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Discrete regularisation of localised kinetic terms†*

F. del Aguila[a], M. Pérez-Victoria[a,b] and J. Santiago[c]

[a]*Departamento de Física Teórica y del Cosmos and CAFPE, Universidad de Granada, E-18071 Granada, Spain*

[b]*Dipartimento di Fisica "G. Galilei", Università di Padova and INFN, Sezzione di Padova, Via Marzolo 8, I-35131 Padua, Italy*

[c]*IPPP,University of Durham, South Road, Durham DH1 3LE, United Kingdom*

Abstract

We investigate the behaviour of 5d models with general brane kinetic terms by discretising the extra dimension.We show that in the continuum limit the Kaluza-Klein masses and wave functions are in general nonanalytic in the coefficients of brane terms.

1 INTRODUCTION

Field theories with extra dimensions give a new perspective on many important issues in particle physics, such as the hierarchy problem, (super)symmetry breaking or the structure of flavour (for recent reviews see [1, 2, 3]). In general they must be understood as effective theories valid below some scale, above which a more fundamental theory must be at work. It is then important to keep in mind the presence of higher order terms in the low-energy expansion of the effective lagrangian. The situation is particularly interesting when the theory contains lower dimensional defects, called "branes" from now on. In this case one expects localised corrections to the lowest-dimensional terms. In fact, infinite localised corrections are induced by quantum corrections when the bulk fields couple to brane fields [4] and in orbifold compactifications [5, 6]. The divergences must be cancelled by localised counterterms, and the coefficients (couplings) of the corresponding brane terms run with the scale, so that they cannot be chosen to vanish at all scales.

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In particular, the effective action of generic brane models contains brane kinetic terms (BKT). These terms modify the phenomenological predictions of the model [7, 8] and have been studied in different contexts [4, 9] (for a list of references see [10]). In [11] we have studied the implications of the most general BKT for particles of spin 0, $\frac{1}{2}$ and 1 in five dimensions, with the fifth dimension compactified on an orbifold. Some of the allowed BKT have dramatic effects in the spectrum of Kaluza-Klein (KK) masses and wave functions. To understand this one should keep in mind that, in general, the equations of motion (or, equivalently, the differential equations for the KK reduction) are not well-defined in the presence of the delta functions which impose localisation in the BKT. A sensible prescription is to regulate the delta functions using thick branes, perform the KK reduction and then take the thin brane limit. However, it turns out that the limits of thin brane and vanishing BKT do not always commute. This signals a breakdown of the perturbative expansion of the effective lagrangian, raising the question of the consistency of the approach.

Inspired by deconstructed theories, we study here a discretised version of five dimensional models with BKT terms, for fields of spin $0, \frac{1}{2}$ and 1. We take the continuum limit and compare with the corresponding results in (the thin brane limit of) the thick brane regularisation. This lattice regularisation is complementary to the thick brane regularisation, in the sense that it is the bulk which is regularised rather than the brane. From a practical point of view, it turns out that discretisation gives a better handle of analytic computations than thick brane regularisation. In [12] we take this point of view seriously and study a full deconstructed model. In particular we calculate loop corrections and show that BKT are also generated in these models if the discrete analogue of Poincaré invariance is broken at some point. This communication is organised as follows: In the next section we discretise the free lagrangian with general (first order) BKT for even scalars. In Section 3 we study its KK decomposition and describe the corresponding results for odd scalars, and for fermions and gauge bosons. Some conclusions are presented in the last section.

