Scientometrics manuscript No.
(will be inserted by the editor)

Ranking of Research Output of Spanish Universities on the Basis of the Multidimensional Prestige of Influential Fields of Study

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the date of receipt and acceptance should be inserted later

Abstract A university may be considered as having dimension-specific prestige in a field of study (e.g., Computer Science) when a particular bibliometric research performance indicator exceeds a threshold value.

But a university has multidimensional prestige in a field of study only if it is influential with respect to a number of dimensions. The multidimensional prestige of influential fields at a given university takes into account that several prestige

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indicators should be used for a distinct analysis of the influence of a university in a particular field of study.

After having identified the multidimensionally influential fields of study at a university their prestige scores can be aggregated to produce a summary measure of the multidimensional prestige of influential fields at this university, which satisfies numerous properties. Here we use this summary measure of multidimensional prestige to assess the comparative performance of Spanish Universities during the period 2006-2010.

Keywords Publication-Based Ranking; Spanish Universities; Bibliometrics; Multidimensional Prestige; Influential Fields of Study

1 Introduction

The interest in the ranking of universities stems from the need to evaluate research output using to this aim some kind of objective metrics. For example, it may guide student choice of a university to pursue a graduate degree (Dridi et al., 2010).

The comparison of research output among universities has been raising an increasing amount of interest in the last few years (Liu and Cheng, 2005; Buela-Casal et al., 2007; Aguillo et al., 2010; Torres-Salinas et al., 2011), since the help it provides to allocate limited funds as fairly as possible. However funding agencies often make their decisions based on partial measures, resulting in unfair assessments of the research output of some of the studied universities (Billaut et al., 2010).

The number of papers produced in a year by each member of staff in an academic institution particularly in the EU and USA is regarded as an indication of their career success. Rankings based on publication in peer-reviewed journals are objective, and many faculty believe academic journals remain the fairest measure of the quality of our research (Dusansky and Vernon, 1998). Since publication-based performance evaluations underlie the work of funding agencies, there are already mechanisms to ensure high levels of accuracy of these data (Dusansky and Vernon, 1998).

In (Torres-Salinas et al., 2011), it was presented a bidimensional quantitative-qualitative index to compare the research output of a group of universities using different dimensions of analysis: (1) The quantitative dimension which shows the net production of a university in a given field during a period of time by using raw indicators that may be correlated with staff of the institution; and (2) the qualitative dimension which can be seen as a measurement for academic excellence, focusing on the ratio of high-quality production on each university in a particular field during the same period of time, and is mostly independent of the size of the institution. A combination of both dimensions provides a robust and objective way to compare research outputs.

In this paper we provide an overall ranking of research production in different fields of main Spanish universities based on a multivariate performance indicator space which integrates both quantitative and qualitative dimensions.

To this aim, we extend the one-dimensional measure developed by (Garcia et al., 2011a,b) to a multidimensional case following (Peichl and Pestel, 2010) who proposes a class of economic measures of richness in Germany. Thus our approach identifies those fields of study at a university that are considered to be multidimensionally influential. Furthermore, the multidimensional prestige of influential fields is to be sensitive to changes in the score distribution of each dimension, which allows us to investigate inequality among multidimensionally influential fields.

For example, let $U = \{s_1, s_2, \ldots, s_n\}$ be the set of fields of study at a given university of example. From (Torres-Salinas et al., 2011) we have that research output and impact of field s_i at this university may be graded on the basis of the raw number of publications, citations, h-index, as well as relative measures of impact and visibility (e.g., JCR journal first quartile, average citations and ratio of highly cited papers).

Regarding the number of dimensions (prestige indicators) to be used in a multidimensional setting in order to measure research output and impact of influential fields at a particular university, we may consider several indicators with different degrees of correlation among them, but which should be used for a distinct analysis of structural changes at the score distribution of prestige in a given field of study, (Torres-Salinas et al., 2011):

– NDOC: Basic indicator for total amount of raw production, it may depend on the number of researchers in the institution focused on the field of study, and how active they are.
– NCIT, ACIT, TOPCIT: According to (Bornmann and Daniel, 2008), in bibliometrics the resonance, or impact, of a scientific work is measured via the number of citations. It can be assumed that the more important a work is for the further development of a field, the more frequently it is cited. That is, NCIT is a raw indicator of scientific relevance, and ACIT and TOPCIT indicate quality of the research output and ratio of very high-quality papers (Aksnes, 2003; Aksnes and Sivertsen, 2004), respectively.

- H-index: Probably the better known index in current bibliometrics, it has proven to be a robust measure of impact, (Hirsch, 2005). By limiting its scope to the period of study, we avoid the seniority dependence the basic h-index usually presents.
- %1Q: The impact factor is widely considered a reliable measure of journal quality (Bornmann and Daniel, 2008), so centering the analysis in the top quartile provides an indicator of top-quality papers. The ratio of citable papers that are top-quality serves as a relative size-independent indicator, %1Q.

In this paper, a field of study s_i at a given university is considered as having dimension-specific prestige when its score based on a given ranking model (e.g., either NDOC or %1Q) exceeds a threshold value. Then, we can define which fields s_i at a given university are considered to be prestigious in a multidimensional setting. Thus, a field of study at this university has multidimensional prestige only if it is an influential field with respect to a number of dimensions. Finally, after having identified the multidimensionally influential fields at a particular university, their prestige scores are aggregated to a summary measure of multidimensional prestige. The summary measure is not only sensitive to the number of dimensions but also takes into account changes in the ranking scores of influential fields of study at the university.

