

UAB-FT-393
UGR-FT-64
May 1996

SPACE-DEPENDENT PROBABILITIES FOR $K^0 - \bar{K}^0$ OSCILLATIONS

B. Ancochea^a, A. Bramon^a, R. Muñoz-Tapia^b and M. Nowakowski^a

^a*Grup de Física Teòrica,
Universitat Autònoma de Barcelona, 08193 Bellaterra, Spain*

^b*Dpto Física Teórica y del Cosmos
Universidad de Granada, 18071 Granada, Spain*

Abstract

We analyze $K^0 - \bar{K}^0$ oscillations in space in terms of propagating wave packets with coherent K_S and K_L components. The oscillation probabilities $P_{K^0 \rightarrow K^0}(x)$ and $P_{K^0 \rightarrow \bar{K}^0}(x)$ depending only on the distance x , are defined through the time integration of a current density $j(x, t)$. The definition is such that it coincides with the experimental setting, thus avoiding some ambiguities and clarifying some controversies that have been discussed recently.

1 Introduction

Due to ongoing and planned experiments in kaon physics there has been a renewed interest in space–time oscillations of neutral meson systems like $K^0\text{--}\bar{K}^0$. In particular, in [1, 2, 3] an effort has been made to compute oscillation probabilities, like $P_{K^0\rightarrow K^0}(r)$ and $P_{K^0\rightarrow\bar{K}^0}(r)$, which depend now solely on the spatial distance r between the K^0 production point and the position where K^0 or \bar{K}^0 are detected. The expression $P(r)$ mimics then closely the usual experimental circumstances where *distances* rather than *times* are measured. This issue can be studied in *single* neutral kaon production in reactions like $\pi^-p \rightarrow \Lambda K^0$, $K^+n \rightarrow pK^0$, $p\bar{p} \rightarrow K^+\pi^-\bar{K}^0$, or in kaon pair production in reactions such as $e^+e^- \rightarrow K_S K_L$ at Φ factories like Daphne [4, 5, 6]. We will concentrate in the present paper on single kaon production — a problem which finds its analogy in recent discussions on neutrino oscillations [7, 8] — and point out the differences to a Φ -factory at an appropriate place.

The methods and arguments given in [1, 2, 3] for the *single* kaon oscillations (for the analogous neutrino case, see [7, 8]) to justify a particular derivation of the spatial probability $P(r)$ are rather different from each other and so are partially the results. In our opinion, the origin of the discrepancies (in both, the derivation and the results) can be traced back to the absence of a clean quantum mechanical definition of a probability which depends only on the distance r . If the question of what is the probability to detect either a K^0 or a \bar{K}^0 at a distance r from the kaon production point is a legitimate one, then a quantum mechanical definition should indeed exist. Sometimes the classical formula $r = vt$ is invoked to transform $P(t)$ into $P(r)$. However, strictly speaking, this procedure might not be appropriate in a quantum mechanical context due to the finite spread of the wave packets. Furthermore, in the neutral kaon system the existence of a different mass for each component K_L and K_S implies different mean velocities.

From the point of view of quantum mechanics a probability has to be defined in terms of the wave function $\Psi(x, t)$ which encodes all the information of the system. This probability should then be constructed according to our experimental requirements. A position dependent probability $P(r)$ for propagating states can only be obtained by integrating an appropriately defined quantity over the time variable. Following the above arguments one concludes that the quantity to be constructed in the first place in terms of $\Psi(x, t)$ should be a probability density in time (dependent on x and t) which corresponds to the current density. Indeed a physical detector at a distance r from

the production point measures the time integrated flux of probability flowing across its surface , i.e.,

$$P(r) = \int_A d\vec{A} \int dt \vec{j}(\vec{x}, t) \quad (1)$$

with

$$\vec{j}(\vec{x}, t) = \frac{dP}{dt dA} \vec{n}, \quad (2)$$

where A is the surface and \vec{n} is a unit vector pointing outwards.

