# Engineering Applications of Artificial Intelligence


## Non-uniform allocation of information granularity to improve consistency and consensus in multi-criteria group decision-making: Application to building refurbishment

--Manuscript Draft--

| | |
|---|---|
| **Manuscript Number:** | EAAI-23-7749R1 |
| **Article Type:** | Research paper |
| **Keywords:** | consensus; consistency; fuzzy preference relations; information granularity; multi-criteria group decision-making |
| **Corresponding Author:** | Francisco Javier Cabrerizo<br>University of Granada<br>Granada, Granada Spain |
| **First Author:** | Juan Carlos González-Quesada |
| **Order of Authors:** | Juan Carlos González-Quesada |
| | Anastasiia Velykorusova, Dr |
| | Audrius Banaitis, Dr |
| | Arturas Kaklauskas, Dr |
| | Francisco Javier Cabrerizo |
| **Abstract:** | If a decision depends on the views of a group of people, it is crucial that these views are consistent, in particular if they are represented in terms of fuzzy preference relations. Moreover, before making the decision, the agreement achieved must be as high as possible. In most cases, to achieve it, the initial viewpoint of each group member must be modified during the decision-making process, which causes an information loss. To handle this issue, some conditions have been added to the modification process. Concretely, some approaches have turned to an allocation of information granularity to control it. However, these approaches either allocate the information granularity in a uniform way or not consider the issues of consistency and consensus in a scenario of multi-criteria group decision-making. To enhance these approaches, we elaborate a novel granular-based approach composed of two processes. Firstly, an automatic process based upon a non-uniform allocation of information granularity is developed to maximize both the consensus and the consistency in multi-criteria group decision-making where fuzzy preference relations are used to model the group members' views. Based upon it, secondly, an interactive process requiring the implication of the group members is introduced, which also intends to maximize both the consensus and the consistency. Some experimental studies are completed to show the essence of this new approach and to demonstrate its performance and flexibility. A case study on building refurbishment is also conducted to validate its effectiveness and feasibility in practical real-world problem-solving. |
| **Suggested Reviewers:** | Francisco Chiclana, Dr<br>Professor, De Montfort University<br>chiclana@dmu.ac.uk<br>Expert in group decision making and consensus. |
| | Xiangrui Chao<br>University of Electronic Science and Technology of China<br>chaoxiangrui@163.com<br>Expert in multi-criteria decision making, consensus and consistency. |

# Reply to Reviewers (EAAI-23-7749)

We would like to express thanks to the four reviewers for their valuable and constructive comments concerning our submission entitled "Non-uniform allocation of information granularity to improve consistency and consensus in multi-criteria group decision-making: Application to building refurbishment".

The manuscript has been revised to fully address the issues raised in the reviews. We have spared no efforts to revise and improve the manuscript according to the constructive comments provided by the four reviewers. In what follows, we present in detail on how the manuscript has been modified. To enhance readability of the presentation, the reviewers' comments are shown in italics. Excerpts taken from the revised manuscript are shown in smaller fonts (11 pts).

## Reviewer 1

*This paper focuses on introducing a novel granular-based approach improving both consistency and consensus in multi-criteria GDM problems. The paper is interesting and written well.*

Thank you for your positive comments.

*I think authors still need to make some minor revisions for its publication in this journal.*

*1. The linguistic quality should be improved to guarantee a smooth reading.*

Following the reviewer's suggestion, the manuscript has been revised again and some typographical and grammatical errors have been corrected. Furthermore, expressions have been corrected to better clarify the content and explanation of the manuscript.

*2. The motivations and contributions of this study should be clearly described in Introduction Section.*

Thank you for your remark. In the paradigm of Granular Computing, an information granularity level can be uniformly or non-uniformly distributed. It has been shown that a non-uniform distribution of information granularity achieves better results than a uniform distribution of information granularity. However, in multi-criteria group decision-making, the models based on a non-uniform distribution of information granularity only improve the consensus or the consistency, but not both. In addition, they are based on an automatic process. On the other hand, in the practical group decision-making, the evaluations are modified and reassessed with the essential involvement of the participants. Therefore, to enhance the existing multi-criteria group decision-making models based on the paradigm of Granular Computing, in this study, we develop the first group decision-making model based on a non-uniform distribution of information granularity that improves both consensus and consistency in multi-criteria environments allowing the involvement of the participants in the process of modification and reassessment of the evaluations. This has been clarified in the Introduction Section as follows:

"Their common characteristic is the adoption of a uniform allocation of information granularity. Although they allow to increase the consistency, the consensus, or both, while they restrict the range where the evaluations can be changed, thanks to the information granules, some studies have demonstrated that the performance and flexibility of these models could be enhanced through a non-uniform allocation of a different information granularity to the entries of the preference relations (Cabrerizo et al., 2023; Zhang et al., 2022).

**There are however unresolved issues in the models based on a non-uniform allocation of information granularity that need to be addressed. The models developed by Cabrerizo et al. (2022a) and Cabrerizo et al. (2023) only deal with, and the model proposed by Cabrerizo et al. (2022b) only deals with the improvement of consensus. In addition, they are based on an automatic modification process that does not involve the participation of the group members. However, in real-world GDM, the evaluations should be modified and reassessed with the essential involvement of the participants. The models introduced by Zhang et al. (2022) and Zhang et al. (2023) improve both consensus and consistency but cannot deal with multi-criteria GDM problems (Boix-Cots et al., 2023), in which the preference of an alternative over other is measured keeping in mind diverse criteria instead of evaluating it as a whole (Huang et al., 2023; Liao et al., 2022). And the models presented by Qin et al. (2023a) and Qin et al. (2023b) are extensions of the best-worst method that improve both consensus and consistency in multi-criteria scenarios, but they do not involve the participation of the group members in the modification process. In order to enhance the existing models based on a non-uniform allocation of information granularity, this study aims to build the first granular-based approach that improves both consistency and consensus in multi-criteria GDM problems and that allows the participation of the group members in the modification process. Assuming fuzzy preference relations to model the group members' evaluations, this general objective leads to the following main contributions of this study:**

- **We develop an automatic process of consensus and consistency improvement based upon a non-uniform allocation of information granularity. Assuming an average level of information granularity, it preserves the initial fuzzy preference relations while improving the consensus and the consistency to the greatest extent.**

- **We introduce an interactive process that involves the participation of the group members. Based on the above automatic process of consensus and consistency improvement, it provides a complete application framework for real-world multi-criteria GDM problem-solving."**

In addition, the following references have been added to the bibliography:

- Cabrerizo, F.J., González-Quesada, J.C., Herrera-Viedma, E., Kaklauskas, A., Pedrycz, W., 2022a. Managing inconsistency with an optimal distribution of information granularity in fuzzy preference relations, in: Proc. IEEE Int. Conf. Syst. Man Cybern. (SMC), Prague, Czech Republic. pp. 359-364.
- Cabrerizo, F.J., González-Quesada, J.C., Morente-Molinera, J.A., Pérez, I.J., Herrera-Viedma, E., Pedrycz, W., 2022b. An improvement of consensus in group decision-making through an optimal distribution of information granularity, in: Proc. IEEE Symp. Ser. Comput. Intell. (SSCI), Singapur. pp. 119-124.
- Qin, J., Ma, X., Liang, Y., 2023a. Building a consensus for the best-worst method in group decision-making with an optimal allocation of information granularity. Inf. Sci. 619, 630-653.

- Qin, J., Ma, X., Pedrycz, W., 2023b. A granular computing-driven best-worst method for supporting group decision making. IEEE Trans. Syst. Man Cybern. Syst. 53, 5591-5603.

*3. Some comments on "multi-criteria GDM methods" are not accurate. Authors need to read the following papers carefully: DOI: 10.1016/j.omega.2023.102839; 10.1016/j.cie.2022.108281; 10.1109/TEM.2021.3138970.*

Thank you for your remark. We have addressed this issue by defining a multi-criteria group decision-making as described in reference (Liu et al., in press). This has been done at the beginning of Section 2.1 as follows:

**"It refers to the process in which multiple decision-makers evaluate a set of alternatives under different criteria (attributes). Its purpose is to select the best one from the set by considering that different decision-makers can have diverse viewpoints on the decision problem, which potentiates the appearance of additional conflicting decision criteria (Amiri et al., 2021; Boix-Cots et al., 2023; Jin et al., 2022, 2023; Liu et al., in press)."**

Furthermore, the following related references have been added to the bibliography:

- Jin, F., Cai, Y., Zhou, L., Ding, T., 2023. Regret-rejoice two-stage multiplicative DEA models-driven cross-efficiency evaluation with probabilistic linguistic information. Omega 117, 102839.
- Jin, F., Cai, Y., Pedrycz, W., Liu, J., 2022. Efficiency evaluation with regret-rejoice cross-efficiency DEA models under distributed linguistic environment. Comput. Ind. Eng. 169, 108281.
- Liu, J., Shao, L., Jin, F., Tao, Z., in press. A multi-attribute group decision-making method based on trust relationship and DEA regret cross-efficiency. IEEE Trans. Eng. Manag.
- Cai, Y., Jin, F., Liu, J., Zhou, L., Tao, Z., 2023. A survey of collaborative decision-making: Bibliometrics, preliminaries, methodologies, applications and future directions. Eng. Appl. Artif. Intell. 122, 106064.
- Liu, J., Bao, A., Jin, F., Zhou, L., Shao, L., 2023. Group decision-making with multiplicative probabilistic linguistic preference relations based on consistency improvement and upgraded multiplicative DEA cross-efficiency. J. Intell. Fuzzy Syst. 44, 9395-9410.

*4. Can you assure that Model (14) has optimal solutions? Whether the optimal solution is unique?*

Thank you for your observation. To solve the optimization model presented in (14), the PSO has been employed as it has also been used in similar related approaches. However, it is true that we should note that while this framework maximizes the values reported by the fitness function, it does not guarantee an optimal result, rather than we may refer to it as the best solution that is produced by the PSO algorithm. It has been clarified in the Concluding remarks and future research directions Section as follows:

**"The proposed granular-based multi-criteria GDM approach makes use of the PSO. This algorithm has been utilized as some related approaches have shown that it serves an appropriate optimization framework (Cabrerizo et al., 2014; Liu et al., 2018a). Nevertheless, even though this algorithm maximizes the value returned by the fitness function, it does not guarantee an optimal result (we may refer to it as the best solution obtained by the PSO). Other related approaches have employed the differential evolution (Cabrerizo et al., 2022b; Zhang et al., 2022). However, like the PSO, it is metaheuristic algorithm that may also end up in a local optimum. Therefore, to solve this shortcoming, it is necessary to make use of an optimization algorithm guaranteeing that the global optimum is reached."**

In addition, we have revised the use of the word "optimal" in the manuscript to clarify that the distribution of information granularity is non-uniform instead of optimal as it cannot been proved.

*5. The necessary time complexity analysis of Algorithm 2 is missing.*

Thank you for your comment. Algorithm 2 consists in solving model (16) to obtain the modified fuzzy preference relations that maximizes both the consensus and the consistency according to a non-uniform distribution of an average level of information granularity. Then, these modified fuzzy preference relations, which are viewed as a decision support, are presented to the decision-makers, who use them as an aid for changing the fuzzy preference relations that they provided. In the experimental studies performed, the computation time of the PSO to solve model (16) is low (about 5-6 seconds). Therefore, most of the time depends on the time spent by the decision-makers to modify their fuzzy preference relations, which is difficult to calculate, as some decision-makers spends 1 or 2 minutes and others spend more than 5. It means that the computation time spent by the PSO to solve model (16) is lower than the time spent by the decision-makers to modify their fuzzy preference relations. In fact, the computation time associated with solving model (16) is low, so it is not important here.

*6. The authors had better enhance the discussion for limitations of this proposed approaches and the future directions in the conclusion section.*

We have addressed this question by modifying the future directions in the Concluding remarks and future research directions Section according to the limitations of the proposal. In such a way, the future directions have been rewritten as follows:

**"Based upon this study, these future research directions emerge as a first step at attempting to improve it:**

- **The proposed granular-based multi-criteria GDM approach makes use of the PSO. This algorithm has been utilized as some related approaches have shown that it serves an appropriate optimization framework (Cabrerizo et al., 2014; Liu et al., 2018a). Nevertheless, even though this algorithm maximizes the value returned by the fitness function, it does not guarantee an optimal result (we may refer to it as the best solution obtained by the PSO). Other related approaches have employed the differential evolution (Cabrerizo et al., 2022b; Zhang et al., 2022). However, like the PSO, it is metaheuristic algorithm that may also end up in a local optimum. Therefore, to solve this shortcoming, it is necessary to make use of an optimization algorithm guaranteeing that the global optimum is reached.**

- **The average level of information granularity admitted by the decision-maker is a synonym of her/his flexibility, i.e., the higher the admitted value, the higher the decision-maker's cooperation to improve consensus and consistency. In the examples performed in this study, we have assumed that all decision-makers admit the same average level of information granularity. However, a complete analysis of the decision-makers' behaviour should be performed to recognize the factors that influence the decision-maker's behaviour and deal with non-cooperative behaviours (Wu et al., 2021; You et al., 2023)."**

In addition, the following references have been added to the bibliography:

- Wu, J., Cao, M., Chiclana, F., Dong, Y.C., Herrera-Viedma, E., 2021. An optimal feedback model to prevent manipulation behavior in consensus under social network group decision making. IEEE Trans. Fuzzy Syst. 29, 1750-1763.
- You, X., Hou, F., Chiclana, F., 2023. Consensus reaching process with noncooperative behaviors in large-scale group social network environment. Appl. Soft Comput. 144, 110454.

*7. I think that authors should enhanced the comparison with recent methods.*

A comparison of the results of a group decision-making model with other group decision-making approaches is not a straightforward task. There are two main reasons. First, the structure and the domain used to model the decision-makers' evaluations (frequently, the models allow the decision-makers to express their evaluations in a different way). Second, the characteristics considered by the models, which are often different. Consequently, a quantitative comparison would be not either fair or meaningful. Anyway, we have added the following paragraph at the end of Section 4.5 (Comparative analysis) to clarify this fact and to compare the proposed approach with the existing approaches based on a non-uniform distribution of information granularity:

**“To put the obtained results by the proposed granular-based approach in a certain context, we have compared them with the results obtained by a granular-based approach based on a uniform distribution of information granularity. It has been possible because both approaches use fuzzy preference relations to represent the decision-makers’ evaluations and try to improve both the consistency and the consensus in multi-criteria GDM scenarios. However, a comparison of the results of a GDM approach with other approaches usually is not a straightforward task because the approaches use different structures and domains of evaluation representation, or they deal with different aspects of the GDM process. As a result, a quantitative comparison would not be meaningful. Anyway, we next elaborate on the advantages of the proposed granular-based approach in comparison with the existing granular-based approaches based on a non-uniform distribution of information granularity. The approaches presented by Cabrerizo et al. (2022a), Cabrerizo et al. (2022b), and Cabrerizo et al. (2023), neither improve both the consistency and the consensus nor allow the participation of the decision-makers in the modification process. The approaches developed by Zhang et al. (2022) and Zhang et al. (2023), even though they improve both the consistency and the consensus and allow the participation of the decision-makers in the modification process, cannot handle multi-criteria GDM settings. The approaches introduced by Qin et al. (2023b) and Qin et al. (2023a), even though they improve both the consensus and the consistency in multi-criteria settings, do not allow the participation of the decision-makers in the modification process. Different from these approaches, the proposed granular-based approach improves both the consistency and the consensus in multi-criteria GDM scenarios and allow the participation of the decision-makers in the modification process.”**

*8. The data of in Section 5, how did you get these data? Or what is the basis to set values of these indicators.*

Thank for your remark. The data of Section 5 (A case study of building refurbishment) are obtained from the technical task of the building, which is available at the following reference (in annex B):

- Velykorusova, A., 2023. Development of a method and intelligent decision support system for sustainable renovation of the built environment. Phd thesis. Vilnius Gediminas Technical University. Available at http://dspace.vgtu.lt/bitstream/1/4360/3/AVelykorusovos%20disertacija.pdf.

To clarify it, we have added that reference to the bibliography and we have cited it at the beginning of Section 5 (A case study of building refurbishment).

*9. The flow chart of the proposed method should be added in the paper.*

Thank you for your remark. We have addressed it by adding two figures depicting the flow chart of the automatic process (Figure 1) and the flow chart of the interactive process (Figure 2). You can find them in pages 14 and 16.

## Reviewer 2

*This article proposes a group decision-making process based on optimal information distribution and demonstrates it with a case study on changing the use of a building. Overall, the article has high quality and solves the problem of improving consensus and consistency in a fuzzy environment.*

Thank you for your positive comments.

