Multiobjective Optimization for Railway Maintenance Plans

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Abstract: Railway track maintenance is a critical problem for any railway administrator. More precisely, preventive maintenance scheduling is a nondeterministic polynomial time (NP)–hard problem, which additionally involves multiple objectives such as economic cost, maximum capacity, serviceability, safety, and passenger comfort. This paper proposes a multiobjective optimization approach to this problem, combined with a track deterioration model that takes into account the degradation caused by maintenance operations. The track behavior is simulated by an exponential deterioration model based on a two-level segmentation. The maintenance schedule is built using a Pareto-based algorithm with two objectives (cost and delay) and three constraints, on top of an initialization heuristic based on expert knowledge. The proposed approach has been tested with two different algorithms (NSGA-II and AMOSA) over a model of a real track to create schedules for different horizons ranging between 3 and 20 years. The solutions obtained by AMOSA outperform those designed by human experts both in terms of time delay and economic cost, demonstrating the capability of the proposal to produce near-optimal long-term maintenance schedules. **DOI: 10.1061/(ASCE)CP.1943-5487.0000757.** *© 2018 American Society of Civil Engineers.*

Introduction

Railway track maintenance represents an important challenge for stakeholders in the railway sector, such as railway contractors and infrastructure administrators, both in terms of money, resources, and safety (Ferreira and López-Pita 2015). The economic cost of railway infrastructure maintenance is up to $150,000/km, two-thirds of which are associated with the track maintenance. Additionally, the nonredundancy of railway tracks implies that maintenance has a direct impact on the level of service and safety that can be provided by the trains. Therefore, the elaboration of feasible maintenance plans is a critical issue for railway infrastructure administrators.

Traditionally, track maintenance can be corrective or preventive. Preventive maintenance is sought after by the maintenance policies in the industry world, and can lead to smaller costs and better quality of the track, while providing a higher flexibility and better management of the resources (Kong and Frangopol 2003). However, the preventive maintenance scheduling problem is nondeterministic polynomial time (NP)–hard (Budai et al. 2006; Gustavsson 2015). A problem H belongs to the NP-hard family when every NP problem (i.e., problems for which a solution can be verified in polynomial time) can be reduced in polynomial time to H, meaning that H is at least as complex as any NP problem (Garey and Johnson 1979). In practice, this implies that a globally optimal preventive maintenance schedule cannot be computed in a feasible time. Moreover, the difficulty of this task increases along with the time span of the schedule. Therefore, it is crucial to develop algorithms that can find near-optimal approximate solutions to this problem in an acceptable time.

An adequate preventive maintenance requires an accurate track deterioration model to anticipate future failures and demands. The specialized literature includes several proposals to model the track based on the workload of the rails and the ballast, either using linear (Esveld 2001; Ramos and Fonseca 2011b; Wen et al. 2016) or nonlinear models (Jovanovic 2004; Zhao et al. 2006; Andrade and Teixeira 2016). Other studies describe how maintenance operations affect the degradation rate of the track (Ramos and Fonseca 2013; Audley and Andrews 2013; Andrade and Teixeira 2016).

Traditional optimization algorithms aim at finding the solution that minimizes (or maximizes) the value of a function for a given problem. However, many real-world problems involve several objective functions. Multiobjective algorithms have been an important research topic for the last decades, as they attempt to optimize several objective functions altogether, allowing them to handle a set of nondominated solutions (Deb 2001; Deb et al. 2002; Bandyopadhyay et al. 2008).

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Multiobjective algorithms have been successfully applied on the railway maintenance scheduling problem. In Caetano and Fonseca (2013), the authors optimize the track lifecycle cost and the track availability for scheduling the renewal strategy. In Ramos and Fonseca (2011a), a biobjective approach optimizes the economic cost and railway capacity after applying a maintenance plan, while complying with some constraints. Some authors propose a different way to tackle a similar problem, translating all the objectives into terms of economic cost (Higgins et al. 1996; Arasteh Khouy et al. 2014). Another multiobjective approach is presented in Podofillini et al. (2006), based on risks and on a Markov model to model the inspection operations. Finally, Caetano and Teixeira (2016) apply multiobjective algorithm to schedule tamping operations. However, the authors have been unable to find in the literature any attempt to combine a multiobjective strategy with a track deterioration model that involves the degradation caused by both tamping and renewal operations.

This paper describes a multiobjective optimization approach for preventive track maintenance scheduling. Two objective functions (cost and delay) and three sets of constraints (one for safety and two for resources) are defined in order to model the problem. Two multiobjective algorithms are considered, to obtain a nondominated set of maintenance plans that satisfy all constraints while minimizing both cost and delay. Two possible initializations of the solution set, based on expert knowledge, are proposed. Each candidate solution to the problem is encoded into a binary vector that represents the maintenance plan of a track over an arbitrary number of trimesters. A nonlinear deterioration model that simulates the behavior of a real track under the effects of time, tamping, and renewal operations underlies the entire optimization process.

This paper is structured as follows. First, the background information about railway track maintenance and multiobjective algorithms is presented. Then, the proposal is described. The experiments performed and their results are then detailed. Finally, the conclusions that can be reached through the current study are explained.

Background

Railway Maintenance

In compliance with the European standard (CEN 2010), there are two possible reactions to insufficient track quality: lowering the maximum speed of service, and carrying out maintenance operations. Although the former is cheaper in the short-term, eventually the quality would decrease under the minimum allowed by the law and the safety constraints. The quality of the service could also deteriorate. Additionally, lowering the speed lowers the maximum capacity of the track. Therefore, an adequate maintenance plan aims at finding a trade-off between maintenance costs and service capacity loss. This trade-off strongly depends on the particular perspective of the decision maker: maintenance subcontractors pursue a low cost, while in general train companies seek to maximize the capacity.

The specialized literature shows two groups of methods to optimize railway maintenance operations (Budai 2009). The first approach starts from a fixed set of necessary operations and aims at organizing them in an optimal schedule, taking into account resource restrictions (technological, production-related, human, and organizational) (Budai et al. 2006; Macedo et al. 2017). The second approach is more complex as it also involves modeling the deterioration process and computing the necessary operations before doing the scheduling (Vale and Ribeiro 2014; Wen et al. 2016). Therefore, both the maintenance operations and their scheduling have to be computed and optimized as a whole. The research carried out in this paper falls within the second category. Some recent proposals tackle the problem of scheduling the railway maintenance and traffic altogether (Lidén and Joborn 2017; Luan et al. 2017). However, in practice they fall very often under the responsibility of different agents (namely the maintenance contractor and the railway operator). This paper focuses on the maintenance scheduling, and takes into account an estimation of the total train delays that arise from this scheduling in combination with the track deterioration.

