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Interference Bound for Local Channel Allocation

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Abstract A lower bound of the total co-channel interference is proposed for the channel allocation problem when applied to a reduced set of nodes. The rest of the network nodes remain unaffected. This bound is independent of the particular channel allocation algorithm employed and no assumptions are made about the propagation model or the deployment scenario. Assuming that the bound is tight to the interference generated by the optimal channel allocation, its computation may help, for example, to estimate the minimum set of nodes for which channel allocation performs nearly-optimal while minimizing node reconfigurations. Another example of usage is the estimation of the minimum number of channels required for a given performance. The tightness of the proposed bound is evaluated through simulations, with a difference lower than 1% in the conducted simulations. In addition, a sample use case -*adaptive local channel allocation*- is also provided.

Keywords Spectrum etiquette · Channel allocation · Interference · Max k-cut

1 Introduction

The exponential mobile data traffic increase [1] and the unprecedent growth in Wi-Fi hotspots [2] have accentuated the spectrum shortage in both licensed and unlicensed bands. This growing scarcity of the available spectrum demands an efficient channel assignment to increase spectrum utilization while avoiding interference among closely located nodes.

In this paper we derive a lower bound of the total co-channel interference when channel allocation is performed in a reduced set of network nodes. The rest of the network nodes remain unaffected. Notably, the obtained bound is independent of the particular channel allocation algorithm employed. Our simulations show (see Sect. 6) that the proposed computation is very tight to the total interference with

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optimal channel allocation. Thus, it may be used to estimate how far any channel allocation algorithm is from the optimal solution. Additionally, it may be used to calculate the minimum number of nodes so that channel allocation performs nearly-optimal. Another example is the estimation of the minimum number of channels required in a certain region to achieve a given performance.

It is noteworthy that, as the problem of channel allocation has been formulated taking into account interference measurements (see Sect. 3), no complex performance metrics or Radio Environment Maps [3] are required for its application.

Althought our system model makes no assumptions about the particular scenario, path loss model or radio technology, we compare our theoretical lower bound to the aggregated co-channel interference achieved by the optimal channel assignment in a dual stripe scenario [4] (typical scenario for small cell deployments). Simulation results show that the derived lower bound is very tight. Additionally, it is also shown that, when a new node is deployed, the optimal local channel allocation -applied to a small set of nodes- produces marginal degradation compared to the optimal global channel allocation -applied to the whole network-.

Finally, a sample use case -*adaptive local channel allocation*- is presented and evaluated as an example of using the proposed interference bound. When a new node is deployed, we utilize the channel allocation proposed in [5] but only performed over a small set of nodes. The size of this set is dynamically computed based on our proposed bound. The previous conclusion for optimal channel allocation also holds in this case, i.e. the performance of both global and local channel allocation procedures are very similar.

To the best of our knowledge, there are no other works that propose a lower bound of the aggregated co-channel interference for local channel allocation. Thus, we believe that our proposal is novel and may be helpful 1) to assess the optimality of existing or new channel allocation solutions when applied to a restricted region, and 2) to produce an online feedback that can be incorporated to improve those solutions.

The rest of the paper is organized as follows. Sect. 2 presents the system model. The local channel allocation problem is formulated in Sect. 3. Sect. 4 summarizes the related work. Sect. 5 explains how a lower bound of the total co-channel interference for the aforementioned problem is derived. Sect. 6 evaluates the tightness of the bound and analyzes the performance of a sample use case. The paper is concluded in Sect. 7.

2 System Model

Let us consider a network which consists of a set $R = \{1, ..., r\}$ of nodes. We do not assume any particular network topology or technology, e.g. set R may be composed of independent Wi-Fi access points, small cells managed by the same operator, or nodes pertaining to a cognitive radio network.

Let us denote the set of available channels for transmission as $K = \{1, ..., k\}$. Binary vector $\bar{x_i} = \{x_{i1}, ..., x_{if}, ..., x_{ik}\}^T$ represents the channel allocation of node i, where $x_{if} \in \{0, 1\}$ indicates whether node i is using channel f or not.

