{-1}}{R_1^{-1}+R_2^{-1}} - \frac{R_3^{-1}R_4^{-1}}{R_3^{-1}+R_4^{-1}}\right)$ the state matrix,

$\mathbf{B}_S = \frac{1}{C_2}\left[\frac{R_1^{-1}R_2^{-1}}{R_1^{-1}+R_2^{-1}} \quad \frac{-R_2^{-2}}{R_1^{-1}+R_2^{-1}} + R_2^{-1} \quad \frac{R_3^{-2}}{R_3^{-1}+R_4^{-1}} - R_3^{-1} \quad -\frac{R_3^{-1}R_4^{-1}}{R_3^{-1}+R_4^{-1}} \quad \frac{R_2^{-1}}{R_1^{-1}+R_2^{-1}} \quad \frac{R_3^{-1}}{R_3^{-1}+R_4^{-1}} \quad 1\right]$ the input matrix and,

$\mathbf{u} = [T_o \;\; 0 \;\; 0 \;\; -T_i \;\; \alpha I_s \;\; 0 \;\; 0]^T$ the input vector.

Then, since the state vector is $\boldsymbol{\theta}_C = \theta$, the next relation can be obtained from Eq. (6.13):

$$\dot{\theta} = \frac{1}{C_2}\left(\frac{R_1^{-1}R_2^{-1}}{R_1^{-1}+R_2^{-1}}(T_o - \theta) + \frac{R_3^{-1}R_4^{-1}}{R_3^{-1}+R_4^{-1}}(T_i - \theta) + \frac{R_2^{-1}}{R_1^{-1}+R_2^{-1}}\,\alpha SI_s\right) \tag{6.18}$$

On the other hand, by introducing the block matrices given by Eq. (6.15), for $N = 3$, into:

$$\mathbf{C}_S = -\mathbf{K}_{11}^{-1}\mathbf{K}_{12} \tag{6.19}$$

$$\mathbf{D}_S = -\mathbf{K}_{11}^{-1}[\mathbf{K}_{b1} \quad \mathbf{I}_{11} \quad \mathbf{0}] \tag{6.20}$$

it is obtained:

$\mathbf{C}_S = \begin{bmatrix} \frac{R_2^{-1}}{R_1^{-1}+R_2^{-1}} \\ \frac{R_3^{-1}}{R_3^{-1}+R_4^{-1}} \end{bmatrix}$ the output matrix, and

$\mathbf{D}_S = \begin{bmatrix} \frac{R_1^{-1}}{R_1^{-1}+R_2^{-1}} & \frac{R_2^{-1}}{R_1^{-1}+R_2^{-1}} & 0 & 0 & \frac{1}{R_1^{-1}+R_2^{-1}} & 0 & 0 \\ 0 & 0 & \frac{R_3^{-1}}{R_3^{-1}+R_4^{-1}} & \frac{-R_4^{-1}}{R_3^{-1}+R_4^{-1}} & 0 & \frac{1}{R_3^{-1}+R_4^{-1}} & 0 \end{bmatrix}$ the feed through matrix.

Furthermore, from Eq. (6.13) the next two measurement equations can be obtained:

$$\theta_{so} = \frac{R_2^{-1}}{R_1^{-1}+R_2^{-1}}\theta + \frac{R_1^{-1}}{R_1^{-1}+R_2^{-1}}T_o + \frac{1}{R_1^{-1}+R_2^{-1}}\,\alpha SI_s \tag{6.21}$$

$$\theta_{si} = \frac{R_3^{-1}}{R_3^{-1}+R_4^{-1}}\theta + \frac{R_4^{-1}}{R_3^{-1}+R_4^{-1}}T_i \tag{6.22}$$

On another hand, an extra measurement equation, Eq. (6.25), can be obtained by defining a new output matrix:

$$\mathbf{C'}_S = -\frac{R_3^{-1}R_4^{-1}}{R_3^{-1}+R_4^{-1}} \tag{6.23}$$

and a new feed through matrix:

$$\mathbf{D'}_S = \begin{bmatrix} 0 & 0 & 0 & \dfrac{R_3^{-1}R_4^{-1}}{R_3^{-1}+R_4^{-1}} & 0 & 0 & 0 \end{bmatrix} \tag{6.24}$$

This extra measurement equation is:

$$Q_i = \frac{1}{S}\frac{R_3^{-1}R_4^{-1}}{R_3^{-1}+R_4^{-1}}(T_i - \theta) \tag{6.25}$$

and it is related to the use of a heat flux meter placed in the inner face of the wall (Naveros, et al., 2014). In Eq. (6.25) the heat flux density is defined as positive when it exits the room (Figure 6.2e).

Finally, the transfer functions related to the different measured outputs can be obtained, as it is shown in Chapter 3 using the Laplace transform, by introducing the state-space matrices obtained in this section into:

$$\mathbf{H}_S = \mathbf{C}_S(s\mathbf{I} - \mathbf{A}_S)^{-1}\mathbf{B}_S + \mathbf{D}_S \tag{6.26}$$

6.3.2. Estimating the ARX model parameters

Once the transfer function is obtained using the Laplace transform, Eq. (6.26), it can be obtained the discrete transfer function as it is shown in Chapter 3:

$$\mathbf{H}_d = \mathbf{C}_S(z\mathbf{I} - \mathbf{A}_d)^{-1}\mathbf{B}_d + \mathbf{D}_S \tag{6.27}$$

and from Eq. (6.27) it can be obtained an ARX model similar to Eq. (6.1).