2 DISCRETISED SCALAR MODELS

As an example we write down a discretised version of the free lagrangian for an even massless scalar ϕ with arbitrary BKT at one brane in a five dimensional model with the fifth coordinate compactified on an S^1/Z_2 orbifold (compare with Eq. (2.1) of Ref. [11]):

$$\begin{aligned}\mathcal{L} &= \sum_{i=0}^{N}(1-\frac{1}{2}(\delta_{i0}+\delta_{iN})+\alpha\delta_{i0})\partial_\mu\phi_i^\dagger\partial^\mu\phi_i \\ &- \frac{1}{s^2}\sum_{i=0}^{N-1}(\phi_{i+1}^\dagger-\phi_i^\dagger)(\phi_{i+1}-\phi_i) \\ &+ \frac{\beta}{s^2}\Big[\phi_0^\dagger(\phi_1-\phi_0)+\text{h.c.}\Big] \\ &- \frac{\gamma}{s^2}(\phi_1^\dagger-\phi_0^\dagger)(\phi_1-\phi_0), \qquad (1)\end{aligned}$$

where s is the lattice spacing and α, β and γ are the dimensionless coefficients of the discretised BKT. The number of sites is $N+1$ and the boundary sites at $i=0,N$ play the role of branes. The spacing is the only dimensionful coefficient. The relation of these parameters with the parameters of the continuum theory is given by

$$s = \frac{\pi R}{N}, \quad \frac{\alpha}{a} = \frac{\beta}{b} = \frac{\gamma}{c} = \frac{N}{2\pi R}, \tag{2}$$

where R is the compactification radius and a, b and c are defined in [11]. We shall take R constant, such that the continuum limit $s \to 0$ corresponds to $N \to \infty$.

In order to define the BKT correctly, we consider a discrete version of the theory with BKT compactified on a circle, compatible with the parity symmetry $i \leftrightarrow -i$, and then identify $\phi_{-i} \sim \phi_i$. In this way we can separate modifications of the generalised mass matrix (which incorporates the discrete five dimensional derivatives) near the branes due to BKT from the ones which impose Neumann boundary conditions in the fundamental region for even fields in the absence of BKT. More details are given in [12]. There are, of course, discretisation ambiguities in the definition of the discrete delta functions. Usually, one expects that the continuum limit is independent of discretisation details, but keeping in mind the singular behaviour associated to BKT in the continuum, this is an issue to be checked. We have written above the minimal versions of the discrete BKT compatible with the orbifold parity, but have checked that the results, apart from small side-effects in some cases, remain the same for discrete delta functions involving a small number of sites in the vicinity of the branes.

In matrix notation, Eq. (1) reads (sum over i,j is understood)

$$\mathcal{L} = \partial_\mu \phi_i^\dagger \mathcal{K}_{ij} \partial^\mu \phi_j - \phi_i^\dagger \mathcal{M}_{ij}^2 \phi_j, \tag{3}$$

where the $(N+1)\times(N+1)$ kinetic matrix $\mathcal{K}$ is

$$\mathrm{diag}(\frac{1}{2}+\alpha, 1, \ldots, 1, \frac{1}{2})$$

and the $(N+1)\times(N+1)$ mass matrix $s^2\mathcal{M}^2$ is

$$\begin{pmatrix} 1+2\beta+\gamma & -1-\beta-\gamma & 0 & \ldots & 0 & 0 \\ -1-\beta-\gamma & 2+\gamma & -1 & \ldots & 0 & 0 \\ 0 & -1 & 2 & \ldots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \ldots & 2 & -1 \\ 0 & 0 & 0 & \ldots & -1 & 1 \end{pmatrix}.$$

One can discretise the lagrangians for odd scalars, and for fermions and gauge bosons in a similar way (Eqs. (2.25) and (2.14) in Ref. [11], respectively).

Solving this theory amounts to diagonalising the mass matrix after canonically normalising the kinetic term. Then

$$\mathcal{L} = \partial_\mu \varphi_i^\dagger \partial^\mu \varphi_i - m_i^2 \varphi_i^\dagger \varphi_i, \tag{4}$$

with KK modes φ_i and masses m_i given by

$$\mathcal{K}_{ij}^{\frac{1}{2}}\phi_j = a_{in}\varphi_n \tag{5}$$

and

$$\mathcal{M}_{ij}^2 = \mathcal{K}_{ik}^{\frac{1}{2}} a_{kn} m_n^2 a_{nl} \mathcal{K}_{lj}^{\frac{1}{2}}. \tag{6}$$

The orthogonal matrix a_{in} is the discrete analogue of the KK wave functions. We have assumed that $\mathcal{K}$ is positive definite, in agreement with the usual positivity requirements of quantum field theory. Then, $\mathcal{K}^{\frac{1}{2}}$ is real and Eq. (4) follows directly from Eqs. (3,5,6). The masses and wave functions can be found analytically or numerically from Eq. (6). Although we shall not discuss gravity here, we note in passing that the localised kinetic terms in DGP models [4] are of the a type.

