Numerical determination of frequency behavior in cloaking structures based on *L-C* distributed networks with TLM method

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Abstract: The increasing interest in metamaterials with negative refractive index has been prompted by a variety of promising optical and microwave applications. Often, the resulting electromagnetic problems to be solve are not analytically derivable; therefore, numerical modeling must be employed and the Transmission Line Modeling (TLM) method constitutes a possible choice. After having greatly simplified the existing TLM techniques for the modeling of metamaterials, we propose in this paper to carry out a frequency study of cloaking structure.

OCIS codes: (230.3205) Invisibility cloaks; (050.1755) Computational electromagnetic methods; (160.3918) Metamaterials.

References and links

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1. Introduction

TLM is a well-known time-domain numerical method which can be regarded as a pure modeling procedure rather than a Maxwell's equations solver. Indeed, TLM consists of modeling a certain medium and its electromagnetic (EM) properties by filling the field space with a network of transmission lines formed by interconnecting unitary circuits or cells,

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termed nodes [1]. Any variation of the EM parameters, such as permittivity, permeability, or conductivity is achieved by adding extra stub transmission lines to the nodes [2]. This characteristic confers to TLM an interesting versatility, which allows simple and elegant modeling of non-homogeneous media [3].

The increasing interest in the study of metamaterials (MM) has made necessary to adapt TLM to them. So *et al*. have presented a novel technique for modeling two- (2D) and three-dimensional (3D) MM with TLM [4]. However, as it has been pointed out by the authors themselves, the new node presented in this innovative paper requires a significant modification of the impulse scattering process when compared to the classical simple version. Considering that these artificial media are on the cutting-edge of current research, it is of first interest to render more comfortable the modeling involving MM. In this sense, it has been shown recently that such a drastic modification is not necessary for the 2D case; a simple 2D condensed node with only three new stubs is perfectly able to model materials with negative parameters [5]. This new technique offers the possibility of easily modeling complex structures involving MM whilst maintaining the potentiality of a time domain method. After a complete presentation of the new nodes (TE and TM modes), we propose in this article to model cloaking structures in order to determine their behavior in terms of the frequency [6,7]. It is worth noting that the TLM technique presented here is nothing more than the numerical incarnation of the well-known dual transmission line approach of MM [8]; this renders the frequency study particularly interesting since the TLM mesh scheme is exactly the same as the real dual network that could be employed to construct an effective cloaking structure.

First, we propose in this paper a solid description of the series node for MM associated with a TM mode, the scattering matrix is in particular entirely given. Second, the description, already given in [5], of the parallel node associated with a TE mode is completed. Third, by using the approximation presented by Huang *et al*. [9], a cloaked Perfectly Electric Conducting (PEC) cylinder is modeled, while the time-domain nature of TLM is exploited so as to study, with a single simulation run, its frequency behavior over a wide spectral range. Although the far field pattern for a simple PEC cylinder is different depending on the TE or TM mode nature of the illuminating EM wave, the results are shown to be the same for the two polarizations once the cylinder is concealed in the cloaking shell.

2. TLM modeling of metamaterials

2.1 Series node for TM modes

For propagation on the *x*-*y* plane, the only non-zero field components for a TM mode are H_z, E_x, and E_y. This polarization may be simulated by a TLM series node formed with seven transmission lines [10]. The link lines 1 to 4 are of identical impedance $Z_0=1/Y_0$. Line 5, with impedance Z_zZ_0, adds extra inductance (shown to be $L_z= Z_zZ_0\Delta t/2$) to the node, allowing an independent control of the relative permeability μ_z; while lines 6 and 7, with admittance Y_xY_0 and Y_yY_0 respectively, are stubs which add extra capacitance (shown to be $C_x = Y_xY_0\Delta t/2$ and $C_y = Y_yY_0\Delta t/2$ respectively) to the node, allowing an independent control of the relative permittivity ε_x and ε_y respectively, Δt being the TLM time-step.

