

Neutrino Masses from an A_4 Symmetry in Holographic Composite Higgs Models

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Abstract

We show that holographic composite Higgs Models with a discrete A_4 symmetry naturally predict hierarchical charged lepton masses and an approximate tri-bimaximal lepton mixing with the correct scale of neutrino masses. They also satisfy current constraints from electroweak precision tests, lepton flavor violation and lepton mixing in a large region of parameter space. Two phenomenologically relevant features arise in these models. First, an extra suppression on the lepton Yukawa couplings makes the τ lepton more composite than naively expected from its mass. As a consequence new light *leptonic* resonances, with masses as low as few hundreds of GeV, large couplings to τ and a very characteristic collider phenomenology, are quite likely. Second, the discrete symmetry A_4 together with the model structure provide a double-layer of flavor protection that allows to keep tree-level mediated processes below present experimental limits. One-loop processes violating lepton flavor, like $\mu \rightarrow e\gamma$, may be however observable at future experiments.

I. INTRODUCTION

The primary goal of the Large Hadron Collider (LHC) is the study of the precise mechanism of electroweak symmetry breaking (EWSB). One interesting possibility is that new gauge interactions become strongly coupled at the TeV scale, breaking some global symmetries but not necessarily the electroweak gauge symmetry. The Higgs boson can then arise as a composite Goldstone boson of the spontaneous global symmetry breaking. The coupling to an elementary sector (external to the strongly coupled theory) breaks explicitly the global symmetries and generates a potential for the Higgs at the loop level [1]. The AdS/CFT correspondence [2] suggests that models with warped extra dimensions [3] are weakly coupled duals to strongly coupled four-dimensional (4D) conformal theories [4], and therefore they provide a calculable framework for composite Higgs models [5].¹ A recent review of tools employed to study models with warped extra dimensions and their phenomenological implications can be found in [7].

Using a custodially symmetric set-up [8] and a fermionic content that guarantees protection of the $Z\bar{b}_L b_L$ coupling [9] (see [10] for an alternative), minimal composite Higgs models from warped extra dimensions [11] have been shown to dynamically generate EWSB at the expense of a modest fine-tuning [11–13]. Furthermore, they are fully compatible with electroweak precision tests (EWPT) [14, 15] and can even be easily extended to accommodate dark matter [16–18]. (How relevant is the fine-tuning is a debatable matter as it has been shown that there is a large intersection in these models among the regions with a good pattern of EWSB, the correct top mass and a good behaviour under EWPT [12, 17].) Most of the studies related to the five-dimensional (5D) realization of composite Higgs models have only focused on the quark sector. In fact, although some of the first studies of bulk fermion phenomenology in models with warped extra dimensions were made with the leptonic sector in mind [19–21], not much progress has been made until quite recently. In particular, older proposals for models of lepton masses have, with few exceptions, not been updated to make them compatible with new, realistic models in warped extra dimensions. One possible reason is that the generation of Yukawa couplings by fermion splitting [22] seemed to naturally lead to a hierarchical pattern of fermion masses and mixing angles, like the one observed in

¹ See [6] for a discussion of composite Higgs models from the effective 4D point of view.

the quark sector, but not to large mixing angles like those observed in the neutrino sector. This was recently shown not to be necessarily true [23], and a realization of neutrino masses within this framework and with a realistic dark matter candidate was presented in [18].

An alternative approach to differentiate the quark and lepton spectra is to assume a global symmetry acting on the leptonic sector. 4D models of neutrino masses with an A_4 symmetry [24, 25] can predict a tri-bimaximal (TBM) [26] pattern of lepton mixing to leading order (LO), what agrees quite well with observation [27, 28]. This global symmetry can be also implemented in simple models with warped extra dimensions [29].² Such a construction presents an advantage over 4D models since the mass hierarchy follows from wave-function overlapping, geometrically realizing the required Frogatt-Nielsen mass generation in 4D models. Besides, it also improves other 5D models that solely rely on the former for it has an extra built-in flavor protection due to the discrete A_4 symmetry. Our goal is to extend this set-up to models of gauge-Higgs unification (GHU), which are arguably the most natural models of EWSB in warped extra dimensions. We will show that, despite some subtleties related to the way fermions acquire non-trivial Yukawa couplings in GHU models [13], it is easy to find examples that naturally generate a realistic fermion spectrum also in the lepton sector.

Two new features phenomenologically relevant come out from our analysis. First, due to an extra suppression of the leptonic Yukawa couplings implied by the A_4 symmetry, the τ lepton is typically more composite than one would naively expect from its mass. This makes new leptonic resonances at the electroweak scale a likely occurrence in these models. Besides, as they come in two almost degenerate doublets with hypercharges $-1/2$ and $-3/2$, respectively, and mainly couple to τ , they provide a very distinctive signature at LHC for they only decay through definite channels and into τ leptons. This structure is dictated by the same symmetry that protects the $Z\bar{b}_L b_L$ coupling in this type of models [9], which in the leptonic sector protects the Standard Model (SM) lepton couplings despite the large new lepton couplings to τ [33]. Second, the A_4 symmetry together with the protecting mechanism above [34, 35] result in a double-layer flavor protection. Thus, lepton flavor violation (LFV) mediated by tree-level exchange of heavy modes is further suppressed,

² Other symmetries that can simultaneously accommodate the pattern of quark and lepton mixing have been also considered in 5D contexts [30, 31] and in models compatible with an underlying GUT structure [32].

and typically below current experimental limits. The main constraints result from one-loop processes, like $\mu \rightarrow e\gamma$, which is close but quite often below the present experimental sensitivity, being then within the reach of future experiments.

The outline of the paper is as follows. We describe the model in Section II. The leptonic spectrum is computed in Section III, where the LO implications of the A_4 symmetry are discussed in detail. The corrections to these LO results are classified in Section IV; and the constraints from EWPT and flavor observables are considered in Section V, where we also give an explicit example of a realistic model. Section VI is left to our conclusions, and some technical details are collected in the appendices.

II. THE MODEL

We consider a 5D model in a slice of AdS_5 with metric

$$ds^2 = a^2(z)(\eta_{\mu\nu}dx^\mu dx^\nu - dz^2) = \left(\frac{R}{z}\right)^2 (\eta_{\mu\nu}dx^\mu dx^\nu - dz^2) \quad (1)$$

and $R \leq z \leq R'$, where $R \sim M_P^{-1}$ and $R' \sim \text{TeV}^{-1}$ are the location of the UV and IR brane, respectively. Following [11], we assume an $SO(5) \times U(1)_X$ bulk gauge symmetry broken by boundary conditions to $SO(4) \times U(1)_X$ on the IR brane and to $SU(2)_L \times U(1)_Y$ on the UV brane. These read

$$\begin{aligned} L_\mu^a(+, +), & \quad B_\mu(+, +), & \quad C_\mu^{\hat{a}}(-, -), \\ R_\mu^b(-, +), & \quad Z'_\mu(-, +), \end{aligned} \quad (2)$$

where $- (+)$ stands for Dirichlet (Neumann) boundary conditions at the corresponding brane. The superscripts $a = 1, 2, 3$, $b = 1, 2$ label the $SO(4)$ gauge bosons in explicit $SU(2)_L \times SU(2)_R$ notation, and

$$B_\mu = \frac{g_X R_\mu^3 + g_5 X_\mu}{\sqrt{g_5^2 + g_X^2}}, \quad Z'_\mu = \frac{g_5 R_\mu^3 - g_X X_\mu}{\sqrt{g_5^2 + g_X^2}}, \quad (3)$$

with g_5 and g_X the 5D $SO(5)$ and $U(1)_X$ gauge couplings, respectively. (The electric charge $Q = T_L^3 + Y = T_L^3 + T_R^3 + Q_X$ with this normalization.) Finally, $C_\mu^{\hat{a}}$, $\hat{a} = 1, \dots, 4$, are the gauge bosons corresponding to the $SO(5)/SO(4)$ coset space.

The gauge directions along $SO(5)/SO(4)$ are broken on both branes and there is a massless zero mode along the 5-th component,

$$C_5^{\hat{a}}(x, z) = \sqrt{\frac{2/R}{1 - (R/R')^2}} \frac{z}{R'} h^{\hat{a}}(x) + \dots \approx \sqrt{\frac{2}{R}} \frac{z}{R'} h^{\hat{a}}(x) + \dots, \quad (4)$$

where the dots denote massive modes. (We have chosen the normalization constant to obtain a canonically normalized scalar, and in the second equality we have used $R \ll R'$.) These four scalars transform as a $\mathbf{4}$ of $SO(4)$ and are identified with the SM Higgs. 5D gauge invariance guarantees that any potential generated for these scalars has to arise from non-local contributions and therefore, it is finite to all orders in perturbation theory [36].

Regarding the matter content of the model, there are several possibilities. We consider here all fermions to be in fundamental representations of $SO(5)$. Thus, four multiplets per family are required in order to have independent localizations for left and right-handed zero modes. This construction is parallel to the one giving rise to realistic composite Higgs models in the quark sector [17, 37]; and as we will show, a similar matter content transforming non-trivially under a global A_4 symmetry generates the observed leptonic spectrum in a natural way, without conflict with present experimental data. Hence, there are four 5D fermion representations per generation transforming as the fundamental $SO(5)$ representation $\mathbf{5}$, with boundary conditions

$$\begin{aligned} \zeta_1 &= \begin{pmatrix} \tilde{X}_1[-+] & \nu_1[++] \\ \tilde{\nu}_1[-+] & e_1[++] \end{pmatrix} \oplus \nu'_1[-+], & \zeta_2 &= \begin{pmatrix} \tilde{X}_2[+-] & \nu_2[+-] \\ \tilde{\nu}_2[+-] & e_2[+-] \end{pmatrix} \oplus \nu'_2[--], \\ \zeta_3 &= \begin{pmatrix} \nu_3[-+] & \tilde{e}_3[-+] \\ e_3[-+] & \tilde{Y}_3[-+] \end{pmatrix} \oplus e'_3[-+], & \zeta_\alpha &= \begin{pmatrix} \nu_\alpha[+-] & \tilde{e}_\alpha[+-] \\ e_\alpha[+-] & \tilde{Y}_\alpha[+-] \end{pmatrix} \oplus e'_\alpha[--], \end{aligned} \quad (5)$$

where $\zeta_{1,2}$ and $\zeta_{3,\alpha}$ have $U(1)_X$ charge 0 and -1 , respectively. Note that there are three copies for each $\zeta_{1,2,3}$ because there are three families, but only one ζ_α set with α running over the three lepton flavors e, μ, τ . We explicitly show the decomposition under $SU(2)_L \times SU(2)_R$, $\mathbf{5} = (\mathbf{2}, \mathbf{2}) \oplus (\mathbf{1}, \mathbf{1})$. The bi-doublet is represented by a 2×2 matrix with the $SU(2)_L$ rotation acting vertically and the $SU(2)_R$ one horizontally (*i.e.* the left and right columns correspond to fields with $T_R^3 = \pm 1/2$, whereas the upper and lower components have $T_L^3 = \pm 1/2$, respectively). The bi-doublets in $\zeta_{1,2}$ contain two $SU(2)_L$ doublets of hypercharge $\frac{1}{2}$ and $-\frac{1}{2}$, and those in $\zeta_{3,\alpha}$ two $SU(2)_L$ doublets of hypercharge $-\frac{1}{2}$ and $-\frac{3}{2}$, respectively. The corresponding electric charges read

$$Q(\nu.) = Q(\tilde{\nu}.) = Q(\nu'.) = 0, \quad Q(e.) = Q(e'.) = Q(\tilde{e}.) = -1, \quad Q(\tilde{X}.) = +1, \quad Q(\tilde{Y}.) = -2, \quad (6)$$

where the dot denotes all possible values of the corresponding subscript. The signs in square brackets are a shorthand for the boundary conditions. A Dirichlet boundary condition

for the right-handed (RH) component is denoted by $[+]$, whereas $[-]$ denotes a Dirichlet boundary condition for the left-handed (LH) chirality. Finally, the first sign corresponds to the boundary condition at the UV brane and the second one at the IR brane. The chosen boundary conditions allow for a LH zero mode transforming as an $SU(2)_L$ doublet with hypercharge $-1/2$ in ζ_1 , a RH singlet of charge -1 in ζ_α , and a RH neutral singlet in ζ_2 .