The setup of the paper is organized as follows: Section 2 defines the multidimensionally influential fields of study at a given university. The Section 3 introduces a summary measure of multidimensional prestige of influential fields, which satisfies numerous properties. Then in Section 4 we shall apply our approach to main universities in Spain in order to analyse the comparative multidimensional prestige of influential fields during the period 2006-2010. The data we employ is from (Torres-Salinas et al., 2011). Section 5 concludes.

2 Multidimensionally influential fields of study at a given university

The number of fields of study at a given university is denoted with n as given above, and let $d \geq 2$ be the number of dimensions in the multivariate indicator space.

Let $\mathbf{X}$ be the matrix of dimension-specific scores x_{ij} which denote the score of field of study s_i at the particular university, with $1 \leq i \leq n$, in ranking model corresponding to dimension j, with $1 \leq j \leq d$:

$$\mathbf{X} = [x_{ij}]_{n \times d} \tag{1}$$

For each dimension j, there is a threshold z_j such that fields s_i at this university with score x_{ij} above threshold z_j are to be considered dimension-specific influential fields of study.

Let $\mathbf{z}$ be the $1 \times d$ vector of dimension-specific thresholds. Using this vector it is possible identify whether field s_i is influential with respect to dimension j or not. Let θ_{ij} be a function defined as:

$$\theta_{ij} = \begin{cases} 1 \text{ if } x_{ij} > z_j \\ 0 \text{ otherwise} \end{cases} \tag{2}$$

Using function θ_{ij} it is possible to construct a matrix Θ^{0-1} which provides information about whether a field of study s_i at the given university is influential regarding dimension j or not:

$$\Theta^{0-1} = [\theta_{ij}]_{n \times d} \tag{3}$$

where each row vector θ_i of Θ^{0-1} gives us a vector of prestige counts which can be denoted as $\mathbf{c} = (c_1, \cdots, c_n)'$ whose elements $c_i = \sum_{j=1}^{d} \theta_{ij}$ are equal to the number of dimensions in which field of study s_i is found to be prestigious.

We can now define which fields of study at a university are considered to be influential in a multidimensional sense: A field of study s_i at the given university is a multidimensionally influential field if it is prestigious for a number of dimensions which is greater than or equal to a certain integer k, with $1 \leq k \leq d$.

That is, a field s_i is multidimensionally influential if $c_i \geq k$, with c_i being the number of dimensions in which field of study s_i at the university was found to be influential.

For a given integer k, we can define a function $\phi_i(\mathbf{z}; k)$ which equals to one if field s_i is multidimensionally influential, and is zero otherwise:

$$\phi_i(\mathbf{z}; k) = \begin{cases} 1 \text{ if } c_i \geq k \\ 0 \text{ otherwise} \end{cases} \tag{4}$$

with $\mathbf{z}$ being the $1 \times d$ vector of dimension-specific thresholds.

Therefore the subset of fields of study at the university which are multidimensionally influential is given by:

$$\Phi(\mathbf{z}; k) = \{s_i, 1 \leq i \leq n | \phi_i(\mathbf{z}; k) = 1\} \tag{5}$$

For a given integer k, let $w(k)$ be the number of multidimensionally influential fields at this university. From equation (5) it follows that $w(k)$ is given by the cardinal of the subset $\Phi(\mathbf{z}; k)$:

$$w(k) = |\Phi(\mathbf{z}; k)| \tag{6}$$

where $|\cdot|$ is the cardinality (size) of a set.

In case of $k = 1$, field of study s_i is multidimensionally influential when it is considered prestigious in only one single dimension (e.g., %1Q). But prestige in one single dimension may be something dangerous (Garcia et al., 2011c).

Second, in case of $k = d$, it is only considered as multidimensionally influential if it is prestigious for all dimensions under consideration. But this is a demanding requirement, especially if the number of dimensions d of the multivariate indicator space is large, which often identifies a very narrow slice of fields at the university under consideration.

In case of $1 < k < d$ we have an intermediate approach as proposed in (Alkire and Foster, 2008).

3 A summary measure of multidimensional prestige

Recall that the vector of prestige counts denoted as $\mathbf{c}$ was defined such that $\mathbf{c} = (c_1, \cdots, c_n)^{'}$, where $c_i = \sum_{j=1}^{d} \theta_{ij}$ is the number of dimensions in which field of study s_i is found to be prestigious, with θ_{ij} being equal to one if field s_i is prestigious with respect to dimension j and zero otherwise as given in equation (2). Since a summary measure of the multidimensional prestige of influential fields at the university must take into account information on multidimensionally influential fields of study only, we must replace the elements of $\mathbf{c}$ as follows:

$$c_i^k = \begin{cases} c_i \text{ if } c_i \geq k \\ 0 \ \text{ otherwise} \end{cases} \tag{7}$$

From equation (7), we have that $\mathbf{c^k} = (c_1^k, \cdots, c_i^k, \cdots, c_n^k)'$ contains zeros for fields s_i not considered to be multidimensionally prestigious, that is, when a field of study s_i is not multidimensionally influential, $c_i < k$, its entry in $\mathbf{c^k}$ is zero.

