Journal of Economic Behavior and Organization 232 (2025) 106941

Contents lists available at ScienceDirect

Journal of Economic Behavior and Organization

journal homepage: www.elsevier.com/locate/jebo

Measuring success in streaming platforms☆

Juan Carlos Gonçalves-Dosantos [a], Ricardo Martínez [b],*, Joaquín Sánchez-Soriano [a]

[a] Universidad Miguel Hernández de Elche, Spain
[b] Universidad de Granada, Spain

ARTICLE INFO

Keywords:
Measure
Relevance
Proportionality
Streaming
Axiom

ABSTRACT

Digital streaming platforms , including Twitch, Spotify, Netflix, Disney+, and Kindle, have emerged as major sources of entertainment with significant growth potential. Many of these platforms distribute royalties among streamers, artists, producers, or writers based on their impact. In this paper, we measure the relevance of each of these contributors to the overall success of the platform, which can play a key role in revenue allocation. We perform an axiomatic analysis to provide normative foundations for four relevance metrics: the uniform, the subscriber-uniform, the proportional, and the subscriber-proportional indicators. The last two indicators implement the so-called pro-rata and user-centric models, which are extensively applied to distribute revenues in the music streaming market. The axioms we propose formalize different principles of fairness, stability, and non-manipulability, and are tailor-made for the streaming context. We complete our analysis with a case study that measures the influence of the 19 most-followed streamers worldwide on the Twitch platform.

1. Introduction

In 2022, Netflix, Amazon Prime Video, Disney+, and HBO MAX – four of the most successful streaming platforms – reported a joint annual revenue of $103,5 billion. These companies operate so-called over-the-top broadcasting services by offering to their customers a media library of movies, series, and documentaries. Viewers must pay a monthly fee to obtain unlimited access to the catalog. As Table 1 shows, the subscription prices for these platforms are similar, but the number of subscribers and earned revenue differ significantly.

Two elements are crucial for determining the success (or revenue) of a platform: the fee and the catalog. Some companies have only two or three shows each year that subscribers are interested in, but those shows may be massive hits. Other platforms choose a different model, offering a wider catalog based on the variety and quantity of content. Be that as it may, it is clear that not all titles offered by these companies are equally in demand, and the available titles may be decisive for consumers when deciding which platform to subscribe to. Therefore, if one platform offers content that is more relevant than others, a worthwhile question to ask is: what is the impact of each single show on the overall success of a platform? This question can be answered by applying the theoretical model presented in this paper.

☆ Juan Carlos Gonçalves-Dosantos acknowledges the grant PID2021-12403030NB-C31 funded by MCIN/AEI/10.13039/501100011033 and by "ERDF A way of making Europe/EU". Ricardo Martínez acknowledges the R&D&I project grant PID2023-147391NB-I00 funded by MCIU/AEI/10.13039/501100011033 and by the FSE+, Spain. Joaquín Sánchez-Soriano acknowledges the R&D&I project grant PID2022-137211NB-I00 funded by MCIN AEI/10.13039/501100011033 and by "ERDF A way of making Europe/EU", and he also acknowledges financial support from Generalitat Valenciana, Spain under project PROMETEO/2021/063.

* Corresponding author.
E-mail address: ricardomartinez@ugr.es (R. Martínez).

https://doi.org/10.1016/j.jebo.2025.106941
Received 13 May 2024; Received in revised form 22 November 2024; Accepted 12 February 2025
Available online 27 February 2025

Table 1
Fees, subscribers, and revenues of the main streaming platforms.

Company	Subscription price	Subscribers	Revenue
Netflix	$7	231 million	$31.6 billion
Amazon Prime Video	$9	200 million	$25.2 billion
Disney+	$8	164 million	$7.4 billion
HBO+	$10	95 million	$39.3 billion

The previous setting is not unique to over-the-top subscription video-on-demand providers; it can be extended to many similar situations, in which selling products in a package may be more profitable than selling them independently (see Adams and Yellen (1976)). Spotify, Twitch, YouTube, and other web-based services are only a few examples. In all these cases, subscribers pay a subscription fee to obtain unlimited access to a set of services, and it is only natural to wonder what the individual contribution of each of those services is to the success of the whole package.

In our model, a *platform* is described by four elements: the set of *services* the platform provides (TV shows, artists, streamers, books, etc.), the set of *subscribers* who have unlimited access to those services (viewers, users, readers, etc.), the *subscription price* paid by each subscriber, and the *consumption matrix* that indicates the quantity of each service consumed by each subscriber. An *indicator* is a measure that determines the relevance of each single service to the success of the platform, which is the total revenue generated from selling subscriptions

In this study, we introduce four natural indicators to determine the share of the platform's success attributable to each service. The *uniform indicator* stipulates that all services are equally relevant. The *subscriber-uniform indicator* states that the relevance of each individual subscriber is divided uniformly among the services that the subscriber consumes. The *proportional indicator* assigns to each service a relevance proportional to its aggregate consumption. The last proposal, the *subscriber-proportional indicator*, also considers a proportional perspective but applied to each single subscriber. More precisely, the success of each single subscriber is proportionally assigned among the services that the subscriber consumes. The relevance of each service is the lump sum of those proportions across subscribers.

In the music streaming industry, the two most relevant remuneration methods are the *pro-rata* and the *user-centric* mechanisms. The former is based on an aggregate approach, where the overall revenue from all music subscriptions is distributed among the artists proportionally to their total number of streams. In the latter, the calculation is based on the listening habits of each individual subscriber; the subscription fee they pay is proportionally distributed, but only among the artists whose music this subscriber consumes. The *proportional indicator* and the *subscriber-proportional indicator* implement the *pro-rata* and *user-centric* principles, respectively.

As in Manshadi et al. (2023) and Singal et al. (2022), we follow the axiomatic methodology to investigate the existence of indicators that satisfy combinations of properties (called *axioms*) that are suitable for our setting. The axioms that we propose formalize different principles related to fairness, stability, and absence of manipulability that an indicator of relevance should satisfy. Regarding fairness, *symmetry* requires that if two services are equal (i.e., they are equally consumed by all the subscribers), then they must be equally relevant to the success of the platform. *Evenness* requires that if two services are consumed by the same subscribers, then both must be equally relevant. *Nullity service* states that services not consumed have no relevance to the platform. With regard to stability, *homogeneity* states that the relevance must not be affected by the units (thousands or millions, for example) in which the consumption matrix is expressed. *Consumption sensitivity* requires that small mistakes in the measurement of the consumption do not drastically alter the significance of each service. *Composition* states that when data is combined from two subgroups, it is possible to determine the indicator of the total group by a suitable composition of the subgroups' indicators. *Sharing-proofness* and *non-manipulability* belong to the third type of principles. The former states that if subscribers share their subscriptions with friends and relatives, then the relevance of the services remains unaltered. The latter axiom requires that the indicator should not be able to be manipulated by merging or splitting services.

Regarding our findings, we obtain some characterizations of the aforementioned indicators. More precisely, if we require both *composition* and *non-manipulability* to be satisfied, we must apply the *subscriber-proportional indicator*. Theorem 2 shows that the *proportional indicator* is the unique indicator that fulfills *sharing-proofness* and *non-manipulability* together. In Theorem 3, we prove that the combination of *symmetry*, *homogeneity*, and *consumption sensitivity* unambiguously yields the *uniform indicator*. Finally, Theorem 4 shows that if *evenness*, *nullity*, and *composition* are demanded, then the *subscriber-uniform indicator* should be implemented. As in our setting, the *proportional* and the *subscriber-proportional* indicators reflect the *pro-rata* and *user-centric* schemes. Theorems 1 and 2 also reveal characterizations of these two mechanisms. In other words, based on several principles of fairness, stability, and non-manipulability, we provide normative foundations for the two most-used revenue allocation methods in the music streaming industry.

We conclude our paper with an illustrative application of the indicators that we characterize. In particular, we measure the relevance of each of the top 19 most-watched Twitch streamers. This information could play a key role in determining the distribution of revenues. We highlight and explain the differences and implications of the four indicators that we analyze in this work. Based on the obtained results, we conclude that the mere use of either the number of viewers or exclusive viewers, which is the usual standard in this industry, is not the most effective approach for determining the impact of each streamer on the platform.

1.1. Related literature

Our work provides a novel approach to determining and valuing the impact of each single item, service, or contributor to the collective success of a platform or bundling product. Other papers have also addressed the question of how to isolate and measure the merit of single agents in the collective success of online frameworks. Singal et al. (2022) provide an axiomatic justification of the *counterfactual adjusted Shapley value*, which measures the contribution of individual advertiser actions (emails, display ads, search ads, etc.) to eventual customer acquisition. The authors show that this Shapley-based metric coincides with an *adjusted unique-uniform* attribution scheme. In contrast, the indicators that we characterize in this work are based on proportionality rather than uniformity. Proportional schemes are usually perceived by society as fair allocation methods and, therefore, as less controversial.

The discussion around how to divide the revenue obtained by subscriptions to music streaming platforms among artists has become particularly relevant in recent years. In this context, the *pro-rata* and the *user-centric* schemes have emerged as the most prominent and applied methods (see, for example, Dimont (2017), Page and Safir (2018b), Page and Safir (2018a), Jari (2018), and Meyn et al. (2023)). Alaei et al. (2022) study the strategic implications of these two schemes in a model with a two-sided streaming service platform that generates revenues by charging users a subscription fee for unlimited access to the content. We contribute to this discussion by providing normative foundations of the *pro-rata* and *user-centric* rules.

