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Referring under Uncertainty

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Abstract: In this paper, we study the referring expression generation problem (REG) when the available information about the properties of objects is uncertain, in the sense that we are not sure about the actual properties an object has. We formalize the problem by extending the conventional REG framework through the use of possibility distributions, represented by fuzzy sets. We show the potential benefits of this proposal in the assessment of the referential success for referring expressions. This approach opens a new line of research in the application of fuzzy sets to the REG problem, complementary to those approaches that use fuzzy sets as a suitable bridge between language and raw data.

Index termns: referring expression, uncertainty, referential success, possibility theory.

1 Introduction

The work of this paper is placed within the Natural Language Generation (NLG) field, where data-to-text systems[RD00] play an important role. These systems try to generate a description of a set of data, similar to a description generated by a human.

In generating such linguistic descriptions, fuzzy sets have been used as a suitable bridge between linguistic terms and expressions, on the one side, and raw data on the other [ABR17; KZ10; MS16]. In this paper, we explore a different use of fuzzy sets in this setting: the use of fuzzy sets for representing our lack of knowledge about the actual properties of objects, in the context we want to describe. From a general point of view, in this work we refer to this lack of knowlewdge as uncertainty.

We manage this uncertainty in one of the most important problems in the data-to-text systems: the referring expression generation problem (REG). As far as we know, fuzzy sets have not been previously applied to solve this problem, and they are also a suitable tool to produce reasonable results. As we will see, the use of our approach produces a gradation in the space of possible referring expressions, which is of particular interest when the main goal of referential success coexists with other objectives, like minimizing the lenght of the expression, among others.

In the next Section 2.1, we present the problem in a formal way. Section 2.2 is devoted to include the mentioned type of uncertainty in the REG problem and in Section 2.3, we present a graded version of this new approach. Finally, some conclusions and future work are presented in Section 3.

2 Formalization of the problem

One of the main problems of data-to-text systems is the REG problem [KVD12; VD16]. This problem, in the classic version, consists in, given a set of objects and a set of properties that objects can satisfy, finding a subset of properties that univocally identifies a target object within the set of objects.

As an example of this, consider the following situation: in a Spanish classroom there are five foreign students, as shown in Figure 1. In this paper, we shall denote them by $\{a, b, c, d, e\}$, so that we can refer to each student without ambiguity when needed. Now, let us consider that two other students have found a lost backpack without any documentation, but one of them (s_1) knows which student the backpack belongs to. She describes the owner of the backpack to her classmate (s_2) in order to look for him in the campus. However, the only information she remembers about the foreign students is the colour of their sweaters, and the nationality of each one (inferred from the language that each of them speaks). Then, she could generate a description like *the backpack belongs to the British student that was wearing a green sweater*, which is a referring expression that allows s_2 to identify the actual owner of the backpack as the student we denote by a. In the next section, we present a formalization of this classical version of the REG problem.

