Deviations of Lambert-Beer's law affect corneal refractive parameters after refractive surgery

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Abstract: We calculate whether deviations of Lambert-Beer's law, which regulates depth ablation during corneal ablation, significantly influence corneal refractive parameters after refractive surgery and whether they influence visual performance. For this, we compute a point-to-point correction on the cornea while assuming a non-linear (including a quadratic term) fit for depth ablation. Post-surgical equations for refractive parameters using a non-linear fit show significant differences with respect to parameters obtained from a linear fit (Lambert-Beer's law). Differences were also significant for corneal aberrations. These results show that corneal-ablation algorithms should include analytical information on deviations from Lambert-Beer's law for achieving an accurate eye correction

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1. Introduction

Discrepancies between real and expected post-surgical corneal shape can limit the outcome of corneal refractive surgery, avoiding an effective correction of eye-aberrations, improvements of visual performance, and even emmetropization [1-2]. Physical aspects of corneal ablation, among other factors (e.g. corneal biomechanics), cause such discrepancies [3-7].

An essential point in corneal ablation is to quantify with high accuracy the stroma removed per pulse. Usually, Lambert-Beer's law [8-13] (also called the blow-off model [13]) is assumed for photo-ablation of corneal tissue:

$$d_p = m \cdot \ln\left(\frac{F_{inc}}{F_{th}}\right) \qquad (1)$$

where d_p is the ablation depth per pulse, m is the slope efficiency of the ablation, F_{inc} is the incident exposure of the laser (energy per illuminated area) pulse and F_{th} is the threshold exposure for the ablation. The quantification of this law would not be very important if the incident exposure at the cornea did not vary during ablation. Reflection losses and non-normal incidence on the cornea are two factors [5-7] that cause the incident exposure differ at each point of the cornea. Therefore, deviations of Lambert-Beer's law are crucial if the incident exposure on the cornea, F_{inc}, varies during the ablation.

In recent years, a number of works [3,5,6] quantify the effect that these factors exert on the ablation while assuming Lambert-Beer's law, but no analysis has been made of how deviations from this law may affect the corneal ablation. In the present work, we evaluate the influence of these deviations and the consequences for post-surgical corneal parameters, corneal spherical aberration and the design of new ablation algorithms.
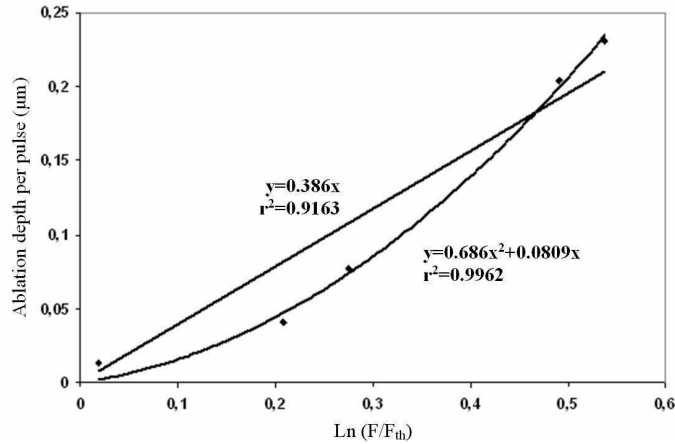
2. Method

Fig. 1. Linear and quadratic fit of experimental data on ablation depth corresponding to Krueger et al. [11]. F and F_{th} indicate the intensity exposure on the cornea and the threshold exposure, respectively.

A numerical review [10-12] of the data provided by experimental works shows that Lambert-Beer's law does not accurately fit experimental results. Most papers perform a linear fit to the equation $y=mx$, with y being the ablation depth per pulse and $x=\ln(F_{inc}/F_{th})$. As an example, Fig. 1 shows experimental data from Krueger et al. [11] on depth ablation in which we have made a linear (Lambert-Beer) fit $y=mx$ and a non-linear fit by including a quadratic term that

quantifies the linear deviation—that is, $y = ax + bx^2$. As can be seen in Fig. 1, the correlation is higher when including a non-linear term. This tendency is similar with other experimental data [10-12].

The point is to know whether the assumption of a non-linear fit significantly alters visual performance as compared to the use of a linear fit. We shall calculate the effects on corneal refractive parameters when Eq. (1), used in practical surgery, is replaced by the following equation, which includes a quadratic term:

$$d_p^{NL} = a \cdot \ln\left(\frac{F_{inc}}{F_{th}}\right) + b \cdot \left(\ln\left(\frac{F_{inc}}{F_{th}}\right)\right)^2 \tag{2}$$

First, we provide an analytic factor which quantifies the point-to-point corneal deviation by using Eq. (2) and, from it we will examine the changes in two important refractive parameters of the cornea (curvature radius and corneal asphericity), evaluating whether these changes are significant for visual performance. Other possibilities of non-linear fit [13] of experimental data will be discussed below.

