Consensus Reaching in Group Decision Making with Linear Uncertain Preferences and Asymmetric Costs

Xiaoxia Xu, Zaiwu Gong, Enrique Herrera-Viedma, *Fellow, IEEE*, Gang Kou and Francisco Javier Cabrerizo

***Abstract*—Consensus reaching process (CRP), as an essential part of group decision making (GDM), can facilitate more effective consensus by taking into account human behaviors. Extending the established research on uncertain minimum cost consensus models (MCCMs), this paper continues to adopt linear uncertainty distribution (LUD) to fit decision-maker's (DM's) preference, but considers asymmetric costs into a new framework of CRP, where DM's preference and weight are both adjusted according to democratic consensus. Moreover, on the basis of uncertain distance measure, two novel optimization-based consensus models are built in this paper: one is to obtain the minimum cost consensus by simultaneously considering asymmetric costs, aggregation function and consensus measure; while the other is to address more flexible GDM scenarios without pre-setting specific consensus level (CL) threshold. Some 0-1 binary variables are further introduced to reduce the calculation complexity resulted from piecewise functions in the new multi-coefficient goal programming models. Finally, an illustrative example and further discussion reveal the feasibility and superiority of our new method.**

***Index Terms*—Group decision making (GDM); Consensus reaching process (CRP); Asymmetric costs; Uncertainty theory; Feedback mechanism.**

I. INTRODUCTION

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DECISION making is important for human daily life, featured by the motivation to select an optimal solution from at least two alternatives [1] or achieve a unanimous agreement where decision-makers (DMs) express various feelings as to the values in question [2]. Practically speaking, multiple DMs participate in a dynamic and negotiable process so as to derive a common solution on behalf of the collective interest, thereby constituting a group decision making (GDM) problem [3]. In other words, eliminating disagreements among DMs becomes necessary to obtain an agreed solution, which is widely appreciated by all stakeholders in real-life scenarios [4]. Without loss of generality, a consensus reaching process (CRP) introduced in GDM can effectively bring DMs' views closer to each other through limited rounds of compromises and adjustments [5]. In fact, Herrera-Viedma et al. [2] concluded that consensus boils down to cooperation, while most GDM boils down to competition.

Generally, a CRP consists of three steps as consensus measure, feedback process and selection [6], while an effective CRP relies heavily on its feedback mechanism, where identification rule (IR) and direction rule (DR) are exploited [7]. To date, developing an automatic feedback mechanism that excludes the moderator's influence has been increasingly crucial to consensus [2]. On the other hand, several indicators are commonly used to measure the CRP performance, such as the minimum deviation or cost [8], the maximum number of experts adjusted under a limited budget [9] or the minimum number of adjusted DMs [10]. Moreover, Labella et al. [11] proposed a new metric to comprehensively evaluate the performance. Given the significance of deriving a consensus for GDM, see [2], [5] for more studies on feedback mechanisms.

The concept of the minimum cost consensus (MCC) was originally proposed by Ben-Arieh and Easton [12] for solving single and multi-criteria GDM problems via linear-time algorithms. Later, a quadratic cost function was adopted to discuss the influence of different factors (e.g., cost, opinion elasticity, the number of adjusted experts) on the consensus [9]. At the same period, Dong et al.'s optimization-based consensus models [13], recognized as the minimum adjustment consensus models (MACMs), aim to maximize the retention of DM's original preference, rather than pursuing a minimum resource consumption. Afterwards, although abundant studies have been conducted [14]–[16], most are based on traditional preference structures (e.g., crisp numbers, intervals or linguistic information), neglecting the characteristics of stochastic distribution in DM's preference. In contrast, linear uncertainty distribution

(LUD) with belief degree provides a feasible way to better simulate DM's uncertainty and ambiguous behaviors in actual GDM problems [17].

When building the minimum cost consensus models (MCCMs), unit costs (i.e., the resources used to persuade DMs to adjust their preference one unit towards the consensus) are usually preset as fixed values [9], [12], which have been criticized due to over-idealization. Therefore, under the premise of unit adjustment costs being intervals, Li et al. [7] facilitated a consensus through optimization modeling, while Zhang et al. [8] further took Stackelberg game into consideration. Moreover, [18] associated the variable unit costs with the incompleteness degree of the DM's linguistic distribution assessments. In contrast, Cheng et al. [19] analyzed the impact of experts' compromise limit and tolerance behaviors on MCCMs based on one hypothesis that cost coefficients are asymmetric due to individual different adjustment directions. Subsequently, the work in [19] was extended through stochastic programming [20] or data-driven robust optimization [21]. As a result, how to make the best use of unit costs to portray DM's psychological behavior in GDM is worth investigating.

In addition to the above unit costs, weights also play a vital role in the CRP, measuring the relative importance of each criterion/DM [22]. Traditionally, weights attached to DMs in the MCCMs and MACMs are either assigned directly by the moderator or automatically preset according to their preferences [23]. However, individuals involved in GDM prefer their influence (i.e., the weight coefficients) to change over time as they compromise to achieve the consensus, which raises a new question of how to determine these coefficients to avoid strategic manipulation [24]. Hence, we adopt the idea that important DMs contribute more to the consensus [11]; meanwhile, the concept of democratic consensus [25] is also incorporated into our feedback mechanism during the CRP, where DMs' weights are variable with the proceeding of GDM. Specifically, DMs' initial weights are equally assigned, but once an initial consensus derived, their weights will be automatically reallocated based on their own contribution to the consensus level (CL). Briefly, a new feedback mechanism excluding the moderator is proposed, by sufficiently respecting individual values via DM's established preference information.

In other words, the final results obtained from GDM are always expected to be as objective as possible; but it's tough to accurately predict the outcomes in advance, due to inherent subjectivity, imprecision and vagueness in DM's preference articulation [2]. Pragmatically, valid information in real-life GDM is insufficient to derive the probability, not to mention that information sources frequently conflict with each other [26], leaving the reliability of the occurrence of certain events to be determined by domain experts. As a result, uncertainty theory proposed by Liu [27] becomes a new mathematical tool to address uncertain phenomenon characterized by non-randomness and non-fuzziness. So far, Liu's theory has been verified feasible to solve GDM problems with low frequency or small sample [17], where the probability cannot be fitted by frequency. In fact, the LUD provides a more inclusive and richer expression form for individuals in GDM [28]. Hence, this paper continues to adopt the LUD to fit DM's preference under a new framework of CRP.

Existing studies greatly contributed to the development of GDM theory, but none has comprehensively explored the CRP framework combined with uncertain MCCMs, asymmetric costs, aggregation function, consensus measure and feedback mechanism. Specifically,

- Prior uncertain MCCMs do not consider asymmetric costs nor the dynamic characteristic of GDM [28], [29]. In other words, they primarily focused on optimization-based consensus modeling while neglecting DM's unbalanced willingness to adjust.
- Extant MCCMs with asymmetric costs do not consider aggregation functions nor feedback mechanisms [19], [20]. Namely, they don't aggregate DMs' choices into a collective wisdom [14], thereby failing to portray social choices or individual values.
- Current studies on the CRP do not take into account DMs' changeable influence during neither the consensus measure nor the feedback mechanism [15], [18], making the importance of different individuals unable to be fully demonstrated.

To conclude, this paper extends the established uncertainty theory-based MCCMs into a general CRP framework. Specifically, our main contributions are threefold: (1) new optimization-based consensus models are built by simultaneously considering aggregation functions, asymmetric costs and consensus measure; (2) a novel CRP framework is proposed by respecting individual values with democratic consensus and simultaneously pursuing a minimum resource consumption based on uncertain MCCMs; and (3) binary variables are introduced to reduce the computation complexity of piecewise functions in the new multi-coefficient programming models.

The rest is organized as follows. Section II recalls preliminaries regarding uncertainty theory and traditional consensus GDM theory. Section III reviews methods of consensus measure and a general CRP framework. Section IV develops two new uncertain consensus models with asymmetric costs. Moreover, Section V gives a numerical example, while further discussion is provided in Section VI. Finally, Section VII gives concluding remarks and subsequent research directions.

II. Preliminaries

This section recalls basic knowledge of uncertainty theory (see II-A) and classic consensus models (see II-B).

A. Uncertainty theory

When no samples are available or only poor information obtained from historical data, the estimated distribution function will deviate far from the actual frequency, causing the law of large numbers invalid, and further obtaining some counterintuitive results. Then, some domain experts are invited to evaluate the belief degree that certain event will happen. Distinguished from probability theory dealing with randomness of frequency, uncertainty theory was proposed to address the uncertainty of belief degrees.

1) Uncertain variable and uncertainty distribution:

Definition *1:* [27] Let Γ be a nonempty set and $\mathcal{L}$ be a σ-algebra over Γ. Each element $\Lambda \in \mathcal{L}$ is called an event. A set function $M : \mathcal{L} \to [0,1]$ is called an uncertainty measure if and only if (i) $M\{\Gamma\} = 1$; (ii) $M\{\Lambda\} + M\{\Lambda^c\} = 1$; (iii) $M\{\bigcup_{i=1}^{\infty} \Lambda_i\} \leq \sum_{i=1}^{\infty} M\{\Lambda_i\}$; and (iv) $M\{\prod_{k=1}^{\infty} \Lambda_k\} = \bigwedge_{k=1}^{\infty} M\{\Lambda_k\}$.