*1. For this article, I have some minor questions. Firstly, the author used PSO to solve model 14, which is a local optimization method and relies on the selection of initial values. In cases where there is not a unique solution, will the local solution provided by the author cause changes in decision outcomes? In other words, further explanation is needed for the issue of result stability.*

Thank you for your observation. We have applied the PSO because it has been used in other similar approaches (for example, Cabrerizo et al., 2014; Liu et al., 2018), where it has been shown that the PSO serves as an appropriate optimization framework for this type of optimization problems. However, as you note, this framework maximizes the values reported by the fitness function, but it does not guarantee an optimal result, rather than we may refer to it as the best solution that is generated by the PSO. Therefore, to solve this limitation, it could be convenient to apply an optimization algorithm guaranteeing that the global optimal solution is achieved. This has been clarified by adding the following future research direction at the Concluding remarks and future research directions Section:

**"The proposed granular-based multi-criteria GDM approach makes use of the PSO. This algorithm has been utilized as some related approaches have shown that it serves an appropriate optimization framework (Cabrerizo et al., 2014; Liu et al., 2018a). Nevertheless, even though this algorithm maximizes the value returned by the fitness function, it does not guarantee an optimal result (we may refer to it as the best solution obtained by the PSO). Other related approaches have employed the differential evolution (Cabrerizo et al., 2022b; Zhang et al., 2022). However, like the PSO, it is metaheuristic algorithm that may also end up in a local optimum. Therefore, to solve this shortcoming, it is necessary to make use of an optimization algorithm guaranteeing that the global optimum is reached."**

*2. Secondly, the author compares the group decision results of the average information distribution model. It needs to be explained whether this comparison is comprehensive, i.e., whether the purpose of this article is only to improve on the average information distribution, or whether it applies to other similar methods.*

The aim of this study is to enhance the existing approaches based on a non-uniform distribution of information granularity by proposing the first granular-based approach improving both consistency and consensus in multi-criteria GDM problems and allowing the participation of the decision-makers in the modification process of the evaluations. Given the fact that the existing approaches based on a non-uniform distribution of information granularity deal with different features of the GDM problem, it is difficult to perform a quantitative comparison. However, to put the obtained results in a certain context, we have compared them with the results obtained by the approaches dealing with the same features of the GDM problem, but assuming a uniform distribution of information granularity. It helps understand the superiority of the approaches based on a non-uniform allocation of information granularity over those ones based on a uniform distribution of information granularity. To clarify it, we have added the following paragraph at the end of Section 4.5 (Comparative analysis):

**"To put the obtained results by the proposed granular-based approach in a certain context, we have compared them with the results obtained by a granular-based approach based on a uniform distribution of information granularity. It has been possible because both approaches use fuzzy preference relations to represent the decision-makers' evaluations and try to improve both the consistency and the consensus in multi-criteria GDM scenarios. However, a comparison of the results of a GDM approach with other approaches usually is not a straightforward task because the approaches use different structures and domains of evaluation representation, or they deal with different aspects of the GDM process. As a result, a quantitative comparison would not be meaningful. Anyway, we next elaborate on the advantages of the proposed granular-based approach in comparison with the existing granular-based approaches based on a non-uniform distribution of information granularity. The approaches presented by Cabrerizo et al. (2022a), Cabrerizo et al. (2022b), and Cabrerizo et al. (2023), neither improve both the consistency and the consensus nor allow the participation of the decision-makers in the modification process. The approaches developed by Zhang et al. (2022) and Zhang et al. (2023), even though they improve both the consistency and the consensus and allow the participation of the decision-makers in the modification process, cannot handle multi-criteria GDM settings. The approaches introduced by Qin et al. (2023b) and Qin et al. (2023a), even though they improve both the consensus and the consistency in multi-criteria settings, do not allow the participation of the decision-makers in the modification process. Different from these approaches, the proposed granular-based approach improves both the consistency and the consensus in multi-criteria GDM scenarios and allow the participation of the decision-makers in the modification process."**

*3. Finally, the reference list is quite lengthy, and it is recommended to be more concise and include more up-to-date references. e.g., Managing Overconfidence Behaviors from Heterogeneous Preference Relations in Linguistic Group Decision Making. IEEE Transactions on Fuzzy Systems, 2023, DOI: 10.1109/TFUZZ.2022.3226321, ACM Transactions on Internet Technology, 2022, DOI: 10.1145/3533432.*

Following the reviewer's comment, some references have been removed. In addition, more recent references have been included. As an example, the following references have been added to the bibliography:

- Ran, Q., Chao, X., Cabrerizo, F.J., Herrera-Viedma, E., 2023. Managing overconfidence behaviors from heterogeneous preference relations in linguistic group decision making. IEEE Trans. Fuzzy Syst. 31, 2435-2449.
- Dong, Y.C., Ran, Q., Chao, X., Li, C.-C., Yu, S., 2023. Personalized individual semantics learning to support a large-scale linguistic consensus process. ACM Trans. Internet Technol. 23, 26.
- Chao, X., Dong, Y.C., Kou, G., Peng, Y., 2022. How to determine the consensus threshold in group decision making: a method based on efficiency benchmark using benefit and cost insight. Ann. Oper. Res. 316, 143-177.
- Qin, J., Ma, X., Pedrycz, W., 2023b. A granular computing-driven best-worst method for supporting group decision making. IEEE Trans. Syst. Man Cybern. Syst. 53, 5591-5603.
- Qin, J., Martinez, L., Pedrycz, W., Ma, X., Liang, Y., 2023c. An overview of granular computing in decision-making: Extensions, applications, and challenges. Inf. Fusion 98, 101833.
- Qin, J., Ma, X., Liang, Y., 2023a. Building a consensus for the best-worst method in group decision-making with an optimal allocation of information granularity. Inf. Sci. 619, 630-653.
- Cabrerizo, F.J., González-Quesada, J.C., Herrera-Viedma, E., Kaklauskas, A., Pedrycz, W., 2022a. Managing inconsistency with an optimal distribution of information granularity in fuzzy preference relations, in: Proc. IEEE Int. Conf. Syst. Man Cybern. (SMC), Prague, Czech Republic. pp. 359-364.
- Cabrerizo, F.J., González-Quesada, J.C., Morente-Molinera, J.A., Pérez, I.J., Herrera-Viedma, E., Pedrycz, W., 2022b. An improvement of consensus in group decision-making through an optimal distribution of information granularity, in: Proc. IEEE Symp. Ser. Comput. Intell. (SSCI), Singapur. pp. 119-124.
- Wu, J., Cao, M., Chiclana, F., Dong, Y.C., Herrera-Viedma, E., 2021. An optimal feedback model to prevent manipulation behavior in consensus under social network group decision making. IEEE Trans. Fuzzy Syst. 29, 1750-1763.
- You, X., Hou, F., Chiclana, F., 2023. Consensus reaching process with noncooperative behaviors in large-scale group social network environment. Appl. Soft Comput. 144, 110454.
- Cabrerizo, F.J., Morente-Molinera, J.A., Pedrycz, W., Taghavi, A., Herrera-Viedma, E., 2018b. Granulating linguistic information in decision making under consensus and consistency. Expert Syst. Appl. 99, 83-92.
- Callejas, E.A., Cerrada, J.A., Cerrada, C., Cabrerizo, F.J., 2019. Group decision making based on a framework of granular computing for multi-criteria and linguistic contexts. IEEE Access 7, 54670-54681.

# Reviewer 3

*This manuscript reports on a new granular computing-based model to improve consensus and consistency in multi-criteria group decision-making problems with experts' opinions represented using fuzzy preference relations. An automatic process of consensus and consistency improvement is developed, which is used to build an interactive process involving the participation of the experts.*

*The paper is interesting and deals with a topic that is attractive for the journal's readership. The methodology, results, and applicability of the approach in real-world problems are supported with convincing arguments and evidence.*

Thank you for your positive comments.

*1. If only, I would like the authors to address/answer whether the proposed methodology is also applicable to representation of preferences other than fuzzy preference relations, and if so, what would be the main changes in the processing of preferences that would need to be affected by a change of preference representation format.*

Thank you for your comment. Yes, it is applicable to other representation of preferences. As mentioned in the Introduction Section, there exist approaches based on a uniform distribution of information granularity dealing with intuitionistic reciprocal preference relations (for example, Cabrerizo et al. (2021) and Cabrerizo et al. (2018)). In fact, to adapt the proposed approach to other representation of preferences, we must only adapt Equation (5) to that way of preference representation (in addition to the methods that calculate the consensus and the consistency). To highlight the fact that a granular-based approach can be applied to other representation of preferences (in addition to fuzzy preference relations), we have added some related studies using linguistic preference relations to model the decision-makers' preferences. In such a way, this part of the Introduction Section has been rewritten as follows:

**"Information granularity (Pedrycz, 2014), which is a very important concept within the granular computing paradigm (Bargiela and Pedrycz, 2003), has recently been used to control the difference between the initial and modified preferences. The key notion consists in modeling the evaluations in terms of information granules, which make the required flexibility degree available with the aim of increasing the agreement and the degree of consistency (Qin et al., 2023c). Following this reasoning, various GDM models have been proposed, e.g., Pedrycz and Song (2011) and Liu et al. (2018a) presented methods improving the consistency and the consensus, respectively, in the analytic hierarchy process; Cabrerizo et al. (2014) introduced a model improving consistency and consensus with fuzzy preference relations; Cabrerizo et al. (2021) and Cabrerizo et al. (2018a) developed models improving the consistency and the consensus, respectively, with intuitionistic reciprocal preference relations; and Cabrerizo et al. (2018b) and Callejas et al. (2019) presented models improving**

**both the consensus and the consistency with linguistic preference relations, to cite some examples."**

In addition, the following related references have been added to the bibliography:

- Cabrerizo, F.J., Morente-Molinera, J.A., Pedrycz, W., Taghavi, A., Herrera-Viedma, E., 2018b. Granulating linguistic information in decision making under consensus and consistency. Expert Syst. Appl. 99, 83-92.
- Callejas, E.A., Cerrada, J.A., Cerrada, C., Cabrerizo, F.J., 2019. Group decision making based on a framework of granular computing for multi-criteria and linguistic contexts. IEEE Access 7, 54670-54681.

*2. Also, I would like to know whether the PSO approach proposed in the methodology is able to achieve global optimum values or just local optimum values. If the second case, could the authors suggest/consider alternatives to PSO that aim at achieving global optimum values? Since O is a linear combination of weighted averages (are continuous functions?), it maybe possible that if the feasible region is proved to be compact, the existence of a global maximum exists, and therefore alternatives to PSO (if it is only possible to guarantee that the solution obtained could be local and not global) may exists.*

Thank you for your remark. As PSO is a metaheuristic, we cannot guarantee that it reaches the global optimum value. On the contraty, we must refer to it as the best solution achieved by the PSO. This issue has been clarified by adding the following future research direction at the Concluding remarks and future research directions Section:

**"The proposed granular-based multi-criteria GDM approach makes use of the PSO. This algorithm has been utilized as some related approaches have shown that it serves an appropriate optimization framework (Cabrerizo et al., 2014; Liu et al., 2018a). Nevertheless, even though this algorithm maximizes the value returned by the fitness function, it does not guarantee an optimal result (we may refer to it as the best solution obtained by the PSO). Other related approaches have employed the differential evolution (Cabrerizo et al., 2022b; Zhang et al., 2022). However, like the PSO, it is metaheuristic algorithm that may also end up in a local optimum. Therefore, to solve this shortcoming, it is necessary to make use of an optimization algorithm guaranteeing that the global optimum is reached."**

*3. Can the authors also comment on the assumptions they chose in their experimental studies reported in Section 4.*

In the experimental studies reported in Section 4 (Experimental studies and comparative analysis), we have assumed that all decision-makers admit the same average level of information granularity, which is a synonym of the flexibility exhibited by the decision-maker. It means the higher its value, the higher the cooperation of the decision-maker to build consensus (and to improve his/her consistency degree). Therefore, to enhance the analysis, it could be interesting to carry out experimental studies in which each decision-maker admits a different average level of information granularity. Based on it, a detailed analysis of the behaviour of decision-makers in practical applications is required. Although the decision-maker is allowed to refuse or partially follow the suggestion provided by the proposed approach, it is promising to identify the factors influencing the behaviour of decision-maker and manage the non-cooperative behaviour in the practical applications. We have incorporated it as future research direction:

**"The average level of information granularity admitted by the decision-maker is a synonym of her/his flexibility, i.e., the higher the admitted value, the higher the decision-maker's cooperation to improve consensus and consistency. In the examples performed in this study, we have assumed that all decision-makers admit the same average level of information granularity. However, a complete analysis of the decision-makers' behaviour should be performed to recognize the factors that influence the decision-maker's behaviour and deal with non-cooperative behaviours (Wu et al., 2021; You et al., 2023)."**

In addition, the following references have been added to the bibliography:

- Wu, J., Cao, M., Chiclana, F., Dong, Y.C., Herrera-Viedma, E., 2021. An optimal feedback model to prevent manipulation behavior in consensus under social network group decision making. IEEE Trans. Fuzzy Syst. 29, 1750-1763.
- You, X., Hou, F., Chiclana, F., 2023. Consensus reaching process with noncooperative behaviors in large-scale group social network environment. Appl. Soft Comput. 144, 110454.

# Reviewer 4

*I would like to begin by thanking the authors for their contribution to EAAI. This manuscript presents a novel approach for addressing the challenge of achieving consensus and consistency in multi-criteria group decision-making, especially when dealing with fuzzy preference relations.*

Thank you for your positive comments.

*Below is my evaluation of the manuscript, highlighting its strengths and areas for potential improvement:*

*1. It would be important to make clear what the novelty and unique addition of the manuscript is.*

Thank you for your suggestion. Following it, and the comment provided by the first reviewer, we have clarified the novelty and the contribution of this study by rewriting that part of the Introduction Section as follows:

"Their common characteristic is the adoption of a uniform allocation of information granularity. Although they allow to increase the consistency, the consensus, or both, while they restrict the range where the evaluations can be changed, thanks to the information granules, some studies have demonstrated that the performance and flexibility of these models could be enhanced through a non-uniform allocation of a different information granularity to the entries of the preference relations (Cabrerizo et al., 2023; Zhang et al., 2022).

**There are however unresolved issues in the models based on a non-uniform allocation of information granularity that need to be addressed. The models developed by Cabrerizo et al. (2022a) and Cabrerizo et al. (2023) only deal with, and the model proposed by Cabrerizo et al. (2022b) only deals with the improvement of consensus. In addition, they are based on an automatic modification process that does not involve the participation of the group members. However, in real-world GDM, the evaluations should be modified and reassessed with the essential involvement of the participants. The models introduced by Zhang et al. (2022) and Zhang et al. (2023) improve both consensus and consistency but cannot deal with multi-criteria GDM problems (Boix-Cots et al., 2023), in which the preference of an alternative over other is measured keeping in mind diverse criteria instead of evaluating it as a whole (Huang et al., 2023; Liao et al., 2022). And the models presented by Qin et al. (2023a) and Qin et al. (2023b) are extensions of the best-worst method that improve both consensus and consistency in multi-criteria scenarios, but they do not involve the participation of the group members in the modification process. In order to enhance the existing models based on a non-uniform allocation of information granularity, this study aims to build the first granular-based approach that improves both consistency and consensus in multi-criteria GDM problems and that allows the participation of the group members in the modification process. Assuming fuzzy preference relations to model the group members' evaluations, this general objective leads to the following main contributions of this study:**

- **We develop an automatic process of consensus and consistency improvement based upon a non-uniform allocation of information granularity. Assuming an average level of information granularity, it preserves the initial fuzzy preference relations while improving the consensus and the consistency to the greatest extent.**

- **We introduce an interactive process that involves the participation of the group members. Based on the above automatic process of consensus and consistency improvement, it provides a complete application framework for real-world multi-criteria GDM problem-solving."**

In addition, the following references have been added to the bibliography:

- Cabrerizo, F.J., González-Quesada, J.C., Herrera-Viedma, E., Kaklauskas, A., Pedrycz, W., 2022a. Managing inconsistency with an optimal distribution of information granularity in fuzzy preference relations, in: Proc. IEEE Int. Conf. Syst. Man Cybern. (SMC), Prague, Czech Republic. pp. 359-364.
- Cabrerizo, F.J., González-Quesada, J.C., Morente-Molinera, J.A., Pérez, I.J., Herrera-Viedma, E., Pedrycz, W., 2022b. An improvement of consensus in group decision-making through an optimal distribution of information granularity, in: Proc. IEEE Symp. Ser. Comput. Intell. (SSCI), Singapur. pp. 119-124.
- Qin, J., Ma, X., Liang, Y., 2023a. Building a consensus for the best-worst method in group decision-making with an optimal allocation of information granularity. Inf. Sci. 619, 630-653.
- Qin, J., Ma, X., Pedrycz, W., 2023b. A granular computing-driven best-worst method for supporting group decision making. IEEE Trans. Syst. Man Cybern. Syst. 53, 5591-5603.

*2. The literature review is has little depth to it. It would be good to strengthen it with more meaningful discussions of previous work and update the list of references.*

We have addressed this comment by improving the discussion about related works in the Introduction Section as follows:

**"Information granularity (Pedrycz, 2014), which is a very important concept within the granular computing paradigm (Bargiela and Pedrycz, 2003), has recently been used to control the difference between the initial and modified preferences. The key notion consists in modeling the evaluations in terms of information granules, which make the required flexibility degree available with the aim of increasing the agreement and the degree of consistency (Qin et al., 2023c). Following this reasoning, various GDM models have been proposed, e.g., Pedrycz and Song (2011) and Liu et al. (2018a) presented methods improving the consistency and the consensus, respectively, in the analytic hierarchy process; Cabrerizo et al. (2014) introduced a model improving consistency and consensus with fuzzy preference relations; Cabrerizo et al. (2021) and Cabrerizo et al. (2018a) developed models improving**

**the consistency and the consensus, respectively, with intuitionistic reciprocal preference relations; and Cabrerizo et al. (2018b) and Callejas et al. (2019) presented models improving both the consensus and the consistency with linguistic preference relations, to cite some examples. Their common characteristic is the adoption of a uniform allocation of information granularity. Although they allow to increase the consistency, the consensus, or both, while they restrict the range where the evaluations can be changed, thanks to the information granules, some studies have demonstrated that the performance and flexibility of these models could be enhanced through a non-uniform allocation of a different information granularity to the entries of the preference relations (Cabrerizo et al., 2023; Zhang et al., 2022).**

**There are however unresolved issues in the models based on a non-uniform allocation of information granularity that need to be addressed. The models developed by Cabrerizo et al. (2022a) and Cabrerizo et al. (2023) only deal with, and the model proposed by Cabrerizo et al. (2022b) only deals with the improvement of consensus. In addition, they are based on an automatic modification process that does not involve the participation of the group members. However, in real-world GDM, the evaluations should be modified and reassessed with the essential involvement of the participants. The models introduced by Zhang et al. (2022) and Zhang et al. (2023) improve both consensus and consistency but cannot deal with multi-criteria GDM problems (Boix-Cots et al., 2023), in which the preference of an alternative over other is measured keeping in mind diverse criteria instead of evaluating it as a whole (Huang et al., 2023; Liao et al., 2022). And the models presented by Qin et al. (2023a) and Qin et al. (2023b) are extensions of the best-worst method that improve both consensus and consistency in multi-criteria scenarios, but they do not involve the participation of the group members in the modification process. In order to enhance the existing models based on a non-uniform allocation of information granularity, this study aims to build the first granular-based approach that improves both consistency and consensus in multi-criteria GDM problems and that allows the participation of the group members in the modification process."**

In addition, the following related previous works have been added to the bibliography:

- Cabrerizo, F.J., Morente-Molinera, J.A., Pedrycz, W., Taghavi, A., Herrera-Viedma, E., 2018b. Granulating linguistic information in decision making under consensus and consistency. Expert Syst. Appl. 99, 83-92.
- Callejas, E.A., Cerrada, J.A., Cerrada, C., Cabrerizo, F.J., 2019. Group decision making based on a framework of granular computing for multi-criteria and linguistic contexts. IEEE Access 7, 54670-54681.
- Qin, J., Ma, X., Pedrycz, W., 2023b. A granular computing-driven best-worst method for supporting group decision making. IEEE Trans. Syst. Man Cybern. Syst. 53, 5591–5603.
- Qin, J., Martinez, L., Pedrycz, W., Ma, X., Liang, Y., 2023c. An overview of granular computing in decision-making: Extensions, applications, and challenges. Inf. Fusion 98, 101833.
- Qin, J., Ma, X., Liang, Y., 2023a. Building a consensus for the best-worst method in group decision-making with an optimal allocation of information granularity. Inf. Sci. 619, 630–653.

