Universidad de Granada

Departamento de Ciencias de la Computación
e Inteligencia Artificial

New Group Decision Making Models with Heterogeneous Information Based on Different Frameworks: Changeable Contexts, Non-Homogeneous Experts and Web 2.0

Tesis Doctoral

Ignacio Javier Pérez Gálvez

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MEMORIA QUE PRESENTA

Ignacio Javier Pérez Gálvez

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Dr. Enrique Herrera Viedma

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La memoria titulada "*New Group Decision Making Models with Heterogeneous Information Based on Different Frameworks: Changeable Contexts, Non-Homogeneous Experts and Web 2.0*", que presenta D. Ignacio Javier Pérez Gálvez para optar al grado de doctor, ha sido realizada dentro del Máster Oficial de Doctorado "*Soft Computing y Sistemas Inteligentes*" del Departamento de Ciencias de la Computación e Inteligencia Artificial de la Universidad de Granada bajo la dirección del doctor D. Enrique Herrera Viedma.

Granada, Abril de 2011

El Doctorando   El Director

Fdo: Ignacio Javier Pérez Gálvez   Fdo: Dr. Enrique Herrera Viedma

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1. Introducción

La Toma de Decisión, es decir, el proceso de seleccionar la mejor alternativa/as de entre un conjunto es una tarea muy común en todas nuestras actividades diarias. Por tanto, el estudio de situaciones de Toma de Decisión y de los mecanismos que permiten resolver esta clase de problemas es fundamental no sólo en Teoría de Decisión, sino también en otras áreas de investigación como la Inteligencia Artificial, Economía, Sociología, Ingeniería, etc.

Sin embargo, los modelos de decisión básicos tienen poco en común con los modelos de decisión reales. Muchos procesos de Toma de Decisión reales se desarrollan en ambientes donde los objetivos, restricciones y alternativas no son conocidos con precisión o no están bien definidos, y es necesario modelar esta incertidumbre. El profesor Zadeh propuso en 1965 una manera práctica y poderosa de tratar dicha incertidumbre en el conocimiento humano: La Teoría de Conjuntos Difusos [Zad65]. La aplicación de la Teoría de Conjuntos Difusos, para resolver la incertidumbre en la información en los procesos de Toma de Decisión, fue propuesta por Bellman y Zadeh en 1970 [BZ70] y, desde ese momento, se ha utilizado con frecuencia debido a su contrastada utilidad en este campo. Su principal cualidad es la de presentar un entorno de trabajo mucho más flexible, donde es posible representar la imprecisión, tanto cualitativa como cuantitativa, de los juicios humanos. Esto permite solucionar satisfactoriamente muchos de los problemas derivados de la pérdida de información.

Por otro lado, es normal que los problemas de decisión necesiten de un análisis de las diferentes alternativas y del problema al que nos enfrentamos. La Toma de Decisión intenta ayudar a los individuos a tomar decisiones difíciles y complejas de una forma racional. Esta racionalidad implica el desarrollo de métodos y modelos que permitan representar fielmente cada problema y analizar las distintas alternativas con criterios objetivos. Sin embargo, no todo problema de decisión se resuelve por medio de un proceso completamente racional. De hecho, muchos factores externos y subjetivos afectan a los procesos de decisión y, por lo tanto, la solución final puede variar si las condiciones en las que se presenta el problema cambian.

Un proceso de Toma de Decisión en el que participen varios individuos o expertos, cada uno de ellos aportando sus propios conocimientos, experiencia y creatividad, proporcionará una decisión de mayor calidad que aquella aportada por un único experto. Por esta razón, el estudio de problemas de Toma de Decisión en Grupo (TDG) [KF90, Rou97] ha sido ampliamente tratado en la literatura.

Un problema de TDG se da en aquellas situaciones donde hay una cuestión común a resolver, un conjunto de opciones o alternativas posibles a escoger, $X = \{x_1, \ldots, x_n\}$, $(n \geq 2)$, y un conjunto de individuos (expertos, jueces, etc.), $E = \{e_1, \ldots, e_m\}$, $(m \geq 2)$, que expresan sus opiniones o preferencias sobre el conjunto de opciones o alternativas. El objetivo es encontrar una solución, que será una o un conjunto de alternativas, que sea la de mayor aceptación por parte de todo el grupo de expertos. A veces, existe una persona, llamada moderador [HHVV96b, KFN92], que no participa en el proceso de discusión y que se encarga de dirigir todo el proceso de resolución del problema de TDG y de ayudar a los expertos a aproximar sus preferencias sobre las alternativas hasta que éstos logran un acuerdo sobre la solución a escoger.

En todo proceso de TDG, son dos los procesos a desarrollar antes de obtener una solución [HHVV96b]: el proceso de consenso y el proceso de selección. Ambos procesos han sido objeto de estudio por diversos autores en diferentes contextos de TDG [FR94, KF90]. El primero, conocido también con el nombre de consenso topológico, hace referencia a cómo alcanzar el máximo grado de consenso o acuerdo entre los individuos o expertos sobre el conjunto de alternativas solución. Este proceso suele estar coordinado por el moderador [HHVV96b, KFN92], que se encarga de controlar el proceso de negociación y de ayudar a los expertos a aproximar sus preferencias. El segundo, conocido también con el nombre de consenso algebraico, hace referencia a cómo obtener el conjunto de alternativas solución a partir de las opiniones expresadas por los individuos o expertos. Ambos procesos actúan conjuntamente de forma secuencial. Primero, el proceso de consenso actúa para lograr alcanzar el máximo grado de consenso posible entre las opiniones de los individuos o expertos. Cuando los expertos han expresado sus opiniones, el moderador calcula el grado de consenso existente. Si el grado es satisfactorio, entonces el proceso de selección se aplica de cara a obtener la solución. Por el contrario, si el grado de consenso medido no es satisfactorio, entonces el moderador insta a los individuos o expertos a modificar sus opiniones de cara a aumentar la proximidad en sus planteamientos. De este modo, un proceso de TDG se puede definir como un proceso dinámico e iterativo en el que los individuos o expertos van cambiando sus opiniones hasta que sus planteamientos sobre la solución son lo suficientemente próximos, momento en el que se obtiene la solución de consenso mediante la aplicación del proceso de selección. Este procedimiento puede observarse gráficamente en la Figura 1.

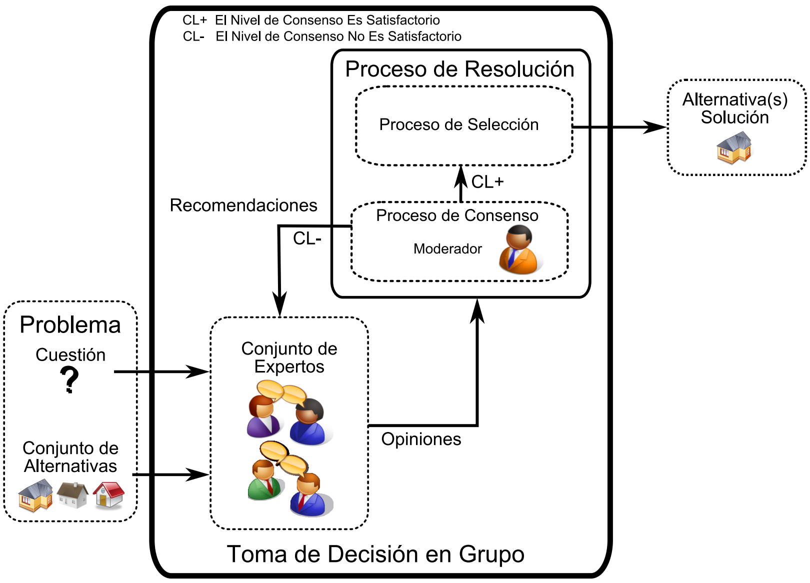

Figure 1: Esquema del proceso de toma de decisión en grupo

Tanto el proceso de consenso como el proceso de selección se describen con mayor detalle a continuación.

1. **Proceso de Consenso:**

   El proceso de consenso hace referencia a cómo alcanzar el mayor grado de acuerdo o coincidencia entre los individuos o expertos sobre el conjunto solución de alternativas, y constituye un área de investigación importante en el campo de la Toma de Decisión en Grupo [AHVC+07, AHVCH10, BAC06, SSB80, CMPHV10, HHVV96b, HHVV97b, HVHC02, HVACH07, KF88, KFN92, MMHV09].

   Normalmente, al inicio de todo problema de TDG, las opiniones de los expertos suelen diferir sustancialmente. En esta situación, consideramos que es apropiado que los expertos cambien sus preferencias y tiendan a aproximar sus opiniones. De esta forma, se consigue que todos los expertos cedan en sus pretensiones iniciales en pos de la búsqueda del consenso y que ninguno de ellos rechace la solución obtenida por considerar que él sí ha cambiado sus preferencias y el resto no. Por tanto, es importante desarrollar procesos de consenso en un intento de obtener una solución al problema sobre la que dicho conjunto de expertos muestre cierto grado de aceptación.

   El proceso de consenso puede dividirse en varios pasos que están representados en la Figura 2:

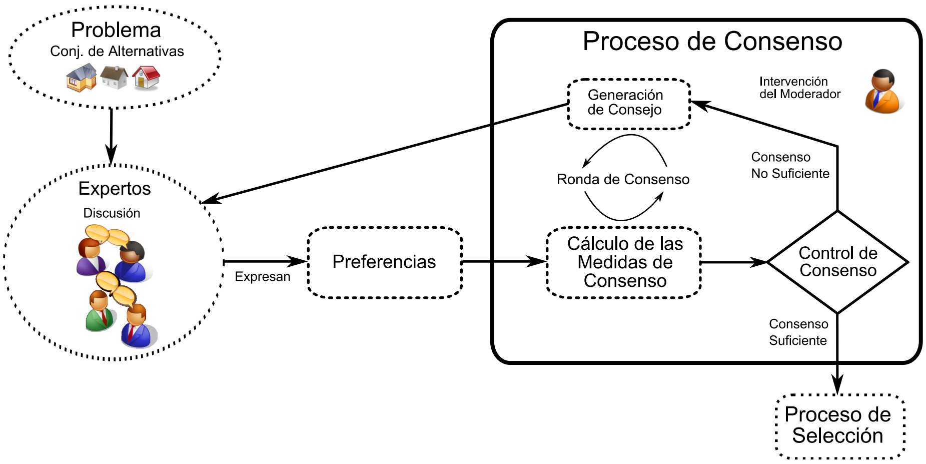

Figure 2: Fases del proceso de consenso

   a) En primer lugar, se debe presentar el problema a resolver a los expertos, junto con el conjunto de las distintas alternativas sobre las cuales debe elegir la mejor (o mejores).
   b) A continuación los expertos pueden discutir y compartir su conocimiento sobre el problema y las alternativas para hacer más fácil el siguiente paso en el que tienen que expresar sus opiniones.
   c) Los expertos expresan sus preferencias sobre las alternativas en un formato de representación de preferencias específico.
   d) El moderador recibe todas las preferencias de los expertos y calcula algunas medidas de consenso que le permitirán saber si se ha alcanzado un nivel de consenso suficiente o no.
   e) Si se ha alcanzado suficiente grado de consenso el proceso de consenso finaliza y comienza el proceso de selección. En caso contrario, se puede aplicar un mecanismo de generación de recomendaciones donde el moderador, con toda la información que posee (las preferencias expresadas por los expertos, medidas de consenso, etc.) puede preparar algunas recomendaciones o pistas para los expertos sobre cómo deben cambiar sus preferencias para alcanzar más fácilmente un estado de consenso. Hay que hacer notar que este paso es opcional y no tiene por tanto que estar presente en todos los modelos de consenso.

f) Por último, se les presentan a los expertos los consejos o recomendaciones del moderador y acaba la primera ronda de consenso. Otra vez los expertos deben discutir acerca de las alternativas para acercar sus puntos de vista (paso b).

2. **Proceso de Selección:**

Una vez que el proceso de consenso ha finalizado, esto es, se ha alcanzado un nivel de consenso suficiente, se aplica el proceso de selección [BGKA09, HHVV95, SMY10, FR94, Tan84].

La selección de alternativas es el proceso mediante el cual se obtiene el conjunto de alternativas solución a partir de las preferencias individuales sobre el conjunto de alternativas de cada uno de los expertos implicados en el proceso de TDG. Para conseguir este objetivo, se ha de tener claro el criterio global o de conjunto a aplicar en la elección de las alternativas que formarán parte del conjunto solución. El proceso de selección se puede dividir en dos fases distintas [Rou97](ver Figura 3):

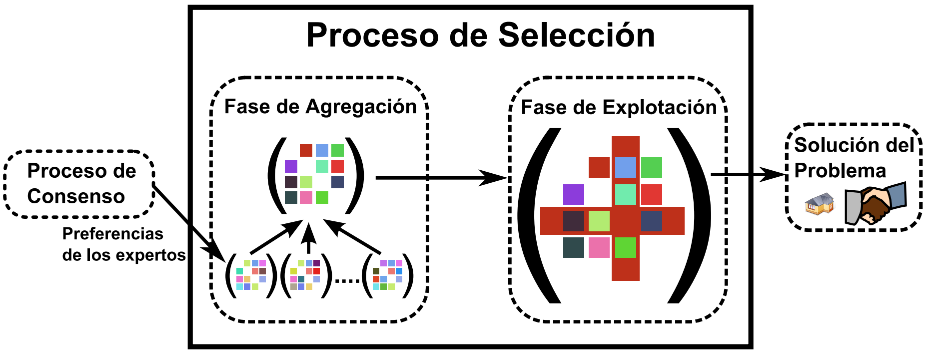

Figure 3: Fases del proceso de selección

a) *Fase de Agregación:* En esta fase todas las preferencias dadas por los expertos deben agregarse en una sola estructura de preferencia colectiva. Esta agregación se suele llevar a cabo por medio de operadores de agregación que se definen específicamente para esta tarea. Este paso puede ser más complicado si nos encontramos ante una situación de TDG heterogénea (ya sea porque tengamos expertos con distinto grado de importancia o porque tengamos distintos formatos de representación de preferencias), ya que se hace necesario algún tipo de homogeneización que transforme todas las estructuras de representación de preferencias en una concreta que sirva como base para la agregación, y además el operador de agregación debe ser capaz de tratar adecuadamente los pesos asignados a los expertos (es decir, dar más importancia a las preferencias de algunos expertos que a las de otros).

b) *Fase de Explotación:* En este paso final se usa la información obtenida en la fase de agregación de preferencias para identificar el conjunto de alternativas solución para el problema. Para hacerlo se debe aplicar algún mecanismo que permita obtener un orden parcial de las alternativas y posteriormente seleccionar la mejor (o mejores). Existen diversas formas de conseguir esto, pero la más común es asociar un cierto valor de utilidad a cada alternativa (basándonos en la información agregada), y por lo tanto produciendo un orden natural entre las alternativas.

A lo largo de esta memoria prestaremos atención a distintas situaciones de TDG, estudiando y analizando los modelos de TDG actuales y tratando de mejorarlos. Para llevar a cabo este estudio, esta memoria se divide en dos partes. En la primera de ellas se puede encontrar el planteamiento general del problema y la discusión conjunta de resultados. La segunda parte recopila las publicaciones asociadas a este estudio.

En la primera parte comenzamos presentando el desarrollo de algunas situaciones del problema de TDG introducido en esta sección junto con las posibles técnicas propuestas para resolver cada una de ellas. En la subsección 1.1 se presentan los diferentes enfoques existentes para cuantificar el consenso en problemas de TDG. La subsección 1.2 describe los problemas de TDG con información heterogénea en contextos variables. En la subsección 1.3 se tratan los problemas TDG que requieren la expresión de opiniones por parte de expertos con diferente nivel de importancia y la subsección 1.4 plantea las situaciones de TDG en comunidades Web 2.0. Seguidamente, en la sección 2, indicamos las razones que justifican la realización de este estudio. Los objetivos perseguidos en esta memoria los podemos encontrar en la sección 3. La sección 4 contiene un resumen de las diferentes propuestas y los resultados más relevantes obtenidos para cada situación estudiada. La sección 5 muestra algunos comentarios finales y las conclusiones obtenidas y, para finalizar la primera parte de la memoria, la seccion 6 presenta varios posibles trabajos futuros que han surgido a partir de los resultados más interesantes que se han obtenido en este estudio.

Finalmente, para desarrollar más amplia y detalladamente los planteamientos y resultados de este estudio, la segunda parte de esta memoria incluye 8 publicaciones distribuidas en 4 secciones:

1. Análisis de las Medidas de Consenso en la Toma de Decisión en Grupo: Ventajas e Inconvenientes.

2. Un Sistema de Apoyo a la Decisión Móvil Basado en Información Heterogénea y en Contextos Variables.

3. Un Modelo de Consenso para problemas de Toma de Decisión en Grupo con Expertos no Homogéneos.

4. Un Modelo de Consenso Lingüístico para Comunidades Web 2.0.

1.1. Propuestas para Medir el Consenso en Problemas de Toma de Decisión en Grupo

Normalmente, en un problema de TDG, el proceso de consenso es dirigido por el moderador [HHVV96b, KFN92], que es una persona que no participa en el proceso de discusión pero que tiene un profundo conocimiento sobre el problema, conoce el estado del acuerdo en cada momento del proceso y está a cargo de supervisar la discusión para que tenga éxito, intentando alcanzar el máximo grado de acuerdo posible y reducir el número de expertos que están en desacuerdo en cada ronda del proceso. Al comienzo de cada problema de TDG, el conjunto de expertos tiene distintas opiniones, entonces se aplica el ya mencionado proceso de consenso y, en cada paso, se mide el grado de acuerdo existente entre los expertos. Si el grado de acuerdo es menor que un límite específico, el moderador instará a los expertos a discutir sobre sus opiniones y hacer un esfuerzo para hacerlas más parecidas entre sí. En caso contrario, el moderador aplicará el proceso de selección para obtener la solución final del problema. De este modo, los expertos, a través del intercambio de información y argumentos racionales, van modificando sus opiniones hasta alcanzar suficiente grado de similitud entre ellas.

La pregunta que surge en este momento es cómo podemos medir la cercanía entre las opiniones de los expertos para obtener el nivel de consenso. Para ello se han propuesto distintas alternativas, por ejemplo, varios autores han propuesto que el consenso se puede medir de forma estricta como 0 (consenso parcial o ausencia de consenso) ó 1 (consenso total) [BSS78]. Utilizando estas medidas de consenso, en [SBS79] se propone una forma de cuantificar lo que falta para alcanzar el consenso total haciendo uso de una extensión del coeficiente de Tanimoto. Además, en [SSB80] se presenta una nueva medida de consenso basada en los a-cortes de las matrices de preferencias.

Sin embargo, el consenso total, visto como un acuerdo unánime de los expertos, no suele ser alcanzado en las situaciones reales de TDG, incluso cuando esto pudiera ocurrir, el proceso para conseguirlo sería demasiado costoso. Por tanto, en la práctica se usa un enfoque más realista como el uso de medidas más suaves de consenso [Kac86, KF88], que evalúan el grado de consenso de forma más flexible y, consecuentemente, consideran varios posibles grados de acuerdo parcial. Estas medidas están basadas en el concepto de coincidencia [HHVV97a], y pueden ser calculadas a partir de las opiniones de los expertos definiendo algunos criterios de similitud.

El propósito de esta parte del estudio es identificar los diferentes enfoques para calcular medidas suaves de consenso en problemas de TDG propuestos en la literatura y analizar sus ventajas e inconvenientes. Para ello, comenzamos identificando tres criterios de similitud diferentes: similitud entre preferencias estricta, similitud entre preferencias suave y similitud entre soluciones. Seguidamente continuamos estudiando la aplicación de estas medidas en procesos de consenso y analizamos sus ventajas e inconvenientes. Además, para finalizar esta parte del estudio, describiremos las nuevas tendencias, con las que haciendo uso de los anteriores criterios de similitud, se consigue generar recomendaciones para ayudar a los expertos a cambiar sus opiniones de forma que se alcance el mayor grado de consenso posible. Dando un paso más, se pueden diseñar procesos de consenso adaptativos que reduzcan el número de cambios de opinión necesarios por parte de los expertos en cada ronda del proceso según se va incrementando el nivel de acuerdo alcanzado.

1.2. Toma de Decisión en Grupo con Información Heterogenea y Contextos Variables

Recientemente, varios autores han estudiado y abordado los problemas de TDG desde diferentes puntos de vista, demostrando que este tipo de problemas no son siempre homogéneos. De echo, podemos establecer una clasificación en tres niveles de heterogeneidad distintos:

1. El primer nivel de heterogeneidad estudiado en la literatura [CHHV98, CHHV01] se centra en las estructuras de representación de preferencias. Normalmente, cada experto e_h expresa sus preferencias sobre las alternativas $X = \{x_1, x_2, ..., x_n\}$, utilizando un formato específico. Los más frecuentemente utilizados son:

    - *Ordenes de Preferencia de Alternativas:* $O^h = \{o^h(1), ..., o^h(n)\}$, donde $o^h(\cdot)$ es una función de permutación sobre el conjunto de índices, $\{1, ..., n\}$, para el experto e_h, definiendo un vector de alternativas ordenado, de mejor a peor.
    - *Funciones de Utilidad:* $U^h = \{u_1^h, ..., u_n^h\}$, $u_i^h \in [0, 1]$, donde u_i^h representa el valor de utilidad de x_i para el experto e_h.
    - *Relaciones de Preferencia Difusas:* $P^h \subset X\text{x}X$, con una función de pertenencia, $\mu_{P^h} : X\text{x}X \to [0, 1]$, donde $\mu_{P^h}(x_i, x_j) = p_{ij}^h$ determina el grado de preferencia de x_i sobre x_j para el experto e_h.
    - *Relaciones de Preferencia Multiplicativas:* $A^h \subset X\text{x}X$, donde la intensidad de preferencia, a_{ij}^h, se mide con una escala de razón, concretamente entre 1/9 y 9.

Las relaciones de preferencia son las más utilizadas en este tipo de problemas debido a que contienen más cantidad de información que los órdenes de preferencia o los valores de utilidad [CHHV98], permitiendo la comparación por pares de alternativas. Por tanto, los usuarios tienen más libertad para expresar sus preferencias que si usan otro tipo de formato de representación. Cuando el número de alternativas es pequeño, la relación de preferencias puede ser representada como una matriz cuadrada $n \times n$ tal que $P^k = (p_{ij}^k)$.

2. El segundo nivel de heterogeneidad tiene que ver con el dominio de representación de preferencias (numérico, lingüístico, multigranular, intervalar, etc.) [ACC$^+$09, CAHV09, HHVM08, MMHV09].

   Existen situaciones en las que la información no puede ser representada de forma cuantitativa. Cuando evaluamos algunos fenómenos que no son exactos y dependen de la percepción de cada uno, solemos utilizar palabras en lenguaje natural en lugar de valores numéricos. Por ejemplo, para evaluar la calidad de un jugador de fútbol, podemos usar términos como *bueno*, *regular* o *malo*.

   El modelado lingüístico ordinal [HHV97, HHVV96a] es una herramienta basada en el concepto de variable lingüística [Zad75] para expresar evaluaciones de forma cuantitativa. Este tipo de modelado simplifica el proceso de operar con palabras y su utilidad ha sido demostrada en diferentes tipos de problemas, por ejemplo, en toma de decisiones [BAC06, CAHV09, HHV00b, HVMMC05], evaluación de calidad web [HVP03, HVPLHP06, HVPM$^+$07], recuperación de información [BP01, HV01, HVLH07], sistemas de recomendaciones [PLHHV09, PMHV09], análisis político [Arf05], etc. El modelado lingüístico ordinal se define considerando un conjunto de etiquetas finito y ordenado $S = \{s_i\}, i \in \{0, ..., g\}$ con cardinalidad impar (normalmente 7 o 9 etiquetas). Por ejemplo, podríamos usar el siguiente conjunto $S = \{s_0 = N, s_1 = VL, s_2 = L, s_3 = M, s_4 = H, s_5 = VH, s_6 = P\}$, donde N=*Null*, VL=*Very Low*, L=*Low*, M=*Medium*, H=*Hight*, VH=*Very Hight* and P=*Perfect*.

   De esta forma, si los expertos deciden expresar sus preferencias utilizando palabras en lenguaje natural, suelen usar relaciones de preferencia lingüísticas [HVMMC05] como formato de representación de preferencias:

   - Una Relación de Preferencia Lingüística, P^h, dada por un experto e_h es un conjunto difuso definido en $X \times X$, caracterizado por una función de pertenencia lingüística $\mu_{P^h} : X \times X \longrightarrow S$, donde el valor $\mu_{P^h}(x_i, x_j) = p_{ij}^h$ es interpretado como el grado de preferencia lingüístico de la alternativa x_i sobre x_j para el experto e_h.

3. Finalmente, de acuerdo con el tercer nivel de heterogeneidad propuesto, existen situaciones de TDG donde no todos los expertos tienen la misma importancia (heterogeneidad entre expertos) [KZR, PCHV10]. Además, podemos encontrar problemas de TDG multicriterio donde unos criterios son más importantes que otros (heterogeneidad entre criterios)[LZZ$^+$09].

El propósito de esta parte de la tesis es el de incluir en los modelos actuales de TDG algunos mecanismos para tratar con los dos primeros tipos de heterogeneidad presentados. Para ello, permitimos a los usuarios elegir tanto la estructura como el dominio de representación de preferencias que más les convenga en cada caso, y proponemos no solo herramientas para hacer esta información homogénea, sino también un procedimiento para gestionar las situaciones en las que los expertos no son capaces de evaluar alguna de las alternativas y por tanto nos encontramos ante un problema de falta de información.

Además, hemos observado que los modelos actuales de TDG son estáticos, es decir, en ellos se asume que los elementos del problema (alternativas y expertos) son fijos a lo largo del proceso de decisión. Sin embargo, en las situaciones reales de TDG, podemos encontrar contextos variables en los que, debido a diferentes razones, algunas características del problema pueden variar mientras se toma la decisión final[PCHV10, PCHV11]. Por ejemplo, en decisiones relativas al comercio electrónico, donde las alternativas son los productos que están a la venta, es posible que la disponibilidad de alguno de estos productos cambie en cualquier momento mientras los expertos discuten sobre qué se va a comprar, incluso podrían aparecer nuevas alternativas mejores que las anteriores antes de tomar la decisión final. Por tanto, proponemos un modelo de TDG más flexible que permita tener en cuenta todos estos cambios que podrían afectar al desarrollo del proceso de decisión.

1.3. Toma de Decisión en Grupo con Expertos de Diferente Importancia

Como hemos mostrado en la sección anterior, en la literatura especializada podemos encontrar distintos enfoques para modelar la heterogeneidad en los problemas de TDG. Algunas de estas propuestas se centran en la heterogeneidad en el formato y el dominio de representación de preferencias, aunque también hay propuestas para modelar la heterogeneidad entre expertos.

El enfoque más utilizado para tratar estas situaciones propone la asignación de pesos a los expertos para modelar el grado de importancia de cada uno y asi calcular una agregación ponderada de las preferencias individuales [HHV97, HHVV96a, KFN92, KZR, Yag88, Yag09]. Según este enfoque, la discusión debería centrarse alrededor de una preferencia colectiva ponderada y, de esta manera, los expertos con mayor peso son los que dirigen el proceso estando al frente de la negociación para alcanzar un acuerdo con los demás expertos. Sin embargo, hay situaciones en las que muchos expertos de poca importancia pueden asociarse, y la suma de sus pesos les hace importantes como grupo. En este caso, los mecanismos actuales de generación de recomendaciones enviarían muchas sugerencias de cambio a los pocos expertos importantes, que suelen disponer de mucha información acerca del problema, y muy pocas a los expertos menos importantes que son los que realmente necesitan ser aconsejados. Como consecuencia, el grupo de expertos con menos experiencia o importancia es el que dirige la negociación, incluso cuando hay expertos mucho más cualificados para ello participando en el proceso. Para solucionar este problema, proponemos un nuevo modelo de consenso en el que se tendrán en cuenta los pesos no solo a la hora de agregar las preferencias, sino también cuando generamos las recomendaciones de cambio para aconsejar a los expertos. Para ello hemos diseñado un nuevo mecanismo de generación de recomendaciones que ajusta la cantidad de consejos que recibirá un experto a la importancia de éste. Este mecanismo se basa en la premisa de que aquellos expertos que son menos importantes o tienen menos experiencia, necesitaran mas consejo que los que ya disponen de suficiente información acerca del problema. Por tanto, esta nueva propuesta calcula las recomendaciones de diferente manera dependiendo de la importancia del experto al que van dirigidas.

1.4. Toma de Decisión en Grupo en Comunidades Web 2.0

En los últimos años, la rápida expansión del uso de Internet ha dado lugar a la creación de muchos tipos de servicios web, con los cuales, usuarios de cualquier parte del mundo pueden asociarse e interactuar con la web añadiendo nuevos contenidos. Una de las últimas tendencias en este campo, conocida como Web 2.0, incluye diferentes tipos de desarrollo web y técnicas de diseño no solo para permitir una mejor comunicación y compartir información de forma más sencilla sino también para mejorar la accesibilidad de los recursos y la colaboración entre los usuarios de este nuevo entorno virtual.

Las comunidades Web 2.0, que pueden tener distintos formatos como foros, blogs, redes sociales, etc., ponen a disposición de sus usuarios una plataforma virtual que estos pueden usar para asociarse y compartir información de forma que cada usuario pueda contribuir al contenido de los recursos, generando, gracias a la colaboración virtual, una inmensa cantidad de información colectiva [Lin08]. De hecho, la Web 2.0 representa un cambio de paradigma en cómo la gente utiliza internet, hoy en día, cualquier usuario puede contribuir a los contenidos online. Entre las diferentes actividades que realizan los miembros de las comunidades virtuales podemos destacar las siguientes:

- **Generar nuevos contenidos y documentos online.** Esta tarea ha tenido mucho éxito gracias a la diversidad y al grado de conocimiento de las personas implicadas. Uno de los ejemplos más claros del éxito de este tipo de colaboración es Wikipedia [wik], donde se han escrito millones de artículos en docenas de idiomas diferentes por los miembros de la comunidad web.

- **Proporcionar recomendaciones sobre diferentes productos o servicios.** Los sistemas de recomendaciones clásicos están incrementando su potencia y exactitud haciendo uso del conocimiento tanto explícito como implícito que producen los usuarios de la Web 2.0 [PLHHV09]. Un claro ejemplo del éxito de los sistemas de recomendaciones, que utiliza las valoraciones de su comunidad de usuarios para generar recomendaciones personalizadas es el almacén online Amazon [ama].

- **Tomar decisiones sobre problemas específicos.**

  Muchas de las comunidades virtuales han crecido alrededor de un foro de discusión donde los usuarios comparten información o discuten sobre distintos temas. En muchas de esas comunidades se utilizan frecuentemente algunos sistemas sencillos para tomar decisiones en grupo como sistemas de votación o referéndum, por ejemplo, servicios como PollDaddy [pol] permiten la creación de encuestas o sondeos online donde los usuarios pueden votar la mejor alternativa como solución a un problema concreto. Gestionar un proceso de decisión entre un grupo muy grande de individuos siempre ha sido una tarea difícil, pero con la aparición de las nuevas tecnologías, posiblemente estemos comenzando una nueva etapa en la que los modelos de democracia tradicionales están dejando paso a otros que implican una participación más directa de los ciudadanos. En la literatura especializada podemos encontrar algunos ejemplos de uso de estas nuevas tecnologías como la e-democracia [Pet09], la e-participación [PTZT09], el e-gobierno [RB09] o la deliberación pública [BR05, Muh09].

Normalmente, en los problemas de TDG en comunidades web se suelen presentar situaciones en las que el grupo de usuarios no es fijo a lo largo de todo el proceso de resolución: un nuevo experto podría conectarse a la red e incorporarse al proceso en cualquier momento o algunos expertos podrían desconectarse y dejar el proceso antes de que termine. Por otra parte, un gran grupo de expertos podría ser reducido para facilitar el cálculo de la solución. Este comportamiento es fácil de encontrar en sistemas de democracia donde los individuos delegan en pequeños grupos de expertos que son los que toman las decisiones (normalmente no es posible involucrar a todo el mundo en cada decisión). En [BGL08] podemos encontrar un ejemplo donde se ha intentado modelar esta situacion presentando un procedimiento recursivo para seleccionar subgrupos de individuos cualificados. Sin embargo, como las comunidades Web 2.0 son un fenómeno bastante reciente con unas características y particularidades propias, todavía es necesario el desarrollo de nuevas herramientas que permitan alcanzar decisiones en grupo con un alto grado de consenso entre sus usuarios. En esta memoria, presentamos un nuevo modelo de consenso diseñado para incorporar los beneficios que ofrece la Web 2.0 (amplio y diverso conocimiento gracias al alto número de usuarios, comunicación en tiempo real, etc.) e intenta minimizar los principales problemas que presentan este tipo de organizaciones (tasas de participación bajas e intermitentes, dificultad de establecer relaciones de confianza, etc.). El modelo incluye un mecanismo de delegación y otro de retroalimentación para incrementar la velocidad del proceso y su convergencia hacia una solución consensuada.

1. Introduction

Decision Making, that is, selecting the best alternative (or alternatives) from a feasible set, is a very common task present in almost every human activity. Thus, it provokes a great interest in the study of decision making situations and mechanisms that allow to solve decision making problems, not only in Decision Theory, but also in other disciplines as Artificial Intelligence, Economy, Sociology, Engineering and so on.

However, basic decision models have little in common with real decision making. Many real decision making processes are developed in environments where objectives, restrictions and feasible options are not exactly known and defined. Thus, it is necessary to study and refine those decision models in order to be able to represent this uncertainty. A practical and powerful way to handle uncertainty in human knowledge was proposed by professor Zadeh in 1965: Fuzzy Sets Theory [Zad65]. The application of Fuzzy Sets Theory to solve information uncertainty in decision making was proposed by Bellman and Zadeh in 1970 [BZ70] and since that moment it has been widely used because of its utility. Fuzzy Sets Theory has provided a much more flexible framework where it is possible to easily represent and tackle imprecision of human judgements.

Usually, decision problems require to make some analysis of the different alternatives and the problem that we face. However, not every decision problem is solved by means of a completely rational process. In fact, many external and subjective factors affect the decision processes, and thus, the final solution for a decision problem can change if the conditions in which the problem is presented vary.

It is obvious that the comparison of different actions according to their desirability in decision problems, in many cases, cannot be done by using a single criterion or a unique person. Thus, we interpret the decision process in the framework of group decision making (GDM) [KF90, Rou97]. This approach has led to numerous evaluation schemes and has become a major concern of research in decision making.

A GDM problem appears when there is a question to be solved, a set of alternatives from where to choose, $X = \{x_1, \ldots, x_n\}$, $(n \geq 2)$, and a set of persons (experts) , $E = \{e_1, \ldots, e_m\}$, $(m \geq 2)$, which express their opinions or preferences about the available options. Experts should have the intention of reaching a collective decision about the problem. Sometimes there exists a particular person, called moderator, which is in charge of the direction of the whole resolution process until the experts reach an agreement about the solution to choose.

To correctly solve a GDM problem two main different processes have to be carried out [HHVV96b]: The consensus process and the selection process. Both have been widely studied by different authors and in different GDM contexts [FR94, KF88, KF90]. The first one refers to how to obtain the highest consensus or agreement among experts about the set of alternatives. The second one (which is also called the algebraic consensus process) refers to how to obtain the final solution set of alternatives from the opinions expressed by experts. Both processes work together sequentially. First of all, the consensus process is developed to reach the maximum consensus degree among experts' preferences. In every step of the process the current consensus degree is measured, and if it does not reach an acceptable level, experts are encouraged to discuss their points of view and change their opinions to increase the proximity of their preferences. Once a certain level of consensus have been reached the selection process is applied and the final solution is obtained. Thus, a GDM process can be defined as a dynamic and iterative process in which experts change their opinions until their preferences about the solution are close enough, therefore allowing the obtention of a solution of consensus by means of the application of the selection process. This is graphically represented in Figure 4. In the following, we will describe both processes with more detail.

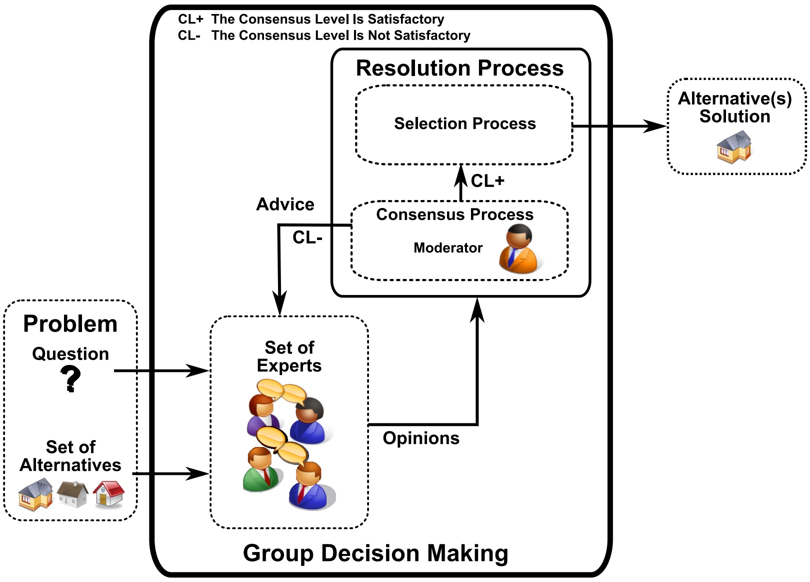

Figure 4: Resolution process of a GDM problem

1. **Consensus Process:**

   A consensus process is an iterative process which is composed by several consensus rounds, where the experts accept to change their preferences following the advice given by a moderator. The moderator knows the agreement in each moment of the consensus process by means of the computation of some consensus measures. As aforementioned, most of the consensus models are guided and controlled by means of consensus measures [AHVC+07, AHVCH10, BAC06, SSB80, CMPHV10, HHVV96b, HHVV97b, HVHC02, HVACH07, KF88, KFN92, MMHV09]. The consensus process can be divided in several steps which are graphically depicted in Figure 5:

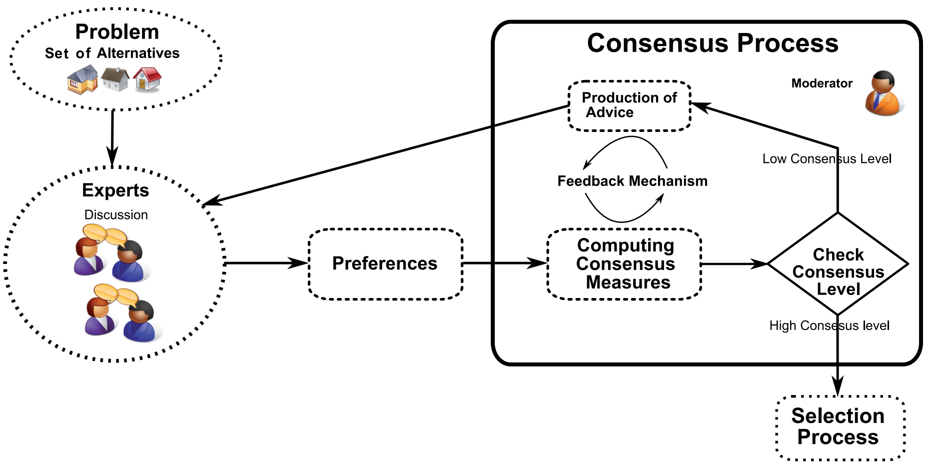

Figure 5: Classical consensus reaching process scheme

a) First of all, the problem to be solved is presented to the experts, along with the different alternatives among they have to choose the best one(s).

b) Then, experts can discuss and share their knowledge about the problem and alternatives in order to facilitate the next step of expressing their opinions.

c) Experts provide their preferences about the alternatives in a particular preference representation format.

d) The moderator receives all the experts' preferences and computes some consensus measures that will allow to identify if a consensus enough state has been reached or not.

e) If a consensued enough state has been reached the consensus process stops and the selection process begins. Otherwise, we can apply an advice generation step where the moderator, with all the information that he/she has (all preferences expressed by experts, consensus measures and so on) can prepare some guidance and advice for experts to more easily reach consensus. Note that this step is optional and is not present in every consensus model.

f) Finally, the advice is given to the experts and the first round of consensus is finished. Again, experts must discuss their opinions and preferences in order to approach their points of view (step b).

2. **Selection Process:**

   Once the consensus process has been carried out, (that is, experts' opinions are close enough) the selection process takes place. This process main aim is to select the final solution set of alternatives for the problem from the preferences given by the experts [BGKA09, HHVV95, SMY10, FR94, Tan84]. The selection process can be splitted in two different phases [Rou97]: (See Figure 6).

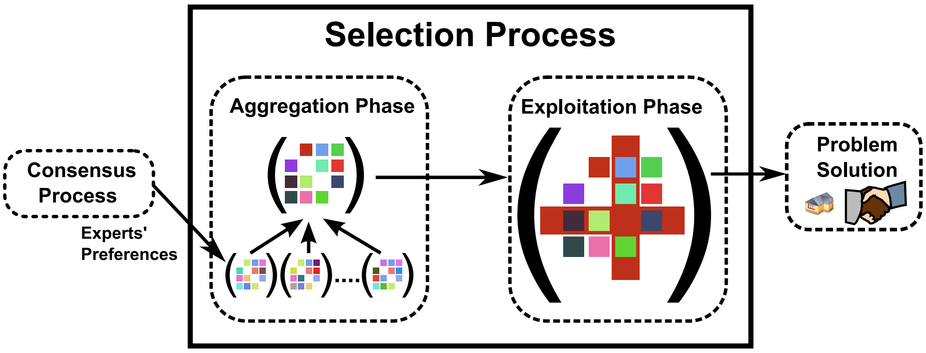

Figure 6: Selection process scheme

a) *Aggregation Phase:* In this phase all preferences given by the experts must be aggregated into only one preference structure. This aggregation is usually carried out by means of particular aggregation operators that are usually defined for this purpose. This step can be more complicated if we have an heterogeneous decision making situation (not equally important experts or different preference representation formats), as some kind of homogenization must be carried out to transform all different preference representation formats into a particular one which acts as a base for the aggregation, and the aggregation operator must be able to handle the weights assigned to the experts (that is, giving more importance to some experts' preferences than others).

b) *Exploitation Phase:* This final step uses the information produced in the aggregation phase to identify the solution set of alternatives for the problem. To do so we must apply some mechanism to obtain a partial order of the alternatives and thus select the best alternative(s). There are several different ways to do this, but a usual one is to associate a certain utility value to each alternative (based on the aggregated information), thus producing a natural order of the alternatives.

We will pay attention to different kinds of GDM situations along this memory, analyzing the current GDM models and trying to improve them. In order to carry out this study, this memory is divided in two parts. First one is devoted to the problem statement and the discussion of the results. Second one corresponds to the publications associated to this study.

In Part I we begin by developing the problem statement introduced in this section and the techniques proposed to solve it with the following subsections: subsection 1.1 introduces the consensus approaches in fuzzy Group Decision Making situations, subsection 1.2 describes GDM problems with heterogeneous information in changeable contexts, subsection 1.3 presents GDM problems with and different experts' importance and subsection 1.4 illustrate GDM situations in web 2.0 communities. Next we indicate the open problems which justify the realization of this memory in section 2. The objectives pursued in this memory are described in Section 3. Section 4 provides a summarized information about the proposals and most interesting results obtained in each part. Section 5 summarizes the results obtained in this memory and present several conclusions about them, moreover, in section 6 we point out several open future works which remain open from the results of the present memory.

Finally, in order to develop the goals set, this memory is constituted by eight publications distributed in four sections which will be developed in Part II. They are the following:

1. Analyzing Consensus Approaches in Fuzzy Group Decision Making: Advantages and Drawbacks.

2. Mobile Decision Support Systems Based on Heterogeneous Information and Changeable Contexts.

3. A New Consensus Model for Group Decision Making Problems with Non Homogeneous Experts.

4. A Linguistic Consensus Model for Web 2.0 Communities.

1.1. Consensus Approaches in Fuzzy Group Decision Making

Normally, in a GDM problem, the consensus process is guided by a human figure called moderator [HHVV96b, KFN92] who is a person that does not participate in the discussion but has a deep knowledge of the problem and is in charge of supervising and addressing the consensus process toward success, i.e., to achieve the maximum possible agreement and to reduce the number of experts outside of the consensus in each new consensus round. At the beginning of every GDM problem, the set of experts have diverging opinions, then, the consensus process is applied, and in each step, the degree of existing consensus among experts' opinions is measured. If the consensus degree is lower than a specified threshold, the moderator would urge experts to discuss their opinions further in an effort to bring them closer. Otherwise, the moderator would apply the selection process in order to obtain the final consensus solution to the GDM problem. In such a way, a GDM problem may be defined as a dynamic and iterative process, in which the experts, via the exchange of information and rational arguments, agree to update their opinions until they become sufficiently similar, and then, the solution alternative(s) is/are obtained.

A natural question in the consensus process is how to measure the closeness among experts' opinions in order to obtain the consensus level. To do so, different approaches have been proposed. For instance, several authors have introduced *hard consensus measures* varying between 0 (no consensus or partial consensus) and 1 (full consensus o complete agreement) [BSS78]. Thus, using *hard consensus measures*, in [BSS78, SBS79], a distance from consensus as a difference between some average preference matrix and one of several possible consensus preference matrices is determined. In [SBS79] some measures of attitudinal similarity between individuals that is an extension of the classical Tanimoto coefficient are derived. And, in [SSB80], a consensus measure based on a-cuts of the respective individual fuzzy preference matrices is derived. However, consensus as a full and unanimous agreement is far from being achieved in real situations, and even if it is, in such a situation, the consensus reaching process could be unacceptably costly. So, in practice, a more realistic approach is to use *softer consensus measures* [Kac86, KF88], which assess the consensus degree in a more flexible way, and therefore reflect the large spectrum of possible partial agreements, and guide the consensus process until widespread agreement (not always full) is achieved among experts. The soft consensus measures are based on the concept of coincidence [HHVV97a], measured by means of similarity criteria defined among experts' opinions.

The aim of this part of the study is to identify the different existing approaches in the literature to compute soft consensus measures in fuzzy GDM problems and analyze their advantages and drawbacks. To do so, firstly, we identify three different coincidence criteria to compute soft consensus measures: *strict coincidence among preferences*, *soft coincidence among preferences* and *coincidence among solutions*. Then, we analyze their application in consensus processes of fuzzy GDM problems and study their drawbacks and advantages. Furthermore, we describe the new advanced approaches, which use the above coincidence criteria, allowing to generate recommendations to help experts change their opinions in order to obtain the highest degree of consensus possible and adapt the consensus process to increase the agreement and to reduce the number of experts' preferences that should be changed after each consensus round.

1.2. Group Decision Making with Heterogeneous Information and Changeable Contexts

Recently, several authors have studied and approached GDM problems from different angles, showing that this kind of problems are not always homogeneous. We can classify them into three different heterogeneity levels.

1. The first heterogeneity level studied in the literature [CHHV98, CHHV01] is focused on the preference representation structures. Usually, each expert e_h provides his/her preferences on the alternatives $X = \{x_1, x_2, ..., x_n\}$, by means of an specific preference's representation format, the most commonly used are:

    - *Preference orderings of alternatives:* $O^h = \{o^h(1), ..., o^h(n)\}$, where $o^h(\cdot)$ is a permutation function over the index set, $\{1, ..., n\}$, for the expert e_h, defining an ordered vector of alternatives, from best to worst.
    - *Utility functions:* $U^h = \{u_1^h, ..., u_n^h\}$, $u_i^h \in [0, 1]$, where u_i^h represents the utility evaluation given by the expert e_h to x_i.

- *Fuzzy preference relations:* $P^h \subset X\text{x}X$, with a membership function, $\mu_{P^h} : X\text{x}X \to [0,1]$, where $\mu_{P^h}(x_i, x_j) = p_{ij}^h$ denotes the preference degree of x_i over x_j for the expert e_h.
- *Multiplicative preference relations:* $A^h \subset X\text{x}X$, where the intensity of preference, a_{ij}^h, is measured using a ratio scale, particularly the 1/9 to 9 scale.

Fuzzy preference relations are widely used in this kind of problems because they are more informative than preference orderings or utility functions [CHHV98], allowing the comparison of the alternatives in a pair by pair basis. Thus, users have much more freedom at giving their preferences and they can gain expressivity against other preference representations. When cardinality of X is small, the preference relation may be conveniently represented by an $n \times n$ matrix $P^h = (p_{ij}^h)$.

2. The second heterogeneity level is focused on the preference representation domain (numeric, linguistic, multi-granular, interval numbers) [ACC+09, CAHV09, HHVM08, MMHV09].

   There are situations in which the information cannot be assessed precisely in a quantitative form but may be in a qualitative one. For example, when attempting to qualify phenomena related to human perception, we are often led to use words in natural language instead of numerical values, e.g. when evaluating quality of a football player, terms like *good*, *medium* or *bad* can be used.

   The ordinal fuzzy linguistic approach [HHV97, HHVV96a] is a tool based on the concept of linguistic variable [Zad75] to deal with qualitative assessments. It is a very useful kind of fuzzy linguistic approach because its use simplifies the processes of computing with words as well as linguistic representation aspects of problems. It has proven its usefulness in many problems, e.g., in decision making [BAC06, CAHV09, HHV00b, HVMMC05], web quality evaluation [HVP03, HVPLHP06, HVPM+07], information retrieval [BP01, HV01, HVLH07], recommender systems [PLHHV09, PMHV09], political analysis [Arf05], etc.

   It is defined by considering a finite and totally ordered label set $S = \{s_i\}, i \in \{0, ..., g\}$ in the usual sense, i.e., $s_i \geq s_j$ if $i \geq j$, and with odd cardinality (usually 7 or 9 labels). The mid term represents an assessment of "approximately 0.5", and the rest of the terms are placed symmetrically around it. The semantics of the label set is established from the ordered structure of the label set by considering that each label for the pair (s_i, s_{g-i}) is equally informative [HHV00b]. For example, we can use the following set of seven labels to represent the linguistic information: $S = \{s_0 = N, s_1 = VL, s_2 = L, s_3 = M, s_4 = H, s_5 = VH, s_6 = P\}$, where N=*Null*, VL=*Very Low*, L=*Low*, M=*Medium*, H=*Hight*, VH=*Very Hight* and P=*Perfect*.