In order to obtain the transfer function in discrete time using Eq. (6.27), it is needed to note that the output matrix, $\mathbf{C}_S$, and the feed through matrix, $\mathbf{D}_S$, are the same obtained for continuous time. And the state matrix, $\mathbf{A}_d$, and the input matrix, $\mathbf{B}_d$, in discrete time are derived, from the state matrix, $\mathbf{A}_S$, and the input matrix, $\mathbf{B}_S$, in continuous time obtained, by using the relations: $\mathbf{A}_d = \mathbf{I} + \mathbf{A}_S\Delta t$ and $\mathbf{B}_d = \mathbf{B}_S\Delta t$

Particularly, in the present example it is obtained:

$$\mathbf{A}_d = 1 + \frac{\Delta t}{C_2}\left(\frac{R_1^{-1}R_2^{-1}}{R_1^{-1}+R_2^{-1}} + \frac{R_3^{-1}R_4^{-1}}{R_3^{-1}+R_4^{-1}}\right)$$

$$\mathbf{B}_d = \frac{\Delta t}{C_2}\left[\frac{R_1^{-1}R_2^{-1}}{R_1^{-1}+R_2^{-1}} \quad \frac{R_2^{-2}}{R_1^{-1}+R_2^{-1}} + R_2^{-1} \quad \frac{R_3^{-2}}{R_3^{-1}+R_4^{-1}} - R_3^{-1} \quad -\frac{R_3^{-1}R_4^{-1}}{R_3^{-1}+R_4^{-1}} \quad \frac{R_2^{-1}}{R_1^{-1}+R_2^{-1}} \quad 0 \quad 1\right]$$

Then, introducing the state space matrices in discrete time into Eq. (6.27), and expressing the result as it is shown in Eq. (6.1), the next ARX models are obtained:

- for the output, θ_{so}:

$$\theta_{so}(t_k) = -m'T_{so}(t_{k-1}) + n'_{10}T_o(t_k) + n'_{11}T_o(t_{k-1}) + n'_{41}T_i(t_{k-1}) + n'_{50}\alpha I_s(t_k) + n'_{51}\alpha I_s(t_{k-1}) \tag{6.28}$$

- for the output, θ_{si}:

$$\theta_{si}(t_k) = -m''T_{si}(t_{k-1}) + n''_{11}T_o(t_{k-1}) + n''_{40}T_i(t_k) + n''_{41}T_i(t_{k-1}) + n''_{51}\alpha I_s(t_{k-1}) \tag{6.29}$$

In the case in which the inside heat flux density, Q_i, is considered as an output, the state matrix, $\mathbf{A}_d$, and the input matrix, $\mathbf{B}_d$, are the same obtained for the outputs θ_{so} and θ_{si}, while the output matrix is $\mathbf{C'}_S$ and the feed through matrix is $\mathbf{D'}_S$ as it is shown in the previous section. The ARX model obtained is:

$$Q_i(t_k) = -mQ_i(t_{k-1}) + n_{11}T_o(t_{k-1}) + n_{40}T_i(t_k) + n_{41}T_i(t_{k-1}) + n_{51}\alpha I_s(t_{k-1}) \tag{6.30}$$

Eq. (6.28)-(6.30) take into account that long wave radiation exchange can be assumed as negligible (Naveros, et al., 2014).

Then, parameters n, n', n'' and m, m', m'' may be estimated from measured data using linear least squares (Appendix A6). It must be noted that this is the first step towards the identification of parameters with physical significance.

In the next section the thermal parameters, R, C are obtained using the estimated ARX model parameters of the inside heat flux density output, Eq. (6.30).

6.3.3. Obtaining the physical parameters

Once the ARX model parameters, m, n, are estimated using Eq. (6.30), it is possible to obtain coefficients of the matrices in discrete time $\mathbf{A}_d$, $\mathbf{B}_d$, $\mathbf{C'}_S$ and $\mathbf{D'}_S$. The relations between parameters that can be found for this case are by comparing Eq. (6.27) with Eq. (6.30):

$$\left.\begin{array}{c} a_{d11} = m \\ c'_{11}b_{d11} = n_{11} \\ d'_{14} = n_{40} \\ (c'_{11}b_{d14} - a_{d11}d'_{14}) = n_{41} \\ c'_{11}b_{d15} = n_{51} \end{array}\right\} \tag{6.31}$$

And, it is also possible to find the relation:

$$c'_{11} = -d'_{14} \tag{6.32}$$

From Eq. (6.31)-(6.32) there are 6 equations to obtain 6 unknowns.

The coefficients of the matrices in continuous time, $\mathbf{A}_S$ and $\mathbf{B}_S$, may be derived from the discrete parameters, $\mathbf{A}_d$ and $\mathbf{B}_d$. It is possible to obtain the lumped parameters (resistances and capacitances, R, C), which characterize the thermal behavior of the wall, thanks to the relation between the matrices $\mathbf{A}_S, \mathbf{B}_s, \mathbf{C}_s$ and $\mathbf{D}_s$ and the block matrices, $\mathbf{K}$, $\mathbf{K}_b$, which contain the physical parameters R, C.

6.4. Results and discussion

6.4.1. Results

This section presents the results of the identification process which are the values obtained for the U-value($\mathrm{W/m^2K}$), the dynamic solar energy transmittance $g(-)$ and the effective heat capacity of the wall, $C_{wall}(\mathrm{J/K})$.

The parameters of the ARX model and their uncertainties are presented besides the *t*-test utilized to check the statistical significance of the parameter values. Then, the physical parameters and their uncertainties are presented. The standard deviations of the physical parameters are obtained by using

the Monte Carlo method for propagation of uncertainties (GUM, 2008). The best estimates of the ARX model parameters and the covariance matrix are considered (Box, 2005).

The parameters of the ARX model may be obtained by fitting Eq. (6.30) to data using, for example, the method of ordinary least squares (Appendix A.6). The *t*-test is applied to check the null hypothesis, $P\,(t_{m-n} < |t|)$, which gives the probability of the parameter value to be zero instead of the estimated parameter value. The *t*-test supposes that $|t|$, as shown in Eq. (6.33), is distributed as a *t*-student distribution with $m - n$ degrees of freedom. A 95% confidence interval is considered, i.e. the estimated parameters are considered significant for $P\,(t_{m-n} < |t|) < 0.05$ (GUM, 2008).

$$|t| = \left|\frac{\hat{x}_i}{\sqrt{\sigma_{ii}}}\right| \sim t_{m-n} \tag{6.33}$$

where σ_{ii} is the i diagonal element of the covariance matrix, $\boldsymbol{\Sigma}_{\hat{\mathbf{x}}}$, obtained using the best likely unbiased estimator of the variance, $s_{\hat{x}}^2$, obtained from ordinary least squares (Box, 2005).

Tables 6.3-6.7 show the parameters obtained from data when Eq. (6.30) is used to do the linear regression using ordinary least squares (Appendix A.6).

