

A minimal set of top anomalous couplings

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Abstract

We simplify the general form of the fermion-fermion-gauge boson interactions generated by dimension-six gauge-invariant effective operators by using the equations of motion to remove redundant operators. It is found that the most general vertex for off-shell fermions f_i, f_j and an off-shell boson $V = W, Z, \gamma, g$ only involves γ^μ and $\sigma^{\mu\nu}q_\nu$ terms, with $q = p_i - p_j$. Examples are given for the $Wtb, Ztt, \gamma tt$ and $g tt$ interactions, whose general expression is greatly simplified with respect to previous results in the literature. The same arguments apply to top flavour-changing neutral interactions with the Z boson, the photon or the gluon, which can also be parameterised in full generality with only γ^μ and $\sigma^{\mu\nu}q_\nu$ couplings. Explicit expressions are given for these vertices in terms of dimension-six gauge-invariant operators. We also discuss how effective operator coefficients might be determined from eventual measurements of anomalous couplings.

1 Introduction

The precise measurement of the couplings among the known fermions and bosons is a standard tool for the search of new physics beyond the Standard Model (SM). In particular, at the Large Hadron Collider (LHC), top quarks will be produced in large numbers, allowing to probe the top couplings with a great precision. Such a high precision is most welcome because, being the top the heaviest quark, effects of new physics on its couplings are expected to be larger than for any other fermion, and deviations with respect to the SM predictions might be detectable. An adequate parameterisation of the most general interactions of the top quark (or any other fermion) with the gauge bosons is compulsory in order to search for new physics and to interpret the results of experimental measurements. In particular, it is important to avoid the appearance of redundant parameters which only lead to a complication of the analyses, both from the theoretical and experimental side, without making them more general.

The on-shell interaction between two fermions f_i, f_j and a gauge boson $V =$

W, Z, γ, g can be parameterised in full generality as

$$\begin{aligned} \mathcal{L}_{Vf_i f_j}^{\text{OS}} = & \bar{f}_j \gamma^\mu (\mathcal{A}_L P_L + \mathcal{A}_R P_R) f_i V_\mu \\ & + \bar{f}_j i \sigma^{\mu\nu} q_\nu (\mathcal{B}_L P_L + \mathcal{B}_R P_R) f_i V_\mu + \text{H.c.}, \end{aligned} \quad (1)$$

where $q = p_i - p_j$ is the outgoing boson momentum and $\mathcal{A}_{L,R}, \mathcal{B}_{L,R}$ are form factors, which in general may depend on q^2 . (For the flavour-conserving photon and gluon vertices $\mathcal{A}_L = \mathcal{A}_R$ and for the flavour-changing ones $\mathcal{A}_{L,R} = 0$ due to gauge symmetry.) A term proportional to q^μ does not give any contribution to the amplitudes for on-shell V , because in this case the vector boson polarisation ϵ_μ satisfies $q^\mu \epsilon_\mu = 0$.¹ Additional terms with different Lorentz structures can be brought into this form by using the on-shell conditions, namely, the Dirac equation. For off-shell fermions f_i, f_j the situation might seem quite different because the Dirac equation cannot be used to restrict the number and structure of the Lagrangian terms. However, as we will show here, if the new anomalous couplings arise from dimension-six gauge-invariant effective operators, then the Lagrangian in Eq. (1) is still the most general one. We recall here that effects of new physics at a high scale Λ can be described by an effective Lagrangian [1–3]

$$\mathcal{L}^{\text{eff}} = \sum \frac{C_x}{\Lambda^2} O_x + \dots, \quad (2)$$

where O_x are dimension-six gauge-invariant operators and C_x are complex constants. (Higher-order corrections from operators of higher dimension, suppressed by higher powers of Λ , are neglected in this work.) Among the operators listed in Ref. [3], fourteen contribute to top electroweak anomalous couplings,

$$\begin{aligned} O_{\phi q}^{(3,ij)} &= i(\phi^\dagger \tau^I D_\mu \phi)(\bar{q}_{Li} \gamma^\mu \tau^I q_{Lj}), & O_{Du}^{ij} &= (\bar{q}_{Li} D_\mu u_{Rj}) D^\mu \tilde{\phi}, \\ O_{\phi q}^{(1,ij)} &= i(\phi^\dagger D_\mu \phi)(\bar{q}_{Li} \gamma^\mu q_{Lj}), & O_{Du}^{ij} &= (D_\mu \bar{q}_{Li} u_{Rj}) D^\mu \tilde{\phi}, \\ O_{\phi\phi}^{ij} &= i(\tilde{\phi}^\dagger D_\mu \phi)(\bar{u}_{Ri} \gamma^\mu d_{Rj}), & O_{Dd}^{ij} &= (\bar{q}_{Li} D_\mu d_{Rj}) D^\mu \phi, \\ O_{\phi u}^{ij} &= i(\phi^\dagger D_\mu \phi)(\bar{u}_{Ri} \gamma^\mu u_{Rj}), & O_{Dd}^{ij} &= (D_\mu \bar{q}_{Li} d_{Rj}) D^\mu \phi, \\ O_{uW}^{ij} &= (\bar{q}_{Li} \sigma^{\mu\nu} \tau^I u_{Rj}) \tilde{\phi} W_{\mu\nu}^I, & O_{qW}^{ij} &= \bar{q}_{Li} \gamma^\mu \tau^I D^\nu q_{Lj} W_{\mu\nu}^I, \\ O_{dW}^{ij} &= (\bar{q}_{Li} \sigma^{\mu\nu} \tau^I d_{Rj}) \phi W_{\mu\nu}^I, & O_{qB}^{ij} &= \bar{q}_{Li} \gamma^\mu D^\nu q_{Lj} B_{\mu\nu}, \\ O_{uB\phi}^{ij} &= (\bar{q}_{Li} \sigma^{\mu\nu} u_{Rj}) \tilde{\phi} B_{\mu\nu}, & O_{uB}^{ij} &= \bar{u}_{Ri} \gamma^\mu D^\nu u_{Rj} B_{\mu\nu}, \end{aligned} \quad (3)$$

up to different values of the flavour indices $i, j = 1, 2, 3$. Here \bar{q}_{Li}, u_{Ri} and d_{Ri} are the quark fields in standard notation (for details see the next section). Operators with

¹A q^μ term can also be dropped if V couples to external massless fermions, in which case its contribution to the amplitude vanishes by application of the Dirac equation. This is indeed the case in several processes of interest at LHC and Tevatron, like for example single top production in t and s channels.

$i = j = 3$ contribute to the Wtb , Ztt or γtt vertices, while operators involving two up-type quarks with $i, j = 1, 3/3, 1$ or $i, j = 2, 3/3, 2$ contribute to flavour-changing neutral (FCN) top-up and top-charm interactions, respectively. Only three operators (up to flavour indices) contribute to strong interactions,

$$\begin{aligned} O_{uG\phi}^{ij} &= (\bar{q}_{Li}\lambda^a\sigma^{\mu\nu}u_{Rj})\tilde{\phi}G_{\mu\nu}^a, & O_{qG}^{ij} &= \bar{q}_{Li}\lambda^a\gamma^\mu D^\nu q_{Lj}G_{\mu\nu}^a, \\ O_{uG}^{ij} &= \bar{u}_{Ri}\lambda^a\gamma^\mu D^\nu u_{Rj}G_{\mu\nu}^a. \end{aligned} \quad (4)$$

For $i = j = 3$ they give diagonal g_{tt} couplings whereas for $i \neq j$ the interactions are flavour-changing, as the electroweak ones.

All operators in the left columns of Eqs. (3), (4) yield γ^μ and $\sigma^{\mu\nu}q_\nu$ terms, while those in the right columns give $k^\mu \equiv (p_i+p_j)^\mu$ and q^μ terms or more complicated Lorentz structures. Not all these contributions to top couplings are independent, however. In Ref. [4] it was pointed out that O_{qW}^{33} , O_{qB}^{33} , O_{uB}^{33} are redundant and can be expressed in terms of other operators in Eqs. (3) with $i = j = 3$, plus four-fermion interactions. This implies in particular that their contributions to the Wtb , Ztt and γtt couplings can be expressed in terms of other operator contributions both for on-shell and off-shell external particles. Here we will generalise this result for operators with $i \neq j$, including also strong interactions. We will find expressions which allow to write: (i) O_{qW}^{ij} , O_{qB}^{ij} and O_{uB}^{ij} in terms of operators in the left column of Eqs. (3) plus four-fermion interactions, extending the results in Ref. [4] to the case of $i \neq j$; (ii) O_{qG}^{ij} and O_{uG}^{ij} in terms of $O_{uG\phi}^{ij}$ plus four-fermion interactions. After proving that these operators are redundant, they can be excluded from further consideration in the same way as several other gauge-invariant dimension-six redundant operators one may construct [3] are not considered.

Concerning the remaining operators, it has been previously noted that using the equations of motion O_{Du}^{33} and O_{Dd}^{33} can be expressed in terms of O_{Du}^{33} , O_{Dd}^{33} , respectively, plus additional terms. More recently, in Ref. [5] it has been shown with a direct calculation of the amplitudes that, for the specific case of the Wtb vertex, the contributions of O_{Du}^{33} , $O_{\bar{D}u}^{33}$, O_{Dd}^{33} and $O_{\bar{D}d}^{33}$ can be rewritten in terms of γ^μ and $\sigma^{\mu\nu}q_\nu$ terms. Here we will show that the underlying reason for this simplification is that the four operators O_{Du}^{ij} , $O_{\bar{D}u}^{ij}$, O_{Dd}^{ij} and $O_{\bar{D}d}^{ij}$ are actually redundant. Therefore, only the operators in the left columns of Eqs. (3), (4) are independent, and all of them give γ^μ or $\sigma^{\mu\nu}q_\nu$ contributions to the vertices.

We must emphasise here that the fundamental principle which allows to rewrite vertex contributions into γ^μ and $\sigma^{\mu\nu}q_\nu$ terms for off-shell particles is gauge symmetry. It is well-known that for off-shell fermions the Lorentz structure in Eq. (1) is not the most general one and, for example, a k^μ term cannot be rewritten into $\sigma^{\mu\nu}q_\nu$

plus γ^μ terms using the Gordon identities if the fermions are off-shell. But when these trilinear terms are generated from gauge-invariant operators, they have associated quartic interactions (among several others) which also contribute to the amplitudes. In this way, the contribution of a k^μ term *plus* the additional contributions related by gauge symmetry are equivalent to the one from a combination of $\sigma^{\mu\nu}q_\nu$ and γ^μ terms. This fact will be explicitly shown here with examples of amplitude calculations.

The aim of this paper is to find general expressions of the electroweak top anomalous interactions generated by dimension-six gauge-invariant operators, which are also minimal in the sense that they involve a set of couplings as small as possible. Section 2 is devoted to obtain relations which will eventually prove that the operators listed in the right columns of Eqs. (3), (4) are redundant and may be safely excluded. The results obtained are then applied in section 3 to the Wtb , Ztt , γtt and $g tt$ vertices. We find much simpler vertex structures in comparison to previous works [6–9]. Altogether, these interactions can be described with only twelve independent anomalous couplings which are the coefficients of the effective γ^μ and $\sigma^{\mu\nu}q_\nu$ interactions. The eventual measurement of these anomalous couplings might be used to determine effective operator coefficients. In section 4 we present results for Ztu/Ztc , $\gamma tq/\gamma tc$ and gtu/gtc interactions, simplifying previous results [10, 11]. For example, the Ztu and Ztc interactions can be each described by only four quantities and the γtu and γtc vertices only need two parameters, which are the most convenient ones to express observables such as cross sections and branching ratios for FCN decays. As we will see, these parameters turn out to be independent, despite the gauge relation between the Z boson and photon fields. In appendix A we give explicit examples to show how gauge symmetry ensures that the contributions to the amplitudes are the same in all cases when operators are rewritten. In appendix B we collect the effective operator contributions to the Wtb , Ztt , γtt and $g tt$ vertices, while in appendix C we do the same for the flavour-changing ones.