The point-to-point corneal deviation can be calculated applying the following correction factor [3,5,6]:

$$\rho = \frac{d_p^{NL}}{d_p} = \frac{a \cdot \ln\left(\frac{F_{inc}}{F_{th}}\right) + b \cdot \left(\ln\left(\frac{F_{inc}}{F_{th}}\right)\right)^2}{m \cdot \ln\left(\frac{F_{inc}}{F_{th}}\right)} \tag{3}$$

The parameter ρ in Eq. (3) provides the deviation in the ablation when assuming Lambert-Beer's law while the real ablation depth more closely approaches a non-linear quadratic fit. The parameters a, b and m are determined from the fits of the particular experimental data of the corneal ablation for each laser system. To deduce a general expression that can be evaluated in practice, we need [3,5,6] to calculate Eq. (3) as a function of the geometric corneal parameters, corneal radius, and p-factor ($p=1+Q$, with Q being the corneal asphericity). To determine ρ, we will introduce into the term F_{inc} of the numerator the variations being due to reflection losses and non-normal incidence on the cornea [5]:

$$F_{inc} = F_0\left(\cos\alpha \cdot (1 - \tilde{R})\right) \tag{4}$$

where F_0 is a constant that indicates the maximum exposure, the factor $(1 - \tilde{R})$ provides the information concerning the reflection losses [5] and $\cos\alpha$ concerns non-normal incidence on the cornea [5]. Equations (3-4) depend on the incident point on the cornea and vary across the cornea [3,5] since factor $(1 - \tilde{R})$ and $\cos\alpha$ depend on the incidence height of the laser from the optical axis [3].

There are two possibilities to compute the denominator of ρ in Eq. (3), depending on the variables that the ablation algorithm takes into account. Lambert-Beer's law is assumed for ablation depth but we do not know whether corrections for reflection losses and non-normal incidence are also applied in ablation algorithms (they are proprietary). Therefore, if in the denominator of Eq. (3) we take F_{inc} as given by Eq. (4), the factor ρ would compute the deviation with an algorithm that takes into account reflection losses and non-normal incidence but not any deviation of Lambert-Beer's law. If in the denominator of ρ we assume $F_{inc}=F_0$, the parameter ρ will enable us to compare the deviation with algorithms that assume the Lambert-Beer's law and the incidence exposure on the cornea to be constant for a laser device. In the mathematical procedure described in this paper, we will consider this second

option, $F_{inc}=F_0$, since, as we will discuss below, the first possibility implies the same procedure but mathematically simpler.

To calculate ρ, we will apply a numerical procedure (described extensively in different papers [3,5,6]), in which we assume the conicoid model [3,5,6] and revolution geometry for the anterior cornea. The point is to obtain ρ as a function of laser parameters and the distance from the optical axis [3,5]. The factor ρ can be expressed analytically as a series expansion in the variable y/R up to order 4 [3], where y indicates the distance from the optical axis and R is the corneal radius. Thus, it is necessary to compute the coefficients of the following factor:

$$\rho = \rho_0 + \rho_1\,(y/R)^2 + \rho_2(\,y/R)^4 \tag{5}$$

After computations [3], we get:

$$\rho_0 = \frac{a(-0.0435+t)+b(0.0018-0.087t+t^2)}{mt} \qquad \rho_1 = \frac{-0.5a+b(0.0435-t)}{mt}$$

$$\rho_2 = \frac{a(0.23424-0.5p)+b(0.223+p(0.0435-t)+0.4648t)}{mt} \tag{6}$$

with R and p being the pre-surgery radius and p-factor, respectively, and $t = \ln(F_0/F_{th})$. Although this factor allows us to evaluate a point-to-point correction, we will evaluate its effect on refractive corneal parameters. We will calculate the post-surgical radius, R' and p-factor, p', by applying the standard paraxial Munnerlyn formula for ablation depth corrected (multiplied) by the factor ρ, given by Eq. (3) [5-6]. Therefore, we would obtain the expected refractive parameters after refractive surgery when considering the effects included in the factor ρ. The paraxial Munnerlyn formula, $c(y)$, used in non-customized refractive surgery [1,5] is given by:

$$c(y) = \frac{4Dy^2}{3} - \frac{Ds^2}{3} \tag{7}$$

where s is the ablation diameter and D the number of diopters to correct. Other equations could be tested but algorithms are proprietary and cannot be explicity known. We used the paraxial formula, as it is usual in different works, given that most non-customized algorithms are based on the paraxial formula. We also computed the procedure shown here with the non-paraxial Munnerlyn formula [1] obtaining similar results.