As shown in Fig. 1, this node can be split into three parts: a series circuit made up of five transmission lines describing the H_z component, and two parallel circuits, each one constituted by three transmission lines, for the E_x and E_y components. Each sub-circuit is described by an equation providing Y_x, Y_y, and Z_z:

$$4Z_0\frac{\Delta t}{2}+Z_zZ_0\frac{\Delta t}{2}=\mu_z\mu_0\frac{\Delta x\Delta y}{\Delta z},\quad 2Y_0\frac{\Delta t}{2}+Y_yY_0\frac{\Delta t}{2}=\varepsilon_y\varepsilon_0\frac{\Delta x\Delta z}{\Delta y},\quad 2Y_0\frac{\Delta t}{2}+Y_xY_0\frac{\Delta t}{2}=\varepsilon_x\varepsilon_0\frac{\Delta y\Delta z}{\Delta x}. \tag{1}$$

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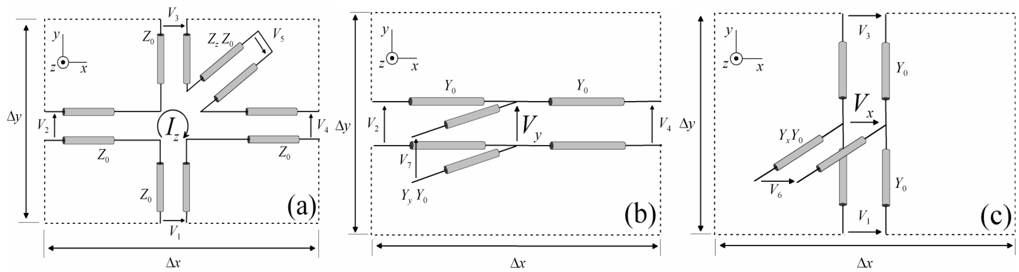

Fig. 1. Splitting of the TLM series node into 3 sub-circuits: (a) series sub-circuit for H_z, (b) parallel sub-circuits for E_y, and (c) parallel sub-circuits for E_x.

Usually, the impedance Z_0 is chosen to make Z_z of Eq. (1) equal to zero in free space (with impedance η_0) in the case $\Delta x = \Delta y = \Delta z$. This yields $Z_0 = \eta_0/\sqrt{2}$.

The scattering matrix $\boldsymbol{S}$ is obtained by following the procedure described in [11]:

$$\begin{bmatrix} a_x & c & b_x & -c & g & i_x & 0 \\ c & a_y & -c & b_y & -g & 0 & i_y \\ b_x & -c & a_x & c & -g & i_x & 0 \\ -c & b_y & c & a_y & g & 0 & i_y \\ d & -d & -d & d & f & 0 & 0 \\ e_x & 0 & e_x & 0 & 0 & h_x & 0 \\ 0 & e_y & 0 & e_y & 0 & 0 & h_y \end{bmatrix}, \tag{2}$$

$$\begin{aligned} a_j &= \frac{2}{2+Y_j} - \frac{2}{4+Z_z}, \quad c = \frac{2}{4+Z_z}, \quad e_j = \frac{2}{2+Y_j}, \quad g = \frac{2}{4+Z_z}, \quad i_j = \frac{2Y_j}{Y_j+2}. \\ b_j &= -\frac{Y_j}{2+Y_j} + \frac{2}{4+Z_z}, \quad d = \frac{2Z_z}{4+Z_z}, \quad f = \frac{4-Z_z}{4+Z_z}, \quad h_j = \frac{Y_j-2}{Y_j+2}, \quad \left(\text{with } j = \{x, y\}\right) \end{aligned} \tag{3}$$