The fit of the linear least regression of the ARX model is estimated as presented in Eq. (6.34), where $\|\cdot\|$ is the Euclidean norm of a vector, $\widehat{\boldsymbol{y}}$ is the vector of estimated output, $\widetilde{\boldsymbol{y}}$ is the vector of measured output and $\overline{\boldsymbol{y}}$ is a vector with all components equal to the mean value of the measured output vector elements (Box, 2005):

$$Fit = 1 - \frac{\|\widehat{\boldsymbol{y}} - \widetilde{\boldsymbol{y}}\|}{\|\widetilde{\boldsymbol{y}} - \overline{\boldsymbol{y}}\|} \tag{6.34}$$

Tables 6.9–6.13 present the specific outside and inside conductances, U_1, U_2 ($\mathrm{W/m^2K}$), the *U*-value, $U(\mathrm{W/m^2K})$, the dynamic solar energy transmittance, $g(-)$, and the effective heat capacity, $C_{wall}(\mathrm{J/K})$. Their standard deviations are also shown.

The time series which pass the statistical test are drawn in Figures 6.5-6.7. In these figures, the estimated error for each time series is taken as two times their standard deviation, $2s_{\hat{x}}^2$, i.e., the error bars represent the 95% confidence interval for every time series.

6.4.2. Discussion

6.4.2.1. Model order

A first order model is used to illustrate the methodology although higher order models are tractable since any order model can be obtained straightforward from the presented methodology. Nonetheless, it is important to notice that the term in the model regression grows with the order of the model and all inputs and outputs are measured data with an associated measurement error, i.e. a greater order model implies greater number of uncertain independent variables to regress in the ARX model. Then, the use of least squares would not give the best likely unbiased parameters since ordinary least squares needs to accomplish with the assumption of no errors, or negligible errors, in the independent variables (Appendix A6), and the regressed parameters of the ARX model could be different in practice of those which would be obtained in theory. The choice of a first order model is supported by the results obtained in previous studies ((Naveros & Ghiaus, 2015), (Naveros, et al., 2014)).

Table 6.3. Regressed parameter of the ARX model corresponding to the input $Q_i(t_{k-1})$

Series	m_1	s_{m_1}	$\|t\|$	$P\ (t_{m-n} < \|t\|)$
01	-0,843	0,006	133	0,01
02	-0,795	0,006	123	0,01
03	-0,850	0,005	155	0,01
04	-0,810	0,008	99	0,01
05	-0,836	0,009	93	0,01
06	-0,838	0,006	133	0,01
07	-0,816	0,010	83	0,01
08	-0,813	0,007	121	0,01
09	-0,812	0,005	150	0,01
10	-0,851	0,008	112	0,01
11	-0,815	0,007	121	0,01
12	-0,806	0,007	119	0,01
13	-0,839	0,005	169	0,01
14	-0,832	0,006	146	0,01
15	-0,827	0,007	116	0,01
16	-0,828	0,009	90	0,01
17	-0,834	0,007	128	0,01
18	-0,831	0,007	124	0,01
19	-0,819	0,007	117	0,01
20	-0,809	0,007	121	0,01
21	-0,820	0,005	151	0,01

Table 6.4. Regressed parameter of the ARX model corresponding to the input $T_o(t_{k-1})$

Series	n_{11}	$s_{n_{11}}$	$\|t\|$	$P\ (t_{m-n} < \|t\|)$
01	-0,21	0,03	7	0,01
02	-0,48	0,02	20	0,01
03	-0,05	0,03	2	0,13
04	-0,42	0,05	8	0,01
05	-0,33	0,04	9	0,01
06	-0,33	0,03	10	0,01
07	-0,32	0,04	9	0,01
08	-0,34	0,02	14	0,01
09	-0,31	0,02	18	0,01
10	-0,24	0,03	9	0,01
11	-0,40	0,03	14	0,01
12	-0,48	0,03	14	0,01
13	-0,26	0,02	11	0,01
14	-0,28	0,03	9	0,01
15	-0,33	0,03	10	0,01
16	-0,34	0,05	7	0,01
17	-0,23	0,03	7	0,01
18	-0,24	0,05	5	0,01
19	-0,47	0,05	9	0,01
20	-0,43	0,05	9	0,01
21	-0,41	0,04	11	0,01

Table 6.5. Regressed parameter of the ARX model corresponding to the input $T_i(t_k)$

Series	n_{40}	$s_{n_{40}}$	$\|t\|$	$P\ (t_{m-n} < \|t\|)$
01	3,0	0,7	4	0,01
02	4,5	0,4	11	0,01
03	5,8	0,5	12	0,01
04	6,2	0,4	14	0,01
05	4,7	0,3	18	0,01
06	3,5	0,3	13	0,01
07	4,7	0,2	27	0,01
08	1,3	0,2	5	0,01
09	1,5	0,2	7	0,01
10	2,3	0,4	7	0,01
11	6,4	0,2	33	0,01
12	6,6	0,3	26	0,01
13	5,5	0,3	21	0,01
14	5,0	0,3	18	0,01
15	3,0	0,3	10	0,01
16	3,5	0,2	14	0,01
17	4,5	0,3	15	0,01
18	3,9	0,3	14	0,01
19	3,7	0,3	11	0,01
20	4,4	0,4	12	0,01
21	5,9	0,4	15	0,01

Table 6.6. Regressed parameter of the ARX model corresponding to the input $T_i(t_{k-1})$

Series	n_{41}	$s_{n_{41}}$	$\|t\|$	$P\ (t_{m-n} < \|t\|)$
01	-2,7	0,7	4	0,01
02	-4,0	0,4	10	0,01
03	-5,6	0,5	12	0,01
04	-5,8	0,4	13	0,01
05	-4,4	0,3	17	0,01
06	-3,2	0,3	12	0,01
07	-4,4	0,2	25	0,01
08	-0,9	0,2	4	0,01
09	-1,2	0,2	5	0,01
10	-2,0	0,4	6	0,01
11	-5,9	0,2	30	0,01
12	-6,0	0,2	24	0,01
13	-5,2	0,3	20	0,01
14	-4,7	0,3	17	0,01
15	-2,6	0,3	9	0,01
16	-3,1	0,2	13	0,01
17	-4,2	0,3	14	0,01
18	-3,6	0,3	13	0,01
19	-3,2	0,3	10	0,01
20	-3,9	0,4	11	0,01
21	-5,4	0,4	14	0,01

Table 6.7. Regressed parameter of the ARX model corresponding to the input $I_{sv}(t_{k-1})$