3 FIELD SPECTRA

Let us now compare the results with the ones obtained with a thick brane regularisation in Ref. [11]. We focus mainly on the relation between the corresponding large N and thin brane limits. Before doing that, we clarify one important point regarding the continuum limit: The coefficients of the BKT in the continuum have dimensions of length and are suppressed by one power of the spacing, which acts as the inverse UV cutoff of the effective theory describing the finite-N deconstructed theory at low energies. Note that the same applies, for instance, to gauge coupling constants. Here we simply want to use the discrete theory as a regulator to understand the classical behaviour of the lowest dimensional lagrangian with BKT in the continuum. Then, in taking the continuum limit, we need to keep the parameters a, b and c fixed. This implies that the discrete parameters α, β and γ are taken to be proportional to N.

3.1 Scalars

For even scalars and a type BKT ($b = c = 0$) the results for the leading term in the $1/N$ expansion (continuum limit), which can be easily obtained analytically, are the same as for the thin brane regularisation [11]. There is always a constant massless mode, and for large positive a the non-zero masses approach $\frac{1}{2}, \frac{3}{2}, ...$, in units of R. For negative a Eqs. (4,5,6) must be generalised to deal with ghosts, possibility which is not discussed here.

As in the thin brane limit of the thick brane regularisation, the large N limit commutes with $a \to 0^+$. However, this is not so for b as is apparent from Fig. 1, where we draw the masses of the lightest KK modes for $N = 10, 25$, as well as for the continuum limit, as a function of $b(a, c = 0)$. For $b = 0$ the masses are $0, 1, 2, ...$ (in units of R), but for a non-zero b, independently of how small it is, the zero mode becomes tachyonic and the massive modes decrease to $\frac{1}{2}, \frac{3}{2}, ...$ for large N. The tachyonic mass squared tends to $-\infty$ for any non-zero value of b for large N, and the corresponding eigenfunction to a normalizable function peaked only at the origin (see Fig. 2). The other eigenfunctions

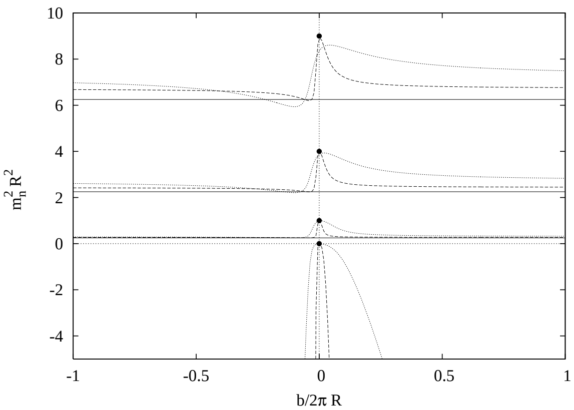

Figure 1: Squared masses of the lightest modes in units of R^2 as a function of $\frac{b}{2\pi R}$ in the range $[-1, 1]$ for $N = 10$ (dots), 25 (dashes) and the thin brane limit (solid). The dots correspond to $b = 0$ and $m_n = \frac{n}{R}$.

