PHYSICS OF BRANE KINETIC TERMS*

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Models with extra dimensions may give new effects visible at future experiments. In these models, bulk fields can develop localized corrections to their kinetic terms which can modify the phenomenological predictions in a sizeable way. We review the case in which both gauge bosons and fermions propagate in the bulk, and discuss the limits on the parameter space arising from electroweak precision data.

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1. Introduction

Models with extra dimensions have received a lot of attention in the last few years (see [1] for a review of their phenomenology). Besides being motivated by string theory, they provide new mechanisms to face longstanding problems, such as the Planck to electroweak and electroweak to cosmological constant hierarchy problems, the fermion flavour problem, symmetry and supersymmetry breaking among others. The original brane world idea in which only gravity propagates in the extra dimensions and matter and gauge fields are bounded to a four-dimensional brane was soon extended to

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allow for gauge fields and even all matter fields propagating in the bulk of extra dimensions. Even in this case, lower dimensional submanifolds (branes, domain walls, orbifold fixed points or planes, . . .) are usually present in the models. In Ref. [2] it was argued that in the original brane world model, matter and gauge loop corrections to the graviton self-energy generate a curvature term in the action localized at the position of the brane, which in turn has striking phenomenological implications, with brane scientists observing four-dimensional gravity up to cosmological distances even in the case of an infinite flat bulk. Even if this particular model has strong coupling problems problems [3], the idea of Brane localized Kinetic Terms (BKT) may have very important phenomenological consequences not only for gravitational physics but for any kind of bulk field. As a matter of fact, these terms are unavoidable in interacting theories with extra dimensions and lower-dimensional defects. The reason is that translational invariance is broken by the defects, and this allows for localized divergent radiative corrections, which must be cancelled by the corresponding localized counterterms. In particular, BKT are induced in this way. Their coefficients run with the scale, so they cannot be set to zero at all energies. This suggests that one should include them already at tree level, which means that they are not necessarily loop suppressed. The generation of BKT by radiative corrections in orbifolds without brane couplings at tree level was explicitly shown in a particular example in [4].

A detailed study of the implications of general BKT for bulk fields of spin 0, 1/2 and 1 has been carried out in Ref. [5]. There, it was shown that some of the BKT one can write have a smooth behaviour as they get small, whereas others give rise to a singular behaviour in the spectrum and have to be dealt with using classical renormalization order by order in perturbation theory. In the following, we discuss in a pedagogical manner the properties and phenomenology of fermions [5, 6] and gauge bosons [7, 8], studying in some detail the case in which both have BKT. (For a compendious review of BKT with a more complete list of references, see [9].) For simplicity, we will focus on BKT which do not need classical renormalization.

2. Brane kinetic terms for bulk fermions and gauge bosons

We consider a five-dimensional model with the fifth dimension $y$ compactified on an orbifold $S^1/Z_2$, that is to say, a circle of radius $R$ with opposite points identified: $y \sim -y$. The four dimensional subspaces $y = 0, \pi R$ are fixed under the $Z_2$ action. From now on we will call these fixed hyperplanes “branes”, although they are static non-fluctuating objects. As argued in the introduction, the action contains in general kinetic terms localized on these branes, besides the usual Poincaré invariant ones.
Let us discuss fermions first. In five dimensions they are Dirac spinors with two chiral components from the four-dimensional point of view: \( \Psi = \Psi_L + \Psi_R \), \( \gamma^5 \Psi_{L,R} = \mp \Psi_{L,R} \). Invariance of the bulk kinetic term requires that the left-handed (LH) and right-handed (RH) components have opposite \( Z_2 \) parities. We choose the LH and RH components to be even and odd, respectively. Taking into account possible BKT, the general kinetic Lagrangian (with gauge couplings) for a fermion reads

\[
L = (1 + a_I^L \delta_I) \bar{\psi}_L \gamma^\mu \partial_\mu \psi_L + (1 + a_I^R \delta_I) \bar{\psi}_R i \gamma_5 \partial_\mu \psi_R - \left( 1 + \frac{b_I}{2} \delta_I \right) \bar{\psi}_L D_y \psi_R
- \frac{b_I}{2} \delta_I (D_y \bar{\psi}_R) \psi_L + \left( 1 + c_I \delta_I \right) \bar{\psi}_R D_y \psi_L + \frac{c_I}{2} \delta_I (D_y \bar{\psi}_L) \psi_R,
\]