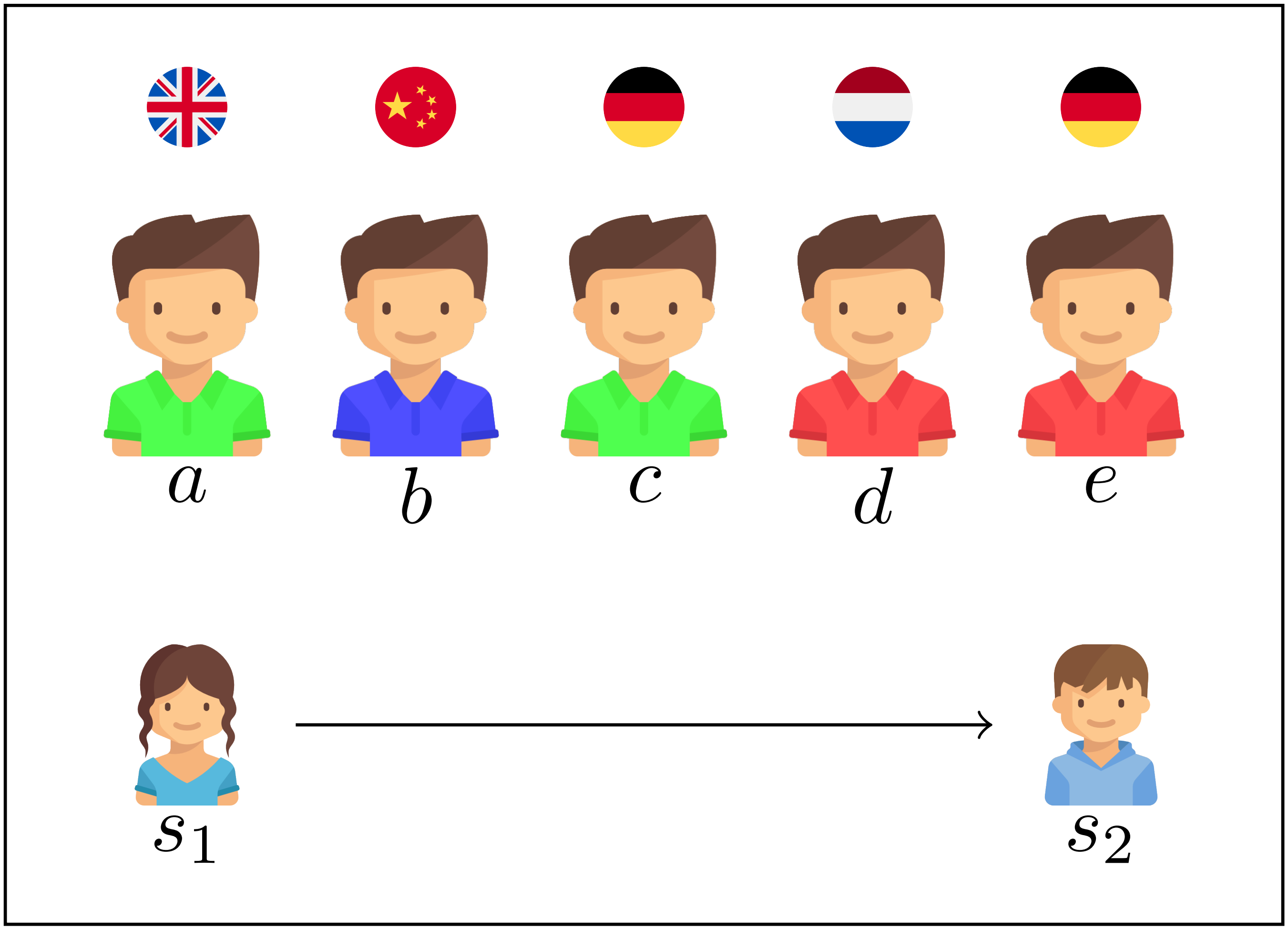

Figure 1. s_1 describes an international student from $\{a, b, c, d, e\}$ to s_2. The person and flag icons in the images of this paper are made by www.freepik.com from www.flaticon.com.

Table 1. Information of the example in Figure1

student	$P(o)$
a	$\{green, British\}$
b	$\{blue, Chinese\}$
c	$\{green, German\}$
d	$\{red, Dutch\}$
e	$\{red, German\}$

2.1 The classical REG problem

Let $\mathcal{O} = \{o_1, \dots, o_n\}$ be a set of objects, and $\mathcal{P} = \{p_1, \dots, p_m\}$ a set of properties. For each $o \in \mathcal{O}$, we shall denote by $P(o) \subseteq \mathcal{P}$ the set of properties satisfied by object o. In the case of our previous example, it is $\mathcal{P} = \{red, blue, green, British, Chinese, German, Dutch\}$, and the information available is shown in Table 1.

A referring expression is a noun phrase whose aim is to univocally identify an object within a collection of objects. Usually, a referring expression takes the form of a conjunction of properties, in which case it is represented as a subset $re \subseteq \mathcal{P}$. In the previous example, the target is student a, and the referring expression could be *The British student with the green sweater*, represented by the set of properties $re = \{green, British\}$.

Table 2. information of the example in Figure2

student	$P(o)$
a	$\{\{green\}, \{British, American\}\}$
b	$\{\{blue, purple\}, \{Chinese\}\}$
c	$\{\{green\}, \{Dutch, German\}\}$
d	$\{\{red\}, \{Dutch\}\}$
e	$\{\{red\}, \{German\}\}$

An expression re is said to have *full referential success* for a certain object o, when re satisfies the aim of univocally identifying o within $\mathcal{O}$. Let $[\![p_i]\!] = \{o_j \in \mathcal{O} : p_i \in P(o_j)\}$. Then, re has full referential success for o iff

$$O_{re} = \bigcap_{p \in re} [\![p]\!] = \{o\} \tag{1}$$

Thus, for example, $re = \{green\}$ has not full referential success for the student a since $O_{re} = \{a, c\}$. Even though student a is in O_{re}, there is another student, c, that is wearing a green sweater, so we need to use additional information in order to successfully identify student a. As an example, the expression $re = \{green, British\}$ has full referential success for a, since $O_{re} = \{a\}$.