López-Navarrete et al. (2019) study the problem of how to allocate the revenues generated in a Smart TV ecosystem between the provider and the content producers. They characterize the core of the game associated with the situation and provide simple formulas for their Shapley and Tijs values, which belong to the core of the game. López-Navarrete et al. (2023) study how to allocate the revenues generated in a video platform such as YouTube, considering the navigation of the users on the platform. For this, they define dynamic games associated with the problem and provide several allocation schemes based on the structure of the Shapley value. Our approach differs from that of López-Navarrete et al. (2019, 2023) since they approach the matter from a game-theoretical perspective and, in our case, from an axiomatic perspective. Bergantiños and Moreno-Ternero (2024, 2025) also study the problem of how to share revenue in the streaming industry, with a particular focus on music streaming services such as Spotify. They develop and characterize allocation mechanisms based on the *pro-rata* and *user-centric* principles, among others. The axioms we propose in this paper differ from those presented in Bergantiños and Moreno-Ternero (2024, 2025). In particular, the results obtained by these authors require properties such as *proportionality on streams*, *equal importance of similar users*, or *non-manipulability by leaving the platform*, which we do not explore in this work. The definition of indicator is also different.

A related paper to this, but from the perspective of claim problems and generalizations, is Ju et al. (2007). The authors describe a general resource allocation problem where an infinitely divisible good must be distributed among a set of entities. Each entity is characterized by a vector, with each value reflecting the entity's relevance to a specific set of issues. In contrast to the problem discussed in Ju et al. (2007), where one infinitely divisible good is allocated, in our model we consider a vector of infinitely divisible goods, one for each subscriber, depending on the subscription fee of each individual subscriber. In addition to the proportional rules explored in Ju et al. (2007), our approach allows us to consider rules that are applied to each subscriber independently. Based on the normative framework proposed by Ju et al. (2007), which encompasses a wide range of resource allocation problems, we will introduce several axioms that are relevant to the context at hand.

Other authors have addressed similar questions from different perspectives in the literature on allocation and attribution problems. Ginsburgh and Zang (2003), Bergantiños and Moreno-Ternero (2015), and Martínez and Sánchez-Soriano (2023) analyze the so-called museum pass problem to distribute the revenue obtained by selling museum passes that allow entrance into a set of museums. Martínez and Sánchez-Soriano (2021) develop a theoretical model to measure the relative relevance of different pathologies to the lethality of a disease in a society. Additionally, in the context of biology applications, Moretti et al. (2007), Albino et al. (2008), and Lucchetti et al. (2010) propose indices to identify the genes that are useful for the diagnosis and prognosis of specific diseases and cancers, while (Martínez and Moreno-Ternero, 2022) report that the weighted averages of the *incidence rate*, *morbidity rate*, and *mortality rate* are the appropriate methods to evaluate the impact of pandemics. Brander et al. (2011) offer an account of the importance of the attribution problem in climate change, and Burger et al. (2020) conduct an in-depth review of the attribution problem in the context of climate change, both from a technical perspective and its legal and policy applications. Manshadi et al. (2023) analyze how governmental and non-profit organizations must face the task of obtaining equitable and efficient rationing of a social good among individuals whose needs (as was the case in the recent pandemic) emerge sequentially and are possibly correlated. Finally, Martínez et al. (2022) characterize several methods to evaluate the multidimensional proposals of bidders in public procurement tenders. However, our approach differs from the above-mentioned papers in the problem we study, in the structure of the elements that define the problem, and in several of the properties used to characterize the solutions presented.

The rest of the paper is organized as follows. In Section 2, we present the model and introduce four indicators to measure the relevance of services in a platform. In Section 3, we propose some axioms particularly suitable for this framework. In Section 4, we present the normative analysis and characterization results. In Section 5, we provide an application of our theoretical model. Concluding remarks are provided in Section 6.

2. Model and indicators

Let $\mathbb{N}$ represent the set of natural numbers, and let $\mathcal{N}$ be the set of all finite and non-empty subsets of $\mathbb{N}$. A **platform** is described by a 4-tuple (N, S, p, C), where $N = \{1, \dots, |N|\} \in \mathcal{N}$ $(|N| \geq 3)$[1] is the set of **services** provided by the platform, $S = \{1, \dots, |S|\} \in \mathcal{N}$

[1] Just as Ju et al. (2007) require this condition for its results, we also require it for the results presented here. However, it is not a limiting condition, as the number of services is typically greater than or equal to 3 on most platforms.

is the set of **subscribers** who have unlimited access to the services in N, $p = (p_1, \ldots, p_{|S|}) \in \mathbb{R}^{|S|}_{++}$ represent the **subscription prices** paid by each subscriber, and C is the **consumption matrix**, each of whose entries $C_{is} \in \mathbb{R}_+$ indicates the quantity of a service i consumed by subscriber s. The **success** of the platform is the total revenue generated from selling subscriptions, $\sigma = \|p\|$.[2] We denote by $\mathcal{D}$ the set of all platforms.

We denote by $C_{i\cdot}$ the ith row of C, which represents the consumption of service i. We also denote by $C_{\cdot s}$ the sth column of C, which represents the consumption of subscriber s. Therefore, $\|C_{i\cdot}\| = \sum_{s\in S} C_{is}$ and $\|C_{\cdot s}\| = \sum_{i\in N} C_{is}$ are the total consumption of service $i \in N$ and subscriber $s \in S$, respectively. We restrict ourselves to consumption matrices such that $\|C_{\cdot s}\| > 0$ for all $s \in S$, that is, we assume that every subscriber has paid the subscription fee with the willingness to consume part of the content that the platform offers. Given a set of services $N' \subseteq N$, or a set of subscribers $S' \subseteq S$, the matrices resulting from removing the rows in N' and the columns in S' are denoted by $C_{N\setminus N'}$ and $C_{S\setminus S'}$, respectively.

The relevance of each service to the success of the platform is measured using an **indicator** R, which is a mapping $R : \mathcal{D} \longrightarrow \mathbb{R}^{|N|}_+$ such that

$$\sum_{i\in N} R_i(N, S, p, C) = \sigma.$$

For each $i \in N$, $R_i(N, S, p, C)$ indicates the share of the success of the platform that is due to i.

Now, we present some examples of possible indicators. The first one is straightforward; it attributes equal responsibility to all services.

Uniform indicator. For each $(N, S, p, C) \in \mathcal{D}$ and each $i \in N$,

$$R^U_i(N, S, p, C) = \frac{\sigma}{|N|}.$$

The second indicator uniformly distributes the success of each subscriber among the services that have been consumed, regardless of the consumption intensity. If subscriber $s \in S$ has not used the service supplied by $i \in N$, that is, if $C_{is} = 0$, then this has no influence on subscriber s.

Subscriber-uniform indicator. For each $(N, S, p, C) \in \mathcal{D}$ and each $i \in N$,

$$R^{SU}_i(N, S, p, C) = \sum_{s\in S\,:\, i\in N_s} \frac{1}{|N_s|} p_s,$$

where $N_s = \{ j \in N : C_{js} \neq 0 \}$.

The next indicator divides the success proportionally to the consumption of each service. It implements the so-called *pro-rata rule*, which is one of the two most prominent remuneration methods in the literature on the music streaming industry.

Proportional indicator. For each $(N, S, p, C) \in \mathcal{D}$ and each $i \in N$,

$$R^P_i(N, S, p, C) = \frac{\|C_{i\cdot}\|}{\sum_{j\in N} \|C_{j\cdot}\|}\sigma.$$

While the previous indicator distributes the success as a whole, the following measure is based on determining the role of each service in the success of attracting the interest of each single subscriber. Consider a subscriber $s \in S$ who has paid the subscription fee p_s. At this individual level, the success of the platform is simply p_s. Now, divide p_s among all the services consumed by s in proportion to the consumption; proceed in the same way for all subscribers and aggregate across them. This indicator mimics the principle behind the other focal remuneration method for music streaming: namely, the *user-centric rule*.

Subscriber-proportional indicator. For each $(N, S, p, C) \in \mathcal{D}$ and each $i \in N$,

$$R^{SP}_i(N, S, p, C) = \sum_{s\in S} \frac{C_{is}}{\|C_{\cdot s}\|} p_s.$$

In the following example, we illustrate the application of the four aforementioned indicators.

Example 1. Consider the platform where $N = \{1, 2, 3\}$, $S = \{1, 2, 3, 4, 5, 6\}$, $p = \left(2, 4, \frac{5}{2}, 2, 1, \frac{7}{2}\right)$, and C is given by

$$\begin{pmatrix} 0 & 5 & 0 & 1 & 2 & 3 \\ 1 & 1 & 2 & 3 & 6 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Notice that, in this case, the success is $\sigma = 15$. The next table shows the relevance of each service according to the four previous indicators:

[2] The notation $\|\cdot\|$ refers to the sum of all absolute values of all entries of a vector or matrix.

Indicator	Services		
	1	2	3
R^U	5	5	5
R^{SU}	7	8	0
R^P	$\frac{55}{8}$	$\frac{65}{8}$	0
R^{SP}	$\frac{91}{12}$	$\frac{89}{12}$	0

3. Axioms

The axiomatic approach to analyzing allocation mechanisms is one of the common methods for determining the most appropriate mechanism, depending on the properties considered most relevant to the problem or context in question.

The following two axioms are the minimum requirements for impartiality when considering different levels of information. Both require symmetric services to be treated symmetrically, but they differ on when two services should be considered as such. *Evenness* aligns with the concept of *share* in television broadcasts, which represents the percentage of viewers watching a particular TV program relative to the total number of viewers watching TV at that time. This measure is the most renowned and significant metric for assessing the relative popularity of a specific broadcast program compared to others. Share simply counts the number of viewers watching each TV program, regardless of whether they have watched the entire program or just a part of it. We adapt this approach to our impartiality principle. Under evenness, two services (TV programs) are considered symmetric if they have been consumed (watched) by the same set of subscribers, independently of the intensity of use (or the time spent on each program). If that happens, then both programs should be equally relevant.