where a sum over \( I = 0, \pi \) is understood and \( \delta_I \equiv \delta(y - IR) \). The BKT which contain \( y \)-derivatives, \( i.e., \) those proportional to \( b_I \) and \( c_I \), give rise to the non-analytical behaviour we have mentioned in the introduction, which has its origin in the fact that the branes in our orbifold are lower-dimensional subspaces, \( i.e., \) they have zero width. This situation may change at small distances in a string theory construction, as the brane gets effectively a microscopic substructure. Within an effective field-theoretical approach, a smooth, well-behaved theory can be recovered implementing at the classical level a renormalization which takes care of \( \delta_0^2 \)-like singularities. We direct the interested reader to Ref. [5] for the details. Here we simply disregard these BKT setting \( b_{0,\pi} = c_{0,\pi} = 0 \). On the other hand, the terms proportional to \( a_I^R \) might naively be argued to vanish, based on the odd character of \( \Psi_R \). However, this is not necessarily so if \( \Psi_R \) is discontinuous at the branes, and it turns out that this is the case when \( a_I^R \) does not vanish. Nevertheless, in the following we also take \( a_I^R = 0 \), which is stable under radiative corrections, to reduce the number of independent parameters in our analysis. Finally, we work with massless fermions\(^1\). The spectrum in four dimensions can be computed by inserting the Kaluza–Klein (KK) expansion of the field,

\[
\Psi_{L,R}(x,y) = \sum_{n=0}^{\infty} \frac{f_{n,R}^L(y)}{2\pi R} \psi^{(n)}(x),
\]

into the action, and requiring the kinetic terms to be diagonal and canonically normalized and the mass terms to be diagonal. This is achieved by the following normalization and eigenvalue conditions:

\[
\int_{-\pi R}^{\pi R} dy \left( 1 + a_0^L \delta_0 + a_2^L \delta_2 \right) \left( \frac{f_{n}^L}{2\pi R} \right)^2 = \int_{-\pi R}^{\pi R} dy \left( \frac{f_{n}^R}{2\pi R} \right)^2 = 1,
\]

\(^1\) In general, one could write Dirac mass terms with masses which are odd functions of \( y \). Such masses may arise from the vev of an odd scalar.
The eigenvalue equations can be solved by iteration, which leads to a quadratic equation for the even component,

$$\partial_y f^L_n = m_n^2(1 + a_0^L \delta_0 + a_\pi^L \delta_\pi) f^L_n,$$

while the odd component can be obtained directly from Eq. (5). The result is a massless zero mode only for the even component (therefore chirality is recovered thanks to the orbifold projection) with a flat wave function,

$$f^L_0 = \frac{1}{\sqrt{1 + \frac{a_0^L + a_\pi^L}{2\pi R}}},$$

plus a tower of vector-like massive KK modes with oscillating wave functions

$$f^L_n(y) = A_n[\cos(m_n y) - \frac{a_0^L m_n}{2} \sin(m_n y)],$$

and

$$f^R_n(y) = A_n[\sin(m_n y) + \frac{a_0^L m_n}{2} \cos(m_n y)].$$

As anticipated in the introduction, the presence of BKT modifies the spectrum. The modifications depend to a great extent on whether the sizes of the BKT at both fixed points are comparable or not. If they are, the first mode can be made arbitrarily light, with mass

$$m_1^2 \sim 2 \frac{a_0^L + a_\pi^L}{a_0^L a_\pi^L \pi R},$$

and couplings to brane fields equal in size to the one of the zero mode for $a_{0,\pi}^L \gg R$, whereas the rest of the massive modes have masses which approach $m_n \sim (n - 1)/R$ and decouple from the branes in that limit. The wave function of the LH and RH components of the first KK mode and the masses and relative couplings to the brane of the first four KK
excitations of the even component are represented, respectively in Figs. 1 and 2 for the symmetric case $a_0^L = a_0^R$. When one of the two BKT is negligible with respect to the other, all KK modes behave similarly, with masses approaching $m_n \sim (n - 1/2)/R$ and couplings to the corresponding brane increasingly small for large values of the BKT. In Figs. 3 and 4, we represent the wave function of the first mode and the masses and couplings to the brane of the first four KK excitations of the even component as a function of $a_0^L$ for $a_0^R = 0$.

![Figure 1](image1.png)

**Fig. 1.** Wave function of the LH (left) and RH (right) components of the first KK mode for different values of $a_0^L = a_0^R$. The different lines correspond to $a_0/R = 0, 1, 2, 4$ for $f_1(0)$ from top to bottom (on the left) for the LH component, and the opposite for the RH one.