The problem then is to find a definition for the current in the case of *mixed* states such as K^0 and \bar{K}^0 . Note that in quantum mechanics the current $\vec{j}(\vec{x}, t)$ is usually defined only for states of definite mass, nevertheless we will show that the corresponding generalization is possible in our case. In doing so we will restrict ourselves for simplicity to 1 + 1 dimensions. Moreover, since the controversial point in connection with $P_{K^0 \rightarrow K^0}(r)$ and $P_{K^0 \rightarrow \bar{K}^0}(r)$ refers to neutral kaon propagation in space–time and it has neither to do with relativistic effects nor with CP–violation in the neutral kaon system, we can work in *i)* a non relativistic limit for which the current take simple expressions, *ii)* a CP conserving theory, and *iii)* a situation with stable kaons, $\Gamma_S = \Gamma_L = 0$. The last two points are consistent with the Bell–Steinberger relation in the limiting case of $\langle K_S | K_L \rangle = 0$ [9]. Note that the non-relativistic limit refers to retaining the rest mass in the energy–momentum relation, while neglecting for the moment higher orders of v/c . The modified Schrödinger equation is then:

$$i \frac{\partial}{\partial t} \Psi_{S/L}(x, t) = \frac{-1}{2m_{S/L}} \frac{\partial^2}{\partial x^2} \Psi_{S/L}(x, t) + m_{S/L} \Psi_{S/L}(x, t), \quad (3)$$

which is the appropriate non-relativistic limit of either Klein-Gordon or Dirac equations. The relativistic counterparts including finite width effects ($\Gamma_{S/L} \neq 0$) will be easily inferred from our results.

In next section we recall some basic formulae of the neutral kaon states and of wave packets. In section 3 the current density is obtained and in section 4 we compute the oscillation probabilities. Some brief conclusions follow.

2 Wave packets for K^0 and \bar{K}^0

In the neutral kaon system one can distinguish two sets of states. The “strong–interaction” basis is given by the states $|K^0\rangle$ and $|\bar{K}^0\rangle$, with well defined strangeness but undefined masses, fulfilling the orthogonality condition $\langle K^0 | \bar{K}^0 \rangle = 0$. In contrast,

the “free-space” propagating states $|K_S\rangle$ and $|K_L\rangle$ have well defined masses, $m_{S/L}$, but are not strangeness eigenstates. This is the appropriate set to describe time evolution in free space given by $|K_{S/L}(t)\rangle = e^{-i\lambda_{S/L}t}|K_{S/L}\rangle$ with $\lambda_{S/L} \equiv m_{S/L} - i/2\Gamma_{S/L} \rightarrow m_{S/L}$ in our limit. The relation between both sets is given by

$$|K_{S/L}\rangle = p |K^0\rangle \pm q |\bar{K}^0\rangle \quad (4)$$

with $|p|^2 + |q|^2 = 1$ and $|p|^2 - |q|^2 \neq 0$ if CP is violated in the mixing.

A general space-time dependent state $|K_{S/L}(x, t)\rangle$ can be written as

$$|K_{S/L}(x, t)\rangle = \Psi_{S/L}(x, t) |K_{S/L}\rangle. \quad (5)$$

From Eqs. (4)-(5) one can now obtain the time evolution of an initially produced K^0 .

In the two dimensional space spanned by $|K^0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $|\bar{K}^0\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ one has

$$\begin{aligned} |K^0(x, t)\rangle &= \begin{pmatrix} \Psi_{K^0}(x, t) \\ \Psi_{\bar{K}^0}(x, t) \end{pmatrix} = \frac{1}{2} \begin{pmatrix} \Psi_S(x, t) + \Psi_L(x, t) \\ \frac{q}{p} (\Psi_S(x, t) - \Psi_L(x, t)) \end{pmatrix} \\ &\longrightarrow \frac{1}{2} \begin{pmatrix} \Psi_S(x, t) + \Psi_L(x, t) \\ \Psi_S(x, t) - \Psi_L(x, t) \end{pmatrix}, \end{aligned} \quad (6)$$

where the last term in (6) holds only in the CP conserving limit $p = q = 1/\sqrt{2}$. In case of an initially produced \bar{K}^0 one similarly has $|\bar{K}^0(x, t)\rangle \rightarrow 1/2 \begin{pmatrix} \Psi_S - \Psi_L \\ -\Psi_S - \Psi_L \end{pmatrix}$.

