Master in Economics · University of Granada

Microeconomics

Teaching Material

Guadalupe Correa-Lopera | University of Granada

Granada, 26 September 2025

Scope & Organization

This document brings together teaching materials prepared for the course *Microeconomics* (Master in Economics, University of Granada).

It covers:

- **Part 1: Consumption Theory**
- **Part 2: Production Theory**

Each chapter includes lecture slides and its corresponding problem set.

Contents

Course: Microeconomics — Master in Economics

PART 1: Consumption Theory

Chapter 1: Consumers' Choice: Preferences and Budget Constraint

Guadalupe Correa-Lopera | University of Granada

Outline

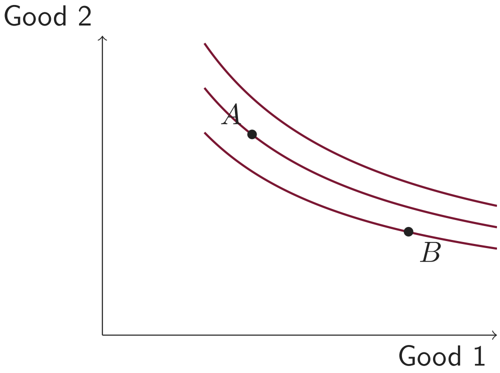
Bundles of Goods

- A **bundle** (or basket) is a list of quantities of each good a consumer might consume.
- With two goods, we write a bundle as (x_1, x_2) where x_1 is the amount of good 1 and x_2 is the amount of good 2.
- Example: Bundle $A = (3, 2)$ means 3 slices of pizza (x_1) and 2 pieces of sushi (x_2); bundle $B = (1, 4)$ means 1 slice of pizza and 4 pieces of sushi.
- Consumers compare bundles to decide which they like more, which they like less, or whether they are indifferent.

Consumer Preferences

- A **preference relation** $\succeq$ compares any two bundles.
- Using $A = (3, 2)$ and $B = (1, 4)$:
  - $A \succeq B$: A is at least as good as B (the consumer weakly prefers A).
  - $A \sim B$: the consumer is indifferent between A and B.
  - $A \succ B$: the consumer strictly prefers A to B.
- The goal is to have a ranking that is defined for any pair of bundles and is internally consistent.

Indifference Curves



- An indifference curve is the set of bundles that are **equally preferred**.
- Curves further to the northeast represent bundles that are **more preferred**.
- The convex shape reflects a taste for balanced baskets over extremes.
- Here, A and B lie on **different** curves, so they are **not** indifferent.

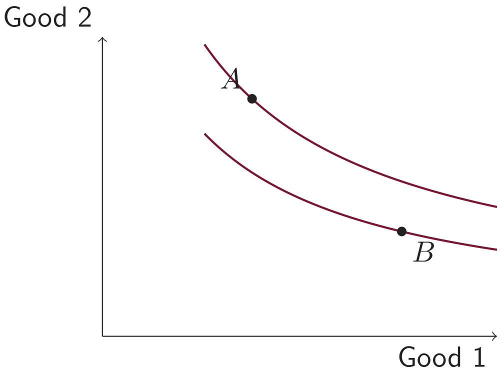
Axioms of Preferences

- **Completeness**: Between any two baskets you can always make a judgment — either you prefer A to B, prefer B to A, or consider them tied. There are no "incomparable" pairs.
- **Transitivity**: Your ranking has no cycles. If you rate A at least as good as B and B at least as good as C, then A must rank at least as good as C; otherwise choices would contradict each other.
- **Monotonicity**: "More is better." If a basket contains at least as much of every good as another and more of at least one good, it is strictly preferred.
- **Continuity**: Tiny changes in quantities do not flip preferences abruptly; evaluations change smoothly with small variations in the goods.

Axioms of Preferences: Statements

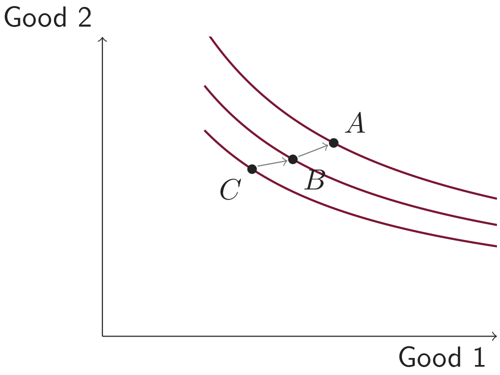
- **Completeness:** For any bundles A and B, either $A \succeq B$ or $B \succeq A$ (or both).
- **Transitivity:** If $A \succeq B$ and $B \succeq C$, then $A \succeq C$.
- **Monotonicity (two goods):** Let $A = (a_1, a_2)$ and $B = (b_1, b_2)$. If $a_1 \geq b_1$, $a_2 \geq b_2$ and at least one inequality is strict, then $A \succ B$.
- **Continuity:** If $A \succeq B$, then any bundle sufficiently close to A is also (weakly) preferred to B.

Completeness (Axioms)



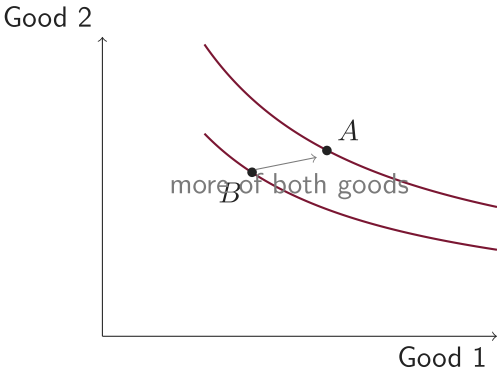
- Any two bundles can be compared.
- In the figure, A lies on a curve located further to the northeast than B, so $A \succ B$.
- If both were on the same curve, then $A \sim B$.

Transitivity (Axioms)

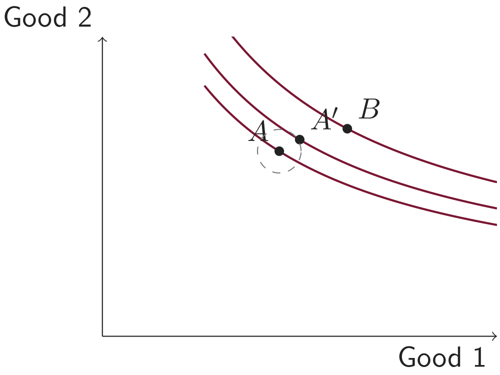


- If A lies on a curve above B, and B lies on a curve above C, then $A \succ B \succ C$.
- Consistency implies $A \succ C$ (no ranking cycles).
- Transitivity keeps the ranking coherent across bundles.

Monotonicity (Axioms)



- A has at least as much of each good as B, and strictly more of one good.
- Then A must lie on a curve located further to the northeast than B.
- Hence, $A \succ B$ ("more is better").



- Preferences vary smoothly: tiny moves around A (e.g., A') do not flip rankings against distant bundles.
- If $B \succ A$, then for any A' sufficiently close to A we still have $B \succ A'$.
- Graphically: B remains on a curve located further to the northeast than both A and A'.

- Let $X \subseteq \mathbb{R}^n_+$ be the consumption set: all bundles with non-negative quantities of n goods.
- A utility function $u : X \to \mathbb{R}$ assigns a number to each bundle in order to represent an existing preference ranking $\succeq$.
- It represents that ranking when, for any bundles $A, B \in X$, if A is at least as preferred as B then $u(A) \geq u(B)$; and conversely, if $u(A) \geq u(B)$ then $A \succeq B$. In particular, $u(A) > u(B)$ implies $A \succ B$, and $u(A) = u(B)$ implies $A \sim B$.
- What matters is the order, not the absolute numbers. For example, $u(A) = 4$ and $u(B) = 2$ does *not* mean that A is "twice as satisfying" as B; it only says that A ranks above B.

Monotone Transformations

- Only the ranking matters, so applying any strictly increasing function f to a utility u leaves the order of bundles unchanged.
- If u represents the preferences and f is strictly increasing, then $v = f \circ u$ represents the same ranking because $u(A) \geq u(B)$ implies $f(u(A)) \geq f(u(B))$.
- As a quick check, take $u(A) = 10$ and $u(B) = 6$; with $f(u) = 3u + 2$ we get $v(A) = 32$ and $v(B) = 20$, and with $f(u) = \sqrt{u}$ (for $u > 0$) we get $v(A) \approx 3.16$ and $v(B) \approx 2.45$, so $A \succ B$ in both cases.
- A decreasing transformation reverses the order and therefore does not represent the same preferences; for instance $f(u) = -u$ gives $v(A) = -10$ and $v(B) = -6$ with $B \succ A$.
- Utility numbers carry only ordinal meaning, so absolute values or ratios should not be interpreted as intensities.

Ordinal vs. Cardinal Utility

- Ordinal utility records only the ranking of bundles, and any strictly increasing transformation of u represents the same preferences.
- Cardinal utility captures intensity so that differences and ratios of utility values have meaning, which is not required for our analysis of consumer choice.
- In what follows we treat utility as ordinal unless explicitly stated otherwise.

Marginal Utility

- We denote the utility function by $u(x_1, x_2)$, which assigns a number to each bundle (x_1, x_2).
- The marginal utility of good i at (x_1, x_2) is the extra satisfaction from an infinitesimal increase in the consumption of x_i, holding the other good fixed.
- Formally, marginal utilities are the partial derivatives of $u(x_1, x_2)$:

$$MU_1(x_1, x_2) = \frac{\partial u(x_1, x_2)}{\partial x_1}, \qquad MU_2(x_1, x_2) = \frac{\partial u(x_1, x_2)}{\partial x_2}.$$

- Near a given bundle (x_1, x_2), a first-order (local) approximation is

$$\Delta u \approx MU_1(x_1, x_2)\,\Delta x_1 + MU_2(x_1, x_2)\,\Delta x_2.$$

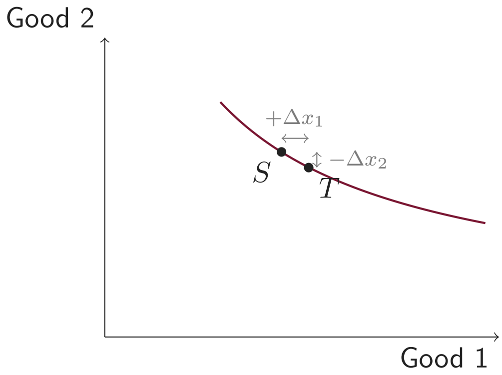
- Example: for $u(x_1, x_2) = x_1^{1/2} x_2^{1/2}$ (Cobb–Douglas), we obtain

$$MU_1 = \tfrac{1}{2}\frac{x_2^{1/2}}{x_1^{1/2}}, \qquad MU_2 = \tfrac{1}{2}\frac{x_1^{1/2}}{x_2^{1/2}}.$$

Diminishing Marginal Utility

- Marginal utilities depend on the current bundle: as the consumption of a good grows while holding the other good fixed, the extra satisfaction from one more unit becomes smaller.
- This property is called **diminishing marginal utility** (DMU). It is a widely used assumption and an empirical regularity in many contexts.
- A simple one–good illustration is $u(x_1) = \sqrt{x_1}$, whose marginal utility $MU_1 = \frac{1}{2\sqrt{x_1}}$ decreases as x_1 increases.
- For two goods, a common specification with DMU is $u(x_1, x_2) = x_1^{\alpha} x_2^{\beta}$ with $0 < \alpha, \beta < 1$, where $MU_1 = \alpha\, x_1^{\alpha-1} x_2^{\beta}$ decreases in x_1 when x_2 is held fixed (and symmetrically for MU_2).
- DMU concerns each good separately.

Indifference Curve: Slope



- Read the slope as "change in Good 2 per unit change in Good 1" along the same indifference curve.
- Discrete reading:

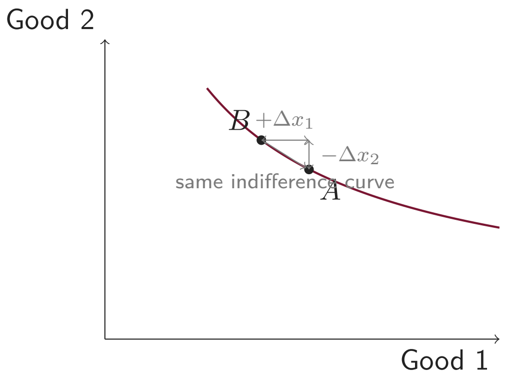
$$\text{slope} \approx \frac{\Delta x_2}{\Delta x_1}.$$

- In the limit (calculus):

$$\text{slope} = \left.\frac{dx_2}{dx_1}\right|_{u=\text{const}} < 0,$$

because increasing Good 1 requires decreasing Good 2 to remain on the same curve.