The mathematical procedure, an analytical minimum-squares analysis, can be found elsewhere [1,3,5,6]. After computations, the post-surgical corneal radius and p-factor are given by:

$$\frac{1}{R'} = \frac{(8bD/3)\cdot(-0.053+t)\cdot(-0.034+t)+129.87mt+aD\cdot(-0.116+8t/3)}{mt} \tag{8}$$

$$p' = \frac{am^2t^2DR^3\left(-144+s^2(-16.7328+36p)\right)+27pm^3t^3}{\left\{3mt+DR\left[a\left(s^2-0.348+8t\right)+b\left(0.144-0.87s^2+\left(2s^2-0.696\right)t+8t^2\right)\right]\right\}^3} +$$
$$+ \frac{bsm^2t^2R^3\left(12.528-288t+s^2(-16.538-33.4656t+p(72t-3.132))\right)}{\left\{3mt+DR\left[a\left(s^2-0.348+8t\right)+b\left(0.144-0.87s^2+\left(2s^2-0.696\right)t+8t^2\right)\right]\right\}^3} \tag{9}$$

3. Results and discussion

These general equations provide the final refractive parameters when considering a correcting quadratic term for ablation depth per pulse that includes the deviation of Lambert-Beer's laws and reflection losses and non-normal incidence on the cornea. The point is to compare refractive parameters given by Eqs. (8) and (9) with the values expected after using the standard paraxial Munnerlyn formula but while assuming the Lambert Beer's law and without considering reflection losses and non-normal incidence on the cornea. These values, R'_{Munn} and p'_{Munn} are given by [5]:

$$\frac{8D}{3} = \frac{1}{R'_{Munn}} - \frac{1}{R} \qquad p'_{Munn} = \frac{R'^{3}_{Munn}}{R^3} \, p \qquad (10)$$

For a computational simulation, we used the coefficients a, b and m from Krueger et al. [11] (see numerical values in Fig. 1) and average initial values [3] d=7 mm, R=7.7 mm, p=0.74, t=0.69. We calculated the corneal-power difference $\Delta\varphi=\varphi'-\varphi_{Munn}$ obtained from Eqs. (8) and (10). Corneal power is calculated as $\varphi=\Delta n/R$, with R being the corneal radius and Δn=0.375 the refraction-index difference between the air and cornea. After computations, we get $\Delta\varphi$=0.14D. The results show that the differences are significant for visual performance. For example, from D=-2 (diopters) of initial ametropia, the difference is greater than 0.28D (diopters), a value that clearly reduces the effective visual acuity [6,14]. In addition, a contrast-sensitivity reduction is expected [6,14]. We checked that from t=0.38 (120mJ/cm^2) to t=0.90 (400mJ/cm^2), the difference in corneal power ranged from $\Delta\varphi$=0.14D to $\Delta\varphi$=0.064D, respectively, being significant for visual performance for a wide range of laser fluences.

It is possible to calculate the difference in profile, to provide a quantitative measure of the magnitude of the error. If the paraxial Munnerlyn formula is given by c(y), the ablation profile corrected by the factor ρ is given by $\rho\cdot c$(y), and therefore the ablation profile difference is Δc(y)=(ρ-1)$\cdot c$(y). Δc(y), as a function of t and D is :

$$\Delta c(y) = \frac{0.476D + 3.06Dy^2}{t} + \left(5.61D - 22.384Dy^2\right) + \left(-4.76D + 23.06Dy^2\right)t \qquad (11)$$

Equation (11) is computed in microns if D is given in diopters and y in meters.

Computational simulations can be made with any ablation data [11-13]. We selected Krueger et al. [11] data because their experimental data include low values (x<1) for x=ln(F_{inc}/F_{th}) although no data with x>0.6. These data are more appropriate to our aim, since our calculations include the cosine law and the effects of reflection losses and, it would be expected that the incident exposure on the cornea would give lower values than the maximum exposure , F_0, the farther we get from the optical axis. Data provided by other authors [12-13] show higher values for x, x>1, but only few experimental data for x<1.