Table 1. Maintenance Operations and Their Triggers

Maintenance operation	Trigger
Rail grinding	—
Rail lubrication	Time
Track inspection	—
Tamping	—
Ballast cleaning	—
Rail renewal	Condition
Ballast renewal	—
Sleeper renewal	—
Fasteners renewal	—
Rail replacement	Failure

Table 1 shows an overview of the different maintenance operations and how they are triggered (Patra et al. 2009). The operations that are performed on a time or failure basis do not need any special considerations to be scheduled; therefore, this paper focuses on operations that are triggered by a certain condition, namely tamping, ballast cleaning, and component renewal.

The effect of tamping has already been modeled in previous research (Jovanovic 2004; Zhao et al. 2006). This modeling is based on geometric data gathered from the tracks, which must be properly aligned prior its use (Xu et al. 2015). However, modeling the exact effect of renewal operations that only involve certain components of the infrastructure proves to be more difficult (Lévi 2001). Following the approach of other studies on the topic, in this paper a single renewal operation is assumed for all the elements of the track, which leaves it in an as-good-as-new condition (Ramos and Fonseca 2011a). Consequently, the remainder of this paper considers two maintenance operations: tamping and renewal.

In this context, the aim of a maintenance schedule is to determine when and where to perform tamping and renewal operations in an optimal way. This optimality can depend on many criteria that may be contradictory of conflicting, and the exact criterion remains in hands of the final decision maker, which is usually the railway administrator. It is not desirable to automatically build a schedule that optimizes a single criterion, or even a fixed combination of them. The next section describes how multiobjective algorithms can overcome this problem.

Multiobjective Algorithms

Let $\mathcal{S}$ be the set of all possible solutions to a given problem. Single-objective optimization consists of looking for a solution $S^* \in \mathcal{S}$ that yields the best value of a function f, which can be the minimum or the maximum, depending on the context (Deb 2001). Hence the problem is called minimization or maximization, respectively. For the sake of simplicity, this paper focuses on minimization problems

$$f(S^*) \leq f(S) \quad \forall S \in \mathcal{S} \tag{1}$$

Conversely, a multiobjective problem involves a set of n objective functions $\mathcal{F} = \{f_1, \ldots, f_n\}$. Thus, the optimization becomes much more difficult, especially when these functions have conflicting behaviors, as it happens in most cases. Considering a single objective at a time is not feasible; the remaining objectives would get extremely bad values. There are two primary ways to achieve multiobjective optimization (Deb 2001):

- Aggregating the objectives into a single function, thus converting the problem to a single-objective one; and
- Looking for nondominated solutions. A solution S_a dominates S_b if $f_i(S_a) \leq f_i(S_b)$, $\forall\ f_i \in \mathcal{F}$. In that case, S_b can be safely discarded because S_a is undoubtedly better. However, if a solution S_c is better than S_a for some functions but not for all of them, S_c and S_a do not dominate each other and none of them can be said to be better than the other. A set of nondominated solutions is called a Pareto front.

Some proposals use the first approach to model the railway maintenance problem. For example, in Arasteh Khouy et al. (2014) all the objective functions are translated into an overall cost C_T that is optimized. Although this simplifies the handling of the objectives, it forces one to establish a balance factor between the objectives prior the execution of the algorithm, fixing their priority. However, the decision criteria for railway maintenance can change according to many factors, and such an approach could avoid reaching potentially interesting solutions (Das and Dennis 1997). Therefore, this paper focuses on approaches that use a Pareto front, which have been proven to yield good results in similar problems (Caetano and Teixeira 2016; Aminbakhsh and Sonmez 2017). The primary advantage of this alternative is the flexibility of the result, namely a set of solutions is made available under different balances of the objectives, and the decision maker can select one of them according to their specific needs.

Many real-world problems include constraints that restrict the solution space. A solution that does not comply with the constraints is said to be *nonfeasible*, and in general terms should not be taken into account as a valid solution for the problem. Algorithms based on Pareto front usually include the constraints into the dominance criterion, so that a feasible solution always dominates a nonfeasible one, independently of the value of the objective functions (Deb et al. 2002; Bandyopadhyay et al. 2008).

The number of objectives is one of many categorizations that can be done of optimization algorithms. Another popular manner to group them is according to how many solutions they handle at a time (Blum and Roli 2003). Trajectory-based algorithms start from a single solution and modify it looking for improvements in the objective function(s). One of the most well-known algorithms in this category for multiobjective optimization is the Simulated Annealing-Based Multiobjective Optimization Algorithm (AMOSA) (Bandyopadhyay et al. 2008). Conversely, population-based algorithms maintain a pool of solutions and generate new solutions from them, increasing the diversification of the search. One of the most used ones is the Nondominated Sorting Genetic Algorithm (NSGA-II) (Deb et al. 2002).

AMOSA

Simulated annealing (SA) (Kirkpatrick et al. 1983) is one of the most popular trajectory-based algorithms. It starts with a randomly generated solution S_c. Then, a new solution S'_c is generated by slightly modifying S_c. If S'_c is better than S_c it is selected as current solution; otherwise, it can still be picked according to a certain probability on the basis of a temperature value, which is gradually reduced as the search goes on until it reaches a minimum value, signaling the end of the search.

The AMOSA (Bandyopadhyay et al. 2008) is a multiobjective adaptation of SA. Instead of using a single current solution, it maintains a so-called archive of nondominated solutions. Therefore, the archive is the Pareto front of the search. First, the archive is randomly initialized, a hill-climbing algorithm is applied to its members, and only the nondominated solutions are kept. Then, a random solution is picked and SA is applied, introducing the domination criterion. In addition to the basic domination definition described earlier, AMOSA defines an *amount of domination*, which takes into account the numeric difference between the values of the objective functions. When the archive gets too large, similar solutions are clustered to reduce its size.

The primary advantage of AMOSA is its capability to intensify the search toward promising areas of the search space. This is achieved first by the hill-climbing algorithm, which quickly improves the fitness of the initial solutions. Then, SA is also based on a hill-climbing procedure, although allowing for more exploratory capabilities thanks to the probability generated by the temperature.