Marginal Rate of Substitution (MRS)



- The **MRS of good 1 for good 2** is the rate at which the consumer is willing to trade good 2 for a little more of good 1 while staying on the same indifference curve.
- Between B and A, the consumer gives up Δx_2 units of good 2 to gain Δx_1 of good 1 with no change in satisfaction.
- The MRS is measured in "units of good 2 per unit of good 1".

Marginal Rate of Substitution

- Along a given indifference curve the satisfaction level is constant, so the total differential satisfies

$$du = MU_1\, dx_1 + MU_2\, dx_2 = 0.$$

- Solving for the slope along the curve,

$$\left.\frac{dx_2}{dx_1}\right|_{u=\text{const}} = -\frac{MU_1}{MU_2}.$$

- The **MRS of good 1 for good 2** is

$$\text{MRS} = \frac{MU_1}{MU_2} \quad \text{and therefore} \quad \text{MRS} = -(\text{slope of the indifference curve}).$$

- Interpretation: the absolute value of the slope tells how many units of good 2 compensate for one extra unit of good 1 at that bundle.

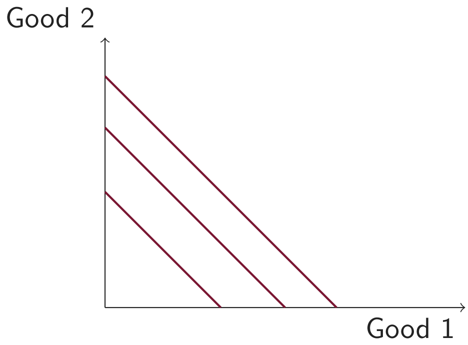
MRS: Numerical Illustration

- Consider $u(x_1, x_2) = x_1^{1/2} x_2^{1/2}$, so

$$MU_1 = \frac{1}{2}\frac{x_2^{1/2}}{x_1^{1/2}}, \qquad MU_2 = \frac{1}{2}\frac{x_1^{1/2}}{x_2^{1/2}}, \qquad \text{MRS} = \frac{MU_1}{MU_2} = \frac{x_2}{x_1}.$$

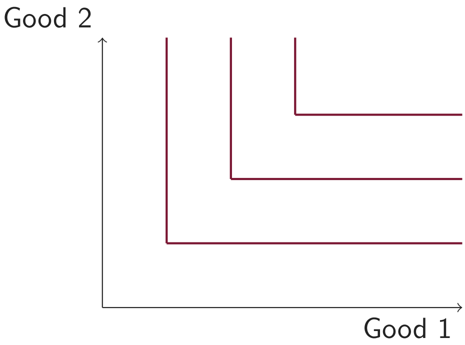
- At $(x_1, x_2) = (4, 2)$ we get MRS $= 0.5$: giving up 0.5 units of good 2 compensates for gaining one unit of good 1 at that point.
- At $(x_1, x_2) = (2, 4)$ we get MRS $= 2$: at that point, the consumer would give up 2 units of good 2 for one more unit of good 1.
- The MRS varies across bundles; it depends on where you are on the indifference curve.

Special Utility Forms: Perfect Substitutes



- The goods replace each other at a fixed rate, so the consumer is willing to swap one for the other at a constant rate.
- Indifference curves are straight, parallel lines with negative slope; curves further to the northeast are strictly preferred.
- A standard utility form is $u(x_1, x_2) = a\,x_1 + b\,x_2$ with $a, b > 0$. In this case, the slope of each indifference curve is constant and equal to $-\frac{a}{b}$.
- The marginal rate of substitution is constant: $\text{MRS} = \frac{a}{b}$.

Special Utility Forms: Perfect Complements



- The goods are consumed together in fixed proportions (e.g., left and right shoes).
- Indifference curves are L-shaped (90° corner) with corners on a ray; curves further to the northeast are strictly preferred.
- A standard utility form is $u(x_1, x_2) = \min\{ax_1, bx_2\}$ with $a, b > 0$; for $a = b = 1$ the corners lie on $x_2 = x_1$.
- Substitution is not desired: along the horizontal arm $MRS = 0$ and along the vertical arm $MRS = \infty$ (undefined at the corner).

Budget Constraint

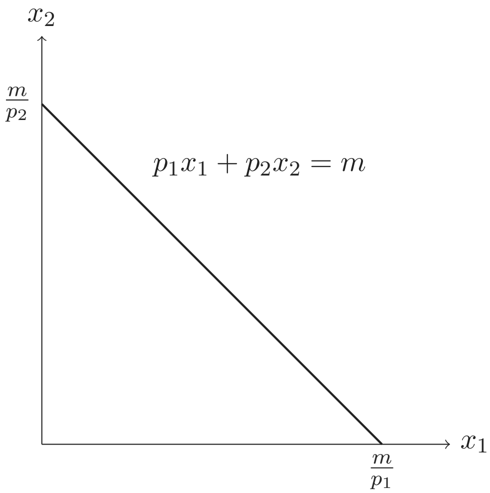
- The **budget constraint** shows all combinations of goods (x_1, x_2) that the consumer can afford given income m and prices p_1, p_2.
- Analytical form:

$$p_1 x_1 + p_2 x_2 \leq m$$

- Where:
  - m: income
  - p_1, p_2: prices of good 1 and good 2
  - x_1, x_2: quantities of good 1 and good 2
- The **budget line** refers to the case where the consumer spends all income:

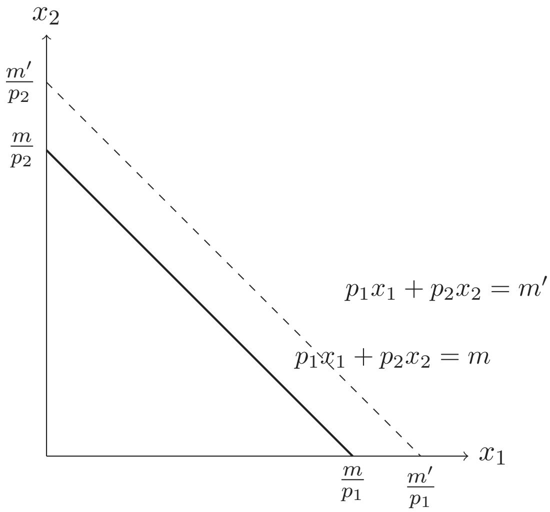
$$p_1 x_1 + p_2 x_2 = m$$

Budget Constraint



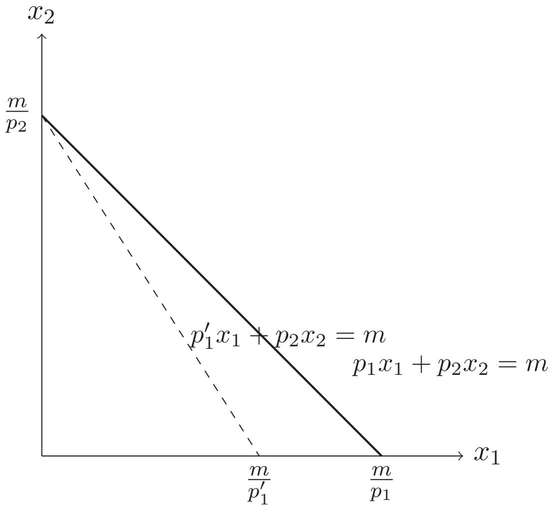
- **Slope:** $-\frac{p_1}{p_2}$
- **Example:** Suppose income and prices are given by $m = 100,\ p_1 = 10,\ p_2 = 20.$
  - Budget line: $10x_1 + 20x_2 = 100.$
  - Intercepts: $x_1 = \frac{100}{10} = 10$, $x_2 = \frac{100}{20} = 5.$
  - Slope: $-\frac{10}{20} = -0.5$

Budget Line: Income Change (Parallel Shift)



- **Definition:** For given prices (p_1, p_2), **increasing** income from m to m' shifts the budget line *outward* in parallel.
- **Intercepts:** move from $\left(\frac{m}{p_1}, 0\right)$ and $\left(0, \frac{m}{p_2}\right)$ to $\left(\frac{m'}{p_1}, 0\right)$ and $\left(0, \frac{m'}{p_2}\right)$.
- **Slope:** unchanged at $-\frac{p_1}{p_2}$ (prices do not change).
- **If $m' < m$:** the shift is inward, also parallel.

Budget Line: Price Change (Rotation)



- **Setup:** Income m and p_2 fixed; consider an **increase** in p_1 (so $p_1' > p_1$).
- **Rotation:** the line *pivots* around the x_2-intercept $\left(0, \frac{m}{p_2}\right)$.
- **Intercepts:** x_2-intercept unchanged $\left(0, \frac{m}{p_2}\right)$; x_1-intercept moves from $\frac{m}{p_1}$ to $\frac{m}{p_1'}$.
- **Slope:** changes from $-\frac{p_1}{p_2}$ to $-\frac{p_1'}{p_2}$.
- **Direction:** with $p_1' > p_1$, the line gets *steeper* (smaller x_1-intercept). If p_1 decreased instead, it would flatten.

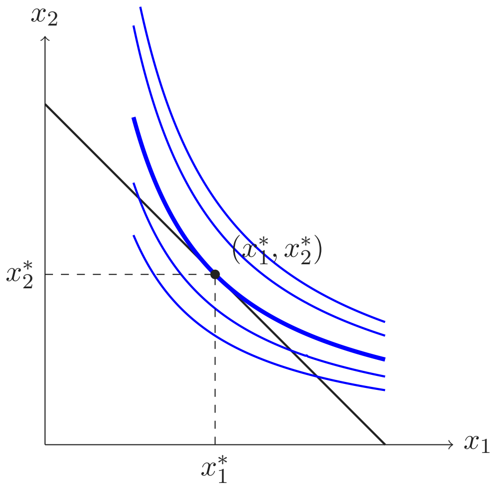
Optimal Choice

- We already have the key elements:
  - **Preferences**: represented by indifference curves
  - **Budget constraint**: represented by the budget line
- **Optimal choice**: the consumer chooses the basket that maximizes satisfaction subject to the budget constraint.

Consumer's Problem

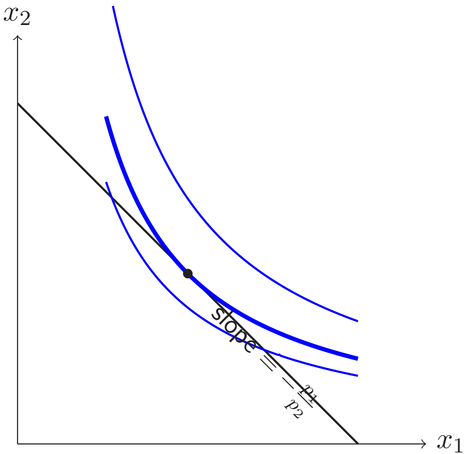
$$\max_{x_1, x_2} U(x_1, x_2) \quad \text{s.t.} \quad p_1 x_1 + p_2 x_2 \leq m$$

Optimal Choice



- The optimal choice occurs at the point of **tangency** between the budget line and the highest attainable indifference curve.
- At this point, the consumer maximizes satisfaction given income and prices.

Optimal Choice



- At the **interior optimum**, the slopes are equal:

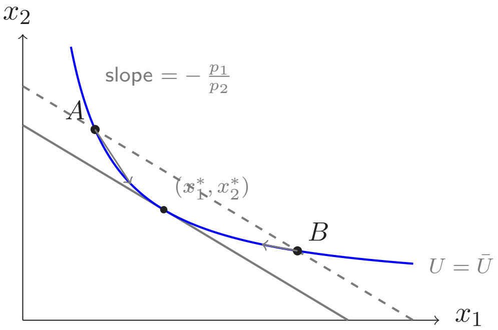
$$-MRS = -\frac{p_1}{p_2}$$

- Which implies:

$$MRS = \frac{p_1}{p_2}$$

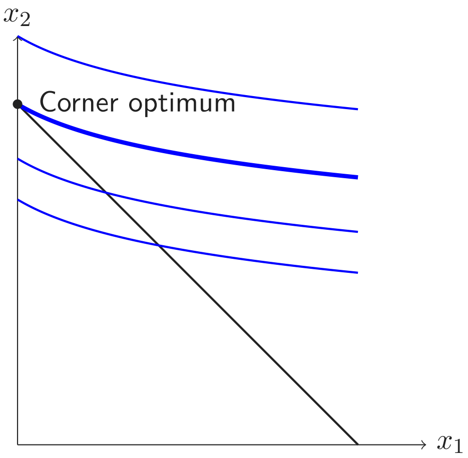
- With $MRS = \dfrac{MU_1}{MU_2} = \dfrac{\partial U/\partial x_1}{\partial U/\partial x_2}$.
- Interpretation: the consumer's willingness to trade goods (MRS) equals the market trade-off (price ratio).