   In such a way, if experts decide to express his/her preferences using words in natural language instead of numerical values, they normally use fuzzy linguistic preference relations [HVMMC05]:

   - A Fuzzy Linguistic Preference Relation (FLPR) P^h given by an expert e_h is a fuzzy set defined on the product set $X \times X$, that is characterized by a linguistic membership function $\mu_{P^h} : X \times X \longrightarrow S$, where the value $\mu_{P^h}(x_i, x_j) = p_{ij}^h$ is interpreted as the linguistic preference degree of the alternative x_i over x_j for the expert e_h.

3. Finally, the third heterogeneity level [KZR, PCHV10], deals with some classical heterogeneous decision scenarios, where every expert has an associated weight value in order to model their different importance levels or knowledge degrees (heterogeneity among experts). Furthermore, in some multi-criteria decision scenarios, we can find criteria with different weight values (heterogeneity among criteria) [LZZ+09].

The aim of this part of the study is to improve the current GDM models with the addition of some mechanism in order to manage the heterogeneity among the preference structures an domain. To do so, users can choose the most suitable structure and domain of preferences' representation and we propose not only tools to make uniform this information but also a procedure to manage situations where the experts are not able to assess some alternatives and they express their preferences with incomplete information.

In addition, we have realized that the proposed resolution methods for GDM problems are frequently static, that is, it is assumed that the problem's elements (alternatives and experts acting in the problem) remains fixed throughout the decision making process. However, in real decision situations we find several changeable decision contexts in which due to different reasons, some information of the problem could vary through decision process. In such cases, the set of alternatives could vary during the decision making process [PCHV10, PCHV11]. Sometimes, when the decision process is slow or it takes a long time, the set of feasible alternatives has to be changeable because his availability or feasibility could change during the decision making time. For example, in e-commerce decision frameworks, where the alternatives are the items that could be bought, it is possible that the availability of some of these items changes while experts are discussing and making the decision, even, new good items might become available. Thus, a model of decision making should present a flexible and adaptive structure to include those changes that could happen through decision process so that we can constantly revise our decision and the parameters of the problem.

1.3. Group Decision Making with Different Experts' Importance

As we have studied in the previous subsection, different approaches for heterogeneity modelling in GDM problems have been proposed in the literature. Some instances of these approaches are focused on heterogeneous domains and structures to represent the preferences. On the other hand, there are some proposals to model the heterogeneity among experts.

The most usual approach in the literature to model this last kind of situations deals with the assignation of weight values to the experts in order to compute a weighted aggregation of their preferences [HHV97, HHVV96a, KFN92, KZR, Yag88, Yag09]. This approach allows to model GDM problems with heterogeneous experts in order to give the necessary importance to every opinion in each case. Thus, the discussion is focussed on a weighted collective preference and, in such a way, the most weighted experts are the main leaders of the discussion and they have to be at front of the negotiation to persuade the remaining experts in order to reach agreement. However, there exists situations with many low-important experts, whose weights' addition makes them important as a group, where this mechanism could miss the target resulting in the opposite effect to the desired. In such a context, the feedback mechanism of the current approaches could send a lot of recommendations to the high-important experts, who have at their disposal a larger amount of knowledge of the problem, in order to change their preferences to narrow them to the remaining experts' opinions instead of send the recommendations to those experts who really need to be advised. Consequently, the less considerable experts become the leaders of the discussion, even, when there are some highly qualified experts taking part in the decision process. In order to overcome such problem, we propose a new consensus approach. We suggest to take into account the importance weights not only to aggregate the experts' preferences but also when advising experts to change their preferences. To do so, we propose an importance based feedback mechanism that adjust the amount of advice required by each expert depending on his own weight value. It seems reasonable that the experts with lower importance or knowledge level will need more advice than those experts that previously have at their disposal a large amount of information to make good decisions. Therefore, this new approach computes the recommendations in a different way depending on the experts' importance level.

1.4. Group Decision Making in Web 2.0 Communities

In the last years, the World Wide Web has allowed the creation of many different services in which users from all over the world can join, interact and produce new contents and resources. One of the most recent trends, the so called Web 2.0, which comprises a set of different web development and design techniques, allows the easy communication, information sharing, interpretability and collaboration in this new virtual environment. Web 2.0 Communities, that can take different forms as Internet forums, groups of blogs, social network services and so on, provide a platform in which, users can collectively contribute to a Web presence and generate massive content behind their virtual collaboration [Lin08]. In fact,Web 2.0 represents a paradigm shift in how people use the web as nowadays, everyone can actively contribute content online. Among the different activities that the users of Web Communities usually perform we can cite:

- **Generate online contents and documents**, which is greatly beneficiated with the great diversity and knowledge of the involved people. One of the clearest examples of this kind of collaboration success is Wikipedia [wik], where millions of articles have been produced by its web community in dozens of different languages.
- **Provide recommendations about different products and services**. Usual recommender systems are increasing their power and accuracy by exploiting their user bases and the explicit and implicit knowledge that they produce [PLHHV09]. A clear example of recommender systems success, which exploits its users community knowledge to provide personalized recommendations, is the Amazon online store [ama].
- **Make decisions about particular problems.** Many online communities have grown around a web forum or some discussion boards where users share information or discuss about selected topics. In many of these communities some simple GDM schemes, as referendum or voting systems are usually used. For example, services like PollDaddy [pol] allow to create online surveys and polls where users can vote about the best alternative to choose for a given decision problem. It is clear that involving a very large number of individuals in a decision process is a difficult task but, with the appearance of new electronic technologies, we are in the beginning of a new stage where traditional democratic models may leave some space to a more direct participation of the citizens. In the specialized literature we can found some efforts about the use of these new technologies in what it is being called e-democracy [Pet09], e-participation [PTZT09], e-Governance [RB09] and public deliberation [BR05, Muh09].

In particular, GDM in Web Communities usually presents some dynamic situations in which the group of experts vary over time: a new expert could incorporate to the process, some experts could leave it or a large group of experts could be simplified in order to minimize communications and to ease the computation of solutions. This behavior is usually found in democratic systems where the individuals delegate into a smaller group of experts to make decisions (it is usually not possible to involve everyone in each decision). There have been some efforts to model this kind of situations. For example, in [BGL08] a recursive procedure to select a qualified subgroups of individuals taking into account their own opinions about the group is presented. However, as Web 2.0 Communities are a quite recent phenomenon with its own characteristics and particularities, there is still a necessity of developing new tools that allow to reach decisions with a high enough consensus level among their users. In this memory we present a new consensus reaching model designed to incorporate the benefits that a Web 2.0 Community offers (rich and diverse knowledge due to a large number of users, real-time communication...) and that tries to minimize the main problems that this kind of organization presents (low and intermittent participation rates, difficulty of establishing trust relations and so on). The model includes some delegation and feedback mechanisms to improve the speed of the process and its convergence towards a solution of consensus.

2. Justification

Decision making is the cognitive process of selecting the best alternative (or alternatives) from among multiple different alternatives. It begins when we need to do something but we do not know what. Therefore decision making is a reasoning process which can be rational or irrational, and can be based on explicit assumptions (usually presented with the alternatives) or tacit assumptions (those which are not explicitly voiced nor necessarily understood by the decision maker).

Decision making situations are very common in every person's daily life: Usual examples include shopping, deciding what to eat, and deciding whom or what to vote for in an election or referendum. However, decision making not only occur for isolated individuals. Usually some decision problems have to be solved by a group of persons (usually experts), which together have to decide which alternative among the given ones is better or more preferable in a particular situation. The existence of multiple persons in a decision process implies several additional difficulties that have to be solved. For example, opinions of the individuals about the alternatives can be very different, and thus, to reach some kind of agreement (or consensus) among experts in the decision process is necessary prior to the actual selection of the best alternative(s).

To properly model GDM situations several aspects have to be taken into account:

- The preference representation formats and domains that experts can use to express their opinions and preferences. This kind of representation can greatly affect the whole decision process. For example, some representation formats as preference orderings of the alternatives are simple representation formats that experts which are not familiar with them can easily learn to use effectively. However, their simplicity usually implies that the amount of information that can be modelled using them and its granularity is quite small. On the contrary, other preference representation formats as preference relations offer a higher level expressivity, and thus, a lot more of information (and more complex information) can be modelled with them. On the other hand, the preferences representation domain used by the experts is also an important factor to represent the preferences in the most appropriate way.

- Lack of information. Although it is desirable for experts who face a decision problem to have a wide and exhaustive knowledge about the different alternatives, this is a requirement that is not often fulfilled. Many different cultural and personal factors can lead to lack of information situations in decision making. For example, experts may not be familiar with some of the alternatives (specially if the set of feasible alternatives is large), or maybe they are not able to properly differentiate among some similar alternatives.

- Changeable contexts. As the decision making process has not an immediate solution, the problem's elements (alternatives and experts) can change during the process time. Thus, it would be desirable to take into account these changes in order to make decisions having in mind the most updated information.

- Heterogeneity among experts. We say we have non homogeneous experts when the opinion of the different experts are not equally important. This constraint has to be taken into account not only to aggregate the preferences in the selection process but also in the generation of advice process in order to compute customized recommendations.

Thus, the study of those aspects is a key point to develop reliable and realistic GDM models and processes.

3. Objectives

The main aim of the thesis is to develop group decision making models with incomplete information which address both problems on the consensus and selection of alternatives. To achieve this aim we have set the following objectives:

- To analyze the current consensus approaches to compute soft consensus measures in fuzzy GDM problems
- To design a new model for GDM problems based on heterogeneous information and changeable contexts that allows experts to choose the best way to express their preferences and, if necessary, to manage the changes of the problem's elements (experts and alternatives) during the decision time.
- To develop a prototype of the previous model by using mobile technologies in order to improve the user-system interaction and to make decisions anytime and anywhere.
- To develop a new adaptive GDM model which manages the heterogeneity among experts not only in the selection process but also in the consensus process.
- To design a new consensus model to reach solutions in GDM environments of Web 2.0 communities. The model has to take into account the different features of this kind of communities as a large user base, low participation and contribute rates, real time communication, intermittent contributions and difficulty of establishing trust relations.

4. Joint Discussion of Results

This section shows a summary of the different proposals presented in this dissertation, and it presents a brief discussion about the obtained results by each one.

4.1. Analyzing Consensus Approaches in Fuzzy Group Decision Making: Advantages and Drawbacks

In this section, some important aspects of the use of the different approaches to obtain soft consensus degrees within the decision making process are analyzed. To do so, we show the advantages and drawbacks of each one of them.

1. **Strict coincidence among preferences:**

   The advantage of this approach is that the computation of the consensus degrees is simple and easy because it assumed only two possible values: 1 if the opinions are equal and otherwise a value of 0. However, the drawback of this approach is that the consensus degrees obtained do not reflect the real consensus situation because it only assigns values of 1 or 0 when comparing the experts' opinions, and, for example, we obtain a consensus value 0 for two different preference situations as (very high, high) and (very high,low), when clearly in the second case the consensus value should be lower than in the first case.

2. **Soft coincidence among preferences:**

   The advantage of this approach is that the consensus degrees obtained are similar to the real consensus situation because they are obtained using similarity functions that assign values between 0 and 1, which are not so strict as in the above approach. The drawback of this approach is that the computation of the consensus degrees is more difficult than in the above approach because we need to define similarity criteria [HVMMC05, HVACH07].

3. **Coincidence among solutions:**

   The advantage of this approach is that the consensus degrees are obtained comparing not the opinions or choice degrees but the position of the alternatives in each solution, what allows us to reflect the real consensus situation in each moment of the consensus reaching process. The drawback of this approach is that the computation of the consensus degrees is more difficult than in the above approaches because we need to define similarity criteria and it is necessary to apply a selection process before obtaining the consensus degrees.

In the following, we describe the new advanced soft consensus approaches which have been developed using the above concepts of coincidence. These approaches allow to generate recommendations to help experts change their opinions in order to obtain the highest degree of consensus possible [HVHC02, HVMMC05, HVACH07] and adapt the consensus process to increase the agreement and to reduce the number of experts' preferences that should be changed after each consensus round [MMHV09].

1. **Approaches generating recommendations to help experts:**

   These approaches generate simple and easy rules to help experts change their opinions in order to obtain the highest degree of consensus possible. To do so, they are based on two consensus criteria, consensus degrees indicating the agreement between experts' opinions and proximity measures used to find out how far the individual opinions are from the group opinion. Thus, proximity measures are used in conjunction with the consensus degrees to build a guidance advice system, which acts as a feedback mechanism that generates advice so that experts can change their opinions. Furthermore, these consensus criteria are computed at the three different levels of representation of information of a preference relation: pair of alternatives, alternative, and relation. It allows us to know the current state of consensus from different viewpoints, and therefore, to guide more correctly the consensus reaching processes. Thus, as these measures are given on three different levels for a preference relation, this measure structure will allow us to find out the consensus state of the process at different levels. For example, we will be able to identify which experts are close to the consensus solution, or in which alternatives the experts are having more trouble to reach consensus.

   Once both consensus and proximity measures are calculated, the recommendations are generated. The production of advice to achieve a solution with the highest degree of consensus possible is carried out in two steps [HVMMC05]: *Identification rules* to identify the experts, alternatives and pairs of alternatives that are contributing less to reach a high degree of consensus and, therefore, should participate in the change process, and *Direction rules* to find out the direction of the change to be recommended in each case.

2. **Adaptive approaches:**

   These approaches are based on a refinement process of the consensus process that allows to increase the agreement and to reduce the number of experts' preferences that should be changed after each consensus round [MMHV09]. The refinement process adapts the search for the furthest experts' preferences to the existent agreement in each round of consensus. So, when the agreement is very low (initial rounds of the consensus process), the number of changes of preferences should be bigger than when the agreement is medium or high (final rounds) (see Figure 7).

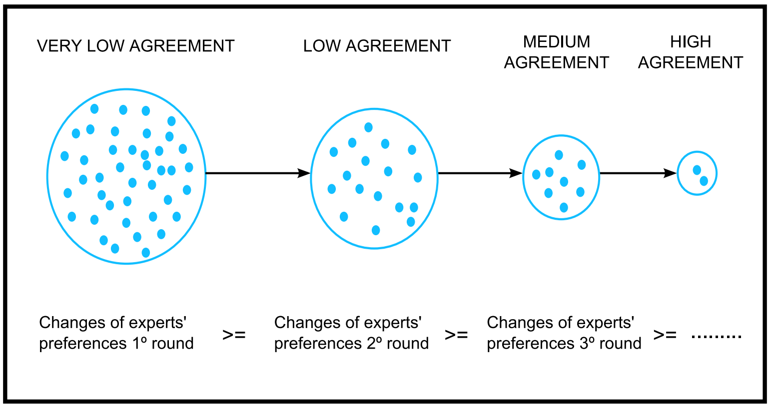

Figure 7: Reduction of the number of changes of preferences into the consensus process

These approaches consider that in the first rounds of the consensus process, the agreement is usually very low and it seems logic that many experts' preferences should be changed. However, after several rounds, the agreement should have improved and then just the furthest experts' preferences from the collective preference should be changed. It involves that the procedure to search for the furthest experts' preferences from collective preference should be different according to the achieved agreement in each round. Each Preference Search Procedure (PSp) should have a different behavior and should return a different set of preferences that each expert should change in order to improve the agreement in the next consensus round. In consequence of the adaptation of the consensus process to the existent agreement in each round, the number of changes of preferences suggested to experts after each consensus round will be smaller according to the favorable evolution of the level of agreement. In this way, in the consensus process, if the agreement among experts is low, i.e, there are a lot of experts' preferences with different assessments, the number of experts which should change their preferences in order to make them closer to collective preference should be great. However, if the agreement is medium or high, it means that the majority of preferences are similar and therefore there exist a low number of experts' preferences far from the collective preference. In this case, only these experts should change them in order to improve the agreement.

The associated journal article to this part is:

- F.J. Cabrerizo, J.M. Moreno , I.J. Pérez, E. Herrera-viedma, Analyzing Consensus Approaches in Fuzzy Group Decision Making: Advantages and Drawbacks. **Soft Computing 14:5 (2010) 451-463. doi:10.1007/s00500-009-0453-x.**

4.2. Mobile Decision Support Systems Based on Heterogeneous Information and Changeable Contexts

In this section, we present a new GDM model specifically designed to give freedom to the experts in the way that they provide their preferences with heterogeneous information, that is, by means of any preference representation format and domain. Furthermore, the model incorporates some mechanisms to manage the changes of the context that might happen during the decision process. Thus, it has to adapt not only to the initial circumstances but also to the changes of the context. In such a way, changeable GDM processes with heterogeneous information could be developed and we can simulate with more accuracy level the real processes of human decision making which are carried out in changeable environments as the Web, commerce, financial investment, health, navigation, natural resources management and so on.

This new GDM model is composed of the following five processes (see Figure 8):

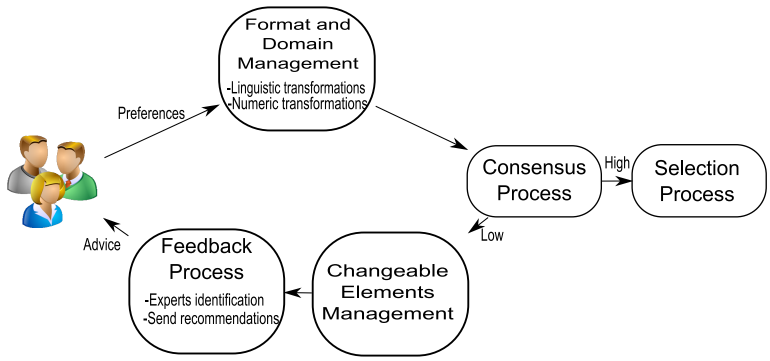

Figure 8: Structure of the new GDM model for heterogeneous information and changeable contexts

1. **Format and domain management:**

   To give a higher degree of freedom to the system, we assume that experts can present their preferences using any of the preference representations presented in section 1.2. Therefore, it is necessary to make the information uniform before applying the consensus and selection processes. We propose to use fuzzy preference relations as the base element to uniform numeric experts' preferences. The following transformation functions are used [CHHV98]: $f^1\left(o_i^k, o_j^k\right) = \frac{1}{2}\left(1 + \frac{o_j^k - o_i^k}{n-1}\right)$, $f^2\left(u_i^k, u_j^k\right) = \frac{\left(u_i^k\right)^2}{\left(u_i^k\right)^2 + \left(u_j^k\right)^2}$, $f^3\left(a_{ij}^k\right) = \frac{1}{2}\left(1 + \log_9 a_{ij}^k\right)$.

   Moreover, we use an iterative complete procedure to estimate the missing values in an incomplete FLPR, which it is based on the linguistic additive consistency property. This procedure estimates missing values in an expert's incomplete FLPR using only the preference values provided by that particular expert. The procedure estimates missing values by means of two different tasks: i) to choose those elements to be estimated in each iteration of the procedure and ii) to estimate a particular missing value.

2. **Consensus process:**

   Initially, in this consensus model we consider that in any nontrivial GDM problem the experts disagree in their opinions so that decision has to be viewed as an iterative process. This means that agreement is obtained only after some rounds of consultation. In each round, we calculate the consensus measures and check the current agreement existing among experts.

   We assume that the consensus as a measurable parameter whose highest value corresponds to unanimity and lowest one to complete disagreement. We use some consensus degrees to measure the current level of consensus in the decision process. They are given at three different levels [HHVV96b, HHVV97a, MMHV09]: pairs of alternatives, alternatives and relations.

3. **Selection process:**

   The selection has two different phases [HHVV95]:

   a) Aggregation:
   This phase defines a collective preference relation, $P^c = \left(p_{ij}^c\right)$, obtained by means of the aggregation of all individual preference relations $\left\{P^1, P^2, \ldots, P^m\right\}$. It indicates the global preference between every pair of alternatives according to the majority of experts' opinions. The aggregation is carried out by means of an aggregation operator ϕ_Q guided by a fuzzy linguistic non-decreasing quantifier Q [HHVV96a]:

   $$p_{ij}^c = \phi_Q(p_{ij}^1, \ldots, p_{ij}^m) \tag{I.1}$$

   b) Exploitation:
   This phase transforms the global information about the alternatives into a global ranking of them, from which the set of solution alternatives is obtained. The global ranking is obtained applying two choice degrees of alternatives to the collective fuzzy preference relation [HHV00a]: the *quantifier guided dominance degree* (QGDD) and the *quantifier guided non dominance degree* (QGNDD).
   Finally, the solution X_{sol} is obtained by selecting the alternatives with maximum choice degrees.

4. **Changeable context management:**

   Classical GDM models are defined in a static framework. In order to make the decision making process more realistic, we provide a new tool to deal with dynamic parameters in decision making, as for example the set of alternatives or the group of experts. In this section we focus on the changes produced in the set of alternatives because it could depend on dynamical external factors like the traffic [Dia02, KSJY09], or the meteorological conditions [Cla08], and so on, and this kind of change is more usual. In such a way, we consider dynamic decision problems in which, at every stage of the process, the discussion is centered on different alternatives.

   We define a method which allows us to introduce new alternatives in the discussion process. Firstly, the system identifies those new alternatives to include in the set of discussion alternatives and the worst alternatives to eliminate. And then, the system asks experts their opinion about such changes, i.e., if they agree or not.

   To identify the new alternatives we can have two particular cases: (see Figures 9 and 10)

   - This first case happens when a good new alternative appears in the set because some dynamic external factors changed during the decision process, and this new alternative deserves to be in the discussion subset. Before including the new alternative in the discussion subset, the system has to identify the worst alternative of the current discussion subset. To find this bad alternative x_i we compare the dominance and non dominance degrees $QGDD_i$ and $QGNDD_i$ of all the alternatives, and choose the less evaluated as the worst alternative.

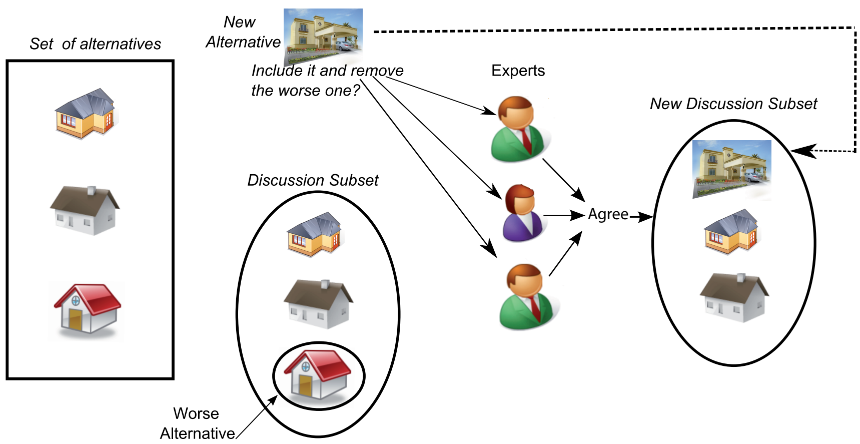

Figure 9: Dynamic choice process of alternatives: case 1

   - This second case is when we observe that an alternative x_i always receives low dominance and non dominance degrees $QGDD_i$ and $QGNDD_i$ due to the changes of the some dynamic external factors during the decision process. Then we could decide to substitute it by another alternative of the initial set of alternatives that was not included in the discussion set of alternatives. This strategy of replacement is commonly used when there is a big set of possible alternatives and they can not be evaluated at the same time. So, we can decide to replace the bad alternatives in the discussion subset in order to evaluate a major number of alternatives. The new alternative to be considered is obtained from the initial list of alternatives that were not included in the discussion subset initially, but now they can be used to replace a bad alternative.

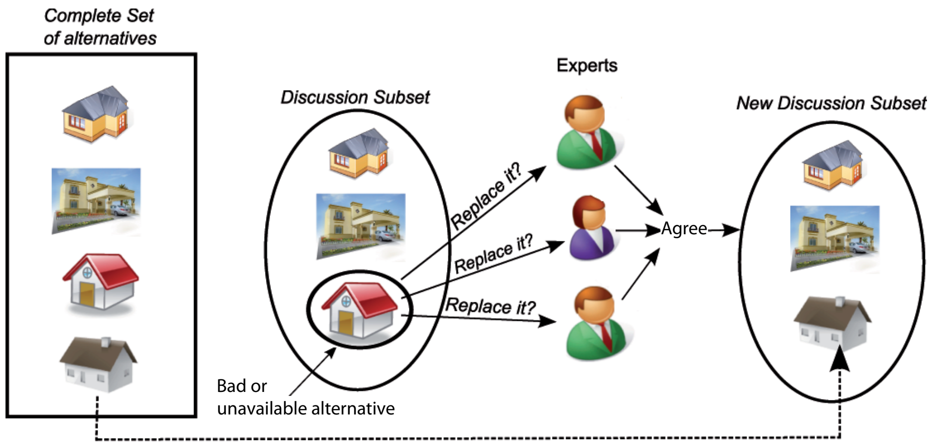

Figure 10: Dynamic choice process of alternatives: case 2

Once the alternatives to be interchanged have been identified, the system gives experts the option to accept or decline the proposed changes. They must provide their their degrees of agreement with the proposed changes using a set of linguistic assessments, as for example:

$\{Completely\ Agree,\ Agree,\ Nor\ Agree/Nor\ Disagree,\ Disagree,\ Completely\ Disagree\}$.

Then, we aggregate degrees of agreement provided by experts using the LOWA operator [HHVV96a]). If we obtain a high degree of agreement (more than nor agree/nor disagree) then the system removes the bad alternative from the discussion subset of alternatives and the new one is incorporated into this discussion subset.

5. **Feedback process**

To guide the change of the experts' opinions, the model simulates a group discussion session in which a feedback mechanism is applied to quickly obtain a high consensus level. This mechanism is able to substitute the moderator's actions in the consensus reaching process. The main problem for the feedback mechanism is how to find a way of making individual positions converge and, therefore, how to support the experts in obtaining and agreeing with a particular solution. To do that, we compute others consensus measures, called proximity measures [CMPHV10, HHVV96b].

These measures evaluate the agreement between the individual experts' opinions and the group opinion and let us to build a feedback mechanism so that experts change their preferences and narrow their positions. To do so, the production of advice to achieve a solution with the highest possible degree of consensus is carried out in two phases: *Identification phase* and *Recommendation phase.*

a) *Identification phase.* We must identify the experts, alternatives and pairs of alternatives that are contributing less to reach a high degree of consensus.

b) *Recommendation phase.* In this phase we recommend expert changes of their preferences according to two kinds of rules:

1) *Rules to change the opinions.* We must find out the direction of change to be applied to the preferences of the experts that are hindering the agreement.

2) *Rules to complete missing values.* Additionally, the feedback process must provide rules for missing preferences values. Thus, the lack of information decreases and in this way better solutions can be obtained. To do so, it has to take into account all missing values that were not provided by the experts and were calculated at the estimation process

Nowadays, organizations have moved from face-to-face group environments to virtual group environment using communication technologies and tools to coordinate and share information with other people. The main objective of these new approaches is that the members of the group could work in an ideal way no matter where they are, having all the necessary information to take the most guessed right decisions. Using the mobile technologies, besides increasing the productivity and the satisfaction of the user, allows to save the operational costs of having to bring together to the complete group in the same place at the same time.

To support the new generation of decision makers and to add real-time process in the GDM field, many authors have proposed to develop decision support systems (DSSs) based on mobile technologies [DR00, ESC08, WCP08]. Similarly, we have developed a prototype specifically designed to incorporate mobile technologies in our model obtaining a Mobile DSS (MDSS) (see Figure 11). Using such a technologies should enable users to maximize the advantages and minimize the drawbacks of DSSs.

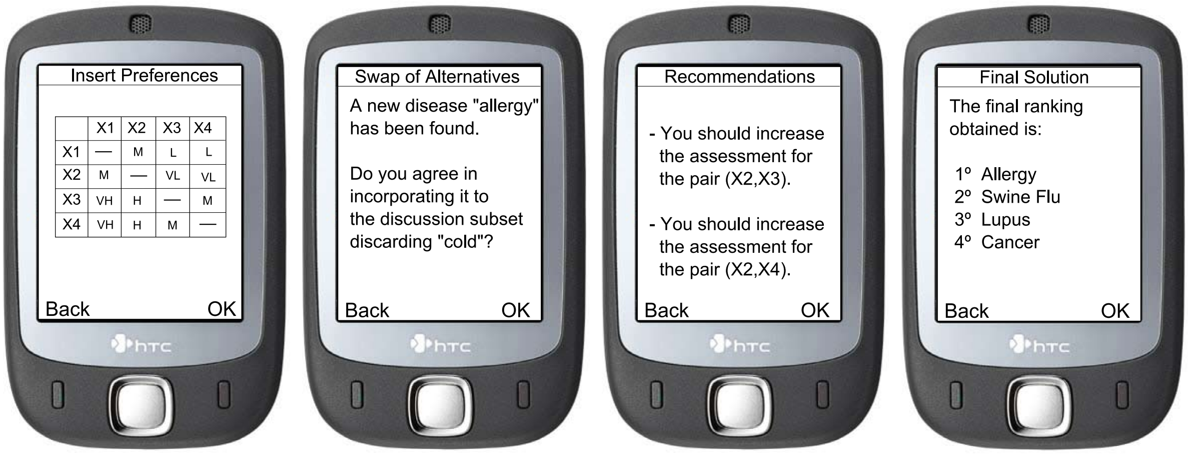

Figure 11: Prototype interfaces

The associated journal articles to this part are:

- I.J. Pérez, F.J. Cabrerizo, E. Herrera-Viedma, A Mobile Decision Support System for Dynamic Group Decision Making Problems. **IEEE Transactions on Systems, Man and Cybernetics - Part A: Systems and Humans 40:6 (2010) 1244-1256 doi:10.1109/TSMCA.2010.2046732.**

- F.J. Cabrerizo, R. Heradio, I.J. Pérez, E. Herrera-Viedma, A Selection Process Based on Additive Consistency to Deal with Incomplete Fuzzy Linguistic Information. **Journal of Universal Computer Science 16:1 (2010) 62-81, doi:10.3217/jucs-016-01-0062.**

- I.J. Pérez, F.J. Cabrerizo, E. Herrera-Viedma, Group Decision Making Problems in a Linguistic and Dynamic Context. **Expert Systems With Applications 38:3 (2011), 1675-1688 doi:10.1016/j.eswa.2010.07.092.**

4.3. A New Consensus Model for Group Decision Making Problems with Non Homogeneous Experts

Usually, when the field of the decision is large and non homogeneous, there are different kinds of experts together in the problem framework. Therefore, the experts' opinions management becomes an important task. To do so in an appropriate way, we need to know the experts' typology or the kind of specialization of each expert before starting the decision making process. Usually experts' heterogeneity is managed in the selection process in order to obtain a weighted collective preference, however, the current approaches do not take into account the experts' importance in the consensus process.

When the agreement of the experts is low, it seems reasonable to send more advice information to those experts with less importance or knowledge level [KZR, MMHV09]. In such a case, in order to bring the preferences closer to each other, we propose a new importance-based feedback mechanism that replaces and automates the moderator's tasks computing and sending different recommendations to the experts according to their own importance degrees. In such a way, we use the experts' importance on the discussion phase (consensus process) to generate importance-based recommendations.

Consequently, we present an importance-based consensus reaching process in order to compute more suitable advice composed of two stages (see Figure 12).

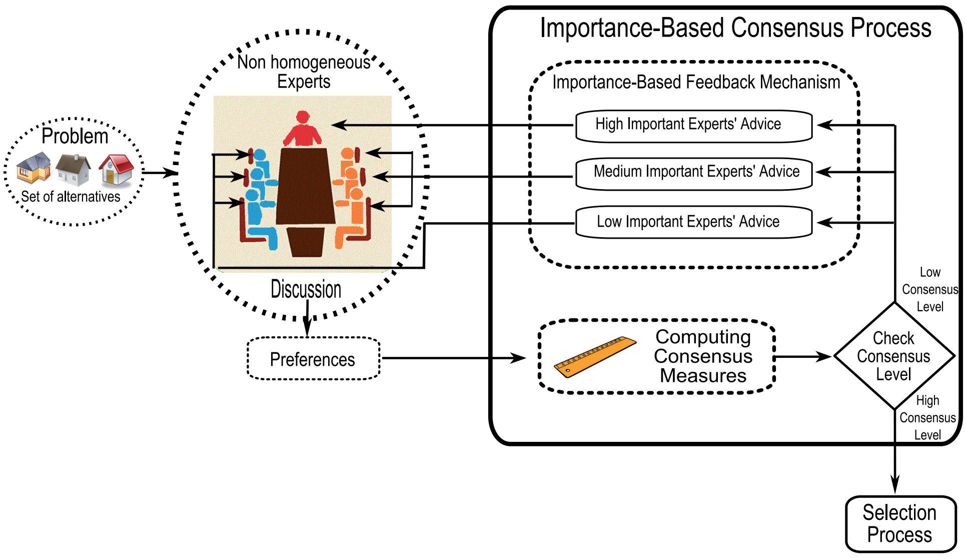

Figure 12: Importance-based consensus reaching process

1. **Computing Consensus Measures and Control the Consensus Process:**

   Once the preferences have been given by the experts, we can compute the level of agreement achieved in the current round. To do so, we obtain the consensus degrees at three different levels to obtain a global consensus degree, called consensus on the relation.

   The consensus indicators make it possible to point out the most controversial alternatives and/or experts isolated in their opinions. Thus, we propose a new importance-based search for preferences to obtain customized recommendations that can narrow the experts' minds.

2. **Importance-Based Feedback Mechanism:**

   If the agreement of the experts is low, then there exists some experts' preferences in disagreement. In such a case, in order to bring the preferences closer to each others, we have to identify the preferences and experts that are hindering the agreement and send them some advice trying to change their mind. This phase is known as feedback process.

   In this section, we propose a new feedback mechanism to guide the change of the controversial experts' opinions. This mechanism is based on the supposition that those experts with lower knowledge level on the problem will need more advice than others with higher importance. In summary, we try to adapt the search for preferences in disagreement to the different kinds of experts. When we are dealing with hight-important experts, it is obvious that their opinions belong to a wider knowledge than the remaining ones. In such a case, only a few number of changes of opinions might lead to consensus. Similarly, when the experts have lower importance, a high number of changes of opinions might be necessary to achieve good consensual solutions. Therefore, in this approach, we propose to compute a customized amount of advice which varies in accordance with the experts' weight values. To do so, we define three different advising strategies to identify the preferences that each expert should modify, in order to increase the consensus level in the next consensus round,

   a) *Advising high-important experts,*

   b) *Advising medium-important experts* and

   c) *Advising low-important experts.*

The associated journal article to this part is:

- I.J. Pérez, F.J. Cabrerizo, S. Alonso, E. Herrera-Viedma, A New Consensus Model for Group Decision Making Problems with Non Homogeneous Experts. **Submitted to IEEE Transactions on Fuzzy Systems**.

4.4. A Linguistic Consensus Model for Web 2.0 Communities

In this section we present a new consensus model that can be applied in Web 2.0 Communities to reach solutions in GDM environments. It takes into account the different characteristics of this kind of communities in order to increase the consensus level of the users when making a decision on a set of alternatives. Some of the main properties of the model are:

- It does not require the existence of a moderator,
- It allows to work in highly dynamical environments where participation and contribution rates change,
- It uses linguistic information to model user preferences and trust relations,
- It allows to weight the contributions of each user according to some degree of expertise,
- It offers a feedback mechanism to help experts to change their preferences about the alternatives and
- It can be easily adapted to real world Web 2.0 communities.

Its operation implies several different steps that are repeated in each consensus round (see Figure 13):

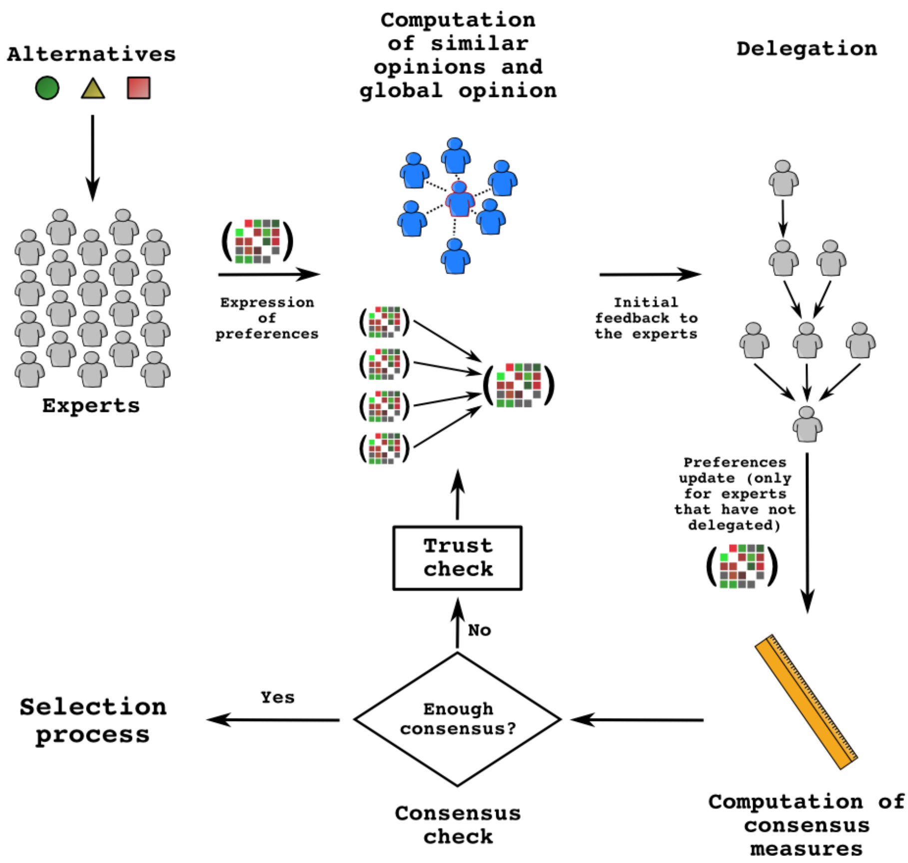

Figure 13: Consensus process scheme for web 2.0 communities

1. **First preferences expression, computation of similar opinions and first global opinion and feedback:**

   In this first step the different alternatives in the problem are presented to the experts (note than in figure 13 we have represented only a small amount of experts, but when applied to a Web 2.0 Community the number of users will usually be larger). Once they know the feasible alternatives, each expert $e^h \in E$ is asked to provide a fuzzy linguistic preference relation P^h that represent his opinions about the alternatives. Although every single member of the community has the opportunity of expressing his preferences about the alternatives only a subset of those experts $\tilde{E}$ will really provide preference relations. We will note $\tilde{e}^h$ to the experts that have provided a preference relation. It is important to note that if an expert at this stage does not provide a preference relation the model will still allow him to contribute in the consensus process in a later stage. Once a certain amount of time has passed (to allow a sufficient number of preferences to be provided) we compute the distance among each pair of experts. These distances will be used to provide information to each expert about the experts that share a similar opinion about of the alternatives. In this step we also compute the current global preference as an aggregation of all the provided preference relations.

Once the distances among experts, the neighbours of each expert and the global preference relation have been computed, this information will be presented to the experts. After receiving this feedback, an expert will know if his opinions are very different to the current global preferences and he will also know which are the experts that share similar opinions. Apart from just his neighbour list, an expert is also able to check the particular preference relations that his neighbours have introduced in order to really check the preferences expressed by his neighbourhood.

2. **Delegation:**

   In this second step the model incorporates a delegation scheme in which experts may choose to delegate into other experts (typically experts from their neighbourhood, with similar opinions). This mechanism is introduced to soften the intermittent contributions problem (because an expert who knows that he will not be able to continue the resolution process may choose to delegate into other experts instead of just leaving the process) and to decrease the number of preference relations involved in the problem. To make the delegation scheme flexible enough and to be able to cover a wide range of different delegation proposals, an expert $\tilde{e}^h$ that decides to delegate has to provide a set of trust evaluations of the other experts. Therefore, this mechanism is based on a particular kind of trust network that simplifies the computations and the time needed to obtain the users preferences.

3. **Change of preferences (feedback mechanism):**

   Once the trust values have been received the system will ask the remaining experts to update their linguistic preference relations P^h in order to achieve a greater level of consensus. This experts will conform the new $\tilde{E}$ subset. As in some cases changing the linguistic preference relations may not be an easy task, the model includes a feedback mechanism that identifies which experts and preference values should be changed to increase the level of consensus and which advices the corresponding experts about it. To do so, the system computes several proximity measures at three different levels: pair of alternatives, alternatives and relations levels.

4. **Computation of consensus measures:**

   Once the updated preferences have been given we can compute some consensus degrees at the same three different levels: pair of alternatives, alternatives and relations.

5. **Consensus and trust checks:**

   In the end of each consensus round we must check the current consensus state. If it is considered a high enough consensus value the consensus process would finish and a selection process would be applied to obtain the final solution for the decision problem. In the case that the level of consensus is not high enough we would continue with the trust check that is introduced to avoid some of the problems that can be derived to one of the characteristics of Web Communities: the difficulty of establishing real trust relations. It is not difficult to imagine an scenario where some experts delegate into another that shares a common point of view on the decision that has to be made and in a certain consensus round, this expert decides to drastically change his preferences, probably not reflecting the other experts opinions anymore. To avoid this kind of situations the trust check will compare the last preference relation expressed by expert $\tilde{e}^h$ with the last preference relations of the experts that delegated in him (direct or indirectly). If the expert has changed his preferences so much, the expert that delegated in $\tilde{e}^h$ would be informed with a special message to warn him about this problematic situation and thus allowing him to take a different course of action in the next consensus round if appropriate. At this point a new consensus round begins. In this new round the current global preference will be computed as a weighted mean of the preferences expressed by the experts in $\tilde{E}$. The weights to be used in this aggregation operation are the accumulated trust values of the trust network obtained in the delegation process.

The associated journal article to this part is:

- S. Alonso, I.J. Pérez, F.J. Cabrerizo, E. Herrera-Viedma, A Linguistic Consensus Model for Web 2.0 Communities. **Submitted to IEEE Transactions on Systems, Man and Cybernetics - Part A: Systems and Humans**.

5. Comentarios Finales: Resultados Obtenidos y Conclusiones

En esta sección se resumen brevemente los resultados y conclusiones obtenidas en cada una de las etapas del estudio.

1. **Análisis de las diferentes propuestas existentes para medir el consenso en problemas de TDG:**

   Hemos analizado las distintas alternativas existentes para medir el consenso en problemas de TDG, observando las ventajas e inconvenientes de cada una de ellas. Además, hemos descrito las nuevas tendencias que se centran en la generación de recomendaciones para ayudar a los expertos a modificar sus preferencias de forma que se alcance un alto grado de consenso. Por otra parte, estas nuevas propuestas se pueden extender intentando adaptar el proceso de consenso para reducir el número de preferencias que deberían ser modificadas tras cada ronda del proceso, haciendo más rápida y efectiva la convergencia hacia una solución consensuada.

2. **Un sistema de apoyo a la decisión móvil basado en información heterogénea y en contextos variables:**

   Hemos presentado un prototipo de sistema de apoyo a la decisión móvil para problemas de TDG basado en contextos de decisión dinámicos que permite el uso de cuatro formatos de representación de preferencias distintos para adaptarse de la mejor forma posible al tipo de problema y al nivel del experto. Además, el sistema incorpora una nueva herramienta para gestionar los cambios (entradas y salidas) que se pueden producir en el conjunto de alternativas durante el proceso de decisión. El prototipo hace uso de las ventajas que nos ofrecen los dispositivos móviles, pudiendo ser utilizado en cualquier momento desde cualquier lugar, aumentando así la satisfacción del usuario. Para realizar las pruebas del prototipo hemos utilizado un teléfono móvil, pero su estructura está diseñada para que pueda ser ejecutado desde cualquier dispositivo móvil como smartphones o PDAs.

   También hemos propuesto un nuevo proceso de selección para tratar problemas de TDG con información lingüística incompleta, este problema aparece cuando alguno de los expertos es incapaz de evaluar alguna de las alternativas y por tanto no puede expresar una opinión sobre ella. Para resolverlo, proponemos un mecanismo para completar estas opiniones con el uso de algunas medidas de consistencia. Este proceso se divide en tres fases: i) estimación de valores perdidos, ii) agregación y iii) explotación. La principal novedad de esta propuesta es la posibilidad de gestionar las preferencias de los usuarios cuando son incompletas, dando lugar a una agregación que otorga más importancia a las opiniones más consistentes.

   Además hemos presentado un nuevo modelo de TDG con información lingüística basado en contextos variables con falta de información. Este modelo ha sido implementado a partir del anterior prototipo, permitiendo el manejo de conjuntos de alternativas variables y ampliando su funcionalidad para tratar con información lingüística y gestionar la falta de información cuando los expertos no son capaces de expresar sus preferencias acerca de cada una de las alternativas. La ejecución también se hará desde dispositivos móviles para conseguir una mejor interacción de usuario con el proceso de decisión.

3. **Un Mecanismo de Generación de Recomendaciones Basado en la Importancia de los Expertos para Problemas de Toma de Decisión en Grupo para Alcanzar Consenso entre Expertos No Homogeneos:**

   La siguiente propuesta se basa en un nuevo modelo de consenso diseñado con la finalidad de gestionar la heterogeneidad entre expertos. Este modelo incorpora un mecanismo de generación de recomendaciones que ajusta la cantidad de consejo que un experto recibirá atendiendo a su nivel de importancia. De esta forma, las opiniones de los expertos más importantes se modificarán menos que las de los expertos con un conocimiento más limitado acerca del problema.

4. **Un modelo de consenso lingüístico para comunidades web 2.0:**

   Para finalizar, hemos presentado un nuevo modelo de consenso diseñado específicamente para ser utilizado por las comunidades Web 2.0. El modelo está pensado para gestionar un amplio conjunto de usuarios gracias a un mecanismo de delegación. Esta herramienta se basa e una especie de red de confianza que simplifica tanto los cálculos como el tiempo necesario para obtener las opiniones de todos los usuarios. Además, este mecanismo no solo resuelve el problema que se presenta cuando los usuarios contribuyen con su opinión de forma intermitente sino que también permite la incorporación de nuevos usuarios una vez ha comenzado el proceso de decisión. Por último, el modelo incorpora otro mecanismo para controlar que la red de confianza no se corrompa, evitando las situaciones en las que un usuario que goza de la confianza de otros, pueda cambiar su opinión por completo y seguir aprovechándose de esta confianza que posiblemente ya no merezca.

5. Concluding Remarks: Summary of the Obtained Results and Conclusions

The following section briefly summarize the obtained results and present several conclusions.

1. **Analyzing consensus approaches in fuzzy group decision making:**

   We have analyzed different consensus approaches to compute soft consensus measures in fuzzy GDM problems. Additionally, we have described the new advanced approaches, i.e., those approaches allowing to generate recommendations to help experts change their opinions in order to obtain the highest degree of consensus possible, and, on the other hand, those approaches adapting the consensus process to increase the agreement and reduce the number of experts' preferences that should be changed after each consensus round.

2. **Mobile decision support systems based on heterogeneous information and changeable contexts:**

   We have presented a prototype of MDSS for GDM problems based on dynamic decision environments, which can be used with four different formats to represent the preferences in the best way according to the kind of problem and the experts' knowledge level. Moreover, the prototype incorporates a new tool for managing dynamic inputs and outputs of alternatives in the set of solution alternatives throughout the decision process. The prototype uses the advantages of M-Internet technologies to improve user satisfaction with the decision process and develop decision processes anytime and anywhere. We have used mobile phones as the device used by the experts to send their preferences, but the structure of the prototype is designed to use any other mobile device, such as PDAs.

   Secondly, we have proposed a new selection process based on additive consistency to deal with GDM problems under incomplete fuzzy linguistic information. The lack of information is a problem when some of the experts are not able to assess some of the alternatives and consequently they can not express a complete opinion. This new selection process is composed of three phases: estimation of missing values, aggregation and exploitation. The main improvements of this selection process is that it supports the management of incomplete fuzzy linguistic information and it allows the aggregation of the experts' preferences in such a way that more importance is given to the most consistent ones.

   Finally, we have presented a new model of linguistic GDM based on dynamic information and mobile technologies. We have also implemented a prototype of this system. As in our previous system, it is designed to deal with linguistic GDM problems based on dynamic sets of alternatives, which uses the advantages of mobile Internet technologies to improve the user-system interaction through decision process. Moreover, the experts can use FLPR to express their preferences and it provides a tool that manages lack of information when an expert is not able to give a complete FLPR. Shortly, with this new GDM model we shall be able to model linguistic GDM problems in which experts could interact in anywhere and anytime, quickly, in a flexible way, and dynamic frameworks.

3. **A new consensus model for group decision making problems with non homogeneous experts:**

   We have proposed a novel consensus approach which has been specially designed to model non homogeneous decision frameworks in the sense of heterogeneity among experts. Assuming three different levels of importance, we have presented a new feedback mechanism that computes different amount of advice according to the experts' importance level. Consequently, the most considerable experts' opinions never will be strongly modified during the consensus reaching process.

4. **A linguistic consensus model for web 2.0 communities:**

   Finally, we have presented a novel consensus model which has been specially designed to be applied in Web 2.0 Communities. Particularly, it has been designed to manage a large users base by means of a delegation scheme. This delegation scheme is based in a particular kind of trust network that simplifies the computations and the time needed to obtain the users preferences. Moreover, this delegation scheme also solves the intermittent contributions problem which is present in almost any online community (that is, many of the users will not continuously collaborate but will do it from time to time). In addition, the model allows to incorporate new experts to the consensus process, that is, the model is able to handle some of the dynamic properties that real Web Communities have. Finally, the model incorporates a trust check mechanism that allow to detect some abnormal situations in which an expert may try to take advantage of others by drastically changing his opinion and benefiting from the trust that the other experts might have deposited in him in previous consensus rounds.

6. Future Work

In this dissertation we have analyzed some new models that help to solve GDM problems with heterogeneous information based on different frameworks. In the previous section we have shortly mentioned some different results that we have obtained, but still there exists more work to be done. Next we present some future open research lines raised from the proposals made in this memory.