- Cabrerizo, F.J., González-Quesada, J.C., Herrera-Viedma, E., Kaklauskas, A., Pedrycz, W., 2022a. Managing inconsistency with an optimal distribution of information granularity in fuzzy preference relations, in: Proc. IEEE Int. Conf. Syst. Man Cybern. (SMC), Prague, Czech Republic. pp. 359-364.
- Cabrerizo, F.J., González-Quesada, J.C., Morente-Molinera, J.A., Pérez, I.J., Herrera-Viedma, E., Pedrycz, W., 2022b. An improvement of consensus in group decision-making through an optimal distribution of information granularity, in: Proc. IEEE Symp. Ser. Comput. Intell. (SSCI), Singapur. pp. 119-124.

*3. It is not clear why the authors did not follow an alternative methodology. No comparative discussion is presented. Please elaborate further on this aspect. In addition, please provide (better) support for the fact that different stakeholders would have different perspectives on the decision problem, potentiating the appearance of additional conflicting decision criteria.*

Thank you for your remark. As mentioned in the manuscript, the objective is to enhance the existing approaches based on a non-uniform distribution of information granularity. However, a comparison of the results of a group decision-making model with other approaches is not a straightforward task. There are two main reasons. First, the structure and the domain used to model the decision-makers' evaluations (frequently, the models allow the decision-makers to express their evaluations in a different way). Second, the characteristics considered by the models, which are often different. Consequently, a quantitative comparison would be not either fair or meaningful. Anyway, according to your comment and the suggestion provided by first reviewer, we have added the following paragraph at the end of Section 4.5 (Comparative analysis) to clarify this fact and to compare the proposed approach with the existing approaches based on a non-uniform distribution of information granularity:

**"To put the obtained results by the proposed granular-based approach in a certain context, we have compared them with the results obtained by a granular-based approach based on a uniform distribution of information granularity. It has been possible because both approaches use fuzzy preference relations to represent the decision-makers' evaluations and try to improve both the consistency and the consensus in multi-criteria GDM scenarios. However, a comparison of the results of a GDM approach with other approaches usually is not a straightforward task because the approaches use different structures and domains of evaluation representation, or they deal with different aspects of the GDM process. As a result, a quantitative comparison would not be meaningful. Anyway, we next elaborate on the advantages of the proposed granular-based approach in comparison with the existing granular-based approaches based on a non-uniform distribution of information granularity. The approaches presented by Cabrerizo et al. (2022a), Cabrerizo et al. (2022b), and Cabrerizo et al. (2023), neither improve both the consistency and the consensus nor allow the participation of the decision-makers in the modification process. The approaches**

**developed by Zhang et al. (2022) and Zhang et al. (2023), even though they improve both the consistency and the consensus and allow the participation of the decision-makers in the modification process, cannot handle multi-criteria GDM settings. The approaches introduced by Qin et al. (2023b) and Qin et al. (2023a), even though they improve both the consensus and the consistency in multi-criteria settings, do not allow the participation of the decision-makers in the modification process. Different from these approaches, the proposed granular-based approach improves both the consistency and the consensus in multi-criteria GDM scenarios and allow the participation of the decision-makers in the modification process."**

On the other hand, we have also clarified that different stakeholders can have different perspectives on the decision problem, potentiating the appearance of additional conflicting decision criteria. Concretely, we have rewritten the beginning of Section 2.1 as follows:

**"It refers to the process in which multiple decision-makers evaluate a set of alternatives under different criteria (attributes). Its purpose is to select the best one from the set by considering that different decision-makers can have diverse viewpoints on the decision problem, which potentiates the appearance of additional conflicting decision criteria (Amiri et al., 2021; Boix-Cots et al., 2023; Jin et al., 2022, 2023; Liu et al., in press)."**

Furthermore, to support it, the following references about multi-criteria GDM have been added to the bibliography:

- Jin, F., Cai, Y., Zhou, L., Ding, T., 2023. Regret-rejoice two-stage multiplicative DEA models-driven cross-efficiency evaluation with probabilistic linguistic information. Omega 117, 102839.
- Jin, F., Cai, Y., Pedrycz, W., Liu, J., 2022. Efficiency evaluation with regret-rejoice cross-efficiency DEA models under distributed linguistic environment. Comput. Ind. Eng. 169, 108281.
- Liu, J., Shao, L., Jin, F., Tao, Z., in press. A multi-attribute group decision-making method based on trust relationship and DEA regret cross-efficiency. IEEE Trans. Eng. Manag.

*4. Please elaborate further on the extrapolations of the results presented to other contexts.*

The results reported by the experimental studies confirm that a non-uniform distribution of information granularity can achieve better values of consensus and consistency in multi-criteria group decision-making problems than the approaches based on a uniform distribution of information granularity. This can be extrapolated to other granular models applied to other contexts (for example, time series, neural networks, and data compression), which can take advantage of a non-uniform distribution of information granularity. This have been described in the Concluding remarks and future research directions Section as follows:

“The numerical experiments and the case study conducted have illustrated the performance and effectiveness of the proposed granular-based approach to solve multi-criteria GDM problems. In comparison with the approaches based upon a uniform distribution of information granularity, the proposed granular-based approach achieves greater values for the consensus and consistency. It is due to a non-uniform allocation allows for a more efficient use of the resources by allocating higher levels of granularity to the entries of the fuzzy preference relations that require it the most. It leads to better decision-making outcomes as higher consistency and consensus degrees are achieved. **These results can be extrapolated to other contexts in which the granular models can take advantage of a non-uniform distribution of information granularity to offer better results, e.g., in time series, neural networks, or data compression (Liu et al., 2018b; Pedrycz, 2023; Song et al., 2019).**”

Furthermore, to support it, the following references have been added to the bibliography:

- Liu, S., Pedrycz, W., Gacek, A., Dai, Y., 2018b. Development of information granules of higher type and their applications to granular models of time series. Eng. Appl. Artif. Intell. 71, 60-72.
- Pedrycz, W., 2023. Granular data compression and representation. IEEE Trans. Fuzzy Syst. 31, 1497-1505.
- Song, M., Jing, Y., Pedrycz, W., 2019. Granular neural networks: A study of optimizing allocation of information granularity in input space. Soft Computing 77, 67-75.

*5. The conclusion should be expanded. What would be a welcome addition is to have a deeper discussion about the practical and managerial implications of the study. In addition, please try to compare the results obtained with previous studies, highlighting the theoretical and managerial implications of your contribution to the fields of Artificial Intelligence and Building Refurbishment.*

Thank you for your suggestions. As we have already commented, a quantitative comparison, in terms of results obtained, of a group decision-making model with others would be not either fair or meaningful as they allow the decision-makers to express their evaluations in a different way and consider different aspects of the decision-making process. Anyway, we have compared our proposal with those multi-criteria GDM models that assume fuzzy preference relations and are based on a uniform distribution of information granularity. In addition, we have described the advantages of our proposal in comparison with the existing ones based on a non-uniform distribution of information granularity. This has been clarified both in the Introduction Section, in the Section 4.5 (Comparative analysis) and in the Concluding remarks and future research directions Section.

*I have made a few comments, but with the intent that they should help the authors improve the manuscript, and make it not only apt for publication, but more closely aligned to its potential and importance in terms of topic. I wish the authors the best of luck in this endeavour.*

Thank you for your comments. We have tried to improve the manuscript according to them. We hope we have achieved it.

# Non-uniform allocation of information granularity to improve consistency and consensus in multi-criteria group decision-making: Application to building refurbishment

Juan Carlos González-Quesada[a], Anastasiia Velykorusova[b], Audrius Banaitis[b], Artūras Kaklauskas[b], Francisco Javier Cabrerizo[a,*]

[a] *Department of Computer Science and Artificial Intelligence, Andalusian Research Institute in Data Science and Computational Intelligence, DaSCI, University of Granada, Calle Periodista Daniel Saucedo Aranda s/n, Granada, 18071, Granada, Spain*
[b] *Department of Construction Management and Real Estate, Vilnius Gediminas Technical University, Sauletekio al. 11, Vilnius, LT-10223, Vilnius, Lithuania*

---

**Abstract**

If a decision depends on the views of a group of people, it is crucial that these views are consistent, in particular if they are represented in terms of fuzzy preference relations. Moreover, before making the decision, the agreement achieved must be as high as possible. In most cases, to achieve it, the initial viewpoint of each group member must be modified during the decision-making process, which causes an information loss. To handle this issue, some conditions have been added to the modification process. Concretely, some approaches have turned to an allocation of information granularity to control it. However, these approaches either allocate the information granularity in a uniform way or not consider the issues of consistency and consensus in a scenario of multi-criteria group decision-making. To enhance these approaches, we elaborate a novel granular-based approach composed of two processes. Firstly, an automatic process based upon a non-uniform allocation of infor-

*Corresponding author.
*Email addresses:* juancarlosgq@ugr.es (Juan Carlos González-Quesada), anastasiia.velykorusova@gmail.com (Anastasiia Velykorusova), audrius.banaitis@vilniustech.lt (Audrius Banaitis), arturas.kaklauskas@vilniustech.lt (Artūras Kaklauskas), cabrerizo@decsai.ugr.es (Francisco Javier Cabrerizo)

mation granularity is developed to maximize both the consensus and the consistency in multi-criteria group decision-making where fuzzy preference relations are used to model the group members' views. Based upon it, secondly, an interactive process requiring the implication of the group members is introduced, which also intends to maximize both the consensus and the consistency. Some experimental studies are completed to show the essence of this new approach and to demonstrate its performance and flexibility. A case study on building refurbishment is also conducted to validate its effectiveness and feasibility in practical real-world problem-solving.

*Keywords:* consensus, consistency, fuzzy preference relations, information granularity, multi-criteria group decision-making

---

## 1. Introduction

Every day people make decisions that have notable consequences both for business and life. As a result, in today's world, to make good decisions is an important need. Decision-making consists in a sequence of actions conducted by a person or a group of people to decide between two or more alternatives the best one that meets the problem requirements (Jain and Lim, 2010). In one of these actions, each person must evaluate the quality of every alternative to solve the problem. To model these evaluations, different structures have been proposed (Millet, 1997), e.g., an ordered vector of alternatives can be used, the fulfilment degree of the alternative as a solution to the problem can be determined via a utility value, or a pairwise comparison representing the preference degree of an alternative over other one can also be used. The latter, when applying repeatedly, produces a preference relation. This structure is the most-used in decision-making because it allows more accurate evaluations, and it makes easy the aggregation of individual evaluations intro group ones, which is particularly important in group decision-making (GDM) (Cai et al., 2023; Mousavi et al., 2020). Despite these good properties, due to this structure produces more information than the one that is required, inconsistent evaluations can be given on account of the current knowledge, which could be insufficient, and the challenging essence of the GDM process itself (Liu et al., 2023). Consistency was perceived as a rationality measure allowing performance degrees. It means the quality of a preference relation may be determined by a consistency degree. Therefore, to ensure the validity of the outcomes, it is essential to give consistent preference relations.

Although the participation of several individuals presents some advantages (each one brings different knowledge to the group, as well as unique viewpoints and ideas on the problem), it requires additional effort to reach a consensual decision (Truman, 2017). When a decision is made based upon the opinion of several people, consensus can be seen as a process of achieving agreement dynamically and creatively between them (Gelderloos, 2006). A group of people participating in a consensus process tries to make decisions that everyone is at least somewhat in agreement with or can accept instead of just voting for an option and letting the majority decide (Butler and Rothstein, 2006). This ensures that all views are taken into account. However, consensus is often difficult to reach (Gong et al., 2021).

In GDM, we must cope with both consistency and consensus. In view of the fact that it is unusual to get an agreement and preference relations with high consistency degrees at the beginning of the GDM process, diverse models intending to improve consistency, consensus, or both, have been constructed, particularly in fuzzy scenarios, i.e., environments in which the fuzzy sets and their extensions are employed to characterize the evaluations given by the individuals (Bustince et al., 2016; Amiri et al., 2023). These methods strive for improving the consistency degrees and the agreement, but most ones reach this target so that they produce high deviations between the initial evaluations expressed by the individuals and the altered ones, which leads to a remarkable loss of information. How can we solve this issue of information loss associated with modifying the initial evaluations? To solve it, diverse restrictions have been integrated into the alteration process.

Information granularity (Pedrycz, 2014), which is a very important concept within the granular computing paradigm (Bargiela and Pedrycz, 2003), has recently been used to control the difference between the initial and modified preferences. The key notion consists in modeling the evaluations in terms of information granules, which make the required flexibility degree available with the aim of increasing the agreement and the degree of consistency (Qin et al., 2023c). Following this reasoning, various GDM models have been proposed, e.g., Pedrycz and Song (2011) and Liu et al. (2018a) presented methods improving the consistency and the consensus, respectively, in the analytic hierarchy process; Cabrerizo et al. (2014) introduced a model improving consistency and consensus with fuzzy preference relations; Cabrerizo et al. (2021) and Cabrerizo et al. (2018a) developed models improving the consistency and the consensus, respectively, with intuitionistic reciprocal preference relations; and Cabrerizo et al. (2018b) and Callejas et al. (2019) presented models improving both the consensus and the consistency with linguistic preference relations, to cite some examples. Their common characteristic is the adoption of a uniform allocation of information granularity. Although they allow to increase the consistency, the consensus, or both, while they restrict the range where the evaluations can be changed, thanks to the information granules, some studies have demonstrated that the performance and flexibility of these models could be enhanced through a non-uniform allocation of a different information granularity to the entries of the preference relations (Cabrerizo et al., 2023; Zhang et al., 2022).

There are however unresolved issues in the models based on a non-uniform allocation of information granularity that need to be addressed. The models developed by Cabrerizo et al. (2022a) and Cabrerizo et al. (2023) only deal with the improvement of consistency, and the model proposed by Cabrerizo et al. (2022b) only deals with the improvement of consensus. In addition, they are based on an automatic modification process that does not involve the participation of the group members. However, in real-world GDM, the evaluations should be modified and reassessed with the essential involvement of the participants. The models introduced by Zhang et al. (2022) and Zhang et al. (2023) improve both consensus and consistency but cannot deal with multi-criteria GDM problems (Boix-Cots et al., 2023), in which the preference of an alternative over other is measured keeping in mind diverse criteria instead of evaluating it as a whole (Huang et al., 2023; Liao et al., 2022). And the models presented by Qin et al. (2023a) and Qin et al. (2023b) are extensions of the best-worst method that improve both consensus and consistency in multi-criteria scenarios, but they do not involve the participation of the group members in the modification process. In order to enhance the existing models based on a non-uniform allocation of information granularity, this study aims to build the first granular-based approach that improves both consistency and consensus in multi-criteria GDM problems and that allows the participation of the group members in the modification process. Assuming fuzzy preference relations to model the group members' evaluations, this general objective leads to the following main contributions of this study:

- We develop an automatic process of consensus and consistency improvement based upon a non-uniform allocation of information granularity. Assuming an average level of information granularity, it preserves the initial fuzzy preference relations while improving the consensus and the consistency to the greatest extent.

- We introduce an interactive process that involves the participation of the group members. Based on the above automatic process of consensus and consistency improvement, it provides a complete application framework for real-world multi-criteria GDM problem-solving.

The structure of this study is as follows. Section 2 offers some background knowledge. The proposed granular-based approach is presented in Section 3. Strong attention is given to the automatic process of consensus and consistency improvement and the interactive process. Some experimental studies are completed and a comparative analysis is carried out in Section 4 to illustrate the essence of the proposed approach and to demonstrate its flexibility and performance. Section 5 provides a case study on reconstructing a non-residential building into a residential building that helps understand the effectiveness and applicability of this novel granular-based approach in practical real-world problem-solving. Finally, Section 6 draws attention to some concluding remarks and future research directions.

## 2. Preliminaries

This section introduces some prerequisites that help understand the proposed approach. This includes the formulation of a multi-criteria GDM problem with fuzzy preference relations, the importance of the consistency and the consensus in such kind of decision-making scenarios, some aggregation operators widely used in GDM problems, and the particle swarm optimization (PSO) algorithm.

### *2.1. Multi-criteria GDM*

It refers to the process in which multiple decision-makers evaluate a set of alternatives under different criteria (attributes). Its purpose is to select the best one from the set by considering that different decision-makers can have diverse viewpoints on the decision problem, which potentiates the appearance of additional conflicting decision criteria (Amiri et al., 2021; Boix-Cots et al., 2023; Jin et al., 2022, 2023; Liu et al., in press). Formally, let $x_i$, $e_h$, and $a_l$, be the alternative, the decision-maker, and the criterion (attribute), respectively, where $i = 1, 2, \ldots, m$ ($m \geq 2$), $h = 1, 2, \ldots, n$ ($n \geq 2$), and $l = 1, 2, \ldots, o$ ($o \geq 2$). Generally, an importance weight $w_l \in [0, 1]$ is assigned to $a_l$ verifying that $\sum_{l=1}^{o} w_l = 1$. The objective is to assign to each alternative a value representing the collective preference of the decision-makers considering all the criteria.

Among the diverse structures and domains of evaluation representation (Herrera-Viedma et al., 2021), in this study, we assume fuzzy preference relations, i.e., preference relations as representation structure and values in-between $[0, 1]$ as representation domain. We have selected them as they are the most popular ones in fuzzy GDM processes.

**Definition 2.1.** *(Kacprzyk and Fedrizzi, 1986) "An individual fuzzy preference relation of decision-maker $e_h$, $P^h$, is given by its membership function $\mu_{P^h} : X \times X \to [0, 1]$."*

In multi-criteria GDM, each group member $e_h$ must provide a fuzzy preference relation for each criteria $a_l$, which is symbolized as $P^{h,l}$ and typically modeled by a matrix $P^{h,l} = [p_{ij}^{h,l}]$, whose component $p_{ij}^{h,l} = \mu_{P^{h,l}}(x_i, x_j)$ is such that the higher $p_{ij}^{h,l}$, the higher the preference of decision-maker $e_h$ of $x_i$ over $x_j$ on the criterion $a_l$: from $p_{ij}^{h,l} = 1$ indicating a definite preference of $x_i$ over $x_j$ on the criterion $a_l$, through $p_{ij}^{h,l} = 0.5$ indicating indifference between $x_i$ over $x_j$ on the criterion $a_l$, to $p_{ij}^{h,l} = 0$ indicating a definite preference of $x_j$ over $x_i$ on the criterion $a_l$. Due to the components of the leading diagonal, i.e., $p_{ii}^{h,l}$, are not taken into account, they are denoted as "–" (Kacprzyk and Fedrizzi, 1986).