When the Consumer is not at the Optimum



- **At A (IC steeper than budget line):** $-\frac{dx_2}{dx_1}\Big|_{U=\bar{U}} < -\frac{p_1}{p_2} \Leftrightarrow \frac{MU_1}{MU_2} > \frac{p_1}{p_2}$. Good 1 yields more utility per euro: $\frac{MU_1}{p_1} > \frac{MU_2}{p_2}$. Utility rises by moving to more x_1 and less x_2 along $U = \bar{U}$.
- **At B (IC flatter than budget line):** $-\frac{dx_2}{dx_1}\Big|_{U=\bar{U}} > -\frac{p_1}{p_2} \Leftrightarrow \frac{MU_1}{MU_2} < \frac{p_1}{p_2}$. Good 2 yields more utility per euro: $\frac{MU_1}{p_1} < \frac{MU_2}{p_2}$. Utility rises by moving to less x_1 and more x_2 along $U = \bar{U}$.
- **Only at tangency:** $\frac{MU_1}{MU_2} = \frac{p_1}{p_2} \Leftrightarrow \frac{MU_1}{p_1} = \frac{MU_2}{p_2}$.

Corner Solution



- A **corner solution** occurs when the optimal bundle is located at a vertex of the budget line (all income on a single good).
- **Here:** the optimum is at $(0, \frac{m}{p_2})$, consuming only good 2. The highest attainable IC *touches* the BL only at that point.
- **Condition:** the IC is *flatter* than the BL along the feasible set:

$$-MRS \geq -\frac{p_1}{p_2} \iff MRS \leq \frac{p_1}{p_2}.$$

- **Intuition:** the consumer values good 2 relatively more than good 1, so the best affordable choice is to spend all income on x_2.

Mathematical Solution to the Consumer's Problem

Consumer's problem:

$$\max_{x_1,x_2} u(x_1,x_2) \quad \text{s.t.} \quad p_1x_1 + p_2x_2 \leq m$$

Lagrangian:

$$\mathcal{L} = u(x_1,x_2) - \lambda\left(p_1x_1 + p_2x_2 - m\right)$$

First-Order Conditions (FOCs):

$$\frac{\partial \mathcal{L}}{\partial x_1} = u_{x_1} - \lambda p_1 = 0, \quad \frac{\partial \mathcal{L}}{\partial x_2} = u_{x_2} - \lambda p_2 = 0, \quad \frac{\partial \mathcal{L}}{\partial \lambda} = -p_1x_1 - p_2x_2 + m = 0$$

Rearranging (interior solution):

$$\frac{u_{x_1}}{u_{x_2}} = \frac{p_1}{p_2} \qquad \Longleftrightarrow \qquad \text{MRS}_{12} = \frac{p_1}{p_2}$$

Also:

$$\frac{u_{x_1}}{p_1} = \frac{u_{x_2}}{p_2} = \lambda$$

λ: Lagrange multiplier.

Lagrange Multiplier

Interpretation:

- λ measures how much *maximum utility* increases when income rises by 1 monetary unit.
- Also called the *shadow price* of the budget constraint.
- At the utility-maximizing point, spending the next unit of money on any good raises utility by the same amount.

Condition:

$$\frac{u_{x_1}}{p_1} = \frac{u_{x_2}}{p_2} = \lambda$$

Example:

- If $\lambda = 0.5$, an extra \$1 of income increases the consumer's maximum utility by 0.5 utils.

Marshallian Demand Functions

Definition:

- The Marshallian (or uncompensated) demand functions give the *optimal quantities* of each good as functions of prices and income.
- They are the general solution to the consumer's problem.
- Allow us to predict how demand reacts to changes in prices or income.

Properties:

- Homogeneous of degree 0: if all prices and income are scaled by the same factor, demand remains unchanged.

Formally:

$$(x_1^*, x_2^*) \quad = \quad \big(x_1^*(p_1, p_2, m),\ x_2^*(p_1, p_2, m)\big)$$

where (x_1^*, x_2^*) solves

$$\max_{x_1, x_2} \ u(x_1, x_2) \quad \text{s.t.} \quad p_1 x_1 + p_2 x_2 \leq m$$

Indirect Utility Function

Definition:

- The indirect utility function gives the *maximum level of utility* attainable for a consumer, given prices (p_1, p_2) and income m.
- It links market variables (prices and income) directly to satisfaction, without explicitly showing quantities.

Formally:

$$v(p_1, p_2, m) = u\big(x_1^*(p_1, p_2, m),\ x_2^*(p_1, p_2, m)\big)$$

How it is obtained:

- Take the Marshallian demand functions $x_1^*(\cdot), x_2^*(\cdot)$.
- Plug them into the original utility function $u(\cdot)$.

Demand Elasticities

Price elasticity of demand (e_{x_1,p_1}):

- Measures the percentage change in the quantity demanded of good x_1 in response to a percentage change in its own price p_1.

$$e_{x_1,p_1} = \frac{\Delta x_1/x_1}{\Delta p_1/p_1} = \frac{\partial x_1(p_1,p_2,m)}{\partial p_1}\frac{p_1}{x_1}$$

Income elasticity of demand ($e_{x_1,m}$):

- Measures the percentage change in the quantity demanded of good x_1 in response to a percentage change in income m.

$$e_{x_1,m} = \frac{\Delta x_1/x_1}{\Delta m/m} = \frac{\partial x_1(p_1,p_2,m)}{\partial m}\frac{m}{x_1}$$

PART 1: Consumption Theory

Chapter 1: Consumers' Choice: Preferences and Budget Constraint

Course: Microeconomics — Master in Economics
University of Granada

Instructor: Guadalupe Correa-Lopera

Instructions. Select the single correct answer (A–D).

Q1. Consider two bundles: $A = (3, 2)$ and $B = (1, 4)$. A consumer's preferences satisfy the standard axioms. Suppose $A \succeq B$ and $B \succeq A$. Which conclusion is correct?

A. A is strictly preferred to B.

B. B is strictly preferred to A.

C. The consumer is indifferent between A and B.

D. Such a situation contradicts the transitivity axiom.

Q2. Which of the following best illustrates a violation of the axiom of continuity?

A. A consumer prefers $A = (3, 3)$ to $B = (2, 2)$, and B to $C = (1, 1)$, yet also prefers C to A.

B. A consumer prefers $A = (2, 2)$ to $B = (1, 1)$, but for every bundle A' arbitrarily close to A, the ranking flips to $B \succ A'$.

C. A consumer cannot decide between $A = (3, 1)$ and $B = (1, 3)$.

D. A consumer claims $A \succeq B$ and $B \succeq A$, but also that $A \succ B$.

Q3. Suppose preferences satisfy monotonicity. Let $A = (4, 3)$ and $B = (2, 5)$. Which statement is correct?

A. $A \succ B$ because A has more of good 1.

B. $B \succ A$ because B has more of good 2.

C. Monotonicity alone does not allow ranking A and B.

D. $A \sim B$ because the totals $4 + 3$ and $2 + 5$ are equal.

Q4. Let $u(x_1, x_2) = x_1^{0.4} x_2^{0.6}$ on $X = \mathbb{R}^2_+ \setminus \{(0, 0)\}$. Which transformation *does not* represent the same preferences as u?

A. $w(x) = u(x)^2$

B. $v(x) = \ln u(x)$

C. $\tilde{u}(x) = u(x) + 5$

D. $z(x) = -\,u(x)$

Q5. For $u(x_1, x_2) = x_1^{1/2} x_2^{1/2}$, compute the marginal rate of substitution MRS at the bundle $(x_1, x_2) = (3, 12)$.

A. $\frac{1}{4}$

B. 4

C. $\frac{12}{\sqrt{3}}$

D. $\frac{\sqrt{12}}{3}$

Q6. Suppose $u(A) = 10$, $u(B) = 6$, and $u(C) = 5$ for an ordinal utility function on $X \subseteq \mathbb{R}^2_+$. Which statement is valid?

A. A is "twice as satisfying" as C because $10 = 2 \times 5$.

B. The utility differences $u(A) - u(B)$ and $u(B) - u(C)$ measure intensity of preference.

C. The ranking $A \succ B \succ C$ is preserved under any strictly increasing transformation of u.

D. $B \sim C$ since 6 is close to 5 in absolute value.

Q7. Let income $m = 120$, prices $p_1 = 10$ and $p_2 = 20$. Which equation represents the consumer's budget line?

A. $10x_1 + 20x_2 \leq 120$

B. $10x_1 + 20x_2 = 120$

C. $p_1x_1 + p_2x_2 \leq m + 20$

D. $10x_1 + 20x_2 \geq 120$

Q8. With $p_1 = 5$ and $p_2 = 10$, the budget line is $5x_1 + 10x_2 = m$. If income doubles from $m = 100$ to $m = 200$, what happens to the budget line?

A. It rotates around the x_1-intercept.

B. It shifts outward in parallel, intercepts doubling in both axes.

C. It rotates around the x_2-intercept.

D. Its slope changes from -0.5 to -1.

Q9. Suppose $m = 100$, $p_1 = 10$, $p_2 = 20$. Now p_1 increases to 20 while p_2 and m remain fixed. What happens to the budget line?

A. It shifts outward in parallel.

B. It pivots inward around the x_2-intercept, becoming steeper.

C. It pivots inward around the x_1-intercept, becoming flatter.

D. It remains unchanged.

Q10. Consider $u(x_1, x_2) = x_1^{1/2} x_2^{1/2}$, prices $p_1 = 2$, $p_2 = 1$, and income $m = 60$. What is the optimal bundle?

A. $(x_1, x_2) = (10, 40)$

B. $(x_1, x_2) = (15, 30)$

C. $(x_1, x_2) = (12, 36)$

D. $(x_1, x_2) = (20, 20)$

Q11. A consumer faces $p_1/p_2 = 2$ and is currently at a feasible bundle where MRS $= 3$. Holding the budget line fixed, which local move along the budget line strictly increases utility?

A. Move toward more x_1 and less x_2.

B. Move toward less x_1 and more x_2.

C. No move along the budget line can improve utility: already optimal.

D. Any small move along the budget line leaves utility unchanged.

Q12. Let $u(x_1, x_2) = x_1 + 2x_2$ (perfect substitutes), with $p_1 = 3$, $p_2 = 4$ and income $m = 60$. What is the utility-maximizing choice?

A. Spend all income on good 1: $(x_1, x_2) = (20, 0)$.

B. Spend all income on good 2: $(x_1, x_2) = (0, 15)$.

C. Split income equally across goods: $(x_1, x_2) = (10, 7.5)$.

D. Any interior split with MRS $= p_1/p_2$ is optimal.

Q13. Let $u(x_1, x_2) = x_1^{\frac{1}{4}} x_2^{\frac{3}{4}}$, prices $p_1 = 4$, $p_2 = 1$, and income $m = 120$. Assuming an interior solution, what is the optimal bundle (x_1^*, x_2^*)?

A. $(x_1^*, x_2^*) = (30, 90)$

B. $(x_1^*, x_2^*) = (7.5, 90)$

C. $(x_1^*, x_2^*) = (15, 60)$

D. $(x_1^*, x_2^*) = (22.5, 30)$

Q14. Consider $u(x_1, x_2) = x_1^{1/2} x_2^{1/2}$ with $p_1 = 1$, $p_2 = 1$, income $m = 20$. Let $\mathcal{L} = u(x_1, x_2) - \lambda(p_1 x_1 + p_2 x_2 - m)$. At the optimum, what is the value of λ?

A. $\lambda = \frac{1}{2}$

B. $\lambda = 1$

C. $\lambda = \frac{\sqrt{2}}{2}$

D. $\lambda = \frac{1}{4}$

Q15. Suppose the Marshallian demand for good 1 is $x_1^*(p_1, p_2, m) = \alpha \dfrac{m}{p_1}$ with $0 < \alpha < 1$. What are the own-price elasticity and income elasticity of demand for x_1?