The wave functions $\Psi_{S/L}(x, t)$ are taken to be wave packets which are known to give a realistic description of the space-time evolution. We choose for simplicity a Gaussian form

$$\Psi_{S/L}(x, t) = \frac{\sqrt{a}}{(2\pi)^{3/4}} \int_{-\infty}^{\infty} dk e^{-\frac{a^2}{4}(k-k_{S/L})^2} e^{i(kx - \omega_{S/L}(k)t)}, \quad (7)$$

where according to (3)

$$E_{S/L} = \omega_{S/L}(k) = m_{S/L} + \frac{k^2}{2m_{S/L}}, \quad (8)$$

and $k_{S/L}$ are the mean values of the momenta of the short/long kaon states. Recall also that a is related to the width of the packet, $\Delta x = a/2$, and that the uncertainty relation gives $\Delta k \Delta x = 1/2$, i.e., $\Delta k = 1/a$ for both K_S and K_L . Upon integration over k an explicit formula for $\Psi_{S/L}(x, t)$ can be obtained

$$\Psi_{S/L}(x, t) = \left(\frac{2}{\pi a^2}\right)^{1/4} \frac{1}{(1 + \gamma_{S/L}^2 t^2)^{1/4}} \exp\left\{-\frac{(x - v_{S/L}t)^2}{a^2(1 + \gamma_{S/L}^2 t^2)}\right\} e^{i\varphi_{S/L}}, \quad (9)$$

with

$$\varphi_i = -\theta_i + k_i x - \omega_i(k_i)t + \frac{(x - v_i t)^2 \gamma_i t}{a^2(1 + \gamma_i^2 t^2)}, \quad i = S, L \quad (10)$$

$\omega(k)$ defined in Eq. (8), and

$$\gamma_i \equiv \frac{2}{m_i a^2}, \quad v_i \equiv \frac{k_i}{m_i}, \quad \tan(2\theta_i) \equiv \gamma_i t. \quad (11)$$

Eqs. (9)-(11) represent the usual Gaussian wave packet displaying a well-known spread in time for definite mass eigenstates K_S or K_L .

Note that in our single K^0 production processes, energy-momentum conservation requires that the two $K_{S/L}$ components of the K^0 wave packet should have slightly different mean energies $E_{S/L}$ and momenta $k_{S/L}$ due to the mass difference $\delta m = m_L - m_S > 0$. In this sense we fully agree with the main point raised by Srivastava et al.[1], also advocated in [7], concerning the presence of two different energies and momenta. Both mass eigenstate components have to be produced in the coherent superposition $1/2[\Psi_S(x, t = 0) + \Psi_L(x, t = 0)]$ corresponding to an initial K^0 . This coherent production of overlapping wave packets of K_S and K_L also requires

$$\Delta k \gg \delta k = k_L - k_S \quad (12)$$

In this respect the situation is very similar to that considered in neutrino oscillations whose wave packet treatment has been discussed in [8]. It is also interesting to observe that in our treatment single K^0 production somehow mimics a two-slit experiment in momentum space, δm playing the role of the “separation” between the K_S and K_L “slits”.

There is a subtle point worth mentioning here referring to the Bargmann superselection rule [10],[11]. This rule states that in non-relativistic quantum mechanics Galilean invariance forbids a coherent superposition of states with different masses. We do not consider this a serious problem here. Beyond the non-relativistic limit such restrictions do not hold as is clearly experienced in the neutral kaon system. Note that Eq. (3) is a limit of a relativistic equation and that we retain the different rest masses. A relativistic current has then to reduce to its non-relativistic counterpart which is based on Eq. (3). Notice also that Galilean invariance cannot be strictly maintained if, for instance, we would use time dependent potentials in the Schrödinger equation (like space-time dependent electromagnetic potentials) [11].

3 Currents for K^0 and \bar{K}^0

The current density for the $K_{S/L}$ mass eigenstates whose wave functions obey Eq. (3) is defined as

$$j_{S/L}(x, t) = \frac{1}{m_{S/L}} \text{Im} \left(\Psi_{S/L}^*(x, t) \frac{\partial}{\partial x} \Psi_{S/L}(x, t) \right), \quad (13)$$

and satisfies the usual continuity equation $\frac{\partial}{\partial t} \rho_{S/L} + \frac{\partial}{\partial x} j_{S/L} = 0$, with $\rho_{S/L}(x, t) = \Psi_{S/L}^* \Psi_{S/L}$. For the K^0 and \bar{K}^0 states we expect that the corresponding currents are not conserved as these states mix in time due to the mass difference δm in their K_S and K_L components. Thus, the modified continuity equation should read