Evenness. For each (N,S,p,C) and each pair $\{i,j\} \subset N$, if $C_{is} > 0 \Leftrightarrow C_{js} > 0$ for all $s \in S$, then $R_i(N,S,p,C) = R_j(N,S,p,C)$.

As an alternative to the previous formulation, the next axiom considers that two services are symmetric, not only if they have been consumed by the same set of subscribers, but if the intensity of that consumption is also the same. If this happens, then both services should be considered equally relevant.

Symmetry. For each (N,S,p,C) and each pair $\{i,j\} \subset N$, if $C_{is} = C_{js}$ for all $s \in S$, then $R_i(N,S,p,C) = R_j(N,S,p,C)$.

Obviously, *evenness* implies *symmetry*. The distinction between these two properties is particularly relevant in this setting. In other contexts, such as museum pass problems (Ginsburgh and Zang, 2003; Martínez and Sánchez-Soriano, 2023) or risk factor problems in epidemiology (Martínez and Sánchez-Soriano, 2021; Martínez and Moreno-Ternero, 2022), *evenness* becomes equivalent to the property of *symmetry*.

The following axiom states that if a service is not consumed by any subscriber, then it has no relevance within the platform, and then its relevance must be equal to zero.

Nullity. For each $(N,S,p,C) \in \mathcal{D}$ and each $i \in N$ such that $C_{is} = 0$ for all $s \in S$, then $R_i(N,S,p,C) = 0$.

The next axiom states that changing units or magnitudes does not impact the relevance of services in determining their success on the platform. In simpler terms, multiplying the consumption matrix by a positive number does not affect the relevance of the services.

Homogeneity. For each (N,S,p,C) and each $\lambda \in \mathbb{R}_{++}$, $R(N,S,p,\lambda C) = R(N,S,p,C)$.

Suppose that there are two disjoint groups of subscribers S and S' (e.g., subscribers based in two regions of a country). Now consider a larger society resulting from combining S and S'. The question is how to recalculate the indicator for $S \cup S'$ from the indicators for S and S'. *Composition*[3] states that the relevance to the success of each service in the large population, $S \cup S'$, is the sum of the relevance in S and S'.

Composition. For each $(N,S,p,C),(N,S',p',C') \in \mathcal{D}$ such that $S \cap S' = \emptyset$,

$$R(N,S \cup S',p \oplus p',C \oplus C') = R(N,S,p,C) + R(N,S',p',C'),$$

where $C \oplus C'$ is the matrix resulting from concatenating C and C' by rows, and $p \oplus p'$ is the vector resulting from concatenating p and p'.

The upcoming axiom, which we will term *consumption sensitivity*, states that minor alterations in the consumption matrix of the problem do not result in significant changes in the relevance of the services.

Consumption sensitivity. For each $\delta \geq 0$, if $\|C - C'\| < \delta$, then $\|R(N,S,p,C) - R(N,S,p,C')\| < \varepsilon(\delta)$, so that $\lim_{\delta \to 0} \varepsilon(\delta) = 0$.

This property implies that if we have two successions of problems $\{(N,S,p,C^n)\}$ and $\{(N,S,p,C'^n)\}$, such that for every $\delta > 0$, there exists n_0 such that $\forall n \geq n_0$, $\|C^n - C'^n\| < \delta$, then $\|R(N,S,p,C^n) - R(N,S,p,C'^n)\| < \varepsilon(\delta)$. Therefore, as $\|C^n - C'^n\|$ goes to 0, $\|R(N,S,p,C^n) - R(N,S,p,C'^n)\|$ also goes to 0.

[3] This property is related to the additivity or decomposability into parts of a problem, which is also common in the resource allocation literature (see, for example, Bergantiños and Moreno-Ternero (2015), Martínez and Moreno-Ternero (2022), López-Navarrete et al. (2023), Martínez and Sánchez-Soriano (2021, 2023)).

Remark 1. Consumption sensitivity should not be confused with the *continuity* property that is usually used in the resource allocation literature. This axiom states that if there is a sequence of problems convergent to a given one, then the associated sequence of allocations converges to the allocation of the limit problem. Formally, let $(N, S, p^n, C^n) \in \mathcal{D}$ be a succession of problems and $(N, S, p, C) \in \mathcal{D}$ a problem such that for every $\delta > 0$, there exists n_0 such that $\forall n \geq n_0$, $\|C - C^n\| < \delta$ and $\|p - p^n\| < \delta$, then $\|R(N, S, p, C) - R(N, S, p^n, C^n)\| < \varepsilon(\delta)$.

On the one hand, the proportional indicator trivially satisfies continuity. However, it violates consumption sensitivity (see Proposition 1).

On the other hand, the following rule satisfies consumption sensitivity but violates continuity. For each $(N, S, p, C) \in \mathcal{D}$ and each $i \in N$,

$$R_i(N, S, p, C) = \begin{cases} \dfrac{\sigma}{|N|} & \text{if } p_s = p_t \ \forall s, t \in S \\ \dfrac{2\sigma}{|N|} & \text{if } i = 1,\ p_s \neq p_t \text{ for some } s, t \in S \\ \dfrac{(|N| - 2)\sigma}{|N|(|N| - 1)} & i \neq 1,\ p_s \neq p_t \text{ for some } s, t \in S. \end{cases}$$

Let us consider the following succession of consumption matrices and price vectors

$$C^n = \begin{pmatrix} 1 + \frac{1}{n} & 1 + \frac{1}{n} & 1 + \frac{1}{n} \\ 2 + \frac{1}{n} & 2 + \frac{1}{n} & 2 + \frac{1}{n} \\ 3 + \frac{1}{n} & 3 + \frac{1}{n} & 3 + \frac{1}{n} \end{pmatrix} \text{ and } p^n = (1 + \frac{1}{n}, 1 + \frac{2}{n}, 1 + \frac{1}{n});$$

and the following consumption matrix and price vector

$$C = \begin{pmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 3 & 3 & 3 \end{pmatrix} \text{ and } p = (1, 1, 1).$$

It is obvious that $\|C - C^n\|$ and $\|p - p^n\|$ goes to 0 when n goes to infinity, but $\|R(N, S, p, C) - R(N, S, p, C^n)\| = \|(1, 1, 1) - (\frac{2(3+\frac{4}{n})}{3}, \frac{3+\frac{4}{n}}{6}, \frac{3+\frac{4}{n}}{6})\| = 2 + \frac{8}{6n} > 0$ for all $n > 1$.

Therefore, both properties are independent each other.

It is a common practice for subscribers of streaming platforms to share their subscriptions with friends and relatives. They divide the subscription cost so that each individual can access the platform's services. *Sharing-proofness* requires that the indicator remains unaffected by this type of behavior.

Sharing-proofness. For each $(N, S, p, C) \in \mathcal{D}$, each non-empty $S' \subseteq S$, and each $s \in S'$, where $p'_s = \sum_{t \in S'} p_t$ and $C'_{is} = \sum_{t \in S'} C_{it}$ for all $i \in N$, then

$$R(N, S, p, C) = R\left(N, \{s\} \cup S \backslash S', (p'_s, p_{S \backslash S'}), (C'_{\cdot s}, C_{S \backslash S'})\right),$$

where $(C'_{\cdot s}, C_{S \backslash S'})$ is the matrix such that $(C'_{\cdot s}, C_{S \backslash S'})_{is} = C'_{is}$ and $(C'_{\cdot s}, C_{S \backslash S'})_{it} = C_{it}$ for all $t \in S \backslash S'$ and all $i \in N$.

Our next axiom states that the relevance to success cannot be manipulated by merging or splitting services. In other words, this implies that no group of services can enhance its success on the platform by consolidating their consumption, and similarly, no individual service can increase its success by creating additional services and dividing its consumption among them. This axiom was introduced by O'Neill (1982) in the context of bankruptcy problems and has been recurrently used in resource allocation problems (e.g. Ju et al. (2007)).

Non-manipulability. For each $(N, S, p, C) \in \mathcal{D}$, each non-empty $N' \subseteq N$, and each $i \in N'$, where $C'_{is} = \sum_{j \in N'} C_{js}$ for all $s \in S$, then

$$R_i\left(\{i\} \cup N \backslash N', S, p, (C'_{i\cdot}, C_{N \backslash N'})\right) = \sum_{j \in N'} R_j(N, S, p, C),$$

where $(C'_{i\cdot}, C_{N \backslash N'})$ is the matrix such that $(C'_{i\cdot}, C_{N \backslash N'})_{is} = C'_{is}$, and $(C'_{i\cdot}, C_{N \backslash N'})_{js} = C_{js}$ for all $s \in S$ and all $j \in N \backslash N'$.

4. Normative analysis

In this section, we present our main results, which are characterizations of the four indicators presented in Section 2. These characterizations are given in terms of the axioms introduced in the previous section and, in some way, show which properties are associated with each of the indicators proposed to measure the relevance of the services to platform success. The next theorem states that the *subscriber-proportional indicator* is the unique indicator that fulfills both *composition* and *non-manipulability*. Based on the connection between the *subscriber-proportional indicator* and the *user-centric principle*, Theorem 1 also provides instrumental and normative justifications for the application of the latter, as it provides insights into its stability and the prevention of strategic behaviors to manipulate the measure of the success. All proofs are relagated to the appendix.

Theorem 1. *An indicator satisfies composition and non-manipulability if and only if it is the subscriber-proportional indicator.*

Table 2
Summary of theoretical findings. Y means that the indicator in the column does satisfy the axiom in the row, N means that the indicator does not satisfy the axiom, and Y* means that the axiom is used to characterize the indicator. Proofs and counterexamples of all properties for all studied relevance indicators that have not been proven in Theorem 1 to 4 are provided in Proposition 1.