As generally in A_4 models, an extra global symmetry must be imposed to forbid dangerous operators. A discrete Z_8 group does the job in our case. Both global symmetries will be broken at the two branes by localized scalars transforming as gauge singlets, ϕ and η at the UV brane and ϕ' and η' at the IR one. The fermion and scalar transformation properties under $A_4 \times Z_8$ are gathered in Table I. The three copies of $\zeta_{1,2,3}$ span the A_4 triplet

	A_4	Z_8		A_4	Z_8
ζ_1	3	1	$\phi(\text{UV})$	3	4
ζ_2	3	2	$\eta(\text{UV})$	1	4
ζ_3	3	1	$\phi'(\text{IR})$	3	5
ζ_α	1, 1', 1''	4	$\eta'(\text{IR})$	1	7

TABLE I: Bulk fermion (left) and localized scalar (right) quantum number assignments under the discrete group $A_4 \times Z_8$.

representation, whereas each ζ_α transforms as one of the three different A_4 one-dimensional representations (see Appendix A1 for a summary of the A_4 representations).

Once the matter content is fixed, we can write down the most general Lagrangian compatible with the symmetries. The bulk Lagrangian reads

$$\mathcal{L} = \int_R^{R'} dz a^4 \left\{ \bar{\zeta}_k \left[i\not{D} + \left(D_z + 2\frac{a'}{a} \right) \gamma^5 - aM_k \right] \zeta_k + \bar{\zeta}_\alpha \left[i\not{D} + \left(D_z + 2\frac{a'}{a} \right) \gamma^5 - aM_\alpha \right] \zeta_\alpha \right\}, \quad (7)$$

where summation on repeated indices $k \in \{1, 2, 3\}$, $\alpha \in \{e, \mu, \tau\}$ is understood. $D_{\mu, z}$ are the gauge covariant derivatives and the bulk Dirac masses are conventionally parametrized in terms of the fundamental scale R ,

$$M_{k, \alpha} = \frac{c_{k, \alpha}}{R}. \quad (8)$$

Note that the A_4 symmetry implies a family independent bulk mass for ζ_k . The most general localized Lagrangians, excluding kinetic terms (discussed below), compatible with

the boundary conditions, and local and global symmetries, can be written

$$\begin{aligned}
-\mathcal{L}_{UV} &= \frac{x_\eta}{2\Lambda} \eta \bar{\nu}'_{2R} \nu'_{2R} + \frac{x_\nu}{2\Lambda} \phi \bar{\nu}'_{2R} \nu'_{2R} + x_l \bar{l}_{1L} l_{3R} + \text{h.c.} + \dots, \\
-\mathcal{L}_{IR} &= \left(\frac{R}{R'}\right)^4 \left\{ \frac{y_b^\alpha}{\Lambda'} \left[(\bar{l}_{3L} \phi')^\alpha l_{\alpha R} + (\tilde{l}_{3L} \phi')^\alpha \tilde{l}_{\alpha R} \right] + \frac{y_s^\alpha}{\Lambda'} (\bar{e}'_{3L} \phi')^\alpha e'_{\alpha R} \right. \\
&\quad \left. + \frac{y_b}{\Lambda'} \left[\eta' \bar{l}_{1L} l_{2R} + \eta' \tilde{l}_{1L} \tilde{l}_{2R} \right] + \frac{y_s}{\Lambda'} \eta' \bar{\nu}'_{1L} \nu'_{2R} \right\} + \text{h.c.} + \dots, \quad (9)
\end{aligned}$$

where we have assumed that lepton number is only violated on the UV brane.³ l denotes the SM-like doublet and \tilde{l} stands for the other $SU(2)_L$ doublet within the given bi-doublet, whereas the dots correspond to higher dimensional operators. We have also introduced the LH and RH chirality projections $\zeta_{L,R} \equiv [(1 \mp \gamma^5)/2]\zeta$, recovering the standard 4D notation. Finally, $(\)^\alpha$, $\alpha = e, \mu, \tau$, are the $\mathbf{3} \times \mathbf{3}$ combinations transforming under A_4 as $\mathbf{1}, \mathbf{1}''$ and $\mathbf{1}'$, respectively.

As usually in these models, we shall assume that $A_4 \times Z_8$ is spontaneously broken by the boundary scalar vacuum expectation values (v.e.v.)

$$\langle \phi \rangle = (v, 0, 0), \quad \langle \eta \rangle = v_\eta, \quad \langle \phi' \rangle = (v', v', v') \quad \text{and} \quad \langle \eta' \rangle = v'_\eta, \quad (10)$$

resulting in the brane localized terms

$$\begin{aligned}
-\mathcal{L}_{UV} &= \frac{1}{2} \bar{\nu}'_{2R} \theta_M \nu'_{2R} + x_l \bar{l}_{1L} l_{3R} + \text{h.c.} + \dots, \\
-\mathcal{L}_{IR} &= \left(\frac{R}{R'}\right)^4 \left\{ \sqrt{3} \frac{v'}{\Lambda'} \left[\bar{l}_{3L} \Omega \begin{pmatrix} y_b^e & 0 & 0 \\ 0 & y_b^\mu & 0 \\ 0 & 0 & y_b^\tau \end{pmatrix} l_R + [l_3, l \rightarrow \tilde{l}_3, \tilde{l}] + \bar{e}'_{3L} \Omega \begin{pmatrix} y_s^e & 0 & 0 \\ 0 & y_s^\mu & 0 \\ 0 & 0 & y_s^\tau \end{pmatrix} e'_R \right] \right. \\
&\quad \left. + y_b \frac{v'_\eta}{\Lambda'} [\bar{l}_{1L} l_{2R} + \tilde{l}_{1L} \tilde{l}_{2R}] + y_s \frac{v'_\eta}{\Lambda'} \bar{\nu}'_{1L} \nu'_{2R} \right\} + \text{h.c.} + \dots, \quad (11)
\end{aligned}$$

with the Majorana mass matrix

$$\theta_M \equiv \begin{pmatrix} \frac{x_\eta v_\eta}{\Lambda} & 0 & 0 \\ 0 & \frac{x_\eta v_\eta}{\Lambda} & \frac{x_\nu v}{\Lambda} \\ 0 & \frac{x_\nu v}{\Lambda} & \frac{x_\eta v_\eta}{\Lambda} \end{pmatrix} = \begin{pmatrix} \epsilon_s & 0 & 0 \\ 0 & \epsilon_s & \epsilon_t \\ 0 & \epsilon_t & \epsilon_s \end{pmatrix}, \quad (12)$$

³ This assumption, which corresponds to the strong sector preserving lepton number, can be obtained as an accidental symmetry by introducing, for instance, larger $SO(5)$ representations.

and the unitary matrix

$$\Omega \equiv \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix}, \quad \omega = e^{2\pi i/3}. \quad (13)$$

In order to simplify Eq. (11), we can rotate the matter fields

$$\zeta_k \rightarrow \Omega \zeta_k, \quad \forall k, \quad (14)$$

leaving the bulk lagrangian \mathcal{L} invariant. However, the localized terms

$$\begin{aligned} -\mathcal{L}_{UV} &= \frac{1}{2} \bar{\nu}'_{2R} \hat{\theta}_M \nu'_{2R} + x_l \bar{l}_{1L} l_{3R} + \text{h.c.} + \dots, \\ -\mathcal{L}_{IR} &= \left(\frac{R}{R'}\right)^4 \left[\sqrt{3} \frac{v'}{\Lambda'} (y_b^\alpha \bar{l}_{3\alpha L} l_{\alpha R} + y_b^\alpha \bar{l}_{3\alpha L} \tilde{l}_{\alpha R} + y_s^\alpha \bar{e}'_{3\alpha L} e'_{\alpha R}) \right. \\ &\quad \left. + y_b \frac{v'_\eta}{\Lambda'} (\bar{l}_{1L} l_{2R} + \bar{l}_{1L} \tilde{l}_{2R}) + y_s \frac{v'_\eta}{\Lambda'} \bar{\nu}'_{1L} \nu'_{2R} \right] + \text{h.c.} + \dots, \end{aligned} \quad (15)$$

become diagonal in flavor space (the terms proportional to x_l and $y_{b,s}$ are actually flavor independent), except for the Majorana masses

$$\hat{\theta}_M \equiv \Omega \theta_M \Omega = \begin{pmatrix} \epsilon_s + \frac{2\epsilon_t}{3} & -\frac{\epsilon_t}{3} & -\frac{\epsilon_t}{3} \\ -\frac{\epsilon_t}{3} & \frac{2\epsilon_t}{3} & \epsilon_s - \frac{\epsilon_t}{3} \\ -\frac{\epsilon_t}{3} & \epsilon_s - \frac{\epsilon_t}{3} & \frac{2\epsilon_t}{3} \end{pmatrix}. \quad (16)$$

Dirichlet boundary conditions are modified in the presence of these boundary terms. Thus, on the UV brane

$$l_{1R} - x_l l_{3R} = 0, \quad l_{3L} + x_l^* l_{1L} = 0, \quad \nu'_{2L} + \hat{\theta}_M^\dagger \nu'_{2R} = 0, \quad (17)$$

and on the IR one

$$\begin{aligned} l_{3\alpha R} + \sqrt{3} \frac{v'}{\Lambda'} y_b^\alpha l_{\alpha R} &= 0, & \tilde{l}_{3\alpha R} + \sqrt{3} \frac{v'}{\Lambda'} y_b^\alpha \tilde{l}_{\alpha R} &= 0, & e'_{3\alpha R} + \sqrt{3} \frac{v'}{\Lambda'} y_s^\alpha e'_{\alpha R} &= 0, \\ l_{\alpha L} - \sqrt{3} \frac{v'}{\Lambda'} y_b^{\alpha*} l_{3\alpha L} &= 0, & \tilde{l}_{\alpha L} - \sqrt{3} \frac{v'}{\Lambda'} y_b^{\alpha*} \tilde{l}_{3\alpha L} &= 0, & e'_{\alpha L} - \sqrt{3} \frac{v'}{\Lambda'} y_s^{\alpha*} e'_{3\alpha L} &= 0, \\ l_{1R} + y_b \frac{v'_\eta}{\Lambda'} l_{2R} &= 0, & \tilde{l}_{1R} + y_b \frac{v'_\eta}{\Lambda'} \tilde{l}_{2R} &= 0, & \nu'_{1R} + y_s \frac{v'_\eta}{\Lambda'} \nu'_{2R} &= 0, \\ l_{2L} - y_b^* \frac{v'_\eta}{\Lambda'} l_{1L} &= 0, & \tilde{l}_{2L} - y_b^* \frac{v'_\eta}{\Lambda'} \tilde{l}_{1L} &= 0, & \nu'_{2L} - y_s^* \frac{v'_\eta}{\Lambda'} \nu'_{1L} &= 0. \end{aligned} \quad (18)$$

From these equations we observe that the lepton doublet zero mode is shared by all multiplets due to the non-zero values of x_l , y_b and y_b^α , whereas y_s splits the RH neutrino zero

mode between ζ_1 and ζ_2 , and y_s^α splits the RH charge -1 singlet between ζ_3 and ζ_α . This splitting is crucial in models of GHU, since the Higgs being part of a gauge multiplet can only mix fermion fields within the same $SO(5)$ multiplet, coupling to them with the same (gauge) strength. The non-trivial flavor structure is then only due to the brane terms above. Thus, the only source of flavor violation in the rotated basis comes from $\hat{\theta}_M$ in Eq. (16), whose particular form will eventually lead to TBM mixing in the leptonic sector. This flavor universality is a welcome consequence of the A_4 symmetry, for it will also prevent flavor violating operators mediated by heavy KK gauge bosons to exceed current experimental bounds. This observation, which was made in simpler models with warped extra dimensions [29], is maintained at this order in the more realistic models with GHU under study here. Incidentally, the extra fields required to complete the $SO(5)$ representations imply that simpler Z_2 or Z_3 symmetries are not suitable to banish operators violating this mixing pattern.