For an uncertain variable ξ, its uncertainty distribution Φ is defined as $\Phi(x) = M\{\xi \leq x\}$, where $M\{\xi \leq x\}$ is the belief degree of the event $\{\xi \leq x\}$. Meanwhile, belief degree is also measured by α, $\alpha \in [0,1]$. Thus, $\Phi(x) = M\{\xi \leq x\} = \alpha$ holds for any real number x.

Definition *2:* [30] An uncertainty distribution $\Phi(x)$ is regular if it is a continuous and strictly increasing function with regard to x at which $0 < \Phi(x) < 1$, and satisfies

$$\lim_{x \to -\infty} \Phi(x) = 0, \quad \lim_{x \to +\infty} \Phi(x) = 1 \tag{1}$$

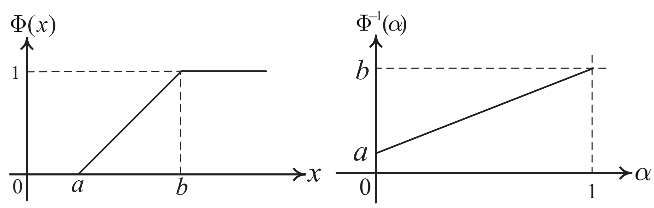
Uncertainty distribution Φ is regular iff its inverse function $\Phi^{-1}(\alpha)$ exists and is unique for each $\alpha \in (0,1)$.

Example *1:* An uncertain variable ξ is called linear if it has a LUD $\Phi(x)$ (see Fig. 1(a)), denoted by $\xi \sim \mathcal{L}(a,b)$, where real number a and b satisfy $a < b$.

$$\Phi(x) = \begin{cases} 0, & \text{if } x \leq a \\ \dfrac{x-a}{b-a}, & \text{if } a \leq x \leq b \\ 1, & \text{if } x \geq b \end{cases} \tag{2}$$

And its inverse uncertainty distribution (see Fig. 1(b)) is

$$\Phi^{-1}(\alpha) = (1-\alpha)a + \alpha b \tag{3}$$



(a) The linear uncertainty distribution (b) The inverse linear uncertainty distribution

Fig. 1: Schematic diagram of a linear uncertain variable

2) Basic properties of uncertain variables:

Theorem *1:* [30] Let ξ be an uncertain variable with uncertainty distribution Φ, then for any real number x (i.e., $x \in \mathcal{R}$), we have

$$M\{\xi \leq x\} = \Phi(x), \quad M\{\xi > x\} = 1 - \Phi(x) \tag{4}$$

To be noted, when the uncertainty distribution $\Phi(x)$ is a continuous function, we have $M\{\xi \leq x\} = M\{\xi < x\} = \Phi(x)$, and $M\{\xi > x\} = M\{\xi \geq x\} = 1 - \Phi(x)$.

Theorem *2:* [30] Let $\xi_1, \xi_2 \cdots \xi_n$ be independent uncertain variables with regular uncertainty distribution $\Phi_1, \Phi_2 \cdots \Phi_n$. If $f(\xi_1 \cdots \xi_m, \xi_{m+1} \cdots \xi_n)$ strictly increases with $\xi_1 \cdots \xi_m$ and decreases with $\xi_{m+1} \cdots \xi_n$, then $f(\xi_1 \cdots \xi_m, \xi_{m+1} \cdots \xi_n)$ has an inverse uncertainty distribution $f(\Phi_1^{-1}(\alpha) \cdots \Phi_m^{-1}(\alpha), \Phi_{m+1}^{-1}(1-\alpha) \cdots \Phi_n^{-1}(1-\alpha))$.

Example *2:* Let ξ_1 and ξ_2 be independent uncertain variables with regular uncertainty distribution Φ_1 and Φ_2, respectively. Then the inverse uncertainty distribution of $\xi_1 - \xi_2$ is

$$\Psi^{-1}(\alpha) = \Phi_1^{-1}(\alpha) - \Phi_2^{-1}(1-\alpha) \tag{5}$$

Theorem *3:* [31] Let ξ and ς be independent uncertain variables with regular uncertainty distributions Φ and Ψ, respectively. Then the distance between ξ and ς is

$$d(\xi, \varsigma) = \int_0^1 |\Phi^{-1}(\alpha) - \Psi^{-1}(1-\alpha)| d\alpha. \tag{6}$$

Example *3:* Let ξ and ς be independent uncertain variables obeying LUDs as $\xi \sim \mathcal{L}(a,b)$ and $\varsigma \sim \mathcal{L}(c,d)$, where $a \leq b$ and $c \leq d$. Based on Example 1, the distance between ξ and ς becomes

$$d(\xi, \varsigma) = \int_0^1 |(b+d-a-c)\alpha + a - d| d\alpha \tag{7}$$

B. Traditional consensus GDM theory

Suppose m DMs participate in a GDM problem, and a finite set $D = \{d_1, d_2, \cdots, d_m\}$ denotes all individuals. Let $o_i \in \mathcal{R}$ denote d_i's original preference, $\bar{o_i}$ be d_i's adjusted preference, and o^c be their reached consensus. In addition, $c_i \in \mathcal{R}^+$ denotes the unit cost of adjusting d_i's preference closer to the consensus, while $w_i \in \mathcal{R}^+$ reflects d_i's importance degree (i.e., weight), $i \in M = \{1, 2, \cdots, m\}$.

1) Traditional consensus models: DMs are normally willing to change opinions after repetitive negotiation efforts, though it escalates the costs of reaching a consensus. Adopting p-norm distance measure (i.e., $||\ ||_p, p \geq 1$), Ben-Arieh and Easton [12] provided a linear-time algorithm to seek the optimal MCC o^{c*} by minimizing the weighted total cost $f_c(o^c) = \sum_{i=1}^{m} w_i c_i ||o^c - o_i||_p$. Later, they discussed such scenarios with/without an ε-consensus (denoted as $|o^c - \bar{o_i}| \leq \varepsilon, \varepsilon > 0$) [9]. For brevity, Ref. [14] develops an optimization model to represent their ideas.

$$\begin{aligned} &\min \phi = \sum_{i=1}^{m} c_i |\bar{o_i} - o_i| \\ &\text{s.t.} \begin{cases} |o^c - \bar{o_i}| \leq \varepsilon_i,\ i \in M \\ o^c \in \mathcal{R},\ \bar{o_i} \in \mathcal{R} \end{cases} \end{aligned} \tag{8}$$

Solving Model (8) yields the optimal consensus o^{c*} and DM's optimal adjusted preference $\bar{o_i}^*$. The first constraint denotes a tolerance behavior, and if $\varepsilon_i = 0$ ($\forall i \in M$), a hard consensus is achieved (i.e., $\bar{o_i}^* = o^{c*}$), which is unrealistic and uneconomical for most GDM problems [2].

Meanwhile, Dong et al. [13] utilized the ordered weighted averaging (OWA) operator and a deviation measure to handle the consensus problems under a 2-tuple fuzzy linguistic environment, so as to preserve DMs' original preferences as much as possible. Similarly, their main ideas can be mathematically described as follows.

$$\begin{aligned} &\min \phi = \sum_{i=1}^{m} d(\bar{o_i}, o_i) \\ &\text{s.t.} \begin{cases} d(o^c, \bar{o_i}) \leq \varepsilon,\ i \in M \\ o^c = F_w(\bar{o_1}, \bar{o_2}, \cdots, \bar{o_m}) \end{cases} \end{aligned} \tag{9}$$

where $d(\cdot)$ represents the rectilinear or Euclidean deviation measure, F_w denotes an aggregation function, and the objective function is to minimize all DMs' adjustments.

Although Model (8) and Model (9) were proposed by different consensus mechanisms, Zhang et al. [14] found that the two models could actually be merged as

$$\min \phi = \sum_{i=1}^{m} c_i * d(\bar{o_i}, o_i)$$
$$\text{s.t.} \begin{cases} d(o^c, \bar{o_i}) \leq \varepsilon, \ i \in M \\ o^c = F_w(\bar{o_1}, \bar{o_2}, \cdots, \bar{o_m}) \end{cases} \tag{10}$$

It turns out that once F_w takes the OWA operator as $(\frac{1}{2} \cdots 0 \cdots \frac{1}{2})^{\mathrm{T}}$, Model (10) reduces to Model (8); and if the unit costs of adjusting DMs' opinions satisfy $c_i = c_j, \forall i, j \in M$, then Model (10) equals Model (9).

2) Consensus models with asymmetric costs: Cheng et al. [19], [32] explored GDM problems with cost constraints based on DM's different adjustment directions, further extending Model (10). In specific, Fig. 2 depicts their cost functions under a symmetric or asymmetric scenario by considering DM's tolerance and compromise behaviors, where the horizontal axis is DM's original preference (i.e., o_i), and the vertical axis represents a unit cost (i.e., c_i).