Property	R^U	R^{SU}	R^P	R^{SP}
Symmetry	Y*	Y	Y	Y
Evenness	Y	Y*	N	N
Homogeneity	Y*	Y	Y	Y
Consumption sensitivity	Y*	N	N	N
Composition	Y	Y*	N	Y*
Sharing-proofness	Y	N	Y*	N
Non-manipulability	N	N	Y*	Y*
Nullity	N	Y*	Y	Y

Our second characterization shows that if we require *sharing-proofness* and *non-manipulability* to be satisfied, then the relevance of each service must be measured using the *proportional indicator*. As mentioned in the introduction, in our setting, the *proportional indicator* implements the *pro-rata principle*. Therefore, Theorem 2 can also be interpreted as a way to justify, from a normative perspective, the application of this principle in the music streaming industry.

Theorem 2. *An indicator satisfies sharing-proofness and non-manipulability if and only if it is the proportional indicator.*

Our third characterization is given in Theorem 3. This result states that the combination of *symmetry*, *homogeneity*, and *consumption sensitivity* unambiguously yield to the application of the *uniform indicator*.

Theorem 3. *An indicator satisfies symmetry, homogeneity, and consumption sensitivity if and only if it is the uniform indicator.*

The final characterization shows that if *composition*, *nullity*, and *evenness* must be satisfied for an indicator, then the relevance of each service must be measured using the *subscriber-uniform indicator*.

Theorem 4. *An indicator satisfies composition, nullity, and evenness if and only if it is the subscriber-uniform indicator.*

We will now examine which properties, in addition to those used in each of the proposed characterizations, are satisfied by each rule. Given the mechanical and simple nature of the proofs, these are relegated to Appendix.

Proposition 1. *The following statements hold:*

(a) The uniform indicator satisfies evenness, composition and sharing-proofness, but violates non-manipulability and nullity.
(b) The subscriber-uniform indicator satisfies symmetry and homogeneity, but violates consumption sensitivity and sharing-proofness.
(c) The proportional indicator satisfies symmetry, homogeneity and nullity, but violates evenness, consumption sensitivity and composition.
(d) The subscriber-proportional indicator satisfies symmetry, homogeneity and nullity, but violates evenness, consumption sensitivity and sharing-proofness.

Table 2 summarizes our theoretical findings. As we can observe, the four indicators satisfy several interesting properties, even though not all properties are required for their characterizations. The *uniform indicator* fulfills all the axioms except *non-manipulability* and *nullity*. The *subscriber-uniform indicator* satisfies all the axioms except *consumption sensitivity*, *sharing-proofness*, and *non-manipulability*, and it is characterized by *composition*, *evenness*, and *nullity*. *Sharing-proofness* and *non-manipulability* unambiguously yield to the *proportional indicator*, but it also satisfies *symmetry*, *homogeneity*, and *nullity*. The *subscriber-proportional indicator* is characterized by *composition* and *non-manipulability*, but it also fulfills *symmetry*, *homogeneity*, and *nullity*. From the standpoint of the axiom, two axioms – (*symmetry* and *homogeneity*) – are satisfied by all indicators in the paper. Moreover, all axioms except *consumption sensitivity* are satisfied by at least two of the four indicators.

Finally, we analyze whether the properties used in each of the theorems are independent, that is, whether all of them are necessary to characterize each of the rules.

Remark 2. The following counterexamples prove the independence of the properties used in Theorems 1 to 4.

1. The axioms of Theorem 1 are independent.
   (a) The uniform indicator satisfies composition, but not non-manipulability.
   (b) The proportional indicator satisfies non-manipulability, but not composition.
2. The axioms of Theorem 2 are independent.
   (a) The uniform indicator satisfies sharing-proofness, but not non-manipulability.
   (b) The subscriber-proportional indicator satisfies non-manipulability, but not sharing-proofness.

3. The axioms of Theorem 3 are independent.

   (a) The proportional indicator satisfies symmetry and homogeneity, but not consumption sensitivity.
   (b) Let R^1 be defined as follows. For each $i \in N$,

$$R_i^1(N,S,p,C) = \frac{\sigma}{|N|} - \min\left\{\frac{1}{\|C_{i\cdot}\|}, \|C_{i\cdot}\|\right\} + \frac{\sum_{j\in N} \min\left\{\frac{1}{\|C_{j\cdot}\|}, \|C_{j\cdot}\|\right\}}{|N|}.$$

   The indicator R^1 satisfies symmetry and consumption sensitivity, but not homogeneity.
   (c) Let R^2 be defined as follows. For each $i \in N$,

$$R_i^2(N,S,p,C) = \begin{cases} \dfrac{2\sigma}{|N|} & \text{if } i = 1 \\ \dfrac{(|N|-2)\sigma}{|N|(|N|-1)} & \text{otherwise.} \end{cases}$$

   The indicator R^2 satisfies homogeneity and consumption sensitivity, but not symmetry.

4. The axioms of Theorem 4 are independent.

   (a) The subscriber-proportional indicator satisfies composition and nullity, but not evenness.
   (b) The uniform indicator satisfies composition and evenness, but not nullity.
   (c) Let R^1 be defined as follows. For each $i \in N$,

$$R_i^1(N,S,p,C) = \begin{cases} \dfrac{\sigma}{|M|} & \text{if } M \neq \emptyset \\ R_i^P(N,S,p,C) & \text{otherwise,} \end{cases}$$

   where $M \subseteq N$ such that if $i \in M$, there exists at least one service $j \in M$, $C_{is} \cdot C_{js} \neq 0$ or $C_{is} = C_{js} = 0$ for all $s \in S$. The indicator R^1 satisfies nullity and evenness, but not composition.

5. A case study: Twitch

In the previous sections, we characterized and analyzed the *uniform*, *subscriber-uniform*, *proportional*, and *subscriber-proportional* indicators in the context of streaming platforms. In this section, we propose an application of these four indicators to measure the relevance of creators on the Twitch streaming platform.[4]

We screened the 19 most-watched streamers worldwide during a three-week period from 23 December 2022 to 15 January 2023. In particular, we collected data on the viewers (identified through their nicknames) who were watching each streamer every streaming hour. In terms of the theoretical model introduced in Section 2, the services N are the streamers, the subscribers S are the viewers, and the consumption C_{is} is the time each viewer spent watching the online content provided by each streamer. The subscription price is assumed to be fixed (and normalized to 1).[5]

The success of the Twitch platform is $\sigma = 5,164,910$, which coincides with the number of viewers. Our database contains approximately five million unique users who watched one or more of the streamers. For the analyzed period, Table 3 shows the country, streaming language, number of viewers, overall share of viewers, exclusive viewers (those who exclusively consume the content of one streamer), and the percentage they represent of the total amount of exclusive viewers. In terms of the number of users, AuronPlay, Ibai, and XQc are the three most relevant streamers, significantly outperforming the others. In terms of exclusive viewers, AuronPlay and XQc perform relatively well, but Ibai drops significantly.

Beyond the mere number of users or exclusive users, what is the specific impact of each streamer on the success of Twitch? Table 4 illustrates the application of the *uniform*, *subscriber-uniform*, *proportional*, and *subscriber-proportional* indicators to our case study. For each metric, we indicate both the value and the percentage of success that is due to each streamer. Not surprisingly, for the uniform indicator, they are all equally relevant. Although this indicator may seem uninformative, it serves as a useful benchmark for comparisons since, by definition, it coincides with the average share. The *subscriber-uniform* indicator assigns the success of each individual viewer among the streamers whose content the subscriber viewed. Under this indicator, exclusive viewers have a significant impact on the distribution of success to each streamer. The subscription fee is fully attributed to that specific streamer. For the remaining viewers, success is distributed uniformly among all the streamers that they viewed. However, exclusive viewers do not completely determine the success rate. For example, Tarik is the third streamer with the most exclusive viewers, yet he is the sixth most successful streamer by this indicator.

[4] https://www.twitch.tv/

[5] Information on the individual subscription fee that each viewer pays is not public. However, the platform has access to this information, which it can easily use to compute the distribution of the revenue that corresponds to each of the indicators.

Table 3
Viewers and exclusive viewers.

Streamer	Country	Language	Viewers		Exclusive viewers	
			Number	%	Number	%
Adin Ross	United States	English	522 425	6.97	279 905	7.49
AuronPlay	Spain	Spanish	758 108	10.11	302 403	8.09
Casimito	Brazil	Portuguese	224 771	3.00	143 199	3.83
ElSpreen	Argentina	Spanish	354 882	4.73	160 886	4.30
elXokas	Spain	Spanish	237 442	3.17	57 847	1.55
Fextralife	United States	English	254 907	3.40	228 050	6.10
Gaules	Brazil	Portuguese	239 381	3.19	162 742	4.35
HasanAbi	United States	English	372 542	4.97	238 815	6.39
Ibai	Spain	Spanish	738 317	9.85	237 515	6.35
IlloJuan	Spain	Spanish	476 256	6.35	157 036	4.20
Juansguarnizo	Mexico	Spanish	400 161	5.34	128 466	3.44
Kai Cenat	United States	English	501 301	6.69	270 001	7.22
Loltyler1	United States	English	299 832	4.00	201 687	5.40
Loud_coringa	Brazil	Portuguese	279 470	3.73	236 475	6.33
Roshtein	Malta	English	4451	0.06	2782	0.07
Rubius	Spain	Spanish	378 760	5.05	115 037	3.08
Tarik	United States	English	410 166	5.47	299 966	8.02
TheGrefg	Spain	Spanish	318 345	4.25	92 534	2.48
XQc	Canada	English	725 958	9.68	422 948	11.31
			7 497 475	100	3 738 294	100

The *proportional indicator* (which implements the *pro-rata* principle in Alaei et al. (2022)) distributes the success proportionally to the overall consumption of each streamer. Notice that according to RPRP, only a few streamers are above average (Adin Ross, AuronPlay, Ibai, IlloJuan, and XQc). Comparing the percentage of viewers with the *proportional indicator*, we observe that the impact of Adin Ross is substantially higher than one might expect by focusing only on the users. His relevance increases by more than 51% according to the RPRP indicator. IlloJuan, for example, is another streamer whose influence is undervalued. The other side of the coin is that the impact of TheGrefg is 35% lower when considering the *proportional indicator*.