Dealing with fuzzy preference relations implies to keep three strictly required and hierarchic rationality levels in mind (Chiclana et al., 2009): (i) Between $x_i$ and itself, indifference is required, (ii) $x_j$ cannot be preferred to $x_i$ if $x_i$ is preferred to $x_j$, and (iii) $x_i$ should be preferred to $x_k$ if $x_j$ is preferred to $x_k$ and $x_i$ is preferred to $x_j$. The first rationality level is the only one guaranteed by the definition of a fuzzy preference relation. The second and the third ones are associated with the reciprocity in the pairwise comparison and the transitivity in the pairwise comparisons between any three alternatives, respectively. The three rationality levels must be satisfied by the fuzzy preference relations to be considered as consistent. For this reason, diverse properties to be satisfied by them have been proposed (Herrera-Viedma et al., 2004). Specifically, the additive transitivity property has widely been used because it makes easy the verification of the consistency (Li et al., 2019).

On the other hand, given the fact that a consensus reaching process leads to a high-quality decision, which has a strong engagement to application, uncountable consensus reaching processes have been designed in fuzzy GDM (Cabrerizo et al., 2015; Chao et al., 2022; Herrera-Viedma et al., 2014; Meng et al., 2023). In them, to evaluate the agreement achieved, consensus measures must be defined. Particularly, soft consensus measures assuming values in-between $[0, 1]$ have been proposed (Kacprzyk and Fedrizzi, 1986; Herrera-Viedma et al., 2014) (values close to 0 determine low agreement and values close to 1 denote high agreement). To compute them, the similarity between the fuzzy preference relations given by the group members has commonly been measured by using three different approaches (Cabrerizo et al., 2010, 2017): (i) Coincidence among solutions, (ii) soft coincidence among preferences, and (iii) strict coincidence among preferences. The second one has been the most used in GDM under fuzzy preference relations because it allows to consider diverse partial coincidence levels that can be assessed in $[0, 1]$. It means a gradual conception of the coincidence concept is assumed.

### *2.2. Aggregation*

Information aggregation is a fundamental concern of interest for all kinds of knowledge-based systems, particularly, for multi-criteria GDM (Yager, 1988). Because it is composed of a stage in which the evaluations given by the group members must be combined, numerical aggregation operators must be utilized. They are mathematical objects that reduce several numerical values to an only one representing them. In the following, two families of aggregation operators, which will be employed by the proposed granular-based approach, are recalled.

#### *2.2.1. Ordered weighted averaging operators*

To aggregate a collection of evaluations related to the satisfaction of a number of criteria, Yager (1988) provided this family of aggregation operators unifying the conjunctive and disjunctive behaviour in only one operator.

**Definition 2.2.** *(Yager, 1988) "A mapping $\phi$ from $\mathbb{R}^n \to \mathbb{R}$ is an ordered weighted averaging (OWA) operator of dimension $n$ if, related to $\phi$, there exists a weighting vector $w = (w_1, w_2, \ldots, w_n)$ such that $w_i \in [0, 1]$, $\sum_{i=1}^{n} w_i = 1$, and where:*

$$\phi(a_1, a_2, \ldots, a_n) = \sum_{j=1}^{n} w_j b_j \tag{1}$$

*being $b_j$ the $j$th largest component in the collection $a_1, a_2, \ldots, a_n$."*

The maximum, the arithmetic mean, the minimum, and the median, are some well-known operators that can be characterized by this family of aggregation operators by means of appropriate weights (Yager, 1993). Actually, a matter of interest associated with the OWA operators is how to create a suitable method for calculating the weights. A way to do this is to draw upon the linguistic quantifiers (e.g. "most", "almost all", "few", or "nearly half") and its application to GDM (Kacprzyk et al., 1992). Based upon these linguistic quantifiers, Yager introduced the following expression to calculate the components, $w_i$, $i = 1, 2, \dots, n$, of the weighting vector $w$:

$$w_i = Q\left(\frac{i}{n}\right) - Q\left(\frac{i-1}{n}\right) \tag{2}$$

where $Q$ is the linguistic quantifier modeled (Yager, 1988). In such a case, $\phi_Q$ is used to represent that $w$ is computed through a linguistic quantifier.

*2.2.2. Induced ordered weighted averaging operators*

It is a more general family of aggregation operators whose arguments are characterized in terms of pairs, called OWA pairs. The underlying idea is that the first components of the pairs are employed to generate an ordering of the second ones, which are then aggregated.

**Definition 2.3.** *(Yager and Filev, 1999) "A mapping $\Phi$ from $\mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}$ is an induced ordered weighted averaging (IOWA) operator of dimension $n$ if, related to $\Phi$, there exists a weighting vector $w = (w_1, w_2, \dots, w_n)$, such that $w_i \in [0, 1]$, $\sum_{i=1}^{n} w_i = 1$, and where:*

$$\Phi(\langle u_1, a_1\rangle, \langle u_1, a_2\rangle, \dots, \langle u_n, a_n\rangle) = \sum_{j=1}^{n} w_j b_j \tag{3}$$

*where $b_j$ is the $a_i$ component of the OWA pair $\langle u_i, a_i\rangle$ that has the $j$th largest $u_i$ value. Due to the role of the components of the OWA pairs, $a_i$ is known as the argument variable and $u_i$ is known as the order inducing variable."*

Through Eq. (2), the components of the weighting vector of the IOWA operator can also be obtained. Again, it is symbolized by $\Phi_Q$.

*2.3. PSO algorithm*

The goal of this algorithm, created by Kennedy and Eberhart (1995), is to emulate social behavior of bird flocks. It is a stochastic optimization method that, in a series of iterations, tries to solve an optimization problem. In each iteration, it tries to find the most promising candidate solutions according

**Algorithm 1** PSO

**for** each particle $i = 1, \ldots, n$ **do**
  Initializing its initial position $\mathbf{x}_i(0)$ within the lower and upper limits, $l_{\text{inf}}$ and $l_{\text{sup}}$, of the search space with a uniformly distributed random vector: $\mathbf{x}_i(0) \sim U(l_{\text{inf}}, l_{\text{sup}})$
  Initializing its best position $\mathbf{xb}_i$ with its initial position: $\mathbf{xb}_i \leftarrow \mathbf{x}_i(0)$
  **if** $(f(\mathbf{xb}_i) > f(\mathbf{xg}))$ **then**
    Updating the global best position $\mathbf{xg}$ with the particle's best position: $\mathbf{xg} \leftarrow \mathbf{xb}_i$
  **end if**
  Initializing its velocity: $\mathbf{v}_i(0) \sim U(-|l_{\text{sup}} - l_{\text{inf}}|, |l_{\text{sup}} - l_{\text{inf}}|)$
**end for**
**for** each iteration $t = 1, \ldots, m$ **do**
  **for** each particle $i = 1, \ldots, n$ **do**
    **for** each dimension $h = 1, \ldots, d$ **do**
      Generating two random numbers: $r_h, s_h \sim U(0, 1)$
      Updating its velocity: $v_{i,h}(t) \leftarrow \omega(t) v_{i,h}(t-1) + c_1 r_h (xb_{i,h} - x_{i,h}(t-1)) + c_2 s_h (xg_h - x_{i,h}(t-1))$
    **end for**
    Updating its position: $\mathbf{x}_i(t) \leftarrow \mathbf{x}_i(t-1) + \mathbf{v}_i(t)$
    **if** $(f(\mathbf{x}_i(t)) > f(\mathbf{xb}_i))$ **then**
      Updating its best position: $\mathbf{xb}_i \leftarrow \mathbf{x}_i(t)$
      **if** $(f(\mathbf{xb}_i) > f(\mathbf{xg}))$ **then**
        Updating the global best position: $\mathbf{xg} \leftarrow \mathbf{xb}_i$
      **end if**
    **end if**
  **end for**
**end for**
**return** $\mathbf{xg}$

to a certain quality measure (fitness function $f$). To do this, it starts by considering a swarm (population) of particles (candidate solutions in a $d$-dimensional search space). To optimize the problem, the particles are moved to different directions of the search space according to a series of not complex mathematical formulas. In each iteration, the displacement of each particle to a new position is guided by a certain velocity, which depends on its best location so far in the search space and the best location reached by any of the other particles. The goal is to guide the particles towards the best location (solution).

Although there are various variants of this algorithm (Flori et al., 2022; Wang et al., 2018), which could be applied in the granular-based approach that we propose in this study, we will use the generic version provided by Kennedy and Eberhart (1995), whose steps are described in Algorithm 1. In terms of the coefficients, $c_1$ and $c_2$ are two acceleration coefficients that influence the step size taken by the particle towards its best position and the global best position, respectively, and $\omega$ stands for the inertia weight. The assignment of low values to $\omega$ implies exploitation (local search). Due to this, the value assigned to $\omega$ is usually decremented as the number of iterations increases. This ensures global search at the beginning of the algorithm and local search at the end. Following this reasoning, its value is usually altered according to:

$$\omega(t) = (\omega(1) - \omega(m))\frac{m-t}{m} + \omega(m) \tag{4}$$

where $\omega(1)$, $\omega(t)$, and $\omega(m)$ are the initial, the current iteration, and the final values assigned to $\omega$; $m$ and $t$ represent the maximum number of iterations and the present iteration.

## 3. The granular-based multi-criteria GDM approach

This section describes a novel granular proposal for modeling multi-criteria GDM problems in which the participants' evaluations are modeled by fuzzy preference relations. Considering the paradigm of granular computing, the proposed approach, on the one hand, includes a process for improving consensus and consistency through a non-uniform allocation of information granularity. On the other hand, using this process, it also includes an interactive process for improving consensus and consistency that involves the participation of the group members. Once both the consensus and the consistency have been improved, the proposed approach activates the selection process, which assigns a value in-between $[0, 1]$ to each alternative to rank them. Next, we describe the approach in detail, along with the optimization framework that allows us to solve the underlying optimization problem.

### 3.1. *Consistency and consensus improvement*

From a practical viewpoint, it is very common that the group members' opinions diverge widely at the beginning of the process. It is also usual, particularly if the opinions are modeled by fuzzy preference relations, that each individual opinion is inconsistent to some extent. This is why the participants must accept the modification of their first preferences if they want to cooperate with the group and to increase their consistency degrees. To improve consensus and consistency in GDM with fuzzy preference relations, two classes of approaches have been considered (Li et al., 2019; Herrera-Viedma et al., 2014), viz., (i) automatic approaches, and (ii) interactive approaches. According to it, we firstly develop a process of consensus and consistency improvement belonging to the first class, i.e., both the consensus and the consistency are improved through an optimization model that does not require the involvement of the group members. However, since in real-world GDM the participation of the group members during the modification of the fuzzy preference relations is vital, based upon this automatic process, we further introduce an interactive process for consensus and consistency improvement in multi-criteria GDM that implicates the group members.

*3.1.1. Automatic process*

To reach an enough agreement, each group member must obviously accept a specific flexibility degree and agree freely to "soften" his/her position. This is a good example where the paradigm of granular computing (concretely, the information granularity concept) becomes a pivotal element to support it (Pedrycz and Song, 2011). In short, by allowing a granular realization of the group members' evaluations, rather than precise numerical ones, collaboration between them is facilitated and consensus can be reached. The same reasoning can be followed when it comes to improving consistency. Unfortunately, by changing the initial evaluations expressed by the participants, a loss of information could be produced. Fortunately, the granularity of information can also be used to control to some extent the difference between the initial and modified opinions (Pedrycz and Song, 2011).

Among the different granular realizations, we assume intervals, i.e., the pairwise comparison performed on each component of the fuzzy preference relation is represented by an interval instead of an exact value in-between $[0, 1]$. This gives rise to a granular (interval-valued) preference relation that supports a moderate information loss during the consensus and consistency improvement process. This is reached by introducing a different information granularity level $\chi_{ij}^{h,l}$ to each entry $p_{ij}^{h,l}$ of the fuzzy preference relation $P^{h,l}$, which helps impose a constraint on the values that $p_{ij}^{h,l}$ can take.

**Definition 3.1.** *Let* $P^{h,l} = [p_{ij}^{h,l}]$ *be the fuzzy preference relation of the* *decision-maker* $e_h$ *on the criterion* $a_l$. *Let* $G^{h,l}(\cdot)$ *be the family of granular (interval-valued) preference relations. Then,* $G^{h,l}(P^{h,l}) = [g_{ij}^{h,l}]$ *is the granular (interval-valued) preference relation linked to* $P^{h,l}$*, and* $g_{ij}^{h,l} = [p_{ij}^{h,l} - 0.5\chi_{ij}^{h,l}, p_{ij}^{h,l} + 0.5\chi_{ij}^{h,l}]$.

Given $x_i$, $x_j$, $e_h$, and $a_l$, the focus is on $p_{ij}^{h,l} \in [0,1]$, whose value must be adjusted within the interval:

$$g_{ij}^{h,l} = [\max(0, p_{ij}^{h,l} - 0.5\chi_{ij}^{h,l}), \min(1, p_{ij}^{h,l} + 0.5\chi_{ij}^{h,l})] \tag{5}$$

The decision-maker's flexibility required during the multi-criteria GDM process is simulated by the information granularity level. Nevertheless, because it could produce a loss of information, an average level of information granularity $\chi^h$ is fixed for each decision-maker $e_h$. It helps guarantee that a defined level of information accuracy is satisfied for each decision-maker. This is reached in the following way:

$$\chi^h = \frac{1}{o(m^2 - m)} \sum_{l=1}^{o} \sum_{i=1}^{m} \sum_{j=1; j \neq i}^{m} \chi_{ij}^{h,l} \tag{6}$$

The value of $\chi^h$, determined by each decision-maker $e_h$, has the following interpretation: Values close to 1 indicate higher compromise and adaptability of the decision-maker $e_h$ to cooperate in the improvement of consensus and consistency.

Then, the global average level of information granularity $\chi$ inserted into the multi-criteria GDM problem is:

$$\chi = \frac{1}{n} \sum_{h=1}^{n} \chi^h \tag{7}$$

Let $\overline{P}^{h,l} \in G^{h,l}(P^{h,l})$ be a particular interval-valued preference relation of the family $G^{h,l}(\cdot)$:

$$\begin{cases} \overline{p}_{ij}^{h,l} \in g_{ij}^{h,l}, & \text{if } i \neq j \\ \overline{p}_{ij}^{h,l} = \text{“–”}, & \text{otherwise} \end{cases} \tag{8}$$

To calculate the values associated with the entries of $\overline{P}^{h,l}$, we must keep in mind that the aim is to produce altered fuzzy preference relations achieving the greatest value of a composite optimization criterion $O$, which incorporates the effect of both consensus and consistency, i.e.:

$$O = \rho O_1 + (1 - \rho) O_2 \tag{9}$$

where $O_1$ stands for the consistency achieved, $O_2$ stands for the consensus reached, and $\rho$ stands for a parameter determining a trade-off between them.

To calculate $O_1$, the mean of the decision-makers' consistency degrees is employed, i.e.:

$$O_1 = \frac{1}{n} \sum_{h=1}^{n} cd^h \tag{10}$$

where $cd^h$ stands for the consistency degree associated with the fuzzy preference relations of the decision-maker $e_h$, which is computed as follows:

$$cd^h = \sum_{l=1}^{o} w_l cd^{hl} \tag{11}$$

where $cd^{hl}$ stands for the consistency degree of the fuzzy preference relation $P^{h,l}$. The procedure presented by Herrera-Viedma et al. (2007) is used to calculate this consistency degree, but other approaches could also be used (Li et al., 2019).

To compute $O_2$, a method based upon the concept of fuzzy coincidence is assumed (Herrera-Viedma et al., 2014). Let $P^{h,l}$ $(h = 1, \ldots, n;\ l = 1, \ldots, o)$ be a set of fuzzy preference relations, the consensus achieved according to them is measured as follows:

$$O_2 = \sum_{l=1}^{o} w_l c_l \tag{12}$$

where $c_l$ stands for the consensus achieved on the criterion $c_l$, which is computed as follows:

$$c_l = 1 - \frac{2 \sum_{h=1}^{n-1} \sum_{q=h+1}^{n} \frac{\sum_{i=1}^{m} \sum_{j=1;\, j \neq i}^{m} |p_{ij}^{h,l} - p_{ij}^{q,l}|}{m^2 - m}}{n^2 - n} \tag{13}$$

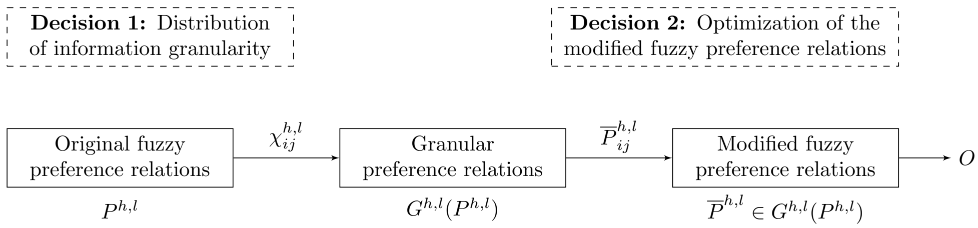



Figure 1: Decision framework of the automatic process.

Based on Eqs. (6), (8) and (9), the next optimization model is constructed. Its aim is to maximize $O$ according to a non-uniform distribution of information granularity:

$$\begin{cases} \max\limits_{\overline{p}_{ij}^{h,l},\chi_{ij}^{h,l}} O \\ \text{s.t.} \begin{cases} \overline{P}^{h,l} \in G^{h,l}(P^{h,l}) \\ \chi^h = \frac{1}{o(m^2-m)} \sum_{l=1}^{o} \sum_{i=1}^{m} \sum_{j=1;j\neq i}^{m} \chi_{ij}^{h,l} \end{cases} \end{cases} \quad (14)$$

where the decision variables are $\overline{p}_{ij}^{h,l}$ and $\chi_{ij}^{h,l}$. Owing to $\overline{p}_{ij}^{h,l}$ is determined by $\chi_{ij}^{h,l}$, this optimization model can be seen as a process of decision-making consisting in two stages executed in order (see Fig. 1).