Series	n_{51}	$s_{n_{51}}$	$\|t\|$	$P\ (t_{m-n} < \|t\|)$
01	-0,0142	0,0004	33	0,01
02	-0,0134	0,0004	34	0,01
03	-0,0183	0,0004	43	0,01
04	-0,0147	0,0006	25	0,01
05	-0,0137	0,0006	24	0,01
06	-0,0134	0,0006	22	0,01
07	-0,0139	0,0006	22	0,01
08	-0,0101	0,0005	19	0,01
09	-0,0104	0,0005	21	0,01
10	-0,0103	0,0005	19	0,01
11	-0,0109	0,0007	15	0,01
12	-0,0106	0,0008	13	0,01
13	-0,0157	0,0006	28	0,01
14	-0,0157	0,0006	27	0,01
15	-0,0152	0,0007	22	0,01
16	-0,0145	0,0007	20	0,01
17	-0,0184	0,0007	26	0,01
18	-0,0183	0,0007	25	0,01
19	-0,0145	0,0006	23	0,01
20	-0,0161	0,0006	29	0,01
21	-0,0157	0,0005	34	0,01

Table 6.8. Fit of the linear regression of the ARX model for each data series

Series	Fit	Fit(%)
01	0,91	91,31%
02	0,92	92,25%
03	0,94	94,24%
04	0,92	92,22%
05	0,93	92,62%
06	0,94	93,84%
07	0,92	91,90%
08	0,93	93,29%
09	0,95	94,67%
10	0,92	91,86%
11	0,93	93,40%
12	0,93	92,98%
13	0,95	94,76%
14	0,95	95,10%
15	0,93	92,82%
16	0,90	89,94%
17	0,93	92,54%
18	0,93	93,05%
19	0,93	92,81%
20	0,94	93,61%
21	0,94	93,88%

Table 6.9. The specific outside conductance of the dynamic physical model for each data series, U_1(W/m^{2}K)

Series	$U_1(\mathbf{W/m^2K})$	$s_{U_1}(\mathbf{W/m^2K})$	$\|t\|$	$P\ (t_{m-n} < \|t\|)$
01	3,5	213,0	0,02	0,99
02	4,3	0,4	10,02	0,01
03	0,5	0,3	1,53	0,13
04	3,3	0,4	8,77	0,01
05	3,5	0,3	10,44	0,01
06	5,0	0,8	6,00	0,01
07	2,9	0,3	10,02	0,01
08	-3,8	318,4	0,01	0,99
09	-10,2	270,5	0,04	0,97
10	7,5	383,8	0,02	0,98
11	3,6	0,3	12,32	0,01
12	4,3	0,4	10,09	0,01
13	2,4	0,3	9,12	0,01
14	2,7	0,4	6,95	0,01
15	6,8	432,3	0,02	0,99
16	5,4	2,8	1,93	0,05
17	2,1	0,5	4,59	0,01
18	2,5	0,9	2,96	0,01
19	11,3	398,1	0,03	0,98
20	5,1	1,4	3,67	0,01
21	4,0	0,6	7,21	0,01

Table 6.10. The specific inside conductance of the dynamic physical model for each data series, U_2(W/m^{2}K)

Series	$U_2(\mathbf{W/m^2K})$	$s_{U_2}(\mathbf{W/m^2K})$	$\|t\|$	$P\ (t_{m-n} < \|t\|)$
01	3,0	0,7	4,16	0,01
02	4,5	0,4	10,84	0,01
03	5,8	0,5	12,12	0,01
04	6,2	0,4	14,43	0,01
05	4,7	0,3	17,91	0,01
06	3,5	0,3	13,40	0,01
07	4,7	0,2	26,69	0,01
08	1,3	0,2	5,39	0,01
09	1,5	0,2	6,64	0,01
10	2,3	0,4	6,49	0,01
11	6,4	0,2	32,97	0,01
12	6,6	0,3	26,18	0,01
13	5,5	0,3	21,18	0,01
14	5,0	0,3	17,89	0,01
15	3,0	0,3	10,16	0,01
16	3,5	0,2	14,04	0,01
17	4,5	0,3	15,12	0,01
18	3,9	0,3	14,48	0,01
19	3,7	0,3	11,46	0,01
20	4,4	0,4	12,24	0,01
21	5,9	0,4	14,97	0,01

Table 6.11. *U*-value of the dynamic physical model for each data series

Series	$U(\mathbf{W/m^2K})$	$s_U(\mathbf{W/m^2K})$	$\|t\|$	$P\ (t_{m-n} < \|t\|)$
01	1,62	0,34	4,80	0,01
02	2,20	0,05	43,09	0,01
03	0,45	0,27	1,64	0,10
04	2,17	0,15	14,26	0,01
05	2,02	0,10	20,72	0,01
06	2,05	0,10	21,46	0,01
07	1,81	0,11	16,65	0,01
08	1,94	0,04	49,14	0,01
09	1,82	0,04	47,53	0,01
10	1,77	0,09	18,71	0,01
11	2,28	0,12	19,64	0,01
12	2,61	0,15	17,29	0,01
13	1,65	0,12	13,30	0,01
14	1,75	0,16	11,10	0,01
15	2,06	0,20	10,14	0,01
16	2,11	0,26	7,99	0,01
17	1,44	0,20	7,07	0,01
18	1,53	0,29	5,23	0,01
19	2,78	0,25	11,07	0,01
20	2,36	0,22	10,63	0,01
21	2,37	0,18	13,11	0,01

Table 6.12. Dynamic g-value of the dynamic physical model for each data series

Series	g	s_g	$\|t\|$	$P\ (t_{m-n} < \|t\|)$
01	0,23	15,90	0,01	0,99
02	0,12	0,01	12,07	0,01
03	0,17	0,01	20,11	0,01
04	0,12	0,01	15,50	0,01
05	0,15	0,01	13,31	0,01
06	0,20	0,02	8,78	0,01
07	0,13	0,01	14,51	0,01
08	-0,11	10,56	0,01	0,99
09	-0,34	8,62	0,04	0,97
10	0,32	14,30	0,02	0,98
11	0,10	0,01	14,03	0,01
12	0,10	0,01	13,75	0,01
13	0,14	0,01	23,78	0,01
14	0,15	0,01	21,69	0,01
15	0,31	16,34	0,02	0,98
16	0,23	0,08	2,85	0,01
17	0,17	0,01	13,92	0,01
18	0,19	0,02	8,75	0,01
19	0,35	9,09	0,04	0,97
20	0,19	0,03	5,92	0,01
21	0,15	0,01	16,23	0,01