We assume that nodes can listen to the environment, determine the interference temperature of the channels and estimate their neighbors' interference contributions. See definition of interference temperature in [6]. With these contributions, node i can estimate the aggregated level of received interference I_i, which is mathematically described as:

$$I_i = \sum_{\substack{\forall j \in R \\ j \neq i}} P \cdot G_{ij} \cdot \bar{x_i}^T \cdot \bar{x_j} \tag{1}$$

where P is the transmission power, G_{ij} is the link gain from node i to node j, and the scalar product $\bar{x_i}^T \cdot \bar{x_j}$ equals one if nodes i and j transmit on the same channel and zero otherwise. G_{ij} includes the effect of both path loss and shadowing. For simplicity reasons, we assume that G_{ij} is equal to G_{ji}. The aggregated level of received interference I_i is used as an estimation of the co-channel interference that the remaining nodes generate to the coverage area of the i-th node.

Based on the interference temperature measurements, nodes decide on the transmission channel. By distributively selecting a transmitting frequency, the radios effectively construct a channel reuse distribution map with reduced co-channel interference.

The considered system model makes no assumptions about the propagation model or the deployment scenario at all. However, our performance evaluation employs the path loss model and the dual stripe scenario defined in [4], as a typical environment for small cell deployment.

3 Problem Formulation: Local Channel Allocation

We formulate the channel allocation problem as an optimization problem to minimize the overall system interference, since this metric has a major impact on throughput and coverage. We assume that each node utilizes one radio, which is applicable to many wireless technologies such as IEEE 802.11, UMTS/HSPA or LTE. In addition, the problem may be generalized to several radios by adding several co-located nodes with one radio.

Whenever a new node n turns on [1], the interference suffered by any node i can be estimated using Eq. (1). The total co-channel interference in the system can be expressed as:

$$CI = \sum_{\forall i \in R} I_i = \sum_{\forall i \in R} \sum_{\substack{\forall j \in R \\ j \neq i}} P \cdot G_{ij} \cdot \bar{x_i}^T \cdot \bar{x_j} = 2 \cdot \sum_{\substack{\forall (i,j) \in R \\ i < j}} P \cdot G_{ij} \cdot \bar{x_i}^T \cdot \bar{x_j} \tag{2}$$

assuming that $G_{ij} \cdot \bar{x_i}^T \cdot \bar{x_j} = G_{ji} \cdot \bar{x_j}^T \cdot \bar{x_i}$.

A channel allocation problem can be formulated as the minimization of the total co-channel interference CI:

[1] Although we argue that channel allocation is triggered by a new node being turned on, it is only an example and the problem formulation makes no assumptions about when channel allocation is executed.

$$
\begin{aligned}
&\min_{\bar{x_i}, \forall i \in R} \Big\{ 2 \cdot \sum_{\substack{\forall (i,j) \in R \\ i<j}} P \cdot G_{ij} \cdot \bar{x_i}^T \cdot \bar{x_j} \Big\} \\
&\text{subject to } \sum_{f=1}^{k} x_{if} = 1, x_{if} \in \{0,1\}
\end{aligned} \tag{3}
$$

Hereafter, we denote CI^O the total co-channel interference achieved by the optimal solution of problem (3). We denote problem (3) as global channel allocation.

The minimization problem (3) implies that the deployment of a new node n causes the execution of channel allocation over all previously deployed nodes. Thus, the channel allocation of the new node n will affect its neighbors, which, in turn, may affect their neighbors, and so forth, propagating the changes.

For this reason, we propose a new optimization problem in which only a reduced set of nodes $M = \{1, .., m\}$, $M \subseteq R$, as well as the new node are subject to change their channels. Additionally, only the network nodes belonging to a set $S = \{1, ..s\}$, $S \subseteq R$, are considered as potential interferers. The remaining nodes $\in R \setminus S$ (i.e. that belongs to set R but not to set S) are considered to have a negligible interference impact and are therefore ignored:

$$
\begin{aligned}
&\min_{\bar{x_i}, \forall i \in N} \Big\{ 2 \cdot \sum_{\substack{\forall (i,j) \in S \\ i<j}} P \cdot G_{ij} \cdot \bar{x_i}^T \cdot \bar{x_j} \Big\} \\
&\text{subject to } \sum_{f=1}^{k} x_{if} = 1, x_{if} \in \{0,1\}
\end{aligned} \tag{4}
$$

where $N = M \cup \{n\}$[2]. We assume that set M is composed by the $|M|$ nodes with the highest G_{ni}, which are the $|M|$ nodes that could potentially interfere most to the new node n. Accordingly, we assume that set S is composed by all the nodes for which the product $P \cdot G_{ni}$ does not exceed a certain margin below the noise floor. Hereafter, we denote $CI^O_{N,S}$ the total co-channel interference achieved by the optimal solution of problem (4). We denote problem (4) as local channel allocation.