1. **Theoretical:**

   - To model more complex real consensus processes, several changes and additions have to be developed. For example, in real consensus processes, experts are not independent, that is, they can be influenced or persuaded by some of their college to change their opinions. Therefore, it seems necessary to obtain new measures in order to quantify and simulate the weapons of influence of each experts on the remaining ones.
   - GDM models are usually defined for static frameworks, in such a way, it is necessary to adapt them to solve dynamic problems where the decision information (including attribute weights and attribute values) are provided at different periods.
   - To increment the number of real GDM situations that can be modelled we will study problems where incomplete information could be expressed by experts in different preference representation formats (type-2 fuzzy sets, unbalanced linguistic term sets, incomplete utility values, and so on).

2. **Practical:**

   - It is important to implement all the models presented to be able to use them in different frameworks. We have to take advantage of the great power of communication that the mobile technologies provides today. To do so, implement these models using these technologies will allow to carry out GDM processes in every country of the world and anytime.
   - We will study the development of more powerful feedback mechanisms to produce complete and useful recommendations to the experts and thus help them to solve the decision problem more efficiently.
   - We will also develop new aiding tools to help experts to reach good decisions. In particular, some graphical interfaces could be useful to easily understand the current consensus state.

Part II. Publications: Published, Accepted and Submitted Papers

1. Analyzing Consensus Approaches in Fuzzy Group Decision Making: Advantages and Drawbacks

The journal paper associated to this part is:

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Analyzing consensus approaches in fuzzy group decision making: advantages and drawbacks

F. J. Cabrerizo · J. M. Moreno · I. J. Pérez · E. Herrera-Viedma

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Abstract Two processes are necessary to solve group decision making problems: a consensus process and a selection process. The consensus process is necessary to obtain a final solution with a certain level of agreement between the experts, while the selection process is necessary to obtain such a final solution. Clearly, it is preferable that the set of experts reach a high degree of consensus before applying the selection process. In order to measure the degree of consensus, different approaches have been proposed. For example, we can use hard consensus measures, which vary between 0 (no consensus or partial consensus) and 1 (full consensus), or soft consensus measures, which assess the consensus degree in a more flexible way. The aim of this paper is to analyze the different consensus approaches in fuzzy group decision making problems and discuss their advantages and drawbacks. Additionally, we study the future trends.

F. J. Cabrerizo (✉)
Department of Software Engineering and Computer Systems, Distance Learning University of Spain, 28040 Madrid, Spain
e-mail: cabrerizo@issi.uned.es

J. M. Moreno
Department of Information and Communication Engineering, University of Murcia, 30100 Murcia, Spain
e-mail: jmmoreno@um.es

I. J. Pérez · E. Herrera-Viedma
Department of Computer Science and Artificial Intelligence, University of Granada, 18071 Granada, Spain

I. J. Pérez
e-mail: ijperez@decsai.ugr.es

E. Herrera-Viedma
e-mail: viedma@decsai.ugr.es

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1 Introduction

In a classical group decision making (GDM) situation there is a problem to solve, a solution set of possible alternatives, $X = \{x_1, \ldots, x_n\}$, and a group of two or more experts, $E = \{e_1, \ldots, e_m\}$, characterized by their own ideas, attitudes, motivations and knowledge, who express their opinions about this set of alternatives to achieve a common solution (Lu et al. 2008; Montero 2008; Nurmi 2008). To do this, experts have to express their preferences by means of a set of evaluations over the set of alternatives.

GDM problems arise from many real-world situations (Chen and Hwang 1992). To solve these problems, experts apply two processes before obtaining a final solution (Herrera-Viedma et al. 2005; Kacprzyk et al. 1992; Kacprzyk et al. 1997): *consensus process* and *selection process* (see Fig. 1). The former consists in how to obtain the maximum degree of consensus or agreement between the set of experts on the solution set of alternatives. Normally, the consensus process is guided by a human figure called moderator (Herrera et al. 1996; Kacprzyk et al. 1992) who does not participate in the discussion but knows the agreement in each moment of the consensus process and is in charge of supervising and addressing the consensus process toward success, i.e., to achieve the maximum possible agreement and to reduce the number of experts outside the consensus in each new consensus round. The latter refers to how to obtain the solution set of alternatives from the opinions on the alternatives given by the experts. It involves two different steps (Herrera et al. 1998; Roubens 1997): aggregation of individual opinions and exploitation of the collective opinion. Clearly, it is preferable that the set of experts achieves a great agreement among their opinions before applying the selection process.

A consensus process is defined as a dynamic and iterative group discussion process, coordinated by a moderator helping experts bring their opinions closer. At the beginning of every GDM problem, the set of experts has diverging opinions, then the consensus process is applied and, in each step, the degree of existing consensus among experts' opinions is measured. If the consensus degree is lower than a specified threshold, the moderator would urge experts to discuss their opinions further in an effort to bring them closer. Otherwise, the moderator would apply the selection process in order to obtain the final consensus solution to the GDM problem.

A natural question in the consensus process is how to measure the closeness among experts' opinions in order to obtain the consensus level. To do so, different approaches have been proposed. For instance, several authors have introduced *hard consensus measures* varying between 0 (no consensus or partial consensus) and 1 (full consensus) (Bezdek et al. 1977, 1978; Spillman et al. 1979, 1980). In this way, using *hard consensus measures*, a distance from consensus as a difference between some average preference matrix and one of several possible consensus preference matrices is determined in Bezdek et al. (1977, 1978). In Spillman et al. (1979), some measures of attitudinal similarity between individuals that is an extension of the classical Tanimoto coefficient are derived. Finally, a consensus measure based on a-cuts of the respective individual fuzzy preference matrices is derived in Spillman et al. (1980). However, consensus as a full and unanimous agreement is far from being achieved in real situations and, even if it is, in such a situation, the consensus reaching process could be unacceptably expensive. A more realistic approach is to use *soft consensus measures* (Kacprzyk 1987; Kacprzyk and Fedrizzi 1986, 1988), which assess the consensus degree in a more flexible way and, therefore, reflect the large spectrum of possible partial agreements and guide the consensus process until widespread agreement (not always full) is achieved among experts. Soft consensus measures are based on the concept of coincidence (Herrera et al. 1997), measured by means of similarity criteria defined among experts' opinions.

The aim of this paper is to analyze consensus approaches in fuzzy GDM problems to compute soft consensus measures and discuss their advantages and drawbacks. We identify three different coincidence criteria to compute soft consensus measures: (1) *strict coincidence among preferences*, (2) *soft coincidence among preferences* and (3) *coincidence among solutions*. Using these coincidence criteria, two advanced consensus approaches have been proposed:

– Approaches allowing to generate recommendations to help experts change their opinions in order to obtain the highest degree of consensus possible (Herrera-Viedma et al. 2002, 2005, 2007), and
– approaches adapting the consensus process to increase the agreement and to reduce the number of experts' preferences that should be changed after each consensus round (Mata et al. 2009).

The rest of the paper is organized as follows. In Sect. 2, we analyze the different approaches to obtain soft

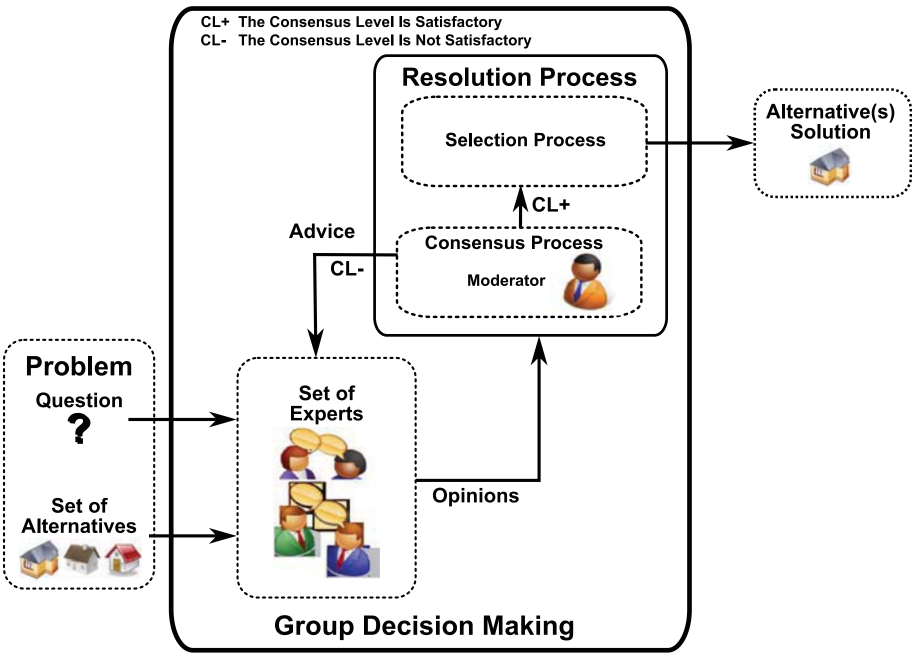

Fig. 1 Resolution process of a GDM problem

consensus measures in fuzzy GDM problems and illustrate an example of application. In Sect. 3, we discuss their advantages and drawbacks. The advanced consensus approaches are shown in Sect. 4. Finally, some concluding remarks are pointed out in Sect. 5.

2 Approaches to obtain soft consensus measures in fuzzy GDM problems

As aforementioned, soft consensus measures are based on the coincidence concept (Herrera et al. 1997), and we can identify three different consensus approaches to compute them: (1) *consensus models based on strict coincidence among preferences*, (2) *consensus models based on soft coincidence among preferences*, and (3) *consensus models based on coincidence among solutions*. We describe them in more detail in the following subsections.

2.1 Consensus models based on strict coincidence among preferences

In this case, similarity criteria among preferences are used to compute the coincidence concept. Only two possible results are assumed: the total coincidence (value 1) or null coincidence (value 0). Some examples of this approach are the following:

- In Kacprzyk (1987), assuming fuzzy preference relations to represent experts' preferences, the first consensus model based on strict coincidence was defined. Given a particular alternative pair and two experts, if their preferences are equal, then they are in agreement (value 1), and, otherwise, they are in disagreement (value 0). Then, consensus measures are calculated across the global set of the alternatives in a hierarchical pooling process from the coincidence measured on experts' preferences and using the fuzzy majority concept represented by a linguistic quantifier (Zadeh 1983).
- In Herrera et al. (1996, 1997), different consensus measures based on strict coincidence were presented assuming that experts' preferences are provided by means of linguistic preference relations. Applying the strict coincidence on preferences provided by the experts for each alternative pair, the expert group is divided into subsets, one subset for each possible linguistic label used to qualify the preference on the alternative pair. Then, using the cardinalities of the subsets of experts, three kinds of consensus measures are defined, each one associated with the three different levels of representation of a preference relation: alternative pair, individual alternative and global relation.

Assume a fuzzy GDM problem based on linguistic preference relations as in Herrera et al. (1996, 1997), i.e., a GDM problem where the experts $E = \{e_1, \ldots, e_m\}$ express their preferences relations $P = \{P^1, \ldots, P^m\}$ on the set of alternatives X, using a linguistic term set $S = \{s_0, \ldots, s_g\}$ whose cardinality or granularity $\#S = g + 1$, being $p_{ik}^h \in S$ the preference degree of alternative x_i over alternative x_k for the expert e_h. Additionally, the following properties are assumed (Herrera-Viedma 2001, 2006):

1. The set S is ordered: $s_i \geq s_j$ if $i \geq j$.
2. Negation operator: $\mathrm{Neg}(s_i) = s_j$ such that $j = g - i$.
3. Min operator: $\mathrm{Min}(s_i, s_j) = s_i$ if $s_i \leq s_j$.
4. Max operator: $\mathrm{Max}(s_i, s_j) = s_i$ if $s_i \geq s_j$.

Then, a consensus model based on strict coincidence could be carried out in the following steps:

1. First, for each pair of experts (e_h, e_l) $(h = 1, \ldots, m-1,\ l = h+1, \ldots, m)$, a strict similarity matrix $\mathrm{SM}^{hl} = [sm_{ik}^{hl}], i, k = 1, \ldots, n$, is obtained as follows:

$$sm_{ik}^{hl} = \begin{cases} 1, & \text{if } p_{ik}^h = p_{ik}^l \\ 0, & \text{otherwise} \end{cases}. \tag{1}$$

2. Then, a collective similarity matrix, $\mathrm{SM} = [sm_{ik}]$, is obtained by aggregating all the similarity matrices using the arithmetic mean ϕ as the aggregation function:

$$sm_{ik} = \phi(sm_{ik}^{hl}, h = 1, \ldots, m-1, l = h+1, \ldots, m). \tag{2}$$

Note 1: In this case, we have used the arithmetic mean as aggregation function ϕ, although, different aggregation operators could be used according to the particular properties that we want to implement.

3. Computing the consensus degrees and proximity measures as in Herrera et al. (1996):

(a) **Consensus degrees**: once the similarity matrices are computed, the consensus degrees are calculated as follows:

1. **Level 1**. *Consensus degree on pairs of alternatives*. The consensus degree, cop_{ik}, on a pair of alternatives, (x_i, x_k), is defined to measure the consensus degree among all the experts on that pair of alternatives. In this case, this is expressed by the element of the collective similarity matrix SM:

$$\mathrm{cop}_{ik} = sm_{ik} \tag{3}$$

The closer cop_{ik} is to 1, the greater the agreement among all the experts on the pair of alternatives (x_i, x_k). This measure will allow the identification of those pairs of alternatives with a poor level of consensus.

2. **Level 2**. *Consensus degree on alternatives*. The consensus degree on the alternative x_i, called ca_i, is defined

to measure the consensus degree among all the experts on that alternative:

$$ca_i = \frac{\sum_{k=1;k\neq i}^{n} (\text{cop}_{ik} + \text{cop}_{ki})}{2n-2} \tag{4}$$

These values can be used to propose the modification of preferences associated with those alternatives with a consensus degree lower than a minimal consensus threshold γ.

3. **Level 3**. *Consensus degree on the relation*. The consensus degree on the relation, called CR, is defined to measure the global consensus degree among all the experts' opinions. It is computed as the average of all the consensus degrees on the alternatives:

$$\text{CR} = \frac{\sum_{i=1}^{n} ca_i}{n}. \tag{5}$$

This is the value used to control the consensus situation.

Note 2: In Herrera et al. (1996) three kinds of consensus are proposed because they allow us to know the current state of consensus from different viewpoints and, therefore, to guide more correctly the consensus reaching process.

(b) **Proximity measures**: to compute the proximity measures for each expert, we need to obtain the collective preference relation, $P^c = [p^c_{ik}]$, which summarizes preferences given by all the experts and is calculated by means of the aggregation of the set of individual preference relations $\{P^1, \ldots, P^m\}$ as follows:

$$p^c_{ik} = \phi(p^1_{ik}, \ldots, p^m_{ik}). \tag{6}$$

To do so, the *linguistic ordered weighted averaging* (LOWA) operator (Herrera et al. 1996) can be used. The LOWA operator is based on the ordered weighted averaging (OWA) operator defined in Yager (1988), and on the convex combination of linguistic labels defined in Delgado et al. (1993). In Herrera et al. (1996), it was shown that it is a rational operator to aggregate linguistic information that satisfies some important properties as commutativity, monotony, unanimity and neutrality.

Definition 1 Let $A = \{a_1, \ldots, a_m\}$ be a set of labels to be aggregated, then the LOWA operator, ϕ, is defined as:

$$\begin{aligned}\phi(a_1, \ldots, a_m) &= W \cdot B^T = \mathcal{C}^m\{w_k, b_k, k = 1, \ldots, m\} \\ &= w_1 \odot b_1 \oplus (1 - w_1) \\ &\quad \odot \mathcal{C}^{m-1}\{\beta_h, b_h, h = 2, \ldots, m\}\end{aligned} \tag{7}$$

where $W = [w_1, \ldots, w_m]$ is a weighting vector, such that $w_i \in [0,1]$ and $\Sigma_i w_i = 1$. $\beta_h = w_h / \Sigma_2^m w_k$, $h = 2, \ldots, m$, and $B = \{b_1, \ldots, b_m\}$ is a vector associated with A, such that $B = \sigma(A) = \{a_{\sigma(1)}, \ldots, a_{\sigma(m)}\}$, where, $a_{\sigma(j)} \leq a_{\sigma(i)}$ $\forall i \leq j$, with σ being a permutation over the set of labels A. $\mathcal{C}^m$ is the convex combination operator of m labels and if $m = 2$, then it is defined as $\mathcal{C}^2\{w_i, b_i, i = 1, 2\} = w_1 \odot s_j \oplus (1 - w_1) \odot s_i = s_k$, such that $k = \min\{\mathcal{T}, i + \text{round}(w_1 \cdot (j - i))\} s_j$, $s_i \in S$, $(j \geq i)$, where "round" is the usual round operation, and $b_1 = s_j$, $b_2 = s_i$. If $w_j = 1$ and $w_i = 0\, with\, i \neq j\, \forall i$, then the convex combination is defined as: $\mathcal{C}^m\{w_i, b_i, i = 1, \ldots, m\} = b_j$.

Using P^c, for each expert, e_h, a proximity matrix, $\text{PM}^h = [pm^h_{ik}]$, is obtained:

$$pm^h_{ik} = \begin{cases} 1, & \text{if } p^h_{ik} = p^c_{ik} \\ 0, & \text{otherwise} \end{cases}. \tag{8}$$

Finally, the computation of the proximity measures is carried out at three different level as follows:

1. **Level 1**. *Proximity measure on pairs of alternatives*. The proximity measure of an expert e_h on a pair of alternatives (x_i, x_k) to the group's one, called pp^h_{ik}, is expressed by the element (i,k) of the proximity matrix PM^h:
$$pp^h_{ik} = pm^h_{ik}. \tag{9}$$
2. **Level 2**. *Proximity measure on alternatives*. The proximity measure of an expert e_h on an alternative x_i to the group's one, called pa^h_i, is calculated as follows:
$$pa^h_i = \frac{\sum_{k=1,k\neq i}^{n} (pp^h_{ik} + pp^h_{ki})}{2n-2}. \tag{10}$$
3. **Level 3**. *Proximity measure on the relation*. The proximity measure of an expert e_h on his/her preference relation to the group's one, called pr^h, is calculated as the average of all proximity measures on the alternatives:
$$pr^h = \frac{\sum_{i=1}^{n} pa^h_i}{n}. \tag{11}$$

Given an expert, if his or her proximity measure is close to 1, then he or she has a positive contribution for the consensus to be high, while if it is close to 0, then he or she has a negative contribution to the consensus.

Example 1 Suppose four experts $E = \{e_1, e_2, e_3, e_4\}$ use the linguistic term set $S = \{Null(N), Very\,Low(VL), Low(L), Medium(M), High(H), Very\,High(VH), Total(T)\}$ to provide their linguistic preference relations on a set of four alternatives:

$$P^1 = \begin{pmatrix} - & H & VH & L \\ L & - & T & VH \\ L & N & - & L \\ H & L & VH & - \end{pmatrix}; \quad P^2 = \begin{pmatrix} - & H & H & M \\ L & - & VH & T \\ VL & L & - & H \\ M & N & L & - \end{pmatrix}$$

$$P^3 = \begin{pmatrix} - & H & M & VH \\ L & - & M & L \\ L & L & - & T \\ VL & H & N & - \end{pmatrix}; \quad P^4 = \begin{pmatrix} - & L & H & M \\ VH & - & M & VH \\ L & M & - & L \\ M & L & T & - \end{pmatrix}.$$

As aforementioned, to obtain the consensus degrees, we compute the different strict similarity matrix for each pair of experts using Eq. (1):

$$\mathrm{SM}^{12} = \begin{pmatrix} - & 1.0 & 0.0 & 0.0 \\ 1.0 & - & 0.0 & 0.0 \\ 0.0 & 0.0 & - & 0.0 \\ 0.0 & 0.0 & 0.0 & - \end{pmatrix}; \quad \mathrm{SM}^{13} = \begin{pmatrix} - & 1.0 & 0.0 & 0.0 \\ 1.0 & - & 0.0 & 0.0 \\ 1.0 & 0.0 & - & 0.0 \\ 0.0 & 0.0 & 0.0 & - \end{pmatrix}$$

$$\mathrm{SM}^{14} = \begin{pmatrix} - & 0.0 & 0.0 & 0.0 \\ 0.0 & - & 0.0 & 1.0 \\ 1.0 & 0.0 & - & 1.0 \\ 0.0 & 1.0 & 0.0 & - \end{pmatrix}; \quad \mathrm{SM}^{23} = \begin{pmatrix} - & 1.0 & 0.0 & 0.0 \\ 1.0 & - & 0.0 & 0.0 \\ 0.0 & 1.0 & - & 0.0 \\ 0.0 & 0.0 & 0.0 & - \end{pmatrix}$$

$$\mathrm{SM}^{24} = \begin{pmatrix} - & 0.0 & 1.0 & 1.0 \\ 0.0 & - & 0.0 & 0.0 \\ 0.0 & 0.0 & - & 0.0 \\ 1.0 & 0.0 & 0.0 & - \end{pmatrix}; \quad \mathrm{SM}^{34} = \begin{pmatrix} - & 0.0 & 0.0 & 0.0 \\ 0.0 & - & 1.0 & 0.0 \\ 1.0 & 0.0 & - & 0.0 \\ 0.0 & 0.0 & 0.0 & - \end{pmatrix}.$$

Then, we compute the collective similarity matrix using the ϕ:

$$\mathrm{SM} = \begin{pmatrix} - & 0.50 & 0.17 & 0.17 \\ 0.50 & - & 0.17 & 0.17 \\ 0.50 & 0.17 & - & 0.17 \\ 0.17 & 0.17 & 0.00 & - \end{pmatrix}.$$

From SM, we obtain the following consensus degrees:

1. *Consensus degrees on pairs of alternatives*. The element (i, k) of SM represents the consensus degrees, cop_{ik}, on the pair of alternatives (x_i, x_k).
2. *Consensus degrees on alternatives*:

   $ca_1 = 0.34,\ ca_2 = 0.28,\ ca_3 = 0.20,\ ca_4 = 0.09.$
3. *Consensus degrees on the relation*:
   $\mathrm{CR} = 0.23$.

Clearly, we have a low consensus degree among experts and, therefore, in a decision situation we would have to continue the negotiation process. To do so, as in Herrera-Viedma et al. (2002, 2005, 2007), we could guide the negotiation process by means of the proximities measure. To obtain the proximity measures, we need to compute the collective fuzzy linguistic preference relation by aggregating all individual linguistic preference relations.

Using the LOWA operator (Herrera et al. 1996) with the weighting vector $W = \{0.5, 0.20, 0.16, 0.14\}$, we obtain the following P^c

$$P^c = \begin{pmatrix} - & H & H & M \\ M & - & VH & VH \\ L & L & - & H \\ M & L & H & - \end{pmatrix}.$$

The proximity matrices for each expert are:

$$\mathrm{PM}^{1} = \begin{pmatrix} - & 1.0 & 0.0 & 0.0 \\ 0.0 & - & 0.0 & 1.0 \\ 1.0 & 0.0 & - & 0.0 \\ 0.0 & 1.0 & 0.0 & - \end{pmatrix}; \quad \mathrm{PM}^{2} = \begin{pmatrix} - & 1.0 & 1.0 & 1.0 \\ 0.0 & - & 1.0 & 0.0 \\ 0.0 & 1.0 & - & 1.0 \\ 1.0 & 0.0 & 0.0 & - \end{pmatrix}$$

$$\mathrm{PM}^{3} = \begin{pmatrix} - & 1.0 & 0.0 & 0.0 \\ 0.0 & - & 0.0 & 0.0 \\ 1.0 & 1.0 & - & 0.0 \\ 0.0 & 0.0 & 0.0 & - \end{pmatrix}; \quad \mathrm{PM}^{4} = \begin{pmatrix} - & 0.0 & 1.0 & 1.0 \\ 0.0 & - & 0.0 & 1.0 \\ 1.0 & 0.0 & - & 0.0 \\ 1.0 & 1.0 & 0.0 & - \end{pmatrix}.$$

And then, the proximity measures are:

1. *Proximity measure on pairs of alternatives*. The proximity measure of an expert e_h on a pair of alternatives (x_i, x_k) to the group's one, pp^h_{ik}, is expressed by the element (i, k) of the proximity matrix PM^h.
2. *Proximity measure on alternatives*:

$$\{pa^1_1, pa^1_2, pa^1_3, pa^1_4\} = \{0.33, 0.50, 0.17, 0.33\}$$
$$\{pa^2_1, pa^2_2, pa^2_3, pa^2_4\} = \{0.67, 0.50, 0.67, 0.50\}$$
$$\{pa^3_1, pa^3_2, pa^3_3, pa^3_4\} = \{0.33, 0.33, 0.33, 0.00\}$$
$$\{pa^4_1, pa^4_2, pa^4_3, pa^4_4\} = \{0.67, 0.33, 0.33, 0.67\}.$$

3. *Proximity measure on the relation*:

$$pr^1 = 0.33,\ pr^2 = 0.58,\ pr^3 = 0.25,\ pr^4 = 0.50.$$

With these scores, the experts 1 and 3 should change highly their positions to increase the level of consensus in the next consensus rounds.

2.2 Consensus models based on soft coincidence among preferences

As above, similarity criteria among preferences are used to compute the coincidence concept. However, in this case, a major number of possible coincidence degrees is considered. It is assumed that the coincidence concept is a gradual concept, which could be assessed with different degrees defined in the unit interval [0,1]. These are the more popular consensus models. Some examples of this approach are the following:

– In Kacprzyk (1987), a first consensus model based on soft coincidence was also defined. But in this case, given a particular alternative pair and two experts, the coincidence among their preference is measured using a closeness function $s : [0, 1] \to [0, 1]$.
– In Kacprzyk and Fedrizzi (1986, 1988), some soft consensus measures are introduced and defined as extensions of those presented in Kacprzyk (1987), considering GDM problems with heterogeneous set of alternatives and heterogeneous groups of experts, respectively.
– An extension of these models is presented in Fedrizzi et al. (1993), which consists in the computation of consensus measures using the ordered weighted averaging (OWA) operator (Yager 1988).
– In Bordogna et al. (1997), a soft consensus model for multi-criteria GDM problems defined in a ordinal fuzzy linguistic approach was defined. In this case, coincidence values are obtained by means of a linguistic similarity function defined directly on linguistic assessments given on the alternatives.
– In Herrera et al. (1997), the fuzzification of soft coincidence concept was presented. The soft coincidence is defined in each alternative pair of a linguistic preference relation as a fuzzy set defined on the set of expert pairs and characterized by closeness observed among their preferences. The closeness among preferences is established by means of ad hoc closeness table defined among all the possible labels of linguistic term set used to represent the preferences.
– In Herrera-Viedma et al. (2005), a soft consensus approach is presented to deal with GDM problems in a multi-granular fuzzy linguistic context. Three kinds of soft consensus measures are considered as in Herrera et al. (1996, 1997, 1997). In this case, the soft coincidence among multi-granular linguistic preferences is obtained using a similarity function defined on transformation of such preferences in a basic linguistic term set.
– In Herrera-Viedma et al. (2007), a soft consensus model based on three consensus measures was also proposed. In this case, experts provide their preferences by means of incomplete fuzzy preference relations assessed in [0,1] and the soft coincidence is defined using a similarity function among preferences in [0,1].
– Finally, in Cabrerizo et al. (2009), a soft consensus model is presented for GDM problems in an unbalanced fuzzy linguistic context (Herrera et al. 2008; Herrera-Viedma and López-Herrera 2007). In this case, as in Herrera et al. (1996, 1997, 1997), the soft coincidence is computed using a similarity function defined on transformation of unbalanced fuzzy linguistic preferences in a basic linguistic term set.

In the framework used previously, we could apply a consensus model based on soft coincidence in a fuzzy GDM problem based on linguistic preference relations as follows:

1. Compute the similarity matrices $\mathrm{SM}^{hl} = [sm^{hl}_{ik}]$, $i, k = 1, \ldots, n$,:

$$sm^{hl}_{ik} = s(p^h_{ik}, p^l_{ik}) \tag{12}$$

where $s(p^h_{ik}, p^l_{ik})$ is a similarity function which measures the coincidence between the opinions p^h_{ik} and p^l_{ik}. Depending on the fuzzy context, different similarity functions can be used (Herrera-Viedma et al. 2005, 2007).

2. Then, a collective similarity matrix, $\mathrm{SM} = [sm_{ik}]$, is obtained by aggregating all the similarity matrices using the arithmetic mean ϕ:

$$sm_{ik} = \phi(sm^{hl}_{ik}, h=1,\ldots,m-1, l=h+1,\ldots,m). \tag{13}$$

3. Compute the consensus degrees and proximity measures:

(a) **Consensus degrees**: once the similarity matrices are computed, the consensus degrees are calculated at three different levels as in the consensus models based on strict coincidence among preferences:

1. **Level 1**. *Consensus degree on pairs of alternatives*:

$$\mathrm{cop}_{ik} = sm_{ik}. \tag{14}$$

2. **Level 2**. *Consensus degree on alternatives*:

$$ca_i = \frac{\sum_{k=1;k\neq i}^{n}(cop_{ik} + cop_{ki})}{2n-2}. \tag{15}$$

3. **Level 3**. *Consensus degree on the relation*:

$$\mathrm{CR} = \frac{\sum_{i=1}^{n} ca_i}{n}. \tag{16}$$

(b) **Proximity measures**: to compute the proximity measures for each expert, we need to obtain the collective preference relation, $P^c = [p_{ik}^c]$, which is computed as follows:

$$p_{ik}^c = \phi(p_{ik}^1, \ldots, p_{ik}^m). \tag{17}$$

To do so, the LOWA operator (Herrera et al. 1996) can be used.

Using P^c, for each expert, e_h, a proximity matrix, $PM^h = [pm_{ik}^h]$, is obtained:

$$pm_{ik}^h = s(p_{ik}^h, p_{ik}^c). \tag{18}$$

Finally, the computation of the proximity measures is carried out at three different levels as follows:

1. **Level 1**. *Proximity measure on pairs of alternatives*:

$$pp_{ik}^h = pm_{ik}^h. \tag{19}$$

2. **Level 2**. *Proximity measure on alternatives*:

$$pa_i^h = \frac{\sum_{k=1,k\neq i}^{n}(pp_{ik}^h + pp_{ki}^h)}{2n-2}. \tag{20}$$

3. **Level 3**. *Proximity measure on the relation*:

$$pr^h = \frac{\sum_{i=1}^{n} pa_i^h}{n}. \tag{21}$$

Example 2 Assuming the same linguistic preference relations provided by the experts in the above example, the soft consensus degrees are obtained as follows.

To obtain the consensus degrees, first, we compute the different similarity matrix for each pair of experts. In this case, we need to define a similarity function. As we assume a fuzzy linguistic framework, the following similarity function can be used:

$$s(s_i, s_j) = 1 - \frac{|i-j|}{g}. \tag{22}$$

Using this similarity function, the following similarity matrices are obtained:

$$\mathrm{SM}^{12} = \begin{pmatrix} - & 1.00 & 0.83 & 0.83 \\ 1.00 & - & 0.83 & 0.83 \\ 0.83 & 0.67 & - & 0.67 \\ 0.83 & 0.67 & 0.50 & - \end{pmatrix}; \quad \mathrm{SM}^{13} = \begin{pmatrix} - & 1.00 & 0.67 & 0.50 \\ 1.00 & - & 0.50 & 0.50 \\ 1.00 & 0.67 & - & 0.33 \\ 0.50 & 0.67 & 0.17 & - \end{pmatrix}$$

$$\mathrm{SM}^{14} = \begin{pmatrix} - & 0.67 & 0.83 & 0.83 \\ 0.50 & - & 0.50 & 1.00 \\ 1.00 & 0.50 & - & 1.00 \\ 0.83 & 1.00 & 0.83 & - \end{pmatrix}; \quad \mathrm{SM}^{23} = \begin{pmatrix} - & 1.00 & 0.83 & 0.67 \\ 1.00 & - & 0.67 & 0.33 \\ 0.83 & 1.00 & - & 0.67 \\ 0.67 & 0.33 & 0.67 & - \end{pmatrix}$$

$$\mathrm{SM}^{24} = \begin{pmatrix} - & 0.50 & 1.00 & 1.00 \\ 0.33 & - & 0.67 & 0.83 \\ 0.83 & 0.83 & - & 0.67 \\ 1.00 & 0.67 & 0.33 & - \end{pmatrix}; \quad \mathrm{SM}^{34} = \begin{pmatrix} - & 0.67 & 0.83 & 0.67 \\ 0.50 & - & 1.00 & 0.50 \\ 1.00 & 0.83 & - & 0.33 \\ 0.67 & 0.67 & 0.00 & - \end{pmatrix}.$$

Then, we compute the collective similarity matrix:

$$\mathrm{SM} = \begin{pmatrix} - & 0.81 & 0.83 & 0.75 \\ 0.72 & - & 0.62 & 0.66 \\ 0.92 & 0.75 & - & 0.27 \\ 0.75 & 0.67 & 0.42 & - \end{pmatrix}.$$

Finally, we obtain the following consensus degrees:

1. *Consensus degrees on pairs of alternatives*. The element (i,k) of sm represents the consensus degrees on the pair of alternatives (x_i, x_k).
2. *Consensus degrees on alternatives*:

   $ca_1 = 0.80,\ ca_2 = 0.71,\ ca_3 = 0.63,\ ca_4 = 0.59.$

3. *Consensus degrees on the relation*:

CR = 0.68.

According to this score, we can affirm that the consensus level is acceptable in contrast to Example 1 based on the strict coincidence.

Proximity measures are obtained from the collective fuzzy linguistic preference relation, which using the LOWA operator with the weighting vector $W = \{0.5, 0.20, 0.16, 0.14\}$, is the following:

$$P^c = \begin{pmatrix} - & H & H & M \\ M & - & VH & VH \\ L & L & - & H \\ M & L & H & - \end{pmatrix}$$

From P^c, the proximity matrices for each expert are:

$$\mathrm{PM}^1 = \begin{pmatrix} - & 1.00 & 0.83 & 0.83 \\ 0.83 & - & 0.83 & 1.00 \\ 1.00 & 0.67 & - & 0.67 \\ 0.83 & 1.00 & 0.83 & - \end{pmatrix}; \quad \mathrm{PM}^2 = \begin{pmatrix} - & 1.00 & 1.00 & 1.00 \\ 0.83 & - & 1.00 & 0.83 \\ 0.83 & 1.00 & - & 1.00 \\ 1.00 & 0.67 & 0.67 & - \end{pmatrix}$$

$$\mathrm{PM}^3 = \begin{pmatrix} - & 1.00 & 0.83 & 0.67 \\ 0.83 & - & 0.67 & 0.50 \\ 1.00 & 1.00 & - & 0.67 \\ 0.67 & 0.67 & 0.33 & - \end{pmatrix}; \quad \mathrm{PM}^4 = \begin{pmatrix} - & 0.67 & 1.00 & 1.00 \\ 0.67 & - & 0.67 & 1.00 \\ 1.00 & 0.83 & - & 0.67 \\ 1.00 & 1.00 & 0.67 & - \end{pmatrix}.$$

1. *Proximity measure on pairs of alternatives*. The proximity measure of an expert e_h on a pair of alternatives (x_i, x_k) to the group's one, pp^h_{ik}, is expressed by the element (i, k) of the proximity matrix PM^h.
2. *Proximity measure on alternatives*:

$$\{pa^1_1, pa^1_2, pa^1_3, pa^1_4\} = \{0.89, 0.89, 0.80, 0.86\}$$
$$\{pa^2_1, pa^2_2, pa^2_3, pa^2_4\} = \{0.94, 0.89, 0.92, 0.86\}$$
$$\{pa^3_1, pa^3_2, pa^3_3, pa^3_4\} = \{0.83, 0.79, 0.75, 0.58\}$$
$$\{pa^4_1, pa^4_2, pa^4_3, pa^4_4\} = \{0.89, 0.81, 0.81, 0.89\}.$$

3. *Proximity measure on the relation*:

$$pr^1 = 0.86, \quad pr^2 = 0.90, \quad pr^3 = 0.74, \quad pr^4 = 0.85$$

In this case, unlike Example 1 all experts present adequate proximity measures, and the experts with worse scores are e_3 and e_4.

2.3 Consensus models based on coincidence among solutions

In this case, similarity criteria among the solutions obtained from the experts' preferences are used to compute the coincidence concept and different degrees assessed in [0,1] are assumed (Ben-Arieh and Chen 2006; Herrera-Viedma et al. 2002). Basically, we compare the positions of the alternatives between the individual solutions and the collective solution, which allows to know better the real consensus situation in each moment of the consensus process. Some examples of this approach are the following:

– In Herrera-Viedma et al. (2002) was defined the first consensus model based on the measurement of the coincidence degree between individual solutions and collective solution. It is assumed that experts represent their preferences by means of different elements of representation (relation, ordering and utilities) and then it is not possible to compare preferences. To overcome this problem, authors propose to compare solutions to obtain the coincidence degrees. This means that the first step of the consensus process to measure coincidence degrees is to apply a selection process to obtain a temporary collective solution and temporary individual solutions, and measure the closeness among them. An important characteristic of this consensus model was the introduction of a recommendation system to aid experts to change their preferences in the consensus reaching process and, in such a way, to substitute the moderator's actions.
– In Ben-Arieh and Chen (2006), a similar consensus model is presented but assuming heterogeneous GDM problems, i.e., experts with different importance degrees.

Following the consensus model defined in Herrera-Viedma et al. (2002), which is based only on consensus degrees not proximity measures, we can define a consensus model based on coincidence among solutions for fuzzy GDM problems with linguistic preference relations as follows:

1. To obtain the collective ordered vector of alternatives (temporary collective solution) V^c. To do so, we apply a selection process in two steps the selection process (Alonso et al. 2009; Chiclana et al. 1998; Roubens 1997):
   (a) *Aggregation*. In this step, a collective preference relation $P^c = (p^c_{ik})$ is obtained by means of the aggregation of all individual preference relations $\{P^1, P^2, \ldots, P^m\}$. This collective relation indicates the global preference between every ordered pair of alternatives according to the majority of experts' opinions.
   (b) *Exploitation*. In this step, the set of solution alternatives is obtained from the collective preference relation. In this consensus model, we call it as the collective ordered vector of alternatives. To do so, different choice degrees of alternatives could be used (Herrera and Herrera-Viedma 2000; Herrera-Viedma et al. 2007).
2. Calculating the individual ordered vector of alternatives (individual solution) V^h for every expert e_h. To do so, we apply directly the exploitation step on each individual linguistic preference relation P^h.
3. Calculating the proximity of each expert e_h for each alternative x_i, called $p^h(x_i)$, by comparing the ranking positions of that alternative in the experts' individual solution V^h (symbolized by V^h_i) and in the collective solution V^c (symbolized by V^c_i) as $p^h(x_i) = p(V^h, V^c)(x_i) = f(|V^c_i - V^h_i|)$. As a general dissimilarity function, $f(x) = (a \cdot x)^b, 1 \geq b \geq 0$ may be considered, and, in particular, the function taking $a = 1/(n-1)$ may be used, and then

$$p^h(x_i) = p(V^h, V^c)(x_i) = f(|V^c_i - V^h_i|) = \left(\frac{|V^c_j - V^h_i|}{n-1}\right)^b \in [0,1]. \quad (23)$$

The parameter b controls the rigorousness of the consensus process, in such a way, that values of b close to one decrease the rigorousness and, therefore, the number of rounds to develop in the group discussion process, and values of b close to zero increase the rigorousness and, therefore, the number of rounds. Appropriate values for b are: 0.5, 0.7, 0.9, 1.

4. Calculating the consensus degree of all experts on each alternative x_i using the following expression:

$$C(x_i) = 1 - \sum_{h=1}^{m} \frac{p^h(x_i)}{m} \quad (24)$$

5. The consensus measure over the set of alternatives, called C_X, will be calculated by the aggregation of the above consensus degrees on the alternatives. It is considered that the consensus degrees about the solution set of alternatives has to take a more important weight in this aggregation. To do so, in Herrera-Viedma et al. (2002) the S-OWA OR-LIKE operator defined by Yager and Filev (1994) was used:

$$\begin{aligned} C_X &= S_{\mathrm{OWAOR-LIKE}}(\{C(x_s);\ x_s \in X_{\mathrm{sol}}\}, \\ &\quad \{C(x_t); x_t \in X - X_{\mathrm{sol}}\}) \\ &= (1-\beta) \cdot \sum_{t=1}^{v} \frac{C(x_t)}{v} + \beta \cdot \sum_{s=1}^{\gamma} \frac{C(x_s)}{\gamma} \end{aligned} \quad (25)$$

where γ is the cardinal of the set X_{sol}; v is the cardinal of the set $X - X_{\mathrm{sol}}$; $\beta \in [0,1]$. β is a parameter to control the OR-LIKE behavior of the aggregation operator. The higher the value of β, the higher is the influence of the consensus degrees of the solution alternatives on the global consensus degree.

Example 3 Assuming the same linguistic preference relations provided by the experts in the above examples, the soft consensus degrees based on coincidence among solutions are obtained as follows:

1. Obtaining the collective ordered vector of alternatives V^c:
   (a) *Aggregation*: Using the LOWA operator (Herrera and Herrera-Viedma 2000) and the weighting vector $W = \{0.5, 0.20, 0.16, 0.14\}$, the following collective linguistic preference relation is obtained:

$$P^c = \begin{pmatrix} - & H & H & M \\ M & - & VH & VH \\ L & L & - & H \\ M & L & H & - \end{pmatrix}.$$

   (b) *Exploitation*: We use a choice degree called *dominance degree* (Herrera and Herrera-Viedma 2000) to characterize the alternatives and compute the ordered vector of alternatives:

$$\mathrm{DD}_i = \phi(p^c_{i1}, p^c_{i2}, \ldots, p^c_{i(i-1)}, p^c_{i(i+1)}, \ldots, p^c_{in}) \quad (26)$$

   To do so, we use the LOWA operator with the weighting vector $W = \{0.54, 0.28, 0.18\}$. Then the dominance degrees $\{DD_1, \ldots, DD_4\}$ are the following:

$$\mathrm{DD}_1 = M, \quad \mathrm{DD}_2 = H$$
$$\mathrm{DD}_3 = M, \quad \mathrm{DD}_4 = M.$$

And thus, the collective ordered vector of alternatives is $\{x_2, x_1, x_3, x_4\}$.

2. Calculating $\{V^h;\ h = 1, \ldots, m\}$:

$$e_1 : \{x_2, x_1, x_4, x_3\}, \quad e_2 : \{x_2, x_1, x_3, x_4\}$$
$$e_3 : \{x_1, x_3, x_2, x_4\}, \quad e_4 : \{x_2, x_4, x_1, x_3\}.$$

3. The differences between the ranking of alternatives in the temporary collective solution and the individual are as follows:

$V_i^c - V_i^h$	x_1	x_2	x_3	x_4
e_1	0	0	−1	1
e_2	0	0	0	0
e_3	1	1	−2	0
e_4	0	2	−1	−1

4. Consensus degrees on alternatives calculated for $b = 1$:

$$(C(x_1), C(x_2), C(x_3), C(x_4)) = (0.83, 1.0, 0.67, 0.67).$$

5. Consensus measure calculated for $b = 1$ and $\beta = 0.6$ is:

$$C_X = 0.88.$$

As we observe, assuming the same framework considered in Examples 1 and 2, we obtain a higher consensus level with this consensus model, which reflects better the actual decision situation.

3 Discussion

In this section, we analyze the advantages and drawbacks of the different fuzzy soft consensus approaches.

1. *Strict coincidence among preferences*. This consensus approach assumes only two possible values: 1 if the opinions are equal and, otherwise, a value of 0. Therefore, as we have seen in Example 1, the advantage of this approach is that the computation of the consensus degrees is simple and easy. However, the drawback of this approach is that the consensus degrees obtained do not reflect the real consensus situation because it only assigns values of 1 or 0 when comparing the experts' opinions, and, for example, we obtain a consensus value 0 for two different preference situations as (very high, high) and (very high, low), when clearly in the second case the consensus value should be lower than in the first case. It can be seen in Example 1, where the degree of consensus obtained is very low (0.23) although checking the preference relations provided by the experts, we can observe that the consensus among the experts is higher.
2. *Soft coincidence among preferences*. In this approach, similarity criteria among preferences are used to compute the coincidence concept but, in this case, it is assumed that the coincidence concept is a gradual concept, which could be assessed with different degrees defined in [0,1]. The advantage of this approach is that the consensus degrees obtained reflect better the real consensus situation. Comparing Examples 1 and 2, this is clearly observed. However, the drawback of this approach is that the computation of the consensus degrees is more difficult because we need to define similarity criteria to compute the consensus measures, and, sometimes, as it happens in Cabrerizo et al. (2009) and Herrera-Viedma et al. (2005), it is not possible to define these similarity measures directly.
3. *Coincidence among solutions*. The advantage of this approach is that the consensus degrees are obtained comparing not the opinions, but the position of the alternatives in each solution, which allows us to reflect the real consensus situation in each moment of the consensus reaching process, as it happens in the Example 3. However, the drawback of this approach is that the computation of the consensus degrees is more difficult than in the above approaches because we need to define similarity criteria and it is necessary to apply a selection process before obtaining the consensus degrees. As we show in Example 3, the computation of the consensus degrees is more complex.

4 Advanced consensus approaches

In this section, we describe the soft advanced consensus approaches, which have been developed using the above concepts of coincidence. These consensus approaches are mainly two: ones that generate recommendations to help experts and others that develop adaptive consensus processes. We present them in the following subsections in depth.

4.1 Consensus approaches generating recommendations to help experts

These approaches generate simple and easy rules to help experts change their opinions and find out which direction that change should follow in order to obtain the highest degree of consensus possible (Herrera-Viedma et al. 2002, 2007).

To do so, they are based on two consensus criteria: consensus degrees indicating the agreement between experts' opinions and proximity measures used to find out how far the individual opinions are from the group opinion. Thus, proximity measures are used in conjunction with the consensus degrees to build a guidance advice system, which acts as a feedback mechanism that generates recommendations, so that experts can change their opinions. Furthermore, these consensus criteria are computed at three different levels of representation of information of a preference relation: pair of alternatives, alternative and relation. Therefore, we will be able to identify which experts are close to the consensus solution, or in which alternatives the experts have more trouble to reach consensus.

So, the computation of the consensus degrees in this advanced consensus approaches is carried out using Eqs. (3)–(5), i.e., as in the above consensus models. Once consensus degrees are calculated, the proximity measures are obtained. To compute them for each expert, Eqs. (9)–(11) are used.

As aforementioned, if the proximity measures are close to 1, then they have a positive contribution for the consensus to be high, while if they are close to 0, then they have a negative contribution to the consensus. Therefore, we can use them to provide advice to the experts to change their opinions and to find out which direction that change has to follow in order to obtain the highest degree of consensus possible.

Thus, once proximity measures are calculated, the recommendations to help experts change their opinions are generated. The production of advice to achieve a solution with the highest degree of consensus possible is carried out using two kinds of rules (Herrera-Viedma et al. 2005): *identification rules* and *direction rules*.

1. **Identification rules (IR)**. We must identify the experts, alternatives and pairs of alternatives contributing less to reach a high degree of consensus and, therefore, should participate in the change process.
   (a) *Identification rule of experts (IR.1)*. It identifies the set of experts who should receive advice on how to change some of their preference values. This set of experts, called $EXPCH$, who should change their opinions are those whose satisfaction degree on the relation is lower than the minimum consensus threshold γ. Therefore, the identification rule of experts, IR.1, is the following:
   $$EXPCH = \{e_h \mid pr^h < \gamma\}. \tag{27}$$
   (b) *Identification rule of alternatives (IR.2)*. It identifies the alternatives, the associated assessments of which should be taken into account by the above experts in the change process of their preferences. This set of alternatives is denoted as ALT. The identification rule of alternatives, IR.2, is the following:
   $$ALT = \{x_i \in X \mid pa_i^h < \gamma \wedge e_h \in EXPCH\}. \tag{28}$$
   (c) *Identification rule of pairs of alternatives (IR.3)*. It identifies the pairs of alternatives (x_i, x_k) whose associate assessments p_{ik}^h should be changed by expert e_h. This set of pairs of alternatives is denoted as $PALT^h$. The identification rule of pairs of alternatives, IR.3, is the following:
   $$PALT^h = \{(x_i, x_k) \mid x_i \in ALT \wedge e_h \in EXPCH \wedge pp_{ik}^h < \gamma\}. \tag{29}$$
2. **Direction rules (DR)**. We must find out the direction of the change to be recommended in each case, i.e., the direction of change to be applied to the preference assessment p_{ik}^h, with $(x_i, x_k) \in PALT^h$. To do this, we define the following two direction rules.
   (a) *DR.1*. If $p_{ik}^h > p_{ik}^c$, the expert e_h should decrease the assessment associated with the pair of alternatives (x_i, x_k), i.e., p_{ik}^h.
   (b) *DR.2*. If $p_{ik}^h < p_{ik}^c$, the expert e_h should increase the assessment associated with the pair of alternatives (x_i, x_k), i.e., p_{ik}^h.

4.2 Adaptive consensus approaches

These consensus approaches are based on a refinement process of the consensus process that allows to increase the agreement and to reduce the number of experts' preferences that should be changed after each consensus round (Mata et al. 2009). The refinement process adapts the search for the furthest experts' preferences to the existent agreement in each round of consensus. So, when the agreement is very low (initial rounds of the consensus process), the number of changes of preferences should be bigger than when the agreement is medium or high (final rounds) (see Fig. 2).

These approaches consider that in the first rounds of the consensus process, the agreement is usually very low and it seems logic that many experts' preferences should be changed. However, after several rounds, the agreement should have improved and then just the furthest experts' preferences from the collective preference should be

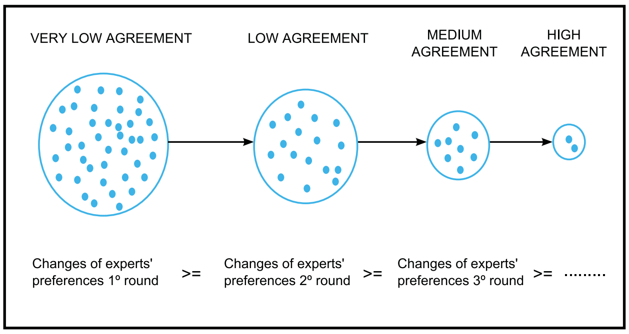

Fig. 2 Reduction in the number of changes of preferences in the consensus process

changed. The procedure to search for the furthest experts' preferences from collective preference should be different according to the achieved agreement in each round. Each Preference Search Procedure (PSp) should have a different behavior and should return a different set of preferences that each expert should change in order to improve the agreement in the next consensus round. In consequence of the adaptation of the consensus process to the existent agreement in each round, the number of changes of preferences suggested to experts after each consensus round will be smaller according to the favorable evolution of the level of agreement.