Table 6.13. Effective heat capacity of the dynamic physical model for each data series

Series	C_{wall}(J/K)	$s_{C_{wall}}$(J/K)	$\|t\|$	$P\ (t_{m-n} < \|t\|)$
01	$8{,}05 \cdot 10^{05}$	$3{,}36 \cdot 10^{07}$	0,02	0,98
02	$6{,}53 \cdot 10^{05}$	$2{,}43 \cdot 10^{04}$	26,92	0,01
03	$8{,}92 \cdot 10^{05}$	$5{,}12 \cdot 10^{04}$	17,42	0,01
04	$7{,}98 \cdot 10^{05}$	$4{,}08 \cdot 10^{04}$	19,58	0,01
05	$8{,}13 \cdot 10^{05}$	$4{,}47 \cdot 10^{04}$	18,17	0,01
06	$8{,}42 \cdot 10^{05}$	$4{,}92 \cdot 10^{04}$	17,13	0,01
07	$7{,}04 \cdot 10^{05}$	$3{,}67 \cdot 10^{04}$	19,20	0,01
08	$-2{,}35 \cdot 10^{05}$	$3{,}09 \cdot 10^{07}$	0,01	0,99
09	$-8{,}08 \cdot 10^{05}$	$2{,}47 \cdot 10^{07}$	0,03	0,97
10	$1{,}17 \cdot 10^{06}$	$4{,}49 \cdot 10^{07}$	0,03	0,98
11	$9{,}06 \cdot 10^{05}$	$3{,}92 \cdot 10^{04}$	23,10	0,01
12	$9{,}66 \cdot 10^{05}$	$4{,}92 \cdot 10^{04}$	19,62	0,01
13	$8{,}15 \cdot 10^{05}$	$3{,}90 \cdot 10^{04}$	20,91	0,01
14	$7{,}68 \cdot 10^{05}$	$3{,}89 \cdot 10^{04}$	19,73	0,01
15	$9{,}86 \cdot 10^{05}$	$4{,}35 \cdot 10^{07}$	0,02	0,98
16	$8{,}84 \cdot 10^{05}$	$2{,}86 \cdot 10^{05}$	3,09	0,01
17	$6{,}69 \cdot 10^{05}$	$6{,}00 \cdot 10^{04}$	11,15	0,01
18	$6{,}52 \cdot 10^{05}$	$8{,}82 \cdot 10^{04}$	7,39	0,01
19	$1{,}44 \cdot 10^{06}$	$3{,}69 \cdot 10^{07}$	0,04	0,97
20	$8{,}44 \cdot 10^{05}$	$1{,}14 \cdot 10^{05}$	7,42	0,01
21	$9{,}19 \cdot 10^{05}$	$5{,}29 \cdot 10^{04}$	17,36	0,01

The order of the model is related to the number of differential equations and the number of differential equations depends on the number of nodes with non-negligible heat capacities. Naveros et al. showed that models which considered a negligible heat capacity in nodes placed at wall surfaces predicted better the surface temperature (Naveros, et al., 2014). Fraisse et al. also reported the advantage of using negligible heat capacities in wall surfaces nodes for a second order model (Fraisse, et al., 2002). More recently, Berthou et al. reported that, for a building, a second order model, which considers a first order model for walls and supposes negligible heat capacities in wall surface nodes is a better candidate than a third order model, which considers a second order model for walls and supposes non-negligible heat capacities in wall surface nodes (Berthou, et al., 2014).

6.4.2.2. Results analysis

The present methodology is used to estimate the *U*-value ($\mathrm{W/m^2K}$), the dynamic solar energy transmittance, $g(-)$, and the effective heat capacity, C_{wall}(J/K), for a simple wall supposed homogeneous. The novelty is the use of an ARX model, deduced from a thermal network, to recover the physical parameters without using the final value theorem ((Jiménez, et al., 2008a), (Jiménez, et al., 2008b), (Jiménez & Madsen, 2008)).