$$\frac{\partial}{\partial t} \rho_{K^0, \bar{K}^0} + \frac{\partial}{\partial x} j_{K^0, \bar{K}^0} = d_{K^0, \bar{K}^0}, \quad (14)$$

where $\rho_{K^0} = \Psi_{K^0}^* \Psi_{K^0}$, $\rho_{\bar{K}^0} = \Psi_{\bar{K}^0}^* \Psi_{\bar{K}^0}$ and d_{K^0, \bar{K}^0} is a kind of ‘‘diffusion term’’ to be determined.

Let us list some general requirements for the current density j_{K^0, \bar{K}^0} : *i*) It should contain only ‘‘velocity terms’’, i.e., terms of the form $1/m_i \left(\Psi_j^* \frac{\partial}{\partial x} \Psi_k \right)$, with $i, j, k = S, L$, whereas one has no similar condition for d_{K^0, \bar{K}^0} . *ii*) As the mass difference goes to zero, $\delta m = m_L - m_S \rightarrow 0$, we should have $d_{K^0, \bar{K}^0} = 0$, independently of the form of the wave packets $\Psi_{S/L}$. In this limit obviously one has $j_{K^0} = j_{\bar{K}^0} = j_S = j_L$. *iii*) In the zero decay width limit the total number of neutral kaons must be a conserved quantity, i.e.,

$$\frac{\partial}{\partial t} (\rho_{K^0} + \rho_{\bar{K}^0}) + \frac{\partial}{\partial x} (j_{K^0} + j_{\bar{K}^0}) = 0 \quad (15)$$

Hence, $d_{K^0} = -d_{\bar{K}^0}$ should hold.

We can now derive the expression for the current density j_{K^0, \bar{K}^0} . First, we take the time derivative of the densities $\frac{\partial}{\partial t} \left(\Psi_{K^0, \bar{K}^0}^* \Psi_{K^0, \bar{K}^0} \right)$, where the wave functions are given in Eq. (6). We then use the Schrödinger equation (3) and separate the ‘‘velocity terms’’ according to point *i*) above. The linear combination of such terms will be called j_{K^0, \bar{K}^0} . All other terms that are not of velocity type are included in d_{K^0, \bar{K}^0} . We thus obtain

$$j_{K^0, \bar{K}^0} = \frac{1}{4} (j_S + j_L) + \frac{1}{2} j_{K^0, \bar{K}^0}^{int}, \quad (16)$$

where $j_{S/L}$ are the usual current densities (13) corresponding to the $K_{S/L}$ states, and j_{K^0, \bar{K}^0}^{int} contains the interference terms

$$j_{K^0}^{int} = \frac{1}{2} \left[\frac{1}{m_S} \text{Im} \left(\Psi_S^* \frac{\partial}{\partial x} \Psi_L \right) + \frac{1}{m_L} \text{Im} \left(\Psi_L^* \frac{\partial}{\partial x} \Psi_S \right) \right] = -j_{\bar{K}^0}^{int}. \quad (17)$$

For the ‘‘diffusion terms’’ we get

$$d_{K^0} = \frac{1}{4} \left(\frac{1}{m_S} - \frac{1}{m_L} \right) \text{Im} \left(\frac{\partial}{\partial x} \Psi_L^* \frac{\partial}{\partial x} \Psi_S \right) + \frac{1}{2} (m_L - m_S) \text{Im}(\Psi_S^* \Psi_L) = -d_{\bar{K}^0} \quad (18)$$

It is now easy to check that the three aforementioned requirements on the current densities are indeed fulfilled. In particular, $d_{K^0, \bar{K}^0} \rightarrow 0$ as $\delta m \rightarrow 0$, and Eq. (15) is also trivially satisfied due to the antisymmetry properties of j_{K^0, \bar{K}^0}^{int} and d_{K^0, \bar{K}^0} in (17) and (18), respectively.