In the consensus process, if the agreement among experts is low, i.e, there are a lot of experts' preferences with different assessments, the number of experts who should change their preferences in order to make them closer to the collective preference should be great. However, if the agreement is medium or high, it means that the majority of preferences are similar and therefore there exists a low number of experts' preferences far from the collective preference. In this case, only these experts should change them in order to improve the agreement. Keeping in mind this idea, these approaches propose distinguishing among three levels of agreement: very low, low and medium consensus. Each level of consensus involves carryying out the search for the furthest preferences in a different way. So when the consensus degree CR is very low, these approaches will search for the furthest preferences on all experts, while if *CR* is medium, the search will be limited to the furthest experts. To do so, these approaches carry out three different PSps:

– PSp for very low consensus,
– PSp for low consensus, and
– PSp for medium consensus.

The possibility of carrying out different PSps according to the existent consensus degree in each round defines the adaptive character of the model.

5 Concluding remarks

We have analyzed different consensus approaches to compute soft consensus measures in fuzzy GDM problems. Additionally, we have described the new advanced approaches, i.e., those approaches allowing to generate recommendations to help experts change their opinions in order to obtain the highest degree of consensus possible, and, on the other hand, those approaches adapting the consensus process to increase the agreement and reduce the number of experts' preferences that should be changed after each consensus round.

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2. Mobile Decision Support Systems Based on Heterogeneous Information and Changeable Contexts

The journal papers associated to this part are:

2.1. A Mobile Decision Support System for Dynamic Group Decision Making Problems

- I.J. Pérez, F.J. Cabrerizo, E. Herrera-Viedma, A Mobile Decision Support System for Dynamic Group Decision Making Problems. IEEE Transactions on Systems, Man and Cybernetics - Part A: Systems and Humans 40:6 (2010) 1244-1256 doi:10.1109/TSMCA.2010.2046732

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A Mobile Decision Support System for Dynamic Group Decision-Making Problems

Ignacio Javier Pérez, Francisco Javier Cabrerizo, and Enrique Herrera-Viedma

***Abstract*—The aim of this paper is to present a decision support system model with two important characteristic: 1) mobile technologies are applied in the decision process and 2) the set of alternatives is not fixed over time to address dynamic decision situations in which the set of solution alternatives could change throughout the decision-making process. We implement a prototype of such mobile decision support system in which experts use mobile phones to provide their preferences anywhere and anytime. To get a general system, experts' preferences are assumed to be represented by different preference representations: 1) fuzzy preference relations; 2) orderings; 3) utility functions; and 4) multiplicative preference relations. Because this prototype incorporates both selection and consensus processes, it allows us to model group decision-making situations. The prototype incorporates a tool for managing the changes on the set of feasible alternatives that could happen throughout the decision process. This way, the prototype provides a new approach to deal with dynamic group decision-making situations to help make decisions anywhere and anytime.**

***Index Terms*—Decision support system (DSS), group decision making (GDM), mobile Internet (M-Internet).**

I. Introduction

A DECISION-MAKING process, which consists of deriving the best option from a feasible set, is present in just about every conceivable human task. As a result, the study of decision making is necessary and important not only in decision theory but also in areas such as management science, operations research, politics, social psychology, artificial intelligence, and soft computing.

It is obvious that the comparison of different actions according to their desirability in decision problems, in many cases, cannot be done by using a single criterion or a unique person. Thus, we interpret the decision process in the framework of group decision making (GDM) [1], [2]. This approach has led to numerous evaluation schemes and has become a major concern of research in decision making. Several authors have provided interesting results on GDM with the help of fuzzy theory, and the reader is referred to the following references [1], [3]–[11].

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I. J. Pérez and E. Herrera-Viedma are with the Department of Computer Science and Artificial Intelligence, University of Granada, 18071 Granada, Spain (e-mail: ijperez@decsai.ugr.es; viedma@decsai.ugr.es).

F. J. Cabrerizo is with the Department of Software Engineering and Computer Systems, Distance Learning University of Spain (UNED), 28040 Madrid, Spain (e-mail: cabrerizo@issi.uned.es).

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The central goal of decision support systems (DSSs) [12]–[14] is to process and provide suitable information to support individuals or organizations in their decision-making tasks. Nowadays, information can be supplied, received, and/or used anywhere, and as such, appropriate mobile DSSs can bridge the gap between theory and practice in decision making. It can also provide additional value to users, which can eventually lead to an increase in the number of successful transactions [15].

The application of the latest technologies extends opportunities in decision making and allows us to carry out consensus processes in situations that we cannot correctly address previously. For example, nowadays, it is possible to carry out consensus processes among several experts that are located in different countries around the world. However, it is important to remark that, even with the adoption of mobile technologies [16], [17], new collaboration and information tools are still needed so that the experts can solve decision-making problems when they cannot meet together.

In the cases where direct communication is not possible and experts do not have the possibility of gathering together, a problem arises in many consensus processes for GDM: experts may not have a clear idea about the current consensus status among all the experts involved in the decision process. In these cases, experts will probably need assistance to establish connections among them and to obtain a clear view of the consensus process progress. This help can be provided through mobile technologies, because it can be considered an efficient way for a continuous communication flow: it allows experts to always have dynamic and updated information to determine the current consensus process status, and at the same time, it provides mechanisms for sending expert preferences in real time, i.e., to simulate real discussion processes. With proper DSS tools, it is possible to determine which experts have similar opinions, and thus, experts may join or form different groups to better discuss every alternative and to try to influence other experts.

The incorporation of mobile technologies in GDM processes is based on the assumption that, if the communications are improved, the decisions will improve, because the discussion can be focused on the problem, with less time spent on unimportant issues.

The aim of this paper is to present a prototype of mobile DSSs (MDSS) to deal automatically with GDM problems, assuming different preference representations and based on mobile technologies. MDSS allows us to develop dynamic GDM processes. In fact, at every stage of the decision process, the users can achieve the following benefits: 1) be informed with updated data about the current stage of the decision process; 2) receive recommendations to help them to change their preferences; and 3) send their updated preferences at any moment, thus improving the user participation in the GDM process. In addition, to better simulate real decision-making processes usually carried out in these cases, the proposed model incorporates both consensus and selection processes. Another innovation introduced in the prototype is a tool for managing not only dynamic inputs of new alternatives that, due to some dynamic external factors, can appear during the decision process but also the outputs of some of them considered good alternatives at the beginning of the process but not so later on or are unavailable at the time. This way, a new approach for dealing with dynamic GDM problems is presented. To build a flexible framework and give a high degree of freedom to represent the preferences, experts are allowed to provide their preferences in any of the following four ways: 1) as a preference ordering of the alternatives; 2) as a utility function; 3) as a fuzzy preference relation; or 4) as a multiplicative preference relation.

To achieve this goal, the paper is set out as follows. General considerations about GDM models and mobile technologies are presented in Section II. Section III defines the prototype of a mobile DSS, including a practical experiment. In Section IV, we discuss some of its drawbacks and advantages. Finally, in Section V, conclusions are drawn.

II. Preliminaries

In this section, we present the classical GDM model and the advantages of using mobile technology in GDM problems.

A. GDM Models

In a GDM problem, we have a finite set of feasible alternatives, $X = \{x_1, x_2, \ldots, x_n\}$, $(n \geq 2)$, to be classified from best to worst by using the information given by a set of experts, $E = \{e_1, e_2, \ldots, e_m\}$, $(m \geq 2)$.

Usual resolution methods for GDM problems include two different processes [8], [18] (see Fig. 1).

1) *Consensus process*. Clearly, in any decision process, it is preferred that the experts reach a high degree of consensus on the solution set of alternatives. Thus, this process refers to how we can obtain the maximum degree of consensus or agreement between the set of experts on the solution alternatives.
2) *Selection process*. This process consists of how we can obtain the solution set of alternatives from the opinions on the alternatives given by the experts.

Usually, resolution methods for GDM problems are static, i.e., it is assumed that the number of alternatives and experts that act in the GDM problem remains fixed throughout the decision-making process. However, in real decision-making situations, we find dynamic GDM problems in which the number of alternatives and/or experts varies during the decision-making process. In this paper, we assume dynamic GDM problems with possible changes on the set of alternatives.

On the other hand, because each expert $e_k \in E$ has his own ideas, attitudes, motivations, and personality, it is quite natural to think that different experts can express their preferences in a different way. This fact has led some authors [19]–[24] to assume that experts' preferences over the set of alternatives may be represented in different ways. The most frequently used alternatives in decision-making theory are given as follows.

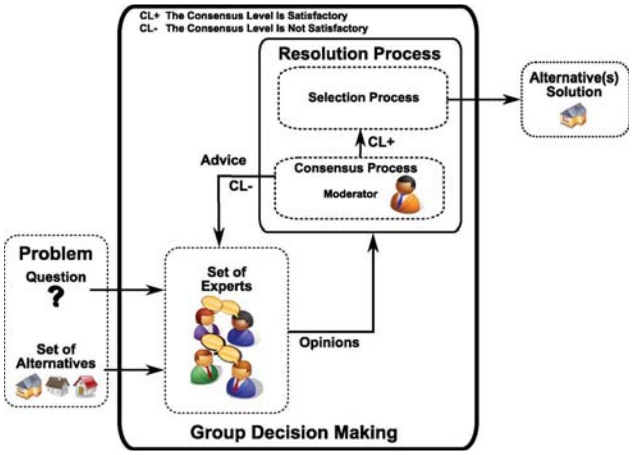

Fig. 1. Resolution process of a GDM.

- *Preference orderings of alternatives.* $O^k = \{o^k(1), \ldots, o^k(n)\}$, where $o^k(\cdot)$ is a permutation function over the index set, $\{1, \ldots, n\}$, for the expert e_k, defining an ordered vector of alternatives, from best to worst.
- *Utility functions.* $U^k = \{u_1^k, \ldots, u_n^k\}$, $u_i^k \in [0, 1]$, where u_i^k represents the utility evaluation given by the expert e_k to x_i.
- *Fuzzy preference relations.* $P^k \subset X\text{x}X$, with a membership function, $\mu_{P^k} : X\text{x}X \rightarrow [0, 1]$, where $\mu_{P^k}(x_i, x_j) = p_{ij}^k$ denotes the preference degree of x_i over x_j.
- *Multiplicative preference relations.* $A^k \subset X\text{x}X$, where the intensity of preference a_{ij}^k, is measured using a ratio scale, particularly the 1/9-to-9 scale.

B. Mobile Technologies in GDM Problems

In this section, we present the advantages and limitations of new mobile technologies, and we discuss the use of mobile devices to solve GDM problems.

1) Advantages and Limitations: Mobile communication systems are characterized by a variety of features [16], [17]. They differ from each other in the degree of their complexity, the level of their offered services, and their operational costs.

Mobile web refers to the World Wide Web accessed from mobile devices such as cell phones, personal digital assistants (PDAs), and other portable gadgets connected to a network. Thus, access to web services no longer requires a desktop computer. The following list shows the different advantages that mobile technologies can provide [16], [17].

- The Internet has provided an easy and effective way of delivering information and services to millions of users who are connected to wired network. Evidently, this wired network addresses two major constraints: 1) time and

2) place. These limitations have raised the issue of the mobile Internet (M-Internet), which enables users to access information from any place at any moment by using a mobile wireless device. The possibility of gaining access to this kind of services in wireless environments provides a great mobility to the users. This mobility can increase productivity due to the increasing agility of some tasks, allow users to save displacements and infrastructure costs, improve business processes, ease decision-making processes by obtaining more dynamic and precise solutions, and even improve the offered services.
- The mobile computing paradigm has several interesting and important applications for business, telecommunications, real-time control systems, and remote operations [15], [25], [26].
- Recently, the fast technological innovation has made it possible to provide secure, fast, and quality communications through the wireless network. Moreover, devices that used to deliver limited information can now provide a wide range of information and services such as email, banking, entertainment, and even games.

However, current mobile web access still suffers from interoperability and usability problems. This condition is partly due to the small physical size of the screens of mobile devices and the incompatibility of many mobile devices with both computer operating systems and the format of much of the available information on the Internet.

Some of the limitations that current mobile services have to face are given as follows.

- *Small screen size*. It is difficult or impossible to properly adapt text and graphics prepared for the standard size of a desktop computer screen with current information standards.
- *Lack of windows*. On the mobile web, only one page can be displayed at a time, and pages usually can only be viewed in the sequence that they were originally accessed.
- *Navigation*. Usual mobile devices do not use a mouse-like pointer but simply an up and down function for scrolling, thereby limiting the flexibility of navigation.
- *Format of accessible pages*. Many sites that can be accessed on a desktop cannot be accessed on a mobile device. Many devices cannot show pages with secured connection, Flash or other similar elements, Portable Document Format (PDF) files, or video sites.
- *Speed*. On most mobile devices, the speed of service is very slow, often slower than dial-up Internet access.
- *Size of messages*. Many devices have limits on the number of characters that can be sent in a single message.

To make use of mobile technology in the best way, several conditions need to be fulfilled. The first condition, nowadays achieved, is the widespread use of mobile devices that connect individuals to the mobile network and the contents that provide useful information and services to users. In addition, the technological support in terms of speed, communication quality, and security are also important in the development of the mobile technology [13].

The mobile web mainly uses lightweight pages written in Extensible Hypertext Markup Language (XHTML) or Wireless Markup Language (WML) to deliver content to mobile devices. However, new tools such as Macromedia's Flash Lite or Sun's J2ME enable the production of richer user interfaces customized for mobile devices.

2) Use of Mobile Technology in GDM Problems: During the last decade, organizations have moved from face-to-face group environments to virtual group environments by using communication technology. Many more workers use mobile devices to coordinate and share information with other people. The main objective is that the members of the group can work in an ideal way where they are, having all the necessary information to take the right decisions [16], [17], [27], [28].

To support the new generation of decision makers and to add real-time processes in the GDM problem field, many authors have proposed to develop DSSs based on mobile technologies [29], [30]. Similarly, we propose to incorporate mobile technologies in a DSS obtaining MDSS. Using such a technology should enable a user to maximize the advantages and minimize the drawbacks of DSSs.

The need of a face-to-face meeting disappears with the use of this model, because the own computer system acts as the moderator. Experts can directly communicate with the system by using their mobile device from any place in the world and at any time. Hereby, a continuous information flow among the system and each member of the group is produced, which can help reach a consensus between the experts in a faster way and to obtain better decisions.

In addition, MDSS can help reduce the time constraint in the decision process. Thus, the time saved by using the MDSS can be used for an exhaustive analysis of the problem and to obtain a better problem definition. This time can also be used to identify more feasible alternative solutions to the problem, and thus, the evaluation of a large set of alternatives can increase the possibility of finding a better solution. The MDSS helps in the resolution of GDM problems by providing a propitious environment for the communication, increasing the satisfaction of the user, and, this way, improving the final decisions.

III. MDSS Based on Dynamic Choice of Alternatives

Although DSSs have typically been associated with desktop systems and involve considerable processing, the development of new compact and mobile technologies provides new opportunities to develop this kind of DSSs over M-Internet [12], [16], [17].

In this section, we describe the implemented GDM model that incorporates a tool for managing dynamic decision models in which the alternatives of the set of solution alternatives can change throughout the decision process and uses different formats to represent preferences. It allows us to develop GDM processes at any time and anywhere and to simulate with more accuracy level the real processes of human decision making, which are developed in dynamic environments such as the web, financial investment, and health. Finally, the prototype of the MDSS is presented.

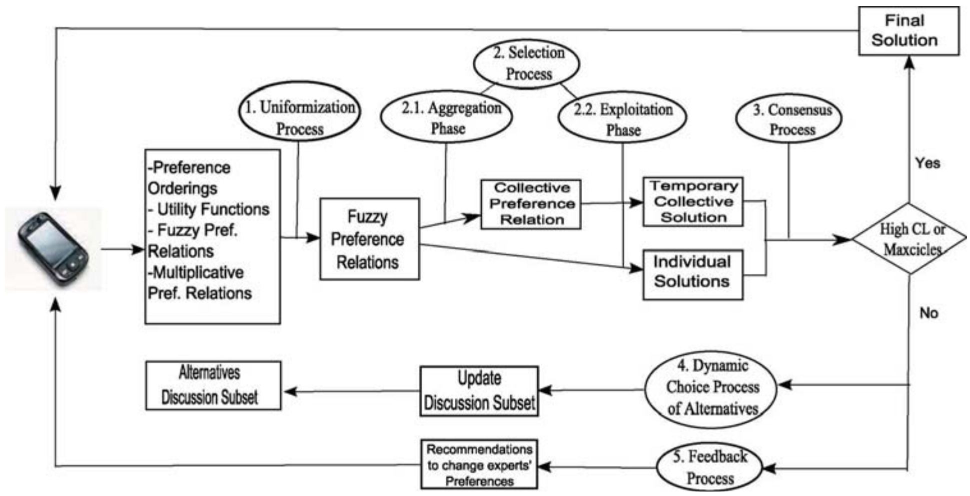

Fig. 2. Operation of the GDM model with multiple preference representation structures.

A. Structure of the Implemented GDM Model

The structure of the proposed MDSS model is composed of the following five processes: 1) uniformization; 2) selection; 3) consensus; 4) dynamic choice process of alternatives; and 5) feedback (see Fig. 2).

1) Uniformization: To give a higher degree of freedom to the system, we assume that experts can present their preferences by using any of the preference representations presented in Section II-A. Therefore, it is necessary to make the information uniform before applying consensus and selection. Similar to [20], we propose to use fuzzy preference relations as the base element to uniform experts' preferences, and the following transformation functions are used [20]: $f^1(o_i^k, o_j^k) = (1/2)(1 + ((o_j^k - o_i^k)/n - 1)$, $f^2(u_i^k, u_j^k) = (u_i^k)^2/((u_i^k)^2 + (u_j^k)^2)$, and $f^3(a_{ij}^k) = (1/2)(1 + \log_9 a_{ij}^k)$.

2) Selection: Once the information is made uniform, we have a set of m individual fuzzy preference relations, and then, we apply a selection process with two phases [2], [31]: 1) *aggregation* and 2) *exploitation*.

- *Aggregation*. This phase defines a collective preference relation, $P^c = (p_{ij}^c)$, obtained with the aggregation of all individual fuzzy preference relations $\{P^1, P^2, \ldots, P^m\}$. It indicates the global preference between every pair of alternatives according to the opinions of the majority of experts. For example, aggregation can be carried out through an ordered weighted averaging (OWA) operator [32], [33].
- *Exploitation*. This phase transforms the global information about the alternatives into a global ranking of them, from which the set of solution alternatives is obtained. The global ranking is obtained by applying two choice degrees of alternatives to the collective fuzzy preference relation [7]: 1) the *quantifier-guided dominance degree* (QGDD) and 2) the *quantifier-guided nondominance degree* (QGNDD). Finally, the solution X_{sol} is obtained by applying these two choice degrees and, thus, selecting the alternatives with maximum choice degrees.

3) Consensus Process: In our MDSS, we use a consensus model for GDM problems with different preference representations similar to [34]. This model presents the following main characteristics.

- It is based on two soft consensus criteria: 1) global consensus measure on the set of alternatives X, symbolized as C_X, and 2) the proximity measures of each expert e_i on X, called P_X^i.
- Both consensus criteria are defined by comparing the individual solutions with the collective solution using as comparison criterion the positions of the alternatives in each solution.

Initially, in this consensus model, we consider that, in any nontrivial GDM problem, the experts disagree in their opinions so that consensus has to be viewed as an iterated process. This approach means that agreement is obtained only after rounds of consultation. In each round, the DSS calculates both the consensus and the proximity measures. The consensus measures evaluate the agreement that exists among experts, and the proximity measures are used in the feedback mechanism to support the group discussion phase of the consensus process.

4) Dynamic Choice Process of Alternatives: In real world, we find many dynamic decision frameworks: 1) health; 2) financial investment; 3) military operations; and 4) Web. In such cases, due to different factors, the set of solution alternatives can vary throughout the decision process. One typical example of this situation is the medical diagnosis. This environment is dynamic in the sense that a patient can present new symptoms, or he can set better due to the medication, and thus, any change in state of the patient should be taken into account by the doctors.

Classical GDM models are defined within static frameworks. To make the decision-making process more realistic, we provide a new tool to deal with dynamic alternatives in decision making. This way, we can solve dynamic decision problems in which, at every stage of the process, the discussion can be centered at different alternatives.

To do so, we define a method that allows us to remove and insert new alternatives into the discussion process. First, the

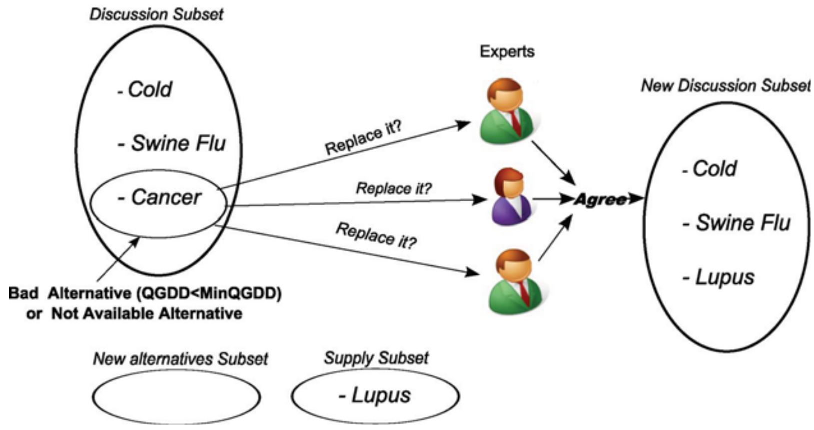

Fig. 3. Dynamic choice process of alternatives: Case 1.

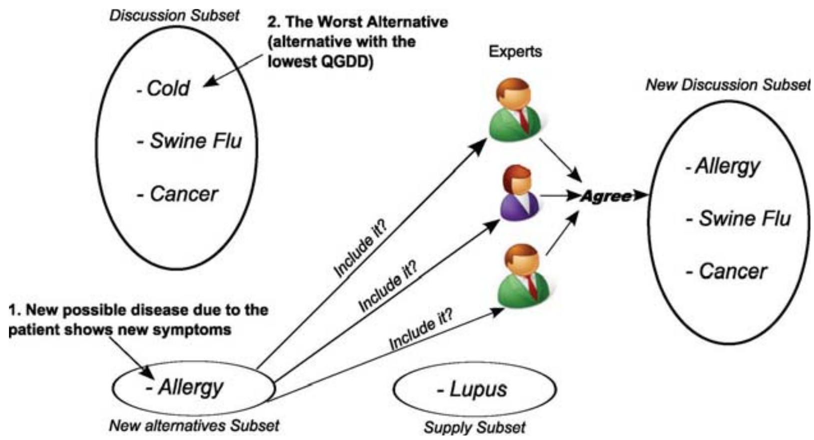

Fig. 4. Dynamic choice process of alternatives: Case 2.

system identifies the worst alternatives that might be removed and the new alternatives to include in the set. These new alternatives can be obtained from a set of new alternatives that appeared at a time or from the supply set of alternatives that includes all the alternatives that we had at the beginning of the process but were not included in the discussion subset because of limitations due to specific parameters of the problem.

Thus, the method has two different phases.

1) *Remove old bad alternatives*. The first phase manages situations in which alternatives of the discussion subset are not available at the moment due to dynamic external factors or because the experts have evaluated them poorly and they have a low dominance degree $(QGDD)$. Therefore, the system checks the availability and the $QGDD$ of each alternative in the current discussion subset. If an alternative is not available or has a $QGDD$ lower than a threshold $(minQGDD)$, the system looks for a new good alternative in the new alternatives subset. If this subset is empty, the system uses the supply subset of alternatives provided by the expert at the beginning of the decision process and that were not taken into account then because of the impossibility of comparing all the alternatives at the same time. Then, the system asks for the experts' opinions about the replacement and acts according to them (see Fig. 3).
2) *Insert new good alternatives*. The second case manages the opposite situation, i.e., when new alternatives have emerged. The system checks if new good alternatives have appeared in the new alternatives subset due to dynamic external factors. If this is the case, the system has to identify the worst alternatives of the current discussion subset. To do this, the system again uses the dominance degree $QGDD$ of all alternatives to choose the worst alternatives. Then, the system asks for the experts' opinions about the replacement and acts according to them (see Fig. 4).

To avoid stagnation at this point, a *maxTime* threshold is established. If the majority of experts that answered the question in *maxTime* think that the changes are appropriate, the system updates the discussion subset according to the aforementioned cases. The possibility of these changes makes experts more involved in the process and improves their satisfaction with the final results.

5) Feedback Process: To guide the change of the experts' opinions, the DSS simulates a group discussion session in which a feedback mechanism is applied to quickly obtain a high level of consensus. This mechanism can substitute the moderator's actions in the consensus process. The main problem is how the experts can find a way of making individual positions converge and, therefore, how it can support the experts in obtaining and agreeing with a particular solution.

When the consensus measure C_X has not reached the required consensus level (CL) and the number of rounds has not reached a maximum number of iterations (MAXCYCLE), defined before the decision process begins, the experts' opinions must be modified. As aforementioned, we use the proximity measures to build a feedback mechanism so that experts can change their opinions and narrow their positions.

This feedback mechanism uses the proximity measures to give simple rules on how experts' preferences can be changed.

- *Rules for changing the preferences*. The rules provided by the feedback mechanism are easy to understand and apply, because they are provided in a natural language.
    1) Each expert e_i is classified by associating experts to their respective total proximity measure P_X^i. Each expert is given his position and his proximity in each alternative.
    2) If the expert's position in the ranking is high (first, second, etc.), then that expert should not change his opinion much, but if it is low, then that expert has to substantially change his opinion. In other words, experts who will change their opinions are those whose individual solutions are farthest from the collective temporary solution. At this point, we have to calculate, using a threshold defined at the beginning of the decision process, how many experts have to change their opinions.

    The rules for changing opinions are given as follows.
    - If the proximity of alternative $p_i(x_j)$ is positive, then we have the rule "decrease values associated to alternative x_j."
    - If the proximity of alternative $p_i(x_j)$ is negative, then we have the rule "increase values associated to alternative x_j."

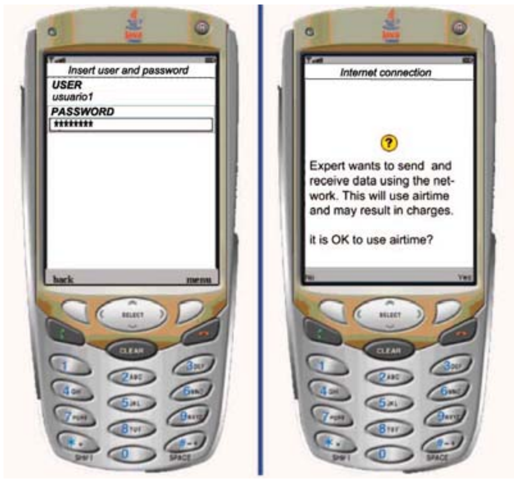

Fig. 5. Authentication and M-Internet connection.

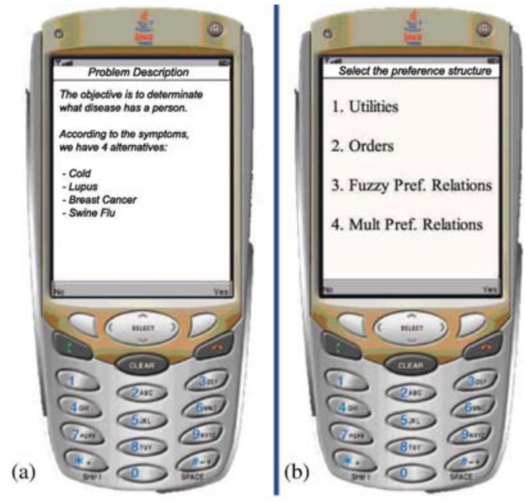

Fig. 6. Problem description and selection of preference representations.

B. Prototype of MDSS

Here, we present the prototype of MDSS, explaining the architecture of the system and the communication and workflow that summarizes the functions of the DSS.

A DSS can be built in several ways, and the technology that was used determines how a DSS has to be developed [14], [15]. The chosen architecture for our prototype of MDSS is a "client/server" architecture, where the client is a mobile device. The client/server paradigm is founded on the concept that clients (such as personal computers or mobile devices) and servers (computer) are both connected by a network that enables servers to provide different services for the clients. Furthermore, the technologies that we have used to implement the prototype of the MDSS comprise Java and Java Midlets for the client software, PHP for the server functions, and MySQL for the database management.

According to the GDM model proposed in the previous section, the prototype lets the user send his/her preferences to the DSS through a mobile device, and the system returns to the expert the final solution or recommendations to increase the CL, depending on the stage of the decision process. One important aspect is that the user–system interaction can be done anytime and anywhere, which facilitates expert's participation and the resolution of the decision process.

In what follows, we describe in detail the client and server of the MDSS prototype.

1) Client: For the implementation of the DSS, we have chosen a thin client model. This model primarily depends on the central server for the processing activities. This prototype is designed to operate on mobile devices with Internet connection.

The client software has to show to the experts the following eight interfaces.

- *Connection*. The device must be connected to the network to send/receive information to the server.
- *Authentication*. The device will ask for a user and password data to access the system (see Fig. 5).
- *Problem description*. When a decision process is started, the device shows to the experts a brief description of the problem and the discussion subset of alternatives [see Fig. 6(a)].
- *Selection of preference representations* [see Fig. 6(b)].
- *Insertion of preferences*. The device will have four different interfaces, one for each different format of preference representation (see Fig. 7).

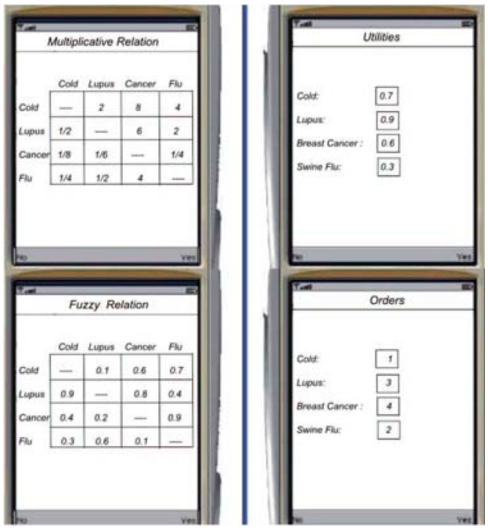

Fig. 7. Insertion of preferences.

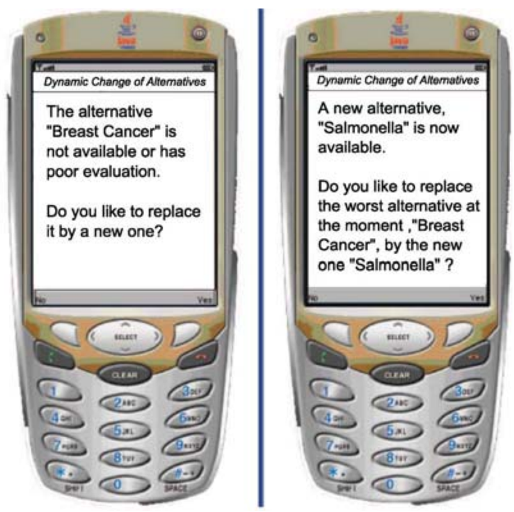

Fig. 8. Change of alternatives question.

- *Change of alternatives*. When a bad or unavailable alternative deserves to be removed from the discussion subset or a new alternative deserves be inserted in the discussion subset, using the new management process of alternatives, the experts can assess if they want to update the discussion subset by changing these alternatives (see Fig. 8).
- *Feedback*. When opinions should be modified, the device shows to the experts the recommendations and lets them send their new preferences [see Fig. 9(a)].
- *Output*. At the end of the decision process, the device will show to the experts the set of solution alternatives as an ordered set of alternatives, marking the most relevant ones [see Fig. 9(b)].

On the technical side of the development of the client part of the DSS, it is worth noting that the client application complies with the MIDP 2.0 specifications [35] and that the J2ME Wireless Toolkit 2.2 [36] provided by Sun was used in the development phase. This wireless toolkit is a set of tools that provide J2ME developers with emulation environments,

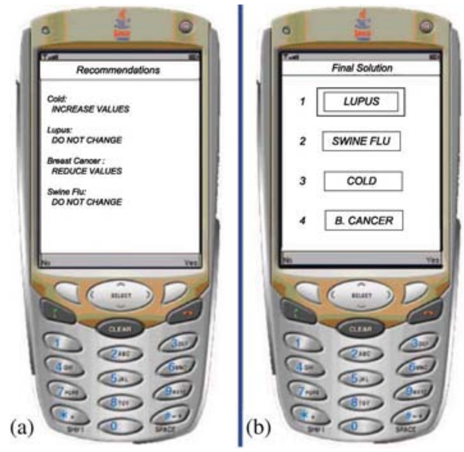

Fig. 9. Recommendations and final solution.

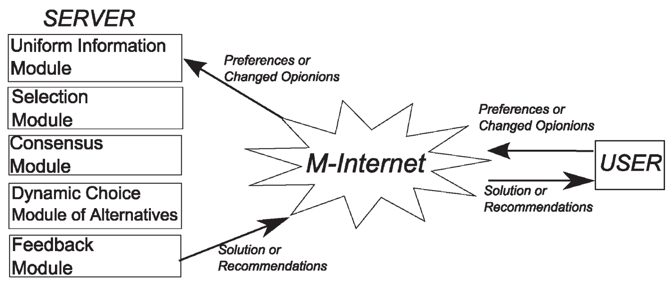

Fig. 10. Operation structure of the MDSS prototype.

documentation, and examples to develop MIDP-compliant applications. The application was later tested using a Java-enabled mobile phone on a Global System for Mobile Communications (GSM) network using a general packet radio service (GPRS)-enabled subscriber identity module (SIM) card. The MIDP application is packaged inside a Java archive (JAR) file, which contains the applications classes and resource files. This JAR file is downloaded to the physical device (mobile phone), along with the Java application descriptor file, when an expert wants to use the MDSS.

2) Server: The server is the other fundamental part of the DSS. It is based on five main modules, which receive/send information from/to the experts through M-Internet technologies (see Fig. 10).

- *Uniform information module*. This module makes expert preferences uniform by using the transformation functions in Section III-A1 to convert all different types of preferences into fuzzy preference relations.
- *Selection module*. After the information is made uniform, the server applies the selection process to obtain a temporary solution of the problem. This process has two phases: 1) aggregation and 2) exploitation. In the aggregation phase, the collective fuzzy preference relation is obtained. In the exploitation phase, the server obtains the QGDDs of alternatives acting over the collective fuzzy preference relation. This degree allows us to establish an order in the alternatives to obtain the ranking of the temporary alternative solutions, from best to worse.

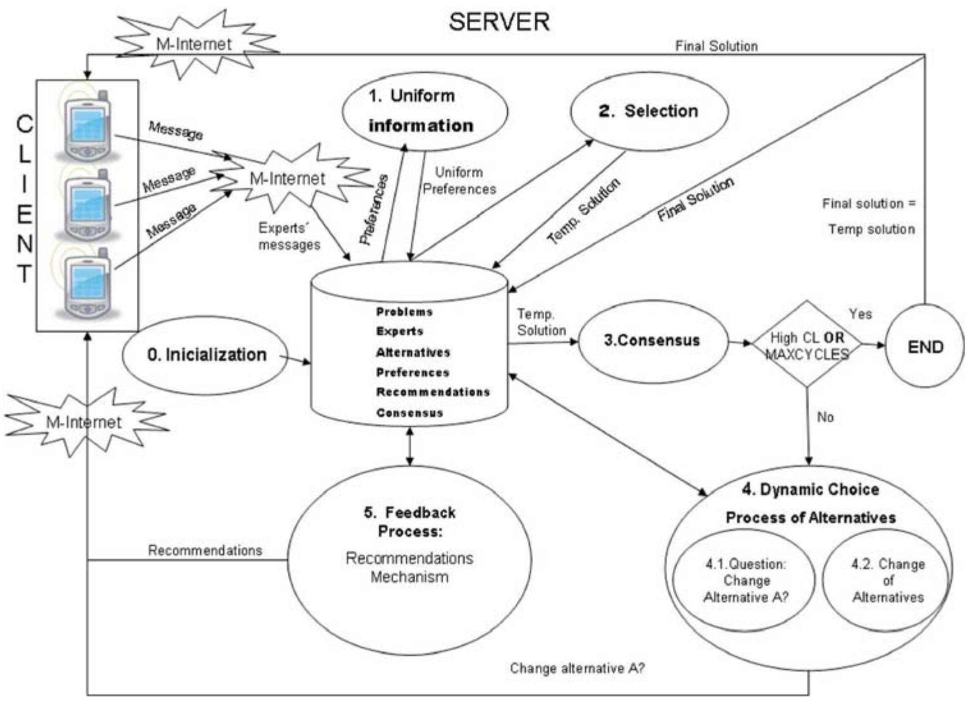

Fig. 11. Functions scheme of the system.

- *Consensus module*. In this module, the consensus and proximity measures are calculated by the server. If the consensus measure has reached the minimum CL defined as a parameter of the problem, the consensus process stops. This temporary collective solution becomes the final consensual solution and is sent to the experts. In other cases, the consensus process should continue.
- *Dynamic choice module of alternatives*. If an old alternative has to be removed from the discussion subset or a new alternative deserves to be inserted in the discussion subset and the minimum CL has not been reached, the server applies the management process of alternatives to determine if the replacement should be done. To do that, the server asks the experts if they agree with the proposed change. If the majority of the experts accept it, the *discussion subset* of alternatives is updated by changing the worst alternative of the set by the new one or by the first one in the supply list.
- *Feedback module*. When a consensus stage is finished without reaching the minimum CL, the server starts a feedback mechanism that generates recommendations rules. These recommendations demand the experts to change their preferences and explain how they will do it (increasing or decreasing preferences).

  This way, the consensus process will converge, and eventually, the solution will reach a high consensus degree.

The server also implements a database that stores all the data of the problem, as well as the experts' data, alternatives data, preferences, consensus measures, recommendations, consensus parameters, and selection parameters.

3) Communication and Work Flow: The DSS has to carry out the following functions, also represented in Fig. 11. In the diagram, we can see all the functions of the system, the form in which they are connected together with the database, and the order in which each of them is executed.

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0) **Initialization**. The first step to the start of the execution of the system consists of the insertion in the database of all the initial parameters of the problem, the experts, and the set of alternatives. Before starting the decision process, it is necessary to set suitable values for all of the parameters according to the problem, particularly those that limit the time that will be spent in its resolution. It is not the same as an urgent medical situation where experts have to quickly decide the best medical treatment to choose a country to visit during holidays. In the first case, the MAXCYCLE of the consensus process and the maximum time of waiting for the expert opinions should be shorter than the second one, because the final solution is required as soon as possible. Therefore, these values are very dependent on the problem at hand, and they have to be established according to the special needs of each situation.

1) **Verify the user messages and store the main information**. When an expert wants to access the system, he/she has to send a message through M-Internet by using his mobile device. The user can send the following two kinds of messages.

i) *Preferences message*. It is composed of authentication information (login and password) and the user's preferences about the problem, using any of these four available formats: 1) *preference orderings*; 2) *utility functions*; 3) *fuzzy preference relations*; or 4) *multiplicative preference relations.*

ii) *Change of alternatives message*. It is composed of authentication information (login and password) and the answer to the change of alternatives question.The message is verified by the server, which checks the login and password in the database. If the authentication process is correct, the rest of the information of the message is stored in the database, and the server decides when the consensus stage can start (if all experts have provided their preferences) or when the change of alternatives mechanism can be finished (if enough experts answer the change of alternatives question).

2) **Make the experts' preferences uniform**. The server makes the information uniform by using fuzzy preference relations as the base element of preferences representation. The server saves this information in the database.

3) **Computation of the set of solution alternatives**. The selection module returns the solution set of alternatives in each stage of the decision process. All the information about the temporary solution is saved in the database.

4) **Computation of the consensus measures**. In this step, the consensus and proximity measures are computed by the server and saved in the database.

5) **Control the consensus state**. In this step, the server determines if the required agreement degree has been reached (and thus, the decision process must finish by applying the selection process) or if a new round of consensus using the feedback mechanism that generates recommendations to change the experts' preferences should begin.

6) **Control the change of alternatives**. When the minimum CL has not been reached and alternatives deserve to be removed or inserted in the discussion subset, the system offers the possibility to update the discussion subset on time.

7) **Generate the recommendations**. In this step, the server generates the recommendations and sends a message to the experts advising that they can use the software again for reading the recommendations and start a new consensus stage. To avoid that the collective solution does not converge after several discussion rounds, the prototype stops if the number of rounds reaches MAXCYCLES.

The results are saved in the database and are sent to the experts through M-Internet to help them change their preferences.

8) **Go to Step 1**. A new round of the decision-making process starts.

The system operation will be illustrated in more detail in the next section, with a practical example.

C. Practical Example of MDSS

In this section, we will illustrate a simple real example of use of the DSS. Take note of the behavior of the system under complex problems, because the prototype allows dynamic sets of alternatives, it manages their inputs and outputs in real time, and it can also address problems with large sets of alternatives them. When all the alternatives cannot be displayed on a mobile screen at the same time, the remaining ones can be ordered in a supply list and be evaluated later in the process. Therefore, the system can support a big number of experts and alternatives to solve complex problems. To illustrate how the prototype works, we will follow the communication flow presented in the previous section.

TABLE I
ALTERNATIVES OF THE PROBLEM

$Code$	$Name$	$Capacity$	$Prize$	$City$
r_1	Las Tinajas	75	20-50 Euros	Granada
r_2	La Pataleta	45	20-40 Euros	Granada
r_3	La Ermita	55	22-35 Euros	Granada
r_4	Kudam	60	25-55 Euros	Granada
r_5	Casa Ramon	60	30-52 Euros	Granada
r_6	Il Gondoliere	45	31-41 Euros	Granada

TABLE II
EXPERTS OF THE PROBLEM AND MOBILE DEVICES USED

$Code$	$Name$	$City$	$MobileDevice$
e_1	Enrique	Granada (Spain)	Nokia N70
e_2	Paco	Leicester (UK)	Nokia 6234
e_3	Javier	Madrid (Spain)	HTC Touch
e_4	Sergio	Granada (Spain)	LG Viewty

TABLE III
INITIAL PARAMETERS OF THE PROBLEM

$Name$	$Value$	$Description$
b	1	Control the proximity measures
β	0.5	Control the S-OWA operator
$minConsDegree$	0.8	Minimum consensus level
$minProxDegree$	0.7	Minimum proximity level
$MAXCYCLES$	4	Maximum number of iterations
$maxTime$	12 (hours)	Maximum waiting time
$minQGDD$	0.2	Minimum dominance level
$DSsize$	4	Discussion subset size

The experiment dealt with the choice of the best restaurant for a Christmas dinner by four members (experts) of a work group. They used their last generation mobile devices, because they live in different countries and cannot gather together to plan the meeting.

In the beginning, the secretary of the work group had to look for a set of available restaurants. Later, a list of six of these available restaurants was created as the feasible candidates to celebrate the dinner. These candidates, arranged according to prize, made up the initial set of alternatives for the problem.

The first step to solve a problem using our prototype is to insert all the parameters of the problem (experts, alternatives, thresholds, timing, and so on) in the database. (See Tables I–III.)

When the initial parameters were defined according to the problem requirements, the decision-making process starts.

Note that the set of alternatives has six restaurants $X = \{R_1, \ldots, R_6\}$, but we suppose that the experts cannot compare all of them altogether. Thus, they will evaluate only four of them $(\mathrm{DSsize} = 4)$, i.e., the initial discussion subset will consist of the first four, $X' = \{R_1, \ldots, R_4\}$. The remaining restaurants are included in the supply set to support changes in the discussion subset at the following iterations of the decision process. These changes can be made when some of the current restaurants obtain a low evaluation or are no longer available for booking.

The first four restaurants are presented to the group of four experts, $E = \{e_1, \ldots, e_4\}$. They are asked to give their opinions about them using our MDSS.

The experiment was carried out using a real set of the latest technology mobile devices (see Table II). Therefore, we have

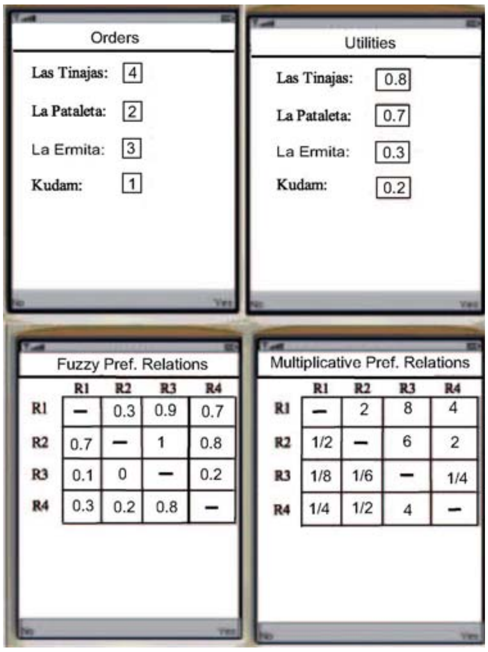

Fig. 12. Expert preferences.

to illustrate the input and output interfaces by using a mobile emulator provided by Sun Microsystem. The input and output data sets are the same as in the real experiment. The interfaces depend on the device screen but are very similar.

Expert e_1 gave his opinions by using preference orderings, e_2 by using utility values, e_3 by using fuzzy preference relations, and, finally, e_4 by using multiplicative preference relations. Experts' initial opinions are shown in Fig. 12.

These preferences and the authentication information are sent to the server by each expert, and if the authentication process is correct, the preferences are stored in the table *preferences* of the database. When the last expert has sent his message, the decision process is started by the server.

1) First Stage in the Decision Process:

a) Uniform information module: Using the transformation functions presented in Section III-A, the system obtains the following individual fuzzy preference relations:

$$P^1 = \begin{pmatrix} 0.5 & 0.16 & 0.33 & 0 \\ 0.83 & 0.5 & 0.66 & 0.33 \\ 0.66 & 0.33 & 0.5 & 0.16 \\ 1 & 0.66 & 0.83 & 0.5 \end{pmatrix}$$

$$P^2 = \begin{pmatrix} 0.5 & 0.57 & 0.88 & 0.94 \\ 0.43 & 0.5 & 0.84 & 0.92 \\ 0.22 & 0.16 & 0.5 & 0.69 \\ 0.06 & 0.08 & 0.21 & 0.5 \end{pmatrix}$$

$$P^3 = \begin{pmatrix} 0.5 & 0.3 & 0.9 & 0.7 \\ 0.7 & 0.5 & 1 & 0.8 \\ 0.1 & 0 & 0.5 & 0.2 \\ 0.3 & 0.2 & 0.8 & 0.5 \end{pmatrix}$$

$$P^4 = \begin{pmatrix} 0.5 & 0.66 & 0.97 & 0.82 \\ 0.34 & 0.5 & 0.91 & 0.66 \\ 0.03 & 0.09 & 0.5 & 0.18 \\ 0.18 & 0.34 & 0.82 & 0.5 \end{pmatrix}.$$

These four relations are also stored in the table *preferences* of the database.

b) Selection module: Using the fuzzy majority criterion with the corresponding OWA operator with the weighting vector $W = [0.5, 0.2, 0.17, 0.13]$ ("most of"), the collective fuzzy preference relation is computed as

$$P^c = \begin{pmatrix} 0.5 & 0.52 & 0.86 & 0.75 \\ 0.48 & 0.5 & 0.91 & 0.77 \\ 0.14 & 0.09 & 0.5 & 0.44 \\ 0.25 & 0.23 & 0.56 & 0.5 \end{pmatrix}.$$

We apply the exploitation process with the corresponding OWA operator with the weighting vector $W = [0.07, 0.67, 0.26]$ ("most of") and compute the dominance choice degree $(QGDD_i)$ over the collective fuzzy preference relation: $QGDD_1 = 0.696$, $QGDD_2 = 0.702$, $QGDD_3 = 0.146$, $QGDD_4 = 0.265$.

These values represent the *dominance that one alternative has over "most of" the alternatives according to "most of" the experts.*

We can see that the best current candidate is R_2, and the collective order of restaurants is $\{R_2, R_1, R_4, R_3\}$. This order is shown as our temporary solution in this first consensus stage.

c) Consensus module: The system computes the individual orders for each expert in a way similar to the global solution, i.e.,

$$e_1 : \{R_4, R_2, R_3, R_1\}$$
$$e_2 : \{R_1, R_2, R_3, R_4\}$$
$$e_3 : \{R_1, R_2, R_4, R_3\}$$
$$e_4 : \{R_2, R_1, R_4, R_3\}.$$

Consensus degrees of the set of experts over the individual alternatives are given as follows: $C(R_1) = 0.55$, $C(R_2) = 0.66$, $C(R_3) = 0.77$, $C(R_4) = 0.66$.

The global consensus measure is computed using an OWA operator, and we obtain the following value: $C_X = 0.67$.

The proximity measures are also computed using an OWA operator: $P_X^1 = 0.55$, $P_X^2 = 0.67$, $P_X^3 = 0.78$, $P_X^4 = 1$.

As we can see, the consensus has not reached the minimum required by the problem $(C_X < 0.8)$, and consequently, the decision process should continue applying both the dynamic choice process of alternatives and the feedback process.

d) Dynamic Choice Process of Alternatives: As soon as the system has verified that the minimum CL among the experts has not been reached and before beginning a new round of consensus, it is necessary to update all the information of the problem that could be changed during the process.

To do so, the system tries to remove and replace the restaurants that cannot be booked at the moment due to theirs being already fully booked or whose dominance degree is below the required minimum value, i.e., $QGDD_i < MinQGDD = 0.2$. New restaurants or restaurants in waiting in the supply list are given as replacement alternatives. In this case, all the restaurants are available for booking; however, La Ermita restaurant has a choice degree $QGDD_3$ lower than $MinQGDD$. Due to external factors, e.g., bookings cancelled, a new good restaurant

Fig. 13. Change of alternative question.

called "Rodizio" is now available to celebrate dinner. Therefore, the list of new alternatives has a new element, and the system suggests that the bad restaurant is removed and the new one is inserted in the discussion subset.