Now we propose a number of constraints which an axiomatic measure of the multidimensional prestige of influential fields at a given university must satisfy. But first, following the approach given in (Garcia et al., 2011a), we define a summary measure MW of the multidimensional prestige of influential fields at the university as the normalized weighted sum of the field contribution to the overall prestige as follows:

Definition 1 Given a configuration $\mathbf{X} = [x_{ij}]_{n \times d}$ of dimension-specific scores of size $n \times d$, and a $1 \times d$ vector $\mathbf{z} = (\mathbf{z_1}, \ldots, \mathbf{z_j}, \ldots, \mathbf{z_d})$ of dimension-specific thresholds, a summary measure of the overall prestige MW of multidimensionally influential fields at a given university is defined by a normalized weighted sum of field contributions to the overall prestige using weighting function f, as follows:

$$MW = \frac{1}{n \times d} \sum_{i=1}^{n} \sum_{j=1}^{d} f\left(\frac{x_{ij}}{z_j}\right), \tag{8}$$

where the mathematical form of f depends on a set of axioms to be proposed.

Appendix A presents a set of axioms in order to define the exact form of a summary measure as that given in Definition 1 which shall have some desirable properties. To this aim we reformulate to the study of the multidimensional prestige of influential fields a number of constraints which were first used in an axiomatic approach to economic poverty measurement (Sen, 1976; Takayama, 1979; Peichl et al., 2008).

Next, following (Garcia et al., 2011c), a theorem states that five axioms given in Appendix A determine an axiomatic measure of multidimensional prestige of influential fields for a given domain-specific score configuration.

Theorem 1 *Let k be such that field of study s_i at a given university is multidimensionally influential if $c_i \geq k$, with c_i being the number of dimensions in which field s_i was found to be influential. Then, a summary measure of the multidimensional prestige of influential fields, given by a normalized weighted sum of domain-specific scores in the configuration $\mathbf{X}$ of size $n \times d$, using a weighting function f as follows:*

$$\frac{1}{n \times d} \sum_{i=1}^{n} \sum_{j=1}^{d} f\left(\frac{x_{ij}}{z_j}\right) \tag{9}$$

and such that satisfies Axioms 1 through 5 in Appendix A, it can be defined as:

$$MW(k) = \frac{1}{n \times d} \sum_{i=1}^{n} \sum_{j=1}^{d} \left(1 - \left(\frac{z_j}{x_{ij}}\right)^{\beta}\right)_{+} \cdot \phi_i(\mathbf{z}; k) \tag{10}$$

with $\beta > 0$ being a sensitivity parameter for the intensity of field prestige (for smaller values of β more weight is put on more intense prestige); $(y)_+ = \max(y, 0)$; and where function $\phi_i(\mathbf{z}; k)$ equals to one if field s_i is multidimensionally influential, and is zero otherwise.

Proof See Appendix B.

Spanish Universities			
Alcala	Alicante	Almeria	Aut. Barcelona
Aut. Madrid	Barcelona	Burgos	Cadiz
Cantabria	Card. Herrera CEU	Carlos III	Cartagena
Castilla la Mancha	Complutense Madrid	Córdoba	Coruña
Deusto	Europea de Madrid	Extremadura	Girona
Granada	Huelva	Baleares	Jaen
Jaume I	La Laguna	La Rioja	Las Palmas
León	Lleida	Mondragón	Oviedo
Pais Vasco	Polit. Cataluña	Polit. Madrid	Polit. Valencia
Pontificia de Comillas	Málaga	Miguel Hernndez	Murcia
Navarra	P. Navarra	Pablo Olavide	Pompeu Fabra
Salamanca	Ramón Llull	Rey Juan Carlos	Rovira i Virgil
San Pablo-Ceu	Santiago Compostela	Sevilla	UNED
Valencia	Valladolid	Vigo	Zaragoza

Table 1 Set of 56 Spanish universities which was chosen to perform the comparison of research output during the period 2006-2010

4 Ranking of Spanish universities

Here we show the ranking of research output of Spanish universities during the period 2006-2010. To this aim we compute the multidimensional prestige of influential fields of study at each institution using a multivariate indicator space.

4.1 Dimensions of the multivariate indicator space

Six variables are candidates to be used in this analysis, (Torres-Salinas et al., 2011): 1) Raw number of citable papers published in scientific journals (NDOC); 2) Number of citations received by all impact citable papers (NCIT); 3) H-Index

(H); 4) Ratio of papers published in journals in the top JCR quartile $\frac{100\times N1Q}{NDOC}$ (%1Q); 5) Average number of citations received by all citable papers (ACIT); and 6) Ratio of papers that belong to the top 10% most cited (TOPCIT).

Once the set of Spanish universities was chosen (see Table 1), along with a period of time (2006-2010), the research output of each university indexed in the Science Citation Index of the ISI-Web of Knowledge (http://isiknowledge.co) was retrieved using the field “Address” as a filter and taking into account all the different names each university receives, (Torres-Salinas et al., 2011). Next, the production of each one of the universities within different fields of study is extracted. The number n of fields at each university may be lesser than or equal to 19 ($n \leq 19$). Table 2 illustrates the 19 fields of study which were used in this analysis.

A scientific work is considered to be part of a field if it was published in a journal indexed in one of the JCR journal categories in this particular field of study. In order to calculate the indicators related to journal Impact Factor, the editions of the JCRs for the period of time of interest should be used. The data were downloaded in September 2011.