The current densities (16) can also be written in terms of the wave functions Ψ_{K^0} and $\Psi_{\bar{K}^0}$

$$\begin{aligned} j_{K^0} &= \frac{1}{2} \frac{1}{m^2 - \frac{1}{4} \delta m^2} \left(2m \text{Im}(\Psi_{K^0}^* \frac{\partial}{\partial x} \Psi_{K^0}) - \delta m \text{Im}(\Psi_{K^0}^* \frac{\partial}{\partial x} \Psi_{\bar{K}^0}) \right) \\ j_{\bar{K}^0} &= \frac{1}{2} \frac{1}{m^2 - \frac{1}{4} \delta m^2} \left(2m \text{Im}(\Psi_{\bar{K}^0}^* \frac{\partial}{\partial x} \Psi_{\bar{K}^0}) - \delta m \text{Im}(\Psi_{\bar{K}^0}^* \frac{\partial}{\partial x} \Psi_{K^0}) \right), \end{aligned} \quad (19)$$

where m is the mean value of the mass, $m = (m_S + m_L)/2$. Note that in contrast to the densities ρ_{K^0, \bar{K}^0} each of the two currents j_{K^0, \bar{K}^0} contains both wave functions Ψ_{K^0, \bar{K}^0} . This means that the constructed currents are unique in the sense that all ‘‘velocity terms’’ are contained in j_{K^0, \bar{K}^0} and not in d_{K^0, \bar{K}^0} .

4 Distance Dependent Probabilities $P_{K^0 \rightarrow K^0, \bar{K}^0}(r)$

Given the currents j_{K^0, \bar{K}^0} in (16) the x -dependent probabilities

$$\begin{aligned} P_{K^0 \rightarrow K^0}(x) &= \int_0^\infty dt j_{K^0}(x, t), \\ P_{K^0 \rightarrow \bar{K}^0}(x) &= \int_0^\infty dt j_{\bar{K}^0}(x, t) \end{aligned} \quad (20)$$

can now be calculated in analogy to Eq. (1) (recall that for simplicity we are working in 1+1 dimensions). Before evaluating the integrals of Eq. (20) it is convenient to elaborate on the kinematics involved. Note that we will use consistently the non-relativistic limit, Eq. (8), and restrict ourselves to the case of single kaon production in ‘‘two-to-two’’ reactions like $\pi^- p \rightarrow \Lambda K^0$, $K^+ n \rightarrow K^0 p$, etc. Working in the CM-system with the total energy $\sqrt{s} = m + M + Q$, where M is the final baryon mass, one has

$$\begin{aligned} \delta k &\equiv k_L - k_S, & k &\equiv \frac{1}{2}(k_S + k_L) \\ \delta E &\equiv E_L - E_S \simeq \frac{k \delta k}{m} + \delta m \left(1 - \frac{k^2}{2m^2} \right), & E &\equiv \frac{1}{2}(E_L + E_S) \\ \delta v &\equiv v_L - v_S \simeq \frac{\delta k}{m} - \frac{k \delta m}{m^2}, & v &\equiv \frac{1}{2}(v_S + v_L) \end{aligned} \quad (21)$$

The Q -value of the reaction can be chosen in the very wide range

$$m \gg Q \gg \delta m \quad (22)$$

imposed by our non-relativistic treatment and the need to produce a K^0 in a coherent $K_{S/L}$ superposition. Expanding the relevant variables one finds

$$\begin{aligned} k^2 &\simeq \frac{2Mm}{M+m}Q, & k\delta k &\simeq -\delta m \left[\frac{Mm}{M+m} - \left(\frac{M}{M+m} \right)^2 Q \right] \\ E &\simeq m + \frac{m}{M+m}Q, & \delta E &\simeq \delta m \left[1 - \frac{M}{M+m} - \frac{M}{(M+m)^2}Q \right] \end{aligned} \quad (23)$$

and, more importantly,

$$\frac{\delta v}{v} \simeq -\frac{\delta m}{m} \left[\frac{M+2m}{M+m} + \frac{m}{Q} \right] \sim O\left(\frac{\delta m}{Q}\right) \sim O\left(\frac{\delta m}{mv^2}\right). \quad (24)$$

We stress that in spite of working in the non-relativistic limit v cannot be arbitrarily small as to imply a Q -value (or a kaon kinetic energy) of the order of the mass difference δm . In such unrealistic situation one would be favouring the production of the K_S component over the slightly heavier K_L one. Taking into account that $\delta m/m \sim 10^{-15}$ Eqs. (22) and (24) imply $v \gg 10^{-7}$. Notice, however, that even velocities of the order of 10^{-3} would require an unrealistic high degree of fine tuning of the CMS energy \sqrt{s} ($Q \sim keV$).