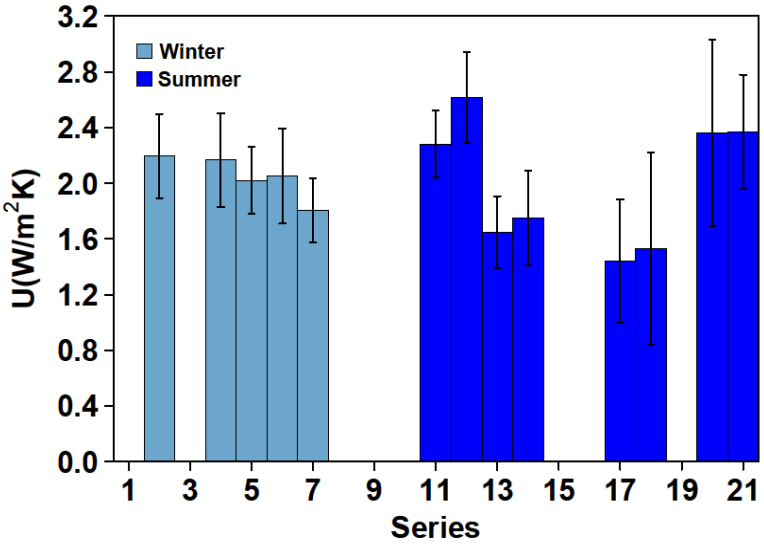


Figure 6.5. U-value of the dynamic physical model for each data series

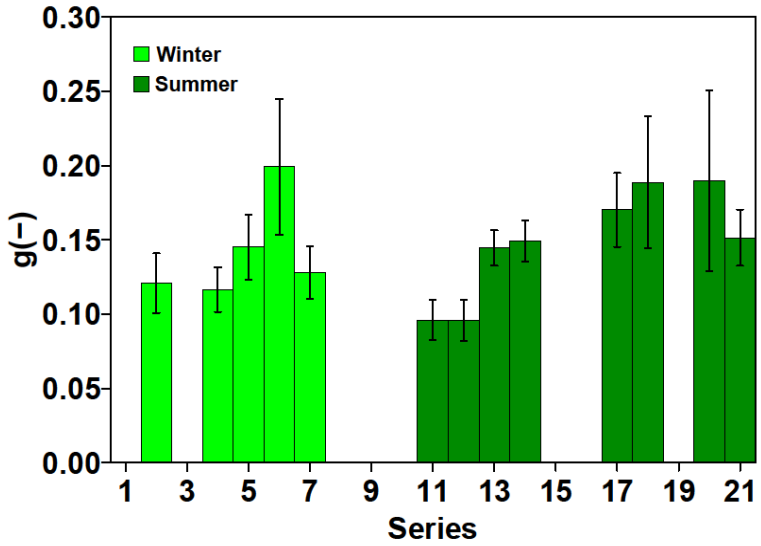


Figure 6.6. g-value of the dynamic physical model for each data series

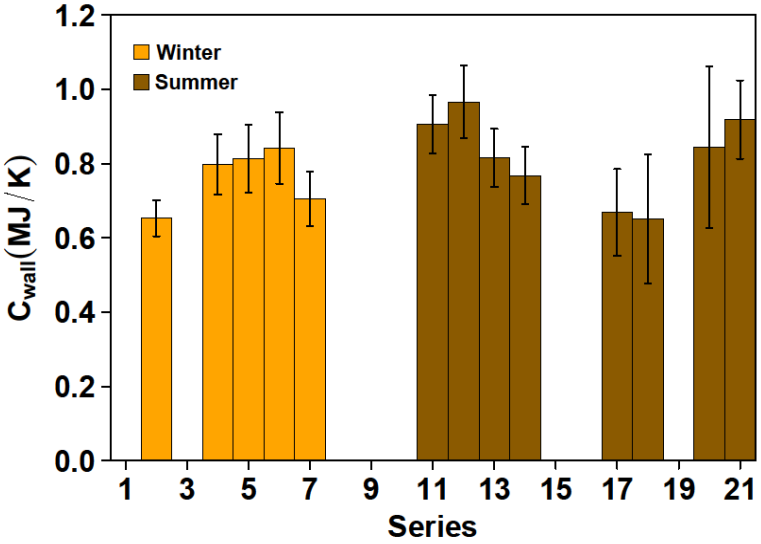


Figure 6.7. Effective heat capacity of the dynamic physical model for each data series

Dynamic solar energy transmittance and *U*-value may be considered constant within their confidence interval, although theoretically, they may be considered dependent on the wind speed. Future studies may consider the influence on wind speed linearizing the parameters (Naveros, et al., 2012).

In this work, the effective heat capacity is defined as $C_{wall} = \rho c S d$, where $\rho(\mathrm{kg/m^3})$ is the wall density, $c(\mathrm{J/kgK})$ is the wall specific heat capacity, $S(\mathrm{m^2})$ is the wall surface and $d(\mathrm{m})$ is the wall width. Since wall physical properties are supposed constants, the estimated effective heat capacity needs to be constant within its confidence interval. The term "effective heat capacity" is used to distinguish it from the term "heat capacity" as it is defined in the EN-ISO-13786:2007 (ISO, 13786:2007). EN-ISO-13786:2007 defines "heat capacity" as a function of the input frequency.

In the case of the dynamic solar energy transmittance, it may be observed in Eq. (6.30), among others, that the absorptivity cannot be separated from the thermal resistances by fitting this model. This problem was observed by Naveros et al. as a high uncertainty in the estimation of the absorptivity (Naveros, et al., 2014). A combination of different models using different inputs and outputs may give information to separate the variables. It could be also supposed that the absorptivity is known by other means as part of the input (Hazyuk, et al., 2012).

Moreover, results also depend on boundary conditions of the experiment. It may be observed, Figures 5-7, that the summer period (Table 6.2, series 11-21) presents results with a higher variance and uncertainty as compared with the winter period (Table 6.2, series 01-10). This may be explained by the noise of the inside heat flux density signal, which is amplified by the model and is higher in the second period (Naveros, et al., 2014).

The results of this work are consistent with those obtained for the same wall by using an extended Kalman filtering approach and maximum likelihood criterion (Naveros, et al., 2014), and by using a method based on averages (Naveros, et al., 2012). They are also consistent with a study by Biddulph et al. about a wall built with similar materials (Biddulph, et al., 2014).

6.5. Conclusions

This chapter has shown how physical parameters of walls can be identified using ARX models without using the final value theorem. For this purpose, an ARX model has been built from a thermal network by deduction as it was shown in previous chapters. Once the ARX model was deduced from a thermal network passing through a state-space model, the parameters of the ARX model are obtained (by using least squares in this particular example) from measured data and, by knowing the structure of the state-space model and the chain of transformations, they were used for identifying the physical parameters (conductance and capacitance). An advantage of this methodology regarding previous works which used the final value theorem is that, in addition to the *U*-value, the effective heat capacity of the wall is recovered.

The application of the methodology, proposed in this dissertation to a simple wall, provides consistent results with previous works which used different methodologies, which can be considered particular cases of the here presented, and statistical criteria to reject not consistent physical parameters. In theory, the unicity of the physical parameters is guaranteed. In practice, modelling and measurement errors occur and the method is limited by them. The different model structures could be used to solve the parameter identification problem and then, they should be compared in future theoretical studies by using different methods. The methodology is exemplified for a simple well-

controlled volume but, as networks in general, it may be scalable and may be applied to complex systems: composed walls, rooms, buildings, etc., in the way that it is presented in this dissertation.

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APPENDIX A6. Parameters estimation and measurement errors using least squares

This appendix is inspired by Crassidis and it shows how the use of ARX models using ordinary linear least squares is sensitive to measurement errors (Crassidis & Junkins, 2011).

Let a batch of measured output values, $\tilde{y}_j \equiv \tilde{y}(t_j)$, of a mathematical process $y_j \equiv y(t_j)$, taken at known discrete instants of time t_j:

$$\{\tilde{y}_1, t_1; \tilde{y}_2, t_2; \dots; \tilde{y}_m, t_m\}$$

and let a linear model of the form:

$$y_j = \sum_{i=1}^{n} x_i h_i(t_j), \qquad m \geq n \tag{A6.1}$$

where:

$$h_i(t_j) \in \{h_1(t_j), h_2(t_j), \dots, h_n(t_j)\}$$

is a set of independent basis functions which is usually formed by measured inputs. It should be noticed that outputs at previous time steps may be introduced as inputs in the set of basis functions when an autoregressive model is considered. In practice, the parameters x_i are a set of constants whose numerical values are unknown and the true values of the output y_j and the true values of the inputs $h_i(t_j)$ are also unknown.