Table 4 in (Torres-Salinas et al., 2011) shows correlation analysis among six bibliometric indicators (i.e., NDOC, NCIT, H, %1Q, ACIT, and TOPCIT) using data from the top 75% Spanish universities in 2000-2009. In general, it turns out that the quantitative indicators (i.e., NDOT, NCIT, and H) are positively correlated as expected, and also, but to a lesser degree, there are correlations within the qualitative ones (i.e., %1Q, ACIT, and TOPCIT). The correlations between a quantitative indicator and a qualitative one are in general very low. Thus, following (Torres-Salinas et al., 2011) we consider that this correlation is low

Fields of Study	
i	*Name*
1	Agriculture
2	Biology
3	Biochemistry, Cell and Molecular Biology
4	Food Science and Technology
5	Materials Science
6	Ecology and Environmental Sciences
7	Pharmacoloy and Toxicology
8	Genetics and Evolutionary Biology
9	Geosciences
10	Computer Science
11	Chemical Engineering
12	Medicine
13	Microbiology and Virology
14	Multidisciplinary Sciences
15	Neurosciences
16	Psychology
17	Chemistry
18	Public Health
19	Veterinary

Table 2 Fields of study which were considered in the analysis of research output of each university

enough to conclude that quantitative and qualitative indicators describe different aspects of information without loss of interpretability, as happens when using variables obtained from a Principal Component Analysis.

From these results, we define the six dimensions of the multivariate indicator space as follows: (j=1) NDOC ; (j=2) NCIT; (j=3) H-index; (j=4) %1Q; (j=5)

ACIT; and (j=6) TOPCIT. Then we have that the number of dimensions in the multivariate indicator space is $d = 6$.

For each dimension of the multivariate indicator space we must define a threshold such that fields of study at a given university with ranking score above this threshold are to be considered dimension-specific influential fields. More precisely, given a dimension-specific threshold z_j as well as scores x_{ij} which denote the ranking score of field s_i corresponding to dimension j, we have that fields of study s_i with ranking score x_{ij} above threshold z_j are dimension-specific influential fields.

For example, thresholds z_j, with $j = 1, \cdots, 6$, can be defined such that the top 30 % of the score distribution given by the corresponding ranking model (over all Spanish universities under consideration) are dimension-specific influential.

Recall that a field of study s_i at a given university is defined multidimensionally influential if it is prestigious with respect to a number of dimensions which is greater than or equal to a certain integer k, with $1 \leq k \leq d$. But in case of $k = 1$, s_i is multidimensionally prestigious when it is considered prestigious in only one dimension which can be something dangerous, (Garcia et al., 2011c). On the other hand, in case of $k = d$, it is only considered as multidimensionally influential if it is prestigious in all dimensions under consideration which is a demanding requirement and often identifies a very narrow slice of fields.

If we choose larger values for thresholds z_j and integer k (e.g., $k = 4$ and thresholds z_j are such that the top 10 % of the score distribution given by the corresponding ranking model are prestigious), we have that the ranking of Spanish universities will be based on more elitist principles. By the contrary if the values of thresholds z_j and k decrease (e.g., $k = 2$ and the top 40 % of the score distribution), it follows a more comprehensive analysis.

University of Granada														
i	*NDOC*	*NCIT*	*H*	%1Q	*ACIT*	*TOPCIT*	Θ^{0-1}						c_i	$\phi_i(z;k)$
1	144	729	13	0.7290	5.0630	0.1250	0	0	0	1	0	1	2	1
2	324	1310	15	0.4040	4.0430	0.1110	1	1	1	0	0	0	3	1
3	353	2399	23	0.3140	6.7960	0.0930	1	1	1	0	1	0	4	1
4	161	776	15	0.6960	4.8200	0.1060	1	1	1	1	0	0	4	1
5	156	934	16	0.6600	5.9870	0.1790	1	1	1	1	0	1	5	1
6	375	2096	18	0.4720	5.5890	0.1200	1	1	1	0	0	1	4	1
7	190	1373	17	0.3160	7.2260	0.1370	1	1	1	0	1	1	5	1
8	100	487	11	0.3500	4.8700	0.0300	0	0	0	0	0	0	0	0
9	705	2962	19	0.5480	4.2010	0.0950	1	1	1	0	0	0	3	1
10	484	2165	20	0.3310	4.4730	0.1710	1	1	1	0	0	1	4	1
11	102	374	10	0.6080	3.6670	0.0690	0	0	0	1	0	0	1	0
12	815	5298	30	0.4090	6.5010	0.0960	1	1	1	0	1	0	4	1
13	207	1297	17	0.3140	6.2660	0.0920	1	1	1	0	1	0	4	1
14	36	709	13	0.8330	19.6940	0.0280	0	0	0	1	1	0	2	1
15	232	1308	17	0.3230	5.6380	0.0780	1	1	1	0	0	0	3	1
16	362	1176	15	0.2210	3.2490	0.1220	1	1	1	0	0	1	4	1
17	857	5019	25	0.5640	5.8560	0.0690	1	1	1	0	0	0	3	1
18	94	378	9	0.3620	4.0210	0.0960	0	0	0	0	0	0	0	0
19	15	47	3	0.8000	3.1330	0.1330	0	0	0	1	0	1	2	1

Table 3 (First column) lists fields of study ordered as given in Table 2; (second column) NDOC, NCIT, H, %1Q, ACIT, and TOPCIT; (third column) θ_{ij} equals to one if field s_i is prestigious with respect to dimension j and zero otherwise; (fourth column) lists prestige counts $c_i = \sum_{j=1}^{d} \theta_{ij}$ that represents the number of dimensions in which field s_i is found to be influential; (fifth column) shows $\phi_i(\mathbf{z}; k)$ values which equal to one if field s_i is multidimensionally influential and is zero otherwise.

An intermediate approach corresponds to the situation in which, for example, $k = 2$ and thresholds z_j are such that the top 30 % of the score distribution given by the corresponding ranking model are dimension-specific influential.

4.2 Multidimensional prestige of influential fields at the University of Granada

In this section, we illustrate the measurement of the multidimensional prestige of influential fields at the University of Granada.

Table 3 (second column) provides information on the one-dimensional score distributions of the six dimensions under consideration: (j=1) NDOC ; (j=2) NCIT; (j=3) H-index; (j=4) %1Q; (j=5) ACIT; and (j=6) TOPCIT. Table 3 (first column) lists the 19 fields of study ordered as given in Table 2.

The multidimensional prestige $MW(k)$ was computed for $k = 2$ and thresholds z_j, with $j = 1, \cdots, 6$, such that only the top 30 % of the score distribution given by the corresponding ranking model in each dimension are dimension-specific influential. In this case we have that $z_1 = 148; z_2 = 748; z_3 = 13; z_4 = 0.57; z_5 = 6.13;$ and $z_6 = 0.11$. The value of β in equation (10) is $\beta = 3$ following the results presented in (Garcia et al., 2011a,b).

For this same university, Table 3 (fourth column) lists prestige counts $c_i = \sum_{j=1}^{d} \theta_{ij}$ which represent the number of dimensions in which field of study s_i is found to be influential, with θ_{ij} being equal to one if field s_i is prestigious with respect to dimension j and zero otherwise as given in equation (2) (third column in Table 3).

Table 3 (fifth column) shows $\phi_i(\mathbf{z}; k)$ values which equal to one if field of study s_i (at the University of Granada) is multidimensionally influential and is zero otherwise, as given in equation (4). Recall that we select $k = 2$ for this example of application.

Table 4 lists the $\theta_{ij}^{\beta}(k)$ values which are defined as:

$$\theta_{ij}^{\beta}(k) = \left(1 - \left(\frac{z_j}{x_{ij}}\right)^{\beta}\right)_{+} \cdot \phi_i(\mathbf{z}; k) \tag{11}$$

Fig. 1 illustrates the computation of the elements $\theta_{ij}^{\beta}(k)$ for a field of study s_i at a given university.

University of Granada						
	${\theta^{\beta}}_{ij}(k)$					
i	$j=1$	$j=2$	$j=3$	$j=4$	$j=5$	$j=6$
1	0.0000	0.0000	0.0000	0.5220	0.0000	0.1800
2	0.9047	0.8138	0.3490	0.0000	0.0000	0.0000
3	0.9263	0.9697	0.8194	0.0000	0.2650	0.0000
4	0.2232	0.1044	0.3490	0.4507	0.0000	0.0000
5	0.1461	0.4864	0.4636	0.3558	0.0000	0.7207
6	0.9385	0.9546	0.6233	0.0000	0.0000	0.0731
7	0.5274	0.8383	0.5528	0.0000	0.3886	0.3771
8	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
9	0.9907	0.9839	0.6797	0.0000	0.0000	0.0000
10	0.9714	0.9588	0.7254	0.0000	0.0000	0.6797
11	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
12	0.9940	0.9972	0.9186	0.0000	0.1604	0.0000
13	0.6345	0.8082	0.5528	0.0000	0.0623	0.0000
14	0.0000	0.0000	0.0000	0.6796	0.9698	0.0000
15	0.7404	0.8130	0.5528	0.0000	0.0000	0.0000
16	0.9317	0.7427	0.3490	0.0000	0.0000	0.1180
17	0.9948	0.9967	0.8594	0.0000	0.0000	0.0000
18	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
19	0.0000	0.0000	0.0000	0.6383	0.0000	0.3192
$\frac{1}{n}\sum_{i=1}^{n}\theta^{\beta}_{ij}(k)$	0.5223	0.5509	0.4103	0.1393	0.0972	0.1299

Table 4 (First column) lists fields of study at the University of Granada ordered as given in Table 2; (second column) lists $\theta^{\beta}_{ij}(k)$ elements given in equation (11).

Since, from equation (10), the summary measure $MW(k)$ of multidimensional prestige of influential fields at a given university is equal to the sum of elements $\theta^{\beta}_{ij}(k)$ divided by the value $n\times d$, it follows that $MW(k)=0.3083$ for the University

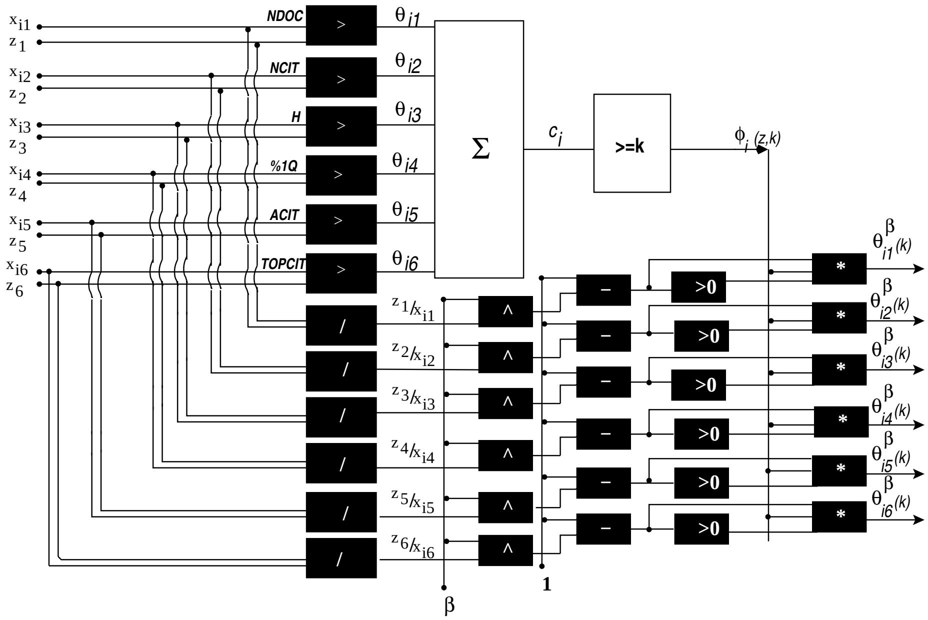

Fig. 1 Computation of $\theta_{ij}^{\beta}(k)$ values for a field of study s_i at a given university.

of Granada. Again, Table 4 (first column) lists fields of study ordered as given in Table 2 .