![Figure 2](image2.png)

**Fig. 2.** Masses (left) and LH couplings to the brane at $y = 0$ normalized to the zero mode coupling (right) for the first few KK modes ($n = 1, 2, 3, 4$ from bottom to top (left) and from top to bottom (right)) as a function of $a_0^L = a_0^R$.

Let us now discuss the situation for gauge bosons in the $S^1/Z_2$ orbifold [7]. The fifth component of a vector boson is forced to have $Z_2$ parity opposite to the one of the other components. Unless one is interested in breaking (part of) the gauge group, the first four components must be taken to be even. Breaking the gauge group by orbifold projections is a very
Fig. 3. Wave function of the LH (left) and RH (right) components of the first KK mode for \( a_L = 0 \) and different values of \( a_L^I \). The different lines correspond to \( a_L^I/R = 0, 1, 2, 4 \) for \( f_1(0) \) from top to bottom (on the left) for the LH component, and the opposite for the RH one.

Fig. 4. Masses (left) and LH couplings to the brane at \( y = 0 \) normalized to the zero mode coupling (right) for the first few KK modes \( (n = 1, 2, 3, 4 \) from bottom to top (left) and from top to bottom (right)) as a function of \( a_L^I \), when \( a_L^I = 0 \).

An interesting possibility which has been exploited in GUT models [10], but here we stick to even \( A_\mu \) for definiteness. The most general kinetic Lagrangian is

\[
\mathcal{L} = -\frac{1}{4}(1 + a_I^A \delta_I) \text{tr} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2}(1 + c_I^A \delta_I) \text{tr} F_{5\mu\nu} F^{5\mu\nu}. \tag{12}
\]

In this case, gauge invariance forbids the most singular “\( b \)-like” BKT. The terms proportional to \( c_I^A \) have no effect at all when treated in a non-perturbative way, but give rise to singularities at high orders of perturbation theory, which may be substracted by (classical) counterterms [5]. In this second approach, a non-vanishing finite effect survives. Here, we simply set \( c_0^A = 0 \) and focus on arbitrary coefficients \( a_I^A \). The gauge invariant KK decomposition of (12) has been carried out in [5]. Working instead in the
“axial” gauge $A_5 = 0$ and expanding in KK modes the even components,

$$A_\mu(x,y) = \sum_n \frac{f_n^A(y)}{\sqrt{2\pi R}} A_\mu^{(n)}(x), \quad (13)$$

we arrive at the same eigenvalue and orthonormality equations as the ones we have obtained above for the even component of the bulk fermion when only $a^L_{0,\pi}$ are non-vanishing [7]. The spectrum and wave functions are, therefore, identical to the one we have just discussed for (even) fermions.

3. Phenomenology of brane kinetic terms

We have seen in the previous section how the presence of BKT affects the masses and wave functions of the KK modes of bulk fermions and bosons. To extract phenomenological implications, we need to compute the couplings of the four-dimensional effective theory, which are given by the overlap of wave functions (with are delta functions for fields on the branes) times the five-dimensional couplings. The impact of BKT on Yukawa couplings can be very relevant phenomenologically, as has been shown in [6]. Nonetheless we will mainly concentrate on gauge couplings, as they affect directly the very precisely measured electroweak observables, and have not been previously studied when both bulk gauge bosons and bulk fermions have non-vanishing BKT.

The gauge interactions of fermions can be obtained from the KK reduction of the fermionic Lagrangian, Eq. (1). They read

$$\mathcal{L}_{\text{int}} = - \sum_{mnr} g g_{mnr} \bar{\psi}_L^{(m)} \gamma A^{(r)} \psi_L^{(n)}, \quad (14)$$

with effective gauge couplings given by

$$g_{mnr} = \frac{g_5}{\sqrt{2\pi R}} \int_{-\pi R}^{\pi R} dy \left( 1 + a_0^L \delta f \right) \frac{f_m^L f_n^L f_r^A}{2\pi R} \cdot (15)$$

The phenomenologically relevant couplings are the ones of two fermion zero modes to the KK excitations of gauge bosons,

$$g^{(00n)}(000) = \sqrt{\frac{1 + a_0^L + a_0^A}{2\pi R}} \left[ \frac{a_0^L - a_0^A}{2\pi R} f_n^A(0) + \frac{a_0^L - a_0^A}{2\pi R} f_n^A(\pi R) \right] \cdot (16)$$