Concerning the calculation of the EM field at the center of a stubbed SCN, its three components may be obtained by using Thevenin equivalents of the circuits shown in Fig. 1 [2],

$$H_z = \frac{2\left(-V_1^i + V_2^i + V_3^i - V_4^i + V_5^i\right)}{\Delta z Z_0 \left(4+Z_z\right)}, \qquad E_y = \frac{2\left(V_2^i + V_4^i + Y_y V_7^i\right)}{\Delta y \left(2+Y_y\right)}, \qquad E_x = \frac{2\left(V_1^i + V_3^i + Y_x V_6^i\right)}{\Delta x \left(2+Y_x\right)}. \tag{4}$$

A particular field component at a node is excited by injecting voltage into the appropriate lines. The hereunder set of voltages excites the desired EM field:

$$\begin{aligned} V_1^i &= 0.5\left(\Delta x E_x - Z_0 \Delta z H_z\right) \qquad & V_5^i &= 0.5 Z_z Z_0 \Delta z H_z \\ V_2^i &= 0.5\left(\Delta y E_y + Z_0 \Delta z H_z\right) \qquad & V_6^i &= 0.5 \Delta x E_x \\ V_3^i &= 0.5\left(\Delta x E_x + Z_0 \Delta z H_z\right) \qquad & V_7^i &= 0.5 \Delta y E_y. \\ V_4^i &= 0.5\left(\Delta y E_y - Z_0 \Delta z H_z\right) & & \end{aligned} \tag{5}$$

2.2 Metamaterials modeling with a series node

The above development allows the modeling of usual material. Adapting it to MM is very natural. Instead of equipping the series sub-circuit of Fig. 1(a) with an inductive stub, we provide it with a capacitive stub. As a result, the introduced capacitance into the node, $C = Y_z Y_0 \Delta t/2$, is equivalent to a frequency-dependent negative inductance, $L_{eq} = -\frac{1}{C\omega^2} = -\frac{Z_z Z_0}{\omega^2}\frac{2}{\Delta t}$.

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In the same way, the capacitive stubs of the parallel sub-circuits of Fig. 1 are substituted by inductive stubs, similar expressions applying for these other two lines. Finally, Y_x, Y_y, and Z_z of Eq. (1) are now given by:

$$Z_z = -\frac{\Delta t^2\omega^2}{4}\left[\frac{2\mu_z\mu_0}{\Delta t\, Z_0}\frac{\Delta x\Delta y}{\Delta z}-4\right],\quad Y_y = -\frac{\Delta t^2\omega^2}{4}\left[\frac{\varepsilon_y\varepsilon_0}{\Delta t Y_0}\frac{\Delta x\Delta z}{\Delta y}-4\right],\quad Y_x = -\frac{\Delta t^2\omega^2}{4}\left[\frac{\varepsilon_x\varepsilon_0}{\Delta t Y_0}\frac{\Delta y\Delta z}{\Delta x}-4\right]. \qquad (6)$$

This set of equations allows modeling, for a certain frequency, relative ε and μ below unity and even the zero value, which becomes a natural value with this approach. It is worth noting that the impedance and admittances given by Eq. (1) diverge only by the factor $-\Delta t^2\omega^2/4$ respectively to those of Eq. (6). The scattering matrix, as well as Eqs. (3-5), is absolutely unaltered with respect to the classical node, which renders the technique very comfortable to use.

2.3 Parallel node for TE modes

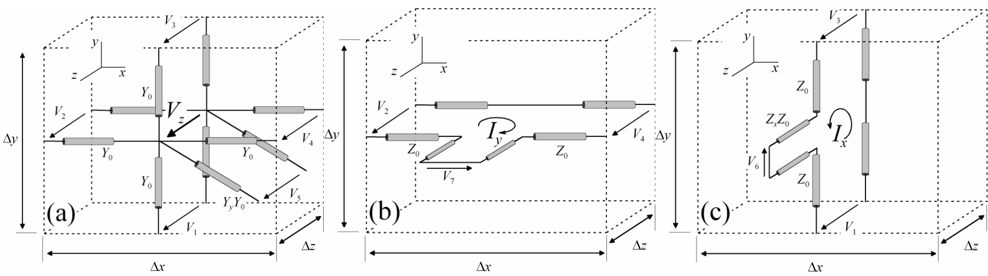

Fig. 2. Splitting of the TLM parallel node into 3 sub-circuits: (a) parallel sub-circuit for E_z, (b) series sub-circuits for H_y, and (c) series sub-circuits for H_x.