III. THE LEPTONIC SPECTRUM

In order to find the lepton masses and mixings we have to solve the equations of motion derived from the bulk action with the boundary conditions in Eqs. (17) and (18). This can be actually carried out exactly in the case of GHU models because the Higgs, which is part of a higher-dimensional gauge field, can be eliminated from the bulk by a rotation in gauge space, thus reducing the Higgs effect to the modification of the boundary conditions. This is essential, for otherwise the Higgs would mix different multiplets in the bulk, and the corresponding equations of motion would be forbiddingly difficult to solve. Still, the large number of fields involved makes the solution of the full system technically challenging. An alternative approach is to perform a Kaluza-Klein (KK) expansion without including the Higgs, and then to take into account its effects by diagonalizing the corresponding mass matrix. In this case one must include the Majorana masses not in the KK expansion but as a contribution to the mass matrix. Otherwise we would have to incorporate the effect of all physical modes up to the Majorana mass scale (which is $\sim R^{-1}$) in order to obtain an accurate enough approximation [21]. Furthermore, the leading contribution to the *light* neutrino masses and mixing angles can be obtained by simply considering the zero modes in the KK expansion (thus including the *heavy* Majorana RH neutrinos), for heavier KK modes

give a suppressed contribution. This so called zero mode approximation (ZMA) is convenient because of the transparent way the flavor structure leading to TBM mixing is realized. We will thus proceed in three steps, first we will compute the light lepton masses and mixing angles in the ZMA. Then, we will include the massive KK modes but still incorporating the localized Majorana masses and the Higgs effects in the mass matrix. Finally, we will take these into account considering the boundary conditions directly in the KK expansion.

The Yukawa couplings, being originally gauge couplings, are flavor diagonal and do not mix different 5D multiplets

$$\begin{aligned}\mathcal{L}_Y &= g_5 h^{\hat{a}}(x) \sqrt{\frac{2}{R}} \frac{1}{\sqrt{1-(R/R')^2}} \int_R^{R'} dz \left(\frac{R}{z}\right)^4 \frac{z}{R'} (\bar{\zeta}_k T_C^{\hat{a}} \Gamma^5 \zeta_k + \bar{\zeta}_\alpha T_C^{\hat{a}} \Gamma^5 \zeta_\alpha) \\ &= -i g_5 v_H \sqrt{\frac{2}{R}} \frac{1}{\sqrt{1-(R/R')^2}} \int_R^{R'} dz \left(\frac{R}{z}\right)^4 \frac{z}{R'} (\bar{\zeta}_{kL} T_C^4 \zeta_{kR} + \bar{\zeta}_{\alpha L} T_C^4 \zeta_{\alpha R}) + \text{h.c.},\end{aligned}\quad (19)$$

where we have used in the last equality $\Gamma^5 = -i\gamma^5$, and assumed that the Higgs takes a v.e.v. $\langle h^{\hat{a}} \rangle = v_H \delta^{\hat{a},4}$. Neglecting $R/R' \ll 1$ and inserting the expression for T_C^4 in Appendix A 2, we get the Yukawa Lagrangian from the bulk

$$\begin{aligned}\mathcal{L}_Y &= \frac{g_5 v_H}{2} \sqrt{\frac{2}{R}} \int_R^{R'} dz \left(\frac{R}{z}\right)^4 \frac{z}{R'} \left\{ \sum_{s=1,2} \left[\bar{\nu}'_{sL} (\nu_{sR} + \tilde{\nu}_{sR}) - (\bar{\nu}_{sL} + \tilde{\bar{\nu}}_{sL}) \nu'_{sR} \right] \right. \\ &\quad \left. + \sum_{s=3,\alpha} \left[\bar{e}'_{sL} (e_{sR} + \tilde{e}_{sR}) - (\bar{e}_{sL} + \tilde{\bar{e}}_{sL}) e'_{sR} \right] \right\} + \text{h.c.}\end{aligned}\quad (20)$$

A. Lepton spectrum in the ZMA

In this section we only consider the leptonic zero modes. The localized Majorana masses and the Higgs couplings will be incorporated as mass terms to be diagonalized. The localized Dirac masses, on the other hand, have to be taken into account exactly. Since they mix different multiplets through the boundary conditions, the physical zero modes (the same will happen for massive modes) are split among all multiplets mixed by them. In particular, the LH lepton doublets live in all four multiplets. Note that as we do not include in the expansion the Majorana mass term, which is the only source of flavor violation, different generations do not mix. The properly normalized zero modes satisfying the corresponding

boundary conditions read

$$l_{1\alpha L}(x, z) = \frac{1}{\sqrt{R'}} \left(\frac{z}{R}\right)^2 \left(\frac{z}{R'}\right)^{-c_1} \frac{f_{c_1}}{\sqrt{l_\alpha}} l_{\alpha L}(x) + \dots, \quad (21)$$

$$l_{2\alpha L}(x, z) = y_b^* \frac{v'_\eta}{\Lambda'} \frac{1}{\sqrt{R'}} \left(\frac{z}{R}\right)^2 \left(\frac{z}{R'}\right)^{-c_2} \frac{f_{c_1}}{\sqrt{l_\alpha}} l_{\alpha L}(x) + \dots, \quad (22)$$

$$l_{3\alpha L}(x, z) = -x_l^* \frac{1}{\sqrt{R'}} \left(\frac{R}{R'}\right)^{c_3-c_1} \left(\frac{z}{R}\right)^2 \left(\frac{z}{R'}\right)^{-c_3} \frac{f_{c_1}}{\sqrt{l_\alpha}} l_{\alpha L}(x) + \dots, \quad (23)$$

$$l_{\alpha L}(x, z) = -\sqrt{3} x_l^* \frac{v'}{\Lambda'} \frac{1}{\sqrt{R'}} \left(\frac{R}{R'}\right)^{c_3-c_1} \left(\frac{z}{R}\right)^2 \left(\frac{z}{R'}\right)^{-c_\alpha} y_b^{\alpha*} \frac{f_{c_1}}{\sqrt{l_\alpha}} l_{\alpha L}(x) + \dots, \quad (24)$$

where $\alpha = e, \mu, \tau$ denote the lepton flavor and $l_{1,2,3}(x, z)$, $l_\alpha(x, z)$ stand for the doublet component of hypercharge $-1/2$ within each $\zeta_{1,2,3,\alpha}$, respectively. Then, $l_\alpha(x)$ are the *physical* zero modes; and the dots correspond to heavy KK modes. The flavor dependent term takes the form

$$l_\alpha \equiv 1 + |y_b|^2 \frac{v'^2_\eta}{\Lambda'^2} \frac{f_{c_1}^2}{f_{c_2}^2} + |x_l|^2 \left(\frac{R}{R'}\right)^{2(c_3-c_1)} \left[\frac{f_{c_1}^2}{f_{c_3}^2} + |y_b^\alpha|^2 \frac{3v'^2_\eta}{\Lambda'^2} \frac{f_{c_1}^2}{f_{c_\alpha}^2} \right], \quad (25)$$

with

$$f_c \equiv \left[\frac{1-2c}{1-\left(\frac{R}{R'}\right)^{1-2c}} \right]^{\frac{1}{2}} \quad (26)$$

defined as usual. Eqs. (21-24) show that x_l governs the splitting of the LH lepton doublet zero mode between $\zeta_{1,2}$ and $\zeta_{3,\alpha}$. Similarly, the splitting between ζ_1 and ζ_2 and the one between ζ_3 and ζ_α are governed by y_b and y_b^α , respectively. Also note that for $c_3 > c_1$ the zero mode components along $\zeta_{3,\alpha}$ have an extra suppression proportional to $(R/R')^{c_3-c_1}$.

The RH charged lepton zero modes live in the $SO(4)$ singlet component of ζ_3 and ζ_α ,

$$e'_{3\alpha R}(x, z) = -\sqrt{3} y_s^\alpha \frac{v'}{\Lambda'} \frac{1}{\sqrt{R'}} \left(\frac{z}{R}\right)^2 \left(\frac{z}{R'}\right)^{c_3} \frac{f_{-c_\alpha}}{\sqrt{\rho_\alpha}} e_{\alpha R}(x) + \dots, \quad (27)$$

$$e'_{\alpha R}(x, z) = \frac{1}{\sqrt{R'}} \left(\frac{z}{R}\right)^2 \left(\frac{z}{R'}\right)^{c_\alpha} \frac{f_{-c_\alpha}}{\sqrt{\rho_\alpha}} e_{\alpha R}(x) + \dots, \quad (28)$$

with

$$\rho_\alpha \equiv 1 + |y_s^\alpha|^2 \frac{3v'^2_\eta}{\Lambda'^2} \frac{f_{-c_\alpha}^2}{f_{-c_3}^2}. \quad (29)$$

Finally, there are RH neutrino zero modes living in the $SO(4)$ singlet components of $\zeta_{1,2}$, which read

$$\nu'_{1\alpha R}(x, z) = -y_s \frac{v'_\eta}{\Lambda'} \frac{1}{\sqrt{R'}} \left(\frac{z}{R}\right)^2 \left(\frac{z}{R'}\right)^{c_1} \frac{f_{-c_2}}{\sqrt{\lambda}} \nu_{\alpha R}(x) + \dots, \quad (30)$$

$$\nu'_{2\alpha R}(x, z) = \frac{1}{\sqrt{R'}} \left(\frac{z}{R}\right)^2 \left(\frac{z}{R'}\right)^{c_2} \frac{f_{-c_2}}{\sqrt{\lambda}} \nu_{\alpha R}(x) + \dots, \quad (31)$$

with

$$\lambda \equiv 1 + |y_s|^2 \frac{v'_\eta{}^2}{\Lambda'^2} \frac{f_{-c_2}^2}{f_{-c_1}^2}. \quad (32)$$

Note that these profiles are not only flavor diagonal but flavor independent.

We can now insert the former expressions in the general Yukawa Lagrangian, Eq. (20), and get the corresponding zero mode mass term

$$- \mathcal{L}_Y = \bar{e}_L \mathcal{M}_D^e e_R + \bar{\nu}_L \mathcal{M}_D^\nu \nu_R + \text{h.c.}, \quad (33)$$

with

$$(\mathcal{M}_D^e)_{\alpha\beta} = \frac{\sqrt{3} g_5 v_H x_l}{2\sqrt{2R}} \frac{v'}{\Lambda'} \left(\frac{R}{R'}\right)^{c_3 - c_1} (y_s^\alpha - y_b^\alpha) \frac{f_{c_1} f_{-c_\alpha}}{\sqrt{l_\alpha \rho_\alpha}} \delta_{\alpha\beta}, \quad (34)$$

$$(\mathcal{M}_D^\nu)_{\alpha\beta} = -\frac{g_5 v_H v'_\eta}{2\sqrt{2R}\Lambda'} (y_s - y_b) \frac{f_{c_1} f_{-c_2}}{\sqrt{\lambda l_\alpha}} \delta_{\alpha\beta}. \quad (35)$$

On the other hand, the UV brane term

$$- \mathcal{L}_M = \frac{1}{2} \bar{\nu}'_{2R}{}^c \hat{\theta}_M \nu'_{2R} \Big|_R + \text{h.c.} \quad (36)$$

gives a Majorana mass contribution to the three RH neutrinos, so that the total zero mode mass Lagrangian writes

$$- \mathcal{L}_m = \bar{e}_L \mathcal{M}_D^e e_R + \bar{\nu}_L \mathcal{M}_D^\nu \nu_R + \frac{1}{2} \bar{\nu}'_R{}^c \mathcal{M}'_M \nu'_R + \text{h.c.}, \quad (37)$$

with

$$\mathcal{M}'_M \equiv \frac{f_{-c_2}^2}{\lambda R'} \left(\frac{R}{R'}\right)^{2c_2} \hat{\theta}_M. \quad (38)$$

Assuming $\lambda \approx 1$ and $R/R' \approx 10^{-16}$, the Majorana mass scale is in the range $(10^{-2} - 10^{-5})/R \approx 10^{17} - 10^{14}$ GeV for $-0.5 \leq c_2 \leq -0.35$. \mathcal{L}_m is already diagonal for charged leptons (see Eq. (34)). Furthermore, the localization parameters f_{-c_α} naturally explain a hierarchical pattern of charged lepton masses.⁴ The electron and muon masses are easily obtained with the corresponding zero modes localized towards the UV brane. The tau mass induces some tension that requires c_1 and c_3 to be relatively close to $1/2$, $c_{1,3} \lesssim 0.6$, and the RH tau to be somewhat localized towards the IR brane, $c_\tau \geq -1/2$. This tension is

⁴ The A_4 symmetry forces the LH charged leptons to share a common localization thus naturally explaining why the mass hierarchy in this sector is smaller than the one in the charge $2/3$ quark sector [38].

stronger the smaller the factor $(y_s^\alpha - y_b^\alpha)x_l v'/\Lambda'$ is. Note the v'/Λ' suppression due to the A_4 structure. This suppression makes the RH tau generically more composite ($c_\tau > -1/2$) than naively expected from its mass. What generically implies *light* leptonic resonances accessible at the LHC, as discussed in section V. The $c_3 - c_1$ difference also controls how the LH zero modes are split between the $\zeta_{1,2}$ and $\zeta_{3,\alpha}$ multiplets (see [37] for a related discussion). This becomes essential to protect the τ (LFV) couplings to the Z when it is near the IR brane.