A. Price elasticity $= 0$; income elasticity $= 1$.

B. Price elasticity $= -\alpha$; income elasticity $= \alpha$.

C. Price elasticity $= -1$; income elasticity $= 1$.

D. Price elasticity $= -1$; income elasticity $= \alpha/(1 - \alpha)$.

Course: Microeconomics — Master in Economics

PART 1: Consumption Theory

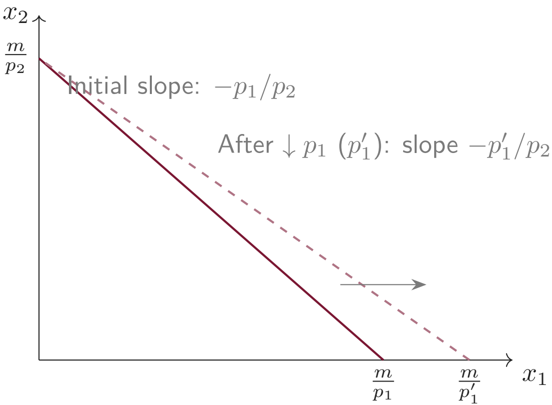
Chapter 2: Price Changes and Consumer Choice. Individual and Market Demand Function

Guadalupe Correa-Lopera | University of Granada

Outline

Price Changes and the Budget Line

Ceteris paribus: Fix m and p_2. Consider a decrease in p_1 to $p_1' < p_1$.



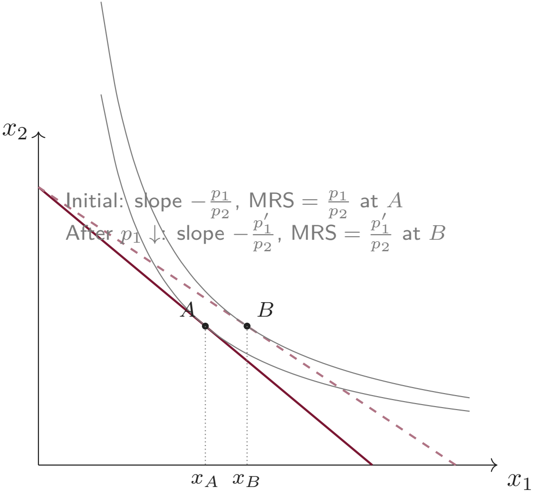
What happens?

- The budget line *rotates outward* around the x_2-intercept $(0, m/p_2)$.
- The x_1-intercept increases from m/p_1 to m/p_1'.
- The feasible set expands along x_1; the optimum will generally move rightward.

At the optimum: tangency (if interior) satisfies MRS $= \frac{p_1}{p_2}$ initially, and MRS $= \frac{p_1'}{p_2}$ after the price change.

From Indifference Curves and Budget Constraints to the Demand Curve

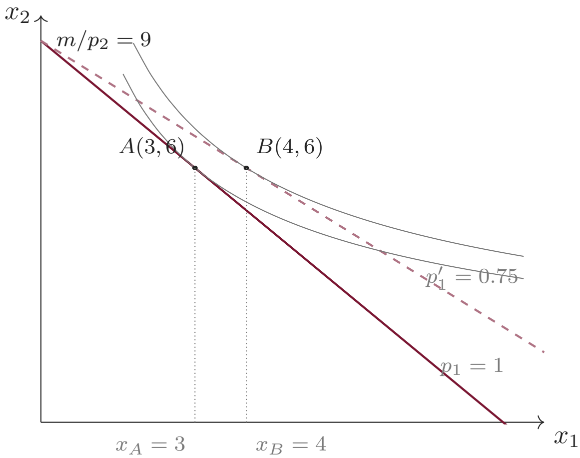
At each price p_1, the consumer maximizes $u(x_1, x_2)$ subject to $p_1x_1 + p_2x_2 \leq m$. An interior optimum satisfies the tangency condition MRS $= \frac{p_1}{p_2}$ and the budget binds.



- Each budget line selects a unique optimum (here A then B) by tangency to an indifference curve.
- When p_1 falls to p_1', the optimum moves rightward to a higher indifference curve.
- The x_1-coordinates, x_A and x_B, are the demanded quantities at those prices.

Numerical Example (I): Tangency Between ICs and the Budget Line

Preferences: $u(x_1, x_2) = \ln x_1 + 2\ln x_2$. **Budget:** $p_1x_1 + p_2x_2 = m$. Let $m = 9$, $p_2 = 1$, initial $p_1 = 1$, new $p_1' = 0.75$.



- Lower p_1 pivots the budget line outward around $(0,\, m/p_2) = (0, 9)$, moving the optimum from A to B.
- A and B are tangency points between an indifference curve and the relevant budget line.

$$\max_{x_1, x_2} \; \ln x_1 + 2\ln x_2 \quad \text{s.t.} \quad p_1x_1 + p_2x_2 = m$$

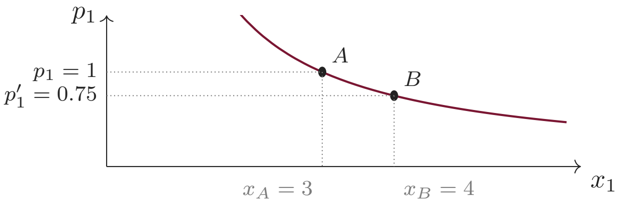
$$\Rightarrow \; \frac{1}{x_1} = \lambda p_1, \; \frac{2}{x_2} = \lambda p_2 \; \Rightarrow \; x_2 = 2\frac{p_1}{p_2}x_1$$

$$p_1x_1 + p_2x_2 = p_1x_1 + p_2\left(2\frac{p_1}{p_2}x_1\right) = 3p_1x_1 = m$$

$$\boxed{x_1^* = \frac{m}{3p_1}}, \qquad \boxed{x_2^* = \frac{2m}{3p_2}}.$$

Numerical Example (II): Individual Demand in the (x_1, p_1) Plane

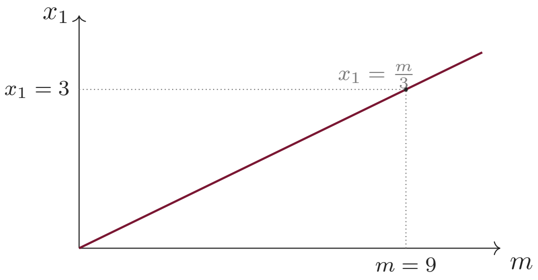
From $u(x_1, x_2) = \ln x_1 + 2 \ln x_2$ we obtained $x_1^* = \frac{m}{3p_1}$, hence the inverse demand is $p_1(x_1) = \frac{m}{3x_1}$; with $m = 9$ this becomes $p_1(x_1) = \frac{3}{x_1}$.

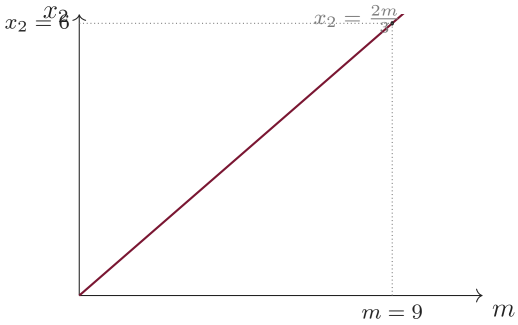


- The individual demand collects all pairs (x_1^*, p_1) solving the consumer problem.
- Points $A = (3, 1)$ and $B = (4, 0.75)$ match the optimal bundles from the previous slide.
- Lower price $\Rightarrow$ higher quantity (downward slope).

Numerical Example (III): Engel Curves for Both Goods (Prices Fixed)

Definition. The Engel curve for a good shows how the optimal quantity changes with income m when prices are fixed. From our utility: $x_1^*(m \mid p) = \frac{m}{3p_1}$ and $x_2^*(m \mid p) = \frac{2m}{3p_2}$. In the plots we set $(p_1, p_2) = (1, 1)$, so $x_1^* = \frac{m}{3}$ and $x_2^* = \frac{2m}{3}$; we mark $m = 9$ to connect with the numerical example.



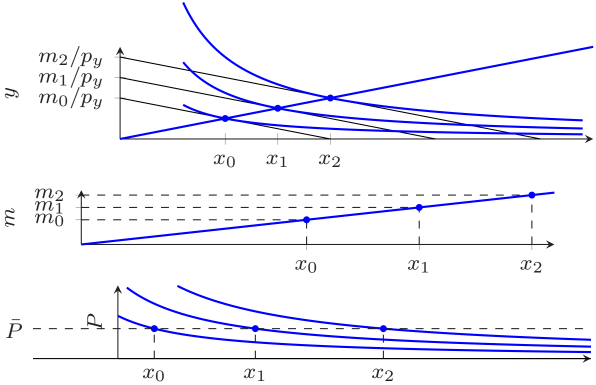


With these preferences and fixed prices, both Engel curves are straight lines through the origin, so both goods are *normal*.

Effects of an Increase in Income

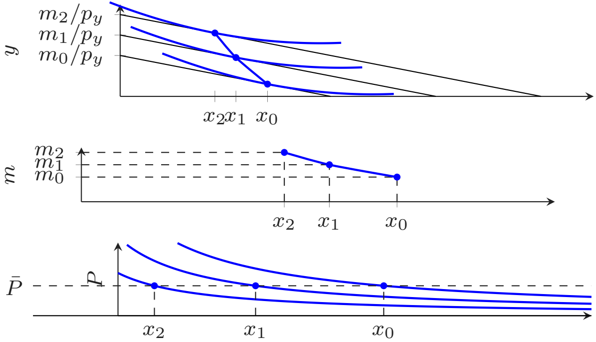
- We increase income m holding prices (p_1, p_2) fixed (*ceteris paribus*).
- The budget line shifts out in a parallel way; the optimum moves along higher indifference curves.
- **Income elasticity of demand** (good x): $\varepsilon_{x,m} = \frac{\partial x}{\partial m} \cdot \frac{m}{x}$
- Classifications by $\varepsilon_{x,m}$:
  - **Normal good**: $\varepsilon_{x,m} > 0$ (demand rises with m). Within normal goods:
    - **Necessary good**: $0 < \varepsilon_{x,m} < 1$ (demand rises less than proportionally with m)
    - **Luxury good**: $\varepsilon_{x,m} > 1$ (demand rises more than proportionally with m)
  - **Inferior good**: $\varepsilon_{x,m} < 0$ (demand falls with m)

Income Increase: Normal Good



- **(Graph 1)** Budget expansion: $m_2 > m_1 > m_0$ (parallel shift). The **Income-Consumption Curve** connects the optimal bundles; both axis intercepts rise with m.
- **(Graph 2) Engel Curve** (normal good): projects (x_i, m_i); upward $\Rightarrow \varepsilon_{x,m} > 0$.
- **(Graph 3) Demand Curve** shifts: for fixed $\bar{P}$, $x_0 < x_1 < x_2$ (rightward shift as income increases).

Income Increase: Inferior Good



- **(Graph 1)** Budget expansion with $m_2 > m_1 > m_0$. The ICC is backward in x: as income rises, the chosen x falls (inferior good).
- **(Graph 2)** Engel Curve is *downward*: $\varepsilon_{x,m} < 0$; higher m implies lower demand for x.
- **(Graph 3)** At a fixed price $\bar{P}$, quantities satisfy $x_0 > x_1 > x_2$: leftward shift of the Demand Curve as income increases.

Price Responsiveness

- Classify goods by how Marshallian demand $x(p_x, p_y, m)$ reacts to its *own* price, holding m and other prices fixed.
- **Ordinary good**:
$$\frac{\partial x}{\partial p_x} < 0$$
Interpretation: when p_x rises, quantity demanded of x falls (law of demand).
- **Giffen good**:
$$\frac{\partial x}{\partial p_x} > 0$$
Interpretation: when p_x rises, quantity demanded of x increases (rare; violates the law of demand).

A price change has two effects

Example: The monthly price of a **streaming service** (x_1) falls, while the price of **cinema tickets** (x_2) remains the same.

- You may watch more streaming because it is now **cheaper relative to** cinema.
- At the same time, your overall **purchasing power increases**: with the same budget, you can afford more leisure in total.

Definitions:

- **Substitution Effect:** the change in consumption caused by a change in **relative prices**, keeping satisfaction level constant.
- **Income Effect:** the change in consumption due to a change in **purchasing power**, similar to the effect of a change in income.