The *subscriber-proportional indicator* (which captures the idea behind the *user-centric* principle in Alaei et al. (2022)) assigns the success of each single viewer among the streamers whose content she has consumed. Therefore, by definition, the impact of an exclusive user is completely attributed to the unique streamer she watches. It is for this reason that the number of exclusive viewers has a higher impact in the *subscriber-proportional indicator* than in the *proportional indicator*. However, RSPRSP and exclusive viewers are not completely correlated. For instance, Ibai is the seventh streamer in terms of the most exclusive viewers (6.35%), but he rises to the third position for the *subscriber-proportional indicator* (8.32%), with an increase of 31%. This is because, in addition to his exclusive followers, his content is consumed by many other users.

We also observe several differences by comparing the *proportional* and *subscriber-proportional* indicators. The most significant change is the relevance of the streamer IlloJuan. He represents 10.15% of the total success of the platform for the former, but only 5.80% for the latter. This is due to the fact that IlloJuan is one of the most watched streamers, relative to the total number of hours viewed, but the number of exclusive viewers (approximately 33%) is substantially lower than in other streamers with higher benefits. In contrast, Gaules is more relevant according to the *subscriber-proportional indicator* compared to the *proportional indicator*. The reason is that Gaules does not have many viewing hours compared to the total number of hours viewed by the users. However, he is a Portuguese streamer, while the others are Spanish or English, so he has a significant number of exclusive viewers, around 68%

At this point, a question that arises is how the relevance of each streamer can be translated into revenue allocation. One way to do this is to distribute the part of the total revenue from subscription fees that the platform dedicates to rewarding streamers in proportion to the relevance of the platform's success that the indicators attribute to them. Note that this form of distribution will not modify in any way the structure of the relevance indicators presented, since the percentages obtained in Table 4 have been obtained using these indicators. In this sense, the comparative analysis carried out in the previous paragraphs on the indicators would also be valid for the allocation of revenue among streamers in proportion to these indicators.

6. Concluding remarks

In recent years, streaming services have risen significantly in popularity. In return for a subscription, consumers have unlimited access to the catalog of services that platforms offer, such as movies, TV shows, music, and books. Evidently, not all services are equally relevant for attracting users' interest to pay for a subscription. In this paper, we have presented a model to measure the relevance of each service with respect to the global success of the platform. We have addressed this problem using an axiomatic approach, and we have provided normative foundations for four basic indicators: the *uniform*, *subscriber-uniform*, *proportional*, and *subscriber-proportional* indicators. These four metrics are characterized by means of several axioms, which represent different principles of fairness and stability. Table 4 summarizes the properties fulfilled by each indicator and those necessary for the characterization.

Table 4
Relevance of streamers according to the proposed indicators.

Streamer	R^U		R^{SU}		R^P		R^{SP}	
	Value	%	Value	%	Value	%	Value	%
Adin Ross	271 837.4	5.26	384 314.07	7.44	545 333.40	10.56	399 648.12	7.74
AuronPlay	271 837.4	5.26	467 563.86	9.05	448 855.80	8.69	462 333.00	8.95
Casimito	271 837.4	5.26	181 757.52	3.52	219 242.70	4.24	193 421.99	3.74
ElSpreen	271 837.4	5.26	228 803.63	4.43	189 751.10	3.67	228 175.61	4.42
elXokas	271 837.4	5.26	117 556.95	2.28	106 974.10	2.07	100 438.95	1.94
Fextralife	271 837.4	5.26	239 775.26	4.64	172 821.40	3.35	238 549.53	4.62
Gaules	271 837.4	5.26	198 698.27	3.85	105 343.10	2.04	185 148.12	3.58
HasanAbi	271 837.4	5.26	294 519.76	5.70	191 565.80	3.71	277 481.56	5.37
Ibai	271 837.4	5.26	417 746.54	8.09	466 978.00	9.04	429 475.25	8.32
IlloJuan	271 837.4	5.26	269 565.17	5.22	524 492.80	10.15	299 687.49	5.80
Juansguarnizo	271 837.4	5.26	223 595.42	4.33	255 009.10	4.94	222 511.11	4.31
Kai Cenat	271 837.4	5.26	369 516.65	7.15	266 601.80	5.16	353 599.40	6.85
Loltyler1	271 837.4	5.26	243 739.64	4.72	188 135.50	3.64	244 417.87	4.73
Loud_coringa	271 837.4	5.26	256 230.97	4.96	252 097.30	4.88	258 610.57	5.00
Roshtein	271 837.4	5.26	3409.45	0.07	5992.60	0.11	3537.18	0.06
Rubius	271 837.4	5.26	204 089.17	3.95	253 755.40	4.91	198 031.44	3.83
Tarik	271 837.4	5.26	346 841.69	6.72	219 131.10	4.24	351 297.07	6.80
TheGrefg	271 837.4	5.26	166 702.99	3.23	141 868.60	2.75	160 103.74	3.10
XQc	271 837.4	5.26	550 482.98	10.66	610 960.50	11.83	558 442.00	10.81

In the particular context of music streaming platforms and the distribution of revenues among artists, two mechanisms have emerged as focal allocation rules: the *pro-rata* and the *user-centric* rules (see Meyn et al. (2023)). In our model, the *pro-rata* and *user-centric* schemes coincide with the *proportional* and *subscriber-proportional* indicators, respectively. Therefore, Theorems 1 and 2 also reveal characterizations of these two allocation rules. In other words, based on principles of fairness, stability, and non-manipulability, we provide normative foundations for the two most prominent revenue allocation mechanisms in the music streaming market.

To illustrate the performance of the four indicators, we have analyzed the streaming platform Twitch. Over a three-week period, we recorded the number of viewers and viewing times of the 19 most-followed Twitch streamers worldwide. Based on the normative approach presented in Section 4, we have concluded that the mere number of viewers is not a suitable metric to capture the impact of each streamer. We have also found that exclusive viewers (i.e., users who consume the content of a unique streamer) have a higher influence in the *subscriber-proportional* indicator than in the *proportional* indicator.

Two of the four characterizations we present use the axiom of non-manipulability. These results rely on the work by Ju et al. (2007). However, these authors did not explore the connections between non-manipulability and composition and sharing-proofness. The latter axiom is tailor-made for the streaming setting and may not be as meaningful in a broader context. Nevertheless, it could be adapted to the model in Ju et al. (2007). If we do so, we would find that the weighted functions in their Theorem 9 are the relative weight of each issue among all of them, corresponding to the counterpart of the proportional indicator we characterize in this paper. With regard to composition, the connection is less evident. On the one hand, its interpretation is not as obvious in different contexts. On the other hand, to properly apply this axiom, the endowment cannot be as general as in Ju et al. (2007), as it must coincide with the sum of the subscription prices. Taking this caveat into account, it can be shown that composition imposes that the weighted functions in their Theorem 9 must be the weight of each subscription price relative to the overall revenue.

In this analysis, we have adopted the viewpoint of a central authority tasked with selecting an appropriate indicator for application. However, we recognize the value of exploring an alternative cooperative approach (Schlicher et al., 2024). The four indicators we characterize can be expressed as the Shapley values (Shapley, 1953) derived from tailored cooperative games designed for each specific case.

We acknowledge that there are still some issues that deserve a deeper analysis. This paper focuses on measuring the influence of each service from the point of view of consumption. However, there are cases where the perspective of the production should also be considered. For instance, considering TV series, the viewers' reaction is important, but the production cost cannot be ignored. In this respect, a natural extension of the proposed model is to define indicators that consider both parts of the market. The literature on resource allocation and attribution problems is quite extensive, and other metrics and axioms can also be explored.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Appendix. Proofs

Proof of Theorem 1.
We start by showing that the subscriber-proportional indicator satisfies the axioms in the statement.

- Composition. Let $(N,S,p,C),(N,S',p',C')\in\mathcal{D}$ such that $S\cap S'=\emptyset$. Let $i\in N$. We have that

$$\begin{aligned}R_i^{SP}(N,S\cup S',p\oplus p',C\oplus C') &= \sum_{s\in S\cup S'}\frac{(C\oplus C')_{is}}{\|(C\oplus C')_{\cdot s}\|}(p\oplus p')_s\\ &= \sum_{s\in S}\frac{C_{is}}{\|C_{\cdot s}\|}p_s+\sum_{s\in S'}\frac{C'_{is}}{\|C'_{\cdot s}\|}p'_s\\ &= R_i^{SP}(N,S,p,C)+R_i^{SP}(N,S',p',C').\end{aligned}$$

- Non-manipulability. Let $(N,S,p,C)\in\mathcal{D}$, and let $N'\subseteq N$ and $i\in N'$ such that $C'_{is}=\sum_{j\in N'}C_{js}$ for all $s\in S$. Then

$$\begin{aligned}R_i^{SP}\left(\{i\}\cup N\backslash N',S,p,(C'_{i\cdot},C_{N\backslash N'})\right) &= \sum_{s\in S}\frac{C'_{is}}{C'_{is}+\sum_{k\in N\backslash N'}C_{ks}}p_s\\ &= \sum_{s\in S}\frac{\sum_{j\in N'}C_{js}}{\sum_{j\in N'}C_{js}+\sum_{k\in N\backslash N'}C_{ks}}p_s\\ &= \sum_{s\in S}\frac{\sum_{j\in N'}C_{js}}{\sum_{j\in N}C_{js}}p_s\\ &= \sum_{j\in N'}\sum_{s\in S}\frac{C_{js}}{\|C_{\cdot s}\|}p_s\\ &= \sum_{j\in N'}R_j^{SP}(N,S,p,C).\end{aligned}$$