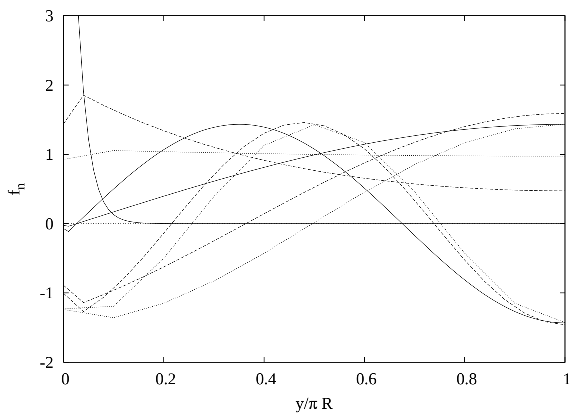

Figure 2: Eigenfunctions of the lightest modes for $b = \frac{R}{10}$ and $N = 10$ (dots), 25 (dashes) and 100 (solid). We plot $f_n(\frac{y}{\pi R} \equiv \frac{is}{\pi R}) \equiv \sqrt{\frac{\pi R}{s}} a_{in}$ for $n = 0$ (tachyon), 1 and 2.

approach $\sim \sin(\frac{2n+1}{2R}|y|)$. (For small b and N they look like $\sim -\cos(\frac{n}{R}y)$.) This implies that in the continuum limit all the bulk modes but the tachyon are decoupled from the brane fields. These modes are also decoupled from the tachyon, which has support only at the origin. All these results are qualitatively independent of the sign of b, in contrast with the results in the thick brane regularisation. There, the same behaviour is obtained for $b \geq 0$ (although the presence of a tachyon was overlooked in [11]), but for negative b the eigenfunctions diverge at some point near $y = 0$. One can define a *principal value* (PV) sort of limit adding a tiny imaginary part to b in the equation of motion, and taking it to zero after performing the thin brane limit. This is presumably equivalent to excluding a symmetric interval around the divergent points, and taking the thin brane limit first and the interval to zero afterwards; this explains the notation PV. We have found that with this prescription the large N limit result in Fig. 1 is recovered also for negative b. Therefore the singularity near the origin may be an artifact of the thick brane regularisation. It is worth emphasizing at this point that the discrete regularisation is easier to handle as it does not have the convergence and numerical problems of the thick brane regularisation.

In Fig. 3 we plot the masses of the lightest KK modes for $N = 10, 25$, as well as for the continuum limit, as a function of $c(a, b = 0)$. For $c < 0$ the zero mode is not affected but

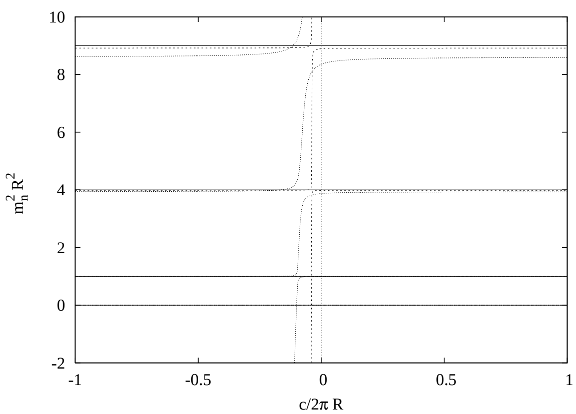

Figure 3: Squared masses of the lightest modes in units of R^2 as a function of $\frac{c}{2\pi R}$ in the range $[-1, 1]$ for $N = 10$ (dots), 25 (dashes) and the thin brane limit (solid).

the first massive mode becomes tachyonic in a similar way as the zero mode does for $b \neq 0$. The other massive modes decrease their masses in one unit. Using the PV prescription, the thin brane limit of the continuum theory coincides with the large N limit of the discretised theory. For non-negative c both limits give no dependence on c.

Odd scalars are not sensitive to BKT [11].

3.2 Fermions

Let us now deal with fermions. From the four-dimensional point of view they are Dirac spinors with two chiral components: $\Psi = \Psi_L + \Psi_R$, $\gamma^5 \Psi_{L,R} = \mp \Psi_{L,R}$. The invariance of the bulk kinetic term requires that the left-handed (LH) and right-handed (RH) components have opposite Z_2 parities. We assume without loss of generality that the LH (RH) component is even (odd), and consider the general BKT a^L, a^R, b and c in Eq. (2.25) of Ref. [11]. In order to avoid the (in)famous doubling problem we include a Wilson term. More details are given in [12]. Here we directly discuss the results [1].