Because $\overline{P}^{h,l} \in G^{h,l}(P^{h,l})$, the interval calculated by Eq. (5) is the one in which the value of $\overline{p}_{ij}^{h,l}$ is located. In consideration of that, we have:

$$\begin{cases} 0 \leq 2|p_{ij}^{h,l} - \overline{p}_{ij}^{h,l}| \leq \chi_{ij}^{h,l} \\ \overline{p}_{ij}^{h,l} \in [0,1] \end{cases} \quad (15)$$

Then, model (14) can be converted into the following one:

$$\begin{cases} \max\limits_{\overline{p}_{ij}^{h,l}} O \\ \text{s.t.} \begin{cases} \sum_{l=1}^{o} \sum_{i=1}^{m} \sum_{j=1;j\neq i}^{m} 2|p_{ij}^{h,l} - \overline{p}_{ij}^{h,l}| \leq o(m^2-m)\chi^h \\ \overline{p}_{ij}^{h,l} \in [0,1], & \text{if } i \neq j \\ \overline{p}_{ij}^{h,l} = \text{“–”}, & \text{otherwise} \end{cases} \end{cases} \quad (16)$$

where $\overline{p}_{ij}^{h,l}$ are the decision variables whose number is $on(m^2-m)$ instead of $2on(m^2-m)$, which is the number of decision variables of model (14).

Thanks to $\chi^h$, the average of the absolute distance between $p_{ij}^{h,l}$ and $\overline{p}_{ij}^{h,l}$ in $l_1$-norm is bounded in model (16). It means a controlled information dissimilarity between $\overline{P}^{h,l}$ and $P^{h,l}$ is ensured for each decision-maker $e_h$ on each criterion $a_l$.

This consensus and consistency improvement process is resolved by performing two steps: (i) according to model (16), we calculate the value that corresponds to $\overline{p}_{ij}^{h,l}$, and (ii) we then calculate the value that corresponds to $\chi_{ij}^{h,l}$. Let $\Delta^{h,l} = o \cdot (m^2 - m) \cdot \chi^h - \sum_{i=1}^{m} \sum_{j=1;j\neq i}^{m} 2|p_{ij}^{h,l} - \overline{p}_{ij}^{h,l}|$ in this second step. Based upon Eq. (15), if $\Delta^{h,l}$ takes values greater than 0, too many values may be given to $\chi_{ij}^{h,l}$ as solutions. Without loss of generality, $\chi_{ij}^{h,l}$ is distributed to the consensus and consistency improvement process in the following way:

$$\chi_{ij}^{h,l} = \begin{cases} 2|p_{ij}^{h,l} - \overline{p}_{ij}^{h,l}| + \frac{1}{\#\Upsilon^{h,l}}\Delta^{h,l}, & \text{if } (i,j) \in \Upsilon^h \\ 0, & \text{otherwise} \end{cases} \tag{17}$$

where $\Upsilon^{h,l} = \{(i,j) \mid p_{ij}^{h,l} \neq \overline{p}_{ij}^{h,l}\}$ and $\#\Upsilon^h$ is its cardinality. Remarkably, $\chi_{ij}^{h,l} = 2|p_{ij}^{h,l} - \overline{p}_{ij}^{h,l}|$ is the only one solution if $\Delta^{h,l} = 0$.

To resolve model (16), the PSO algorithm is applied (see Section 2.3). The reasons of the use of this algorithm are that its implementation by means of programming languages is very easy, it has few parameters to adjust, and its convergence is very fast (Wang et al., 2018). In light of the number of decision variables is $on(m^2 - m)$ in model (16), this number corresponds to the dimension $d$ of the particles. Let $\mathbf{x}_i(t) = (x_{i,1}(t), \ldots, x_{i,d}(t))$ be the $i$th particle at iteration $t$. Since $p_{ij}^{h,l} \in [0,1]$, the search space is $x_{i,h}(t) \in [0,1]$, $h = 1, 2, \ldots, d$. In model (16), to manage the constraints, the fitness function $f$ is defined as:

$$f(\mathbf{x}_i(t)) = \begin{cases} O, & \text{if } e(\mathbf{x}_i(t)) = 1 \\ 0, & \text{otherwise} \end{cases} \tag{18}$$

where $O$ is the value of the consensus and consistency achieved considering the fuzzy preference relations generated by the vector received as parameter and computed using Eq. (9), and $e$ is a function that controls that the information granularity level injected is lower than the one allowed by the decision-makers. If one of the modified fuzzy preference relations constructed based upon the vector that is received as parameter has an average information granularity level higher than the one allowed by that decision-maker,

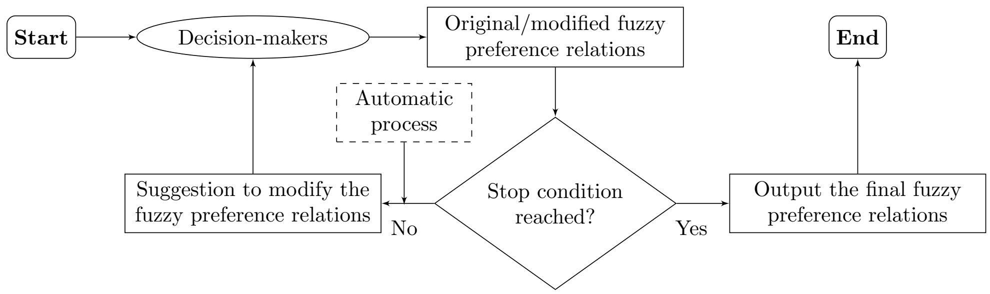



Figure 2: Decision framework of the interactive process.

this function returns 0 as there exists a fuzzy preference relation that is not valid. In other case, it returns 1.

To guarantee that the value of each component of the particle, $\mathbf{x}_i(t)$, is in-between $[0, 1]$, after the update of the particle's position, the boundary constraints must be tested. Then, if $x_{i,h}(t) > 1$, then $x_{i,h}(t) = 1$, and if $x_{i,h}(t) < 0$, then $x_{i,h}(t) = 0$. Using the PSO, the entries of the fuzzy preference relations are modified to maximize the value of $O$ and, then, using Eq. (17), the value of $\chi_{ij}^{h,l}$ can be computed.

*3.1.2. Interactive process*

This process aims to return the optimized fuzzy preference relations, $\overline{P}^{h,l}$, to the decision-makers, who must generate new modified fuzzy preference relations, $R^{h,l}$, based upon them. That is, the optimized fuzzy preference relations returned by the automatic approach are viewed as a decision support that the group members utilize as an aid for changing the fuzzy preference relations that they provided (see Fig. 2).

To build $R^{h,l} = [r_{ij}^{h,l}]$, these interval-valued elements are recommended:

$$r_{ij}^{h,l} \in [\min(p_{ij}^{h,l} \overline{p}_{ij}^{h,l}), \max(p_{ij}^{h,l}, \overline{p}_{ij}^{h,l})] \tag{19}$$

As a result, we have:

$$r_{ij}^{h,l} = \min(p_{ij}^{h,l}, \overline{p}_{ij}^{h,l})\beta_{ij}^{h,l} + \max(p_{ij}^{h,l}, \overline{p}_{ij}^{h,l})(1 - \beta_{ij}^{h,l}) \tag{20}$$

where $\beta_{ij}^{h,l} \in [0, 1]$ is a value that is determined by $e_h$.

The interactive process for consensus and consistency improvement stops is one of the following two stop requisites is satisfied: (i) if the current iteration reaches the maximum number of iterations, $\eta$, or (ii) if the value

**Algorithm 2**

**Input:** $P^{h,l,0}$, $\chi^h$, $\eta$, $\theta$
**Output:** $R^{h,l}$

$k = 0$
Calculating $O$ through Eq. (9)
**while** $k < \eta$ **and** $O < \theta$ **do**
  **for** each decision-maker $h = 1, \ldots, m$ **do**
    **for** each criterion $l = 1, \ldots, o$ **do**
      Obtaining $\overline{P}^{h,l,k}$ through model (16)
      Constructing $P^{h,l,k+1}$ through Eq. (20)
    **end for**
  **end for**
  Calculating $O$ through Eq. (9)
  $k = k + 1$
**end while**
**return** $P^{h,l,k}$

of the optimization criterion, $O$, achieved according to the modified fuzzy preference relations, is equal or higher than a determinate threshold, $\theta$, establishing a minimum required value for this criterion. Provided that $O \geq \theta$ and the present iteration is lower than $\eta$, the recommendation is provided to the group members according to the optimized fuzzy preference relations returned by the automatic process. Based on it, the group members can modify their fuzzy preference relations. Most notably, each group member can refuse, entirely accept, or partially accept, the feedback received. Let $k$ and $P^{h,l,k}$ be the iteration number and the fuzzy preference relation provided by the decision-maker $e_h$ on the criterion $a_l$ at the iteration $k$. The steps of the interactive process are described in Algorithm 2.

### *3.2. Selection process*

This process is applied once the consistency and consensus improvement process has finalized. It consists in assigning a value within the interval $[0, 1]$ to each alternative taking into account the fuzzy preference relations provided by the group members on each criterion. Once a value is assigned to each alternative, a ranking of them can be determined. This is achieved in two phases, which are detailed next.

*3.2.1. Aggregation*

The objective of this phase is to combine all the fuzzy preference relations provided by the group members in a collective fuzzy preference relation that summarizes all them. This is done in the following way:

- Applying the OWA operator, a collective fuzzy preference relation, $CP^l = [cp_{ij}^l]$, is computed for every criterion, $a_l$:

$$cp_{ij}^l = \phi_Q(p_{ij}^{1,l}, p_{ij}^{2,l}, \ldots, p_{ij}^{n,l}) \tag{21}$$

- Considering that each criterion presents a different importance weight, the IOWA operator is applied to compute the collective fuzzy preference relation, $CP = [cp_{ij}]$:

$$cp_{ij} = \Phi_Q(\langle w_1, cp_{ij}^1\rangle, \langle w_2, cp_{ij}^2\rangle, \ldots, \langle w_o, cp_{ij}^o\rangle) \tag{22}$$

*3.2.2. Exploitation*

The objective of this phase is to make use of the values contained in the collective fuzzy preference relation to select the alternative that best solves the multi-criteria GDM problem. To do so, a choice degree assigning a value in-between $[0, 1]$ to each alternative is utilized. We use the quantifier-guided non-dominance degree ($QGNDD$) (Herrera-Viedma et al., 2007), which is based upon the OWA operator and the concept of fuzzy majority. It measures the degree in which an alternative is not dominated by a fuzzy majority of the others as follows:

$$QGNDD_i = \phi_Q(1 - p_{1i}^s, \ldots, 1 - p_{(i-1)i}^s, 1 - p_{(i+1)i}^s, \ldots, 1 - p_{mi}^s) \tag{23}$$

where the degree in which $x_i$ is strictly dominanted by $x_j$ is defined by $p_{ji}^s = max\{cp_{ji} - cp_{ij}, 0\}$.

## 4. Experimental studies and comparative analysis

We offer some numerical experiments to examine the performance and effectiveness of the proposed granular-based approach in the improvement of the consistency degrees and the consensus in multi-criteria GDM. We also conduct several comparative experiments to demonstrate the superiority of this granular-based approach in relation to the ones based upon a uniform distribution of information granularity.

*4.1. Numerical example*

Let us suppose a multi-criteria GDM problem consisting in four decision-makers, $\{e_1, e_2, e_3, e_4\}$, that must rank four alternatives, $\{x_1, x_2, x_3, x_4\}$, considering three criteria, $\{a_1, a_2, a_3\}$, whose importance weights are equal, i.e., $w_1 = w_2 = w_3 = 0.33$. The initial fuzzy preference relations, $P^{h,l}$ ($h = 1, 2, 3, 4; l = 1, 2, 3$), expressed by the decision-makers are randomly generated. Commonly, to generate random data, the normal and the uniform distributions are employed. However, because the data generated by means of the normal distribution leads to a high consensus degree, in this numerical example, the uniform distribution is utilized, and the following preference relations are obtained:

$$P^{1,1} = \begin{bmatrix} - & 0.29 & 0.69 & 0.86 \\ 0.61 & - & 0.09 & 0.14 \\ 0.02 & 0.72 & - & 0.54 \\ 0.03 & 0.63 & 0.03 & - \end{bmatrix} P^{1,2} = \begin{bmatrix} - & 0.12 & 0.54 & 0.02 \\ 0.54 & - & 0.11 & 0.01 \\ 0.06 & 0.86 & - & 0.73 \\ 0.95 & 0.56 & 0.06 & - \end{bmatrix}$$

$$P^{1,3} = \begin{bmatrix} - & 0.24 & 0.58 & 0.88 \\ 0.64 & - & 0.78 & 0.04 \\ 0.11 & 0.12 & - & 0.84 \\ 0.04 & 0.79 & 0.09 & - \end{bmatrix} P^{2,1} = \begin{bmatrix} - & 0.02 & 0.80 & 0.08 \\ 0.79 & - & 0.12 & 0.11 \\ 0.07 & 0.84 & - & 0.53 \\ 0.53 & 0.69 & 0.11 & - \end{bmatrix}$$

$$P^{2,2} = \begin{bmatrix} - & 0.66 & 0.60 & 0.00 \\ 0.16 & - & 0.03 & 0.59 \\ 0.07 & 0.74 & - & 0.56 \\ 0.65 & 0.39 & 0.29 & - \end{bmatrix} P^{2,3} = \begin{bmatrix} - & 0.54 & 0.33 & 0.59 \\ 0.21 & - & 0.00 & 0.82 \\ 0.59 & 0.71 & - & 0.02 \\ 0.09 & 0.06 & 0.62 & - \end{bmatrix}$$

$$P^{3,1} = \begin{bmatrix} - & 0.02 & 0.10 & 0.01 \\ 0.72 & - & 0.77 & 0.11 \\ 0.83 & 0.17 & - & 0.59 \\ 0.55 & 0.86 & 0.09 & - \end{bmatrix} P^{3,2} = \begin{bmatrix} - & 0.03 & 0.19 & 0.76 \\ 0.91 & - & 0.16 & 0.73 \\ 0.80 & 0.69 & - & 0.25 \\ 0.17 & 0.00 & 0.73 & - \end{bmatrix}$$

$$P^{3,3} = \begin{bmatrix} - & 0.21 & 0.72 & 0.79 \\ 0.67 & - & 0.18 & 0.18 \\ 0.17 & 0.55 & - & 0.72 \\ 0.07 & 0.67 & 0.25 & - \end{bmatrix} P^{4,1} = \begin{bmatrix} - & 0.17 & 0.73 & 0.34 \\ 0.80 & - & 0.67 & 0.37 \\ 0.20 & 0.26 & - & 0.66 \\ 0.61 & 0.57 & 0.26 & - \end{bmatrix}$$

$$P^{4,2} = \begin{bmatrix} - & 0.51 & 0.54 & 0.34 \\ 0.12 & - & 0.21 & 0.77 \\ 0.22 & 0.77 & - & 0.08 \\ 0.59 & 0.21 & 0.68 & - \end{bmatrix} P^{4,3} = \begin{bmatrix} - & 0.61 & 0.41 & 0.82 \\ 0.19 & - & 0.58 & 0.10 \\ 0.52 & 0.00 & - & 0.31 \\ 0.04 & 0.81 & 0.58 & - \end{bmatrix}$$

With these fuzzy preference relations, through Eq. (11) the decision-makers achieve the following initial consistency degrees: $cd^1 = 0.672$, $cd^2 =$

0.692, $cd^3 = 0.717$, and $cd^4 = 0.758$. It means the global consistency degree achieved is equal to 0.710. Regarding the consensus, using Eqs. (12) and (13), it is equal to 0.699. To improve these values, the automatic process presented in Section 3.1 is applied. After an intense experimentation, the PSO is executed with the following values: The inertia weight coefficients, $\omega(1)$ and $\omega(m)$, are set to 0.9 and 0.4, respectively; the acceleration parameters, $c_1$ and $c_2$, are set to 2; the size of swarm is set to $10on(m^2 - m)$; and the number of iterations is set to 1000. In addition, the average level of information granularity is set to 0.2 for the four decision-makers, i.e., $\chi^1 = \chi^2 = \chi^3 = \chi^4 = \chi = 0.2$, and for the calculation of $f$, the parameter $\rho$ is set to 0.5 in Eq. (9), i.e., the same importance is assigned to the consistency and the consensus.