From Eq. (A6.1) it follows that the variables x and y are related according to a simple linear regression model. It seems reasonable to select the optimum x_i based upon a measure of "how well" the proposed model, Eq. (A6.1), predicts the output measured-values given a basis of input measured-values. For this end, we seek a set of estimates, denoted by $\{\hat{x}_1, \hat{x}_2, \dots, \hat{x}_n\}$, which can be used in Eq. (A6.1) to predict y_j. Errors, however, can arise between the true value y_j and the predicted value $\hat{y}_j \equiv \hat{y}(t_j)$ from a number of sources, including:

- measurement errors in outputs and inputs;
- modelling errors, i.e., the actual process being observed may not be accurately modelled by Eq. (A6.1).

In virtually every application, some combination of these error sources is present and they cannot be separated.

Firstly, it is not possible to know the true value, y_j, of the output since there is always a measurement error, v_j, supposed Gaussian noise and associated to each measured output, $\tilde{y}_j$.

$$\tilde{y}_j = \sum_{i=1}^{n} x_i h_i(t_j) + v_j \ , \qquad j = 1,2, \dots, m \tag{A6.2}$$

Secondly, the true values of the basis $h_i(t_j)$ are also unknown since the measured inputs, $\tilde{h}_i(t_j)$, also have measurement errors. Therefore, the model to estimate the predicted value, $\hat{y}_j$, is given by:

$$\hat{y}_j = \sum_{i=1}^{n} \hat{x}_i \tilde{h}_i(t_j) \ , \qquad j = 1,2,\dots,m \tag{A6.3}$$

which, by splitting the error of the basis $\tilde{h}_i = h_i + w_i$, may be expressed as:

$$\hat{y}_j = \sum_{i=1}^{n} \left(\hat{x}_i h_i(t_j) + \hat{x}_i w_{ij} \right) \ , \qquad j = 1,2,\dots,m \tag{A6.4}$$

or:

$$\hat{y}_j = \sum_{i=1}^{n} \left(\hat{x}_i h_i(t_j) + u_{ij} \right) \ , \qquad j = 1,2,\dots,m \tag{A6.5}$$

where the error due to the inputs $\tilde{h}_i(t_j)$, i.e. the term $\sum_{i=1}^{n} u_{ij}$, will grow with the number of parameters since it implies more terms to add. If the parameters are constant, this error will be also distributed as Gaussian noise while w_{ij} is distributed as Gaussian noise

Taking into account Eq. (A6.2) and (A6.5), the residual error, ε_j, is obtained:

$$\tilde{y}_j - \hat{y}_j = \sum_{i=1}^{n} x_i h_i(t) + v_j - \sum_{i=1}^{n} \hat{x}_i \tilde{h}_i(t_j) = \varepsilon_j, \qquad j = 1,2,\dots,m \tag{A6.6}$$

The residual error, ε_j, includes: 1) the measurement errors of output, v_j; 2) the error of the model, considering the difference between the unknown parameters x_i, and the estimated parameters $\hat{x}_i$; 3) the measurement errors of inputs, $\sum_{i=1}^{n} u_{ij}$.

Finally, using Eq. (A6.3) and (A6.6) the measured output can be written as:

$$\tilde{y}(t_j) = \sum_{i=1}^{n} \hat{x}_i \tilde{h}_i(t_j) + \varepsilon(t_j) \ , \qquad j = 1,2,\dots,m \tag{A6.7}$$

Eq. (A6.7) is equivalent to Eq. (6.1) and ordinary least squares may be applied for estimating the parameters of the ARX model (Crassidis & Junkins, 2011). These estimated parameters are sensitive to both the measurement input-output errors and the model error. In theory, the optimal use of ordinary least squares requires no errors in independent variables (inputs) to obtain the best likely unbiased estimators. In practice, the method becomes not optimal if inputs have non-negligible errors.

CONCLUSIONS AND PERSPECTIVES

The improvement of energy efficiency of buildings requires the quantification of the energy exchange between buildings and their surroundings. This improvement is needed for saving energy and reducing the greenhouse effect and the Earth global warming. Energy efficiency in buildings is a wide field and this dissertation has been focused on heat exchange only. The heat exchange in buildings has been related to the classic heat equation, which is a partial differential equation (PDE), usually utilized for studying the energy flow in form of heat (or heat flow) within solids by conduction as well as convective and radiative heat flow at solid surfaces. The heat equation is a distributed in parameter model which is obtained using the principle of energy conservation and considering that heat flow, from a system to another system at different temperature, follows a linear (or step-wise linearizable) law. Moreover, the linear law for heat flow between systems may be used whatever is the mode of energy exchange: conduction, convection or radiation. On another hand, it is also shown that the heat flow between systems at different temperature may be studied using thermal networks; in this case, systems will be represented by nodes. The most important principle in thermal networks is the principle of energy conservation. Furthermore, the interactions between the nodes of thermal networks are described by a linear law in the range of validity of such assumption. Thermal networks are not introduced from the analogy with the electrical networks, as usual, since that assumption has important limitations; the main problem observed is that Kirchhoff's voltage law does not apply for thermal networks since temperatures cannot be added algebraically as it is done with electrical voltage. Hence, this dissertation starts with graph theory by using directed graph (digraphs) as basis for constructing and stating thermal networks. This work considers that the construction of thermal networks is simplified by using graph theory and thermodynamics laws directly instead of adapting electrical networks. The use of the analogy requires first a whole knowledge of electrical networks which is not necessary for studying thermal networks. Moreover, thermal networks may be considered as lumped parameter models which are usually considered non-distributed parameter models. Nonetheless, a lumped model considers also the distribution of the variables in space implicitly; the only theoretical difference regarding to a distributed parameter model is based on the assumption that the distributed parameter model uses the exact value of the variables at every space point, while the lumped parameter model will only use the exact value of the variable at a finite number of space points while for the other values of the variable it will be used an approximation, which may be obtained by interpolation between exact values of the variables. In the case of a thermal network as defined in this text, the distribution of the variables in space is present in the number of nodes of the thermal network, i.e. in the number of differential and algebraic equations (DAE) which will be used for describing a particular thermal system.

