Standard Model Prediction of the Muon Anomalous Magnetic Moment [1]

Joaquim Prades

CAFPE and Departamento de Física Teórica y del Cosmos, Universidad de Granada, Campus de Fuente Nueva, E-18002 Granada, Spain.

I review the present Standard Model prediction of the muon anomalous magnetic moment. The discrepancy with its experimental determination is $(25.5 \pm 8.0) \times 10^{-10}$, i.e., 3.2 standard deviations.

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1. Introduction

The general vertex Γ_μ between a fermion f and an external electromagnetic (EM) field $A^\mu(q = p - p')$ can be written as

$$\overline{u}(p')\Gamma_\mu u(p) = \overline{u}(p')\left[\gamma_\mu\, F_1(q^2) + i\,\frac{\sigma_{\mu\nu}q^\nu}{2m_f}\, F(q^2) + \cdots\right] \qquad (1)$$

The magnetic dipole moment for a charged fermion ($f = e, \mu, \tau, \cdots$) is proportional to the spin $\vec{s}$ through the gyromagnetic factor $g_f \equiv 2(F_1(0) + F_2(0))$

$$\vec{\mu} = g_f\frac{e}{2m_f}\,\vec{s}. \qquad (2)$$

The Dirac vertex predicts $F_1(0) = 1$ and $F_2(0) = 0$ at tree-level, quantum loops modify this prediction to $F_2(0) \equiv a_f \neq 0$ while $F_1(0)$ is a conserved charge. The quantity $a_f = (g_f - 2)/2$ is commonly called the fermion anomaly.

The muon anomalous magnetic moment a_μ has been measured by the E821 experiment (Muon g-2 Collaboration) at BNL with an impressive accuracy of 0.72 ppm [1] yielding the present world average

$$a_\mu^{\text{exp}} = (11, 659, 208.9 \pm 6.3) \times 10^{-10} \qquad (3)$$

with an accuracy of 0.54 ppm. New experiments [2,3] are being designed to measure a_μ with an accuracy better than 0.14 ppm. Here, I'm interested in what is the present status of the Standard Model (SM) prediction for this very precise measurement. Is there room for new physics in a_μ? Recent reviews can be found in [4–9].

First, I will shortly recall recent advances in the electron anomaly a_e which is a necessary ingredient to predict the muon anomaly a_μ. I'll discuss then briefly the main contribution to a_e that is QED, and the much smaller hadronic and weak contributions. Secondly, I'll discuss briefly the main contribution to a_μ that is QED and the smaller weak contribution. Then I discuss the also smaller but dominant in the uncertainty hadronic contribution to a_μ. Finally, I'll give the conclusions and prospects for improving a_μ both theoretic and experimentally.

2. The electron anomaly a_e

The Harvard group [10] has very recently made a very precise measurement of the electron anomaly

$$a_e^{\text{exp}} = (11, 596, 521, 807.3 \pm 2.8) \times 10^{-13} \qquad (4)$$

improving the 1987 measurement of the University of Washington group [11]

$$a_e^{\text{exp}} = (11{,}596{,}521{,}883 \pm 42) \times 10^{-13} \tag{5}$$

by more than a factor 15.

The new a_e measurement provides the most accurate value for the fine structure constant [1]

$$\alpha^{-1} = 137.035\,999\,084(33)(39) = 137.035\,999\,084(51) \tag{6}$$

which is one order of magnitude more precise than the best previous determination[2]. The best non-a_e based determinations of α come from atomic physics, and in particular from the precise measurement of Cs [16] and Rb [17] atomic masses,

$$\text{Cs atom } (2002): \alpha^{-1} = 137.036\,000\,000(1\,100)$$
$$\text{Rb atom } (2008): \alpha^{-1} = 137.035\,999\,450(620)\,. \tag{7}$$

It is expected that the Rb atomic mass measurement can reach the level of accuracy of a_e determinations soon. At present, a_e together from α from Cs and Rb atomic mass determinations checks QED at 3-loops ! If both determinations were at the same level of accuracy, it would check QED at 4-loops and a possible substructure of the electron.

3. Standard Model Contributions to a_e

The dominant SM contribution comes from QED

$$a_e^{\text{QED}} = \sum_{n=1} C_{2n} \left(\frac{\alpha}{\pi}\right)^n \tag{8}$$

with $C_2 = 1/2$ calculated by Schwinger [18] 61 years ago; one can find the values of the rest of the coefficients in different reviews [4–9]. The coefficients C_4 [19] and C_6 [20] are known analytically including lepton mass corrections. C_8 is only numerically known and being continuously improved since early 1980s [21]. Actually, it was corrected recently in 2007 [13]. For C_{10}, one uses 0.0 ± 4.6 as an estimate where the error is based on the C_{2n} values growing. Its full numerical calculation is in progress, it

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[1] For a very recent discussion of the extraction of α from a_e and other methods, see [12].

[2] Notice that due to the 2007 corrected value for the four-loop order QED coefficient [13], the value quoted in both PDG 2008 [14] and CODATA 2006 [15] $\alpha^{-1} = 137.035\,999\,679(94)$ is not correct.

involves 12,672 diagrams and some partial results are already available [22]. Recently, specific classes of the QED eighth and tenth orders have been calculated analytically using Mellin-Barnes transform techniques [23] and agree with the numerical results in [13, 21, 22].

There are other much smaller contributions to a_e but that at the level of present precision start to be needed. The largest is the leading order hadronic vacuum polarization contribution (see Fig 1) [24]

$$a_e^{\text{LO Hadronic}} = (18.75 \pm 0.18) \times 10^{-13} \,. \tag{9}$$

while higher order hadronic vacuum polarization contribution is [25]

$$a_e^{\text{HO Hadronic}} = -(2.25 \pm 0.05) \times 10^{-13} \tag{10}$$

and the hadronic light-by-light contribution (see Fig 2) is [26]

$$a_e^{\text{Hadronic LbL}} = (0.35 \pm 0.10) \times 10^{-13} \,. \tag{11}$$

The total hadronic contribution to a_e is

$$a_e^{\text{Hadronic}} = (16.85 \pm 0.21) \times 10^{-13} \,. \tag{12}$$

Notice that this is the first time that a_e^{exp} is sensitive to the hadronic contribution. The electroweak contribution is much smaller [5]

$$a_e^{\text{EW}} = (0.385 \pm 0.004) \times 10^{-13} \,. \tag{13}$$

4. Standard Model Contributions to a_μ

4.1. QED

A precise value of the fine structure constant is very important to the determination of the muon anomaly since QED is again the dominant SM contribution. One can write the QED contribution as

$$a_\mu^{\text{QED}} = \sum_{n=1} C_{2n} \left(\frac{\alpha}{\pi} \right)^n \tag{14}$$

and the same comments and authors can be quoted for the C_{2n} coefficients that can be found in the reviews [4–9]. For C_{10}, one uses 663 ± 20 which is also estimate and its calculation is progress in parallel to C_{10} for a_e [22]. Using those coefficients and α from the latest a_e in (6), one gets

$$a_\mu^{\text{QED}} = (11, 658, 471.810 \pm 0.015) \times 10^{-10} \tag{15}$$

where the largest uncertainty comes from C_{10}. Given that precision, the difference between the experimental value (3) and its QED prediction

$$\Delta_{\text{Not QED}}^{\text{exp}} = (736.2 \pm 6.3) \times 10^{-10} \tag{16}$$

can be considered as a "measurement". In fact, a_μ has been sensitive to the hadronic contribution since 1975 and to the electroweak ones since 2001.
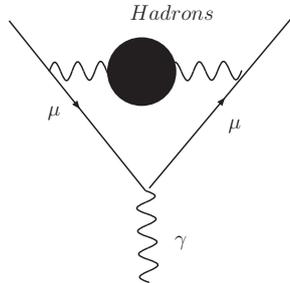
Fig. 1. Lowest order hadronic vacuum polarization contribution to muon $g - 2$.

4.2. Electroweak

The one loop electroweak contribution is fully known since 1972 [28]

$$
\begin{aligned}
a_\mu^{\text{EW one-loop}} &= \frac{5G_\mu m^2}{24\sqrt{2}\pi^2}\left[1 + \frac{1}{5}\left(1 - 4\sin^2(\theta_W)\right)^2 + \mathcal{O}\left(m/M_{Z,W,H}\right)\right] \\
&= (19.5 \pm 0.2) \times 10^{-10}\,.
\end{aligned}
\tag{17}
$$

The full two-loops electroweak contribution has been finished much more recently (in 2004) though its potential importance was already pointed out in 1992 [29]. The full result can be found in [30].

The final result for the two-loop electroweak contribution [27, 30] is $-(4.1 \pm 0.1) \times 10^{-10}$ and the total electroweak contribution

$$
a_\mu^{\text{EW}} = [(19.5 \pm 0.2) - (4.1 \pm 0.1)] \times 10^{-10} = (15.4 \pm 0.2) \times 10^{-10}\,.
\tag{18}
$$

Again, given the experimental precision, the difference between the precise experimental value in (3) and its QED plus EW prediction

$$
\Delta^{\text{exp}}_{\text{Not QED+EW}} = (720.8 \pm 6.3) \times 10^{-10}
\tag{19}
$$

can be considered as a "measurement". This also tells us that we need to know the hadronic contribution to a_μ with an accuracy below the 1 % level!

4.3. Hadronic Contributions

4.3.1. Vacuum Polarization Contribution

This contribution is depicted in Fig. 1. Its leading-order (LO) contribution is order α^2 and can be written in terms of the one-photon $e^+e^- \to \gamma^* \to$ hadrons cross-section, $\sigma^{(0)}(s)$ [31].

$$
a_\mu^{\text{LO Hadronic}} = \frac{1}{4\pi^3}\int_{4m_\pi^2}^\infty \mathrm{d}s\, K(s)\, \sigma^{(0)}(s)
\tag{20}
$$

with

$$K(s) = \int_0^1 dx \, \frac{x^2(1-x)}{x^2 + \frac{s}{m^2}(1-x)} \, . \tag{21}$$

The kernel $K(s)$ behaves as m^2/s making the low-energy region (ρ-region) dominate in the integrand. In fact, the $\pi^+\pi^-$ channel below 1 GeV gives 72 % of the total of $a_\mu^{\text{LO Hadronic}}$.

The one-photon cross-section $\sigma^{(0)}(s)$ is (almost) experimentally obtained. One has to correct the experimental cross-section to include final state photons and exclude the running of $\alpha(s)$ not to make double counting with higher orders. It is also welcome to eliminate systematics to have the cross-section $\sigma^{(0)}$ normalized to the $e^+e^- \to \mu^+\mu^-$ cross-section measured by the same experiment. In the case of the initial state radiation (ISR) method, this is done by BaBar which measures $e^+e^- \to \pi^+\pi^-\gamma(\gamma)$ normalized to $e^+e^- \to \mu^+\mu^-\gamma(\gamma)$ cross-section measured by the same experiment and is planed to be done in the forthcoming KLOE-2 measurement.

For the low energy contribution to $\sigma^{(0)}$, we have very precise e^+e^- data from several low-energy experiments, namely, from CMD-2 and SND detectors at VEPP-2M (Novosibirsk), KLOE at DAΦNE (Frascati) and BaBar at PEP-II (SLAC). The last results from CMD-2 [32, 33] and SND [34] are in nice agreement. Final KLOE data [35] are at the same level of accuracy as CMD-2 and SND data and there is an overall agreement at energies between 0.630 and 0.958 GeV with CMD-2 and SND though KLOE data lies somewhat lower. Very recently, BaBar using the ISR method has also released e^+e^- results for the dominant $\pi^+\pi^-$ at energies between 0.3 GeV and 3 GeV [36]. Belle at KEK (Tsukuba) which has larger statistics, will do so soon.

At high energy, higher than 13 GeV, and in between $\overline{c}c$ and $\overline{b}b$ thresholds, perturbative QCD is used to calculate $\sigma^{(0)}(s)$.

Combining all precision e^+e^- data, including the latest BaBar data [36], the authors [37] quote

$$a_\mu^{\text{LO Hadronic}} = (695.5 \pm 4.0_{\text{exp}} \pm 0.7_{\text{QCD}}) \times 10^{-10} = (695.5 \pm 4.1) \times 10^{-10} \, . \tag{22}$$

A new analysis of isospin corrections to $\tau^+ \to \pi^+\pi^0\nu$ data when compared to the the CVC $e^+e^- \to \pi^+\pi^-$ data has been also recently released [38]. The discrepancy between τ data and the combined e^+e^- data for the dominant $\pi^+\pi^-$ contribution to $\sigma^{(0)}$ of the measurements quoted above is reduced to 1.5 σ from the previous 4.5 σ discrepancy [37, 38].

There are also hadronic vacuum polarization contributions at higher order included in Fig 1. The most recent evaluation including α^3 corrections
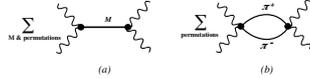
Fig. 2. Hadronic light-by-light contribution to a_μ is in Fig. 2.

is in [39] using also $e^+ e^-$ data,

$$a_\mu^{\text{HO Hadronic}} = -(9.79 \pm 0.08_{\text{exp}} \pm 0.03_{\text{rad}}) \times 10^{-10} = -(9.79 \pm 0.09) \times 10^{-10}\,.$$

Where the uncertainty also takes into account non-included radiative corrections [39].

4.3.2. Light-by-Light Contribution

One of the six possible momenta routing to the hadronic light-by-light contribution to a_μ is in Fig. 2. Recent work on a^{HLbL} can be found in [26, 40–47]. For reviews on the status, prospects of a_μ^{HLbL} and references see [4, 5, 48]. In particular, the discussion in [26] lead the authors to give the following value

$$a_\mu^{\text{Hadronic LbL}} = (10.5 \pm 2.6) \times 10^{-10} \tag{23}$$

for their present best estimate.

5. Result for a_μ^{SM}

Adding all pieces discussed above contributing to a_μ, one gets

$$\begin{aligned}
10^{10} a_\mu^{\text{SM}} &= \overbrace{(11,\,658,\,471.810 \pm 0.015)}^{\text{QED}} + \overbrace{(15.4 \pm 0.2)}^{\text{EW}} \\
&+ \underbrace{(695.5 \pm 4.1)}_{\text{LO Had}} \underbrace{-(9.79 \pm 0.09)}_{\text{HO Had}} + \underbrace{(10.5 \pm 2.6)}_{\text{Had LbL}} \\
&= \underbrace{(11,\,658,\,487.2 \pm 0.2)}_{\text{QED+EW}} + \underbrace{(696.2 \pm 4.9)}_{\text{Hadronic}} \\
&= 11,\,659,\,183.4 \pm 4.9\,.
\end{aligned} \tag{24}$$

Using this result and the experimental value of a_μ in (3), one gets

$$a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = (25.5 \pm 8.0) \times 10^{-10}\,. \tag{25}$$

6. Conclusions and Prospects

Combining all recent precise e^+e^- data obtained with different methods (energy scanning and ISR) in different experiments to calculate $\sigma^{(0)}$ [37] and using the new evaluation of the hadronic light-by-light contribution in [26, 48] one gets more than 3 σ of discrepancy between the SM value for a_μ (24) and its experimental determination (3). This discrepancy has been slowly growing due to impressive theory and experiment recent achievements. In fact, both theory and experiment uncertainties have been reduced by more than a factor two in the last eight years [3].

There are planned new e^+e^- experiments at Novosibirsk (VEPP-2000) and Frascati (DAΦNE-2) which will cross-check present results and reduce the present uncertainty in $a_\mu^{\mathrm{LO\,Had}}$ which dominates the final SM uncertainty now. There are also new τ data at B-factories and a new τ-charm factory at Beijing which will cross-check the τ result. It will also help to understand better the isospin violation corrections in order to use the forthcoming precise τ data. In addition, a new full calculation of $a_\mu^{\mathrm{Had\,LbL}}$ is desirable and possible. The goal in this case is to reduce its present uncertainty to the level of $(1.5 \sim 2.0) \times 10^{-10}$. With all these eventual theory improvements the uncertainty of the SM prediction can be reduced soon enough for the new experiments being designed. As said before, these experiments are being designed to measure a_μ with an accuracy better than 0.14 ppm in parallel to the expected theory advances. The proposed experiment at Fermilab is designed to reduce the a_μ uncertainty to the level of 1.6×10^{-10} [2] and the one at J-PARC to somewhere between 1.2×10^{-10} and 0.6×10^{-10} [3].

With all the theory activity detailed above and the tantalizing more than 3 σ discrepancy in a_μ (25) between the SM prediction and its experimental determination, I believe that those new $g - 2$ experiments are very timely, necessarily complementary to direct searches like LHC and ILC and should be done as soon as possible. Its very likely that they give the first new physics discovery !

Acknowledgments

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[3] In 2001 , this discrepancy was $a_\mu^{\mathrm{exp}} - a_\mu^{\mathrm{SM}} = (23.1 \pm 16.9) \times 10^{-10}$. [49]

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