At this stage it is instructive to point out some differences to a Φ -factory like Daphne. In the latter case one usually studies correlation probabilities of K_S and K_L decays depending on two space-time points [4]. The kinematics dictates in this case $\delta v/v = -(\delta m/m)$ rather than (24).

In terms of the Gaussian wave packets (9), the currents $j_{S/L}$ appearing in (16) can be cast into a simple form

$$j_i = \sqrt{\frac{2}{\pi}} \frac{v_i + x\gamma_i^2 t}{a(1 + \gamma_i^2 t^2)^{3/2}} \exp \left[-2 \frac{(x - v_i t)^2}{a^2(1 + \gamma_i^2 t^2)} \right], \quad i = L, S \quad (25)$$

Introducing the new variables

$$az_i \equiv \frac{x - v_i t}{(1 + \gamma_i^2 t^2)^{1/2}} \quad (26)$$

we can compute to a very good approximation

$$\int_0^\infty dt j_i(x, t) = \sqrt{\frac{2}{\pi}} \int_{-\frac{1}{2}ak_i \ll -1}^{\frac{x}{a} \gg 1} dz_i e^{-2z_i^2} \simeq 1 \quad (27)$$

Indeed in the upper limit $x \gg a$ since the position x of the detector is considered to be far from the K^0 production point, far in comparison to the width of the packet $a/2$. Otherwise, one would be measuring the inner wave packet effects rather than its space-time propagation which is our interest here. In the lower limit of the integral the momentum k is much larger than its dispersion $\Delta k = 1/a$ for any realistic velocity $v \sim 10^{-3} - 10^{-1}$.

Neglecting in (17) polynomial terms of order $\delta m/m$ and expanding further the remaining expression in leading order of $\delta m/m$, we can write

$$j_{K^0}^{int} \simeq \sqrt{\frac{2}{\pi}} \frac{v + x\gamma^2 t}{a(1 + \gamma^2 t^2)^{3/2}} \exp \left\{ -2 \frac{(x - vt)^2}{a^2(1 + \gamma^2 t^2)} \right\} \exp \left\{ -\frac{t^2 \delta v^2}{2a^2(1 + \gamma^2 t^2)} \right\} \\ \times \left[\cos \theta(x, t) + \sin \theta(x, t) \frac{\delta v \gamma t}{2(v + x\gamma^2 t)} \right] \quad (28)$$

where

$$\theta(x, t) \equiv -\delta E t + \delta k x + \frac{2t\delta v(vt - x)}{a^2} \frac{\gamma t}{(1 + \gamma^2 t^2)} \quad (29)$$

is the crucial phase and the mean $\gamma \equiv 2/ma^2$, $v \equiv k/m$ have been defined in analogy to Eq. (11). Observing the similarities between the Eqs. (25) and (28), we introduce a variable as in (26), $az = \frac{x-vt}{(1+\gamma^2 t^2)^{1/2}}$, whose inverse in the region of interest $0 \leq t \leq \infty$ is

$$t(z) = \frac{vx - az\sqrt{v^2 + \gamma^2(x^2 - a^2 z^2)}}{(v^2 - \gamma^2 a^2 z^2)} \quad (30)$$

for $v > 0$ and $x \geq 0$. The integrand has now a form of $e^{-2zz^2} f(z)$ which can be expanded in a Taylor series around $z = 0$. Note that $t(0) = x/v$ and that the coefficient of $\sin \theta$ in (28) is bounded by $1/2(\delta v/v) \frac{\gamma t(0)}{1+\gamma^2 t^2(0)} \leq 1/4(\delta v/v) \sim O(\delta m/Q)$ and hence small by virtue of Eq. (22). In leading order we then obtain

$$\int_0^\infty dt j_{K^0}^{int}(x, t) \simeq \sqrt{\frac{2}{\pi}} \cos \theta(x, t(0)) \exp \left\{ -\frac{t^2(0) \delta v^2}{2a^2(1 + \gamma^2 t^2(0))v^2} \right\} \int_{-\infty}^\infty dz e^{-2zz^2} \\ \simeq \cos \theta(x, x/v) \quad (31)$$

since the additional Gaussian factor has a negligible exponent, $1/2(\delta v^2/\gamma^2) \frac{\gamma^2 x^2/v^2}{1+\gamma^2 x^2/v^2} \leq 1/2(\delta v^2/\gamma^2) \sim 1/2(\delta k/\Delta k) \ll 1$ due to (12).