2 Effective operator equalities

In this paper we follow the notation of Ref. [3] for gauge invariant effective operators with slight normalisation changes and sign differences, also introducing flavour indices. We denote by

$$q_{Li} = \begin{pmatrix} u_{Li} \\ d_{Li} \end{pmatrix}, \quad u_{Ri}, \quad d_{Ri} \quad (i = 1, 2, 3) \quad (5)$$

the quark weak interaction eigenstates, with $(u_1, u_2, u_3) = (u, c, t)$ and $(d_1, d_2, d_3) = (d, s, b)$ in the usual notation. Analogously, ℓ_{Li} and e_{Ri} the lepton doublets and singlets, respectively. The covariant derivative is

$$D_\mu = \partial_\mu + ig_s \frac{\lambda^a}{2} G_\mu^a + ig \frac{\tau^I}{2} W_\mu^I + ig' Y B_\mu, \quad (6)$$

where G_μ^a , W_μ^I and B_μ are the gauge fields for SU(3), SU(2)_L and U(1)_Y, λ^a are the Gell-Mann matrices with $a = 1 \dots 8$, τ^I the Pauli matrices for $I = 1, 2, 3$ and Y is the hypercharge (with $Q = T_3 + Y$) of the field to which D_μ is applied. The charged W boson fields are

$$W_\mu^\pm = \frac{1}{\sqrt{2}} (W_\mu^1 \mp iW_\mu^2) \quad (7)$$

and the Z and photon are related to the W^3 , B fields by

$$\begin{aligned} Z_\mu &= c_W W_\mu^3 - s_W B_\mu, \\ A_\mu &= s_W W_\mu^3 + c_W B_\mu, \end{aligned} \quad (8)$$

where s_W and c_W are the sine and cosine of the weak angle θ_W , respectively. The field strength tensors for SU(3), SU(2)_L and U(1)_Y are

$$\begin{aligned} G_{\mu\nu}^a &= \partial_\mu G_\nu^a - \partial_\nu G_\mu^a - g_s f_{abc} G_\mu^b G_\nu^c, \\ W_{\mu\nu}^I &= \partial_\mu W_\nu^I - \partial_\nu W_\mu^I - g \epsilon_{IJK} W_\mu^J W_\nu^K, \\ B_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu, \end{aligned} \quad (9)$$

and the dual tensors are

$$\tilde{F}^{\mu\nu} = \frac{1}{2} \epsilon^{\mu\nu\tau\rho} F_{\tau\rho}, \quad (10)$$

for $F = B, W^I, G^a$ with $\epsilon_{0123} = 1$. The SM Higgs doublet ϕ has vacuum expectation value

$$\langle \phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}, \quad (11)$$

with $v = 246$ GeV, and we define $\tilde{\phi} = \epsilon \phi^*$, $\epsilon = i\tau^2$. We will use the dimension-four equations of motion of the quark fields

$$\begin{aligned} i\mathcal{D}q_{Li} &= Y_{ij}^u u_{Rj} \tilde{\phi} + Y_{ij}^d d_{Rj} \phi, \\ i\mathcal{D}u_{Ri} &= Y_{ij}^{u\dagger} \tilde{\phi}^\dagger q_{Lj}, \\ i\mathcal{D}d_{Ri} &= Y_{ij}^{d\dagger} \phi^\dagger q_{Lj}, \end{aligned} \quad (12)$$

with Y^u, Y^d the 3×3 matrices of up- and down-type quark Yukawa couplings. The equations of motion of gauge fields are

$$\begin{aligned}
\partial_\nu B^{\nu\mu} &- g' \left\{ -\frac{1}{2} \bar{\ell}_{Li} \gamma^\mu \ell_{Li} - \bar{e}_{Ri} \gamma^\mu e_{Ri} + \frac{1}{6} \bar{q}_{Li} \gamma^\mu q_{Li} + \frac{2}{3} \bar{u}_{Ri} \gamma^\mu u_{Ri} - \frac{1}{3} \bar{d}_{Ri} \gamma^\mu d_{Ri} \right. \\
&\quad \left. + \frac{i}{2} \phi^\dagger \overleftrightarrow{D}^\mu \phi \right\} = 0, \\
(D_\nu W^{\nu\mu})^I &- g \left\{ \bar{\ell}_{Li} \gamma^\mu \frac{\tau^I}{2} \ell_{Li} + \bar{q}_{Li} \gamma^\mu \frac{\tau^I}{2} q_{Li} + i \left[\phi^\dagger \frac{\tau^I}{2} D^\mu \phi - (D^\mu \phi)^\dagger \frac{\tau^I}{2} \phi \right] \right\} = 0, \\
(D_\nu G^{\nu\mu})^a &- g_s \left\{ \bar{q}_{Li} \frac{\lambda^a}{2} \gamma^\mu q_{Li} + \bar{u}_{Ri} \frac{\lambda^a}{2} \gamma^\mu u_{Ri} + \bar{d}_{Ri} \frac{\lambda^a}{2} \gamma^\mu d_{Ri} \right\} = 0, \tag{13}
\end{aligned}$$

summing over flavours $i = 1, 2, 3$, with

$$\begin{aligned}
(D_\mu W_{\nu\sigma})^I &= \partial_\mu W_{\nu\sigma}^I - g \epsilon_{IJK} W_\mu^J W_{\nu\sigma}^K, \\
(D_\mu G_{\nu\sigma})^a &= \partial_\mu G_{\nu\sigma}^a - g_s f_{abc} G_\mu^b G_{\nu\sigma}^c. \tag{14}
\end{aligned}$$

The equation of motion for the scalar field is

$$D_\mu D^\mu \phi - m^2 \phi + \lambda (\phi^\dagger \phi) \phi + Y_{ij}^{e\dagger} \bar{e}_{Ri} \ell_{Lj} + Y_{ij}^u (\bar{q}_{Li} \epsilon)^T u_{Rj} + Y_{ij}^{d\dagger} \bar{d}_{Ri} q_{Lj}, \tag{15}$$

and we will also use

$$D_\mu D^\mu \tilde{\phi} - m^2 \tilde{\phi} + \lambda (\tilde{\phi}^\dagger \tilde{\phi}) \tilde{\phi} + Y_{ij}^e (\bar{\ell}_{Li} \epsilon)^T \bar{e}_{Rj} + Y_{ij}^{u\dagger} \bar{u}_{Ri} q_{Lj} + Y_{ij}^d (\bar{q}_{Li} \epsilon)^T d_{Rj}. \tag{16}$$

In the rest of this section we will use the dimension-four equations of motion for the interacting SM fields to obtain relations among the effective operators. These equations can be used to remove redundant operators even for off-shell external particles [12]. Although for definiteness we restrict ourselves to operators involving quarks, it is evident that the same arguments apply to the lepton sector where the fields have the same isospin structure but are singlets under SU(3), and the equivalent leptonic operators are redundant as well. In appendix A we provide examples showing that the rewriting of contributions implied by the operator equalities give the same result in amplitude calculations even with off-shell fermions and bosons.

2.1 Equalities for O_{qW}^{ij} , O_{qB}^{ij} , O_{uB}^{ij} , O_{qG}^{ij} and O_{uG}^{ij}

In Ref. [4] it was pointed out that O_{qW}^{33} , O_{qB}^{33} and O_{uB}^{33} are redundant and can be written in terms of other gauge-invariant operators. We extend this result to the non-diagonal case $i \neq j$ also including O_{qG}^{ij} and O_{uG}^{ij} . The desired relations can be obtained by writing all these operators as

$$O_x^{ij} = \frac{1}{2} [O_x^{ij} + (O_x^{ji})^\dagger] + \frac{1}{2} [O_x^{ij} - (O_x^{ji})^\dagger], \tag{17}$$

and relating each of the terms between brackets to other gauge invariant operators. Note that for $i = j$ the first term is hermitian while the second one is anti-hermitian. For the first one we have

$$\begin{aligned}
O_{qW}^{ij} + (O_{qW}^{ji})^\dagger &= (\bar{q}_{Li}\gamma^\mu\tau^I q_{Lj})(D^\nu W_{\nu\mu})^I, \\
O_{qB}^{ij} + (O_{qB}^{ji})^\dagger &= (\bar{q}_{Li}\gamma^\mu q_{Lj})\partial^\nu B_{\nu\mu}, \\
O_{uB}^{ij} + (O_{uB}^{ji})^\dagger &= (\bar{u}_{Ri}\gamma^\mu u_{Rj})\partial^\nu B_{\nu\mu}, \\
O_{qG}^{ij} + (O_{qG}^{ji})^\dagger &= (\bar{q}_{Li}\lambda^a\gamma^\mu q_{Lj})(D^\nu G_{\nu\mu})^a, \\
O_{uG}^{ij} + (O_{uG}^{ji})^\dagger &= (\bar{u}_{Ri}\lambda^a\gamma^\mu u_{Rj})(D^\nu G_{\nu\mu})^a,
\end{aligned} \tag{18}$$

up to a total derivative. These sums can then be transformed using the gauge field equations of motion. For the second term we make use of the operators involving dual field strengths, which we define with an extra i factor,

$$\begin{aligned}
O_{q\tilde{W}}^{ij} &= i\bar{q}_{Li}\gamma^\mu D^\nu\tau^I q_{Lj}\tilde{W}_{\mu\nu}^I, & O_{q\tilde{G}}^{ij} &= i\bar{q}_{Li}\lambda^a\gamma^\mu D^\nu q_{Lj}\tilde{G}_{\mu\nu}^a, \\
O_{q\tilde{B}}^{ij} &= i\bar{q}_{Li}\gamma^\mu D^\nu q_{Lj}\tilde{B}_{\mu\nu}, & O_{u\tilde{G}}^{ij} &= i\bar{u}_{Ri}\lambda^a\gamma^\mu D^\nu u_{Rj}\tilde{G}_{\mu\nu}^a, \\
O_{u\tilde{B}}^{ij} &= i\bar{u}_{Ri}\gamma^\mu D^\nu u_{Rj}\tilde{B}_{\mu\nu}.
\end{aligned} \tag{19}$$

Their relation with the ones involving B , W^I , G^a ,

$$\begin{aligned}
O_{q\tilde{W}}^{ij} &= O_{qW}^{ij} + \frac{1}{2}\bar{q}_{Li}\sigma^{\mu\nu}\tau^I i\rlap{/}{D}q_{Lj}W_{\mu\nu}^I, \\
O_{q\tilde{B}}^{ij} &= O_{qB}^{ij} + \frac{1}{2}\bar{q}_{Li}\sigma^{\mu\nu} i\rlap{/}{D}q_{Lj}B_{\mu\nu}, \\
O_{u\tilde{B}}^{ij} &= -O_{uB}^{ij} - \frac{1}{2}\bar{u}_{Ri}\sigma^{\mu\nu} i\rlap{/}{D}u_{Rj}B_{\mu\nu}, \\
O_{q\tilde{G}}^{ij} &= O_{qG}^{ij} + \frac{1}{2}\bar{q}_{Li}\lambda^a\sigma^{\mu\nu} i\rlap{/}{D}q_{Lj}G_{\mu\nu}^a, \\
O_{u\tilde{G}}^{ij} &= -O_{uG}^{ij} + \frac{1}{2}\bar{u}_{Ri}\lambda^a\sigma^{\mu\nu} i\rlap{/}{D}u_{Rj}G_{\mu\nu}^a,
\end{aligned} \tag{20}$$

can be trivially obtained from the equality [3]

$$\tilde{F}^{\mu\nu}\gamma_\mu D_\nu\psi_\pm = \pm \left(iF^{\mu\nu}\gamma_\mu D_\nu - \frac{1}{2}F^{\mu\nu}\sigma_{\mu\nu}\rlap{/}{D} \right) \psi_\pm, \tag{21}$$

where the spinors ψ_\pm satisfy $\gamma_5\psi_\pm = \pm\psi_\pm$.² The quark equations of motion can then be used in the last terms in Eqs. (20). Moreover, using the Bianchi identities it can be

²In Refs. [8,9] relations equivalent to those in Eqs. (20) plus the hermitian conjugate are quoted but without the $1/2$ factors in the terms with the $\sigma^{\mu\nu}$ matrices. In order to clarify this discrepancy, we have confirmed Eqs. (20) with a direct calculation using the property $\sigma^{\mu\nu}\gamma^\sigma = -\epsilon^{\mu\nu\sigma\rho}\gamma_5\gamma_\rho + i\gamma^\mu g^{\nu\sigma} - i\gamma^\nu g^{\mu\sigma}$. Notice also a different sign in Ref. [3] when writing Eq. (21).

easily seen that

$$\begin{aligned}
O_{q\tilde{W}}^{ij} - (O_{q\tilde{W}}^{ji})^\dagger &= O_{q\tilde{B}}^{ij} - (O_{q\tilde{B}}^{ji})^\dagger = O_{u\tilde{B}}^{ij} - (O_{u\tilde{B}}^{ji})^\dagger = 0, \\
O_{q\tilde{G}}^{ij} - (O_{q\tilde{G}}^{ji})^\dagger &= O_{u\tilde{G}}^{ij} - (O_{u\tilde{G}}^{ji})^\dagger = 0,
\end{aligned} \tag{22}$$