As in the case of problem (3), problem (4) is also a non-linear optimization problem which cannot be solved in polynomial time unless P=NP [7]. Problem (4) can be solved by exhaustive search if set N is small enough, although not in real time due to its computational complexity.

4 Related work

The literature on channel allocation in wireless and mobile networks is abundant. [8] and [9] give comprehensive surveys on this topic. Among others, several

[2] Although we argue that only the new node and the most interfering nodes are subject to modify their channels, this is only a typical scenario explained for the sake of readability. The problem formulation requires no assumptions about the nodes belonging to sets N and S.

heuristic solutions using local search, tabu search, simulated annealing, genetic algorithms, neural networks, graph theory, and game theory have been proposed to solve this problem.

Algorithms based on local search methods start with an initial solution and try to find better solutions by iteratively doing small moves. For example, in [10] the authors utilize a centralized stochastic local search algorithm to find a channel assignment that minimizes the network interference in wireless mesh networks. For that purpose they developed their algorithm on top of *Kangaroo*, a constraint-based local search system. The *interface constraint* ensures that the number of channels assigned to a node does not exceed the number of radios available. The *network interference* is the number of interfering pairs of links in the channel assignment. The local search tries to minimize the violation of interface constraints or minimize the network interference. Since a binary interference model is used, the performance is only computed in terms of active radio interfaces per node and no other performance indicators -such as interference levels or system throughput- are computed.

Standard local search solutions only allow moves that produce improvements. In order to achieve better local optima, other strategies allow worsening moves.

For example, tabu search allows non-improving moves by modifying the possible new solutions (*neighborhood*) from the current solution. To avoid cycling, the solutions selected in the last iterations are declared *tabu solutions* and cannot be selected again. In [11], the authors present a tabu search algorithm for dynamic spectrum allocation in cellular networks, with the objective of maximizing the operator's reward. Revenue is modeled as a function of the achieved throughput, and cost is proportional to the bandwidth of the spectrum leased to a spectrum broker. A solution is feasible if each cell has at least one frequency block. Starting from an initial solution, a *neighbor* solution that maximizes the reward is selected and added to the tabu list, so it is not used in next iterations. Although results are sound, new cell deployments may lead to the reconfiguration of the whole network.

Simulated annealing (SA) also allows for worsening moves, accepting a new solution if it is an improvement move or with a certain probability. This probability depends on the value of the solution and on a *temperature* parameter, which decreases (*cooling*) in each iteration. In [12] the authors propose a simulated annealing algorithm with a utility function that computes the average effective channel utilization (ECU) of the access point (AP) divided by the number of users connected. The objective of this utility function is to minimize the co-channel interference between APs. This work assumes a constant temperature function that decreases after reaching a steady-state. In order to reduce complexity, the Gibbs sampling technique is used to convert the global optimization problem to a series of local optimization problems. The co-channel interference model assumes that two APs interfere each other if they utilize the same channel and they are within their *interference range*, i.e. the distance between them is lower than a predefined threshold. Therefore, realistic interference levels between APs are not considered and the performance is only evaluated as the average ECU of different variations of the algorithm.

Genetic algorithms start with a whole set of solutions (*generation*), and iteratively builds new generations by recombination and mutations of solutions (*chromosomes*) from the previous generation. [13] presents two evolutionary algorithms to solve the channel assignment problem for IEEE 802.11ac networks. The first algorithm assumes that each gene i represents the channel assigned to the access point i, and one individual is a candidate solution of the channels to be assigned to the APs. Selection is performed using *binary tournament* and *elitism*, ensuring that the best individual is within the population at each generation. The second is a *differential evolution* algorithm, which generates mutant vectors using the weighted difference of random individuals from the population. Results show that the achieved aggregated interference is low compared to some reference solutions. However, this work does not consider dynamic situations in which access points may appear or dissapear from the network.

Artificial neural networks generate new solutions by emulating the behavior of a grid of *neurons* which try to minimize the objective of the problem (*energy function*). In these networks, the neurons are connected (*synapses*) using a set of given weights, and their states change dynamically depending on their neighbor states and these weights. As an example, [14] presents a solution based on competitive Hopfield neural networks for the frequency assignment problem in satellite communications. This work assumes that the frequency band is divided into a number of segments so that every carrier can be described as a collection of consecutive unit segments. Then, the N-carrier M-segment assignment problem is mapped onto a 2-dimensional neural network with $N \times M$ neurons. In this case the energy function is defined to represent the constraints of the segments between satellite systems. Additionally, stochastics dynamics are introduced in order to help the network escape from local optima. The performance evaluation shows that the largest and the total interference levels decrease through the different iterations. However, the scenario proposed in this work is very restrictive since it is composed by only two satellite systems, and the frequency assignment of the first system is fixed.