Let us now turn our attention to the neutrino sector. The matrix elements in Eq. (37) satisfy $\|\mathcal{M}_D^\nu\| \sim \mathcal{O}(\text{TeV}) \ll \|\mathcal{M}_M^\nu\| \lesssim \mathcal{O}(M_{\text{Pl}})$ for natural values of the model parameters. Then, integrating out the *heavy* RH neutrinos we obtain the standard see-saw type Majorana mass matrix for the LH neutrinos

$$\begin{aligned} \tilde{\mathcal{M}}^\nu &= -\mathcal{M}_D^\nu \mathcal{M}_M^\nu^{-1} (\mathcal{M}_D^\nu)^T \\ &= -\frac{\tilde{m}}{3} \begin{pmatrix} \frac{1}{\iota_e} \left[\frac{1}{\epsilon_s} + \frac{2}{\epsilon_s + \epsilon_t} \right] & \frac{1}{\sqrt{\iota_e \iota_\mu}} \left[\frac{1}{\epsilon_s} - \frac{1}{\epsilon_s + \epsilon_t} \right] & \frac{1}{\sqrt{\iota_e \iota_\tau}} \left[\frac{1}{\epsilon_s} - \frac{1}{\epsilon_s + \epsilon_t} \right] \\ \frac{1}{\sqrt{\iota_e \iota_\mu}} \left[\frac{1}{\epsilon_s} - \frac{1}{\epsilon_s + \epsilon_t} \right] & \frac{1}{\iota_\mu} \left[\frac{1}{\epsilon_s} - \frac{1}{\epsilon_s - \epsilon_t} - \frac{\epsilon_t}{\Delta} \right] & \frac{1}{\sqrt{\iota_\mu \iota_\tau}} \left[\frac{1}{\epsilon_s} + \frac{1}{\epsilon_s - \epsilon_t} + \frac{\epsilon_s}{\Delta} \right] \\ \frac{1}{\sqrt{\iota_e \iota_\tau}} \left[\frac{1}{\epsilon_s} - \frac{1}{\epsilon_s + \epsilon_t} \right] & \frac{1}{\sqrt{\iota_\mu \iota_\tau}} \left[\frac{1}{\epsilon_s} + \frac{1}{\epsilon_s - \epsilon_t} + \frac{\epsilon_s}{\Delta} \right] & \frac{1}{\iota_\tau} \left[\frac{1}{\epsilon_s} - \frac{1}{\epsilon_s - \epsilon_t} - \frac{\epsilon_t}{\Delta} \right] \end{pmatrix}, \end{aligned} \quad (39)$$

where we have defined $\Delta = \epsilon_s^2 - \epsilon_t^2$ and

$$\tilde{m} \equiv \frac{g_5^2 (y_s - y_b)^2 v_\eta'^2}{R} v_H^2 R' f_{c_1}^2 \left(\frac{R}{R'} \right)^{-2c_2} = g^2 \log \left(\frac{R'}{R} \right) \frac{(y_s - y_b)^2 v_\eta'^2}{8\Lambda'^2} v_H^2 R' f_{c_1}^2 \left(\frac{R}{R'} \right)^{-2c_2}. \quad (40)$$

In the last equality we have used the tree-level matching of the 5D and 4D gauge coupling constants (in the absence of brane kinetic terms)

$$g_5 = g \sqrt{R \log(R'/R)}, \quad (41)$$

with $g \approx 0.65$ the 4D $SU(2)_L$ coupling constant. If we choose $c_{1,3} \geq \frac{1}{2}$, $c_3 > c_\alpha$, we can take $\iota_\alpha \cong \iota$ independent of α and then

$$\tilde{\mathcal{M}}^\nu \cong -\frac{\tilde{m}}{3\iota} \begin{pmatrix} \frac{1}{\epsilon_s} + \frac{2}{\epsilon_s + \epsilon_t} & \frac{1}{\epsilon_s} - \frac{1}{\epsilon_s + \epsilon_t} & \frac{1}{\epsilon_s} - \frac{1}{\epsilon_s + \epsilon_t} \\ \frac{1}{\epsilon_s} - \frac{1}{\epsilon_s + \epsilon_t} & \frac{1}{\epsilon_s} - \frac{1}{\epsilon_s - \epsilon_t} - \frac{\epsilon_t}{\Delta} & \frac{1}{\epsilon_s} + \frac{1}{\epsilon_s - \epsilon_t} + \frac{\epsilon_s}{\Delta} \\ \frac{1}{\epsilon_s} - \frac{1}{\epsilon_s + \epsilon_t} & \frac{1}{\epsilon_s} + \frac{1}{\epsilon_s - \epsilon_t} + \frac{\epsilon_s}{\Delta} & \frac{1}{\epsilon_s} - \frac{1}{\epsilon_s - \epsilon_t} - \frac{\epsilon_t}{\Delta} \end{pmatrix}, \quad (42)$$

which can be diagonalized by the Harrison-Perkins-Scott matrix [26]

$$U_{HPS} = \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix}. \quad (43)$$

Recall that the charged lepton sector is already diagonal in this basis and therefore, U_{HPS} gives the PMNS mixing matrix with the predicted TBM form. The resulting neutrino mass spectrum reads

$$U_{HPS}^T \tilde{\mathcal{M}}^\nu U_{HPS} = -\frac{\tilde{m}}{l} \begin{pmatrix} \frac{1}{\epsilon_s + \epsilon_t} & 0 & 0 \\ 0 & \frac{1}{\epsilon_s} & 0 \\ 0 & 0 & \frac{1}{\epsilon_t - \epsilon_s} \end{pmatrix}, \quad (44)$$

implying the neutrino mass-squared differences

$$\Delta m_{21}^2 \equiv |m_2|^2 - |m_1|^2 = \left| \frac{\tilde{m}}{\nu \epsilon_s} \right|^2 \left[1 - \frac{1}{(1+r)^2} \right], \quad (45)$$

$$\Delta m_{31}^2 \equiv |m_3|^2 - |m_1|^2 = \left| \frac{\tilde{m}}{\nu \epsilon_s} \right|^2 \left[\frac{4r}{(1-r^2)^2} \right], \quad (46)$$

where $r \equiv \epsilon_t/\epsilon_s$. From Eq. (45) we see that Δm_{21}^2 is positive, as conventionally assumed, for $r < -2$ or $r > 0$. (For $-2 < r < 0$ we would have to exchange the ordering of the first two neutrinos, thus ruining the TBM prediction.) Hence, the neutrino spectrum is normal ($\Delta m_{31}^2 > 0$) for $r > 0$ and inverted ($\Delta m_{31}^2 < 0$) for $r < -2$ (see Eq. (46)). There are three solutions to Eqs. (45) and (46) reproducing the observed mass-squared differences, $\Delta m_{21}^2 \approx 7.67 \times 10^{-5} \text{ eV}^2$ and $\Delta m_{31}^2 \approx 2.46 \text{ } (-2.37) \times 10^{-3} \text{ eV}^2$ for normal (inverted) hierarchy [27], in the allowed r range,

$$r \approx -2.01, 0.79, 1.20. \quad (47)$$

The other solution $r \approx -1.99$ does not give the correct mixing pattern and is therefore ignored. However, both, the normal ($r = 0.79, 1.20$) and the inverted ($r = -2.01$) mass hierarchy, can be realized in these models, with similar phenomenology in either case. On the other hand, the correct scale of neutrino masses is easily obtained varying the localization parameter c_2 , which lies in the interval $-0.4 \lesssim c_2 \lesssim -0.2$ for $c_{1,3}$ values giving the τ mass and $|\epsilon_{t,s}| \sim \mathcal{O}(10^{-2} - 10^{-1})$.

These results receive three types of corrections. First, there are bulk lepton KK modes with masses $\sim \text{TeV}$ which mix with the zero modes. This mixing is small for leptons localized near the UV brane, and therefore the modifications they induce on the fermion masses and mixings are small too. However since the inter-generational mixing is large in the lepton sector, it is important to check that no large LFV is introduced. The second source of corrections is related to the perturbative treatment of the Higgs effects. This is justified

for the scales allowed by EWPT, but in GHU models we can actually test how good this approximation is because in this case it is possible to get a solution to all orders in the Higgs v.e.v.. These two types of corrections, which do not significantly modify the picture drawn above, are studied in the next two subsections. Finally, we have only included the LO $A_4 \times Z_8$ breaking terms. Higher orders, although suppressed by extra powers of $1/\Lambda^{(i)}$, could destabilize the TBM mixing pattern and introduce new sources of LFV. We will consider these higher order corrections in the following section.

B. Inclusion of massive KK modes

The lepton mass Lagrangian contains a Dirac part that includes the Yukawa Lagrangian plus the KK mass terms,

$$\mathcal{L}_D = \mathcal{L}_Y - \sum_{n \geq 1} \left[m_n^l \tilde{l}_L^{(n)} l_R^{(n)} + m_n^{\tilde{l}_{1/2}} \tilde{l}_{\frac{1}{2}L}^{(n)} \tilde{l}_{\frac{1}{2}R}^{(n)} + m_n^{\tilde{l}_{-3/2}} \tilde{l}_{-\frac{3}{2}L}^{(n)} \tilde{l}_{-\frac{3}{2}R}^{(n)} + m_n^{\nu'} \bar{\nu}'_L^{(n)} \nu'_R^{(n)} + m_n^{e'} \bar{e}'_L^{(n)} e'_R^{(n)} + \text{h.c.} \right], \quad (48)$$

where the $SU(2)_L$ doublets with hypercharges $\frac{1}{2}$ and $-\frac{3}{2}$ are denoted by $\tilde{l}_{\frac{1}{2}}$ and $\tilde{l}_{-\frac{3}{2}}$, respectively, and the SM-like (hypercharge $-\frac{1}{2}$) doublets which participate from all $SO(5)$ multiplets by l . Obviously, \mathcal{L}_Y also includes Yukawa couplings with the massive KK modes.

The Dirac mass Lagrangian can be written in matrix form

$$- \mathcal{L}_D = \bar{e}_L \mathcal{M}_D^e e_R + \bar{\nu}_L \mathcal{M}_D^\nu \nu_R + \sum_{n \geq 1} m_n^{\tilde{l}_{1/2}} \bar{\tilde{X}}_L^{(n)} \tilde{X}_R^{(n)} + \sum_{n \geq 1} m_n^{\tilde{l}_{-3/2}} \bar{\tilde{Y}}_L^{(n)} \tilde{Y}_R^{(n)} + \text{h.c.}, \quad (49)$$

where we have grouped together the charge -1 leptons into $e_{L,R}$ and the neutral ones into $\nu_{L,R}$.⁵ The UV brane term, Eq. (36), induces a Majorana mass term that now involves all KK modes of the RH neutrinos

$$- \mathcal{L}_M = \frac{1}{2} \bar{\nu}_R^c \mathcal{M}_M^\nu \nu_R + \text{h.c.} . \quad (50)$$

The mass Lagrangian is diagonal for the charge $+1$ and -2 sectors but not for the charge -1 and neutral ones. However, it is still true that it is family diagonal except for the terms involving the Majorana neutrino masses. Thus, although we have now to diagonalize the charged lepton mass term, this diagonalization does not mix different generations and then

⁵ In this subsection we use the same calligraphic notation to denote matrices although they have a larger size here because they also include massive KK modes.

does not introduce flavor changing neutral currents (FCNC). The corresponding modification of the diagonal Z couplings is proportional to the charged lepton masses and therefore relatively small [39]. On the other hand, the neutrino mass matrix

$$\mathcal{M}^\nu = \begin{pmatrix} 0 & \mathcal{M}_D^\nu \\ (\mathcal{M}_D^\nu)^T & \mathcal{M}_M^\nu \end{pmatrix} \quad (51)$$

is not family diagonal, and the required rotation could in principle induce modifications of the TBM mixing and introduce dangerous non-diagonal couplings between the SM charged leptons and the neutrino KK modes of mass $\sim \text{TeV}$, implying large LFV processes at the loop level. We have numerically checked that neither of these two possibilities is actually realized. The inclusion of massive KK modes does not appreciably modify the TBM mixing pattern and furthermore, although there are non-negligible charged couplings between the SM charged leptons and the neutrino KK modes, they are, to an excellent approximation, family diagonal, *i.e.* if the coupling eN is sizable for some heavy N , then the couplings μN and τN are extremely suppressed. This can be easily understood observing that flavor violation (and also light neutrino masses and thus TBM mixing) is induced by the corresponding Majorana mass, which being localized at the UV brane is much larger than the TeV scale. (For a detailed discussion of the effect on neutrino masses and mixing see [21].)