up to total derivatives. Joining the two terms it is found that

$$\begin{aligned}
O_{qW}^{ij} &= -\frac{1}{4} \left[Y_{jk}^u O_{uW}^{ik} + Y_{jk}^d O_{dW}^{ik} - Y_{ki}^{u\dagger} (O_{uW}^{jk})^\dagger - Y_{ki}^{d\dagger} (O_{dW}^{jk})^\dagger \right] + \frac{g}{4} \left[O_{\phi q}^{(3,i,j)} + O_{\phi q}^{(3,j,i)\dagger} \right] \\
&\quad + \frac{g}{4} (\bar{q}_{Li} \gamma^\mu \tau^I q_{Lj}) (\bar{\ell}_{Lk} \gamma_\mu \tau^I \ell_{Lk}) + \frac{g}{4} (\bar{q}_{Li} \gamma^\mu q_{Lj}) (\bar{q}_{Lk} \gamma_\mu \tau^I q_{Lk}), \\
O_{qB}^{ij} &= -\frac{1}{4} \left[Y_{jk}^u O_{uB\phi}^{ik} + Y_{jk}^d O_{dB\phi}^{ik} - Y_{ki}^{u\dagger} (O_{uB\phi}^{jk})^\dagger - Y_{ki}^{d\dagger} (O_{dB\phi}^{jk})^\dagger \right] + \frac{g'}{4} \left[O_{\phi q}^{(1,i,j)} + O_{\phi q}^{(1,j,i)\dagger} \right] \\
&\quad - \frac{g'}{4} (\bar{q}_{Li} \gamma^\mu q_{Lj}) (\bar{\ell}_{Lk} \gamma_\mu \ell_{Lk}) - \frac{g'}{2} (\bar{q}_{Li} \gamma^\mu q_{Lj}) (\bar{e}_{Rk} \gamma_\mu e_{Rk}) + \frac{g'}{12} (\bar{q}_{Li} \gamma^\mu q_{Lj}) (\bar{q}_{Lk} \gamma_\mu q_{Lk}) \\
&\quad + \frac{g'}{3} (\bar{q}_{Li} \gamma^\mu q_{Lj}) (\bar{u}_{Rk} \gamma_\mu u_{Rk}) - \frac{g'}{6} (\bar{q}_{Li} \gamma^\mu q_{Lj}) (\bar{d}_{Rk} \gamma_\mu d_{Rk}), \\
O_{uB}^{ij} &= \frac{1}{4} \left[Y_{ki}^u O_{uB\phi}^{kj} - Y_{jk}^{u\dagger} (O_{uB\phi}^{ki})^\dagger \right] + \frac{g'}{4} \left[O_{\phi u}^{ij} + (O_{\phi u}^{ji})^\dagger \right] - \frac{g'}{4} (\bar{u}_{Ri} \gamma^\mu u_{Rj}) (\bar{\ell}_{Lk} \gamma_\mu \ell_{Lk}) \\
&\quad - \frac{g'}{2} (\bar{u}_{Ri} \gamma^\mu u_{Rj}) (\bar{e}_{Rk} \gamma_\mu e_{Rk}) + \frac{g'}{12} (\bar{u}_{Ri} \gamma^\mu u_{Rj}) (\bar{q}_{Lk} \gamma_\mu q_{Lk}) \\
&\quad + \frac{g'}{3} (\bar{u}_{Ri} \gamma^\mu u_{Rj}) (\bar{u}_{Rk} \gamma_\mu u_{Rk}) - \frac{g'}{6} (\bar{u}_{Ri} \gamma^\mu u_{Rj}) (\bar{d}_{Rk} \gamma_\mu d_{Rk}), \\
O_{qG}^{ij} &= -\frac{1}{4} \left[Y_{jk}^u O_{uG\phi}^{ik} + Y_{jk}^d O_{dG\phi}^{ik} - Y_{ki}^{u\dagger} (O_{uG\phi}^{jk})^\dagger - Y_{ki}^{d\dagger} (O_{dG\phi}^{jk})^\dagger \right] \\
&\quad + \frac{g_s}{4} (\bar{q}_{Li} \lambda^a \gamma^\mu q_{Lj}) (\bar{q}_{Lk} \lambda^a \gamma_\mu q_{Lk}) + \frac{g_s}{4} (\bar{q}_{Li} \lambda^a \gamma^\mu q_{Lj}) (\bar{u}_{Rk} \lambda^a \gamma_\mu u_{Rk}) \\
&\quad + \frac{g_s}{4} (\bar{q}_{Li} \lambda^a \gamma^\mu q_{Lj}) (\bar{d}_{Rk} \lambda^a \gamma_\mu d_{Rk}), \\
O_{uG}^{ij} &= \frac{1}{4} \left[Y_{ki}^u O_{uG\phi}^{kj} - Y_{jk}^{u\dagger} (O_{uG\phi}^{ki})^\dagger \right] + \frac{g_s}{4} (\bar{u}_{Ri} \lambda^a \gamma^\mu u_{Rj}) (\bar{q}_{Lk} \lambda^a \gamma_\mu q_{Lk}) \\
&\quad + \frac{g_s}{4} (\bar{u}_{Ri} \lambda^a \gamma^\mu u_{Rj}) (\bar{u}_{Rk} \lambda^a \gamma_\mu u_{Rk}) + \frac{g_s}{4} (\bar{u}_{Ri} \lambda^a \gamma^\mu u_{Rj}) (\bar{d}_{Rk} \lambda^a \gamma_\mu d_{Rk}).
\end{aligned} \tag{23}$$

A sum over $k = 1, 2, 3$ is understood. The operators

$$\begin{aligned}
O_{dB\phi}^{ij} &= (\bar{q}_{Li} \sigma^{\mu\nu} d_{Rj}) \phi B_{\mu\nu}, \\
O_{dG\phi}^{ij} &= (\bar{q}_{Li} \lambda^a \sigma^{\mu\nu} d_{Rj}) \phi G_{\mu\nu}^a
\end{aligned} \tag{24}$$

appearing in the above equations do not contribute to top couplings.

2.2 Equalities for O_{Du}^{ij} , $O_{\tilde{D}u}^{ij}$, O_{Dd}^{ij} and $O_{\tilde{D}d}^{ij}$

In order to show that these operators are redundant, it is convenient to consider their sums $O_{Du}^{ij} + O_{\tilde{D}u}^{ij}$, $O_{Dd}^{ij} + O_{\tilde{D}d}^{ij}$ and differences $O_{Du}^{ij} - O_{\tilde{D}u}^{ij}$, $O_{Dd}^{ij} - O_{\tilde{D}d}^{ij}$. The sums can

be written as

$$\begin{aligned}
O_{Du}^{ij} + O_{\bar{D}u}^{ij} &= D_\mu(\bar{q}_{Li}u_{Rj})D^\mu\tilde{\phi} = -\bar{q}_{Li}u_{Rj}D_\mu D^\mu\tilde{\phi}, \\
O_{Dd}^{ij} + O_{\bar{D}d}^{ij} &= D_\mu(\bar{q}_{Li}d_{Rj})D^\mu\phi = -\bar{q}_{Li}d_{Rj}D_\mu D^\mu\phi.
\end{aligned} \tag{25}$$

Using the scalar equations of motion it is found that these sums are equivalent to

$$\begin{aligned}
O_{Du}^{ij} + O_{\bar{D}u}^{ij} &= -m^2\bar{q}_{Li}u_{Rj}\tilde{\phi} + \lambda O_{u\phi}^{ij} + Y_{kl}^e(\bar{q}_{Li}u_{Rj}) [(\bar{\ell}_{Lk}\epsilon)^T e_{Rl}] \\
&\quad + Y_{kl}^{u\dagger}(\bar{q}_{Li}u_{Rj})(\bar{u}_{Rk}q_{Ll}) + Y_{kl}^d(\bar{q}_{Li}u_{Rj}) [(\bar{q}_{Lk}\epsilon)^T d_{Rl}], \\
O_{Dd}^{ij} + O_{\bar{D}d}^{ij} &= -m^2\bar{q}_{Li}d_{Rj}\phi + \lambda O_{d\phi}^{ij} + Y_{kl}^{e\dagger}(\bar{q}_{Li}d_{Rj})(\bar{e}_{Rk}\ell_{Ll}), \\
&\quad + Y_{kl}^u(\bar{q}_{Li}d_{Rj}) [(\bar{q}_{Lk}\epsilon)^T u_{Rl}] + Y_{kl}^{d\dagger}(\bar{q}_{Li}d_{Rj})(\bar{d}_{Rk}q_{Ll}),
\end{aligned} \tag{26}$$

summing over k, l , with

$$\begin{aligned}
O_{u\phi}^{ij} &= (\phi^\dagger\phi)\bar{q}_{Li}u_{Rj}\tilde{\phi}, \\
O_{d\phi}^{ij} &= (\phi^\dagger\phi)\bar{q}_{Li}d_{Rj}\phi,
\end{aligned} \tag{27}$$

and therefore redundant. Note that the rewritten terms on the right-hand side of these equations do not contribute to the gauge boson vertices. This result is not surprising since the contribution before rewriting is proportional to q^μ and vanishes if the gauge boson is on-shell or coupling to massless external fermions.

In order to rewrite the differences $O_{Du}^{ij} - O_{\bar{D}u}^{ij}$ and $O_{Dd}^{ij} - O_{\bar{D}d}^{ij}$, we introduce the auxiliary operators

$$\begin{aligned}
O'_{Du}{}^{ij} &= i(\bar{q}_{Li}\sigma^{\mu\nu}D_\nu u_{Rj})D_\mu\tilde{\phi}, \\
O'_{\bar{D}u}{}^{ij} &= i(D_\nu\bar{q}_{Li}\sigma^{\mu\nu}u_{Rj})D_\mu\tilde{\phi}, \\
O'_{Dd}{}^{ij} &= i(\bar{q}_{Li}\sigma^{\mu\nu}D_\nu d_{Rj})D_\mu\phi, \\
O'_{\bar{D}d}{}^{ij} &= i(D_\nu\bar{q}_{Li}\sigma^{\mu\nu}d_{Rj})D_\mu\phi,
\end{aligned} \tag{28}$$

which are gauge invariant. Using the definition of the $\sigma^{\mu\nu}$ matrices and $\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}$, it is easy to see that these two sets of operators are related by

$$\begin{aligned}
O_{Du}^{ij} &= O'_{Du}{}^{ij} - i\bar{q}_{Li}\gamma^\mu(i\cancel{D}u_{Rj})D_\mu\tilde{\phi} && \equiv O'_{Du}{}^{ij} - \Delta O_{Du}^{ij}, \\
O_{\bar{D}u}^{ij} &= -O'_{\bar{D}u}{}^{ij} + i\overline{(i\cancel{D}q_{Li})}\gamma^\mu u_{Rj}D_\mu\tilde{\phi} && \equiv -O'_{\bar{D}u}{}^{ij} + \Delta O_{\bar{D}u}^{ij}, \\
O_{Dd}^{ij} &= O'_{Dd}{}^{ij} - i\bar{q}_{Li}\gamma^\mu(i\cancel{D}d_{Rj})D_\mu\phi && \equiv O'_{Dd}{}^{ij} - \Delta O_{Dd}^{ij}, \\
O_{\bar{D}d}^{ij} &= -O'_{\bar{D}d}{}^{ij} + i\overline{(i\cancel{D}q_{Li})}\gamma^\mu d_{Rj}D_\mu\phi && \equiv -O'_{\bar{D}d}{}^{ij} + \Delta O_{\bar{D}d}^{ij}.
\end{aligned} \tag{29}$$

Using the fermion equations of motion we can obtain after a little algebra that

$$\begin{aligned}
\Delta O_{Du}^{ij} &= \frac{1}{2} Y_{jk}^{u\dagger} \left[(O_{\phi q}^{(3,ki)})^\dagger - (O_{\phi q}^{(1,ki)})^\dagger \right], \\
\Delta O_{\bar{D}u}^{ij} &= -Y_{ki}^{u\dagger} (O_{\phi u}^{jk})^\dagger + Y_{ki}^{d\dagger} (O_{\phi\phi}^{jk})^\dagger, \\
\Delta O_{Dd}^{ij} &= \frac{1}{2} Y_{jk}^{d\dagger} \left[O_{\phi q}^{(3,ik)} + O_{\phi q}^{(1,ik)} \right], \\
\Delta O_{\bar{D}d}^{ij} &= Y_{ki}^{u\dagger} O_{\phi\phi}^{kj} + Y_{ki}^{d\dagger} O_{\phi d}^{kj},
\end{aligned} \tag{30}$$

with

$$O_{\phi d}^{ij} = i(\phi^\dagger D_\mu \phi)(\bar{d}_{Ri} \gamma^\mu d_{Rj}). \tag{31}$$

On the other hand, we have

$$\begin{aligned}
O'_{Du}{}^{ij} + O'_{\bar{D}u}{}^{ij} &= iD_\nu (\bar{q}_{Li} \sigma^{\mu\nu} u_{Rj}) D_\mu \tilde{\phi} = -i\bar{q}_{Li} \sigma^{\mu\nu} u_{Rj} D_\nu D_\mu \tilde{\phi}, \\
O'_{Dd}{}^{ij} + O'_{\bar{D}d}{}^{ij} &= iD_\nu (\bar{q}_{Li} \sigma^{\mu\nu} d_{Rj}) D_\mu \phi = -i\bar{q}_{Li} \sigma^{\mu\nu} d_{Rj} D_\nu D_\mu \phi.
\end{aligned} \tag{32}$$

Since

$$[D_\mu, D_\nu] \phi = ig \frac{\tau^I}{2} W_{\mu\nu}^I \phi + ig' Y B_{\mu\nu} \phi, \tag{33}$$

these sums can also be written in terms of already known operators,

$$\begin{aligned}
O'_{Du}{}^{ij} + O'_{\bar{D}u}{}^{ij} &= -\frac{g}{4} O_{uW}^{ij} + \frac{g'}{4} O_{uB\phi}^{ij}, \\
O'_{Dd}{}^{ij} + O'_{\bar{D}d}{}^{ij} &= -\frac{g}{4} O_{dW}^{ij} - \frac{g'}{4} O_{dB\phi}^{ij}.
\end{aligned} \tag{34}$$

Finally, using Eqs. (29), (30) and (34) we arrive at the desired result,

$$\begin{aligned}
O_{Du}^{ij} - O_{\bar{D}u}^{ij} &= -\frac{g}{4} O_{uW}^{ij} + \frac{g'}{4} O_{uB\phi}^{ij} - \frac{1}{2} Y_{jk}^{u\dagger} \left[(O_{\phi q}^{(3,ki)})^\dagger - (O_{\phi q}^{(1,ki)})^\dagger \right] \\
&\quad + Y_{ki}^{u\dagger} (O_{\phi u}^{jk})^\dagger - Y_{ki}^{d\dagger} (O_{\phi\phi}^{jk})^\dagger, \\
O_{Dd}^{ij} - O_{\bar{D}d}^{ij} &= -\frac{g}{4} O_{dW}^{ij} - \frac{g'}{4} O_{dB\phi}^{ij} - \frac{1}{2} Y_{jk}^{d\dagger} \left[O_{\phi q}^{(3,ik)} + O_{\phi q}^{(1,ik)} \right] \\
&\quad - Y_{ki}^{u\dagger} O_{\phi\phi}^{kj} - Y_{ki}^{d\dagger} O_{\phi d}^{kj}.
\end{aligned} \tag{35}$$

This, together with Eqs. (26), shows that the four operators O_{Du}^{ij} , $O_{\bar{D}u}^{ij}$, O_{Dd}^{ij} and $O_{\bar{D}d}^{ij}$ are redundant.