Graph coloring approaches are mainly based on variations of the *max k-cut* problem, in which network nodes (vertices) are linked by edges. These edges may include a weight, which is related to the co-channel interference between nodes (or to another performance metric). The objective of this problem is to partition the graph so that the aggregated weight of the edges that cross different partitions is maximized. This maximization is equivalent to the minimization of the aggregated weight of the internal edges, which represents the co-channel interference between nodes. [15] includes a survey of graph-based models and algorithms applied to the channel allocation problem. One solution within this category is presented in [16], which proposes a dynamic channel assignment for femtocells networks. This work first uses a graph coloring algorithm to group *femtocell access points* (FAPs) and then dynamically assigns channels according to the channel state of *femtocell user equipments* (FUEs). The interference graph is constructed by adding an edge between FAPs if the ratio between the received power and the interference is lower than a given threshold Γ_{th}. The graph coloring algorithm utilize the saturation degree of a vertex (number of different colors adjacent to the vertex). Based on this coloring algorithm, the *femtocell network controller* (FNC) clusters FAPs into groups. Then another algorithm assigns spare channels to those FAPs with highest interference. Results are given in terms of FUE throughput for different values of Γ_{th} and FUE traffic load. These results highly depend on the threshold used to compute the interference graph, whose optimization depends on the scenario and the propagation model.

Game theoretical solutions assume that network nodes are the players who are trying to maximize their benefit, e.g. to minimize their co-channel interference. Their strategies consist in selecting the best channel for that purpose. There are different kind of games (e.g. based on cooperative and non-cooperative strategies) and they iterate until some kind of equilibrium (e.g. Nash -no possible improvements due to unilateral decisions- or Pareto -no possible improvements without worsening other players- equilibria) is reached. [17] presents an extensive overview about game theory applied to the spectrum sharing problem. One of these solutions [5] is explained in Sect. 6.4, which is used as the basis for our sample use case.

Although these approaches are very interesting, to the best of our knowledge, none of the existing works compare the current aggregated co-channel interference to a theoretical lower bound in order to iteratively improve the channel assignment.

5 A Lower Bound of the Total Co-Channel Interference

In this section we derive a lower bound of the total co-channel interference for the optimization problems (3) and (4).

5.1 Lower Bound for Global Channel Allocation

It is known that the minimization problem (3) is equivalent to the *max k-cut* problem in graph theory [16]. Let us consider an undirected weighted graph $\mathcal{G} = (V, E)$ where the vertex set $V = \{1, ..., r\}$ represents the set of nodes $R = \{1, ..., r\}$, and each edge $(i, j) \in E$ has a weight $\omega_{ij} = P \cdot G_{ij} = P \cdot G_{ji} = \omega_{ji}$. Note that $\mathcal{G} = (V, E)$ is a fully interconnected graph because there exists an associated weight $\omega_{ij}\ \forall i, j \in R$.

Let us define the *External Weight* of a given channel assignment for graph $\mathcal{G}$ as the sum of the weights of all edges with their endpoints in different channels. The *max k-cut* problem aims at finding a partition of the vertex set V into k subsets (i.e. channels) such that External Weight is maximized [18]. This is equivalent to partition V into k subsets such that the sum of the weights of all edges with their endpoints in the same partitions is minimized (i.e. problem (3)). The *max k-cut* problem is mathematically formulated in [18] as the following integer program:

$$\begin{aligned} &\max_{\bar{y_i}, \forall i \in R} \Big\{ 2 \cdot \frac{k-1}{k} \cdot \sum_{\substack{\forall (i,j) \in R \\ i<j}} P \cdot G_{ij} \cdot (1 - \bar{y_i}^T \cdot \bar{y_j}) \Big\} \\ &\text{subject to } \bar{y_j} \in \{\bar{a_1}, \bar{a_2}, ..., \bar{a_k}\} \forall j \end{aligned} \tag{5}$$

where a_i are the vertices of an equilateral simplex σ_k in $\mathbb{R}^{k-1}$ with centroid $c_k = 0$ and scaled so that $|\bar{a_i}| = 1$. The vectors $\bar{a_1}, \bar{a_2}, ..., \bar{a_k}$ fulfill the property $\bar{a_i}^T \cdot \bar{a_j} = -1/(k-1), \forall i \neq j$. Note that the factor $(1 - \bar{y_i}^T \cdot \bar{y_j})$ contributes to the sum if nodes i, j are allocated different channels:

$$1 - \bar{y_i}^T \cdot \bar{y_j} = \begin{cases} 0, & \text{if } \bar{y_i} = \bar{y_j} \\ \frac{k}{k-1}, & \text{if } \bar{y_i} \neq \bar{y_j} \end{cases} \tag{6}$$