C. Exact Higgs treatment

GHU models like the one we are considering are among the best motivated models with warped extra dimensions, due to the extra protection of the Higgs potential. They are also interesting because they allow us to solve the bulk equations of motion in the presence of a bulk Higgs. We can perform a field redefinition identical to a gauge transformation which locally removes the Higgs from the action, except at one of the branes. Then, the Higgs does not enter in the bulk action for rotated fields but only as a boundary condition, which can be implemented numerically. We can, therefore, compute non-linear effects of the Higgs due to its Goldstone boson nature. These effects are typically small for the values of the KK scale allowed by EWPT, but this exact treatment will allow us to test our approximation. Besides, we can also include the UV localized Majorana masses as exact boundary conditions, instead of perturbatively.

The field transformation that removes the Higgs locally except at the IR brane is identical to a gauge transformation with gauge parameter

$$\rho(z, v_H) = \exp \left[i \frac{\sqrt{2} g_5 v_H T_C^4}{\sqrt{R(R'^2 - R^2)}} \int_R^z dz' z' \right] \approx \exp \left[i g v_H T_C^4 \sqrt{\log(R'/R)/2} \left(\frac{z^2 - R^2}{R'} \right) \right]. \quad (52)$$

This is not an actual gauge transformation because this gauge parameter does not satisfy the corresponding boundary conditions, but it eliminates the Higgs boson locally except at the IR brane. The bulk action for the rotated fields

$$\zeta' = \rho \zeta \quad (53)$$

is then free of the Higgs v.e.v. v_H , and it can be solved analytically as we did before. The boundary conditions at the UV brane remain the same, since $\rho(R) = 1$. However, the boundary conditions at the IR brane in Eq. (18) apply to the original fields and when written in terms of the rotated ones, they will explicitly include the Higgs effects. Note that the physical modes will now participate from all multiplets, not only from those mixed by localized terms but from those mixed by the Higgs, too. This makes the corresponding boundary conditions much more challenging. Also note that, since we are imposing now as boundary conditions the UV localized Majorana masses, we necessarily have to deal with all three generations simultaneously in the neutrino expansion. For instance, once we impose the UV boundary conditions, the Higgs dependent IR boundary conditions give a system of 8 equations with 8 unknowns (per family) for the charge -1 leptons and two independent systems of 24 equations with 24 unknowns for the neutral ones (due to the Majorana boundary condition the three families mix and the corresponding system of 24 equations with 24 complex unknowns splits into real and imaginary parts, as discussed in Appendix B). Requiring a non-trivial solution of the corresponding systems fixes the mass of the physical states and determines all unknowns in terms of one normalization constant, which is then fixed by the normalization condition involving all relevant multiplets. The exact expression for these boundary conditions are too large to be included here but we have checked that the masses of the charged and neutral leptons (for simplicity we have neglected inter-generational mixing) are in excellent agreement with those obtained with a perturbative treatment of Higgs and UV Majorana mass effects.

IV. HIGHER ORDER EFFECTS

We have seen in the previous section that a global A_4 symmetry can naturally explain the observed lepton masses and TBM neutrino mixing at LO in the breaking of this discrete symmetry along the appropriate direction. The zero mode pattern remains almost unchanged when lepton KK modes or bulk Higgs effects are included. Furthermore, this global symmetry provides an extra level of flavor protection that makes the model compatible with experimental data despite the large number of new particles. The nearly exact realization of TBM mixing, the very precise cancellation of flavor violations and the τ mass preference for a not too small value of v'/Λ' (or alternatively a large degree of compositeness) must be also verified at higher orders in the global symmetry breaking. The structure of higher order contributions is greatly simplified because ϕ (ϕ') preserves a Z_2 (Z_3) subgroup of A_4 [13]. In practice, this means that

$$\langle \phi \rangle^3 \sim \langle \phi \rangle, \quad \langle \phi' \rangle^2 \sim 1 + \langle \phi' \rangle, \quad (54)$$

where \sim means that they have the same A_4 transformation properties. Hence, only operators with one or two powers of ϕ on the UV brane and operators with none or one power of ϕ' on the IR brane give rise to independent flavor structures. The allowed operators are further constrained by the Z_8 symmetry.

The Majorana neutrino masses on the UV brane already have terms with none and one power of ϕ , so the only new structure comes from operators with two powers of ϕ . The lowest order contribution compatible with Z_8 has of the form

$$\frac{\eta\phi^2}{\Lambda^3} \bar{\nu}'_{2R}{}^c \nu'_{2R} + \text{h.c.} \rightarrow \bar{\nu}'_{2R}{}^c \begin{pmatrix} \delta_1 + \delta_2 + \delta_3 & 0 & 0 \\ 0 & \delta_1 + \omega\delta_2 + \omega^2\delta_3 & 0 \\ 0 & 0 & \delta_1 + \omega^2\delta_2 + \omega\delta_3 \end{pmatrix} \nu'_{2R} + \text{h.c.}, \quad (55)$$

with $\delta_i \sim v_\eta v^2/\Lambda^3$ arbitrary. The boundary coupling between the ζ_1 and ζ_3 bi-doublets gets new structures from terms with one or two powers of ϕ . The latter gives a similar contribution to the previous one for neutrinos, whereas the former gives a 2 – 3 mixing,

$$\left[\frac{\eta\phi}{\Lambda^2} + \frac{\phi^2}{\Lambda^2} \right] \bar{l}_{1L} l_{3R} + \text{h.c.} \rightarrow \bar{l}_{1L} \begin{pmatrix} \rho_1 + \rho_2 + \rho_3 & 0 & 0 \\ 0 & \rho_1 + \omega\rho_2 + \omega^2\rho_3 & \gamma_1 \\ 0 & \gamma_2 & \rho_1 + \omega^2\rho_2 + \omega\rho_3 \end{pmatrix} l_{3R} + \text{h.c.}, \quad (56)$$

where $\rho_i \sim v^2/\Lambda^2$ and $\gamma_i \sim v_\eta v/\Lambda^2$.

Let us discuss now the terms on the IR brane. The leading term mixing ζ_1 with ζ_2 contains no power of ϕ' , so the only new structure corresponds to one factor of ϕ' . The first such term comes at order $1/\Lambda'^3$, due to the Z_8 symmetry. At this order we have

$$\left[\frac{\eta'^{*2}\phi'}{\Lambda'^3} + \frac{\eta'|\phi'|^2}{\Lambda'^3} + \frac{\eta'^*\phi'^{\dagger 2}}{\Lambda'^3} \right] \left(\bar{l}_{1L}l_{2R} + \bar{l}_{1L}\tilde{l}_{2R} \right) + \text{h.c.} \rightarrow \bar{l}_{1L} \begin{pmatrix} \epsilon_1 & \epsilon_2 & \epsilon_3 \\ \epsilon_3 & \epsilon_1 & \epsilon_2 \\ \epsilon_2 & \epsilon_3 & \epsilon_1 \end{pmatrix} l_{2R} + [l_{1,2} \rightarrow \tilde{l}_{1,2}] + \text{h.c.}, \quad (57)$$

where $\epsilon_1 \sim v'_\eta v'^2/\Lambda'^3$ and $\epsilon_{2,3}$ get contributions $\sim v'_\eta v'^2/\Lambda'^3$ and $\sim v'^2 v'/\Lambda'^3$, and similarly for $\bar{\nu}'_{1L}\nu'_{2R}$. Finally, the coupling between ζ_3 and ζ_α is not modified by higher order terms, because we already have a term with one power of ϕ' and the singlet contribution cannot result from a singlet structure under A_4 . No further structures are generated at higher orders.

Therefore the higher order effects in the A_4 breaking can be summarized, after the rotation $\zeta_k \rightarrow \Omega \zeta_k$ in Eq. (14), by the following replacements

$$\hat{\theta}_M \rightarrow \hat{\theta}_M + \begin{pmatrix} \delta_1 & \delta_3 & \delta_2 \\ \delta_3 & \delta_2 & \delta_1 \\ \delta_2 & \delta_1 & \delta_3 \end{pmatrix} \quad (58)$$

for the Majorana masses,

$$x_l \rightarrow x_l + X_l, \quad (59)$$

with

$$X_l = \Omega^\dagger \begin{pmatrix} \rho_1 + \rho_2 + \rho_3 & 0 & 0 \\ 0 & \rho_1 + \omega\rho_2 + \omega^2\rho_3 & \gamma_1 \\ 0 & \gamma_2 & \rho_1 + \omega^2\rho_2 + \omega\rho_3 \end{pmatrix} \Omega \quad (60)$$

for the mixing between the SM LH doublets in ζ_1 and ζ_3 , and

$$y_{b,s} \frac{v'}{\Lambda'} \rightarrow y_{b,s} \frac{v'}{\Lambda'} + \begin{pmatrix} \epsilon_1^{b,s} + \epsilon_2^{b,s} + \epsilon_3^{b,s} & 0 & 0 \\ 0 & \epsilon_1^{b,s} + \omega\epsilon_2^{b,s} + \omega^2\epsilon_3^{b,s} & \\ 0 & 0 & \epsilon_1^{b,s} + \omega^2\epsilon_2^{b,s} + \omega\epsilon_3^{b,s} \end{pmatrix} \quad (61)$$

for the mixing between the bi-doublets or the singlets in ζ_1 and ζ_2 . The IR terms remain diagonal whereas the UV terms receive non-diagonal corrections. All three effects are a

source of violation of TBM and the non-diagonal X_l a source of FCNC for the charged leptons. This implies some constraint on $v_{(\eta)}^{(\prime)}/\Lambda^{(\prime)}$ that will be discussed in the next section.