3 General Wtb , Ztt , γtt and $g tt$ interactions

We consider new physics contributions to the third generation interactions, which are described by the dimension-six effective operators in Eqs. (3) and (4) with flavour indices $i = j = 3$. These contributions are collected in appendix B for complex coefficients C_x . Here we exclude from the analysis the redundant operators in these equations, in the same way as many other possible redundant operators one may construct [3] are ignored. Thus, we provide completely general expressions for the Wtb , Ztt , γtt and $g tt$ vertices for off-shell fermions and bosons which also involve a minimal set of couplings. We remark that the expressions presented here do not make any assumption about quark masses and mixings. (In fact, the operator equalities in the previous section involve arbitrary Yukawa matrices, with arbitrary masses and mixings.) Moreover, we take into account all contributing operators independently of whether they also give corrections to the Zbb vertex, which is very constrained by present experimental data, or not. Cancellations among effective operator contributions are possible and occur in minimal SM extensions [13] as we will show later in more detail. (The same remarks apply to the results in the next section.) We finally show that anomalous coupling measurements might be used to determine effective operator coefficients.

3.1 Wtb vertex

The effective Wtb vertex including SM contributions and those from dimension-six operators can be parameterised as

$$\begin{aligned} \mathcal{L}_{Wtb} = & -\frac{g}{\sqrt{2}} \bar{b} \gamma^\mu (V_L P_L + V_R P_R) t W_\mu^- \\ & -\frac{g}{\sqrt{2}} \bar{b} \frac{i\sigma^{\mu\nu} q_\nu}{M_W} (g_L P_L + g_R P_R) t W_\mu^- + \text{H.c.} . \end{aligned} \quad (36)$$

The mass scale normalising the $\sigma^{\mu\nu} q_\nu$ term has been taken as M_W because this choice considerably simplifies the algebraic expressions of observables calculated from this vertex [14]. Additionally, with this normalisation the relation between g_L , g_R and effective operator coefficients is simpler and involves the ratio of scales v^2/Λ^2 . Within the SM, V_L equals the Cabibbo-Kobayashi-Maskawa matrix element $V_{tb} \simeq 1$, while the rest of couplings V_R , g_L and g_R vanish at the tree level. The contributions to these

couplings from the operators in Eqs. (3) are³

$$\begin{aligned}\delta V_L &= C_{\phi q}^{(3,33)*} \frac{v^2}{\Lambda^2}, & \delta g_L &= \sqrt{2} C_{dW}^{33*} \frac{v^2}{\Lambda^2}, \\ \delta V_R &= \frac{1}{2} C_{\phi\phi}^{33} \frac{v^2}{\Lambda^2}, & \delta g_R &= \sqrt{2} C_{uW}^{33} \frac{v^2}{\Lambda^2}.\end{aligned}\quad (37)$$

After removing redundant operators the structure of the Lagrangian in Eq. (36) is rather simple. All the new physics effects on the Wtb vertex can be described by four parameters, which have a direct connection with effective operator coefficients.

3.2 Ztt vertex

We parameterise the Ztt vertex including the SM contributions as well as those from dimension-six effective operators as

$$\begin{aligned}\mathcal{L}_{Ztt} &= -\frac{g}{2c_W} \bar{t} \gamma^\mu (X_{tt}^L P_L + X_{tt}^R P_R - 2s_W^2 Q_t) t Z_\mu \\ &\quad -\frac{g}{2c_W} \bar{t} \frac{i\sigma^{\mu\nu} q_\nu}{M_Z} (d_V^Z + id_A^Z \gamma_5) t Z_\mu,\end{aligned}\quad (38)$$

with $Q_t = 2/3$ the top quark electric charge. The mass scale for the $\sigma^{\mu\nu} q_\nu$ term is taken as M_Z in analogy with the Wtb vertex but, on the other hand, we have parameterised this coupling in terms of the vector and axial parts. The former is real while the latter is purely imaginary and CP-violating. They are the weak analogous to the top quark magnetic and electric dipole moment, respectively (see next subsection), up to normalisation. Within the SM, these couplings take the values $X_{tt}^L = 2T_3(t_L) = 1$, $X_{tt}^R = 2T_3(t_R) = 0$, where T_3 denotes the third isospin component, and $d_V^Z = d_A^Z = 0$ at the tree level. The contributions from dimension-six operators are

$$\begin{aligned}\delta X_{tt}^L &= \text{Re} \left[C_{\phi q}^{(3,33)} - C_{\phi q}^{(1,33)} \right] \frac{v^2}{\Lambda^2}, & \delta d_V^Z &= \sqrt{2} \text{Re} \left[c_W C_{uW}^{33} - s_W C_{uB\phi}^{33} \right] \frac{v^2}{\Lambda^2}, \\ \delta X_{tt}^R &= -\text{Re} C_{\phi u}^{33} \frac{v^2}{\Lambda^2}, & \delta d_A^Z &= \sqrt{2} \text{Im} \left[c_W C_{uW}^{33} - s_W C_{uB\phi}^{33} \right] \frac{v^2}{\Lambda^2}.\end{aligned}\quad (39)$$

At this point it is worthwhile to discuss the relation between the Ztt and Zbb vertices, the latter very constrained by LEP data. Some authors drop from their analyses the operators $O_{\phi q}^{(3)}$ and $O_{\phi q}^{(1)}$ for this reason, because they give contributions

$$\delta X_{bb}^L = \text{Re} \left[C_{\phi q}^{(3,33)} + C_{\phi q}^{(1,33)} \right] \frac{v^2}{\Lambda^2}.\quad (40)$$

However, cancellations are possible and take place in minimal models. For example, in a SM extension with a $Q = 2/3$ singlet we have $C_{\phi q}^{(3,33)} = -C_{\phi q}^{(1,33)}$ [13], so that

³Notice a typo in Eqs. (8) of Ref. [5], where a minus sign should multiply O_{uW} .

the contribution to the Zbb vertex identically cancels but there can be deviations in the Ztt interaction. One can still wonder about the corrections to the Wtb vertex from $O_{\phi q}^{(3,33)}$, which may affect low-energy B physics [15]. However, in this particular model additional contributions can (at least partly) make up for the difference as it has been shown with a global analysis of precision electroweak data and low energy constraints from B and K physics [16]. The reason behind the (partial) cancellation among new physics contributions in this simple, particular model is precisely the Glashow-Iliopoulos-Maiani (GIM) mechanism [17].

3.3 γtt vertex

The γtt vertex including the SM coupling (given by the top electric charge Q_t) and contributions from dimension-six effective operators can be parameterised as

$$\mathcal{L}_{\gamma tt} = -eQ_t \bar{t} \gamma^\mu t A_\mu - e \bar{t} \frac{i\sigma^{\mu\nu} q_\nu}{m_t} (d_V^\gamma + id_A^\gamma \gamma_5) t A_\mu. \quad (41)$$

The couplings d_V^γ, d_A^γ are real and related to the top quark magnetic and electric dipole moment, respectively, by a multiplicative factor, and the latter is CP-violating. For this interaction we have

$$\begin{aligned} \delta d_V^\gamma &= \frac{\sqrt{2}}{e} \text{Re} [c_W C_{uB\phi}^{33} + s_W C_{uW}^{33}] \frac{vm_t}{\Lambda^2}, \\ \delta d_A^\gamma &= \frac{\sqrt{2}}{e} \text{Im} [c_W C_{uB\phi}^{33} + s_W C_{uW}^{33}] \frac{vm_t}{\Lambda^2}. \end{aligned} \quad (42)$$

We note that the γ^μ term does not receive corrections from dimension-six operators (this also applies to the $g tt, \gamma tu/\gamma tc$ and gtu/gtc vertices). If we had included the redundant operators O_{qW}, O_{qB} and O_{uB} , the first two would yield corrections $\sim q^2 \bar{t}_L \gamma^\mu t_L A_\mu$ and the latter $\sim q^2 \bar{t}_R \gamma^\mu t_R A_\mu$, non-vanishing only when the photon is off-shell. The operator rewriting in Eqs. (23) eliminates such terms, so that corrections to the electromagnetic coupling are absent even for off-shell photons. In particular, as dictated by Eqs. (23) the contribution to the amplitudes of a q^2 -dependent γ^μ term can be reproduced by a constant γ^μ term (proportional to the square of the gauge boson mass) plus four-fermion interactions. An explicit example can be found in appendix A.

3.4 $g tt$ vertex

This vertex, including the SM contribution, is written as

$$\mathcal{L}_{g tt} = -g_s \bar{t} \frac{\lambda^a}{2} \gamma^\mu t G_\mu^a - g_s \bar{t} \lambda^a \frac{i\sigma^{\mu\nu} q_\nu}{m_t} (d_V^g + id_A^g \gamma_5) t G_\mu^a, \quad (43)$$

The couplings d_V^g , d_A^g are real and related to the top chromomagnetic and chromoelectric dipole moments, respectively. They vanish in the SM at the tree level. The new physics contributions from effective operators are

$$\begin{aligned}\delta d_V^g &= \frac{\sqrt{2}}{g_s} \text{Re} C_{uG\phi}^{33} \frac{vm_t}{\Lambda^2}, \\ \delta d_A^g &= \frac{\sqrt{2}}{g_s} \text{Im} C_{uG\phi}^{33} \frac{vm_t}{\Lambda^2}.\end{aligned}\tag{44}$$

As in the case of the photon, the γ^μ term does not include corrections from dimension-six operators.

3.5 Determination of effective operator coefficients

One may finally wonder whether hypothetical measurements of these anomalous couplings might provide any insight on the effective operators generating them. The answer is affirmative, since there are 12 anomalous couplings and only 8 operator coefficients. (Notice, however, that some of the anomalous couplings correspond to the real or imaginary parts of an effective operator coefficient or combination of them.) The measurement of Wtb anomalous couplings would translate into a measurement of $C_{\phi q}^{3,33}$, $C_{\phi\phi}^{33}$, C_{dW}^{33} and C_{uW}^{33} . In the Ztt vertex, the anomalous contributions to X_{tt}^L and X_{tt}^R would then determine the real parts of $C_{\phi q}^{(1,33)}$ and $C_{\phi q}^{33}$. From d_V^Z and d_V^γ the real parts of C_{uW}^{33} and $C_{uB\phi}^{33}$ might be obtained, while from d_A^Z and d_A^γ one would obtain the imaginary parts. In the gluon vertex, d_V^g and d_A^g determine the real and imaginary parts, respectively, of $C_{uG\phi}^{33}$. The only coefficient for which two independent determinations are possible is C_{uW}^{33} , which could be obtained from g_R and also from the combined measurements of d_V^Z , d_V^γ (the real part) and d_A^Z , d_A^γ (the imaginary part). Of course, this is a tremendously optimistic picture, since obtaining these measurements in a real detector is very challenging (see for instance Ref. [18]) and finding some evidence of physics beyond the SM would already be very positive.

4 General top flavour-changing interactions

In this section we collect the general Lagrangians for Ztc , γtc and gtc interactions with off-shell t , c quarks and gauge bosons, and the relation between the respective terms and coefficients of dimension-six gauge-invariant operators. For Ztu , γtu and gtu vertices the Lagrangian structure is the same and the coefficients are obtained by replacing the generation index ($2 \rightarrow 1$). The contributions of all operators are collected in appendix C.

4.1 Ztc vertex

This interaction, as the remaining flavour-changing ones, vanishes in the SM at the tree level due to the GIM mechanism. The contributions from dimension-six operators can be parameterised with the Lagrangian

$$\begin{aligned} \mathcal{L}_{Ztc} = & -\frac{g}{2c_W} \bar{c} \gamma^\mu (X_{ct}^L P_L + X_{ct}^R P_R) t Z_\mu \\ & -\frac{g}{2c_W} \bar{c} \frac{i\sigma^{\mu\nu} q_\nu}{M_Z} (\kappa_{ct}^L P_L + \kappa_{ct}^R P_R) t Z_\mu + \text{H.c.}, \end{aligned} \quad (45)$$

including only γ^μ and $\sigma^{\mu\nu} q_\nu$ terms and involving four anomalous couplings whose contributions from effective operators read

$$\begin{aligned} \delta X_{ct}^L &= \frac{1}{2} \left[C_{\phi q}^{(3,23)} + C_{\phi q}^{(3,32)*} - C_{\phi q}^{(1,23)} - C_{\phi q}^{(1,32)*} \right] \frac{v^2}{\Lambda^2}, \\ \delta X_{ct}^R &= -\frac{1}{2} \left[C_{\phi u}^{23} + C_{\phi u}^{32*} \right] \frac{v^2}{\Lambda^2}, \\ \delta \kappa_{ct}^L &= \sqrt{2} \left[c_W C_{uW}^{32*} - s_W C_{uB\phi}^{32*} \right] \frac{v^2}{\Lambda^2}, \\ \delta \kappa_{ct}^R &= \sqrt{2} \left[c_W C_{uW}^{23} - s_W C_{uB\phi}^{23} \right] \frac{v^2}{\Lambda^2}. \end{aligned} \quad (46)$$

A few comments are now in order. The Lagrangian in Eq. (45) for Ztc interactions involves all contributing dimension-six effective operators, including $O_{\phi q}^{(3,ij)}$ and $O_{\phi q}^{(1,ij)}$ that were discarded in Refs. [10, 11] because cancellations were banned there. These cancellations naturally happen in some SM extensions, for example with an extra $Q = 2/3$ quark singlet (see the discussion in section 3.2 and Refs. [13, 16]). Hence, this simplification does not have a solid phenomenological basis and turns out to be restrictive. We also observe that, although there are several possible contributing operators, the number of relevant parameters necessary to describe the Ztc interaction is only four instead of seven as considered before [10, 11]. We also emphasise that these parameters are independent, as we will observe by comparing with the results in the next subsection. We finally remark that a Lagrangian equivalent to the one in Eq. (45) has been used for previous phenomenological analyses [19, 20]. The results presented here show that those analyses are completely general also if regarded from the framework of dimension-six gauge-invariant effective operators.