Let us express the External Weight of the optimal solution as:

$$EW^O = 2 \cdot \frac{k-1}{k} \sum_{\forall (i,j) \in R, i<j} P \cdot G_{ij} \cdot (1 - \bar{y_i}^T \cdot \bar{y_j}) \tag{7}$$

Additionally, let us define the *Total Weight* of graph $\mathcal{G}$ as the sum of the weights of all edges $(i,j) \in E$. The total weight can be calculated as:

$$TW = 2 \sum_{\forall (i,j) \in R, i<j} P \cdot G_{ij} \tag{8}$$

TW is the sum of the co-channel interference and the external weight for any given solution, e.g. $TW = EW^O + CI^O$.

To solve problem (5), the authors in [18] propose the following positive SemiDefinite Program (SDP) relaxation which can be computed in polynomial time:

$$\begin{aligned} &\max_{\bar{v_i}, \forall i \in R} \Big\{ 2 \cdot \frac{k-1}{k} \cdot \sum_{\substack{\forall (i,j) \in R \\ i<j}} P \cdot G_{ij} \cdot (1 - \bar{v_i}^T \cdot \bar{v_j}) \Big\} \\ &\text{subject to } \bar{v_j} \in \mathbb{S}_{n-1}, \mathbb{S}_{n-1} = \{\bar{v} \in \mathbb{R}^n : |\bar{v}| = 1\} \\ &\qquad\qquad \bar{v_i}^T \cdot \bar{v_j} \geq -1/(k-1) \forall i \neq j \end{aligned} \tag{9}$$

where the products $\bar{v_i}^T \cdot \bar{v_j}$ can be replaced by the components Y_{ij} of a positive semidefinite matrix Y. Due to the relaxation in the SDP program, the solution of (9) provides an upper bound of the External Weight EW^*, i.e.:

$$EW^* = 2 \cdot \frac{k-1}{k} \cdot \sum_{\substack{\forall (i,j) \in R \\ i<j}} P \cdot G_{ij} \cdot (1 - \bar{v_i}^T \cdot \bar{v_j}) \geq EW^O \tag{10}$$

Upper bound EW^* can be used to obtain a lower bound of the co-channel interference in the optimization problem (3):

$$CI^* = TW - EW^* \leq TW - EW^O = CI^O \tag{11}$$

Results in Section 6 show that CI^* lower bound is tight compared to the optimal solution of problem (3).

5.2 Lower Bound for Local Channel Allocation (Problem (4))

Next, we derive the lower bound for the optimization problem (4). The derivation includes additional constrains to the SDP program (9) so channels modifications are restricted to the local set of nodes $\in N$. Additionally, only the nodes belonging to set S are considered as potential interferers.

First, we identify that two nodes $\in S \setminus N$ (i.e. that belongs to set S but not to set N) that are allocated the same channel do not contribute to the External Weight if their associated vectors have an angle separation equal to zero. This imposes the first set of constraints:

$$\forall i,j \in S \setminus N | \bar{x_i}^T \cdot \bar{x_j} = 1 \Rightarrow \bar{y_i}^T \cdot \bar{y_j} = 1 \tag{12}$$

Second, the maximum contribution to the External Weight of two nodes belonging to $S \setminus N$ with different channel allocation occurs when their associated vectors have a scalar product equal to $-1/(k-1)$. This imposes the second set of constraints:

$$\forall i,j \in S \setminus N | \bar{x_i}^T \cdot \bar{x_j} = 0 \Rightarrow \bar{y_i}^T \cdot \bar{y_j} = -1/(k-1) \tag{13}$$

Third, we only consider nodes $\in S$ as potential interferers, i.e. that can contribute to the External and Internal Weights. Hereafter we respectively denote $TW_{N,S}$, $EW_{N,S}$ and $CI_{N,S}$ the Total Weight, the External Weight and the aggregated Co-channel Interference of problem (4).