In general, the REG problem consists in, given an object o, finding a referring expression re that has *full referential success* for o. This problem has been widely studied in the literature [Dal89; Dee+12; KEV03; KVD12; VD16]. In addition, some extensions to this problem have been proposed, like the reference to sets [KVD12], or the use of gradual properties in the referring expressions[Gat+16; RS+16]. As an example of the latter, the property *red* can be defined in a gradual way, since an object could be considered *red* to a certain degree, and *orange* to another degree.

In this paper, we address another type of uncertainty in the properties, which derives from a lack of knowledge about the properties that hold for some objects.

2.2 Uncertainty in properties

Sometimes, the collection of properties satisfied by a certain object is affected by uncertainty, in the sense that we know that the object satisfies one property, among a given subset of properties, but we do not know exactly which property in the subset is satisfied.

As an example, and following with the scenario in Figure 1, let us suppose that the student s_1 (the transmitter) knows that s_2 (the receiver) is not able to distinguish between blue and purple properly, thereby the sweater of the student b could be *blue or purple* from the point of view of student s_2. In addition, s_1 thinks that s_2 is not able to elucidate whether a person is *British or American* from his/her accent, or whether a person is speaking *German or Dutch*. These facts are depicted in Figure 2. Hence, in addition to individual properties that are known to hold for the considered object (pieces of information about the object without uncertainty), we can have pieces of uncertain information that, in this example, can be represented as *sets of properties*, such sets having a *disjunctive meaning*. Thus, the information about the properties that an object satisfies is no longer represented as a crisp set of individual properties, as in the classical REG problem, but as a crisp set of subsets of properties, each subset representing a (possibly uncertain) available piece of information about the considered object.

Now we have, formally:

$$P(o) = \{S_1, \dots, S_k\} \subset 2^{\mathcal{P}} \backslash \emptyset \tag{2}$$

that is, $S_i \subseteq \mathcal{P}$, and we require additionally that $S_i \nsubseteq S_j \ \forall S_i, S_j \in P(o)$ with $i \neq j$, in order to avoid redundancy by subsumption. For example, we can represent the information in Figure 2 in the form of Table 2, where the properties *purple* and *American* are added to $\mathcal{P}$, since now we consider them as properties that objects in $\mathcal{O}$ can fulfil.

Note that the classical REG problem can be formulated equivalently using only pieces of (certain) information as in Eq. (2). This can be done by considering that all the properties satisfied by an object are represented by