In addition to looking at the overall value of multidimensional prestige of influential fields at the University of Granada, we can provide information on how different dimensions of the multivariate indicator space contribute to the measure $MW(k)$ of multidimensional prestige. To this aim, we rewrite equation (10) as follows:

$$MW(k) = \frac{1}{d}\sum_{j=1}^{d}\frac{\sum_{i=1}^{n}\theta_{ij}^{\beta}(k)}{n} = \frac{1}{d}\sum_{j=1}^{d}\Pi_j^{\beta}(k) \qquad (12)$$

where $\Pi_j^{\beta}(k) = \frac{1}{n}\sum_{i=1}^{n}\theta_{ij}^{\beta}(k)$ represents the contribution of each dimension j (multiplied by the number d of dimensions) to the measurement of multidimensional prestige of influential fields.

To the University of Granada, from Table 4 (bottom) we have that the contribution $\Pi_j^\beta(k)$ of the NCIT dimension ($j = 2$) is about 29.7% of the multidimensional prestige, and taken together, the NDOC, NCIT, and H dimensions make up about 80.19% of the multidimensional prestige of influential fields of study at this university. Hence, the NDOC, NCIT and H dimensions play a dominant role to the measurement of the multidimensional prestige $MW(k)$ for the University of Granada.

4.3 Results

In this section, we use the summary measure of multidimensional prestige $MW(k)$ to assess the comparative performance of selected Spanish universities during the period 2006-2010. Fifty-six main universities in Spain are considered in this experiment.

Tables 5 and 6 show the ranking of the 56 Spanish universities according to the multidimensional prestige $MW(k)$ of influential fields of study, for different selections of k and thresholds z_j with $j = 1, \cdots, 6$. For our analysis the university with the best value of the multidimensional prestige of influential fields is assigned the rank #1, the second best #2, and so.

In order to produce the results given in Tables 5 and 6, thresholds z_j with $j = 1, \cdots, 6$ were defined such that the top 20% (or alternatively 30%, and 40%) of the score distribution given by the corresponding journal ranking model (over all selected Spanish universities) are dimension-specific influential. For example, in case of the top 30% we have that $z_1 = 148; z_2 = 748; z_3 = 13; z_4 = 0.57; z_5 = 6.13;$ and $z_6 = 0.11$.

Ranking of Spanish Universities										
University	$k=2$			$k=3$			$k=4$			*Median*
	20%	30%	40%	20%	30%	40%	20%	30%	40%	
BARCELONA	1	1	1	1	1	1	1	1	1	1
AUTÓNOMA DE BARCELONA	2	2	2	2	2	2	2	2	2	2
COMPLUTENSE DE MADRID	3	3	3	5	3	3	6	4	5	3
VALENCIA	4	4	4	3	4	4	3	3	4	4
AUTÓNOMA DE MADRID	5	5	5	4	5	5	5	5	3	5
POMPEU FABRA	6	7	8	6	6	8	4	6	8	6
GRANADA	7	6	7	7	7	7	25	10	6	7
SANTIAGO DE COMPOSTELA	8	8	6	8	8	6	15	9	7	8
SEVILLA	10	10	9	9	9	9	21	7	9	9
ZARAGOZA	11	9	10	10	10	10	12	8	10	10
POLITÉCNICA DE VALENCIA	9	11	11	11	11	12	8	11	12	11
ROVIRA I VIRGILI	16	13	13	15	12	13	20	15	13	13
CASTILLA-LA MANCHA	19	14	12	19	15	11	32	14	11	14
CÓRDOBA	12	18	16	12	16	16	7	12	14	14
VIGO	13	12	14	16	13	15	10	16	15	14
PAÍS VASCO	17	16	15	13	14	14	27	21	19	16
MURCIA	20	22	22	17	19	17	16	13	16	17
REY JUAN CARLOS	14	15	18	14	17	18	14	17	21	17
ISLAS BALEARES	23	19	17	24	18	19	17	20	20	19
MIGUEL HERNÁNDEZ	22	21	19	18	21	20	19	18	17	19
OVIEDO	18	23	21	20	23	22	9	22	23	22
JAUME I DE CASTELLÓN	15	17	23	27	22	24	18	23	31	23
NAVARRA	28	24	25	23	20	25	11	19	22	23
LLEIDA	25	20	20	36	26	21	46	24	25	25
SALAMANCA	24	26	26	21	25	26	13	25	18	25
POLITÉCNICA DE CATALUÑA	21	25	28	22	24	27	49	40	30	27
ALICANTE	29	29	30	28	28	32	22	26	27	28

Table 5 Ranking of Spanish universities during the period 2006-2010 according to the multidimensional prestige $MW(k)$ of influential fields, for different selections of k and thresholds z_j.

Regarding the value of k, here we follow an intermediate approach, and thus, a field s_i at a given university is defined multidimensionally influential if it is prestigious with respect to a number of dimensions which is greater than or equal to a certain integer k with $1 < k < 6$. Thus the multidimensional prestige $MW(k)$ was computed for different values of k, with $k = 2, 3$, and 4.

Ranking of Spanish Universities										
University	$k = 2$			$k = 3$			$k = 4$			*Median*
	20%	30%	40%	20%	30%	40%	20%	30%	40%	
LA LAGUNA	31	28	24	26	29	23	43	31	24	28
ALCALÁ DE HENARES	26	27	29	29	27	29	31	27	32	29
MÁLAGA	33	34	27	33	36	28	30	29	26	30
EXTREMADURA	30	32	32	30	31	31	40	28	28	31
PABLO DE OLAVIDE	32	31	35	38	32	34	24	30	34	32
GIRONA	39	36	33	39	33	33	26	34	33	33
CANTABRIA	27	30	34	25	34	37	33	38	35	34
ALMERÍA	41	38	36	37	35	35	23	32	29	35
BURGOS	34	37	39	34	37	40	28	33	44	37
LA RIOJA	37	35	38	35	39	41	44	35	43	38
CÁDIZ	52	42	37	43	41	36	37	39	36	39
POLITÉCNICA DE MADRID	45	33	31	42	30	30	50	41	40	40
CORUÑA. A	38	43	44	31	40	42	36	44	42	42
JAÉN	50	45	41	41	44	38	42	47	37	42
VALLADOLID	40	44	42	32	38	39	56	56	46	42
PALMAS (LAS)	46	47	48	40	43	45	29	37	39	43
HUELVA	54	51	47	49	45	43	41	36	38	45
CARLOS III	42	49	46	46	51	46	35	43	49	46
LEÓN	49	46	43	50	47	44	45	48	41	46
POLITÉCNICA DE CARTAGENA	44	39	40	52	54	47	48	50	47	47
U.N.E.D.	48	48	45	56	42	48	55	55	56	48
CARDENAL HERRERA-CEU	55	55	55	45	50	53	34	42	48	50
DEUSTO	43	50	53	47	52	54	38	45	50	50
EUROPEA DE MADRID	53	54	54	48	46	50	39	46	51	50
PÚBLICA DE NAVARRA	51	52	51	44	48	49	52	52	45	51
RAMÓN LLULL	36	40	49	54	49	51	53	53	54	51
MONDRAGÓN	56	56	56	51	53	55	47	49	52	53
PONTIFICIA DE COMILLAS	47	53	52	53	55	56	51	51	53	53
SAN PABLO-CEU	35	41	50	55	56	52	54	54	55	54

Table 6 Ranking of Spanish universities during the period 2006-2010 according to the multidimensional prestige $MW(k)$ of influential fields, for different selections of k and thresholds z_j.

Recall that if we choose larger values for thresholds z_j (e.g., only the top 20% of the score distribution are dimension-specific influential), we have that the ranking of Spanish universities will be based on more elitist principles.

By the contrary if the values of thresholds z_j decrease (e.g., the top 40% of the score distribution are dimension-specific influential), it follows a more comprehensive analysis.

Looking for a general pattern of rankings across all the above selections for k and thresholds z_j, from Table 5 we have that the top ten Spanish universities were (based on the Median rank): (1) Barcelona; (2) Autónoma de Barcelona; (3) Complutense de Madrid; (4) Valencia; (5) Autónoma de Madrid; (6) Pompue Fabra; (7) Granada; (8) Santiago de Compostela; (9) Sevilla; and (10) Zaragoza.

This result is congruent with those from other academic ranking studies (Shanghai Jiao Tong University, 2011). It should be pointed out that we have been able to report these results without assigning weights, since the various scores on different dimensions can be combined into a single score that reflects overall quality of a given university. Our ranking follows rigorous methodological criteria and thus may constitute an effective instrument for quality assessment of universities. The three main characteristics of our data were: (1) Internationally comparable data; (2) quantitative and qualitative indicators; and (3) open to verification.

5 Conclusions

Here we have presented a comparison of 56 Spanish universities based on the measurement of multidimensional prestige of influential fields of study during the period 2006-2010.

The multidimensional prestige takes into account that several indicators should be used for a distinct analysis of structural changes at the score distribution of field prestige. We argue that the prestige of influential field of study at a given university should not only consider one indicator as a single dimension, but in addition take into account further dimensions.

After having identified the multidimensionally influential fields of study at a given university, their prestige scores can be aggregated to produce a summary measure of multidimensional prestige for this university which satisfies numerous properties (following an axiomatic approach).

What are the limitations of the proposed approach? It is not rare that one would like to impose more axioms that are jointly compatible. It may also happen that the summary measure resulting from the original list of axioms is found to react very bad to some significant institution. One must then formalize the characteristics of the particular institution and state an additional axiom that specifies how the criterion should behave in this situation, and finally determine the greatest subset of axioms from the original list that are compatible with the new axiom. Of course, compatibility may hold for several distinct such subsets.

From the results showed in this paper, the top three Spanish universities (during the period 2006-2010) were: (1) Barcelona; (2) Autónoma de Barcelona; and (3) Complutense de Madrid.

In this paper we argue that this type of analysis, for example, may be relevant to the evaluation of research output using objective metrics in several quantitative and qualitative dimensions, which may guide student choice of a university to pursue a graduate degree or funding agencies to make their decisions regarding the allocation of limited funds.

We are developing a publicly available suite of Web-based tools designed to facilitate analysis of Spanish universities using the proposed approach. It will be freely available at: http://cvg.ugr.es/scientometrics

Acknowledgments. This research was sponsored by the Spanish Board for Science and Technology (MICINN) under grant TIN2010-15157 cofinanced with European FEDER funds.

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A Appendix: Set of Axioms

A first axiom states that a field of study at the given university which is not multidimensionally prestigious should not influence a summary measure of the overall prestige of multidimensionally influential fields.

Axiom 1. *Given two configurations of dimension-specific scores* $\mathbf{X}$ *and* $\mathbf{X}'$ *of the same size* $n \times d$ *where the scores of multidimensionally influential fields at the university are the same in both cases, the summary measure of the multidimensional prestige of influential fields measured on either configuration should give the same value.*

Now, a second axiom can be justified on the idea that small changes in the configuration of dimension-specific scores for multidimensionally influential fields of study shall not lead to discontinuously large changes in the summary measure of multidimensional prestige.

Axiom 2. *The summary measure of the multidimensional prestige of influential fields at a given university should be a continuous function of dimension-specific scores for multidimensionally influential fields.*

In the following, a third axiom states than an increment in some dimension-specific score (above the corresponding threshold z_j) for a multidimensionally influential field of study shall increase the summary measure.

Axiom 3. *An index of multidimensional prestige of influential fields should increase whenever some dimension-specific score (above threshold z_j corresponding to that dimension) rises for a multidimensionally influential field of study.*

Next an axiom states a property of subgroup decomposability. That is, the index has to be additively decomposable, i.e., the index of overall prestige is a weighted sum over several subgroups of fields of study in which the complete set U can be partitioned.

Axiom 4 *The overall prestige of multidimensionally influential fields can be decomposed into the weighted sum of subgroup-prestige indices.*

And the following axiom requires that the summary measure of multidimensional prestige of influential fields shall increase after a progressive transfer (from a more influential field of study to a less prestigious one) of domain-specific scores above the corresponding threshold z_j between two multidimensionally influential fields at the university.

Axiom 5 *An overall prestige index should increase when a rank-preserving progressive transfer (above the corresponding domain-specific threshold) between two multidimensionally influential fields at a given university takes place.*

B Appendix: Proof of Theorem 1

Proof Given a configuration $\mathbf{X}$, let MW be a normalized weighted sum of the dimension-specific scores in $\mathbf{X}$ using weighting function f

$$MW = \frac{1}{n \times d} \sum_{i=1}^{n} \sum_{j=1}^{d} f\left(\frac{x_{ij}}{z_j}\right) \tag{13}$$

where we have that f should be a continuous function for multidimensionally influential fields of study in order to satisfy Axiom 2, i.e., to verify that small changes in the configuration of dimension-specific scores (for multidimensionally influential fields at the university) shall not lead to discontinuously large changes in the summary measure MW.

But also it follows that weighting function f should be a strictly increasing function for multidimensionally influential fields of study at the university, since Axiom 3 states that an increment in some dimension-specific score (above the corresponding threshold z_j) for a multidimensionally influential field shall increase the summary measure of multidimensional prestige MW.

From Axiom 1, a field of study which is not multidimensionally prestigious should not influence the overall prestige MW, i.e., MW is independent of the dimension-specific scores for fields of study at the given university which are not multidimensionally influential. Hence to fulfill Axiom 1 we have that

$$f\left(\frac{x_{ij}}{z_j}\right) = 0 \tag{14}$$

for all i such that $\phi_i(\mathbf{z}; k) = 0$; where $\phi_i(\mathbf{z}; k)$ equals to one if field s_i is multidimensionally prestigious and zero otherwise, as given in equation (4).

Now, from Axiom 4, the summary measure MW can be decomposed into the weighted sum of subgroup prestige indices. Thus it follows that the measure MW has to be additively decomposable.

Finally, following Axiom 5, the summary measure of multidimensional prestige MW should increase after a progressive transfer (from a more influential field of study to a less prestigious one) of domain-specific scores above the corresponding threshold z_j between two multidimensionally influential fields at the university under consideration. Hence we have that weighting function f has to be concave for multidimensionally influential fields, and thus, the relative dimension-specific scores $\frac{x_{ij}}{z_j}$ then have to be transformed by a function that is concave on $(1, \infty)$ for multidimensionally influential fields of study.

For example, given a multidimensionally influential field s_i, we have that

$$f\left(\frac{x_{ij}}{z_j}\right) = \left(1 - \left(\frac{z_j}{x_{ij}}\right)^{\beta}\right) \cdot \phi_i(\mathbf{z}; k)$$

is concave for $x_{ij} > z_j$ and $\beta > 0$.

To sum up, following Axiom 1 through Axiom 5, the summary measure MW

$$MW = \frac{1}{n \times d} \sum_{i=1}^{n} \sum_{j=1}^{d} f\left(\frac{x_{ij}}{z_j}\right) \tag{15}$$

shall satisfy that $f : R_+ \rightarrow [0,1]$ is a strictly increasing and concave function on $(1,\infty)$ for multidimensionally influential fields s_i at the given university.

Following (Peichl and Pestel, 2010), if we define weighting function f as:

$$f\left(\frac{x_{ij}}{z_j}\right) = \left(1 - \left(\frac{z_j}{x_{ij}}\right)^{\beta}\right)_+ \cdot \phi_i(\mathbf{z};k) \tag{16}$$

where $(v)_+ = \max(v,0)$, we obtain a summary measure of the multidimensional prestige of influential fields, that resembles equation (10) satisfying Axiom 1 through Axiom 5, since f being defined as given in equation (16) it is a strictly increasing and concave function $f : R_+ \rightarrow [0,1]$ on $(1,\infty)$ for multidimensionally influential fields s_i. □