Because there are no more new alternatives, the question (Fig. 13) is sent to all the experts, and the system waits for the experts' answers to update the discussion subset. Experts e_1, e_3, and e_4 answer that they agree with the change. e_2 does not answer the question within the threshold waiting time $maxTime$. Thus, the restaurant R_3 is replaced with the new restaurant R_7 into the discussion subset of alternatives.

e) Feedback Process: Next, the feedback process is applied, and recommendation to the experts are given on their preference values to change to improve the CL. This approach is done in the following two steps.

- *Classification of experts*. The system ranks the experts according to their proximity measures: e_4, e_3, e_2, and e_1.
- *Changing the opinions*. At this point, two of the experts, e_1 and e_2, whose proximity measures are lower than the parameter $minProxDegree$, are asked to change their opinions. They are not requested to change preferences on the restaurant R_3, because it is replaced by R_7. Obviously, all the experts were asked to introduce their preferences about the new alternative R_7.

We can see the recommendations received by the experts in their mobile devices in Fig. 14.

2) Second Stage in the Decision Process: In this stage, all the experts have to send their preferences again, because the alternative set has been modified (the candidate R_7 replaced the candidate R_3). Experts e_1 and e_2 also received recommendations to change their preferences, because their proximity levels were low in the previous round.

The experts' opinions given in the second round are shown in Fig. 15.

The *uniform information module* transforms these preferences to fuzzy preference relations, and the *selection module*, with the same operations that in the previous stage, obtains a new temporary solution. The new collective ranking of restaurants is given as follows: $\{R_2, R_1, R_7, R_4\}$.

Fig. 14. Recommendations.

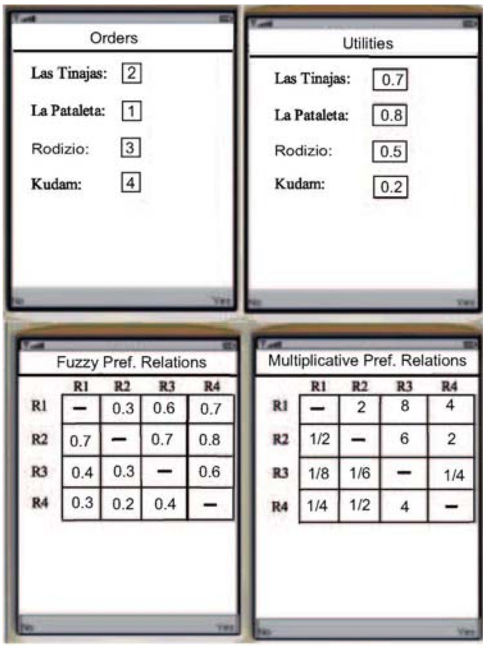

Fig. 15. New experts' preferences.

The next module, i.e, the *"Consensus module,"* obtains the CL: $C_X = 0.88$.

This CL has reached the minimum level required by the problem $(C_X > 0.8)$, and in this case, the decision-making process has finished, with R_2 being the best alternative. The restaurants R_1, R_7, and R_4 make up the supply list, and the solution is stored in the table *consensus* of the database. All this information is sent to experts by their mobile phones (see Fig. 16).

Fig. 16. Final solution.

IV. Discussion: Drawbacks and Advantages

In this section, we point out some drawbacks and advantages of the implemented MDSS.

- *Drawbacks*. We find the following drawbacks of our system.
  1) To take part in the GDM process the users need a last-generation mobile device to install the MDSS, and this condition can be very expensive for them.
  2) The user interfaces have to be easy and very simple, because the mobile device screen is very small.
  3) The MDSS prototype can only be applied in numerical decision contexts, and it would be desirable to use other more flexible frameworks such as linguistic contexts.
  4) Studies on the incorporation of consistency measures and dealing with missing values would be desirable.
- *Advantages*. On the other hand, we find the following advantages.
  1) MDSS allows us to develop a distributed GDM process, because the experts do not have to gather together to discuss the problem to solve.
  2) MDSS improves the speed of the classical DSSs, because the experts receive and send the information using their mobile devices, which are carried at all times.
  3) MDSS provides a higher flexibility degree in the representation of preferences, because the experts can use different preference representations formats to express their opinions. This way, we allow experts to provide their preferences anywhere, anytime, and in multiple formats.
  4) MDSS incorporates a feedback mechanism that provides linguistic recommendations to the experts to quickly obtain a high consensus degree.
  5) MDSS allows us to address large sets of alternatives in decision problems, because it incorporates the management of dynamic sets of alternatives.

V. Concluding Remarks

We have presented a prototype of MDSS for GDM problems based on dynamic decision environments, which incorporates a new tool for managing dynamic inputs and outputs of alternatives in the set of solution alternatives throughout the decision process. The prototype uses the advantages of M-Internet technologies to improve user satisfaction with the decision process and develop decision processes anytime and anywhere. We have used mobile phones as the device used by the experts to send their preferences, but the structure of the prototype is designed to use any other mobile device, such as PDAs. The prototype can be used with four different formats to represent the preferences in the best way according to the kind of problem and the experts' knowledge level.

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Ignacio Javier Pérez was born in Granada, Spain, on June 8, 1984. He received the M.Sc. degree in computer sciences from the University of Granada, Granada, in 2007 and is currently working toward the Ph.D. degree in the Department of Computer Science and Artificial Intelligence, University of Granada.

He is currently also a Research Fellow with the Department of Computer Science and Artificial Intelligence, University of Granada. His research interests include group decision making, decision support systems, consensus models, linguistic modeling, modeling situations with missing/incomplete information, aggregation of information, digital libraries, and web quality evaluation.

Francisco Javier Cabrerizo was born in Jódar, Spain, on February 8, 1983. He received the M.Sc. and Ph.D. degrees in computer sciences from the University of Granada, Granada, Spain, in 2006 and 2008, respectively.

He is currently an Associate Professor with the Department of Software Engineering and Computer Systems, Distance Learning University of Spain (UNED), Madrid, Spain. He is a member of the editorial board of the *Journal of Universal Computer Science*. His research interests include group decision making, decision support systems, consensus models, linguistic modeling, modeling situations with missing/incomplete information, aggregation of information, digital libraries, web quality evaluation, and bibliometric measures.

Enrique Herrera-Viedma was born in 1969. He received the M.Sc. and Ph.D. degrees in computer science from the University of Granada, Granada, Spain, in 1993 and 1996, respectively.

He is currently a Professor with the Department of Computer Science and Artificial Intelligence, University of Granada. He is a member of the editorial board of the journals *Fuzzy Sets and Systems, Soft Computing,* and the *International Journal of Information Technology and Decision Making*. He is the author or coauthor of more than 80 papers published in international journals. Six of his papers are classed as highly cited as well as being in the top 1% of the most cited papers in its field. His current research interests include group decision making, consensus models, linguistic modeling, aggregation of information, information retrieval, bibliometric measures, digital libraries, web quality evaluation, and recommender systems.

Dr. Herrera-Viedma's h-index is 26, and he is classed as one of the Most Cited Scientists in Engineering in the ISI Web of Science—Essential Science Indicators, i.e., he is in the top 1% of the most cited scientists in the field of engineering.

2.2. A Selection Process Based on Additive Consistency to Deal with Incomplete Fuzzy Linguistic Information

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A Selection Process Based on Additive Consistency to Deal with Incomplete Fuzzy Linguistic Information

Francisco Javier Cabrerizo, Rubén Heradio
(Dept. of Software Engineering and Computer Systems, UNED, Madrid, Spain
{cabrerizo,rheradio}@issi.uned.es)

Ignacio Javier Pérez, Enrique Herrera-Viedma
(Dept. of Computer Science and A.I., University of Granada, Granada, Spain
{ijperez,viedma}@decsai.ugr.es)

Abstract: In group decision making situations, there may be cases in which experts do not have an in-depth knowledge of the problem to be solved and, as a result, they may present incomplete information. In this paper, we present a new selection process to deal with incomplete fuzzy linguistic information. As part of it, we use an iterative procedure to estimate the missing information. This procedure is guided by the additive consistency property and only uses the preference values provided by the experts. In addition, the additive consistency property is also used to measure the level of consistency of the information provided by the experts. The main novelties of this selection process are both the possibility to manage decision situations under incomplete fuzzy linguistic information and the importance of the experts' preferences in the aggregation processes is modeled by means of the experts' consistency.

Key Words: group decision making, incomplete information, fuzzy linguistic information, consistency, aggregation
Category: H.0, I.2, I.6, J.6

1 Introduction

In Group Decision Making (GDM) problems there are a set of alternatives to solve a problem and a group of experts, characterized by their own ideas, attitudes, motivations and knowledge, trying to achieve a common solution. To do this, experts have to express their preferences by means of a set of evaluations over the set of alternatives.

Preference relations are usually assumed to model experts' preferences in GDM problems [Orlovski 1978, Saaty 1980, Tanino 1984]. According to the nature of the information expressed for every pair of alternatives, there exist many different representation formats of preference relations. In this paper, we use fuzzy linguistic preference relations (FLPRs) because of most GDM problems present qualitative aspects that are complex to assess by means of precise and exact values and, in such cases, an ordinal fuzzy linguistic approach can be used to obtain a better solution [Herrera et al. 1997a, Herrera et al. 1998, Herrera-Viedma 2001, Herrera-Viedma et al. 2005, Herrera-Viedma et al. 2006, Zadeh 1975a, Zadeh 1975b, Zadeh 1975c]. FLPRs assessed on a 2-tuple fuzzy linguistic modelling [Herrera and Martínez 2000] are assumed because it provides some advantages with respect to the ordinal fuzzy linguistic modelling [Cabrerizo et al. 2009, Herrera and Martínez 2001]. The main advantage of pairwise comparison is that of focusing exclusively on two alternatives at a time, which facilitates experts when expressing their preferences. However, this way of providing preferences limits experts in their global perception of the alternatives and, as a consequence, the provided preferences could be not rational. Usually, rationality is related the consistency concept, which is associated with the *transitivity property*. Many properties have been suggested to model transitivity of a fuzzy preference relation [Herrera-Viedma et al. 2004]. One of these properties is the *additive consistency*, which, as it was shown in [Herrera-Viedma et al. 2004], can be seen as the parallel concept of Saaty's consistency property in the case of multiplicative preference relations [Saaty 1980].

It is obvious that consistent information, i.e., information which does not imply any kind of contradiction, is more relevant or important than information containing some contradictions. The general procedure for the inclusion of importance degrees in GDM problems involves the transformation of the preference values under the importance degrees to generate new values. This activity is carried out by means of a transformation function [Herrera and Herrera-Viedma 1997, Yager 1978, Yager 1994] or by using the importance degrees to induce the ordering of the preference values prior to their aggregation as in Induced Ordered Weighted Averaging (IOWA) operator [Yager and Filev 1999].

As aforementioned, each expert has his/her own knowledge concerning the problem being studied, which also may imply a major drawback, that of an expert not having a perfect knowledge of the problem to be solved. Indeed, experts could not be able to efficiently express any kind of preference degree between two or more of the available options. This may be due to an expert not possessing a precise or sufficient level of knowledge of part of the problem, or because that expert is unable to discriminate the degree to which some options are better than others. Experts would rather not guess those preference degrees in these situations and, as a consequence, they might provide incomplete information [Alonso et al. 2008, Kim et al. 1999, Herrera-Viedma et al. 2007a, Herrera-Viedma et al. 2007, Xu 2005]. In this way, a difficulty that has to be addressed is the lack of information in the experts' opinions and, therefore, it would be of great importance to provide experts some tools that allow them to express this lack of knowledge in their opinions.

The aim of this paper is to present a new selection process based on additive consistency property to deal with GDM problems with incomplete FLPRs. This new selection process is composed of three steps: (1) *estimation of missing preference values*, (2) *aggregation* and (3) *exploitation*. So, we define an *additive consistency* measure for FLPRs that is based on the additive transitivity property [Tanino 1984]. In the first step we use an iterative complete procedure to estimate missing information in the case of incomplete FLPRs. It is based on the linguistic extension of Tanino's consistency principle and it carries out the completion of a particular expert's incomplete FLPRs using only the information he/she provides. The, following the choice scheme proposed in [Fodor and Roubens 1994], *aggregation* following by *exploitation*, this new selection process is completed. Furthermore, we use the additive consistency measure to propose a new IOWA operator, which we call the additive-consistency 2-tuple linguistic IOWA operator. The aggregation step of a selection process consists in combining the experts' individual preferences into a collective one, in such a way, that it summarizes or reflects the properties contained in all the individual preferences. This aggregation is carried out by using that new linguistic IOWA operator. The exploitation phase transforms the global information about the alternatives into a global ranking of them. To do this, two quantifier guided choice degrees of alternatives are used: the dominance and non-dominance degrees. The main improvements of this new selection process is that it supports the management of incomplete fuzzy linguistic information and allows the aggregation of the experts' preferences, in such a way, that more importance is given to the most consistent ones.

The rest of the paper is set out as follows. Section 2 deals with the preliminaries necessary to develop the new selection process. Section 3 presents the new selection process based on additive consistency to deal with incomplete FLPR. Section 4 shows an example as to how to apply it. Finally, in Section 5, we draw our conclusions.

2 Preliminaries

In this section, we present those tools necessary to design the new selection process, that is, the concept of incomplete 2-tuple FLPR, consistency measures and the iterative procedure to estimate missing values.

2.1 Incomplete 2-tuple FLPRs

A preference relation is defined as $P^h \subset X \times X$, where the value $\mu_{P^h}(x_i, x_k) = p^h_{ik}$ is interpreted as the preference degree of the alternative x_i over x_k for the expert e_h. According to the nature of the information expressed for every pair of alternatives, there exist many different representation domains of preference relations. As aforementioned, we use the 2-tuple fuzzy linguistic model [Herrera and Martínez 2000] to represent experts' preferences.

Definition 1. Let $S = \{s_0, \ldots, s_g\}$ be a linguistic term set and $\beta \in [0, g]$ a value representing the result of a symbolic aggregation operation, being $g + 1$

the cardinality of S, then the 2-tuple that expresses the equivalent information to β is obtained with the following function:

$$\Delta\colon [0,g] \longrightarrow S \times [-0.5, 0.5)$$
$$\Delta(\beta) = (s_i, \alpha), \text{ with } \begin{cases} s_i, & i = round(\beta) \\ \alpha = \beta - i, & \alpha \in [-0.5, 0.5), \end{cases} \tag{1}$$

where $round(\cdot)$ is the usual round operation, s_i has the closest index label to "β", and "α" is the value of the symbolic translation.

Proposition 2. *Let $S = \{s_0, \ldots, s_g\}$ be a linguistic term set and (s_i, α) be a 2-tuple. There is always a Δ^{-1} function such that from a 2-tuple it returns its equivalent numerical value $\beta \in [0, g]$.*

$$\Delta^{-1}\colon S \times [-0.5, 0.5) \longrightarrow [0, g]$$
$$\Delta^{-1}(s_i, \alpha) = i + \alpha = \beta. \tag{2}$$

A 2-tuple linguistic computational model to combine linguistic information is composed of following operators: h

1. *A 2-tuple comparison operator.* The comparison of linguistic information represented by linguistic 2-tuples is carried out according to an ordinary lexicographic order (see [Herrera and Martínez 2000] for more details).

2. *A 2-tuple negation operator.*

$$Neg(s_i, \alpha) = \Delta(g - (\Delta^{-1}(s_i, \alpha))). \tag{3}$$

3. *2-tuple aggregation operators.* Extending the classical aggregation operators, such as the Linguistic Ordered Weighted Averaging (LOWA) operator [Herrera et al. 1996], the weighted average operator, the Ordered Weighted Averaging (OWA) operator, etc., (see [Herrera and Martínez 2000]).

A linguistic term $s_i \in S$ can be seen as a linguistic 2-tuple by adding to it the value 0 as symbolic translation, i.e., $s_i \in S \equiv (s_i, 0)$.

Definition 3. A 2-tuple FLPR P^h on a set of alternatives $X = \{x_1, \ldots, s_n\}$ is a fuzzy set defined on the product set $X \times X$, whic is characterized by a 2-tuple linguistic membership function

$$\mu_{P^h} : X \times X \longrightarrow S \times [-0.5, 0.5). \tag{4}$$

When cardinality of X is small, the preference relation may be conveniently represented by a $n \times n$ matrix $P^h = (p^h_{ik})$, being $p^h_{ik} = \mu_{P^h}(x_i, x_k)$, $\forall i, k \in \{1, \ldots, n\}$ and $p^h_{ik} \in (S \times [-0.5, 0.5))$.

Usual models to solve GDM problems assume that experts are always able to provide all the preferences required. However, this situation is not always possible to achieve. Experts could have some difficulties in giving all their preferences due to lack of knowledge about part of the problem, or simply because they may not be able to quantify some of their degree of preference. It must be clear then that when an expert e_h is not able to express the particular value p^h_{ik}, this does not mean that he/she prefers both options with the same intensity.

In order to model these situations, in the following definitions we express the concept of an incomplete 2-tuple FLPR:

Definition 4. A function $f : X \times Y$ is partial when not every element in the set X necessarily maps to an element in the set Y. When every element from the set X maps to one element of the set Y then we have a total function.

Definition 5. A 2-tuple FLPR P^h on a set of alternatives X with a partial 2-tuple linguistic membership function is an incomplete 2-tuple FLPR.

Obviously, a 2-tuple FLPR is complete when its membership function is totally defined. Clearly, definition (3) includes both definitions of complete and incomplete 2-tuple FLPRs.

2.2 Consistency measures

The previous definition of a 2-tuple FLPR does not imply any kind of consistency property. In fact, preference values of a preference relation can be contradictory. Obviously, an inconsistent source of information is not as useful as a consistent one and, thus, it would be quite important to be able to measure the consistency of the information provided by experts for a particular problem.

To make a rational choice, properties to be satisfied by such preference relations have been suggested. One of these properties is the *transitivity property*, which represents the idea that the preference value obtained by directly two alternatives should be equal to or greater than the preference value between those two alternatives obtained using an indirect chain of alternatives. There are several possible characterizations for the transitivity property (see [Herrera-Viedma et al. 2004]). In this paper, we make use of the *additive transitivity property*, which can be seen for fuzzy preference relations as the parallel concept of Saaty's consistency property for multiplicative preference relations [Saaty 1980]. The mathematical formulation of the *additive transitivity* was given by Tanino in [Tanino 1984]:

$$(p_{ij}^h - 0.5) + (p_{jk}^h - 0.5) = (p_{ik}^h - 0.5),\ \forall i,j,k \in \{1,\ldots,n\}. \tag{5}$$

Using the transformation functions Δ and Δ^{-1}, we define the linguistic additive transitivity property for 2-tuple FLPR as follows:

$$\begin{aligned}&\Delta[(\Delta^{-1}(p_{ij}^h) - \Delta^{-1}(s_{g/2},0)) + (\Delta^{-1}(p_{jk}^h) - \Delta^{-1}(s_{g/2},0))] = \\ &\Delta[(\Delta^{-1}(p_{ik}^h) - \Delta^{-1}(s_{g/2},0)],\ \ \forall i,j,k \in \{1,\ldots,n\}.\end{aligned} \tag{6}$$

As in the case of additive transitivity, the linguistic additive transitivity implies linguistic additive reciprocity. Indeed, because $p_{ii}^h = (s_{g/2},0),\ \forall i$, if we make $k = i$ in (6), then we have: $\Delta(\Delta^{-1}(p_{ij}^h) + \Delta^{-1}(p_{ji}^h)) = (s_g,0),\ \forall i,j \in \{1,\ldots,n\}$. Then, expression (6) could be rewritten as:

$$p_{ik}^h = \Delta(\Delta^{-1}(p_{ij}^h) + \Delta^{-1}(p_{jk}^h) - \Delta^{-1}(s_{g/2},0)),\ \ \forall i,j,k \in \{1,\ldots,n\}. \tag{7}$$

A 2-tuple FLPR will be considered "additive consistent" when for every three options, $x_i, x_j, x_k \in X$, their associated 2-tuple fuzzy linguistic preference degrees, $p_{ij}^h, p_{jk}^h, p_{ik}^h$, fulfil (7). An additive consistent 2-tuple FLPR will be referred as consistent throughout the paper, as this is the only transitivity property we are considering.

Expression (7) could be used to calculate an estimated value of a preference degree using other preference degrees. Indeed, the preference value p_{ik}^h $(i \neq k)$ can be estimated using an intermediate alternative x_j in three different ways:

1. From $p_{ik}^h = \Delta(\Delta^{-1}(p_{ij}^h) + \Delta^{-1}(p_{jk}^h) - \Delta^{-1}(s_{g/2},0))$ we obtain the estimate
$$(cp_{ik}^h)^{j1} = \Delta(\Delta^{-1}(p_{ij}^h) + \Delta^{-1}(p_{jk}^h) - \Delta^{-1}(s_{g/2},0)). \tag{8}$$
2. From $p_{jk}^h = \Delta(\Delta^{-1}(p_{ji}^h) + \Delta^{-1}(p_{ik}^h) - \Delta^{-1}(s_{g/2},0))$ we obtain the estimate
$$(cp_{ik}^h)^{j2} = \Delta(\Delta^{-1}(p_{jk}^h) - \Delta^{-1}(p_{ji}^h) + \Delta^{-1}(s_{g/2},0)). \tag{9}$$
3. From $p_{ij}^h = \Delta(\Delta^{-1}(p_{ik}^h) + \Delta^{-1}(p_{kj}^h) - \Delta^{-1}(s_{g/2},0))$ we obtain the estimate
$$(cp_{ik}^h)^{j3} = \Delta(\Delta^{-1}(p_{ij}^h) - \Delta^{-1}(p_{kj}^h) + \Delta^{-1}(s_{g/2},0)). \tag{10}$$

The overall estimated value cp_{ik}^h of p_{ik}^h is obtained as the average of all possible $(cp_{ik}^h)^{j1}$, $(cp_{ik}^h)^{j2}$ and $(cp_{ik}^h)^{j3}$ values:

$$cp_{ik}^h = \Delta\left(\frac{\sum_{j=1;i\neq k\neq j}^{n} (\Delta^{-1}((cp_{ik}^h)^{j1}) + \Delta^{-1}((cp_{ik}^h)^{j2}) + \Delta^{-1}((cp_{ik}^h)^{j3}))}{3(n-2)}\right). \tag{11}$$

We should point out that in expressions (8), (9) and (10), we could find that the value of argument of the function Δ could lie outside the interval $[0, g]$. In order to avoid this problem, the following function is used on the arguments of Δ:

$$f(y) = \begin{cases} 0, & \text{if } y < 0 \\ g, & \text{if } y > g \\ y, & \text{otherwise,} \end{cases} \tag{12}$$

When the information provided is completely consistent, then $(cp_{ik}^h)^{jl} = p_{ik}^h$, $\forall j, l$. The error between a preference value and its estimated one is defined as follows.

Definition 6. The error between a preference value and its estimated one in $[0, 1]$ is computed as:

$$\varepsilon p_{ik}^h = \frac{|\Delta^{-1}(cp_{ik}^h) - \Delta^{-1}(p_{ik}^h)|}{g}. \tag{13}$$

Thus, it can be used to define the consistency level of the preference degree p_{ik}^h.

Definition 7. The consistency level associated to p_{ik}^h is defined as:

$$cl_{ik}^h = 1 - \varepsilon p_{ik}^h. \tag{14}$$

When $cl_{ik}^h = 1$, then $\varepsilon p_{ik}^h = 0$ and there is no inconsistency at all. The lower the value of cl_{ik}^h, the higher the value of εp_{ik}^h and the more inconsistent is p_{ik}^h with respect to the rest of information.

In the following, we define the consistency levels associated with individual alternatives and the whole 2-tuple FLPR:

Definition 8. The consistency level, $cl_i^h \in [0, 1]$, associated to a particular alternative x_i of a 2-tuple FLPR, P^h, is defined as:

$$cl_i^h = \frac{\sum_{k=1; i \neq k}^{n} (cl_{ik}^h + cl_{ki}^h)}{2(n-1)}. \tag{15}$$

Definition 9. The consistency level, $cl^h \in [0, 1]$, of a 2-tuple FLPR, P^h, is defined as follows:

$$cl^h = \frac{\sum_{i=1}^{n} cl_i^h}{n}. \tag{16}$$

When working with an incomplete 2-tuple FLPR, expression (11) cannot be used to obtain the estimate of a known preference value. In these cases, the following sets can be defined [Herrera-Viedma et al. 2007]:

$$\begin{array}{l} A = \{(i,j) \mid i,j \in \{1,\ldots,n\} \wedge i \neq j\} \\ MV^h = \{(i,j) \in A \mid p_{ij}^h \text{ is unknown}\} \\ EV^h = A \setminus MV^h \\ H_{ik}^{h1} = \{j \neq i,k \mid (i,j),(j,k) \in EV^h\} \\ H_{ik}^{h2} = \{j \neq i,k \mid (j,i),(j,k) \in EV^h\} \\ H_{ik}^{h3} = \{j \neq i,k \mid (i,j),(k,j) \in EV^h\} \\ EV_i^h = \{(a,b) \mid (a,b) \in EV^h \wedge (a = i \vee b = i)\}, \end{array} \tag{17}$$

where MV^h is the set of pairs of alternatives whose preference degrees are not given by expert e_h, EV^h is the set of pairs of alternatives whose preference degrees are given by the expert e_h; H_{ik}^{h1}, H_{ik}^{h2}, H_{ik}^{h3} are the sets of intermediate alternative x_j $(j \neq i,k)$ that can be used to estimate the preference value p_{ik}^h $(i \neq k)$ using (8)–(10), respectively; and EV_i^h is the set of pairs of alternatives whose preference degrees involving the alternative x_i are given by the expert e_h. Then, the estimated value of a particular preference degree p_{ik}^h $((i,k) \in EV^h)$ can be calculated as [Herrera-Viedma et al. 2007a, Herrera-Viedma et al. 2007]:

$$\text{if } (\#H_{ik}^{h1} + \#H_{ik}^{h2} + \#H_{ik}^{h3}) \neq 0 \Rightarrow$$

$$cp_{ik}^h = \Delta\left(\frac{\sum_{j\in H_{ik}^{h1}} \Delta^{-1}((cp_{ik}^h)^{j1}) + \sum_{j\in H_{ik}^{h2}} \Delta^{-1}((cp_{ik}^h)^{j2}) + \sum_{j\in H_{ik}^{h3}} \Delta^{-1}((cp_{ik}^h)^{j3})}{(\#H_{ik}^{h1} + \#H_{ik}^{h2} + \#H_{ik}^{h3})}\right). \tag{18}$$

An important factor to take into account when analyzing the consistency in decision making situations with incomplete information is the notion of completeness. Clearly, the higher the number of preference values provided by an expert the higher the chance of inconsistency [Herrera-Viedma et al. 2007]. Therefore, a degree of completeness associated with the number or preference values provided should also be taken into account to produce a fairer measure of consistency of an incomplete 2-tuple FLPR.

Given an incomplete 2-tuple FLPR, we can easily characterize two completeness levels, *the completeness level of a relation* and *the completeness level of an alternative*. For an incomplete 2-tuple FLPR P^h, its completeness level, C^h, can be defined as the ratio of the number of preference values known, $\#EV^h$, to the total possible number of preference values, $n^2 - n$:

$$C^h = \frac{\#EV^h}{n^2 - n}. \tag{19}$$

For an alternative x_i, we can define its completeness level according to the preferences provided by the expert e_h, C_i^h, as the ratio between the actual number of preference values known for x_i, $\#EV_i^h$, and the total number of possible preference values in which x_i is involved with a different alternative, $2(n-1)$:

$$C_i^h = \frac{\#EV_i^h}{2(n-1)}. \tag{20}$$

So, we can define the consistency level associated to a preference value in an incomplete 2-tuple FLPR as follows.

Definition 10. The consistency level cl_{ik}^h associated to p_{ik}^h $(i,k) \in EV^h$ is defined as a linear combination of its associated error and the average of the completeness values associated to the two alternatives involved in that preference degree

$$cl_{ik}^h = (1-\alpha_{ik}^h)\cdot(1-\varepsilon p_{ik}^h) + \alpha_{ik}^h \cdot \frac{C_i^h + C_k^h}{2};\ \alpha_{ik}^h \in [0,1], \tag{21}$$

where α_{ik}^h is a parameter to control the influence of completeness in the evaluation of the consistency levels for e_h defined as:

$$\alpha_{ik}^h = 1 - \frac{\#EV_i^h + \#EV_k^h - \#(EV_i^h \cap EV_k^h)}{4(n-1)-2}. \tag{22}$$

Clearly, expression (21) is an extension of expression (14), because when P^h is complete both EV^h and A coincide and $\alpha_{ik}^h = 0$, $\forall i,k$.

Definition 11. The consistency level of an incomplete 2-tuple FLPR is defined as follows:

$$cl^h = \frac{\sum_{(i,k)\in EV^h} cl_{ik}^h}{\#EV^h}. \tag{23}$$

2.3 Estimation procedure of missing values for incomplete 2-tuple FLPRs

We use an iterative complete procedure to estimate the missing values in an incomplete 2-tuple FLPR, which it is based on the linguistic additive consistency property. This procedure estimates missing values in an expert's incomplete 2-tuple FLPR using only the preference values provided by that particular expert. The procedure estimates missing values by means of two different tasks:

A) Choose those elements to be estimated in each iteration of the procedure

The subset of missing values MV^h that can be estimated in step t of our procedure is denoted by EMV_t^h and defined as follows:

$$EMV_t^h = \left\{ (i,k) \in MV^h \setminus \bigcup_{l=0}^{t-1} EMV_l^h \mid i \neq k \wedge \exists j \in \{H_{ik}^{h1} \cup H_{ik}^{h2} \cup H_{ik}^{h3}\} \right\}, \tag{24}$$

and $EMV_0^h = \emptyset$ (by definition). When $EMV_{maxIter}^h = \emptyset$, with $maxIter > 0$, the procedure will stop as there will not be any more missing values to be estimated. Furthermore, if $\bigcup_{l=0}^{maxIter} EMV_l^h = MV^h$, then all missing values are estimated, and, consequently, the procedure is said to be successful in the completion of the incomplete 2-tuple FLPR.

B) Estimate a particular missing value

In order to estimate a particular value p_{ik}^h with $(i,k) \in EMV_t^h$, the following function $estimate\_p(h,i,k)$ is proposed:

function estimate_p(h,i,k)
1) $(cp_{ik}^h)^1 = (s_0, 0),\ (cp_{ik}^h)^2 = (s_0, 0),\ (cp_{ik}^h)^3 = (s_0, 0),\ \mathcal{K} = 0.$
2) if $\#H_{ik}^{h1} \neq 0$, then $(cp_{ik}^h)^1 = \Delta((\sum_{j \in H_{ik}^{h1}} \Delta^{-1}((cp_{ik}^h)^{j1}))/\#H_{ik}^{h1}),\ \ \mathcal{K}{+}{+}.$
3) if $\#H_{ik}^{h2} \neq 0$, then $(cp_{ik}^h)^2 = \Delta((\sum_{j \in H_{ik}^{h2}} \Delta^{-1}((cp_{ik}^h)^{j2}))/\#H_{ik}^{h2}),\ \ \mathcal{K}{+}{+}.$
4) if $\#H_{ik}^{h3} \neq 0$, then $(cp_{ik}^h)^3 = \Delta((\sum_{j \in H_{ik}^{h3}} \Delta^{-1}((cp_{ik}^h)^{j3}))/\#H_{ik}^{h3}),\ \ \mathcal{K}{+}{+}.$
5) Calculate $cp_{ik}^h = \Delta\left(\frac{\Delta^{-1}(cp_{ik}^h)^1 + \Delta^{-1}(cp_{ik}^h)^2 + \Delta^{-1}(cp_{ik}^h)^3}{\mathcal{K}}\right).$
end function

Then, the complete iterative estimation procedure is the following:

0. $EMV_0^h = \emptyset$
1. $t = 1$
2. while $EMV_t^h \neq \emptyset$ {
3. for every $(i,k) \in EMV_t^h$ {
4. estimate_p(h,i,k)
5. }
6. $t{+}{+}$
7. }

3 A selection process based on additive consistency to deal with incomplete fuzzy linguistic information

In this section, we present a new selection process based on additive consistency to deal with incomplete fuzzy linguistic information. It consists of three phases: (1) *estimation of missing information*, (2) *aggregation* and (3) *exploitation*. The estimation of missing information completes the opinions provided by the experts. To do so, it uses the consistency based procedure to estimate missing information shown in Section 2.3. The aggregation phase defines a collective 2-tuple FLPR indicating the global preference between every ordered pair of alternatives, while the exploitation phase transforms the global information about the alternatives into a global ranking of them, from which a choice set of alternatives is derived.

3.1 Estimation of missing information

In this phase, each incomplete 2-tuple FLPR is completed following the consistency based procedure to estimate missing information shown in Section 2.3. In such a way, we allow to solve GDM situations with incomplete information because of if the missing information is not completed, we could find that some preference degrees of the collective preference relation cannot be computed in the aggregation phase and, consequently, the ordering of some alternatives cannot be computed in the exploitation phase. Therefore, for each incomplete 2-tuple FLPR, P^h, we obtain its corresponding complete 2-tuple FLPR, $\bar{P}^h$.

3.2 Aggregation

Once we have estimated all the missing values in every incomplete 2-tuple FLPR, we have a set of m individual 2-tuple FLPRs $\{\bar{P}^1, \ldots, \bar{P}^m\}$. From this set, a collective 2-tuple FLPR, $P^c = (p^c_{ik})$, must be obtained by means of an aggregation procedure. In our case, each value $p^c_{ik} \in S \times [-0.5, 0.5)$ will represent the preference of alternative x_i over alternative x_k according to the majority of the most consistent experts' opinions. To do that, we use a 2-tuple linguistic OWA operator to aggregate the experts' opinions.

Definition 12. A 2-tuple linguistic OWA operator of dimension n is a function $\phi : (S \times [-0.5, 0.5))^n \longrightarrow S \times [-0.5, 0.5)$, that has a weighting vector associated with it, $W = (w_1, \ldots, w_n)$, with $w_i \in [0, 1]$, $\sum_{i=1}^n w_i = 1$, and it is defined according to the following expression:

$$\phi_W(p_1, \ldots, p_n) = \Delta(\sum_{i=1}^{n} w_i \cdot \Delta^{-1}(p_{\sigma(i)})),\ p_i \in S \times [-0.5, 0.5), \tag{25}$$

being $\sigma : \{1, \ldots, n\} \longrightarrow \{1, \ldots, n\}$ a permutation defined on 2-tuple linguistic values, such that $p_{\sigma(i)} \geq p_{\sigma(i+1)}$, $\forall i = 1, \ldots, n-1$, i.e., $p_{\sigma(i)}$ is the i-highest 2-tuple linguistic value in the set $\{p_1, \ldots, p_n\}$.

A natural question in the definition of the OWA operator is how to obtain the associated weighting vector. In [Yager 1988], it was defined an expression to obtain W that allows to represent the concept of fuzzy majority [Kacprzyk 1986] by means of a fuzzy linguistic non-decreasing quantifier Q [Zadeh 1983]:

$$w_i = Q(i/n) - Q((i-1)/n), \quad i = 1, \ldots, n. \tag{26}$$

The 2-tuple linguistic OWA operator does not take into account the importance of the experts. However, a rational assumption in the resolution process of a GDM problem is that of associating more importance to the experts who provide the most "consistent" information. This assumption implies that GDM problems should be viewed as heterogeneous. Indeed, in any GDM problem with incomplete information, each expert e_h can have an importance degree associated with him/her, which, for example, can be his/her own consistency level of the relation cl^h or consistency levels of the preference values cl^h_{ik} in each preference value p^h_{ik}.

Usually, procedures for the inclusion of these importance values in the aggregation process involve the transformation of the preference values, p^h_{ik}, under the importance degree I^h, to generate a new value, $\tilde{p}^h_{ik}$ [Herrera et al. 1998, Herrera and Herrera-Viedma 1997]. Usually, this process is carried out by means of a transformation function g, $\tilde{p}^h_{ik} = g(p^h_{ik}, I^h)$ [Herrera et al. 1998, Yager 1978]. One alternative possibility could consist of using importance degrees or consistency levels as the order inducing values of the IOWA operator to be applied in the aggregation phase of the selection process. Yager and Filev defined the IOWA operator as an extension of the OWA operator [Yager 1988] to allow a different reordering of the values to be aggregated [Yager and Filev 1999].

Definition 13. A 2-tuple linguistic IOWA operator of dimension n is a function

$$\Phi_W(\langle u_1, p_1\rangle, \ldots, \langle u_n, p_n\rangle) = \Delta(\sum_{i=1}^{n} w_i \cdot \Delta^{-1}(p_{\sigma(i)})),\ p_i \in S \times [-0.5, 0.5), \quad (27)$$

being σ a permutation of $\{1, \ldots, n\}$ such that $u_{\sigma(i)} \geq u_{\sigma(i+1)}$, $\forall i = 1, \ldots, n-1$, i.e., $\langle u_{\sigma(1)}, p_{\sigma(1)}\rangle$ is the 2-tuple with $u_{\sigma(i)}$ the i-th highest value in the set $\{u_1, \ldots, u_n\}$.

In the above definition, the reordering of the set of values to be aggregated, $\{p_1, \ldots, p_n\}$, is induced by the reordering of the set of values $\{u_1, \ldots, u_n\}$ associated with them, which is based upon their magnitude. Due to this use of the set of values $\{u_1, \ldots, u_n\}$, Yager and Filev called them the values of an order inducing variable $\{p_1, \ldots, p_n\}$ the values of the argument variable [Yager and Filev 1999].

In this case, to obtain the associated weighting vector, in [Yager 1996], Yager also proposed a procedure to evaluate the overall satisfaction of Q important (u_k) criteria (or experts) (e_k) by the alternative x_j. In this procedure, once the satisfaction values to be aggregated have been ordered, the weighting vector associated with an IOWA operator using a linguistic quantifier Q are calculated following the expression

$$w_i = Q\left(\frac{\sum_{k=1}^{i} u_{\sigma(k)}}{T}\right) - Q\left(\frac{\sum_{k=1}^{i-1} u_{\sigma(k)}}{T}\right), \quad (28)$$

being $T = \sum_{k=1}^{n} u_k$ the total sum of importance, and σ the permutation used to produce the ordering of the values to be aggregated. This approach for the inclusion of importance degrees associates a zero weight to those experts with a zero importance degree. In our case, the consistency levels of the 2-tuple FLPRs are used to obtain the importance degrees associated with the experts.

Definition (13) allows the construction of many different operators. Indeed, the set of consistency levels of the preference values, $\{cl_{ik}^1, \ldots, cl_{ik}^m\}$, may be used to define an IOWA operator, i.e., and the ordering of the preference values to be aggregated $\{\bar{p}_{ik}^1, \ldots, \bar{p}_{ik}^m\}$ can be induced by ordering the experts from the most to the least consistent one. In such a way, we obtain an IOWA operator that we call the additive-consistency 2-tuple IOWA operator, which can be viewed as an extension of the AC-IOWA operator [Chiclana et al. 2007, Chiclana et al. 2004, Chiclana et al. 2004a, Herrera-Viedma et al. 2007a].

Definition 14. The additive-consistency 2-tuple linguistic IOWA operator of dimension m, Φ_W^{AC}, is a 2-tuple linguistic IOWA operator whose set of order inducing values is $\{cl_{ik}^1, \ldots, cl_{ik}^m\}$.

Then, the collective 2-tuple FLPR is obtained as follows:

$$p_{ik}^c = \Phi_Q^{AC}(\langle cl_{ik}^1, \bar{p}_{ik}^1\rangle, \ldots, \langle cl_{ik}^m, \bar{p}_{ik}^m\rangle), \tag{29}$$

where Q is the fuzzy quantifier used to implement the fuzzy majority concept and, using (28), to compute the weighting vector of the additive-consistency 2-tuple IOWA operator, Φ_Q^{AC}.

3.3 Exploitation

In this phase, in order to select the "best" alternative(s) acceptable for the majority of the most consistent experts, we can use two different quantifier-guided choice degrees of alternatives [Herrera-Viedma et al. 2007a]:

- $QGDD_i$: This quantifier guided dominance degree quantifies the dominance that one alternative has over all the others in a fuzzy majority sense and is defined as follows:

$$QGDD_i = \phi_Q(p_{i1}^c, p_{i2}^c, \ldots, p_{i(i-1)}^c, p_{i(i+1)}^c, \ldots, p_{in}^c). \tag{30}$$

- $QGNDD_i$: This quantifier guided non-dominance degree gives the degree in which each alternative is not dominated by a fuzzy majority of the remaining alternatives and is defined as follows:

$$QGNDD_i = \phi_Q(Neg(p_{1i}^s), Neg(p_{2i}^s), \ldots, Neg(p_{(i-1)i}^s), Neg(p_{(i+1)i}^s), \ldots, Neg(p_{ni}^s)), \tag{31}$$

  where

$$p_{ki}^s = \begin{cases} (s_0, 0), & \text{if } p_{ki}^c < p_{ik}^c \\ \Delta(\Delta^{-1}(p_{ki}^c) - \Delta^{-1}(p_{ik}^c)), & \text{if } p_{ki}^c \geq p_{ik}^c, \end{cases}$$

represents the degree in which x_i is strictly dominated by x_k.

The application of the above choice degrees of alternatives over X may be carried out according to two different policies: *sequential policy* and *conjunctive policy* [Herrera-Viedma et al. 2007a]. Thus, in a complete selection process, the choice degrees can be applied in three steps:

1. **Step 1.** The application of each choice degree of alternatives over X to obtain the following sets of alternatives:

$$X^{QGDD} = \{x_i \in X \mid QGDD_i = sup_{x_j \in X} QGDD_j\}, \tag{32}$$

$$X^{QGNDD} = \{x_i \in X \mid QGNDD_i = sup_{x_j \in X} QGNDD_j\}, \tag{33}$$

   whose elements are called maximum dominance elements on the fuzzy majority of X quantified by Q and maximal non-dominated elements by the fuzzy majority of X quantified by Q, respectively.

2. **Step 2.** The application of the conjunction selection policy, obtaining the following set of alternatives:

$$X^{QGCP} = X^{QGDD} \cap X^{QGNDD}. \tag{34}$$

   If $X^{QGCP} \neq \emptyset$, then End. Otherwise, continue.

3. **Step 3.** The application of the one of the two sequential selection policies, according to either a dominance or non-dominance criterion, i.e.:

   – *Dominance based sequential selection process QG-DD-NDD.* To apply the quantifier guided dominance degree over X, and obtain X^{QGDD}. If $\#(X^{QGDD}) = 1$, then End, and this is the solution set. Otherwise, continue obtaining

$$X^{QG-DD-NDD} = \{x_i \in X^{QGDD} \mid QGNDD_i = sup_{x_j \in X^{QGDD}} QGNDD_j\}. \tag{35}$$

   This is the selection set of alternatives.

   – *Non-dominance based sequential selection process QG-NDD-DD.* To apply the quantifier guided non-dominance degree over X, and obtain X^{QGNDD}. If $\#(X^{QGNDD}) = 1$, then End, and this is the solution set. Otherwise, continue obtaining

$$X^{QG-NDD-DD} = \{x_i \in X^{QGNDD} \mid QGDD_i = sup_{x_j \in X^{QGNDD}} QGDD_j\}. \tag{36}$$

   This is the selection set of alternatives.

4 Example of application

Let $X = \{x_1, x_2, x_3, x_4\}$ be a set of four alternatives and $S = \{N, MW, W, E, B, MB, T\}$ a set of seven linguistic labels with the following meaning:

$$N = Null \quad MW = Much\ Worse \quad W = Worse \quad E = Equally\ Preferred$$
$$B = Better \quad MB = Much\ Better \quad T = Total$$

Let us suppose that three different experts $E = \{e_1, e_2, e_3\}$ provide the following incomplete FLPRs using the linguistic expression domain S:

$$P^1 = \begin{pmatrix} - & x & \mathbf{W} & x \\ x & - & x & \mathbf{MW} \\ \mathbf{MB} & x & - & \mathbf{E} \\ x & \mathbf{B} & \mathbf{E} & - \end{pmatrix}; P^2 = \begin{pmatrix} - & \mathbf{W} & \mathbf{B} & \mathbf{B} \\ \mathbf{T} & - & \mathbf{B} & \mathbf{T} \\ \mathbf{W} & \mathbf{W} & - & \mathbf{MW} \\ \mathbf{N} & \mathbf{W} & \mathbf{MB} & - \end{pmatrix}; P^3 = \begin{pmatrix} - & \mathbf{MW} & x & x \\ \mathbf{B} & - & \mathbf{MB} & \mathbf{B} \\ \mathbf{W} & x & - & \mathbf{W} \\ \mathbf{W} & \mathbf{MW} & \mathbf{B} & - \end{pmatrix}.$$

Then, the respective 2-tuple FLPRs are the following:

$$P^1 = \begin{pmatrix} - & x & (\mathbf{W},\mathbf{0}) & x \\ x & - & x & (\mathbf{MW},\mathbf{0}) \\ (\mathbf{MB},\mathbf{0}) & x & - & (\mathbf{E},\mathbf{0}) \\ x & (\mathbf{B},\mathbf{0}) & (\mathbf{E},\mathbf{0}) & - \end{pmatrix};$$

$$P^2 = \begin{pmatrix} - & (\mathbf{W},\mathbf{0}) & (\mathbf{B},\mathbf{0}) & (\mathbf{B},\mathbf{0}) \\ (\mathbf{T},\mathbf{0}) & - & (\mathbf{B},\mathbf{0}) & (\mathbf{T},\mathbf{0}) \\ (\mathbf{W},\mathbf{0}) & (\mathbf{W},\mathbf{0}) & - & (\mathbf{MW},\mathbf{0}) \\ (\mathbf{N},\mathbf{0}) & (\mathbf{W},\mathbf{0}) & (\mathbf{MB},\mathbf{0}) & - \end{pmatrix};$$

$$P^3 = \begin{pmatrix} - & (\mathbf{MW},\mathbf{0}) & x & x \\ (\mathbf{B},\mathbf{0}) & - & (\mathbf{MB},\mathbf{0}) & (\mathbf{B},\mathbf{0}) \\ (\mathbf{W},\mathbf{0}) & x & - & (\mathbf{W},\mathbf{0}) \\ (\mathbf{W},\mathbf{0}) & (\mathbf{MW},\mathbf{0}) & (\mathbf{B},\mathbf{0}) & - \end{pmatrix}.$$

(A) Estimation of missing information

As we observe two 2-tuple FLPRs are incomplete $\{P^1, P^3\}$. As an example, we show how to complete P^1 using the consistency based procedure to estimate missing information shown in Section 2.3:

Step 1: The set of elements that can be estimated are:

$$EMV_1^1 = \{(1,4), (2,3), (3,2), (4,1)\}.$$

After these elements have been estimated, we have:

$$P^1 = \begin{pmatrix} - & x & (\mathbf{W},\mathbf{0}) & (W,-0.33) \\ x & - & (MW,0.33) & (\mathbf{MW},\mathbf{0}) \\ (\mathbf{MB},\mathbf{0}) & (B,0.33) & - & (\mathbf{E},\mathbf{0}) \\ (MB,-0.33) & (\mathbf{B},\mathbf{0}) & (\mathbf{E},\mathbf{0}) & - \end{pmatrix}.$$

As an example, to estimate p_{14}^1 the procedure is as follows:
$H_{14}^{11} = \{3\} \Rightarrow (cp_{14}^1)^1 = \Delta(\Delta^{-1}(cp_{14}^1)^{31}) = \Delta(\Delta^{-1}(\Delta(\Delta^{-1}(p_{13}^1) + \Delta^{-1}(p_{34}^1) - \Delta^{-1}(s_{g/2}, 0)))) = \Delta(\Delta^{-1}(\Delta(2+3-3))) = \Delta(\Delta^{-1}(\Delta(2))) = (W, 0).$

$H_{14}^{12} = \{3\} \Rightarrow (cp_{14}^1)^2 = \Delta(\Delta^{-1}(cp_{14}^1)^{32}) = \Delta(\Delta^{-1}(\Delta(\Delta^{-1}(p_{34}^1) - \Delta^{-1}(p_{31}^1) + \Delta^{-1}(s_{g/2}, 0)))) = \Delta(\Delta^{-1}(\Delta(3 - 5 + 3))) = \Delta(\Delta^{-1}(\Delta(1))) = (MW, 0)$.
$H_{14}^{13} = \{3\} \Rightarrow (cp_{14}^1)^3 = \Delta(\Delta^{-1}(cp_{14}^1)^{33}) = \Delta(\Delta^{-1}(\Delta(\Delta^{-1}(p_{13}^1) - \Delta^{-1}(p_{43}^1) + \Delta^{-1}(s_{g/2}, 0)))) = \Delta(\Delta^{-1}(\Delta(2 - 3 + 3))) = \Delta(\Delta^{-1}(\Delta(2))) = (W, 0)$.
$cp_{14}^1 = \Delta\left(\frac{\Delta^{-1}(cp_{14}^1)^1 + \Delta^{-1}(cp_{14}^1)^2 + \Delta^{-1}(cp_{14}^1)^3}{3}\right) = \Delta\left(\frac{2+1+2}{3}\right) = (W, -0.33)$.

Step 2: The set of elements that can be estimated are:

$$EMV_2^1 = \{(1,2), (2,1)\}.$$

After these elements have been estimated, we have the following complete 2-tuple FLPR:

$$\bar{P}^1 = \begin{pmatrix} - & (E,0) & (\mathbf{W},\mathbf{0}) & (W,-0.33) \\ (E,0) & - & (MW,0.33) & (\mathbf{MW},\mathbf{0}) \\ (\mathbf{MB},\mathbf{0}) & (B,0.33) & - & (\mathbf{E},\mathbf{0}) \\ (MB,-0.33) & (\mathbf{B},\mathbf{0}) & (\mathbf{E},\mathbf{0}) & - \end{pmatrix}.$$

As an example, to estimate p_{12}^1 the procedure is as follows:
$H_{12}^{11} = \{3,4\} \Rightarrow (cp_{12}^1)^1 = \Delta\left(\frac{\Delta^{-1}(cp_{12}^1)^{31} + \Delta^{-1}(cp_{12}^1)^{41}}{2}\right) = (E, 0)$.
$H_{12}^{12} = \{3,4\} \Rightarrow (cp_{12}^1)^2 = \Delta\left(\frac{\Delta^{-1}(cp_{12}^1)^{32} + \Delta^{-1}(cp_{12}^1)^{42}}{2}\right) = (W, 0.33)$.
$H_{12}^{13} = \{3,4\} \Rightarrow (cp_{12}^1)^3 = \Delta\left(\frac{\Delta^{-1}(cp_{12}^1)^{33} + \Delta^{-1}(cp_{12}^1)^{43}}{2}\right) = (B, -0.33)$.
$cp_{12}^1 = \Delta\left(\frac{\Delta^{-1}(cp_{12}^1)^1 + \Delta^{-1}(cp_{12}^1)^2 + \Delta^{-1}(cp_{12}^1)^3}{3}\right) = \Delta\left(\frac{3+2.33+3.67}{3}\right) = (E, 0)$.