4.2 γtc vertex

The γtc vertex arising from dimension-six effective operators can be parameterised in full generality as

$$\mathcal{L}_{\gamma tc} = -e \bar{c} \frac{i\sigma^{\mu\nu} q_\nu}{m_t} (\lambda_{ct}^L P_L + \lambda_{ct}^R P_R) t A_\mu + \text{H.c.}, \quad (47)$$

where

$$\begin{aligned}\delta\lambda_{ct}^L &= \frac{\sqrt{2}}{e} [s_W C_{uW}^{32*} + c_W C_{uB\phi}^{32*}] \frac{vm_t}{\Lambda^2}, \\ \delta\lambda_{ct}^R &= \frac{\sqrt{2}}{e} [s_W C_{uW}^{23} + c_W C_{uB\phi}^{23}] \frac{vm_t}{\Lambda^2}.\end{aligned}\tag{48}$$

This Lagrangian is completely general but only involves two independent parameters instead of the four ones considered in Refs. [10, 11]. As we have anticipated, these parameters are independent from the corresponding ones involving a Ztc interaction. (For example, a combination of operators with $C_{uB\phi}^{32} = -\tan\theta_W C_{uW}^{23}$ does not contribute to λ_{ct}^R but contributes to κ_{ct}^R .) An equivalent Lagrangian has been used in previous phenomenological analyses [19, 20] which are completely general, as we have shown here.

4.3 gtc vertex

Finally, the gtc vertex arising from dimension-six effective operators is written as

$$\mathcal{L}_{gtc} = -g_s \bar{c} \lambda^a \frac{i\sigma^{\mu\nu} q_\nu}{m_t} (\zeta_{ct}^L P_L + \zeta_{ct}^R P_R) t G_\mu^a,\tag{49}$$

where the contributions to the two relevant couplings $\zeta_{ct}^L, \zeta_{ct}^R$ are

$$\begin{aligned}\delta\zeta_{ct}^L &= \frac{\sqrt{2}}{g_s} C_{uG\phi}^{32*} \frac{vm_t}{\Lambda^2}, \\ \delta\zeta_{ct}^R &= \frac{\sqrt{2}}{g_s} C_{uG\phi}^{23} \frac{vm_t}{\Lambda^2}.\end{aligned}\tag{50}$$

This parameterisation seems more convenient than other ones [21] because the rewriting of O_{qG} and O_{uG} eliminates quartic terms which otherwise would have to be included in amplitude calculations. Of course, the physical results are independent of the parameterisation but the effort needed for computations may be reduced if an adequate parameter set is chosen.

4.4 Determination of effective operator coefficients

A hypothetical measurement of FCN couplings might eventually be used to determine effective operator coefficients but not completely, because there are 8 anomalous couplings involved in the vertices and 9 effective operator coefficients. Let us focus on top-charm interactions for definiteness. The simultaneous measurement of κ_{ct}^L and λ_{ct}^L would determine C_{uW}^{32} and $C_{uB\phi}^{32}$, while κ_{ct}^R and λ_{ct}^R would do the same for C_{uW}^{23} and

$C_{uB\phi}^{23}$. From the gluon couplings, $C_{uG\phi}^{32}$ and $C_{uG\phi}^{23}$ might be obtained. On the other hand, the various coefficients appearing in X_{ct}^L and X_{ct}^R cannot be disentangled using only measurements of FCN couplings. As we have pointed out, the FCN anomalous couplings are all independent, although they could be related to anomalous couplings appearing in the Wtd and Wts vertices, which are not addressed here.

5 Summary

New physics at a higher scale can be described within the framework of gauge-invariant effective operators which result after integrating the heavy degrees of freedom. These operators can induce corrections to SM couplings and, in particular, may originate anomalous couplings of the top quark to the gauge bosons. The large number and variety of dimension-six gauge-invariant effective operators [3] leads to the appearance of many possible Lorentz structures for the top trilinear vertices involving a large number of parameters.

In this work we have used the equations of motion to remove redundant operators, arriving at the gratifying conclusion that *all* effective operator contributions to the trilinear $Vf_i f_j$ vertices involving a W or Z boson, a photon or a gluon, can be parameterised in full generality using only γ^μ and $\sigma^{\mu\nu}q_\nu$ terms, with $q = p_j - p_i$. This result, which is well-known for an on-shell boson V and on-shell fermions f_i, f_j , is also valid if they are off-shell due to the gauge structure of the theory, *i.e.* the fact that gauge-invariant effective operators include not only $Vf_i f_j$ vertices but also other ones as for example $gVf_i f_j$ and four-fermion interactions. In this way, phenomenological analyses involving anomalous couplings can be considerably simplified. Compared to previous literature, we find that new physics contributions to top interactions can be described with a smaller number of parameters and a simpler Lorentz structure. The reduction in the number of effective operator coefficients can be read in Table 1 for each of the couplings studied. The number of anomalous couplings involved in the vertices is included as well. Note that some of the anomalous couplings are real or purely imaginary by definition. For example, in the g_{tt} vertex the two anomalous couplings involve the real and imaginary part of the coefficient $C_{uG\phi}^{33}$.

A second important point which must be noted is that in most cases the relation between anomalous couplings and effective operator coefficients is direct, for example in the Wtb vertex. This would allow for the determination of effective operator coefficients if these anomalous couplings were measured at LHC [5]. In some other cases effective operator coefficients can be determined by simultaneous measurements of two

	Wtb	Ztt	γtt	$g tt$	Ztu/c	$\gamma tu/c$	gtu/c
C_x^0	9	10	5	3	20	10	6
C_x	4	5	2	1	10	4	2
N_{\min}	4	4	2	2	4	2	2

Table 1: For each interaction: number of effective operator coefficients C_x^0 , C_x contributing to the trilinear vertex before and after removing redundant operators; number of anomalous couplings N_{\min} necessary to describe the vertex.

anomalous couplings, *e.g.* involving the Z boson and the photon couplings. Moreover, if all anomalous couplings in the Wtb , Ztt and γtt vertices might be determined (which is an extremely optimistic assumption) a consistency check could be performed by comparing the determination of C_{uW}^{33} from the Wtb coupling g_R and from the simultaneous measurement of the Ztt couplings d_V^Z , d_A^Z and γtt couplings d_V^{γ} , d_A^{γ} . Analogous tests are not possible in FCN interactions where all anomalous couplings involve independent combinations of operator coefficients, but could be possible if anomalous Wtd and Wts interactions were included.

Working within the framework of gauge-invariant effective operators, phenomenological studies of the influence of top anomalous couplings can be carried out using the simple Lagrangians given in sections 3 and 4. (This is valid for any other fermion, since the results are general.) Of course, in a given process there may be further contributions to the amplitudes apart from those originating from trilinear vertices, related by gauge symmetry. But in this respect the operator rewriting performed also proves to be very useful. The redundant operators removed include associated interactions, for example $gZtc$ and $g\gamma tc$ vertices, which should otherwise be taken into account in amplitude calculations. After rewriting these operators and showing that they are redundant, not only the extra terms in the vertex are unnecessary but also the quartic interactions. This fact greatly simplifies the theoretical setup and the development of Monte Carlo generators involving top anomalous couplings.

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A Gauge symmetry and amplitude calculations

For a better understanding of the results obtained, we show in this appendix how the relations among operators obtained actually translate into the rewriting of the vertices, and why this rewriting gives the same result in amplitude calculations. For simplicity we restrict ourselves to charged current Wtb interactions and study the implications for two single top processes in hadron collisions: s -channel production $u\bar{d} \rightarrow t\bar{b}$ and tW^- associated production, $gb \rightarrow tW^-$, depicted in Fig. 1. Analogous results hold for the rest of operators and other processes, as it is expected, once that all contributions from gauge invariant operators are included.

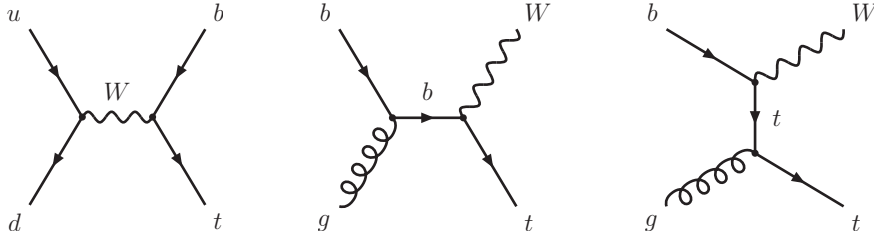


Figure 1: Left: Feynman diagram for single top production in the $u\bar{d} \rightarrow t\bar{b}$ process. Center, right: diagrams contributing to $gb \rightarrow tW^-$.

We begin with the operator O_{qW}^{33} , whose contribution to these two processes is given by the Wtb and $gWtb$ vertices,

$$\begin{aligned} \alpha O_{qW}^{33} + \alpha^* (O_{qW}^{33})^\dagger \supset & -\sqrt{2} \operatorname{Re} \alpha q^2 \bar{b}_L \gamma^\mu t_L W_\mu^- \\ & + i\sqrt{2} \operatorname{Im} \alpha [\bar{b}_L (\not{q} k^\mu - k \cdot q \gamma^\mu) t_L \\ & + 2g_s \bar{t}_L \frac{\lambda^a}{2} (\gamma^\mu \not{p}_W^\nu - \not{p}_W g^{\mu\nu}) b_L G_\nu^a] W_\mu^- + \text{H.c.} \end{aligned} \quad (51)$$

where $q = p_t - p_b$ and p_W is the outgoing W boson momentum, which are equal in the triple vertex. On the other hand, using Eqs. (23) to write O_{qW}^{33} in terms of other operators, the relevant contributions are Wtb vertices and a four-fermion interaction,

$$\begin{aligned} \alpha O_{qW}^{33} + \alpha^* (O_{qW}^{33})^\dagger \supset & -\sqrt{2} \operatorname{Re} \alpha \left[M_W^2 \bar{b}_L \gamma^\mu t_L W_\mu^- - \frac{g}{\sqrt{2}} (\bar{b}_L \gamma^\mu t_L) (\bar{u}_L \gamma_\mu d_L) + \dots \right] \\ & + i\sqrt{2} \operatorname{Im} \alpha \bar{b} i\sigma^{\mu\nu} q_\nu (m_t P_R - m_b P_L) t W_\mu^- + \text{H.c.} \end{aligned} \quad (52)$$

This rewriting gives the same results in amplitude calculations even for off-shell fermions or bosons. In the s -channel process, the t and b quarks involved in the Wtb vertex are on-shell but the W boson is not. The non-trivial substitution is in this case

$$q^2 \bar{b}_L \gamma^\mu t_L W_\mu^- \rightarrow M_W^2 \bar{b}_L \gamma^\mu t_L W_\mu^- - \frac{g}{\sqrt{2}} (\bar{b}_L \gamma^\mu t_L) (\bar{u}_L \gamma_\mu d_L) \quad (53)$$

As we will find with the explicit calculation below, the resulting amplitude is the same if we use (i) the $q^2 \bar{b}_L \gamma^\mu t_L$ interaction on the left-hand side of this equation, or (ii) the $M_W^2 \bar{b}_L \gamma^\mu t_L$ term on the right-hand side plus the four-fermion contribution. In tW^- associated production the W boson is on-shell but the top and bottom quarks are not. For this process, the non-trivial substitution is

$$\begin{aligned} & \left[\bar{b}_L (\not{q} k^\mu - k \cdot q \gamma^\mu) t_L + 2g_s \bar{t}_L \frac{\lambda^a}{2} (\gamma^\mu \not{p}_W^\nu - \not{p}_W g^{\mu\nu}) b_L G_\nu^a \right] W_\mu^- \\ & \rightarrow \bar{b} i \sigma^{\mu\nu} q_\nu (m_t P_R - m_b P_L) t W_\mu^-, \end{aligned} \quad (54)$$

which gives the same result for the $gb \rightarrow tW^-$ amplitude once that the contribution of the Wtb vertex to both diagrams and the new diagram involving the quartic vertex, which is only present before the rewriting, are summed. The same reckoning obviously applies to the Z boson, the photon and the gluon, namely the operators O_{qB}^{ij} , O_{uB}^{ij} , O_{qG}^{ij} and O_{uG}^{ij} . Note that for this latter case a different approach has been taken in Ref. [21] and subsequent works, doing the opposite replacement to Eq. (54). We find that performing the substitutions as suggested here has the added advantage of removing quartic interactions from the analysis.