Substitution and Income Effects

Suppose the price of good x_1 **falls** (while income and p_2 stay constant). We distinguish three consumption levels:

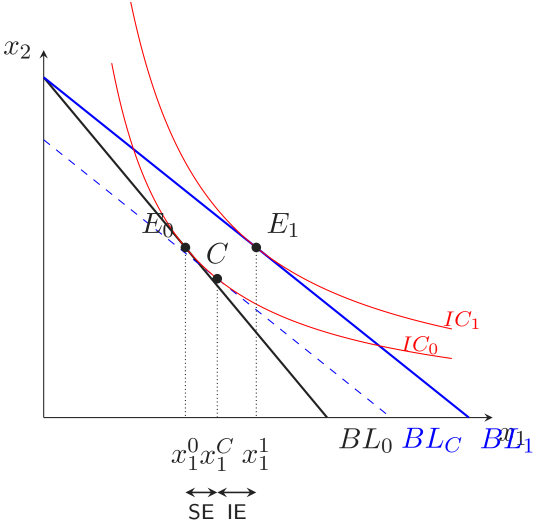
- x_1^0: initial quantity consumed (before the price change).
- x_1^C: quantity consumed after the price change, **if only the substitution effect operates** (utility held constant, but with the new relative prices).
- x_1^1: final quantity consumed (after the price change, with full adjustment).

$$SE = x_1^C - x_1^0, \quad IE = x_1^1 - x_1^C, \quad TE = x_1^1 - x_1^0$$

Signs when p_1 falls:

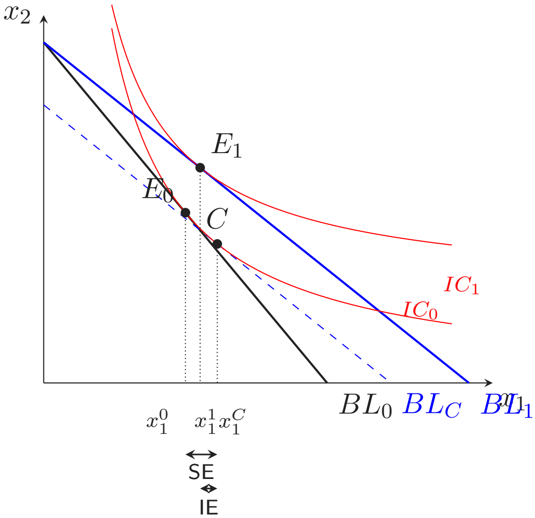
Type of good	Substitution Effect	Income Effect	Total Effect
Normal	$SE > 0$	$IE > 0$	$TE > 0$ (reinforce)
Inferior	$SE > 0$	$IE < 0$	Ambiguous (depends on size)
Giffen	$SE > 0$	$IE < 0$, dominates	$TE < 0$ (demand falls)

Normal Good: Substitution and Income Effects



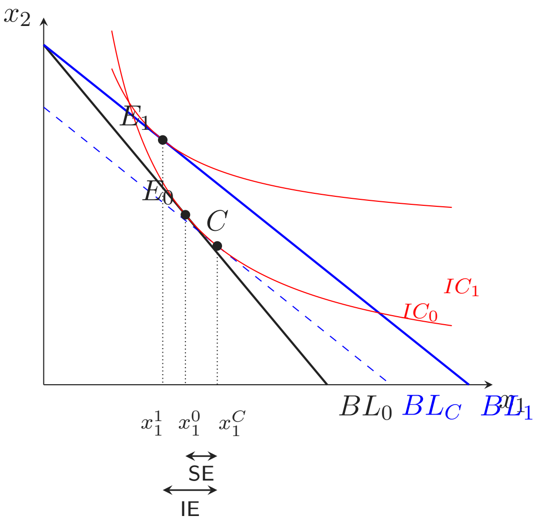
- **Price drop in x_1:** the budget line pivots out ($BL_0 \to BL_1$).
- **SE ($x_1^0 \leftrightarrow x_1^C$):** pure relative–price response; the consumer substitutes toward the cheaper x_1 while staying on IC_0.
- **IE ($x_1^C \leftrightarrow x_1^1$):** with the new prices, higher purchasing power moves the optimum to E on IC_1; for a normal good, x_1 rises.
- **Result:** for a normal good, SE and IE reinforce $\Rightarrow$ total demand for x_1 increases.

Inferior Good: Substitution and Income Effects



- **Price drop in x_1:** budget pivots out $(BL_0 \to BL_1)$.
- **SE $(E_0 \to C$ on $IC_0)$:** with new relative prices, the consumer substitutes toward the cheaper x_1.
- **IE $(C \to E_1$ on $BL_1)$:** because x_1 is *inferior*, higher purchasing power reduces x_1.
- **Net effect:** *ambiguous*. Here SE still dominates $(x_1^1 > x_1^0)$; a larger $|IE|$ could reverse it.

Giffen Good: Substitution and Income Effects



- **Price drop in x_1:** the budget pivots out ($BL_0 \to BL_1$).
- **SE ($E_0 \to C$ on IC_0):** as x_1 becomes cheaper, the consumer substitutes toward x_1.
- **IE ($C \to E_1$ on BL_1):** at the new prices, higher purchasing power *reduces* x_1 *because* x_1 *is inferior*.
- **Giffen result:** the *negative IE dominates* the positive SE $\Rightarrow$ $x_1^1 < x_1^0$ (demand falls when price falls).

Market Demand Function

- The **market demand** for a good at price P is the *total quantity* consumers want to buy, holding other prices and incomes fixed.
- Notation: consumer i demands $x_i(P)$. The market demand is

$$X^M(P) = \sum_{i=1}^{N} x_i(P).$$

- With n identical consumers, $X^M(P) = n\, x_i(P)$.
- The **reservation price** (often called *choke price*) is the highest price at which demand falls to zero.

From Individual to Market Demand: An Example

Two consumers with linear demands:

$$x_A(P) = \max\{0, 20 - 2P\}, \qquad x_B(P) = \max\{0, 15 - P\}.$$

- Consumer A's reservation price: $P = 10$.
- Consumer B's reservation price: $P = 15$.
- Below $P = 10$, both buy $\Rightarrow$ add both demands.
- Between $10 < P \leq 15$, only B buys.
- Above 15, no demand.

Step-by-Step Arithmetic

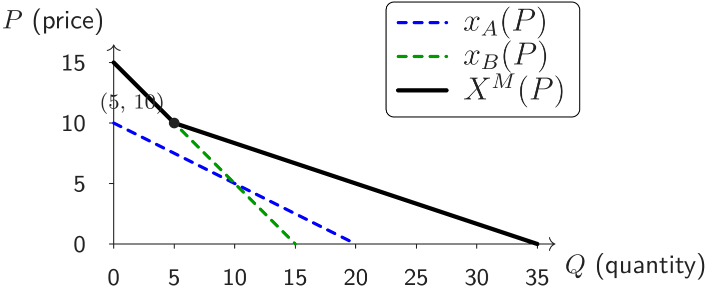
At selected prices:

Price P	5	8	10	12	15
$x_A(P) = 20 - 2P$ (if $P \leq 10$)	10	4	0	0	0
$x_B(P) = 15 - P$ (if $P \leq 15$)	10	7	5	3	0
$X^M(P) = x_A + x_B$	20	11	5	3	0

- At $P = 10$: consumer A stops buying $\Rightarrow$ kink.
- At $P = 15$: consumer B stops buying $\Rightarrow$ demand is zero.
- **Market demand** is obtained by horizontal summation of individual demands.

Graphical Aggregation of Individual Demands

- For $0 \leq P \leq 10$:
  $X^M(P) = 35 - 3P$.
- For $10 < P \leq 15$:
  $X^M(P) = 15 - P$.
- For $P > 15$:
  $X^M(P) = 0$.



Market Demand Function (Piecewise Form)

$$X^M(P) = \begin{cases} 35 - 3P, & 0 \leq P \leq 10, \\ 15 - P, & 10 < P \leq 15, \\ 0, & P > 15. \end{cases}$$

- The slope of market demand changes at $P = 10$ when consumer A exits.
- Beyond $P = 15$ nobody buys, so demand is zero.

PART 1: Consumption Theory

Chapter 2: Price Changes and Consumer Choice. Individual and Market Demand Function

Course: Microeconomics — Master in Economics
University of Granada

Instructor: Guadalupe Correa-Lopera

Instructions. Select the single correct answer (A–D).

Q1. Assume an interior solution. Preferences $u(x_1, x_2) = x_1 + \ln x_2$ with budget $p_1x_1 + p_2x_2 = m$. What is the Marshallian demand for x_1?

A. $x_1 = \dfrac{m}{p_1 + p_2}$

B. $x_1 = \dfrac{m}{p_1} - 1$

C. $x_1 = \dfrac{m - p_2}{p_1}$

D. $x_1 = \dfrac{m}{p_1} - \dfrac{p_2}{p_1}$

Q2. Let preferences be CES: $u(x_1, x_2) = (x_1^\rho + x_2^\rho)^{1/\rho}$ with $\rho = 0.5$. Budget: $m = 120$, $p_1 = 2$, $p_2 = 4$. What is the optimal demand for x_1?

A. 25

B. 30

C. 20

D. 40

Q3. Suppose $u(x_1, x_2) = \min\{2x_1,\ x_2\}$, income $m = 60$, prices $p_1 = 2$, $p_2 = 4$. What is the optimal bundle?

A. $(15, 30)$

B. $(10, 40)$

C. $(6, 12)$

D. $(20, 20)$

Q4. Let preferences be Cobb–Douglas with share $\alpha \in (0, 1)$: the Marshallian demand is $x_1^*(p_1, p_2, m) = \alpha m/p_1$. Which statement is correct about the income response of x_1?

A. x_1 is inferior and $\varepsilon_{x_1,m} < 0$.

B. x_1 is a necessity with $0 < \varepsilon_{x_1,m} < 1$.

C. x_1 has unit income elasticity $\varepsilon_{x_1,m} = 1$ (normal, neither necessity nor luxury).

D. x_1 is a luxury with $\varepsilon_{x_1,m} > 1$.

Q5. Consider $u(x_1, x_2) = \ln x_1 + x_2$ with prices $p_1, p_2 > 0$. Holding prices fixed, what is the income elasticity of demand for x_1?

A. $\varepsilon_{x_1,m} = 1$

B. $\varepsilon_{x_1,m} = 0$

C. $\varepsilon_{x_1,m} < 0$

D. $\varepsilon_{x_1,m} > 1$

Q6. Suppose the Engel curve is $x(m) = 0.1m^{0.8}$, $m > 0$. How is the good classified?

A. Inferior

B. Necessity

C. Luxury

D. Normal with unit income elasticity

Q7. Assume preferences are monotone and strictly convex and the initial optimum is interior. If p_1 falls (with m and p_2 fixed) and we hold utility at the initial level (Hicksian compensation), what happens to x_1 due to the substitution effect?

A. It may decrease if the good is inferior.

B. It is zero for normal goods and negative for inferior goods.

C. It strictly increases: $SE > 0$.

D. It is always ambiguous, depending on income effects.

Q8. Let $u(x_1, x_2) = x_1^{1/3} x_2^{2/3}$, income $m = 120$, p_2 fixed, and p_1 falls from 8 to 1. Compute the substitution effect SE and the income effect IE.

A. $SE = 10,\ IE = 25$

B. $SE = 12,\ IE = 18$

C. $SE = 15,\ IE = 20$

D. $SE = 20,\ IE = 15$

Q9. After a fall in p_1, suppose good x_1 is Giffen. Let x_1^0 be the initial consumption, x_1^C the compensated level (same utility as initially, at new prices), and x_1^1 the final consumption. Which ordering must hold?

A. $x_1^0 < x_1^C < x_1^1$

B. $x_1^1 < x_1^C < x_1^0$

C. $x_1^1 < x_1^0 < x_1^C$

D. $x_1^C < x_1^1 < x_1^0$

Q10. Two consumers have individual demands (in units) at price $P \geq 0$:

$$x_A(P) = \max\{0,\, 24 - 2P\}, \qquad x_B(P) = \max\{0,\, 18 - 3P\}.$$

Let $X^M(P)$ denote market demand. Which expression is correct?