Hence, we reformulate the SemiDefinite Program (9) by including the constraints (12) and (13) and considering sets S and N:

$$\begin{aligned}
&\max_{\bar{v_i}, \forall i \in S} \Big\{ 2 \cdot \frac{k-1}{k} \sum_{\substack{\forall (i,j) \in S \\ i<j}} P \cdot G_{ij} \cdot (1 - \bar{v_i}^T \cdot \bar{v_j}) \Big\} \\
&\text{subject to } \bar{v_j} \in \mathbb{S}_{n-1}, \mathbb{S}_{n-1} = \{\bar{v} \in \mathbb{R}^n : |\bar{v}| = 1\} \\
&\qquad \bar{v_i}^T \cdot \bar{v_j} \geq -1/(k-1) \forall i,j \in N \\
&\qquad \bar{v_i}^T \cdot \bar{v_j} = 1, \forall i,j \in S \setminus N | \bar{x_i}^T \cdot \bar{x_j} = 1 \\
&\qquad \bar{v_i}^T \cdot \bar{v_j} = -1/(k-1), \forall i,j \in S \setminus N | \bar{x_i}^T \cdot \bar{x_j} = 0
\end{aligned} \tag{14}$$

As in the case of (9), the solution to the SDP program (14) provides an upper bound of the External Weight EW^*_N, i.e:

$$EW^*_{N,S} = 2 \cdot \frac{k-1}{k} \cdot \sum_{\substack{\forall (i,j) \in S \\ i<j}} P \cdot G_{ij} \cdot (1 - \bar{v_i}^T \cdot \bar{v_j}) \geq EW^O_{N,S} \tag{15}$$

where $EW^O_{N,S}$ is the External Weight of the optimal solution of problem (4). $EW^*_{N,S}$ can be used to obtain a lower bound of the co-channel interference in the optimization problem (4):

$$CI^*_{N,S} = TW_{N,S} - EW^*_{N,S} \leq TW_{N,S} - EW^O_{N,S} = CI^O_{N,S} \tag{16}$$

Results in Sect. 6 show that $CI^*_{N,S}$ lower bound is tight -with a difference lower than 1%- compared to the optimal solution of problem (4).

6 Performance evaluation

6.1 Simulation setup

We have implemented a static network simulator in MATLAB that computes the path loss and the interference between nodes in order to evaluate the tightness of the proposed lower bounds of the total co-channel interference. Without loss of generality, the deployment scenario is the dual stripe model defined in [4]. We have selected 15 apartments per stripe, thus totaling 60 apartments. The path loss follows the model defined in [4]. A single node is located inside each apartment with a deployment ratio of 66%, which accounts for 40 nodes. The product $P \cdot G_{ij}$ is computed $\forall(i,j) \in R$, where P=100 mW. Given a node n, set S is composed by all the nodes for which the product $P \cdot G_{ni}$ is at maximum 10 dB below the noise floor (-110dBm).

Unless otherwise stated, each simulation starts with a clean deployment and nodes are switched on, one by one, in a random empty apartment. Each time a new node is deployed, set N is derived and channel allocation is performed. Several simulation campaigns have been carried out with different objectives. 50 snapshots have been simulated for each case and results have been averaged over all snapshots.

6.2 Evaluation of the lower bound CI^*

The purpose of this set of simulations is to evaluate the tightness of the lower bound CI^* of the optimization problem (3), i.e. assuming that all nodes may reconfigure their channels.

Due to the NP-hardness of problem (3), it is not feasible to compute the optimal channel allocation through exhaustive search by testing all possible channel combinations in all the nodes. For that reason, we compute the optimal channel allocation of problem (4) instead, i.e. assuming that only a reduced set of neighbor nodes may reconfigure their channels. Since the channel combination space is reduced, it is possible -although not in real time- to find the optimal solution $CI^O_{N,S}$. In this case, $|N|$ remains constant during the simulation.

Fig. 1 presents the lower bound CI^* and the normalized optimal co-channel interference ($CI^O_{N,S}/TW_{N,S}$) for all 50 snapshots with 3 channels, considering sets M (i.e. not including the node being deployed) of sizes from 0 to 6. Results show that, when $|M|$ is 4 or 6, $CI^O_{N,S}$ is very close to CI^*. Since $CI^* \leq CI^O \leq CI^O_{N,S}, \forall N$, CI^O is also very close to both CI^* and $CI^O_{N,S}$. We have observed a similar trend for the case of 5 channels.

Thus, we can conclude that 1) the lower bound in Eq. (11) is tight to the interference with the optimal solution, i.e. $CI^* \approx CI^O$, and 2) the optimal channel allocation over a small set of nodes produces low degradation compared to the optimal channel allocation over the whole network, i.e. $CI^O_{N,S} \approx CI^O$.