The heat equation (PDE) is approximated by a system of DAE (thermal network) by space discretization. This implies that the heat equation may be considered the starting point for the study of heat transfer processes in buildings, further considering thermodynamics principles and the graph theory for building thermal networks (system of DAE). The connection is important for showing explicitly the equivalence of solving the heat equation or using thermal networks directly. Moreover, it is demonstrated that the choice of the discretisation method for solving the heat equation (i.e. finite element, finite differences or finite volume) does not change the analysis that may be done a posteriori using thermal networks. In fact, the idea remaining is that, in practice, we do not know continuous in space and time functions but only a finite number of function values (discrete in space and time functions). For this reason, even the resolution of the heat equation as the starting point derives for

every virtual application in the resolution of a system of DAE. One of the advantages of utilizing thermal networks is the representation of the heat equation in matrix form, in which the boundary conditions appear explicitly in one expression that is similar for strong (infinite system of DAE) and weak (finite system of DAE) forms. The assumption of space linearity with temperatures changing linearly between two space points, or more in general nodes, with a known value (measured or estimated) allows us the use of the work by Gilbert Strang for studying thermal networks using linear algebra.

From a thermal network, which is the graphical representation of a system of DAE, is built a chain of transformations between different models representing heat exchange. First, the system of DAE is transformed into state-space representation. This step may be seen as a change of variables of the physical parameters, i.e. the parameters of the state-space representation have physical meaning since they depend on the physical parameters. But the physical meaning is shared between the parameters and the knowledge of the change of variables (transformations). The following transformation is from state-space to transfer function representation, which implies going from time domain to frequency domain and may be the first step for the integration in time of the differential equations. This step may be seen as a second change of variables of the physical parameters. For completing the time integration, the transfer function is discretized and finally, by going back to time domain, it is transformed in an autoregressive with exogenous model (ARX). The parameters of the ARX are the same with those of the discrete transfer function, which have been obtained through a new transformation (change of variables). Therefore, the ARX parameters have physical meaning but their physical meaning is shared with the knowledge of all transformations (change of variables). The transformations may be done in the direct and in the inverse way. When the aim is to solve the heat equation, the whole chain of transformations is not usually presented explicitly. This chain covers all the possible kinds of model structures which may be used and any of them is favoured in this work. The importance of deducing all the model structures from physical principles resides in the novelty of doing it explicitly since the literature usually highlights the part of mathematical methods instead of the part related to physical modelling of heat transfer. The different model representations may be used indistinctly for simulation, control or system identification, but this work dealt with system identification only. The problem has been stated from a deterministic point of view without considering error propagation in the transformations.

The different classes of models, or model structures, for the heat transfer processes are deduced from first principles and the procedure connects bi-directionally thermal networks, state-space and ARX model parameters in a general and automatic way. The order of the different model structures is connected to each other and it is chosen finite. For practical applications, the choice of model order may be done following a-priori physical knowledge on the thermal system, or it may be determined by using estimation methods related to the identification procedure which follows the modelling. Moreover, it has been demonstrated, independently of the estimation of the model parameters, the bi-directional connection between the parameters of different model structure for giving a new insight on the identifiability of the thermal network parameters from a deterministic point of view. The proposed procedure determines if there exists a solution to identify the thermal network parameters, i.e. thermal resistances and capacities, and if such solution is unique without requiring the estimation of the parameter values. Any of the model structures deduced from the heat equation (i.e. thermal networks, state-space, transfer function or ARX models), may be employed as:

- white-box models for system simulation, i.e. the output is estimated using a known model with known parameters and measured input data;

- grey-box models for system identification, i.e. the model and its parameters are estimated in part from measured data using any estimation method.

This work considers a deterministic point of view that implies that high order models are closer to the exact description of the heat equation. From a practical point of view, uncertainty and numerical issues need to be considered after modelling since they are a limit for system simulation as well as for system identification. In the particular case of simulation-based estimation of energy efficiency in buildings, very large errors are obtained and that is the reason for seeking the development of estimation methods by using measurements. Much advancement was achieved in experimental identification of building thermal parameters in the last years. The choice of the order of the model is an important problem which still needs to be studied; the heat transfer analysis starts from the heat equation which may be expressed as a model in matrix form of infinite order; for practical application the order of the model needs to be finite. In matrix representation, the continuous and the discrete models have the same formalism and the choice of the model structure is based on the discretization of the differential and algebraic equations (DAE) which models the physical phenomena. The aim of the model is to reproduce the measurable input-output relation of the variables related by the physical system. Then, the precision of the model does not need to be higher than the achievable accuracy of the measurements and the model order needs to take into account the domain of variation of the variables. The transfer of the signal from input to output implies a change of amplitude in function of frequency. Spectrum analysis of the inputs of the system reveals the frequency domain on which the model needs to be valid. The domain of validity of the model is limited by the length of the measured signal and the sampling time step. This domain is reduced by considering the frequency range in which the amplitudes of input signals are large enough to be measured. Furthermore, the frequency domain is reduced by taking into account the measurable variation of output signals as a function of the amplitude variation of the input signals and the attenuation done by the system. The present chapter gives a criterion, based on the frequency spectrum of the inputs and on the frequency response of thermal models, to be applied for choosing the order of the model used on building thermal parameter identification. This can optimize the characterization of building energy performance.

Finally, it has been shown how physical parameters of walls can be identified using ARX models without using the final value theorem. For this purpose, an ARX model has been built from a thermal network by deduction, as it was shown in previous chapters. Once the ARX model was deduced from a thermal network passing through a state-space model, the parameters of the ARX model are obtained (by using least squares in this particular example) from measured data and, by knowing the structure of the state-space model and the chain of transformations, they were used for identifying the physical parameters (conductance and capacitance). An advantage of this methodology regarding previous works which used the final value theorem is that, in addition to the *U*-value, the effective heat capacity of the wall is recovered. The application to a simple wall of the methodology, proposed in this dissertation, provides:

1) consistent results with previous works, which used different methodologies which can be considered particular cases of the one presented here; and

2) statistical criteria to reject not consistent physical parameters.

In theory, the unicity of the physical parameters is guaranteed. In practice, modelling and measurement errors occur and the method is limited by them. The different model structures could be used to solve the parameter identification problem and then, they should be compared in future theoretical studies by using different methods. The methodology is exemplified for a simple well-

controlled volume but it may be scalable and may be applied to complex systems: composed walls, rooms, buildings, etc., in the way that it is presented in this dissertation.

In short, thermal networks obtained from graph theory and thermodynamics have been used for modelling heat transfer connecting different model structures which can be used in the resolution of the direct and the inverse problems. The use of the methodology in the resolution of the inverse problem has been shown; the use of the methodology in the resolution of the direct problem is left for future studies, as the inclusion of user behavior.