A. $X^M(P) = \begin{cases} 42 - 5P, & 0 \leq P \leq 6, \\ 24 - 2P, & 6 < P \leq 12, \\ 0, & P > 12. \end{cases}$

B. $X^M(P) = \begin{cases} 42 - 5P, & 0 \leq P \leq 6, \\ 18 - 3P, & 6 < P \leq 12, \\ 0, & P > 12. \end{cases}$

C. $X^M(P) = \begin{cases} 24 - 2P, & 0 \leq P \leq 12, \\ 18 - 3P, & 12 < P \leq 18, \\ 0, & P > 18. \end{cases}$

D. $X^M(P) = \begin{cases} 42 - 5P, & 0 \leq P \leq 12, \\ 0, & P > 12. \end{cases}$

Q11. Consider a market with two consumers. Consumer A has individual demand $x_A(P) = \max\{0,\, 12 - P\}$, and consumer B has individual demand $x_B(P) = \max\{0,\, 9 - 2P\}$. Determine the price elasticity of aggregate market demand when the price is $P = 4$.

A. -0.75

B. -0.44

C. -2.00

D. -1.33

Q12. There are 40 identical consumers. Each has individual demand $x_i(P) = \max\{0,\, 10 - 2P\}$. What is the market demand $X^M(P)$?

A. $X^M(P) = \begin{cases} 400 - 80P, & 0 \leq P \leq 5, \\ 0, & P > 5. \end{cases}$

B. $X^M(P) = \begin{cases} 400 - 80P, & 0 \leq P \leq 10, \\ 0, & P > 10. \end{cases}$

C. $X^M(P) = \begin{cases} 400 - 2P, & 0 \leq P \leq 5, \\ 0, & P > 5. \end{cases}$

D. $X^M(P) = \begin{cases} 200 - 40P, & 0 \leq P \leq 5, \\ 0, & P > 5. \end{cases}$

Course: Microeconomics — Master in Economics

PART 2: Production Theory

Chapter 3: Technology and Production

Guadalupe Correa-Lopera | University of Granada

Outline

Basics of Production

- **Production** is the technological process that transforms inputs into an output (good or service).
- We study a setup with **two inputs**:
  - K: *capital* (buildings, machines, equipment),
  - L: *labor* (hours/effort of workers).
- Firms are **price takers in input markets**: they can hire K and L at given rental/wage rates.
- Output increases with inputs, with **diminishing marginal returns**.
- **Short run (SR):** K fixed, firm chooses L. **Long run (LR):** K and L both variable.
- Given a target output $\bar{Q}$, firms will later choose inputs to **minimize cost** (isoquants/isocosts).

Production Function and Marginal Products

- The firm's **technology** (its production process) is summarized by a production function:

$$Q = f(K, L),$$

  mapping each input bundle to the *maximum* attainable output.
- **Marginal products** (local extra output from a tiny increase in one input):

$$MP_L(K, L) = \frac{\partial f(K, L)}{\partial L}, \qquad MP_K(K, L) = \frac{\partial f(K, L)}{\partial K}.$$

- **Diminishing marginal products (DMPL/DMPK):**

$$\frac{\partial^2 f}{\partial L^2} < 0, \qquad \frac{\partial^2 f}{\partial K^2} < 0.$$

Example (Cobb–Douglas): $Q = A\,K^{\alpha} L^{\beta}$ with $A, \alpha, \beta > 0$. Then

$$MP_L = \beta A\,K^{\alpha} L^{\beta-1}, \qquad MP_K = \alpha A\,K^{\alpha-1} L^{\beta}.$$

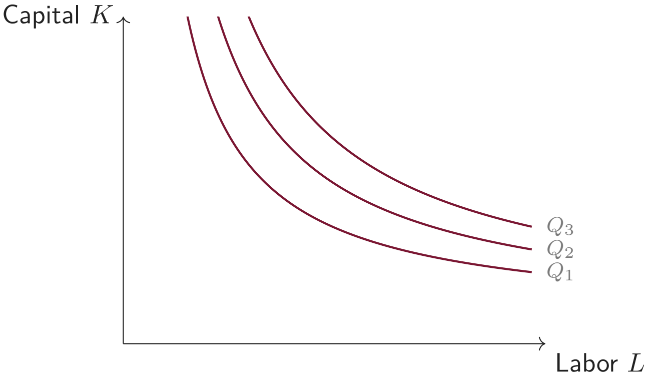
If $0 < \alpha, \beta < 1$ we have DMPL and DMPK.

Isoquants

- For any output level $\bar{Q}$, the **isoquant** collects all input pairs producing $\bar{Q}$:

$$\mathcal{I}(\bar{Q}) = \{(K, L) \in \mathbb{R}^2_+ : f(K, L) = \bar{Q}\}.$$

- Graphically: higher isoquants $\Rightarrow$ higher Q.
- Isoquants illustrate different input combinations that yield the same Q.



MRTS: Slope of an Isoquant

- Along an isoquant $Q = \bar{Q}$, total differential is zero:

$$dQ = MP_L\, dL + MP_K\, dK = 0.$$

- Rearranging:

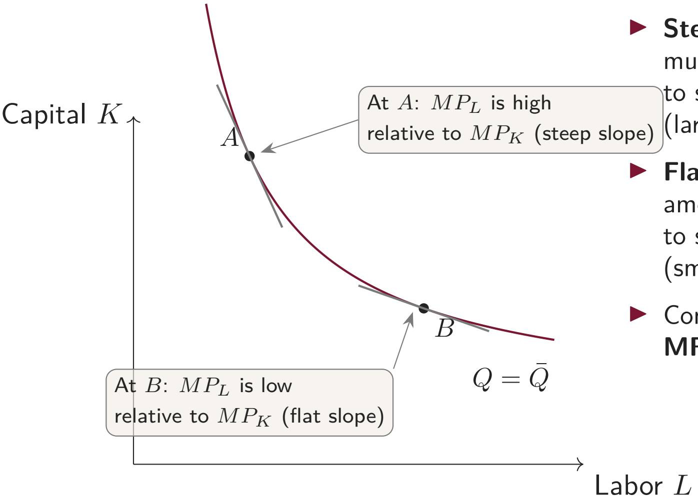
$$\left.\frac{dK}{dL}\right|_{Q=\bar{Q}} = -\frac{MP_L}{MP_K}.$$

- Hence:

$$\text{slope of isoquant} < 0, \qquad MRTS_{L,K} = -\frac{dK}{dL} = \frac{MP_L}{MP_K} > 0.$$

- **Interpretation:** The MRTS of L for K measures how many units of capital the firm can give up for one more unit of labor, while keeping Q constant.

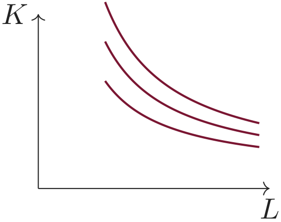
MRTS: Slope of an Isoquant



- **Steeper** isoquant at $A \Rightarrow$ the firm must give up a large amount of K to substitute one more unit of L (large MRTS).
- **Flatter** at $B \Rightarrow$ only a small amount of K needs to be given up to substitute one more unit of L (small MRTS).
- Convexity captures a **diminishing MRTS** as L increases.

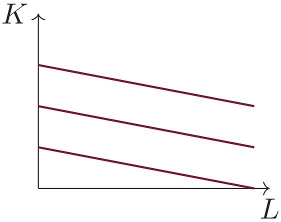
Common Isoquant Shapes

Smooth substitution



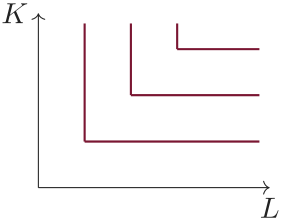
Gradual trade-off between inputs; convex isoquants, diminishing MRTS.

Perfect substitutes



Inputs are interchangeable at a constant rate; isoquants are parallel straight lines.

Fixed proportions



Inputs used in rigid ratios; L-shaped isoquants, no substitution at the kink.

Short-Run: Fixed vs. Variable Inputs

- **Short-Run (SR):** at least one input is fixed.
  - K: typically considered *fixed* (plant, machines).
  - L: typically *variable* (hours of labor).
- Output depends on how much the variable input L is combined with fixed K:

$$Q = f(\bar{K}, L).$$

- In the SR, firms cannot adjust K but can vary L to respond to demand.
- Key question: how does Q change as L increases while K is fixed?

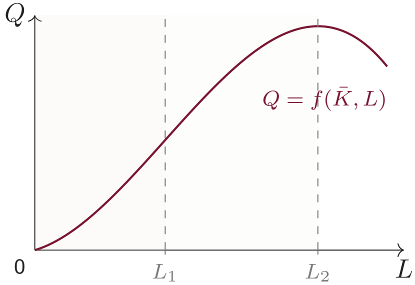
Law of Diminishing Returns

- **Definition:** As more of the variable input L is added to a fixed amount of K, the **marginal product of** L eventually declines.

$$MP_L(L) = \frac{\Delta Q}{\Delta L} \quad \text{falls as } L \text{ rises (holding } K \text{ fixed).}$$

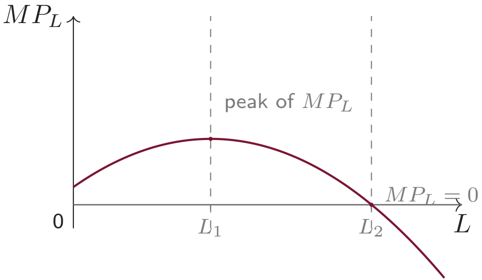
- Intuition:
  - With few workers, adding extra labor makes production much more efficient (specialization).
  - After some point, workers crowd the fixed capital: each additional worker adds less output.
  - Eventually, MP_L can even become **negative** if overcrowding occurs.
- This principle is also called the *Law of Diminishing Marginal Returns*.

SR Production Function $Q(L\,|\,\bar{K})$



Note: Slope of $Q(\cdot)$ equals MP_L.

Marginal Product of Labor $MP_L(L)$



Note: $MP_L = \frac{\partial Q}{\partial L}\Big|_{\bar{K}}$.

- **For** $0 < L < L_1$**: Increasing marginal returns** $(\frac{d^2Q}{dL^2} > 0)$. Equivalently: $MP_L > 0$ and $\frac{dMP_L}{dL} > 0$.
- **For** $L_1 < L < L_2$**: Diminishing marginal returns** $(\frac{d^2Q}{dL^2} < 0)$. Equivalently: $MP_L > 0$ and $\frac{dMP_L}{dL} < 0$.
- **For** $L > L_2$**: Diminishing total returns** $(\frac{dQ}{dL} = MP_L < 0)$. Additional L reduces total Q.

Long-Run Production

- In the **long-run**, all inputs are variable: both K and L can be adjusted.
- We ask: what happens to output Q if we scale *all inputs* by the same factor $t > 0$?
- **Returns to scale (RTS):** the relation between input scaling and output scaling.

$$f(tK, tL) \;\; \text{vs.} \;\; t\, f(K, L).$$

- Three possible cases: increasing, constant, or decreasing returns to scale.

Types of Returns to Scale

- **Increasing RTS (IRS):**

$$f(tK, tL) > t\, f(K, L)$$

Output grows more than proportionally. Example: specialization and efficiency gains $\Rightarrow$ doubling inputs yields **more than double** output.

- **Constant RTS (CRS):**

$$f(tK, tL) = t\, f(K, L)$$

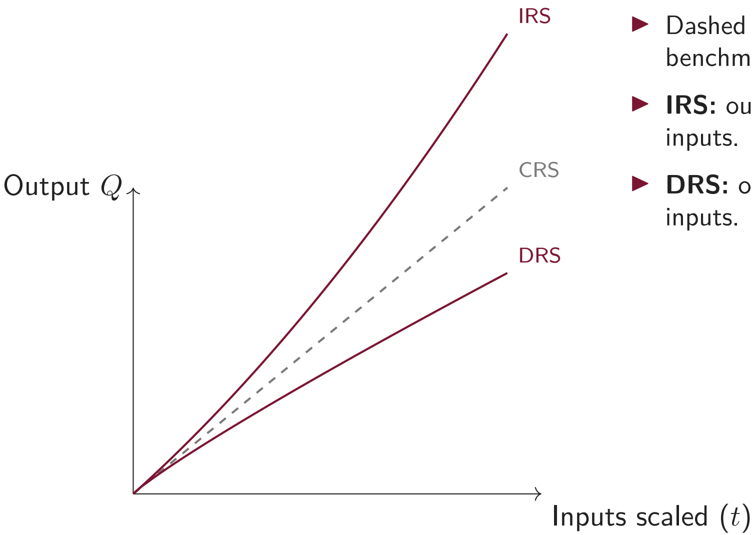
Output grows in the same proportion. Example: replicating a plant $\Rightarrow$ doubling inputs yields **exactly double** output.

- **Decreasing RTS (DRS):**

$$f(tK, tL) < t\, f(K, L)$$

Output grows less than proportionally. Example: coordination problems in large firms $\Rightarrow$ doubling inputs yields **less than double** output.

Illustration of Returns to Scale



- Dashed line = **CRS** $(Q = t)$ benchmark.
- **IRS:** output grows faster than inputs.
- **DRS:** output grows slower than inputs.

Example: Cobb–Douglas and RTS

- Cobb–Douglas production function: $Q = f(K, L) = AK^{\alpha}L^{\beta}$.
- Scale all inputs by t:

$$f(tK, tL) = A\,(tK)^{\alpha}(tL)^{\beta} = t^{\alpha+\beta} f(K, L).$$

- Classification:

$$\alpha + \beta \begin{cases} > 1 & \Rightarrow \text{IRS}, \\ = 1 & \Rightarrow \text{CRS}, \\ < 1 & \Rightarrow \text{DRS}. \end{cases}$$

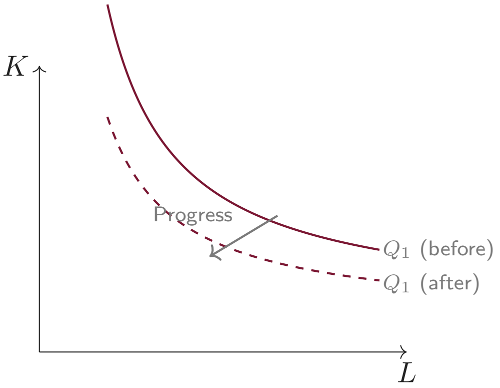
- **Example:** If $Q = K^{0.4}L^{0.6}$ then $\alpha + \beta = 1 \Rightarrow$ CRS.

Technological Progress

- **Technological progress:** improvements in the production process that allow the same inputs to produce more output.
- Can be represented as an **upward shift of the production function** or an **inward shift of isoquants**.
- Two main types:
  - **Neutral progress:** improves productivity of all inputs proportionally.
  - **Biased progress:** favors the productivity of one input relative to the other.
- **Technological progress** has important implications for long-run growth and for the relative use of inputs.

Neutral Technological Progress

- **Neutral progress:** productivity improves proportionally for all inputs.
- Isoquants for a given Q shift **inward toward the origin**, keeping the same shape.
- Interpretation: the same output can now be produced with *less of both inputs*.
- The **MRTS is unchanged** at every point.



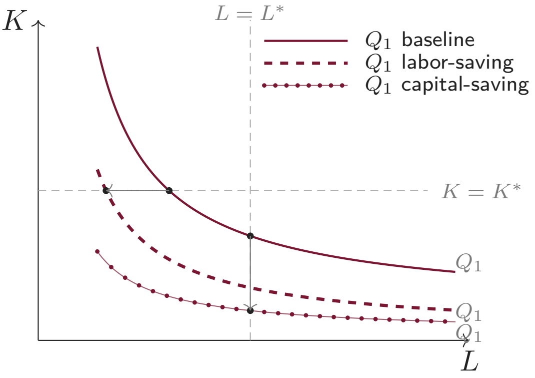
Biased progress: the productivity gain is stronger for one input.

- **Labor-saving:** for the same Q, the isoquant pivots inward toward the L-axis (needs much less L at a given K).

  MRTS increases: $MRTS_{L,K} = \frac{MP_L}{MP_K} \uparrow \Rightarrow$ isoquant **steeper**.

- **Capital-saving:** for the same Q, the isoquant pivots inward toward the K-axis (needs much less K at a given L).

  MRTS decreases: $MRTS_{L,K} = \frac{MP_L}{MP_K} \downarrow \Rightarrow$ isoquant **flatter**.



PART 2: Production Theory
Chapter 3: Technology and Production

Course: Microeconomics — Master in Economics
University of Granada
Instructor: Guadalupe Correa-Lopera

Instructions. Select the single correct answer (A–D).

Q1. Consider the Cobb–Douglas technology $f(K, L) = K^{0.4}L^{0.6}$. At $(K, L) = (16, 9)$, what is the marginal rate of technical substitution of L for K, i.e., $\text{MRTS}_{L,K}$?

- **A.** $\frac{3}{8}$
- **B.** $\frac{8}{3}$
- **C.** $\frac{2}{3}$
- **D.** $\frac{3}{2}$

Q2. For $f(K, L) = K^{1/2}L^{1/2}$, both bundles $A = (1, 4)$ and $B = (4, 1)$ lie on the same isoquant. Which point has a *larger* $\text{MRTS}_{L,K}$?

- **A.** A has larger $\text{MRTS}_{L,K}$.
- **B.** B has larger $\text{MRTS}_{L,K}$.
- **C.** Both have the same $\text{MRTS}_{L,K}$.
- **D.** It is impossible to compare without prices.

Q3. Which statement about isoquants is *correct*?

- **A.** For perfect substitutes $f(K, L) = K + L$, isoquants have a constant slope (MRTS is constant).
- **B.** For fixed proportions $f(K, L) = \min\{K, 2L\}$, isoquants are straight lines with constant slope.
- **C.** For Cobb–Douglas $f(K, L) = K^{\alpha}L^{\beta}$, isoquants are concave to the origin.
- **D.** For fixed proportions, the MRTS is finite and well-defined at the kink.

Q4. In the short run with $\bar{K}$ fixed, let $Q(L) = 10L - 0.5L^2$. At which L does the marginal product MP_L become zero?

- **A.** $L = 5$
- **B.** $L = 10$
- **C.** $L = 15$

D. $L = 20$

Q5. With $Q(L) = 10L - 0.5L^2$, which pair (L^*, Q^*) maximizes total output?

A. (5, 37.5)

B. (10, 50)

C. (15, 37.5)

D. (20, 0)

Q6. Suppose $Q(L) = 2\sqrt{L}$ (with $\bar{K}$ fixed). Compute AP_L and MP_L at $L = 4$. Which statement is correct?

A. $AP_L = 1$, $MP_L = 0.5$, hence $MP_L < AP_L$.

B. $AP_L = 2$, $MP_L = 0.5$, hence $MP_L < AP_L$.

C. $AP_L = 1$, $MP_L = 1$, hence $MP_L = AP_L$.

D. $AP_L = 2$, $MP_L = 1$, hence $MP_L < AP_L$.

Q7. Consider $Q(K, L) = 2K^{0.3}L^{0.4}$. If all inputs are doubled, what happens to output?

A. It less than doubles (DRS).

B. It doubles exactly (CRS).

C. It more than doubles (IRS).

D. It quadruples.

Q8. With $Q(K, L) = K^{0.5}L^{0.5}$, both inputs are scaled by $t = 3$. The new output Q' relative to the initial Q is:

A. Q' is exactly 3 times Q (CRS).

B. Q' is more than 3 times Q (IRS).

C. Q' is less than 3 times Q (DRS).

D. Q' equals Q (no scaling effect).

Q9. Consider $Q(K, L) = K^{0.6}L^{0.3}$. When both inputs are scaled by a common factor $t > 1$, which statement best describes the technology's returns to scale?

A. $Q(tK, tL) > t\,Q(K, L)$ for all $t > 1$ (increasing returns to scale).

B. $Q(tK, tL) = t\,Q(K, L)$ for all $t > 1$ (constant returns to scale).

C. $Q(tK, tL) < t\,Q(K, L)$ for all $t > 1$ (decreasing returns to scale).

D. The returns to scale cannot be determined without prices.

Q10. Starting from $Q = K^{0.5}L^{0.5}$, neutral technological progress doubles productivity: $Q' = 2K^{0.5}L^{0.5}$. For any given (K, L):

A. Output remains unchanged.

B. Output doubles; isoquants shift inward; MRTS is unchanged.

C. Output more than doubles; MRTS increases.

D. Isoquants shift outward, requiring more inputs.

Q11. Suppose technological progress is *labor-saving*. For a given Q, the isoquant:

A. Shifts inward in parallel, keeping the same slope.

B. Pivots inward toward the K-axis; isoquant becomes steeper (MRTS increases).

C. Pivots inward toward the L-axis; isoquant becomes steeper (MRTS increases).

D. Does not move; only costs change.

Q12. Two firms share $Q = K^{0.5}L^{0.5}$ and have $K = L = 4$. Firm A gets neutral progress (factor $t = 2$); Firm B gets capital-saving progress that doubles the effectiveness of K (i.e., K is replaced by $2K$). Which firm attains higher output?

A. Firm A, with output $Q_A = 8$.

B. Firm B, with output $Q_B = 16$.

C. Both firms obtain the same output.

D. Firm B benefits more because capital-saving progress always dominates neutral progress.

Course: Microeconomics — Master in Economics

PART 2: Production Theory

Chapter 4: Producer's Choice: Benefits Maximization and Cost Minimization

Guadalupe Correa-Lopera | University of Granada

Outline

Cost Minimization Problem (Long-Run)

- In the previous chapter, we studied the **technology of production**: how inputs K (capital) and L (labor) combine to generate output Q.
- Now we turn to the **firm's decision problem**: given input prices, how should a firm choose inputs in order to **produce at the lowest possible cost**?
- This is the starting point of the **producer's choice**:

$$\min_{L,K} C = wL + rK \quad \text{s.t.} \quad f(K,L) \geq \bar{Q},$$

$$w = \text{wage (price of labor)}, \quad r = \text{rental rate of capital}.$$

- We first develop the tools to represent this problem graphically, and then derive the firm's optimal input choice.

Isocost Lines

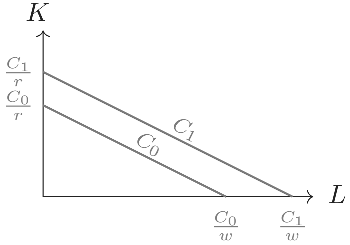
- To study cost minimization, we first define the **isocost line**.
- An *isocost* shows all combinations of labor L and capital K that can be purchased with the same total cost C.

$$C = wL + rK \quad \Rightarrow \quad K = \frac{C}{r} - \frac{w}{r}L.$$

- Its slope is

$$\left.\frac{dK}{dL}\right|_C = -\frac{w}{r},$$

which represents the **market trade-off** between inputs (relative price of labor vs. capital).

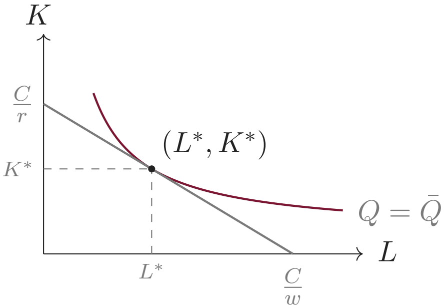


Optimal Input Choice: Tangency Condition

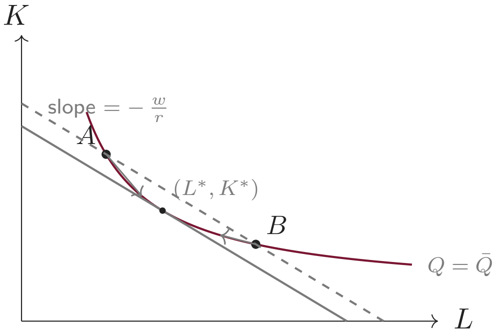
- At the cost-minimizing bundle (L^*, K^*):

$$\frac{MP_L}{MP_K} = \frac{w}{r}.$$

- **Tangency**: slope of isoquant $(-MP_L/MP_K)$ equals slope of isocost $(-w/r)$.
- Intuition: the firm equalizes the **technological trade-off** (MRTS) with the **market trade-off** (price ratio).



When the Firm is not at the Optimum

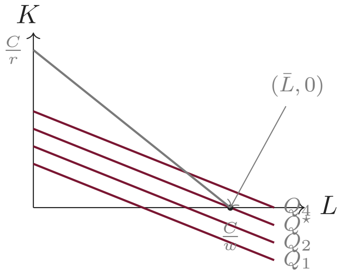


- **At A (isoquant steeper than isocost):**
  $$-\left.\frac{dK}{dL}\right|_{Q=\bar{Q}} < -\frac{w}{r} \iff \frac{MP_L}{MP_K} > \frac{w}{r}.$$
  Labor yields more output per euro: $\frac{MP_L}{w} > \frac{MP_K}{r}$. Cost falls by moving to more L and less K along $Q = \bar{Q}$.
- **At B (isoquant flatter than isocost):**
  $$-\left.\frac{dK}{dL}\right|_{Q=\bar{Q}} > -\frac{w}{r} \iff \frac{MP_L}{MP_K} < \frac{w}{r}.$$
  Capital yields more output per euro: $\frac{MP_L}{w} < \frac{MP_K}{r}$. Cost falls by moving to less L and more K along $Q = \bar{Q}$.
- **Only at tangency:**
  $$\frac{MP_L}{MP_K} = \frac{w}{r} \iff \frac{MP_L}{w} = \frac{MP_K}{r}.$$

Corner Solutions (When Tangency Fails)

- With **perfect substitutes**, isoquants are straight and parallel. For $Q = aL + bK$, isoquants have slope $-\frac{a}{b}$.
- If this slope differs from the isocost slope $-\frac{w}{r}$ (and one is steeper), the optimum is a **corner**: use only the cheaper input.

$$(L^*, K^*) = (\bar{L}, 0) \text{ or } (0, \bar{K}).$$



- In contrast, with strictly convex isoquants, the optimum is interior (tangency holds).

Expansion Path and Total Cost Curve

- So far: for each target $\bar{Q}$, cost minimization yields an **optimal input bundle** (L^*, K^*).
- As $\bar{Q}$ increases, these bundles trace the **expansion path**: a curve showing how the cost-minimizing mix of inputs varies with output.
- For each output $\bar{Q}$, we define the **total cost function (TC)** as

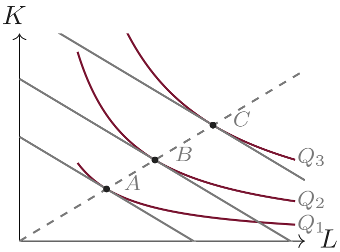
$$TC(\bar{Q}) \;=\; \min_{L,K} \{ \, wL + rK \,:\; f(K,L) \geq \bar{Q} \, \}.$$

  It gives the minimum expenditure required to produce $\bar{Q}$.
- Plotting $(\bar{Q}, TC(\bar{Q}))$ yields the **total cost curve (TC)**: the least cost of producing different output levels.

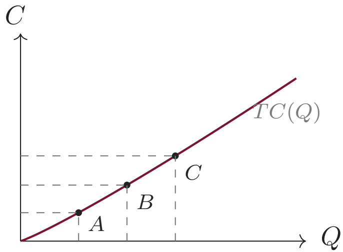
From Expansion Path to Total Cost Curve

1. For each target output $\bar{Q}$, solve the **cost minimization** problem $\Rightarrow$ optimal bundle (L^*, K^*) and minimum cost $C(\bar{Q}) = wL^*(\bar{Q}) + rK^*(\bar{Q})$.
2. As $\bar{Q}$ increases, (L^*, K^*) traces the **expansion path** (with fixed prices, often a ray for Cobb–Douglas).
3. Plot the pairs $(\bar{Q}, C(\bar{Q}))$ in (Q, C) space $\Rightarrow$ the **total cost curve** $TC(Q)$.
4. Typically $TC'(Q) > 0$ and $TC''(Q) \geq 0$ when marginal products are diminishing.

From Expansion Path to Total Cost Curve



Expansion path: optimal input bundles A, B, C as Q grows.



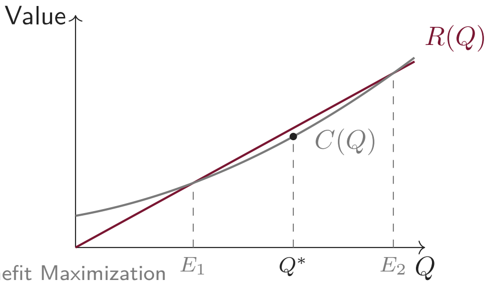
Total cost curve: minimum cost C for each Q, points A, B, C.

Firm's Objective: Benefit Maximization

- So far, we studied how firms minimize the cost of producing a given output.
- Now we turn to the firm's **ultimate goal**: maximizing **profit** π.

$$\pi(Q) = R(Q) - C(Q),$$

  where $R(Q) = p \cdot Q$ is total revenue and $C(Q) = TC(Q)$ is total cost.
- Key trade-off:
  - More output $\Rightarrow$ more revenue, but also higher cost.
  - The firm chooses the Q^* that maximizes $\pi(Q)$.



- **Zero-profit outputs** E_1 and E_2: where $R(Q) = C(Q)$ (i.e., $\pi = 0$).
- **Profits > 0** for $Q \in (E_1, E_2)$; losses outside.
- **Maximum profit at** Q^*: the vertical gap $R(Q) - C(Q)$ is largest at that output.

Mathematical Solution to the Firm's Problem

Firm's profit maximization:

$$\max_{L,K\geq 0} \ \pi(L,K) = p\,Q - wL - rK \quad \text{s.t.} \quad Q = f(L,K)$$

Substitute $Q = f(L,K)$:

$$\max_{L,K} \ \pi(L,K) = p\,f(L,K) - wL - rK$$

First-Order Conditions (FOCs):

$$\frac{\partial \pi}{\partial L} = p \cdot \frac{\partial f}{\partial L} - w = 0 \quad \Longrightarrow \quad p \cdot MP_L = w,$$

$$\frac{\partial \pi}{\partial K} = p \cdot \frac{\partial f}{\partial K} - r = 0 \quad \Longrightarrow \quad p \cdot MP_K = r.$$

Interpretation: the revenue contributed by the last unit of each input exactly equals the cost of hiring that unit.

Accounting vs. Economic Costs

- **Accounting costs:** actual monetary outlays recorded in the books (e.g., wages, rent, materials).
- **Economic costs:** accounting costs *plus* opportunity costs of alternatives that are **given up**.
- Example: if a student spends one year studying full-time, the *accounting cost* is tuition and books, while the *economic cost* also includes the salary they give up by not working that year.
- In decision-making, **economic costs** are the relevant ones.

Opportunity and Sunk Costs

- **Opportunity cost:** value of the best alternative given up when making a choice.
- **Sunk cost:** expenditure that has already been incurred and cannot be recovered.
- **Rule:** sunk costs should not affect current decisions, but opportunity costs must be considered.

Fixed, Variable, and Total Costs

- **Fixed cost (FC):** independent of output Q (e.g., rent, equipment).
- **Variable cost (VC):** varies with output (e.g., labor, materials).
- **Total cost (TC):**

$$TC(Q) = FC + VC(Q).$$

Average and Marginal Costs

- **Average fixed cost (AFC):** fixed cost per unit of output.

$$AFC(Q) = \frac{FC}{Q}, \quad \text{decreases as } Q \text{ increases.}$$

- **Average variable cost (AVC):** variable cost per unit of output.

$$AVC(Q) = \frac{VC(Q)}{Q}.$$

- **Average total cost (ATC):** total cost per unit of output.

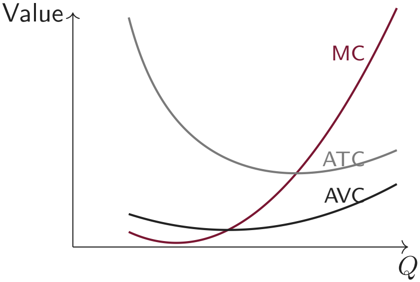
$$ATC(Q) = \frac{TC(Q)}{Q} = AFC(Q) + AVC(Q).$$

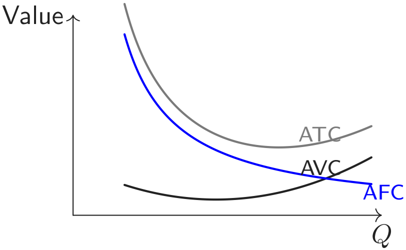
- **Marginal cost (MC):** extra cost of producing *one more unit of output*.

$$MC(Q) = \frac{dTC(Q)}{dQ}.$$

(In discrete terms: $MC \approx \frac{\Delta TC}{\Delta Q}$).

Relationship between Average and Marginal Cost





- **Crossing rule:** The MC curve intersects the AVC and ATC curves exactly at their *minimum* points.
- **Direction:** If $MC < ATC$ (or AVC), that average is *decreasing*; If $MC > ATC$ (or AVC), that average is *increasing*.
- Where MC touches AVC (resp. ATC) we have the output level that minimizes AVC (resp. ATC).
- **Why ATC lies above AVC:** The vertical gap equals the average fixed cost (AFC); as output grows, AFC shrinks and the gap narrows.

PART 2: Production Theory

Chapter 4: Producer's Choice: Benefits Maximization and Cost Minimization

Course: Microeconomics — Master in Economics
University of Granada

Instructor: Guadalupe Correa-Lopera

Instructions. Select the single correct answer (A–D).

Q1. A firm produces with $f(K, L) = K^{1/2}L^{1/2}$. Input prices are $w = 4$ and $r = 1$. The firm wants to produce exactly $\bar{Q} = 10$ at minimum cost. Which bundle is cost-minimizing?

- **A.** $(L, K) = (5, 20)$ with total cost $C = 40$.
- **B.** $(L, K) = (10, 10)$ with total cost $C = 50$.
- **C.** $(L, K) = (4, 25)$ with total cost $C = 41$.
- **D.** $(L, K) = (6, 16)$ with total cost $C = 40$.

Q2. Two input bundles $A = (L, K) = (10, 20)$ and $B = (25, 10)$ lie on the *same* isocost line for some strictly positive input prices (w, r). Which statement is correct about that isocost in (L, K) space?

- **A.** Slope $= -\frac{2}{3}$; isocost can be written as $2L + 3K = C$.
- **B.** Slope $= -\frac{3}{2}$; isocost can be written as $3L + 2K = C$.
- **C.** Slope $= -1$; isocost can be written as $L + K = C$.
- **D.** Slope $= -2$; isocost can be written as $2L + K = C$.

Q3. Output is $Q = 2L + K$. Input prices are $w = 5$ and $r = 3$. At the cost-minimizing bundle for any target $\bar{Q}$, which input(s) does the firm use?

- **A.** Only capital, because its unit price $r = 3$ is lower than the unit price of labor $w = 5$.
- **B.** Only labor, because labor has a lower cost per effective unit of output.
- **C.** A positive mix of inputs, because cost minimization requires tangency whenever isoquants are linear.
- **D.** A positive mix of inputs, because the cost per effective unit is identical for labor and capital.

Q4. Suppose total revenue is $R(Q) = 20Q$ and total cost is $C(Q) = 50 + 6Q + Q^2$. At which output Q is profit $\pi(Q)$ maximized?

- **A.** $Q = 5$
- **B.** $Q = 6$

C. $Q = 7$

D. $Q = 10$

Q5. A firm maximizes $\pi(L, K) = pf(L, K) - wL - rK$ with $p > 0$, $w > 0$, $r > 0$. Suppose the optimum is interior. Which condition correctly characterizes the choice of inputs?

A. Each input is hired up to the point where its value of marginal product equals its price: $p \cdot MP_L = w$ and $p \cdot MP_K = r$.

B. Inputs are chosen so that the marginal rate of technical substitution equals the output price: $\frac{MP_L}{MP_K} = p$.

C. Profit maximization requires equalizing the marginal products of labor and capital: $MP_L = MP_K$.

D. The optimal input mix occurs where total revenue equals total cost: $pf(L, K) = wL + rK$.

Q6. A firm faces input prices $w = 4$, $r = 2$, and output price $p = 16$. Its production function is $f(L, K) = L^{1/2}K^{1/4}$. Which input bundle (L, K) maximizes profit?

A. $(16, 16)$

B. $(8, 8)$

C. $(10, 40)$

D. $(40, 10)$

Q7. A student pays €1,000 in tuition and €200 in books to study for one year. If the student gives up a €15,000 job opportunity, what is the *economic cost* of studying?

A. €1,200

B. €15,000

C. €16,200

D. €15,200

Q8. Suppose a firm has fixed cost $FC = 100$ and variable cost $VC(Q) = 4Q$. Which of the following statements is correct about its cost functions?

A. $TC(Q) = 100 + 4Q$; $MC = 4$; $AFC(Q) = 100/Q$.

B. $TC(Q) = 4Q$; $MC = Q$; $ATC(Q) = 4 + 100/Q$.

C. $TC(Q) = 100 + 4Q^2$; $MC = 8Q$; $AFC(Q) = 100/Q$.

D. $TC(Q) = 100 + 4Q$; $MC = 100$; $AFC(Q) = 4/Q$.

Q9. Which relationship between average and marginal cost is *always true*?

A. If $MC < ATC$, then ATC is decreasing.

B. If $MC > AVC$, then ATC must also be increasing.

C. MC intersects ATC at the maximum point of ATC.

D. MC and AVC never intersect.