Concerning the p-factor, the results lead to the same conclusion: a significant influence in visual performance. An easy computation shows that the post-surgery p-factor difference, $\Delta p'$=$p'-p'_{Munn}$, calculated from Eqs. (9) and (10) is higher than $\Delta p'$=0.01 from 1D (diopters) of initial myopia. Values of $\Delta p'$ higher than 0.01 diminish significantly contrast-sensitivity. Figure 2 shows the corneal primary spherical-aberration difference as a function of the initial degree of myopia computed from the post-surgery p-factor differences. Primary spherical-aberration, S, is given by [14] S=((p-1)$y^4\Delta$n)/R^3. Spherical-aberration differences exceed the quarter-wavelength criterion [8] for aberration from 1D (diopters).

Our results show that deviations of Lambert-Beer's law exert an influence when determining refractive parameters after surgery. During corneal ablation, changes occur in incident exposure on the cornea, and therefore a highly accurate quantification of ablation

depth per pulse is necessary. The analysis shown in this paper can be extended in a similar procedure including more terms in the non-linear fit.
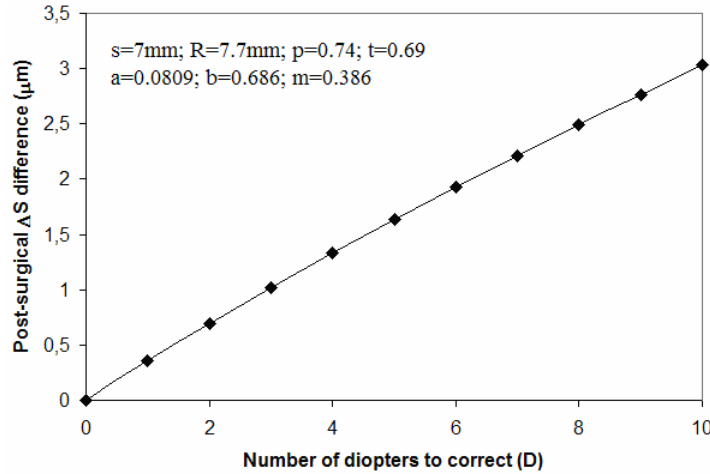

Fig. 2. Post-surgical corneal spherical-aberration difference (ΔS) between computations when considering or not a quadratic deviation from Lambert-Beer's law as a function of the initial degree of myopia.

As commented above, other models could be applied for determining the best fit of the experimental data corresponding to ablation per pulse. For example, the steady-state model for which, the ablation rate, d_p, is given by $d_p = a(x-1)/x$, with $x = F_{inc}/F_{th}$ and a constant, and a model that unifies the blow-off and the steady-state model for which [13], $d_p = a + b\ln((F_{inc}/F_{th}) + c)$ with a, b and c as constants. We computed fits from experimental data [10-12] to these models obtaining lower correlation coefficients ($r < 0.93$) than the non-linear quadratic fit. In any case, for each laser device the important point is to achieve the best analytical adjustment for ablation rate and to apply it jointly to the computation of reflection losses and non-normal incidence.

An additional point is that algorithms are proprietary and it is not possible to know exactly which aspects the companies take into account in their algorithm designs. As indicated above, the analysis shown here assumes a comparison with algorithms that do not consider exposure changes (reflection losses and non-normal incidence on the cornea) and deviations of Lambert-Beer's laws. Equations including reflection losses and non-normal incidence have been published in recent years [3,5,6], and thus it is possible that recent algorithms consider these two effects but not the deviations of Lambert-Beer's law. In this case, an easy computation in Eq. (3) shows that the point-to-point correction in the cornea, ρ', is given by a linear equation type $\rho' = cte_1 + cte_2\left(\cos\alpha \cdot (1 - \tilde{R})\right)$ with cte_1 and cte_2 being constants and depending on laser parameters. For obtaining ρ', using the same procedure shown in methods, we get:

$$\rho' = (\alpha - 0.043\beta) - 0.5\beta y^2 + (0.232 - 0.5p)\beta y^4$$

$$with \quad \alpha = \frac{1}{m}[a + bt] \quad and \quad \beta = \frac{b}{m} \tag{12}$$

Computing the effect on refractive parameters and aberrations for Krueger et al. data [11], we get significant differences for corneal parameters (radius and p-factor) and corneal spherical-aberration although we consider only the deviations of Lambert-Beer's law.

We should indicate that manufacturers could use their corresponding experimentally measured data (with interpolation to obtain a continuous function) for ablation depth as a

function of laser fluence. In that case, the non-lineal behaviour of the ablation depth does not necessarily imply errors in the ablation algorithm.

From these results, our proposal is that ablation algorithms should not assumed Lambert-Beer's law, and should include analytical correction factors or new corneal-ablation laws after experimentally quantifying deviations from Lambert-Beer's law. This could help minimize experimental corneal-shape differences found between the real ablation and the predicted one, a necessary step towards more effective eye correction during surgery.