A. A comment on brane kinetic terms

We have neglected so far in our discussion the effect of brane kinetic terms (BKT). These are generated by quantum corrections and therefore cannot be set to zero at arbitrary scales [40]. The global symmetries of our model, however, strongly constrain them. In particular, all possible BKT are proportional to the identity at leading order in A_4 and Z_8 breaking, except those involving ζ_α fields, which are diagonal but flavor dependent. Corrections to this pattern are of order v^2/Λ^2 , where v and Λ stand here for any v, v_η, v', v'_η and Λ, Λ' , respectively. Since at leading order there is no flavor violation in the charged lepton sector, these flavor dependent (but diagonal) BKT do not generate FCNC. Once higher order terms are included, there is a small flavor violation in the charged lepton sector and therefore the flavor dependent BKT will induce FCNC. Higher order contributions to BKT breaking A_4 are also a potential source of flavor violation. However, the A_4 and Z_8 symmetries ensure that the v'^2/Λ'^2 corrections are diagonal. Therefore, their effect is subleading and we will disregard them here. Hence, we include higher order effects in the localized mass terms but not in the BKT. The effects of diagonal BKT are well-known (see for instance [41]). They do not change the functional dependence of the fermion zero modes and only affect their normalization. The ones leading to flavor violation are the BKTs for the RH component of the $SO(4)$ singlet in ζ_α (all the other ones are proportional to the identity, up to tiny corrections). The corresponding BKT can be written as

$$\delta S = \int d^4x dz a^4 \delta(z - R') R' \kappa^\alpha \bar{e}'_{\alpha R} i \not{D} e'_{\alpha R} + \dots, \quad (62)$$

where κ^α is a dimensionless coefficients parametrizing the BKTs. The fermion zero modes for the RH charge -1 leptons have the same functional form as in the absence of BKT, Eqs. (27) and (28), except for the normalization that is now

$$\rho_\alpha^{\text{BKT}} = \rho_\alpha + \kappa^\alpha f_{-c_\alpha}^2. \quad (63)$$

Note that $f_{-c}^2 \ll 1$ for $c \lesssim -0.4$ so this effect in the normalization can be only relevant for the tau lepton. A more significant effect regarding flavor is that the covariant derivative

in Eq. (62) contains the KK expansion of the corresponding gauge bosons. This implies the following flavor dependent coupling of the fermion zero modes to the gauge boson KK modes

$$\delta S = \int d^4x \left[g \sqrt{R \log(R'/R)} \kappa^\alpha \frac{f_{-c_\alpha}^2}{\rho_{\text{BKT}}^\alpha} f_n^A(R') \right] \bar{e}_{\alpha R} \mathcal{A}_n e_{\alpha R} + \dots, \quad (64)$$

which has to be added to the bulk contribution. We have used again the tree level matching of the coupling constant Eq. (41) and assumed a KK expansion of the gauge bosons

$$A_\mu(x, z) = \sum_n f_n^A(z) A_\mu^n(x), \quad (65)$$

with $A_\mu(x, z)$ a generic gauge field (we have left the group structure implicit). After the inclusion of higher order terms in the brane mass terms discussed in the previous section, the charged lepton sector is no longer flavor diagonal in the current eigenstate basis. The rotation of the RH charged leptons required to go to the physical basis will then induce flavor violating couplings to the gauge boson KK modes. Recall however that the charged lepton mass hierarchy is obtained by means of the localization of the RH charged lepton zero modes. This implies that the RH rotation to go to the physical mass is strongly hierarchical and therefore the FCNC induced by the BKT suppressed by the charged lepton masses. We have indeed numerically checked that BKTs do not impose any significant constraint in the model and we will therefore neglect them in the discussion about electroweak and flavor constraints in the next section.

V. ELECTROWEAK AND FLAVOR CONSTRAINTS

Once we have classified all possible higher order terms in the $A_4 \times Z_8$ breaking expansion, we can discuss their effects on the leptonic mixing, *i.e.* departures from TBM mixing, as well as LFV. All three new flavor structures, Eqs. (58-61), are a source of departure from TBM mixing; whereas LFV is mostly affected by Eq. (60). Given the large number of parameters in our model, it is difficult to establish detailed bounds on each one. However, there are some general tendencies that are easy to understand. We have performed a detailed scanning to test these tendencies. The main conclusion is that a large region of parameter space is allowed by all current electroweak and flavor data for an IR scale $1/R' = 1.5$ TeV, provided v/Λ is not too large ($\lesssim 0.1$) and the LH charged leptons are close to the UV brane ($c_{3,1} \gtrsim 0.5$). This conclusion might seem a bit surprising, given previous studies of LFV in

models with warped extra dimensions [42]. The reason our model works so well regarding LFV is a combination of two types of flavor protection. The first one is the protection provided by the A_4 symmetry, which in simpler models with warped extra dimensions is enough to ensure agreement with experimental data [29]. In our case, due to the richer structure imposed by GHU models, this protection is not enough. This is where the second layer of flavor protection enters. Our model naturally falls in the optimal configuration discussed in [34]. The custodial symmetry, together with a LR symmetry originally proposed to protect the $Z\bar{b}_L b_L$ coupling [9], and the splitting of the SM fields in two separate sectors ($\zeta_{1,2}$ and $\zeta_{3,\alpha}$ in our case) reduce LFV in our model to values compatible with current data, despite the low scale of new physics.⁶

We must require for the model to be realistic that it satisfies all experimental constraints. We can classify them in four types: those from EWPT, which we will estimate requiring small deviations from the SM tree-level couplings; limits on LFV processes which can proceed at tree-level; bounds on LFV processes which are banished to higher orders; and constraints from neutrino oscillations. The first three types of restrictions are mainly related to the heavy spectrum, whereas the latter one depends more directly on the discrete flavor symmetry breaking. Thus, although it involves less precisely determined parameters, it does restrict the model. The following phenomenological analysis must be understood as an existence proof. A refined analysis, which is outside the scope of this paper, should consistently include all contributions to a given order. We have done this for tree-level processes, but not for one-loop contributions which have been only estimated with the typically larger amplitude bearing, for instance, possible cancellations. On the other hand, we have not considered one-loop corrections to $Z\bar{e}\mu$ [43]. A detailed study of this type of constraints will be presented elsewhere, for they require a precise enough (numerical) treatment of fermion mixing to recover the proper behaviour of the different contributions, and then of decoupling [44]. The restrictions we explicitly consider are:

- **Electroweak precision tests.** We have required the gauge couplings of the SM charged leptons to be in agreement with the SM prediction within 2 per mille accuracy [29], both for neutral $Z\bar{l}_\alpha l_\alpha$ and charged $W\bar{l}_\alpha \nu_\alpha$ currents. This is typically the present limit on the mixing of the electroweak gauge bosons with new resonances [45],

⁶ Recent analysis of LFV in 4D supersymmetric models with an A_4 symmetry can be found in [25].

and on the square of the SM lepton mixing with heavier vector-like fermions [46].

- **Tree-level LFV.** We have included the most relevant constraints following [42]. Explicitly, we have studied the decays $\mu \rightarrow e^-e^+e^-$, $\tau \rightarrow \mu^-\mu^+\mu^-$, $\tau \rightarrow e^-e^+e^-$, $\tau \rightarrow \mu^-e^+e^-$, $\tau \rightarrow \mu^-e^+\mu^-$ and the $\mu - e$ nuclear conversion rate. The tri-lepton decays $l \rightarrow l_1\bar{l}_2l_3$ are mediated by LFV tree-level couplings to the physical Z gauge boson and its KK excitations. (The effects due to fermion mixing are negligible).⁷ At low energies, these contributions can be parameterized by the following effective Lagrangian,

$$\begin{aligned}
-\mathcal{L}_{\text{eff}} = & \frac{4G_F}{\sqrt{2}} \left[g_3^{\alpha\beta} \left(\bar{l}_R^\beta \gamma^\mu l_R^\alpha \right) \left(\bar{l}_R^\beta \gamma^\mu l_R^\beta \right) + g_4^{\alpha\beta} \left(\bar{l}_L^\beta \gamma^\mu l_L^\alpha \right) \left(\bar{l}_L^\beta \gamma^\mu l_L^\beta \right) \right. \\
& \left. + g_5^{\alpha\beta} \left(\bar{l}_R^\beta \gamma^\mu l_R^\alpha \right) \left(\bar{l}_L^\beta \gamma^\mu l_L^\beta \right) + g_6^{\alpha\beta} \left(\bar{l}_L^\beta \gamma^\mu l_L^\alpha \right) \left(\bar{l}_R^\beta \gamma^\mu l_R^\beta \right) \right] + \text{h.c.}, \quad (66)
\end{aligned}$$

where $\alpha = e, \mu, \tau$. In terms of this effective Lagrangian, the branching ratios for these decays read

$$\begin{aligned}
\mathcal{B}(\mu \rightarrow eee) &= 2 \left(|g_3^{\mu e}|^2 + |g_4^{\mu e}|^2 \right) + |g_5^{\mu e}|^2 + |g_6^{\mu e}|^2, \\
\mathcal{B}(\tau \rightarrow \mu\mu\mu) &= \left\{ 2 \left(|g_3^{\tau\mu}|^2 + |g_4^{\tau\mu}|^2 \right) + |g_5^{\tau\mu}|^2 + |g_6^{\tau\mu}|^2 \right\} \mathcal{B}(\tau \rightarrow e\nu\nu), \\
\mathcal{B}(\tau \rightarrow eee) &= \left\{ 2 \left(|g_3^{\tau e}|^2 + |g_4^{\tau e}|^2 \right) + |g_5^{\tau e}|^2 + |g_6^{\tau e}|^2 \right\} \mathcal{B}(\tau \rightarrow e\nu\nu), \\
\mathcal{B}(\tau \rightarrow ee\mu) &= \left\{ |g_3^{\tau\mu}|^2 + |g_4^{\tau\mu}|^2 + |g_5^{\tau\mu}|^2 + |g_6^{\tau\mu}|^2 \right\} \mathcal{B}(\tau \rightarrow e\nu\nu), \\
\mathcal{B}(\tau \rightarrow e\mu\mu) &= \left\{ |g_3^{\tau e}|^2 + |g_4^{\tau e}|^2 + |g_5^{\tau e}|^2 + |g_6^{\tau e}|^2 \right\} \mathcal{B}(\tau \rightarrow e\nu\nu). \quad (67)
\end{aligned}$$

For the $\mu - e$ conversion rate we have applied the usual expression

$$B_{\text{conv}} = \frac{2p_e E_e G_F^2 m_\mu^3 \alpha^3 |F_q|^2 Z_{\text{eff}}^4 Q_N^2}{\pi^2 Z \Gamma_{\text{capt}}} \left[|g_R^{\mu e}|^2 + |g_L^{\mu e}|^2 \right], \quad (68)$$

where $g_{L,R}^{\mu e}$ are the corresponding off-diagonal $Z\bar{e}\mu$ couplings, G_F is the Fermi constant and α the QED coupling strength, while the other terms are atomic physics constants defined in [48]. We shall use the current PDG [45] bounds for the tri-lepton decays and the titanium bound $B_{\text{conv}} < 6.1 \times 10^{-13}$ from the SINDRUM II experiment [49] for $\mu - e$ conversion.

⁷ Higgs mediated contributions [47] are suppressed by the A_4 symmetry and the SM lepton masses, and then very small in this class of models.

- **One-loop LFV.** We have also considered the constraints on gauge boson [29] and Higgs [42] mediated amplitudes for $\mu \rightarrow e\gamma$. The charged boson contributions to this branching ratio read

$$\mathcal{B}_G(\mu \rightarrow e\gamma) = \frac{3\alpha}{8\pi} \left[\left| \sum_V \sum_i U_{\mu i}^{VL*} U_{ei}^{VL} F_1 \left(\frac{m_i^2}{M_V^2} \right) \right|^2 + L \rightarrow R \right], \quad (69)$$

where V denotes the gauge boson running in the loop, including the W_L zero mode and its lightest KK excitation, and the first W_R KK mode (the charged gauge boson in $SU(2)_R$). The subscript i indicates the massive fermion running in the loop, and $U_{e,\mu i}^{VL,R}$ stand for the electron and muon couplings to the corresponding gauge boson and heavy lepton (in units of $g/\sqrt{2}$). Finally, the function F_1 is given by

$$F_1(z) = \frac{1}{6(1-z)^4} (10 - 43z + 78z^2 - 49z^3 + 4z^4 + 18z^3 \log z). \quad (70)$$

There is a comparable contribution from neutral gauge boson exchange, typically of opposite sign [50, 51]. The Higgs mediated branching ratio reads [42, 50]

$$\mathcal{B}_H(\mu \rightarrow e\gamma) = \frac{3\alpha}{8\pi} \left[\left| \sum_i \Lambda_{\bar{e}_L i_R} \Lambda_{i_L \mu_R} \frac{v_H^2}{2m_\mu m_i} F_2 \left(\frac{m_H^2}{m_i^2} \right) \right|^2 + L \leftrightarrow R \right], \quad (71)$$

where Λ is the corresponding Yukawa matrix, $v_H \approx 246$ GeV,

$$F_2(x) = \frac{1}{(1-x)^3} (1 - 4x + 3x^2 - 2x^2 \log x), \quad (72)$$

and the sum runs over the leptonic KK modes. The contributions in Eqs. (69) and (71) are of similar order when the mixing between light (SM) and heavy (vector-like) leptons, which is encoded in U and Λ , respectively, is explicitly taken into account, despite the apparently large enhancement factor v_H/m_μ in the latter case [52]. We will use the current limit $\mathcal{B}(\mu \rightarrow e\gamma) < 1.2 \times 10^{-11}$ [45], as well as the expected bound $\sim 10^{-13}$ from the on-going MEG experiment [53] in the quantitative discussion below.