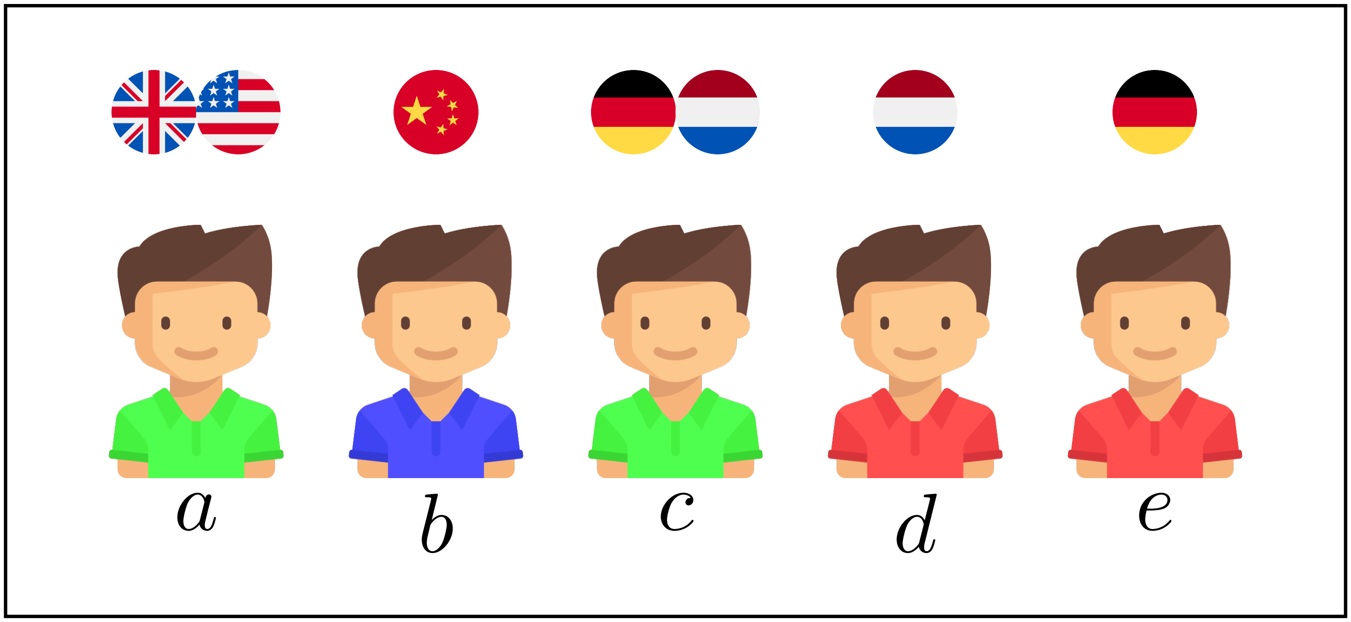

Figure 2. Student s_2 has uncertain information about the nationalities of students a and c, and about the colour of sweater that student b is wearing.

singletons, so that $P(o) = \{S_1, \ldots, S_k\}$ with $S_i = \{p_j\}$, $p_j \in \mathcal{P}$. In such case, $S_i \nsubseteq S_j$ holds trivially, since it is equivalent to $S_i \neq S_j$ when all S_i are singletons, and this property is guaranteed by the initial definition we gave in this paper of $P(o)$ as a set. Referring expressions can be also equivalently represented as a set of singletons from $2^{\mathcal{P}}$ in the same way.

Finally, the notion of full referential success can be maintained as in Eq. (1) by considering $[\![p_i]\!] = \{o_j \in \mathcal{O} : \{p_i\} \in P(o_j)\}$, which completes our equivalent formalization of the classical REG problem, and shows that this problem can be seen as a particular case of the problem of generating referring expressions under uncertainty.

Once we have shown that the new formulation can be used for the classical REG problem, let us explain how it can be used in order to solve the REG problem, when we consider uncertain pieces of information in $P(o)$. First, it is obvious that the uncertainty in the information induces, in general, uncertainty in the set of objects that satisfy a given referring expression. In order to represent this uncertainty about the objects, let us introduce the following set-valued functions on $2^{\mathcal{P}} \backslash \emptyset$:

$$[\![S]\!]^{\Pi} = \{o_j \in \mathcal{O} : \exists S' \in P(o_j) \text{ such that } S' \cap S \neq \emptyset\} \tag{3}$$

$$[\![S]\!]^{N} = \{o_j \in \mathcal{O} : \exists S' \in P(o_j) \text{ such that } S' \subseteq S\} \tag{4}$$

The set $[\![S]\!]^{\Pi}$ represents the collection of objects that *possibly* satisfy S, whilst the set $[\![S]\!]^{N}$ represents the collection of objects that *necessarily* satisfy S, according to possibility theory, and it is $[\![S]\!]^{N} \subseteq [\![S]\!]^{\Pi}$.

For instance, for the property $S = \{Dutch, German\}$, according to the information in Figure 2, we have that $[\![S]\!]^{\Pi} = \{c, d, e\}$ and $[\![S]\!]^{N} = \{c, d, e\}$. Although in this particular example $[\![S]\!]^{\Pi} = [\![S]\!]^{N}$, it is easy to find examples where the inclusion is proper. As an example, when $S = \{Dutch\}$ we have $[\![S]\!]^{\Pi} = \{c, d\}$ and $[\![S]\!]^{N} = \{d\}$.

It is easy to show that in the classical REG framework (that is, if we consider that all pieces of information are represented by singletons), it is $S' \cap S \neq \emptyset$ iff $S' = S$ iff $S' \subseteq S$, and hence $[\![S]\!]^{\Pi} = [\![S]\!]^{N} = [\![S]\!] = [\![\{p\}]\!] = [\![p]\!]$ for $S = \{p\}$.

Let us now consider that the extended form of a referring expression is $re \subset 2^{\mathcal{P}} \backslash \emptyset$, satisfying the same conditions that we impose for $P(o)$. Using the above-defined functions, we can define two different sets:

$$O^{\Pi}_{re} = \bigcap_{S \in re} [\![S]\!]^{\Pi} \tag{5}$$

Table 3. Possible and necessary referents of different referring expressions according to Table 2

re	O^{Π}_{re}	O^{N}_{re}
$\{\{Dutch\}\}$	$\{c, d\}$	$\{d\}$
$\{\{German\}\}$	$\{c, e\}$	$\{e\}$
$\{\{Dutch, German\}\}$	$\{c, d, e\}$	$\{c, d, e\}$
$\{\{green\}, \{Dutch\}\}$	$\{c\}$	$\emptyset$
$\{\{green\}, \{German\}\}$	$\{c\}$	$\emptyset$
$\{\{green\}, \{Dutch, German\}\}$	$\{c\}$	$\{c\}$
$\{\{blue\}\}$	$\{b\}$	$\emptyset$
$\{\{purple\}\}$	$\{b\}$	$\emptyset$
$\{\{blue, purple\}\}$	$\{b\}$	$\{b\}$

Table 4. Full referential success of the referring expressions for the students in Figure 2

re	a	b	c	d	e
$\{\{Dutch\}\}$	0	0	0	0	0
$\{\{German\}\}$	0	0	0	0	0
$\{\{Dutch, German\}\}$	0	0	0	0	0
$\{\{green\}, \{Dutch\}\}$	0	0	0	0	0
$\{\{green\}, \{German\}\}$	0	0	0	0	0
$\{\{green\}, \{Dutch, German\}\}$	0	0	**1**	0	0
$\{\{blue\}\}$	0	0	0	0	0
$\{\{purple\}\}$	0	0	0	0	0
$\{\{blue, purple\}\}$	0	**1**	0	0	0

$$O^{N}_{re} = \bigcap_{S \in re} [\![S]\!]^{N} \tag{6}$$

where S represents each piece of information of re. Then, we call *possible referent of* re to each $o \in O^{\Pi}_{re}$, so O^{Π}_{re} is the set of objects that possibly fulfil the information in re. On the other hand, each $o \in O^{N}_{re}$ is named *necessary referent of* re, since each element of O^{N}_{re} necessarily satisfies the information of re. It is clear that, according to the definitions, $O^{N}_{re} \subseteq O^{\Pi}_{re}$. Also, in the classical case it is $O^{\Pi}_{re} = O^{N}_{re} = O_{re}$ since, as we have shown, in that case it is $[\![S]\!]^{\Pi} = [\![S]\!]^{N}$.

Additionally, using the above defined sets, we can extend the definition of full referential success in Equation (1) to a setting with uncertainty (in the sense of this paper): a referring expression re has full referential success for an object o iff

$$O^{N}_{re} = O^{\Pi}_{re} = \{o\} \tag{7}$$

In other words, o is a necessary referent, and there is no other possible referent in $\mathcal{O}$. Note that, since in the classical case it is $O_{re} = O^{N}_{re} = O^{\Pi}_{re}$, this definition generalizes the classical definition of full referential success in Eq. (1).

In Table 3, we compute the set of possible and necessary referents of different referring expressions, according to the information of the example in Figure 2. Table 4 summarizes whether those referring expressions have full referential success (1) or not (0), for each student in Figure 2.

As can be seen, as new pieces of information are added to the referring expression, the number of possible referents decreases or remains the same, as expected. The same happens to the number of necessary referents, always preserving the inclusion restriction.

2.3 Representing uncertainty as possibility distributions

In the previous section, we consider uncertainty represented by a crisp version of possibility theory [DNP00], that is, a conventional disjunction of possible values. This idea can be generalized using *graded* possibility distributions. This way, a more powerful model is available for those contexts that require a gradual management of uncertainty in the sense of [DNP00].

Possibility distributions, represented by fuzzy sets with a disjunctive use, can be obtained in different ways. For example, we could get different descriptions of a piece of information for the same object from different sources, each description represented by a crisp set (either a singleton or a non-empty set with a disjunctive meaning), and we need to "aggregate" the information from those sources. One possibility is to consider the union of all descriptions into a set representing the piece of information (a situation that can be dealt with by using our proposal in the previous section). Another alternative, able to provide a more informative aggregation, is to consider a possibility distribution, where the frequency of appearance of each individual property, in the set of descriptions, is interpreted as the possibility that the property is the actual value for the object. We shall see an example of this alternative later.

As additional examples, possibility distributions can arise from a single information source that produces different graded alternatives in a natural way due to its measuring mechanisms or, when the source of information is a person, due to his/her ability to provide values to each alternative, that can be interpreted in this case as the degree to which the source (e.g. a person) beliefs that the alternative is possible.

Whatever the origin of the degrees of uncertainty, we have formally:

$$P(o) = \{\tilde{S}_1, \dots, \tilde{S}_k\} \subset [0,1]^{\mathcal{P}} \backslash \emptyset \tag{8}$$

where $\tilde{S}_i$ is a fuzzy subset of $\mathcal{P}$, representing a possibility distribution, with the following requirements:

- We assume that each $\tilde{S}_i$ is a normal fuzzy set, that is, there is always at least one property that is fully possible.
- As in the crisp case, we require $\tilde{S}_i \nsubseteq \tilde{S}_j$, $\forall \tilde{S}_i, \tilde{S}_j \in P(o)$ with $i \neq j$.

Note that Eq. (2) is a particular case of Eq. (8). Now we generalize our set-valued functions to the case $[0,1]^{\mathcal{P}} \backslash \emptyset$:

$$[\![\tilde{S}]\!]^{\Pi}(o) = \max_{\tilde{S}' \in P(o)} \max_{p \in \mathcal{P}} \min\{\tilde{S}'(p), \tilde{S}(p)\} \tag{9}$$

$$[\![\tilde{S}]\!]^{N}(o) = \max_{\tilde{S}' \in P(o)} \min_{p \in \mathcal{P}} \max\{1 - \tilde{S}'(p), \tilde{S}(p)\} \tag{10}$$

Note that the previous equations represent the possibility, $[\![\tilde{S}]\!]^{\Pi}(o)$ (resp. necessity $[\![\tilde{S}]\!]^{N}(o)$), that object o satisfies the property represented by the possibility distribution $\tilde{S}$, according to possibility theory. These equations give the same result as Eqs. (3) and (4) when the possibility distributions are represented by crisp sets, so they generalize the proposal in the previous section. Following with the generalization, we can define the following fuzzy sets:

$$O^{\Pi}_{re}(o) = \min_{\tilde{S} \in re} [\![\tilde{S}]\!]^{\Pi}(o) \tag{11}$$

$$O^{N}_{re}(o) = \min_{\tilde{S} \in re} [\![\tilde{S}]\!]^{N}(o) \tag{12}$$

that correspond to the degree of possibility $O^{\Pi}_{re}(o)$ and necessity $O^{N}_{re}(o)$ that object o is a referent for re. These equations have similar semantics to those of Eqs. (5) and (6), and reduce to them in the case of crisp possibility distributions.

In this paper, we are going to use examples where there are not degrees in the referring expressions, therefore we shall keep $re \subset 2^{\mathcal{P}} \backslash \emptyset$ as in the previous section. For that particular case, we can redefine the set-valued functions in Eqs. (9) and (10) as follows:

$$[\![S]\!]^{\Pi}(o) = \max_{\tilde{S}' \in P(o)} \max_{p \in S} \tilde{S}'(p) \tag{13}$$

$$[\![S]\!]^{N}(o) = \max_{\tilde{S}' \in P(o)} \min_{p \notin S} (1 - \tilde{S}'(p)) \tag{14}$$

assuming in Eq. (14) that $S \neq \mathcal{P}$, being $[\![S]\!]^{N}(o) = 1$ otherwise.

Additionally, we can extend the notion of referential success in Eq. (7) to the case where we have graded disjunctive information: a referring expression re has referential success for an object o to a degree given by:

$$rs^{\Pi N}(re, o) = \min\{rs^{\Pi}(re, o), rs^{N}(re, o)\} \tag{15}$$

where rs^{Π} (resp. rs^{N}) represents any fuzzy referential success measure, that were first proposed in [MRGS16a], calculated using the fuzzy set O^{Π}_{re} (resp. O^{N}_{re}). That is, we consider *degrees of referential success*, that can be calculated using Eq. (15) on the basis of any measure of referential success (we shall see an example later).

We can show that the notion of *full referential success* that we have been employing in this paper, which is not gradual but crisp, can be formulated in terms of any measure of referential success $rs^{\Pi N}(re, o)$:

Proposition 2.1
A referring expression re has full referential success for o iff $rs^{\Pi N}(re, o) = 1$ for any fuzzy referential success measure $rs^{\Pi N}$.

Proof: By definition, it is obvious that $rs^{\Pi N}(re, o) = 1$ iff $rs^{\Pi}(re, o) = rs^{N}(re, o) = 1$ and, by the properties of any fuzzy referential success measure [MRGS16b; Mar+18], it is $rs(re, o) = 1$ iff $O_{re} = \{o\}$. Hence, $rs^{\Pi N}(re, o) = 1$ iff $rs^{N}(re, o) = rs^{\Pi}(re, o) = 1$ iff $O^{N}_{re} = O^{\Pi}_{re} = \{o\}$, which corresponds to the definition of full referential success according to Eq. (7). ∎

In previous works, we have shown that there are infinite measures of referential success. Among them, an important family of measures can be derived from different measures of specificity [MRGS16b; Mar+18]. As an example, given a referring expression re, consider that $\alpha_1 \geq \cdots \geq \alpha_k$ are the membership degrees of objects in O_{re} in nonincreasing order. Then

$$Sp(O_{re}) = \alpha_1 - \alpha_2 \tag{16}$$

is a specificity measure for O_{re}. The corresponding referential success measure is defined as follows:

$$rs(re, o) = min(O_{re}(o), Sp(O^{*}_{re})) \tag{17}$$

where O^{*}_{re} is obtained by normalizing O_{re} (i.e., by dividing all degrees in O_{re} by the maximum degree α_1).

2.4 Examples with graded information

In Table 5, we present an example with information regarding nationalities affected by graded uncertainty. In this table, some degrees represent the transmitter beliefs about the nationalities of the students, according to their language. In Table 5, we can see that, for instance, the transmitter is almost convinced that student a is British, but she also thinks that he may be American. In the same table, graded uncertainty about the sweater's colour is also presented, with the same interpretation of degrees.

In Tables 6, 7 and 8, we compute the sets O^{Π}_{re} and O^{N}_{re}, and the degree of referential success, respectively, for different referring expressions according to the information in Table 5. As can be seen, the presence of graded uncertainty produces fuzzy versions of the set of possible and necessary referents, and also a graded assessment in the computation of the referential success.

In order to show the influence of degrees in the final results, let us consider a slight modification regarding the available information about student b in Table 5. Let us suppose that in the classroom there are n Spanish students, and only one of them is not able to distinguish blue from purple. In addition, the transmitter that

Table 5. Graded information of the example in Figure 2

student	$P(o)$
a	$\{^{1.0}/_{green},\ ^{0.6}/_{American} + {}^{1.0}/_{British}\}$
b	$\{^{1.0}/_{blue} + {}^{1.0}/_{purple},\ ^{1.0}/_{Chinese}\}$
c	$\{^{1.0}/_{green},\ ^{0.7}/_{Dutch} + {}^{1.0}/_{German}\}$
d	$\{^{1.0}/_{red},\ ^{1.0}/_{Dutch}\}$
e	$\{^{1.0}/_{red},\ ^{1.0}/_{German}\}$

Table 6. Membership degree of the students in Figure 2 to O^{Π}_{re} for different referring expressions

re	a	b	c	d	e
$\{\{Dutch\}\}$	0	0	0.7	1.0	0
$\{\{German\}\}$	0	0	1.0	0	1.0
$\{\{Dutch, German\}\}$	0	0	1.0	1.0	1.0
$\{\{green\}, \{Dutch\}\}$	0	0	0.7	0	0
$\{\{green\}, \{German\}\}$	0	0	1.0	0	0
$\{\{green\}, \{Dutch, German\}\}$	0	0	1.0	0	0
$\{\{blue\}\}$	0	1.0	0	0	0
$\{\{purple\}\}$	0	1.0	0	0	0
$\{\{blue, purple\}\}$	0	1.0	0	0	0

has to produce the referring expression does not know who, among the Spanish students in the class, will be the receiver. According to this situation, we can define the aggregated information about the foreign student b as $P(b) = \{^1/_{Chinese},\ ^1/_{blue} + {}^{1/n}/_{purple}\}$. Note that, in this case, uncertainty arises from the mentioned fact that the transmitter does not know which of the students will be the receiver. In this situation, we can obtain the possibility for a certain colour according to the probability that the colour is in the set of colours associated to a Spanish student, that is taken at random in the classroom.

It is evident that, depending on the value of n, we should expect different results. For instance, when $n = 100$ our available information is $P(b) = \{^1/_{Chinese},\ ^1/_{blue} + {}^{0.01}/_{purple}\}$, whilst for $n = 3$ the information can be approximated as $P(b) = \{^1/_{Chinese},\ ^1/_{blue} + {}^{0.33}/_{purple}\}$. It is immediate that the information in Table 5 would correspond to the case $n = 1$. Tables 9, 10 and 11 show the sets O^{Π}_{re} and O^{N}_{re}, and the degree of referential success, respectively, for $n = 100$. The same information is provided in Tables 12, 13 and 14, for $n = 3$. As can be seen in the tables, degrees vary significantly, providing the intuitively expected results.