The automatic process returns the following modified fuzzy preference relations:

$$\overline{P}^{1,1} = \begin{bmatrix} - & 0.19 & 0.64 & 0.73 \\ 0.50 & - & 0.16 & 0.25 \\ 0.23 & 0.50 & - & 0.54 \\ 0.10 & 0.68 & 0.02 & - \end{bmatrix} \overline{P}^{1,2} = \begin{bmatrix} - & 0.16 & 0.52 & 0.19 \\ 0.53 & - & 0.17 & 0.00 \\ 0.24 & 0.78 & - & 0.53 \\ 0.68 & 0.45 & 0.11 & - \end{bmatrix}$$

$$\overline{P}^{1,3} = \begin{bmatrix} - & 0.40 & 0.48 & 0.77 \\ 0.53 & - & 0.66 & 0.14 \\ 0.19 & 0.17 & - & 0.63 \\ 0.03 & 0.69 & 0.16 & - \end{bmatrix} \overline{P}^{2,1} = \begin{bmatrix} - & 0.20 & 0.66 & 0.13 \\ 0.62 & - & 0.18 & 0.21 \\ 0.31 & 0.74 & - & 0.55 \\ 0.52 & 0.71 & 0.16 & - \end{bmatrix}$$

$$\overline{P}^{2,2} = \begin{bmatrix} - & 0.55 & 0.53 & 0.23 \\ 0.33 & - & 0.14 & 0.63 \\ 0.23 & 0.67 & - & 0.59 \\ 0.65 & 0.48 & 0.38 & - \end{bmatrix} \overline{P}^{2,3} = \begin{bmatrix} - & 0.55 & 0.36 & 0.63 \\ 0.24 & - & 0.20 & 0.66 \\ 0.46 & 0.54 & - & 0.16 \\ 0.08 & 0.27 & 0.56 & - \end{bmatrix}$$

$$\overline{P}^{3,1} = \begin{bmatrix} - & 0.18 & 0.19 & 0.00 \\ 0.60 & - & 0.71 & 0.23 \\ 0.70 & 0.41 & - & 0.59 \\ 0.47 & 0.75 & 0.08 & - \end{bmatrix} \overline{P}^{3,2} = \begin{bmatrix} - & 0.07 & 0.25 & 0.42 \\ 0.60 & - & 0.18 & 0.67 \\ 0.67 & 0.71 & - & 0.32 \\ 0.23 & 0.02 & 0.67 & - \end{bmatrix}$$

$$\overline{P}^{3,3} = \begin{bmatrix} - & 0.34 & 0.54 & 0.75 \\ 0.48 & - & 0.31 & 0.27 \\ 0.33 & 0.50 & - & 0.57 \\ 0.09 & 0.66 & 0.25 & - \end{bmatrix} \overline{P}^{4,1} = \begin{bmatrix} - & 0.19 & 0.64 & 0.33 \\ 0.71 & - & 0.50 & 0.32 \\ 0.30 & 0.40 & - & 0.51 \\ 0.48 & 0.69 & 0.32 & - \end{bmatrix}$$

$$\overline{P}^{4,2} = \begin{bmatrix} - & 0.49 & 0.49 & 0.29 \\ 0.45 & - & 0.17 & 0.61 \\ 0.24 & 0.69 & - & 0.26 \\ 0.56 & 0.31 & 0.59 & - \end{bmatrix} \overline{P}^{4,3} = \begin{bmatrix} - & 0.56 & 0.51 & 0.74 \\ 0.27 & - & 0.48 & 0.22 \\ 0.41 & 0.29 & - & 0.35 \\ 0.10 & 0.69 & 0.53 & - \end{bmatrix}$$

Based upon these modified fuzzy preference relations, the matrices, $D^{h,l}$ ($h = 1, 2, 3, 4;\ l = 1, 2, 3$), are constructed, which contains the information granularity distributed to each entry of $\overline{P}^{h,l}$:

$$D^{1,1} = \begin{bmatrix} - & 0.202 & 0.106 & 0.267 \\ 0.226 & - & 0.156 & 0.224 \\ 0.429 & 0.453 & - & 0.019 \\ 0.140 & 0.103 & 0.070 & - \end{bmatrix} D^{1,2} = \begin{bmatrix} - & 0.070 & 0.033 & 0.346 \\ 0.037 & - & 0.116 & 0.004 \\ 0.369 & 0.158 & - & 0.396 \\ 0.539 & 0.220 & 0.107 & - \end{bmatrix}$$

$$D^{1,3} = \begin{bmatrix} - & 0.323 & 0.184 & 0.217 \\ 0.230 & - & 0.238 & 0.200 \\ 0.161 & 0.087 & - & 0.401 \\ 0.015 & 0.203 & 0.135 & - \end{bmatrix} D^{2,1} = \begin{bmatrix} - & 0.369 & 0.276 & 0.118 \\ 0.347 & - & 0.134 & 0.204 \\ 0.493 & 0.216 & - & 0.047 \\ 0.029 & 0.060 & 0.100 & - \end{bmatrix}$$

$$D^{2,2} = \begin{bmatrix} - & 0.223 & 0.147 & 0.470 \\ 0.339 & - & 0.213 & 0.084 \\ 0.331 & 0.139 & - & 0.063 \\ 0.002 & 0.196 & 0.188 & - \end{bmatrix} D^{2,3} = \begin{bmatrix} - & 0.017 & 0.068 & 0.065 \\ 0.052 & - & 0.403 & 0.318 \\ 0.270 & 0.345 & - & 0.272 \\ 0.012 & 0.434 & 0.136 & - \end{bmatrix}$$

$$D^{3,1} = \begin{bmatrix} - & 0.333 & 0.181 & 0.016 \\ 0.266 & - & 0.131 & 0.258 \\ 0.267 & 0.487 & - & 0.011 \\ 0.175 & 0.239 & 0.029 & - \end{bmatrix} D^{3,2} = \begin{bmatrix} - & 0.084 & 0.126 & 0.688 \\ 0.617 & - & 0.055 & 0.120 \\ 0.244 & 0.030 & - & 0.140 \\ 0.123 & 0.041 & 0.126 & - \end{bmatrix}$$

$$D^{3,3} = \begin{bmatrix} - & 0.269 & 0.369 & 0.089 \\ 0.375 & - & 0.264 & 0.190 \\ 0.335 & 0.106 & - & 0.313 \\ 0.046 & 0.024 & 0.013 & - \end{bmatrix} D^{4,1} = \begin{bmatrix} - & 0.060 & 0.187 & 0.019 \\ 0.189 & - & 0.351 & 0.122 \\ 0.209 & 0.301 & - & 0.295 \\ 0.270 & 0.257 & 0.134 & - \end{bmatrix}$$

$$D^{4,2} = \begin{bmatrix} - & 0.040 & 0.114 & 0.122 \\ 0.666 & - & 0.083 & 0.317 \\ 0.049 & 0.171 & - & 0.368 \\ 0.078 & 0.199 & 0.187 & - \end{bmatrix} D^{4,3} = \begin{bmatrix} - & 0.099 & 0.205 & 0.161 \\ 0.155 & - & 0.195 & 0.228 \\ 0.218 & 0.595 & - & 0.085 \\ 0.120 & 0.248 & 0.087 & - \end{bmatrix}$$

With these modified fuzzy preference relations, the global consistency degree achieved is now 0.803 while the consensus reached is 0.818. Even though the average level of information granularity allowed is low (0.2), both the consensus achieved within the whole group and the consistency degrees of the individual decision-makers have notably improved, which means that the non-uniform distribution of an average level of information granularity has a great impact on the consensus and consistency improvement. This demonstrates the effectiveness of the automatic process for improving the consistency and the consensus.

### *4.2. Sensitive analysis for values of $\chi$*

To investigate the impact of the average level of information granularity on the performance of the proposed granular-based approach, we execute the automatic process with different values of $\chi^h$. Particularly, we set $\chi^1 = \chi^2 = \chi^3 = \chi^4$ to 0.1, 0.2, 0.3, 0.4, 0.5, and 0.6. Replicating the approach performed in Section 4.1, we distribute the different average levels of information granularity to the fuzzy preference relations to analyze the impact of the granularity on the improvement of the consensus and the consistency. Table 1 shows the evolution of the values for $O$ (optimization criterion), $O_1$ (consistency), and $O_2$ (consensus), for different granule sizes.

On the one hand, we note that when an average level of information granularity is allowed, the value of the consensus and the consistency increases significantly compared to when precise numerical values are utilized (0.699 and 0.710, respectively). On the other hand, it seems easy to understand that the greater the average level of information granularity injected into the proposed granular-based approach, the higher the probability of reaching higher values of $O$. This can be observed in Table 1, where a clear increasing trend in the values of the optimization criterion $O$ is visible in line with the increment of the values of $\chi^h$. This is due to the greater the average level of information granularity allowed, the higher the length of the intervals in which the preference degree can be located and, therefore, the possibility of maximizing the optimization criterion is greater.

Table 1: Values of $O, O_1$, and $O_2$, for different values of $\chi^h$.

| $\chi^h$ | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 |
|---|---|---|---|---|---|---|
| $O$ | 0.748 | 0.811 | 0.855 | 0.907 | 0.951 | 0.973 |
| $O_1$ | 0.753 | 0.803 | 0.853 | 0.906 | 0.955 | 0.969 |
| $O_2$ | 0.744 | 0.818 | 0.857 | 0.908 | 0.948 | 0.978 |

Table 2: Values of $O_1$ and $O_2$ for different values of $\rho$.

| $\rho$ | 0.0 | 0.25 | 0.50 | 0.75 | 1.0 |
|---|---|---|---|---|---|
| $O_1$ | 0.772 | 0.787 | 0.804 | 0.821 | 0.836 |
| $O_2$ | 0.835 | 0.828 | 0.818 | 0.784 | 0.730 |

*4.3. Sensitive analysis for values of $\rho$*

Another interesting analysis is to examine how the parameter $\rho$ impacts on the composite optimization criterion $O$. Because this parameter defines a compromise between consensus and consistency, we only pay attention to the values reached by $O_1$ and $O_2$. Replicating again the approach performed in Section 4.1, with $\rho$ assuming values equal to 0.0, 0.25, 0.50, 0.75, and 1.0, Table 2 shows the impact of $\rho$ on the results obtained. As it is expected, we note that low values of $\rho$ imply high values of consensus, $O_2$, and low values of consistency, $O_1$. On the contrary, high values of $\rho$ imply low values of consensus, $O_2$, and high values of consistency, $O_1$. In such a way, the best value of the consistency and the worst value of the consensus are achieved for $\rho = 1.0$, and vice versa, the best value of the consensus and the worst value of the consistency are achieved for $\rho = 0.0$. It seems intuitive as when values close to 1.0 are assigned to $\rho$, the PSO principally focuses on maximizing the consistency. Meanwhile, when values close to 0.0 are assigned to $\rho$, the PSO pays attention to maximize the consensus.

*4.4. Interactive process*

The performance and efficacy of the interactive process are exemplified by using the numerical example presented in Section 4.1. We assume that the behavior of the group members is characterized by randomly accepting the modified fuzzy preference relations returned by the automatic approach, i.e., for each entry of the fuzzy preference relations that a group member has provided, he/she randomly accepts the value provided by the automatic approach or keeps the value that he/she had provided.

We set $\eta$ to 6, $\chi^1$, $\chi^2$, $\chi^3$, and $\chi^4$ to 0.2, and $\theta$ to 1 in the interactive process. Let $\sigma$ be the probability that the decision-makers accept the value provided by the automatic process. Fig. 3 shows the values of the optimization criterion, $O$, in successive iterations of the interactive process. With the increasing numbers of iterations, we can note that the values of $O$ increase, which illustrates the efficacy of the interactive process. Furthermore, the

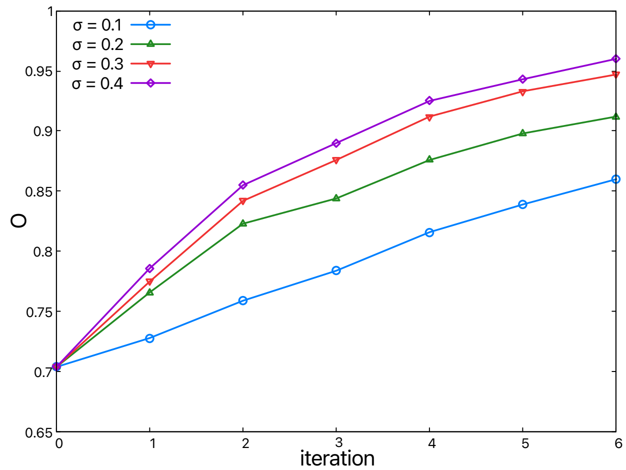


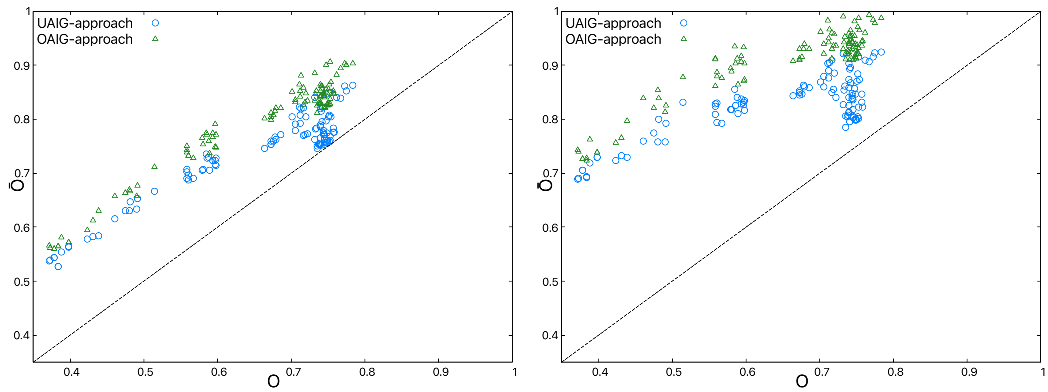

Figure 3: Performance of the interactive process for different values of $\sigma$.

higher the probability of accepting the values returned by the automatic approach, the greater the increment of the values of $O$, i.e., fewer iterations are required to achieve great values of $O$. It seems intuitive as the values returned by the automatic process are the ones that better optimize the fuzzy preference relations to maximize the value of the optimization criterion. Therefore, the higher the number of modifications accepted by the decision-makers, the greater the probability of achieving a greater value of $O$.

### *4.5. Comparative analysis*

The effectiveness and superiority of the granular-based approach based upon a non-uniform allocation of information granularity, referred to OAIG-approach for comparison purposes, is corroborated by conducting two experiments in which this approach is compared with the granularity-based approach presented by Cabrerizo et al. (2014), which improves the consistency and consensus in GDM by distributing a uniform level of information granularity in each component of the fuzzy preference relations (viz., UAIG-approach).

Firstly, we conduct an experiment in which 100 multi-criteria GDM problems with different number of alternatives, decision-makers, and criteria, are randomly generated. We set $\chi^h$ to 0.2 and 0.4. With the objective of improving the consensus and the consistency in these multi-criteria GDM problems, both the UAIG-approach and the OAIG-approach are executed to obtain the modified fuzzy preference relations. Figs. 4a and 4b depict the relation



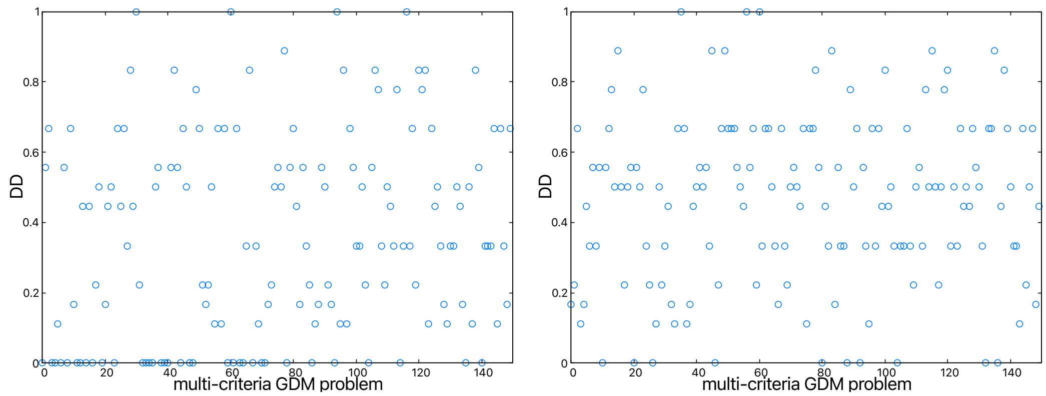

(a) $\chi^h = 0.2$. (b) $\chi^h = 0.4$.

Figure 4: Results achieved by the proposed OAIG-approach vs UAIG-approach.

between the optimized and the original result of the optimization criterion, i.e., $\overline{O}$ and $O$, respectively. On the one hand, it is clear that the values of $\overline{O}$ reached by the UAIG- and the OAIG-approaches are greater than the values of $O$. It means both approaches are able to improve both the consensus and the consistency. On the other hand, it can also be noted that the values achieved by the proposed OAIG-approach are clearly greater than the ones achieved by the UAIG-approach. This verifies the superiority of the OAIG-approach.

Secondly, to verify if the rankings of the alternatives produced by the proposed OAIG-approach and the UAIG-approach are different, we have simulated 150 multi-criteria GDM problems with different number of alternatives, decision-makers, and criteria. In addition, we set $\chi^h$ to 0.2 and 0.3. Let $R^1$ and $R^2$ be the rankings of the alternatives according to the modified fuzzy preference relations obtained by the proposed OAIG-approach and the UAIG-approach, respectively, when applying the selection process presented in Section 3.2 with $Q(a) = a^{1/2}$ representing the linguistic quantifier "most" utilized in Eq. (2) to produce the weights of the OWA and IOWA operators. The $l_1$-norm distance between $R^1$ and $R^2$ is calculated to measure the decision discrepancy, $DD$, between the proposed OAIG-approach and the UAIG-approach:

$$DD = \frac{1}{m} \sum_{i=0}^{m} \left( \left| r_i^1 - r_i^2 \right| \right) \tag{24}$$




(a) $\chi^h = 0.2$.        (b) $\chi^h = 0.3$.

Figure 5: Decision discrepancy between the OAIG- and UAIG- approaches.

Figs. 5a and 5b give evidence of the decision discrepancy between the proposed OAIG-approach and the UAIG-approach. As it can be noticed, the decision discrepancy values are greater than 0. It means the proposed OAIG-approach can change the decisions achieved by the UAIG-approach. Particularly, the greater the value of $\chi^h$, the lower the number of multi-criteria GDM problems having a $DD$ equal to 0, i.e., problems with the same decisions achieved by both approaches.

To put the obtained results by the proposed granular-based approach in a certain context, we have compared them with the results obtained by a granular-based approach based on a uniform distribution of information granularity. It has been possible because both approaches use fuzzy preference relations to represent the decision-makers' evaluations and try to improve both the consistency and the consensus in multi-criteria GDM scenarios. However, a comparison of the results of a GDM approach with other approaches usually is not a straightforward task because the approaches use different structures and domains of evaluation representation, or they deal with different aspects of the GDM process. As a result, a quantitative comparison would not be meaningful. Anyway, we next elaborate on the advantages of the proposed granular-based approach in comparison with the existing granular-based approaches based on a non-uniform distribution of information granularity. The approaches presented by Cabrerizo et al. (2022a), Cabrerizo et al. (2022b), and Cabrerizo et al. (2023), neither improve both the consistency and the consensus nor allow the participation of the decision-makers in the modification process. The approaches developed by Zhang et al. (2022) and Zhang et al. (2023), even though they improve both the consistency and the consensus and allow the participation of the decision-makers in the modification process, cannot handle multi-criteria GDM settings. The approaches introduced by Qin et al. (2023b) and Qin et al. (2023a), even though they improve both the consensus and the consistency in multi-criteria settings, do not allow the participation of the decision-makers in the modification process. Different from these approaches, the proposed granular-based approach improves both the consistency and the consensus in multi-criteria GDM scenarios and allow the participation of the decision-makers in the modification process.

## 5. A case study of building refurbishment

This section presents a case study on reconstructing a non-residential building into a residential building (Velykorusova, 2023), which is conducted by using the proposed approach. The building under analysis was built before 1917 and was reconstructed by LLC "Engineering and Construction" in 2019 (see Fig. 4). It is located at Nizhniy Val Street in the Podilskyi district of Kyiv, a densely built urban central historical territory that falls within the archaeological protection zone. The object of study is the reconstruction of three floors of a brick building (out of eight floors) with an unambiguous mountain roof type and metal tiles. The maximum public dimensions are 46,940 x 9,400 m. The building structure consists of load-bearing living and transverse brick walls, monolithic on metal beams, wooden beams, and cork floor. The partitions of the building are brick.