For the rewriting of the combinations $O_{Du}^{ij} - O_{Du}^{ij}$, $O_{Dd}^{ij} - O_{Dd}^{ij}$ the arguments are analogous. The operator equalities imply for these processes the replacements

$$\begin{aligned} [\bar{b}_L k^\mu t_R - g_s \bar{b}_L \lambda^a g^{\mu\nu} t_R G_\nu^a] W_\mu^- & \rightarrow \bar{b}_L i \sigma^{\mu\nu} q_\nu t_R + m_t \bar{b}_L \gamma^\mu t_L + m_b \bar{b}_R \gamma^\mu t_R, \\ [\bar{t}_L k^\mu b_R - g_s \bar{t}_L \lambda^a g^{\mu\nu} b_R G_\nu^a] W_\mu^+ & \rightarrow -\bar{t}_L i \sigma^{\mu\nu} q_\nu b_R + m_b \bar{t}_L \gamma^\mu b_L + m_t \bar{t}_R \gamma^\mu b_R, \end{aligned} \quad (55)$$

which give the same result in amplitude calculations [5]. Notice that the trilinear term substitutions in these equations exactly correspond to the Gordon identities that can be applied for on-shell fermions. In the off-shell case, *e.g.* in tW^- production, the trilinear terms in both sides are not equal but their difference is compensated precisely by the $gWtb$ quartic vertex, which is not present on the right-hand side. Besides, it must be remarked that the rewritten expressions bring the advantage of not only removing the k^μ terms from the effective Wtb vertex, but also the associated quartic $gWtb$ interactions which otherwise should be included in some of the amplitudes.

In the following we carry out the amplitude calculations to check that the replacements in Eqs. (53), (54) give the same results in $u\bar{d} \rightarrow t\bar{b}$ and $gb \rightarrow tW^-$. The same has been done in Ref. [5] for the replacements in Eqs. (55).

A.1 Amplitude for $u\bar{d} \rightarrow t\bar{b}$

We denote by p_1 , p_2 , p_3 and p_4 the momenta of the u , \bar{d} , t and \bar{b} quarks, respectively. Using the Wtb interaction on the left-hand side of Eq. (53) and the standard Wud

vertex, the amplitude reads

$$\mathcal{M}_1 = -g \frac{q^2}{q^2 - M_W^2} \bar{u}(p_3) \gamma^\mu P_L v(p_4) \bar{v}(p_2) \gamma_\mu P_L u(p_1), \quad (56)$$

while using the Wtb interaction on the right-hand side the amplitude is

$$\mathcal{M}_2 = -g \frac{M_W^2}{q^2 - M_W^2} \bar{u}(p_3) \gamma^\mu P_L v(p_4) \bar{v}(p_2) \gamma_\mu P_L u(p_1). \quad (57)$$

The amplitude corresponding to the four-fermion interaction is

$$\mathcal{M}_3 = -g \bar{u}(p_3) \gamma^\mu P_L v(p_4) \bar{v}(p_2) \gamma_\mu P_L u(p_1), \quad (58)$$

so that it is evident that $\mathcal{M}_1 = \mathcal{M}_2 + \mathcal{M}_3$, as it should be.

A.2 Amplitude for $gb \rightarrow tW^-$

Checking that the substitution in Eq. (54) gives the same result in the $gb \rightarrow tW^-$ amplitude is algebraically much more involved. The computations can be considerably simplified if we define an “off-shell” operator subtracting the trilinear terms in this equation,

$$\mathcal{O}_3 = [\bar{b}(\not{q}k^\mu - k \cdot q \gamma^\mu) P_L t - \bar{b}i\sigma^{\mu\nu} q_\nu (m_t P_R - m_b P_L) t] W_\mu^- + \text{H.c.} \quad (59)$$

Then, to prove the validity of substitution in Eq. (54) we only have to show that the contribution of \mathcal{O}_3 plus the quartic term identically vanish. Using the anticommutation relation for γ matrices and the definition of $\sigma^{\mu\nu}$, \mathcal{O}_3 can be written in a much more convenient form,

$$\begin{aligned} \mathcal{O}_3 &= [\bar{b}\gamma^\mu (m_t P_L - m_b P_R) (\not{p}_t - m_t) t - \bar{b}(\not{p}_b - m_b) \gamma^\mu (m_b P_L - m_t P_R) t] W_\mu^- \\ &+ [\bar{b}k^\mu P_R (\not{p}_t - m_t) t - \bar{b}(\not{p}_b - m_b) k^\mu P_L t] W_\mu^- \\ &+ [-\bar{b}\gamma^\mu P_L (p_t^2 - m_t^2) t + \bar{b}(p_b^2 - m_b^2) \gamma^\mu P_L t] W_\mu^- + \text{H.c.} \end{aligned} \quad (60)$$

This expression makes it apparent that \mathcal{O}_3 vanishes for both t, b on-shell.

We denote by p_1, p_2, p_3 and p_4 the momenta of the gluon, b, t and W^- boson, respectively. We use superscripts a, b, c to label the contributions to the amplitudes of the three terms in Eq. (60), in the order shown, and subscripts 1, 2 corresponding to the s - and t -channel diagrams. After trivial simplifications using $(\not{p} - m)(\not{p} + m) = p^2 - m^2$, the first term gives

$$\begin{aligned} \mathcal{M}_1^a &= -\frac{g_s}{2} \bar{u}(p_3) \lambda^a \gamma^\mu \gamma^\nu (m_t P_L - m_b P_R) u(p_2) \times \varepsilon, \\ \mathcal{M}_2^a &= -\frac{g_s}{2} \bar{u}(p_3) \lambda^a \gamma^\nu \gamma^\mu (m_t P_L - m_b P_R) u(p_2) \times \varepsilon, \end{aligned} \quad (61)$$

where ε stands for the product of polarisation vectors $\varepsilon_\mu^*(p_4)\varepsilon_\nu(p_1)$. The sum of both diagrams is

$$\mathcal{M}_{1+2}^a = -g_s \bar{u}(p_3) \lambda^a g^{\mu\nu} (m_t P_L - m_b P_R) u(p_2) \times \varepsilon. \quad (62)$$

The second term gives

$$\begin{aligned} \mathcal{M}_1^b &= \frac{g_s}{2} \bar{u}(p_3) \lambda^a (p_5 + p_3)^\mu \gamma^\nu P_L u(p_2) \times \varepsilon, \\ \mathcal{M}_2^b &= \frac{g_s}{2} \bar{u}(p_3) \lambda^a (p_2 + p_6)^\mu \gamma^\nu P_L u(p_2) \times \varepsilon, \end{aligned} \quad (63)$$

where $p_5 = p_1 + p_2$ and $p_6 = p_3 - p_1$ are the momenta of the internal b , t quarks in the s - and t -channel diagrams, respectively. The sum of both is

$$\mathcal{M}_{1+2}^b = g_s \bar{u}(p_3) \lambda^a p_1^\mu \gamma^\nu P_L u(p_2) \times \varepsilon. \quad (64)$$

The third term yields the contributions

$$\begin{aligned} \mathcal{M}_1^c &= -\frac{g_s}{2} \bar{u}(p_3) \lambda^a [P_R \gamma^\mu (\not{p}_2 + m_b) \gamma^\nu + \gamma^\mu \not{p}_1 \gamma^\nu P_L] u(p_2) \times \varepsilon, \\ \mathcal{M}_2^c &= \frac{g_s}{2} \bar{u}(p_3) \lambda^a [\gamma^\nu (\not{p}_3 + m_t) \gamma^\mu P_L - \gamma^\nu \not{p}_1 \gamma^\mu P_L] u(p_2) \times \varepsilon. \end{aligned} \quad (65)$$

Using the γ anticommutation relations and the Dirac equation for external fermions, the sum of both is

$$\begin{aligned} \mathcal{M}_{1+2}^c &= -g_s \bar{u}(p_3) \lambda^a p_1^\mu \gamma^\nu P_L u(p_2) \times \varepsilon - g_s \bar{u}(p_3) \lambda^a \gamma^\mu p_4^\nu P_L u(p_2) \times \varepsilon \\ &\quad + g_s \bar{u}(p_3) \lambda^a \not{p}_1 g^{\mu\nu} P_L u(p_2) \times \varepsilon. \end{aligned} \quad (66)$$

Notice that the first of these terms already cancels \mathcal{M}_{1+2}^b . Finally, the contribution of the quartic $gWtb$ coupling is

$$\begin{aligned} \mathcal{M}_3 &= -g_s \bar{u}(p_3) \lambda^a \not{p}_1 g^{\mu\nu} P_L u(p_2) \times \varepsilon + g_s \bar{u}(p_3) \lambda^a (m_t P_L - m_b P_R) g^{\mu\nu} u(p_2) \times \varepsilon \\ &\quad + g_s \bar{u}(p_3) \lambda^a \gamma^\mu p_4^\nu P_L u(p_2) \times \varepsilon. \end{aligned} \quad (67)$$

The sum

$$\mathcal{M}_{1+2}^a + \mathcal{M}_{1+2}^b + \mathcal{M}_{1+2}^c + \mathcal{M}_3 = 0 \quad (68)$$

vanishes, as expected.

B Operator contributions to Wtb , Ztt , γtt and $g tt$

We collect here the contribution to the effective Wtb , Ztt , γtt and $g tt$ vertices of the operators in Eqs. (3), (4), including also those from the operators which are redundant. We also collect the contributions to the associated $gWtb$, $gZtt$, $g\gamma tt$ and $gg tt$ quartic couplings. We use the shorthand $\alpha_x = C_x/\Lambda^2$ and drop the indices in the α constants. Our expressions coincide with those in Refs. [8,9] except for sign differences originating from the different definitions of the covariant derivative and the Z field, and also coincide with Ref. [22]. For the Wtb interaction we have

$$\begin{aligned}
\alpha O_{\phi q}^{(3,33)} + \alpha^*(O_{\phi q}^{(3,33)})^\dagger &\supset -\alpha \frac{gv^2}{\sqrt{2}} \bar{t}_L \gamma^\mu b_L W_\mu^+ - \alpha^* \frac{gv^2}{\sqrt{2}} \bar{b}_L \gamma^\mu t_L W_\mu^-, \\
\alpha O_{\phi\phi}^{33} + \alpha^*(O_{\phi\phi}^{33})^\dagger &\supset -\alpha \frac{gv^2}{2\sqrt{2}} \bar{t}_R \gamma^\mu b_R W_\mu^+ - \alpha^* \frac{gv^2}{2\sqrt{2}} \bar{b}_R \gamma^\mu t_R W_\mu^-, \\
\alpha O_{uW}^{33} + \alpha^*(O_{uW}^{33})^\dagger &\supset \alpha v \bar{b}_L \sigma^{\mu\nu} t_R W_{\mu\nu}^- + \alpha^* v \bar{t}_R \sigma^{\mu\nu} b_L W_{\mu\nu}^+, \\
\alpha O_{dW}^{33} + \alpha^*(O_{dW}^{33})^\dagger &\supset \alpha v \bar{t}_L \sigma^{\mu\nu} b_R W_{\mu\nu}^+ + \alpha^* v \bar{b}_R \sigma^{\mu\nu} t_L W_{\mu\nu}^-, \\
\alpha O_{Du}^{33} + \alpha^*(O_{Du}^{33})^\dagger &\supset \alpha \frac{gv}{2} i \bar{b}_L \partial^\mu t_R W_\mu^- - \alpha^* \frac{gv}{2} i \partial^\mu \bar{t}_R b_L W_\mu^+, \\
\alpha O_{\bar{D}u}^{33} + \alpha^*(O_{\bar{D}u}^{33})^\dagger &\supset \alpha \frac{gv}{2} i \partial^\mu \bar{b}_L t_R W_\mu^- - \alpha^* \frac{gv}{2} i \bar{t}_R \partial^\mu b_L W_\mu^+, \\
\alpha O_{Dd}^{33} + \alpha^*(O_{Dd}^{33})^\dagger &\supset \alpha \frac{gv}{2} i \bar{t}_L \partial^\mu b_R W_\mu^+ - \alpha^* \frac{gv}{2} i \partial^\mu \bar{b}_R t_L W_\mu^-, \\
\alpha O_{\bar{D}d}^{33} + \alpha^*(O_{\bar{D}d}^{33})^\dagger &\supset \alpha \frac{gv}{2} i \partial^\mu \bar{t}_L b_R W_\mu^+ - \alpha^* \frac{gv}{2} i \bar{b}_R \partial^\mu t_L W_\mu^-, \\
\alpha O_{qW}^{33} + \alpha^*(O_{qW}^{33})^\dagger &\supset \sqrt{2} \left[\text{Re } \alpha \partial^\nu (\bar{b}_L \gamma^\mu t_L) + i \text{Im } \alpha \bar{b}_L \gamma^\mu \overleftrightarrow{\partial}^\nu t_L \right] W_{\mu\nu}^- \\
&\quad + \sqrt{2} \left[\text{Re } \alpha \partial^\nu (\bar{t}_L \gamma^\mu b_L) + i \text{Im } \alpha \bar{t}_L \gamma^\mu \overleftrightarrow{\partial}^\nu b_L \right] W_{\mu\nu}^+. \quad (69)
\end{aligned}$$