Note that these couplings vanish when $a_0^L = a_0^A$ and $a_\pi^L = a_\pi^A$, and in particular in the limit of no BKT (due to KK number conservation).
A fit to electroweak precision observables taking into account the modifications induced by the presence of the gauge boson KK modes results in bounds on the compactification scale, as a function of the fermion and gauge boson BKT. We adapt the methods of Ref. [8] to our particular case. Considering all gauge bosons to share the same BKT on each brane, with common coefficient \( a^A_I \), and similarly for fermions, with coefficient \( a^L_I \), but allowing for independent \( a^A_I \) and \( a^L_I \), our model is described by the following parameters: \( R, a^A_I, a^L_I, g_5, g'_5 \) and \( v \), where \( g_5, g'_5 \) are the gauge couplings of SU(2)_L and U(1)_Y, respectively, and \( v \) is the vev of the Higgs boson, which we consider localized at \( y = 0 \). The idea is to compute the corrections due to the extra dimensions to all observables, fix three of our independent parameters in terms of three observables and perform the fit as a function of the remaining parameters. We fix \( g_5, g'_5 \) and \( v \) using \( \alpha(M_Z) \simeq 1/129 \), \( M_Z \simeq 91.2 \) GeV and \( G_F \simeq 1 \times 10^{-5} \) GeV\(^{-2} \). All the observables are then expressed in terms of them plus the compactification scale and the BKT. There are two types of modifications due to the presence of extra dimensions, one is the usual four-fermion interactions mediated by gauge boson KK excitations, the other is due to the localized Higgs and can be traced to the different mixing of the \( W \) and \( Z \) with their respective KK towers or, alternatively, to the fact that if the KK expansion is performed including the localized Higgs vev, the \( Z \) and \( W \) zero modes are no longer flat. All these modifications can be captured in the definition of effective oblique parameters \( T_{\text{eff}}, S_{\text{eff}}, U_{\text{eff}} \) (we call them effective because they actually contain the leading non-oblique effects as well). The localized Higgs effect only affects \( T_{\text{eff}} \) whereas the others depend on the couplings in Eq. (16) and therefore vanish in the limit \( a^L_I = a^A_I \). The KK contributions to \( S, T, U \) are determined by matching the effective Lagrangian for zero modes, with fermion interactions rescaled to unity, to the following generic Lagrangian with oblique corrections:

\[
\mathcal{L} = - \frac{1}{2g^2} \Pi_{WW}(0) W^\mu W^\nu - \frac{1}{4(g^2 + g'^2)} Z^\mu Z^\nu - \frac{1}{4\sqrt{2}G_F} \Pi_{WW}(0) - \frac{1}{2} \left( \frac{1}{4\sqrt{2}G_F} + \Pi_{ZZ}(0) \right) Z^\mu Z^\nu,
\]

where \( g \equiv g_5f_0^A/\sqrt{2\pi R} \), \( g' \equiv g'_5f_0^A/\sqrt{2\pi R} \) and we have only represented the relevant terms. The oblique parameters are defined in terms of the “self-energies” above (including non-standard tree-level contributions) as

\[
S = 16\pi \left( \frac{\Pi_{ZZ}(0)}{g^2 + g'^2} - \Pi_{3Q}^L \right),
\]

(18)
\[ T = \frac{4\pi}{s^2 c^2 m_Z^2} \left( \frac{\Pi_{WW}(0)}{g^2} - \frac{\Pi_{ZZ}(0)}{g^2 + g'^2} \right), \]

\[ U = 16\pi \left( \frac{\Pi_{WW}'}{g^2} + \frac{\Pi_{ZZ}'}{g^2 + g'^2} \right), \]

where \( \Pi_{3Q}' \) is related to the \( Z - \gamma \) kinetic mixing which is zero in our case and \( s \equiv g_5 / \sqrt{g_5^2 + g'_5} = \sin \theta_W + \mathcal{O}(v^2/\mu_n^2) \) and \( c \equiv g_5 / \sqrt{g_5^2 + g'_5} = \cos \theta_W + \mathcal{O}(v^2/\mu_n^2) \), with \( \theta_W \) the Weinberg angle. We also have to take into account non-oblique four fermion interactions, which can be parameterized by

\[ \Delta T = -\frac{1}{\alpha} \frac{\delta G_F}{G_F} = -\frac{1}{\alpha} \sum_n \left( \frac{g^{(00n)} m_W}{m_n^2} \right)^2. \]