Taking into account the kinematics given in Eqs. (22)-(24) the phase turn out to be

$$\theta(x, x/v) = -\delta m \left(1 - \frac{1}{2}v^2 \right) \frac{x}{v}. \quad (32)$$

Putting now everything together we obtain

$$P_{K^0 \rightarrow K^0, \bar{K}^0}(x) = \frac{1}{2} \left(1 \pm \cos \left[\delta m \frac{x}{v} \left(1 - \frac{1}{2} v^2 \right) \right] \right) \quad (33)$$

which is our final result in non-relativistic approximation and for stable kaons. Reintroducing the non-vanishing $\Gamma_{S/L}$ decay widths and higher orders in v/c , Eq. (33) can be cast into its final form as

$$P_{K^0 \rightarrow K^0, \bar{K}^0}(\tau) = \frac{1}{4} \left(e^{-\Gamma_S \tau} + e^{-\Gamma_L \tau} \pm e^{-\frac{1}{2}(\Gamma_S + \Gamma_L)} \cos[\delta m \tau] \right), \quad (34)$$

which corresponds to the usual expression in terms of the proper time τ .

5 Conclusions

We have argued that in order to avoid theoretical ambiguities it is essential to use a proper definition of space-dependent probability $P(r)$: the time and surface integrated current density, $j(\vec{x}, t)$. We have shown that it is possible to construct such a current density for coherent superposition of different mass eigenstates. Further, we have used throughout wave packets which describe the physical space-time evolution and take into account the necessary overlapping of wave functions to obtain interference effects. Wave packets have also been used in [4] to compute space-dependent decay correlation probabilities at a Φ -factory. Their results are quite in line with those obtained in the present paper.

In our treatment the two components K_S and K_L of the neutral kaon evolve in space-time with different momenta and energies as dictated by energy-momentum conservation in a situation that can be visualized as a two slit experiment in momentum space. We have then obtained the oscillatory term $\cos[\delta m \tau]$, which corresponds to the usual expression but without the recourse to any classical formula to transform time probabilities into space probabilities.

Acknowledgments

This paper has been partially supported by CICYT contracts AEN94-936 and AEN95-815 and EURODAPHNE, HCHP, EEC contract no. CHRX-CT 920026. R.M.T. acknowledges a EC post-doctoral fellowship ERBC HBCICT-941777. M.N. thanks the Spanish Ministerio de Educacion y Ciencias for a post-doctoral fellowship.

References

- [1] Y.N. Srivastava, A. Widom and E. Sassaroli, Phys. Lett. **344B** (1995) 436 .
- [2] H. J. Lipkin, Phys. Lett. **348B** (1995) 604 .
- [3] J. Lowe et al., “No Λ Oscillations”, hep-ph 9605234.
- [4] G. D’Ambrosio, G. Isidori and A. Pugliese in “The Second Dafne Physics Handbook”, Vol. I & II, Ed. by L. Maiani, G. Panzeri and N. Paver, Frascati (Italy) 1995.
- [5] Y.N. Srivastava, A. Widom and E. Sassaroli, Zeit. f. Phys., **C 66** (1995) 601
- [6] B. Kayser and L. Stodolsky, Phys. Lett. **359B** (1995) 343 .
- [7] T. Goldman, “Source Dependence of Neutrino Oscillations”, Los Alamos preprint LA-UR-96-1349, hep-ph 9604357.
- [8] Chung Wook Kim and Aihud Pevsner, ”Neutrinos in Physics and Astrophysics”, Harwood Academic Publishers 1993, Chapter 9 and references therein.
- [9] J. S. Bell and J. Steinberger in Proc. of the Oxford Conference on Elem. Part. Phys. 1965, p.195; P.K. Kabir, “The CP Puzzle”, Academic Press, London 1968.
- [10] V. Bargmann, Annals of Math. **59** (1954) 1.
- [11] A. Galindo and P. Pascual, “Quantum Mechanics”, Springer Verlag 1990.