[1] The KK reduction proceeds as in the scalar case, but in this case one must diagonalise the matrix $\mathcal{M}\mathcal{M}^\dagger$, where $\mathcal{M}$ is the discretised fermion mass matrix.

As in the thick brane regularisation, there is always a LH chiral zero mode. The solutions for $a^L(a^R, b, c = 0)$ coincide with those for scalars of the same parity. a^R is irrelevant when $a^L = 0$, and when both are non-zero the limits $a^L \to 0$, $a^R \to 0$ and $N \to \infty$ commute. For non-zero b the behaviour of fermions is quite different from the one of scalars. In the continuum limit the masses and the eigenfunctions away from the brane are the same as for vanishing b. At the brane, the even wave functions are discontinuous and zero. This behaviour is independent of the sign of b. Again, one must use the PV prescription in the thick brane case to reproduce it for negative b.

It is worth comparing now the scalar and fermion spectra for non-zero b. Although in both cases the bulk tends to decouple from the brane, in the scalar case there is a tachyon of infinite mass, localised at the brane, which decouples from the KK tower because all the other eigenfunctions vanish at the origin. In the fermionic case there are, of course, no tachyons but the even functions become discontinuous at the origin, where they vanish. Moreover, scalar masses are shifted down to half-integer values, whereas fermion masses remain unchanged. In both cases there is no explicit dependence on the b value, as long as it is non-zero, and the $b \to 0$ limit is always singular. Imposing supersymmetry, which links scalars to fermions, one gets rid of tachyons. This is technically so because the supersymmetric scalar lagrangian includes $b^2\delta^2$ terms which drastically modify the scalar behaviour, resulting in the same solutions as for fermions [11].

For fermions, the only effect of a non-zero c in the continuum limit, independently of its sign, is to cancel the effect of a^R. This holds in the thin brane limit of the thick brane regularisation (with the PV prescription) as well. This behaviour is also singular for $c \to 0$ if $a^R \neq 0$ and $a^L \neq 0$. The parameter b is always determinant, since for $b \neq 0$ the bulk fields decouple from the brane and the other BKT become irrelevant.

3.3 Gauge bosons

The situation for gauge bosons is a particular case of the one for scalars. The only new feature is that gauge invariance requires $b = 0$ for the four-dimensional components of the gauge field, and $b = c = 0$ for the fifth-dimensional one.

4 CONCLUSIONS

We have used a discrete regularisation to analyse the behaviour of BKT for bulk fields in brane models with extra dimensions. This regularisation has proven to be very efficient as compared with regularisations of BKT in the continuum used before, and has allowed us to identify some tachyonic modes which had evaded our previous numerical computations. Moreover, discretisation is free of problems which are present in the thick brane regularisation when some parameters take negative values. This has led us to propose a PV prescription for the thick brane, such that the results with both regularisations agree in all cases. Except for positive a type terms, the BKT we have studied show a singular behaviour when they approach 0. It is clear that theories without "dangerous" BKT are distinguished, since they do not present these problems. In principle, supersymmetric models with $b = c$ chosen to be zero at tree level are stable under quantum corrections and belong to this class. Putting $b = c = 0$ could be justified in the context of string theory. More generally, one can construct critical theories by including a tower of higher-dimensional operators such that the singular behaviour is smoothed down, as proposed in Ref. [11]. This is equivalent to a classical renormalisation of the theory.

Even in the noncritical case, it is plaussible that the microscopic theory takes care of the singular behaviour via some mechanism which introduces a new scale (possibly related to the cutoff scale controlling the dimensionful coefficients of the BKT). In fact, both the thick brane and the lattice regularisations suggest that this is unavoidable, as they both introduce a length parameter: the brane width and the spacing, respectively. From a low energy perspective, in organising the perturbative series one should take into account that there are two scales at play, and make sure that the effective lagrangian to be used describes the actual physics in the regime of interest.

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