Now, we prove the converse. Let R be an indicator that fulfills composition and non-manipulability. Let $(N,S,p,C)\in\mathcal{D}$. Suppose that S is a singleton (i.e., there is only one subscriber $S=\{s\}$), and let $p^{(s)}$ and $C^{(s)}=\left(C^{(s)}_{1s},\dots,C^{(s)}_{|N|s}\right)^T$ denote the corresponding subscription price and consumption matrix of this platform, respectively. Since R satisfies *non-manipulability*, we can apply Theorem 9 from Ju et al. (2007)[6] to obtain that, for each $i\in N$,

$$R_i\left(N,\{s\},p^{(s)},C^{(s)}\right)=\frac{C^{(s)}_{is}}{\sum_{i\in N}C^{(s)}_{is}}p^{(s)}=R_i^{SP}\left(N,\{s\},p^{(s)},C^{(s)}\right).$$

Now, suppose that S is such that $|S|\ge 2$. Notice that

$$p=p^{(1)}\oplus\cdots\oplus p^{(|S|)}\text{ and }C=C^{(1)}\oplus\cdots\oplus C^{(|S|)},$$

where each pair $p^{(s)}$ and $C^{(s)}$ corresponds to the subscription price and the consumption matrix with only one subscriber. Since R satisfies *composition*, it follows that, for each $i\in N$,

$$R_i(N,S,p,C)=\sum_{s\in S}R_i\left(N,\{s\},p^{(s)},C^{(s)}\right)=\sum_{s\in S}R_i^{SP}\left(N,\{s\},p^{(s)},C^{(s)}\right)=R_i^{SP}(N,S,p,C).\quad\square$$

Proof of Theorem 2.

We start by showing that the proportional indicator satisfies the axioms in the statement.

- Sharing-proofness. Let $(N,S,p,C)\in\mathcal{D}$. Let $S'\subseteq S$ and $s\in S'$ such that $p'_s=\sum_{t\in S'}p_t$ and $C'_{is}=\sum_{t\in S'}C_{it}$ for all $i\in N$ and $C'_{is}=C_{is}$ for all $i\in N$ and $s\in S\backslash S'$. Then

$$\begin{aligned}R_i^{P}\left(N,\{s\}\cup S\backslash S',(p'_s,p_{S\backslash S'}),C'\right) &= \frac{\|C'_{i\cdot}\|}{\sum_{j\in N}\|C'_{j\cdot}\|}\sigma\\ &= \frac{\sum_{t\in\{s\}\cup S\backslash S'}C'_{it}}{\sum_{j\in N}\sum_{t\in\{s\}\cup S\backslash S'}C'_{jt}}\sigma\\ &= \frac{C'_{is}+\sum_{t\in S\backslash S'}C'_{it}}{\sum_{j\in N}\left(C'_{js}+\sum_{t\in S\backslash S'}C'_{jt}\right)}\sigma\\ &= \frac{\sum_{t\in S'}C_{it}+\sum_{t\in S\backslash S'}C_{it}}{\sum_{j\in N}\left(\sum_{t\in S'}C_{jt}+\sum_{t\in S\backslash S'}C_{jt}\right)}\sigma\\ &= \frac{\sum_{t\in S}C_{it}}{\sum_{j\in N}\sum_{t\in S}C_{jt}}\sigma\\ &= R_i^{P}(N,S,p,C).\end{aligned}$$

[6] It is easy to check that when $|S|=1$, our model coincides with the resource allocation model proposed by Ju et al. (2007).

- Non-manipulability. Let $(N,S,p,C) \in \mathcal{D}$. Let $N' \subseteq N$ and $i \in N'$ such that $C'_{is} = \sum_{j\in N'} C_{js}$ for all $s \in S$. Then

$$\begin{aligned} R_i^P\left(\{i\}\cup N\backslash N', S, p, (C'_{i\cdot}, C_{N\backslash N'})\right) &= \frac{\|C'_{i\cdot}\|}{\|C'_{i\cdot}\| + \sum_{k\in N\backslash N'} \|C_{k\cdot}\|}\sigma \\ &= \frac{\sum_{j\in N'} \|C_{j\cdot}\|}{\sum_{j\in N'} \|C_{j\cdot}\| + \sum_{k\in N\backslash N'} \|C_{k\cdot}\|}\sigma \\ &= \sum_{j\in N'} \frac{\|C_{j\cdot}\|}{\sum_{k\in N} \|C_{k\cdot}\|}\sigma \\ &= \sum_{j\in N'} R_j^P(N,S,p,C). \end{aligned}$$

Now, we prove the converse. Let R be an indicator that fulfills sharing proofness and non-manipulability. Let $(N,S,p,C) \in \mathcal{D}$. Now, consider the platform $(N,\{1\},p^{(1)},C^{(1)})$ where $p^{(1)} = p_1 + \dots + p_{|S|}$ and $C_i^{(1)} = \sum_{s\in S} C_{is}$ for all $i \in N$. *Sharing-proofness* requires that

$$R(N,S,p,C) = R\left(N,\{1\},p^{(1)},C^{(1)}\right).$$

Notice that in $\left(N,\{1\},p^{(1)},C^{(1)}\right)$ there is only one subscriber. Applying Theorem 9 in Ju et al. (2007), *non-manipulability* implies that for each $i \in N$,

$$R_i\left(N,\{1\},p^{(1)},C^{(1)}\right) = \frac{C_i^{(1)}}{\|C^{(1)}\|}\sigma = \frac{\|C_{i\cdot}\|}{\|C\|}\sigma.$$

Therefore, for each $i \in N$, we conclude that

$$R_i(N,S,p,C) = \frac{\|C_{i\cdot}\|}{\|C\|}\sigma = R_i^P(N,S,p,C). \quad \square$$

Proof of Theorem 3.

We start by showing that the uniform indicator satisfies the axioms in the statement.

- Symmetry. Let $(N,S,p,C) \in \mathcal{D}$. Let $i,j \in N$ such that $C_{is} = C_{js}$ for all $s \in S$. It follows that

$$R_i^U(N,S,p,C) = \frac{\sigma}{|N|} = R_j^U(N,S,p,C).$$

- Homogeneity. Let $(N,S,p,C) \in \mathcal{D}$ and $\lambda \in \mathbb{R}_{++}$. Let $i \in N$. It follows that

$$R_i^U(N,S,p,C) = \frac{\sigma}{|N|} = R_i^U(N,S,p,\lambda C).$$

- Consumption sensitivity. Since $R_i^U(N,S,p,C) = \frac{\sigma}{|N|}$ for all $i \in N$, it is easy to see that small changes in the consumption matrix do not alter the relevance of the services given by the uniform indicator.

Next, we prove the converse. Let R be an indicator that satisfies symmetry, homogeneity, and consumption sensitivity, and $R \neq R^U$. Therefore, there exists a problem (N,S,p,C) such that $R(N,S,p,C) \neq R^U(N,S,p,C)$ and there are at least $i,j \in N$ such that $R_i(N,S,p,C) \neq R_i^U(N,S,p,C)$ and $R_j(N,S,p,C) \neq R_j^U(N,S,p,C)$.

For an arbitrary $n \in \mathbb{N}_{++}$, let C^{0n} be the consumption matrix for which all coordinates are exactly equal to $\frac{1}{n}$. On the one hand, by *symmetry* we know that $R_i(N,S,p,C^{0n}) = \frac{\sigma}{|N|}$ for all $i \in N$ and for all $n \in \mathbb{N}_{++}$. On the other hand, by *homogeneity*, $R(N,S,p,\frac{1}{n}C) = R(N,S,p,C)$ for all $n \in \mathbb{N}_{++}$.

Now, it is straightforward to prove that $\|C^{0n} - \frac{1}{n}C\|$ goes to 0 when n goes to infinity, but since $R(N,S,p,C) \neq R^U(N,S,p,C)$, it follows that $\|R(N,S,p,C^{0n}) - R(N,S,p,\frac{1}{n}C)\| = \|R^U(N,S,p,C^{0n}) - R(N,S,p,\frac{1}{n}C)\| = \|R^U(N,S,p,C^{0n}) - R(N,S,p,C)\| = \|R^U(N,S,p,C) - R(N,S,p,C)\| \geq |R_i^U(N,S,p,C) - R_i(N,S,p,C)| + |R_j^U(N,S,p,C) - R_j(N,S,p,C)| > 0$ for all $n \in \mathbb{N}_{++}$. This contradicts the fact that R satisfies consumption sensitivity. $\square$

Proof of Theorem 4.

We start by showing that the *subscriber-uniform indicator* satisfies the axioms in the statement.

- Composition. Let $(N,S,p,C),(N,S',p',C') \in \mathcal{D}$ such that $S \cap S' = \emptyset$. Let $i \in N$. It follows that

$$\begin{aligned} R_i^{SU}(N,S\cup S',p\oplus p',C\oplus C') &= \sum_{s\in S\cup S'\,:\,i\in N_s} \frac{1}{|N_s|}(p\oplus p')_s \\ &= \sum_{s\in S\,:\,i\in N_s} \frac{1}{|N_s|}p_s + \sum_{s\in S'\,:\,i\in N_s} \frac{1}{|N_s|}p'_s \\ &= R_i^{SU}(N,S,p,C) + R_i^{SP}(N,S',p',C'). \end{aligned}$$

- Nullity. Let $(N,S,p,C) \in \mathcal{D}$ and $i \in N$ such that $C_{is} = 0$ for all $s \in S$, then $i \notin N_s = \{j \in N : C_{js} \neq 0\}$ for all $s \in S$ and

$$R_i^{SU}(N,S,p,C) = \sum_{s\in S\,:\,i\in N_s} \frac{1}{|N_s|}p_s = 0.$$

- Evenness. Let $(N, S, p, C) \in \mathcal{D}$. Let $i, j \in N$ such that $C_{is} \cdot C_{js} \neq 0$ or $C_{is} = C_{js} = 0$ for all $s \in S$. Then $i \in N_s$ if and only if $j \in N_s$ for all $s \in S$, and it follows that
$$R_i^{SU}(N, S, p, C) = \sum_{s \in S\,:\, i \in N_s} \frac{1}{|N_s|} p_s = \sum_{s \in S\,:\, j \in N_s} \frac{1}{|N_s|} p_s = R_j^{SU}(N, S, p, C).$$

Now, we prove the converse. Let R be an indicator that fulfills *composition*, *nullity*, and *evenness*. Let $(N, S, p, C) \in \mathcal{D}$. Suppose that S is a singleton (i.e., there is only one subscriber $S = \{s\}$), and let $p^{(s)}$ and $C^{(s)} = \left(C_{1s}^{(s)}, \dots, C_{|N|s}^{(s)}\right)^T$ be the corresponding subscription price and consumption matrix of this platform, respectively. Since R satisfies *nullity*, it follows that
$$R_i\left(N, \{s\}, p^{(s)}, C^{(s)}\right) = 0,$$
for all $i \in N$ such that $C_{is}^{(s)} = 0$. Additionally, since R satisfies *evenness*, it follows that
$$R_i\left(N, \{s\}, p^{(s)}, C^{(s)}\right) = \frac{p^{(s)}}{|N_s|},$$
for all $i \in N$ such that $C_{is}^{(s)} > 0$, where $N_s = \{j \in N : C_{js}^{(s)} > 0\}$.

Now, suppose that S is such that $|S| \geq 2$. Notice that
$$p = p^{(1)} \oplus \cdots \oplus p^{(|S|)} \text{ and } C = C^{(1)} \oplus \cdots \oplus C^{(|S|)},$$
where each pair $p^{(s)}$ and $C^{(s)}$ corresponds to the subscription price and the consumption matrix with only one subscriber. Since R satisfies *composition*, it follows that for each $i \in N$,
$$R_i(N, S, p, C) = \sum_{s \in S} R_i\left(N, \{s\}, p^{(s)}, C^{(s)}\right) = \sum_{s \in S} R_i^{SU}\left(N, \{s\}, p^{(s)}, C^{(s)}\right) = R_i^{SU}(N, S, p, C). \quad \square$$

Proof of Proposition 1.

(a) We show the fulfillment or violation of each property:

- Evenness. Let $(N, S, p, C) \in \mathcal{D}$. Let $i, j \in N$ such that $C_{is} \cdot C_{js} \neq 0$ or $C_{is} = C_{js} = 0$ for all $s \in S$. Then, we have $R_i^U(N, S, p, C) = \frac{\sigma}{|N|} = R_j^U(N, S, p, C)$.
- Composition. Let $(N, S, p, C), (N, S', p', C') \in \mathcal{D}$ such that $S \cap S' = \emptyset$. Let $i \in N$. Then we have that
$$\begin{aligned} R_i^U(N, S \cup S', p \oplus p', C \oplus C') &= \sum_{s \in S \cup S'} \frac{(p \oplus p')_s}{|N|} \\ &= \sum_{s \in S} \frac{p_s}{|N|} + \sum_{s \in S'} \frac{p'_s}{|N|} \\ &= R_i^U(N, S, p, C) + R_i^U(N, S', p', C'). \end{aligned}$$
- Sharing-proofness. Let $(N, S, p, C) \in \mathcal{D}$. Let $S' \subseteq S$ and $s \in S'$ such that $p'_s = \sum_{t \in S'} p_t$ and $C'_{is} = \sum_{t \in S'} C_{it}$ for all $i \in N$ and $C'_{is} = C_{is}$ for all $i \in N$ and $s \in S \backslash S'$. Then
$$\begin{aligned} R_i^U\left(N, \{s\} \cup S \backslash S', (p'_s, p_{S \backslash S'}), C'\right) &= \frac{p'_s + \sum_{t \in S \backslash S'} p_t}{|N|} \\ &= \frac{\sum_{t \in S'} p_t + \sum_{t \in S \backslash S'} p_t}{|N|} \\ &= \frac{\sum_{t \in S} p_t}{|N|} \\ &= R_i^U(N, S, p, C). \end{aligned}$$
- Non-manipulability. Consider a platform where $N = \{1, 2, 3\}$, $S = \{1\}$, $p_1 = 1$, and C is given by
$$\begin{pmatrix} 0 \\ 1 \\ 2.5 \end{pmatrix}.$$
The uniform indicator is equal to $R_i^U(N, S, p, C) = \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$. Now, consider $N' = \{1, 2\}$ and $C'_{11} = C_{11} + C_{21} = 1$. Then
$$R_1^U\left(\{1\} \cup N \backslash N', S, p, (C'_{1\cdot}, C_{N \backslash N'})\right) = \frac{1}{2} \neq \sum_{j \in N'} R_j(N, S, p, C) = \frac{2}{3}.$$
- Nullity. Let $(N, S, p, C) \in \mathcal{D}$. Let $i \in N$ such that $C_{is} = 0$ for all $s \in S$. Then
$$R_i^U(N, S, p, C) = \frac{\sigma}{|N|} \neq 0.$$

(b) We show the fulfillment or violation of each property:

- Symmetry. Let $(N,S,p,C) \in \mathcal{D}$. Let $i,j \in N$ such that $C_{is} = C_{js}$ for all $s \in S$. Then $i \in N_s$ if and only if $j \in N_s$. Therefore, we have $R_i^{SU}(N,S,p,C) = R_j^{SU}(N,S,p,C)$.
- Homogeneity. Let $(N,S,p,C) \in \mathcal{D}$ and $\lambda \in \mathbb{R}_{++}$. Let $i \in N$. We have that

$$R_i^{SU}(N,S,p,\lambda C) = \sum_{s\in S\,:\,i\in N_s} \frac{1}{|N_s|} p_s = R_i^{SU}(N,S,p,C),$$

  where $N_s = \{j \in N : C_{js} \neq 0\}$.
- Consumption sensitivity. Let $(N,S,p,C^{1n}),(N,S,p,C^{2n}) \in \mathcal{D}$ where

$$C^{1n} = \begin{pmatrix} \frac{1}{n} \\ \frac{1}{n} \\ \frac{1}{n} \end{pmatrix} \text{ and } C^{2n} = \begin{pmatrix} 0 \\ \frac{1}{n} \\ \frac{2}{n} \end{pmatrix}.$$

  It is obvious that $\|C^{1n} - C^{2n}\|$ goes to 0 when n goes to infinity, but $\|R^{SU}(N,S,p,C^{1n}) - R^{SU}(N,S,p,C^{2n})\| = \|(\frac{p}{3},\frac{p}{3},\frac{p}{3}) - (0,\frac{p}{2},\frac{p}{2})\| = \frac{2p}{3} > 0$ for all n.
- Sharing-proofness. Consider a platform where $N = \{1,2,3\}$, $S = \{1,2\}$, $p = (1,1)$, and C is given by

$$\begin{pmatrix} 0 & 2 \\ 1 & 0 \\ 1 & 0 \end{pmatrix}.$$

  Then $R^{SU}(N,S,p,C) = \left(1,\frac{1}{2},\frac{1}{2}\right)$. Let $S' = \{1,2\} \subset S$ and $s = 1 \in S'$ such that $p'_s = \sum_{t\in S'} p_t$ and $C'_{is} = \sum_{t\in S'} C_{it}$ for all $i \in N$. Then $R^{SU}(N,S',p',C') = \left(\frac{2}{3},\frac{2}{3},\frac{2}{3}\right) \neq R^{SU}(N,S,p,C)$.

(c) We show the fulfillment or violation of each property:

- Symmetry. Let $(N,S,p,C) \in \mathcal{D}$. Let $i,j \in N$ such that $C_{is} = C_{js}$ for all $s \in S$. We have that

$$R_i^P(N,S,p,C) = \frac{\|C_{i\cdot}\|}{\sum_{t\in N}\|C_{t\cdot}\|}\sigma = \frac{\sum_{s\in S} C_{is}}{\sum_{t\in N}\|C_{t\cdot}\|}\sigma = \frac{\sum_{s\in S} C_{js}}{\sum_{t\in N}\|C_{t\cdot}\|}\sigma = \frac{\|C_{j\cdot}\|}{\sum_{t\in N}\|C_{t\cdot}\|}\sigma = R_j^P(N,S,p,C).$$

- Homogeneity. Let $(N,S,p,C) \in \mathcal{D}$, $\lambda \in \mathbb{R}_{++}$ and $i \in N$. We have that

$$R_i^P(N,S,p,\lambda C) = \frac{\|\lambda C_{i\cdot}\|}{\sum_{j\in N}\|\lambda C_{j\cdot}\|}\sigma = \frac{\lambda\|C_{i\cdot}\|}{\sum_{j\in N}\lambda\|C_{j\cdot}\|}\sigma = R_i^P(N,S,p,C).$$

- Nullity. Let $(N,S,p,C) \in \mathcal{D}$. Let $i \in N$ such that $C_{is} = 0$ for all $s \in S$. Then

$$R_i^P(N,S,p,C) = \frac{\|C_{i\cdot}\|}{\sum_{j\in N}\|C_{j\cdot}\|}\sigma = \frac{\sum_{s\in S} C_{is}}{\sum_{j\in N}\|C_{j\cdot}\|}\sigma = 0.$$

- Evenness. Consider a platform where $N = \{1,2,3\}$, $S = \{1\}$, p_1, and C is given by

$$\begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}.$$

  Then $C_{21} \cdot C_{31} \neq 0$ and $R_2^P(N,S,p,C) = \frac{p_1}{3} \neq \frac{2p_1}{3} = R_3^P(N,S,p,C)$.
- Composition. Consider a platform where $N = \{1,2,3\}$, $S = \{1\}$, $S' = \{2\}$, $p_1 = p_2 = 1$, where C^1 and C^2 are given by

$$C^1 = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \quad \text{and} \quad C^2 = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}.$$

  Then, $R^P(N,\{1\},p_1,C^1) = \left(0,\frac{1}{2},\frac{1}{2}\right)$ and $R^P(N,\{2\},p_2,C^2) = \left(\frac{1}{2},\frac{1}{4},\frac{1}{4}\right)$. By other way, $R^P(N,\{1\}\cup\{2\},p_1\oplus p_2,C^1\oplus C^2) = \left(\frac{2}{3},\frac{2}{3},\frac{2}{3}\right) \neq \left(\frac{1}{2},\frac{3}{4},\frac{3}{4}\right) = R^P(N,\{1\},p_1,C^1) + R^P(N,\{2\},p_2,C^2)$.
- Consumption sensitivity. Let $(N,S,p,C^{1n}),(N,S,p,C^{2n}) \in \mathcal{D}$ where

$$C^{1n} = \begin{pmatrix} \frac{1}{n} \\ \frac{1}{n} \\ \frac{1}{n} \end{pmatrix} \text{ and } C^{2n} = \begin{pmatrix} 0 \\ \frac{1}{n} \\ \frac{2}{n} \end{pmatrix}.$$

It is obvious that $\|C^{1n}-C^{2n}\|$ goes to 0 when n goes to infinity, but $\|R^{P}(N,S,p,C^{1n})-R^{P}(N,S,p,C^{2n})\| = \|(\frac{p}{3},\frac{p}{3},\frac{p}{3})-(0,\frac{p}{3},\frac{2p}{3})\| = \frac{2p}{3} > 0$ for all n.

(d) We show the fulfillment or violation of each property:

- Symmetry. Let $(N,S,p,C)\in\mathcal{D}$. Let $i,j\in N$ such that $C_{is}=C_{js}$ for all $s\in S$. We have that
$$R_i^{SP}(N,S,p,C)=\sum_{s\in S}\frac{C_{is}}{\|C_{\cdot s}\|}p_s=\sum_{s\in S}\frac{C_{js}}{\|C_{\cdot s}\|}p_s=R_j^{SP}(N,S,p,C).$$
- Homogeneity. Let $(N,S,p,C)\in\mathcal{D}$, $\lambda\in\mathbb{R}_{++}$ and $i\in N$. We have that
$$R_i^{SP}(N,S,p,\lambda C)=\sum_{s\in S}\frac{\lambda C_{is}}{\|\lambda C_{\cdot s}\|}p_s=\sum_{s\in S}\frac{\lambda C_{is}}{\lambda\|C_{\cdot s}\|}p_s=R_i^{SP}(N,S,p,C).$$
- Nullity. Let $(N,S,p,C)\in\mathcal{D}$. Let $i\in N$ such that $C_{is}=0$ for all $s\in S$, then
$$R_i^{SP}(N,S,p,C)=\sum_{s\in S}\frac{C_{is}}{\|C_{\cdot s}\|}p_s=0.$$
- Evenness. Consider a platform where $N=\{1,2,3\}$, $S=\{1\}$, p_1, and C is given by
$$\begin{pmatrix}0\\1\\2\end{pmatrix}.$$
Then $C_{21}\cdot C_{31}\neq 0$ but $R_2^{SP}(N,S,p,C)=\frac{p_1}{3}\neq\frac{2p_1}{3}=R_3^{SP}(N,S,p,C)$.
- Consumption sensitivity. Let $(N,S,p,C^{1n}),(N,S,p,C^{2n})\in\mathcal{D}$ where
$$C^{1n}=\begin{pmatrix}\frac{1}{n}\\\frac{1}{n}\\\frac{1}{n}\end{pmatrix}\text{ and }C^{2n}=\begin{pmatrix}0\\\frac{1}{n}\\\frac{2}{n}\end{pmatrix}.$$
It is obvious that $\|C^{1n}-C^{2n}\|$ goes to 0 when n goes to infinity, but $\|R^{SP}(N,S,p,C^{1n})-R^{SP}(N,S,p,C^{2n})\| = \|(\frac{p}{3},\frac{p}{3},\frac{p}{3})-(0,\frac{p}{3},\frac{2p}{3})\| = \frac{2p}{3} > 0$ for all n.
- Sharing-proofness. Consider a platform where $N=\{1,2,3\}$, $S=\{1,2\}$, $p=(1,1)$, and C is given by
$$\begin{pmatrix}0&2\\1&1\\1&1\end{pmatrix}$$
Then, $R^{SP}(N,S,p,C)=\left(\frac{1}{2},\frac{3}{4},\frac{3}{4}\right)$. Let $S'=\{1,2\}\subset S$ and $s=1\in S'$ such that $p'_s=\sum_{t\in S'}p_t$ and $C'_{is}=\sum_{t\in S'}C_{it}$ for all $i\in N$, then $R^{SP}(N,S',p',C')=\left(\frac{2}{3},\frac{2}{3},\frac{2}{3}\right)\neq R^{SP}(N,S,p,C)$. □

Data availability

The authors do not have permission to share data.

References

Adams, W.J., Yellen, J.L., 1976. Commodity bundling and the burden of monopoly. Q. J. Econ. 90, 475–497.
Alaei, S., Makhdoumi, A., Malekian, A., Pekeč, S., 2022. Revenue-sharing allocation strategies for two-sided media platforms: pro-rata vs. user-centric. Manag. Sci. 68, 8699–8721.
Albino, D., Scaruffi, P., Moretti, S., Coco, S., Truini, M., Cristofano, C.D., Cavazzana, A., Stigliani, S., Bonassi, S., Tonini, G.P., 2008. Identification of low intratumoral gene expression heterogeneity in neuroblastic tumors by genome-wide expression analysis and game theory. Cancer 113, 1412–1422.
Bergantiños, G., Moreno-Ternero, J.D., 2015. The axiomatic approach to the problem of sharing the revenue from museum passes. Games Econom. Behav. 89, 78–92.
Bergantiños, G., Moreno-Ternero, J.D., 2024. The Shapley index for music streaming platforms. arXiv [econ.TH], 2411.07166.
Bergantiños, G., Moreno-Ternero, J.D., 2025. Revenue sharing at music streaming platforms. Manag. Sci. forthcoming.
Brander, K., Bruno, J., Hobday, A., Schoeman, D., 2011. The value of attribution. Nat. Clim. Chang. 1, 70–71.
Burger, M., Wentz, J., Horton, R., 2020. The law and science of climate change attribution. Columbia J. Environ. Law 5, 57–240.
Dimont, J., 2017. Royalty inequity: Why music streaming services should switch to a per-subscriber model. CU Hast. Law J. 69, 675–700.
Ginsburgh, V., Zang, I., 2003. The museum pass game and its value. Games Econom. Behav. 43, 322–325.
Jari, M., 2018. Pro-rata and user-centric distribution models: A comparative study. Technical report.
Ju, B.G., Miyagawa, E., Sakai, T., 2007. Non-manipulable division rules in claim problems and generalizations. J. Econom. Theory 132, 1–16.
López-Navarrete, F., Sánchez-Soriano, J., Bonastre, O.M., 2019. Allocating revenues in a smart TV ecosystem. Int. Trans. Oper. Res. 26, 1611–1632.

López-Navarrete, F., Sánchez-Soriano, J., Bonastre, O.M., 2023. Dynamic generation and attribution of revenues in a video platform. ArXiv, 4859841 [math.OC].
Lucchetti, R., Moretti, S., Patrone, F., Radrizzani, P., 2010. The Shapley and banzhaf values in microarray games. Comput. Oper. Res. 37, 1406–1412.
Manshadi, V., Niazadeh, R., Rodilitz, S., 2023. Fair dynamic rationing. Manag. Sci. 69, 6818–6836.
Martínez, R., Moreno-Ternero, J.D., 2022. An axiomatic approach towards pandemic performance indicators. Econ. Model. 116, 105983.
Martínez, R., Sánchez-Soriano, J., 2021. Mathematical indices for the influence of risk factors on the lethality of a disease. J. Math. Biol. 83, 74.
Martínez, R., Sánchez-Soriano, J., 2023. Order preservation with dummies in the museum pass problem. Ann. Oper. Res. forthcoming.
Martínez, R., Sánchez-Soriano, J., Llorca, N., 2022. Assessments in public procurement procedures. Omega 111, 102660.
Meyn, J., Kandziora, M., Albers, S., Clement, M., 2023. Consequences of platforms' remuneration models for digital content: initial evidence and a research agenda for streaming services. J. Acad. Mark. Sci. 51, 114–131.
Moretti, S., Patrone, F., Bonassi, S., 2007. The class of microarray games and the relevance index for genes. TOP 15, 256–280.
O'Neill, B., 1982. A problem of rights arbitration from the talmud. Math. Soc. Sci. 2, 345–371.
Page, W., Safir, D., 2018a. Money in, money out: lessons from cmos in allocating and distributing licensing revenue. Technical report.
Page, W., Safir, D., 2018b. User-centric revisited: The unintended consequences ofroyalty distribution. Technical report.
Schlicher, L., Dietzenbacher, B., Musegaas, M., 2024. Stable streaming platforms: a cooperative game approach. Omega 125, 103020.
Shapley, L.S., 1953. A value for n-person games. Contrib. To Theory Games 2, 307–317.
Singal, R., Besbes, O., Desir, A., Goyal, V., Iyengar, G., 2022. Shapley meets uniform: An axiomatic framework for attribution in online advertising. Manag. Sci. 68, 7457–7479.