NSGA-II

Evolutionary algorithms use a population of solutions (called individuals) that evolve together. New individuals are obtained by combining (crossing) several individuals (generally two) and introducing random mutations. A number of multiobjective evolutionary algorithms have been suggested in the literature (Knowles and Corne 2000). One of the most well-known is NSGA-II (Deb et al. 2002).

The NSGA-II is based on the concept of nondominated sorting; when a new population is generated, the individuals are grouped into fronts according their domination. The first front corresponds to the Pareto front; the second one includes the solutions that would form the Pareto front if the first front was removed; and so forth. The following steps summarize the NSGA-II algorithm [for the full description, please refer to the original publication (Deb et al. 2002)]:

1. Population initialization with N randomly generated individuals;
2. Binary tournament: Select N random pairs of individuals and pick the best of each pair;
3. Crossover: The N selected individuals are grouped in pairs. Each pair is combined by a crossover operator that generates two new individuals, for a total of N new individuals;
4. Mutation: Each new individual suffers a random mutation with a given probability;
5. Evaluation of the new individuals;
6. Nondominated sorting: Sort old and new individuals together;
7. Selection of the new population: The fronts are included in the new population in order until size N is reached. If the last selected front does not fit entirely, select the individuals so that they are as spread as possible across the front; and
8. Go to Step 2 until a stop criterion (typically a fixed number of generations) is met.

The algorithm design is focused on reducing the computational complexity of the nondominated and crowding sorting. The NSGA-II favors a wide exploration of the search space rather than a deep intensification toward already known areas. This makes NSGA-II especially powerful when dealing with problems of which little knowledge is possessed, or where the structure of the search space is unknown or highly complex (El-Abbasy et al. 2017).

Proposal

This section describes the proposal for building maintenance plans using optimization on the basis of multiobjective algorithms, including a modeling of track to simulate the whole maintenance process, the encoding of the generated maintenance plans, the evaluation of the cost and delay functions, and the safety and resource constraints that are used to model the problem, the solution initialization process, the operators, and other particular considerations for the design and implementation of the algorithms and the proof that the problem is NP-hard.

Railway Modeling

The core of a good optimization framework for any real-world problem is an adequate representation. In this case, it must simulate the response of the track over time and the different maintenance operations that are performed on it. This section describes the railway segmentation process, the deterioration model, and the modeling of maintenance operations used in this paper.

Railway Segmentation

The behavior of the track depends on a wide variety of factors such as curvature, traffic, ballast type, and previously applied maintenance. Thus, the track cannot be modeled as a whole, it must be segmented and each segment must be treated separately (Jovanovic 2004). There are two main types of segmentation strategies: static segmentation divides the track into segments of the same length, and dynamic segmentation takes into account the factors that affect its behavior.

This paper describes a two-level segmentation procedure that combines both approaches. First, the track is dynamically divided into *sections*, according to the curvature, age, and type of the track, previously applied maintenance operations, and the presence of elements such as switches, bridges, or tunnels. This design ensures that the characteristics of quality, deterioration, and maximum allowed speed remain constant within each section. Then, each section is statically divided into *segments* of lengths between 25 and 100 m. This approach allows one to accurately model a real track where tamping and renewal operations have different ranges. For instance, tamping is carried out throughout a segment while the renewal is performed on an entire section. Note that the number of segments within each section is variable because there is no constraint on the length of the sections.

Deterioration Model

Deterioration models can be categorized into mechanistic and stochastic approaches (Cárdenas-Gallo et al. 2017). Mechanistic models are based on a simulation of the track geometry taking into account physical factors such as ballast and sleeper type, weather conditions, workload, and wheel geometry. These models provide insight into the behavior of different components of the railway infrastructure from a physical point of view; however, their use for predictive modeling is hindered by large uncertainties (Nguyen et al. 2016). Stochastic approaches produce a model from data measured from the tracks themselves. These can be broadly classified into linear (Esveld 2001; Ramos and Fonseca 2011b; Wen et al. 2016) and nonlinear models (Jovanovic 2004; Zhao et al. 2006; Andrade and Teixeira 2016). The latter assume the deterioration of the track to be inversely proportional to the current quality, which reflects the behavior measured from the tracks more accurately (Hummitzsch 2009). Furthermore, maintenance operations also affect this degradation rate (Ramos and Fonseca 2013; Audley and Andrews 2013; Andrade and Teixeira 2016).

For this approach, the authors consider an exponential fitting model (Hummitzsch 2009) combined with a mixed maintenance model where tamping operations restore the quality of the track while increasing the deterioration rate and renewals restore the track to its maximum quality, as suggested in Ramos and Fonseca (2011a). This is shown in Eq. (2), where Q_0 = initial quality; b = deterioration rate; and t = time (in days). Although all track segments are based on the same exponential model, the parameters Q_0 and b are different for each segment. These parameters can be estimated from geometric auscultation data

$$\frac{dQ(t)}{dt} = b \cdot Q(t) \Leftrightarrow Q(t) = Q_0 \cdot e^{bt} \tag{2}$$

This study considers the standard deviation of longitudinal Level D1 (σ) as the quality measure, following European regulations (CEN 2010). Therefore, Eq. (3) gives the quality σ_{ijk} of segment j of section i in trimester k, considering that no maintenance operations have been performed in that time period. Fig. 1 shows the exponential behavior of a segment between successive tamping operations

$$\sigma_{ijk} = \sigma_{ij0} \cdot e^{b_{ijk}(90t)} \tag{3}$$

Maintenance Operations Modeling

When the quality level attains a certain threshold, maintenance operations are performed in order to take it to an appropriate value. This introduces a break in the model, as the quality is changed. Moreover, maintenance operations also change the deterioration rate (Ramos and Fonseca 2013; Audley and Andrews 2013), which makes the modeling problem much more difficult, in particular with respect to the estimation of Q_0 and b.

Previous studies in the literature consider that tamping induces a constant change in the first derivative of the exponential deterioration model curve (Hummitzsch 2009). This model starts from the first derivative of σ in trimester $k = 0$ [Eq. (4)] and assumes a constant ratio c between this value before and after a tamping [Eq. (5)].