For propagation on the *x*-*y* plane, the only non-zero field components for a TE mode are E_z, H_x, and H_y. The parallel node is required for such a polarization, and it is obtained as it has been done hereinbefore for the series node. The three circuits describing each component of the EM field are depicted in Fig. 2. Similarly to Eq. (1), the corresponding equations giving Y_z, Z_x, and Z_y are:

$$4Y_0\frac{\Delta t}{2}+Y_zY_0\frac{\Delta t}{2}=\varepsilon_z\varepsilon_0\frac{\Delta x\Delta y}{\Delta z},\qquad 2Z_0\frac{\Delta t}{2}+Z_yZ_0\frac{\Delta t}{2}=\mu_y\mu_0\frac{\Delta x\Delta z}{\Delta y},\qquad 2Z_0\frac{\Delta t}{2}+Z_xZ_0\frac{\Delta t}{2}=\mu_x\mu_0\frac{\Delta y\Delta z}{\Delta x}. \qquad (7)$$

In Eq. (7), Z_0 is usually chosen to be $Z_0 = \eta_0\sqrt{2}$, so that there is no stub if free space is modeled with $\Delta x = \Delta y = \Delta z$.

The scattering matrix, as well as its elements, may in this case be written as

$$\begin{bmatrix} a_x & c & b_x & c & g & -i_x & 0 \\ c & a_y & c & b_y & g & 0 & i_y \\ b_x & c & a_x & c & g & i_x & 0 \\ c & b_y & c & a_y & g & 0 & -i_y \\ c & c & c & c & c & f & 0 \\ -e_x & 0 & e_x & 0 & 0 & h_x & 0 \\ 0 & e_y & 0 & -e_y & 0 & 0 & h_y \end{bmatrix}, \qquad (8)$$

$$\begin{aligned} a_j &= \frac{2}{4+Y_z}-\frac{2}{2+Z_j} & c &= \frac{2}{4+Y_z} & f &= \frac{Y_z-4}{Y_z+4} & h_j &= \frac{2-Z_j}{2+Z_j} \\ b_j &= \frac{Y_z}{2+Y_z}-\frac{Z_j}{4+Z_j} & e_j &= \frac{2Z_j}{2+Z_j} & g &= \frac{2Y_z}{Y_z+4} & i_j &= \frac{2}{2+Z_j}. \quad (\text{with } j=\{x,y\}) \end{aligned} \qquad (9)$$

The three components of the EM fields at the center of each node are

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$$E_z = \frac{2\left(V_1^i + V_2^i + V_3^i + V_4^i + Y_z V_5^i\right)}{\Delta z\left(4+Y_z\right)}, \qquad H_y = \frac{2\left(-V_2^i + V_4^i + V_7^i\right)}{\Delta y Z_0\left(2+Z_y\right)}, \qquad H_x = \frac{2\left(V_1^i - V_3^i + V_6^i\right)}{\Delta x Z_0\left(2+Z_x\right)}, \tag{10}$$

while the following set of incident pulses allows exciting the desired EM source:

$$\begin{aligned} V_1^i &= 0.5\left(\Delta z E_z + Z_0 \Delta x H_x\right) & V_5^i &= 0.5\Delta z E_z \\ V_2^i &= 0.5\left(\Delta z E_z - Z_0 \Delta y H_y\right) & V_6^i &= 0.5 Z_x Z_0 \Delta x H_x \\ V_3^i &= 0.5\left(\Delta z E_z - Z_0 \Delta x H_x\right) & V_7^i &= 0.5 Z_y Z_0 \Delta y H_y. \\ V_4^i &= 0.5\left(\Delta z E_z + Z_0 \Delta y H_y\right) & & \end{aligned} \tag{11}$$