In order to include this non-oblique effect in the electroweak fit we redefine \( T \rightarrow T + \Delta T \) and \( U \rightarrow U - 4s^2\Delta T \) [8]. The tree-level contribution of extra dimensional physics to the resulting effective oblique parameters reads

\[ S_{\text{eff}} = \tilde{S} = -\frac{8s^2 c^2}{\alpha} \sum_n \frac{g^{(00n)} f_n^A(0) m_Z^2}{g^{(000)} f_0^A(0) m_n^2}, \]

\[ T_{\text{eff}} = \tilde{T} + \Delta T = \frac{1}{\alpha} \sum_n \left[ \frac{f_n^A(0)}{f_0^A(0)} - 2\frac{g^{(00n)}}{g^{(000)}} \right] \frac{f_n^A(0) m_Z^2 - m_W^2}{m_n^2} \]

\[ - \left( \frac{g^{(000)}}{g^{(000)}} \right)^2 \frac{m_W^2}{m_n}, \]

\[ U_{\text{eff}} = \tilde{U} - 4s^2\Delta T = \frac{4s^2}{\alpha} \sum_n \left( \frac{g^{(00n)}}{g^{(000)}} \right)^2 \frac{m_n^2}{m_Z^2}, \]

where the bars indicate that only tree level contributions are included. Note that, being already of order \( v^2/\mu_n^2 \), we can plug the experimental results for \( s, c, m_Z \) and \( m_W \) in the above expressions. A global fit to the electroweak observables gives [11]

\[ S_{\text{new}} = -0.03 \pm 0.11, \]

\[ T_{\text{new}} = -0.02 \pm 0.13, \]

\[ U_{\text{new}} = 0.24 \pm 0.13, \]

for \( m_t = 173 \text{ GeV} \) and \( m_H = 115 \text{ GeV} \) and “new” here stands for beyond the SM contribution. Requiring that the contribution to each of the three effective oblique parameters remains within the 1-\( \sigma \) interval, we obtain the bounds on the compactification scale shown in Fig. 5 as a function of the
gauge boson (horizontal axis) and fermion (vertical axis) BKT, for BKT equal at both branes and for BKT only at \( y = 0 \). In both cases there is a band along the diagonal \( a^L = a^A \) where \( U_{\text{eff}} \) is too small to give a meaningful bound on the compactification scale. The rest of the bands correspond to \( M_c \geq 2, 3, 3.5, 4 \) TeV for the case that \( a^L_0 = a^L_3, a^A_0 = a^A_3 \) (left plot) and \( M_c \geq 1, 2, 3, 4, 5, 6 \) TeV when \( a^L_2 = a^A_3 = 0 \) (right plot). Note that in the symmetric case the bounds are less strict, even though in that case the first KK excitation is lighter. The reason is that the first KK mode decouples when \( a^L_0 = a^L_\pi \) (see Eq. (16) and note that \( f^4_1(\pi R) = -f^4_1(0) \) when \( a^A_0 = a^A_\pi \)). Therefore, only a suppressed contribution to \( T_{\text{eff}} \approx (m_Z^2 - m_W^2)/m^2_1 \) effectively bounds \( M_c \). In the case of BKT just at one brane the added effect of all modes is much stronger. It is also apparent from the figures that the dependence on the fermion and gauge boson BKT is highly asymmetric in this case, with a stronger dependence on the gauge boson BKT than on the fermion ones. Finally, we observe that the effect of decoupling from the brane with BKT, which we pointed out in Section 2, is not very important, as far as gauge interactions are concerned, in models in which both gauge bosons and fermions live in the bulk.

As we mentioned at the beginning of this section, Yukawa couplings are also modified by the presence of BKT for fermions. Models with bulk fermions have a non-unitary CKM matrix due to the mixing of zero and massive KK modes [12]. In many models the Higgs lives on one of the branes, say the one at \( y = 0 \), and then the departure from the SM is pro-
portional to the mass of the quarks involved, i.e., it is most relevant for top physics [13]. These effects give rise to stringent limits from the $T$ parameter, which become weaker when $a_0 \gg a_\pi$. Details can be found in [6].

4. Conclusions

We have seen that the BKT for bulk fields have a significant impact on the phenomenology of compact extra dimensional theories when the corresponding coefficients are comparable to or larger than the compactification scale. This means that the parameter space of the theory without BKT is enlarged, and this allows for a greater freedom in the construction of models which are consistent with present data (and possibly with implications observable in future experiments). Of course, the price to pay is a reduction of predictivity in a general effective approach.

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