Universidad de Granada

Departamento de Ciencias de la Computación
e Inteligencia Artificial

Sistemas de Ayuda a la Toma de Decisiones en Grupo Basados en Información Lingüistica Difusa

Programa Oficial de Doctorado en Tecnologías de la Información y la Comunicación

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Juan Antonio Morente Molinera

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La memoria titulada "*Sistemas de Ayuda a la Toma de Decisiones en Grupo Basados en Información Lingüistica Difusa*", que presenta D. Juan Antonio Morente Molinera para optar al grado de doctor, ha sido realizada dentro del Máster Oficial de Doctorado "*Soft Computing y Sistemas Inteligentes*" del Departamento de Ciencias de la Computación e Inteligencia Artificial de la Universidad de Granada bajo la dirección de los doctores D. Enrique Herrera Viedma y D. Ignacio Javier Pérez Gálvez.

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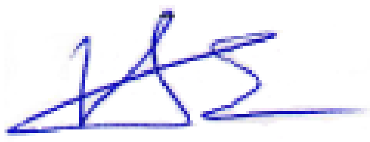

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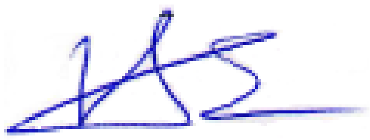

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1. Approach, Objectives and Structure of this Thesis

1.1. Approach

Decision making is a task constantly present in human's life. Its goal consists in choosing, from an alternative set, the best option. Consequently, research on the decision making area is necessary and important not only in decision theory but also in areas as diverse as management science [ASW$^+$15], politics [Bur14], social psychology [dMCRG14], artificial intelligence [LFYL14], soft computing [CAWHV15] and so on.

Most of the decisions that humans make during their whole life are made in groups. This usually occurs because, when a decision result is going to affect a set of people, all of their opinions are important and should be taken into account. This specific case of decision making is known as group decision making. A typical group decision making problem consists in choosing, from a specific set of alternatives, the one that is most popular among a set of experts. For this purpose, each expert provides a set of preferences. According to the importance given to the experts as a whole group, group decision making methods can be classified from directive to participatory. The closer a group decision making method is to a directive range, the more importance an expert minority might have. On the other hand, participative group decision making methods try to make decisions in a consensual way, that is, the decision results must be supported by all the experts participating in the decision making process.

Decision Support Systems have been an important research area since its first appearance in the 60s [Sim60, Ant65]. Their main purpose is to aid users in the sometimes difficult process of decision making. Thanks to Decision Support Systems, users can carry out the decision making task in an organized way due to the fact that a computational system leads the whole process. Moreover, there exist some approaches, called consensus approaches that are able to compute soft consensus measures [CMPHV10]. In such a way, the consensus level can be obtained and users could be informed about how the group decision making process is going on. Since the 60s, computational environments, in which group decision making processes are held, have evolved drastically. It is very important that Decision Support Systems evolve in the same way as the computing environments where they are used since each environment has its own characteristics and peculiarities. Consequently, methods must be adapted to the new circumstances. Traditional group decision making methods environment assumed that the experts are reunited in a single room and carry out a face-to-face discussion. Nevertheless, more recent environments demand the experts to make decisions without having to reunite in the same place. In this thesis, we have created novel Decision Support Systems that works and try to get the most out from the most recent computational environment: Web 2.0 and smartphones. Although there have been some attempt of making group decisions making models that works in this kind of environments [APCHV12], there is still a high amount of work to do in order to adapt them into the Web 2.0 and smartphones environment. The main objective of this thesis is to cover this research gap.

The appearance of Web 2.0 technologies [And10, CF08] and smartphones [BPSF15, CB10] have profoundly altered the way that Internet was traditionally conceived. In its first days, Internet was used as a way of consulting information where every webpage was static, that is, Internet users could only consult the information stored on it and not promote any changes. All the stored data was provided by a small minority of the overall users. Nowadays, Internet users have assumed the main role being the ones providing and consuming, at the same time, all the information available on the Internet. Therefore, Internet purpose has changed completely. Originally, Internet was used almost strictly for scientific and education purposes. Internet was the place where users could obtain information about a high amount of topics. Nevertheless, at present, Internet is used in almost every aspect of our lifes. This includes chatting, buying, listening music, watching movies, and a long etc. With the appearance of smartphones and 3G/4G technologies [HY03], the last access restrictions imposed by the Internet have been dissipated. This is due to the fact that smartphones allow us to access the Internet independently of the place where we are located. This way, users can obtain information at any time in any location.

The increase on users and the active participative role that they have assumed in the new era of Internet have triggered an exponential growth on the information available on the Internet generating the phenomenom called Big Data [MB12]. All the public information that all the users share is stored and can be used for other users to get benefit from it. An example of this fact is the increase use of webpages such as tripadvisor or amazon. This webpages let the users to provide information about their experience with certain products that are consulted by other users that are tempted to obtain the same products. This way, they can benefit from the other users experience. Although this is clearly positive, we cannot avoid the fact that information veracity must be proved. In its recent days, when only a few users were able to provide information, the veracity of the information was easier to prove. Nowadays, it is difficult to stablish the origin of most of the information available on the Web. Thus, it is difficult to establish its veracity. Moreover, the high increase of the information highlights the need of tools that categorize and organize the available information in order for users to take advantage of it. If the information is disorganized, users get lost in the high amount of data and it is impossible for them to get any benefit from it. In our designed Decision Support Systems, Fuzzy Ontologies have been added as a support tool since they are quite useful for managing information tasks. Fuzzy Ontologies [CC07] allow us to store and manage information in an organized way. Unlike relational databases, they are able to stablish and model the connections among the different elements that compose them. Therefore, they are more suitable when dealing with a high amount of information. Web 2.0 technologies can take advantage of Fuzzy Ontologies in order to store, in an ordered way, all the information that is provided by the Internet users. Since Fuzzy Ontologies provide means that allow us to carry out searches and retrieve specific pieces of data, it is an interesting way for users to deal with the information stored on the Internet.

The high increase on users participating in Internet activities also entails several problems. Since they have to communicate and share information, if rules are not set, due to the large number of users, the situation may get out of control causing failure in communications, loss of information and misunderstandings. Consequently, methods that allow users to communicate in a organized and efficient way are needed. Another issue that should be taken into account when a high user participation rate is expected is the user-system communication means. When a high amount of users are communicating with the same system, it is quite probable that not all of them feel comfortable with the communications means provided by the system. It is necessary to find ways to solve this issue since if we want users to use the system, they have to find themselves comfortable or, otherwise, data provided accuracy will decrease and users will eventually get tired and stop using the Web application. Therefore, methods that ease the way that users express themselves are needed. Group decision making methods [HACHV09, PCHV11a] are used to carry out organized decisions among a set of experts. If these kind of methods are used over Web 2.0 technologies, it is possible to build Group Decision Support Systems that are able to coordinate a high amount of experts and allow them to work together in order to reach a common goal. In order to ease the way that experts use to express their preferences, multi-granular fuzzy linguistic modelling methods [HHVM00, MMPUHV15] are used. They allow the use of different linguistic label sets when communicating with the same computational system. They are also quite useful in environments where a high amount of users have to provide information to the same system since they can select the linguistic label set that better fulfil their necessities.

We have used the developed Decision Support Systems to allow Internet users to get the most out of Web 2.0 technologies and smartphones. We have centered our efforts in designing systems which allow users to communicate, make decisions together and allow the information that they provide to be stored, organized and consulted in an ordered way.

1.2. Planteamiento

La toma de decisiones es una tarea que está constantemente presente en la vida del ser humano. Su principal objetivo consiste en elegir, a partir de un conjunto de alternativas, la mejor opción. En consecuencia, la investigación en el área de la toma de decisiones es necesaria e importante no sólo en el campo de la teoría de la decisión, sino también en áreas tan diversas como la ciencia de la administración [ASW+15], la política [Bur14], la psicología social [dMCRG14], la inteligencia artificial [LFYL14], el Soft Computing [CAWHV15], etc.

La mayor parte de las decisiones que los seres humanos tomamos durante toda nuestra vida se realiza en grupos. Esto generalmente se debe a que, cuando el resultado de la decisión va a afectar a un conjunto de personas, todas sus opiniones son importantes y deben tenerse en cuenta. Este caso específico de toma de decisiones se conoce como la toma de decisiones en grupo. Un problema típico de toma de decisiones en grupo consiste en elegir, a partir de un conjunto específico de alternativas, la que es más aceptada entre un conjunto de expertos. Para llevar a cabo este proceso, cada experto proporciona un conjunto de preferencias. Dependiendo de la importancia que se de a los expertos dentro del grupo, los métodos de toma de decisiones en grupo pueden ir de directivos a participativos. En los métodos directivos, una minoría de expertos puede ser capaz de llevar el liderazgo en la decisión. Por otra parte, los métodos más participativos tratan de tomar las decisiones de manera consensuada, es decir, los resultados del proceso de decisión deben ser apoyados por todos los expertos que participan en dicho proceso.

Los Sistemas de Soporte para la Toma de Decisiones han sido un área importante de investigación desde su primera aparición en los años 60 [Sim60, Ant65]. Su propósito principal es ayudar a los usuarios en el a veces difícil proceso de tomar una buena decisión. Gracias a los Sistema de Soporte para la Toma de Decisiones, los usuarios pueden llevar a cabo la tarea de tomar decisiones de una manera organizada debido al hecho de que un sistema informático se encarga de dirigir todo el proceso. Por otra parte, existen algunos enfoques, denominados enfoques de consenso, que son capaces de medir el consenso alcanzado en un proceso de decisión [CMPHV10]. De esta manera, es posible informar a los usuarios acerca del estado en que se encuentra el proceso de toma de decisiones. Desde los años 60, los entornos informáticos en los que se llevan a cabo los procesos de toma de decisiones en grupo han evolucionado drásticamente. Es muy importante que los Sistemas de Soporte para la Toma de Decisiones evolucionen de la misma manera que los entornos de computación en donde son utilizados ya que cada uno tiene sus propias características y peculiaridades. En consecuencia, los métodos deben adaptarse a las nuevas circunstancias. Los entornos computacionales en los que se llevan a cabo los métodos tradicionales de toma de decisiones están pensados para que los expertos se reúnan en un mismo lugar y lleven a cabo un debate presencial. Sin embargo, los entornos más recientes exigen que los expertos puedan tomar decisiones sin tener que reunirse. En esta tesis, nos hemos centrado en diseñar Sistemas de Soporte para la Toma de Decisiones que trabajen y traten de sacar el máximo provecho del más reciente entorno computacional: la Web 2.0 y los smartphones. Aunque ya hay en la literatura científica algún diseño de este tipo de sistemas que trabaja en entornos Web 2.0 [APCHV12], todavía hay una gran cantidad de trabajo por hacer a fin de adaptarlos al entorno Web 2.0 y los smartphones. El principal objetivo de esta tesis es arrojar un poco de luz sobre la inminente evolución y adaptación a los nuevos entornos de los modelos actuales de toma de decisiones en grupo.

La aparición de las tecnologías Web 2.0 [And10, CF08] y los smarphones [BPSF15, CB10] han alterado profundamente las bases de lo que tradicionalmente había sido Internet. En sus primeros momentos, Internet se utilizaba como un medio de consulta de información donde cada página Web era estática, es decir, los usuarios de Internet únicamente tenían acceso a la información almacenada y no podían participar ni proponer ningún cambio. Toda la información almacenada provenía de una pequeña minoría de usuarios considerados expertos en la temática. Actualmente, este paradigma ha cambiado radicalmente. Los usuarios de Internet han asumido el papel principal siendo los que proporcionan y, a la vez, consumen toda la información disponible en Internet. En sus comienzos, el uso principal de Internet era científico y académico. Su propósito era almacenar información sobre una amplia variedad de temáticas. Sin embargo, actualmente, Internet se usa en casi cada aspecto de nuestra vida entre los que incluimos hablar, comprar, vender, escuchar música, ver películas y un largo etc. Con la aparición de los smartphones y las tecnologías 3G/4G [HY03], los últimos requisitos remanentes de acceso a Internet han desparecido. Gracias a estos dispositivos de bolsillo, es posible acceder a Internet independientemente del lugar en el que nos encontremos. De esta manera, los usuarios pueden utilizar los servicios proporcionados por Internet en cualquier momento y lugar.

El aumento de usuarios con acceso a Internet y el rol participativo que han asumido en la nueva era de Internet ha promovido un crecimiento exponencial de la cantidad de información disponible para su uso y consumo. A esto se le denomina el fenómeno Big Data [MB12]. La información pública proporcionada por los usuarios se almacena y puede ser usada en beneficio de otros usuarios. Un ejemplo de esto es el aumento de popularidad y uso de páginas Webs tales como Tripadvisor o Amazon. En estas páginas, los usuarios pueden comentar y hablar de sus experiencias con los productos que ofertan. De esta forma, si otro usuario se plantea obtener el producto puede consultar los comentarios para tener una idea más clara de sus beneficios y problemas. Aunque es indudable que el nuevo paradigma de Internet viene acompañado de multitud de ventajas y posibilidades, también plantea ciertos retos y problemas. Entre ellos se encuentra el problema de comprobar la veracidad de la información. En el paradigma anterior, dado que sólo un pequeño grupo de expertos se encargaba de proporcionar la información, la veracidad de esta era fácil de comprobar. Actualmente, debido a la ingente cantidad de proveedores de información, es casi imposible establecer el origen de la mayor parte de los datos disponibles. Por tanto, es difícil conocer su veracidad. Además, el alto crecimiento de la información almacenada en Internet hace que sea necesario el desarrollo de herramientas que permitan categorizar y organizar la información disponible de forma que los usuarios puedan aprovecharla. Si no se organiza adecuadamente la información, los usuarios de Internet pueden perderse fácilmente ante tal cantidad de datos con lo que no encontrarán lo que necesitan. Para solucionar esto, hemos usado las Ontologías Difusas [CC07] como herramienta de soporte de los Sistemas de Soporte para la Toma de Decisiones diseñados. Las Ontologías Difusas son herramientas que permiten almacenar y manejar información de forma organizada. A diferencia de las bases de datos relacionales, las Ontologías pueden establecer y modelar las conexiones entre los distintos elementos que las componen. Por tanto, son mucho más adecuadas para el manejo de cantidades ingentes de información. Las herramientas Web 2.0 pueden utilizarse para almacenar de forma ordenada toda la información que proviene de los usuarios de Internet. Debido a que las Ontologías Difusas proporcionan medios que nos permiten realizar búsquedas y obtener datos específicos, su uso nos puede ayudar a controlar la información almacenada en Internet.

El aumento del número usuarios que participa en este nuevo paradigma de Internet también puede generar algunos problemas. Dado que todos ellos tienen que comunicarse y compartir su propia información, si este proceso no se regula adecuadamente, la situación podría descontrolarse y causar fallos en las comunicaciones, pérdida de información y malentendidos entre los usuarios. Por este motivo, es necesario el desarrollo de métodos que permitan a los usuarios comunicarse de manera eficiente y organizada. Por otro lado, es importante que los usuarios dispongan de medios adecuados para interactuar de forma cómoda con el sistema que de soporte a la acción que quieran realizar. Cuando muchos usuarios tienen que interactuar con el mismo sistema, es muy probable que no todos se sientan cómodos con la interfaz propuesta. Dado que los usuarios dejarán de utilizar la interfaz sino se sienten cómodos con ella, es necesario desarrollar herramientas que permitan a cada usuario comunicarse de la forma que le resulte más sencilla. El objetivo de los métodos de toma de decisiones en grupo [HACHV09, PCHV11a] es asistir a varios usuarios, denominados expertos, a llevar a cabo procesos de toma de decisiónes sobre un conjunto de alternativas. Usando este tipo de métodos sobre las tecnologías Web 2.0, es posible construir aplicaciones web que sean capaces de coordinar a un alto número de usuarios de forma que trabajen juntos para alcanzar un mismo objetivo. Además, con el objetivo de facilitar al usuario el uso de estas herramientas, hemos utilizado métodos de modelado ligüístico multi-granular [HHVM00, MMPUHV15]. Este tipo de métodos permiten el uso de diferentes conjuntos lingüísticos difusos a la hora de comunicarse con el mismo sistema computacional. Este tipo de modelado es muy usado en entornos con un alto número de usuarios que tienen que proporcionar información al mismo sistema. De esta forma, cada usuario puede elegir el conjunto lingüístico difuso que más se adapta a sus necesidades.

Los sistemas de Soporte para la Toma de Decisiones desarrollados han sido utilizados para permitir a los usuarios de Internet sacar el máximo partido a las tecnologías Web 2.0 y a los smartphones. Hemos centrado nuestro esfuerzo en diseñar sistemas que permitan a los usuarios comunicarse, tomar decisiones conjuntas y que almacenen la información generada de forma organizada y fácil de consultar.

1.3. Objectives

The objective of this thesis consists in the designing of Decision Support Systems that help users to carry out decisions and retrieve information using Web 2.0 technologies and smartphones. For this purpose, we will use multi-granular fuzzy linguistic modelling, Fuzzy Ontologies and group decision making methods.

Concretely, the objectives of this thesis are exposed below:

1. The first objective consists on study the state-of-the-art of group decision making methods. We are interested on the benefits that the use of these kind of tools can offer in order to take advantage of them in our designed systems.

2. Afterwards, the same process is carried out with Fuzzy Ontologies. We study the state-of-the-art of these kind of methods and their properties.

3. Next multi-granular fuzzy linguistic modelling methods are reviewed. We have carried out a detailed comparison among the different multi-granular fuzzy linguistic methods in order to point out the advantages and drawbacks of each method. After that process, we are able to select the most adequate method for the procedures that we are going to develop.

4. Although, as the review carried out in the previous step suggests, multi-granular fuzzy linguistic modelling methods have been widely used in group decision making area, they have not been used in Fuzzy Ontologies. For that reason, we have carried out a study about the benefits that Fuzzy Ontologies can obtain if multi-granular fuzzy linguistic modellings methods are used in the Fuzzy Ontology designing and consulting procedures.

5. Once that we have studied the state-of-the-art of the tools that we want to

employ and how they can interact, we can start designing the methods that will achieve our goals. In total, three different methods have been designed. They are briefly described below:

- **A Decision Support System for decision making in changeable and multi-granular contexts**: This method uses multi-granular linguistic modellings along with several techniques in order to carry out group decision making processes using Web 2.0 technologies and smartphones. Our method is able to work correctly in dynamic contexts where experts can join and leave the process at any time. Also, it allows alternatives to be added and removed in any moment.
- **A linguistic mobile group Decision Support System based on Fuzzy Ontologies**: This method uses Fuzzy Ontologies in order to create a novel group decision making method that works on smartphones and is able to deal with a high amount of alternatives. Fuzzy Ontologies are used in order to reduce the high amount of alternatives into a feasible set that experts can use to carry out the discussion. Alternatives available can depend on the location of the users. The GPS included in most of smartphones is used to delimit an specific zone. This way, if, for example, the group decision making experts want to select a wine in a restaurant, they can choose among the wines available on the location where they are eating.
- **Creating knowledge databases for storing and share people knowledge automatically using group decision making and Fuzzy Ontologies**: This method purpose is to extract knowledge from a high amount of users and store in an organized way using Fuzzy Ontologies. This way, other users can get benefit from this common users knowledge. Veracity of the information is ensured due to the fact that

a high amount of users support the information that is retrieved from the process. For this method, Fuzzy Ontologies, group decision making methods and multi-granular fuzzy linguistic modelling methods are used.

1.4. Structure of the Thesis

In this subsection, we show the research plan followed during the realization of this thesis. In such a way, this is reflected in the chapter structure of this memory. The first chapter consists in these introductory sections, the next three chapters explain in detail how multi-granular linguistic modelling, group decision making and Fuzzy Ontologies methods work and the last five chapters describe all the original contributions of this memory.

Chapter 1 is dedicated to provide a brief introduction about the thesis purpose. The approach, objectives and structure of it are highlighted.

In Chapter 2, concepts that are needed to understand the novel developed methods are exposed. Concretely, basis of multi-granular fuzzy linguistic modelling methods, linguistic modelling and Fuzzy Ontologies are exposed. The section starts exposing what linguistic modelling is and continues with the definition of a multi-granular fuzzy linguistic modelling environment and how the situation is usually resolved. Next, basis of group decision making methods are described. Furthermore, reasons why group decision making can be useful in Web 2.0 environments are exposed. Next, the steps followed by these methods to solve a group decision making problem are exposed. A brief explanation of what can be used to implement each of them are also exposed. Finally, Fuzzy Ontologies are presented.

In Chapter 3, a review on the most used multi-granular fuzzy linguistic modelling methods used in group decision making field is carried out. They are divided into six different categories. Finally, advantages and drawbacks of each method are highlighted.

Once that we are familiar with the state of the art of the required tools, on chapter 4, a novel method that uses group decision making methods and multi-granular linguistic information is presented in order to show the advantages that can be obtained when combining both methodologies. This method is designed to work over the Internet using smartphones and Web 2.0 technologies. The designed system focuses on removing the traditional group decision making system problem of having static alternatives and experts.

Nevertheless, although we managed to design an effective method, it has the drawback of not being able to deal with the high amount of information problem that the use of Web 2.0 technologies entails. In order to overcome this limitation for future researches, we have decided to use Fuzzy Ontologies. Since multi-granular linguistic modelling methods are one of the main used tools for our Decision Support System designs, we have decided to carry out an study on how they can be applied to Fuzzy Ontologies in order to improve them first. Different innovative ways of combining both methodologies and advantages obtained are presented. Results of this study are shown in chapter 5.

Afterwards, our next step has been to solve the high amount of information problem by carrying out the design of a novel Decision Support System that is capable of dealing with a high amount of alternatives using Fuzzy Ontologies. Thanks to them, we are able to reduce the high amount of initial alternatives into a feasible set. GPS mobile information has also been used for discarding alternatives that are not available in the current situation. Results of this research are exposed in chapter 6.

As it has been exposed in the introduction, Web 2.0 technologies are born in order for users to share information. Nevertheless, both traditional Decision Support Systems and the novel developed ones do not make use of this feature and discard the group decision making results achieved. We believe that this information is valuable and should not be discarded. In order to overcome this limitation, a novel Group Decision Support System that is capable of storing the decision results into a knowledge database has been designed. Thanks to Fuzzy Ontologies, the decision results are stored in a way that other users can benefit from the stored information after the finalization of the decision making process. group decision making methods are used to extract users information and multi-granular fuzzy linguistic modelling methods are used to guarantee a user-friendly communication with the system.

Step by step, we have been adapting traditional Decision Support Systems to Web 2.0 technologies. In such a way, this kind of methods can improve and use the benefits that Web 2.0 computational environment provides.

Finally, in Chapter 8, some conclusions and some possible future lines derived from the thesis research results are exposed.

2. Preliminares

Esta sección está dedicada a exponer todos los conceptos necesarios para entender de forma adecuada la investigación llevada a cabo en esta tesis. En la subsección 2.1, se exponen las bases del modelado ligüístico multi-granular. En la subsección 2.2, explicamos qué es un sistema de toma de decisiones. Finalmente, en la subsección 2.3, se definen las ontologías.

2.1. Modelado lingüístico difuso multi-granular

En esta subsección se presentan las bases del modelado lingüístico multi-granular. En la subsección 2.1.1, se exponen las bases del modelado lingüístico tradicional. En la subsección 2.1.2, se definen los procesos de modelado lingüístico difuso multi-granular.

2.1.1. Modelado lingüístico

En la actualidad, no dejan de aparecer sistemas informáticos en donde el usuario se comunica con el sistema con el objetivo de obtener una información determinada. Debido a que las personas nos comunicamos entre nosotras usando palabras que representan conceptos y a que los ordenadores sólo son capaces de tratar con números, esta comunicación puede complicarse. Para que la comunicación sea correcta, es necesario implementar métodos que mejoren esta comunicación usuario-sistema. Estos métodos deben ayudar a los usuarios a expresar sus ideas de forma cómoda y al ordenador a entenderlas y manejarlas de la mejor manera posible.

Una manera de mejorar esta comunicación usuario-sistema es mediante el uso del modelado lingüístico [Zad75a, Zad75b, Zad75c]. Gracias al modelado lingüístico, los usuarios pueden expresarse mediante el uso de palabras que representen conceptos y, a su vez, el sistema es capaz de trabajar con la información proporcionada mediante el uso del entorno matemático que tiene asociado.

Formalmente, se define una variable lingüística como una quintupla $< L, T(L), U, S, M >$ donde:

- L es el nombre de la variable.
- $T(L)$ es un conjunto finito de etiquetas lingüísticas o palabras.
- U es el universo de discurso.
- S es la regla sintáctica que genera los términos de $T(L)$.
- M es una regla semántica que asocia a cada valor lingüístico X su significado $M(X)$ donde $M(X)$ es un subconjunto difuso [Zad65] de U.

En la Figura 1, podemos ver un ejemplo gráfico de como puede definirse la variable *altura* [HACHV09].

El modelado lingüístico obtiene toda su flexibilidad representativa gracias al uso de los conjuntos difusos propuestos por Zadeh en 1975 [Zad65]. Siendo X el universo de discurso, un conjunto difuso A en X es un objeto de la forma:

$$A = \{(x, \mu(x)) : x \in X\} \tag{1}$$

donde $\mu(x) : X \to [0, 1]$ representa la función de pertenencia de A.

Gráficamente, la diferencia entre un conjunto difuso y uno regular se muestra en la Figura 2.

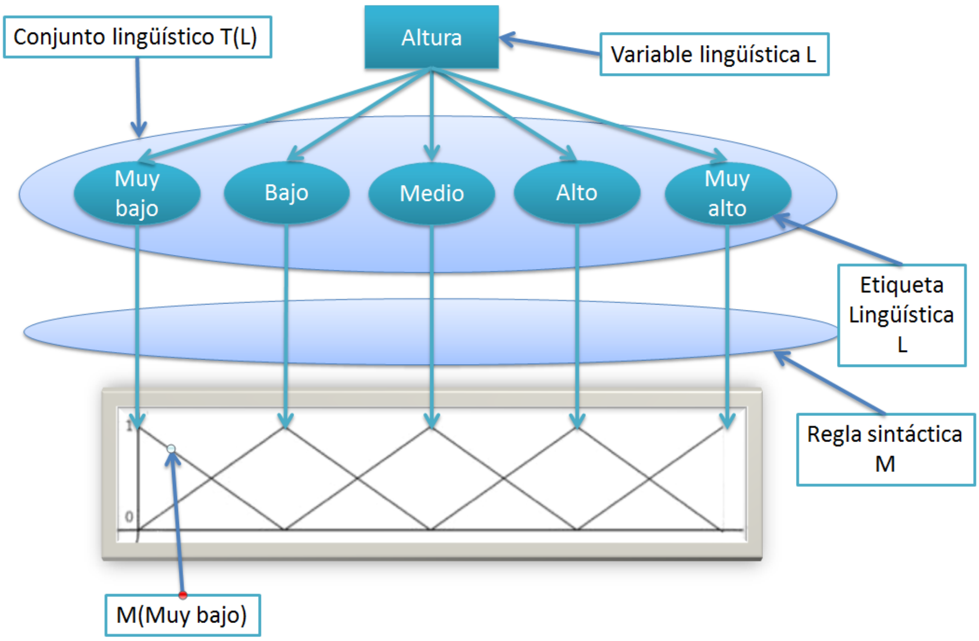


Figure 1: Esquema de la variable lingüística *altura*.

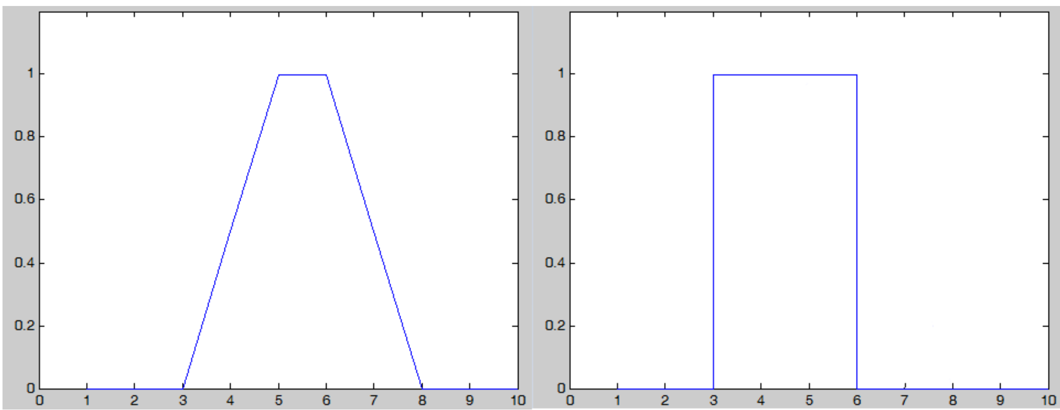


Figure 2: Diferencias entre un conjunto difuso y un conjunto regular. En la primera imagen vemos la representación de un conjunto difuso y en la segunda la de un conjunto no difuso.

El modelado lingüístico presenta las siguientes ventajas:

- **Comunicación usuario-sistema más sencilla**: El modelado lingüístico permite a los usuarios expresarse utilizando conceptos en vez de números. Para los usuarios es mucho más sencillo expresarse usando conceptos imprecisos tales como *bajo* o *alto* que valores numéricos como lo serían, por ejemplo, los valores 5.6 o 8.6 del intervalo [0,10]. Por tanto, el modelado lingüístico mejora notablemente la comunicación usuario-sistema permitiendo a los usuarios expresarse de la forma más cómoda para ellos y, a su vez, proporcionando herramientas al sistema que le permita poder trabajar con el tipo de información proporcionada.

- **Permite el manejo de información imprecisa**: Una de las mayores ventajas de utilizar modelado lingüístico es que nos permite crear sistemas computacionales que trabajan con conceptos de naturaleza imprecisa. Al ser imprecisos, cada persona puede tener su propia definición del concepto significando, por tanto, cosas diferentes para personas diferentes. Por ejemplo, si afirmamos que *Juan es muy alto*, algunas personas pueden pensar que la altura de Juan se sitúa en torno a los 2 metros mientras que otros pueden imaginar a Juan como alguien que mide más de 2 metros y medio. Como podemos observar, no es posible trabajar a nivel numérico con valores como *alto* o *muy alto* sin conocer los valores reales asociados en su definición. Sería necesario, por tanto, restringir cada concepto a un rango fijo de valores. Sin embargo, los rangos numéricos, debido a sus límites estrictos, no nos permiten realizar representaciones fiables de un concepto impreciso. Por ejemplo, si definimos la etiqueta *muy alto* como el conjunto de valores numéricos que está por encima de 2.5 metros, entonces se da el caso de que una persona que mida 2.49 metros no es considerada muy alta mientras que una de 2.5 metros sí que lo sería. Gracias al modelado lingüístico y su uso de los conjuntos difusos [Zad65], es posible dar una buena solución este tipo de situaciones. El valor de pertenencia a un conjunto difuso no es binario, $\{0, 1\}$, sino que viene determinado por una serie de grados, normalmente, un valor numérico perteneciente al intervalo $[0, 1]$. Usando este tipo de representación, podríamos afirmar que una persona es muy alta con grado uno si mide 2.5 metros y muy alta con grado 0.98 si midiera 2.49. Vemos entonces como la representación obtenida es mucho más adecuada para modelar este tipo de situaciones que el uso de intervalos simples cerrados. Por tanto, podemos concluir que el modelado lingüístico nos permite trabajar de forma ágil y eficiente con información imprecisa sin tener que tener en cuenta la representación numérica asociada a los conceptos con los que estemos tratando.

- **Manera de razonar similar a la del ser humano**: Cuando los seres humanos tomamos decisiones y realizamos razonamientos, lo hacemos de forma imprecisa y conceptual, sin conocer mediciones exactas. Por ejemplo, si queremos averiguar quien es más alto, si Juan o Enrique, medimos a ojo su estatura y afirmamos que *Juan es más alto que Enrique*. Los seres humanos somos capaces de establecer este tipo de hechos sin necesidad de conocer la altura real de Juan y Enrique ni la diferencia entre ellas. Una lectura visual aproximada es mas que suficiente para obtener la respuesta a nuestra pregunta ya que no precisamos de mas información. Gracias al modelado lingüístico, podemos crear sistemas que lleven a cabo este tipo de razonamientos, similares a los humanos, sin necesidad de obtener medidas exactas de ningún tipo.

En la literatura, hay tradicionalmente dos manera distintas de trabajar con variables lingüísticas: el enfoque simbólico y el no simbólico [HACHV09, HHV00]:

- **Modelado lingüístico simbólico**: Este tipo de modelado trabaja con las etiquetas lingüísticas usando sus índices y orden dentro del conjunto. Por lo general, para trabajar usando el modelado lingüístico simbólico, se suelen considerar conjuntos de etiquetas que tengan las siguientes características:

    - *Orden total*: Debe definirse un orden total entre las etiquetas que forman la variable de la siguiente manera:

    $$S = \{s_1, \ldots, s_i, \ldots, s_n\} \tag{2}$$

    Donde s_i ¡s_j si $i < j$.

    - *Conjuntos balanceados*: Se puede definir un conjunto balanceado como aquel que está compuesto por un número de elementos impar donde uno de ellos representa el término medio y el resto de ellos están equidistantes unos de otros. Gracias a esta distribución, es posible asociar, para cada término, su inverso.

- **Modelado lingüístico no simbólico:** Este tipo de modelado trabaja con los conjuntos difusos que cada etiqueta lingüística tiene asociados. Por ello, permite operar de forma sencilla gracias a que los conjuntos difusos tienen un entorno matemático con el que se puede operar. Sin embargo, el conjunto difuso final obtenido tras las operaciones requeridas no suele parecerse a ninguno asociado a las etiquetas disponibles dentro del conjunto de etiquetas lingüísticas. Por tanto, si queremos expresar los resultados lingüísticamente usando este tipo de modelado, es necesario realizar procesos de aproximación que conllevarán una pérdida de precisión en los cálculos.

El modelado lingüístico nos permite desarrollar métodos de toma de decisiones que utilicen conceptos y lleven a cabo razonamientos de forma similar a como lo hacemos los humanos [CHVP13, PCHV11a].

2.1.2. Modelado lingüístico difuso multi-granular

Aunque el modelado lingüístico mejora notablemente la comunicación usuario-sistema, posee ciertas limitaciones. Cuando definimos un conjunto de etiquetas lingüístico, su granularidad, esto es, su número de elementos, es fijo y no puede variar. Cuando tratamos con sistemas multi-usuario, es decir, sistemas computacionales que requieren de la participación de más de un usuario para llevar a cabo su tarea, esta limitación puede presentar problemas. Esto se debe a que el conjunto de etiquetas definido puede no ser adecuado para todos los usuarios que participan en el sistema. Tomando como ejemplo un sistema en donde los usuarios tienen que proporcionar su opinión acerca de una serie de productos, es fácil observar como obligar a todos los usuarios a usar el mismo conjunto de etiquetas puede producir problemas. De esta forma, algunos usuarios pueden percibir que el conjunto de etiquetas elegido tiene demasiadas etiquetas entre las que elegir. A su vez, otros pueden notar que el conjunto no tiene suficientes etiquetas para poder proporcionar de forma precisa sus preferencias.

Por tanto, sería deseable que cada usuario pudiera elegir el conjunto lingüístico que le fuera más cómodo para poder expresarse. De esta forma, si un usuario no tiene mucho conocimiento del tema tratado puede escoger un conjunto de etiquetas lingüísticas con un valor de granularidad bajo con el objetivo de dar una valoración más imprecisa. Por otro lado, si el usuario quiere proporcionar información más precisa, puede utilizar un conjunto de etiquetas lingüístico con un valor de granularidad alta para, de esta forma, poder elegir entre más etiquetas. El uso de varios conjuntos de etiquetas lingüísticas al mismo tiempo implica que deben implementarse métodos que permitan al sistema computacional tratar con este tipo de situaciones. Los métodos que se encargan de esta tarea son denominados métodos de modelado lingüístico difuso multi-granular [HHVM00, MMHV09, MMPUHV15]. El esquema que suele seguir un sistema que utiliza este tipo de técnicas es el siguiente:

1. **Elección del conjunto de etiquetas lingüístico**: Los usuarios deciden qué conjuntos de etiquetas lingüísticas quieren usar para proporcionar la información necesaria al sistema.

2. **Aportación de la información**: Los usuarios facilitan la información al sistema para que lleve a cabo las operaciones necesarias.

3. **Selección de un conjunto básico de etiquetas lingüístico**: El sistema elige el conjunto de etiquetas lingüístico con el que quiere operar. A este conjunto se le denomina conjunto básico de etiquetas lingüístico o BLTS.

4. **Transformación de los conjuntos de etiquetas al conjunto básico**: El sistema expresa todas las etiquetas lingüísticas en función del conjunto básico de etiquetas. Para ello suele usarse una función de transformación que varía según el método de modelado lingüístico multi-granular elegido.

5. **Realización de las operaciones necesarias**: Una vez que toda la información está expresada usando el mismo conjunto de etiquetas lingüístico, se pueden utilizar el entorno normal de computación asociado al modelado lingüístico para llevar a cabo las operaciones necesarias.

El esquema de este proceso puede verse gráficamente en la Figura 3. En la sección 3, realizaremos un repaso de los métodos de manejo de información multi-granular más utilizados.

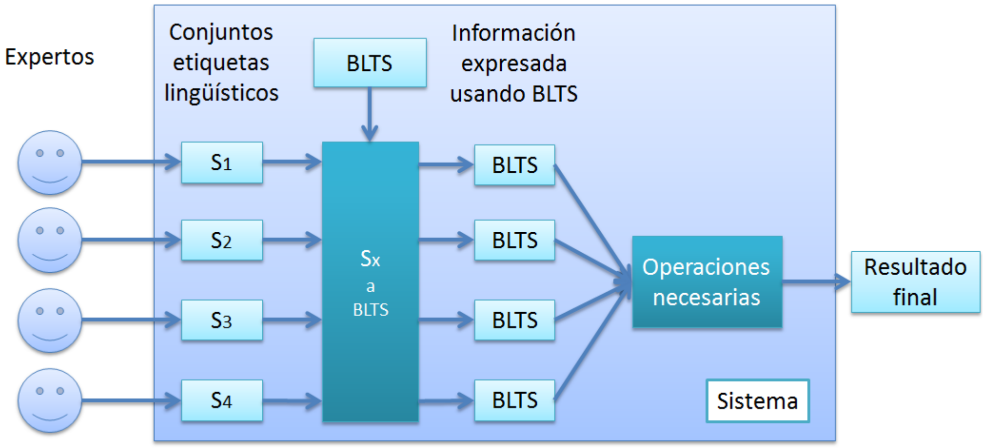


Figure 3: Esquema de un proceso de modelado lingüístico difuso multi-granular. S_1, S_2, S_3, S_4 son los conjuntos de etiquetas lingüísticas elegidos por el usuario y $BLTS$ es el conjunto básico de etiquetas lingüísticas.

2.2. Sistemas de Tomas de Decisiones en Grupo

2.2.1. Introducción

En la vida diaria de cualquier persona y empresa es ineludible la toma constante de decisiones. Por lo general, el problema de tomar decisiones puede definirse como la selección de una alternativa o la creación de un ranking a partir de un conjunto finito de alternativas [Orl78]. Cada alternativa conlleva una serie de ventajas e inconvenientes por lo que es necesario que el experto que lleve a cabo el proceso de toma de decisiones escoja o proporcione buenas posiciones en el ranking a las alternativas que proporcionen soluciones satisfactorias al problema.

Con el objetivo de asistir al decisor en el proceso de toma de decisiones, se diseñan los primeros Sistemas de Soporte a la Tomas de Decisiones [Mar03]. Los Sistemas de Soporte a la Toma de Decisiones son aplicaciones que nos ayudan a llevar a cabo procesos de toma de decisiones de forma guiada a través de un algoritmo informático. Su uso garantiza un proceso de toma de decisiones ordenado, eficiente y racional, lo que ayudara al decisor a encontrar las mejores alternativas que resuelvan su problema.

En las grandes empresas y, en general, en cualquier organización, la toma de decisiones no recae sobre una única persona. En su lugar, es una serie de personas las que deben de tomar la decisión. A esto se le llama *toma de decisiones en grupo* [Kac86]. Por lo general, las personas que deben de tomar una decisión se reúnen, debaten y, finalmente, llegan a una conclusión [PCAHV14]. Por lo general, un debate organizado se lleva a cabo siguiendo los siguientes pasos:

1. **Proposición de alternativas**: Los decisores proponen diferentes alternativas para resolver el problema planteado.

2. **Discusión**: Cuando el conjunto inicial de alternativas está fijado, los decisores discuten acerca de los pros y contras de escoger entre unas u otras.

3. **Actualización del conjunto de alternativas planteadas**: Durante la discusión, es posible que algunas de las alternativas se descarten por ser inviables. También es posible que aparezcan nuevas alternativas. Por tanto, el conjunto de posibles alternativas se va modificando conforme avanza la discusión.

4. **Reducción del conjunto de alternativas**: Conforme la discusión avanza, se produce un proceso de descarte en el que las alternativas menos populares se van descartando hasta que queda un conjunto reducido con las alternativas que proporcionan las soluciones más satisfactorias.

5. **Consenso y elección de la alternativa**: Tras centrar la discusión sobre el conjunto reducido de alternativas, las diferentes facciones acercan posturas

hasta que al final se llega a un consenso [CMPHV10] y se escoge una única alternativa o se define finalmente el ranking de alternativas final.

Lamentablemente, cuando un debate como el que se ha descrito arriba se lleva a cabo en la práctica, la situación dista mucho de ser tal ideal como en la teoría. Generalmente, los debates cara a cara tienen las desventajas que enumeramos a continuación:

- El debate obliga a los decisores a reunirse en un lugar y hora específicos. Por lo general, los decisores tienen otras responsabilidades a las que deben atender con lo que, para reunirse y tomar una decisión, éstos deben de dejar a un lado sus obligaciones. Esto no siempre es posible con lo que muchas veces las reuniones deben realizarse a horas intempestivas, con fechas lejanas en el tiempo o, en el caso de que sean decisiones urgentes, se realizan con la ausencia de algún miembro que, por tanto, no participa en el proceso de decisión.
- Las reuniones son largas, desperdiciándose mucho tiempo. Una de las razones se debe a que, cuando hay mucha gente reunida, es fácil perder el hilo principal de la discusión y acabar hablando cosas que no tienen nada que ver con lo que se pretendía discutir en un principio. Cuando los decisores hablan, tienden a hacerlo de forma emotiva lo que, muchas veces, conlleva cierta pérdida de racionalización de la conversación. Esta pérdida de racionalización facilita el desvío del hilo principal de la conversación dificultando la comunicación entre los asistentes al debate que acaban hablando de cosas que no tienen nada que ver con la decisión que deben tomar.
- Debido a que la decisión debe de tomarse en el tiempo estipulado para la reunión, el tiempo para pensar acerca de las ventajas e inconvenientes de las diferentes alternativas es limitado. Esto evita que los decisores piensen

con claridad y acaben tomando decisiones precipitadas. Para que en un proceso de toma de decisiones se tomen buenas decisiones, es necesario dar a los decisores tiempo para que reflexionen profundamente acerca de las consecuencias de cada una de las alternativas de forma que no erren en sus decisiones.

- En los debates cara a cara, el turno de palabra no siempre se respeta. Hay decisores que son más propensos a hablar interrumpiendo a los demás y otros más tímidos a los que les cuesta participar en reuniones en público. En los debates es muy importante escuchar los puntos de vista de todo el mundo si se quiere encontrar la mejor solución al problema planteado. Por lo tanto, es importante encontrar formas de evitar este tipo de situaciones en las que sólo una porción de los decisores participa en el debate.

- Si la reunión acaba sin que se llegue a ningún consenso, debe prepararse otra reunión en la que seguir con el debate lo que conlleva una gran pérdida extra de tiempo y la búsqueda de otro espacio de tiempo en la que los decisores puedan reunirse.

Como podemos ver, la toma de decisiones en grupo tradicional plantea varios problemas que deben tratar de solucionarse. Una manera de solucionarlos consiste en implementar procesos de toma de decisiones en grupo asistidos por ordenador usando las herramientas que proporciona la Web 2.0 [O'r09].

En sus inicios, Internet era un medio pensando exclusivamente para consultar información. Un pequeño grupo de expertos proporcionaba en Internet la información que era consultada por todo aquel que podía permitirse tener acceso. Actualmente, la situación ha cambiado completamente. Hoy en día, la mayor parte de la población tiene acceso a Internet. Además, cada uno de los usuarios con acceso es a la vez consumidor y proveedor de información. Por este motivo, la cantidad de información disponible en Internet ha aumentado exponencialmente en los últimos años. Aunque la cantidad de información es mayor, no se debe olvidar que la información es menos fiable y se encuentra de forma más desorganizada. Las herramientas que han permitido llevar a cabo este cambio de paradigma son denominadas tecnologías Web 2.0. Entre las aplicaciones Web que implementan estás características encontramos Webs tan conocidas como Facebook [ESL07], Youtube [BG13] o Twitter [HP09]. La Web 2.0 ha hecho que Internet pase de ser un medio de consulta de información dirigido a una minoría a una herramienta con la que cualquiera puede comunicarse y compartir información.

Aun mas reciente que la Web 2.0 es la aparición de los llamados smartphones [BWTJ11, CB10]. Los primeros teléfonos móviles que solo servían para realizar llamadas han evolucionado hasta el punto de convertirse en pequeños ordenadores portátiles que nos asisten y están con nosotros a lo largo del día. Gracias a las conexiones de datos y Wifis, los smartphones nos permiten acceder a Internet desde cualquier lugar en cualquier momento. Gracias a ellos, los usuarios pueden comunicarse y tener acceso constante a cualquier tipo de información almacenada en Internet. Además, con la aparición de los sistemas operativos Android [Dev11] e IOS, la programación de aplicaciones para estos dispositivos se ha simplificado apareciendo en el mercado un enorme elenco de programas de asistencia al usuario. Por lo general, estos programas usan el acceso a Internet, que proporcionan los smartphones, para cubrir las necesidades del usuario.

Gracias a la Web 2.0 y los smartphones, es posible definir sistemas de toma de decisiones asistidos por ordenador que permitan solucionar los problemas de los debates mediante reunión. Gracias a estos sistemas, podemos definir procesos de toma de decisiones en grupo con las siguientes características:

- Se puede participar en los procesos de toma de decisiones desde cualquier parte y en cualquier momento. Gracias a Internet, ya no es necesario para los decisores reunirse en el mismo lugar para poder llevar a cabo procesos de toma de decisiones. Esto evita que los decisores tengan que dar de lado sus obligaciones y puedan participar en el proceso cuando tengan un hueco libre en su agenda.
- Se desperdicia muy poco tiempo en el proceso de toma de decisiones. Ya que, por lo general, se usará una herramienta de tipo chat o foro para llevar a cabo la discusión, es más difícil para los decisores irse por las ramas. Al usar texto escrito para la comunicación, es mucho más fácil para los decisores centrarse en el tema de la discusión y presentar sus ideas y opiniones de forma ordenada. Esto mejora la comunicación y hace que el proceso de toma de decisiones sea mucho más fluido.
- Ya que, usando un sistema guiado por ordenador, no es necesario reunirse, los decisores tienen tiempo para leer y reflexionar tranquilamente sobre las preferencias de los demás. Esto hace que los decisores tengan las ideas más claras y entiendan mejor las ventajas e inconvenientes de cada una de las alternativas lo que, a su vez, conlleva la toma de mejores decisiones.
- Dado que los decisores comunican su opinión por escrito, desaparecen las interrupciones y cada decisor puede expresarse libremente y ser leído por el resto. La comunicación por escrito usando foros elimina también el miedo escénico que pueda sufrir algún decisor al hablar en público.
- Los procesos de toma de decisiones asistidos permiten poner fechas límite en la que se debe tomar la decisión. De esta forma, si es urgente decidirse,

se proporciona menos tiempo a los decisores para debatir mientras que si la decisión no corre prisa se puede dejar todo el tiempo que los decisores necesiten para ponerse de acuerdo. Tanto en un caso como en otro, dado que los decisores pueden debatir desde cualquier lugar y en cualquier momento sin necesidad de reunirse, el tiempo disponible se aprovecha mucho más que si se tuvieran que realizar varias reuniones.

Tal y como podemos ver, los procesos de toma de decisiones en grupo asistidos permiten a los decisores debatir y comunicarse entre ellos de forma sencilla, eficiente, ordenada y racional. Gracias a ellos se consiguen evitar todos los inconvenientes de las reuniones de debate tradicionales.

Los procesos de toma de decisiones guiados por computador requieren de una comunicación usuario-sistema fluida [HACHV09]. Sin embargo, debido a que los sistemas informáticos están acostumbrados a comunicarse usando números y las personas conceptos, este tipo de comunicación no siempre es del todo satisfactoria. Conseguir una comunicación fluida y cómoda para el usuario es un factor crítico a la hora de que el proceso que el sistema esté llevando a cabo tenga éxito. Si la comunicación no es cómoda, el usuario se negará a usar el sistema o proporcionara información inexacta que, a su vez, producirá resultados de peor calidad. Para solucionar este problema, se pueden utilizar técnicas de modelado lingüístico [HHVV96] y modelado lingüístico multigranular [HHVM00].

Este capítulo está organizado de la siguiente forma. En la subsección 2.2.2, se explica la base y los conceptos básicos de los métodos de toma de decisiones en grupo. En la subsección 2.2.3, se definen las medidas básicas de cálculo del consenso y la proximidad. En la subsección 2.2.4, se exponen los métodos de agregación de información más utilizados en los métodos de toma de decisiones en grupo. En la subsección 2.2.5, se comentan algunos algoritmos de selección. Finalmente, el capítulo termina con un ejemplo de uso.

2.2.2. Conceptos Básicos

Formalmente, el problema subyacente a un proceso de toma de decisiones en grupo se puede definir de la siguiente manera:

Sea $X = \{x_1, x_2, \ldots, x_n\}(n \geq 1)$ un conjunto de alternativas posibles y, teniendo en cuenta los valores de preferencia, $P = \{p_1, \ldots, p_m\}$, proporcionados por un grupo de expertos $E = \{e_1, \ldots, e_m\}$, ¿cómo deben ordenarse los valores del conjunto X de mejor a peor alternativa posible?

Por lo general, para resolver el problema, los procesos de toma de decisiones en grupo siguen los siguientes pasos [HACHV09]:

1. **Introducción de preferencias en el sistema**: Los expertos proporcionan sus preferencias al sistema. Las preferencias definen directa o indirectamente un orden sobre el conjunto de alternativas.

2. **Cálculo de la matriz colectiva de preferencias**: La información de las preferencias proporcionadas por todos los decisores es agregada en una sola pieza de información. La matriz colectiva representa la media de las preferencias proporcionadas.

3. **Proceso de selección de alternativas**: Usando la matriz colectiva y los operadores de selección deseados, se genera el ranking final de las alternativas.

El esquema comentado arriba tiene la desventaja de que no permite a los decisores

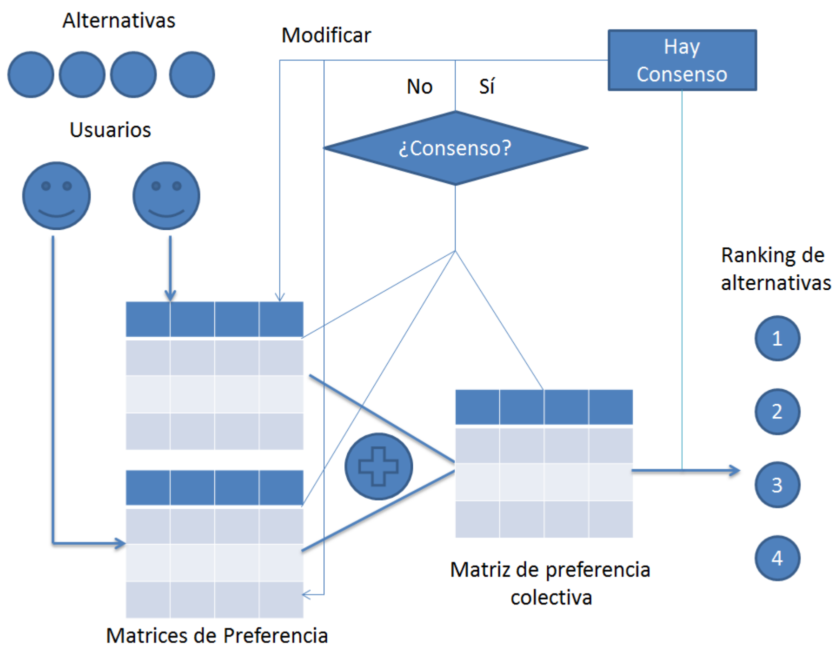


Figure 4: Proceso de toma de decisiones con medidas de consenso.

debatir ni llegar a ningún consenso antes de tomar la decisión final. Para solucionar este problema se utilizan las *medidas de consenso* [CMPHV10]. Usando las matrices de preferencia de los expertos involucrados en el proceso de decisión, las medidas de consenso permiten determinar si los expertos opinan de forma parecida o si, por el contrario, tienen opiniones encontradas. De esta forma, si los expertos no llegan a un consenso, se les puede permitir que hablen y modifiquen sus preferencias con el objetivo de que se pongan de acuerdo. Si, por el contrario, todos están de acuerdo, se calcula el ranking de alternativas y el proceso de decisión termina. En la Figura 4, podemos ver un esquema de como se definiría un proceso de toma de decisiones con medidas de consenso.

En un proceso de toma de decisiones, los expertos pueden proporcionar sus preferencias de diferentes formas. El procedimiento elegido es muy importante ya que establecerá la forma en que se deben realizar las operaciones necesarias para la toma de decisiones. Los métodos más comunes en la literatura son los siguientes [CHHV98, Tan90]:

- **Órdenes de preferencia**: El experto e_k proporciona sus preferencias utilizando una lista ordenada de preferencias $O^k = \{o^k(1), \ldots, o^k(n)\}$ donde $o^k(\cdot)$ se define como una función de permutación sobre el conjunto de índices $\{1, \ldots, n\}$ del conjunto de alternativas. De esta forma, las alternativas aparecen ordenadas de mejor a peor opción.

- **Funciones de utilidad**: El experto e_k comunica sus preferencias representadas como un conjunto de n valores de utilidad $U^k = \{u_i^k, i = 1, \ldots, n\}$, $u_i^k \in [0,1]$ donde u_i^k representa la evaluación que el experto e_k proporciona a la alternativa x_i.

- **Relaciones de preferencia difusa**: El experto e_k proporciona sus preferencias mediante una relación $P^k \subset X \times X$ cuya función de pertenencia es $\mu_{P^k} : X \times X \to [0,1]$. $\mu_{P^k} = p_{ij}^k$ establece el grado de pertenencia de la alternativa x_i sobre x_j. Puede asumirse, sin pérdida de generalidad, la reciprocidad de P^k: $p_{ij}^k + p_{ji}^k = 1$ y $p_{ii}^k = -$ (indefinido) $\forall i, j, k$. Una característica importante de esta representación es que permite medir la consistencia de la respuesta del usuario.

2.2.3. Medidas de consenso y proximidad

Para calcular el consenso de un proceso de toma de decisiones que utiliza relaciones de preferencia difusa, podemos seguir los pasos expuestos en el artículo de Mata [MMHV09] y que detallamos a continuación:

1. Para cada par de expertos e_i y e_j, calculamos las matrices de similaridad sm_{ij}. Para ello, aplicamos la siguiente función de similaridad para cada uno

de los valores de preferencia de cada dos expertos:

$$s(p_i^{lk}, p_j^{lk}) = 1 - |(p_i^{lk} - p_j^{lk})/g| \tag{3}$$

donde $s(p_i^{lk}, p_j^{lk})$ muestra la similaridad entre las preferencias de las alternativas x_l sobre x_k para los expertos e_i y e_j.

2. Una vez calculadas todas las matrices se agregan en una única matriz de consenso colectiva. Para ello, podemos utilizar el operador de media:

$$sm_c = \phi(sm_{ij}), \forall i, \forall j, i \neq j, i < j \tag{4}$$

3. Utilizando la matriz de consenso colectiva sm_c, podemos calcular tres medidas distintas de consenso, cada una representativa de un nivel diferente:

   a) *Nivel 1, consenso entre pares de alternativas*: Cada valor de la matriz sm_c nos muestra el consenso alcanzado para cada par de alternativas:

   $$cp^{lk} = cm^{lk} \forall l, k = 1, \ldots, n, \wedge l \neq k \tag{5}$$

   donde n es el número de alternativas del proceso de toma de decisiones.

   b) *Nivel 2, consenso en cada alternativa*: Para cada alternativa x_l, puede calcularse el nivel de consenso alcanzado, ca^l, usando la matriz cp tal y como muestra la siguiente expresión:

   $$ca^l = \frac{\sum_{k=1, l \neq k}^{n} (cp^{lk} + cp^{kl}))}{2(n-1)} \tag{6}$$

   c) *Nivel 3, consenso general del proceso*: Finalmente, podemos agregar los valores de consenso de cada una de las alternativas para obtener un valor de consenso global:

   $$cr = \sum_{l=1}^{n} ca^l / n \tag{7}$$

También es interesante calcular la distancia que hay entre las preferencias de cada uno de los expertos a la matriz colectiva global. De esta forma, podemos ver si las opiniones del experto son similares o no a la de los demás y en que grado. Estas medidas de proximidad [HVMMC05], al igual que las de consenso, se pueden calcular en tres niveles distintos:

1. *Nivel 1, proximidad en cada par de alternativas*: El nivel de proximidad para cada par de alternativas (x_l, x_k), pp_i, del experto e_1, puede calcularse de la siguiente forma:
$$pp_i^{lk} = s(p_i^{lk}, p_c^{lk}) \tag{8}$$
donde p_c es la matriz colectiva.

2. *Nivel 2, proximidad para cada alternativa*: De manera análoga que en el consenso, podemos calcular el nivel de proximidad del experto a cada una de las alternativas mediante la siguiente expresión:
$$pa_i^l = \frac{\sum_{k=1, l \neq k}^{n} (pp_i^{lk} + pp_i^{kl})}{2 \cdot (n-1)} \tag{9}$$

3. *Nivel 3, Proximidad general*: El nivel de proximidad general de las preferencias del experto e_i puede calcularse usando la siguiente expresión:
$$pr_i = \frac{pa_i^l}{n} \tag{10}$$

2.2.4. Métodos de agregación de información

Para calcular la matriz colectiva de preferencias es necesario agregar la información proporcionada por los expertos. Para ello debemos usar algún operador de agregación. A continuación expondremos cuatro operadores distintos que pueden usarse para completar esta tarea:

- el operador de media.

- el de media ponderada.
- el operador de media de pesos ordenados (OWA) [Yag88, Yag96].
- el operador de media de pesos ponderados lingüístico (LOWA) [HHV97].

Para calcular la matriz de preferencias colectiva utilizando el operador de media podemos utilizar la siguiente expresión:

$$C_{ij} = \frac{p_{ij}^1 + \ldots + p_{ij}^n}{m} \tag{11}$$

Si en el proceso de decisión consideramos que la opinión de algunos de los expertos es más importante que las de otros y, por lo tanto, deben de tener más peso en la decisión, podemos usar un operador de agregación con pesos. De esta forma, podemos asignar un peso para cada opinión dando más importancia a las opiniones de los expertos que, por ejemplo, estén mas versados en el tema que se esté tratando. Por tanto, para llevar a cabo este tipo de agregación, introducimos la variable $W = \{w_1, \ldots, w_m\}$ donde el valor w_i indica el peso asignado al experto e_i. Debe tenerse en cuenta que $w_i \in [0, 1]$ y $\sum_{i=0}^{m} w_i = 1$. Por tanto, podemos llevar a cabo la agregación utilizando el operador de media ponderada tal y como se muestra a continuación:

$$C_{ij} = \frac{w_1 \cdot p_{ij}^1 + \ldots + w_n \cdot p_{ij}^n}{m} \tag{12}$$

También es posible realizar la agregación usando el operador de media con pesos ordenados, OWA. Este operador, mediante la manipulación del vector de pesos, nos permite definir su comportamiento. El operador OWA se define de la siguiente manera::

$$OWA(a_1, \ldots, a_n) = \sum_{j=1}^{n} w_j b_j \tag{13}$$

donde b_j es el jotaésimo valor mayor del conjunto $A = \{a_1, \ldots, a_n\}$ y $W = [w_1, \ldots, w_m]$ es un vector de pesos tal que $w_i \in [0, 1]$ y $\sum_{i=1}^{m} w_i = 1$.

Existen varias maneras de definir el conjunto de pesos que debe asociarse al operador OWA. En [Yag96], Yager propone un método que nos permite obtener un conjunto de pesos asociados utilizando cuantificadores difusos [LK98]. De esta forma, podemos calcular un conjunto de pesos usando un cuantificador proporcional no decreciente, Q_{nd}, mediante la siguiente expresión:

$$w_i = Q\left(\frac{i}{n}\right) - Q\left(\frac{i-1}{n}\right), i = 1, \ldots, n. \tag{14}$$

Por tanto, podemos redefinir la expresión de un operador OWA utilizando cuantificadores de la siguiente manera:

$$OWA(a_1, \ldots, a_n) = \sum_{j=1}^{n} \left(Q\left(\frac{i}{n}\right) - Q\left(\frac{i-1}{n}\right) b_j \right) \tag{15}$$

Para que un cuantificador sea proporcional no decreciente debe cumplirse lo siguiente:

$$\forall a, b \quad \text{if} \quad a > b \quad \text{then} \quad Q(a) \geq Q(b)$$

Un cuantificador de pertenencia no decreciente puede representarse mediante la siguiente expresión:

$$Q(r) = \begin{cases} 0 & \text{if } r < a \\ \frac{r-a}{b-a} & \text{if } a \leq r \leq b \\ 1 & \text{if } r > b \end{cases} \tag{16}$$

En la Figura 5, podemos ver algunos ejemplos de cuantificadores proporcionales difusos. Los parámetros utilizados para (a, b) son $(0{,}3, 0{,}8)$,$(0, 0{,}5)$ y $(0{,}5, 0{,}1)$ respectivamente.

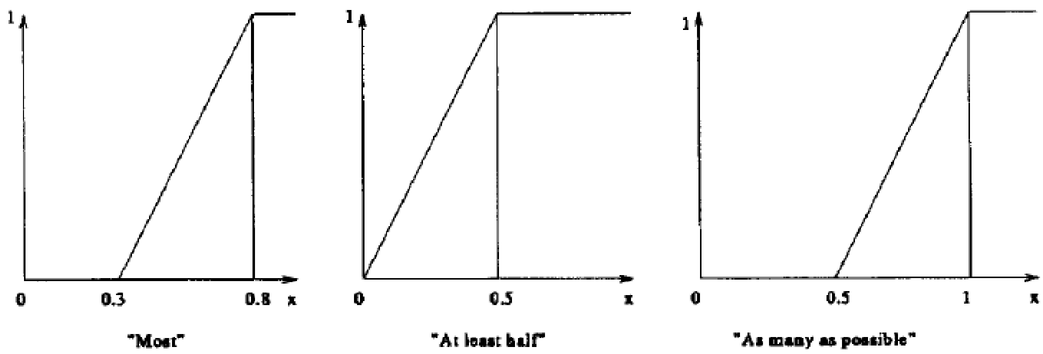


Figure 5: Ejemplos de cuantificadores difusos proporcionales no decrecientes.

Por lo general, en problemas de toma de decisiones en grupo y multi-criterio, se utiliza un operador OWA que defina el concepto de mayoría difusa. De esta forma, se les da mayor prioridad a aquellas alternativas que satisfagan la mayor parte de los criterios o a aquellas elegidas por la mayoría de los expertos.

Yager define la medida de *orness* [Yag88] cuyo objetivo es proporcionar un grado de similaridad del operador OWA definido con una *t-norma* o *t-conorma*. El grado de similaridad, a partir de un vector de pesos W puede calcularse usando la siguiente expresión:

$$orness(W) = \frac{1}{n-1}\sum_{i=1}^{n}(n-i)w_i \tag{17}$$

Donde el resultado se encuentra en el intervalo [0,1]. Analizando el *orness* asociado a un vector de pesos, se pueden alcanzar las siguientes conclusiones:

- Si el valor de *orness* es cercano a 1, entonces el operador se comporta como una *t-conorma*.

- Si el valor de *orness* es cercano a 0, entonces el operador se comporta como una *t-norma*.

- Si el valor de *orness* es 0.5, el operador se comporta como un operador de media.
- Si el valor de *orness* es exactamente 1, el operador OWA se comporta como el operador máx.
- SI el valor de *orness* es 0, el operador OWA definido se comporta como la función mín.

El valor de *orness* de un cuantificador se puede calcular utilizando la siguiente expresión:

$$orness(Q) = \frac{1}{n-1} \sum_{j=1}^{n-1} \left(\frac{j}{n} \right) \tag{18}$$

En el caso de información lingüística, podemos utilizar el operador de agregación lingüístico ordenado, LOWA [HHV97]. Sea $A = \{a_1, \ldots, a_m\}$ un conjunto de etiquetas a agregar, el operador LOWA, se define como

$$\begin{aligned} LOWA(a_1, \ldots, a_m) &= W \cdot B^T = \zeta^m\{w_k, b_k, k = 1, \ldots, m\} \\ &= w_1 \odot b_1 \oplus (1 - w_1) \\ &\odot \zeta^{m-1}\{\beta_h, b_h, h = 2, \ldots, m\} \end{aligned} \tag{19}$$

donde $W = [w_1, \ldots, w_m]$, es un vector de pesos tal que, $w_i \in [0, 1]$, $\sum_i w_i = 1$, $\beta_h = w_h / \sum_2^m w_k, h = 2, \ldots, m$, y $B = \{b_1, \ldots, b_m\}$ es un vector asociado a A tal que,

$$B = \sigma(A) = \{a_{\sigma(1)}, \ldots, a_{\sigma(n)}\} \tag{20}$$

donde, $a_{\sigma(j)} \leq a_{\sigma(i)}$ $\forall i \leq j$ siendo σ una permutación sobre el conjunto de etiquetas A. ζ^m es el operador de combinación convexa i de m etiquetas. Si $m = 2$, entonces se define como:

$$\begin{aligned} &\zeta^2\{w_i, b_i, i = 1, 2\} = w_1 \odot s_j \oplus (1 - w_1) \odot s_i = s_k, \\ &s_j, s_i \in S \ (j \geq i) \end{aligned} \tag{21}$$

tal que, $k = min\{T, i + round(w_1 \cdot (j - i))\}$, donde *round* es el operador de redondeo común y $b_1 = s_j$, $b_2 = s_i$. Si $w_j = 1$ y $w_i = 0$ con $i \neq j$ $\forall i$, entonces la combinación convexa se define como:

$$\zeta^m\{w_i, b_i, i = 1, \ldots, m\} = b_j. \tag{22}$$

Para calcular los valores del vector de pesos, al igual que se hacía con el operador OWA, se utiliza un cuantificador lingüístico difuso que represente el concepto de mayoría difusa.

2.2.5. Operadores de selección

Para el proceso de ranking de alternativas, se utilizan los operadores de selección. Este tipo de operadores son capaces de obtener un ranking de alternativas a partir de una matriz colectiva de preferencias. Dos ejemplos de este tipo de operadores son los operadores de dominancia y no dominancia, GDD y GNDD respectivamente [HHVV95]. El operador GDD calcula el grado en que una alternativa domina a otra mientras que el de no dominancia se encarga de determinar qué alternativas no son dominadas por otras.

El operador GDD se calcula mediante la siguiente expresión:

$$GDD_i = \phi(c_{i1}, c_{i2}, \ldots, c_{i(i-1)}, c_{i(i+1)}, \ldots, c_{in}) \tag{23}$$

donde c es la matriz de preferencia colectiva y ϕ representa el operador de media.

El operador GNDD puede calcularse utilizando la siguiente expresión:

$$GNDD_i = \phi(c^s_{1i}, c^s_{2i}, \ldots, c^s_{(i-1)i}, c^s_{(i+1)i}, \ldots, c^s_{ni}) \tag{24}$$

donde

$$c_{ji}^{s} = \text{máx}\{c_{ji} - c_{ij}, 1\}$$

2.2.6. Estado del arte

En esta sección, haremos un repaso por los artículos más actuales que se han escrito sobre métodos de toma de decisiones en grupo.

Por ejemplo, en [CUPHV14], se define el concepto de relación de preferencia difusa granular. Cada valor de la matriz de preferencia está formado por un gránulo de información que puede ser un intervalo, un conjunto difuso, un conjunto rough, etc. en vez de un valor numérico. Gracias a esto, aumenta la flexibilidad de representación facilitando la manera en que los usuarios proporcionan sus preferencias.

En [PMH14], los autores presentan un modelo de consenso compatible con procesos de toma de decisiones en grupo que manejan un gran número de expertos. Para ello, se define un esquema de clustering difuso que detecta y agrupa comportamientos no cooperativos. Además, el método incluye una herramienta de análisis visual basada en mapas auto-organizativos que facilita la monitorización del proceso.

En [PCAHV14], se define un nuevo modelo de consenso para procesos de toma de decisiones en grupo en entornos heterogéneos. Se consideran entornos heterogéneos aquellos en los que los expertos usan diferentes sistemas de representación para proporcionar sus preferencias y donde los expertos tienen diferente conocimiento acerca del tema que se está tratando. Este modelo usa medidas de consenso y similaridad e introduce un nuevo mecanismo de asistencia para los expertos que tiene en cuenta su relevancia.

En [Liu14], los autores presentan dos métodos de toma de decisiones en grupo multicriterio que utilizan número difusos intuicionistas interválicos. Para agregar la información, se utilizan los operadores de agregación de promedio ponderado Hamacher.

En [XXC13], se estudian los operadores de agregación para información difusa indecisa. Los autores proponen dos métodos para determinar los vectores de pesos en la fase de agregación. Gracias a este nuevo sistema, se obtienen vectores de pesos para los expertos de forma más objetiva.

En [CXX13], los autores presentan un nuevo tipo de estructura para representar las preferencias de los expertos. Para ello, utilizan relaciones de preferencia con conjuntos difusos indecisos con intervalos. Además, definen operadores de agregación y selección que nos permiten llevar a cabo el proceso de toma de decisiones utilizando la estructura definida.

En [CTGdMHV13], se presenta un estudio comparativo de varias medidas de similaridad para métodos de consenso en procesos de toma de decisiones en grupo. Usando el test no paramétrico de Wilcoxon de ranking con signo y unión de pares, los autores demuestran como usar diferentes medidas de similaridad puede afectar significativamente a los resultados obtenidos.

En [RMH13], los autores presentan un nuevo método de toma de decisiones en grupo que utiliza conjuntos indecisos. De esta forma, se presentan nuevas herramientas con las que los expertos pueden expresar de forma imprecisa sus opiniones. Gracias a la versatilidad de los conjuntos difusos indecisos, el sistema puede trabajar correctamente con la información imprecisa proporcionada por los expertos.

En [CCC15] se define un nuevo método de toma de decisiones en grupo difuso multiatributo basado en conjuntos intuicionistas y metodología de razonamiento evidencial. Primero, el método utiliza un procedimiento de razonamiento evidencial para llevar a cabo el proceso de agregación por pesos. A continuación se calcula, para cada alternativa, un conjunto difuso intuicionista que indica la preferencia global de cada alternativa. Finalmente, se calcula un valor transformado para cada alternativa y se lleva a cabo el proceso de ranking.

En [Zha13], los autores definen varios operadores de agregación de potencia usando conjuntos difusos indecisos y luego las aplican para resolver problemas de toma de decisiones en grupo multiatributo. Para ello, extienden operadores ya existentes y los aplican en el entorno de los conjuntos difusos indecisos.

En [ZZ13] se define un nuevo modelo de toma de decisiones en grupo multiatributo. Para ello, primero, usando números difusos de tipo 2 y la teoría de conjuntos difusos suaves, se define la noción de conjunto difuso trapezoidal interválico de tipo 2 suave. A continuación, los autores describen cómo utilizar esta nueva herramienta dentro de un proceso de toma de decisiones en grupo.

En [WC14b] se define un modelo cuyo objetivo es determinar el grado de importancia de los expertos en un proceso de toma de decisiones en grupo. Para ello, se utiliza el grado de confianza y el nivel de consenso. Los autores desarrollan una metodología de análisis de redes sociales que representa y modela las relaciones de confianza entre expertos. Además, los autores definen nuevas medidas de similaridad y selección utilizando las relaciones de preferencia difusas recíprocas interválicas.

2.2.7. Ejemplo de uso

Con el objetivo de mejorar la comprensión acerca del funcionamiento de los métodos de toma de decisiones en grupo, mostraremos un ejemplo.

Cuatro amigos, $E = \{e_1, e_2, e_3, e_4\}$, tratan de decidir adonde deben de ir en vacaciones. Deben elegir entre tres alternativas,$X = \{x_1 : Londres, x_2 : Roma, x_3 : Edimburgo\}$. Para expresarse, los expertos utilizarán el conjunto lingüístico balanceado $S = \{s_1, s_2, s_3, s_4, s_5\}$. Tras una breve discusión, los expertos proporcionan sus preferencias:

$$P_1 = \begin{pmatrix} - & s_2 & s_1 \\ s_4 & - & s_5 \\ s_2 & s_1 & - \end{pmatrix} P_2 = \begin{pmatrix} - & s_1 & s_1 \\ s_3 & - & s_5 \\ s_3 & s_1 & - \end{pmatrix}$$
$$P_3 = \begin{pmatrix} - & s_2 & s_1 \\ s_4 & - & s_5 \\ s_2 & s_2 & - \end{pmatrix} P_4 = \begin{pmatrix} - & s_2 & s_2 \\ s_4 & - & s_5 \\ s_3 & s_1 & - \end{pmatrix}$$

Tras obtener las preferencias individuales de cada experto, el sistema calcula la matriz colectiva de preferencias. Para ello, ya que el conjunto S es balanceado, agregaremos los índices de las etiquetas. La matriz colectiva de preferencias obtenida se muestra a continuación:

$$P_C = \begin{pmatrix} - & 1{,}75 & 1{,}25 \\ 3{,}75 & - & 5 \\ 2{,}5 & 1{,}25 & - \end{pmatrix}$$

A continuación, mediremos el consenso alcanzado por los expertos. Para ello, primero calcularemos la matriz de consenso sm_ij para cada par de expertos. Para realizar esta operación correctamente normalizaremos los índices de las etiquetas y los expresaremos numéricamente en el intervalo [0,1] obteniendo lo siguiente:

$$P_1 = \begin{pmatrix} - & 0{,}25 & 0 \\ 0{,}75 & - & 1 \\ 0{,}25 & 0 & - \end{pmatrix} P_2 = \begin{pmatrix} - & 0 & 0 \\ 0{,}5 & - & 1 \\ 0{,}5 & 0 & - \end{pmatrix}$$
$$P_3 = \begin{pmatrix} - & 0{,}25 & 0 \\ 0{,}75 & - & 1 \\ 0{,}25 & 0{,}25 & - \end{pmatrix} P_4 = \begin{pmatrix} - & 0{,}25 & 0{,}25 \\ 0{,}75 & - & 1 \\ 0{,}5 & 0 & - \end{pmatrix}$$

Las matrices de similaridad para cada par de expertos se muestran a continuación:

$$sm_{12} = \begin{pmatrix} - & 0{,}75 & 1 \\ 0{,}75 & - & 1 \\ 0{,}75 & 1 & - \end{pmatrix} sm_{13} = \begin{pmatrix} - & 1 & 1 \\ 1 & - & 1 \\ 1 & 0{,}75 & - \end{pmatrix}$$
$$sm_{14} = \begin{pmatrix} - & 1 & 0{,}75 \\ 1 & - & 1 \\ 0{,}75 & 1 & - \end{pmatrix} sm_{23} = \begin{pmatrix} - & 0{,}75 & 1 \\ 0{,}75 & - & 1 \\ 0{,}75 & 0{,}75 & - \end{pmatrix}$$
$$sm_{24} = \begin{pmatrix} - & 0{,}75 & 0{,}75 \\ 0{,}75 & - & 1 \\ 1 & 1 & - \end{pmatrix} sm_{34} = \begin{pmatrix} - & 1 & 0{,}75 \\ 1 & - & 1 \\ 0{,}75 & 0{,}75 & - \end{pmatrix}$$

Table 1: Consensus values for the group decision making example process and for each alternative.

x_1	x_2	x_3	Decision Making process
0.86	0.9	0.89	0.88

A continuación, las agregamos en una única matriz de similaridad colectiva:

$$sm_c = \begin{pmatrix} - & 0{,}875 & 0{,}875 \\ 0{,}875 & - & 1 \\ 0{,}833 & 0{,}875 & - \end{pmatrix}$$

En la Tabla 1, podemos ver el consenso alcanzado en cada una de las alternativas y el consenso general del proceso de toma de decisiones. Podemos observar como el valor de consenso, situado en el intervalo [0,1], es muy alto. Gracias a estas medidas podemos observar claramente como, en este caso, los expertos están de acuerdo entre sí. Dado que el valor de consenso es alto, no es necesario llevar ninguna otra ronda de debate. A continuación, para cada experto, calcularemos las medidas de proximidad a la matriz colectiva de preferencias. Los resultados para cada par de alternativas se muestran a continuación:

$$PP_1 = \begin{pmatrix} - & -0{,}25 & 0{,}25 \\ -0{,}25 & - & 0 \\ 0{,}5 & 0{,}25 & - \end{pmatrix} PP_2 = \begin{pmatrix} - & 0{,}75 & 0{,}25 \\ 0{,}75 & - & 0 \\ -0{,}5 & 0{,}25 & - \end{pmatrix}$$

$$PP_3 = \begin{pmatrix} - & -0{,}25 & 0{,}25 \\ -0{,}25 & - & 0 \\ 0{,}5 & -0{,}75 & - \end{pmatrix} PP_4 = \begin{pmatrix} - & -0{,}25 & -0{,}75 \\ -0{,}25 & - & 0 \\ -0{,}5 & 0{,}25 & - \end{pmatrix}$$

En la tabla 2, podemos ver los valores de proximidad por alternativa y global de cada uno de los expertos. Como podemos ver, los expertos e_1 y e_3 son los que se aproximan más a los valores de preferencias globales. e_2 debería proporcionar valores más altos para la alternativa x_2 y x_4 para aumentar el nivel de consenso.

Table 2: Proximity values for the group decision making example for each expert and alternative and for the global group decision making.

	x_1	x_2	x_3	Decision Making process
e_1	0.06	-0.06	0.25	0.083
e_2	0.31	0.44	0	0.25
e_3	0.06	-0.31	0	-0.084
e_4	-0.43	-0.06	-0.25	-0.25

Table 3: Proximity values for the group decision making example for each expert and alternative and for the global group decision making.

	x_1	x_2	x_3
GDD	0.125	0.843	0
GNDD	0.5925	1	0.53
mean(GDD,GNDD)	0.358	0.9215	0.265
Position in the ranking	2	1	3

e_4, sin embargo, debe proporcionar valores más bajos ya que los valores de proximidad obtenidos son negativos.

Por último, utilizaremos la media de los valores obtenidos por los operadores de selección GDD y GNDD para obtener el ranking de alternativas a partir de la matriz de preferencias colectivas. Los valores obtenidos para ambos operadores y la media pueden verse en la Tabla 3. Para el operador GNDD, necesitamos calcular la matriz P_c^s. El resultado de calcular dicha matriz se muestra a continuación:

$$P_c^s = \begin{pmatrix} - & 0 & 0 \\ 0{,}5 & - & 0{,}94 \\ 0{,}315 & 0 & - \end{pmatrix} \tag{25}$$

Como podemos ver, el ranking obtenido es tal que $x_2 \succ x_1 \succ x_3$. Dado que el consenso general obtenido es muy alto, 0.88, podemos afirmar que la decisión se ha tomado de manera consensuada.

2.3. Ontologías

Debido al rápido crecimiento de la información disponible en Internet, se hace necesario el desarrollo de técnicas que permitan ordenarla y clasificarla. Con este objetivo nació el proyecto de la Web semántica [HKR11, MSZ01]. Este proyecto trata de añadir términos semánticos a los datos disponibles en Internet de forma que puedan implementarse métodos que sean capaces de realizar búsquedas y razonar sobre los datos usando su significado. Dado que, hasta ahora, todos los buscadores de Internet realizan búsquedas sintácticas, esto es, buscan palabras concretas sin preocuparse de su significado, la creación de una Web Semántica supondría un gran avance en la manera en que hasta ahora se maneja la información.

Una de las herramientas que la Web Semántica utiliza para representar la información son las Ontologías [Fen01, JS13, MS01]. Gracias a las Ontologías, es posible definir un universo de elementos referentes a una temática concreta, describirlos mediante atributos y utilizar la información almacenada para realizar procesos de razonamiento inductivo.

Formalmente, una Ontología [BCM+03] puede definirse como una quintupla $O = \{I, C, R, A\}$ donde:

- I es el conjunto de individuos.
- C denota el conjunto de conceptos. Los conceptos se usan para describir individuos.
- R es el conjunto de relaciones. Las relaciones establecen conexiones entre individuos y entre individuos y conceptos. Generalmente, se asigna un 0 cuando un individuo no está relacionado con un concepto y un 1 cuando sí

lo esta.

- A denota el conjunto de axiomas. Los axiomas establecen afirmaciones sobre los datos almacenados en la Ontología.

En la Figura 6, se muestra un árbol jerárquico de conceptos de una Ontología que describe un conjunto de vinos. En la Figura 7 se muestran los individuos, es decir, los vinos, que están relacionados con el concepto Extra_Full de la Ontología de vinos. De esta forma, podemos afirmar que el individuo *Ribon_Crianza* está relacionado con el concepto *Body Extra_Full* pero no con el resto de conceptos de la categoría *Body*. Formalmente podemos afirmar que:

$$\begin{array}{cc} \mu_{Extra\_Full}(Ribon\_Crianza) = 1 & \mu_{Full}(Ribon\_Crianza) = 0 \\ \mu_{Medium}(Ribon\_Crianza) = 0 & \mu_{Light}(Ribon\_Crianza) = 0 \end{array} \tag{26}$$

donde $\mu_{concepto}(individuo)$ vale 1 si existe una relación entre el individuo y el concepto y 0 sino.

Actualmente existen programas como Protégé [TNNM13] que nos permiten diseñar Ontologías de forma sencilla. Estos programas presentan un entorno gráfico que nos permite introducir los conceptos e individuos y establecer relaciones entre ellos. El programa genera un fichero estándar OWL [MVH$^+$04] que puede ser tratado por todos los programas que trabajan con este tipo de estructuras.

2.3.1. Ontologías Difusas

En la subsección 2.3.1.1, se exponen la definición y conceptos básicos de las Ontologías Difusas. En la subsección 2.3.1.2, se expone el estado del arte en este campo. Finalmente, en la subsección 2.3.1.3, se propone un ejemplo de uso de una Ontología Difusa.

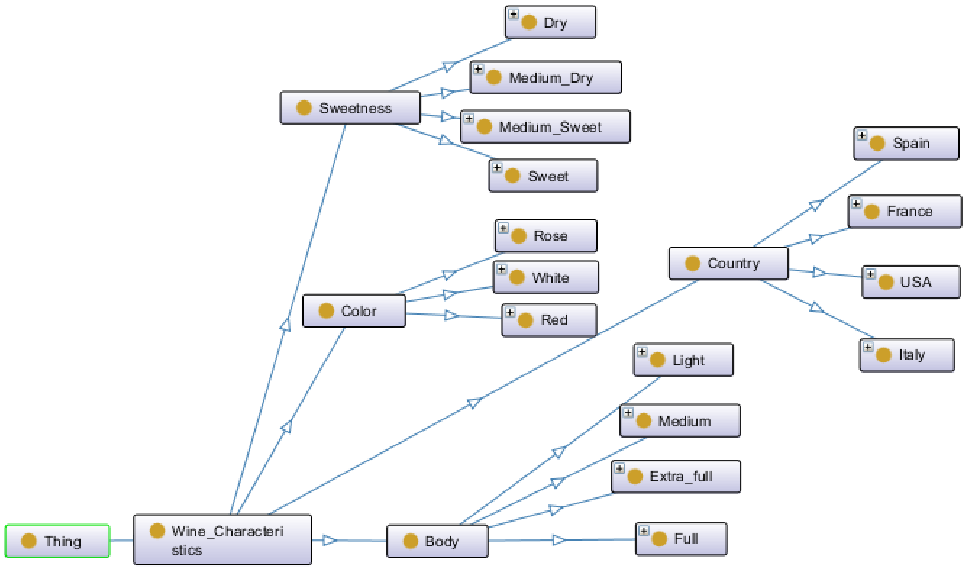


Figure 6: Árbol jerárquico de conceptos de una Ontología de vinos.

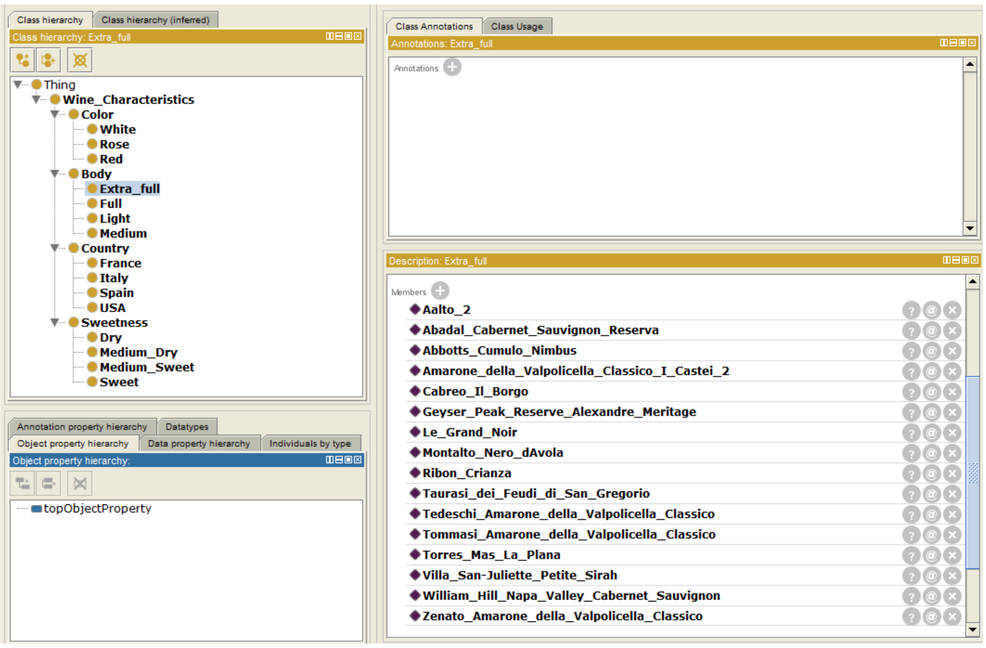


Figure 7: Individuos que estan relacionados con el concepto *Extra_Full* de la Ontología.

2.3.1.1 Conceptos básicos

Las Ontologías permiten a los diseñadores representar individuos y describirlos mediante conceptos. Además, es posible crear programas informáticos que realicen razonamientos inductivos sobre ellas. Sin embargo, cuando intentamos representar usando Ontologías un entorno real, podemos encontrarnos con diversos problemas. Uno de los más importantes consiste en la imposibilidad de asignar correctamente un individuo a dos conceptos relacionados. Tomando el ejemplo de la Ontología de vinos expuesto en la sección 2.3, si queremos indicar que uno de los vinos, por ejemplo el *Le_Grand_Noir*, tiene un cuerpo entre *Extra_full* y *Full* podríamos usar el siguiente esquema formal:

$$\begin{array}{cc} \mu_{Extra\_Full}(Le\_Grand\_Noir) = 1 & \mu_{Full}(Le\_Grand\_Noir) = 1 \\ \mu_{Medium}(Le\_Grand\_Noir) = 0 & \mu_{Light}(Le\_Grand\_Noir) = 0 \end{array} \qquad (27)$$

Sin embargo, usando esta asignación, un programa de razonamiento inductivo trabajará con este vino como si fuera a la vez *Full* y *Extra_Full* que no es lo que queremos indicar.

Para poder solucionar este problema aparecen las Ontologías Difusas [CC07]. Las Ontologías Difusas agregan el conjunto de relaciones difusas F. Formalmente, pueden definirse como la tupla $O = \{I, C, R, F, A\}$ donde:

- I es el conjunto de individuos.
- C denota el conjunto de conceptos.
- R es el conjunto de relaciones no difusas.
- F es el conjunto de relaciones difusas.
- A denota el conjunto de axiomas. Los axiomas establecen afirmaciones sobre los datos almacenados en la Ontología.

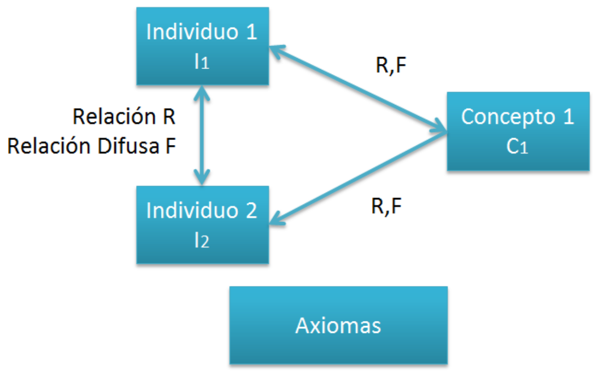


Figure 8: Esquema de una Ontología Difusa.

En la Figura 8, podemos ver un esquema de una Ontología Difusa.

El nuevo elemento introducido, F, nos permite establecer relaciones difusas entre los elementos de la Ontología. De esta forma, en vez de usar $\{0, 1\}$ en la función μ, podemos establecer las relaciones entre individuos y conceptos usando un intervalo, por lo general, $[0, 1]$. De esta forma, volviendo al ejemplo anterior, si queremos expresar que el vino *Le_Grand_Noir*, tiene un cuerpo entre *Extra_full* y *Full* podemos indicar lo siguiente:

$$\begin{array}{cc} \mu_{Extra\_Full}(Le\_Grand\_Noir) = 0{,}8 & \mu_{Full}(Le\_Grand\_Noir) = 0{,}5 \\ \mu_{Medium}(Le\_Grand\_Noir) = 0 & \mu_{Light}(Le\_Grand\_Noir) = 0 \end{array} \tag{28}$$

De esta forma, el razonador puede comprender de manera satisfactoria que el vino tiene un cuerpo que no es completamente *Extra_Full* ni *Full* sino que pertenece con cierto grado a ambas categorías. Este tipo de representación es mucho más cercana a lo que queremos indicar que lo que podíamos conseguir usando Ontologías no difusas.

Los pasos que deben seguirse para obtener información de una Ontología Difusa son los siguientes [CBM12]:

1. **Formulación de la búsqueda:** Primero, el consultor debe proporcionar datos acerca de lo que está buscando. Para ello, enviará al sistema una lista de características que los individuos buscados deben cumplir. Por lo general, esta lista se corresponderá con un conjunto de conceptos y valores que el consultor desea que tengan.

2. **Cálculo del valor de los valores de similaridad:** El razonador, utilizando los valores proporcionados por el usuario, calcula la similaridad existente entre cada uno de los individuos que componen la Ontología Difusa y las características proporcionadas. Para cada uno de los individuos de la Ontología Difusa se asigna un valor de similaridad indicando lo parecido que es cada individuo de la Ontología al individuo ideal definido por el usuario mediante sus datos de búsqueda.

3. **Presentación del ranking al usuario:** Los individuos que obtengan mayor valor de similaridad se presentan ordenados al consultor. Este corte puede hacerse de dos maneras:

    - *Por umbral:* Se establece un umbral de similaridad λ y todos los individuos que lo superen son devueltos en orden al consultor. Este método tiene la ventaja de proporcionar siempre valores de calidad pero tiene la desventaja de que, si ningún individuo supera el umbral, no se devuelve ningún elemento al usuario. Además, si muchos elementos superan el umbral, se devuelven una cantidad enorme de individuos al usuario que éste no podrá manejar con comodidad.
    - *Por número de elementos:* Se devuelven los X elementos que han obtenido el mayor valor de similaridad. Este método tiene la ventaja de que siempre se le dan opciones al usuario y la desventaja de que puede que las soluciones devueltas no sean de calidad.

- *Mixto:* Se devuelven los X mejores elementos que superan el umbral de similaridad λ. De esta forma evitamos el problema del exceso de individuos devueltos que el método por umbral tenía.

2.3.1.2 Estado del arte

Las ontologías, tanto clásicas como en su versión difusa, han sido ampliamente utilizadas en la literatura. A continuación, mostraremos algunos ejemplos que nos ayudarán a entender su importancia. Dichos ejemplos han sido clasificados en cuatro categorías diferentes:

- **Sistemas de razonamiento basados en casos**: Los sistemas de razonamiento basados en casos tratan de encontrar soluciones a problemas basándose en experiencia previa. Las ontologías pueden usarse, en este tipo de sistemas, para almacenar y manejar toda la información relativa a la experiencia previa. En [BAF+11], los autores realizan un repaso sobre los sistemas existentes de razonamiento basados en casos. Además, identifican 10 áreas diferentes en las que los autores consideran que deben enfocarse las futuras investigaciones dentro de este campo. En [AL13], las ontologías difusas se utilizan para desarrollar un sistema de razonamiento basado en casos para servicios de respuestas a emergencias. Este sistema utiliza las ontologías difusas para describir la estructura de casos, definir el vocabulario de búsqueda y facilitar el asesoramiento a la hora de calcular las medidas de similaridad. Esto último se consigue uniendo la terminología de la búsqueda con la de la base de casos.

- **Sistemas de recomendaciones**: El campo de los sistemas de recomendaciones han ido ganando importancia de forma exponencial desde la aparición de las tecnologías Web 2.0. Los modelos más recientes hacen usos de las ontologías para poder clasificar de forma clara la información con la que trabajan. Por ejemplo, en [CHBC12], los autores definen un sistema de recomendaciones basado en ontologías de dominio cuyo objetivo es seleccionar la medicación más adecuada para tratar la diabetes a un determinado paciente. Las ontologías se usan para almacenar la base de conocimiento proporcionada por los especialistas médicos del departamento de salud de Taichung. Primero, el sistema construye una ontología utilizando los atributos de cada una de las posibles medicaciones, cómo se administran, efectos secundarios y síntomas de los pacientes. Después, utilizando el Lenguaje de Reglas de la Web Semántica y la consola del Sistema Experto de Java, se diseñan las posibles prescripciones médicas para los pacientes con diabetes. En [MCPBMHV15], se presenta un nuevo modelo cuyo objetivo es representar los valores de confianza de los usuarios de un sistema de recomendaciones. Gracias a las ontologías, podemos caracterizar los perfiles de usuarios usados para generar las recomendaciones. Por último, en [CKL14], se presenta un sistema de recomendaciones para bibliotecas que utiliza técnicas de Big Data tales como Mapreduce y ontologías. Concretamente, se utilizan las ontologías para asignar a los usuarios palabras clave de interés.

- **Tratamiento del contenido de las ontologías**: En esta categoría se engloban los artículos referidos al manejo del contenido almacenado en las ontologías. Encontrar nuevas maneras de tratar con la información almacenada en las ontologías es muy importante si queremos sacarles el máximo partido. En [Gru93], se describe un mecanismo de definición de ontologías que es compatible con una gran cantidad de sistemas de representación. Ademas, los autores presentan un sistema denominado Ontolingua que es capaz de traducir información representada mediante una forma estándar en datos expresados mediante otros sistemas de representación. En [MZYC14], los autores se centran en la resolución del problema referente a cómo almacenar la información contenida en una ontología difusa. Para ello, se propone el uso de bases de datos difusas y se explica paso a paso como llevar a cabo este proceso. En [ZMFW10], se aborda una propuesta que nos permite construir ontologías difusas usando información almacenada en bases de datos orientadas a objetos. De esta manera, los diseñadores pueden ahorrar tiempo en el proceso de construcción de la ontología. En [SBI11], los autores revisan métodos para el cálculo del contenido informativo de un concepto ontológico, es decir, su grado de generalidad/concreción. Después, proponen varias mejoras que permiten realizar el proceso de manera más efectiva. En [KPK+15], los autores tratan de resolver el problema del mapeo. Cuando se trabaja con varias ontologías a la vez, muchas veces es necesario unirlas y compararlas para poder establecer similitudes entre los diferentes conceptos y encontrar posibles incompatibilidades. Por tanto, son necesarios algoritmos que traten de solucionar estas incompatibilidades terminológicas y conceptuales. Los autores proponen un método que tarda menos en ejecutarse que los ya existentes.

- **Aplicaciones de las ontologías:** En esta categoría se engloban todos aquellos artículos que muestran aplicaciones que se benefician del uso de las ontologías. Por ejemplo, en [PWM+13], los autores presentan un nuevo algoritmo de toma de decisiones en grupo que utiliza ontologías difusas para almacenar una gran cantidad alternativas. De esta manera, es posible realizar búsquedas en las ontologías con el objetivo de reducir el conjunto disponible de alternativas a un subconjunto más manejable que cumpla una serie de características. En [RLT+14], se diseña y valida una ontología sobre la diabetes Mellites que permite diagnosticar esta enfermedad en los pacientes. Usando ontologías, se lleva a cabo un proceso de extracción y valoración de los datos almacenados en los Electronic Health Records. En [DRCLCF14], se utilizan las ontologías difusas para construir un sistema de reconocimiento del comportamiento humano. El propósito de este sistema es reconocer que está haciendo una determinada persona y actuar en consecuencia. En [SA08], se diseña una ontología para investigación biomédica denominada Bio-Zen Plus. Su principal ventaja es que es la primera capaz de trabajar de forma óptima en la Web Semántica. En [SGB+15], se desarrollan una ontología y un método de razonamiento automático para tratar con información farmacogenómica. Los autores proporcionan un formalismo conciso para representar la información almacenada, encontrar errores y definiciones incompletas, asignar alelos y fenotipos a los pacientes, proporcionar un soporte adecuado a los pacientes y encontrar inconsistencias en las guías de tratamiento. Finalmente, en [AKK15], se desarrolla una ontología difusa de tipo 2 cuyo propósito es ayudar a la identificación de obstáculos marítimos. Gracias a esta ontología, es posible obtener información precisa en tiempo real acerca del riesgo de colisión en operaciones marítimas.

2.3.1.3 Ejemplo

Con el objetivo de mejorar la comprensión sobre como funcionan las Ontologías Difusas, terminaremos esta sección con un ejemplo.

Un comprador quiere comprarse un nuevo smartphone. Para ello consulta una Ontología Difusa de smartphones en busca de un modelo que se adapte a sus necesidades. Para ello, utilizara el conjunto de etiquetas lingüístico $B = \{muy\_bajo, bajo, medio, alto, muy\_alto\}$ con valor de granularidad 5. Tras decidir las características que desea que tenga su smartphone, el comprador proporciona las siguientes preferencias:

- *Tamaño de pantalla:* alto.
- *Capacidad:* bajo.
- *precio:* medio.

Dado que el comprador sólo se ha interesado por esas tres características, los valores que alcancen los individuos de la Ontología Difusa en las demás no se tienen en cuenta. De esta forma, podrán seleccionarse smartphones que tengan diferentes valores en esas características. Además, permitimos que el usuario nos indique la importancia que tiene para el cada una de las características. El comprador indica que para él, lo que debe tener más peso es el tamaño de la pantalla, luego la capacidad y por último el precio. Aunque hay muchas maneras de modelar esta situación, en este ejemplo le daremos un peso de $w_1 = 0{,}43$ al tamaño de la pantalla, $w_2 = 0{,}33$ a la capacidad y, por último, $w_3 = 0{,}23$ al precio. Esta configuración supone la misma distancia entre la primera y segunda característica y entre la segunda y la tercera. Dado que usaremos dichos pesos en la fase de agregación, debemos tener en cuenta que $w_1+w_2+w_3 = 1$.

Una vez que el usuario ha proporcionado la información necesaria, el proceso de razonamiento comienza. La búsqueda proporcionada por el usuario, con la que el razonador trabaja, puede resumirse en el valor Q de la siguientes manera:

$$Q = \{0{,}43 \cdot tampantalla\_alto, 0{,}33 \cdot capacidad\_bajo, 0{,}23 \cdot precio\_bajo\} \qquad (29)$$

El razonador, utilizando la búsqueda Q, busca en la Ontología Difusa de Smartphones aquellos individuos que obtienen un valor mayor de similaridad con ella. En este ejemplo, con el objetivo de mejorar la compresión del lector y la claridad del ejemplo, la Ontología Difusa de Smartphones sólo se compone de 6 de estos. Los valores que cada uno de estos individuos tiene asociados para

Table 4: Valores de las características de los individuos $s_1 - s_6$.

Individuo	**Tampantalla**	**Capacidad**	**Precio**
s_1	alto	alto	alto
s_2	muy bajo	bajo	bajo
s_3	bajo	alto	medio
s_4	muy bajo	muy bajo	muy bajo
s_5	muy alto	alto	muy alto
s_6	alto	muy bajo	bajo

cada una de las características disponibles puede verse en la Tabla 4.

Para cada uno de los seis individuos de la Ontología, el razonador calcula el valor de similaridad con los valores de la búsqueda proporcionada por el comprador. Para ello, utilizaremos el operador aritmético de media ponderada. La importancia de cada una de las características especificadas en la búsqueda la dará el valor del peso proporcionado por el usuario. Los cálculos y resultados de este paso pueden verse en la Tabla 6.

Para poder realizar los cálculos con éxito, es necesario definir valores de similaridad entre las etiquetas del conjunto lingüístico utilizado. En la tabla 5, podemos ver los valores de distancia utilizados en este ejemplo. Como podemos ver, los valores asignados respetan la equidistancia entre las etiquetas consecutivas de un conjunto de etiquetas lingüístico balanceado.

Por tanto, los smartphones, ordenados de más adecuados a menos adecuados sería tal que $s_6 \succ s_1 \succ s_2 \succ s_4 \succ s_5 \succ s_3$. Si analizamos los resultados, vemos como el smartphone s_6 es el que se asemeja más a lo que busca el usuario. Aunque s_2 tienen unos valores de capacidad y precio que se adaptan a los del usuario, no

Table 5: Similaridad entre etiquetas del conjunto lingüístico B.

Etiqueta	**muy_bajo**	**bajo**	**medio**	**alto**	**muy_alto**
very_low	1	0.75	0.25	0	0
low	0.75	1	0.75	0.25	0
medium	0.25	0.75	1	0.75	0.25
high	0	0.25	0.75	1	0.75
very_high	0	0	0.25	0.75	1

Table 6: Cálculo de los valores de similaridad.

Individuo	**Operaciones**	**Resultado**
s_1	$0{,}43 \cdot 1 + 0{,}33 \cdot 0{,}25 + 0{,}23 \cdot 0{,}25$	0.57
s_2	$0{,}43 \cdot 0 + 0{,}33 \cdot 1 + 0{,}23 \cdot 1$	0.56
s_3	$0{,}43 \cdot 0{,}25 + 0{,}33 \cdot 0{,}25 + 0{,}23 \cdot 0{,}25$	0.25
s_4	$0{,}43 \cdot 0 + 0{,}33 \cdot 0{,}75 + 0{,}23 \cdot 0{,}75$	0.42
s_5	$0{,}43 \cdot 0{,}75 + 0{,}33 \cdot 0{,}25 + 0{,}23 \cdot 0$	0.385
s_6	$0{,}43 \cdot 1 + 0{,}33 \cdot 0{,}75 + 0{,}23 \cdot 1$	0.9

tiene un tamaño de pantalla adecuado. Dado que esa era la característica mas importante para el usuario, eso explica el bajo valor de similaridad obtenido.

2.3.2. The Fuzzy Wine Ontology

As an example of Fuzzy Ontology, we will mainly work with the Fuzzy Wine Ontology. The Fuzzy Wine Ontology [CBM12, CMB13] was designed to work as a place-holder for applications developed for industrial purposes, and was built with non-classified information. Knowledge about wines is, with its naturally imprecise nature a perfect environment for testing decision support systems. The information included in the ontology modelled has been collected from websites created by and for wine connoisseurs [1]. Also academic publications, non-academic publications and books have been used to complete the ontology. The measurable wine properties were mostly collected from the Finnish alcohol

[1] e.g. www.alko.fi, www.winesfromspain.com, www.snooth.com

distribution monopoly Alko (e.g. alcohol level and price). Currently, the ontology contains over 600 wines, providing an appropriate tool to handle imprecise, expert-based knowledge, to produce precise recommendations.

The evaluation of the ontology was conducted by using an application-based evaluation, where the basic approach is to use the ontology in an application and then evaluate the results, i.e. how well the application meets its objectives [HBZA12]. If the results produced from the application are good and useful, one can conclude that the creation of the ontology is successful. Brank et al. [BGM05] state that this evaluation approach is a bit vague and has several drawbacks, one being the difficulty to clearly pinpoint how the ontology improves the end result, as the quality and design of the ontology is hard to evaluate. Nevertheless, the results produced from implementing the Fuzzy Wine Ontology with the application has proven to produce similar answers as the professional advice given by the wine connoisseurs. For the current application purposes, we feel that it is sufficient enough that the results produced are consistent with the advice given by the connoisseurs.

The Fuzzy Wine Ontology is composed of the following descriptive attributes:

- *Country of origin:* The location where the wine is produced has a strong impact on the final product. The weather and the different grapes give each wine a special character. This implies that different countries and regions have their own supporters. In the Fuzzy Wine Ontology, four countries are included: France, Spain, Italy, and USA.

- *Quality:* Wine quality can be judged based on different criteria, such as color, acidity, alcohol, sweetness and body, which all have an impact on the wine taste. The used concepts are listed below:

- **Alcohol:** Represents the alcohol level of the wine. A linguistic term set of granularity 3 is used, that is $S = \{Low, Medium, High\}$.
- **Acidity:** Represents how acid the wine is. S is used for its representation.
- **Price:** Price of the wine. It is a fuzzy concept represented also using S.
- **Year:** Wine year. It is represented as a fuzzy concept with an linguistic term set whose granularity value is 4, that is,

$$S = \{Novello, Regular, Old, Exclusive\}$$

- **Body:** Wine Body. Treated as a crisp concept. One of the values *Medium*, *Full* or *ExtraFull* can be chosen.
- **Sweetness:** Wine sweetness. It is also treated as a crisp concept whose possible values are *Dry*, *MediumDry* and *Sweet*.
- **Color:** Wine color (*White*, *Red* or *Rose*). It is stored as a crisp concept.

▪ *Context:* Depending on the context, wine drinkers select their wines in order to fit the dinner environment. People will alter their chosen wine depending on the particular context. The ontology includes 6 different contexts: Formal, Candle, Friends, Business, Family and Picnic. Logically, different contexts demand different attributes from the wines.

▪ *Food:* Most recommendations for pairing wines are based on the type of food being eaten. Different attributes fit well to different types of food and spices. There are 11 different food categories included: Lamb, Chicken, Beef, Pork, Fish, Game, Salad, Grilled Food, Shellfish, Pasta, Party.

The Fuzzy Wine Ontology is scalable, meaning that one can add and remove wines without affecting the overall functionality. The querying of the ontology works in the following way. First the wine's membership values to the different categories are calculated, then, with the use of the OWA operator [Yag88], the different values and weights are combined to produce a general value that represents the suitability of the specific wines for the specific scenario. In this way, the most suitable wines for different contexts can be retrieved. In this paper, the results are used as a basis for group decision making.

The Fuzzy Wine Ontology was modelled using the Web Ontology Language (OWL), which offers a family of knowledge representation languages to create ontologies aimed at the Semantic Web. As it is supported by the World Wide Web Consortium (W3C) it can be considered to be the standard language for Semantic Web aimed ontology modelling [Hor09].

3. On multi-granular fuzzy linguistic modelling in group decision making problems: a systematic review and future trends

3.1. Introduction

Decision making is a process that all humans carry out many times in their daily activities and it consists in choosing, among several possible actions, the one that is considered to give better profit. An important part of the decision making process is the way that experts express their preferences about a set of possible alternatives. The chosen method for the recollection and storage of the expert's information is vital because, if it is not intuitive for them, they will not be able to express themselves correctly. In such a case, the decision making process would be hindered. Linguistic modelling and multi-granular fuzzy linguistic modelling methods can be used in order to solve this problem.

The fuzzy linguistic approach proposed by Zadeh in 1975 [Zad75a, Zad75b, Zad75c] has been used satisfactorily to represent linguistic information during the last 40 years. In the current literature, it is possible to find two kinds of fuzzy linguistic approaches in order to represent linguistic information [HACHV09, HHV00]: traditional fuzzy linguistic approach and ordinal fuzzy linguistic approach. The former is more classical and is based on the membership functions associated to each label [Zad75a, Zad75b, Zad75c], while the latter is based on the symbolic ordinal representation of the labels [APCHV12, HM01b, MH12, TGDMMHV12]. The symbolic approximation approach has awakened high interest among the scientific community because of its simplicity and application possibilities [GM12, PRLX12, PTLMHV12, dS11, TLPP+14].

In some environments, using a unique linguistic term set is not enough to give a clear representation of the information. It is very important to use an adequate number of labels to represent each concept because, if the granularity is too low, then loss of precision is produced. On the other hand, if granularity is too high, then too much information is kept in each linguistic term set and to choose the precise label that best resembles the item that is being described could become a tiresome task. In such cases, the use of several linguistic term sets with different granularities and shapes, becomes essential. Thus, a multi-granular linguistic context should be used, i.e., several linguistic term set should be used in order to represent the linguistic information [HHVM00]. The multi-granular fuzzy linguistic modelling is appropriate in cases where several information providers need different criteria to express their preferences. For example, this could happen when they have different knowledge levels and need different expression linguistic domains with a different granularity and/or semantics. Multi-granular fuzzy linguistic modelling has been applied successfully in areas such as information retrieval [HVHM+04, HVLH07], recommender systems [MBPE08, SGHVO+11], consensus [CHVP13, MMHV09], web quality [HVPLHP06, HVPM+07] and decision making [HHVM00, JFM08].

In this chapter, we will show a comprehensive presentation of the state of the art of all known multi-granular fuzzy linguistic modelling approaches, with an in-depth analysis of the respective problems and solutions as well as more relevant applications. Furthermore, in order to give some advice of how the described methods could be improved, new trends and challenges of multi-granular fuzzy linguistic modelling are going to be discussed. From this viewpoint, we will report the results of a systematic literature review of researches published in international journals since 2000, taking into account their importance and impact in nowadays published methods. Methods selected after carrying out the systematic review process have been classified into six different categories:

- **Traditional multi-granular fuzzy linguistic modelling based on fuzzy membership functions:** Methods classified in this category use the semantics associated to each label to carry out the operations among elements of different linguistic term sets [JFM08, ZG12].
- **Ordinal multi-granular fuzzy linguistic modelling based on a basic Linguistic Term Set**: All the labels belonging to different linguistic term sets are uniformed by expressing them using a unique linguistic term set called Basic Linguistic Term Set (BLTS) and working on this special linguistic

term set the required operations are carried out [CBA06, HHVM00, Xu09].

- **Ordinal multi-granular fuzzy linguistic modelling based on 2-tuple fuzzy linguistic modelling**: In this category, methods use the 2-tuple fuzzy linguistic modelling and its properties [HM00] to manage the multi-granular linguistic information [ELM11, HM01b, Zha12].
- **Ordinal multi-granular fuzzy linguistic modelling based on hierarchical trees:** The multi-granular linguistic information is managed using the concept of hierarchical trees [HN05].
- **Multi-granular fuzzy linguistic modelling based on qualitative description spaces**: This method uses the concept of generalized description space to model and manage the multi-granular linguistic information [RSA+11].
- **Ordinal multi-granular fuzzy linguistic modelling based on discrete fuzzy numbers:** Discrete fuzzy numbers mathematical environment [VRT12] is used to deal with the multi-granular linguistic information [MRTHV14].

This chapter is organized as follows. Subsection 3.2 presents the method used for the revision. In subsection 5.2, different multi-granular fuzzy linguistic approaches are described. In subsection 5.4, a comparison among those multi-granular fuzzy linguistic approaches is presented. Finally, some future research lines are discussed.

3.2. Systematic Literature Review process

Guidelines presented by Kitchenham [BKB+07] have been followed in order to carry out the systematic review in an organized, efficient and accurate way.

Kitchenham establishes that to develop a systematic literature review we have to fix three main points: research questions that motivate the review, search process of relevant literature, and inclusion criteria of the retrieved literature.

The research questions that motivate our study are to show the origins and recent trends on the multi-granular fuzzy linguistic modelling in order to provide to the scientific community the basis of multi-granular fuzzy linguistic modelling. To do that, we did a search process in the bibliographic database Web of Science edited and managed by Thomson Reuters. We focused in the relevant papers published on journals indexed in the Journal Citation Reports (papers presented in international conferences were not considered). Then, we applied the following inclusion and exclusion criterion: Only articles that develop novel multi-granular information management methods (primary studies) were taken into account. Application articles were discarded.

3.3. Analysis of multi-granular fuzzy linguistic modelling methods

In this subsection, the main primary studies about multi-granular linguistic approaches are described, by showing their performance, characteristics and some examples of application. As mentioned in the introduction, the multi-granular linguistic approaches are organized into six different methodologies:

- Traditional multi-granular fuzzy linguistic modelling based on fuzzy membership functions.
- Ordinal multi-granular fuzzy linguistic modelling based on a basic Linguistic Term Set.
- Ordinal multi-granular fuzzy linguistic modelling based on 2-tuple fuzzy linguistic modelling.

- Ordinal multi-granular fuzzy linguistic modelling based on hierarchical trees.
- Multi-granular fuzzy linguistic modelling based on qualitative description spaces.
- Ordinal multi-granular fuzzy linguistic modelling based on discrete fuzzy numbers.

3.3.1. Traditional multi-granular fuzzy linguistic modelling based on fuzzy membership functions

This methodology follows a traditional multi-granular fuzzy linguistic modelling based on membership functions approach [JFM08, ZG12]. The next scheme is used in order to deal with multi-granular information:

1. All the labels belonging to different linguistic term sets present an associated semantics represented by membership functions.
2. Computations are carried out on the membership functions of the labels.
3. Unless some kind of transformation is performed, computation results are expressed using the membership functions instead of particular linguistic labels.

Generally, Trapezoidal Fuzzy Numbers (TFNs) are used in order to represent the information and carry out the required computations. They have a strong mathematical environment that let us to work with a wide range of operations. The disadvantage is that it is a troublesome task to express them linguistically making the results interpretation and data providing become difficult tasks for common people. Experts can provide their preferences using an ordinal linguistic term set. In such a way, the experts provide labels that are translated into TFNs in order to carry out the Group Decision Making computations. This solves the data providing problem but not the results interpretation, because we have to translate a membership function into a particular label of the original linguistic term set. Because labels have their own semantics, results that do not match to any of the label semantics are obtained after performing computations. No solutions are presented in the analysed papers to this issue. The authors probably did not considered this to be a problem because they were only calculating an alternative ranking. Nevertheless, if consensus approaches want to be applied to these methods, results cannot be given to the experts using TFNs because they lack interpretation. One way of solving this issue, although it could imply a loss of information, consists in assigning to each obtained TFN that label whose semantics (also a TFN) is the closest one.

Regular linguistic term sets are not the only way for experts to provide their preferences. For example, uncertain linguistic terms (ULTS) can be employed [ZG12]. ULTSs allow users to provide their preferences using a label interval instead of a unique label. This way, preference constructions like *I prefer* x_1 *to* x_2 *with a between high and very high degree* can be used. ULTSs also help in the task of classifying linguistic terms that do not belong to any linguistic term set. So, they can be part of the linguistic term interval that better suits them.

Once that all the preferences are expressed in terms of an unique TFN, calculation of the collective preference information piece is carried out. For this process, goal programming model is a preferred method. Then, the TFN whose distance to all the provided TFNs is minimum is selected.

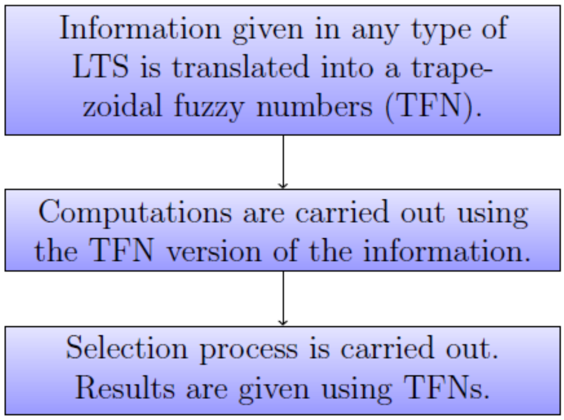


Figure 9: Scheme of traditional multi-granular fuzzy linguistic modelling based on fuzzy membership functions approaches.

Finally, ranking is made through the selection process. Closeness coefficients using the TOPSIS principle [Wei10, ZG12] or the calculation of dominance degree [DP83, JFM08] are two possibilities.

In Figure 9, the explained process can be seen schematically. For the sake of simplicity and a precise understanding by the readers of the presented methodology, a simplified multi-granular Group Decision Making example that follows this methodology is presented below.

Example 1. Three experts, $E = \{e_1, e_2, e_3\}$, have to make a decision among a set of three alternatives, $X = \{x_1, x_2, x_3\}$. Two linguistic term sets, S^1 and S^2 are used to provide their preferences. As in [JFM08], one preference value is provided for each alternative. S^1 and S^2 and their semantics are described in Table 7.

After a brief discussion among the experts, e_1 provides the preference values $P_1 = \{s_5^1, s_1^1, s_2^1\}$, e_2 provides $P_2 = \{s_7^2, s_3^2, s_4^2\}$ and e_3 provides $P_3 = \{s_6^2, s_2^2, s_3^2\}$. Preferences of each expert are aggregated into a collective preference vector using the mean operator among the TFNs associated to each of the labels. Results are showed in Table 8.

S^1	S^1 semantics	S^2	S^2 semantics
s_1^1	(0, 0, 0,25)	s_1^2	(0, 0, 0,16)
s_2^1	(0, 0,25, 0,5)	s_2^2	(0, 0,16, 0,33)
s_3^1	(0,25, 0,5, 0,75)	s_3^2	(0,16, 0,33, 0,5)
s_4^1	(0,5, 0,75, 1)	s_4^2	(0,33, 0,5, 0,66)
s_5^1	(0,75, 1, 1)	s_5^2	(0,5, 0,66, 0,83)
		s_6^2	(0,66, 0,83, 1)
		s_7^2	(0,83, 1, 1)

Table 7: S^1 and S^2 semantics for each label.

x_1	x_2	x_3
(0,7466, 0,9433, 1)	(0,053, 0,1633, 0,36)	(0,1633, 0,36, 0,5533)

Table 8: Collective preference values.

For the selection phase, TOPSIS method described in [ZG12] has been used. Closeness coefficients for each alternative are showed in Table 9. Then, ranking of the alternatives is as follows: $x_1 \succ x_3 \succ x_2$.

3.3.2. Ordinal multi-granular fuzzy linguistic modelling based on a Basic linguistic term set

Multi-granular Group Decision Making methods classified in this category follow the next steps:

1. **Providing preferences**: Experts provide their preferences using the linguistic term set that better fits them.
2. **Making the information uniform**: All the provided information is expressed using a unique linguistic term set that is called BLTS. In such a

x_1	x_2	x_3
0.881	0.1849	0.35915

Table 9: Closeness coefficient of each alternative.

way, the same linguistic term set is employed for all the preference values and any operation can be carried out.

3. **Computing collective values**: All the provided information is aggregated into a collective piece of information.

4. **Selection phase**: Using the collective preference values and any selection criteria, ranking among the alternatives is made.

Different ways of representation of preferences could be used:

- **Balanced ordinal linguistic term sets**: [CBA06, HHVM00]: Balanced ordinal linguistic term sets are linguistic term sets whose number of linguistic terms is odd and they are equally distributed in an ordinal scale. Labels belonging to other linguistic term sets are translated to the linguistic term set that has the highest granularity in order to carry out computations. This could become a disadvantage if operations with a high number of labels are inadequate or not desired. The method presented in [CBA06] established this requirement and allowed the use of any balanced linguistic term set as the BLTS. In such a way, small granularity linguistic term sets can become the BLTS easing the subsequent operations. It should be pointed out that the smaller the BLTS, the less the representation capability is and, consequently, the more loss of information is produced. The main problem of using balanced linguistic term sets is that they have several restrictions that reduce flexibility. If balanced ordinal linguistic term sets are used, information can be aggregated using OWA operator [CHVHA07, Yag88, Yag96] and selection process can be carried out using the non-dominance degree [Orl78].

- **Unbalanced linguistic term sets** [PWGW13, Xu09]: In order to introduce flexibility in the way that experts express their preferences, unbalanced linguistic term sets can also be used in this methodology. The main problem is that not every unbalanced linguistic term set is allowed, only the ones having most of the labels concentrated near the linguistic term set medium term having the same number of labels before and after it. Although this is a clear representational advantage, the allowed unbalanced linguistic term sets are not the most ideal ones. In general, experts are interested in unbalanced linguistic term sets that have labels concentrated in the right side or the left side of the medium label [CAHV09, HVLH07]. For example, they can require more options when trying to give a positive answer than a negative one. They can be more interested and need more specification when trying to rate how positive an alternative is than how negative it is. In the recent literature, this issue is not yet resolved. Another disadvantage is that linguistic terms are indexed using fractions instead of natural numbers. This reduces readability and introduces complexity to the model.

- **Uncertain linguistic term sets** [PWGW13, Xu09] : If uncertain linguistic term sets are used, information can be aggregated using the ULWA operator [Xu04] and selection process can be carried out using, for example, the degree of possibility [Xu06].

A scheme of the explained process can be seen in Figure 10. Finally, for the sake of helping the reader to understand how these multi-granular management methods work, Example 2 is showed.

Example 2. Three experts, $E = \{e_1, e_2, e_3\}$, have to make a decision among a set of three alternatives, $X = \{x_1, x_2, x_3\}$. Two ordinal linguistic term sets, $S^1 = \{s^1_1, \ldots, s^1_5\}$ and $S^2 = \{s^2_1, \ldots, s^2_7\}$ are available for them to provide their preferences. S_1 and S_2 semantics are described in Table 7. First, experts provide their preferences. For the sake of simplicity, each expert will provide one value

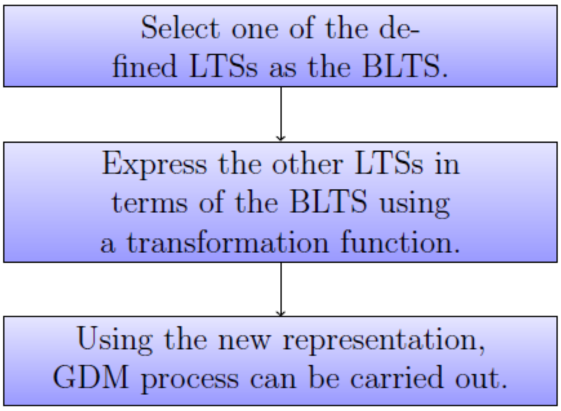


Figure 10: Ordinal multi-granular fuzzy linguistic modelling based on Basic linguistic term set methodology scheme.

of preference for each alternative: $P_1 = \{s^2_7, s^2_3, s^2_1\}$, $P_2 = \{s^1_5, s^1_2, s^1_1\}$ and $P_3 = \{s^2_6, s^2_4, s^2_2\}$. Next, the BLTS has to be chosen among the provided linguistic term sets. Following [HHVM00] and, because S^2 is the linguistic term set having the highest granularity, it is chosen as BLTS. Then, all linguistic preferences are expressed using S^2 by means of the transformation function [HHVM00] that is showed below.

Let $A = \{l_0, \ldots, l_p\}$ and $S_T = \{c_0, \ldots, c_g\}$ be two linguistic term sets, such that, $g \geq p$. Then, a multi-granularity transformation function, τ_{AS_T} is defined as

$$\begin{aligned} &\tau_{AS_T} : A \rightarrow F(S_T), \\ &\tau_{AS_T}(l_i) = \{(c_k, \alpha^i_k)/k \in \{0, \ldots, g\}\}, \forall l_i \in A, \\ &\alpha^i_k = \max_y \min\{\mu_{l_i}(y), \mu_{c_k}(y)\} \end{aligned} \tag{30}$$

where $F(S_T)$ is the set of fuzzy sets defined in S_T, and $\mu_{l_i}(y)$ and $\mu_{c_k}(y)$ are the membership functions of the fuzzy sets associated to the term l_i and c_k, respectively.

In Table 10, final uniformed representation is exposed.

Label	Associated semantics
s^2_7	(0,0,0,0,0,0,1)
s^2_3	(0,0,1,0,0,0,0)
s^2_1	(1,0,0,0,0,0,0)
s^1_5	(0,0,0,0,0.19,0.9,1)
s^1_2	(0.39,0.78,0.80,0.40,0,0,0)
s^1_1	(1,0.60,0.21,0,0,0,0)
s^2_6	(0,0,0,0,0,1,0)
s^2_4	(0,0,0,1,0,0,0)
s^2_2	(0,1,0,0,0,0,0)

Table 10: Final uniformed representation for all the provided lables in the example.

Alternative	Collective value
x_1	(0,0,0,0,0.063,0.63,0.66)
x_2	(0.13,0.26,0.6,0.46,0,0,0)
x_3	(0.66,0.53,0.07,0,0,0,0)

Table 11: Collective preferences values.

After aggregating the information corresponding to each alternative, results exposed on Table 11 are obtained. Mean operator will be the chosen one as we want to give to the experts the same importance. Because high values at the start of a collective tuple indicates a general negative opinion about the alternative and high values at the end of it refers to positive ratings, using the gravity center measure can help us to rank the alternatives from best to worst. After applying it, gravity center values of 6.33, 2.62 and 1.05 for x_1, x_2 and x_3 are obtained, respectively. It is important to point out that gravity center values are located in the interval [1,7]. Consequently, it is possible to conclude that $x_1 \succ x_2 \succ x_3$.

3.3.3. Ordinal multi-granular fuzzy linguistic modelling based on 2-tuple representation model

Ordinal multi-granular fuzzy linguistic modelling based on 2-tuple and Linguistic Hierarchies [ELM11, HM01b, Zha12] uses the 2-tuple representation model in order to deal with multi-granular information. 2-tuple linguistic modelling provides an easy way of dealing with linguistic term sets and operating with them without any loss of information. Furthermore, they are able to represent elements that do not belong to the initial linguistic term set. These properties can be used in order to develop methods that deal with multi-granular linguistic information.

2-tuple linguistic modelling is based on the concept of symbolic translation. Let β be the result of an aggregation of the indexes of a set of terms assessed in a linguistic term set S whose cardinality is $g+1$. Let $i = round(\beta)$ and $\alpha = \beta - i$ be two values such that $i \in [0, g]$ and $\alpha \in [-0{,}5, 0{,}5)$ then α is called a symbolic translation where $round()$ is the usual round operation.

Let $S = \{s_i | i = 0, 1, 2, \ldots, g\}$ be a linguistic term set and $\beta \in [0, g]$ a value representing the result of a symbolic aggregation operation, then the 2-tuple that expresses the equivalent information to β is obtained with the following expression:

$$\Delta : [0, g] \rightarrow S \times [0{,}5, 0{,}5) \tag{31}$$

$$\Delta(\beta) = (s_i, \alpha), \text{ with } \begin{cases} s_i & i = round(\beta) \\ \alpha = \beta - i & \alpha \in [-0{,}5, 0{,}5) \end{cases} \tag{32}$$

where s_i has the closest index label to β and α is the value of the symbolic translation.

Let $S = \{s_i | i = 0, 1, 2, \ldots, g\}$ be a linguistic term set and (s_i, α) be a 2-tuple. There exists a function Δ^{-1} such that from a 2-tuple it returns its equivalent numerical value $\beta \in [0, g] \subset \mathfrak{R}$:

$$\Delta^{-1} : S \times [0{,}5, 0{,}5) \rightarrow [0, g] \tag{33}$$

$$\Delta^{-1}(s_i, \alpha) = i + \alpha = \beta \tag{34}$$

A conversion of any linguistic label into a linguistic 2-tuple can be performed directly adding a zero value as a symbolic translation

$$\Delta(s_i) = (s_i, 0), i = 0, 1, 2, \ldots, g \tag{35}$$

Two different approaches that use 2-tuple representation method can be followed:

- **Use of generalized linguistic 2-tuple variable** [Zha12]: All the 2-tuple linguistic term sets are expressed using the generalized linguistic 2-tuple variable [CT05]. Posterior Group Decision Making operations are carried out using it. The main advantage of this approach is the simplicity. Its main drawback is that all the Group Decision Making processes have to be carried out using an specific unchanging representation. The scheme used by this approach follows the next steps:

  1. Experts express their preferences using their preferred linguistic term sets.
  2. linguistic term sets are translated into 2-tuple linguistic information using equation (35).
  3. 2-tuple linguistic information provided by the users are translated into the generalized linguistic 2-tuple variable.
  4. Information can be then aggregated and selection processes can be applied in order to obtain the alternatives ranking. IVTWA operator [Zha12] can be used for the aggregation and selection process can be carried out comparing the obtained collective values for each alternative.

- **Linguistic hierarchy building** [ELM11, HM01b]: A hierarchy is built using the linguistic term sets. Although this approach is more complex, the hierarchy allows translations among the different linguistic term sets that conforms it. In such a way, any linguistic term set of the hierarchy can be used for carry out computations. The scheme used by this method follows the following steps:

  1. A hierarchy is built using the linguistic term sets that experts use to express themselves.
  2. One of the linguistic term sets that conforms the hierarchy is used as the target linguistic term set for carrying out the computations.
  3. Preferences provided by the users using different linguistic term sets are translated into the target linguistic term set. Thanks to 2-tuple representation model and Linguistic Hierarchies (LHs), it is possible to carry out this process without loss of information.
  4. Information is aggregated and selection process is carried out in order to calculate the alternatives ranking.

  In general, each level of a LH represents a unique linguistic term set. This way, expressions that let us express labels from one level into another one within the hierarchy are defined. These methods tend to be efficient and are able to provide results in a linguistic manner without needing to use, in most of the cases, a defuzzification process. They also avoid loss of precision in the fusion of multi-granular linguistic information. The main drawback of this method is that only the linguistic term sets that define the hierarchy can be used by the experts in the decision process. It is also not possible to use linguistic term sets that are unbalanced [HVLH07] or have atypical characteristics. Another problem is that all the linguistic term sets

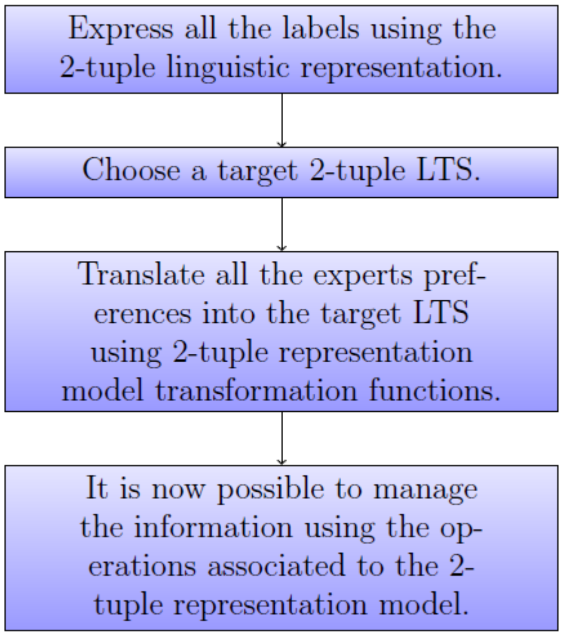


Figure 11: Ordinal multi-granular fuzzy linguistic modelling based on 2-tuple scheme.

that conform the LH have to keep their formal modal points from one level to the next. This problem is partially solved in [ELM11]. Authors allow the creation of hierarchies that do not have to keep all the former modal points of the previous levels. Nevertheless, it requires the creation of a new level that usually has an enormous granularity value. For this reason, several ways of minimizing the granularity level are provided. Nonetheless, the model can become extremely complex and difficult to manage. The granularity increases with the number of levels of the hierarchy making the hierarchies with a high number of levels become unmanageable.

A scheme of the process followed in this methodology can be seen in Figure 11 and Example 3.

Example 3. Three experts, $E = \{e_1, e_2, e_3\}$, have to participate in a Group Decision Making process. They have to rank a set of three alternatives, $X = \{x_1, x_2, x_3\}$. Three ordinal linguistic term sets, $S^1 = \{s_0^3, \ldots, s_2^3\}$, $S^2 = \{s_0^5, \ldots, s_4^5\}$ and $S^3 = \{s_0^9, \ldots, s_8^9\}$ are available for them to provide their preferences. Because an LH approach want to be showed in this example, different linguistic term sets from the previous examples have to be chosen in order for them to fulfil the LH building requirements. The built $LH = \cup_t l(t, n(t))$ is defined as follows:

$$l(1,3)\ \{s_0^3, s_1^3, s_2^3\}$$
$$l(2,5)\ \{s_0^5, s_1^5, s_2^5, s_3^5, s_4^5\}$$
$$l(3,9)\ \{s_0^9, s_1^9, s_2^9, s_3^9, s_4^9, s_5^9, s_6^9, s_7^9, s_8^9\}$$

For the sake of simplicity, each expert will provide a preference value for each alternative in a different linguistic term set. This way, preference values are: $p_1 = \{s_0^3, s_0^3, s_2^3\}$, $p_2 = \{s_0^5, s_2^5, s_4^5\}$ and $p_3 = \{s_1^9, s_4^9, s_7^9\}$. All the information must be expressed in the same level of the LH in order to carry out computations. Level 2 of the hierarchy will be the one chosen for computation purposes. It should be pointed out that the same results will be obtained regardless of the chosen LH level. After translating step, the provided preference results expressed below are obtained:

$$p_1 = \{(s_0^5, 0), (s_0^5, 0), (s_4^5, 0)\}$$
$$p_2 = \{(s_0^5, 0), (s_2^5, 0), (s_4^5, 0)\}$$
$$p_3 = \{(s_0^5, 0{,}5), (s_2^5, 0), (s_3^5, 0{,}5)\}$$

Aggregation can be done if OWA operator is applied over the equivalent numerical values of the tuples. Therefore, the following collective value is obtained:

$$p_c = \{(s_0^5, 0{,}166), (s_1^5, -0{,}33), (s_4^5, -0{,}17)\}$$

Therefore, it can be concluded that $x_3 \succ x_2 \succ x_1$.

3.3.4. Ordinal multi-granular fuzzy linguistic modelling based on hierarchical trees

Hierarchical trees [HN05] are a special hierarchical construction that is built directly over the labels, without taking into account any semantics associated to them. Each level of the tree represents a different linguistic term set. The closer the linguistic term set is to the tree root, the less granularity it has. Comparing to LHs, Hierarchical trees are more flexible because any structure is valid as long as each label is connected to one label from the previous level and at least another one of the next. Its main disadvantage is that translations from labels belonging to lower granularity linguistic term sets to labels belonging to high granularity ones can lead into the creation of a set of transformation rules. This can be seen in the following example.

Example 4. This example shows the transformation of the term sets of level 1 to the level 2 and viceversa of the hierarchical tree of Figure 12.

$$
\begin{array}{ll}
\Phi_2^1(none) = \{none\} & \Phi_2^1(low) = \{very\ low, low\} \\
\Phi_2^1(medium) = \{medium\} & \Phi_2^1(high) = \{high, very\ high\} \\
\Phi_2^1(perfect) = \{perfect\} & \Phi_1^2(none) = \{none\} \\
\Phi_1^2(very\ low) = \{low\} & \Phi_1^2(low) = \{low\} \\
\Phi_1^2(medium) = \{medium\} & \Phi_1^2(high) = \{high\} \\
\Phi_1^2(very\ high) = \{high\} & \Phi_1^2(perfect) = \{perfect\}
\end{array}
$$

where Φ_i^j refers to the transformation function from level i to level j. For example, label *high* from t_1 is transformed into the label set $\{high, very\ high\}$ from t_2. A mathematical framework capable to deal with this situation is needed for carrying out the desired computations.

Aggregation and selection phase can be done using a choice function based on the satisfactory principle [HN05]. Satisfactory principle claims that *it is perfectly*

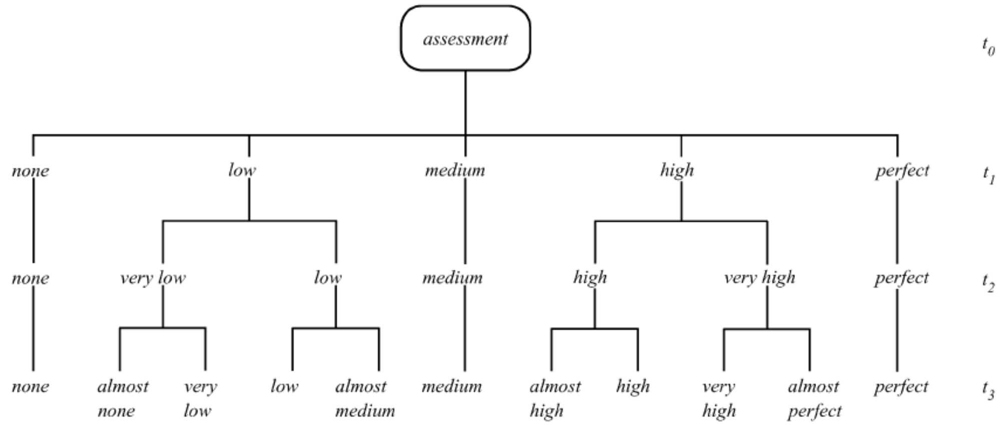


Figure 12: Hierarchical tree of the linguistic variable *assessment* [HN05].

satisfactory to select an alternative as the best if its performance is as at least good as all the others under the same evaluation scheme.

The process carried out by this method can be seen in Figure 13 and Example 5.

Example 5. Three experts, $E = \{e_1, e_2, e_3\}$, participate in a Group Decision Making process. They have to rank three alternatives, $X = \{x_1, x_2, x_3\}$. Using the linguistic term set given in Figure 12, expert e_1 decides to use t_1 to provide its preferences, e_2 prefers t_2 and e_3 uses t_3. For the sake of simplicity, they are asked to provide a linguistic preference value for each of the alternatives. Preferences provided are:

$$P_1 = \{low, medium, perfect\}$$

$$P_2 = \{very\ low, high, very\ high\}$$

$$P_3 = \{almost\ none, almost\ medium, almost\ high\}$$

t_2 is chosen as the main target set. After carrying out translations the following results are obtained:

$$P_1 = \{\{very\ low, low\}, medium, perfect\}$$

$$P_2 = \{very\ low, high, very\ high\}$$

$$P_3 = \{very\ low, low, very\ high\}$$

LOWA operator can be used to aggregate the provided information [HHV00]. T-norm min is used in order to reduce the label sets inside the preference matrix into a single labels. After reduction and aggregation phase are applied, the following collective preference matrix is obtained:

$$P_c = \{very\ low, medium, very\ high\}$$

In such a way, comparing the label indexes, the final ranking is $x_3 \succ x_2 \succ x_1$. It is important to notice that no labels semantic have been used in this example, only the labels ordering. Therefore, ordinal multi-granular fuzzy linguistic modelling based on hierarchical trees helps us to define multi-granular symbolic Group Decision Making methods. It should be pointed out that loss of information has been carried out in the process described in the example. For instance, the label provided by expert e_1 for alternative x_1, *low*, has been translated into the set of labels $\{very\ low, low\}$. In order to solve this issue, the t-norm min has been used in order to obtain a unique label, *very_low*. Label *very_low* from level t_2 is not located at the same distance from the mid term label as the original label that has been provided, that is, *low* from t_1. Therefore, the provided information has been modified by the method and it can be stated that loss of information has been produced.

3.3.5. Multi-granular fuzzy linguistic modelling based on qualitative description spaces

This method uses description spaces in order to represent the information [RSA+11]. A description space is an ordered triple $(\Lambda, \mathbb{S}_n, \mu)$ where

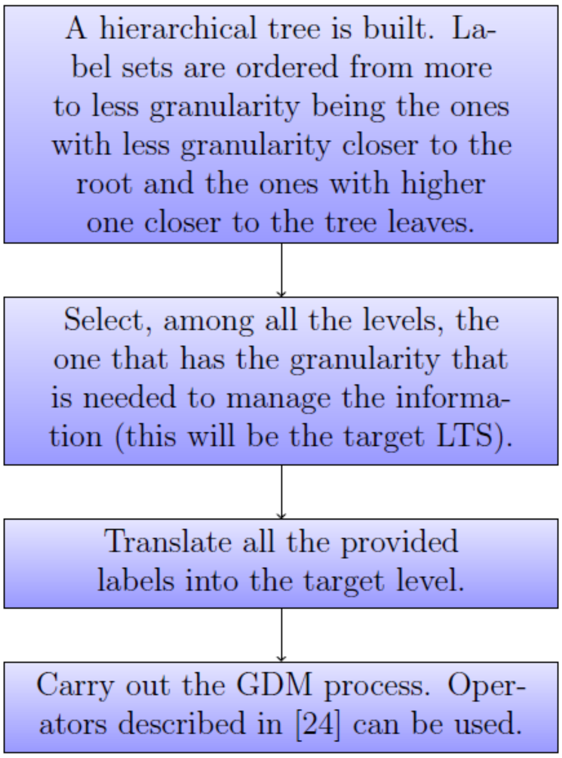


Figure 13: Ordinal multi-granular fuzzy linguistic modelling based on hierarchical trees scheme.

$\Lambda = \{a_t | t \in I, I \subset \mathbb{R}\}$ is a set of features, $\mathbb{S}_n$ is an order-of-magnitude space with granularity n and μ is a normalized measure defined in $\mathbb{S}_n$, such that all features in Λ are assessed by $\mathbb{S}_n$ labels [RSA+11]. In Group Decision Making problems, Λ is used to represent the alternatives or, in the case of multi-criteria Group Decision Making, the different criteria used. $\mathbb{S}_n$ is the label set used by the experts to express their preferences and μ can be used to give weights to the different labels of the linguistic term set. In such a way, unbalanced linguistic term sets with more granularity in one of the extreme sides of the interval can be defined. Consequently, experts could have more specification possibilities and precision when giving positive evaluations than negative, if necessary.

A qualitative assessment Q can be associated to a description space. Given a description space $(\Lambda, \mathbb{S}_n, \mu)$, a qualitative assessment Q is a mapping $Q : \Lambda \to \mathbb{S}_n$. It can be seen straightforward from this definition that qualitative assessments will be used in the Group Decision Making problems in order to represent the preferences provided by the experts. It can be seen that description spaces provide a mathematical representation that fits perfectly a Group Decision Making problem representation.

In order to introduce multi-granularity in the described environment, the generalized description space concept is introduced. A generalized description space is an ordered triple such that

$$\left(\Lambda, \bigcup \mathbb{S}_{n_i}, \{\mu\}\right) = \left(\biguplus_{i=1}^{r} \Lambda_i, \bigcup_{i=1}^{r} \mathbb{S}_{n_i}, \{\mu_1, \ldots, \mu_r\}\right) \tag{36}$$

where $r > 1$. A generalized description space consists in the disjoint union, $\biguplus$, of different sets of features, Λ_i, where each of them admits qualitative descriptions in $\mathbb{S}_{n_i}$ and normalized measures μ_i defined in $\mathbb{S}_{n_i}$. In such a way, in order to carry out a Group Decision Making process, this methodology follows the next steps:

1. Experts give their qualitative assessments using the linguistic term set that they prefer.

2. Find the optimal representative for each of the alternatives. This is done selecting the label that has the less distance to all the provided labels for the same alternative.

3. Distance from the best possible ranking value for each alternative in each description space to the provided one is calculated.

4. Alternatives are ordered by the distances computed to the best possible ranking value.

We should point out that this methodology does not use semantics associated to the labels and, in addition, it does not require any label translation. Therefore, no loss of information is produced in conversions. linguistic term sets with a even number of labels are supported. Its main drawback consists in that multi-granularity is associated to alternatives, that is, the same linguistic term set must be chosen to describe all the preferences provided for an specific alternative. Thus, experts must use the linguistic term set associated with the alternative in order to describe it, not the one that they could prefer. A scheme of the followed process can be seen in Figure 14 and an example is showed below.

Example 6. Three experts, $E = \{e_1, e_2, e_3\}$, participate in a Group Decision Making process. They have to rank three alternatives, $X = \{x_1, x_2, x_3\}$. The following linguistic term sets are available for experts to express their preferences:

$$\mathbb{S}_4 = \{B_1, B_2, B_3, B_4\}$$

$$\mathbb{S}_5 = \{B_1, B_2, B_3, B_4, B_5\}$$

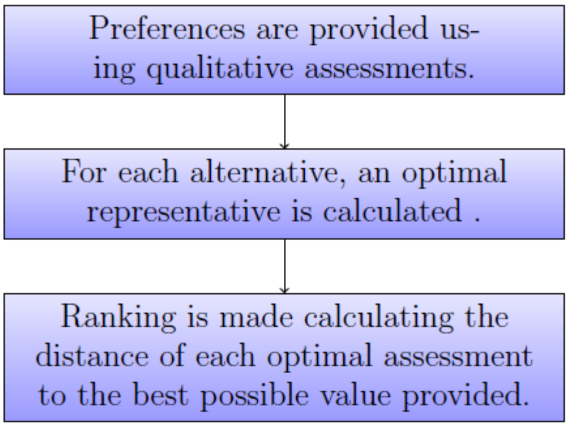


Figure 14: Ordinal multi-granular fuzzy linguistic modelling based on qualitative description spaces methodology scheme.

$\mathbb{S}_4$	B_1	B_2	B_3	B_4
B_1	0	2.8	5.33	8.06
B_2	2.8	0	2.6	5.33
B_3	5.33	2.6	0	2.8
B_4	8.06	5.33	2.8	0

Table 12: $\mathbb{S}_4$ label distance matrix.

Matrices expressing the distances among the different labels in each of the two description spaces are specified in Table 12 and 13 for $\mathbb{S}_4$ and $\mathbb{S}_5$, respectively. For a deeper explanation of how the distances among labels can be calculated, readers can consult [RSA$^+$11]. Alternative x_1 will be described using $\mathbb{S}_4$ and alternatives x_2 and x_3 will use $\mathbb{S}_5$. For the sake of simplicity, each expert will provide a label describing each alternative. After a brief discussion, experts provide the following preferences:

$$P_1 = \{B_2, B_4, B_1\}$$

$$P_2 = \{B_1, B_5, B_2\}$$

$$P_3 = \{B_1, B_4, B_2\}$$

$\mathbb{S}_5$	B_1	B_2	B_3	B_4	B_5
B_1	0	1.6	3.96	6.2	8.4
B_2	1.6	0	1.6	3.96	6.2
B_3	3.96	1.6	0	1.6	3.96
B_4	6.2	3.96	1.6	0	1.6
B_5	8.4	6.2	3.96	1.6	0

Table 13: $\mathbb{S}_5$ label distance matrix.

For the aggregation process, label that is closer to all the labels used for describing the alternative is chosen. Then, the collective preference matrix obtained is showed below:

$$P_c = \{B_1, B_4, B_2\}$$

$B_1 \in \mathbb{S}_4$ and $B_4, B_2 \in \mathbb{S}_5$. Because of that, no direct comparison is allowed. In order to make the ranking, distance to the best possible value of each linguistic term set is computed and a ranking is made according to the obtained values:

$$B_1 = 8{,}04, B_4 = 1{,}6, B_2 = 6{,}2$$

According to the obtained values, $x_2 \succ x_1 \succ x_3$.

3.3.6. Ordinal multi-granular fuzzy linguistic modelling based on discrete fuzzy numbers

In [MRTHV14], the concept of Subjective Linguistic Hierarchy (SLH) is introduced. A SLH is a LH that is built using linguistic term sets whose linguistic terms are represented by Discrete Fuzzy Numbers [MRTHV14, Vox01]. Therefore, experts do not provide a single label, they provide a list of all the labels with an associated number in the interval [0,1] that represents the level of agreement that the expert has with that label in the corresponding description. It should be pointed out that value 1 must be assigned to at least one of the labels and monotonicity properties must be fulfilled. For example, taking into account the linguistic term set $S = \{s_1, \ldots, s_7\}$, a preference value of $\{0{,}3/s_5, 1/s_6, 0/s_7\}$ indicates that the preference provided by the expert matches perfectly the label s_6, it has something to do with label s_5 and s_7 should not be considered as a provided label for that description. This type of description is really flexible because it allows the addition of degrees to the labels. Nevertheless, this flexibility reduces the methodology simplicity. Thus, it is more complicated for experts to provide their preferences because they have to think on the numerical degree that they want to give to the labels. Of course, this can be solved if experts are provided with a set of labels that are lately translated into discrete fuzzy numbers. As most of the methodologies explained in previous subsections, this methodology follows the next scheme:

1. **Providing preferences:** Experts express their preferences by means of DFNs. It is also possible to provide single labels due to the fact that a label s_i can be expressed as the DFN $\{1/s_i\}$. This reduces complexity allowing experts to express themselves using labels instead of complex DFNs.

2. **Uniforming preferences:** An linguistic term set is chosen as the target for computations. Sometimes, it can be necessary to carry out translations from linguistic term sets with small granularity to linguistic term sets with bigger granularities. To do so, completions are employed [MRTHV14]. That is, some labels of the origin linguistic term set are translated directly into labels from the target linguistic term set and others are inferred using the surrounding valuations of the labels belonging to the target linguistic term set. This way of solving this problem is based in the assumption that labels that are close have similar valuations. For example, if an expert has provided a high valuation value to a high position linguistic term it is not probable that they provide a high intensity value to a term located in a low position. Thanks to completions, it is possible to carry out translations from one

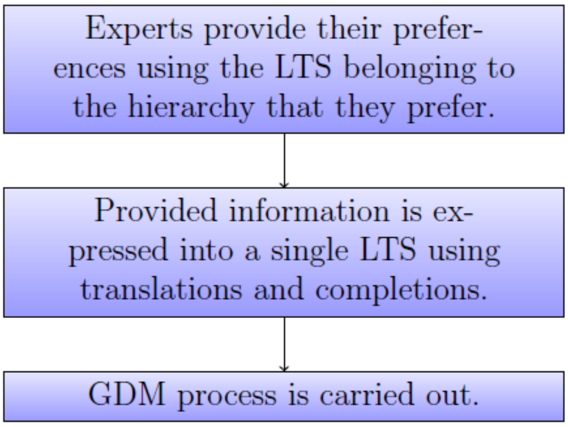


Figure 15: Ordinal multi-granular fuzzy linguistic modelling based on discrete fuzzy numbers methodology scheme.

level of the hierarchy to another one, allowing us to manage linguistic terms belonging to different linguistic term sets from the hierarchy.

3. **Aggregation phase:** Aggregation functions are used to add the discrete fuzzy numbers into a collective value. Several aggregation functions for discrete fuzzy numbers are exposed in [MMT13].

4. **Selection phase:** Method described in [CL01] can be used. According to it, left and right dominance concept is applied over the collective matrix calculated in the previous step in order to obtain the ranking of alternatives.

A scheme of the followed process can be seen in Figure 15. In order to provide a clear understanding to the reader, a brief example is showed below.

Example 7. Three experts, $E = \{e_1, e_2, e_3\}$, participate in a Group Decision Making process. They have to rank three alternatives, $X = \{x_1, x_2, x_3\}$. Ordinal linguistic term sets $S^5 = \{s_0^5, \ldots, s_4^5\}$ and $S^9 = \{s_0^9, \ldots, s_8^9\}$ of granularity values 5 and 9, respectively, are used to provide their preferences. Expert e_1 decides to use S^5 to provide his/her preferences, e_2 and e_3 prefer S^9. Expert preferences

	x_1	x_2
e_1	$\{0{,}5/s_0^5, 0{,}8/s_1^5, 1/s_2^5, 0{,}6/s_3^5\}$	$\{0{,}5/s_2^5, 0{,}8/s_3^5, 1/s_4^5\}$
e_2	$\{1/s_0^9, 0{,}8/s_1^9, 0{,}5/s_2^9, 0{,}1/s_3^9\}$	$\{0{,}1/s_6^9, 0{,}5/s_7^9, 1/s_8^9\}$
e_3	$\{1/s_1^9, 0{,}6/s_2^9, 0{,}5/s_3^9, 0{,}2/s_4^9\}$	$\{0{,}1/s_5^9, 0{,}8/s_6^9, 0{,}9/s_7^9, 1/s_8^9\}$

	x_3
e_1	$\{0{,}6/s_1^5, 1/s_2^5, 1/s_3^5, 0{,}8/s_4^5\}$
e_2	$\{0{,}5/s_3^9, 1/s_4^9, 0{,}8/s_5^9, 0{,}5/s_6^9\}$
e_3	$\{0{,}6/s_2^9, 0{,}9/s_3^9, 1/s_4^9, 0{,}7/s_5^9\}$

Table 14: Preferences provided by users.

are exposed in Table 14. Labels belonging to S^5 are translated into S^9 taking into account that they are equally distributed over the same range. The linguistic assessments provided by e_1 are transformed as:

$$\{s_0^5, s_1^5, s_2^5, s_3^5\} \to \{s_0^9, s_2^9, s_4^9, s_6^9\}$$
$$\{s_2^5, s_3^5, s_4^5\} \to \{s_4^9, s_6^9, s_8^9\}$$
$$\{s_1^5, s_2^5, s_3^5, s_4^5\} \to \{s_2^9, s_4^9, s_6^9, s_8^9\}$$

Valuation values of the labels that are located between the estimated labels are calculated using the t-norm mín. Then, we obtain the following linguistic assessments for the expert e_1:

$$p_1^1 = \{0{,}5/s_0^9, 0{,}5/s_1^9, 0{,}8/s_2^9, 0{,}8/s_3^9, 1/s_4^9, 0{,}6/s_5^9, 0{,}6/s_6^9\}$$
$$p_1^2 = \{0{,}5/s_4^9, 0{,}5/s_5^9, 0{,}8/s_6^9, 0{,}8/s_7^9, 1/s_8^9\}$$
$$p_1^3 = \{0{,}6/s_2^9, 0{,}6/s_3^9, 1/s_4^9, 1/s_5^9, 1/s_6^9, 0{,}8/s_7^9, 0{,}8/s_8^9\}$$

where p_i^j indicates the provided preference value of expert i for alternative j. The form v/L indicates the valuation value v for the label L.

Once that all the preferences have been expressed using the same linguistic term set, information has to be aggregated. This model uses kernel aggregation

	x_1	x_2	x_3
e_1	3.14	6.36	5.17
e_2	0.875	7.56	4.89
e_3	1.95	7	3.56

Table 15: Preferences gravity centers.

functions [MMT13]. Nevertheless, for the sake of simplicity and in order to show the reader that the models exposed here are flexible, another aggregation approach will be used. First, gravity center value of every preference provided will be calculated. Afterwards, mean over the obtained values will be computed. In Table 15 gravity values are showed. Collective ranking values are showed below:

$$P_c = \{1{,}98, 6{,}97, 4{,}54\}$$

Straightforward from the collective matrix, it can be seen that $x_2 \succ x_3 \succ x_1$.

3.4. Discussion and Future Trends

All the presented methods have their own advantages and drawbacks, that is, some work better in certain environments than others. Therefore, choosing the best approach in each situation is critical for obtaining good quality results. In this subsection, a discussion on the different fuzzy multi-granular modellings is presented in order to provide the user a brief advice of what method should be chosen depending on the problem and the quality of results that the user expects to obtain.

In multi-granular fuzzy linguistic modelling based on fuzzy membership functions, operations are made with semantics associated to the labels and not in a symbolic way. In this case, all the labels are translated into fuzzy numbers and operations are made using them. This model, although it reduces complexity and does not have any problems of loss of information, needs a defuzzification process if the results want to be expressed by means of linguistic labels instead of numerical values. The complexity of this process depends directly on the output fuzzy sets. Thus, in some cases, it can become difficult to assign a label to a fuzzy number.

In the ordinal multi-granular fuzzy linguistic modelling based on a basic linguistic term set, all the labels of the term sets are expressed using the labels of an unique term set. A good characteristic of this methodology is that, if a accurate result is looked for, there is no loss of information. Although linguistic terms of the BLTS are used in order to express results, they just describe a fuzzy set defined using those linguistic terms. Methods enclosed in this category do not show results using labels in a symbolic manner.

In ordinal multi-granular fuzzy linguistic modelling based on 2-tuple and Linguistic Hierarchies, methods use a combination of linguistic hierarchies and 2-tuple representation model. 2-tuple linguistic modelling has the capability of representing elements within a linguistic term set when they do not fit to any linguistic term. This is a great advantage in multi-granular information representation because expressions that allow a label to be expressed in terms of labels of another linguistic term set can be defined. A hierarchy where each level represents a linguistic term set is defined. Expressions to translate elements from one level to another one are also defined. Methods that use hierarchies use linguistic terms and they are capable of expressing them in terms of any other linguistic term set of the hierarchy. Thus, any linguistic term set that belongs to the hierarchy can be used in order to resolve the problem. Furthermore, 2-tuple linguistic model has the capability of expressing results using labels of a term set although the result does not match to any of them. Therefore, there is no loss of information in computations. Nevertheless, results are expressed in a half-linguistic way because the result is a 2-tuple linguistic value, not a label of the original term set. In such a way, if only the associated label of the 2-tuple is provided, loss of information is produced because that label alone is not the obtained result.

Ordinal multigranular fuzzy linguistic modelling based on hierarchical trees uses trees in order to give a way of translating linguistic terms from one linguistic term set into labels of other linguistic term set of the tree. This method is totally symbolic, it only uses the labels from the term sets to make the uniforming computations making it a good method to use in environments where no precise information is given, that is, with vague information. Nevertheless, it has two main drawbacks:

1. There is loss of information. When translating labels from one level to another one, if the target level has less granularity, several labels can be translated into the same label. If the target level has more granularity, it is possible that the translation derives in a set of labels instead of one. Therefore, methods that are able to work with sets of labels should be used in this case.

2. The decision making method that has been designed for its use is not symbolic. Thus, the ranking is made calculating numerical values.

In multi-granular fuzzy linguistic modelling based on qualitative description spaces methodology, general description spaces are used in order to represent the information. The main advantage is that description spaces have more representation capability than normal linguistic term sets making them able to model complex situations. Nevertheless, this method can become inefficient if linguistic term sets with high granularity and a high amount of features are used because, in these situations, the used trees in the decision making algorithm can become enormous and distances are difficult to calculate. Furthermore, multi-granularity is applied over the alternatives and not on the experts. Therefore, experts cannot choose any linguistic term set to use.

In ordinal multi-granular fuzzy linguistic modelling based on discrete fuzzy numbers, discrete fuzzy numbers are used to define a novel multi-granular fuzzy linguistic modelling approach that uses hierarchies. Its main advantage is that the methodology works exclusively using discrete fuzzy numbers environment allowing it to carry out operations in a symbolic way. The main drawback about this methodology is that, in translations from low granularity linguistic term sets to high granularity ones, information is estimated.

Table 16 summarizes all the techniques used for representing multi-granular information and shows their respective advantages and drawbacks. Furthermore, features of each of them are exposed in Table 17.

Technique	**Advantages**	**Drawbacks**

MFLM based on fuzzy membership functions	• There is no loss of information. • They are flexible models that allow a wide range of computations types.	• Semantics have to be associated to the labels in order to make computations. • Computations results are not given using linguistic labels. • Some mathematical models can become complex.
FLM based on a Basic LTS	• There is no loss of information. • It is possible to choose the LTS used to uniform the information.	• Difficulties in expressing the operations results using linguistic labels. • Operations are carried out using the fuzzy sets mathematical environment.

MFLM based on 2-tuple and Linguistic Hierarchies	▪ There is no loss of information. ▪ LHs allow you to choose the target LTS for carrying out the computations. ▪ Results can be expressed linguistically.	▪ It carry out computations in a semantic way using numbers. ▪ When using LHs, only LTSs from the hierarchy can be chosen. ▪ When not using LHs, results that do not belong to any of the labels of the LTSs can be obtained.
MFLM based on hierarchical trees	▪ No association of semantics is required. ▪ Hierarchical trees are more flexible than hierarchies. Any LTS is accepted. ▪ The model is simple.	▪ There is loss of information. ▪ One label can have several translations into another level of the hierarchical tree.

MFLM based on qualitative description spaces	- No loss of information. - It works symbolically. - Great representation capability.	- Complex model. - Experts cannot decide which LTS to use.
MFLM based on discrete fuzzy numbers	- It is a symbolic approach. - Results are given using linguistic labels. - All the operations are carried out using labels ordering. - Any LTS can be used. - No loss of information is produced if transformations are done into a higher granularity scale.	Estimated information is used.

Table 16: Advantages and drawbacks of the multi-granular fuzzy linguistic modelling approaches.

Current methods focus their attention especially on the six considered

Technique	**Refs**	**Loss of data**	**Repr. type**	**Complexity**	**Set restrictions**	**Results in input sets**
MFLM based on fuzzy membership functions	[JFM08] [ZG12]	No	Semantic	Medium	Medium	No
FLM based on a Basic LTS	[CBA06] [HHVM00] [Xu09]	No	Semantic	Medium	Medium	No
MFLM based on 2-tuple	[ELM11][HM01b] [Zha12]	No	Symbolic	Low	High	Yes
MFLM based on Hierarchical trees	[HN05]	Yes	Symbolic	Low	Low	Yes
MFLM based on description spaces	[RSA+11]	No	Symbolic	High	Low	Yes
MFLM based on discrete fuzzy numbers	[MRTHV14]	No	Symbolic	Low	Low	Yes

Table 17: Comparative about techniques used for dealing with multi-granular information. MFLM refers to multi-granular fuzzy linguistic modelling and LTS to linguistic label. set.

categories. Nevertheless, there are several techniques that can be used in order to develop new linguistic multi-granular decision making models as for example:

1. *Hesitant Fuzzy sets.*
2. *Type-2 fuzzy sets.*

3.4.1. Fuzzy Linguistic Multi-granular Modelling based on Hesitant Fuzzy Linguistic Sets

Torra [Tor10] introduced a new extension for fuzzy sets to manage those situations in which several values are possible for the definition of a membership function of a fuzzy set. Though this situation might be modelled by fuzzy multisets, they are not completely suitable for these situations.

A HFS is defined in terms of a function that returns a set of membership values for each element in the domain. Let X be a reference set, a hesitant fuzzy set on X is a function h that returns a subset of values in $[0, 1]$.

$$h : X \to \{[0, 1]\} \tag{37}$$

Therefore, given a set of fuzzy sets, a hesitant fuzzy set is defined as the union of their membership functions.

Let $M = \{\mu_1, \mu_2, \ldots, \mu_n\}$ be a set of n membership functions. The hesitant fuzzy set associated with M, h_M, is defined as:

$$\begin{aligned} h_M &: M \to \{[0, 1]\} \\ h_M(x) &= \bigcup_{\mu \in M} \{\mu(x)\} \end{aligned} \tag{38}$$

It should be noticed that hesitant fuzzy sets are similar to uncertain fuzzy sets. Their main difference consists in that a hesitant fuzzy set is a set of membership functions and an uncertain linguistic fuzzy set is an interval. Given a hesitant fuzzy set, h, its lower and upper bounds are:

$$\begin{aligned} h^-(x) &= \text{mín}\, h(x) \\ h^+(x) &= \text{máx}\, h(x) \end{aligned} \tag{39}$$

It should be pointed out that $h^-(x)$ and $h^+(x)$ can define an uncertain linguistic fuzzy set.

HFSs have been applied successfully in decision making field [TN09, Wei12, XX11]. However, no multi-granular fuzzy linguistic modelling have been yet designed.

Let S be a linguistic term set, $S = \{s_0, \ldots, s_g\}$, an Hesitant Fuzzy Linguistic Term Set (HFLTS), H_S, is an ordered finite subset of consecutive linguistic terms of S [RMH12].

Let S be a linguistic term set, $S = \{s_0, \ldots, s_g\}$, the empty HFLTS and the full HFLTS for a linguistic variable ϑ, are defined as follows:

- Empty HFLTS: $H_S(\vartheta) = \{\}$.
- Full HFLTS: $H_S(\vartheta) = S$.

Any other HFLTS is formed with at least one linguistic term in S. For example, for the linguistic term set $S = \{s_0 : nothing, s_1 : very\_low, s_2 : low, s_3 : medium, s_4 : high, s_5 : very\_high, s_6 : perfect\}$, a possible HFLTS can be:

$$H_S(\vartheta) = \{s_1 : very\_low, s_2 : low, s_3 : medium\} \tag{40}$$

Let S be a linguistic term set, $S = \{s_0, \ldots, s_g\}$ and H_S a HFTS:

- The upper bound, H_{S^+}, and the lower bound, H_{S^-}, of the HFLTS, H_S are defined as:

$$H_{S^+} = \text{máx}\{s_i | s_i \in H_s\} \tag{41}$$

$$H_{S^-} = \text{mín}\{s_i | s_i \in H_s\} \tag{42}$$

- The envelope of the HFLTS, $env(H_S)$, is a linguistic interval whose limits are obtained by means of upper bound (máx) and lower bound (mín), hence:

$$env(H_S) = [H_{S^-}, H_{S^-}], H_{S^-} \leq H_{S^+} \tag{43}$$

HFLTS can be generated using a context-free grammar, G_H. These grammars are close to the linguistic structures used by human beings to provide their assessments in real world problems where they are not sure about one single value to assess the criteria of the alternatives. Therefore, the hesitant situation is modelled by means of linguistic structures generated by the production rules, $P \in G_H$ being necessary to model semantically such information. To do it, the use of HFLTS is proposed.

HFLTSs allow the use of several labels in order to provide the information avoiding the necessity of choosing just one. Furthermore, it is possible to attach them a context-free grammar making the communication between human beings and the computer became better because a language close to the natural one is used. HFTLSs have a strong mathematical environment that let us operate between different sets. Thus, it would be possible to create a multi-granular modelling that inherit all the advantages that this type of sets brings us.

3.4.2. Fuzzy Linguistic Multi-granular Modelling based on Type-2 Fuzzy sets

A type-2 fuzzy set, denoted $\widetilde{A}$ is characterized by a type-2 membership function $\mu_{\widetilde{A}}(x, u)$, where $x \in X$ and $u \in J_x \subseteq [0, 1]$ [MJ02], i.e.

$$\widetilde{A} = \{((x, u), \mu_{\widetilde{A}}(x, u)) | \forall x \in X, \forall u \in J_x \subseteq [0, 1]\} \tag{44}$$

in which $0 \leq \mu_{\widetilde{A}}(x, u) \leq 1$. $\widetilde{A}$ can also be expressed as

$$\widetilde{A} = \int_{x \in X} \int_{u \in J_x} \mu_{\widetilde{A}}(x, u)/(x, u) \;\; J_x \subseteq [0, 1] \tag{45}$$

where $\int \int$ denotes union over all admissible x and u. For discrete universes of discourse, $\int$ is replaced by $\sum$. In expressions (44) and (45), the restriction ($\forall u \in J_x \subseteq [0, 1]$) is consistent with the type-1 constraint that $0 \leq \mu_A(x) \leq 1$ i.e., when uncertainties disappear, a type-2 membership function must reduce to a type-1 membership function, in which case $0 \leq \mu_A(x) \leq 1$. This restriction is consistent with the fact that the amplitudes of a membership should lie between or be equal to 0 and 1.

Uncertainty in the primary memberships of a type-2 fuzzy set, $\widetilde{A}$, consists of a bounded region that is called *footprint of uncertainty* (FOU). It is the union of all primary memberships, i.e.,

$$FOU(\widetilde{A}) = \bigcup_{x \in X} J_x \tag{46}$$

When the third dimension of a type-2 fuzzy set is always set to one or zero, the type-2 fuzzy set is called *interval type-2 fuzzy set.* General type-2 fuzzy sets are computationally intensive. Things simplify a lot when secondary membership functions are either zero or one [LM00].

In [Yag80], Yager gives an example of how to define a fuzzy subset of type-2 using linguistic labels in the secondary membership function.

Using the extension principle, the logical operations of *and*, *not* and *or* was extended to fuzzy sets of type-2 by Mizumoto and Tanaka [MT76].

Mendel, on [MJ02], explains that type-2 fuzzy sets, because they not have a crisp membership value, have more representation capability than type-1 fuzzy sets. Type-2 fuzzy sets have a strong mathematical environment that let us operate with them. Thus, if labels whose associated semantics are represented by this type of sets are used, it is possible to create a multi-granular modelling that allow us to take advantage of all the representation benefits of type-2 fuzzy sets.

In the recent literature, some applications of the interval type-2 fuzzy sets in the decision making field are available [CL10, OG04]. However, no multi-granular fuzzy linguistic modellings have been designed.

4. A decision support system for decision making in changeable and multi-granular fuzzy linguistic contexts

4.1. Introduction

A decision making process consits in selecting, among a set of alternatives, the one that is considered to be the one giving the best profit. Decision Making is part of our everyday life and is a critical field of study in areas such as operations research [XLB$^+$13, ZKK$^+$14], politics [HKPV13, RHBK13], social psychology [CBS13, KPA13], artificial intelligence [CLN13, DRCLCF14] and soft computing [PCAHV14, PWM+13]. When decisions are not made by an unique person but, instead, by a group of experts, it is called Group Decision Making [CUPHV14, CTGdMHV13, Kac86, MMHV09, WC14b, WC14c].

Fuzzy set theory [Zad65] and the fuzzy linguistic modelling [Zad75a, Zad75b, Zad75c] have become really useful for researchers in the creation of approaches that help experts to communicate with the decision support systems (DSSs) in a userfriendly way. In particular, multigranularity fuzzy linguistic modelling methods [HHVM00, MRTHV14, MPZC14] ease the expert-system communication by allowing experts to choose the linguistic term set that better fit them and providing the system with tools that allow it to manage the heterogeneous received information. Having a flexible and comfortable way for experts to provide their preferences is critical because the better the experts can express themselves, the more reliable information is received in the system and, consequently, the more accurate and trustful results are obtained in the Group Decision Making process. In such a way, experts that have a wide knowledge of the problem, can choose an linguistic term set that let them be very specific in the providing information step. On the other hand, experts that want to provide less specific information, can choose linguistic term sets with less labels that let them provide information with a higher level of uncertainty.

DSSs [AO14, AHVCH10] aim is to provide suitable information and assistance to experts in the decision making process. Long ago, when Internet has just appeared, information was static and Internet users were only able to consult and retrieve the information from certain specific data websites. Websites were only managed by a small set of people who possessed all the information. Thanks to mobile technologies [Gog12] and Web 2.0 technologies [APCHV12], information is now provided and consumed worldwide by everyone who want to participate in this process. The amount of information available has increased exponentially and it is possible to surf the Internet regardless of the place and time. Consequently, traditional DSSs have to be enhanced and adapted to this new paradigm.

When experts are involved in a Group Decision Making process but they are not located in the same place, it is necessary to include approaches that help them to reach a consensus. Consensus measures [CMPHV10, HVCKP14, WC14a] allow experts to have a clear idea of how the decision making process is going on. They can, for example, obtain information about the percentage of experts that prefer each alternative, if a final solution has been reached or, on the contrary, everyone is selecting different alternatives. Thanks to this information, experts can debate and center the discussion among their disagreements making it able for the debate to move forward in order to reach an overall agreement.

In this chapter, we present a new linguistic DSS that is designed to work in changeable environments where the experts and alternatives available can be changed at anytime, and assuming multi-granular linguistic information and multiple types of preference representation formats to represent the preferences provided by the experts. Furthermore, this DSS allows experts to make decisions independently of their locations using their smartphones. We introduce new consensus methods to address the consensus reaching process in such a decision contexts.

The chapter is organized as follows. In subsection 4.2, the designed DSS along with its characteristics and architecture is presented. In subsection 4.3, an application example is showed. Finally, in subsection 4.4, drawbacks and advantages of the system are discussed.

4.2. Decision Support System description and architecture

Typically, DSSs have been implemented over desktop computers, but in this subsection, we present a DSS designed for its use in smartphones, and therefore, it can be accessed from anywhere and anytime. Expert preferences are provided using multiple preference representation formats (ordering, utility functions and linguistic preference relations) assessed with multi-granular linguistic information in order to ease the way that experts express their opinions. Furthermore, in order to deal with changeable decision contexts, DSS allows new alternatives to be introduced while the discussion is carried out. Moreover, experts can abandon and enter the discussion in the middle of the Group Decision Making process.

In subsection 4.2.1, the processes implemented by the DSS are explained. In subsection 4.2.2, the architecture that can be used to build the system is exposed. Finally, in subsection 4.2.3, the DSS workflow is showed and explained.

4.2.1. Processes of the DSS

The processes that the designed DSS has to develop in order to carry out its support activity are the following:

1. **Uniforming**: Given that the expert preferences can be provided by means of multiple representations and, in any cases, by using multi-granular linguistic information, then two uniforming processes are developed in the DSS:

- *Uniforming the multi-granular linguistic preference relations*: If experts have chosen different linguistic term sets in order to provide their linguistic preference relations, i.e. they use multi-granular linguistic preference relations, it is necessary to carry out a uniformization process. We use the 2-tuple multi-granular fuzzy linguistic modelling defined in [HM01b] to uniform the linguistic information. As aforementioned in chapter 3, the method uses the 2-tuple representation [HM00, HM01a]. Furthermore, it uses several linguistic term sets in order to build a linguistic hierarchy and establish rules and transformation functions among the different levels of it. In such a way, any label belonging to any linguistic term set of the hierarchy can be expressed using labels of any other hierarchy belonging linguistic term set. Once that all the labels provided by the experts are expressed using the same linguistic term set, it is possible to operate with them.

- *Uniforming the rest of preference representations*: Among all the possible preference representations (preference orderings, utility functions and linguistic preference relations), the linguistic preference relations are chosen as the target ones. If experts have selected preference orderings or utility functions to provide their preferences, then the following transformation functions can be used for uniforming the preferences [CHHV98]:

  - preferences orderings:

$$p_{ij} = f^1(o_i^k, o_j^k) = \Delta\left(\frac{1}{2}\left(1 + \frac{\Delta^{-1}(o_j^k) - \Delta^{-1}(o_i^k)}{n-1}\right)\right) \qquad (47)$$

  where p_{ij} represents the obtained linguistic preference value for alternative x_i over alternative x_j,:

$$\Delta : [0, g] \rightarrow S \times [-0{,}5, 0{,}5)$$
$$\Delta(\beta) = (s_i, \alpha) \ with \begin{cases} s_i & i = round(\beta) \\ \alpha = \beta - i & \alpha \in [-0{,}5, 0{,}5) \end{cases} \tag{48}$$

and

$$\begin{aligned} &\Delta^{-1} : S \times [-0{,}5, 0{,}5) \rightarrow [0, g] \\ &\Delta^{-1}(s_i, \alpha) = i + \alpha = \beta \end{aligned} \tag{49}$$

Example 2. Let $\{s_2, s_1, s_3\}$ be a preference ordering associated to the alternatives set $\{x_1, x_2, x_3\}$. The preference relation associated values are calculated as follows:

$$\begin{aligned} &\Delta^{-1}(\{s_2, s_1, s_3\}) = \{2, 1, 3\} \\ &p_{12} = \left(\frac{1}{2}\left(1 + \frac{1-2}{2}\right)\right) = 0{,}25 \\ &p_{13} = \left(\frac{1}{2}\left(1 + \frac{3-2}{2}\right)\right) = 0{,}75 \\ &\ldots \\ &p_{32} = \left(\frac{1}{2}\left(1 + \frac{1-3}{2}\right)\right) = 0 \end{aligned}$$

After calculating all the values of p, the following matrix is obtained:

$$P = \begin{pmatrix} - & 0{,}25 & 0{,}75 \\ 0{,}75 & - & 1 \\ 0{,}25 & 0 & - \end{pmatrix}$$

Readers should notice that results are expressed in the numerical interval [0,1]. If another range value is being used, values must be converted into it. Δ operator is applied after that if linguistic results are desired.

- utility functions:

$$p_{ij} = f^2(u_i^k, u_j^k) = \Delta\left(\frac{\Delta^{-1}(u_i^k)^2}{\Delta^{-1}(u_i^k)^2 + \Delta^{-1}(u_j^k)^2}\right) \tag{50}$$

where utilities values must be expressed using an linguistic term set whose lowest linguistic value have an index value of 1.

Example 3. Let $\{x_1, x_2, x_3\}$ be a set of alternatives whose associated utility function values are $\{s_5, s_3, s_1\}$ from a balanced linguistic term set whose granularity value is 5. The preference relation associated values are calculated as follows:

$$\Delta^{-1}\{s_5, s_3, s_1\} = \{5, 3, 1\}$$
$$p_{12} = \frac{5^2}{5^2 + 3^2} = 0{,}73$$
$$p_{13} = \frac{3^2}{5^2 + 3^2} = 0{,}26$$
$$\ldots$$
$$p_{32} = \frac{1^2}{1^2 + 3^2} = 0{,}1$$

The preference relation matrix obtained is shown below:

$$P = \begin{pmatrix} - & 0{,}73 & 0{,}96 \\ 0{,}26 & - & 0{,}9 \\ 0{,}03 & 0{,}1 & - \end{pmatrix}$$

As in Example 2, values are expressed in the interval [0,1].

2. **Consensus measuring**: DSS supports the open debate among experts and aids to the experts to achieve an agreement before making a final decision by means of consensus measures. The level of consensus is measured in order to provide information to the experts about how the decision making process is going on. Distances among the preferences provided by the different experts can be used for this purpose. Two different consensus measures are calculated [CMPHV10]:

- *Alternative level*: It measures the consensus reached in each alternative. We use this measure to identify those alternatives that present a minor level of agreement among experts.
- *Global level*: It measures the global level of consensus among the experts. When this value is considered high enough, decision making process can end and a final decision is made.

3. **Selection**: When a high level of consensus is reached then we can apply a selection process to identify the solution alternatives. A selection process presents two steps [HACHV09]:

   - *Aggregation*: All the individual preferences provided by the experts, p^k, $\forall k, k \in [1, m]$, are aggregated into a collective preference matrix, P, that represents all the information recollected. It indicates the preference of all the experts for the alternatives two-by-two. For example, we could use OWA operators [Yag88] or induced OWA operators [CHVHA07] for carrying out this task.
   - *Exploitation*: Using the information contained in P, the ranking of the alternatives is made. This process can be carried out using some of these choice degrees of alternatives [CTGdMHV13]: *quantifier-guided dominance degree* and *quantifier-guided nondominance degree*. These choice degrees can be applied over the matrix P in order to obtain a final ranking of alternaives, i.e, the solution alternatives X_{sol}.

4. **Dynamic Choice of Alternatives and Experts**: Although classical Group Decision Making frameworks present a static number of experts and alternatives, in real world problems there can be changeable decision situations where the number of experts or alternatives could change throughout the decision making process. For example, in medicine field when the experts

are trying to elucidate the most appropriate treatment for a certain patient using his/her symptoms, if a new symptom appears, it is possible that new alternatives should be considered. Also the number of experts could vary, because if new symptoms are detected maybe new doctors would have to be invited to the expert panel to provide new knowledge to process the new symptoms. Then, the new DSS implements the following two processes to deal with changeable aspects of the Group Decision Making problems:

a) *Dynamic choice of alternatives*: If one of the experts considers that some alternative/s must be removed or aggregated, he/she indicates it to the system. Afterwards, the proposal is voted by all of the experts involved in the Group Decision Making process. If a majority is reached, then the set of alternatives is changed into the new one. Next, if new alternatives are added, experts are asked to introduce their preferences values related with the new alternatives. If alternatives are deleted, preferences values related to the alternatives are removed from the preference matrix of each expert. The Group Decision Making process continues using the new set of alternatives.

Example. Let $E = \{e_1, e_2, e_3\}$ be a set of experts and $X = \{x_1, x_2, x_3\}$ be a set of alternatives. Then, using the linguistic term set $S = \{VL, L, M, H, VH\}$ e_1 provided the reference values matrix

$$P_1 = \begin{pmatrix} - & VL & M \\ VL & - & H \\ VL & VL & - \end{pmatrix} \tag{51}$$

Imagine that a new alternative x_4 appears and that all experts agree to include it in the decision making process. In such a way, experts must fill the values in the preference matrix that are indicated by value ?, as for example it happens in

$$P_1 = \begin{pmatrix} - & VL & M & ? \\ VL & - & H & ? \\ VL & VL & - & ? \\ ? & ? & ? & - \end{pmatrix} \tag{52}$$

On the other hand, if the alternative x_2 is considered unnecessary for experts, values related to it should be deleted in all expert preferences, and the preference matrix given in (51) would be simplified as

$$P_1 = \begin{pmatrix} - & M \\ VL & - \end{pmatrix} \tag{53}$$

b) *Dynamic choice of experts*: Also it is possible that the set of experts that are involved in a Group Decision Making process varies throughout Group Decision Making process. The, if an expert wants to be added to the Group Decision Making process, the rest of experts that are already involved should decide on his/her incorporation into the decision process. If a majority is reached, the new expert is now part of the Group Decision Making and he/she can provide his/her own preference values. If an expert wants to abandon the Group Decision Making process, his/her preference values are deleted, collective preference matrix recalculated and Group Decision Making process continues without considering the preferences provided by that expert.

5. **Feedback Process:** DSS implements a feedback process in order to promote consensus to achieve decisions unanimously. With such process the experts can receive a set of suggestions of how to modify his/her preferences in order to increase the overall consensus level. This feedback process is carried out as follows:

- *Calculation of proximity values* [CMPHV10]: The collective preference relations are calculated, and then, we calculate the distance of each expert to the collective preference values. These distances are called proximity values, P_x, and measure the distance of each of the expert to the global consensus opinion.
- *Selection of outlier experts*: Using P_x and a threshold value λ, experts whose distance to consensus is higher than λ are selected in order to provide them several suggestions of how to increase the global consensus level.
- *Providing suggestions*: If the proximity value is positive for a given alternative x_i, expert is asked to decrease the preference values related to it. If the value is negative, the expert is asked to increase it.

4.2.2. DSS architecture

In the previous subsection, the processes carried out by the DSS have been explained. In this one, how the DSS together with its processes can be implemented and software and hardware elements needed for a proper working are exposed.

Although smartphones technology has improved a lot these last years, mobiles are not yet capable of carrying out hard computing tasks. Nevertheless, because client-server architecture [Ber96] can be used, this issue has not prevented the creation of complex mobile applications. The main advantage of client-server architecture is that high intensive computing tasks are carried out in a server while the client is only used to provide data and show results to the users. Client-server architecture also supports the sharing of information among several clients using the server as the computation and data distribution core. Consequently, it is the perfect architecture choice for our DSS.

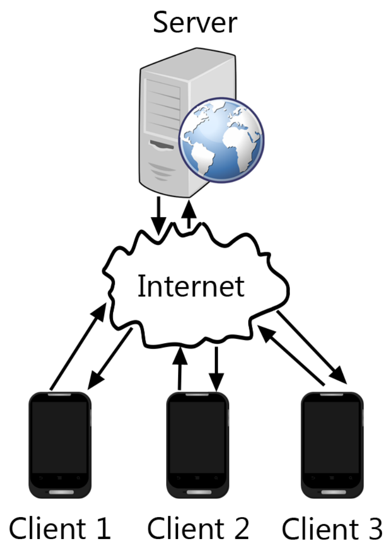


Figure 16: Client-server architecture with 3 clients.

As it can be seen in Figure 16, there is one client per expert and a main server that process the information and carry out all the Group Decision Making computations needed. The client program is installed in the smartphones while the server is located in a static computer with Internet access. Characteristics and how each part works are described below:

- **Client:** The duty of the client part of the DSS is to support the system-user interactions, i.e, to receive the user preferences, to provide the suggestions to the users, and to show the results in each phase of the Group Decision Making process. No further computation is needed because all the required data will be sent to, or received from, the server that will be in charge of computations. For the implementation of the client, either Android or IOS platforms (or both) can be used. The following eight interfaces must be implemented:

  - *Connection*: Device checks if connection to the server is possible. If it is not, it informs the expert.

- *Authentication*: In order to avoid stolen identity issues, the expert is asked to enter a username and a password. Digital certificates can be used in order to improve the system security and guarantee the user identity.
- *Group Decision Making process selection*: It is used to select the desired Group Decision Making process.
- *Problem description:* Description of the problem to solve and possible alternatives are showed to the expert.
- *Selection of preferences representations*: Experts select the means that they want to use to express themselves.
- *Insertion of preferences:* Shows an interface that can be used by experts to provide their preferences.
- *Change of alternatives:* From this interface users can request the addition or deletion of one or several alternatives.
- *Change of experts:* From this interface the expert can abandon the Group Decision Making process or request for a new expert to be added to the Group Decision Making process.
- *Feedback*: Feedback interface provides information about how experts can modify their opinions in order to reach a consensus.
- *Group Decision Making results*: this interface shows final Group Decision Making results.

- **Server:** The server is composed by a set of modules which are in charge of carrying out all the intensive computations of the Group Decision Making process. The server can be implemented using any server scripting language like PHP [GBR04] or JSP [HL03]. The server is composed of the following modules:

- *Uniforming information module:* This module is in charge of transforming all the provided information into the same representation model.
- *Selection module:* The selection module is in charge of carrying out the selection process.
- *Consensus module:* This module is in charge of calculating the consensus measures for the Group Decision Making process.
- *Dynamic choice of alternatives module:* This module is in charge of managing all the alternatives changing petitions.
- *Dynamic choice of experts module:* This module manages all the experts adding and abdication petitions.
- *Feedback module:* This module generates the recommendation rules that are showed to the experts in order to suggest them several ways of reaching a higher consensus.
- *Database module:* The purpose of this module is to store all the data generated in the Group Decision Making process, as for example, experts involved in each Group Decision Making process, preferences provided, alternatives, Group Decision Making problem information and final/temporary results.

A scheme of the interfaces and modules of the client and the server and how they interact among each others can be seen in Figure 17.

4.2.3. DSS workflow

In this subsection, the workflow of a DSS for Group Decision Making problems is exposed. Steps followed are showed below:

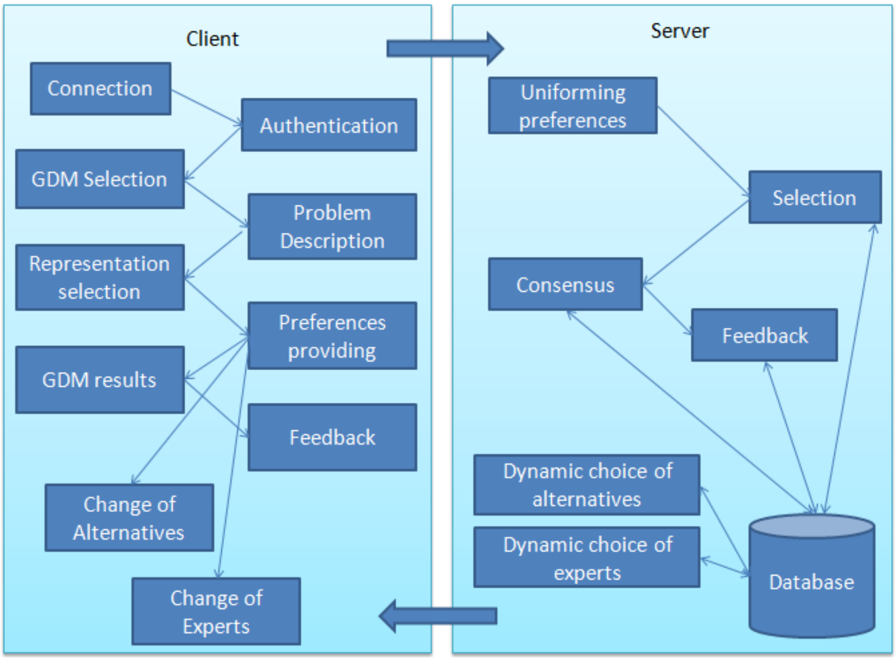


Figure 17: Modules and interfaces from the DSS client-server architecture and its connections.

1. **Initialization**: Initial values of all the parameters needed for carrying out the Group Decision Making process are set and stored in the database. In such a way, initial experts involved in the decision with their usernames and passwords, possible alternatives, Group Decision Making information and maximum number of Group Decision Making rounds until calculating a final result (MAXROUND variable) are set and stored in the database. A proper MAXROUND value is critical in the Group Decision Making process because, if it is too low, it is possible that experts cannot reach a consensus before calculating the final result. Nevertheless, if MAXROUND is too high, then Group Decision Making process can last too long. Another interesting parameter that is initialized in this step is the MAXWAIT variable. Its purpose is to establish a maximum limit time in order for experts to provide their preferences. Experts that do not provide their preferences in a time less

than MAXWAIT are excluded in the Group Decision Making process round. This variable can be discarded if all expert preferences are required. Experts excluded by MAXWAIT variable can join the process later. For example, if an experts is having trouble with his/her connection, the Group Decision Making process continues and he/she can join later when his/her connection problem is solved. Nevertheless, MAXWAIT variable value should be long enough in order for allowing experts to solve their connection issues and participate. The initialization process is carried out by one of the experts involved in the Group Decision Making process.

2. **Sending preferences**: Once that the Group Decision Making process is initiated, experts can log in, select the Group Decision Making process among the Group Decision Making processes list, read the problem description and provide their preferences over the alternatives using the representation method and linguistic term set that they prefer. All this information is sent from the client device to the server in order to be stored in the database.

3. **Uniforming preferences**: All the information provided by the experts are uniformed in the DSS. This process is made in two phases:

   - Preferences provided in a different representation approach than linguistic preference relations are transformed into them using the transformation functions exposed in 4.2.1.
   - Labels belonging to different linguistic term sets are all expressed with a unique linguistic term set using the procedure described in 3.3.3.

4. **Computation of temporary Group Decision Making solution**: Once that all the preferences have been homogenized, selection process is carried out and temporary Group Decision Making results calculated. These results

are stored in the database and sent to the client devices in order to be shown to the experts.

5. **Computation of consensus measures**: Consensus measures are calculated and stored in the database. They are also sent to client devices in order to be shown to the experts. Number of rounds variable is increased.

6. **Group Decision Making status control**: Global consensus value and MAXROUND variable are consulted. If the number of rounds carried out is higher than MAXROUND value or global consensus value is above some predefined threshold α, then the Group Decision Making process is finished and temporary results calculated in step 4 become the final Group Decision Making results.

7. **Generating recommendations**: If the Group Decision Making process must continue one more round, recommendations are generated using proximity values as exposed in subsection 4.2.1. A personal recommendation is generated for each expert and sent to its mobile device. They are not forced to follow any suggestion. Therefore, recommendations can be followed or discarded.

8. **Go to step 2**.

The main workflow can be seen schematically in Figure 18. Optionally, several sub-processes can be started at the preferences sending step of the Group Decision Making process by any expert. Group Decision Making process pauses until the request has been processed. After resolving it, experts continue providing their preferences and Group Decision Making process continue as normal. The two possible subprocesses that can provoke this situation are exposed below:

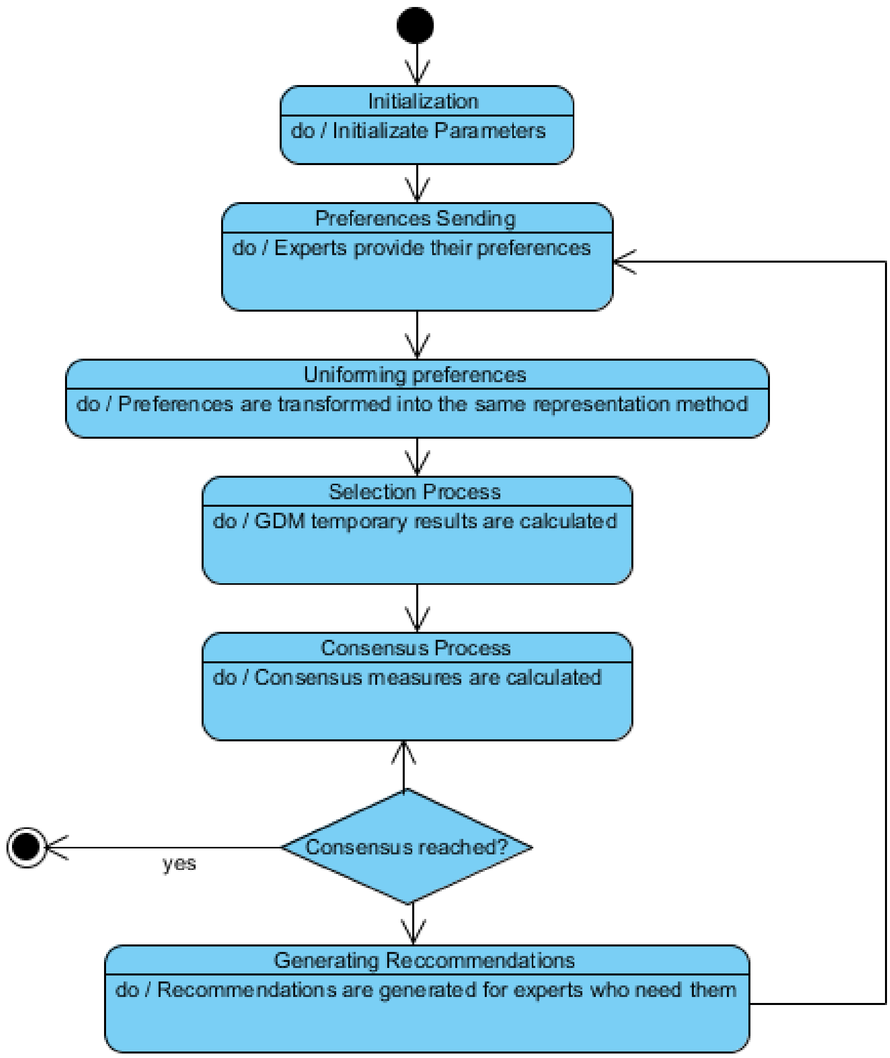


Figure 18: Main workflow state diagram.

1. **Alternatives changing request**: If an expert considers that a new alternative should be added or one of the actual alternatives should be removed, he/she indicates it to the system. The client device sends a request to the server. When the server receives it, it stops the Group Decision Making process and sends a notification to all the experts in order to develop the voting process. The server receives all the expert opinions and, if a majority vote is reached, then the change is made. On the contrary, if most of the experts disagree with the request of alternatives change, the server notifies that the change has not been made and Group Decision Making process restarts without any modification of parameters.

2. **Experts changing requests**: Two different requests can be made:

   - *New expert request:* Experts can suggest new experts to be added to the Group Decision Making process. The expert sends through the client a new expert adding request. When the request is received by the server, the Group Decision Making process is stopped and a request notification is sent to all the experts. The experts vote on the proposal and preferences are sent to the server. If an agreement is reached, the expert is added to the Group Decision Making process. On the contrary, if a majority of the experts disagree, the Group Decision Making process is continued without any change.
   - *Abdication request:* If an expert wants to abandon the Group Decision Making process, he/she indicates it to the server. This server revokes the granted permissions to the expert and his/her preferences are deleted. He/She can not longer access the Group Decision Making process unless another expert asks for his/her inclusion.

Optional requests workflow can be seen schematically in Figure 19.

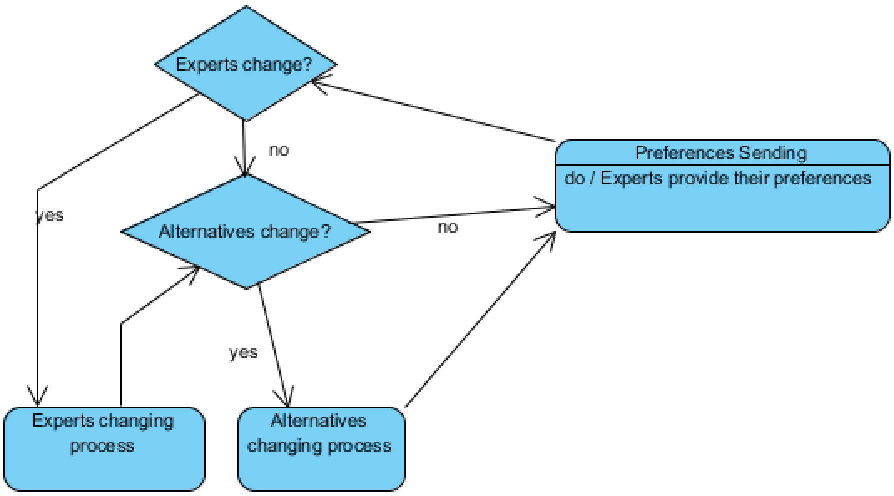


Figure 19: Optional request workflow state diagram.

4.3. Use Example

In this section, an use example of the designed DSS is showed. Let $E = \{e_1, e_2, e_3\}$ be a set of experts that are teachers in a high school and have to decide where to invest a certain amount of money. Initially, three alternatives are discussed: $X = \{x_1 :$ new computers in the computer science room, $x_2 :$ interactive whiteboards in several classrooms, $x_3 :$ build a new football field$\}$. The three experts are registered in the server database and variables initialized. MAXROUND variable is set to 5 and α value managing the consensus threshold is set to 0.8. MAXWAIT variable is not taken into account because it is considered mandatory that the three experts vote.

e_1 decides to provide his/her preferences using utility values while experts e_2 and e_3 prefer linguistic preference relations, but in a multi-granular linguistic context, i.e., e_1 and e_2 decide to use the balanced linguistic term set S_1 for expressing their preferences while e_3 prefer S_2. Both linguistic term sets are exposed below:

$$S_1 = \{s^1_0, s^1_1, s^1_2, s^1_3, s^1_4\}$$
$$S_2 = \{s^2_0, s^2_1, s^2_2, s^2_3, s^2_4, s^2_5, s^2_6, s^2_7, s^2_8\}$$

The provided preferences in the first round are showed below:

$$P_1 = \{s^1_3, s^1_1, s^1_0\}$$
$$P_2 = \begin{pmatrix} - & s^1_2 & s^1_0 \\ s^1_0 & - & s^1_2 \\ s^1_4 & s^1_3 & - \end{pmatrix}$$
$$P_3 = \begin{pmatrix} - & s^2_3 & s^2_5 \\ s^2_7 & - & s^2_8 \\ s^2_6 & s^2_2 & - \end{pmatrix}$$

Before any further computation, information must be unified. First, preferences that are not expressed using linguistic preference relations will be transformed into them. In the example, e_1 is the only expert using another representation approach. Applying one of the transformation functions exposed in subsection 4.2.1, a linguistic preference relation can be obtained. Since the transformation function does not work with linguistic term sets that have labels denoted as 0, first, computations will be held using the equivalent balanced linguistic term set $S^1_{1-5} = \{s^{1'}_1, s^{1'}_2, s^{1'}_3, s^{1'}_4, s^{1'}_5\}$. Since the resulting preference matrix values are expressed in the interval [0,1], transformation operations will be performed in order to express results in the interval [0,4], that is, the one where the linguistic term set S^1 is defined. Therefore, P_1 can be expressed as:

$$P^{[1,5]}_1 = \{s^{1'}_4, s^{1'}_2, s^{1'}_1\}$$

After this transformation, equation (50) can be applied satisfactorily over the utility values provided by e_1 as follows:

$$\Delta^{-1}\{s_4^{1'}, s_2^{1'}, s_1^{1'}\} = \{4, 2, 1\}$$
$$p_{12} = \frac{4^2}{4^2 + 2^2} = 0{,}8$$
$$p_{13} = \frac{4^2}{4^2 + 1^2} = 0{,}9411$$
$$\ldots$$
$$p_{32} = \frac{1^2}{1^2 + 2^2} = 0{,}2$$

After calculating all the elements, the following matrix is obtained:

$$P_1^{[0,1]} = \begin{pmatrix} - & 0{,}8 & 0{,}9411 \\ 0{,}2 & - & 0{,}8 \\ 0{,}058 & 0{,}2 & - \end{pmatrix}$$

For expressing it in in the interval [0,4], each element is multiplied by 4. Results are showed below:

$$P_1^{[0,4]} = \begin{pmatrix} - & 3{,}2 & 3{,}76 \\ 0{,}8 & - & 3{,}2 \\ 0{,}232 & 0{,}8 & - \end{pmatrix} \tag{54}$$

After results are expressed in the desired interval, expression (48) is applied in order to express results linguistically. The following matrix is obtained:

$$P_1 = \begin{pmatrix} - & (s_3^1, 0{,}2) & (s_4^1, -0{,}24) \\ (s_1^1, -0{,}2) & - & (s_3^1, 0{,}2) \\ (s_0^1, 0{,}23) & (s_1^1, -0{,}2) & - \end{pmatrix}$$

Now that all the preferences are expressed using linguistic preference relations, labels have to be expressed using the same linguistic term set. In order to reduce computations, S_1 is chosen as reference linguistic term set. Therefore, multi-granular transformation functions aforementioned must be applied. Since S^1 and S^2 fulfil all the requirement to build a LH, transformation is straightforward. In

Table 18: S^2 to S^1 translation table.

S^2	S^1
s_0^2	s_0^1
s_1^2	$s_{0,5}^1$
s_2^2	s_1^1
s_3^2	$s_{1,5}^1$
s_4^2	s_2^1
s_5^2	$s_{2,5}^1$
s_6^2	s_3^1
s_7^2	$s_{3,5}^1$
s_8^2	s_4^1

Table 18, it is possible to see the label of S^1 that correspond to each label of S^2. Since 2-tuple linguistic representation is being used, it is possible to represent each $s_{x,5}$ label as $(s_x, 0{,}5)$.

Using the transformation Table 18, P_3 can be represented using S^1 as follows:

$$P_3 = \begin{pmatrix} - & (s_1^1, 0{,}5) & (s_2^1, 0{,}5) \\ (s_3, 0{,}5) & - & s_4^1 \\ s_3^1 & s_1^1 & - \end{pmatrix}$$

Now that all the information provided by the experts is finally unified, selection process is carried out and temporary results calculated. First, collective matrix is calculated by adding all the preferences provided by the users:

$$P_c = \begin{pmatrix} - & (s_2^1, 0{,}23) & (s_2^1, 0{,}08) \\ (s_2^1, -0{,}57) & - & (s_3^1, 0{,}06) \\ (s_2^1, 0{,}41) & (s_2^1, -0{,}4) & - \end{pmatrix}$$

After the aggregation step is performed, exploitation is done. GDD and GNDD values are calculated using the collective preference matrix and mean value between them will be taken as the final resulting value to build the ranking:

$$DMR = \{(s^1_3, -0{,}05), (s^1_3, -0{,}06), (s^1_3, -0{,}36)\}$$

and therefore, $x_1 \succ x_2 \succ x_3$.

After calculating the temporary results, consensus measures are computed. Distances among every two matrices of the experts are calculated and the obtained results aggregated. The collective consensus matrix is exposed below:

$$C_c = \begin{pmatrix} - & (s^1_2, -0{,}13) & (s^1_1, 0{,}5) \\ (s^1_2, -0{,}33) & - & (s^1_2, -0{,}33) \\ (s^1_1, 0{,}5) & (s^1_3, -0{,}46) & - \end{pmatrix}$$

Using the collective consensus matrix, global consensus and consensus reached in each alternative can be calculated. Results are showed below:

$$C_{x_1} = (s^1_2, -0{,}115)$$
$$C_{x_2} = (s^1_2, 0{,}437)$$
$$C_{x_3} = (s^1_2, 0{,}05)$$
$$C_G = 0{,}5325 = (s^1_2, 0{,}12)$$

It is easy to notice that consensus is very low. Moreover, there is no consensus among any of the alternatives, that is, they all obtain low consensus values. More debate is still needed to be carried out in order to make a consensual decision.

Proximity measures [CMPHV10] are used to generate recommendations to the experts about how to reach consensus. After applying the required operations, the following values are obtained for each expert:

$$PR_{e_1} = \{0{,}341, 0{,}158, 0{,}299\}$$
$$PR_{e_2} = \{0{,}333, 0{,}2575, 0{,}383\}$$
$$PR_{e_3} = \{0{,}238, 0{,}271, 0{,}159\}$$

If a proximity measure is above a certain threshold, λ, recommendation is sent to the expert. Otherwise, the expert is asked to maintain the provided preference values for that specific alternative. In this example, λ is established as 0.33. This way, every proximity value above this value will encourage the DSS to send the expert a recommendation. In this round, experts e_1 and e_2 have proximity values over λ. e_1 needs recommendation about alternative x_1 while e_3 needs it about x_1 and x_3. In the case of expert e_2, his/her similarity matrix is showed below:

$$SM_2 = \begin{pmatrix} - & 0{,}0575 & 0{,}52 \\ 0{,}3575 & - & 0{,}265 \\ -0{,}3975 & -0{,}35 & - \end{pmatrix}$$

Therefore, because x_1 and x_3 preferences want to be encouraged for modification, the DSS suggests e_2 to provide higher values in p^2_{31} and p^2_{32} preference values and lower ones in p^2_{21} and p^2_{13}. These changes will make e_2 opinions to become closer to the ones that the rest of the experts have, increasing consensus. Same process is carried out for expert e_1.

After recommendations are generated, the Group Decision Making process starts the second round. Experts have to modify or maintain their preferences values. Nevertheless, e_1 indicates that a new expert wants to join the Group Decision Making process. Providing preferences step is interrupted and experts vote on the proposal. The result is positive. This way, server gives the required privileges to the new expert in order to participate in the decision making process. This new expert will be called e_4. Providing preferences step restarts, but, e_4 suggests the adding of a new alternative. Providing preferences step is interrupted again, and the addition of a new alternative is studied by the experts. The new alternative, x_4 : buy new books for the library, is voted on and accepted as a possibility by all of the experts. In such a way, it is included in the set of alternatives. Providing preferences step is restarted. This time, experts have to provide their preference values over 4 alternatives instead of 3. e_4 decides to use S_1 linguistic labels and linguistic preference relation as the format of preference representation. The new preferences provided by the experts are exposed below:

$$P_1 = \{s_2^1, s_1^1, s_0^1, s_4^1\}$$

$$P_2 = \begin{pmatrix} - & s_2^1 & s_3^1 & s_1^1 \\ s_1^1 & - & s_3^1 & s_1^1 \\ s_1^1 & s_2^1 & - & s_0^1 \\ s_3^1 & s_4^1 & s_4^1 & - \end{pmatrix}$$

$$P_3 = \begin{pmatrix} - & s_4^2 & s_6^2 & s_2^2 \\ s_2^2 & - & s_7^2 & s_2^2 \\ s_2^2 & s_4^2 & - & s_0^2 \\ s_7^2 & s_8^2 & s_8^2 & - \end{pmatrix}$$

$$P_4 = \begin{pmatrix} - & s_2^1 & s_3^1 & s_0^1 \\ s_1^1 & - & s_4^1 & s_1^1 \\ s_2^1 & s_2^1 & - & s_1^1 \\ s_4^1 & s_4^1 & s_3^1 & - \end{pmatrix}$$

The selection process is carried out as in the previous round. The final obtained rank is as follows: $x_4 \succ x_1 \succ x_2 \succ x_3$. Global consensus value in this round is 0.866. Because the value is above the specified threshold, $\alpha = 0{,}8$, Group Decision Making process is ended being, x_4 : buy new books for the library, the most voted alternative.

In this example, it can be seen how our DSS is able to satisfactorily introduce new ideas and experts in the middle of a Group Decision Making process. In real world, new people can bring new ideas and perspectives and, consequently, make the debate go forward. This is why it is considered extremely important that

DSSs are able to manage this type of dynamic contexts where alternatives and experts can vary over time during the experts debate.

4.4. Discussion

Here, a DSS that works over mobile phones, with heterogeneous preference representation formats, and multi-granular fuzzy linguistic information, is presented. Their main highlights are the following

- **Flexibility in the experts' location**: Because the DSS is implemented over mobile phones, experts can participate in the Group Decision Making process and provide their preferences from everywhere and anytime. Experts can take advantage of information files and the media (audio and video) in order to expose their points of view.

- **Userfrienly expert-server communication**: Experts are provided with several preference providing possibilities along with the capability of choosing the linguistic term set that better fits them. In such a way, every expert can choose the way that they want to express his/her preferences.

- **Dynamic Group Decision Making contexts allowed**: Our DSS adapts itself to any change in the Group Decision Making process environment. It accepts the adding and removing of alternatives and experts in the middle of the process. This way, the designed DSS is able to manage situations that occur in the real world that are not considered by classical Group Decision Makings.

- **Several Group Decision Making processes can be managed**: Generally, the experts are not faced only with one Group Decision Making process at a time. In companies and public institutions, a high amount of decisions have to be made constantly and every expert belongs to a small part of them. The designed DSS is able to manage all the Group Decision Making processes and provide access to each one only to the experts that are allowed to participate. Consequently, this DSS is a really useful tool to carry out, in a tidy and efficient way, all the decisions that have to be made in an institution or company.

The use of the designed DSS entails the following restrictions and drawbacks:

- **No consistency measuring**: Consistency measures help us to measure the reliability of the preferences provided. For example, if an expert provide a high value to indicate that he/she prefers alternative x_1 to alternative x_2 and he/she also specifies that x_2 is highly preferred to x_1, he/she is being inconsistent. Measuring inconsistency can allow us to detect if the experts are providing reliable preferences or just random values. If an expert is considered to be inconsistent it is possible to advice him/her or simply ignore him/her preferences in the Group Decision Making process.

- **Internet access is needed**: In order to make decisions, Internet connection must be available. Because 3G connection is available almost everywhere this is not a really big restriction. Nevertheless, it would be desirable to allow experts to provide their preferences in an off-line environment in order for them to be sent automatically when Internet connection is reached.

- **The server represents a single point of failure**: The designed architecture charges all the computations and expert-to-expert communication responsibilities over a single server. Consequently, if the server breaks or goes down, then the system is not able to work until the server problem is resolved. To avoid this situation, it would be desirable to have several servers working at the same time because then, if one stop working, the

Table 19: Characteristics summarizing table. GDM refers to Group Decision Making

Characteristic	**Yes**	**No**
Can be accessed from everywhere	X	
Flexible preferences representation	X	
Number of alternatives can vary	X	
Number of experts can vary	X	
Preferences are provided using words	X	
Several set of words can be used	X	
Release mobile phones from hard computations	X	
Manage several GDM process at the same time	X	
Consensus is promoted before making a final decision	X	
Suggestions about reaching consensus are made to experts	X	
Security policies avoid stolen identity issues	X	
Experts do not have access to all the GDM processes	X	
GDM process time consumed is controlled	X	
Works without Internet access		X
Experts consistency is taken into account		X
Several servers are used		X

rest can assume their responsibilities and the system will continue working. Also, if the servers share computations, the system will be able to work faster when a high amount of Group Decision Making processes are carried out at the same time because all the work is shared.

Table 29 summarizes all the highlights and drawbacks of the designed DSS.

5. Building and managing Fuzzy Ontologies by using multi-granular linguistic information

5.1. Introduction

Ontologies have become an important tool in the domain modelling field. Thanks to them, it is possible to carry out real world representations, stablish axioms and obtain conclusions of them [Fen01, LR09]. Ontologies have been wide used in several fields. In biomedicine field [FG13, HGR13, HDG13], ontologies have been employed, for example, to build knowledge databases about genes and proteins characteristics that help researchers to classify and understand how the human body works. In semantic web field [Lan13, JS13, TNNM13], ontologies have been used to classify concepts that can be referred through the web. This way, searches are improved and give better results to the users because a concept, instead of a word that can have different meanings, is used. In the artificial intelligence field [DRCLCF14, PWM+13, PK14], ontologies can also be applied to create knowledge databases to be used in systems that employ the provided information to carry out different tasks.

However, classical Crisp Ontologies have one important drawback, that is, their element descriptions can only be expressed using crisp membership values. Consequently, each described element has a set of fulfilled characteristics and another one with characteristics that do not describe the element. That is, membership value of each element to each concept is represented by the values {0,1} where 0 means that the element does not fulfil the concept and 1 means that the element has the characteristic expressed by the concept. In real world problems, this kind of scenario is not enough to describe correctly certain situations. For solving this issue and being able to provide a more flexible way of carrying out descriptions, Fuzzy Ontologies have been developed. Thanks to Fuzzy Ontologies, it is possible to provide membership values from the defined elements to the concepts using the interval [0,1]. Therefore, each described element can fulfil concepts totally (1 value), do not fulfil it (0 value) or partially fulfil it with a certain degree value (]0,1[). Thanks to this new representation, it is possible to model the uncertainty that is implicit in many real world environments and using fuzzy sets theory [Zad65], it is possible for the ontology to deal with it using its associated mathematical environment. Fuzzy Ontologies is a field that is clearly present in the recent literature as it can be seen in [CMB13, THPG14, TDN13].

Fuzzy Ontologies also open the way for introducing linguistic modelling in this research field [BS11]. Thanks to it, elements can be described by using words instead of numbers. Linguistic modelling and linguistic term sets [Zad75a, Zad75b, Zad75c] in order to describe elements have one main advantage and one main drawback. The advantage is that words are more flexible that numbers. Consequently, this is the best way when trying to model concepts whose meaning is imprecise. They are also easier for humans to use that numbers making them a perfect choice when trying to model people opinions. On the other hand, the main drawback of using linguistic labels is the loss of precision that they produce when trying to represent precise data values.

Fuzzy Ontologies are used to create big knowledge stores whose data can come from different information sources, and therefore, source information is expressed using different representation methods. Due to the heterogeneity of the information, sometimes it is difficult to manage it. In such a way, it is extremely important to be able to work and combine different information expressed using different data types. Consequently, methods that are able to deal with data expressed using different representation models are needed. Thanks to them, data can be expressed in a way that it can be compared and managed together, without having to take into account the origin of the information.

In this kind of scenarios where data is heterogeneous and it is represented using fuzzy sets theory and linguistic modelling, multi-granular fuzzy linguistic methods [MPZC14, MMPUHV15, PCHV11b] become essential. Thanks to them, it is possible to carry out conversion operations in order to homogenize the information. In such a way, the system can easily work with all the information. Multi-granular fuzzy linguistic modelling can also allow users to select the linguistic term sets that better fits them. Therefore, user-system communication is improved.

In this chapter, three new different ways of how multi-granular fuzzy linguistic modelling processes can be applied when fuzzy ontologies are built and managed are proposed and analysed. To do so, advantages, drawbacks and viability of the different processes depending on the type of information we are dealing with, are presented.

In subsection 5.2, some new methods to solve the multigranularity treatment problem that is present in Fuzzy Ontologies are proposed. In subsection 5.3, examples of the exposed approaches described in subsection 5.2 are showed. In subsection 5.4, advantages and drawbacks of the proposed methods are analysed. Finally, some conclusions are pointed out.

5.2. Multi-granular fuzzy linguistic modelling methods for building and managing Fuzzy Ontologies

In this subsection, several different ways of dealing with multi-granular information in the ontology creation process are exposed. Furthermore, a way for users to carry out queries using the linguistic term set that better fits them will also be shown.

Generally, when an ontology is created, these steps are followed:

1. **Information search:** First of all, reliable information sources must be consulted and, afterwards, information is extracted in order to gather the necessary data for the ontology that is being created. When several information sources are consulted, the probability that the information is expressed using different means is very high. Information must be uniformed in order to be able to carry out comparisons.

2. **Information preprocessing step:** Transformation functions are applied over the extracted information in order to express them using the same representation method. This step is mandatory since it would be impossible to carry out any operation if the information is not homogeneous. Afterwards, data is stored in a way that can be used by queries. It should be taken into account in the design that the preprocessing step is carried out only once while queries are made repeatedly. This way, for the sake of efficiency, data computations that are always carried out in all the queries can be pre computed in this step. Consequently, time will be saved in the query process.

3. **Query design:** A method of user-system communication with the Fuzzy Ontology has to be developed. Depending on how the information has been stored in the preprocessing step, the building of possible queries differs. Therefore, the designing of a communication method with the ontology is a critical task. Depending on the representation, it could be possible to allow users to use different linguistic term sets, that is, queries can become multi-granular if users can select the linguistic term sets that better fits them when making a query.

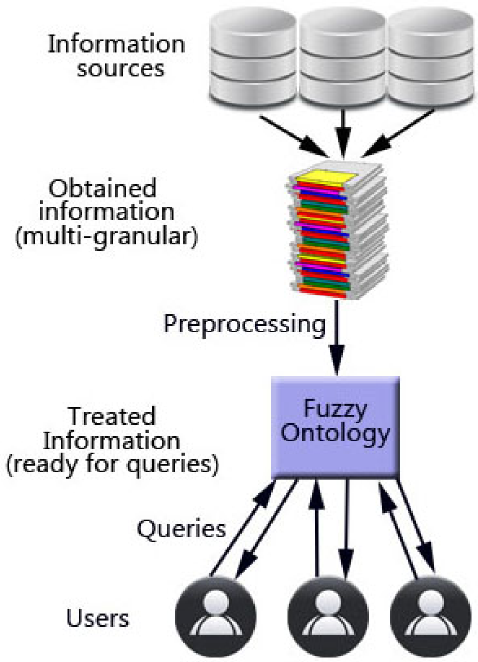


Figure 20: Creation and use ontology scheme.

4. **Validation:** After the ontology is created, a validation process must be carried out in order to confirm that the ontology works correctly and results are the expected ones. Since this chapter deals with multi-granular fuzzy linguistic modellings application in the Fuzzy Ontology building and management processes, in the following, we focuss our efforts on the three previous steps. Fuzzy ontology validation processes can be further studied in the literature [DHBZA13].

In Figure 20, a scheme of how the ontology is created and used is shown. In the case of the Fuzzy Wine Ontology exposed in subsection 2.3.2, the creation process is exposed below:

1. **Information search**: Well-known databases of wines were searched in order to gather all the wines information needed. Data recollected has different representations since several different sources were used.

2. **Preprocessing step**: Information is uniformed and expressed using linguistic term sets or crisp values. More details about the final representation chosen can be seen in subsection 2.3.2.

3. **Query design**: Users can perform queries using labels of the linguistic term set that have been used to represent the Fuzzy Ontology information or, in the case of the crisp values, users indicate the characteristics that they are interested in. For example, if a wine with low alcohol, high acidity and from Spain is needed, the search made by the user have the following form:

$$\begin{aligned} Q_u = \{&Alcohol = LowAlcohol \wedge Acidity = HighAcidity \\ &\wedge Country = Spain\} \end{aligned} \tag{55}$$

   It can be seen that wine searchers are forced to carry out queries using the linguistic term set that have been selected for representing the information in the Fuzzy Ontology. When the Fuzzy Ontology is going to be used by a high amount of people, it would be desirable to let them choose the way of expressing the query that better fits them.

In conclusion, there are two possible ways where it is possible to take advantage of multigranularity treatment methods:

- **Multi-granular source data treatment at the Fuzzy Ontology building process**: Linguistic data belonging to different sources may need multigranular treatment in order to be able to express the information using the same metrics and to work with it.

- **Multi-granular queries design for the Fuzzy Ontology management**: Users carrying out queries to the Fuzzy Ontology may need to have different linguistic term sets for expressing themselves. In such a way, they can choose the most comfortable way to communicate with the system.

In this chapter, both situations are analysed and solutions are suggested. In subsection 5.2.1, how to apply multi-granular fuzzy linguistic modellings to the data recollected from different data sources in order to build a Fuzzy Ontology is studied. In subsection 5.2.2, methods to design queries for managing the Fuzzy Ontology are proposed.

5.2.1. Multi-granular source data treatment at the Fuzzy Ontology building process

In the first step of the creation of an ontology, data is extracted from different sources. Generally, each source has its own way of storing the information making it impossible to carry out comparisons among them directly. Consequently, data transformation operations must be carried out. Two types of data representations can be found in the information sources:

- **Numerical Information**: It is the one referring to concepts that can be accurately measured. The main two operations that can be performed to uniform numerical information are defined below:
  - *Domain change:* When a measure is carried out, it is usual to establish both minimum and maximum range values. Consequently, the minimum range value represents the lowest possible valid value while the maximun range value represents the highest one. It is usual that different information sources choose different range intervals for expressing the numerical information. Before being able to work with this type of information, an unique range value interval must be chosen and all the information must be normalized into it.
  - *Number format:* Different numeric formats can be used to represent the numerical information, for example, real, integer, etc. Transformation rules must be established in order for numbers to use the same format.

For example, if all the information must be expressed using integer values but there are values expressed using real numbers, rules of how to deal with real values must be defined. One possible way of dealing with this situation could be to apply the floor operator. It is important to point out that the best way of carrying out these operations is to express the information using the format that is able to represent more elements. This way, loss of precision is avoided. For example, when dealing with integer and real numbers, it is much better to express integer numbers using the real format.

- **Linguistic Information**: It is the one referring to concepts whose definition entails imprecision and uncertainty. Concepts like beauty, tastiness and sympathy belong to this category. Nevertheless, it is also possible to express numerical nature information using words if an accurate value is not known or they do not want be expressed in a precise way. It is usual that different linguistic information sources use different linguistic term sets with different granularities in order to represent the linguistic information. In order to carry out operations using all of this information, all labels must belong to the same linguistic term set. Thanks to multi-granular fuzzy linguistic modellings [MMPUHV15, MRTHV14], this task can be carried out without any trouble. Several possible options of dealing with multi-granular linguistic information are listed below:

  - *Symbolic multi-granular fuzzy linguistic modellings*: These fuzzy linguistic modellings carry out label translations belonging to different linguistic term sets taking into account the indexes of the labels in each linguistic term set. This way, computations become quite simple and no extra representation framework must be added to the labels. The main drawback of these fuzzy linguistic modellings is that they usually have restrictions, that is, these methods do not usually work with all the possible linguistic term sets. Furthermore, they can produce loss of information. Inside this category, it can be found fuzzy linguistic modellings that use linguistic hierarchies [HM01b, ELM11], discrete fuzzy numbers [MRTHV14] and qualitative descriptive spaces [RSA+11].

- *Semantic multi-granular fuzzy linguistic modellings*: These fuzzy linguistic modellings associate a fuzzy set to each label. In such a way, the initial label representation is lost and all the transforming operations are carried out using the associated fuzzy sets and their mathematical environment. The main advantage of these methods is their flexibility, that is, they can operate with any linguistic term set and do not have any restrictions as long as a fuzzy set is associated to every label in every linguistic term set. Their main drawback is located in the results presentation. To associate a label to the resulting fuzzy set can become a troublesome task due to the fact that, after computations, it is possible that no label fits the result. Carrying out this process entails loss of precision in the process. Inside this category, it can be found fuzzy linguistic modellings that use triangular fuzzy numbers [ZG12, JFM08] and the ones that are based on a Basic Linguistic Term Set. [HHVM00, CBA06].

- *Linguistic to Numeric conversion*: If the Fuzzy Ontology designer considers that there is no need to work with linguistic information, it is possible to use semantic multi-granular fuzzy linguistic modellings. Thus, linguistic information can be converted into numeric one. The main advantage of this approach is to have all the advantages of semantic multi-granular fuzzy linguistic modellings and precision of numerical data without the consequences of having to translate fuzzy sets into labels.

There is not a best way of carrying out this task, depending on the desired results, the most suitable multi-granular fuzzy linguistic modelling should be chosen. Using all the presented processes, it is possible to manage all the recollected heterogeneous information and transform it into what the designer needs for his/her Fuzzy Ontology design. Depending on how the user query is designed, information must be transformed and presented in an specific way. In the following subsection, several user query designs that allow users to select the linguistic term set that they prefer are presented. Each design needs the information to be presented in an specific way.

5.2.2. Multi-granular queries design for the Fuzzy Ontology management

In regular Fuzzy Ontologies, users are forced to express themselves using, for each concept, an unique linguistic term set. It would be desirable to allow users to choose the linguistic term sets that they prefer. A Fuzzy Ontology query process using multi-granular fuzzy linguistic modelling could be held as follows:

1. **Linguistic term set selection:** The user formulates his/her query using, for each characteristic, the linguistic term set labels that better fits his/her expression capacity. Depending on the Fuzzy Ontology design and the multi-granular fuzzy linguistic modelling used, there could be some restrictions, that is, it is possible that the set of chosen linguistic term set must fulfil certain specifications in order to be valid.

2. **Query resolving process:** The Fuzzy Ontology support system carries out the necessary transformations to the user provided information in order

to carry out the Fuzzy Ontology query resolution. For example, if an user provides his/her information using the linguistic term set S_1 but in the Fuzzy Ontology the information is stored using the linguistic term set S_2, a multi-granularity fuzzy linguistic modelling must be applied. Thus, the labels from S_1 provided by the user are expressed using labels of S_2 and comparisons with the Fuzzy Ontology information can be carried out. For this purpose, the method described in subsection 3.3.3 will be used. The transformation function is exposed below:

$$TF_t^{t'} : l(t, n(t)) \rightarrow l(t', n(t'))$$
$$TF_t^{t'} \left(s_i^{n(t)}, \alpha^{n(t)}\right) = \Delta \left(\frac{\Delta^{-1}\left(s_i^{n(t)}, \alpha^{n(t)}\right) \cdot (n(t') - 1)}{n(t) - 1} \right) \quad (56)$$

3. **Result presentation:** The query results consist in a list of elements that are ordered according to their associated matching values. The Fuzzy Ontology user can select if he/she wants to see these results numerically or linguistically and, in the second case, he/she can select the target linguistic term set. Transformation functions must be applied to the obtained results in order to fulfil the result representation requirements asked by the user.

Thanks to multi-granular fuzzy linguistic modellings, user-system communication is improved. Therefore, users have more means to formulate the Fuzzy Ontology queries because the Fuzzy Ontology support system adapts itself to the users communication needs. Users can express themselves better and, consequently, the system receives more reliable information.

In this subsection, several Fuzzy Ontology designs that allow users to express themselves linguistically using the linguistic term set that they prefer are presented:

1. **Semantic approach**: All the gathered information is stored in its numerical value. Linguistic information is also expressed numerically using fuzzy sets mathematical environment. Membership values of the labels associated fuzzy sets are used to carry out this transformation. Linguistic queries provided by users are also expressed numerically in order to carry out computations.

2. **Duplicity approach**: Information is duplicated and stored using different representations. Users provide their queries in any of that representations.

3. **Symbolic approach**: Information is stored linguistically using the same linguistic term set for each of the concepts. Users can provide their queries in any linguistic term set and, in order to carry out comparisons, multi-granular fuzzy linguistic modellings are applied to it.

Semantic approach

This approach expresses all the gathered information in a numeric way. Therefore, semantic multi-granular fuzzy linguistic modellings [ZG12, JFM08] are applied to the user query in order for it to be also expressed numerically for computations to be carried out. To build an Fuzzy Ontology using this approach, the next steps must be followed:

1. **Selecting target numeric interval**: The numeric interval used to represent the information referring to each concept is chosen. The interval can be as wide as desired as long as it has a minimum and a maximum value. This restriction will allow us to transform the linguistic information into numerical one.

2. **Transforming linguistic information**: Gathered linguistic information is expressed using the chosen numerical interval associating a specific number

inside the interval to each label. This way, a high linguistic value will be associated to numerical values close to the maximum interval value. On the other hand, low linguistic values will be associated to positions close to the minimum interval value. It should be pointed out that this process entails a loss of precision that will be traduced in less accurate results.

3. **Transforming numerical information**: It is possible that gathered numeric information is expressed using a different scale or measure that the chosen one. Depending on the case, a transformation function that let us express the numeric information using the chosen representation must be applied. For example, if some piece of information must represent the number of square meters of a house but the information gathered refers to square centimetres, information should be transformed and expressed using metres instead of centimetres. Also, if, for example, a student score in an specific subject is measured using the interval [0,100] but the interval [0,10] want to be used, it is possible to carry out a domain change as exposed in subsection 5.2.1.

Queries using this approach are formulated and resolved as follows:

1. **Linguistic term set selection**: User selects the linguistic term set that he/she want to use for each of the characteristics that he/she will include on the search.

2. **Query providing step**: The user formulates the query linguistically using the linguistic term sets that he/she have chosen.

3. **Transforming linguistic information**: Linguistic information provided is transformed into numeric one associating a fuzzy set to each of the labels. In order to carry out computations, the fuzzy set is defuzzified [LK99] in

order to obtain a single number. One way of achieve this purpose is to calculate the gravity center, GV, of the fuzzy set as follows:

$$GV = \int_x \frac{x \cdot \mu(x)}{\mu(x)} \tag{57}$$

4. **Resolving the query:** Once that the query has been expressed numerically, the query is resolved using the following steps:

   a) For each of the elements of the Fuzzy Ontology, the characteristics that the user has included in his/her query are retrieved.

   b) For all the characteristics, distance value between the user specified value and the one of each element in the Fuzzy Ontology is measured.

   c) Elements are sorted in a way that the elements whose proximity is closer to the one specified by the user are in high positions of the ranking.

   d) Elements located in high positions of the ranking (or only the best element) are returned to the user.

In Figure 21, a scheme of this approach can be seen graphically.

Duplicity approach

This approach stores the same information several times using different linguistic representations in order to allow the user to choose the representation that better fits him/her. This ontology building approach follows the next steps:

1. **Selecting target linguistic term sets:** The set of linguistic term sets that user will be able to choose in order to perform his/her queries are selected. It is important to select linguistic term sets with different granularities in order for users to be able to select among a wide range of possibilities. This

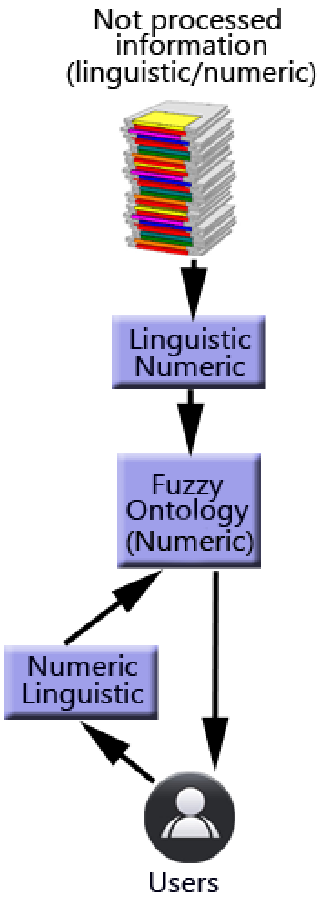


Figure 21: Multi-granular ontology semantic approach scheme.

way, user-system communication will be increased. Otherwise, there can be experts that will not find a suitable linguistic term set for them.

2. **Transforming the information:** All the gathered information that conforms the Fuzzy Ontology is replicated and expressed using, for each replication, a different linguistic term set of the chosen ones in the previous step. Multi-granular fuzzy linguistic modellings can be used to carry out the necessary linguistic transformations. In the case of numeric information, membership function value to each of the labels from all the linguistic term sets is calculated and stored.

Queries that use this Fuzzy Ontology approach are formulated and resolved as follows:

1. **Linguistic term set selection:** For each of the characteristics, user selects one of the linguistic term sets that have been pre-selected in the Fuzzy Ontology building step.

2. **Query providing step:** The chosen set of linguistic term sets are used to formulate the query.

3. **Resolving the query:** Query is resolved using the following steps:

   a) For each element of the Fuzzy Ontology, characteristics expressed using the linguistic term sets provided by the user are retrieved. The rest of redundant information is not taken into account.

   b) Membership function values of each element to each of the labels provided by the user are aggregated. Any aggregation operator such as OWA [Yag96] can be used for this purpose. A ranking of elements is made using the aggregation resulting values, that is, the matching values.

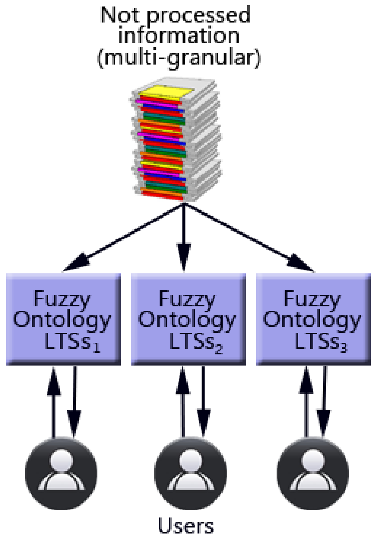


Figure 22: Multi-granular ontology creation scheme.

c) Elements with the best matching values of the raking (or only the best one) are returned to the user.

In Figure 22, a graphical scheme of this approach is shown.

Symbolic approach

This approach stores all the information linguistically and uses symbolic multi-granular fuzzy linguistic modellings in order to express the query provided by the user into the Fuzzy Ontology used linguistic term sets. This ontology building approach follows the following steps:

1. **Linguistic term sets selection:** A linguistic term set is chosen to represent the information of each of the characteristics.

2. **Transforming information:** Gathered information referring to each of the characteristics is expressed using the chosen linguistic term sets. Multi-granular fuzzy linguistic modellings can be used to carry out this task. In the case of numeric information, membership function value to each of the

labels of the chosen linguistic term set are calculated and stored.

With this approach, queries are formulated and resolved as follows:

1. **Linguistic term set selection**: The user selects the linguistic term sets that he/she wants to use to express his/her preferences for each characteristic.

2. **Uniforming linguistic information:** The user formulates his/her query using his/her chosen linguistic term sets. Because user linguistic term sets can be different from the Fuzzy Ontology linguistic term sets chosen to represent the information, multi-granular fuzzy linguistic modellings are used to express the user information in terms of the Fuzzy Ontology stored one.

3. **Resolving the query**: Membership function values of each element to each of the query labels are aggregated. Finally, a ranking of elements is made using the obtained aggregated values. Better elements (or only the best one) are returned to the user.

In Figure 23, a scheme of this approach is showed graphically.

5.3. Illustrative Example

In this section, an example of each approach proposed in subsection 5.2.2 is exposed. Fuzzy Wine Ontology is used in order to test the different designed Fuzzy Ontology support system versions. Specifications of Fuzzy Wine Ontology can be seen in subsection 2.3.2. Fuzzy Ontologies for each example have been built using the techniques that subsection 5.2.2 describes.

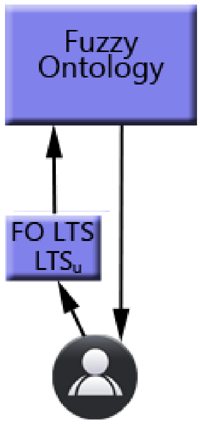


Figure 23: Multi-granular ontology query maker scheme.

Table 20: User selected linguistic term sets. LTS refers to linguistic term set.

Characteristic	LTS	Chosen label
Alcohol	$S_7 = \{s_1^7, s_2^7, s_3^7, s_4^7, s_5^7, s_6^7, s_7^7\}$	s_5^7
Acidity	$S_7 = \{s_1^7, s_2^7, s_3^7, s_4^7, s_5^7, s_6^7, s_7^7\}$	s_4^7
Year	$S_4 = \{s_1^4, s_2^4, s_3^4, s_4^4\}$	s_3^4
Price	$S_3 = \{s_1^3, s_2^3, s_3^3\}$	s_2^3

5.3.1. Example of Semantic Approach

A Fuzzy Ontology user wants to choose, among the 600 possibilities, the wine that better fits his/her desires. He/she focuses his/her query in the alcohol level, the acidity, the year and the price characteristics. For each one, the linguistic term sets used by the user to express his/her desires and the selected label are shown in Table 20. Because the information expressed by the user is linguistic and the one stored in the Fuzzy Ontology is numeric, it is necessary to carry out a linguistic to numeric conversion of the query. Because the user has provided labels and not precise values, imprecision and loss of information is produced during this step. In Table 21, the numeric value associated to each query value after the transformation is showed. Range column indicates the minimum and maximum numerical values accepted by the Fuzzy Ontology as valid for each characteristic.

Table 21: Numeric conversion of the labels.

Characteristic	**Chosen label**	**Numeric conversion**	**Range**
Alcohol	s_5^7	13.333	[0,20]
Acidity	s_4^7	5	[0,10]
Year	s_3^4	2003	[1800,2012]
Price	s_2^3	14	[0,500]

Table 22: Fuzzy Ontology selected wines by semantic approach.

Wines	**Distances**
Campo_Viejo_Reserva	0.8916664
Chateau_Bonnin_Pichon	0.91416645
Cave_de_Tain_Crozes_Hermitage	0.93833363
Tiempo_Briego	0.9391664

Similarity measures among the wanted values and the characteristic values of each element of the Fuzzy Ontology are calculated. First four wines that obtained the lowest distance values are showed in Table 22.

Characteristics values stored in the Fuzzy Ontology for the best selected choice can be seen in Table 23. Distances vector of the numeric conversion of the query and the best choice selected in the Fuzzy Ontology is $\{0{,}167, 0{,}2, 3, 0{,}2\}$. Due to the low distances among the numeric version of the query and the Fuzzy Ontology element it can be stated that Campo_Viejo_Reserva is a great choice for the user. Nevertheless, it should not be forgotten that loss of information has been produced during the linguistic to numeric query conversion.

Table 23: Campo_Viejo_Reserva characteristic values.

Alcohol	13.5
Acidity	5.2
Year	2006
Price	13.8

Table 24: Fuzzy Ontology selected wines by duplicity approach.

Wines	**Matching value**
Tiempo_Briego	0.86962026
Marques_de_Arienzo_Reserva	0.75835556
Castillo_Montroy_Reserva	0.7372372
Beringer_Founders_Estate_Merlot_2	0.73692477

Table 25: Tiempo_Briego characteristic values.

Characteristic	**label**	**membership value**
Alcohol	s_5^7	0.9489
Acidity	s_4^7	0.94012
Year	s_3^4	1.0
Price	s_2^3	0.589412

5.3.2. Example of Duplicity Approach

A user wants to retrieve a wine from the Fuzzy Wine Ontology that has specific characteristics. His/her preferences are described in Table 20. The Fuzzy Ontology process the query, and, without any further conversion, it selects, for each element of the Fuzzy Ontology, the characteristics values that are expressed using the user linguistic term sets. Membership function values for each label provided by the user are aggregated into a one single value used for stablish comparisons among the different wines. The four best wines according to the query and their matching values are showed in Table 24. The wine with best matching value is Tiempo_Briego. Membership function values for each of the labels can be seen in Table 25. It is easy to see that membership function values to the labels provided by the user are quite high for Tiempo_Briego wine making it a excellent choice for the user. It should be also pointed out that Tiempo_Briego was also one of the better choices selected by the semantic approach. It can be estimated that, without taking into account any transformation process, this approach takes 21

Table 26: Fuzzy Ontology selected linguistic term sets for symbolic approach.

Characteristic	LTS
Alcohol	$S_3 = \{s_1^3, s_2^3, s_3^3\}$
Acidity	$S_3 = \{s_1^3, s_2^3, s_3^3\}$
Year	$S_4 = \{s_1^4, s_2^4, s_3^4, s_4^4\}$
Price	$S_3 = \{s_1^3, s_2^3, s_3^3\}$

times more time to execute this example than semantic approach. This is because, while semantic approach makes one comparison, in this approach a comparison per label is carried out.

5.3.3. Example of Symbolic Approach

A user wants to use the Fuzzy Wine Ontology support system in order to retrieve a wine that has certain features. Characteristics that the user is interested in, linguistic term sets used in the query and the selected labels are shown in Table 20. linguistic term sets used by the Fuzzy Ontology for that characteristics are shown in Table 26. It can be seen that, although same linguistic term set is used for representing year and price, the user and Fuzzy Ontology use different linguistic term sets for representing linguistically the alcohol and acidity information of the wine. In such a way, a multi-granular fuzzy linguistic modelling must be used in order to transform the query labels whose representations differ. It has to be pointed out that any multi-granular fuzzy linguistic modelling is valid for carrying out this conversion. In this example, the membership function value of the gravity center of the query label to the Fuzzy Ontology labels is used to carried out the transformation of labels. For example, in the alcohol case, s_5^7 corresponds to $\{0{,}66 : s_2^3, 0{,}416 : s_3^3\}$. Then, wines that have a closer membership value for 0.66 in label s_2^3 and 0.416 for label s_3^3 will be selected as desired characteristics

Table 27: Fuzzy Ontology selected wines for symbolic approach.

Wine	**Matching value**
Tiempo_Briego	0.8672779
Beringer_Founders_Estate_Merlot_2	0.8476215
Jean-Baptiste_Adam_Pinot_Gris_Reserve	0.820538

Table 28: Tiempo_Briego membership values for alcohol and acidity characteristics.

Alcohol	s_1^3	s_2^3	s_3^3
	0	0.6	0.5
Acidity	s_1^3	s_2^3	s_3^3
	0.5	0.2857	0

values. User query expressed using Fuzzy Ontology labels is exposed below:

$$Alcohol : \{0{,}666 : s_2^3, 0{,}416 : s_3^3\}$$

$$Price : \{1{,}0 : s_2^3\}$$

$$Year : \{1{,}0 : s_3^4\}$$

$$Acidity : \{0{,}375 : s_1^3, 0{,}4285 : s_2^3\}$$

After performing the query, the first four better results and respective matching values obtained can be seen in Table 27. Tiempo_Briego is the most appropriate wine for the user. Its membership values for each label in alcohol and acidity concepts can be seen in Table 28. It can be seen that, for the alcohol, the distance values between the wine characteristic and the query are $\{0{,}06, 0{,}084\}$. For the acidity, the distance values are $\{0{,}125, 0{,}1428\}$. Furthermore, for the Year, distance value is 0 and for the price 0.41. Taking into account this results, it can be seen that Tiempo_Briego characteristics are quite close to the one desired by the user. Consequently, it is a good choice for the user to order.

It should be noticed that Tiempo_Briego is also the best choice selected by the duplicity approach and the fourth best wine chosen by semantic approach. Using this approach in this example and ignoring transformation information functions execution time, it can be estimated that the time consumed is 13 times higher than in semantic approach. As in duplicity approach, one comparison must be performed for each label used in the Fuzzy Ontology for the wanted characteristics.

In conclusion, it can be seen that the three approaches produce reliable results. Nevertheless, the obtained results by each one of them differ. This happens because of the use of heterogeneous information and the loss of precision that is present in the transformation functions used for making the information homogeneous.

5.4. Discussion

In this section, advantages and drawbacks of the presented Fuzzy Ontology designs that use multi-granular fuzzy linguistic modellings are exposed. Each proposed method has its own strengths and weaknesses and is not suitable for all the possible scenarios. To analyse the Fuzzy Ontology data environment and select a proper design method is extremely important if good results want to be obtained. The suitability of each proposed approach to every possible scenario is analysed below:

- **Semantic approach:** Semantic approach stores all the gathered information from databases numerically, convert the linguistic labels provided by the user into numeric information and carry out computations numerically. The main advantage about this approach is that is the one requiring less disk space for storing information. This is due to the fact that only one numerical value per concept and element is stored. On the contrary, approaches that use linguistic labels need to store the membership function of each element to each of the labels, that is, several numerical values per concept and element. This approach is also quite efficient because only one number comparison and an unique linguistic to numeric conversion is made per query. The main drawback of this approach is the loss of accurateness that converting linguistic information into numeric one entails. All the imprecision and vagueness related to linguistic labels is lost in all the linguistic to numeric conversions carried out during the Fuzzy Ontology building and the query process. For example, if a *medium_alcohol* wine is searched, the numeric conversion just converts the linguistic value into a numeric one (or an interval) that is, indeed, medium. The problem is that alcohol values that do not belong with degree 1 to *medium_alcohol* values set are discarded, that is, all the imprecision representation capability that linguistic labels have is totally wasted. In conclusion, this approach is appropriate for environments where not much disk space is available or a high amount of information needs to be stored. Although it is the method that has the least number of comparisons per query, transformation functions need to be applied during the query process. On the other hand, it is not the best choice if linguistic nature information is being dealt because results will not be too accurate. This approach is also the best to choose when numerical nature information is dealt.

- **Duplicity approach:** This approach preselects a set of linguistic term sets to represent each Fuzzy Ontology concept and stores the information several times using the selected linguistic term sets. This way, the user can select one of the available linguistic term sets for each concept and expresses his/her query using it. The main advantage of this method is that it does not need to carry out any information transformation during the query process. Nevertheless, it carries out more comparisons per query than the semantic approach. Therefore, it can be considered more efficient than the symbolic approach, which carry out information transformations in the query process, but less efficient than the semantic one. This is due to the fact that less comparisons per individual are carried out in the semantic approach. Another highlight of this method is that the information is stored linguistically making it able to take advantage of the imprecision nature of words. For example, if a *medium_alcohol* wine is searched, membership function values of each Fuzzy Ontology element to the label *medium_alcohol* are consulted. This way, no loss of information is produced. The two main disadvantages of this approach is the disk space requirement and that the linguistic term sets that the user can use are preselected. Because information is replicated using different linguistic term sets for its representation, Fuzzy Ontologies using this approach need a high amount of space. Let $\mathbb{S} = \{S_1, S_2, \ldots, S_a\}$ be the linguistic term sets set used for representing the information and $G = \{g_1, \ldots, g_a\}$ represents the set of granularity values for each set. Then, for each concept and element, $\sum_{i=1}^{a} g_i$ numeric values are needed for a proper representation. Comparing to the unique numerical value used in the semantic approach, it can be seen that far more disk space is needed. Moreover, because the linguistic term sets set $\mathbb{S}$ is fixed in the Fuzzy Ontology building step, only linguistic term sets belonging to it can be used by the user in his/her queries. In conclusion, this approach is the best choice in environments where there is no disk space restrictions and information nature is linguistic.

- **Symbolic approach:** Symbolic approach stores the information linguistically and convert the user query labels into labels from the linguistic term sets used to store the information in the Fuzzy Ontology. The main highlight of this approach is that it allows a proper management of linguistic information without the high requiring of disk space used by duplicity approach. An unique linguistic term set with granularity g is used for representing each concept. Thus, only g numerical values are needed for each concept representation. It should be noticed that this approach still needs more space than semantic approach. The cost of having these advantages is that, in every query made, a multi-granular fuzzy linguistic modelling must be applied to convert labels used by the user in the query to the Fuzzy Ontology labels. Consequently, this approach is the least efficient of the three exposed. In conclusion, symbolic approach is a good choice when a lot of information needs to be represented because it does not waste too much disk space. It also should be used when information nature is linguistic because it deals properly with the imprecision that is inherent to words. Although it is true that having to apply a multi-granular fuzzy linguistic modelling in each query makes this approach the least efficient one, if the chosen method is efficient, then it is possible for this approach to work well in environments where there is a lot of information and a real time response is needed. Nevertheless, if a really high amount of elements are stored in the Fuzzy Ontology, like in big data problems [Bet14, ZE$^+$11], it is possible to experience a response delay.

A summary of this analysis can be seen in Table 29.

Table 29: Characteristics summarizing table.

Characteristic	**Semantic approach**	**Duplicity approach**	**Symbolic approach**
Disk space required for storing	Very Low	High	Low
Efficiency in resolving queries	High	Medium	Low
Number of LTSs for the user to choose	Unlimited	Restricted	Unlimited
Deals properly with imprecision	No	Yes	Yes
Information stored nature	Numeric	Linguistic	Linguistic
Number of conversions	High	Low	High

6. A linguistic mobile group decision support system based on fuzzy ontologies to facilitate knowledge mobilization

6.1. Introduction

Nowadays, users are demanding more assistance applications for helping them with their everyday life. As most users always carry their mobile devices with them, this is the artefact they want to get assistance from. Decision support developed for mobile devices is therefore becoming an increasingly important research area. It is also a critical part of knowledge mobilization [GLCA+14], a movement that will change how knowledge management is conducted. Knowledge mobilization states that knowledge obtained from formal research should be available and usable by every person who is in need of it.

At the same time, developments in the ICT-field have initiated a never ending flow of new technical devices. These technical devices are able to connect to the Internet allowing users to share and consume information regardless of time and location. In order to allow knowledge mobilization to work on these devices, it is necessary to use tools and technologies such as decision support systems [DD13], fuzzy ontologies [CMB13], recommendation systems [TLPP+14], etc. Moreover, all these methods must work together in order to carry out the necessary tasks. Consequently, one of present challenges is to find ways of connecting all these tools and technologies in order to allow mobile phones to provide real-time knowledge to the user whenever and wherever he/she needs it. In other words, to bring knowledge mobilization to mobile phones. Thanks to server languages such as PHP or JSP [Hal01], databases languages such as Oracle and MySQL and mobile operational systems such as IOS and Android [Dev11], it can be stated that, today, the creation of this type of multiple tools that collaborate as Internet applications is possible.

In this chapter, we are going to describe the implementation of a mobile decision support system that is capable of allowing users to get real time knowledge about a certain topic. By applying expert knowledge using ontologies [CMB13], it is possible for non-experts to take advantage of this expert wisdom and use the advice of experts that have plenty of knowledge about the topic that should be dealt. The implemented system use linguistic modelling in order to ease the way in which experts communicate with the system. It has been extensively proven that experts are more comfortable expressing themselves using words instead of numbers. This is because humans are used to deal with concepts. In the Group Decision Making process, consensus measures [CMPHV10] will be used to help users to reach an agreement. In order to increase clarity and provide example of a use case, the implemented application deals with the problem of choosing a wine. In it, a set of users must decide which wine they should order depending on their tastes, the food that they have ordered, the price and the meeting context. By combining the Fuzzy Wine Ontology [CMB13, Wik13], group decision making support algorithms [HACHV09] and the *fuzzyDL* reasoner [BS11] a

Web Platform application and an Android application have been developed and implemented. Thanks to both implementations, every mobile that has an Internet connection will be able to use the application. Therefore, participants can use their mobile devices to reach a decision regarding the choice of wine. Thanks to the mobile implementations, users can get access to the knowledge at any time independently of their location. A GPS or IP location can also be used in order to determine the set of wines that are available at a certain location.

The paper is structured in the following way. Section 6.2 presents the two implementations developed. Next, Section 6.3 presents a Discussion and Analysis of the implemented applications.

6.2. Decision Support Systems for recommending wine

Combining the Fuzzy Wine Ontology with a decision support algorithm, a novel decision support system has been created that aids dinner guests to choose the most suitable wine for the dinner context. Two different versions of the system were developed and implemented:

- *Web platform*: This version was implemented using JavaServer Pages (JSP) and runs over a web browser in any device that has internet access.

- *Android application*: This version consists of an Android app that can be downloaded and installed in any mobile device that supports Android applications.

Both implementations follow the activity diagram showed in Figure 24. As can be seen there, the developed applications have the following steps:

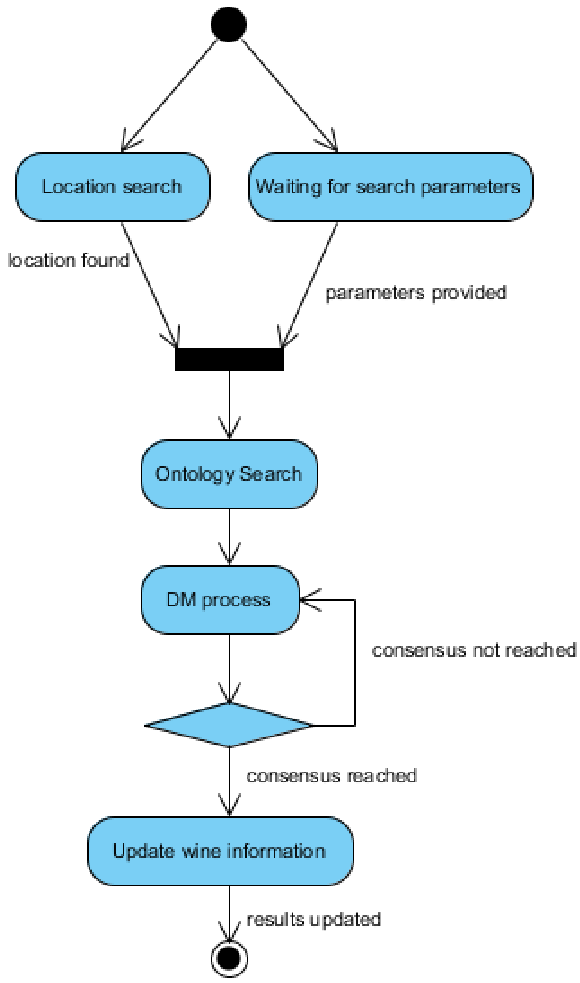


Figure 24: Web platform and Android Application activity diagram.

1. **Location search**: First, the users' location is retrieved using IP location or GPS. This data is sent to Google services in order to retrieve information about the actual location of the used device.

2. **Ontology search**: After location search is performed and search parameters are provided, the fuzzy wine ontology search starts. The parameters for the search are specified below:

   a) *Context*: Refers to the scenario surrounding the dinner. Depending on the purpose of the dinner, some wines can be more suitable than others.

Usually one has different criteria for different contexts, e.g. for a formal dinner few hosts want to be considered a cheap person, so cheap wines can be given a lower importance. Three options are available: Candle, Friends and Formal.

b) *Food*: The type of food that the users are going to eat. Depending on this factor, there are some wines that are more suitable than others. Based on knowledge retrieved from wine experts, different wine properties are suitable for different types of food. For instance, the well known fact that red wines are more suitable together with meat than white wines. Five options are available: Game, Fish, Grilled food, Chicken and Shellfish.

c) *Number of people*: The number of people that are participating in the dinner party. This parameter will only be used in the group decision making process.

d) *Number of wines*: The number of wines that the ontology search must provide. The minimum is four (one for each different criterion). This feature allows users to choose how many wines they want to decide among in the decision making process.

Because different criteria can be equally valid when a wine is chosen, several searches with different criteria are carried out. Thanks to it, users can choose their favourite wine according to the criteria that best fit them. In total, four searches with four different criteria are done:

a) *Most famous wine*: This is the most famous wine of the location where the users are in. It is selected as the best wine produced at the location or the one that is typically consumed among the natives. This criterion allows users to taste a wine that is characteristic of the place that they are visiting.

b) *Lowest price wine*: This option retrieves the lowest priced wine from the ontology. It is a suitable criterion for people who are not fond of wines and just want to choose an economic option.

c) *Best wines according to the context and food*: This option retrieves, using the ontology, a list of the best wines for the context and food specified by the user. The most suitable wine, among the ones available in that location, is chosen.

d) *Most voted wine*: This criteria takes into account results from previous group decision making processes in order to recommend a specific wine. The most voted wine available in the location is selected and added to the search result list. If no wine from that location has been ever selected by any users, then this criterion is not taken into account.

With these four criteria and the number of wine parameters specified by the users, a list of wines and the reason why they were chosen is presented.

3. **Decision Making (DM) process**: Users must decide which wine they want to order among the presented ones. The web platform and Android app implements a group decision algorithm that can assist in the decision.

4. **Updating wine information**: After the group decision making process has ended, the wine-location database is updated adding one to the number of times that the selected wine has been chosen. Thanks to this, posterior decisions will give feedback on what other people have selected on previous occasions. The DM process will be described in more detail in subsection 6.2.1.

Information about which wines are available in each location and how many times a wine has been chosen are stored in a database. Its entity-relation diagram

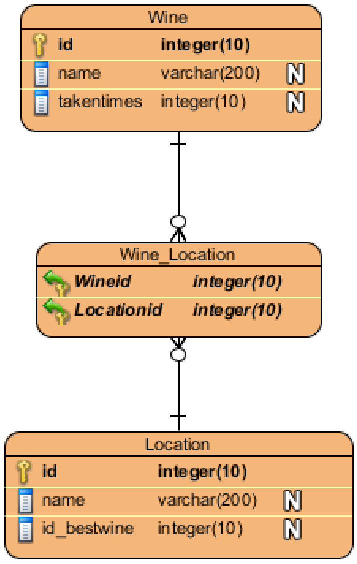


Figure 25: Wine-Location database entity-relation diagram.

can be seen in Figure 25. As can be seen, it has two tables and one relationship:

- **Wine table**: This table stores all the wines that conform with the ontology without taking into account their locations and the number of times that the wine has been previously selected.

- **Location table**: This table stores all the locations available. Because this table is independent of the wine, it is possible to add locations, delete them and modify associated wines at any time. Thanks to this database structure, the wine-location association process is dynamic and scalable.

- **Wine_Location relationship**: It stores which wines are associated with specific locations. A wine can be associated with multiple locations and in each location there are several wines.

To facilitate the database management, an application has been implemented in order to ease the information updating tasks; wines and locations stored in the wine-location database can be updated at any time.

Incoming requests of mobile devices or Web browsers are handled by the server servlet which is in charge of dealing with the ontology API and the wine-location database. When an ontology search is made, the server servlet retrieves the wines that are affiliated with the users' location, and sends the query to the ontology. When the ontology API returns the wine list results, the server servlet sends the resulting information to the mobile device or web browser that has made the request. Both the Web browser and the Android application share the same ontology and wine-location database, that is, the server is the same for both applications. Due to this, decision making results and wine information is shared by the two versions avoiding redundancy issues and easing the information updating task.

In subsection 6.2.1, the used algorithm is explained in more detail and, for the sake of a better understanding, an example is showed. In subsections 6.2.2 and 6.2.3, the Web platform and Android application are described in more detail.

6.2.1. The Implemented Group Decision Making Algorithm

The implemented Group Decision Making algorithm for carrying out the decision making process in the application is based on the one described in [PCHV10]. A questionnaire asking the users about their degree of preference among each possible alternative is filled in and preference relation matrices are built using the provided information.

One way of knowing if experts have reached an agreement is to use consensus measures, which makes it possible to calculate the overall agreement among the ranked alternatives. If the consensus is low, it is reasonable to go for another decision making round, but if consensus is high, it means that almost all experts agreed, making it useless to repeat the process one more time. Consensus measures can also be used to advise users of how to modify their opinions in order to reach a higher consensus [CMPHV10].

The wine selection application that has been created in this article needs human input in order to work. To ease the way that users express themselves, linguistic modelling has been used [Zad75a, Zad75b, Zad75c]. Good overviews of linguistic modelling can be found in [RL09, TL09]. Although the research in this field has generated much publications lately, it is also the case that quite a few application papers in other fields claim to use linguistic modelling. For example, in decision support, [LSCW13] and [APCHV12] use linguistic modelling in order to deal with imprecise information. Linguistic modelling has also been applied satisfactorily to ontologies as can be seen in [Rod12] and [BHRT10]. Concretely, in the implemented application, linguistic values belonging to the balanced linguistic term set $S = \{s_1, \ldots, s_n\}$ are used by the experts to express the preference degrees. $n = 7$ is considered a number high enough to allow experts to express themselves correctly and low enough not to confuse them with unnecessary complexity. When providing labels to the question *How much do you prefer alternative 1 to alternative 2?*, s_1 will indicate that alternative 2 is totally preferred, s_7 will denote that alternative 1 is preferred with the highest possible degree to alternative 2 and s_4 will indicate that they are equally preferred for the user. Using this method, users can communicate with the system in a comfortable way using words. Linguistic modelling is also used for the system to provide recommendation to users of how to modify their opinions in order to reach a consensus. Thanks to linguistic modelling, communication processes become easier to the user who can express himself/herself using methods that are familiar to him/her and also to receive information that will be easy for him/her to understand.

In the Group Decision Making algorithm that was implemented for the mobile application, the consensus and alternative ranking values belong to the interval [0,1], therefore, they can be expressed linguistically using a balanced linguistic term set $S = \{s_1, \ldots, s_n\}$ if the following expression is applied:

$$LR_r = s_{(i+1)} | i = round(r \cdot (n-1)) \tag{58}$$

where r is the numerical value that should be expressed linguistically, LR_r is its linguistic representation, $r \in [0,1]$ and s_i is a linguistic value that belongs to S.

The Group Decision Making process has the following steps:

1. *Providing preferences*: Taking turns, a questionnaire is provided to the users in order to collect their preferences. With the retrieved information, a preference relation matrix is built for each user.

2. *Decision making calculation*: Using the preferences matrices, the group decision making algorithm is executed so that a ranking of the selected wines, together with consensus information, is showed to the users.

3. *Preliminary decision making results*: When the results are showed to the users they can decide, using the consensus information, whether to choose the first ranked wine or to modify their preferences. If they choose the second option, the preference providing step is repeated but, this time, advice is supplied to the users in order to make them reach a consensus.

4. *Final result*: When consensus is high enough or users do not want to continue modifying their preferences, the first ranked wine at this stage is chosen and the group decision making process ends.

For a better understanding of the linguistic Group Decision Making process used, a brief example of how the algorithm works is presented: Imagine that three dinner guests, e_1, e_2 and e_3 should decide what wine to drink for the dinner. A mobile device is used to search for the suitable wines, available in the restaurant, that fit the purpose of the dinner and the ordered food. After performing the search, the wine alternatives w_1, w_2, w_3 and w_4 are provided to the users. A questionnaire is filled in by the attendants using the balanced linguistic term set S for describing the grade of preference between every two wines.

$$S = \{s_1 : very\_low, s_2 : fairly\_low, s_3 : low, s_4 : medium,$$
$$s_5 : high, s_6 : fairly\_high, s_7 : very\_high\}$$

Using the questionnaire results, a preference relation matrix is built for each dinner guest. Results are showed below:

$$P_1 = \begin{pmatrix} - & s_2 & s_1 & s_3 \\ s_7 & - & s_6 & s_5 \\ s_3 & s_4 & - & s_5 \\ s_1 & s_1 & s_2 & - \end{pmatrix} P_2 = \begin{pmatrix} - & s_3 & s_1 & s_2 \\ s_5 & - & s_7 & s_6 \\ s_4 & s_4 & - & s_3 \\ s_2 & s_1 & s_1 & - \end{pmatrix} P_3 = \begin{pmatrix} - & s_1 & s_1 & s_2 \\ s_7 & - & s_6 & s_7 \\ s_5 & s_3 & - & s_2 \\ s_3 & s_1 & s_2 & - \end{pmatrix}$$

Aggregating P matrices, the collective preference matrix (C) is calculated. Although results are given in the interval [0,6], it is possible to make a domain change and express them in the interval [0,1]. Both matrices are showed below:

$$C_{[0,6]} = \begin{pmatrix} - & 1 & 0 & 1{,}33 \\ 5{,}33 & - & 5{,}66 & 5 \\ 3 & 2{,}66 & - & 2 \\ 1 & 0 & 0{,}66 & - \end{pmatrix} C_{[0,1]} = \begin{pmatrix} - & 0{,}16 & 0 & 0{,}22 \\ 0{,}88 & - & 0{,}94 & 0{,}83 \\ 0{,}5 & 0{,}44 & - & 0{,}33 \\ 0{,}16 & 0 & 0{,}11 & - \end{pmatrix}$$

Using C, the selection process is carried out. The t-norm *maximum* has been used to compute the final ranking result. The resulting values for each of the alternatives, belonging to the interval [0,1], are specified in Table 30. According to the table, the final alternatives ranking of the group decision making process is $\{w_2, w_3, w_4, w_1\}$; w_2 is the most preferred wine among the dinner guests and w_1 the least preferred option.

Alternatives	GDD	GNDD	$T(GDD, GNDD)$
w_1	0.1294	0.5927	0.5927
w_2	0.8883	1	1
w_3	0.4255	0.8333	0.8333
w_4	0.0922	0.6294	0.6294

Table 30: Results of the selection process for the decision making example.

6.2.2. Web Platform Application

The web platform application was created for users whose mobile device does not have an Android operating system installed, as it can be used in every device that allows an internet connection and has an web browser installed in it. In the web platform, the server servlet is the element that handles the communicating, presents the results to the user and carries out the group decision making process. In other words, all the computational effort is resolved there. The following software has been used in the web platform implementation:

- The Web platform was implemented using JSP, Javascript and Java languages.
- A Tomcat server is used for running the servlet.
- The Wine-Location database was built using MYSQL.
- The connection between the server and the database uses JDBC.
- Java Netbeans IDE was the development environment used.

Finally, for a better understanding of how the Web platform application works, an example with the Web Platform Application is presented:

Four people are seated in a restaurant in Aguilar de la Frontera, a town in Córdoba, Spain. It is an informal meal among friends and they are going to eat grilled food. They want help to find four wines for them to discuss further about. After filling in all the information on the Web page as Figure 26a shows, ontology results according to where they are located are shown (Figure 26b) and the decision making process starts. First, each one of the friends fills in the questionnaire presented in Figure 27a and, after that, preliminary results are displayed (Figure 27b). Now, the friends can repeat the decision making process pressing the *vote again* link or select the most voted wine by pressing the *select the most voted wine* link. Because consensus is high, they decide not to go for another decision making round and select the wine: *Pedro_Ximenez_1927*.

6.2.3. Android Application

The Android application follows a client-server model in order to make operations that require a high computational effort to execute in an adequate environment. The three client-server requests that must be performed in order to complete the process are described in more detail below:

(a) (b)

Figure 26: Web platform, information form screenshot and ontology results screenshot.

1. **Location request**: Google servers are used to retrieve the mobile device location. An IP address or GPS coordinates can be used for this purpose, it is up to the user to decide which method to use. Furthermore, mobile devices that have an Android operating system but do not have a GPS component can still use the Android version.

2. **Ontology search results request**: Fuzzy ontology searches are computationally intensive and cannot be executed on the mobile platform. Because of that, search data is sent to a server that makes the ontology search and returns the results.

3. **Update Wine-Location database with decision making results**: The mobile device sends the final decision making results to a servlet applet that updates the wine-location database. Because the database contains overall

(a) (b)

Figure 27: Web platform, questionnaire screenshot and decision making temporary results screenshot.

information of all the decision making processes carried out by all the devices that have used the app, it has to be stored on the server in order to enable all the different devices to use the same database. This way, redundancy is avoided and wine-location assignments can be changed, without having to update the application for all the mobile devices separately, modifying only one database.

The following software were used in the Android implementation:

- Because the same server is used, the servlet programming languages are the same as in the web platform version: JSP, Javascript and Java.
- Sockets are used in the Android application-server communication for the

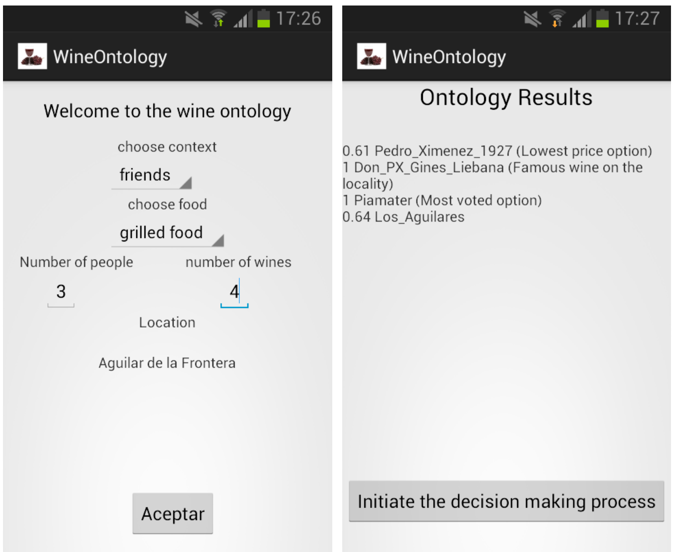
ontology search information sharing.

- Java language was used for programming the Android application.

- For security reasons, the connection between the Android application and the database is made through the server, not directly via a JSP script.

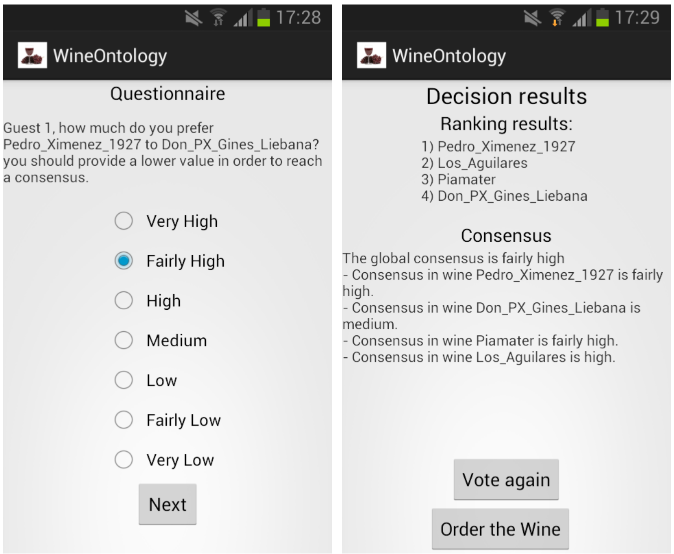
- Eclipse IDE and the Software Development Kit that Android provides were the development environments used.

Finally, for a better understanding of how the Android application works, screenshots of the example presented in the previous section is shown. Figure 28a shows the meal information input part of the Android application and in Figure 28b, the ontology results screen is displayed. Figure 29a shows an example of a poll question. Questions are showed one by one to the dinner guests for better readability. Figure 29b presents an example of the preliminary decision screen.



(a) (b)

Figure 28: Android application, search information screenshot and wine ontology results screenshot.



(a) (b)

Figure 29: Android application, questionnaire screenshot and temporary results decision screenshot.

6.3. Discussion and Analysis

A novel application that combines fuzzy ontologies with a decision support algorithm has been developed and implemented. The goal is to create a decision support system that helps users to choose the wine that best fits them for various types of food in different dinner contexts.

Thanks to fuzzy ontologies, the imprecise knowledge of wine connoisseurs is available for the application users to benefit from. Dinner guests that do not know too much about wines can get support and make a good wine selection. Decision support algorithms provide users with an accurate method to make a decision. Users can discuss and vote for their favourite wines in a efficient and organized way. Consensus measures give them a clear overview of what the other dinner guests have voted and help them to reach an agreement from the advice given.

It can be stated that The Fuzzy Wine Ontology is valid and complete due to the fact that all the characteristics that are used for sommeliers to describe and classify a wine are taken into account as concepts. Also, every individual is related to every concept avoiding missing information. Due to the fact that our ontology is constantly updated, its coverage will increase with every wine added. Now, the most important wines of every European country are included. Data has been retrieved from well-known web-sites with expert knowledge of wines. Because the set of individuals of the ontology and the set of concepts are not interacting, an application based evaluation is sufficient to test the correctness of the ontology. If the data inside is correct, then it can be stated that the ontology is valid.

GPS and/or IP location features are used in order to retrieve the users' location. Therefore, wine recommendations are location dependant, that is, the wine list provided to the guests contains only wines that are available in their actual location. Wines that are not available are omitted in order to avoid impossible choices and to speed up the computations.

The application has been designed to be used in mobile devices. Dinner guests can use it and make decisions in real time at the restaurant where they are going to have dinner. For users that do not have Android installed in their mobile devices, a Web Platform version has been implemented. Thanks to it, every mobile device that has an internet connection can use the developed application. Although the wine selection problem has been the approach used in this implementation, this decision making support scheme can be applied to assist users in a number of other situations. For example:

- Information about loans from banks located in a specific location could be stored in the ontology in order to help users to select the loan that is most adequate for their current situation.

- Data about apartments available for rent or sales in a specific location could be used to advise users about the ones that best fit them.

- If travel information is stored in the ontology, this system can be used to advise a group of friends about where they should go on holidays.

- In companies, if experts‘ knowledge about company management is stored in the ontology, a decision support system to advise non-expert members of the company how to make certain critical decisions can be built.

- For investors, this approach can help them to choose where to invest their money in order to obtain the highest benefit.

Apart from applying the same scheme to other fields, there are other future upgrades that can be used to improve the system. The created wine selection decision support system allows users to select one wine. An interesting approach would be if the application could help them to select one wine for each dish of the dinner.

Methods to increase the speed of the application should be investigated and applied. The speed of the application is directly dependant on the available wines in a specific location. As more wines are included the ontology search time increases dramatically. Because the application has real time requirements, a solution should be found in order to allow it to search among a large number of items.

The actual application searches for wines according to the locality where the dinner guests are having the meal. Using GPS coordinates to get a more precise address, it could be possible to make the ontology search dependant on the wines available in the restaurant instead of the whole locality.

7. Creating knowledge databases for storing and share people knowledge automatically using group decision and fuzzy ontologies

7.1. Introduction

In its recent days, Internet was designed as a consulting mean. Only a few recognized experts were able to provide information while the rest of the people, who could afford to access, were only allowed to carry out consulting tasks. Therefore, Internet was designed for minorities and the information available, comparing to nowadays, was quite limited. Nowadays, situation has dramatically changed. Thanks to Web 2.0 [And10, O'r09], Internet has become a place where users can connect and share high amounts of information. Therefore, Internet users have become providing and consuming information entities. This situation has made information more accessible and available than ever. Nevertheless, in most of the cases, information available is badly structured and, therefore, of little use for users. Users just cannot manage all the available amount of information by themselves. In order to deal with this problem, fields like Big Data [Mad12, MSC13], for extracting conclusions from the data, and semantic web [BLHL01, HvH10, MS01], for sorting it, have arisen.

Ontologies [Fen01, Lan13] are tools that provide a way of sorting, classifying and describing high amounts of information. Knowledge databases created using ontologies are easy to manage and allow users to search information and extract conclusions from it. Because our system needs to work with conceptual information provided by users, imprecision must be dealt with. For this reason, fuzzy ontologies [CC07] will be used. Crisp ontologies allow each element to be described or not, $\{0, 1\}$, by each concept in the ontology. On the contrary, fuzzy ontologies associate each element to each concept using a particular degree located in the interval $[0, 1]$. This way, elements can be associated to contradictory concepts. For example, when referring to a person height, if John measures 1.78 meters, it can be stated in a fuzzy ontology that John has a medium height of 0.7 degree and a high height of 0.3 degree. In a crisp ontology, that measure can be represented with a medium height of 1 and a high height of 0 or the reversal. Therefore, it is easy to see that a fuzzy ontology has more flexible representation capability than a crisp one.

Retrieving information from Internet users is a quite complicated issue. Especially, when subjective information is being dealt. It is mandatory to analyse and verify the reliability of the provided data. In order to carry out this task, Group Decision Making methods [CMPHV10, PCHV10, PCHV11b] can be used. They allow a set of Internet users to provide information, carry out debates and make a final choice. If this approach is used, the final obtained information piece is not an outlier opinion from a unique user but a consensus opinion totally guaranteed by a majority of the users that have dealt with the matter. Consequently, the obtained information can be considered reliable.

As it has been said, information provided by humans are going to be dealt. Consequently, it is quite important to provide users with tools that help them to provide the information. The easier the way that information is provided, the more reliable the obtained data will be. For this reason, multi-granular linguistic information [HHVM00, MRTHV14, MMPUHV15] is used in this chapter. Thanks to it, Internet users can express themselves using words instead of numbers, i.e. they will express themselves as they are more used to do it. Moreover, users will be able to select the linguistic term set that they want to use to express themselves. This way, if the user wants to be very concrete about the provided information, he/she can use a linguistic term set with a high granularity while, if he/she wants to be less concrete, he/she can select an linguistic term set with a lower granularity.

In this chapter, the design of an automatic process for creating knowledge databases using people common knowledge about a certain issue is being presented. Group Decision Making methods are used in order to obtain information totally guaranteed by a majority of users. In order to ease the way that Internet users provide their opinions, multi-granular linguistic information is used. Finally, the trusted information is automatically stored in a fuzzy ontology where other users can get benefit of the obtained knowledge and reach conclusions.

Thanks to the automatic designed method, users can share their subjective knowledge about a certain topic and allow other people to take advantage of it. Retrieved information is sorted in a fuzzy ontology allowing a complete exploitation of the available data. Subjective information provided by human beings is difficult to be dealt due to the fact that it is difficult to measure and validate. Thanks to our system, a tool for dealing with this type of information is presented. Moreover, the used information is validated and make it objective because it is ratified by a majority of users in the Group Decision Making process. In such a way, stored information is not longer a unique person opinion.

On the contrary, it is the opinion of a majority, information worthy of being used and taken into account.

In subsection 7.2, the designed process structure is described. In subsection 7.3, an example is exposed. In subsection 7.4, we expose a comparison among the state of the art and our own proposal by analyzing its advantages and drawbacks. Finally, some conclusions are pointed out.

7.2. Method Scheme and Description

In this subsection, the designed method is described. It creates automatic knowledge databases using Internet users information. In such a way, information is stored in an organized way and completely exploited. In order to create this user knowledge ontology, the next steps are followed:

- **Individuals and concepts definition**: Each designed fuzzy ontology is related to a certain topic. Therefore, first, it is necessary to identify the individuals and concepts that are related with the topic that is being dealt with and the relations among the different elements that compound the fuzzy ontology. From now on, it is considered that every individual is related to every concept. Also, it is assumed that individuals are not related among them. In the case that some of these two statements is false, the exposed method is still valid but the designer must deal with possible inconsistencies and introduce small modifications in the process. This issue will be further discussed in subsection 7.4.

- **Ranking process**: Group Decision Making processes are used in order for Internet users to define the values of the relations between each individual and concept.

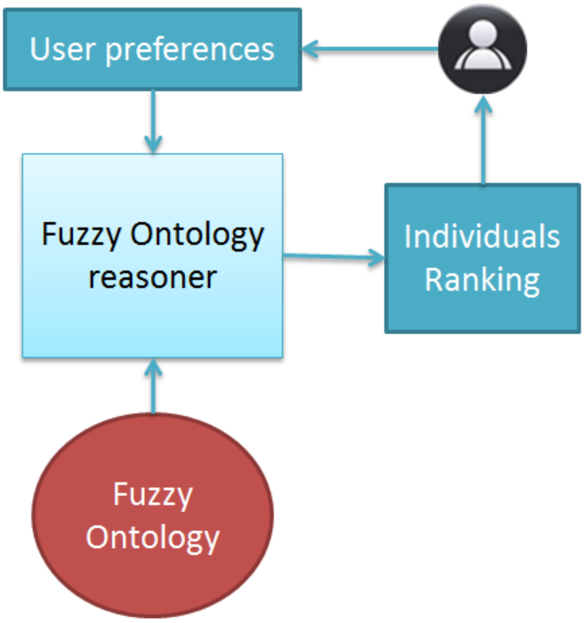


Figure 30: Fuzzy ontology query process.

- **Fuzzy Ontology creation process**: Once that the relation values between every individual with every concept are defined, the fuzzy ontology is created reuniting the information.
- **Fuzzy Ontology consulting process**: Steps exposed in subsection 2.3.1.1 are followed in order for Internet users to retrieve information. A graphical scheme of the process can be seen in Figure 30.

In Figure 31, a graphical scheme of the overall process is exposed. In subsection 7.2.1 the ranking creation process is described in detail. In subsection 7.2.2, the fuzzy ontology creation process is detailed. In subsection 7.2.3, the fuzzy ontology consulting process is exposed.

7.2.1. Ranking process

After defining the fuzzy ontology concepts and individuals, it is necessary to define the individual-concepts relations. In order to accomplish this task, seve-

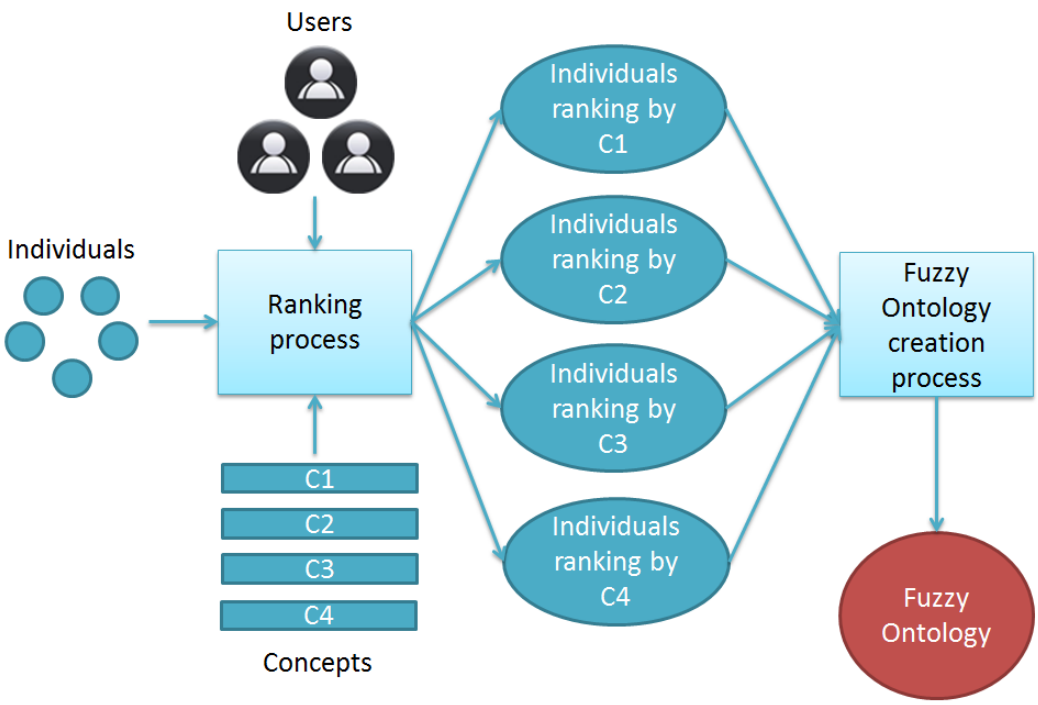


Figure 31: Fuzzy ontology creation process scheme using Group Decision Making.

ral Group Decision Making processes are carried out, one per concept available. Elements of each of them are defined as follows:

- **Alternatives**: Each individual is considered as an alternative.
- **Experts**: Experts are compound by the set of users that want, or are allowed, to participate. In order to retrieve common Internet users knowledge, every Internet user that knows about the dealt problem should be welcomed to participate.