The project provides for the exterior decoration of the house, installation of metal-plastic windows, ventilation (air conditioners), a water supply system, and heating and roof tiles in accordance with the project task. However, the project does not provide for the involvement or transfer of any structure located on the design site or the removal of existing greenery. Therefore, according to the case study of the project, the water supply system, heating, ventilation, windows, and roof, had to be renovated in the building. Among them, we focus here on the ventilation.

The ventilation of a residential building is natural and mechanical. The building is equipped with plastic windows with the S710 profile REHAU, which is already worn out. Therefore, ventilation is poor. For the purpose of creating and ensuring parameters of the air environment in the building established by the current norms, supply and exhaust ventilation are provided,

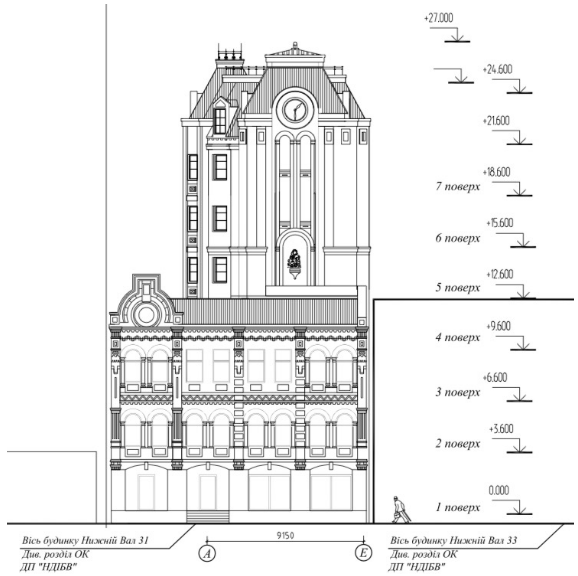



Figure 6: Facade A–E of the building under analysis.

with mechanical motivation. The capacity of ventilation systems is determined by the normative exchange of indoor air and on a per capita basis. The project provides for the possibility of installing outdoor air conditioning units. The air ducts are made of galvanized sheet steel.

Next, the determination of significance, degree of usefulness, and priority of the renovated ventilation is analyzed. The project of ventilation of the object is developed on the basis of the following initial data:

- Architectural - construction drawings;
- Current building codes and regulations:
  - DBN B.2.5-67: 2013 “Heating, ventilation, and air conditioning”;
  - Estimated winter temperature for heating and ventilation design - 22C;
  - Estimated summer temperature for ventilation design +23C;
- Estimated wind speed:
  - in the warm season - 2.1 m/s;
  - in the cold season - 2.8 m/s.

Estimated air circulation in general and auxiliary rooms is determined according to the normative multiplicity of air exchange. In premises with the allocation of harmful air, circulation is designed for their assimilation. The consumption of outdoor air of administrative rooms is calculated in accordance with DBN B.2.5-67: 2013. Air supply in the parking lot is expected to be concentrated in the driveway. Air removal is carried out equally from the lower and upper zones. Ventilation equipment of the parking lot is installed in the ventilation chambers of the basement. Excess consumption of exhaust air over supply according to item 8.39, DBN B.2.3-15-2007 is provided. In administrative rooms, it is envisaged to reduce the productivity of ventilation systems to a single air exchange during non-working hours in order to save energy resources. Air ducts of supply and exhaust systems are laid hidden in ventilation shafts and behind architectural constructions. Air ducts are designed from galvanized steel according to GOST 19904-74 and density class according to DBN B.2.5-67-2013.

Exhaust emissions are carried out in architectural mines. The bottom of the holes in the architectural mines is 1.0 m from the roof level of the building. The air intake is carried out at a height of 2.0 m from ground level, and the distance from the air intake to the air outlet is no less than 8.0 m. To reduce the noise of ventilation systems, the following measures are provided: (i) Installation of pumps and fans on vibration-insulating bases, (ii) connection of fans and air ducts on flexible inserts, and (iii) installation of mufflers.

The alternative ventilation of five companies was analyzed according to four indicators with the same importance weights. The following companies were chosen for ventilation: Danfoss, Denmark ($x_1$); KERMI, Germany ($x_2$); BT-Service, Ukraine ($x_3$); VIKMA LTD, Ukraine ($x_4$); and Behtc, Ukraine ($x_5$). And the following indicators were considered: Level of noise ($a_1$); durability ($a_2$); price ($a_3$); and max pressure ($a_4$). Four ventilation specialists were asked for their preferences. They provided the following ones:

$$P^{1,1} = \begin{bmatrix} - & 0.72 & 0.29 & 0.51 & 0.51 \\ 0.05 & - & 0.57 & 0.50 & 0.94 \\ 0.68 & 0.19 & - & 0.50 & 0.51 \\ 0.82 & 0.29 & 0.28 & - & 0.95 \\ 0.43 & 0.00 & 0.39 & 0.01 & - \end{bmatrix} P^{1,2} = \begin{bmatrix} - & 0.66 & 0.76 & 0.07 & 0.51 \\ 0.28 & - & 0.08 & 0.42 & 0.70 \\ 0.15 & 0.83 & - & 0.31 & 0.32 \\ 0.59 & 0.52 & 0.60 & - & 0.51 \\ 0.47 & 0.05 & 0.65 & 0.47 & - \end{bmatrix}$$

$$P^{1,3} = \begin{bmatrix} - & 0.85 & 0.74 & 0.54 & 0.83 \\ 0.10 & - & 0.54 & 0.72 & 0.74 \\ 0.06 & 0.01 & - & 0.65 & 0.73 \\ 0.19 & 0.15 & 0.29 & - & 0.76 \\ 0.15 & 0.12 & 0.19 & 0.02 & - \end{bmatrix} P^{1,4} = \begin{bmatrix} - & 0.72 & 0.60 & 0.69 & 0.83 \\ 0.09 & - & 0.52 & 0.08 & 0.24 \\ 0.02 & 0.00 & - & 0.58 & 0.88 \\ 0.08 & 0.59 & 0.11 & - & 0.78 \\ 0.11 & 0.64 & 0.04 & 0.12 & - \end{bmatrix}$$

$$P^{2,1} = \begin{bmatrix} - & 0.60 & 0.75 & 0.18 & 0.92 \\ 0.32 & - & 0.50 & 0.68 & 0.35 \\ 0.04 & 0.34 & - & 0.02 & 0.60 \\ 0.51 & 0.08 & 0.50 & - & 0.08 \\ 0.04 & 0.52 & 0.25 & 0.74 & - \end{bmatrix} P^{2,2} = \begin{bmatrix} - & 0.17 & 0.30 & 0.17 & 0.76 \\ 0.71 & - & 0.65 & 0.59 & 0.74 \\ 0.53 & 0.18 & - & 0.66 & 0.80 \\ 0.58 & 0.40 & 0.01 & - & 0.14 \\ 0.21 & 0.16 & 0.02 & 0.51 & - \end{bmatrix}$$

$$P^{2,3} = \begin{bmatrix} - & 0.78 & 0.25 & 0.32 & 0.73 \\ 0.04 & - & 0.60 & 0.12 & 0.10 \\ 0.54 & 0.36 & - & 0.52 & 0.53 \\ 0.54 & 0.50 & 0.01 & - & 0.14 \\ 0.09 & 0.61 & 0.36 & 0.64 & - \end{bmatrix} P^{2,4} = \begin{bmatrix} - & 0.59 & 0.15 & 0.78 & 0.78 \\ 0.34 & - & 0.36 & 0.23 & 0.16 \\ 0.68 & 0.52 & - & 0.50 & 0.29 \\ 0.03 & 0.54 & 0.40 & - & 0.33 \\ 0.16 & 0.75 & 0.56 & 0.62 & - \end{bmatrix}$$

$$P^{3,1} = \begin{bmatrix} - & 0.13 & 0.16 & 0.06 & 0.02 \\ 0.80 & - & 0.73 & 0.02 & 0.79 \\ 0.81 & 0.15 & - & 0.71 & 0.69 \\ 0.85 & 0.63 & 0.14 & - & 0.51 \\ 0.84 & 0.16 & 0.25 & 0.10 & - \end{bmatrix} P^{3,2} = \begin{bmatrix} - & 0.02 & 0.10 & 0.01 & 0.77 \\ 0.72 & - & 0.11 & 0.59 & 0.00 \\ 0.83 & 0.86 & - & 0.01 & 0.77 \\ 0.55 & 0.09 & 0.88 & - & 0.56 \\ 0.17 & 0.93 & 0.12 & 0.07 & - \end{bmatrix}$$

$$P^{3,3} = \begin{bmatrix} - & 0.73 & 0.25 & 0.78 & 0.62 \\ 0.87 & - & 0.67 & 0.82 & 0.59 \\ 0.73 & 0.30 & - & 0.64 & 0.15 \\ 0.01 & 0.03 & 0.15 & - & 0.60 \\ 0.36 & 0.19 & 0.51 & 0.10 & - \end{bmatrix} P^{3,4} = \begin{bmatrix} - & 0.53 & 0.28 & 0.09 & 0.91 \\ 0.16 & - & 0.52 & 0.75 & 0.79 \\ 0.51 & 0.24 & - & 0.12 & 0.05 \\ 0.76 & 0.20 & 0.72 & - & 0.02 \\ 0.00 & 0.15 & 0.61 & 0.78 & - \end{bmatrix}$$

$$P^{4,1} = \begin{bmatrix} - & 0.57 & 0.19 & 0.07 & 0.07 \\ 0.31 & - & 0.52 & 0.21 & 0.12 \\ 0.70 & 0.05 & - & 0.75 & 0.70 \\ 0.54 & 0.62 & 0.11 & - & 0.86 \\ 0.57 & 0.74 & 0.04 & 0.02 & - \end{bmatrix} P^{4,2} = \begin{bmatrix} - & 0.28 & 0.51 & 0.17 & 0.73 \\ 0.62 & - & 0.34 & 0.67 & 0.37 \\ 0.10 & 0.61 & - & 0.66 & 0.35 \\ 0.80 & 0.26 & 0.26 & - & 0.17 \\ 0.20 & 0.57 & 0.63 & 0.75 & - \end{bmatrix}$$

$$P^{4,3} = \begin{bmatrix} - & 0.82 & 0.58 & 0.10 & 0.31 \\ 0.04 & - & 0.32 & 0.84 & 0.19 \\ 0.64 & 0.55 & - & 0.73 & 0.27 \\ 0.81 & 0.02 & 0.01 & - & 0.06 \\ 0.58 & 0.59 & 0.53 & 0.92 & - \end{bmatrix} P^{4,4} = \begin{bmatrix} - & 0.55 & 0.78 & 0.53 & 0.50 \\ 0.41 & - & 0.53 & 0.85 & 0.54 \\ 0.18 & 0.35 & - & 0.17 & 0.00 \\ 0.03 & 0.04 & 0.64 & - & 0.50 \\ 0.22 & 0.13 & 0.64 & 0.02 & - \end{bmatrix}$$

According to these fuzzy preference relations, the consistency degrees of the ventilation specialists are: $cd^1 = 0.823$, $cd^2 = 0.822$, $cd^3 = 0.776$, and

$cd^4 = 0.818$. It means the global consistency degree achieved is equal to 0.809. Meanwhile, the consensus achieved is 0.687. Because the same importance is given to the consensus and the consistency ($\rho = 0.5$), the value of the optimization criterion, $O$, is 0.748. Considering that a minimum value of 0.80 for $O$ is required before applying the selection process, the ventilation specialist must reconsider their preferences to increase it. To do so, the proposed granular-based approach is carried out.

To cooperate with the increasing of the consensus and the consistency, each ventilation specialist admits an average level of information granularity equal to 0.2. Using the same values for the parameters of the PSO than in the example performed in Section 4.1, the following modified fuzzy preference relations are obtained:

$$\overline{P}^{1,1} = \begin{bmatrix} - & 0.43 & 0.29 & 0.38 & 0.50 \\ 0.18 & - & 0.54 & 0.34 & 0.81 \\ 0.67 & 0.22 & - & 0.50 & 0.58 \\ 0.68 & 0.44 & 0.29 & - & 0.79 \\ 0.51 & 0.24 & 0.24 & 0.05 & - \end{bmatrix} \overline{P}^{1,2} = \begin{bmatrix} - & 0.49 & 0.82 & 0.21 & 0.61 \\ 0.42 & - & 0.22 & 0.45 & 0.59 \\ 0.36 & 0.82 & - & 0.36 & 0.41 \\ 0.54 & 0.45 & 0.57 & - & 0.59 \\ 0.23 & 0.08 & 0.48 & 0.40 & - \end{bmatrix}$$

$$\overline{P}^{1,3} = \begin{bmatrix} - & 0.80 & 0.53 & 0.50 & 0.69 \\ 0.28 & - & 0.51 & 0.74 & 0.68 \\ 0.17 & 0.13 & - & 0.65 & 0.53 \\ 0.06 & 0.23 & 0.18 & - & 0.62 \\ 0.23 & 0.15 & 0.32 & 0.00 & - \end{bmatrix} \overline{P}^{1,4} = \begin{bmatrix} - & 0.73 & 0.56 & 0.60 & 0.82 \\ 0.30 & - & 0.49 & 0.13 & 0.35 \\ 0.26 & 0.11 & - & 0.54 & 0.55 \\ 0.20 & 0.58 & 0.26 & - & 0.51 \\ 0.12 & 0.61 & 0.12 & 0.09 & - \end{bmatrix}$$

$$\overline{P}^{2,1} = \begin{bmatrix} - & 0.55 & 0.62 & 0.15 & 0.75 \\ 0.32 & - & 0.46 & 0.65 & 0.56 \\ 0.08 & 0.21 & - & 0.14 & 0.69 \\ 0.51 & 0.26 & 0.29 & - & 0.17 \\ 0.21 & 0.40 & 0.21 & 0.70 & - \end{bmatrix} \overline{P}^{2,2} = \begin{bmatrix} - & 0.25 & 0.22 & 0.24 & 0.54 \\ 0.72 & - & 0.47 & 0.59 & 0.76 \\ 0.58 & 0.32 & - & 0.56 & 0.67 \\ 0.64 & 0.29 & 0.18 & - & 0.45 \\ 0.21 & 0.15 & 0.15 & 0.39 & - \end{bmatrix}$$

$$\overline{P}^{2,3} = \begin{bmatrix} - & 0.78 & 0.25 & 0.50 & 0.73 \\ 0.12 & - & 0.46 & 0.31 & 0.16 \\ 0.43 & 0.48 & - & 0.52 & 0.52 \\ 0.35 & 0.37 & 0.13 & - & 0.27 \\ 0.30 & 0.49 & 0.43 & 0.54 & - \end{bmatrix} \overline{P}^{2,4} = \begin{bmatrix} - & 0.48 & 0.44 & 0.59 & 0.80 \\ 0.22 & - & 0.45 & 0.57 & 0.21 \\ 0.52 & 0.51 & - & 0.51 & 0.23 \\ 0.07 & 0.49 & 0.43 & - & 0.54 \\ 0.19 & 0.61 & 0.55 & 0.58 & - \end{bmatrix}$$

$$\overline{P}^{3,1} = \begin{bmatrix} - & 0.29 & 0.18 & 0.08 & 0.14 \\ 0.71 & - & 0.56 & 0.11 & 0.75 \\ 0.82 & 0.26 & - & 0.36 & 0.59 \\ 0.61 & 0.55 & 0.16 & - & 0.53 \\ 0.77 & 0.26 & 0.27 & 0.04 & - \end{bmatrix} \overline{P}^{3,2} = \begin{bmatrix} - & 0.10 & 0.16 & 0.00 & 0.60 \\ 0.62 & - & 0.11 & 0.59 & 0.28 \\ 0.74 & 0.85 & - & 0.27 & 0.74 \\ 0.59 & 0.02 & 0.63 & - & 0.56 \\ 0.27 & 0.81 & 0.06 & 0.25 & - \end{bmatrix}$$

$$\overline{P}^{3,3} = \begin{bmatrix} - & 0.73 & 0.41 & 0.72 & 0.71 \\ 0.75 & - & 0.46 & 0.66 & 0.58 \\ 0.57 & 0.47 & - & 0.79 & 0.37 \\ 0.04 & 0.10 & 0.12 & - & 0.64 \\ 0.28 & 0.22 & 0.46 & 0.21 & - \end{bmatrix} \quad \overline{P}^{3,4} = \begin{bmatrix} - & 0.35 & 0.32 & 0.22 & 0.80 \\ 0.27 & - & 0.57 & 0.56 & 0.64 \\ 0.44 & 0.25 & - & 0.17 & 0.19 \\ 0.75 & 0.20 & 0.66 & - & 0.17 \\ 0.16 & 0.28 & 0.56 & 0.63 & - \end{bmatrix}$$

$$\overline{P}^{4,1} = \begin{bmatrix} - & 0.50 & 0.19 & 0.13 & 0.21 \\ 0.30 & - & 0.31 & 0.20 & 0.28 \\ 0.76 & 0.26 & - & 0.41 & 0.67 \\ 0.55 & 0.53 & 0.29 & - & 0.82 \\ 0.62 & 0.48 & 0.03 & 0.00 & - \end{bmatrix} \quad \overline{P}^{4,2} = \begin{bmatrix} - & 0.16 & 0.32 & 0.17 & 0.63 \\ 0.49 & - & 0.33 & 0.62 & 0.35 \\ 0.16 & 0.57 & - & 0.37 & 0.39 \\ 0.58 & 0.32 & 0.21 & - & 0.31 \\ 0.24 & 0.45 & 0.51 & 0.56 & - \end{bmatrix}$$

$$\overline{P}^{4,3} = \begin{bmatrix} - & 0.69 & 0.55 & 0.19 & 0.32 \\ 0.19 & - & 0.41 & 0.70 & 0.28 \\ 0.43 & 0.53 & - & 0.74 & 0.30 \\ 0.59 & 0.19 & 0.09 & - & 0.22 \\ 0.51 & 0.54 & 0.52 & 0.78 & - \end{bmatrix} \quad \overline{P}^{4,4} = \begin{bmatrix} - & 0.54 & 0.84 & 0.55 & 0.73 \\ 0.36 & - & 0.54 & 0.49 & 0.54 \\ 0.26 & 0.28 & - & 0.32 & 0.18 \\ 0.02 & 0.10 & 0.53 & - & 0.46 \\ 0.21 & 0.31 & 0.55 & 0.23 & - \end{bmatrix}$$

The ventilation specialists accept the modified fuzzy preference relations returned by the automatic process. With them, the global consistency degree achieved is 0.877 while the consensus reached is 0.797. It means the value of the optimization criterion, $O$, is 0.834, which is greater than the required minimum value (0.8). Therefore, the selection process can be applied.