6.3 Evaluation of the lower bound $CI^*_{N,S}$

The aim of this experiment is to evaluate the tightness of the proposed lower bound of the total co-channel interference when channel allocation is carried out in a reduced set of nodes. The conducted simulations vary $|M|$ from 1 to 8 and $|K|$ (available channels) from 2 to 5.

For every network instance, the experiment starts with an initial deployment where a random channel is allocated to all nodes except the new node n. Once the initial allocation is completed, the new node is turned on and an optimal channel allocation is applied to this node and a reduced number of neighbor nodes by

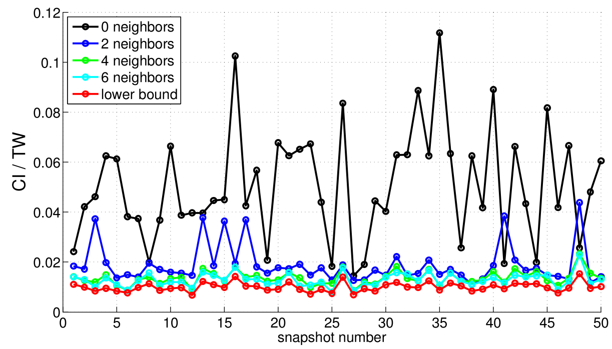

Fig. 1 Normalized optimal co-channel interference $CI^O_{N,S}/TW_{N,S}$ Vs lower bound CI^*/TW using $k = 3$ channels.

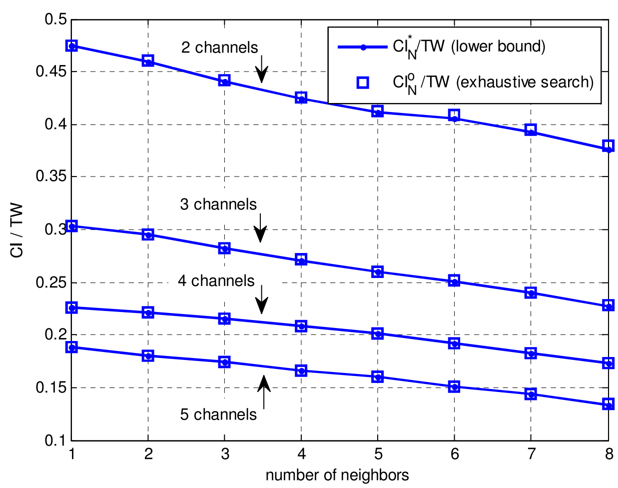

Fig. 2 Optimal allocation $CI^O_{N,S}/TW_{N,S}$ Vs lower bound $CI^*_{N,S}/TW_{N,S}$.

means of an exhaustive search. allowing us to compute $CI^O_{N,S}$. The lower bound of the total co-channel interference $CI^*_{N,S}$ is calculated by solving the SDP program presented in (14).

Fig. 2 shows that the lower bound nearly superimpose on the optimal co-channel interference, i.e. $CI^*_{N,S} \approx CI^O_{N,S}$. The difference is lower than 1% for all the conducted simulations.

6.4 Use case: adaptive local channel allocation

This set of simulations is intended to evaluate a sample use case for the lower bound $CI^*_{N,S}$. With this objective, we propose an adaptive strategy to compute the region over which we shall apply channel allocation. This strategy is independent of the particular channel allocation algorithm employed.

Whenever a new node n is to be deployed, the algorithm starts by selecting the set of nodes $S = \{1, ..s\}$, $S \subseteq R$, for which the product $P \cdot G_{ni}$ does not exceed a given margin below the noise floor. Next, nodes $\in S$ are sorted by descending G_{ni}, i.e. from higher to lower link gain. The resulting set $\hat{S}$ fulfills $\forall \hat{\imath}, \hat{\jmath} \in \hat{S}$, $\hat{\imath} < \hat{\jmath} \Rightarrow G_{n\hat{\imath}} \geq G_{n\hat{\jmath}}$. Then, the algorithm computes the lower bound of the co-channel interference $CI^*_{max_N,S}$ considering max_N number of nodes. This is performed using the SDP program with constraints explained in Section 5. After that, it computes the lower bound $CI^*_{N,S}$ considering fewer nodes, starting from min_N nodes $\in \hat{S}$ and increasing by inc_N. At each step, the calculated $CI^*_{N,S}$ is compared to $CI^*_{max_N,S}$. If the increase between them is lower than a predefined $threshold$, then the current number of nodes is selected. Although nodes are selected using a simple and intuitive criterion -based on potential interference-, this algorithm illustrates that local channel allocation in small regions may achieve near optimal performance. More complex criteria for neighborhood selection are left for further study.