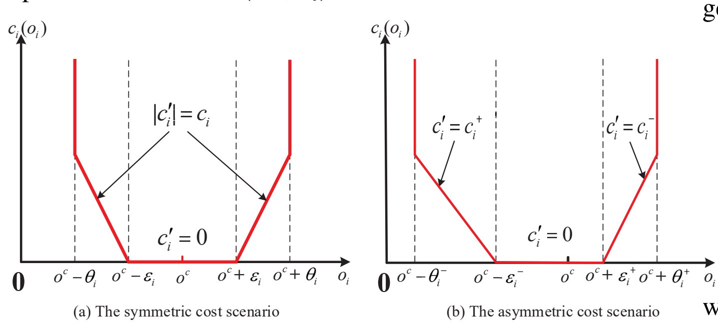


Fig. 2: Cost functions with tolerance and compromise

Due to space limitations, only the asymmetric cost scenario is reported hereafter [19]. That is, once taking the tolerance and compromise limit into account, d_i's optimal adjusted preference is obtained as

$$\bar{o_i}^* = \begin{cases} o^c - \varepsilon_i^-, & \text{if } o_i \in [o^c - \theta_i^-, o^c - \varepsilon_i^-) \\ o_i, & \text{if } o_i \in [o^c - \varepsilon_i^-, o^c + \varepsilon_i^+] \\ o^c + \varepsilon_i^+, & \text{if } o_i \in (o^c + \varepsilon_i^+, o^c + \theta_i^+] \end{cases} \tag{11}$$

The total cost for d_i to adjust preference becomes:

$$c_i(o_i) = \begin{cases} c_i^+(o^c - \varepsilon_i^- - o_i), & \text{if } o_i \in [o^c - \theta_i^-, o^c - \varepsilon_i^-) \\ 0, & \text{if } o_i \in [o^c - \varepsilon_i^-, o^c + \varepsilon_i^+] \\ c_i^-(o_i - o^c - \varepsilon_i^+), & \text{if } o_i \in (o^c + \varepsilon_i^+, o^c + \theta_i^+] \end{cases} \tag{12}$$

where c_i^- denotes d_i's unit cost with a downward adjustment, and c_i^+ conversely indicates the unit cost of an upward adjustment. In addition, ε_i measures d_i's tolerance of the consensus, while θ_i reflects d_i's compromise limit.

To be more specific, Fig. 2(b) shows that once d_i's original preference is located at $[o^c - \varepsilon_i^-, o^c + \varepsilon_i^+]$, a tolerance behavior exists (also known as the soft consensus [2]), namely, any preference within this subinterval is acceptable, thereby requiring no adjustments nor yielding any costs. Moreover, any preference located at $[o^c - \theta_i^-, o^c - \varepsilon_i^-)$ or $(o^c + \varepsilon_i^+, o^c + \theta_i^+]$ corresponds to a compromise limit behavior, inducing a total cost of $c_i^+(o^c - \varepsilon_i^- - o_i)$ or $c_i^-(o_i - o^c - \varepsilon_i^+)$, respectively. In fact, too many preference adjustments go against DM's willingness, and incur unnecessary extra costs, thus, those original preferences smaller than $o^c - \theta_i^-$ or larger than $o^c + \theta_i^+$ won't be discussed, however, DMs can refresh their preferences to rejoin the GDM process. In this regard, the objective function of Model (10) is revised into

$$\begin{aligned} \min \phi = & \sum_{i:\ o_i \in [o^c - \theta_i^-, o^c - \varepsilon_i^-)} c_i^+(o^c - \varepsilon_i^- - o_i) \\ & + \sum_{i:\ o_i \in (o^c + \varepsilon_i^+, o^c + \theta_i^+]} c_i^-(o_i - o^c - \varepsilon_i^+) \end{aligned} \tag{13}$$

III. General CRP framework regarding GDM problems

Methods to obtain the CL are reviewed in III-A, while a general framework of CRP is recalled in III-B.

A. Consensus measure

Consensus level (CL), the current level of unanimous within a group, is often calculated by distance functions [4] and generally measured in two ways [11]:

- The distance between DM's preference and the collective preference, shown as Eq. (14);

$$CL(o_1, \ldots, o_m) = 1 - f_2(f_1(d(o_i, o^c))) \geq \beta \tag{14}$$

- The distance between two arbitrarily chosen individual preferences, shown as Eq. (15).

$$CL(o_1, \ldots, o_m) = 1 - g_2(g_1(d(o_i, o_j))) \geq \beta, i \neq j \tag{15}$$

where β is a preset CL threshold, $d(\cdot)$ represents distance measure, $f_1 : \mathcal{R}^+ \to \mathcal{R}^+$, $f_2 : \mathcal{R}^+ \to [0, 1]$, $g_1 : \mathcal{R}^+ \to \mathcal{R}^+$, $g_2 : \mathcal{R}^+ \to [0, 1]$ are mapping functions, and the remaining notation is defined in Section II-B.

Since DM's influence is directly reflected by the weight w_i with $w_i \geq 0$ and $\sum w_i = 1$, Ref. [11] incorporated individual weights into the calculation of the CL, that is,

$$CL(\bar{o_1}, \ldots, \bar{o_m}) = \sum_{i=1}^{m} w_i |\bar{o_i} - o^c| \leq \gamma \tag{16}$$

$$CL(\bar{o_1}, \ldots, \bar{o_m}) = \sum_{i=1}^{m-1} \sum_{j=i+1}^{m} \frac{w_i + w_j}{m - 1} |\bar{o_i} - \bar{o_j}| \leq \gamma \tag{17}$$

where $\gamma = 1 - \beta$ and $\gamma \in [0, 1]$. In fact, Eqs. (16) and (17) both emphasize that the more important DM contributes more to the CL, though only Eq. (16) is used for subsequent analysis.

B. A general CRP framework

Several basic steps constitute a CRP framework for solving consensus GDM problems, that is, preference expression, preference aggregation, consensus measure, preference adjustment and selection. In addition, Herrera-Viedma et al. [6] provided a general consensus framework, shown as Fig. 3, to address GDM problems with heterogeneous preference structures.

Regarding all the above mentioned steps, feedback mechanism can be initiated based on diverse principles, such as the minimum deviation or cost [33], the maximum number of experts adjusted under a limited budget [9], the minimum number of adjusted DMs [10] or some optimization-based rules [8]. In this paper, a CL threshold is used to initiate DMs' preference modification process.

Table I: Notation and their meanings

Notation	Description
d_i	The i-th DM and $i \in M = \{1, 2, \cdots, m\}$
o_i	d_i's original preference with $o_i \sim \mathcal{L}(a_i, b_i)$ and $0 \le a_i \le b_i \le 1$
$\bar{o}_i$	d_i's adjusted preference with $\bar{o}_i \sim \mathcal{L}(\bar{a}_i, \bar{b}_i)$ and $0 \le \bar{a}_i \le \bar{b}_i \le 1$
o^c	Consensus with $o^c \sim \mathcal{L}(a^c, b^c)$ and $0 \le a^c \le b^c \le 1$
α	Belief degree with $\alpha \in [0, 1]$
Φ, $\Phi^{-1}(\alpha)$	Uncertainty distribution and inverse uncertainty distribution
c_i^+, c_i^-	Unit cost of d_i's upward/downward preference adjustment
ε_i, θ_i	d_i's tolerance/compromise limit threshold over consensus
w_i, $\bar{w}_i$	d_i's original/updated weight with $w_i, \bar{w}_i \ge 0$ and $\sum w_i = 1$, $\sum \bar{w}_i = 1$
η	Adjusted parameter of weights attached to CL (e.g., $\eta = 1.5$)
γ, γ_0, $\Delta\gamma$	Thresholds of the overall preference deviation (Dev) with γ=γ_0-$\Delta\gamma$ and $\Delta\gamma \ge 0$
β	Predefined threshold of CL with $\beta = 1 - \gamma$
λ	Trade-off coefficient between CL and the total cost with $\lambda \in [0, 1]$
δ	d_i's preference adjustment willingness with $\delta \in [0, 1]$
z_{i1}, z_{i2}, z_{i3}, y_{i1}, y_{i2}, y_{i3}, x_i	0-1 binary variable

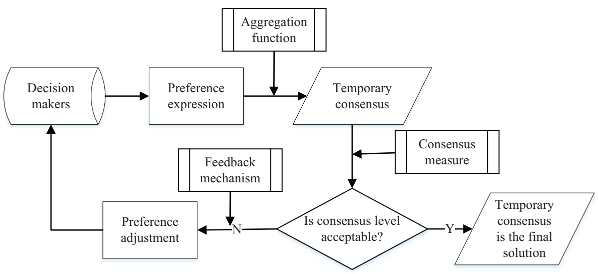


Fig. 3: The general CRP framework in Ref. [6]

IV. Uncertain consensus models considering asymmetric costs and consensus measure

Traditional preference structures (e.g., crisp numbers [9], [12] or linguistic information [13]) are used in Model (8) and Model (9), however, providing exact values is rather difficult. By contrast, DMs are more likely to express their decisions through LUDs, which give sufficient tolerance to their inherent uncertainty and hesitation [17]. Therefore, the LUD is continuously adopted to simulate DMs' preferences in this paper. For brevity, basic notation is listed in Table I, and four assumptions are given as:

- Individual preferences are independent of each other, implying that consensus can be derived through aggregation functions, where variables (e.g., preference and weight) are bounded within [0,1].
- DMs are more sensitive to losses than gains (see prospect theory [34]), so let $|c_i^-| > |c_i^+|$ reflect DM's adjusting willingness, namely, the changing trend of the downward-adjusting subinterval is steeper than that of the upward one (see Fig. 2(b)).
- DM's influence changes with the procedure of GDM [25]. In specific, DMs initially have equal weights, then their influence diversifies due to their own contribution to the CL during the CRP.
- All DMs are committed to reaching a consensus by completely following feedback suggestions, in other words, non-cooperation behavior [3] is neglected.