Associated quartic $gWtb$ terms arise only from the redundant operators:

$$\begin{aligned}
\alpha O_{Du}^{33} + \alpha^*(O_{Du}^{33})^\dagger &\supset -\frac{gg_s v}{4} \left[\alpha \bar{b}_L \lambda^a g^{\mu\nu} t_R W_\mu^- + \alpha^* \bar{t}_R \lambda^a g^{\mu\nu} b_L W_\mu^+ \right] G_\nu^a, \\
\alpha O_{\bar{D}u}^{33} + \alpha^*(O_{\bar{D}u}^{33})^\dagger &\supset \frac{gg_s v}{4} \left[\alpha \bar{b}_L \lambda^a g^{\mu\nu} t_R W_\mu^- + \alpha^* \bar{t}_R \lambda^a g^{\mu\nu} b_L W_\mu^+ \right] G_\nu^a, \\
\alpha O_{Dd}^{33} + \alpha^*(O_{Dd}^{33})^\dagger &\supset -\frac{gg_s v}{4} \left[\alpha \bar{t}_L \lambda^a g^{\mu\nu} b_R W_\mu^+ + \alpha^* \bar{b}_R \lambda^a g^{\mu\nu} t_L W_\mu^- \right] G_\nu^a, \\
\alpha O_{\bar{D}d}^{33} + \alpha^*(O_{\bar{D}d}^{33})^\dagger &\supset \frac{gg_s v}{4} \left[\alpha \bar{t}_L \lambda^a g^{\mu\nu} b_R W_\mu^+ + \alpha^* \bar{b}_R \lambda^a g^{\mu\nu} t_L W_\mu^- \right] G_\nu^a, \\
\alpha O_{qW}^{33} + \alpha^*(O_{qW}^{33})^\dagger &\supset -\sqrt{2} \text{Im } \alpha g_s \left[\bar{b}_L \lambda^a \gamma^\mu g^{\nu\sigma} t_L W_{\mu\nu}^- + \bar{t}_L \lambda^a \gamma^\mu g^{\nu\sigma} b_L W_{\mu\nu}^+ \right] G_\sigma^a. \quad (70)
\end{aligned}$$

The contributions from effective operators to the effective Ztt vertex are

$$\begin{aligned}
\alpha O_{\phi q}^{(3,33)} + \alpha^*(O_{\phi q}^{(3,33)})^\dagger &\supset -\text{Re } \alpha \frac{gv^2}{2c_W} \bar{t}_L \gamma^\mu t_L Z_\mu, \\
\alpha O_{\phi q}^{(1,33)} + \alpha^*(O_{\phi q}^{(1,33)})^\dagger &\supset \text{Re } \alpha \frac{gv^2}{2c_W} \bar{t}_L \gamma^\mu t_L Z_\mu, \\
\alpha O_{\phi u}^{33} + \alpha^*(O_{\phi u}^{33})^\dagger &\supset \text{Re } \alpha \frac{gv^2}{2c_W} \bar{t}_R \gamma^\mu t_R Z_\mu, \\
\alpha O_{uW}^{33} + \alpha^*(O_{uW}^{33})^\dagger &\supset \frac{v}{\sqrt{2}} c_W [\text{Re } \alpha \bar{t} \sigma^{\mu\nu} t + i \text{Im } \alpha \bar{t} \sigma^{\mu\nu} \gamma_5 t] Z_{\mu\nu}, \\
\alpha O_{uB\phi}^{33} + \alpha^*(O_{uB\phi}^{33})^\dagger &\supset -\frac{v}{\sqrt{2}} s_W [\text{Re } \alpha \bar{t} \sigma^{\mu\nu} t + i \text{Im } \alpha \bar{t} \sigma^{\mu\nu} \gamma_5 t] Z_{\mu\nu}, \\
\alpha O_{Du}^{33} + \alpha^*(O_{Du}^{33})^\dagger &\supset \frac{gv}{2\sqrt{2}c_W} [\alpha i \bar{t}_L \partial^\mu t_R - \alpha^* i \partial^\mu \bar{t}_R t_L] Z_\mu, \\
\alpha O_{\bar{D}u}^{33} + \alpha^*(O_{\bar{D}u}^{33})^\dagger &\supset \frac{gv}{2\sqrt{2}c_W} [\alpha i \partial^\mu \bar{t}_L t_R - \alpha^* i \bar{t}_R \partial^\mu t_L] Z_\mu, \\
\alpha O_{qW}^{33} + \alpha^*(O_{qW}^{33})^\dagger &\supset c_W \left[\text{Re } \alpha \partial^\nu (\bar{t}_L \gamma^\mu t_L) + i \text{Im } \alpha \bar{t}_L \gamma^\mu \overleftrightarrow{\partial}^\nu t_L \right] Z_{\mu\nu}, \\
\alpha O_{qB}^{33} + \alpha^*(O_{qB}^{33})^\dagger &\supset -s_W \left[\text{Re } \alpha \partial^\nu (\bar{t}_L \gamma^\mu t_L) + i \text{Im } \alpha \bar{t}_L \gamma^\mu \overleftrightarrow{\partial}^\nu t_L \right] Z_{\mu\nu}, \\
\alpha O_{uB}^{33} + \alpha^*(O_{uB}^{33})^\dagger &\supset -s_W \left[\text{Re } \alpha \partial^\nu (\bar{t}_R \gamma^\mu t_R) + i \text{Im } \alpha \bar{t}_R \gamma^\mu \overleftrightarrow{\partial}^\nu t_R \right] Z_{\mu\nu}. \quad (71)
\end{aligned}$$

Among these operators, the contributions to the $gZtt$ vertex are only from the redundant ones,

$$\begin{aligned}
\alpha O_{Du}^{33} + \alpha^*(O_{Du}^{33})^\dagger &\supset -\frac{gg_s v}{4\sqrt{2}c_W} [\text{Re } \alpha \bar{t} \lambda^a g^{\mu\nu} t + i \text{Im } \alpha \bar{t} \lambda^a g^{\mu\nu} \gamma_5 t] G_\nu^a Z_\mu, \\
\alpha O_{\bar{D}u}^{33} + \alpha^*(O_{\bar{D}u}^{33})^\dagger &\supset \frac{gg_s v}{4\sqrt{2}c_W} [\text{Re } \alpha \bar{t} \lambda^a g^{\mu\nu} t + i \text{Im } \alpha \bar{t} \lambda^a g^{\mu\nu} \gamma_5 t] G_\nu^a Z_\mu, \\
\alpha O_{qW}^{33} + \alpha^*(O_{qW}^{33})^\dagger &\supset -\text{Im } \alpha g_s c_W \bar{t}_L \lambda^a \gamma^\mu g^{\nu\sigma} t_L G_\sigma^a Z_{\mu\nu}, \\
\alpha O_{qB}^{33} + \alpha^*(O_{qB}^{33})^\dagger &\supset \text{Im } \alpha g_s s_W \bar{t}_L \lambda^a \gamma^\mu g^{\nu\sigma} t_L G_\sigma^a Z_{\mu\nu}, \\
\alpha O_{uB}^{33} + \alpha^*(O_{uB}^{33})^\dagger &\supset \text{Im } \alpha g_s s_W \bar{t}_R \lambda^a \gamma^\mu g^{\nu\sigma} t_R G_\sigma^a Z_{\mu\nu}. \quad (72)
\end{aligned}$$

The operators contributing to the γtt vertex are

$$\begin{aligned}
\alpha O_{uW}^{33} + \alpha^*(O_{uW}^{33})^\dagger &\supset \frac{v}{\sqrt{2}} s_W [\text{Re } \alpha \bar{t} \sigma^{\mu\nu} t + i \text{Im } \alpha \bar{t} \sigma^{\mu\nu} \gamma_5 t] A_{\mu\nu}, \\
\alpha O_{uB\phi}^{33} + \alpha^*(O_{uB\phi}^{33})^\dagger &\supset \frac{v}{\sqrt{2}} c_W [\text{Re } \alpha \bar{t} \sigma^{\mu\nu} t + i \text{Im } \alpha \bar{t} \sigma^{\mu\nu} \gamma_5 t] A_{\mu\nu}, \\
\alpha O_{qW}^{33} + \alpha^*(O_{qW}^{33})^\dagger &\supset s_W \left[\text{Re } \alpha \partial^\nu (\bar{t}_L \gamma^\mu t_L) + i \text{Im } \alpha \bar{t}_L \gamma^\mu \overleftrightarrow{\partial}^\nu t_L \right] A_{\mu\nu}, \\
\alpha O_{qB}^{33} + \alpha^*(O_{qB}^{33})^\dagger &\supset c_W \left[\text{Re } \alpha \partial^\nu (\bar{t}_L \gamma^\mu t_L) + i \text{Im } \alpha \bar{t}_L \gamma^\mu \overleftrightarrow{\partial}^\nu t_L \right] A_{\mu\nu}, \\
\alpha O_{uB}^{33} + \alpha^*(O_{uB}^{33})^\dagger &\supset c_W \left[\text{Re } \alpha \partial^\nu (\bar{t}_R \gamma^\mu t_R) + i \text{Im } \alpha \bar{t}_R \gamma^\mu \overleftrightarrow{\partial}^\nu t_R \right] A_{\mu\nu}. \quad (73)
\end{aligned}$$

The associated $g\gamma tt$ quartic vertices are

$$\begin{aligned}
\alpha O_{qW}^{33} + \alpha^*(O_{qW}^{33})^\dagger &\supset -\text{Im } \alpha g_s s_W \bar{t}_L \lambda^a \gamma^\mu g^{\nu\sigma} t_L G_\sigma^a A_{\mu\nu}, \\
\alpha O_{qB}^{33} + \alpha^*(O_{qB}^{33})^\dagger &\supset -\text{Im } \alpha g_s c_W \bar{t}_L \lambda^a \gamma^\mu g^{\nu\sigma} t_L G_\sigma^a A_{\mu\nu}, \\
\alpha O_{uB}^{33} + \alpha^*(O_{uB}^{33})^\dagger &\supset -\text{Im } \alpha g_s c_W \bar{t}_R \lambda^a \gamma^\mu g^{\nu\sigma} t_R G_\sigma^a A_{\mu\nu}.
\end{aligned} \tag{74}$$

Finally, the contributions to the $g\bar{t}t$ interaction are

$$\begin{aligned}
\alpha O_{uG\phi}^{33} + \alpha^*(O_{uG\phi}^{33})^\dagger &\supset \frac{v}{\sqrt{2}} [\text{Re } \alpha \bar{t} \lambda^a \sigma^{\mu\nu} t + i \text{Im } \alpha t \lambda^a \sigma^{\mu\nu} \gamma_5 t] G_{\mu\nu}^a, \\
\alpha O_{qG}^{33} + \alpha^*(O_{qG}^{33})^\dagger &\supset \left[\text{Re } \alpha \partial^\nu (\bar{t}_L \lambda^a \gamma^\mu t_L) + i \text{Im } \alpha \bar{t}_L \lambda^a \gamma^\mu \overleftrightarrow{\partial}^\nu t_L \right] G_{\mu\nu}^a, \\
\alpha O_{uG}^{33} + \alpha^*(O_{uG}^{33})^\dagger &\supset \left[\text{Re } \alpha \partial^\nu (\bar{t}_R \lambda^a \gamma^\mu t_R) + i \text{Im } \alpha \bar{t}_R \lambda^a \gamma^\mu \overleftrightarrow{\partial}^\nu t_R \right] G_{\mu\nu}^a.
\end{aligned} \tag{75}$$

The two redundant operators include several associated quartic vertices with extra W , Z , γ bosons as the ones listed above, as well as new ones with an extra gluon. They are

$$\begin{aligned}
\alpha O_{qG}^{33} + \alpha^*(O_{qG}^{33})^\dagger &\supset i \frac{g_s}{2} [\text{Re } \alpha \bar{t}_L [\lambda^a, \lambda^b] \gamma^\mu g^{\nu\sigma} t_L + i \text{Im } \alpha t_L \{\lambda^a, \lambda^b\} \gamma^\mu g^{\nu\sigma} t_L] G_\sigma^b G_{\mu\nu}^a \\
&\quad - \sqrt{2} g \text{Im } \alpha [\bar{t}_L \lambda^a \gamma^\mu g^{\nu\sigma} b_L W_\sigma^+ + \bar{b}_L \lambda^a \gamma^\mu g^{\nu\sigma} t_L W_\sigma^-] G_{\mu\nu}^a \\
&\quad - \frac{g}{c_W} f_u^L \text{Im } \alpha \bar{t}_L \lambda^a \gamma^\mu g^{\nu\sigma} t_L Z_\sigma G_{\mu\nu}^a \\
&\quad - 2Q_t e \text{Im } \alpha \bar{t}_L \lambda^a \gamma^\mu g^{\nu\sigma} t_L A_\sigma G_{\mu\nu}^a, \\
\alpha O_{uG}^{33} + \alpha^*(O_{uG}^{33})^\dagger &\supset i \frac{g_s}{2} [\text{Re } \alpha \bar{t}_R [\lambda^a, \lambda^b] \gamma^\mu g^{\nu\sigma} t_R + i \text{Im } \alpha t_R \{\lambda^a, \lambda^b\} \gamma^\mu g^{\nu\sigma} t_R] G_\sigma^b G_{\mu\nu}^a \\
&\quad - \frac{g}{c_W} f_u^R \text{Im } \alpha \bar{t}_R \lambda^a \gamma^\mu g^{\nu\sigma} t_R Z_\sigma G_{\mu\nu}^a \\
&\quad - 2Q_t e \text{Im } \alpha \bar{t}_R \lambda^a \gamma^\mu g^{\nu\sigma} t_R A_\sigma G_{\mu\nu}^a,
\end{aligned} \tag{76}$$

with $f_u^L = 1 - 4/3 s_W^2$, $f_u^R = -4/3 s_W^2$, $Q_t = 2/3$.

C Operator contributions to top FCN interactions

In this appendix we give the effective operator contributions to top FCN interactions, also including those operators which are redundant. Contributions for top-charm couplings arise from both combinations $i, j = 2, 3/3, 2$, while for top-up they are obtained setting $i, j = 1, 3/3, 1$. Notice that for $i = j = 3$ we can recover the results for $Z\bar{t}t$ and

γtt trivially. The Ztu , Ztc vertices are

$$\begin{aligned}
\alpha O_{\phi q}^{(3,ij)} + \alpha^*(O_{\phi q}^{(3,ij)})^\dagger &\supset -\frac{gv^2}{4c_W} [\alpha \bar{u}_{Li} \gamma^\mu u_{Lj} + \alpha^* \bar{u}_{Lj} \gamma^\mu u_{Li}] Z_\mu, \\
\alpha O_{\phi q}^{(1,ij)} + \alpha^*(O_{\phi q}^{(1,ij)})^\dagger &\supset \frac{gv^2}{2c_W} [\alpha \bar{u}_{Li} \gamma^\mu u_{Lj} + \alpha^* \bar{u}_{Lj} \gamma^\mu u_{Li}] Z_\mu, \\
\alpha O_{\phi u}^{ij} + \alpha^*(O_{\phi u}^{ij})^\dagger &\supset \frac{gv^2}{4c_W} [\alpha \bar{u}_{Ri} \gamma^\mu u_{Rj} + \alpha^* \bar{u}_{Rj} \gamma^\mu u_{Ri}] Z_\mu, \\
\alpha O_{uW}^{ij} + \alpha^*(O_{uW}^{ij})^\dagger &\supset \frac{v}{\sqrt{2}} c_W [\alpha \bar{u}_{Li} \sigma^{\mu\nu} u_{Rj} + \alpha^* \bar{u}_{Rj} \sigma^{\mu\nu} u_{Li}] Z_{\mu\nu}, \\
\alpha O_{uB\phi}^{ij} + \alpha^*(O_{uB\phi}^{ij})^\dagger &\supset -\frac{v}{\sqrt{2}} s_W [\alpha \bar{u}_{Li} \sigma^{\mu\nu} u_{Rj} + \alpha^* \bar{u}_{Rj} \sigma^{\mu\nu} u_{Li}] Z_{\mu\nu}, \\
\alpha O_{Du}^{ij} + \alpha^*(O_{Du}^{ij})^\dagger &\supset \frac{gv}{2\sqrt{2}c_W} [\alpha i \bar{u}_{Li} \partial^\mu u_{Rj} - \alpha^* i \partial^\mu \bar{u}_{Rj} u_{Li}] Z_\mu, \\
\alpha O_{\bar{D}u}^{ij} + \alpha^*(O_{\bar{D}u}^{ij})^\dagger &\supset \frac{gv}{2\sqrt{2}c_W} [\alpha i \partial^\mu \bar{u}_{Li} u_{Rj} - \alpha^* i \bar{u}_{Rj} \partial^\mu u_{Li}] Z_\mu, \\
\alpha O_{qW}^{ij} + \alpha^*(O_{qW}^{ij})^\dagger &\supset c_W [\alpha \bar{u}_{Li} \gamma^\mu \partial^\nu u_{Lj} + \alpha^* \partial^\nu \bar{u}_{Lj} \gamma^\mu u_{Li}] Z_{\mu\nu}, \\
\alpha O_{qB}^{ij} + \alpha^*(O_{qB}^{ij})^\dagger &\supset -s_W [\alpha \bar{u}_{Li} \gamma^\mu \partial^\nu u_{Lj} + \alpha^* \partial^\nu \bar{u}_{Lj} \gamma^\mu u_{Li}] Z_{\mu\nu}, \\
\alpha O_{uB}^{ij} + \alpha^*(O_{uB}^{ij})^\dagger &\supset -s_W [\alpha \bar{u}_{Ri} \gamma^\mu \partial^\nu u_{Rj} + \alpha^* \partial^\nu \bar{u}_{Rj} \gamma^\mu u_{Ri}] Z_{\mu\nu}. \tag{77}
\end{aligned}$$

The associated quartic couplings with an extra gluon are

$$\begin{aligned}
\alpha O_{Du}^{ij} + \alpha^*(O_{Du}^{ij})^\dagger &\supset -\frac{gg_s v}{4\sqrt{2}c_W} [\alpha \bar{u}_{Li} \lambda^a g^{\mu\nu} u_{Rj} + \alpha^* \bar{u}_{Rj} \lambda^a g^{\mu\nu} u_{Li}] G_\nu^a Z_\mu \\
\alpha O_{\bar{D}u}^{ij} + \alpha^*(O_{\bar{D}u}^{ij})^\dagger &\supset \frac{gg_s v}{4\sqrt{2}c_W} [\alpha \bar{u}_{Li} \lambda^a g^{\mu\nu} u_{Rj} + \alpha^* \bar{u}_{Rj} \lambda^a g^{\mu\nu} u_{Li}] G_\nu^a Z_\mu \\
\alpha O_{qW}^{ij} + \alpha^*(O_{qW}^{ij})^\dagger &\supset i \frac{g_s c_W}{2} [\alpha \bar{u}_{Li} \lambda^a \gamma^\mu g^{\nu\sigma} u_{Lj} - \alpha^* \bar{u}_{Lj} \lambda^a \gamma^\mu g^{\nu\sigma} u_{Li}] G_\sigma^a Z_{\mu\nu} \\
\alpha O_{qB}^{ij} + \alpha^*(O_{qB}^{ij})^\dagger &\supset -i \frac{g_s s_W}{2} [\alpha \bar{u}_{Li} \lambda^a \gamma^\mu g^{\nu\sigma} u_{Lj} - \alpha^* \bar{u}_{Lj} \lambda^a \gamma^\mu g^{\nu\sigma} u_{Li}] G_\sigma^a Z_{\mu\nu} \\
\alpha O_{uB}^{ij} + \alpha^*(O_{uB}^{ij})^\dagger &\supset -i \frac{g_s s_W}{2} [\alpha \bar{u}_{Ri} \lambda^a \gamma^\mu g^{\nu\sigma} u_{Rj} - \alpha^* \bar{u}_{Rj} \lambda^a \gamma^\mu g^{\nu\sigma} u_{Ri}] G_\sigma^a Z_{\mu\nu}. \tag{78}
\end{aligned}$$

For the γtu , γtc vertices the corresponding contributions are

$$\begin{aligned}
\alpha O_{uW}^{ij} + \alpha^*(O_{uW}^{ij})^\dagger &\supset \frac{v}{\sqrt{2}} s_W [\alpha \bar{u}_{Li} \sigma^{\mu\nu} u_{Rj} + \alpha^* \bar{u}_{Rj} \sigma^{\mu\nu} u_{Li}] A_{\mu\nu}, \\
\alpha O_{uB\phi}^{ij} + \alpha^*(O_{uB\phi}^{ij})^\dagger &\supset \frac{v}{\sqrt{2}} c_W [\alpha \bar{u}_{Li} \sigma^{\mu\nu} u_{Rj} + \alpha^* \bar{u}_{Rj} \sigma^{\mu\nu} u_{Li}] A_{\mu\nu}, \\
\alpha O_{qW}^{ij} + \alpha^*(O_{qW}^{ij})^\dagger &\supset s_W [\alpha \bar{u}_{Li} \gamma^\mu \partial^\nu u_{Lj} + \alpha^* \partial^\nu \bar{u}_{Lj} \gamma^\mu u_{Li}] A_{\mu\nu}, \\
\alpha O_{qB}^{ij} + \alpha^*(O_{qB}^{ij})^\dagger &\supset c_W [\alpha \bar{u}_{Li} \gamma^\mu \partial^\nu u_{Lj} + \alpha^* \partial^\nu \bar{u}_{Lj} \gamma^\mu u_{Li}] A_{\mu\nu}, \\
\alpha O_{uB}^{ij} + \alpha^*(O_{uB}^{ij})^\dagger &\supset c_W [\alpha \bar{u}_{Ri} \gamma^\mu \partial^\nu u_{Rj} + \alpha^* \partial^\nu \bar{u}_{Rj} \gamma^\mu u_{Ri}] A_{\mu\nu}, \tag{79}
\end{aligned}$$

whereas the associated $g\gamma tu$, $g\gamma tc$ couplings are

$$\begin{aligned}
\alpha O_{qW}^{ij} + \alpha^*(O_{qW}^{ij})^\dagger &\supset i\frac{g_s s_W}{2} [\alpha \bar{u}_{Li} \lambda^a \gamma^\mu u_{Lj} - \alpha^* \bar{u}_{Lj} \lambda^a \gamma^\mu g^{\nu\sigma} u_{Li}] G_\sigma^a A_{\mu\nu} \\
\alpha O_{qB}^{ij} + \alpha^*(O_{qB}^{ij})^\dagger &\supset i\frac{g_s c_W}{2} [\alpha \bar{u}_{Li} \lambda^a \gamma^\mu u_{Lj} - \alpha^* \bar{u}_{Lj} \lambda^a \gamma^\mu g^{\nu\sigma} u_{Li}] G_\sigma^a A_{\mu\nu} \\
\alpha O_{uB}^{ij} + \alpha^*(O_{uB}^{ij})^\dagger &\supset i\frac{g_s c_W}{2} [\alpha \bar{u}_{Ri} \lambda^a \gamma^\mu u_{Rj} - \alpha^* \bar{u}_{Rj} \lambda^a \gamma^\mu g^{\nu\sigma} u_{Ri}] G_\sigma^a A_{\mu\nu}. \quad (80)
\end{aligned}$$

The gtu and gtc vertices can be obtained from the operators

$$\begin{aligned}
\alpha O_{uG\phi}^{ij} + \alpha^*(O_{uG\phi}^{ij})^\dagger &\supset \frac{v}{\sqrt{2}} [\alpha \bar{u}_{Li} \lambda^a \sigma^{\mu\nu} u_{Rj} + \alpha^* \bar{u}_{Rj} \lambda^a \sigma^{\mu\nu} u_{Li}] G_{\mu\nu}^a, \\
\alpha O_{qG}^{ij} + \alpha^*(O_{qG}^{ij})^\dagger &\supset [\alpha \bar{u}_{Li} \lambda^a \gamma^\mu \partial^\nu u_{Lj} + \alpha^* \partial^\nu \bar{u}_{Lj} \lambda^a \gamma^\mu u_{Li}] G_{\mu\nu}^a, \\
\alpha O_{uG}^{ij} + \alpha^*(O_{uG}^{ij})^\dagger &\supset [\alpha \bar{u}_{Ri} \lambda^a \gamma^\mu \partial^\nu u_{Rj} + \alpha^* \partial^\nu \bar{u}_{Rj} \lambda^a \gamma^\mu u_{Ri}] G_{\mu\nu}^a. \quad (81)
\end{aligned}$$

The last two also include the associated quartic vertices

$$\begin{aligned}
\alpha O_{qG}^{ij} + \alpha^*(O_{qG}^{ij})^\dagger &\supset i\frac{g_s}{2} [\alpha \bar{u}_{Li} \lambda^a \lambda^b \gamma^\mu g^{\nu\sigma} u_{Lj} - \alpha^* \bar{u}_{Lj} \lambda^b \lambda^a \gamma^\mu g^{\nu\sigma} u_{Li}] G_\sigma^b G_{\mu\nu}^a \\
&\quad + i\frac{g}{\sqrt{2}} [\alpha \bar{u}_{Li} \lambda^a \gamma^\mu g^{\nu\sigma} d_{Lj} - \alpha^* \bar{u}_{Lj} \lambda^a \gamma^\mu g^{\nu\sigma} d_{Li}] W_\sigma^+ G_{\mu\nu}^a \\
&\quad + i\frac{g}{\sqrt{2}} [\alpha \bar{d}_{Li} \lambda^a \gamma^\mu g^{\nu\sigma} u_{Lj} - \alpha^* \bar{d}_{Lj} \lambda^a \gamma^\mu g^{\nu\sigma} u_{Li}] W_\sigma^- G_{\mu\nu}^a \\
&\quad + i\frac{g}{2c_W} f_u^L [\alpha \bar{u}_{Li} \lambda^a \gamma^\mu g^{\nu\sigma} u_{Lj} - \alpha^* \bar{u}_{Lj} \lambda^a \gamma^\mu g^{\nu\sigma} u_{Li}] Z_\sigma G_{\mu\nu}^a \\
&\quad + iQ_t e [\alpha \bar{u}_{Li} \lambda^a \gamma^\mu g^{\nu\sigma} u_{Lj} - \alpha^* \bar{u}_{Lj} \lambda^a \gamma^\mu g^{\nu\sigma} u_{Li}] A_\sigma G_{\mu\nu}^a, \\
\alpha O_{uG}^{ij} + \alpha^*(O_{uG}^{ij})^\dagger &\supset i\frac{g_s}{2} [\alpha \bar{u}_{Ri} \lambda^a \lambda^b \gamma^\mu g^{\nu\sigma} u_{Rj} - \alpha^* \bar{u}_{Rj} \lambda^b \lambda^a \gamma^\mu g^{\nu\sigma} u_{Ri}] G_\sigma^b G_{\mu\nu}^a \\
&\quad + i\frac{g}{2c_W} f_u^R [\alpha \bar{u}_{Ri} \lambda^a \gamma^\mu g^{\nu\sigma} u_{Rj} - \alpha^* \bar{u}_{Rj} \lambda^a \gamma^\mu g^{\nu\sigma} u_{Ri}] Z_\sigma G_{\mu\nu}^a \\
&\quad + iQ_t e [\alpha \bar{u}_{Ri} \lambda^a \gamma^\mu g^{\nu\sigma} u_{Rj} - \alpha^* \bar{u}_{Rj} \lambda^a \gamma^\mu g^{\nu\sigma} u_{Ri}] A_\sigma G_{\mu\nu}^a, \quad (82)
\end{aligned}$$

with $f_u^L = 1 - 4/3 s_W^2$, $f_u^R = -4/3 s_W^2$, $Q_t = 2/3$.

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