- **PMNS matrix.** We shall take the constraints on the PMNS mixing matrix from [27]

$$|U|_{3\sigma} = \begin{pmatrix} 0.77 \rightarrow 0.86 & 0.50 \rightarrow 0.63 & 0.00 \rightarrow 0.22 \\ 0.22 \rightarrow 0.56 & 0.44 \rightarrow 0.73 & 0.57 \rightarrow 0.80 \\ 0.21 \rightarrow 0.55 & 0.40 \rightarrow 0.71 & 0.59 \rightarrow 0.82 \end{pmatrix}. \quad (73)$$

Let us discuss the scanning over the model parameters. Electroweak tests are generically satisfied for our choice of IR scale $1/R' = 1.5$ TeV, as expected for UV localized light fermions [15] (with partial protection of universality for Z couplings). The constraints from tree-level LFV are also typically mild, due to the double layer of flavor protection in our model. Among the processes considered only $\mu \rightarrow eee$ and $\mu - e$ conversion are close to current experimental limits. In our general scan up to $\sim 70\%$ and $\sim 51\%$ of the points pass the corresponding bounds, respectively.⁸ The main constraint turns out to arise from $\mu \rightarrow e\gamma$. In general the new contributions are smaller for smaller values of v/Λ and relatively large values of c_3 . On the other hand, the departure from TBM mixing is somewhat sensitive to the value of v'/Λ' , decreasing with this ratio. Thus, in the following we fix $v'/\Lambda' = 0.05$ to ensure a nearly correct neutrino mixing, passing $\sim 82\%$ of the points the PMNS test when varying the other parameters. These are randomly selected with $0 \leq v_\eta/\Lambda, v'_\eta/\Lambda' \leq 0.3$ and $c_3 \geq 0.5$. v/Λ is computed from Eq. (45). In the Figure we show the most restricting observables as a function of this ratio, together with the corresponding current experimental limit (solid line). These are $\mu \rightarrow eee$ (top-left panel), $\mu - e$ conversion in nuclei (top-right panel) and $\mu \rightarrow e\gamma$ (lower panels, with the full range of v/Λ on the left and only small v/Λ values on the right), for which we have also drawn the expected sensitivity from MEG (dashed line). The extra flavor protection of tree-level mediated processes in the top panels relative to the one-loop $\mu \rightarrow e\gamma$ decay can be clearly observed in the Figure. As we see, a large number of points passes the different tests for relatively small v/Λ values. If we restrict ourselves to $v/\Lambda \leq 0.05$, about 97% of the points pass the $\mu \rightarrow eee$ and $\mu - e$ conversion tests, whereas 61% satisfy the $\mu \rightarrow e\gamma$ bound (and only 28% the expected MEG sensitivity). We collect in Table II the percentage of points that satisfies all the experimental tests for different ranges of c_3 and v/Λ . Requiring $c_3 \geq 0.55$ and $v/\Lambda \leq 0.05$ we find that 91% of the points pass all current experimental constraints (53% if we include the projected MEG sensitivity on $\mu \rightarrow e\gamma$). For v/Λ small enough, all tree-level LFV effects are negligible, and the only (mild) constraint comes from $\mu \rightarrow e\gamma$. Note, however, that in our scans we have randomly selected order one values of the dimensionless couplings and fixed the global scale through the ratios $v_{(n)}^{(i)}/\Lambda^{(i)}$. The unbalanced sensitivity of $\mu \rightarrow e\gamma$ forces the global scale to be small, and then all other effects are almost negligible, including deviations from

⁸ $\mu - e$ conversion can be within the reach of projected experiments (see [25]).

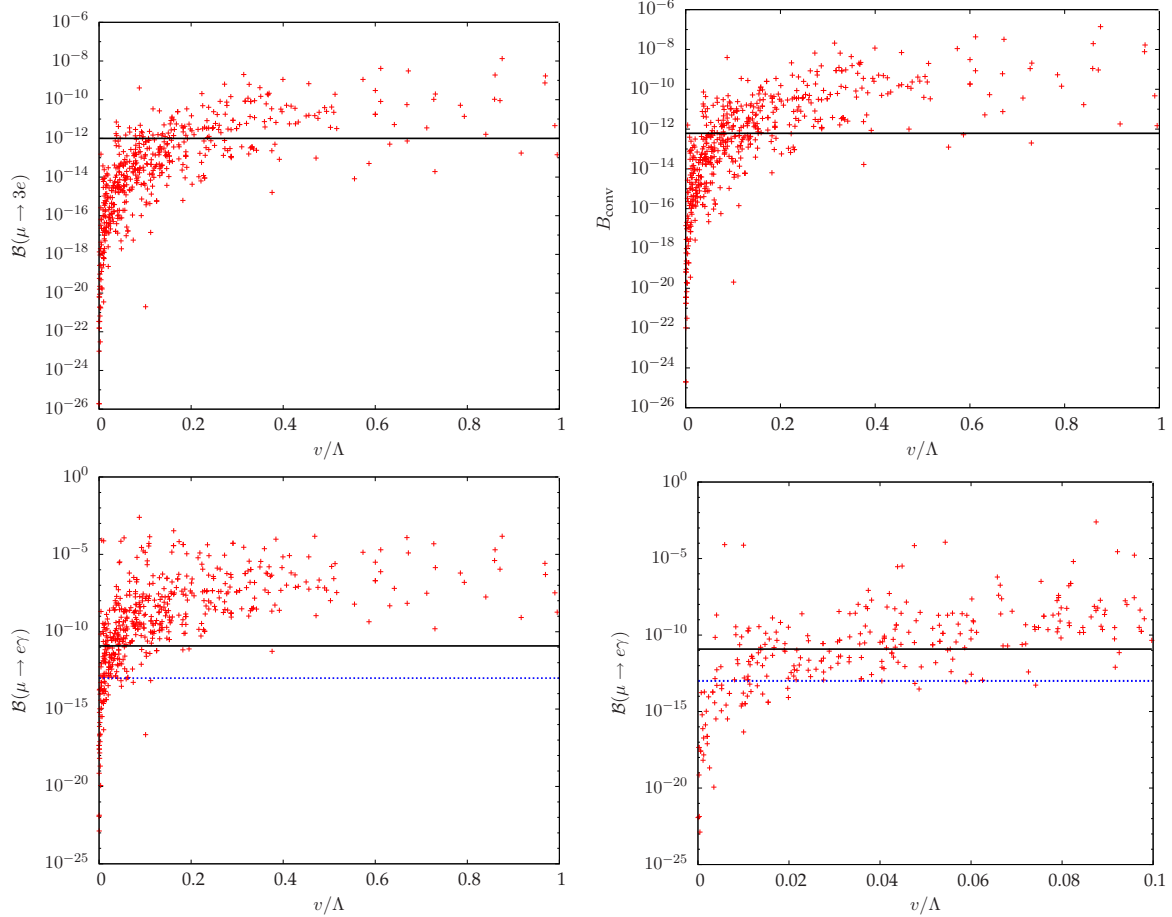


FIG. 1: LFV branching ratios as a function of v/Λ for the scan described in the text. $\mu \rightarrow eee$ is plotted on the top-left panel, $\mu - e$ conversion in nuclei on the top-right one, and $\mu \rightarrow e\gamma$ on the two lower panels (any v/Λ value on the left panel and small v/Λ values on the right one). The horizontal lines correspond to the current experimental upper bound (solid) and future sensitivity (dashed).

constraint	all tests	all tests + MEG
$c_3 \geq 0.5, \quad \frac{v}{\Lambda} \leq 0.05$	60%	28%
$c_3 \geq 0.55, \quad \frac{v}{\Lambda} \leq 0.15$	65%	31%
$c_3 \geq 0.55, \quad \frac{v}{\Lambda} \leq 0.05$	91%	53%

TABLE II: Percentage of points that satisfies all experimental tests (including the projected MEG sensitivity on the last column) for different parameter intervals.

TBM mixing. Of course, it is also possible that some couplings are accidentally larger than others, thus inducing sizable corrections to some observables without being excluded by the $\mu \rightarrow e\gamma$ limit. For example, if we set $v/\Lambda = 0.5$ and all the coefficients of higher dimensional operators equal to zero except $\delta_2 = 8$ (well below its NDA estimate $\delta \lesssim 4\pi^2 x_\eta$ if $x_\eta \sim 1$), we obtain

$$\sin \theta_{13} = 0.18, \tag{74}$$

with all other observables within experimental limits. Thus, in our construction sizable departures from TBM mixing can be still compatible in with all other experimental constraints (although some fine-tuning might be necessary for large departures).

Our analysis shows that, in general, small values of v/Λ and v'/Λ' are preferred by lepton mixing and LFV observables. We have already emphasized the correlation between v'/Λ' and c_τ (the smaller the former, the larger c_τ has to be in order to reproduce the τ mass, implying in turn a more composite τ_R). This has important consequences regarding the spectrum in our model as a larger c_τ value implies light modes. The structure is very generic in this class of models. There is a relatively light, almost degenerate bi-doublet (two charge -1 , one neutral and one charge -2 leptons) that mainly couples to τ_R . This bi-doublet mostly lives in ζ_τ (see Eq. (5)), which is light due to the assigned twisted boundary conditions [54]. These four leptons can be very light and couple strongly to τ_R without being experimentally excluded because they are almost degenerate (see [33] for a recent discussion of this phenomenon in the quark sector). This degeneracy also dictates a very characteristic collider phenomenology as we comment in the following.

A. A numerical example and its collider implications

As we have argued, it is very likely in this class of composite Higgs models that either there are LFV processes close to current experimental limits and then accessible at future flavor experiments or light leptonic resonances in an almost degenerate bi-doublet mainly coupled to τ_R . Let us comment on an example predicting new leptonic resonances accessible

at the LHC. The model is defined by the parameters

$$\begin{aligned}
R^{-1} &= 10^{16} \text{ TeV}, & R'^{-1} &= 1.5 \text{ TeV}, \\
c_1 &= 0.65, & c_2 &= -0.19, & c_3 &= 0.57, & c_{e,\mu,\tau} &= (-0.71, -0.54, 0.49), \\
x_\eta &= 0.98, & x_\nu &= 1.28, & x_l &= 0.65, \\
y_b &= 0.73, & y_s &= 0.44, & y_b^\alpha &= (1.10, -1.37, -0.36), & y_s^\alpha &= (-1.63, 0.93, 1.64), \\
\frac{v}{\Lambda} &= 0.024, & \frac{v_\eta}{\Lambda} &= 0.04, & \frac{v'}{\Lambda'} &= 0.05, & \frac{v'_\eta}{\Lambda'} &= 0.13,
\end{aligned} \tag{75}$$

and random order one dimensionless coefficients for higher order operators. The spectrum reproduces the observed pattern of charged lepton and neutrino masses and mixing angles. Above them, there is an almost degenerate bi-doublet, whose matter content we denote by $N_{L,R}$, $T_{L,R}^1$, $T_{L,R}^2$ and $Y_{L,R}$, all with masses ≈ 410 GeV. Other fermionic resonances have masses $\gtrsim 3$ TeV.⁹ The bosonic resonances are all above 3.5 TeV.

The lepton couplings to the SM bosons can be written

$$\mathcal{L}^W = \frac{g}{\sqrt{2}} W_\mu^+ \bar{\psi}_Q^i \gamma^\mu \left[V_{ij}^{QL} P_L + V_{ij}^{QR} P_R \right] \psi_{(Q-1)}^j + \text{h.c.}, \tag{76}$$

$$\mathcal{L}^Z = \frac{g}{2c_W} Z_\mu \bar{\psi}_Q^i \gamma^\mu \left[X_{ij}^{QL} P_L + X_{ij}^{QR} P_R - 2s_W^2 Q \delta_{ij} \right] \psi_Q^j, \tag{77}$$

$$\mathcal{L}^H = -\frac{H}{\sqrt{2}} \bar{\psi}_Q^i Y_{ij}^Q P_R \psi_Q^j + \text{h.c.}, \tag{78}$$

where $P_{L,R}$ stand for the chirality projectors and ψ_Q is the lepton of charge Q , when it exists. In our case $Q = -2, -1, 0$ (and $+1$ if heavier modes are included).

The relevant couplings, ignoring e and μ , read

$$|V^{(0)L}| \approx \begin{pmatrix} 0.71 & 0 & 0 \\ 0 & 0.71 & 0.71 \end{pmatrix}, \quad |V^{(0)R}| \approx \begin{pmatrix} 0 & 0 & 0 \\ 0.12 & 0.71 & 0.70 \end{pmatrix}, \tag{79}$$

$$|V^{(-1)L}| \approx \begin{pmatrix} 0 & 0.71 & 0.71 \end{pmatrix}^T, \quad |V^{(-1)R}| \approx \begin{pmatrix} 0.12 & 0.71 & 0.70 \end{pmatrix}^T, \tag{80}$$

$$X^{(-1)L} \approx \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad X^{(-1)R} \approx \begin{pmatrix} 0 & 0.16 & 0 \\ 0.16 & 0 & 0.95 \\ 0 & 0.95 & 0 \end{pmatrix}, \tag{81}$$

⁹ $N_{L,R}$ is a quasi-Dirac neutrino, for it has a tiny Majorana mass which is irrelevant for its collider phenomenology and will ignore in the following.

and

$$Y^{(-1)} \approx \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ -0.39 & 0 & 0 \end{pmatrix}, \quad (82)$$

where a “0” entry means $\lesssim 10^{-2}$, and the order of charge -1 leptons is τ, T^1, T^2 . These values are in good agreement with the expectations from degenerate bi-doublets [33] (the small deviations, with no consequence at the LHC, are due to the heavy modes with masses $\gtrsim 3$ TeV). As we see, all four new leptons decay into taus 100 % of the time, $N \rightarrow W^+\tau$, $Y \rightarrow W^-\tau$, $T^1 \rightarrow Z\tau$ and $T^2 \rightarrow H\tau$.

The new leptons are produced in pairs at the LHC. Single production in association with a τ is suppressed because the off-diagonal couplings $\bar{\tau}FV$, with F the new lepton and $V = W, Z$, are small [55]. Drell-Yan pair production results in different final states

$$\tau\bar{\tau}WZ \text{ and } \tau\bar{\tau}WH \quad (83)$$

from W exchange,

$$\tau\bar{\tau}W^+W^-, \tau\bar{\tau}ZZ \text{ and } \tau\bar{\tau}HH \quad (84)$$

from photon and Z exchange, whereas

$$\tau\bar{\tau}ZH, \quad (85)$$

only proceeds through Z exchange. These signals are difficult to disentangle from the background because tau leptons must be reconstructed, but the relative lightness of these new leptons and their very characteristic decay channels help to search for them. A dedicated analysis, that is currently under way, is required to fully assess the LHC reach for vector-like leptonic resonances as predicted in this class of models. (See [56] for generic decay channels.)

VI. CONCLUSIONS

Models with warped extra dimensions provide a neat solution to the hierarchy problem. In this context, models of gauge-Higgs unification in warped extra dimensions are among the most natural models of EWSB, realizing in a calculable way the old idea of composite Higgs. Furthermore, they also offer a rationale for the flavor structure in the quark sector. Mass hierarchies and hierarchical mixing angles are naturally generated by wave function

localization, with an added bonus in the form of a built-in flavor protection that makes new physics at $\sim 2 - 3$ TeV compatible with current EW precision and flavor data.¹⁰ Given the success of GHU models in the quark sector, it is interesting to study their implications for the leptonic one.

In this article, we have studied for the first time the implementation of a global A_4 symmetry in models of GHU. The extra structure implied by the larger gauge symmetry results in some technical differences with respect to the simpler cases studied in the past. Thus, although LFV is generated at tree level, the global symmetry provides a strong enough flavor protection because a subgroup of the custodial symmetry required by EWPT naturally provides the necessary extra suppression. We have also investigated possible deviations from TBM (which is predicted at LO by the assumed discrete symmetry breaking) and the implications of EWPT, LFV and neutrino masses and mixing on the spectrum of new resonances. This requires a precise enough determination of the masses and mixings of particles spreaded by many orders of magnitude, making the numerical analysis rather challenging. We must ensure that the many different types of corrections do not alter the necessary disparity of masses and mixings. In particular, the stringent bounds on LFV processes demands a precise evaluation of the mixing between light and heavy leptons, because this mixing is what renders the one-loop contributions small enough.

The model is compatible with all those experimental constraints for new gauge boson masses

$$M_{KK}^{\text{gauge}} \gtrsim 3.5 \text{ TeV}. \quad (86)$$

Then, KK gluons could be accessible at the LHC [59] (see [60] for KK EW resonances). A new characteristic feature of our construction is the correlation between one-loop LFV (for instance, $\mu \rightarrow e\gamma$) and the presence of light leptonic resonances in the spectrum. In order to keep LFV below current (and expected) experimental bounds, the A_4 breaking has to be relatively small. On the other hand, charged leptons masses are protected by this global flavor symmetry. Thus, the smaller its breaking is, the more composite τ_R has to be in order to predict the correct τ mass. This in turn implies the existence of new leptonic resonances with masses of few hundreds of GeV and large couplings to τ_R . They come in a full almost

¹⁰ Full compatibility with flavor data requires a mild tuning of parameters or some (minimal) structure in the flavor realization [13, 38, 57, 58].

degenerate $SU(2)_L \times SU(2)_R$ bi-doublet with a very distinctive phenomenology at the LHC. Hence, as the discovery of new resonances in the quark sector [33, 61], the observation of LFV processes near present limits or of new vector-like lepton doublets only decaying into taus at LHC would be a strong indication of a strongly coupled realization of EWSB.

Note Added. During the writing of this paper, the possibility of new light leptonic resonances accessible at LHC has been also discussed in models with warped extra dimensions in [62, 63]. However, in our case these new resonances are a consequence of the A_4 symmetry predicting tri-bimaximal mixing, that correlates them to lepton flavor violating processes through the tau mass.

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Appendix A: Group theory summary

In this appendix we summarize the main group theory properties used in the text.

1. A_4 representations

A_4 is the group of even permutations of four elements. It has twelve elements which can be written in terms of two generators, S and T , satisfying

$$S^2 = T^3 = (ST)^3 = \mathbf{1}. \tag{A1}$$

This discrete group has three inequivalent one-dimensional representations

$$\begin{aligned}
\mathbf{1} &: S = 1, \quad T = 1, \\
\mathbf{1}' &: S = 1, \quad T = e^{i2\pi/3} = \omega, \\
\mathbf{1}'' &: S = 1, \quad T = e^{i4\pi/3} = \omega^2,
\end{aligned} \tag{A2}$$

and one three-dimensional irreducible representation, $\mathbf{3}$; being the Clebsch-Gordan series of their non-trivial products

$$\begin{aligned}
\mathbf{1}' \times \mathbf{1}' &= \mathbf{1}'', \quad \mathbf{1}' \times \mathbf{1}'' = \mathbf{1}, \quad \mathbf{1}'' \times \mathbf{1}'' = \mathbf{1}', \\
\mathbf{1}' \times \mathbf{3} &= \mathbf{3}, \quad \mathbf{1}'' \times \mathbf{3} = \mathbf{3}, \\
\mathbf{3}_x \times \mathbf{3}_y &= \mathbf{3}_1 + \mathbf{3}_2 + \mathbf{1} + \mathbf{1}' + \mathbf{1}''.
\end{aligned} \tag{A3}$$

In the basis where S is diagonal

$$S = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}, \quad T = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}, \tag{A4}$$

and the decomposition of $\mathbf{3}_x \times \mathbf{3}_y$ reads

$$\begin{aligned}
\mathbf{1} &= x_1 y_1 + x_2 y_2 + x_3 y_3, \\
\mathbf{1}' &= x_1 y_1 + \omega^2 x_2 y_2 + \omega x_3 y_3, \\
\mathbf{1}'' &= x_1 y_1 + \omega x_2 y_2 + \omega^2 x_3 y_3, \\
\mathbf{3}_1 &= (x_2 y_3, x_3 y_1, x_1 y_2), \\
\mathbf{3}_2 &= (x_3 y_2, x_1 y_3, x_2 y_1),
\end{aligned} \tag{A5}$$

with $\mathbf{3}_x = (x_1, x_2, x_3)$ and $\mathbf{3}_y = (y_1, y_2, y_3)$.

2. $SO(5)$ generators in the fundamental representation

The ten $SO(5)$ generators can be written in the fundamental representation (5)

$$\begin{aligned}
T_{L,ij}^a &= -\frac{i}{2} \left[\frac{1}{2} \epsilon^{abc} (\delta_i^b \delta_j^c - \delta_j^b \delta_i^c) + (\delta_i^a \delta_j^4 - \delta_j^a \delta_i^4) \right], \quad a = 1, 2, 3, \\
T_{R,ij}^a &= -\frac{i}{2} \left[\frac{1}{2} \epsilon^{abc} (\delta_i^b \delta_j^c - \delta_j^b \delta_i^c) - (\delta_i^a \delta_j^4 - \delta_j^a \delta_i^4) \right], \quad a = 1, 2, 3, \\
T_{C,\hat{a}}^{\hat{a}} &= -\frac{i}{\sqrt{2}} [\delta_i^{\hat{a}} \delta_j^5 - \delta_j^{\hat{a}} \delta_i^5], \quad \hat{a} = 1, 2, 3, 4.
\end{aligned} \tag{A6}$$

They are normalized to $\text{Tr}T^\alpha T^\beta = \delta^{\alpha\beta}$. In this basis $\mathbf{5}$, which decomposes into $(\mathbf{2}, \mathbf{2}) \oplus (\mathbf{1}, \mathbf{1})$ under $SU(2)_L \times SU(2)_R$, reads

$$Q = \frac{1}{\sqrt{2}} \begin{pmatrix} q_{++} - q_{--} \\ iq_{++} + iq_{--} \\ -iq_{+-} + iq_{-+} \\ q_{+-} + q_{-+} \\ \sqrt{2}q_{00} \end{pmatrix}, \quad (\text{A7})$$

where the first (second) subscript $\pm, 0$ corresponds to $T_L^3 = \pm\frac{1}{2}, 0$ ($T_R^3 = \pm\frac{1}{2}, 0$), respectively.

Appendix B: KK expansion in the presence of boundary Majorana masses

Let us consider the effect on the KK expansion of a bulk fermion ψ with a UV localized Majorana mass term

$$\theta_{ij} \bar{\psi}_{iR}^c \psi_{jR} + \text{h.c.}, \quad (\text{B1})$$

and then satisfying the UV boundary condition

$$\psi_{iL}(x, R) + \theta_{ij}^\dagger \psi_{jR}^c(x, R) = 0. \quad (\text{B2})$$

The corresponding KK expansion can be written

$$\psi_{iL}(x, z) = \sum_n g_{in}(z) \xi_n(x), \quad \psi_{iR}(x, z) = \sum_n f_{in}(z) \bar{\xi}_n(x), \quad (\text{B3})$$

where the 4D fields are assumed to obey a Majorana equation

$$i\bar{\sigma}^\mu \partial_\mu \xi_n - m_n \bar{\xi}_n = 0, \quad i\sigma^\mu \partial_\mu \bar{\xi}_n - m_n^* \xi_n = 0. \quad (\text{B4})$$

If we insert these expansions in the 5D equations of motion and use the 4D Majorana equations, we get a system of coupled differential equations

$$f'_{in} + m_n g_{in} - \frac{c_i + 2}{z} f_{in} = 0, \quad (\text{B5})$$

$$g'_{in} - m_n^* f_{in} + \frac{c_i - 2}{z} g_{in} = 0. \quad (\text{B6})$$

The solution of this system of equations can be obtained using standard techniques (decoupling by iteration and use of the first order equation to obtain the second solution),

$$g_{in}(z) = z^{5/2} (A_{in} J_{c_i+1/2}(|m_n|z) + B_{in} Y_{c_i+1/2}(|m_n|z)), \quad (\text{B7})$$

$$f_{in}(z) = \frac{m_n}{|m_n|} z^{5/2} (A_{in} J_{c_i-1/2}(|m_n|z) + B_{in} Y_{c_i-1/2}(|m_n|z)). \quad (\text{B8})$$

These two n functions are mixed by the boundary condition in Eq. (B2). If θ_{ij} is real, m_n will be also real and the linear system resulting from imposing the boundary condition will factorize into two simpler ones; one for the real part of the unknowns, and another one for their imaginary parts (obtained changing the sign of θ_{ij}).

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