3 Conclusions and future work

In this paper, we have proposed an approach to manage uncertainty which derives from a lack of knowledge in the referring expression generation (REG) problem. We have considered uncertainty in the sense that we know that the object satisfies one property among a given subset of properties, but we do not know exactly which property in the subset is satisfied; additionally, we may have possibility degrees associated to the alternatives in the subset of properties.

This work starts a new research branch with different open problems. On the one hand, we would like to study the use of graded disjunctive information in the referring expression itself, with particular attention to the possible semantics of these degrees. On the other hand, we have to analyse the process of combination of different sources of information into a single piece of disjunctive information. Finally, we shall work on the development of REG algorithms on the basis of the concepts and measures introduced here.

Table 7. Membership degree of the students in Figure 2 to O^N_{re} for different referring expressions

re	a	b	c	d	e
$\{\{Dutch\}\}$	0	0	0	1.0	0
$\{\{German\}\}$	0	0	0.3	0	1.0
$\{\{Dutch, German\}\}$	0	0	1.0	1.0	1.0
$\{\{green\}, \{Dutch\}\}$	0	0	0	0	0
$\{\{green\}, \{German\}\}$	0	0	0.3	0	0
$\{\{green\}, \{Dutch, German\}\}$	0	0	1.0	0	0
$\{\{blue\}\}$	0	0	0	0	0
$\{\{purple\}\}$	0	0	0	0	0
$\{\{blue, purple\}\}$	0	1.0	0	0	0

Table 8. Referential success of different referring expressions according to the disjunctive graded information in Table 5

re	a	b	c	d	e
$\{\{Dutch\}\}$	0	0	0	0.3	0
$\{\{German\}\}$	0	0	0	0	0
$\{\{Dutch, German\}\}$	0	0	0	0	0
$\{\{green\}, \{Dutch\}\}$	0	0	0	0	0
$\{\{green\}, \{German\}\}$	0	0	0.3	0	0
$\{\{green\}, \{Dutch, German\}\}$	0	0	1.0	0	0
$\{\{blue\}\}$	0	0	0	0	0
$\{\{purple\}\}$	0	0	0	0	0
$\{\{blue, purple\}\}$	0	1.0	0	0	0

Table 9. Membership degree of the students in Figure 2 to O^{Π}_{re} for different referring expressions for $n = 100$

re	a	b	c	d	e
$\{\{blue\}\}$	0	1.0	0	0	0
$\{\{purple\}\}$	0	0.01	0	0	0
$\{\{blue, purple\}\}$	0	1.0	0	0	0

Table 10. Membership degree of the students in Figure 2 to O^N_{re} for different referring expressions for $n = 100$

re	a	b	c	d	e
$\{\{blue\}\}$	0	0.99	0	0	0
$\{\{purple\}\}$	0	0	0	0	0
$\{\{blue, purple\}\}$	0	1.0	0	0	0

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Table 11. Referential success for $n = 100$

re	a	b	c	d	e
$\{\{blue\}\}$	0	0.99	0	0	0
$\{\{purple\}\}$	0	0	0	0	0
$\{\{blue, purple\}\}$	0	1.0	0	0	0

Table 12. Membership degree of the students in Figure 2 to O_{re}^{Π} for different referring expressions for $n = 3$

re	a	b	c	d	e
$\{\{blue\}\}$	0	1.0	0	0	0
$\{\{purple\}\}$	0	0.33	0	0	0
$\{\{blue, purple\}\}$	0	1.0	0	0	0

Table 13. Membership degree of the students in Figure 2 to O_{re}^{N} for different referring expressions for $n = 3$

re	a	b	c	d	e
$\{\{blue\}\}$	0	0.66	0	0	0
$\{\{purple\}\}$	0	0	0	0	0
$\{\{blue, purple\}\}$	0	1.0	0	0	0

Table 14. Referential success for $n = 3$

re	a	b	c	d	e
$\{\{blue\}\}$	0	0.66	0	0	0
$\{\{purple\}\}$	0	0	0	0	0
$\{\{blue, purple\}\}$	0	1.0	0	0	0

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