Since CL is bounded within [0, 1], the GDM problem discussed can be further simplified: boundary values of DM's preference in Fig. 2(b) are preset as $o^c - \theta_i^- = 0$ and $o^c + \theta_i^+ = 1$. In addition, d_i's tolerance behavior is no longer distinguished as ε_i^+ or ε_i^-, that is, only one parameter (i.e., ε_i) is used to denote d_i's tolerance to the consensus o^c. As a result, Eq. (13) evolves into

$$\min \phi = \sum_{o_i \in [0, o^c - \varepsilon_i)} c_i^+ (o^c - \varepsilon_i - o_i) + \sum_{o_i \in (o^c + \varepsilon_i, 1]} c_i^- (o_i - o^c - \varepsilon_i) \tag{18}$$

A. Uncertain MCCM with asymmetric costs

Theorem *4:* [29] Let $o_{i, i \in M}$ be an independent uncertain variable obeying a linear uncertainty distribution (LUD) as $o_i \sim \mathcal{L}(a_i, b_i)$ with $a_i \le b_i$. Then $\sum w_i o_i$ obeys a LUD as $\sum w_i o_i \sim \mathcal{L}(\sum w_i a_i, \sum w_i b_i)$.

Cost is always a crucial metric to measure the GDM quality and efficiency, so we take the total cost minimization as our priority. As stated before, Eq. (16) is used to obtain the CL. Hence, a new uncertain MCCM is built by comprehensively considering asymmetric costs, aggregation function and consensus measure.

$$\begin{aligned} & \min \phi = \textstyle\sum_{i=1}^{m} \{c_i^+, c_i^-\} * d(\bar{o}_i, o_i) \\ & \text{s.t.} \begin{cases} d(\bar{o}_i, o^c) \le \varepsilon_i & (19-1) \\ o^c = \sum_{i=1}^{m} w_i \bar{o}_i & (19-2) \\ \sum_{i=1}^{m} w_i * d(\bar{o}_i, o^c) \le \gamma & (19-3) \\ o_i \sim \mathcal{L}(a_i, b_i), \bar{o}_i \sim \mathcal{L}(\bar{a}_i, \bar{b}_i), o^c \sim \mathcal{L}(a^c, b^c) & (19-4) \\ 0 \le \bar{a}_i \le \bar{b}_i \le 1, 0 \le a^c \le b^c \le 1, i \in M & (19-5) \end{cases} \end{aligned} \tag{19}$$

Solving Model (19) yields the minimum cost ϕ^*, the optimal consensus o^{c*}, and d_i's optimal adjusted preference $\bar{o}_i{}^*$. Note that, $d(\bar{o}_i, o_i)$ is the distance measure between d_i's adjusted preference $\bar{o}_i$ and original preference o_i; the expression $\{c_i^+, c_i^-\}$ means only one coefficient is taken due to d_i's adjustment direction, corresponding to Eq. (18); (19-1) reflects d_i's tolerance behavior; (19-2) uses the arithmetic weighted averaging (AWA) operator to fuse DMs' preferences; (19-3) is the consensus measure, and (19-4) indicates that all involved preferences obey the LUD under (19-5).

Theorem *5:* [29] Distance between any two independent variables with LUDs, denoted as $\xi \sim \mathcal{L}(a, b)$ and $\varsigma \sim \mathcal{L}(c, d)$ with $a \le b, c \le d$, can be transformed into a piecewise function as

$$d(\xi,\varsigma)=\begin{cases}\dfrac{a+b-c-d}{2}, & \text{if } a>d\\ \dfrac{c+d-a-b}{2}, & \text{if } b<c\\ \dfrac{(d-a)^2}{b+d-a-c+\epsilon}+\dfrac{a+b-c-d}{2}, & \text{otherwise}\end{cases}\tag{20}$$

where ϵ is the non-Archimedean infinitesimal. Hereafter, we take $\epsilon=10^{-6}$ to ensure that a rare case of $a=b$ and $c=d$ still holds in Eq. (20), and then the two uncertain variables essentially degenerate to two real numbers.

Basically, piecewise functions seldomly exist in final models due to calculation complexity, meanwhile, distinguished from the method in [29], which remains the form of absolute values, we use the big M method to transform Eq. (20) into a hybrid 0-1 programming model (i.e., Model (21)). Here, let U be a sufficiently large positive number and we take $U=10^6$ in the subsequent analysis.

$$\begin{aligned}&d(\xi,\varsigma)=z_3*\tfrac{(d-a)^2}{b+d-a-c+\epsilon}+(0.5-z_2)*(a+b-c-d)\\&\text{s.t.}\begin{cases}-U(1-z_2)(1-z_3)\le d-a<U(1-z_1)\\-U(1-z_1)(1-z_3)\le b-c<U(1-z_2)\\-U(1-z_1)<\frac{a+b-c-d}{2}<U(1-z_2)\\-U(1-z_2)<\frac{c+d-a-b}{2}<U(1-z_1)\\\frac{(d-a)^2}{b+d-a-c+\epsilon}+\frac{a+b-c-d}{2}>-U*z_2\\z_1+z_2+z_3=1\\z_1,z_2,z_3\in\{0,1\}\end{cases}\end{aligned}\tag{21}$$

where z_1,z_2,z_3 are binary variables with one and only one value of 1. For example, if $z_1=1$, then $z_2=0$, $z_3=0$, we get the first case of $a>b$; and if $z_3=1$, then $z_1=0$, $z_2=0$, we have $d\ge a$ and $b\ge c$, which corresponds to the third case. Detailed transformation from Eq. (20) to Model (21), which focuses on the relative positions of the four parameters (i.e., a,b,c and d), is omitted here due to limited space. In addition, another 0-1 variable x_i is introduced to handle the multi-coefficient problem [35] of Model (19) (i.e., $\{c_i^+,c_i^-\}$), we then have Model (22).

$$\min\ \phi=\sum_{i=1}^m[x_ic_i^++(1-x_i)c_i^-]*[z_{i3}*\tfrac{(b_i-\bar{a}_i)^2}{\bar{b}_i+b_i-\bar{a}_i-a_i+\epsilon}+(0.5-z_{i2})*(\bar{a}_i+\bar{b}_i-a_i-b_i)]$$

$$\text{s.t.}\begin{cases}-U(1-z_{i2})(1-z_{i3})\le b_i-\bar{a}_i<U(1-z_{i1}) & (22-1)\\-U(1-z_{i1})(1-z_{i3})\le \bar{b}_i-a_i<U(1-z_{i2}) & (22-2)\\-U(1-z_{i1})<\frac{\bar{a}_i+\bar{b}_i-a_i-b_i}{2}<U(1-z_{i2}) & (22-3)\\-U(1-z_{i2})<\frac{a_i+b_i-\bar{a}_i-\bar{b}_i}{2}<U(1-z_{i1}) & (22-4)\\\frac{(b_i-\bar{a}_i)^2}{\bar{b}_i+b_i-\bar{a}_i-a_i+\epsilon}+\frac{\bar{a}_i+\bar{b}_i-a_i-b_i}{2}>-U*z_{i2} & (22-5)\\y_{i3}*\frac{(\sum w_i\bar{b}_i-\bar{a}_i)^2}{\bar{b}_i+\sum w_i\bar{b}_i-\bar{a}_i-\sum w_i\bar{a}_i+\epsilon}+(0.5-y_{i2})(\bar{a}_i+\bar{b}_i-\sum w_i\bar{a}_i-\sum w_i\bar{b}_i)\le\varepsilon_i & (22-6)\\-U(1-y_{i2})(1-y_{i3})\le\sum w_i\bar{b}_i-\bar{a}_i<U(1-y_{i1}) & (22-7)\\-U(1-y_{i1})(1-y_{i3})\le\bar{b}_i-\sum w_i\bar{a}_i<U(1-y_{i2}) & (22-8)\\-U(1-y_{i1})<\frac{\bar{a}_i+\bar{b}_i-\sum w_i\bar{a}_i-\sum w_i\bar{b}_i}{2}<U(1-y_{i2}) & (22-9)\\-U(1-y_{i2})<\frac{\sum w_i\bar{a}_i+\sum w_i\bar{b}_i-\bar{a}_i-\bar{b}_i}{2}<U(1-y_{i1}) & (22-10)\\\frac{(\sum w_i\bar{b}_i-\bar{a}_i)^2}{\bar{b}_i+\sum w_i\bar{b}_i-\bar{a}_i-\sum w_i\bar{a}_i+\epsilon}+\frac{\bar{a}_i+\bar{b}_i-\sum w_i\bar{a}_i-\sum w_i\bar{b}_i}{2}>-U*y_{i2} & (22-11)\\z_{i1}+z_{i2}+z_{i3}=1,\ y_{i1}+y_{i2}+y_{i3}=1 & (22-12)\\a^c=\sum w_i\bar{a}_i,\ b^c=\sum w_i\bar{b}_i & (22-13)\\\sum_{i=1}^m w_i*[y_{i3}*\frac{(\sum w_i\bar{b}_i-\bar{a}_i)^2}{\bar{b}_i+\sum w_i\bar{b}_i-\bar{a}_i-\sum w_i\bar{a}_i+\epsilon}+(0.5-y_{i2})(\bar{a}_i+\bar{b}_i-\sum w_i\bar{a}_i-\sum w_i\bar{b}_i)]\le\gamma & (22-14)\\o_i\sim\mathcal{L}(a_i,b_i),\bar{o}_i\sim\mathcal{L}(\bar{a}_i,\bar{b}_i),o^c\sim\mathcal{L}(a^c,b^c) & (22-15)\\z_{i1},z_{i2},z_{i3},y_{i1},y_{i2},y_{i3},x_i\in\{0,1\},0\le\bar{a}_i\le\bar{b}_i\le1,0\le a^c\le b^c\le1,i\in M & (22-16)\end{cases}\tag{22}$$