Due to its good performance, we combined this adaptive strategy with the channel allocation proposed by Comaniciu [5]. Based on that work, we utilize a normal form game defined as $\Gamma = \{S, \{X_i\}_{i \in S}, \{U_i\}_{i \in S}\}$, where S is the finite set of players (nodes), x_i is the set of strategies associated to player i (channel selection), x_{-i} is the current strategy profile of its opponents, and $U_i : \mathbb{S} \rightarrow \mathbb{R}$ is the set of utility functions that the players associate with their strategies:

$$
\begin{gathered}
U_i(x_i, x_{-i}) = -\sum_{\substack{j \neq i \\ j \in S}} P \cdot G_{ij} \cdot f(x_i, x_j) \\
\text{where } f(x_i, x_j) = \bar{x_i}^T \cdot \bar{x_j} = \begin{cases} 0, & \text{if } \bar{x_i} \neq \bar{x_j} \\ 1, & \text{if } \bar{x_i} = \bar{x_j} \end{cases}
\end{gathered}
\tag{17}
$$

This utility function is a particular case of Comaniciu's, who demonstrated that the proposed game converges to Nash equilibrium by following a best response dynamic. In order to apply this game to a reduced set of nodes, we utilize the same utility function (i.e. considering the interference from/to all nodes) but only nodes in set N were allowed to change their strategy (channel).

This use case is evaluated by executing a set of simulations with $min_N = 0$, $max_N = 10$, $inc_N = 2$, and $threshold = 5\%$.

Fig. 3 shows that the co-channel interference using this strategy is similar to that of global channel allocation (problem (3)). Notice that the highest degradation, with 7 channels, produces an increase of the normalized co-channel interference from 3.6e-5 to 7.3e-3 meaning that it grows 10log10(7.3 105/3.6 105) = 3.1 dB. Additionally, it achieves significant less interference compared to a random channel assignment for the new node (from 4.5 to 40.9 dB for 2 to 8 channels, respectively). Moreover, the number of reconfigurations is very low: on average,

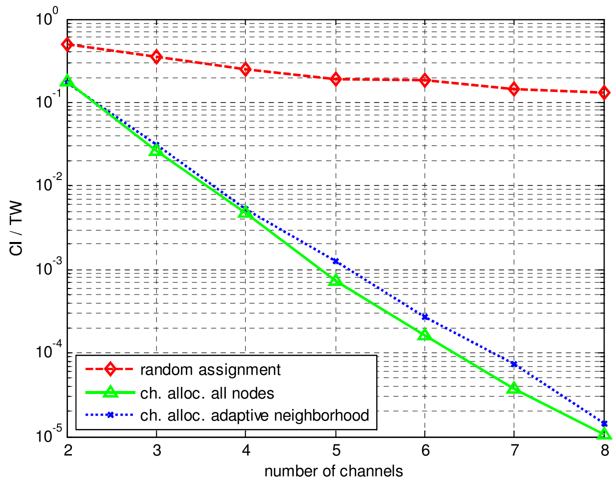

Fig. 3 Comparison of the adaptive solution and other approaches.

less than 0.35 nodes have to change their channels when a new node is deployed, compared to 9.7 nodes in the case of problem (3).

7 Conclusion

In this paper, we have presented a lower bound of the total co-channel interference when channel allocation problem is performed in a reduced set of nodes instead of the whole network. Our lower bound makes no assumptions about scenarios, path loss models or radio technologies and is independent of the particular channel allocation algorithm employed. The bound may be used to estimate the optimality of any solution and, additionally, may be incorporated as an online feedback to improve existing or new channel assignment solutions.

Simulation results have revealed that the proposed lower bound is very tight to the interference of the optimal channel allocation. It is also shown that the optimal channel allocation over a small set of nodes produces similar results compared to the optimal global channel allocation, with a difference lower than 1% in the conducted simulations.

A sample use case -adaptive local channel allocation- has been devised and evaluated by means of simulations. In this sample algorithm, we compute in real time -based on our proposed interference bound- the region in which channel allocation will be applied. This allows operators to reduce the number of node reconfigurations. Results show that the achieved co-channel interference is similar to that of a global channel allocation in which all network nodes may be reconfigured, with a degradation of 3.1 dB in the worst case.

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