For P^3 we get:

$$\bar{P}^3 = \begin{pmatrix} - & (\mathbf{MW},\mathbf{0}) & (B,-0.25) & (E,-0.33) \\ (\mathbf{B},\mathbf{0}) & - & (\mathbf{MB},\mathbf{0}) & (\mathbf{B},\mathbf{0}) \\ (\mathbf{W},\mathbf{0}) & (N,0.33) & - & (\mathbf{W},\mathbf{0}) \\ (\mathbf{W},\mathbf{0}) & (\mathbf{MW},\mathbf{0}) & (\mathbf{B},\mathbf{0}) & - \end{pmatrix}.$$

The corresponding consistency level matrix associated with the incomplete 2-tuple FLPR P^1 is:

$$CL^1 = \begin{pmatrix} - & 0.80 & 0.70 & 0.76 \\ 0.80 & - & 0.78 & 0.70 \\ 0.70 & 0.80 & - & 0.90 \\ 0.80 & 0.70 & 0.90 & - \end{pmatrix}.$$

As an example, to compute cl_{13}^1, the following calculations are needed:

$EV_1^1 = \{(1,3), (3,1)\} \Rightarrow C_1^1 = 2/6$.
$EV_3^1 = \{(1,3), (3,1), (3,4), (4,3)\} \Rightarrow C_3^1 = 4/6$.
$EV_1^1 \cap EV_3^1 = \{(1,3), (3,1)\} \Rightarrow \alpha_{13}^1 = 1 - \frac{2+4-2}{10} = 0.6$.

For p_{13}^1 we have that there is no intermediate alternative to calculate an estimated value and consequently we have:

$$\varepsilon p_{13}^1 = 0 \Rightarrow cl_{13}^1 = (1-0.6)\cdot(1-0) + 0.6 \cdot \frac{\frac{2}{6}+\frac{4}{2}}{2} = 0.7.$$

For P^2 and P^3 we get:

$$CL^2 = \begin{pmatrix} - & 0.83 & 0.92 & 0.80 \\ 0.50 & - & 0.53 & 0.75 \\ 0.92 & 0.70 & - & 0.50 \\ 0.36 & 0.92 & 0.50 & - \end{pmatrix}; CL^3 = \begin{pmatrix} - & 0.80 & 0.81 & 0.81 \\ 0.77 & - & 0.82 & 0.75 \\ 0.78 & 0.81 & - & 0.80 \\ 0.87 & 0.97 & 0.80 & - \end{pmatrix}.$$

(B) Aggregation

Once the incomplete 2-tuple FLPRs are completed, we aggregate them by means of the additive-consistency 2-tuple linguistic IOWA operator and using the consistency level of the preference values as the order inducing variable. We make use of the linguistic quantifier "most of", defined as $Q(r) = r^{1/2}$, which applying (28), generates a weighting vector of three values to obtain each collective preference value p_{ik}^c.
As example, the collective preference value p_{12}^c is obtained as follows:

$cl_{12}^1 = 0.80,\ cl_{12}^2 = 0.83,\ cl_{12}^3 = 0.80.$
$\bar{p}_{12}^1 = (E,0),\ \bar{p}_{12}^2 = (W,0),\ \bar{p}_{12}^3 = (MW,0).$
$\sigma(1) = 2,\ \sigma(2) = 1,\ \sigma(3) = 3.$
$T = cl_{12}^1 + cl_{12}^2 + cl_{12}^3.$
$Q(0) = 0;\ Q\left(\frac{cl_{12}^3}{T}\right) = 0.33; Q\left(\frac{cl_{12}^3 + cl_{12}^2}{T}\right) = 0.67;\ Q\left(\frac{cl_{12}^3 + cl_{12}^2 + cl_{12}^1}{T}\right) = 1.$
$w_1 = 0.33;\ w_2 = 0.34;\ w_3 = 0.33.$
$p_{12}^c = \Delta(w_1 \cdot \Delta^{-1}(\bar{p}_{12}^2) + w_2 \cdot \Delta^{-1}(\bar{p}_{12}^1) + w_3 \cdot \Delta^{-1}(\bar{p}_{12}^3)) = (W, 0.01).$

Then, the collective 2-tuple FLPR is:

$$P^c = \begin{pmatrix} - & (W,0.01) & (E,0.32) & (E,-0.20) \\ (MB,-0.13) & - & (B,-0.29) & (B,-0.28) \\ (E,-0.10) & (W,0.11) & - & (W,0.36) \\ (W,-0.30) & (W,0.09) & (B,0.05) & - \end{pmatrix}.$$

(C) Exploitation

Using again the same linguistic quantifier "most of" and (26), we obtain the weighting vector $W = (w_1, w_2, w_3)$:

$$\begin{aligned} w_1 &= Q(1/3) - Q(0) = 0.58 - 0 = 0.58. \\ w_2 &= Q(2/3) - Q(1/3) = 0.82 - 0.58 = 0.24. \\ w_3 &= Q(1) - Q(2/3) = 1 - 0.82 = 0.18. \end{aligned}$$

and the following quantifier guided dominance and non-dominance degrees of all the alternatives:

$$\begin{array}{lcccc} & x_1 & x_2 & x_3 & x_4 \\ QGDD_i & \text{(E,-0.04)} & \text{(B,0.38)} & \text{(W,0.21)} & \text{(E,0.16)} \\ QGNDD_i & \text{(MB,0.48)} & \text{(T,0.00)} & \text{(MB,0.01)} & \text{(MB,0.26)} \end{array}$$

To calculate the quantifier guided non-dominance degree the following matrix P^s is obtained:

$$P^c = \begin{pmatrix} - & (N, 0.00) & (N, 0.42) & (MW, 0.10) \\ (E, -0.14) & - & (W, -0.40) & (E, -0.37) \\ (N, 0.00) & (N, 0.00) & - & (N, 0.00) \\ (N, 0.00) & (N, 0.00) & (W, -0.31) & - \end{pmatrix}.$$

Clearly, the maximal sets are:

$$X^{QGDD} = \{x_2\} \text{ and } X^{QGNDD} = \{x_2\}.$$

Finally, applying the conjunction selection policy we obtain:

$$X^{QGCP} = X^{QGDD} \cap X^{QGNDD} = \{x_2\}.$$

which means that alternative x_2 is the best alternative according to "most of" the most consistent experts.

5 Conclusions

In this paper we have presented a new selection process based on additive consistency to deal with GDM problems under incomplete fuzzy linguistic information. This new selection process is composed of three phases: estimation of missing values, aggregation and exploitation. The main improvements of this selection process is that it supports the management of incomplete fuzzy linguistic information and it allows the aggregation of the experts' preferences in such a way that more importance is given to the most consistent ones.

In the future we think to research two new challenges: i) Study new strategies to compute the missing value, for example by using consensus criteria [Herrera et al. 1997a, Herrera-Viedma et al. 2005, Mata et al. 2009], and ii) design new selection process for GDM problems under unbalanced fuzzy linguistic information [Cabrerizo et al. 2009, Herrera-Viedma and López-Herrera 2007, Herrera et al. 2008].

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2.3. Group Decision Making Problems in a Linguistic and Dynamic Context

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Group decision making problems in a linguistic and dynamic context

I.J. Pérez [a], F.J. Cabrerizo [b], E. Herrera-Viedma [a,*]

[a] Dept. of Computer Science and Artificial Intelligence, University of Granada, 18071 Granada, Spain
[b] Dept. of Software Engineering and Computer Systems, Distance Learning University of Spain (UNED), 28040 Madrid, Spain

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ABSTRACT

The aim of this paper is to present a new model of decision support system for group decision making problems based on a linguistic approach and dynamic sets of alternatives. The model incorporates a mechanism that allows to manage dynamic decision situations in which some information about the problem is not constant in time. We assume that the set of alternatives can change during the decision making process. The model is presented in a mobile and dynamic context where the experts' preferences can be incomplete. The linguistic approach is used to represent both the experts' preferences about the alternatives and the agreement degrees to manage the change of some alternatives. A prototype of such mobile decision support system in which the experts use mobile devices to provide their linguistic preferences at anytime and anywhere has been implemented. In such a way, we provide a new linguistic group decision making framework that is mobile and dynamic.

1. Introduction

Group decision making (GDM) is used to obtain the best solution(s) for a problem according to the information provided by some decision makers. Usually, each decision maker (expert) may approach the decision process from a different angle, but they have a common interest in reaching an agreement on taking the best decision. Concretely, in a GDM problem we have a set of different alternatives to solve the problem and a set of experts which are usually required to provide their preferences about the alternatives by means of a particular preference format.

Several authors have provided interesting results on GDM with the help of fuzzy theory (Alonso, Herrera-Viedma, Chiclana, & Herrera, 2009; Boran, Gent, Kurt, & Akay, 2009; Cabrerizo, Alonso, & Herrera-Viedma, 2009; Cabrerizo, Moreno, Pérez, & Herrera-Viedma, 2010; Fodors & Roubens, 1994; Herrera, Herrera-Viedma, & Verdegay, 1995; Kacprzyk & Fedrizzi, 1990; Sanayei, Mousavi, & Yazdankhah, 2010). There are decision situations in which the experts' preferences cannot be assessed precisely in a quantitative form but may be in a qualitative one, and thus, the use of a *linguistic approach* is necessary (Alonso, Herrera-Viedma, et al., 2009; Ben-Arieh & Chen, 2006; Herrera, Herrera-Viedma, & Verdegay, 1996b; Herrera, Herrera-Viedma, & Martınez, 2008; Zhang & Chu, 2009). The *linguistic approach* is an approximate technique which represents qualitative aspects as linguistic values by means of *linguistic variables*, that is, variables whose values are not numbers but words or sentences in a natural or artificial language (Herrera & Herrera-Viedma, 2000b).

In this paper we will assume that experts provide their preferences using *linguistic preference relations* (Herrera & Herrera-Viedma, 2000a). Other different issue arises when each expert has his/her own experience concerning the problem being studied and they could have some difficulties in giving all their preferences. This may be due to an expert not possessing a precise or sufficient level of knowledge of the problem, or because that expert is unable to discriminate the degree to which some options are better than others. In such situations, experts are forced to provide incomplete linguistic preference relations (Alonso, Cabrerizo, Chiclana, Herrera, & Herrera-Viedma, 2009; Cabrerizo, Pérez, & Herrera-Viedma, 2010; Herrera-Viedma, Alonso, Chiclana, & Herrera, 2007; Herrera-Viedma, Chiclana, Herrera, & Alonso, 2007; Herrera-Viedma, Herrera, Chiclana, & Luque, 2004; Porcel & Herrera-Viedma, 2010). Therefore, it is of great importance to provide tools to deal with this lack of knowledge in experts' opinions.

However, GDM is still a difficult process by different reasons as the complexity of real problems, which usually deal with large or dynamic sets of alternatives, or as the lack of clear, complete and timely information and also as the geographical dispersion of experts. A GDM process should be agile so that the experts can share and process information rapidly. Furthermore, there are decision situations where some data of decision process can change through the time. For example, this happens in decision making in geographic context in Burigat and Chittaro (2008), in business (Xu, 2007), in navigation applications (Dia, 2002; Kim, Seok, Joo, & Young, 2009), in natural resources management (Clarke, 2008). We focus on the configuration data of a GDM problem that could

* Corresponding author.
E-mail addresses: ijperez@decsai.ugr.es (I.J. Pérez), cabrerizo@issi.uned.es (F.J. Cabrerizo), viedma@decsai.ugr.es (E. Herrera-Viedma).

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change through time, as for example decision criteria, the members of the group of experts and the alternatives of the initial set of alternatives. Therefore, the structure of a GDM model should be flexible to deal with those changes that could alter the initial configuration of a GDM problem while experts make their decision (Pérez, Cabrerizo, & Herrera-Viedma, in press; Saaty, 2007; Xu, 2007).

On the other hand, we find that in many domains, the activities of planning and executing a task are disconnected in the sense that they happen at different places or are carried out by different persons. In order to bridge that gap and make sure that models of the planned activity and its actual realization are identical it is desirable to make further use of the mobile technologies (Tsai, Han, Xu, & Chua, 2009). The incorporation of new communication technologies (Katz, 2008; Schiller, 2003), besides solving the geographical dispersion of experts problem, extends opportunities for decision making (Daume & Robertson, 2000), because if communications are improved, decisions will usually be upgraded as well. This improvement is provided by mobile technologies because it is a good way for a continuous communication flow. It allows that experts always have updated and timely information at anywhere and anytime, obtaining in this way an agile decision process by simulating a real discussion meeting.

In this paper we present a new model of decision support system for GDM problems based on a linguistic approach and dynamic sets of alternatives. Experts provide their preferences by means of linguistic preference relations which could be incomplete, i.e., this new GDM model is user-friendly. We consider that the configuration data of the GDM problem with respect to the initial set of alternatives can change through decision process, i.e., this new model presents a dynamic and flexible structure. Furthermore, we implement a prototype of such GDM model using mobile technologies allowing experts to develop the decision process in anywhere and anytime. In such a way, we present a very complete GDM framework that facilitates the development of GDM processes.

In order to do this, the paper is set out as follows. Preliminaries are presented in Section 2. Section 3 defines the new linguistic GDM model. Section 4 presents the prototype of such linguistic GDM model. A case of use is shown in Section 5, and finally, Section 6 draws our conclusions.

2. Preliminaries

In the following subsections we present the ordinal fuzzy linguistic approach, GDM problems with incomplete fuzzy linguistic preference relations, and the foundations of mobile technologies.

2.1. Ordinal fuzzy linguistic approach

There are situations in which the information cannot be assessed precisely in a quantitative form but may be in a qualitative one. For example, when attempting to qualify phenomena related to human perception, we are often led to use words in natural language instead of numerical values, e.g. when evaluating quality of a football player, terms like *good*, *medium* or *bad* can be used.

The ordinal fuzzy linguistic approach (Herrera & Herrera-Viedma, 1997; Herrera, Herrera-Viedma, & Verdegay, 1996a) is a tool based on the concept of linguistic variable (Zadeh, 1975) to deal with qualitative assessments. It is a very useful kind of fuzzy linguistic approach because its use simplifies the processes of computing with words as well as linguistic representation aspects of problems. It has proven its usefulness in many problems, e.g. in decision making (Ben-Arieh & Chen, 2006; Cabrerizo et al., 2009; Herrera & Herrera-Viedma, 2000b; Herrera-Viedma, Martinez, Mata, & Chiclana, 2005), web quality evaluation (Herrera-Viedma & Peis, 2003; Herrera-Viedma, Pasi, López-Herrera, & Porcel, 2006; Herrera-Viedma, Peis, Morales, Alonso, & Anaya, 2007), information retrieval (Bordogna & Pasi, 2001; Herrera-Viedma, 2001; Herrera-Viedma & López-Herrera, 2007), recommender systems (Porcel, Lopez-Herrera, & Herrera-Viedma, 2009; Porcel, Moreno, & Herrera-Viedma, 2009), evaluation of libraries (Cabrerizo, Lpez-Gijón, Ruíz-Rodrıguez, & Herrera-Viedma, 2010; López-Gijón, Avila, Pérez, & Herrera-Viedma, 2010), political analysis (Arfi, 2005), etc.

It is defined by considering a finite and totally ordered label set $S = \{s_i\}$, $i \in \{0,\ldots,g\}$ in the usual sense, i.e., $s_i \geqslant s_j$ if $i \geqslant j$, and with odd cardinality (usually 7 or 9 labels). The mid term represents an assessment of "approximately 0.5", and the rest of the terms are placed symmetrically around it. The semantics of the label set is established from the ordered structure of the label set by considering that each label for the pair (s_i, s_{g-i}) is equally informative (Herrera & Herrera-Viedma, 2000b). For example, we can use the following set of seven labels to represent the linguistic information: $S = \{s_0 = N,\ s_1 = VL,\ s_2 = L,\ s_3 = M,\ s_4 = H,\ s_5 = VH,\ s_6 = P\}$, where $N = Null$, $VL = Very\ \ Low$, $L = Low$, $M = Medium$, $H = Height$, $VH = Very\ Height$ and $P = Perfect$.

In any linguistic modeling we also need some management operators for linguistic information. An advantage of the ordinal fuzzy linguistic modeling is the simplicity and speed of its computational model. It is based on the symbolic computational model (Herrera & Herrera-Viedma, 1997; Herrera et al., 1996a) and acts by direct computation on labels by taking into account the order of such linguistic assessments in the ordered structure of labels. Usually, the ordinal fuzzy linguistic model for computing with words is defined by establishing (i) a negation operator, (ii) comparison operators based on the ordered structure of linguistic terms, and (iii) adequate aggregation operators of ordinal fuzzy linguistic information. In most ordinal fuzzy linguistic approaches the negation operator is defined from the semantics associated to the linguistic terms as $NEG(s_i) = s_j \mid j = g - i$; and there are defined two comparison operators of linguistic terms:

1. *Maximization operator:* $MAX(s_i,s_j) = s_i$ if $s_i \geqslant s_j$; and
2. *Minimization operator:* $MIN(s_i,s_j) = s_i$ if $s_i \leqslant s_j$.

Using these operators it is possible to define automatic and symbolic aggregation operators of linguistic information, as for example the LOWA operator (Herrera et al., 1996a) and the LWA operator (Herrera & Herrera-Viedma, 1997). Specifically, we will use the LOWA operator.

2.2. GDM problems with incomplete fuzzy linguistic preference relations

In a GDM problem we have a finite set of feasible alternatives $X = \{x_1,x_2,\ldots,x_n\}$, $(n \geqslant 2)$ and the question is to find the best alternatives according to the preferences given by a set of experts, $E = \{e_1,e_2,\ldots,e_m\}$, $(m \geqslant 2)$.

A usual resolution method for a GDM problem is composed of two different processes (Herrera et al., 1996b; Herrera, Herrera-Viedma, & Verdegay, 1997) (see Fig. 1):

1. *Consensus process:* Clearly, in any decision process, it is preferable that the experts reach a high degree of consensus on the solution set of alternatives. Thus, this process refers to how to obtain the maximum degree of consensus or agreement among the experts on the solution alternatives.
2. *Selection process:* This process consists in how to obtain the solution set of alternatives from the opinions on the alternatives given by the experts.

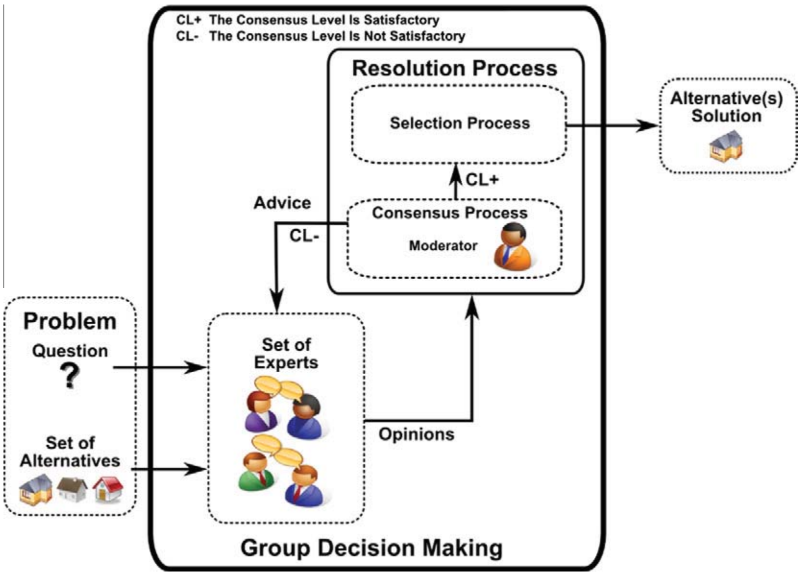


Fig. 1. Resolution process of a GDM.

In a GDM problem the experts can present their opinions using different elements of preference representation (preference orderings, utility functions or preference relations) (Chiclana, Herrera, & Herrera-Viedma, 1998), but in this paper, we assume that the experts give their preferences using incomplete fuzzy linguistic preference relations.

Definition 1. A fuzzy linguistic preference relation (FLPR) P^h given by an expert e_h is a fuzzy set defined on the product set $X \times X$, that is characterized by a linguistic membership function

$$\mu_{P^h} : X \times X \to S$$

where the value $\mu_{P^h}(x_i, x_k) = p^h_{ik}$ is interpreted as the linguistic preference degree of the alternative x_i over x_k for the expert e_h.

Definition 2. A FLPR P^h is complete when $\forall (x_i, x_k) \in (X \times X)$ the expert e_h has provided only one linguistic preference degree $p^h_{ik} \in S$.

Definition 3. A FLPR P^h is incomplete when $\exists (x_i, x_k) \in (X \times X)$ such that the expert e_h has provided no linguistic preference degree $p^h_{ik} \in S$.

2.3. Mobile technologies and GDM

With the fast increase of the new technologies usage (Katz, 2008; Schiller, 2003) and the new services that are offered, the impact in the society of the mobile communication devices is much bigger. Moreover, the growing penetration of mobile devices and the recent technological innovation in the wireless technology field have changed the old wired Internet world to the new wireless mobile Internet world, as known as M-Internet (Cowie & Burstein, 2007; Muntermann, 2008; Varsney & Vetter, 2000).

This recent massive use of wireless technology has strongly modified the organization and management of work and has made critical to gather and estimate a set of decision problem data and to share them among experts in real-time (Pistolesi, 2006, chap. 13).

Nowadays, organizations have moved from face-to-face group environments to virtual group environment using communication technologies and tools to coordinate and share information with other people. The main objective of these new approaches is that the members of the group could work in an ideal way no matter where they are, having all the necessary information to take the most guessed right decisions. Using the mobile technologies, besides increasing the productivity and the satisfaction of the user, allows to save the operational costs of having to bring together to the complete group in the same place at the same time.

To support the new generation of decision makers and to add real-time process in the GDM field, many authors have proposed to develop decision support systems based on mobile technologies (Daume & Robertson, 2000; Eren, Subasi, & Coskun, 2008; Wen, Chen, & Pao, 2008). Similarly, we propose to incorporate mobile technologies in our DSS obtaining a Mobile DSS (MDSS). Using such a technologies should enable users to maximize the advantages and minimize the drawbacks of DSSs.

While DSSs have typically been associated with desktop systems and involve considerable processing, the development of new compact and mobile technologies provides new opportunities to develop this kind of DSSs over M-Internet (Aronson, Liang, & Turban, 2005; Katz, 2008; Schiller, 2003). The incorporation of mobile technologies on the GDM process is based on the supposition that if the communications are improved the decisions will be upgraded (Daume & Robertson, 2000; Imielinski & Badrinath, 1994; Katz, 2008; Schiller, 2003; Wen et al., 2008). If the communications are improved, then the discussion can be focused on the problem and less time is lost with unimportant issues. This saved time can be used to do an exhaustive analysis of the problem and to obtain a better problem definition. This time also could be used to identify more alternatives that can be solutions of the problem.

The language used to compute interfaces and the form of human computer interaction are key obstacles to provide this sort of mobile decision support. The physical limitations and style of use of mobile devices requires a specialized form of decision support system. The design of the decision support models and their customization to particular tasks is more easily done using less restrictive, declarative styles of description (Daume & Robertson, 2000). Mobile computing requires solutions for service discovery in a dynamic environment, establishment of data communication channels between devices and service providers, development tools that allow the integration of devices and distributed resources, and security, such as confidentiality of data and authentication (Navarro, Schulter, Koch, Assuntao, & Westphall, 2006). In Section 4, we describe an architecture, which helps to overcome these obstacles.

The Mobile Web mainly uses lightweight pages written in Extensible Hypertext or Wireless Markup Language (XHTML) or

(WML), to deliver content to mobile devices. However, new tools such as Macromedia's Flash Lite or Sun's J2ME enable the production of richer user interfaces customized for mobile devices. To support these requirements, we develop J2ME applications for mobile devices that implement the new GDM models using the best architecture for each model. These applications can be installed on any mobile device equipped with a Java Virtual Machine (JVM).

3. A new model of linguistic GDM based on mobile technologies and dynamic information

In this section we present a new GDM model that incorporates a mechanism to manage some dynamic information that might change during the decision process, in particular, the alternatives to be analyzed. Furthermore, the model is specifically designed to use mobile devices as the main communication tool and we allow that the experts use incomplete FLPRs to provide their preferences. In such a way, GDM processes could be developed at anytime and anywhere and we can simulate with more accuracy level the real processes of human decision making which are developed in dynamic environments as the Web, financial investment, health, navigation, natural resources management and so on.

This new linguistic, dynamic and mobile GDM model is composed of the following five processes (see Fig. 2):

1. *An estimation process* to complete the incomplete FLPRs.
2. *A selection process* to obtain a temporary solution.
3. *A consensus process* to measure the agreement degree.
4. *A managing process of dynamic information* to deal with the dynamic set of alternatives.
5. *A feedback process* to help experts in the consensus reaching process.

3.1. Estimation process

In Alonso, Cabrerizo, et al. (2009) we develop an additive consistency based estimation process of missing values to deal with incomplete FLPRs defined in a 2-tuple linguistic context. In this paper we adapt it to deal with incomplete FLPRs defined in an ordinal linguistic context.

To deal with incomplete FLPRs we need to define the following sets (Herrera-Viedma, Alonso, et al., 2007):

$$A = \{(i,j) \mid i,j \in \{1,\dots,n\} \wedge i \neq j\}$$
$$MV^h = \left\{(i,j) \in A \middle| p_{ij}^h \text{ is unknown}\right\} \tag{1}$$
$$EV^h = A \setminus MV^h$$

where MV^h is the set of pairs of alternatives whose preference degrees are not given by expert e_h and EV^h is the set of pairs of alternatives whose preference degrees are given by the expert e_h. We do not take into account the preference value of one alternative over itself as this is always assumed to be equal to $s_{g/2}$.

Then, the subset of missing values that could be estimated in step t of the process, called EMV_t^h (*estimated missing values*), is defined as follows:

$$EMV_t^h = \left\{(i,k) \in MV^h \setminus \bigcup_{l=0}^{t-1} EMV_l^h \mid i \neq k \wedge \exists j \in \left\{H_{ik}^{t1} \cup H_{ik}^{t2} \cup H_{ik}^{t3}\right\}\right\} \tag{2}$$

with

$$H_{ik}^{t1} = \left\{j \mid (i,j),(j,k) \in \left\{EV \bigcup_{l=0}^{t-1} EMV_l\right\}\right\} \tag{3}$$

$$H_{ik}^{t2} = \left\{j \mid (j,i),(j,k) \in \left\{EV \bigcup_{l=0}^{t-1} EMV_l\right\}\right\} \tag{4}$$

$$H_{ik}^{t3} = \left\{j \mid (i,j),(k,j) \in \left\{EV \bigcup_{l=0}^{t-1} EMV_l\right\}\right\} \tag{5}$$

and $EMV_0^h = \emptyset$ (by definition). When $EMV_{maxIter}^h = \emptyset$, with $maxIter > 0$, the procedure will stop as there will not be any more missing values to be estimated. Furthermore, if $\bigcup_{l=0}^{maxIter} EMV_l^h = MV^h$, then all missing values are estimated, and, consequently, the procedure is said to be successful in the completion of the incomplete FLPR.

In iteration t, to estimate a particular value p_{ik}^h with $(i,k) \in EMV_t^h$, the following function *estimate_p*(h,i,k) is proposed:

function estimate_p (h,i,k).

(1) $\left(cp_{ik}^h\right)^1 = (s_0), \left(cp_{ik}^h\right)^2 = (s_0), \left(cp_{ik}^h\right)^3 = (s_0), \mathcal{K} = 0$

(2) if $\#(H_{ik}^{t1}) \neq 0$, then $\left(cp_{ik}^h\right)^1 = \frac{\sum_{j \in H_{ik}^{t1}} (cp_{ik}^h)^{j1}}{\#(H_{ik}^{t1})}, \mathcal{K}{+}{+}$

(3) if $\#(H_{ik}^{t2}) \neq 0$, then $\left(cp_{ik}^h\right)^2 = \frac{\sum_{j \in H_{ik}^{t2}} (cp_{ik}^h)^{j2}}{\#(H_{ik}^{t2})}, \mathcal{K}{+}{+}$

(4) if $\#(H_{ik}^{t3}) \neq 0$, then $\left(cp_{ik}^h\right)^3 = \frac{\sum_{j \in H_{ik}^{t3}} (cp_{ik}^h)^{j3}}{\#(H_{ik}^{t3})}, \mathcal{K}{+}{+}$

(5) $cp_{ik}^h = s_q \in S \mid q = round\left(\frac{(cp_{ik}^h)^1 + (cp_{ik}^h)^2 + (cp_{ik}^h)^3}{\mathcal{K}}\right)$

end function

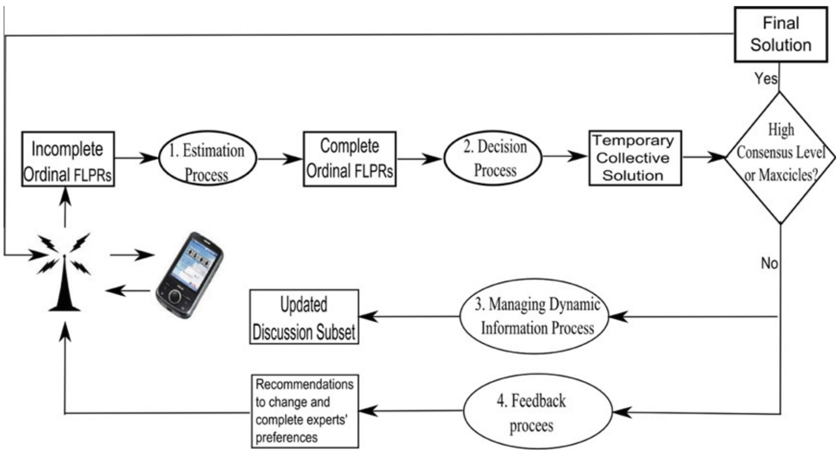


Fig. 2. Structure of the linguistic, dynamic and mobile GDM model.

being

- $(cp_{ik}^h)^{j1} = \begin{cases} g & \text{if } v' \geqslant \frac{3}{2}g \\ 0 & \text{if } v' < 0 \\ v' & \text{othercase} \end{cases}$ with $v' = I(p_{ij}^h) + I(p_{jk}^h) - I(s_{g/2})$.
- $(cp_{ik}^h)^{j2} = \begin{cases} g & \text{if } v' \geqslant \frac{3}{2}g \\ 0 & \text{if } v' < 0 \\ v' & \text{othercase} \end{cases}$ with $v' = I(p_{jk}^h) - I(p_{ji}^h) + I(s_{g/2})$.
- $(cp_{ik}^h)^{j3} = \begin{cases} g & \text{if } v' \geqslant \frac{3}{2}g \\ 0 & \text{if } v' < 0 \\ v' & \text{othercase} \end{cases}$ with $v' = I(p_{ij}^h) - I(p_{kj}^h) + I(s_{g/2})$.
- and $I:S \rightarrow \{0,\ldots,g\} \mid I(s_p) = p \forall s_p \in S$.

Then, the complete iterative estimation procedure is the following:

Iterative estimation procedure.

```
0. EMV_0^h = ∅
1. t = 1
2. while EMV_t^h ≠ ∅ {
3.    for every (i,k) ∈ EMV_t^h {
4.       estimate_p (h,i,k)
5.    }
6.    t++
7. }
```

In Alonso, Cabrerizo, et al. (2009) was demonstrated that an incomplete FLPR can be completed if a set of $n-1$ non-leading diagonal preference values, where each one of the alternatives is compared at least once, is known.

3.2. Selection process

The selection has two different phases (Herrera et al., 1995):

1. Aggregation: This phase defines a collective preference relation, $P^c = \left(p_{ij}^c\right)$, obtained by means of the aggregation of all individual linguistic preference relations $\{P^1, P^2, \ldots, P^m\}$. It indicates the global preference between every pair of alternatives according to the majority of experts' opinions. The aggregation is carried out by means of a LOWA operator ϕ_Q guided by a fuzzy linguistic non-decreasing quantifier Q (Herrera et al., 1996a):

$$p_{ij}^c = \phi_Q(p_{ij}^1, \ldots, p_{ij}^m) \tag{6}$$

2. Exploitation: This phase transforms the global information about the alternatives into a global ranking of them, from which the set of solution alternatives is obtained. The global ranking is obtained applying these two choice degrees of alternatives on the collective preference relation (Herrera & Herrera-Viedma, 2000a):
   (a) $QGDD_i$: This quantifier guided dominance degree quantifies the dominance that one alternative x_i has over all the others in a fuzzy majority sense:

$$QGDD_i = \phi_Q\left(p_{i1}^c, p_{i2}^c, \ldots, p_{i(i-1)}^c, p_{i(i+1)}^c, \ldots, p_{in}^c\right) \tag{7}$$

   This measure allows us to define the set of non-dominated alternatives with maximum linguistic dominance degree:

$$X^{QGDD} = \{x_i \in X \mid QGDD_i = sup_{x_j \in X} QGDD_j\} \tag{8}$$

   (b) $QGNDD_i$: This quantifier guided non-dominance degree gives the degree in which each alternative x_i is not dominated by a fuzzy majority of the remaining alternatives:

$$QGNDD_i = \phi_Q(NEG(p_{1i}^s), NEG(p_{2i}^s), \ldots, NEG(p_{(i-1)i}^s), NEG(p_{(i+1)i}^s), \ldots, NEG(p_{ni}^s)) \tag{9}$$

   where

$$p_{ij}^s = \begin{cases} s_0 & \text{if } p_{ij}^c < p_{ji}^c \\ s_{I(p_{ij}^c) - I(p_{ji}^c)} & \text{if } p_{ij}^c \geqslant p_{ji}^c \end{cases}$$

   represents the degree in which x_i is strictly dominated by x_j. The set of non-dominated alternatives with maximum linguistic non-dominance degree is

$$X^{QGNDD} = \{x_i \in X \mid QGNDD_i = sup_{x_j \in X} QGNDD_j\} \tag{10}$$

   Finally, the solution X_{sol} is obtained as:

$$X_{sol} = X^{QGDD} \cap X^{QGNDD}. \tag{11}$$

3.3. Consensus process

We assume that the consensus as a measurable parameter whose highest value corresponds to unanimity and lowest one to complete disagreement. We use some consensus degrees to measure the current level of consensus in the decision process. They are given at three different levels (Herrera et al., 1996b; Herrera et al., 1997; Mata, Martínez, & Herrera-Viedma, 2009): pairs of alternatives, alternatives and relations. The computation of the consensus degrees is carried out as follows:

1. For each pair of experts, e_i, e_j $(i < j)$, a similarity matrix, $SM^{ij} = (sm_{lk}^{ij})$, is defined where

$$sm_{lk}^{ij} = 1 - \frac{\left|I(p_{lk}^i) - I\left(p_{lk}^j\right)\right|}{g}. \tag{12}$$

2. A consensus matrix, CM, is calculated by aggregating all the similarity matrices using the arithmetic mean as the aggregation function ϕ:

$$cm_{lk} = \phi\left(sm_{lk}^{12}, sm_{lk}^{13}, \ldots, sm_{lk}^{1m}, sm_{lk}^{23}, \ldots, sm_{lk}^{(n-1)n},\right). \tag{13}$$

3. Once the consensus matrix, CM, is computed, we proceed to calculate the consensus degrees:
   (a) **Level 1.** *Consensus degree on pairs of alternatives, cp_{lk}.* It measures the agreement on the pair of alternatives (x_l, x_k) amongst all the experts.

$$cp_{lk} = cm_{lk}. \tag{14}$$

   (b) **Level 2.** *Consensus degree on alternatives, ca_l.* It measures the agreement on an alternative x_l amongst all the experts.

$$ca_l = \frac{\sum_{k=1}^n cp_{lk}}{n}. \tag{15}$$

   (c) **Level 3.** *Consensus degree on the relation, cr.* It measures the global consensus degree amongst the experts' opinions.

$$cr = \frac{\sum_{l=1}^n ca_l}{n}. \tag{16}$$

Initially, in this consensus model we consider that in any non-trivial GDM problem the experts disagree in their opinions so that decision has to be viewed as an iterative process. This means that agreement is obtained only after some rounds of consultation. In each round, we calculate the consensus measures and check the current agreement existing among experts using *cr*.

3.4. Managing process of dynamic information

In the real world we find many dynamic decision frameworks, as health, financial investment, military operations, Web, navigation, natural resources management and so on. In such cases, due to different reasons, some information of the problem could vary through decision process. Thus, a model of decision making should present a flexible and adaptive structure to include those changes that could happen through decision process so that we can constantly revise our decision and the parameters of the problem.

Classical GDM models are defined in a static framework. In order to make the decision making process more realistic, we provide a new tool to deal with dynamic parameters in decision making, as for example the set of alternatives or the group of experts. As aforementioned, in this paper we focus on the changes produced in the set of alternatives because it could depend on dynamical external factors like the traffic (Dia, 2002; Kim et al., 2009), or the meteorological conditions (Clarke, 2008), and so on, and this kind of change is more usual. In such a way, we consider dynamic decision problems in which, at every stage of the process, the discussion is centered on different alternatives.

We define a method which allows us to introduce new alternatives in the discussion process. Firstly, the system identifies those new alternatives to include in the set of discussion alternatives and the worst alternatives to eliminate. And then, the system asks experts their opinion about such changes, i.e., if they agree or not.

To identify the new alternatives we can have two particular cases: (see Figs. 3 and 4)

- This first case happens when a good new alternative appears in the set because some dynamic external factors changed during the decision process, and this new alternative deserves to be in the discussion subset. Before including the new alternative in the discussion subset, the system has to identify the worst alternative of the current discussion subset. To find this bad alternative x_i we compare the dominance and non-dominance degrees $QGDD_i$ and $QGNDD_i$ of all the alternatives, and choose the less evaluated as the worst alternative.
- This second case is when we observe that an alternative x_i always receives low dominance and non-dominance degrees $QGDD_i$ and $QGNDD_i$ due to the changes of the some dynamic external factors during the decision process. Then we could decide to substitute it by another alternative of the initial set of alternatives that was not included in the discussion set of alternatives. This strategy of replacement is commonly used when there is a big set of possible alternatives and they can not be evaluated at the same time. So, we can decide to replace the bad alternatives in the discussion subset in order to evaluate a major number of alternatives. The new alternative to be considered is obtained from the initial list of alternatives that were not included in the discussion subset initially, but now they can be used to replace a bad alternative.

Once the alternatives to be interchanged have been identified, the system gives experts the option to accept or decline the proposed changes. They must provide their degrees of agreement with the proposed changes using a set of linguistic assessments, as for example:

$$\{\textit{Completely Agree},\ \textit{Agree},\ \textit{Nor Agree/Nor Disagree},\ \textit{Disagree},\ \textit{Completely Disagree}\}.$$

At this point, to avoid stagnation a *maxTime* threshold is established.

Then, we aggregate degrees of agreement provided by experts using the LOWA operator (Herrera et al., 1996a). If we obtain a high degree of agreement (more than nor agree/nor disagree) then the system removes the bad alternative from the discussion subset of alternatives and the new one is incorporated into this discussion subset.

3.5. Feedback process

We apply a feedback mechanism to guide the change of the experts' opinions. This mechanism is able to substitute the moderator's actions in the consensus reaching process. It helps experts to change their preferences and to complete their missing values. The main problem for the feedback mechanism is how to find a way of making individual positions converge and, therefore, how to support the experts in obtaining and agreeing with a particular solution (Herrera-Viedma, Herrera, & Chiclana, 2002).

When the consensus measure c_r has not reached the required consensus level (CL) and the number of rounds has not reached a maximum number of iterations (defined prior to the beginning of the decision process) (*MAXCYCLE*), the experts' opinions must be

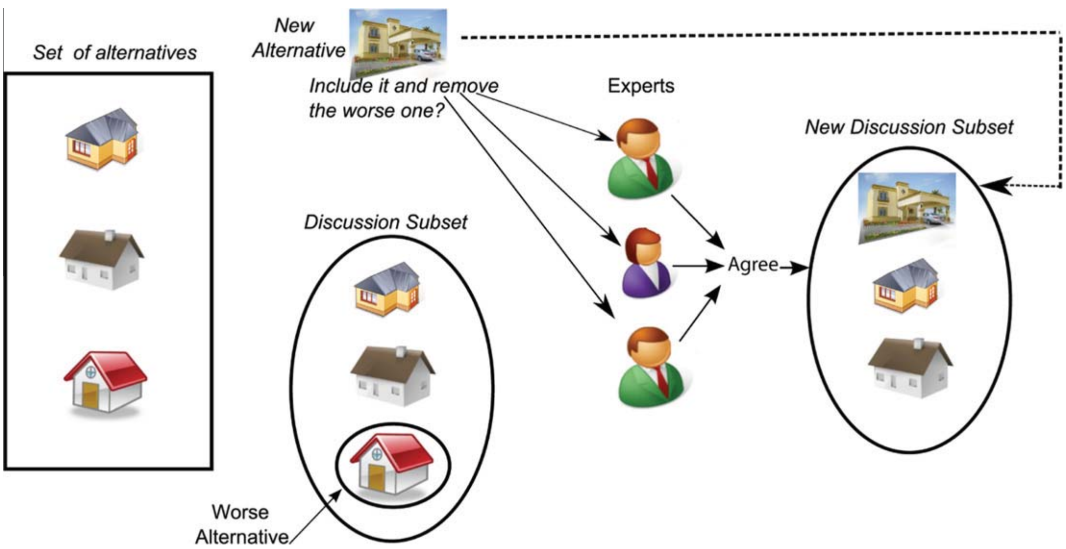


Fig. 3. Dynamic choice process of alternatives: case 1.

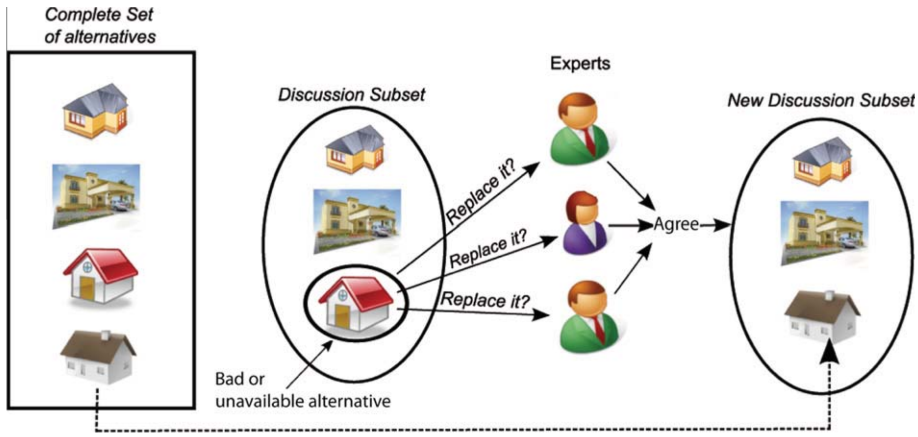


Fig. 4. Dynamic choice process of alternatives: case 2.

modified. To do that, we compute others consensus measures, called proximity measures (Herrera et al., 1996b), which allows us to build a feedback mechanism so that experts change their opinions and narrow their positions (Herrera-Viedma, Alonso, et al., 2007; Mata et al., 2009).

3.5.1. Computation of proximity measures

These measures evaluate the agreement between the individual experts' opinions and the group opinion. To compute them for each expert, we need to use the collective FLPR, $P^c = (p^c_{lk})$, calculated previously.

1. For each expert, e_h, a proximity matrix, $PM^h = (pm^h_{lk})$, is obtained where
$$pm^h_{lk} = 1 - \frac{|I(p^h_{lk}) - I(p^c_{lk})|}{g}. \quad (17)$$
2. Computation of proximity measures at three different levels:
   (a) **Level 1.** *Proximity measure on pairs of alternatives, pp^h_{lk}.* It measures the proximity between the preferences on each pair of alternatives of the expert e_h and the group.
   $$pp^h_{lk} = pm^h_{lk}. \quad (18)$$
   (b) **Level 2.** *Proximity measure on alternatives, pa^h_l.* It measures the proximity between the preferences on each alternative x_l of the expert e_h and the group.
   $$pa^h_l = \frac{\sum_{k=1}^n pp^h_{lk}}{n}. \quad (19)$$
   (c) **Level 3.** *Proximity measure on the relation, pr^h.* It measures the global proximity between the preferences of each expert e_h and the group.
   $$pr^h = \frac{\sum_{l=1}^n pa^h_l}{n}. \quad (20)$$

3.5.2. Production of advice

The production of advice to achieve a solution with the highest possible degree of consensus is carried out in two phases: *Identification phase* and *Recommendation phase*.

1. **Identification phase.** We must identify the experts, alternatives and pairs of alternatives that are contributing less to reach a high degree of consensus.
   (a) *Identification of experts.* We identify the set of experts, *EXPCH*, that should receive advice on how to change some of their preference values:
   $$EXPCH = \{h | pr^h < \gamma\} \quad (21)$$
   where γ is the minimum proximity level required for the expert to be noted to change.
   (b) *Identification of alternatives.* We identify the alternatives whose associated assessments should be taken into account by the above experts in the change process of their preferences;
   $$ALT_h = \{x_l \in X | pa^h_l < \gamma \wedge h \in EXPCH\} \quad (22)$$
   (c) *Identification of pairs of alternatives.* In this step we identify the particular pairs of alternatives (x_l, x_k) whose respective assessments p^h_{lk} the expert e_h should change.
   $$PALT_h = \{(x_l, x_k) | pp^h_{lk} < \gamma \wedge x_l \in ALT_h \wedge h \in EXPCH\} \quad (23)$$
2. **Recommendation phase.** In this phase we recommend expert changes of their preferences according to two kinds of rules:
   (a) **Rules to change the opinions.** We must find out the direction of change to be applied to the preference assessment p^h_{lk}, with $(x_l, x_k) \in PALT_h$. To do this, we define the following two direction rules. It is worth to note that if one of the alternatives of the pair (x_l, x_k) has been replaced in the managing process of dynamic information, that pair has to be removed from the set $PALT_h$, as there is not need to provide rules for alternatives that are being removed from the discussion subset.
   - If $p^h_{lk} > p^c_{lk}$, the expert e_h should decrease the assessment associated to the pair of alternatives (x_l, x_k).
   - If $p^h_{lk} < p^c_{lk}$, the expert e_h should increase the assessment associated to the pair of alternatives (x_l, x_k).

   (b) **Rules to complete missing values.** Additionally, the feedback process must provide rules for missing preferences values. Thus, the lack of information decreases and in this way better solutions can be obtained. To do so, it has to take into account all missing values that were not provided by the experts and were calculated at the estimation process. The advice generated to complete the preferences is the following:
   - "You should provide a value for p^h_{lk} near to cp^h_{lk}".

Furthermore, in order to avoid stagnation and give more dynamism and speed to the consensus process, we do not have to wait for all experts who have received recommendations of change provide new preferences: it is enough that a significant number of experts, different for each problem, send their changed preferences to begin the new round of consensus process after a minimum time threshold has been surpassed.

Obviously, the decision process will depend on the size of the group of experts as well as on the size of the set of alternatives, so that when these sizes are small and when opinions are homogeneous, the required consensus level is easier to obtain. This process has been designed to converge to a final solution with a high consensus degree.

4. Prototype of the new linguistic GDM model

Here we present the implemented prototype of the linguistic GDM model, explaining and the communication and work flow that summarizes the functions of this system.

The most used architecture for mobile devices is the "Client/Server" architecture (see Fig. 5), where the client is a mobile device. The client/server paradigm is founded on the concept that clients (such as personal computers, or mobile devices) and servers (computers) are both connected by a network enabling servers to provide different services for the clients. When a client sends a request to a server, this server processes the request and sends a response back to client.

In addition, we can currently identify two approaches to mobile deployments: thick-client and thin-client.

- *Thin-client* architectures rely entirely on Web technologies to deliver mobile applications. No additional technology investment is required, and there is no risk of client-side software becoming obsolete. There is no need for additional servers, and no unique client-side software or upgrade costs. Its main drawback is that all content cannot be easily delivered to all browsers.
- *Thick-client* deployments run special software on each type of mobile device, fed by special servers that manage the interactions with those devices. The client-side software controls how content is displayed. This was an important factor in the early days of mobile browsers, when each device displayed content differently.

We have chosen a *thick-client* model for our implementation. This allows us to use the software in all the mobile devices without taking into account the kind of browser. Furthermore, the technologies that we have used to implement the prototype comprise Java and Java Midlets for the client software, PHP for the server functions and MySQL for the database management.

According to the model of linguistic GDM model defined in the previous section, the prototype allows user to send his/her preferences by means of a mobile device, and the system returns to the experts the final solution or recommendations to increase the consensus levels, depending of the status of the decision process. An important aspect is that the user-system interaction can be done anytime and anywhere which facilitates expert's participation and the resolution of the decision process.

In what follows, we describe the client and server of the prototype in detail.

4.1. Client

The client software shows the next seven interfaces to the experts:

- *Authentication:* The device asks a user and a password to access the system (see Fig. 6a).
- *Connection:* The device must be connected to the network to send/receive information to the server (see Fig. 6b).
- *Problem description:* When a decision process is started, the device shows to the experts a brief description about the problem and the discussion subset of alternatives (see Fig. 7a).
- *Insertion of preferences:* The device will have a specific interface to insert the linguistic preferences using a set of labels (see Fig. 7b). To introduce or change the preferences on the interface, the user has to use the keys of the device.
- *Swap of Alternatives:* When a new alternative appears in the environment of the problem because some dynamic external factors have changed and this alternative deserves to be a member of the discussion subset or when an alternative have a low dominance degree to the current temporary solution of consensus, the system asks the experts if they want to modify the discussion subset by swapping these alternatives. The experts can assess if they agree to swap the alternatives sending their answer to the question received (Fig. 8a). The user can select the chosen degree by using the cursor key of the device.
- *Feedback:* When opinions should be modified, the device shows experts the recommendations and allows experts to send their new preferences. This system also shows the advice generated to complete the missing values at the last stage (see Fig. 8b).
- *Output:* At the end of the decision process, the device will show the set of solution alternatives as an ordered set of alternatives marking the most relevant ones (see Fig. 8c).