Finally, the procedure adopted in Section 2.2 may be employed for the modeling of MM with the parallel node. This lets Eqs. (8-11) absolutely unchanged, while Eq. (7) becomes:

$$Y_z = -\frac{\Delta t^2\omega^2}{4}\left[\frac{2\varepsilon_z\varepsilon_0}{\Delta t Y_0}\frac{\Delta x\Delta y}{\Delta z} - 4\right], \quad Z_y = -\frac{\Delta t^2\omega^2}{4}\left[\frac{\mu_y\mu_0}{\Delta t\, Z_0}\frac{\Delta x\Delta z}{\Delta y} - 4\right], \quad Z_x = -\frac{\Delta t^2\omega^2}{4}\left[\frac{\mu_x\mu_0}{\Delta t\, Z_0}\frac{\Delta y\Delta z}{\Delta x} - 4\right]. \tag{12}$$

2.4 Modeling of metamaterials with the 3D Symmetrical Condensed Node

Finally, we would like to make a comment on the 3D modeling of MM. The well-known Symmetrical Condensed Node (SCN) is usually employed in TLM for 3D simulation because it offers very high performance [12]. In its original version, the SCN has six extra lines (three capacitive open circuits for the *E*-field, and three inductive short circuits for the *H*-field), in addition to the regular 12 link lines. So *et al.* have claimed that the corresponding scattering matrix for modeling metamaterials with the 3D SCN would be of size 27×27, which is unnecessarily too big to be easily implemented, and pushes the authors to introduce an inter-cell approach to get around the problem [4]. Actually, in the light of the development presented hereinbefore, it appears that the original SCN described by the 18×18 scattering matrix is perfectly well suited for the implementation of MM; there is in particular no need to add neither extra lines nor changes in the scattering matrix or inter-cell positions. The only requirement is to substitute the inductive stubs by capacitive stubs, and vice versa.

3. Numerical results

Employing a time domain method like TLM for the simulation of a cloaked object presents a certain advantages. For instance, the actual dynamic process can be modeled [13], or the frequency behavior can be reached from a simple Fourier Transform. A cloaked PEC infinite cylinder is modeled with the series node for TM mode, following the same approach presented by Huang *et al.* [9]. This approach consists basically in substituting the anisotropic medium of the shell by a concentric layered structure of alternating homogeneous isotropic materials. Furthermore, the permittivity and permeability of the layers are calculated very close to their inner boundary (but not rigorously in the vicinity of the boundary in order to avoid divergence of the EM parameters in the first layer) since this procedure has been shown to give the best results [5]. The cloaking entity is illuminated by a Gaussian pulse, which theoretically contains all the frequencies. Nevertheless, 2 GHz is chosen to be the working frequency in order to fix the parameters of Eq. (6), which are frequency dependent. The inner and the outer radius of the cloaking shell are R_1 = 0.1 m and R_2 = 0.2 m, respectively, and it is made up of 20 layers. The far field pattern for 2 GHz is depicted in Fig. 3; moreover, by using a simple Fourier Transform, the far field in terms of the frequency for five different angles (from 0º to 180º using 45º steps) is obtained and shown in the same Fig. 3. As expected, the cloaking shell is manifestly efficient for the 2 GHz functional frequency, but it is worth noting that a frequency band appears around this frequency for all directions with the noticeable exception of 0º. This last direction is characterized by a very narrow low radiation region, which shows that it is the most conflictive direction.