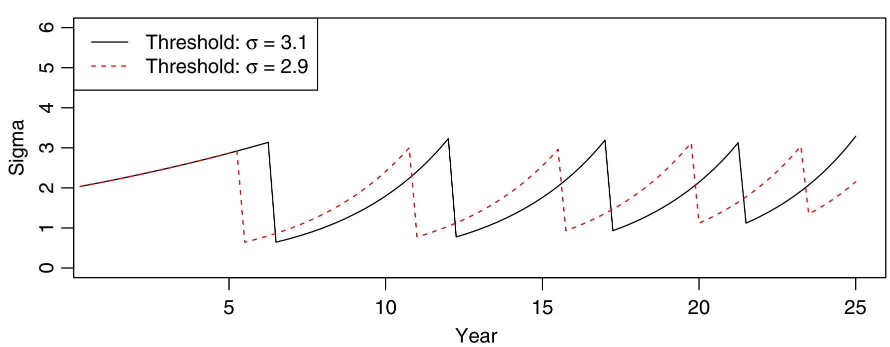

Fig. 1. Example of quality simulation with the deterioration model; both lines simulate the same segment, with a slightly different quality threshold for tamping

Then, it estimates $\sigma_{ij(k+1)}$ with a linear fitting using the age of the track, so that the new deterioration rate is given by Eq. (6)

$$\sigma'_{ij0} = \sigma_{ij0} \cdot b_{ij0} \tag{4}$$

$$\sigma'_{ij(k+1)} = c \cdot \sigma'_{ijk} \tag{5}$$

$$b_{ij(k+1)} = \frac{\sigma'_{ij(k+1)}}{\sigma_{ij(k+1)}} \tag{6}$$

Fig. 1 depicts an example of the quality of a segment over the years after successive tamping operations, for two slightly different quality thresholds. It is shown that the more tampings are performed, the faster the track deteriorates, and the smaller the quality gain is. Moreover, the small difference in the threshold causes serious disturbances of the degradation forecast for large time horizons. This highlights the difficulty of the tackled problem, namely decisions that are made for early stages of the scheduling might have important long-term effects on the track behavior.

The modeling of a renewal operation is simpler. It is considered to be applied to a whole section of the track, whose quality is restored to some level Q_{best}, with a certain deterioration rate b_{best}. This operation resets the deterioration model to the optimal state of a new track.

Solution Modeling

Maintenance operations can be encoded as a vector of binary values that indicate if the operation is performed or not at a certain time and location. Focusing on tamping operations, the vector is of the form $\mathbf{x} = \{x_{ijk}\}$, where i = track section; j = segment within a section; and k = trimester. Likewise, complete renewal operations are represented as a vector $\mathbf{y} = \{y_{ik}\}$.

The length of these vectors is $N_g N_k$ and $N_s N_k$, respectively, where N_s, N_g, and N_k are the number of sections, segments, and trimesters. Each solution to the scheduling problem is represented by the concatenation of $\mathbf{x}$ and $\mathbf{y}$, as shown in Fig. 2, where N_i is the number of segments in section i. Note that each section can be split into a different number of segments, according to the segmentation procedure previously described. This gives an overview of the difficulty of the problem, which involves a very high dimensionality. More precisely, the size of the search space is $2^{N_k(N_s+N_g)}$, making brute force or even exact approaches unfeasible.

Objective Functions and Constraints

The proposed approach to the optimization of maintenance plans uses two different objective functions: economic cost of the maintenance, and time delay of the trains. This design complies with other approaches (Patra et al. 2009; Ramos and Fonseca 2011a). Other secondary objectives are considered to be included within these, such as the durability of the track (reflected as a higher cost), or level of service (which reflects the deterioration state in the same way as the delay). However, two more factors must be taken into account: safety and resources. These have been implemented as constraints, so that a solution that violates any constraint is said to be nonfeasible.

Cost

The economic cost of railway maintenance includes costs of track inspection and maintenance operations. In the literature, various approaches to assess these costs can be found (Patra et al. 2009; Guler 2013), which involve duration and length of the operations and cost of the workforce and equipment. Based on the cost functions defined in Patra et al. (2009) and Guler (2013), the maintenance cost is defined as the sum of tamping cost (C_T) and renewal cost (C_R) for all sections, segments, and trimesters, as shown in Eq. (7), where C_t = cost of a tamping operation per meter; L_{ij} = segment length; C_r = renewal cost per meter; L_i = section length; and r = discount rate (which models the economic impact of the investment). Eq. (7) is the first objective function for the modeling of the problem, and it is to be minimized

$$f_1(\mathbf{x}, \mathbf{y}) = C_T + C_R = \sum_k \frac{\sum_{ij}(C_t \cdot L_{ij} \cdot x_{ijk}) + \sum_i (C_r \cdot L_i \cdot y_{ik})}{(1+r)^k} \tag{7}$$

Delay

The other primary objective for railway maintenance is the maximization of the track availability and capacity. Usually, maintenance operations are performed when no trains are scheduled, so that the availability is not affected. As for the capacity, it can be translated into terms of overall time delay of the trains (Ramos and Fonseca 2011a). Table 2 shows the maximum speed of the track depending on its measured quality, according to European standards (CEN 2010).

In order to calculate the delay, the maximum permissible nominal speed $s_i^{\max}$ of section i is defined as the minimum speed across

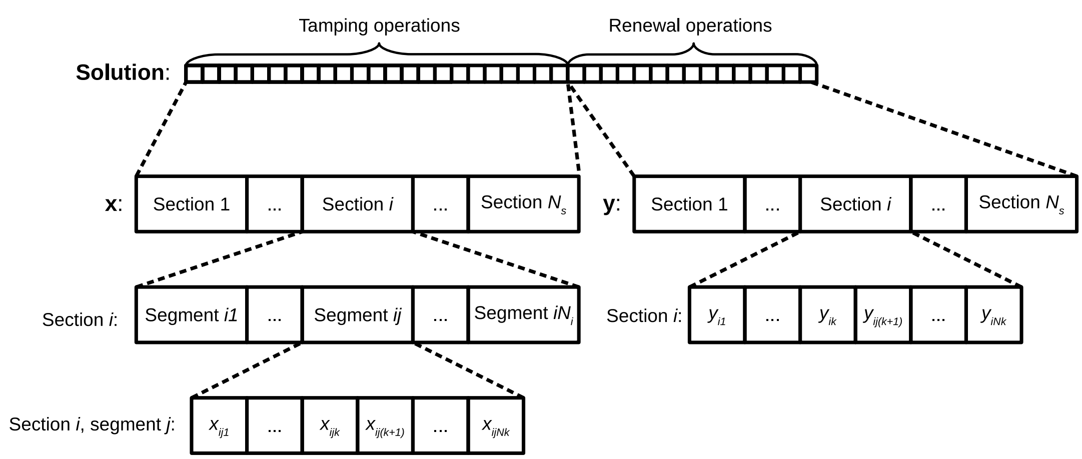

Fig. 2. Representation of a single solution to the maintenance scheduling problem