Solving Model (22) yields d_i's optimal adjusted preference $\bar{o}_i{}^*$ (i.e., $\bar{a}_i{}^*$, $\bar{b}_i{}^*$), the optimal consensus o^{c*} (i.e., a^{c*}, b^{c*}), and the minimum cost ϕ^*. Here, DM's preference is fitted by the LUD as $o_i\sim\mathcal{L}(a_i,b_i)$, with known parameters a_i,b_i satisfying $0\le a_i\le b_i\le 1$. The objective function is constituted by two parts: the former $x_ic_i^++(1-x_i)c_i^-$ determines the exact value from the known asymmetric costs c_i^+,c_i^- due to d_i's adjustment direction; while the latter $z_{i3}*\frac{(b_i-\bar{a}_i)^2}{\bar{b}_i+b_i-\bar{a}_i-a_i+\epsilon}+(0.5-z_{i2})*(\bar{a}_i+\bar{b}_i-a_i-b_i)$ is transformed from the distance measure $d(\bar{o}_i,o_i)$ due to Model (21), satisfying (22-1)-(22-5) and (22-12). In specific, if a DM adjusts the preference upwards, the asymmetric cost takes c_i^+ with $x_i=1$; otherwise, the unit cost is c_i^- with $x_i=0$, where $|c_i^-|>|c_i^+|$ corresponds to the previous second assumption. Similarly, the expression $y_{i3}*\frac{(\sum w_i\bar{b}_i-\bar{a}_i)^2}{\bar{b}_i+\sum w_i\bar{b}_i-\bar{a}_i-\sum w_i\bar{a}_i+\epsilon}+(0.5-y_{i2})(\bar{a}_i+\bar{b}_i-\sum w_i\bar{a}_i-\sum w_i\bar{b}_i)$ in (22-6) and (22-14) is transformed from $d(\bar{o}_i,o^c)$ under (22-7)-(22-12). In addition, (22-6) reflects individual tolerance behavior, where ε_i is predetermined due to individual background, knowledge and experience by each DM, while the unit costs c_i^+ and c_i^- accordingly preset by the moderator; (22-13) is derived from Theorem 4, and (22-14) is the consensus measure (i.e., Eq. (16)). Generally, the higher the CL the better, so let Dev denote the overall preference deviation, rather than the CL defined by [11]. Since $Dev=1-CL$, the smaller Dev the better.

Once all DMs provide initial preferences with LUDs, an initial consensus is reached immediately based on Theorem (4), thereby obtaining the initial Dev (denoted by γ_0) based on Eq. (16) and Eq. (20), which can be viewed as the Dev benchmark of Model (22). Generally, the Dev threshold (i.e., γ) in Model (22) should be smaller than γ_0 to improve the CL, thus $\gamma=\gamma_0-\Delta\gamma$ with $\gamma_0\ge 0$.

B. Uncertain consensus model with asymmetric costs

Inappropriate CL thresholds easily lead to the failure of reaching a consensus, so some maximum CL models are built to aid DM's preference adjustment [15], [16]. Similarly, solving Model (22) may not yield the MCC due to inappropriate γ thresholds. Hence, building a new model that considers both the CL and the budget is of great necessity [36], where the Dev (or CL) threshold no longer needs to be pre-determined. By introducing a trade-off coefficient λ ($\lambda \in [0,1]$), a more flexible model to handle GDM problems is built (i.e., Model (23)), where variables and constraints are defined same as in Model (22). Furthermore, since the existence theorem of optimal solutions [37] (i.e., a single-objective programing model with non-empty and bounded feasible region must have an optimal solution), Model (23) surely has optimal solutions.

In fact, if $\lambda = 0$, Model (23) reduces to Model (22); and if $\lambda = 1$, Model (23) becomes the maximum CL model [16]. By comparison, Model (23) is more objective since it doesn't need to pre-determine a Dev threshold; meanwhile, it better supports GDM by providing a full relationship between CL and the total cost with regard to the trade-off coefficient λ.

$$
\begin{aligned}
&\min \lambda * \gamma + (1-\lambda) * \phi \\
&\text{s.t.}\ \begin{cases}
\phi = \sum_{i=1}^{m} [x_i c_i^{+} + (1-x_i) c_i^{-}] * [z_{i3} * \frac{(b_i - \bar{a_i})^2}{\bar{b_i} + b_i - \bar{a_i} - a_i + \epsilon} + (0.5 - z_{i2}) * (\bar{a_i} + \bar{b_i} - a_i - b_i)] & (23-1) \\
-U(1-z_{i2})(1-z_{i3}) \leq b_i - \bar{a_i} < U(1-z_{i1}) & (23-2) \\
-U(1-z_{i1})(1-z_{i3}) \leq \bar{b_i} - a_i < U(1-z_{i2}) & (23-3) \\
-U(1-z_{i1}) < \frac{\bar{a_i} + \bar{b_i} - a_i - b_i}{2} < U(1-z_{i2}) & (23-4) \\
-U(1-z_{i2}) < \frac{a_i + b_i - \bar{a_i} - \bar{b_i}}{2} < U(1-z_{i1}) & (23-5) \\
\frac{(b_i - \bar{a_i})^2}{\bar{b_i} + b_i - \bar{a_i} - a_i + \epsilon} + \frac{\bar{a_i} + \bar{b_i} - a_i - b_i}{2} > -U * z_{i2} & (23-6) \\
y_{i3} * \frac{(\sum w_i \bar{b_i} - \bar{a_i})^2}{\bar{b_i} + \sum w_i \bar{b_i} - \bar{a_i} - \sum w_i \bar{a_i} + \epsilon} + (0.5 - y_{i2})(\bar{a_i} + \bar{b_i} - \sum w_i \bar{a_i} - \sum w_i \bar{b_i}) \leq \varepsilon_i & (23-7) \\
-U(1-y_{i2})(1-y_{i3}) \leq \sum w_i \bar{b_i} - \bar{a_i} < U(1-y_{i1}) & (23-8) \\
-U(1-y_{i1})(1-y_{i3}) \leq \bar{b_i} - \sum w_i \bar{a_i} < U(1-y_{i2}) & (23-9) \\
-U * (1-y_{i1}) < \frac{\bar{a_i} + \bar{b_i} - \sum w_i \bar{a_i} - \sum w_i \bar{b_i}}{2} < U(1-y_{i2}) & (23-10) \\
-U(1-y_{i2}) < \frac{\sum w_i \bar{a_i} + \sum w_i \bar{b_i} - \bar{a_i} - \bar{b_i}}{2} < U(1-y_{i1}) & (23-11) \\
\frac{(\sum w_i \bar{b_i} - \bar{a_i})^2}{\bar{b_i} + \sum w_i \bar{b_i} - \bar{a_i} - \sum w_i \bar{a_i} + \epsilon} + \frac{\bar{a_i} + \bar{b_i} - \sum w_i \bar{a_i} - \sum w_i \bar{b_i}}{2} > -U * y_{i2} & (23-12) \\
z_{i1} + z_{i2} + z_{i3} = 1,\ y_{i1} + y_{i2} + y_{i3} = 1 & (23-13) \\
a^c = \sum w_i \bar{a_i},\ b^c = \sum w_i \bar{b_i} & (23-14) \\
\sum_{i=1}^{m} w_i * [y_{i3} * \frac{(\sum w_i \bar{b_i} - \bar{a_i})^2}{\bar{b_i} + \sum w_i \bar{b_i} - \bar{a_i} - \sum w_i \bar{a_i} + \epsilon} + (0.5 - y_{i2})(\bar{a_i} + \bar{b_i} - \sum w_i \bar{a_i} - \sum w_i \bar{b_i})] \leq \gamma & (23-15) \\
o_i \sim \mathcal{L}(a_i, b_i), \bar{o_i} \sim \mathcal{L}(\bar{a_i}, \bar{b_i}), o^c \sim \mathcal{L}(a^c, b^c) & (23-16) \\
z_{i1}, z_{i2}, z_{i3}, y_{i1}, y_{i2}, y_{i3}, x_i \in \{0,1\}, 0 \leq \bar{a_i} \leq \bar{b_i} \leq 1, 0 \leq a^c \leq b^c \leq 1, \gamma \in [0,1], i \in M & (23-17)
\end{cases}
\end{aligned}
\tag{23}
$$

C. Feedback mechanism based on democratic consensus

Feedback mechanisms insist the less modified DMs the better, avoiding unnecessary adjustments and preserving DMs' original preferences to the fullest extent [36]. Assume only one DM needs to be adjusted at each iteration, and democratic consensus is implemented during the CRP, which ensures DMs' effective participation and satisfaction by realizing a soft consensus [25]. Specifically, DMs are first assigned with equal weights to protect the interest of minorities, then their influence is updated with their own contribution to the CL.

Definition *3:* DM d_k's contribution to the CL is measured by the deviation between the overall CL and the CL reached by the remaining $m-1$ DMs (i.e., $CL_{\bar{k}}$), thus,

$$CL_k = CL - CL_{\bar{k}}, k \in M \tag{24}$$

Modifying the preference with a maximum deviation from the consensus and moderately decreasing the weight can significantly improve the CL [38]. Next, define the identification rule (IR) and direction rule (DR) as

- IR: DM with a minimum contribution to CL is identified as the one to be adjusted, denoted as d_k. If there exist more than two DMs with the same contribution value, then d_k is randomly chosen.
- DR: Preference adjustment and weight reallocation are both incorporated in the modification process.
  - DM d_k's updated preference is expressed as Eq. (25), where $\delta \in [0,1]$ is the parameter reflecting d_k's self-confidence, and the larger δ, the less he/she is willing to make revisions.

$$\bar{o_k} = \delta * o_k + (1-\delta) * o^c \tag{25}$$

  - Weight is updated by Eq. (26), where η is the variable controlling the impact of d_i's consensus contribution CL_i^t on the weight w_i^{t+1}. Then normalize all weights by Eq. (27).

$$w_i^{t+1} = w_i^t * (1 + CL_i^t)^{\eta} \tag{26}$$

$$\bar{w_i} = \frac{w_i^{t+1}}{\sum_{i=1}^{m} w_i^{t+1}} \tag{27}$$

Explicitly, the larger the parameter η, the stronger modification of the DM d_i [38]. Fig. 4 provides a new CRP framework that combines democratic consensus and the optimization-based uncertain MCCMs. Essentially, the procedure prior to Model (22) is an intra-group self-adjustment that fully respect individual values by adjusting only one DM's preference but updating all weights based on each DM's contribution to the CL. Note that if the required CL threshold is still not reached after Model (22) takes effect, all the adjusted preferences optimized by Model (22) will be used to start the next iteration; meanwhile, whenever any DM has some changes in either the preference or the weight, a new CL is calculated, so as to

Table II: Original data of Section V

DM d_i	Initial weight w_i	Original preference o_i	Upward cost c_i^+	Downward cost c_i^-	Tolerance ε_i
d_1	0.2	$\mathcal{L}(0.24, 0.86)$	6.0	7.4	0.10
d_2	0.2	$\mathcal{L}(0.25, 0.75)$	7.1	8.1	0.12
d_3	0.2	$\mathcal{L}(0.29, 0.90)$	2.8	9.3	0.20
d_4	0.2	$\mathcal{L}(0.05, 0.46)$	1.5	8.8	0.03
d_5	0.2	$\mathcal{L}(0.55, 0.80)$	6.2	9.6	0.08

minimize the overall resource consumption.

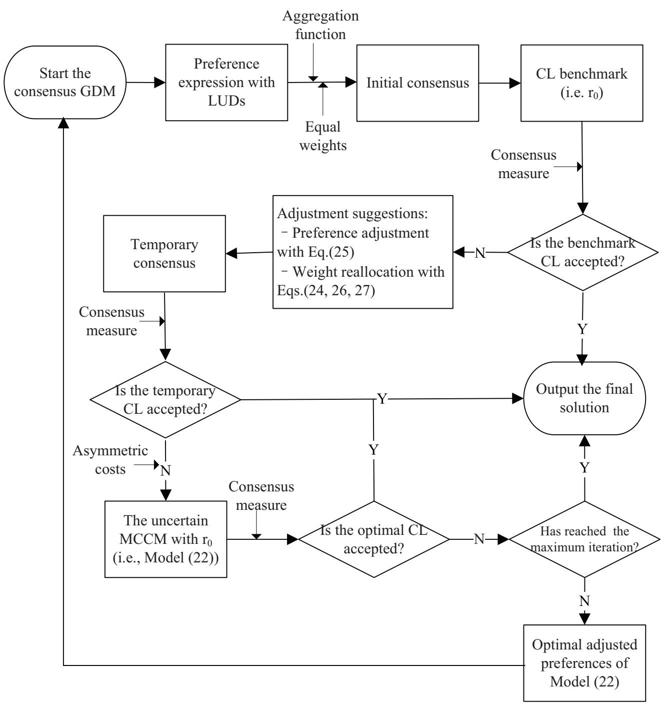


Fig. 4: The new CRP framework of this paper

V. ILLUSTRATIVE EXAMPLE

Suppose five cities (DMs) located in the Yangtze River Delta participate in the trans-boundary water pollution negotiation under the governance of the Ministry of Water Resources (MWR) of China (i.e., the moderator) [19], denoted as d_i, $i \in M = \{1, 2, 3, 4, 5\}$. Considering the differences and flexibility of the environmental capacity, population density, annual financial goals and historical pollution data, original data about the five cities are shown as Table II, including d_i's initial preference as their desired pollution index o_i obeying the LUD (i.e., $o_i \sim \mathcal{L}(a_i, b_i)$), the asymmetric costs c_i^+, c_i^- satisfying $|c_i^-| > |c_i^+|$, and each tolerance ε_i to the final index (i.e., o^c). In pursuit of democracy, the weight of each city is initially assigned with 0.2. Normally, an ideal CL threshold is pre-determined due to specific GDM problem, hereafter let $CL^* = 0.85$ and the maximum iteration be 5 [16]. The new CRP is implemented step by step as follows.

Step 1. Compute the initial CL to determine whether there exists one DM (i.e., d_k) to be adjusted. If yes, go to Step 2; otherwise, terminate the CRP.

As the weight vector is $(0.2, 0.2, 0.2, 0.2, 0.2)^{\mathrm{T}}$, we initially get the consensus $o^c \sim \mathcal{L}(0.28, 0.75)$ with Theorem 4. Next, Eq. (20) is adopted to obtain an initial CL by computing variables in Table III, including the uncertain distance (i.e., $d(o_i, o^c)$), the overall Dev/CL except d_i (i.e., $Dev_{\bar{i}}$ and $CL_{\bar{i}}$), and d_i's contribution to the reached consensus (i.e., CL_i). Hence, we get the benchmark Dev $\gamma_0 = 0.263$, and the benchmark $CL = 0.737 < CL^*$, showing that d_k needs to be identified.

Table III: Results calculated from original data

d_i	$d(o_i, o^c)$	$Dev_{\bar{i}}$	$CL_{\bar{i}}$	CL_i
d_1	0.276	0.260	0.740	-0.003
d_2	0.244	0.268	0.732	0.005
d_3	0.278	0.259	0.741	-0.004
d_4	0.299	0.254	0.746	-0.009
d_5	0.217	0.274	0.726	0.011

Specifically, taking d_1 as an example to obtain Table III, since $o_1 \sim \mathcal{L}(0.24, 0.86)$ and $o^c \sim \mathcal{L}(0.28, 0.75)$, we get $a = 0.24 < d = 0.75$ and $b = 0.86 > c = 0.28$, then the third case of Eq. (20) is employed to obtain their distance as $\frac{(0.75-0.24)^2}{0.86+0.75-0.24-0.28+10^{-6}} + \frac{0.24+0.86-0.28-0.75}{2} = 0.276$. To be noted, when computing $Dev_{\bar{i}}$, the remaining four DMs' weights are changed from 0.2 into 0.25 for $\tilde{w}_j = \frac{w_j}{1-w_i}$, $j \neq i$, $\forall j \in M$. Comparing all the values in the last column of Table III, we have $d_k = d_4$ due to his/her minimum contribution $CL_4 = -0.009$.

Step 2. Recalculate and check whether the new temporary consensus meets the threshold CL^* by adjusting d_k's preference and updating all DMs' weights. If yes, terminate the CRP; otherwise, move to Step 3.

Let the parameter of d_k's adjustment willingness be $\delta = 0.5$, and the controlling parameter in Eq. (26) be $\eta = 1.5$ [25]. Therefore, d_4's adjusted preference is $\bar{o_4} \sim \mathcal{L}(0.165, 0.605)$, and the weight vector is updated as $(0.199, 0.201,\ 0.199, 0.197, 0.203)^{\mathrm{T}}$ (see Table IV). Here, the last five columns of Table IV are the weights of the remaining four DMs except d_i, which are used to calculate $Dev_{\bar{i}}$. For example, the last value of 0.248 is derived by $\tilde{w_4} = \frac{0.197}{1-0.203} = 0.248$. Moreover, using d_3 as an instance, the new weight $\bar{w_3} = 0.199$ is obtained by first calculating Eq. (26) (i.e., $w_3^1 = 0.2 * (1 - 0.004)^{1.5} = 0.1989$), then being normalized with Eq. (27). Repeating the calculation in Step 1, we have $CL = 0.75$, which still fails to meet the threshold of 0.85, so proceed to Step 3.

Step 3. Solve Model (22) with γ_0 from Step 1 to produce optimal solutions, then check if the obtained optimal CL meets CL^*. If yes, terminate the CRP; otherwise, return to Step 1 with derived adjusted preferences.

Use the Dev benchmark in Step 1 (i.e., $\gamma_0 = 0.263$), instead of any smaller values, to pre-determine γ in Model (22),

Table IV: Updated weights due to DM's variable influence

d_i	w_i	$\bar{w_i}$	$w_{\bar{1}}$	$w_{\bar{2}}$	$w_{\bar{3}}$	$w_{\bar{4}}$	$w_{\bar{5}}$
d_1	0.2	0.199		0.249	0.248	0.248	0.250
d_2	0.2	0.201	0.251		0.251	0.251	0.253
d_3	0.2	0.199	0.248	0.249		0.248	0.250
d_4	0.2	0.197	0.246	0.247	0.246		0.248
d_5	0.2	0.203	0.254	0.255	0.254	0.253	

so as to verify if Model (22) is effective. Moreover, solve Model (22) by adopting updated information in Step 2, to fully demonstrate individual influence on the CRP.

Table V: Optimal solution of Model (22) with $\gamma = 0.263$

d_i	$\bar{o_i}^*$	$d(\bar{o_i}^*, o^{c*})$	$Dev_{\bar{i}}$	$CL_{\bar{i}}$	CL_i
d_1	$\mathcal{L}(0.5690, 0.5690)$	0.000	0.051	0.949	0.010
d_2	$\mathcal{L}(0.4991, 0.4991)$	0.070	0.033	0.967	-0.007
d_3	$\mathcal{L}(0.5906, 0.5906)$	0.022	0.045	0.955	0.005
d_4	$\mathcal{L}(0.5390, 0.5390)$	0.030	0.043	0.957	0.003
d_5	$\mathcal{L}(0.6490, 0.6490)$	0.080	0.031	0.969	-0.010

Finally, the reached consensus from Model (22) is $o^{c*} \sim \mathcal{L}(0.569, 0.569)$, and the minimum cost is $\phi^* = 2.8982$. Meanwhile, Table V provides d_i's optimal adjusted preference along with relevant indicators. Obviously, both the achieved consensus and DM's optimal adjusted preferences degenerate to real numbers. In other words, given the constructed form of the LUD, all the final preferences have the same upper and lower bounds. Such findings are consistent with existing studies on uncertain MCCMs [28], [29]. That is, once certain conditions are met (e.g., the belief degree is no less than 0.5 [28]), the original LUDs degenerate to crisp numbers. The obtained final Dev is $\gamma^* = 0.04$, and the optimal $CL = 0.96$ exceeds its threshold of 0.85, so CRP is terminated at this point, which implies that Model (22) works in the new CRP framework. Provided that the current CL still fails to meet its threshold, all the optimal adjusted preferences from Model (22) will be used for the next iteration.

VI. Further discussion

Parametric behavior analysis regarding the two new models is conducted in VI-A, while a comparative analysis with other studies is given in VI-B.

A. Parametric behavior analysis

1) Parametric behavior analysis of λ: The influence of λ (i.e., the trade-off coefficient between the budget and the CL) on the consensus is discussed with a step of 0.1 due to $\lambda \in [0, 1]$. Note that initial adjustments of weights or preference aren't involved in this section. Namely, results are obtained by solving Model (23) with data in Table II. Fig. 5 provides the changing trends of the CL and the consensus cost with λ; and Table VI exhibits the λ-related optimal solutions, including the reached consensus and DMs' optimal adjusted preferences.

Fig. 5 shows the final range of the CL is [0.9612, 0.9766] and the range of total cost is [3.0760, 3.1429] with respect to λ. Once the two objectives (i.e., γ and ϕ) are considered separately: if $\lambda = 0$ (i.e., only the resource is required to be

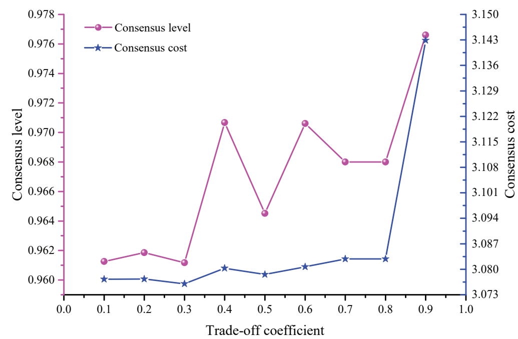


Fig. 5: Parametric behavior analysis of λ in Model (23)

least consumed), the optimal cost is 3.0743 with CL being 0; by contrast, if $\lambda = 1$ (i.e., the maximum CL becomes the priority of the discussed GDM), then CL reaches the maximum value 1 with a cost of 16.596. Since the results of these two extreme scenarios are far away from the rest, they are omitted in Fig. 5. Table VI verifies most optimal preferences end up being LUDs with identical endpoint values, where the optimized preferences still strictly obeying the LUD with two different endpoints are underlined. Moreover, results show that once the trade-off parameter reaches a certain threshold (e.g., 0.7 in Table VI), the GDM involved focuses more on CL rather than the total cost, and all preferences become crisp numbers.

Overall, there are sharp fluctuations when $\lambda \in [0.3, 0.7]$, thus CL is of great necessity for GDM, where detailed reasons remain to be further explored; although such fluctuations are negligible once taking the numerical scale into account. Besides, the consistent volatility trends of the CL and the cost suggest that increasing budget helps improve CL, but their conflict within [0.6, 0.7] might due to the complete transformation of LUDs into crisp numbers with $\lambda = 0.7$ or factors ignored in this paper.

2) Parametric behavior analysis of η: Since DM's weight plays an important role during CRP, this section conducts the parametric behavior analysis of η from 1.5 to 3.5 with a step of 0.5. Fig. 6 provides the changes of weights regarding η, and the optimal solutions of Model (22) are shown in Table VII, where the original data except w_i come from Table II.

Fig. 6 shows that once the controlling variable η of DM's CL contribution to their new weight gets larger, the more obvious the differences in weight distribution, where specific data are shown as the 2nd column of Table VII. In other words, the smaller the value of η, the more even of all DMs' weights, and the less differences among individual influence on the final decision. Results in Table VII show that Model (22) works, since all CLs are significantly improved from the benchmark $CL = 0.737$ from Step 1 in Section V. Here, only d_4's optimal adjusted preference $\bar{o_4}^*$ is listed in Table VII due to limited space and the fact that other four DMs' preferences have all degenerated into crisp numbers. Since only minor changes exist in all the optimal values of ϕ^* and CL, the impact of η on the final results is actually negligible here. Note that the parameter behavior analysis of δ in Eq. (25), which reflects

Table VI: Optimal results of Model (23)

λ	o^{c*}	$\bar{o_1}^*$	$\bar{o_2}^*$	$\bar{o_3}^*$	$\bar{o_4}^*$	$\bar{o_5}^*$
0.0	$\mathcal{L}(0.5599, 0.5699)$	$\mathcal{L}(0.5499, 0.5499)$	$\mathcal{L}(0.4999, 0.4999)$	$\mathcal{L}(0.5948, 0.5948)$	$\mathcal{L}(0.5099, 0.5599)$	$\mathcal{L}(0.6449, 0.6449)$
0.1	$\overline{\mathcal{L}(0.5638, 0.5638)}$	$\mathcal{L}(0.5638, 0.5638)$	$\mathcal{L}(0.4969, 0.4969)$	$\mathcal{L}(0.5807, 0.5807)$	$\overline{\mathcal{L}(0.5338, 0.5338)}$	$\mathcal{L}(0.6438, 0.6438)$
0.2	$\mathcal{L}(0.5642, 0.5642)$	$\mathcal{L}(0.5642, 0.5642)$	$\mathcal{L}(0.4988, 0.4988)$	$\mathcal{L}(0.5795, 0.5795)$	$\mathcal{L}(0.5342, 0.5342)$	$\mathcal{L}(0.6442, 0.6442)$
0.3	$\mathcal{L}(0.5584, 0.5684)$	$\mathcal{L}(0.5584, 0.5584)$	$\mathcal{L}(0.5013, 0.5013)$	$\mathcal{L}(0.5805, 0.5805)$	$\mathcal{L}(0.5084, 0.5584)$	$\mathcal{L}(0.6434, 0.6434)$
0.4	$\overline{\mathcal{L}(0.5572, 0.5705)}$	$\mathcal{L}(0.5572, 0.5572)$	$\mathcal{L}(0.5138, 0.5138)$	$\mathcal{L}(0.5705, 0.5705)$	$\overline{\mathcal{L}(0.5004, 0.5673)}$	$\mathcal{L}(0.6438, 0.6438)$
0.5	$\overline{\mathcal{L}(0.5592, 0.5650)}$	$\mathcal{L}(0.5592, 0.5592)$	$\mathcal{L}(0.5063, 0.5063)$	$\mathcal{L}(0.5708, 0.5708)$	$\overline{\mathcal{L}(0.5176, 0.5465)}$	$\mathcal{L}(0.6421, 0.6421)$
0.6	$\overline{\mathcal{L}(0.5583, 0.5713)}$	$\mathcal{L}(0.5583, 0.5583)$	$\mathcal{L}(0.5148, 0.5148)$	$\mathcal{L}(0.5713, 0.5713)$	$\overline{\mathcal{L}(0.5023, 0.5674)}$	$\mathcal{L}(0.6448, 0.6448)$
0.7	$\overline{\mathcal{L}(0.5607, 0.5607)}$	$\mathcal{L}(0.5607, 0.5607)$	$\mathcal{L}(0.5107, 0.5107)$	$\mathcal{L}(0.5607, 0.5607)$	$\overline{\mathcal{L}(0.5307, 0.5307)}$	$\mathcal{L}(0.6407, 0.6407)$
0.8	$\mathcal{L}(0.5607, 0.5607)$	$\mathcal{L}(0.5607, 0.5607)$	$\mathcal{L}(0.5107, 0.5107)$	$\mathcal{L}(0.5607, 0.5607)$	$\mathcal{L}(0.5307, 0.5307)$	$\mathcal{L}(0.6407, 0.6407)$
0.9	$\mathcal{L}(0.5650, 0.5650)$	$\mathcal{L}(0.5650, 0.5650)$	$\mathcal{L}(0.5364, 0.5364)$	$\mathcal{L}(0.5650, 0.5650)$	$\mathcal{L}(0.5350, 0.5350)$	$\mathcal{L}(0.6235, 0.6235)$
1.0	$\mathcal{L}(0.0500, 0.0500)$	$\mathcal{L}(0.0500, 0.0500)$	$\mathcal{L}(0.0500, 0.0500)$	$\mathcal{L}(0.0500, 0.0500)$	$\mathcal{L}(0.0500, 0.0500)$	$\mathcal{L}(0.0500, 0.0500)$

Table VII: Optimal results of Model (22)

η	$\bar{W}$	$\bar{o_4}^*$	o^{c*}	ϕ^*	CL
1.5	$(0.1990, 0.2014, 0.1989, 0.1973, 0.2034)^{\mathrm{T}}$	$\mathcal{L}(0.5342, 0.5342)$	$\mathcal{L}(0.5642, 0.5642)$	3.0774	0.9605
2.0	$(0.1986, 0.2019, 0.1985, 0.1964, 0.2046)^{\mathrm{T}}$	$\mathcal{L}(0.5349, 0.5350)$	$\mathcal{L}(0.5650, 0.5650)$	3.0774	0.9603
2.5	$(0.1983, 0.2023, 0.1981, 0.1955, 0.2058)^{\mathrm{T}}$	$\mathcal{L}(0.5347, 0.5347)$	$\mathcal{L}(0.5647, 0.5647)$	3.0773	0.9605
3.0	$(0.1980, 0.2028, 0.1977, 0.1946, 0.2069)^{\mathrm{T}}$	$\mathcal{L}(0.5102, 0.5604)$	$\mathcal{L}(0.5604, 0.5702)$	3.0743	0.9555
3.5	$(0.1976, 0.2033, 0.1973, 0.1938, 0.2081)^{\mathrm{T}}$	$\mathcal{L}(0.5102, 0.5605)$	$\mathcal{L}(0.5605, 0.5702)$	3.0743	0.9554

DM's adjustment willingness [33] is no longer discussed.

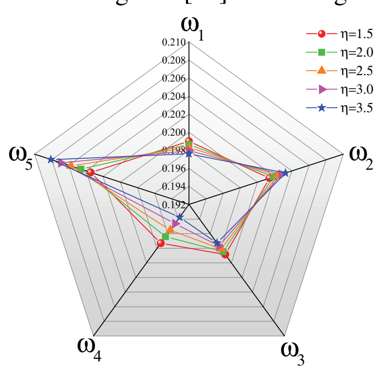


Fig. 6: Changes of weights with regard to η

B. Comparative analysis

1) Comparison between models with/without feedback mechanism: Using data from Table II and directly setting $\gamma = 0.15$ for Model (22), namely, feedback mechanism is no longer considered in this section. Results show that all adjusted preferences degenerate into crisp numbers (see Table VIII), the minimum cost is 3.077, and the consensus reached is $o^{c*} \sim \mathcal{L}(0.564, 0.564)$.

The final $CL = 0.96$ achieves the goal of 0.85, meaning the uncertain MCCM still works without feedback mechanism. However, the cost required to achieve the same CL (i.e., $CL = 0.96$) is less in Section V ($\phi^* = 2.8982$) than here ($\phi^* = 3.077$), implying that the feedback mechanism promotes a higher cost-effective consensus.

Table VIII: Optimal results of Model (22) with $\gamma = 0.15$

d_i	$\bar{o_i}^*$	$d(\bar{o_i}^*, o^{c*})$	$Dev_{\bar{i}}$	$CL_{\bar{i}}$	CL_i
d_1	$\mathcal{L}(0.564, 0.564)$	0.000	0.049	0.951	0.010
d_2	$\mathcal{L}(0.496, 0.496)$	0.068	0.032	0.968	-0.007
d_3	$\mathcal{L}(0.582, 0.582)$	0.018	0.045	0.955	0.005
d_4	$\mathcal{L}(0.534, 0.534)$	0.030	0.042	0.958	0.002
d_5	$\mathcal{L}(0.644, 0.644)$	0.080	0.029	0.971	-0.010

2) Comparison with existing approaches: Table IX provides a comparison of existing research with our new method from several perspectives, such as individual preference structure, unit cost setting, aggregation function, feedback mechanism and solving method. In fact, Gong et al. [28] also looked into the influence of non-cooperation behaviors on the uncertain MCCMs; while [29] incorporated utility function into uncertain MCCMs, and presented the relationships between their new models with traditional MCCMs; Similarly, Cheng et al. [32] also took the satisfaction functions into account, and explored the effect of individual behaviors (i.e., tolerance and compromise limit) on the final decision; additionally, [20], [21] adopted stochastic or robust optimization theory to deepen Cheng et al.'s research with asymmetric costs. Meanwhile, [7], [8] use optimization-based models with CRP to locate specific unit costs from pre-determined intervals. Concisely, this paper extends the MCCMs with LUDs into a new CRP framework by integrating asymmetric costs, aggregation function and consensus measure, which further generalizes the classic GDM theory by considering uncertainty theory and individual behaviors.

VII. Conclusion

Uncertainty theory well depicts DM's inherent subjectivity, imprecision and vagueness, so this paper adopts LUDs to extend the uncertain MCCMs into a new CRP framework. In specific, two new consensus models are built by considering asymmetric costs, aggregation function and consensus measure, where DM's preference is fit by LUD and the setting of asymmetric costs is further rationalized based on prospect theory. Moreover, a new CRP is designed by respecting DM's values with democratic consensus and minimizing resources with new uncertain MCCMs. To avoid the calculation complexity from piecewise functions in the uncertain distance measure, binary variables are introduced to transform the multi-coefficient goal programming models in view of the big M method. Furthermore, we found that (i) the new consensus

Table IX: Comparison with existing consensus studies

Refs	Preferences	Unit cost	Aggregation	Feedback mechanism	Solving method
[28]	LUD	Single fixed value	—	—	OM
[29]	LUD	Single fixed value	AWA	—	OM
[19]	Crisp number	Asymmetric costs	—	—	OM
[32]	Crisp number	Asymmetric costs	AWA	—	OM
[7]	Crisp number	Interval value	OWA	Adjustment of preference & unit costs	OM
[8]	Crisp number	Interval value	AWA	Adaptive differential evolution algorithm	Stackelberg game & OM
[20], [21]	Crisp number	Asymmetric costs	—	—	Stochastic/Robust OM
This paper	LUD	Asymmetric costs	AWA	Preference adjustment & weight reallocation	OM

Note: "—" means not involved; and "OM" is short for optimization-based modeling.

models exclude moderator's influence by setting the Dev threshold of Model (22) with a benchmark from initially provided information, or by providing a full relationship between the CL and the cost via Model (23); (ii) CRP helps promote a higher cost-effective consensus (see Section VI-B); and (iii) once certain conditions are met, DMs' preferences fit by LUDs degenerate into crisp numbers, which is consistent with previous findings [28], [29]. Note that besides transboundary pollution management, our method is feasible to handle other GDM problems featured by non-randomness and non-fuzziness, such as urban demolition negotiation, trust evaluation in social networks or emergency management for natural disasters [39].

It's widely accepted that heterogeneous preferences [6], [23] attract more concerns in real-life situations, so transformation from the LUD to the traditional forms needs to be explored. Moreover, interactions among DMs are truly non-negligible, thereby building uncertain MCCMs combined with social network analysis [36] will be our next focus. Finally, how to integrate well-known decision technology (e.g., survey, datamining) [39], [40] to practically interpret our findings is also worth investigating.

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