Let $C = \{c_1, \ldots, c_l\}$ be the set of concepts, $D = \{d_1, \ldots, d_k\}$ the set of individuals and $E = \{e_1, \ldots, e_n\}$ the set of experts, the ranking process is hold as follows:

1. l Group Decision Making processes are created, one for each element in C. Each element in D is an alternative.

2. Each Group Decision Making process is carried out separately. Their purpose is to create a ranking with the alternatives set. Alternatives for each Group Decision Making process, GDM_i, must be sort according to the following criteria: *the more the alternative fulfil the concept c_i the better ranking value it gets.*

3. For each GDM_i, experts express their preferences using the linguistic term set and the representation method that they prefer. Experts can express their preferences using three different preference providing representations: utility values, preferences orderings and preference relations. For computations, preference relations are used.

4. Using fuzzy multi-granular linguistic methods and the transformation expressions, information is uniformed and expressed using preference relations and labels from the same linguistic term set. All the used transformation functions are described in subsections 4.2.1 and 5.2.2.

5. The uniformed information is aggregated into a single collective preference matrix. This matrix represents the overall opinion of all the experts that participate in GDM_i. For the calculation, Mean operator is used as follows:

$$C_i = \phi(P_{ij}^h), i = \{1, \ldots, l\}, j = \{1, \ldots, k\}, h = \{1, \ldots, n\} \tag{59}$$

6. For each Group Decision Making process, the mean of the selection operators GDD and GNDD ranking values is calculated using the collective preference matrix. GDD and GNDD operators are calculated as follows:

$$GDD_i = \phi(c_{i1}, c_{i2}, \ldots, c_{i(i-1)}, c_{i(i+1)}, \ldots, c_{in}) \tag{60}$$

$$GNDD_i = \phi(c_{1i}^s, c_{2i}^s, \ldots, c_{(i-1)i}^s, c_{(i+1)i}^s, \ldots, c_{ni}^s) \tag{61}$$

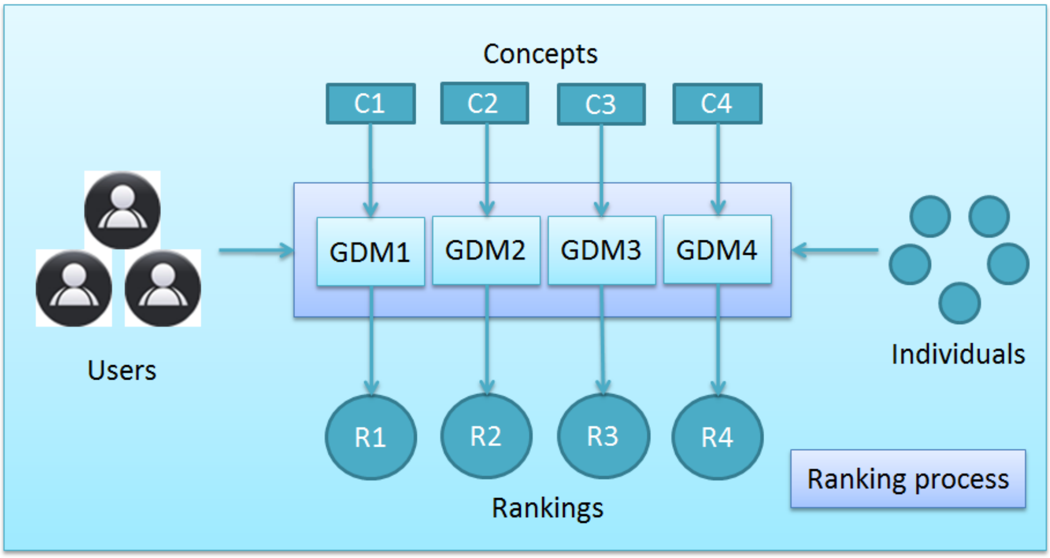


Figure 32: Ranking process scheme.

where

$$c_{ji}^{s} = \text{máx}\{c_{ji} - c_{ij}, 1\}$$

Therefore, the final ranking values are calculated as follows:

$$RV = (GDD_i + GNDD_i)/2, \forall i \in [0, m] \tag{62}$$

Finally, alternatives are sort according to their RV.

7. Once that all the alternatives are sorted for each concept, a set of rankings $R = \{r_1, \ldots, r_l\}$ is obtained, a different one for each concept. Using this ranking set, the fuzzy ontology is created as exposed using the guidelines exposed in subsection 7.2.2.

A scheme of the described process can be seen in Figure 32.

7.2.2. Fuzzy Ontology creation process

Once that the ranking set R has been calculated in the previous step, its information is used in order to build a fuzzy ontology. Thanks to it, the

Individual	c_i
d_1	s_3
d_2	s_2
d_3	s_1
d_4	s_4

Table 31: Fuzzy ontology for concept c_i of the example.

knowledge that has been provided by the users is stored in a organized way. Also, other Internet users can access and have benefit from it. It has to be taken into account that the more users the Group Decision Making processes have hold, the more reliable the recollected information is. This is due to the fact that the obtained conclusions are ratified by more people.

The fuzzy ontology creation process follows the next steps:

1. **Ranking linguistic term set association**: A linguistic term set $S = \{s_1, \dots, s_k\}$ containing the same number of labels as individuals are in the fuzzy ontology is defined. The label indicating the higher value is assigned to the first individual in the ranking, the second higher value to the second position in the ranking and so on. For example, if a set of four individuals is ranked as $R_i = \{d_3, d_2, d_1, d_4\}$ for the concept i, then the linguistic term set $S = \{s_1, s_2, s_3, s_4\}$ is defined. Fuzzy ontology results for concept c_i is exposed in Table 31. This way, the more an individual fulfil each concept, the higher index value its associated label has.

2. **Fuzzy Ontology structure construction**: After applying the ranking linguistic term set association for all the concepts in the fuzzy ontology, the information is reunited and the fuzzy ontology constructed. It must be noticed that, because the number of individuals are the same in each Group Decision Making process, the same linguistic term set is used for all

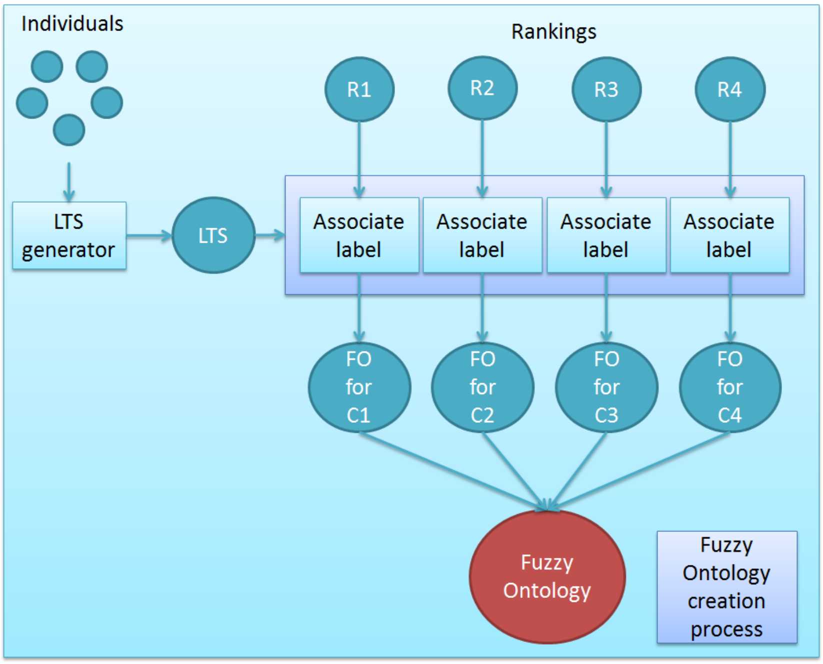


Figure 33: Fuzzy ontology creation process scheme.

the concepts in the ontology. It should also be taken into account that in cases where a lot of individuals are available, the linguistic term set used have an extremely high and unmanageable granularity value. Therefore, it cannot be used by experts for consulting tasks. For solving this issue, multi-granularity linguistic approaches can be used. In subsection 7.2.3, this issue will be further discussed.

A scheme of this process can be seen in Figure 33.

7.2.3. Fuzzy Ontology consulting process

After building the fuzzy ontology using the users knowledge, each consultant that want to retrieve information from it can do it. For this task, the next steps are followed:

1. **Linguistic term set selection**: As stated in the previous subsection, for

representing individuals in the fuzzy ontology, a linguistic term set whose granularity is the number or individuals has been used. Usually, number of individuals is quite high making the linguistic term set unaffordable for users use. In order to solve this issue, users are allowed to express themselves using the linguistic term set that better fit their necessities. The linguistic term set labels used will be expressed into the fuzzy ontology labels using a multi-granular fuzzy linguistic method.

2. **Query providing**: After selecting an appropriate linguistic term set, the user formulates his/her query. The query is constituted by a set of concepts and their desired value. Users do not have to specify values for all the concepts available in the ontology, only for those that he/she is interested in.

3. **Query uniforming**: Labels from the query provided by the user must be transformed into labels from the fuzzy ontology in order to carry out comparisons. For this purpose, a multi-granular fuzzy linguistic method is used. Its transformation function is showed in expression (56).

4. **Fuzzy ontology reasoning**: Once that the query is expressed using the linguistic term set used by the fuzzy ontology, a ranking of the individuals is carried out. First, similarity values between the query and each of the individuals are calculated. Next, individuals are sorted using the similarity values obtained.

5. **Fuzzy ontology results providing**: The top values of the obtained ranking from the previous step are showed to the user. They are the choices that are closer to what the user is looking for.

7.3. Illustrative Example

In this subsection, a fuzzy ontology creation and consulting process examples are showed. A company, such as Filmaffinity, is interested in building a movie fuzzy ontology using the opinions of their users. In such a way, the users can consult it in order to find films that are adapted to their tastes. The company want to classify 20 different movies, $D = \{d_1, \ldots d_{20}\}$ using the seven following concepts, $C = \{c_1, \ldots, c_7\}$:

1. **Action**: Measures the amount of action in the film.
2. **Humour**: Takes into account if the film is comical.
3. **Drama**: Refers to whether the film argument is sad and touching.
4. **Mystery**: Mystery films get high label values in this concept.
5. **Argument**: Quality of the movie argument is measured.
6. **Overall opinion**: Refers to the overall opinion of the users for an specific movie.
7. **Actors performance**: Measures the film actors performance quality.

It has to be noticed that this is a brief movie fuzzy ontology example. Other concepts like science fiction or horror could be added. To create a functional movie fuzzy ontology is out of the scope of this chapter.

Because seven concepts want to be measured using users opinion, seven different Group Decision Making processes must be hold. For example, for the action concept, users are asked to sort the films according to the level of action on them. Because a high amount of individuals is available, it is difficult for experts to carry out a pairwise comparison of all of them. To overcome this issue, two possible paths can be followed:

1. To use a Group Decision Making method that is able to deal with this type of situations.

2. Use another preferences representation method like utility values.

In this example, a Group Decision Making method that allows participation of a high amount of experts and a high number of alternatives is followed. This method allows users to provide information only about the movies that they prefer. In such a way, the number of pairwise comparisons that a user must fill is chosen by him/her. Because participation from a high number of users is expected, enough information to carry out a reliable Group Decision Making process will be recollected. This example focuses in preferences provided by a set of three experts: $E = \{e_1, e_2, e_3\}$. e_1 uses preference relations and the linguistic label set S_1, e_2 uses utility function values and the linguistic label set S^2 and e_3 uses preference relations and the linguistic label set S^2. Linguistic label sets S^1 and S^2 are defined below:

$$S^1 = \{s^1_1, \ldots, s^1_5\}$$
$$S^2 = \{s^2_1, \ldots s^2_9\}$$

All three experts have decided to provide information about the set of movies $\{d_1, d_2, d_3, d_4\}$. Preferences provided are exposed below:

$$P_1 = \begin{pmatrix} - & s_3^1 & s_1^1 & s_2^1 \\ s_5^1 & - & s_4^1 & s_5^1 \\ s_1^1 & s_3^1 & - & s_2^1 \\ s_2^1 & s_3^1 & s_3^1 & - \end{pmatrix}$$

$$P_2 = \left(s_6^2, s_9^2, s_1^2, s_3^2\right)$$

$$P_3 = \begin{pmatrix} - & s_5^2 & s_2^2 & s_3^2 \\ s_9^2 & - & s_8^2 & s_9^2 \\ s_2^2 & s_3^2 & - & s_1^2 \\ s_3^2 & s_5^2 & s_4^2 & - \end{pmatrix}$$

In order to carry out the Group Decision Making process, preferences must be unified. Preference relations and S^2 are the preference representation method and the linguistic term set that will be used for computations. Therefore, P_1 labels must be expressed using labels from S^2 and the utility function vector P_2 must be expressed using preference relations.

After applying the multi-granular transformation function exposed in (56), the following preference relation is obtained for e_1:

$$P_1 = \begin{pmatrix} - & s_5^2 & s_1^2 & s_3^2 \\ s_9^2 & - & s_7^2 & s_9^2 \\ s_1^2 & s_5^2 & - & s_3^2 \\ s_3^2 & s_5^2 & s_5^2 & - \end{pmatrix}$$

After applying the expression (50) over P_2, the following preference relation is obtained for e_2:

$$P_2 = \begin{pmatrix} - & s_3^2 & s_9^2 & s_8^2 \\ s_7^2 & - & s_9^2 & s_9^2 \\ s_1^2 & s_1^2 & - & s_1^2 \\ s_2^2 & s_1^2 & s_9^2 & - \end{pmatrix}$$

For the sake of clarity, α values from the labels have been omitted. It is important to notice that a bit of precision is lost in this simplification process.

After unifying the information, preferences are aggregated into a single collective matrix value. The collective matrix is not built at once using all the recollected preference values. On the contrary, collective matrix is in constant update with every new preference entry. After aggregating the three preference matrix provided by the experts $\{e_1, e_2, e_3\}$, the following collective matrix is obtained:

$$C = \begin{pmatrix} - & 4{,}33 & 4 & 4{,}66 & - & \dots & - \\ 8{,}33 & - & 8 & 9 & - & \dots & - \\ 1{,}33 & 3 & - & 1{,}66 & - & \dots & - \\ 2{,}66 & 3{,}66 & 6 & - & - & \dots & - \\ - & - & - & - & - & \dots & - \\ \dots & \dots & \dots & \dots & \dots & \dots & - \\ - & - & - & - & - & - & - \end{pmatrix}$$

where C has a row and column count value of 20. Because only three experts have participated in the process and they have introduced values for the same alternatives, the rest of the collective matrix values referring to other alternatives remains empty. Collective matrix is filled using numbers based on the labels from the used Basic Linguistic Term Set indexes. Because the collective matrix is only use for computational purposes, there is no need of using labels for enhancing

alternative	GDD	GNDD	Mean	Linguistic
d_1	4.33	7.66	6	s^2_6
d_2	8.44	9	8.72	$(s^2_9, -0{,}22)$
d_3	2	4.98	3.5	$(s^2_3, 0{,}5)$
d_4	4.11	6.55	5.33	$(s^2_5, 0{,}33)$

Table 32: Ranking results for the first four alternatives.

comprehension.

In order to obtain the final ranking, the mean between the GDD and GNDD operators resulting values can be used. If only the first 4 films were taken into account, the ranking would be as follows:

$$R = \{d_2, d_1, d_4, d_3\} \tag{63}$$

GDD and GNDD results for the first four alternatives can be seen in Table 32.

After carrying out the seven Group Decision Making processes over the 20 individuals with all the experts, the ranking set $R = \{R_1, \ldots, R_7\}$ is obtained. An linguistic term set with a granularity value of 20, $S^3 = \{s^3_1, \ldots, s^3_{20}\}$, is chosen for the fuzzy ontology creation. For each concept, each label from the linguistic term set is assigned to each individual according to their position in the ranking. In Table 33, a table representing the final fuzzy ontology obtained is showed.

Once that the fuzzy ontology is created, users can formulate queries and extract knowledge from it. Imagine that a user wants to use the fuzzy ontology with only 5 movies exposed in Table 34. He/She wants to select a film that has a good argument and a lot of action. Although the fuzzy ontology uses the linguistic term set $S^4 = \{s^4_1, \ldots, s^4_5\}$ that has a granularity value of 5, the user wants to use the linguistic term set $S^5 = \{s^5_1, s^5_2, s^5_3\}$ whose granularity value is 3 to perform

Individual	c_1	c_2	c_3	c_4	c_5	c_6	c_7
d_1	s_8^3	s_8^3	s_6^3	s_{19}^3	s_2^3	s_{20}^3	s_3^3
d_2	s_{10}^3	s_4^3	s_3^3	s_8^3	s_5^3	s_{17}^3	s_8^3
d_3	s_{16}^3	s_{19}^3	s_{18}^3	s_9^3	s_3^3	s_{10}^3	s_1^3
d_4	s_{19}^3	s_2^3	s_5^3	s_{12}^3	s_9^3	s_5^3	s_{13}^3
...	...	...	...	...	...	...	...
d_{20}	s_1^3	s_5^3	s_{13}^3	s_4^3	s_{12}^3	s_4^3	s_7^3

Table 33: Fuzzy ontology of 20 elements.

Individual	c_1	c_2	c_3	c_4	c_5	c_6	c_7
d_1	s_5^4	s_4^4	s_5^4	s_3^4	s_5^4	s_1^4	s_2^4
d_2	s_1^4	s_3^4	s_4^4	s_4^4	s_4^4	s_2^4	s_3^4
d_3	s_2^4	s_1^4	s_1^4	s_5^4	s_3^4	s_3^4	s_4^4
d_4	s_4^4	s_2^4	s_2^4	s_2^4	s_2^4	s_5^4	s_1^4
d_5	s_3^4	s_5^4	s_3^4	s_1^4	s_1^4	s_4^4	s_5^4

Table 34: Fuzzy ontology of 5 elements.

the query. Therefore, the user formulates the following query:

$$Q = \{s_3^5 \cdot c_1, s_3^5 \cdot c_5\} \tag{64}$$

First, labels expressed using the linguistic term set S^5 must be expressed in terms of S^4. Once that the multi-granular linguistic transformation process has been carried out, the expert query is expressed as follows:

$$Q = \{s_5^4 \cdot c_1, s_5^4 \cdot c_5\} \tag{65}$$

After that, similarity between every individual in the fuzzy ontology and the query is calculated. Indexes of the labels are used. Results and calculations can be seen in Table 35. Values are expressed in the interval $[1, 5]$. Finally, the following ranking is presented to the user: $\{d_1, d_5, \{d_2, d_3\}, d_4\}$. Consequently, d_1 is the film that better fulfil the requirements suggested by the user.

Individual	Operations	Similarity value
d_1	$5-(\|5-5\|+\|5-5\|)/2$	5
d_2	$5-(\|1-5\|+\|4-5\|)/2$	2.5
d_3	$5-(\|2-5\|+\|3-5\|)/2$	2.5
d_4	$5-(\|4-5\|+\|2-5\|)/2$	2
d_5	$5-(\|3-5\|+\|1-5\|)/2$	3

Table 35: Similarity values calculation.

7.4. Discussion

In this chapter, an automatized method to retrieve and store user knowledge in an organized way has been proposed. First, information is retrieved using a Group Decision Making process. This way, the stored information is not given by an unique user. Instead, each piece of information is supported by the users majority. Next, retrieved information is stored using a fuzzy ontology. Fuzzy ontologies allow information to be stored in a organized way and provide a mathematical environment that let users to perform queries over the stored data.

The presented method has the following advantages:

- **The process is automatized**: One of the most important advantages of this method is that it is automatized. Therefore, it can be easily implemented on a computer that can carry out by itself all the required steps. Consequently, a computational system can create fuzzy ontologies using the users information without any direct human intervention.

- **Allows information sharing**: The designed system allows Internet users to classify and share their own knowledge. Therefore, our system retrieves users information and stored it in a fuzzy ontology. Afterwards, carrying out fuzzy ontology queries, any user can get benefit from that knowledge.

- **The retrieved information is organized**: Because fuzzy ontologies are used, the retrieved user information is dealt in an organized way. A good organization makes easier for users to access the required information. Also, information utility and interpretation are increased.

- **Information is trustworthy**: The employed Group Decision Making processes allow users to provide the fuzzy ontology required information. The final obtained ranking is made taking into account the opinion of all the users that have wanted to participate in the process. Therefore, the obtained rankings are ratified by the majority of the users. This fact is quite important since it proves that the information stored in the fuzzy ontology is, indeed, knowledge that is hold by a important majority of users and not only a by single outlier person.

- **Easy to implement**: The defined process uses mainly Group Decision Making methods and fuzzy ontologies. They are quite known tools which implementations are available thorough the Internet and the research literature. Therefore, the defined method is quite easy to implement.

- **Can be used in mobile phones**: It is important to notice that this method is quite easy to implement in smartphones that use Android or IOS using the available implementation frameworks. This way, users can retrieve and provide information to the system independently of time and location.

- **User-friendly interface**: Because the designed method requires active user participation, it is important to ease human-computer communications. For this purpose, linguistic modelling and multi-granular fuzzy linguistic methods have been used. Thanks to them, the user is able to express himself/herself using words instead of numbers. This way, they can communicate with the system using the means that they are used to employ to communicate with other humans. Multi-granular fuzzy linguistic methods allow users to select the precision of the information that they want to provide. This way, if they do not know much about the dealt topic, they can select a low precision granularity and provide more imprecise information. On the contrary, if they are quite fond on the topic, they can provide accurate results selecting a linguistic term set with a high amount of labels.

Depending on the defined background and the information nature, this method can present several issues. Several possible problems and ways to overcome them are listed below:

- **Individuals and concepts dependencies**: Previously, it has been stated that all the individuals are independent among them. Also, it has been assumed that all the individuals are related to all the concepts. In the case of individual-to-individual relations existence, it would be necessary for users to provide information about them. Also, the final designed fuzzy ontology should be analysed and, in the case that inconsistencies are found, to fix them. We believe that the best way to fix inconsistencies is to present, to the same users that have provided the fuzzy ontology information, all the possible ways to solve the inconsistency and allow them to choose one. For example, imagine that a fuzzy ontology has two individuals, *John* and *Anthony*, and they are related by the relations, *son of* and *father of.* It is easy to see that an scenario where *Anthony is the father of John* and, at the same time, *Anthony is the child of John* is inconsistent. Therefore, in order to solve this issue, users are asked to elucidate whether *Antonio* is the father or the child. Afterwards, the fuzzy ontology is fixed according the results. Because the fuzzy ontology is expected to be fulfilled with users knowledge, to let the users decide is the most trustworthy way. In the case that there are individuals that are not related to some of the concepts, the solution is quite easy since the only thing that should be done is to include, in each concept related Group Decision Making process, only those individuals that are related to the concept.

- **Dealing with non-ordinal concepts**: In the described process, it has been assumed that all the concepts can be represented using ordinal values. Nevertheless, there are concepts that, because of their meaning nature, cannot be dealt this way. For example, in the case of smartphones, a concept indicating the mobile phone brand cannot be dealt using an ordinal linguistic term set. This is because, this concept is composed by a set of elements, for example, $\{Nokia, Sony\ Ericsson, Samsung, Apple\}$ that cannot be sorted using their indexes. In this case, the better approach is to ask the users to select one value for each of the individuals. This way, the most voted value is the one assigned to the relation.

- **Information updating**: One possible disadvantage of the described method is that once that the fuzzy ontology is created, it remains static and no new individual information can be added without having to rebuild the entire fuzzy ontology. Because the individual-concept relations are built using the individuals ordering, the rebuilding process cannot be avoided unless other preference representation method is used. For instance, the use of utility values in the Group Decision Making process allows individuals to be dealt individually. This way, when a new individual is aggregated, users just have to rate it and, afterwards, it is introduced in the ontology. The use of utility values has the disadvantage of making users to manage a high granularity linguistic term set if a large amount of individuals is dealt with and a certain level of precision want to be maintained. It should be noticed that users are not used to deal with high granularity linguistic term sets. Thus, the human-computer communication degree decreases. In conclusion, the use of utility values is recommended in cases when the fuzzy ontology must carry out daily or every hour updates. On the contrary, if only time to time updating is needed, the use of preference relations and rebuilding the fuzzy ontology is recommended. In the case of adding a new concept, the task does not produce any inconveniences. A Group Decision Making process is carried out for the new added concept and the new information is added to the fuzzy ontology.

- **Dealing with objective information**: The described process has assumed that all the information contained in the fuzzy ontology is subjective and, therefore, the relation values depend on people opinion. Nevertheless, there are situations where these values are well known and, thus, there is no need to carry out any Group Decision Making process to assign them. In these cases, the corresponding values can be filled by the fuzzy ontology manager using a trustworthy corpus that contains the required information. This way, the number of Group Decision Making process can be reduced and be dedicated only to fill information about the concepts that refer to subjective information.

In Table 36, a brief summary of the discussed issues and their solutions are exposed.

In subsection 2.3.1.2, state of the art of ontologies and applications are revised. As it can be seen, ontologies have been widely used in a high amount of fields for a variety of purposes. Nevertheless, in most of the cases, ontologies are built manually using experts retrieved information. Because this can be a long and difficult process, it is necessary the creation of automatized methods that are able to ease these procedures. In [RLT+14, SGB+15, ZMFW10], automatized procedures are introduced. For instance, in [RLT+14], an automatized procedure

Issue	**Solution**
Concepts and individuals dependencies	Analyse the fuzzy ontology and carry out GDM processes to solve inconsistencies.
Non-ordinal concepts	Assign, for each individual, the value that is most voted by the users.
Constant updating information demands	Build the fuzzy ontology using utility values.
Objective information	Use a corpus instead of asking the users.

Table 36: Possible design issues and their solutions. GDM acronym refers to Group Decision Making.

to extract information from a specific database is proposed. Moreover, [ZMFW10] proposes an automatic approach for building ontologies using databases. Also, in [SGB$^+$15], some of the reasoning methodologies are automatized. Nevertheless, all of these processes need a database of some previous stored information. In our designed method, the ontology relation values information is directly extracted from users, no intermediate database is needed. Therefore, ontology builders only define the Group Decision Making environment while users fill all the information. Due to the way that the information is retrieved, it is stored and ratified by a high amount of users. Therefore, the extraction process defined also carries out some validation among the received data. Consequently, the designed method is highly recommended when ontology builders want to build a new ontology from scratch and using users knowledge. Methods like [ZMFW10], can be of more use when a proper database containing all the needed information is available.

8. Concluding remarks and future work

8.1. Concluding remarks

In this dissertation, several Decision Support Systems that can be operated by Internet users to make the most out of Web 2.0 technologies have been designed. These tools use group decision making methods in order to allow users to communicate and make consensual decisions independently of their location. Fuzzy Ontologies are used as a supporting tool that helps to store information in an organized way. Finally, multi-granular fuzzy linguistic modelling methods have been used to ease the way that users communicate with the computational system.

As stated in the objectives section, before starting to develop novel methods, basis of the tools that we want to employ had to be revised. Therefore, basis of linguistic modelling and multi-granular fuzzy linguistic modellings have been presented. The process followed by these kind of methods in order to create a user-friendly interface system has been exposed. Next, group decision making basis has been showed. Afterwards, the structure and steps that must be followed in order to carry out a group decision making process has been presented. Examples of aggregation operators that can be used in the collective matrix calculation have been showed. Also, consensus and feedback operators whose purpose is to measure and promote the consensus in group decision making processes have been exposed. Finally, selection operators that can be employed to carry out the ranking calculation step have been disclosed. In order for the reader to understand all the presented tools, a group decision making process example has been performed. One of the issues of Web 2.0 technologies that we wanted to solve in this dissertation is the high amount of available information problem. In order to manage it, we have proposed the use of Fuzzy Ontologies. Therefore, basis of Fuzzy Ontologies have been introduced.

After exposing the basis of the tools that we are going to employ, an analysis of the state-of-the-art methods of multi-granular fuzzy linguistic information management in the group decision making field has been exposed. In total, six different approaches has been elucidated. Advantages and drawbacks of each of them have been showed. Finally, some future proposals has been showed. As we stated in the objectives section, knowing the state of the art of multi-granular fuzzy linguistic modelling methods is critical in order to be able to develop novel methods.

After revising basis on group decision making methods, Fuzzy Ontologies and carrying out a thorough study about multi-granular fuzzy linguistic modelling methods, a new group decision making method has been proposed. This method has been designed to work over the Internet using smartphones and Web 2.0 technologies. The designed method takes into account the fact that experts cannot be connected all the time to the system and that new alternatives can appear at any time during the group decision making process. Also, it is possible that some of the alternatives are discarded during the discussion and, therefore, it is desirable to remove them from the process. In order to be able to work in dynamic environments like this, the group decision making method has been designed in a way that experts and alternatives can be added and removed from the process at any time.

As it has been previously stated, one of our goals has been to use Fuzzy Ontologies in our designed Decision Support Systems for solving the information storage problem. Before applying them to our designed systems, we have carried out a study about how Fuzzy Ontologies can get benefit and become more user-friendly if multi-granular fuzzy linguistic modelling methods are applied.

After carrying out this study, two novel tools that use Fuzzy Ontologies along with group decision making methods and multi-granular fuzzy linguistic modelling have been developed:

- **A linguistic mobile group decision support system based on Fuzzy Ontologies to facilitate knowledge mobilization:** This method effectively uses Fuzzy Ontologies in order to reduce the number of alternatives that the experts have to discuss about. The method has been built in order to work on smartphones. Also, GPS has been used in order to modify the available alternatives depending on the place where the experts are located. Thanks to this method, we have been able to achieve the goal of dealing with a high amount of alternatives in group decision making problems.

- **An automatized process to create knowledge databases for storing and sharing people knowledge:** Traditional group decision making methods and the previous novel developed methods did not store the results for posterior uses. As we have pinpointed in chapter 1, Web 2.0 technologies is about sharing information and making the most out of other users experiences. This method uses group decision making procedures in order to extract information from a high number of experts. Afterwards, the information is stored in an organized way on a Fuzzy Ontology. Thus, other users can get benefit from the stored knowledge if they carry out queries over it.

8.2. Conclusiones

En esta tesis hemos presentado varios Sistemas de Soporte para la Toma de Decisiones novedosos diseñados para que los usuarios de Internet puedan sacar el máximo partido a las tecnologías Web 2.0. Estas herramientas utilizan métodos de toma de decisiones en grupo para permitir a los usuarios comunicarse y tomar decisiones consensuadas independientemente de su localización. Además, se ha hecho uso de las Ontologías Difusas como método de guardado de la información ya que nos permite realizar un proceso de almacenamiento ordenado que nos permite buscar y recuperar la información de forma sencilla. Finalmente, las herramientas que hemos diseñado hacen uso de métodos de modelado lingüístico multi-granular que mejoran significativamente la comunicación usuario-sistema.

Tal y como se indica en la sección de objetivos, antes de desarrollar nuevos métodos es necesario estudiar las bases de las herramientas que vamos a utilizar. Por ello, hemos presentado los conceptos necesarios para comprender los métodos propuestos en esta tesis. Entre estos conceptos se encuentran el modelado lingüístico y el modelado lingüístico multigranular. Concretamente, hemos descrito el proceso por el cual este tipo de métodos son capaces de crear un sistema de comunicación usuario-sistema agradable para el usuario. A continuación, nos hemos centrado en la exposición de las bases de los métodos de toma de decisiones en grupo. Hemos comenzado exponiendo la estructura y pasos que estos algoritmos siguen para llevar a cabo su objetivo. Además, hemos comentado los diversos operadores de agregación que pueden usarse en el cálculo de la matriz colectiva. También hemos mostrado los operadores de consenso y recomendaciones que pueden utilizarse para medir y promover el consenso entre los distintos expertos que participan en el proceso. Finalmente, hemos expuesto varios operadores de selección que pueden usarse para calcular el ranking de alternativas. Para que el lector pueda entender mejor cómo funcionan este tipo de herramientas, hemos realizado un ejemplo práctico de un proceso de toma de decisiones. Otro problema de la Web 2.0 que hemos querido tratar de resolver en esta tesis es el problema del tratamiento de la alta cantidad de información disponible. Para ello, hemos optado por el uso de las Ontologías Difusas. Con el objetivo de entenderlas mejor, hemos llevado a cabo una introducción a la estructura y manejo de esta herramienta.

Tras haber expuesto las bases de los métodos que vamos a emplear, hemos realizado un análisis del estado del arte de la aplicación de los métodos de manejo de información lingüística multi-granular en el campo de la toma de decisiones en grupo. En dicho análisis, los diferentes métodos observados se han clasificado en seis categorías diferentes. A continuación, hemos analizado las ventajas e inconvenientes de cada método. Finalmente, el capítulo acaba exponiendo varias posibles líneas de investigación futuras. Tal y como hemos indicado en la sección de objetivos, conocer el estado del arte de los métodos de modelado lingüístico multigranular ha sido un paso decisivo a la hora de desarrollar nuestros propios métodos de Soporte para la Toma de Decisiones.

Tras revisar los métodos de toma de decisiones en grupo, su estado del arte, las Ontologías Difusas y el manejo de información lingüística multi-granular, se ha propuesto un nuevo método de toma de decisiones en grupo. Este método ha sido diseñado para funcionar usando Internet y las tecnologías Web 2.0. En su desarrollo, se ha tenido en cuenta el hecho de que los expertos no pueden estar conectados todo el tiempo al sistema y, por tanto, pueden abandonar y volver al proceso en cualquier momento. Además, se ha considerado la posibilidad de que algunas de las alternativas se descarten durante el proceso y que algunas nuevas aparezcan. De esta forma, el proceso de toma de decisiones ha sido diseñado para que sea capaz de trabajar en este tipo de entornos dinámicos.

Como hemos comentado antes, uno de nuestros objetivos era usar las Ontologías Difusas para manejar la información generada por los Sistemas de Soporte para la Toma de Decisiones desarrollados. Antes de comenzar a diseñar nuevos métodos siguiendo esta nueva línea, hemos realizado un estudio sobre como las

Ontologías Difusas pueden beneficiarse y volverse más sencillas de utilizar para el usuario si se usan métodos de manejo de información lingüística multi-granular.

Tras llevar a cabo este estudio, hemos desarrollado dos nuevas herramientas que utilizan Ontologías Difusas así como métodos de toma de decisiones en grupo y métodos de manejo de información lingüística multi-granular:

- **Sistema de soporte móvil de toma de decisiones en grupo basado en Ontologías Difusas para facilitar la movilización de conocimiento**: Este método usa Ontologías Difusas para reducir el número de alternativas sobre las que los expertos debaten. Con el objetivo de permitir su uso en cualquier lugar, se ha diseñado de forma que funcione en smartphones. Además, se ha utilizado la señal de GPS del móvil para modificar las alternativas disponibles dependiendo del lugar en donde los expertos se encuentren. Gracias a este método hemos sido capaces de alcanzar una de las metas que nos habíamos propuesto al inicio de la tesis, crear Sistemas de Soporte a la Toma de Decisiones que fueran capaces de tratar con un alto número de alternativas.

- **Un proceso automatizado para crear bases de datos de conocimiento para el almacenamiento e intercambio del conocimiento de la gente:** Tanto los métodos tradicionales de toma de decisiones como los primeros métodos que hemos desarrollado en esta tesis no guardan los resultados del proceso de toma de decisiones para su posterior uso. Tal y como hemos señalado en el capítulo 1, las tecnologías Web 2.0 tratan acerca de compartir información y sacar el máximo provecho de las experiencias de otros usuarios. Por ello, hemos diseñado un método que utiliza procedimientos de toma de decisiones en grupo para extraer información de un gran número de expertos. Después, la información es almacenada de forma ordenada en una Ontología Difusa. De esta forma, otros usuarios pueden beneficiarse del conocimiento almacenado si llevan a cabo procesos de búsqueda de información sobre la Ontología creada.

8.3. Associated publications to the dissertation thesis

In this section, a list of all the scientific journals and national and international conference publications associated to this dissertation thesis are exposed.

The international journal publications associated to this dissertation are exposed below:

- J.A. Morente-Molinera, I.J. Pérez, M.R. Ureña, E. Herrera-Viedma, On multi-granular fuzzy linguistic modelling in group decision making problems: a systematic review and future trends. Knowledge-Based Systems 74 (2015) 49-60.

- J. A. Morente-Molinera, R. Al-Hmouz, A. Morfeq, A. S. Balamash, E. Herrera-Viedma, A decision support system for decision making in changeable and multi-granular fuzzy linguistic contexts. Journal of Multi-valued Logic and Soft Computing (2015) **(in press)**.

- J. A. Morente-Molinera, I. J. Pérez, M. R. Ureña, E. Herrera-Viedma, Building and Managing Fuzzy Ontologies with Heterogeneous Linguistic Information. Knowledge-Based Systems. **(in press)**.

- J. A. Morente-Molinera, I. J. Pérez, M. R. Ureña, E. Herrera-Viedma, Creating knowledge databases for storing and share people knowledge automatically using group decision making and fuzzy ontologies. Information Sciences. **Accepted for publication**.

- J. A. Morente-Molinera, R. Wikström, E. Herrera-Viedma, C. Carlsson, An implementation of a linguistic mobile decision support system based on fuzzy ontologies to facilitate knowledge mobilization. Decision Support Systems **Accepted for publication**.

The international conference publications are showed below:

- J.A. Morente-Molinera, I.J. Pérez, R. Wikström, E. Herrera-Viedma, C. Carlsson, Designing a decision support system for recommending smartphones using fuzzy ontologies. IEEE Intelligent Systems 2014 (IS2014). Advances in Intelligent Systems and Computing 323, 323-334, Warsaw (Poland), September 24-26, 2014.

- J.A. Morente-Molinera, J.D. Castellón Fuentes, E. Herrera-Viedma, A.G. López-Herrera, A decision making web platform for educational centers. International Technology, Education and Development Conference (INTED 2013), INTED2013 Proceedings, 1368-1376, Valencia (Spain), March 4-5, 2013.

- J.A. Morente-Molinera, I. J. Pérez, F. Chiclana, E. Herrera-Viedma, A novel group decision making method to overcome the Web 2.0 challenges. IEEE International Conference on Systems, Man, and Cybernetics (SMC 2015), Hong Kong (China), October 9-12, 2015.

The national conference publications are listed below:

- J.A. Morente-Molinera, M.R. Ureña, E. Herrera-Viedma, Plataforma Web 2.0 para la gestión de toma de decisiones en centros educativos. XVII Congreso Español sobre Tecnologías y Lógica Fuzzy (ESTYLF 2014).

8.4. Future Work

In this section, trends and future work that we are planning to perform regarding to this dissertation topic are shown:

1. One of the point where we will focus our research is in the development of new multi-granular fuzzy linguistic modelling methods. By using hesitant fuzzy sets and type-2 fuzzy sets we will try to overcome the actual problems that these kind of method entails. This way, we will try to increase the representation capability and expressibility of the actual methods. Consequently, we will try to define novel methods that are able to work correctly in the largest number of possible scenarios.

2. Except for the linguistic mobile group decision support system exposed in Chapter 8, there is no working implementation for any of the proposed methods. Therefore, from now on, we will focus our research in carrying out implementations of the exposed systems in order to provide real solutions to the Internet users.

3. We will also continue in our actual research line of designing new methods that help users to make the most out of Web 2.0 technologies and smartphones. In the present days, there is a clear need for mobile applications that help users to get assistance and carry out daily tasks.

4. We will study recommender systems and big data systems like Hadoop and figure it out if they can contribute in the creation of novel systems that can assess users in different matters using Web 2.0 technologies.

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