Table 2. Maximum Speed and Minimum Quality Values according to EN 13848-5

Standard deviation in longitudinal level D1 (mm)	Speed (km/h)	L_{QN3}
2.3–3.0	$s \leq 80$	3.1
1.8–2.7	$80 < s \leq 120$	2.7
1.4–2.4	$120 < s \leq 160$	2.2
1.2–1.9	$160 < s \leq 230$	2.0
1.0–1.5	$230 < s \leq 300$	1.7

every segment j within the section, considering that the track is in perfect condition, and depends primarily on the curvature of the track. Accordingly, the maximum speed that a train t, whose average speed is s_t^{mean}, can attain within section i is denoted s_i^t. Eq. (8) presents the maximum speed for a train t in section i and trimester k, where $s_{ik}(\sigma_{ik})$ is the maximum speed in the section taking into account the deterioration state of the track ($\sigma_{ik} = \max_j\{\sigma_{ijk}\}$), as defined in Table 2

$$s_{ik}^t = \min\{s_i^t, s_{ik}(\sigma_{ik})\}, \qquad s_i^t = \min\{s_i^{\max}, s_t^{\text{mean}}\},$$
$$s_i^{\max} = \min_j\{s_{ij}^{\max}\} \tag{8}$$

Based on these equations, the second objective function is defined by calculating the overall delay in hours, as detailed in Eq. (9), where N_t = number of trains; and L_i = length of the section. Note that $\mathbf{x}$ and $\mathbf{y}$ are not explicitly shown, but they are used to calculate σ_{ik}. For each train, each trimester, and each section, the time difference is calculated with respect to the same track in perfect conditions. Therefore, the time delay would be zero in such a case where all σ_{ik} are low enough to allow $s_{ik}(\sigma_{ik}) \geq s_i^t \ \forall i, k, t$

$$f_2(\mathbf{x}, \mathbf{y}) = \sum_{ik} \sum_{t=1}^{N_t} \frac{L_i}{1000} \left(\frac{1}{s_i^t} - \frac{1}{s_{ik}^t} \right) \tag{9}$$

Safety and Resource Constraints

Even though a low quality of the track can be palliated by reducing the speed, each segment has to be kept above the acceptable minimum determined by the legal and technical normative for safety reasons. Table 2 shows the quality limit values for each speed in the experiments, which were extracted from CEN (2010). Thus, the safety constraint can be represented as shown in Eq. (10)

$$1 - \frac{\sigma_{ijk}}{\max\{L_{QN3}\}} \geq 0 \quad \forall\ i, j, k \tag{10}$$

The other constraint to be included into the model refers to the available resources. In particular, the limits of the resources for tamping and renewal operations [Eqs. (11) and (12), respectively] are modeled by establishing a maximum extent of operations per trimester ($\max_t$ and $\max_r$, respectively), measured in meters

$$1 - \frac{\sum_{ij} L_{ij} x_{ijk}}{\max_t} \geq 0 \quad \forall\ k \tag{11}$$

$$1 - \frac{\sum_i L_i y_{ik}}{\max_r} \geq 0 \quad \forall\ k \tag{12}$$

Proof That Railway Maintenance Planning Is NP-Hard

The problem defined previously can be proven to be NP-hard. Consider a simplification of the problem that involves only tamping operations ($\mathbf{y} = 0$, $C_r = 0$), the cost function f_1 with no discount rate ($r = 0$) as a single objective, and a deterioration model where tamping does not change the deterioration rate ($c = 1$). With these conditions, the safety constraint is held if and only if the period between two consecutive tampings on the same segment is kept under a threshold T_{ij}.

This simplification can be expressed as an integer linear programming problem with a binary decision variable $\mathbf{x}$ [Eq. (13)]. As integer programming problems are known to be NP-hard (Garey and Johnson 1979), this simplified version of railway maintenance scheduling is also NP-hard, and so is the full nonlinear multiobjective problem that is tackled in this paper

$$\begin{aligned}
&\text{Minimize: } C_t \sum_{ijk} L_{ij} x_{ijk} \quad \text{(cost function)} \\
&\text{Subject to: } \sum_{k=l+1}^{N_k - T_{ij}} x_{ijk} \geq 1 \quad \forall\ l = 0, \ldots, N_k \quad \text{(safety constraint)} \\
&\qquad \sum_{ij} L_{ij} x_{ijk} \leq \max{}_t \quad \forall\ k = 1, \ldots, N_k \quad \text{(resource constraint)}
\end{aligned} \tag{13}$$

Solution Initialization

The search space of the tackled optimization problem has two primary difficulties, namely its very high dimensionality [$N_k(N_s + N_g)$ dimensions], and its complexity due to the constraints that restrict the feasibility of the solutions. Moreover, the objectives of a maintenance plan differ depending on the horizon of the schedule; a short-term scheduling usually prioritizes tamping operations while a long-term approach must make an adequate use of renewal operations.

Conversely, there are experts on railway maintenance scheduling that possess information about how to build good maintenance plans. Therefore, this proposal does not use a randomly generated initial set of solutions. Instead, those solutions are generated following certain heuristic rules given by experts to conform an initial set of feasible and reasonably good solutions. Then, it falls on the algorithm to improve those solutions and obtain maintenance plans that are better than those designed by the experts. This design ensures that the quality of the obtained solutions to the problem will be at least as high as that of the human-designed initial set. Furthermore, the improvement can be measured by simply evaluating the differences between the initial solution set and the final Pareto front.

When considering short-term scheduling, each solution is initialized as follows:

1. For the first trimester, tamping is programmed in the segments whose deterioration is above the threshold ($\max\{L_{QN3}\}$);
   - If the tamping capacity is insufficient, a renewal is performed in the section with the largest number of segments needing action;
   - Otherwise, and if there is some remaining tamping capacity, a random number of tampings are programed in the segments with worst quality among those that do not have tamping scheduled;
   - The same operation is performed for the remaining renewal capacity;
2. After the maintenance of the first trimester has been scheduled, the deterioration model simulates the quality for the second trimester, and the operations are scheduled following Step 1. This procedure is iteratively applied for the whole simulation time span; and
3. If no renewal is planned, it is randomly determined if a single renewal should be introduced into the solution.

This procedure aims to ensure the generation of feasible solutions. Note that there may be cases in which the track is in such a bad state that the available resources do not suffice to mend it within a single trimester. This situation can also arise when the first trimesters are assigned a low amount of tampings and renewals. In extremely bad quality tracks, feasible solutions might be entirely nonexistent. However, this kind of solutions could also be interesting as a starting point for the algorithm, because they introduce diversity into the search. Eventually, as nonfeasible solutions are dominated by feasible ones, these solutions will disappear from the population, but their information could have been used to generate new promising solutions.

Different rules apply for long-term horizons, as renewal must often be preferred over tamping in order to obtain feasible schedules. Therefore, a different initialization heuristic was used:

1. The total number of renewals is randomly fixed between the maximum and half of the maximum;
2. These operations are randomly distributed among all the trimesters in the schedule;
3. For each trimester:
   a. The deterioration model is applied;
   b. If this trimester had a renewal operation scheduled, it is performed over the most deteriorated section in terms of $dQ(t)/dt$ [Eq. (2)];
   c. Tamping is applied over any section above the threshold ($\max\{L_{QN3}\}$); and
   d. If there is any remaining tamping capacity, a random fraction of it is used to schedule tamping over the sections with worst quality.

Operators and Implementation Particularities

The NSGA-II uses single-point crossover and bitwise mutation, as suggested in the original paper for binary problems (Deb et al. 2002). The AMOSA uses only the bitwise mutation, as it does not involve any crossover operations.

The primary difference in the implementation with respect to the originally published algorithms lies in the hill-climbing technique for AMOSA. Although the same algorithm was implemented, an additional criterion was added to allow for handling such a high dimension problem [note that the number of dimensions is $2N_k(N_s+N_g)$; see Table 5 for the dimensionality of the track evaluated in this paper]. Instead of performing the hill-climbing procedure until no improvement is reached, the procedure is interrupted when the solution has been improved more than a fixed number of times $\max_{HC}$. Otherwise, the search space for the hill-climbing procedure would be too large to be used as initial greedy algorithm to improve the solutions.

Experiments and Results

Case Study and Parameters

Two multiobjective algorithms have been used for the experimental framework of this paper: NSGA-II and AMOSA. Both algorithms have been executed up to a total of 500,000 evaluations of the objective functions, and the corresponding parameters have been set up accordingly (Table 3). The horizon of the prediction was 3 years, which corresponds to an average contract period for maintenance contractors. Both algorithms started from the same set of initial solutions. The value for $\max_{HC}$ was chosen so as to invest approximately 2,000 evaluations for the hill-climbing procedure, and the remaining evaluations for the simulated annealing optimization.

Table 3. Parameters for the Optimization Algorithms

Algorithm	Parameter	Value
NSGA-II	Size of the population	104
	Number of generations	4,810
	Crossover probability	0.6
	Mutation probability	0.3
AMOSA	HL	104
	SL	104
	γ	1
	α	0.9183544
	$\max_{HC}$	20
	Initial temperature	500
	Minimum temperature	0.1
	Iterations per temperature	5,000

Table 4. Parameters Concerning the Considered Track

Parameter	Description	Value
C_t	Tamping cost[a]	10
C_r	Renewal cost[b]	150
$\max_t$	Maximum tamping (m)	5,100
$\max_r$	Maximum renewal (m)	12,000
s_t^{mean}	Average speed of train (km/h)	60–135
r	Discount rate	0.03
N_s	Number of sections	24
N_g	Number of segments	1,435

[a]Thousands of euros per segment.
[b]Thousands of euros per section.

Table 5. Parameters Concerning the Solutions

Parameter	Description	Horizon: 3 years	5 years	10 years	20 years
N_k	Number of trimesters	12	20	40	80
N_gN_k	Length of **x**	17,220	28,700	57,400	114,800
N_sN_k	Length of **y**	288	480	960	1,920
$N_k(N_s+N_g)$	Length of the solution	17,508	29,180	58,360	116,720

The experiments have been performed on a model of a real railway track from the Swedish Iron Ore Line, which is 152 km long and runs in the northern part of Sweden, subject to temperatures between −40 and 25°C and heavy snowfalls during winter. A total of 19 geometrical auscultations with a resolution of 25 cm performed between 2007 and 2012 are available. These data were spatially aligned to match the measurements taken at different points in time, using correlation-based alignment on the curvature. This information was used to estimate the initial Q_0 and b for every segment of the track by an exponential fitting. Tables 4 and 5 contain the parameters that define the track modeling and the solutions to the problem for this case study, respectively.

To complete the study and give an overview of the potential of the proposed multiobjective approach, a complementary study is presented in a subsequent section, with horizons longer than 3 years for the maintenance plans, namely 5, 10, and 20 years. Because of the computational constraints, the number of evaluations was reduced to 20,000 for these tests.

Scheduling for 3 Years

Tables 6 and 7 present a summary of the solutions in the final Pareto fronts obtained by NSGA-II and AMOSA, respectively. These

Table 6. Summary of the Pareto Front Obtained by NSGA-II

Cost (Euros)	Delay (h)	Tampings	Renewals
6,215,034	334.01	501	9
6,224,578	263.46	517	9
6,235,872	239.43	536	9
6,251,445	212.07	551	9
6,259,503	207.73	568	9
6,259,854	178.66	556	9
6,287,670	172.05	608	9
6,293,251	164.96	601	9
6,299,343	159.31	618	9
6,305,647	158.21	629	9
6,305,650	148.29	631	9
6,318,460	148.12	648	9
6,326,540	138.54	667	9
6,329,879	127.06	672	9
6,345,281	121.64	695	9
6,362,485	100.86	725	9

Table 7. Summary of the Pareto Front Obtained by AMOSA

Cost (Euros)	Delay (h)	Tampings	Renewals
5,689,494	137.98	665	8
5,690,557	126.63	667	8
6,286,985	126.63	623	9
6,289,973	126.14	625	9
6,293,171	119.84	630	9
6,293,924	118.34	630	9
6,297,452	116.68	636	9
6,299,088	113.50	640	9
6,321,768	110.84	663	9
6,323,540	110.83	666	9
6,325,447	109.16	670	9
6,325,871	103.03	669	9
6,326,664	100.72	668	9
6,335,103	99.06	680	9
6,339,205	99.03	689	9
6,343,664	96.25	691	9
6,345,849	93.09	694	9
6,347,311	91.43	696	9
6,350,716	89.45	699	9
6,351,128	86.29	702	9
6,364,458	86.26	723	9
6,364,458	86.26	724	9
6,369,357	84.75	729	9
6,382,535	84.68	758	9
6,393,818	81.53	774	9
6,397,428	78.81	780	9
6,996,854	72.01	839	10
7,006,133	68.85	852	10
7,007,671	62.06	855	10
7,007,671	62.06	856	10

show clearly that renewal and tamping operations increase the maintenance cost and decrease the time delay. They also reflect that renewal improves the track quality more than tamping, allowing for a higher nominal speed.

Table 7 shows the flexibility provided by the Pareto front. The difference between the two extremes of the Pareto (first and last rows of the table) states that the delay can be reduced by 55% by increasing the cost by around 23%. However, railway maintenance companies may be more interested in the intermediate results, seeking a trade-off between cost and delay. The approach proposed in this paper allows for consideration of a wide set of nondominated solutions that provides a rich decision support for railway maintenance companies.

Fig. 3 depicts the initial population and the final Pareto fronts of NSGA-II and AMOSA. At first sight, it is observed that the AMOSA Pareto front outperforms that of NSGA-II. This behavior arises because the initial local search performed by AMOSA proves to be crucial for the algorithm convergence. The initial population of solutions is not random; quite the contrary, it has been generated according to directions and constraints given by experts, so they all have a reasonable quality. The AMOSA's local search focuses on further improving these solutions, rather than exploring entirely new areas of the search space for unknown solutions to the problem, which is the strategy followed by NSGA-II. Thus, AMOSA starts its exploratory search from a set of already optimized solutions, which yields far better results, as demonstrated by the distance between the initial population and the Pareto front in Fig. 3. The solution of minimal cost is reduced from approximately 6.2 to 5.7 million Euro, and minimal delay is improved from 100 to 62 h. Moreover, it is able to explore solutions with different amounts of renewals than initially provided in the expert-based solutions, demonstrating a considerable diversification of the search as well. Note that the solutions with the lowest delays, which involve 10 renewal operations, also involve a high number of tampings. This highlights the heavy maintenance that would be required to keep the track at an optimal quality at all times. Conversely, it can be seen that the combinations of existing solutions favored by NSGA-II do not suffice to reach the performance of AMOSA.

To further illustrate this behavior, Fig. 4 gives an overall view of all the 500,000 solutions explored by AMOSA. It shows that even though AMOSA focuses on improving the good solutions, a good deal of exploration effort is made. This plot also shows the structure of the problems; each of the vertical stripes represents a certain number of renewals (the three stripes with solutions in the Pareto correspond, from left to right, to 8, 9, and 10, respectively), and each additional renewal increases the cost of the maintenance plan, but reduces the delay. It is shown that the search explored feasible maintenance plans with 11 renewals, but they did not yield better delays than solutions with 10 renewals. Some plans with seven or six renewals and a very low cost were also generated, but they did not comply with the constraints and therefore were not included into the final set of solutions. To summarize, the proposed approach has been shown to greatly improve the quality of solutions in both objectives. In addition, by design the obtained solutions will never be worse than those obtained by human experts. While metaheuristics have no guarantee for quality assurance, they are usually better than other simpler methods. In addition, because of the large budgets of the maintenance contracts, the improvement in solutions easily leads to large economic savings.

Long-Term Scheduling

It is well-known that models and solutions for long-term horizons are subject to important uncertainties and therefore cannot be considered as an exact forecast (Ramos and Fonseca 2011b). However, the results presented in this complementary study are useful to illustrate the behavior of the multiobjective approach, and they represent the long-term point of view of the railway owner. A similar study is presented in Ramos and Fonseca (2011a), in which a small custom track is simulated over 30 years; the authors are able to generate nine nondominated feasible solutions. Nevertheless, the results cannot be compared to those obtained in this paper because they do not take into account the deterioration caused by tamping operations, which simplifies the problem and the search space they consider.

This section presents the results obtained after additional executions of the algorithms for a simulation of the track over 5,

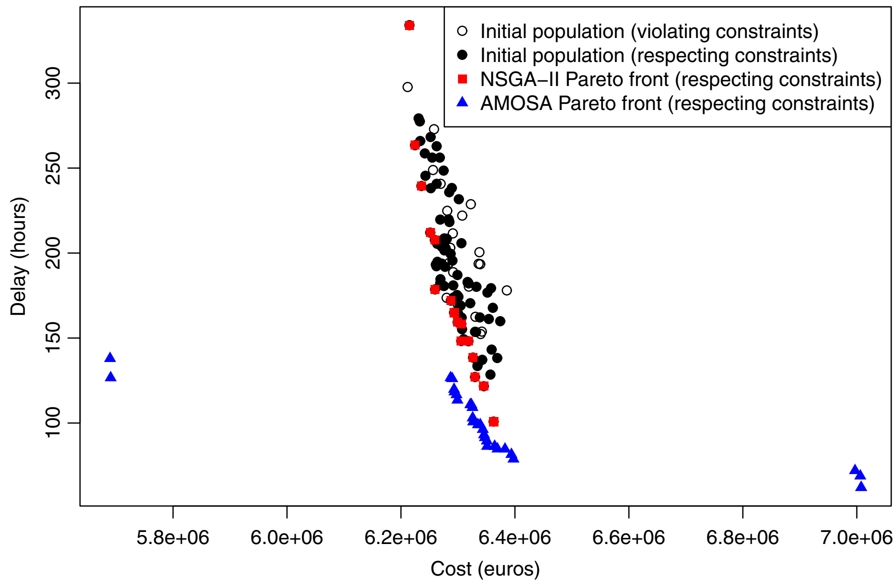

Fig. 3. Initial population and Pareto fronts of NSGA-II and AMOSA

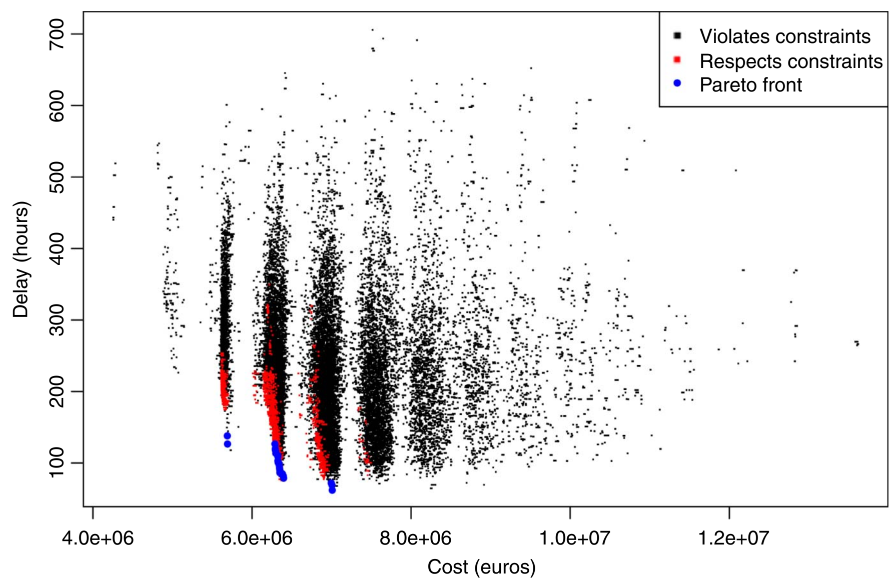

Fig. 4. All the solutions generated by AMOSA

10, and 20 years. This is reflected in a linear increase in the size of the solutions and therefore an exponential growth of the search space. The initialization rules for the population are also different, as human experts follow different scheduling patterns for such long-term situations. Because of the higher computational cost of the objective and constraint functions, only 20,000 evaluations of the objective functions were performed for each horizon and algorithm. Note that the difficulty of the problem is such that NSGA-II did not obtain any improvement with respect to the initial population; therefore, only the results from AMOSA are presented in this paper.

Fig. 5 represents the initial populations and the Pareto fronts obtained by AMOSA, in terms of average cost and delay per year. For the sake of simplicity, only feasible solutions are shown in the initial populations. The plot shows great improvements on both objectives for all three horizons. The initial solutions are in general worse for distant horizons because the complexity of the scheduling (which is an NP-hard problem) increases greatly as the horizon grows.

However, the Pareto fronts surprisingly follow the opposite behavior, namely the larger the horizon, the better the final Pareto front of solutions. This means that the proposed scheduling procedure works best with more distant horizons than with small ones, despite the exponential growth of the search space. This behavior arises because for long-term simulations, the cost of the renewal operations can be amortized over the years, yielding better quality railways at lower costs per year, which in turn leads to lower average delays. In this manner, this approach has been able to improve altogether two objectives that are a priori opposed to each other. Furthermore, it implies an improvement of the average track quality after applying the computed maintenance schedules with respect to the current state of the tracks, which is the result of a maintenance plan carefully designed by experts.

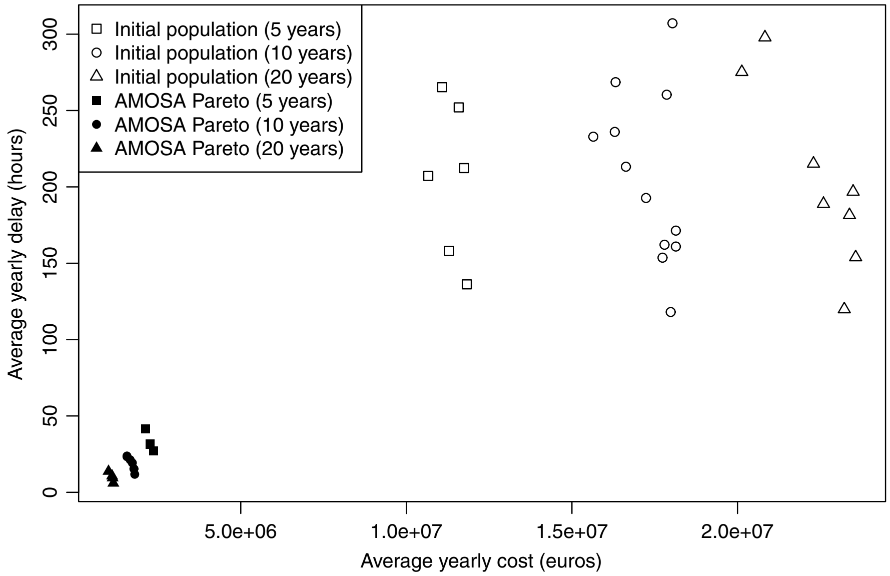

Fig. 5. Feasible solutions of the initial populations and AMOSA Pareto fronts for the three long-term horizons tested

The case with the largest horizon is especially illustrative; both the average cost per year and the average delay are reduced by a factor of at least 20. This reflects the advantages of the proposed metaheuristics over human-designed approaches and assesses the quality of the obtained solutions.

The limiting factors for most optimization algorithms are the size of the solution space and the number of evaluations. The results in this paper demonstrate that the proposal is able to explore very large solution spaces and reach good solutions in very few iterations. As an example, the number of possible solutions for the considered railway for 20 years is more than 10^{35000}, and the proposed method is able to provide high-quality solutions after evaluating only 20,000 of them.

Conclusions

In this paper, a multiobjective approach has been described to tackle the railway track maintenance scheduling problem. Two objective functions have been considered (maintenance costs and train delays), as well as three sets of constraints that model safety limits and resources. The proposal includes a deterioration model based on exponential fitting and a two-level segmentation, that takes into account the variations in the deterioration curve caused by tamping and renewal operations. Two multiobjective algorithms (AMOSA and NSGA-II) have been applied to the problem, starting from an initial population of solutions generated heuristically according to expert knowledge.

The described approach has been tested over a model of a real railway from northern Sweden to generate a maintenance schedule for 3 years. Both algorithms have been run with equivalent parameters and started from the same initial population. Then, an additional set of experiments for longer horizons (namely 5, 10, and 20 years) has been performed.

As for the results obtained, AMOSA outperformed NSGA-II because of its stronger intensification strategy. Furthermore, both the Pareto front and the solution space explored by AMOSA showed that a wide range of solutions were analyzed, providing the decision maker with a fair variety of possible maintenance schedules. All the solutions provided in the Pareto front for the 3 years horizon were nonconstrained, which stresses the adequacy of the proposed scheme. Moreover, the results obtained for long-term horizons show a very important decrease of the cost and delay, and this decrease is higher for more distant horizons, assessing the capabilities of the proposed scheme to schedule railway maintenance plans.

The primary limitation of the proposal is the computational complexity of simulating of the degradation model for each generated schedule, which limits the number of evaluations that can be carried out during the optimization algorithm. Therefore, even though the obtained solutions were of very high quality, it would be of interest to develop new approaches that can make use of parallel computing infrastructures to solve this problem, which would allow the authors to deal with longer railways (which would have an impact on the dimensionality of the search space and the complexity of the problem). Another possibility of extending the work consists of considering more complex maintenance schedules, including availability of human and material resources and time slots.

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