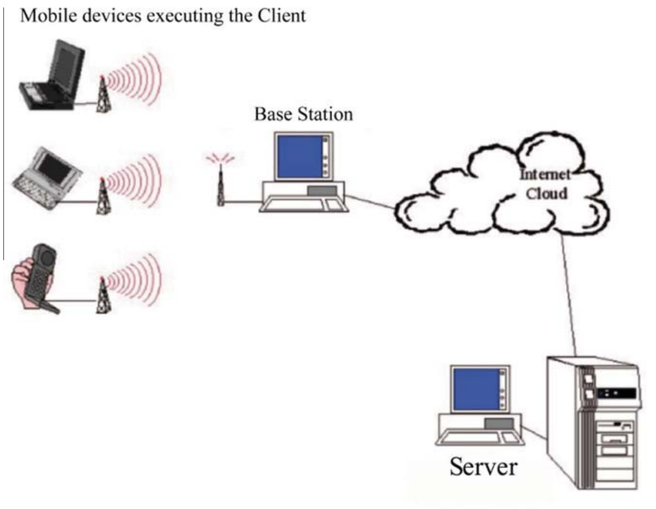


Fig. 5. "Client/Server" architecture.

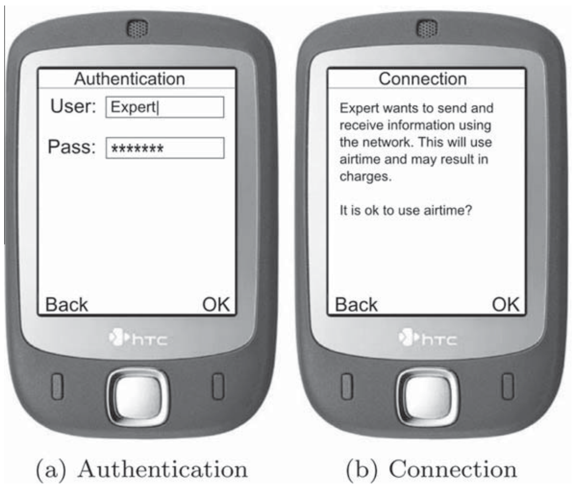
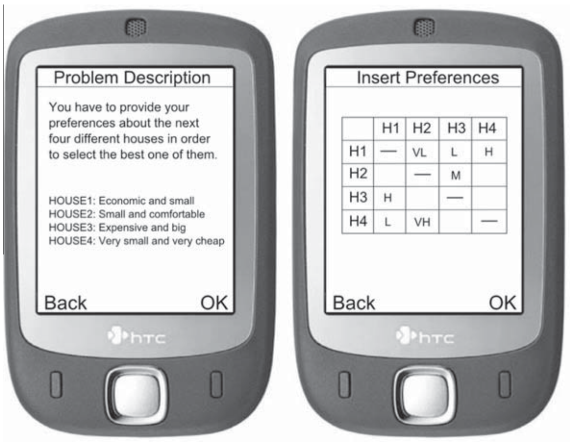


Fig. 6. Connection interfaces.



(a) Problem description (b) Insertion Of preferences

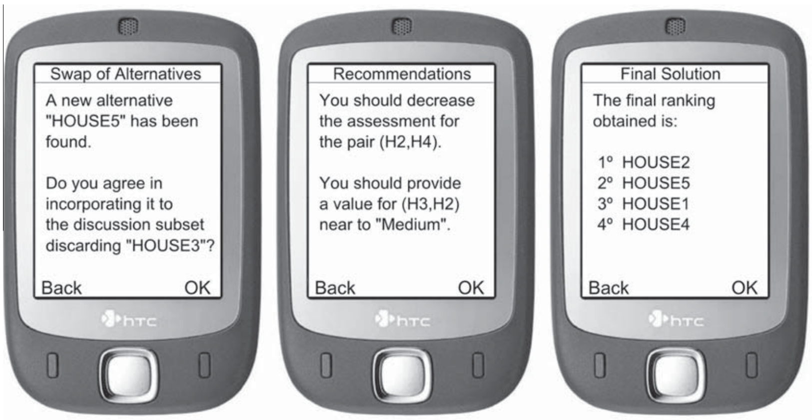
Fig. 7. Input interfaces.

On the technical side of the development of the client part, it is worth to note that the client application complies with the MIDP 2.0 specifications (http://java.sun.com/products/midp/) and that the J2ME Wireless Toolkit 2.2 (http://java.sun.com/products/sjw-toolkit/) provided by SUN was used in the development phase. This wireless toolkit is a set of tools that provide J2ME developers with some emulation environments, documentation, and examples to develop MIDP-compliant applications. The application was later tested using a JAVA-enabled mobile phone on a GSM network using a GPRS-enabled SIM card. The MIDP application is packaged inside a JAVA archive (JAR) file, which contains the applications classes and resource files. This JAR file is the one that actually is downloaded to the physical device (mobile phone) along with the JAVA application descriptor file when an expert wants to use our this prototype.

4.2. Server

The server is the main side of the linguistic GDM prototype. It implements the four main modules and the database that stores the problem data as well as problem parameters and the information generated during the decision process. The communication with the client to receive/send information from/to the experts is supported by mobile Internet (M-Internet) technologies (see Fig. 9). Concretely, the four modules of the server are:

1. *An estimation module:*
   This module completes experts' preferences by using that procedure to estimate missing values presented in Section 3.1.
2. *A decision module:*
   Once the incomplete FLPR are completed, the server starts the decision module to obtain a temporary solution of the problem. In this module the consensus measures are also calculated. If the consensus level has reached the minimum consensus level, the decision process stops and this temporary collective solution is assumed as the final consensus solution. In other case, the decision process should continue.



(a) Swap of alternatives (b) Recommendations (c) Final solution

Fig. 8. Output interfaces.

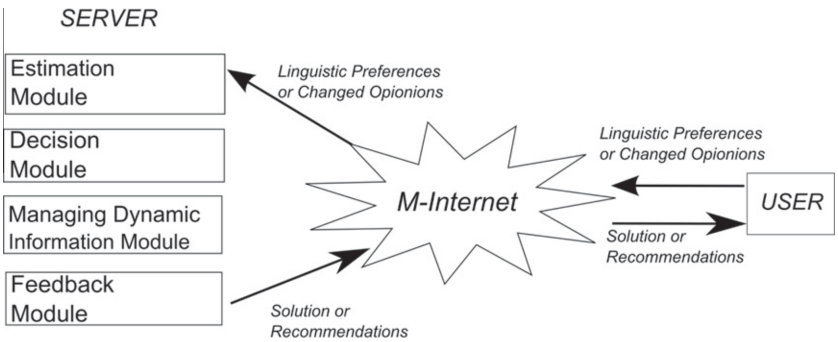


Fig. 9. Server modules.

3. *A managing module of dynamic information:*
   If some external factors change during the decision process, any information of the problem could change too. This module is able to manage the changes in the alternatives set by replacing a bad alternative when it does not deserve to be part of the discussion subset, or including a new alternative dynamically generated during the decision process. To do that, the server asks experts if they agree to change the worst alternatives incorporating the new ones in the process. If the majority of the experts accept the change, the module updates the discussion subset of alternatives.
4. *A feedback module:*
   When a consensus round is finished without reaching the minimum consensus level, the server starts the feedback module, that calculates the proximity measures and generates recommendations rules. These rules help experts to change their preferences about some alternatives to reach consensus. Moreover, some rules are provided to complete the preferences in the cases where missing values exist.

Between client and server some communication functions are developed. In what follows, we present how the modules are connected together with the database, and the order in which each of them is executed.

0. **Initialization:** An initial step is to insert in the database all the initial parameters of the linguistic GDM problem.
1. **Verify user messages and store the main information:** When an expert wants to access to the system, he has to send a message through M-Internet using his/her mobile device. The user can send two kinds of messages:
   (i) *A preferences message:* It is composed by authentication information (login and password) and his/her preferences about the problem, using a set of labels to represent a FLPR.
   (ii) *A change of alternatives message:* It is composed by authentication information (login and password) and his/her linguistic level of agreement with the proposed change of alternatives.

These messages are verified by the server, checking the login and password in the database. If the authentication process is correct, the rest of the information of the message is stored in the database and the server decides if the consensus stage should start (if all experts have provided their preferences) or, if the managing module of dynamic information can be finished (if enough experts provide their agreement degrees on the proposed change of alternatives).

2. **Estimate missing values:** The server estimates the missing values in the FLPR by following the procedure presented in Section 3.1. The server stores this information in the database.
3. **Calculate the set of solution alternatives and the consensus measures:** The decision module returns the solution set of alternatives in each stage of the decision process. All the information about the temporary solution is saved in the database.
4. **Control the consensus state:** In this step, the server determines if the required agreement degree has been reached (and thus, the decision process must be finished) or if we must begin a new round of consensus using the feedback mechanism that generates recommendations to change the experts' preferences.
5. **Management of new alternatives:** When the minimum consensus level has not been reached, the system checks if some new good alternatives appear in the problem environment or an old alternative deserves be removed.
6. **Generate the recommendations:** In this step, the server calculates the proximity measures and generates the recommendations to change and complete the FLPRs. It sends a message to the experts advising that they can use the software again for reading the recommendations and in such a way to start a new consensus stage. In order to avoid that the collective solution does not converge after several discussion rounds, the prototype stops if the number of rounds surpass *MAXCYCLES*. These recommendations are saved in the database and sent to the experts through M-Internet.

In the next section we present a practical example on the use of the prototype to provide more detail about its operation.

5. Example

Medical diagnosis is an example of decision process that can beneficiate by the use of our system. This scenario presents all the characteristics to be a GDM problem. There is a patient who presents some symptoms, but all of them are common to several diseases. These diseases shape the set of alternatives of the problem. In addition, there are some doctors considered specialists in differential diagnosis. They conform the set of experts of the problem and they have to jointly diagnose which is the disease that the patient has contracted. The experts live in different cities of the world and they can not have a meeting to discuss and reach the consensual solution. Moreover, this environment is dynamic in the sense that the patient is now in a hospital and, at any moment, he could present new symptoms or he could set better due to the medication, and thus, any change of state of the patient might be taken into account by the doctors. So, the experts might decide to use our system because they can use the mobile communication technologies to reach the consensus, and they can change some possible diseases in the discussion set of alternatives according with the current patient's state.

The first step to solve a problem using our prototype is to insert all the initial parameters of the problem (experts, alternatives, thresholds, timing...) in the database. We assume a set of three experts (doctors), $\{e_1,e_2,e_3\}$, and a set of four alternatives (possible diseases) $\{x_1,x_2,x_3,x_4\}$. The remaining parameters (see Table 1) are used by the system to obtain the necessary consensus degree among the experts.

When the initial parameters of the problem are defined, the decision making process starts.

Table 1
Initial parameters of the problem.

Name	Value	Description
Ndiseases	4	Number of diseases in the discussion subset
Nexperts	3	Number of experts (doctors)
minConsDegree	0.75	Minimum consensus level required by the problem
minProxDegree	0.75	Minimum proximity level required for the experts to be noted to change
MAXCYCLES	4	Maximum number of iterations of the consensus process
maxTime	12 h	Maximum time of waiting for the experts opinions to change
minQGDD	L	Minimum dominance level that an alternative has to reach to avoid to be changed
DSsize	4	Discussion subset size

Table 2
Choice degrees.

	x_1	x_2	x_3	x_4
$QGDD_i$	M	VH	H	H
$QGNDD_i$	VH	P	P	P

5.1. First round

The three experts send their FLPRs using their mobile devices as show the Fig. 10 with the next set of seven labels:

$S = \{s_0 = N, s_1 = VL, s_2 = L, s_3 = M, s_4 = H, s_5 = VH, s_6 = P\}$.

where N = *Null*, VL = *Very Low*, L = *Low*, M = *Medium*, H = *Height*, VH = *Very Height* and P = *Perfect*.

5.1.1. Estimation process

Two of the experts give incomplete FLPRs $\{P^1, P^3\}$. Then, the estimation process completes the missing values in the following steps:

Step 1: The set of elements that can be estimated are:

$EMV_1^1 = \{(1,4),(2,3),(3,2),(4,1)\}$.

With these estimated preference degrees we have:

$$P^1 = \begin{pmatrix} - & x & \mathbf{L} & L \\ x & - & VL & \mathbf{VL} \\ \mathbf{VH} & H & - & \mathbf{M} \\ VH & \mathbf{H} & \mathbf{M} & - \end{pmatrix}.$$

As an example, to estimate p_{14}^1 the procedure is as follows:

$$H_{14}^{11} = \{3\} \Rightarrow (cp_{14}^1)^1 = I(p_{13}^1) + I(p_{34}^1) - I(s_{g/2}) = 2 + 3 - 3 = 2.$$
$$H_{14}^{12} = \{3\} \Rightarrow (cp_{14}^1)^2 = I(p_{34}^1) - I(p_{31}^1) + I(s_{g/2}) = 3 - 5 + 3 = 1.$$
$$H_{14}^{13} = \{3\} \Rightarrow (cp_{14}^1)^3 = I(p_{13}^1) - I(p_{43}^1) + I(s_{g/2}) = 2 - 3 + 3 = 2.$$

$$\mathcal{K} = 3 \Rightarrow cp_{14}^1 = L \text{ given that round } \left(\frac{(cp_{14}^1)^1 + (cp_{14}^1)^2 + (cp_{14}^1)^3}{3} \right) = 1,67$$

Step 2: The set of elements that can be estimated are:

$EMV_2^1 = \{(1,2),(2,1)\}$.

After these elements have been estimated, we have the following complete FLPR:

$$\overline{P}^1 = \begin{pmatrix} - & M & \mathbf{L} & L \\ M & - & VL & \mathbf{VL} \\ \mathbf{VH} & H & - & \mathbf{M} \\ VH & \mathbf{H} & \mathbf{M} & - \end{pmatrix}.$$

For P^3 we get the following complete FLPR:

$$\overline{P}^3 = \begin{pmatrix} - & \mathbf{VL} & H & M \\ \mathbf{H} & - & \mathbf{VH} & \mathbf{H} \\ \mathbf{L} & N & - & L \\ \mathbf{L} & \mathbf{VL} & \mathbf{H} & - \end{pmatrix}.$$

5.1.2. Selection process

In this phase the system obtains the collective temporary solution by aggregating the experts' preferences.

1. Aggregation: Once the incomplete FLPRs are completed, we aggregate them by means of the LOWA operator. We use the linguistic quantifier *most of* defined as $Q(r) = r^{1/2}$. Then, we obtain the following collective FLPR:

$$P^c = \begin{pmatrix} - & L & H & M \\ VH & - & H & VH \\ H & M & - & M \\ M & M & H & - \end{pmatrix}$$

2. Exploitation: Using again the same linguistic quantifier "most of", we obtain $QGDD_i$ and $QGNDD_i$ $\forall x_i \in X$ (see Table 2) and, the maximal sets are:

$$X^{QGDD} = \{x_2\} \quad \text{and } X^{QGNDD} = \{x_2\}.$$

5.1.3. Consensus process

In this phase the system calculates the consensus measures.

1. *Similarity matrices:*

$$SM_{12} = \begin{pmatrix} - & 0.83 & 0.66 & 0.66 \\ 0.50 & - & 0.50 & 0.16 \\ 0.50 & 0.66 & - & 0.83 \\ 0.16 & 0.66 & 0.66 & - \end{pmatrix}$$

$$SM_{13} = \begin{pmatrix} - & 0.66 & 0.66 & 0.83 \\ 0.83 & - & 0.33 & 0.50 \\ 0.50 & 0.33 & - & 0.83 \\ 0.50 & 0.50 & 0.83 & - \end{pmatrix}$$

(a) Expert 1 (b) Expert 2 (c) Expert 3

Fig. 10. Expert preferences.

$$SM_{23} = \begin{pmatrix} - & 0.83 & 1.00 & 0.83 \\ 0.66 & - & 0.83 & 0.66 \\ 1.00 & 0.66 & - & 1.00 \\ 0.66 & 0.83 & 0.83 & - \end{pmatrix}$$

2. *Consensus matrix:*

$$CM = \begin{pmatrix} - & 0.77 & 0.77 & 0.77 \\ 0.66 & - & 0.55 & 0.44 \\ 0.66 & 0.55 & - & 0.88 \\ 0.44 & 0.66 & 0.77 & - \end{pmatrix}$$

3. *Consensus degrees on pairs of alternatives.* The element (l, k) of CM represents the consensus degrees on the pair of alternatives (x_l, x_k).
4. *Consensus on alternatives:*

$$ca^1 = 0.77 \; ca^2 = 0.55 \; ca^3 = 0.69 \; ca^4 = 0.62$$

5. *Consensus on the relation:*

$$cr = 0.66$$

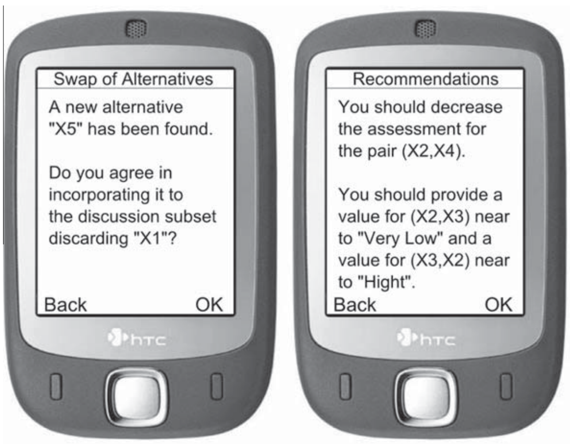
As $cr < minConsDegree = 0.75$ is satisfied, then it is concluded that there is no consensus amongst the experts, and consequently, the system should to continue by executing the next two processes: managing process of dynamic information to replace some alternatives in the discussion subset and feedback process to support the experts' changes in their preferences in order to increase cr.

5.1.4. Managing process of dynamic information

As soon as the system has verified that the minimum consensus level among the experts has not been reached and before beginning a new round of consensus, it is necessary to update all the information of the problem that there could be changed during the process.

In this case, the patient, due to the medication, has started to show a new symptom that is typical of a disease that was not included in the initial discussion subset of the problem. This new situation does not suppose any problem because the system manages the dynamic information. We identify those alternatives with low choice degrees and ask the experts if they agree to replace those identified alternatives by others new ones that has appeared (See Fig. 11a).

The experts' answers were the following: (*Agree, Nor Agree/Nor Disagree and Completely Agree*). The system applies the LOWA operator to aggregate these opinions and obtain a collective agreement degree. In this case we obtain, (Agree), what represents an affirmative position to introduce the changes of alternatives. Therefore, the change of x_1 by x_5 is done. The experts will be informed about it and then they are urged to refill their preferences about the new alternative.



(a) Swap of alternatives (b) Recommendations

Fig. 11. Swapping and recommendation.

5.1.5. Feedback process

- **Computation of proximity measures:**
  1. *Proximity matrices:*

$$PM_1 = \begin{pmatrix} - & 0.83 & 0.66 & 0.83 \\ 0.66 & - & 0.50 & 0.33 \\ 0.83 & 0.83 & - & 1.00 \\ 0.66 & 0.83 & 0.83 & - \end{pmatrix}$$

$$PM_2 = \begin{pmatrix} - & 1.00 & 1.00 & 0.83 \\ 0.83 & - & 1.00 & 0.83 \\ 0.66 & 0.83 & - & 0.83 \\ 0.50 & 0.83 & 0.83 & - \end{pmatrix}$$

$$PM_3 = \begin{pmatrix} - & 0.83 & 1.00 & 1.00 \\ 0.83 & - & 0.83 & 0.83 \\ 0.66 & 0.50 & - & 0.83 \\ 0.83 & 0.66 & 1.00 & - \end{pmatrix}$$

  2. *Proximity on pairs of alternatives*: $PP_i = PM_i$.
  3. *Proximity on alternatives (See Table 3)*:
  4. *Proximity on the relation*:

$$pr_1 = 0.73 \; pr_2 = 0.83 \; pr_3 = 0.81$$

- **Production of advice:**
  1. *Identification phase:*
     (a) Identification of experts:

$$EXPCH = \{e_i | pr_i < minProxDegree\} = \{e_1\}$$

     (b) Identification of alternatives:

$$ALT_1 = \{x_l \in X | pa_l^i < minProxDegree \wedge e_i \in EXPCH\} = \{x_2\}$$

     (c) Identification of pairs of alternatives to generate recommendations:

$$PALT_1 = \{(x_2, x_1), (x_2, x_3), (x_2, x_4)\}$$

  2. *Recommendation phase:*
     In this phase, we have to take into account that alternative x_1 has been replaced in the previous process by x_5. So, x_1 does not need rules to be modified and there is a new alternative in the discussion subset, x_5, that needs new preference values. The recommendations interface is shown in Fig. 11b.
     (a) Rules to change the opinions:
       – Because x_1 has been replaced, p_{21}^1 does not need be modified.
       – Because p_{23}^1 is an estimated element, the expert cannot modify his preference.
       – Because $p_{24}^1 < p_{24}^c$, expert e_1 is advised to increase the assessment of this preference value.

Table 3
Proximity measures on alternatives.

x_1	x_2	x_3	x_4
$pa_1^1 = 0.77$	$pa_1^2 = 0.50$	$pa_1^3 = 0.88$	$pa_1^4 = 0.77$
$pa_2^1 = 0.94$	$pa_2^2 = 0.88$	$pa_2^3 = 0.77$	$pa_2^4 = 0.72$
$pa_3^1 = 0.94$	$pa_3^2 = 0.83$	$pa_3^3 = 0.66$	$pa_3^4 = 0.83$

(a) Expert 1 (b) Expert 2 (c) Expert 3

Fig. 12. Expert preferences.

(b) Rules to complete the missing values:
- Expert e_1 is advised to provide a value for p^1_{23} near to *VL*
- Expert e_1 is advised to provide a value for p^1_{32} near to *H*
- Expert e_3 is advised to provide a value for p^3_{32} near to *N*

5.2. Second round

The experts send their preferences about the new discussion subset to start the second round (see Fig. 12).

As the experts have inserted complete preference relations, the estimation missing values process is avoided and the system continues the decision process.

5.2.1. Selection process

1. Aggregation: The collective FLPR is:

$$P^c = \begin{pmatrix} - & VH & H & H \\ L & - & H & VH \\ M & M & - & M \\ M & M & H & - \end{pmatrix}$$

2. Exploitation: Using again the same linguistic quantifier "most of", we obtain the following choice (see Table 4). Clearly, the maximal sets are:

$$X^{QGDD} = \{x_5\} \quad \text{and} \quad X^{QGNDD} = \{x_5\}.$$

5.2.2. Consensus process

Consensus on the relation:

$cr = 0.79$

Because $cr > minConsDegree$, then it is concluded that there is the required consensus amongst the experts, and consequently, the current solution is the final solution, that is stored and sent to the experts (see Fig. 13).

Table 4
Choice degrees in 2nd round.

	x_5	x_2	x_3	x_4
$QGDD_i$	VH	H	M	H
$QGNDD_i$	P	VH	VH	VH

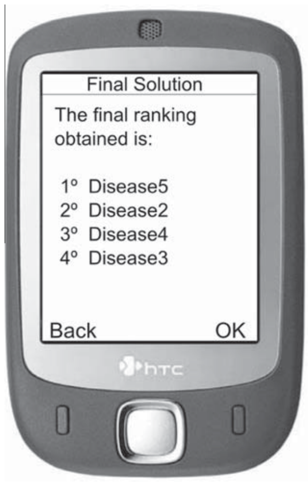


Fig. 13. Final solution.

6. Conclusions

We have presented a new model of linguistic GDM based on dynamic information and mobile technologies. We have also implemented a prototype of this system. It is designed to deal with linguistic GDM problems based on dynamic sets of alternatives, which uses the advantages of mobile Internet technologies to improve the user-system interaction through decision process. The experts can use FLPR to express their preferences and it provides a tool that manages lack of information when an expert is not able to give a complete FLPR. We have used mobile phones as the device used by the experts to send their FLPRs but the structure of the prototype is designed to use any other mobile device as PDAs.

Shortly, with this new GDM model we shall be able to model linguistic GDM problems in which experts could interact in anywhere and anytime, quickly, in a flexible way, and dynamic frameworks.

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3. A New Consensus Model for Group Decision Making Problems with Non Homogeneous Experts

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A New Consensus Model for Group Decision Making Problems with Non Homogeneous Experts

I.J. Pérez, F.J. Cabrerizo, S. Alonso and E. Herrera-Viedma

Abstract

One of the newest trends in group decision making problems management is the change from homogeneous to non homogeneous frameworks. In fact, different approaches for heterogeneity modelling in group decision making problems have been proposed in the literature. Some instances of these approaches are focused on heterogeneous domains and structures to represent preferences. On the other hand, there are some proposals to model the heterogeneity among experts. However, all of them deal with a selection process in which weight values are assigned to the experts and where the global preference is obtained by means of a weighted aggregation of the individual preferences. The aim of this paper is to improve the consensus reaching process among non homogeneous experts. In such a way, we propose a new importance based feedback mechanism specifically designed to undertake group decision making situations in which the experts have different importance or confidence levels. This new approach increases the convergence toward the consensual solution computing and sending just the necessary amount of advice required by each expert depending on his own knowledge of the problem.

Index Terms

Group decision making, consensus process, feedback mechanism, heterogeneous decision frameworks.

I. Introduction

Group decision making (GDM) consists of multiple individuals interacting to reach a decision. Each decision maker (expert) may have unique motivations or goals and may approach the decision process from a different angle, but have a common interest in reaching eventual agreement on selecting the "best" option(s) [1], [2]. To do this, experts have to express their preferences by means of a set of evaluations over a set of alternatives. Several

I.J. Pérez and E. Herrera-Viedma are with the Dept. of Computer Science and Artificial Intelligence, University of Granada, e-mail: {ijperez},{viedma}@decsai.ugr.es.

F.J. Cabrerizo is with the Dept. of Software Engineering and Computer Systems, Distance Learning University of Spain (UNED), e-mail: cabrerizo@issi.uned.es

S. Alonso is with the Software Engineering Dept., University of Granada, e-mail: zerjioi@ugr.es

authors have provided interesting results on GDM with the help of fuzzy theory, which can be further studied in the following references [3], [4], [5], [6], [7], [8], [9], [10].

Usually, two processes are necessary to solve GDM problems: a consensus process and a selection process. The consensus process is used to reach a final solution with a certain level of agreement among the experts. On the other hand, the selection process uses all individual preferences in order to obtain a collective solution. Clearly, it is preferable that the set of experts reach a high degree of consensus before applying the selection process. In order to measure the degree of consensus, different approaches have been proposed [11], [12], [13], [14]. In addition, to achieve a good consensus level among the experts, it is useful to provide the whole group of experts with some advice (feedback information) on how far the group is from consensus, what are the most controversial issues (alternatives), what preferences are in the highest disagreement with the rest of the group, how their change would influence the consensus degree and so on.

Initially, GDM situations were defined in homogeneous decision contexts but, in recent years, due to the wide range of different heterogeneous problems that could be solved with this kind of approaches, new GDM models have been proposed and improved in order to deal with non-homogeneous frameworks. In particular, we can find some heterogeneous GDM models at three different levels in the literature:

1) *Heterogeneity at the preference representation structure level* (orders, utility functions or preference relations...) [15], [16]. Particularly, preference relations [17], [15], [16], [1], [18], [9], [19] have been widely used because they are a very expressive format and also they present good properties that allow to operate with them easily.
2) *Heterogeneity at the preference representation domain level* (numeric, linguistic, multi-granular, interval numbers...) [3], [5], [20], [12], [21], [22], [23]
3) *Heterogeneity at the importance degree of experts level* [24], [25]. To model such situations, the most usual approach in the literature deals with the assignation of weight values to the experts in order to compute a weighted aggregation of their preferences [26], [27], [14], [25], [28], [10]. This approach focus the discussion on a weighted collective preference and, in such a way, the most weighted experts are the main leaders of the discussion. They have to be at front of the negotiation to persuade the remaining experts in order to reach agreement.

However, we can find situations situations in which a group experts with lower importance but whose weights' addition would make them important as a group, where this kind of mechanism could lead to a opposite effect to the desired. In fact, experts with higher importance are suppossed to have deeper knowledge about the problem and thus, they usually require less recommendations about the problem to be solved. This implies that the recommendation mechanism has to be adapted in order to provide a larger amount of recommendations to the lower weighted experts. In fact, some related methods that deal with adaptive consensus models in which the recommendation approach is adapted on the current consensus level have been proposed and used with good results [21].

In this paper we propose a new consensus approach to overcome this issue. We suggest to take into account the importance weights not only to aggregate the experts' preferences but also when advising experts to change their preferences. To do so, we propose an importance based feedback mechanism that adjusts the amount of advice required by each expert depending on his own weight value. It seems reasonable that the experts with lower importance or knowledge level will need more advice than those experts that previously have at their disposal a large amount of information to make good decisions. Therefore, this new approach computes the recommendations in a different way depending on the experts' importance level.

In order to do this, the paper is set out as follows. Some general considerations about GDM and consensus reaching process are presented in Section II. Section III presents the new importance-based consensus reaching process. A case of use is shown in Section IV. Finally, Section V draws our conclusions.

II. Preliminaries

In this section we show some preliminaries about GDM and existing consensus models.

A. Group Decision Making

A decision making process, consisting in deriving the best option from a feasible set, is present in just about every conceivable human task. It is obvious that the comparison of different actions according to their desirability in decision problems, in many cases, cannot be done by using a single criterion or an unique person. Thus, we interpret the decision process in the framework of GDM.

In a classical GDM situation there is a problem to solve, a solution set of possible alternatives, $X = \{x_1, x_2, \ldots, x_n\},\ (n \geq 2)$ and a group of two or more experts, $E = \{e_1, e_2, \ldots, e_m\},\ (m \geq 2)$ characterized by their own ideas, attitudes, motivations and knowledge, who express their opinions about this set of alternatives to achieve a common solution [29], [30], [31].

Usual resolution methods for GDM problems are composed by two different processes [1] (see Figure 1):

1) *Consensus process:* Clearly, in any decision process, it is preferable that the experts reach a high degree of consensus on the solution set of alternatives. Thus, this process refers to how to obtain the maximum degree of agreement among the experts on the solution alternatives.
2) *Selection process:* This process consists in how to obtain the solution set of alternatives from the opinions on the alternatives given by the experts.

Recently, several authors have study and approach GDM problems from different angles, showing that this kind of problems are not always homogeneous. We can classify them into three different heterogeneity levels. based

- The first heterogeneity level studied in the literature focus in the use of different preference representation structures [17], [15], [16], [22], [19] (orders, utility functions, preference relations and so on). In many existing decision models the experts provide his/her preferences on the alternatives by means of preference relations in which every alternative is compared against each other.
- The second heterogeneity level is focused on the preference representation domain (numeric, linguistic, multi-granular, unbalanced information, interval numbers, etc.) [3], [5], [20], [12], [21], [23].
- Finally, the third heterogeneity level deals with the differences of the experts in the process: different experts have different knowledge about the problem and have different perceptions and opinions about them. Some

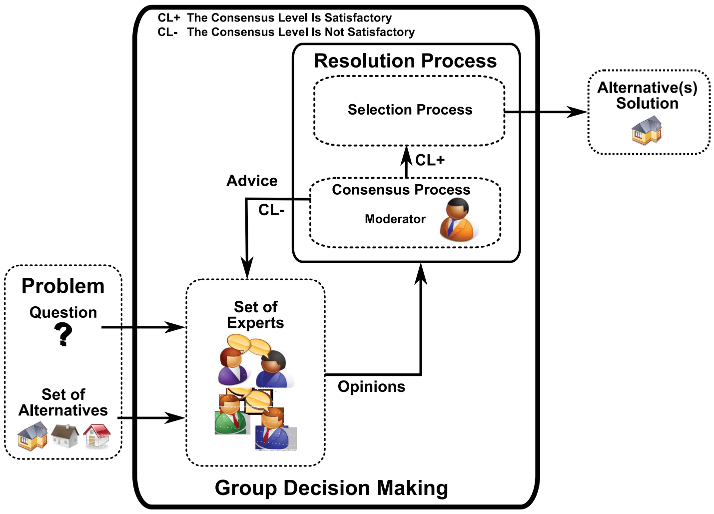

Fig. 1: *Resolution process of a GDM*

clasical models tackle this heterogeneity by assigning a weight value to each expert that is used in the aggregation phases in order to model their different importance levels or knowledge degrees. In fact, the preferred method for several authors to compute these weight values is to use them like an aggregation operator parameter on the selection process [24], [26], [27], [14], [25], [28], [10].

In this paper we will focus on the third heterogeneity level. In fact, we introduce a new scheme for a recommendation mechanism in which the recommedations amount is different according to the importance of the experts. To do so, we assume that the experts provide their preferences using fuzzy preference relations (FPR). Thus, an expert e_k provides his preferences about the alternatives X using a FPR P^k characterized by a membership function [11]:

$$\mu_P : X \times X \longrightarrow [0, 1]$$

where the value $\mu_{P^k}(x_i, x_j) = p^k_{ij}$ is interpreted as the preference degree of the alternative x_i over x_j by the expert e_k [32].

- $p^k_{ij} > 0.5$ indicates that x_i is preferred to x_j by the expert e_k.
- $p^k_{ij} < 0.5$ indicates that x_j is preferred to x_i by the expert e_k.
- $p^k_{ij} = 0.5$ indicates indifference between x_i and x_j by the expert e_k.

We have chosen this preference structure because its good properties: they are more informative than preference orderings or utility functions [15] as they allow the comparison of the alternatives in a pair by pair basis. Thus, users have much more freedom at giving their preferences and they can gain expressivity against other preference representations. Moreover, the use of fuzzy logic in this kind of contexts is a good choice as it allows to express the preferences in an easy and precise way.

B. Classical Consensus Reaching Process

A consensus reaching process in a GDM problem is an iterative process composed by several discussion rounds in which experts are expected to modify their preferences according to the advice given by the moderator. The moderator plays a key role in this process. Normally, the moderator is a person who does not participate in the discussion but knows the preferences of each expert and the level of agreement during the consensus process. He is in charge of supervising and driving the consensus process toward success, i.e., to achieve the maximum possible agreement and reduce the number of experts outside of the consensus in each round.

Usually, the moderator carries out three main tasks: (i) to compute the consensus measures, (ii) to check the level of agreement and (iii) to produce some advice for those experts that should change their minds. (See Figure 2)

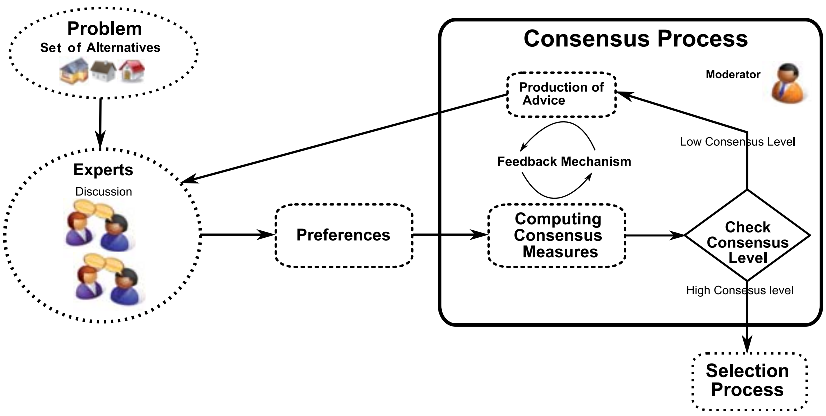

Fig. 2: Classical consensus reaching process

In order to evaluate the agreement some similarity measures among the experts [1], [11], [12], [13], [21] are usually computed. Two types of measurements to guide the consensus reaching process were proposed in [33]:

1) *Consensus measures* to evaluate the level of agreement among all the experts. They are used to identify the preference values where the agreement is not sufficient.
2) *Proximity measures* to evaluate the distance among the experts' individual preferences and the group or collective one. They are used to identify the experts who should change their preferences in the following rounds.

These measurements are computed at the three different levels of representation of a preference relation: pairs of alternatives, alternatives, and relation [33].

III. A CONSENSUS REACHING PROCESS IN GDM WITH NON HOMOGENEOUS EXPERTS

In heterogeneous GDM scenarios that include several non homogeneous experts with different levels and kind of knowledge, it is necessary to take into account the importance degree of each expert in order to reach the global consensus degree in a more appropriate and realistic way. Usually, these situations have been modeled with the inclusion of some weight values in the computation of the global preferences (selection process) [26], [27], [14], [25], [28], [10]. In such a way, a weighted aggregation of the individual preferences is computed to model the influence of the importance levels on the final decision. However, those approaches do not take into account the heterogeneity of the experts in the consensus process: when the agreement of the experts is low, it seems reasonable to send more advice information to those experts with less importance or knowledge level [25], [21]. In order to bring the preferences closer to each other, in the following we propose a new importance-based feedback mechanism that replaces and automates the moderator's tasks (computing and sending different recommendations to the experts) according to their own importance degrees. In such a way, we use the experts' importance on the discussion phase (consensus process) to generate importance-based recommendations.

Consequently, we present an importance-based consensus reaching process in order to compute more suitable advice composed of two different stages (see Figure 3).

1) *Computing Consensus Measures and Consensus Control Process.*
2) *Importance-Based Feedback Mechanism.*

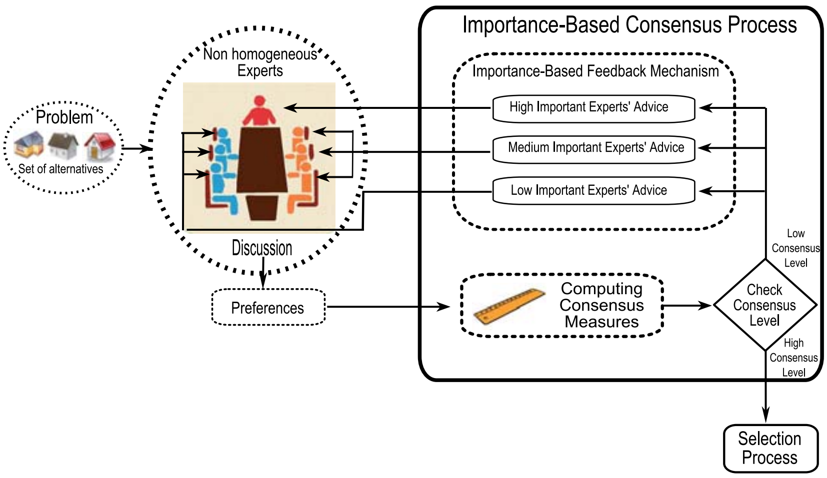

Fig. 3: Consensus reaching process with non homogeneous experts

A. *Computing Consensus Measures and Consensus Control Process*

Once the preferences have been given by the experts, we can compute the level of agreement achieved in the current round. To do so, we firstly define for each pair of experts (e^k, e^l) $(k < l)$ a similarity matrix $SM^{kl} = \left(sm_{ij}^{kl}\right)$ where

$$sm_{ij}^{kl} = \left(1 - \left|p_{ij}^k - p_{ij}^l\right|\right)$$

Then, a consensus matrix, CM, is calculated by aggregating all the similarity matrices using the arithmetic mean as the aggregation function ϕ:

$$cm_{ij} = \phi(sm_{ij}^{12}, sm_{ij}^{13}, \ldots, sm_{ij}^{1m}, sm_{ij}^{23}, \ldots, sm_{ij}^{(m-1)m}).$$

Once the similarity and consensus matrices are computed we proceed to obtain the consensus degrees at the three different levels to obtain a global consensus degree, called consensus on the relation:

1) *Consensus degree on pairs of alternatives*. The consensus degree on a pair of alternatives (x_i, x_j), denoted cp_{ij}, is defined to measure the consensus degree amongst all the experts on that pair of alternatives:

$$cp_{ij} = cm_{ij}$$

2) *Consensus degree on alternatives*. The consensus degree on alternative x_i, denoted ca_i, is defined to measure the consensus degree amongst all the experts on that alternative:

$$ca_i = \frac{\sum_{j=1;j\neq i}^{n}(cp_{ij} + cp_{ji})}{2(n-1)}$$

3) *Consensus degree on the relation*. The consensus degree on the relation, denoted CR, is defined to measure the global consensus degree amongst all the experts' opinions:

$$CR = \frac{\sum_{i=1}^{n} ca_i}{n}$$

When the consensus measure CR has not reached the minimum required consensus level CL and the number of rounds has not reached a maximum number of iterations (defined prior to the beginning of the decision process), the experts' opinions that are hindering the agreement must be modified. The value of CL will obviously depend on the particular problem we are dealing with. When the consequences of the decision to be made are of utmost importance, the minimum level of consensus required to make that decision should be logically as high as possible. At the other extreme, when the decision's consequences are not really serious (but are still important), and it is urgent to obtain a solution of the problem, a lower CL implies an small number of consensus rounds to reach the agreement, and consequently, quicker decisions.

The consensus indicators make it possible to point out the most controversial alternatives and/or experts isolated in their opinions. Thus, in the following we propose a new importance-based search for preferences to obtain customized recommendations that can narrow the experts' minds.

B. Importance-Based Feedback Mechanism

If the agreement of the experts is low, then there exist some experts' preferences in disagreement. In such a case, in order to bring the preferences closer to each other, we have to identify the preferences and experts that are hindering the agreement and send them some advice trying to change their mind. This phase is known as feedback process.

The main problem for the feedback process is how to find a way of making individual positions converge and, therefore, how to support the experts in obtaining and agreeing with a particular solution [34], [35], [6], [11].

In this section, we propose a new feedback mechanism to guide the change of the controversial experts' opinions. This mechanism is based on the supposition that those experts with lower knowledge level on the problem will need more advice than others with higher importance. In summary, we try to adapt the search for preferences in disagreement to the different kinds of experts. When we deal with important experts, it is obvious that their opinions belong to a wider knowledge than the remaining ones. In such a case, only a few number of changes of opinions might lead to consensus. Similarly, when the experts have lower importance, a high number of changes of opinions might be necessary to achieve good consensual solutions.

This new importance-based production of advice is developed with the aim of modeling those group decision making situations in which the experts' knowledge is quite different among each others.

As it has been previously said, in heterogeneous contexts where experts do have different importance levels, weight values are usually assigned to them. Those weight values can be modeled as a fuzzy subset I where the membership function $\mu_I(e_k) \in [0,1]$ denotes a degree of importance of the expert e_k.

In this paper, we use the heterogeneity of the experts in a new way. In fact, we propose to compute a customized amount of advice which varies in accordance with the experts' weight values. To do so, we define three different advising strategies to identify the preferences that each expert should modify, in order to increase the consensus level in the next consensus round, i)*advising high-important experts*, ii)*advising medium-important experts* and iii)*advising low-important experts*.

Firstly, the experts are included by their own importance degree into three different subsets E_{high}, E_{med} and E_{low} in the following way:

- if $\mu_I(e_k) < \lambda_1 \rightarrow e_k \in E_{low}$.
- if $\lambda_1 \leq \mu_I(e_k) < \lambda_2 \rightarrow e_k \in E_{med}$, and
- if $\mu_I(e_k) \leq \lambda_2 \rightarrow e_k \in E_{high}$,

Where λ_1 and λ_2 $(0 \leq \lambda_1 \leq \lambda_2)$ are two threshold parameters whose values depend on the problem dealt with.

1) Computing proximity measures: Each of the previously established experts' subsets implies a different search policy to identify the preferences with low agreement degree (controversial preferences). Low important experts will be advised to modify all the preference values identified in disagreement, while if the weight value is greater, the search will be limited to the controversial preferences of those experts furthest from the group. The proximity among each expert and the collective opinion is measured by others distance degrees, called proximity measures.

To compute proximity measures for each expert we need to obtain the collective fuzzy preference relation, P^c, which summarizes preferences given by all the experts.

These measures evaluate the agreement between the individual experts' opinions and the collective one. The collective preference, $P^c = (p_{ij}^c)$, is computed by means of the aggregation of all individual preference relations $\{P^1, P^2, \ldots, P^m\}$: $p_{ij}^c = \phi(p_{ij}^1, p_{ij}^2, ..., p_{ij}^m)$ with ϕ, an appropriate aggregation operator. It indicates the global preference between every pair of alternatives according to the experts' opinions.

For each expert, e_k, a proximity matrix, $PM^k = (pm_{ij}^k)$, is obtained where

$$pm_{ij}^k = (1 - |p_{ij}^k - p_{ij}^c|).$$

Once we have the proximity matrix, we compute the proximity measures in each level of a fuzzy preference relation:

1) *Proximity measure on pairs of alternatives, pp_{ij}^k.* It measures the proximity between the preferences on each pair of alternatives of the expert e_k and the group.

$$pp_{ij}^k = pm_{ij}^k.$$

2) *Proximity measure on alternatives, pa_i^k.* It measures the proximity between the preferences on each alternative x_i of the expert e_k and the group.

$$pa_i^k = \frac{\sum_{j=1, j \neq i}^{n} (pp_{ij}^k + pp_{ji}^k)}{2(n-1)}.$$

3) *Proximity measure on the relation, pr^k.* It measures the global proximity between the preferences of each expert e_k and the group.

$$pr^k = \frac{\sum_{i=1}^{n} pa_i^k}{n}.$$

2) Search for preferences phase: Once we have computed the proximity measures, and according with the above mentioned importance based classification of the experts, we propose three different identification strategies to find the controversial preferences. All of them have to be carried out at every consensus round and each one will identify the preferences in a different way based on the experts importance and the current consensus and proximity measures previously computed.

1) *Identify Low-Important Experts' Controversial Preferences:*

Taking into account just the experts' subset E_{low}, the system has to advise experts with low knowledge or confidence level. Consequently, it seems reasonable that, a priori, these experts can express less informed opinions. Thus, the agreement should be improved by suggesting important changes in the experts' preferences. To do this, the procedure tries to modify the preference values on all the pairs of alternatives where the agreement is not high enough for all the experts.

In order to find the set of preferences to be changed by each expert $e_k \in E_{low}$, this strategy acts as follows.

a) The pairs of alternatives with a consensus degree smaller than a threshold α_1, P, are identified.

$$P = \{(i,j) | cp_{ij} < \alpha_1\}$$

The value of α_1 may be static and fixed before starting the consensus round or dynamic with respect to the level of consensus reached in each round. We suggest to use a dynamic value computed as the average of the consensus degree at level of all pairs of alternatives.

b) Finally, the set of controversial preferences PCH^k to be changed by each expert $e_k \in E_{low}$ is

$$PCH^k = P.$$

2) *Identify Medium-Important Experts' Controversial Preferences:*

In this case, where we consider just the experts' subset E_{med}, it seems reasonable to reduce the number of changes and modify the point of view for the analysis of the agreement. While in the previous strategy we focused on all the pairs of alternatives in disagreement, now, the agreement is analyzed from the point of view of the alternatives and only the preference values in disagreement of those alternatives where agreement is not high enough will be considered.

Another important difference is the number of experts involved in the change of preferences. While in the previous strategy all experts were required to modify the identified preference values, in this case, just will do it those experts with proximity value at level of alternatives, for those identified alternatives in disagreement, smaller than a proximity threshold β_1. Hence, this new parameter is used to identify the experts that will be required to modify their preferences. As in the previous case, the value of β_1 may be static or dynamic. Again we consider the arithmetic mean of all proximity measures as a possible dynamic value.

This strategy finds out the set of preferences to be changed by each expert $e_k \in E_{med}$, as follows.

a) Initially, alternatives to be changed are identified. A new dynamic threshold α_2 is suggested in this case, such as the average of the consensus degrees at level of alternatives. Then

$$XCH = \{i | ca_i < \alpha_2\}$$

b) Now, pairs of alternatives to be changed are identified as

$$P = \{(i, j) | i \in XCH \wedge cp_{ij} < \alpha_1\}$$

c) Finally, the set of preference values that are required to be modified is

$$PCH^k = \{(i, j) \in P | pa_i^k < \beta_1\}.$$

3) *Identify High-Important Experts' Controversial Preferences:*

In this situation, we are only dealing with experts' subset E_{high}, whose knowledge level is so high that does not need to be strongly modified in order to get a well considered preference. Therefore, the agreement should be improved by suggesting fewer changes than in the previous two cases. In such a way, we only need to change the mind of those experts who have proximity values on the pairs of alternatives identified in disagreement smaller than an specific proximity threshold at level of pairs of alternatives. To do so, we propose a new dynamic threshold β_2 computed as the arithmetic mean of every proximity measures on pairs of alternatives of those pairs of alternatives previously identified to be changed.

a) Initially, alternatives to be changed are identified.

$$XCH = \{i | ca_i < \alpha_2\}$$

b) Now, pairs of alternatives to be changed are identified as

$$P = \{(i,j) | i \in XCH \wedge cp_{ij} < \alpha_1\}$$

c) Finally, the set of preference values that are required to be modified will be

$$PCH^k = \{(i,j) \in P | pa_i^k < \beta_1 \wedge pp_{ij}^k < \beta_2\}.$$

In conclusion, this importance based search for controversial preferences reduces the number of changes as the expert's knowledge level increases.

3) Generation of advice phase: Once the system has isolated the preferences to be changed by the experts depending on the importance degree of each one, the model shows the right direction of the changes in order to achieve the agreement. In this paper, we use a mechanism based on a set of direction rules to suggest the changes. For each preference value identified as controversial, the model will suggest increasing or decreasing the current assessment in the following way:

- if $((i,j) \in PCH^k \wedge (p_{ij}^k - p_{ij}^c) < 0)$, then the expert e_k should increase the assessments associated with the pair of alternatives (x_i, x_j).
- if $((i,j) \in PCH^k \wedge (p_{ij}^k - p_{ij}^c) > 0)$, then the expert e_k should decrease the assessments associated with the pair of alternatives (x_i, x_j).
- if $((i,j) \in PCH^k \wedge (p_{ij}^k - p_{ij}^c) = 0)$, then the expert e_k should not modify the assessments associated with the pair of alternatives (x_i, x_j).

Finally, it is worth noting that the changes suggested are just recommendations presented to the experts to show them the most appropriate way to narrow their positions. Then, each expert must decide, on his own, if and how to take the received advice into account.

IV. Example

Let us suppose that there is a research group composed of three experts $E = \{e_1, e_2, e_3\}$, with different experience level among them. The first one, e_1, is a full professor, the second one, e_2, is an assistant professor and finally, e_3, is a ph.d. student. They have finished a research project and they like to publish their results in the most related international journal. Therefore, they consider four related journals, $X = \{ESWA,\ Soft\ Computing,\ IEEE-TFS,\ Fuzzy\ Sets\ and\ Systems\}$ and they have to reach consensus to submit the paper to the best one.

In this framework, we need a system that help the experts to reach agreement by sending them customized amounts of advice. In such a way, our model achieves that the professors opinions leads the students preferences, avoiding the contrary situation that could be possible using classical models to solve GDM problems with a lot of low important experts.

Due to the fact that the experts involved in the problem have different experience level, they received associated weight values:

$$W = \{\mu_I(e_1) = 0.8,\ \mu_I(e_2) = 0.5,\ \mu_I(e_3) = 0.2\}.$$

Then they express their preferences on the journals using fuzzy preference relations:

$$P^1 = \begin{pmatrix} - & 0.2 & 0.1 & 0.3 \\ 0.8 & - & 0.4 & 0.6 \\ 0.8 & 0.7 & - & 0.7 \\ 0.6 & 0.4 & 0.2 & - \end{pmatrix}$$

$$P^2 = \begin{pmatrix} - & 0.4 & 0 & 0.2 \\ 0.6 & - & 0.2 & 0.3 \\ 1 & 0.8 & - & 0.9 \\ 0.8 & 0.9 & 0.1 & - \end{pmatrix}$$

$$P^3 = \begin{pmatrix} - & 0.5 & 0.6 & 0.7 \\ 0.5 & - & 0.7 & 0.9 \\ 0.4 & 0.3 & - & 0.6 \\ 0.3 & 0.1 & 0.4 & - \end{pmatrix}$$

The static parameters applied in this example are:

$$CL = 0.75,\ MaxRounds = 10,\ \lambda_1 = 0.3,\ \lambda_2 = 0.6$$

In the following, we show how to apply each step of the consensus model.

A. Computing Consensus Degree and Controlling the Consensus Process

1) **Computing consensus degree:** The consensus degree is obtained at the three different levels. First, the similarity matrix for each pair of experts is computed, and so the consensus matrix is obtained. Then, the consensus degrees on pairs of alternatives, alternatives, and global relation are obtained from the consensus matrix.

   a) *Consensus Matrix:*

$$CM = \begin{pmatrix} - & 0.80 & 0.60 & 0.66 \\ 0.60 & - & 0.66 & 0.60 \\ 0.60 & 0.66 & - & 0.80 \\ 0.66 & 0.46 & 0.80 & - \end{pmatrix}$$

   b) *Consensus on pairs of alternatives:* The element (i, j) of CM represents the consensus degrees on the pair of alternatives (x_i, x_j), thus, $cp_{ij} = cm_{ij}$.

   c) *Consensus on alternatives:*

$$ca_1 = 0.68 \quad ca_2 = 0.66 \quad ca_3 = 0.65 \quad ca_4 = 0.66$$

   d) *Consensus on the relation:*

$$CR = 0.66$$

2) **Controlling the consensus process:** In this step of the consensus model, the global consensus value, CR, is compared with the minimum consensus threshold, CL. In this example, we have decided to use the value, $CL = 0.75$. As $CR < CL$ and number of rounds $< MaxRounds$, it is concluded that there is no consensus among the experts, and consequently, the proximity measures are computed in order to start the feedback mechanism and support the experts on the necessary changes in their preferences in order to increase CR.

B. Importance-Based Feedback Process

1) **Computing proximity measures:** In this step, the proximity measures are computed. To do so, first the collective fuzzy linguistic preference relation is obtained by aggregating all individual preference relations. In this case, this is done using the arithmetic mean like aggregation operator.

$$P^c = \begin{pmatrix} - & 0.37 & 0.23 & 0.40 \\ 0.63 & - & 0.43 & 0.60 \\ 0.73 & 0.60 & - & 1.73 \\ 0.57 & 0.47 & 0.23 & - \end{pmatrix}$$

a) *Proximity matrices:*

$$PM_1 = \begin{pmatrix} - & 0.83 & 0.87 & 0.90 \\ 0.83 & - & 0.97 & 1 \\ 0.93 & 0.90 & - & 0.97 \\ 0.97 & 0.93 & 0.97 & - \end{pmatrix}$$

$$PM_2 = \begin{pmatrix} - & 0.97 & 0.77 & 0.80 \\ 0.97 & - & 0.77 & 0.70 \\ 0.73 & 0.80 & - & 0.83 \\ 0.77 & 0.57 & 0.87 & - \end{pmatrix}$$

$$PM_3 = \begin{pmatrix} - & 0.87 & 0.63 & 0.70 \\ 0.87 & - & 0.73 & 0.70 \\ 0.67 & 0.70 & - & 0.87 \\ 0.73 & 0.63 & 0.83 & - \end{pmatrix}$$

b) *Proximity on pairs of alternatives:* $PP_i = PM_i$.

c) *Proximity on alternatives (See Table I):*

TABLE I: Proximity measures on alternatives

x_1	x_2	x_3	x_4
$pa_1^1 = 0.89$	$pa_2^1 = 0.92$	$pa_3^1 = 0.94$	$pa_4^1 = 0.96$
$pa_1^2 = 0.83$	$pa_2^2 = 0.8$	$pa_3^2 = 0.79$	$pa_4^2 = 0.76$
$pa_1^3 = 0.74$	$pa_2^3 = 0.75$	$pa_3^3 = 0.74$	$pa_4^3 = 0.74$

d) *Proximity on the relation:*

$$pr_1 = 0.93 \quad pr_2 = 0.79 \quad pr_3 = 0.74$$

2) **Search for preferences phase:** In order to compute customized recommendations, the experts are included by their own importance degree into three different subsets:

Dynamic threshold values: $\alpha_1 = \bar{cp},\ \alpha_2 = \bar{ca},\ \beta_1 = \bar{pa}_i,\ \beta_2 = \bar{pp}_{ij}$

$$E_{low} = \{e_3\},\ E_{med} = \{e_2\},\ E_{high} = \{e_1\}$$

a) *Identify low important experts' controversial preferences:*

i) Identification of pairs of alternatives in disagreement

$$P = \{(i,j)|cp_{ij} < \alpha_1\} = \{(1,3),(1,4),(2,3),(2,4),(3,1),(3,2),(4,1),(4,2)\}.$$

ii) Set of preferences to be changed by each expert e_k in E_{low}

$$PCH^3 = P.$$

b) *Identify medium important experts' controversial preferences:*

i) Identifying the alternatives with consensus degree not high enough

$$XCH = \{i|ca_i < \alpha_2\} = \{2,3,4\}.$$

ii) For each one of the aforementioned alternatives, the preference values in disagreement are identified

$$P = \{(i,j)|i \in XCH \wedge cp_{ij} < \alpha_1\} = \{(2,3),(2,4),(3,1),(3,2),(4,1),(4,2)\}.$$

iii) Set of preferences to be changed by each expert e_k in E_{med}

$$PCH^2 = \{(i,j) \in P|pa_i^2 < \beta_1\} = \{(2,3),(2,4),(3,1),(3,2),(4,1),(4,2)\}.$$

c) *Identify high important experts' controversial preferences:*

i) Identifying the alternatives with consensus degree not high enough

$$XCH = \{i|ca_i < \alpha_2\} = \{2,3,4\}$$

ii) For each one of the aforementioned alternatives, the preference values in disagreement are identified

$$P = \{(i,j)|i \in XCH \wedge cp_{ij} < \alpha_1\} = \{(2,3),(2,4),(3,1),(3,2),(4,1),(4,2)\}.$$

iii) Set of preferences to be changed by each expert e_k in E_{high}

$$PCH^1 = \{(i,j) \in P|pa_i^1 < \beta_1 \wedge pp_{ij}^1 < \beta_2\} = \{\}.$$

3) **Generation of advice phase:**

- Expert e_1 do not have to change his preferences.

- Expert e_2 has to increase the assessment of his preference values

$$\{p_{23}, p_{24}\}$$

- Expert e_2 has to decrease the assessment of his preference values

$$\{p_{31}, p_{32}, p_{41}, p_{42}\}$$

- Expert e_3 has to increase the assessment of his preference values

$$\{p_{31}, p_{32}, p_{41}, p_{42}\}$$

- Expert e_3 has to decrease the assessment of his preference values

$$\{p_{13}, p_{14}, p_{23}, p_{24}\}$$

V. Concluding Remarks

In this paper, we have presented a novel consensus approach which has been specially designed to model non homogeneous decision frameworks in the sense of heterogeneity among experts. Assuming fuzzy preference relations to express experts' preferences and different levels of importance, we have presented a new feedback mechanism that computes different amount of advice according to the experts' importance level. Consequently, the most considerable experts' opinions never will be strongly modified during the consensus reaching process.

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4. A Linguistic Consensus Model for Web 2.0 Communities

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A Linguistic Consensus Model for Web 2.0 Communities

Sergio Alonso, Ignacio J. Pérez, Francisco J. Cabrerizo and Enrique Herrera-Viedma

Abstract

Web 2.0 Communities are a quite recent phenomenon which involve large numbers of users and where communication between members is carried out in real time. Despite of that good characteristics, there is still a necessity of developing tools to help users to reach decisions with a high level of consensus in those new virtual environments. In this contribution we present a new consensus reaching model with linguistic preferences designed to minimize the main problems that this kind of organization presents (low and intermittent participation rates, difficulty of establishing trust relations and so on) while incorporating the benefits that a Web 2.0 Community offers (rich and diverse knowledge due to a large number of users, real-time communication…). The model includes some delegation and feedback mechanisms to improve the speed of the process and its convergence towards a solution of consensus. We also show its possible application to some of the decision making processes that are carried out in the Wikipedia.

Index Terms

Consensus, Web 2.0, Linguistic Preferences, Group Decision Making

I. Introduction

Making decisions, that is, the cognitive process leading to the selection of a course of action among several alternatives according to a set of criteria, is a common activity that appears in almost any human endeavor: from choosing what to eat, what to wear and what to buy to selecting a representative or voting in an election. Group Decision Making (GDM) is a particular case of decision making where the final selected choice has to be done by multiple persons. GDM presents several special characteristics that distinguishes from individual decision making. For example, on the one hand, the total knowledge about a particular decision problem of a complete group of persons is usually higher than the knowledge of a particular individual, and thus, the group final decision may be better justified. On the other hand, the heterogeneous nature of the persons involved in the decision may introduce additional dificulties like very different points of view, specially on topics where feelings or beliefs are present.

S. Alonso is with the Department of Software Engineering, University of Granada, Spain; e-mail: zerjioi@ugr.es.

I.J. Pérez and E. Herrera-Viedma are with the Department of Computer Science and Artificial Intelligence, University of Granada, Spain; e-mail: {ijperez,viedma}@decsai.ugr.es.

F.J. Cabrerizo is with the Department of Software Engineering and Computer Systems, Distance Learning University of Spain, Spain; email: cabrerizo@issi.uned.es.

One of the fields where GDM is a fundamental matter is politics. As political decisions may influence lots of people, during all history it has been necessary to develop different forms of government to make decisions. One of those forms of goverment is democracy, where usually a set of elected officers undertake to represent the interests and/or views of citizens within a framework of the rule of law. However, as this kind of system only requires a periodic involvement in the elections of the majority of the citizens, the electorate is almost excluded from the political decision making, which can derive into a lack of political interest, knowledge and responsability among the non-participant population [1].

It is clear that involving a very large number of individuals in a decision process is a difficult task but, with the appearance of new electronic technologies, we are in the beginning of a new stage where traditional democratic models may leave some space to a more direct participation of the citizens. In the specialized literature we can found some efforts about the use of these new technologies in what it is being called e-democracy [1], e-participation [2], e-Governance [3] and public deliberation [4], [5].

In fact, new Web technologies have allowed the creation of many different services where users from all over the world can join, interact and produce new contents and resources. One of the most recent trends, the so called *Web 2.0*, which comprises a set of different web developement and design techniques, allows the easy communication, information sharing, interoperatbility and collaboration in this new virtual environment. Web 2.0 Communities, that can take different forms as Internet forums, groups of blogs, social network services and so on, provide a plataform in which users can collectively contribute to a Web presence and generate massive content behind their virtual collaboration [6]. In fact, Web 2.0 represents a paradigm shift in how people use the web as nowadays, everyone can actively contribute content online.

Among the different activities that the users of Web Communities usually perform we can cite:

- **Generate online contents and documents**, which is greatly beneficiated with the diversity and knowledge of the involved people. One of the clearest examples of this kind of collaboration success is Wikipedia [7], where millions of articles have been produced by its web community in dozens of different languages [8]. It is clear that in a massive service as Wikipedia many situations where it is necessary to make decisions about its inner workings and the contents that are being created arise [9].
- **Provide recommendations** about different products and services. Usual recommender systems are increasing their power and accuracy by exploiting their user bases and the explict and implicit knowledge that they produce [10]. This kind of systems represent a quite powerful addition to Web 2.0 systems where decisions have to be made. A clear example of recommeder systems success, which exploits its users community knowledge to provide personalized recommendations, is the Amazon online store [11].
- **Participate in Discussions and Forums**. Many online communities have grown around a web forum or some discussion boards where users share information or discuss about selected topics. In many of these communities some simple group decision making schemes, as referendum or voting systems are usually used. For example, services like PollDaddy [12] allow to create online surveys and polls where users can vote about the best alternative to choose for a given decision problem.

It is thus clear that to develop more sophisticated GDM models and schemes that can be applied into the new Web 2.0 Communities is a current necessity. In fact, there have been several efforts in the specialized literature to create different models to correctly address and solve GDM situations. Some of them make use of fuzzy theory as it is a good tool to model and deal with vague or imprecise opinions (which is a quite common situation in any GDM process) [13], [14]. Many of those models are usually focused on solving GDM situations in which a particular issue or difficulty is present. For example, there have been models that allow to use linguistic assessments instead of numerical ones, thus making it easier for the experts to express their preferences about the alternatives [15]. Other models allow experts to use multiple preference structures (and even multi-granular linguistic information) [16], [17] and other different approaches deal with incomplete information situations if experts are not able to provide all their preferences when solving a GDM problem [18] or when a consensus process is carried out [19].

Moreover, usual GDM models have been complemented with consensus schemes that allow users to interact until there is a certain degree of agreement on the selected solution [20], [21]. This consensus models allow not only to provide better solutions to decision problems, but also to increase the users satisfaction with the decision process as all the opinions are reconsidered to achieve a high enough level of consensus.

However, those approaches are not usually well suited to be used by Web Communities due to some of their inherent properties. For example, due to the diversity of the users backgrounds, using numerical preferences might be not adequate and thus, linguistic assessments should be used [22]. In fact, current online technologies as chatterbots are being developed to interact with users in a linguistic way [23]. Moreover, we can find studies where linguistic concepts in social networks are described by means of fuzzy sets and the computing with words paradigm [24].

Also, dynamic situations in which some of the parameters of the problem, as the set of experts, the set of alternatives and even the set of criteria to select the solutions change, have not been modeled. This kind of situations are quite common in other environments: in [25] the problem of managing time-dependent preferences (that is preferences expressed at different periods) is presented; the problem of dealing with dynamic real-time information to choose the best routes is shown in [26]; a practical example about resource managment where the criteria to make decisions (climate) changes over time can be found in [27] and a decision support system for mobile environments where the alternatives set changes during the decision process is shown in [28]. Thus, it is important to develop new models that take into account this kinds of dynamical situations to solve realistic GDM problems [29].

For the particular case of Web Communities, dynamic situations in which the group of experts vary over time are quite common: a new expert could incorporate to the process, some experts could leave it or a large group of experts could be simplified in order to minimize communications and to ease the computation of solutions. This behaviour is usually found in democratic systems where the individuals delegate into a smaller group of experts to make decisions (it is usually not possible to involve everyone in each decision). There have been some efforts to model this kind of situations. For example, in [30] a recursive procedure to select a qualified subgroups of individuals taking into account their own opinions about the group is presented. However, there is still a big necessity of creating new consensus models that suit Web Communities characterstics appropriately.

In this paper we present a consensus model in which preferences are expressed in a linguistic way and that has been designed taking into account the characteristics of Web 2.0 Communities. In particular, it has been designed considering that the number of users of this kind of communities is usually large [31]. For example, online music communities usually gather hundreds or even thousands of individuals that share an interest about particular bands or music genres. To reach a consensual decision with such a large user base is not an easy task because, for example, not every member of the community is willing to participate and contribute to solve the problem [32] or maybe because the topic being discussed is controversial and involves individual feelings or beliefs [33]. In addition, this model allows dynamic sets of users, that is, the users set to solve the decision problem may change in time. Moreover, by means of a delegation scheme (based on a particular kind of trust network [34]) we may achieve an important simplification in the obtaining of a proper consensus level. The model also incorporates a feedback mechanism that helps the users to change their preferences towards a higher consensus level solution. In addition, a trust checking procedure allows to avoid some of the problems that the delegation scheme could introduce in the consensus reaching model. Finally, a brief discussion about the applicability of the model to increase the consensus level in the decision making processes of the Wikipedia is also presented.

To do so, the paper is set as follows: in section II we present our preliminaries, that is, some of the most important characteristics of Web 2.0 Communities and the basic concepts that we use in our paper. In section III we introduce the new consensus model with linguistic preferences that helps to obtain consensual decisions in Web 2.0 Communities as well as its possible application to the Wikipedia. Finally, in section IV we point out our conclusions.

II. Preliminaries

In this section we present our preliminaries: first we present some of the main characteristics of Web 2.0 Communities that have to be taken into account when designing any tool for them; second, we present some groundwork about the use of linguistic preferences in consensus models.

A. Web 2.0 Communities

New Web 2.0 technologies have provided a new framework in which virtual communities can be created in order to collaborate, communicate, share information and resources and so on. This very recent kind of communities allows people from all over the globe to meet other individuals which share some of their interests. Apart from the obvious advantage of meeting new people with similar interests, Web Communities present some characteristics that make them different from other more usual kinds of organizations. In the following we discuss some of those characteristics and how they can affect in the particular case of GDM situations:

- **Large user base:** Web Communities usually have a large user base [31] (it is easy to find web communities with thousands of users). This can be seen from a double perspective. On the one hand, the total knowledge that a large user base implies is usually greater and more diverse than in a small community. This can be seen as a clear advantage: taking decisions is usually better performed when there is a rich knowledge on the evaluated subject. On the other hand, managing a large and diverse amount of opinions in order to extract and use that knowledge might be a difficult task: for example, some of the users might not find easy to use typical numerical preference representation formats and thus, linguistic ones should be implemented.

- **Heterogeneous user base:** Not only the user base in Web Communities is large, but it is usually heterogeneous. This fact implies that we cannot easily assume that all the individuals may find easy to use the tools that are being developed and introduced in the websites. A clear example is the use of numerical ratings: some users may find difficult to express their preferences about a set of alternatives using numerical ratings and thus, it may be interesting to provide tools which can deal with natural language or linguistic assessments.
- **Low participation and contribution rates:** Although many Web Communities have a quite large user base, many of those users do not directly participate in the community activities. Moreover, encouraging them to do so can be difficult [32]. Many of the users of a web community are mere spectators which make use of the produced resources but that does not (and is not willing to) contribute themselves with additional resources. This can be a serious issue when making decisions if only a small subset of the users contribute to a decision and it does not reflect the overall opinion of the community.
- **Intermittent contributions:** Partially due to the fast communication possibilities and due to a very diverse involvement of the different members, it is a common issue that some of them might not be able to collaborate during a whole decision process, but only in part of it. This phenomenon is well known in web communities: new members are continuously incorporated to the community and existing users leave it or temporarily cease in their contributions.
- **Real time communication:** The technologies that support Web Communities allow near real time communication among its members. This fact let us create models that in traditional scenarios would be quite impractical. For example, in a referendum, it is not easy at all to make a second round if there has been a problem in the first one due to the high amount of resources that it requires.
- **Difficulty of establishing trust relations:** As the main communication schemes in Web Communities use electronic devices and, in the majority of the cases, the members of the community do not know each other personally, it might be difficult to trust in the other members to, for example, delegate votes. This fact implies that it might be necessary to implement control mechanisms to avoid a malicious user taking advantage of others.

B. Consensus Models with Fuzzy Linguistic Preferences

Usual GDM models follow a scheme (see figure 1) in which two phases are differentiated: the first one consists in a *consensus process* in which the users (that we will call *experts* in the following), discuss about the alternatives and express their preferences about them using a particular preference representation format. A special individual (the moderator) checks the different opinions and confirms if there is enough consensus among all the experts. If there is not enough consensus, the moderator urges the experts to re-discuss about the alternatives and to provide a new set of opinions to improve the consensus level in a new consensus round. Once the desired consensus have been reached (or a maximum number of consensus rounds has been reached) the second phase (the *selection process*)

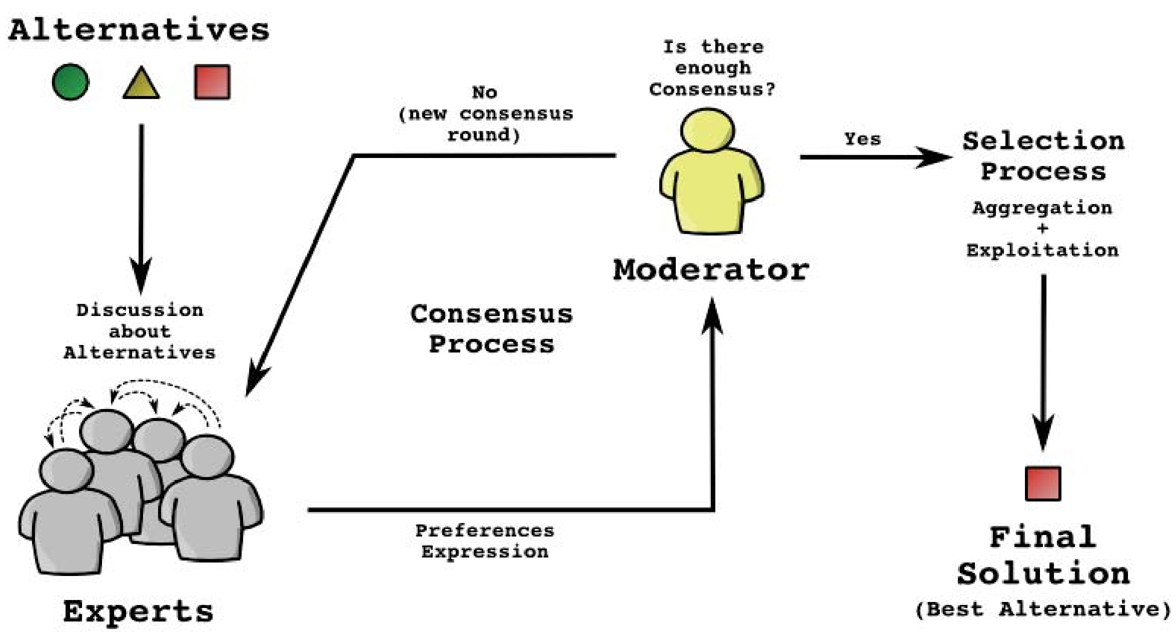

Fig. 1. Typical scheme of GDM models

starts and the best solution is obtained by agreggating the last opinions from the experts and applying an exploitation step which identifies the best alternative from the agreggated information.

In this paper we center our attention only in the consensus process, where the experts are suppossed to narrow their different opinions about the alternatives to obtain a final solution with a high level of consensus. In the consensus model that we propose, the experts $E = \{e^1, \dots, e^m\}$ will provide their preferences about the set of alternatives $X = \{x_1, \dots, x_n\}$ in form of fuzzy linguistic preference relations [35]. In particular, we will use the 2-tuple linguistic computational model [36], in which the linguistic information is represented by a 2-tuple (s, α), $s \in S$, where S is a usual term set with odd cardinality and where the terms are uniformly distributed.

Definition 1: Let $\beta \in [0, q]$ be the result of an aggregation of the indexes of a set of labels assessed in a linguistic term set $S = \{s_0, \dots, s_q\}$, i.e., the result of a symbolic aggregation operation. Let $i = round(\beta)$ and $\alpha = \beta - i$ be two values, such that, $i \in [0, q]$ and $\alpha \in [0.5, 0.5)$, then α is called a symbolic translation.

The model also defines two functions Δ^{-1} and Δ to transform 2-tuples to numerical values and viceversa [36].

Definition 2: A 2-tuple linguistic preference relation P^h given by expert e^h on a set of alternatives X is a set of 2-tuples on the product set $X \times X$, i.e., it is characterized by a membership function $\mu_P^h : X \times X \to S \times [0.5, 0.5)$.

III. A Linguistic Consensus Model for Web 2.0 Communities

In this section we present a new consensus model that can be applied in Web 2.0 Communities to reach solutions in GDM environments and its possible application to the Wikipedia. It takes into account the different characteristics of this kind of communities (see section II-A) in order to increase the consensus level of the users when making a decision on a set of alternatives. Some of the properties of the model are:

- It does not require the existence of a moderator,
- it allows to work in higly dynamical environments where participation and contribution rates change,
- it uses linguistic information to model user preferences and trust relations,
- it allows to weight the contributions of each user according to some degree of expertise,
- it offers a feedback mechanism to help experts to change their preferences about the alternatives and
- it can be easily adapted to real world Web 2.0 communities.

Its operation implies several different steps that are repeated in each consensus round:

1) First preferences expression, computation of similar opinions and first global opinion and feedback,
2) delegation,
3) change of preferences (feedback mechanism),
4) computation of consensus measures and
5) consensus and trust checks.

In figure 2 we have depicted the main steps of the model and in the following we describe them more detail.

A. First Step: First Preferences Expression, Computation of Similar Opinions and First Global Opinion and Feedback

In this first step the different alternatives in the problem are presented to the experts (note than in figure 2 we have represented only a small amount of experts, but when applied to a Web 2.0 Community the number of users will usually be larger). Once they know the feasible alternatives, each expert $e^h \in E$ is asked to provide a fuzzy linguistic preference relation P^h that represent his opinions about the alternatives. Although every single member of the community has the oportunity of expressing his preferences about the alternatives, as we have previously mentioned, only a subset of those experts $\tilde{E}$ will really provide preference relations. We will note $\tilde{e}^h$ to the experts that have provided a preference relation. It is important to note that if an expert at this stage does not provide a preference relation the model will still allow him to contribute in the consensus process in a later stage. Once a certain amount of time has passed (to allow a sufficient number of preferences to be provided) we compute the distance among each pair of experts $\tilde{e}^h$ and $\tilde{e}^g$ in the following way:

$$d^{hg} = d^{gh} = \sqrt{\sum_{i=1} \sum_{\substack{j=1 \\ j \neq i}} \left(\frac{\Delta^{-1}(p_{ij}^h) - \Delta^{-1}(p_{ij}^g)}{q} \right)^2}$$

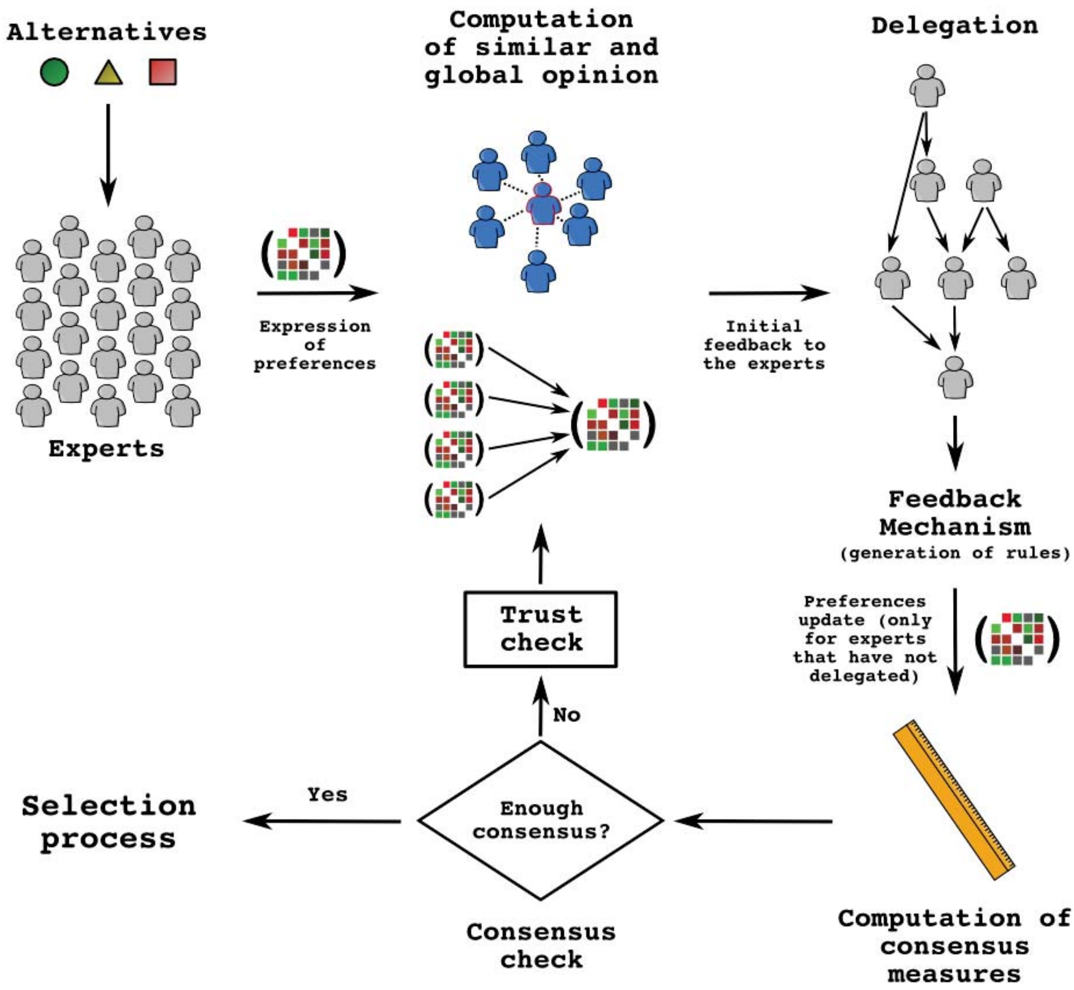

Fig. 2. Scheme of the presented consensus model

This distances will be used to provide information to each expert about the experts that share a similar opinion about of the alternatives. In fact, for each $\tilde{e}^h \in \tilde{E}$ we define his *set of neighbours* as

$$N^h = \{\tilde{e}^{\beta_1}, \ldots, \tilde{e}^{\beta_{nn}}\}$$

where nn is the number of neighbours that each expert will be presented (this parameter is defined prior to the start of the consensus process) and e^{β_i} is the i-th nearest expert to $\tilde{e}^h$ (with lowest $d^{h\beta_i}$).

As it happens in many real world GDM problems, it is possible that the preferences of every different expert may be weighted differently. This may be interestig in situations where some of the experts have a great reputation or expertise in the problem field. Thus, we assume that for every expert in the problem a *trust weight* τ^h is given. If for a particular problem the preferences of every expert are considered equally important, then all the trust weights will be initialized to 1: $\tau^h = 1$.

The last task at this first step is to compute the current global preference as an aggregation of all the provided preference relations. To do so, we will apply a weighted average to compute it:

$$p^c_{ij} = \Delta\left(\frac{\sum\limits_{\tilde{e}^h \in \tilde{E}} \tau^h \cdot \Delta^{-1}(p^h_{ij})}{T}\right) \quad (1)$$

Once the distances among experts, the neighbours of each expert and the global preference relation have been computed, this information will be presented to the experts. After receiving this feedback, an expert will know if his opinions are very different to the current global preferences and he will also know which are the experts that share similar opinions. Apart from just his neighbour list, an expert is also able to check the particular preference relations that his neighbours have introduced in order to really check the preferences expressed by his neighbourhood.

B. Second Step: Delegation

In this second step the model incorporates a delegation scheme in which experts may choose to delegate into other experts (typically experts from their neighbourhood, with similar opinions). This mechanism is introduced to soften the intermittent contributions problem (because an expert who knows that he will not be able to continue the resolution process may choose to delegate into other experts instead of just leaving the process) and to decrease the number of preference relations involved in the problem. To make the delegation scheme flexible enough and to be able to cover a wide range of different delegation proposals, an expert $\tilde{e}^h$ that decides to delegate has to provide a set of trust evaluations of other experts $t^h_j, j \in \{1, \ldots, m\}$. In this proposal, we assume that this trust evaluations are given using a linguist terms set in the form $TS = \{ts_{-3} = total\ distrust, ts_{-2} = high\ distrust, ts_{-1} = low\ distrust, ts_0 = neutral, ts_1 = low\ trust, ts_2 = high\ trust, ts_3 = total\ trust\}$. Note that as a result of the usually large number of experts that may take part in the resolution process, many of the trust evaluations t^h_j of expert $\tilde{e}^h$ will be neutral (ts_0) as the expert may not be able to evaluate all the rest of experts. However, he might know some experts that he trusts or distrusts, and thus, those trust evaluations are the ones that the system will take into account. This implies that the user does not have to provide trust evaluations for all the rest of experts, but only of those that he really trusts or distrusts. Once an experts has provided his trust evaluations for some other experts he will not be required to update his preferences to improve the consensus level.

Once a certain amount of time have passed (enough time for the experts to decide if they wanted to delegate or not), the system will re-compute the trust weights τ^h for every expert according to the trust evaluations of the rest of the experts. To do so, for every expert $\tilde{e}^h$ that has provided his linguistic trust evaluations t^h_j we compute $tt^h = \sum_{j=1}^m |\tilde{t}^h_j|$ where $\tilde{t}^h_j$ is the index of the linguistic term t^h_j in TS. Then, for each $t^h_j \neq ts_0$ we compute an increment of the trust value $\Delta\tau^j = \tau^h \cdot \frac{\tilde{t}^h_j}{tt^h}$. At this point, every trust weight τ^j can be updated adding this increment: $\tau^j = \tau^j + \Delta\tau^j$ and the trust value for the expert that delegated becomes 0: $\tau^h = 0$. If, after the all the trust updates have been done an expert has a new trust value less than 0, the system should round it to 0. A trust value of 0 means that the opinion of that expert is not trusted enough to take part in the process (and, in fact, in expression 1, a trust weight equal to 0 is not taken into account in the global preferences relation).

Example: Suppose that a particular expert participating in the decision process $\tilde{e}^1$ whose current trust weight is $\tau^1 = 2$ decides that he will no longer take part in the process, and thus, he wants to delegate in other experts. He decides that experts $\tilde{e}^2$ and $\tilde{e}^3$ (whose current trust weights are $\tau^2 = 2$ and $\tau^3 = 1$) can be trusted but, on the other way, $\tilde{e}^4$ (whose current trust weight is $\tau^4 = 1.5$) can not. Thus, he provides the following trust evaluations: $t^1_2 = ts_1$, $t^1_3 = ts_2$ and $t^1_4 = ts_{-2}$. Then, the system computes $tt^h = |1| + |2| + |-2| = 5$; $\Delta\tau^2 = 2 \cdot \frac{1}{5} = 0.4$; $\Delta\tau^3 = 2 \cdot \frac{2}{5} = 0.8$ and $\Delta\tau^4 = 2 \cdot \frac{-2}{5} = -0.8$. Finally, the system updates the trust weights of $\tilde{e}^2$, $\tilde{e}^3$ and $\tilde{e}^4$ by adding the increments: $\tau^2 = 2 + 0.4 = 2.4$, $\tau^3 = 1 + 0.8 = 1.8$ and $\tau^4 = 1.5 - 0.8 = 0.7$.

Note that the expressed trust evaluations may be seen as a directed graph structure among the set of experts. This directed graph structure conforms a kind of trust network which can be used to stablish a kind of delegation scheme in which some transitivity conditions occur: if an expert $\tilde{e}^h$ delegates in an expert $\tilde{e}^k$ and $\tilde{e}^k$ delegates in $\tilde{e}^j$ the situation would be similar as if both $\tilde{e}^h$ and $\tilde{e}^k$ would have directly delegated in $\tilde{e}^j$. Note that the model should avoid cicles in the trust network. If an expert tries to delegate in another one and this delegation would produce a cicle in the trust network, the system should alert him about this situation and ask him to reconsider his trust evaluations. In figure 3 we have depicted a group of experts in which some of them have delegated by expressing some trust evaluation over other experts. The two experts on the right have not delegated in any other expert and have neither been chosen by other experts to delegate in them. In addition, a similar situation to the example above has been depicted with experts in the upper left part of the image.

It is clear that with this kind of trust evaluation to delegate it is easy to replicate more typical delgation schemes. For example, if an user wants to delegate its entire trust weight into another expert, he might just provide a positive trust evaluation for that expert. Or, if an expert wants to delegate his opinion into a group of experts equally, he just have to provide equal positive trust values for each one of the delegates. Finally, if an expert is not sure about whom to delegate in, but he knows that he does not trust a particular expert, he can reduce the trust weight of that expert by giving him a negative trust evaluation.

This delegation mechanism provides several advantages to the model: first of all, it allows experts not to provide their preferences in every consensus round. If an expert delegates in another one, he will not have to update his preferences but, in a certain way (through the delegate), his opinion will still influence the consensus state. Thus, the consensus rounds may be carried out faster as only a subset of experts will have to change their preferences. Moreover, the computations will also be reduced as the system will not have to deal with a large amount of preference relations. Additionally, as the mechanism allows to give different trust evaluations to multiple experts, it is possible to delegate into a group of experts that *as a whole* have a similar opinion to the expert, not conferring too much weight to a single person.

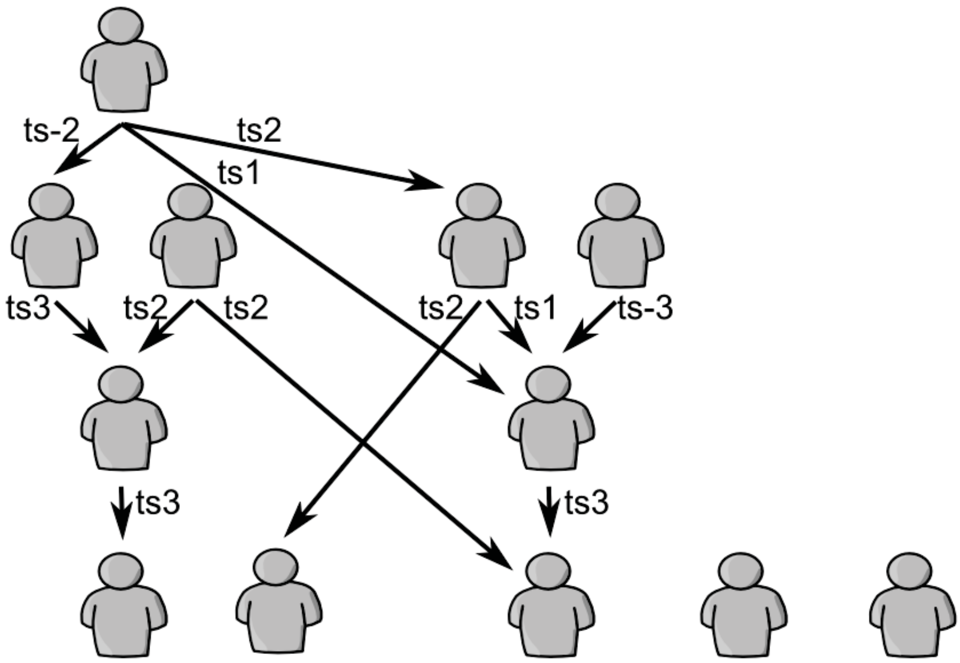

Fig. 3. Example of the delegation scheme

C. Third Step: Change of Preferences (Feedback Mechanism)

Once the trust weights have been re-computed the system will ask the remaining experts to update their linguistic preference relations P^h in order to achieve a greater level of consensus. This experts will conform the new $\tilde{E}$ subset. As in some cases changing the linguistic preference relations may not be an easy task, the model includes a feedback mechanism that identifies which experts and preference values should be changed to increase the level of consensus and which advices the corresponding experts about it. To do so, the system computes several proximity measures [37] at three diferent levels: pair of alternatives, alternatives and relations levels.

Level 1) *Proximity measure on pairs of alternatives:* The proximity measure of an expert $\tilde{e}^h$ on the pair of alternatives to the group one, denoted pp^h_{ik} is calculated as

$$pp^h_{ik} = 1 - \frac{|\Delta^{-1}(p^h_{ik}) - \Delta^{-1}(p^c_{ik})|}{q}$$

Level 2) *Proximity measure on alternatives:* The proximity measure of an expert $\tilde{e}^h$ on alternative x_i to the group one, denoted pa^h_i is calculated as:

$$pa^h_i = \frac{\sum_{k=1;k\neq i}^{n}(pp^h_{ik} + pp^h_{ki})}{2\cdot(n-1)}$$

Level 3) *Proximity measure on the relation:* The proximity measure of an expert $\tilde{e}^h$ on his preference relation to the group one, denoted pr^h, is calculated as:

$$pr^h = \frac{\sum_{i=1}^{n} pa_i^h}{n}$$

Using these proximity measures we define the APS set that contains 3-tuples (h, i, k) symbolizing preference degrees p_{ik}^h that should be changed because they affect badly to the consensus state. To compute the APS set we follow a three simple step process:

Step 1) We identify the set of experts EXPCH that should receive advice on how to change some of their preference values. The experts that should change their opinions are those whose proximity level on the relation is lower than a certain threshold γ (set prior to the beginning of the decision process):

$$EXPCH = \{h \mid pr^h < \gamma\}$$

Step 2) We identify the alternatives that the above experts should consider to change. This set of alternatives is denoted as ALT. To do this, we select the alternatives with a proximity level lower than the γ threshold:

$$ALT = \{(h, i) \mid h \in EXPCH \wedge pa_i^h < \gamma\}$$

Step 3) Finally, we identify preference values for every alternative and expert $(x_i; \tilde{e}_h|(h, i) \in ALT)$ that should be changed according to their proximity measures on the pairs of alternatives:

$$APS = \{(h, i, k) \mid (h, i) \in ALT \wedge pp_{ik}^h < \gamma\}$$

Once the feedback mechanism knows which preference values are contributing less to the consensus state ($p_{ik}^h \mid (h, i, k) \in APS$), it generates some easy to follow rules which are presented to the experts that should change their opinions. For each $(h, i, k) \in APS$ the generated rule for expert $\tilde{e}^h$ has the following form: *"You should change your preference value (i, k) to a value close to p_{ik}^c"*.

Note that this rules are just recommendations that are offered to the experts to increase the consensus level in a fast way but, in any case they are ever forced to follow them.

D. Fourth Step: Computation of Consensus Measures

Once the updated preferences have been given we can compute some consensus degrees. To do so, we firstly define for each pair of experts $(\tilde{e}^h, \tilde{e}^l)$ $(h < l)$ of the new $\tilde{E}$ a similary matrix $SM^{hl} = \left(sm_{ik}^{hl}\right)$ where

$$sm_{ik}^{hl} = \tau^h \cdot \tau^l \cdot \left(1 - \left|\frac{\Delta^{-1}(p_{ik}^h) - \Delta^{-1}(p_{ik}^l)}{q}\right|\right)$$

Then, a collective similarity matrix, $SM = (sm_{ik})$ is obtained by aggregating all the $(\#\tilde{E} - 1) \times (\#\tilde{E} - 2)$ similarity matrices using following expression:

$$sm_{ik} = \frac{\displaystyle\sum_{h,l \in \tilde{E} | h<l} sm_{ik}^{hl}}{T \cdot (T-1)/2}$$

where $T = \sum_{i=1}^{m} \tau^i$.

Once the similarity matrices are computed we proceed to obtain the consensus degrees at the three different levels:

L. 1. *Consensus degree on pairs of alternatives*. The consensus degree on a pair of alternatives (x_i, x_k), denoted cop_{ik}, is defined to measure the consensus degree amongst all the experts on that pair of alternatives:

$$cop_{ik} = sm_{ik}$$

L. 2. *Consensus degree on alternatives*. The consensus degree on alternative x_i, denoted ca_i, is defined to measure the consensus degree amongst all the experts on that alternative:

$$ca_i = \frac{\sum_{k=1;k\neq i}^{n}(cop_{ik} + cop_{ki})}{2(n-1)}$$

L. 3. *Consensus degree on the relation*. The consensus degree on the relation, denoted CR, is defined to measure the global consensus degree amongst all the experts' opinions:

$$CR = \frac{\sum_{i=1}^{n} ca_i}{n}$$

E. Fifth Step: Consensus and Trust Checks

In the end of each consensus round we must check the current consensus state. If it is considered a high enough consensus value the consensus process would finish and a selection process would be applied to obtain the final solution for the decision problem. To do so, we check if $CR > \gamma$, being γ a threshold value fixed prior to the beginning of the GDM process. In the case that the level of consensus is not high enough we would continue with the trust check that is described in the following. Note that in real applications it might be desirable to include a $maximumRounds$ parameter to control the maximum consensus rounds that can be executed in order to avoid stagnation.

The trust check is introduced to avoid some of the problems that can be derived to one of the characteristics of Web Communities: the difficulty of stablishing real trust relations. It is not difficult to imagine an scenario where some experts delegate into another that shares a common point of view on the decision that has to be made and in a certain consensus round, this expert decides to drastically change his preferences, probably not reflecting the other experts opinions anymore. To avoid this kind of situations the trust check will compare the last preference relation expressed by expert $\tilde{e}^h$ with the last preference relations of the experts that delegated in him (direct or indirectly). This comparison can be made by applying a distance operator (as the euclidean or cosine distances) over the preference relations or computing proximity measures similar to the ones presented in section III-C. If this distance is greater than a certain stablished threshold, the expert that delegated in $\tilde{e}^h$ would be informed with a special message to warn him about this problematic situation and thus allowing him to take a different course of action in the next consensus round if apropriate.

At this point a new consensus round begins: a new global preference will be computed with the new preferences of the experts and their new trust weights, and new distance measures will be obtained. New experts may join the new round (by giving their preference relations), some other experts may decide to delegate on others, and all previously involved expert may change their preferences or their trust evaluations over other experts.

We would like to emphasize that in each new consensus round all the members of the Web Community can participate, independently of what they did in the previous rounds. For example, an expert that delegated in a previous consensus round may decide not to continue delegating (maybe because the trust check mechanism has warned him that the expert in which he delegated has drastically changed his preferences) and thus to provide again a new fuzzy linguistic preference relation or to delegate in a different individuals; an expert which had not delegated in any of the previous rounds might decide to delegate in the current consensus round or even an expert which has not participated until this moment in the consensus process (he did not provide any preference relation in the first step of the model) could join the process by providing his initial preferences.

F. Possible Application of the Consensus Model to an Existing Web 2.0 Community: Wikipedia

Wikipedia [7], as almost any other Web 2.0 service is a very recent phenomenon that has attracted a lot of attention from the public and the media. Its main pourpose is to create an online freely available encyclopedia. One of its revolutionary aspects is that the contents, contrary to other more conventional encyclopedias, are created and updated in collaborative way by any of its users. In fact, it follows a similar tendency present in the Web where anyone can freely create and publish content without any need of third-party control, which has not been the case in the traditional models of publishing and broadcasting, which are usually governed by centralized organizations [8]. Despite the uncentralized nature of the Wikipedia, there are currently some studies that analyze the quality of the contents of the Wikipedia that assure that its quality is almost as good as other well reputed encyclopedias [38], [39].

In such a vast environment, where millions of encyclopedical entries and millions of users interact it has been necessary to introduce new tools and features [40] to improve not only the quality of the entries, but the coordination [41], [33], cooperation [42] among the users, the social transparency of the articles [43] and the semantic annotation of the contents [44].

However, it is still necessary to develop new tools to avoid conflic [45] and increase the consensus of the decisions taken in Wikipedia. As the Wikipedia covers conflictive and controversial topics (political and religious ones are a clear example where there is no clear neutral point of view) this kind of tools may help to reach better decisions about the contents presented in such topics.

The consensus model proposed in this contribution may be appropriate for some of those situations. For example, lets imagine a particular conflictive topic covered in the Wikipedia. Suppose that in the discussion page for that topic have been porposed four different alternatives to solve the discussion in the topic: to completely remove the article, to rewrite it according to a particular point of view, to split it in several articles that can be managed separately and that do not provoque too much controversial or to leave it as it is in its present state. If we apply our model, we would allow to choose a solution of consensus among the alternatives in which:

- every user that is willing to participate can do it (thus increasing the level of confidence in the final decision making),
- new users may incorporate in the middle of the consensus process,
- participating users will not be forced to finish the consensus process, as they may choose to delegate into other users,
- some users may have higher weight than others (for example, Wikipedia administrators or the users that have actively contributed to the conflicting article),
- the consensus status may be reached faster than using traditional discussion mechanism (due to the incorporation of the feedback mechanism),
- the preferences of the users are given in a linguistic way increasing their understandability.

IV. Conclusions

In this contribution we have presented a novel consensus model which has been specially designed to be applied in Web 2.0 Communities. Particularly, it uses fuzzy linguistic preference relations for the expression and management of experts' preferences and it has been designed to manage a large users base by means of a delegation scheme. This delegation scheme is based in a particular kind of trust network created from linguistic trust evaluations given by the experts that simplifies the computations and the time needed to obtain the users preferences. Moreover, this delegation scheme also solves the intermittent contributions problem which is present in almost any online community (that is, many of the users will not continuosly collaborate but will do it from time to time). The model also incorporates a feedback mechanism to help the experts in changing their preferences in order to obtain a high level of consensus fastly.

In addition, the model allows to incoporate new experts to the consensus process, that is, the model is able to handle some of the dynamic properties that real Web Communities have. Finally, the model incorporates a trust check mechanism that allow to detect some abnormal situations in which an expert may try to take advantage of others by drastically changing his opinion and benefiting from the trust that the other experts might have deposited in him in previous consensus rounds.

It has also been shown that this model can be used in existing Web 2.0 Communities as the Wikipedia to reach consensus in difficult decision making situations.

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