On the other hand, simulating the TE mode requires the use of the parallel node presented in section 2.3. It turns out that the results for this polarization are exactly the same as for the

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TM mode. This means that, although the cloaking is not perfect, the PEC cylinder is well concealed by the cloaking shell. Indeed, it is well known, and moreover verified by our TLM simulations, that the Scattering Width of a simple circular conducting cylinder depends strongly on the polarization. In order to get a decisive confirmation of this observation, the

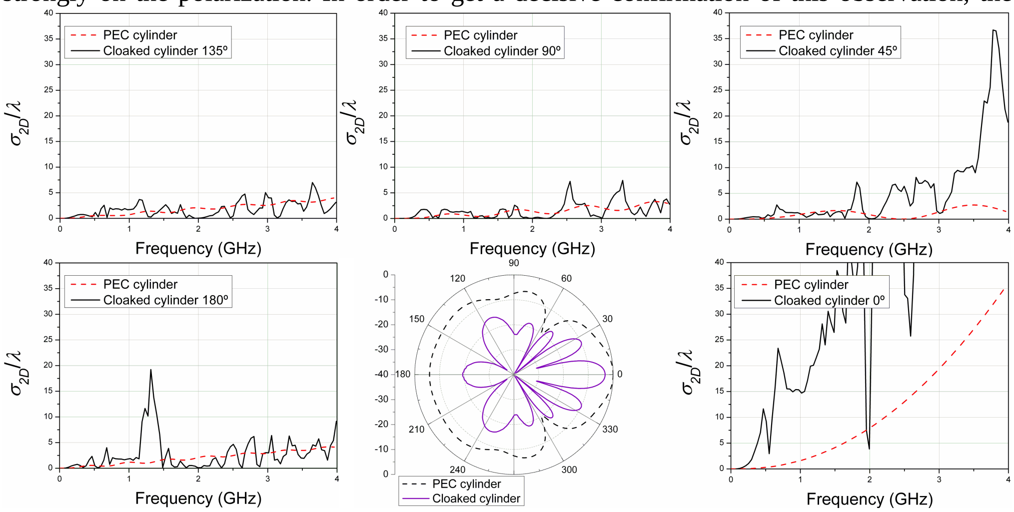

Fig. 3. (Online color) Far field pattern of a cloaking structure at 2GHz, and its Scattering Width versus frequency for 0º, 45º, 90º, 135º, 180º.

same cloaking structure, but without the cylinder inside, is modeled, and once again, the same results are obtained. To explain this, let us note that the first layer of the cloaking shell in our simulation has a relative permittivity $\varepsilon = 2.10^{5}$, and relative permeability $\mu = 4.10^{-5}$. It is reasonable to think that these extreme values shield the central region. Nevertheless, the other layers are not perfectly able to steer the radiation around themselves due to both numerical discretization and to the approximation consisting on substituting the theoretical anisotropic material by isotropic layers, which produces the observable forward scattering.

Finally, it is worth noting that the technique presented above corresponds to an actual situation for the whole frequency range. Indeed, it is well known that a distributed system with series capacitors and shunt inductors is a possible way to build materials with permittivity and permeability values below unity. Therefore, the TLM nodes presented in this paper for modeling MM, are completely equivalent to such a dual *L-C* network, and are thus expected to perfectly describe it, not only at the design frequency, but also for the whole frequency band.

4. Conclusion

TLM nodes for MM modeling have been reported in this article; one for the 2D TM mode, another one for the TE mode. We have proposed a detailed description of each one; the corresponding scattering matrices have been given. These new nodes have the merit to leave the original ones almost unaltered.

As an application, we have built a cloaking structure for a TM polarization. The advantages offered by a time domain method such as TLM have been applied to propose a behavior study in terms of the frequency in the cloaked PEC cylinder. In this sense, it is worth noting that modeling a cloaking structure with TLM is more than simply solving Maxwell's equations, it is indeed equivalent to substituting the cloak shell by a dual distributed *L-C* network, which is well known as an actual technique for implementing the exotic material parameters associated with MM.

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