To generate the weights of the OWA and IOWA operators, the linguistic quantifier "most" defined by $Q(a) = a^{1/2}$ is utilized. Applying the selection process, the following collective fuzzy preference relation is obtained:

$$CP = \begin{bmatrix} - & 0.49 & 0.42 & 0.31 & 0.56 \\ 0.35 & - & 0.45 & 0.43 & 0.58 \\ 0.48 & 0.34 & - & 0.44 & 0.54 \\ 0.49 & 0.38 & 0.31 & - & 0.56 \\ 0.37 & 0.32 & 0.29 & 0.25 & - \end{bmatrix}$$

Based upon this collective fuzzy preference relation, the following choice degrees are obtained after the application of the exploitation phase:

$$QNGDD_1 = 0.89 \qquad QGNDD_2 = 0.92 \qquad QGNDD_3 = 0.94$$
$$QNGDD_2 = 0.92 \qquad QGNDD_2 = 0.72$$

According to these choice degrees, the BT-Service company ($x_3$) was selected for renovating the ventilation system.

## 6. Concluding remarks and future research directions

To resolve multi-criteria GDM problems with fuzzy preference relations, this study has proposed a granular-based multi-criteria GDM approach whose objective is to maximize the consistency and the consensus while minimizing the information loss. To achieve this goal, first, we have built an automatic process of consistency and consensus improvement that assumes an average level of information granularity to allocate it in a non-uniform way among the entries of the fuzzy preference relations. There exists a mismatch between the information loss and the consistency and consensus improvement, i.e., an increment of the consistency and consensus degrees also means an increment of the information loss. Nevertheless, the average level of information granularity, which is bounded, allows to maximize the consensus and the consistency with a limited information loss between the first and the modified fuzzy preference relations. Then, taking as a basis this process, we have introduced an interactive process whose main characteristic is the involvement of the group members when modifying the fuzzy preference relations.

The numerical experiments and the case study conducted have illustrated the performance and effectiveness of the proposed granular-based approach to solve multi-criteria GDM problems. In comparison with the approaches based upon a uniform distribution of information granularity, the proposed granular-based approach achieves greater values for the consensus and consistency. It is due to a non-uniform allocation allows for a more efficient use of the resources by allocating higher levels of granularity to the entries of the fuzzy preference relations that require it the most. It leads to better decision-making outcomes as higher consistency and consensus degrees are achieved. These results can be extrapolated to other contexts in which the granular models can take advantage of a non-uniform distribution of information granularity to offer better results, e.g., in time series, neural networks, or data compression (Liu et al., 2018b; Pedrycz, 2023; Song et al., 2019).

Based upon this study, these future research directions emerge as a first step at attempting to improve it:

- The proposed granular-based multi-criteria GDM approach makes use of the PSO. This algorithm has been utilized as some related approaches have shown that it serves an appropriate optimization framework (Cabrerizo et al., 2014; Liu et al., 2018a). Nevertheless, even though this algorithm maximizes the value returned by the fitness function, it does not guarantee an optimal result (we may refer to it as the best solution obtained by the PSO). Other related approaches have employed the differential evolution (Cabrerizo et al., 2022b; Zhang et al., 2022). However, like the PSO, it is metaheuristic algorithm that may also end up in a local optimum. Therefore, to solve this shortcoming, it is necessary to make use of an optimization algorithm guaranteeing that the global optimum is reached.

- The average level of information granularity admitted by the decision-maker is a synonym of her/his flexibility, i.e., the higher the admitted value, the higher the decision-maker's cooperation to improve consensus and consistency. In the examples performed in this study, we have assumed that all decision-makers admit the same average level of information granularity. However, a complete analysis of the decision-makers' behaviour should be performed to recognize the factors that influence the decision-maker's behaviour and deal with non-cooperative behaviours (Wu et al., 2021; You et al., 2023).

## Acknowledgments

This work was supported by the program "Ayudas para la recualificación del sistema universitario español para 2021–2023" funded by the Ministerio de Universidades and Unión Europea - NextGenerationEU, by the grant PID2022-139297OB-I00 funded by MCIN / AEI / 10.13039/501100011033 and by "ERDF A way of making Europe", and by the project No. 2020-1-LT01-KA203-078100 "Minimizing the influence of coronavirus in a built environment" (MICROBE) from the European Union's Erasmus + program.

## References

Amiri, M., Hashemi-Tabatabaei, M., Ghahremanloo, M., Keshavarz-Ghorabaee, M., Zavadskas, E.K., Antucheviciene, J., 2021. A novel model for multi-criteria assessment based on BWM and possibilistic chance-constrained programming. Comput. Ind. Eng. 156, 107287.

Amiri, M., Hashemi-Tabatabaei, M., Keshavarz-Ghorabaee, M., Kaklauskas, A., Zavadskas, E.K., Antucheviciene, J., 2023. A fuzzy extension of simplified best-worst method (F-SBWM) and its applications to decision-making problems. Symmetry 15, 81.

Bargiela, A., Pedrycz, W., 2003. Granular Computing: An Introduction. Kluwer Academic Publishers, Dordrecht.

Boix-Cots, D., Pardo-Bosch, F., Pujadas, P., 2023. A systematic review on multi-criteria group decision-making methods based on weights: Analysis and classification scheme. Inf. Fusion 96, 16–36.

Bustince, H., Berrenechea, E., Pagola, M., Fernández, J., Xu, Z.S., Bedregal, B., Montero, J., Hagras, H., Herrera, F., De Baets, B., 2016. A historical account of types of fuzzy sets and their relationships. IEEE Trans. Fuzzy Syst. 24, 179–194.

Butler, C.T., Rothstein, A., 2006. On Conflict and Consensus: A Handbook on Formal Consensus Decision Making. Food Not Bombs Publishing, Takoma Park, WA.

Cabrerizo, F.J., Al-Hmouz, R., Morfeq, A., Balamash, A.S., Martínez, M.A., Herrera-Viedma, E., 2017. Soft consensus measures in group decision making using unbalanced fuzzy linguistic information. Soft Comput. 21, 3037–3050.

Cabrerizo, F.J., Chiclana, F., Al-Hmouz, R., Morfeq, A., Balamash, A.S., Herrera-Viedma, E., 2015. Fuzzy decision making and consensus: Challenges. J. Intell. Fuzzy Syst. 29, 1109–1118.

Cabrerizo, F.J., González-Quesada, J.C., Herrera-Viedma, E., Kaklauskas, A., Pedrycz, W., 2022a. Managing inconsistency with an optimal distribution of information granularity in fuzzy preference relations, in: Proc. IEEE Int. Conf. Syst. Man Cybern. (SMC), Prague, Czech Republic. pp. 359–364.

Cabrerizo, F.J., González-Quesada, J.C., Morente-Molinera, J.A., Pérez, I.J., Herrera-Viedma, E., Pedrycz, W., 2022b. An improvement of consensus in group decision-making through an optimal distribution of information granularity, in: Proc. IEEE Symp. Ser. Comput. Intell. (SSCI), Singapur. pp. 119–124.

Cabrerizo, F.J., Kaklauskas, A., Pérez, I.J., Herrera-Viedma, E., 2023. A granular-based approach to address multiplicative consistency of reciprocal preference relations in decision-making, in: Proc. 56th Hawaii Int. Conf. Syst. Sci. (HICSS), Maui, Hawaii, USA. pp. 1541–1550.

Cabrerizo, F.J., Moreno, J.M., Pérez, I.J., Herrera-Viedma, E., 2010. Analyzing consensus approaches in fuzzy group decision making: Advantages and drawbacks. Soft Comput. 14, 451–463.

Cabrerizo, F.J., Morente-Molinera, J.A., Alonso, S., Pedrycz, W., Herrera-Viedma, E., 2018a. Improving consensus in group decision making with intuitionistic reciprocal preference relations: A granular computing approach, in: Proc. IEEE Int. Conf. Syst. Man Cybern. (SMC), Miyazaki, Japan. pp. 1471–1476.

Cabrerizo, F.J., Morente-Molinera, J.A., Pedrycz, W., Taghavi, A., Herrera-Viedma, E., 2018b. Granulating linguistic information in decision making under consensus and consistency. Expert Syst. Appl. 99, 83–92.

Cabrerizo, F.J., Pérez, I.J., Morente-Molinera, J.A., Alonso, S., Herrera-Viedma, E., 2021. A granular computing based approach for improving the consistency of intuitionistic reciprocal preference relations, in: Shahbazova, S., Kacprzyk, J., Balas, V., Kreinovich, V. (Eds.), Recent Developments and the New Direction in Soft-Computing Foundations and Applications. Springer, Cham, pp. 457–469.

Cabrerizo, F.J., Ureña, R., Pedrycz, W., Herrera-Viedma, E., 2014. Building consensus in group decision making with an allocation of information granularity. Fuzzy Sets Syst. 255, 115–127.

Cai, Y., Jin, F., Liu, J., Zhou, L., Tao, Z., 2023. A survey of collaborative decision-making: Bibliometrics, preliminaries, methodologies, applications and future directions. Eng. Appl. Artif. Intell. 122, 106064.

Callejas, E.A., Cerrada, J.A., Cerrada, C., Cabrerizo, F.J., 2019. Group decision making based on a framework of granular computing for multi-criteria and linguistic contexts. IEEE Access 7, 54670–54681.

Chao, X., Dong, Y.C., Kou, G., Peng, Y., 2022. How to determine the consensus threshold in group decision making: a method based on efficiency benchmark using benefit and cost insight. Ann. Oper. Res. 316, 143–177.

Chiclana, F., Herrera-Viedma, E., Alonso, S., Herrera, F., 2009. Cardinal consistency of reciprocal preference relations: A characterization of multiplicative consistency. IEEE Trans. Fuzzy Syst. 17, 14–23.

Flori, A., Oulhadj, H., Siarry, P., 2022. QUAntum Particle Swarm Optimization: an auto-adaptive PSO for local and global optimization. Comput. Optim. Appl. 82, 525–559.

Gelderloos, P., 2006. Consensus: A New Handbook for Grassroots Social, Political, and Environmental Groups. See Sharp Press, Tucson, AZ.

Gong, Z., Xu, X., Guo, W., Herrera-Viedma, E., Cabrerizo, F.J., 2021. Minimum cost consensus modeling under various linear uncertain-constrained scenarios. Inf. Fusion 66, 1–17.

Herrera-Viedma, E., Cabrerizo, F.J., Kacprzyk, J., Pedrycz, W., 2014. A review of soft consensus models in a fuzzy environment. Inf. Fusion 17, 4–13.

Herrera-Viedma, E., Chiclana, F., Herrera, F., Alonso, S., 2007. A group decision-making model with incomplete fuzzy preference relations based on additive consistency. IEEE Trans. Syst. Man Cybern. Part B-Cybern. 37, 176–189.

Herrera-Viedma, E., Herrera, F., Chiclana, F., Luque, M., 2004. Some issues on consistency of fuzzy preference relations. Eur. J. Oper. Res. 154, 98–109.

Herrera-Viedma, E., Palomares, I., Li, C.C., Cabrerizo, F.J., Dong, Y.C., Chiclana, F., Herrera, F., 2021. Revisiting fuzzy and linguistic decision making: Scenarios and challenges for making wiser decisions in a better way. IEEE Trans. Syst. Man Cybern. -Syst. 51, 191–208.

Huang, Z., Yang, C., Zhou, X., Gui, W., 2023. An improved TOPSIS-based multi-criteria decision-making approach for evaluating the working condition of the aluminum reduction cell. Eng. Appl. Artif. Intell. 117, 105599.

Jain, L.C., Lim, C.P., 2010. Handbook on Decision Making: Techniques and Applications. Springer-Verlag, Berlin.

Jin, F., Cai, Y., Pedrycz, W., Liu, J., 2022. Efficiency evaluation with regret-rejoice cross-efficiency DEA models under distributed linguistic environment. Comput. Ind. Eng. 169, 108281.

Jin, F., Cai, Y., Zhou, L., Ding, T., 2023. Regret-rejoice two-stage multiplicative DEA models-driven cross-efficiency evaluation with probabilistic linguistic information. Omega 117, 102839.

Kacprzyk, J., Fedrizzi, M., 1986. Group decision making with a fuzzy linguistic majority. Fuzzy Sets Syst. 18, 105–118.

Kacprzyk, J., Fedrizzi, M., Nurmi, H., 1992. Fuzzy logic with linguistic quantifiers in group decision making, in: Yager, R.R., Zadeh, L.A. (Eds.), An Introduction to Fuzzy Logic Applications in Intelligent Systems. Springer, Boston, MA, pp. 263–280.

Kennedy, J., Eberhart, R.C., 1995. Particle swarm optimization, in: 1995 IEEE International Conference on Neural Networks, Perth, Australia. pp. 1942–1948.

Li, C.C., Dong, Y.C., Xu, Y., Chiclana, F., Herrera-Viedma, E., Herrera, F., 2019. An overview on managing additive consistency of reciprocal preference relations for consistency-driven decision making and fusion: Taxonomy and future directions. Inf. Fusion 52, 143–156.

Liao, H., Zhang, Z., Xu, Z., Banaitis, A., 2022. A heterogeneous regret-theory-based method with Choquet integral to multiattribute reverse auction. IEEE Trans. Eng. Manag. 69, 2248–2259.

Liu, F., Wu, Y., Pedrycz, W., 2018a. A modified consensus model in group decision making with an allocation of information granularity. IEEE Trans. Fuzzy Syst. 26, 3182–3187.

Liu, J., Bao, A., Jin, F., Zhou, L., Shao, L., 2023. Group decision-making with multiplicative probabilistic linguistic preference relations based on consistency improvement and upgraded multiplicative DEA cross-efficiency. J. Intell. Fuzzy Syst. 44, 9395–9410.

Liu, J., Shao, L., Jin, F., Tao, Z., in press. A multi-attribute group decision-making method based on trust relationship and DEA regret cross-efficiency. IEEE Trans. Eng. Manag. .

Liu, S., Pedrycz, W., Gacek, A., Dai, Y., 2018b. Development of information granules of higher type and their applications to granular models of time series. Eng. Appl. Artif. Intell. 71, 60–72.

Meng, F., Zhao, D., Zhang, X., 2023. A fair consensus adjustment mechanism for large-scale group decision making in term of Gini coefficient. Eng. Appl. Artif. Intell. 126, Part C, 106962.

Millet, I., 1997. The effectiveness of alternative preference elicitation methods in the analytic hierarchy process. J. Multi-Criteria Decis. Anal. 6, 41–51.

Mousavi, S.M., Foroozesh, N., Zavadskas, E.K., Antucheviciene, J., 2020. A new soft computing approach for green supplier selection problem with interval type-2 trapezoidal fuzzy statistical group decision and avoidance of information loss. Soft Comput. 24, 12313–12327.

Pedrycz, W., 2014. Allocation of information granularity in optimization and decision-making models: Towards building the foundations of Granular Computing. Eur. J. Oper. Res. 232, 137–145.

Pedrycz, W., 2023. Granular data compression and representation. IEEE Trans. Fuzzy Syst. 31, 1497–1505.

Pedrycz, W., Song, M., 2011. Analytic hierarchy process (AHP) in group decision making and its optimization with an allocation of information granularity. IEEE Trans. Fuzzy Syst. 19, 527–539.

Qin, J., Ma, X., Liang, Y., 2023a. Building a consensus for the best-worst method in group decision-making with an optimal allocation of information granularity. Inf. Sci. 619, 630–653.

Qin, J., Ma, X., Pedrycz, W., 2023b. A granular computing-driven best-worst method for supporting group decision making. IEEE Trans. Syst. Man Cybern. Syst. 53, 5591–5603.

Qin, J., Martinez, L., Pedrycz, W., Ma, X., Liang, Y., 2023c. An overview of granular computing in decision-making: Extensions, applications, and challenges. Inf. Fusion 98, 101833.

Song, M., Jing, Y., Pedrycz, W., 2019. Granular neural networks: A study of optimizing allocation of information granularity in input space. Soft Computing 77, 67–75.

Truman, M.I., 2017. The Art of Consensus Decision Making: Building a Consensus. CreateSpace Independent Publishing Platform, Scotts Valley, CA.

Velykorusova, A., 2023. Development of a method and intelligent decision support system for sustainable renovation of the built environment. Phd thesis. Vilnius Gediminas Technical University. Available at http://dspace.vgtu.lt/bitstream/1/4360/3/A_Velykorusovos%20disertacija.pdf.

Wang, D., Tan, D., Liu, L., 2018. Particle swarm optimization algorithm: An overview. Soft Comput. 22, 387–408.

Wu, J., Cao, M., Chiclana, F., Dong, Y.C., Herrera-Viedma, E., 2021. An optimal feedback model to prevent manipulation behavior in consensus under social network group decision making. IEEE Trans. Fuzzy Syst. 29, 1750–1763.

Yager, R.R., 1988. On ordered weighted averaging aggregation operators in multicriteria decisionmaking. IEEE Trans. Syst. Man Cybern. 18, 183–190.

Yager, R.R., 1993. Families of OWA operators. Fuzzy Sets Syst. 59, 125–148.

Yager, R.R., Filev, D.P., 1999. Induced ordered weighted averaging operators. IEEE Trans. Syst. Man Cybern. Part B-Cybern. 29, 141–150.

You, X., Hou, F., Chiclana, F., 2023. Consensus reaching process with noncooperative behaviors in large-scale group social network environment. Appl. Soft Comput. 144, 110454.

Zhang, B., Dong, Y.C., Pedrycz, W., 2023. Consensus model driven by interpretable rules in large-scale group decision making with optimal allocation of information granularity. IEEE Trans. Syst. Man Cybern. -Syst. 53, 1233–1245.

Zhang, B., Pedrycz, W., Fayek, A.R., Dong, Y.C., 2022. A differential evolution-based consistency improvement method in AHP with an optimal allocation of information granularity. IEEE T. Cybern. 52, 6733–6744.

# CRediT author statement

Juan Carlos González-Quesada: Software, Writing - Original Draft, Writing - Review & Editing, Methodology. Anastasiia Velykorusova: Validation, Writing - Review & Editing. Audrius Banaitis: Conceptualization, Validation. Arturas Kaklauskas: Funding acquisition, Supervision. Francisco Javier Cabrerizo: Funding acquisition, Supervision, Writing - Review & Editing, Conceptualization.

**Declaration of interests**

☒The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

☐The authors declare the following financial interests/personal relationships which may be considered as potential competing interests: