SUPPORTING STUDENTS WITH LEARNING DISABILITIES TO EXPLORE LINEAR RELATIONSHIPS USING ONLINE LEARNING OBJECTS

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The study of linear relationships is foundational for mathematics teaching and learning. However, students' abilities to connect different representations of linear relationships have proven to be challenging. In response, a computer-based instructional sequence was designed to support students' understanding of the connections among representations. In this paper we report on the affordances of this dynamic mode of representation specifically for students with learning disabilities. We outline four results identified by teachers as they implemented the online lessons.

Keywords: Algebraic understanding; Computer-based instruction; Learning disabilities and mathematics

Apoyo a estudiantes con problemas de aprendizaje para explorar relaciones lineales mediante el uso de objetos de aprendizaje en línea

El estudio de las relaciones lineales es fundamental en la enseñanza y el aprendizaje de las matemáticas. Sin embargo, las habilidades de los estudiantes para conectar distintas representaciones de las relaciones lineales han demostrado ser un reto. Ante esto, hemos diseñado una secuencia de enseñanza basada en ordenadores para fomentar en los estudiantes la comprensión de las conexiones entre estas representaciones. Presentamos las potencialidades de este tipo de representación dinámica para estudiantes con dificultades de aprendizaje, destacando cuatro resultados identificados por maestros al implementar las lecciones en línea.

Palabras clave: Comprensión algebraica; Dificultades de aprendizaje y matemáticas; Instrucción basada en ordenadores

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A current focus of mathematics instruction centers on the push for algebra reform and the resulting recommendation from the National Council of Teachers of Mathematics (NCTM, 2000) that algebra become an essential strand of the intermediate (Grade 7-8) curriculum, prior to formal algebraic instruction in high school. An understanding of linear relationships is central to the development of algebraic thinking. Expressing an understanding of a linear relationship can be thought of as describing a systematic variation of instances across some domain. The major characteristic of a linear relationship is the covariation between two sets of data represented by two variables, the independent variable, x, and the dependent variable, y. The nature of the relationship is that for every instance of x there is one corresponding instance of y, determined by the underlying linear rule. The relationship that connects the two variables is one of predictable change or growth.

Linear relationships can be represented symbolically/numerically through equations and algebraic symbols using the form $y = mx + b$, where m is the coefficient, or multiplicative factor, of x, and b is the additive (sometimes known as the constant) term of the relationship. A linear relationship can also be represented graphically, where m represents the gradient of the slope and b represents the y-intercept. These representations are intertwined, such that a change in one representation leads to a change in the other representation. Mathematics education researchers stress that it is the ability to make connections among different representations (e.g., Rico, 2009), specifically symbolic/numeric and graphic ones, that allow students to develop insights for constructing the concept of a linear relationship (e.g., Bloch, 2003; Evan, 1998).

There have been numerous studies that have documented the difficulties students have when exploring the connections among representations of linear relationships (e.g., Evan, 1998; Moschkovich, 1996, 1998, 1999). Students have difficulties shifting between different modes of presentation (Brassel & Rowe, 1993; Yerushalmy, 1991). When graphing a linear relationship of the form $y = mx + b$ researchers have noted that the connections between m and the slope of the line, and b and the y-intercept are not clear (Bardini & Stacey, 2006). Students also have difficulty predicting how changes in one parameter of the equation will affect the graphic representation (Moschkovich, 1996; Moschkovich, Schoenfeld, & Arcavi, 1993).

Learning Disabilities and Mathematics

Although little research to date has been conducted on the algebraic learning of students with learning disabilities, it would seem predictable that these students would also find the conceptual underpinnings of linear relationships as elusive as typically developing students. This is supported by a recent United States finding that 75% of Grade 8 students with learning disabilities earned a scale score on the Algebra and Functions Strand of the National Assessment of Educational Progress (NAEP) Mathematics that was below the mean score for the full sample (Foegen, 2008).

Research on learning disabilities in the domain of mathematics is still in its infancy (Gersten, Jordan, & Flojo, 2005). The vast majority of work done to date has focused almost exclusively on content typical of elementary school classrooms (Geary, 2007) and on basic number concepts and simple arithmetic. Little research has been done on students with learning disabilities understanding of complex concepts like linear functions.

A principal area of consideration with respect to learning disabilities and mathematics is the divide between procedural and conceptual instructional practice and whether explicit inquiry-based instruction can and should be integrated for students with learning disabilities (Pedrotty-Bryant, 2005). Students with learning disabilities often have difficulty retaining facts (Geary, 1993), and so the instructional approach for these students tends toward memorization through repetition rather than the development of conceptual knowledge (Cawley & Parmar, 1992). This rote drill approach may seem a successful strategy as it offers students a means of producing the correct answer, but it is an extremely limited way of understanding complex concepts such as linear relationships.

The National Council of Teachers of Mathematics (NCTM) emphasizes equitable instruction for all students. Given that prevalence figures indicate that between 5% and 10% of school-age children exhibit learning disabilities (Fuchs, Fuchs, & Hollenbeck, 2007; Gross-Tsur, Manor, & Shalev, 1996; Ostad, 1997; Shalev, Auerbach, & Gross-Tsur, 2000), there is a need to examine mathematics education for these students, and how these students can be supported to learn complex mathematical concepts.

New Instructional Approach for Teaching Linear Relationships

This paper reports on a research study conducted to investigate the implementation of a teaching approach designed to address some of the instructional difficulties outlined above. As part of a larger long-term study, we have been investigating the affordances of an instructional approach that prioritizes visual representations of linear relationships—specifically, the building of linear growing patterns and the construction of graphs (Beatty, 2007; Beatty & Bruce, 2008). Previous research on the lesson sequence has shown that it supports students' progression from working with linear growing patterns as an anchoring representation to considering graphical representations of linear relationships. Students also make connections among different representations–pattern rules (equations), patterns and graphs. These are showed in Figure 1 which includes two representations on connecting the linear growing pattern to a graphical representation, both represent the pattern rule "*number of tiles* = *position number* $\times 2 + 3$".

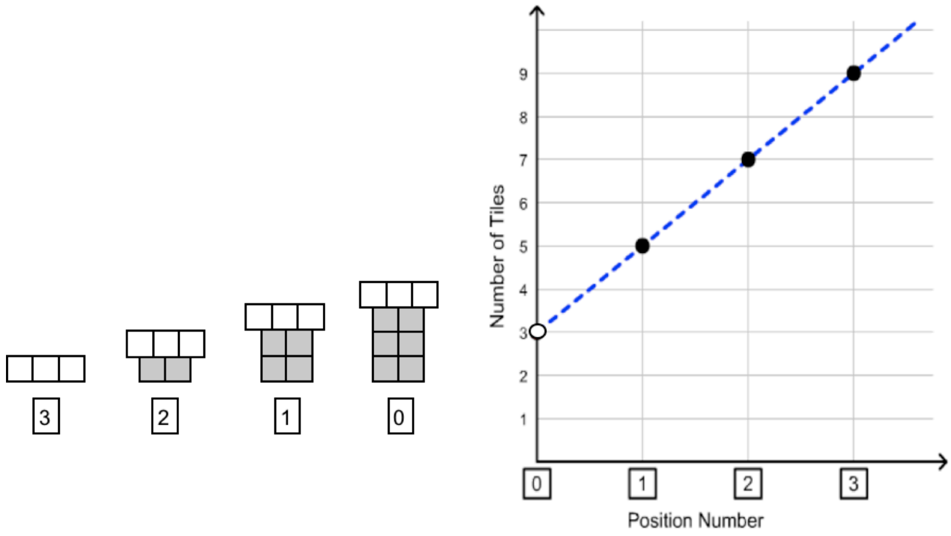

Figure 1. Connecting the linear growing pattern to a graphical representation

In our previous study we evaluated the experimental algebra lesson sequence by conducting quantitative analyses of student learning to determine whether there was an increase in student scores from pre to post intervention. We calculated a total pretest and posttest score comprised of 10 sub-items measuring students' ability to find generalized rules/functions for patterns. The results indicate that the mean posttest score ($M = 6.72, SD = 2.74$) was significantly greater than the mean pre-test score ($M = 4.21, SD = 2.74, t(311) = -14.33, p < .000$). The standardized effect size index d, was .99, a high value. The mean difference was 2.51 points between the two tests. Table 1 outlines the results from pre to posttest.

Table 1
Student Scores Pre to Posttest

Number of students who scored between 1 and 5 out of 10	Number of students who scored 6 or above out of 10
Pretest ($n = 295$)	
204	91
Posttest ($n = 294$)	
91 (12 achieved scores of 9)	228 (114 achieved scores of 9)

Note: n = total number of students.

We then conducted a one-way repeated-measures ANOVA to compare pre and posttest achievement as a function of students' demonstrated achievement level (low *n* = 67, mid *n* = 164, high *n* = 79). Levels were based on teacher rating and report card marks. Students designated as "low" were on Individual Education Plans (IEP) and most had been identified as having some kind of learning disability. The multivariate test indicated a significant effect, $F(1,307) = 159.32, p < .000$, but with no interaction of test results by level, $F(2,307) = 1.723, p =< .18$. These results indicate that students at all three levels increased their test scores from pre to post (see Figure 2). These results suggested that the positive effects for all students, including those identified as having a learning disability, were important enough to pursue further dissemination of the learning sequence by capitalizing on the potential of online learning objects.

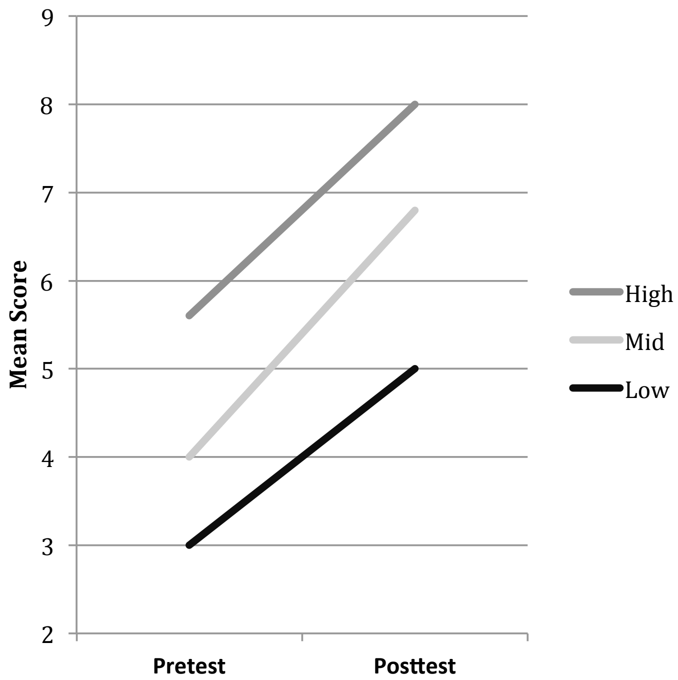

Figure 2. Estimated marginal means by achievement level pre and posttest

In the study reported in this paper we enhanced the original instructional sequence by including computer-based dynamic interactive representations of linear relationships. These online learning objects or Critical Learning Instructional Paths Supports (CLIPS) offered the possibility of combining a proven instructional sequence with unique properties of digital technology.

Unique Support of CLIPS for Students with Learning Disabilities

CLIPS are created using flash animation and incorporate audio narration, offering students the ability to consider mathematical concepts in non-static environments. Although CLIPS was designed for use by all learners, we hypothesized two specific ways that this kind of environment would support students with learning disabilities.

The first affordance is supporting students in focusing their attention. Students naturally focus attention through stressing some features as foreground and ignoring others as background (Mason, 2008). However, previous studies indicate that students with learning disabilities often display attention difficulties, which may adversely affect their learning (Fuchs et al., 2007; Fuchs, Fuchs, Powell, Seethaler, Cirino, & Fletcher, 2008; Montague, 2007). The CLIPS computer animation was designed to direct student's attention in order that they would discern details and recognize relationships that we, as the educational designers of the activities, believe are important to discern and recognize. In each activity the aspect that we want students to notice—for example the connections between the numeric value of the constant in a pattern rule, the number of tiles in a pattern, and the vertical intercept of a trend line on a graph—becomes the focus of students' attention. As the student works through an activity designed to highlight these connections, the constant in the pattern rule flashes red, the red tiles that "stay the same" in the linear growing pattern flash, and the vertical intercept on the graph has a red flashing ring around it (Figure 3). In addition all activities have audio narration that directs students' attention to particular aspects of the task.

The second affordance of the technology is that mathematical connections can be conveyed to the students interactively. Students move through a series of scenes for each activity, so that the mathematical concepts are introduced in a logical order of increasing complexity. The animation creates opportunities for students to interact with the material by providing activities in which the co-action between user and environment can exist. This co-action takes many forms, from filling in numeric values, dragging words to complete sentences, to more sophisticated and rich interactions such as constructing patterns using virtual tiles or graphs using the graphing tool. Each representation is linked to the other representation so that as students create one, they can see the corresponding changes in the other. Thus the mathematical symbols that students work with are dynamic objects that are constructible, manipulable and interactive. This offers the opportunity for students with learning disabilities to construct an understanding of the process of linear co-variation, rather than simply memorizing rote facts.

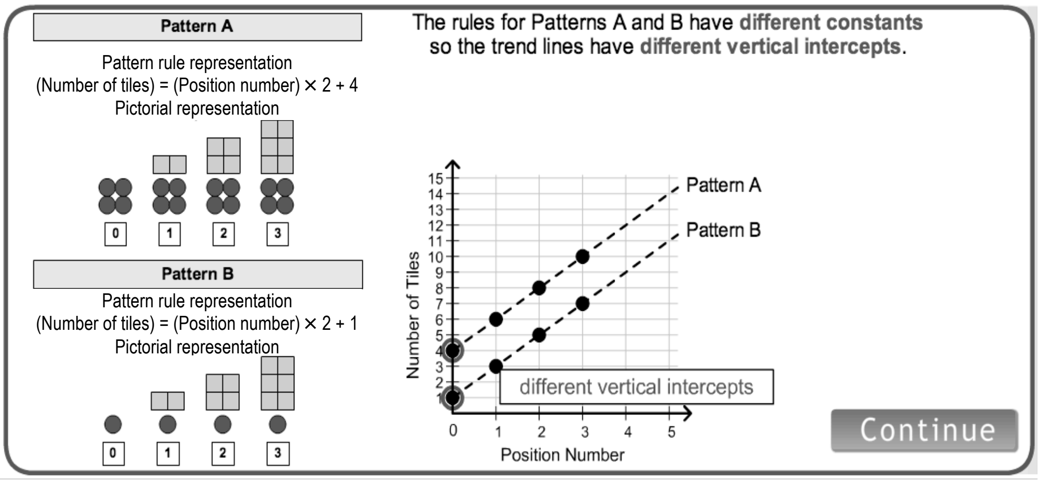

Figure 3. Screen capture of activity to compare pattern rules that have the multiplier but different constants

CLIPS are comprised of sequences of activities, none of which are more than 5 to 10 minutes in length. All activities build on the concepts of the previous activity. The activities are interactive, and offer immediate levelled feedback to participants. Thus, CLIPS offers students opportunities for numerous fast-paced practices, modeling with representative examples, and opportunities to explore concepts with immediate feedback. These instructional components have been identified vital by many researchers for students with learning disabilities to learn abstract mathematical concepts (Anderson-Inman, Knox-Quinn, & Horney, 1996; Fuchs et al., 2008; Swanson & Hoskyn, 1999).

The CLIPS learning objects were designed to alleviate difficulties most students have been shown to have when developing their understanding of linear relationships —particularly the ability to make connections among representations. It was our belief that the interactive nature of CLIPS would also support students with learning disabilities in constructing their understanding (rather than asking them to memorize for example the procedures for constructing a linear graph given a linear equation).

METHOD

This study was part of a larger research project in which we investigated the affordance of the CLIPS algebra sequence for all students. Fifteen Grades 7 and 8 teachers in two different school districts volunteered to participate in a study designed to research the impact of integrating CLIPS as part of their classroom pedagogy. For this paper, we focus specifically on data relating to students with learning disabilities.

Participants

Math coordinators in two school districts invited Grade 7 and 8 teachers to participate in the study. Fifteen teachers attended three full day in-service sessions on the use of CLIPS to promote student understanding of linear patterns using multiple representations.

Overall there were 342 participating students. Of these, approximately 10% were identified with a learning disability and were following an Individual Education Plan (IEP).

Data Sources

On two of the three professional learning sessions we conducted focus group interviews with the teachers. Transcripts from 12 focus group interviews with participating teachers were coded to identify categories and themes. Subsequently, data was transformed to count the frequency of themes and codes in order to identify prevalence of a code or theme.

Interviews were also conducted with participating students identified as having a learning disability. Transcripts of these interviews were coded.

Between the professional learning sessions, researchers observed classroom implementation of CLIPS in the classroom. These sessions were videotaped, and the video was coded for episodes of student engagement with CLIPS. In addition, field notes of each of the observed lessons were taken.

Data Analysis

All field notes and interview transcripts were entered into the software package NVivo-8©, which allowed for the subsequent coding and sub-coding of themes. The data was coded initially using a priori themes (based on a set of start codes generated by the research team). The data were then reviewed to establish an emergent coding system using open coding (identifying key words and phrases directly from transcripts and other data sources) and active axial coding (which isolates one key theme in relation to others). Codes were used to merge categories together to establish thematic categories and sub-categories of processes found within the data sets. These thematic categories underwent both a descriptive analysis in order to discern and understand the elements of the category. Cross-referencing of themes from the open codes identified in focus group interview transcripts to other sources of data (e.g. field notes, classroom observation, videotape and documents) was undertaken for the purpose of complementarity (Green, Caracelli, & Graham, 1989).

Results

We had hypothesized that the dynamic/interactive nature of the CLIPS learning objects would support students with learning disabilities. In fact, all of the teachers we worked with expressed overwhelming surprise at the levels of learning exhibited by their students who had been identified with learning disabilities. In this paper we highlight four major themes that were reported by all 15 teachers during the focus group interviews. These themes were then cross-referenced with student interviews, and episodes observed during classroom observations. Two of the themes relate to our initial hypotheses that the directing of students' attention, and use of interactive sequenced learning episodes would support the learning of students with learning disabilities in two ways: (a) to make connections among representations, and (b) to develop a conceptual understanding of linear relationships. The other two themes were unanticipated, and refer to the social and emotional support provided by working with CLIPS that appeared to increase student engagement, confidence, and ability to integrate into the classroom mathematics community.

Making Connections Among Representations

Teacher's in-class assessments revealed that students with learning disabilities were able to make connections among different representations of linear relationships, and could predict how changes in one representation would affect other representations. Four specific areas of learning were identified. Students were able to:

- ♦ Determine the underpinning explicit functional rules of linear growing patterns.
- ♦ Create a graph of an explicit linear function from a given pattern rule;
- ♦ Determine the explicit linear function, given a graphical representation of a linear relationship.
- ♦ Make connections among three representations of linear relationships—pattern rules, patterns, and graphs. For example, make predictions about the angle of the slope of a trend line given changes in the value of *m* in the pattern rule.

In all the teacher focus group interviews, the teachers reported that their students with learning disabilities were able to make connections among representations, based on conversations they had with students as they engaged with the activities, and also based on individual in-class assessments. For many of the teachers this was the first time they had seen this level of understanding in this population of students.

Teacher 1 (fragment 2.1): The concepts were introduced slowly and accessibly and reinforced so that with confidence I can say all my students on an IEP can look at a graph and tell you the rule for that graph, can build a pattern from that graph, and can give you a story related to that graph. I've never had that experience before. On the quizzes and assessments I've been doing, they've all being getting Level 4 [out of 4].

Teacher 5 (fragment 2.3): I know anecdotally from being in the class and working with the groups and listening to them that every one of my kids can make connections among these representations—look at a graph and find the rule and build a pattern from the rule—every one of my kids can do those things.

This understanding was also reported by the students during their interviews. In particular, students identified the animation of the CLIPS, and the multiple interconnected models as supporting their own understanding of multiple interconnected representations.

Grade 7 student: I liked when the robot was showing a times 2 pattern, and then the robot stacked the tiles and there were dots on the top, and then the tiles disappeared and it turned into a graph line. It made sense! It was really cool. See… the dots on the graph were the tiles in the pattern. So I understood that it was all, like, it was showing the same thing… the same rule.

Conceptual Understanding Versus Rote Understanding

As previously stated, most students with learning disabilities are traditionally taught through rote memorization, resulting in what Skemp (1976, 1979) terms "instrumental understanding", that is, the memorization of steps of mechanical procedures. Thus, students with learning disabilities are typically taught to memorize steps or formulas with little exploration to the underlying meaning of the mathematics with which they are engaged. Conceptual understanding, however, is the thoughtful and connected learning of mathematical principles and concepts leading to an understanding of why and how mathematical concepts are related (Mason & Johnston-Wilder, 2004).

Teachers in this study reported that their students with learning disabilities constructed an understanding of linear relationships through their engagement with the material. For instance, in the interview excerpt below, a teacher outlined a conversation she had with one of her students. The student had been working with linear growing patterns with both a multiplier and a constant, and then creating graphical representations of the patterns. Because each pattern rule was of the form $y = mx + b$ where $b > 0$, the graphical representations had a y-intercept > 0. This initially puzzled the student.

Teacher 3 (fragment 2.3): One of my students said to me the other day "ah, so I get it. The graph doesn't always have to come from 0!" She said, "I thought I'd drawn my line wrong, and then I realized…" because she had thought that all the lines had to start at the origin but then she figured out that this wasn't the case. She figured out that the y-intercept shows the number of tiles that would be at the 0 position of a pattern. I think they are starting to understand position 0 and the *y*-intercept. This is the kind of thing I'm seeing a lot with the identified kids, the kids not working at grade level. They're making comments like, "This makes sense! If math was like this all the time, it'd be great." Because they can figure it out. They can work at it and figure it out themselves.

The students also referred to the opportunity to engage in a variety of tasks to develop mathematical thinking. For instance, the student quoted below highlighted the difference between his previous instruction, during which the teacher "showed you how to do it", and the exploratory nature of CLIPS activities that offered students different kinds of representations with which to experiment.

Grade 8 student: Having the pictures and the animation on CLIPS was good, so there are different kinds of pictures, the graph, and the robot too. So there was more than one way to see things, which helped a lot! I learned mathematically that there are different ways to show things. Especially last year, our teacher was very much, "This is how you do it." And this way [using CLIPS] it showed, oh you can do it like this, you can put it in a graph, you can use just the formula, you can draw a picture. So that's what I really liked. That there was more than one way.

Students also reported that CLIPS offered opportunities to explore different representations and strategies so that they were able to access the material in ways that were meaningful to them.

Grade 7 student: This is a different way to do math. There wasn't a textbook and a million questions. As long as you get to a final answer, it doesn't matter how you got there. I can draw pictures or whatever and it's ok. I can play and make mistakes and then I can figure out what's right and wrong… I'm not being judged. Math is always… do it this way or you're wrong. And with this… there's so many different ways. I get to think about it, and talk about it.

Students were not asked to memorize procedures. Instead they were given the opportunity to explore abstract concepts by starting with concrete representations (linear growing patterns) then moving to more abstract representations (linear graphs) and finally symbolic notation (pattern rules and equations). The benefit of this progression was summed up by one of the teachers.

Teacher 5 (fragment 2.2): I think the biggest thing for kids is, you know you go from a picture of what a pattern looks like, and then there's a pattern rule that comes out of that and now it makes sense. It's not just times 3 plus 1 and numbers that you've manipulated and letters. I think it makes a visual out of algebra and helps them understand it better. And then that the graph goes from the pattern—the connections that they get to see—. I think that they're involved in it because they've made those connections so they understand it better. It makes sense to them.

Inclusive Classroom Community

Teachers reported that through CLIPS they were able to create an inclusive mathematics community in the classroom. There were two main indicators: (a) all students remained in the classroom, and (b) all students participated in all of the same activities.

First, teacher reported that students in pullout remediation programs were no longer removed for math learning, but remained in the classroom. All of the teachers reported that the sequential nature of the lessons and activities allowed their lowest-achieving students to access the material successfully. This was ascribed to the animated, visual nature of the materials, the voice-overs of any written descriptions or instructions, and the capacity for students to repeat any lesson or activity they did not understand.

Students particularly highlighted the importance of being able to work at an individual pace, including replaying scenes and activities in order to fully grasp the material.

Grade 7 student: I find it easier because when you're in a lesson with a teacher, you can't go back and re-play if you don't get it. You have to go up and ask them later and maybe you'll forget. But you can replay this if you don't understand something so that you can understand it better.

Secondly, teachers in this study reported that no modifications of the material were necessary. All students were able to access the material at whatever level was meaningful to them. For the first time during the year, the teachers did not have to program separately for students with learning disabilities,

Teacher 4 (fragment 2.1): All my students with learning disabilities were doing what everyone else was doing—all the same lessons—. And they're doing fine! That's huge! That these kids can engage in the same activities and communicate their thinking to the class! There was no IEP in place for this. They all did the exact same thing and I did not accommodate any student at any time for this. And everyone did well.

This finding was also reported by students during their interviews with the researcher. Students stated that they enjoyed participating in the same activities as their peers. During classroom observations students with learning disabilities were observed to be working alongside typically developing peers and engaging in the same activities (see Figure 4).

Figure 4. Students with learning disabilities working with CLIPS

Increase in Student Confidence

Finally, teachers reported that they perceived an increase in the confidence of students with learning disabilities in math class. They cited both an increase in student attendance and contributions to discussion in math class as indicators.

Teacher 3 (fragment 1.1): The biggest difference for me was seeing IEP kids who are normally petrified of math, and not terribly successful, and believing that they can't do it, actually leading the discussion. One of my self-proclaimed weak math students got the concept and was questioning typically stronger math students in class about their patterns and explaining why it wasn't a linear growing pattern—that the growth wasn't predictable—. Our class is a big class with lots of learning needs and for the first time ever they all get it!

Over the course of the classroom observations, this increased participation was noted. In particular, classroom discussion led by students identified as having an LD increased. For example the discussion cited above referred to a classroom discussion during which Henry (LD) questioned the pattern made by a typically developing student, Caitlyn, and whether or not it represented linear growth (Figure 5).

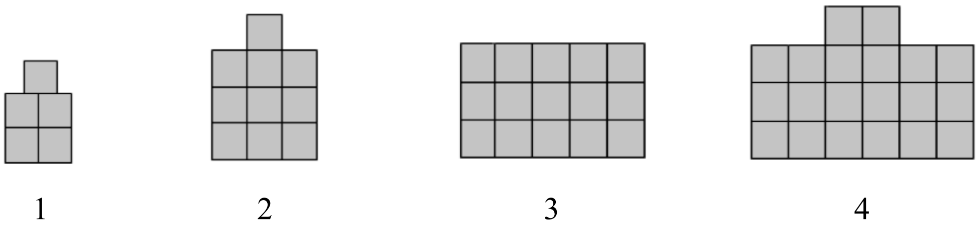

Figure 5. Typically developing student' model

Caitlyn had created the pattern to represent the pattern rule "$tiles = position\ number \times 5$", or $y = 5x$. Henry led the class discussion about the pattern, and pointed out that although the pattern was correct numerically, it did not represent a linear growing pattern.

Henry: I think your pattern is wrong because yours goes, like, one on top, and then just switches. First you had four on the bottom and one on top, but then you have for position 3 none on top, and for position 4 two on top. I wouldn't know how to build the next one. I wouldn't know how to build the 10th position of this!

Caitlyn: But look it goes up by 5 each time. It's growing by the same each time.

Henry: Ya but… well, I think the numbers are right but I don't think it looks like a pattern because the shape's not the same. It needs to be the same so you can see how it grows.

In this instance, Henry was able to think about both the geometric and numeric properties of a linear growing pattern, and recognized the need to be able to predict what iterations of the pattern would look like further down the sequence.

In another instance, a student with LD was able to articulate the need for an accurate scale along the *x*-axis of a graph in order for the trend line to show linear growth. She was part of a classroom discussion during which students were analysing graphs they had created as a supplementary activity to the CLIPS activities.

Grade 8 student: Your graph there has, like, has a kink in the trend line. I think it's because your position numbers [values along the *x*-axis] are not spaced out the same. These two numbers are kind of closer together. It [the trend line] should be straight because the pattern grows by the same each time.

This student was able to reason that if the growth of the pattern remains constant, then the trend line should be straight. This is a fundamental aspect of linear graphs that this student was able to discover based on her experiences with the CLIPS activities connecting linear growing patterns and graphs. In both of these examples the CLIPS activities provided the students with a means of justifying their thinking, and critiquing others' ideas.

EDUCATIONAL CONTRIBUTIONS

In this study, we found that the sequenced dynamic representations of linear relationships had a positive effect on the levels of achievement of students identified as having learning disabilities. This study suggests that students with learning disabilities can use computer-supported activities to learn complex mathematical concepts. Previous research on computer assistive technology to support mathematical learning of students with learning disabilities has tended to focus on software programs designed to provide practice with computational skills in order to increase automaticity in basic mathematics tasks such as carrying out arithmetic operations.

In contrast, we believe that the CLIPS activities allowed students to construct deep conceptual understanding of complex algebraic relationships rather than memorize procedures. CLIPS incorporates a dynamic graduated teaching sequence that proceeds along a continuum from concrete iconic representations (virtual tiles) to representational formats (graphs and diagrams) to more abstract or symbolic representations. The online activities directed students' attention to specific important ideas—particularly the explicit connections between actions on concrete (linear growing patterns) or abstract (graphs) representations and the related symbolic procedures (pattern rules and equations). CLIPS also incorporate immediate levelled corrective feedback, which has been shown to be an important element within an instructional system (e.g., Fuchs, Fuchs, & Hamlett, 1989; Guskey & Gates, 1989). Finally, CLIPS offers the ability to go back and replay if you didn't get something thereby allowing students to work through the material at their own pace.

To summarize, the design features included a slow rate of introduction of new concepts (the rate could be modified by each individual student), a large assortment of support explanations and activities, and multiple opportunities for practice and review. Researchers have concluded that when concepts are introduced rapidly with minimal explanations and sparse practice and review, students with learning disabilities may be overwhelmed by memorization, strategies, vocabulary and language coding (e.g., Fuchs, Compton, Fuchs, Paulsen, Bryant, & Hamlett, 2005). The design features of the CLIPS program seem to have narrowed the typical gap in mathematics attainment between general and special education students.

Support for this comes from our finding that students with learning disabilities became participating members of the classroom mathematics community. They engaged with the same material and constructed similar kinds of understanding as their peers. The initial entry point was accessible for all students. The sequence then subsequently incrementally built in complexity. Each student trusted that they would be able to continue to successfully work on activities, and that the material would not become too complex too quickly. As a result, these students demonstrated an understanding of the connections among multiple representations of linear relationships that has been shown to be difficult for typically developing students.

It also appears that the effects of working on CLIPS resulted in changes in classroom practice beyond the immediate scope of the research study. All teachers reported that, subsequent to working with CLIPS, the students who had been in remedial pullout programs were no longer removed from the classroom for math, but remained as contributing members of the classroom mathematics community for the remainder of the school year.

This leads us to question whether the learning difficulties for many of these students may have been curriculum or instructional difficulties in addition to learning disabilities. If tasks involving complex mathematical thinking are thought to be suitable only for "high achievement" students then the result may be that students with learning disabilities are given only routine and repetitive tasks. However, if all learners are treated as having the capacity to engage with complex concepts, and are supported through flexible, dynamic sequences, it appears that students with learning disabilities who are generally considered "low attainers" can transcend expectations. More research is needed in this important field.

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