Studying the physics of the sling by automating the shooting.

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The sling is a tool that has accompanied humanity since prehistoric times. It consists of a rope with a handle at one end and a cradle in the middle for a stone, clay or metal projectile. It is used in hunting, herding, and it was of great importance in ancient wars, since a skilled shooter can reach targets more than hundred meters away. The majority of works on the topic are of an archaeological nature, probably due to the complexity of modeling the process of shooting. In this work, we present a systematic study comparing manual and automated shootings to understand the physics of this amazing tool with the goal of being easily replicated and expanded by physics students.

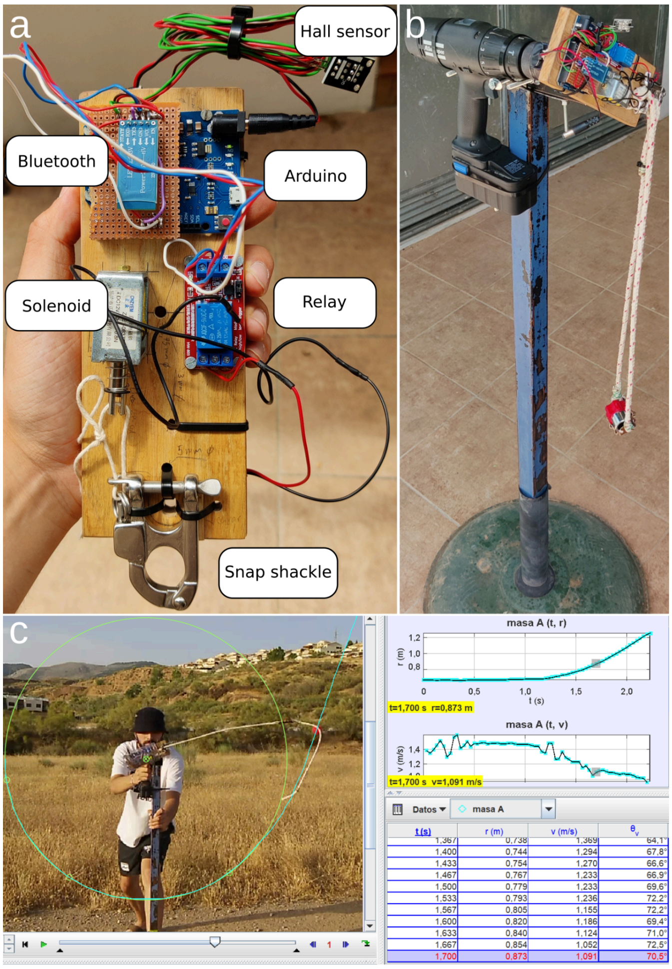
Introduction

The sling is one of the oldest tools used by humanity appearing in the myth of David against Goliath, and with evidence of being used since 40000 years ago, during the Upper Paleolithic period [1]. It was an important warfare tool during the Roman Empire period and even during medieval times, used as a terror weapon thanks to the whistling of the projectiles [1]. Today it survives as a tradition with more peaceful applications such as protecting crops from animals, herding, and as a sport with annual competitions. Other artifacts such as trebuchets have a sling as part of their construction. Trebuchets are easily reproducible machines with limited human factor, therefore there are more sources studying its physics [2-5]. Nevertheless, there are two significant differences between trebuchets and slings: i) the trebuchet accelerates the projectile thanks to the gravitational potential energy of a counterweight in one swing, while sling shooting accelerates the projectile from repetitive sling swinging; and ii) the trebuchet often employs an overhand throw, much like throwing a stone with your arm instinctively, while sling shooting allows to take advantage of changes in the plane of swinging, retracting or extending the arm, and releasing the projectile at a precise time during the swinging. Therefore, studying in a systematic way the physics of sling shooting is a difficult task. Skov studied numerous sling shootings providing a comprehensive description of the physics involved in the process [6], though he stated in this work that “the challenge of evaluating sling capability, like many experiments, can be seen as eliminating as many variables as possible and controlling the others. However, without the ability to separate the sling from its user, as is possible with other weapon systems, it is impossible to account completely for the largest source of variability”. Furthermore, Borovsky et al. studied sling shooting from a forensic perspective [7]. In both cases the authors had a collection of sling shoots that classified a posteriori by speed and reach. Achieving reproducibility a priori is very challenging even for a skilled person. In this work, we address the issue of the reproducibility by comparing sling shooting by hand with an automated machine that significantly reduces this variability and allows to control the velocity and angle of departure, with the goal in mind of studying in a more systematic way the physics of this ancient but still amazing tool.

Automated sling shooting

The automated machine is composed of a control board based on Arduino (see Fig. 1a) mounted on a cordless drill with variable speed that provides the swinging (see Fig. 1b). The back side of the control board holds three 9V batteries: one for the Arduino, and two in parallel to activate the solenoid and release the projectile, a red golf ball. The Hall sensor and a fixed neodymium magnet mounted on the fixed pole are used as a tachometer to measure the rotational speed ω in rpm, and as a point of reference. The control board is operated via bluetooth from a smartphone with three parameters: the target speed ω_0, the tolerance $\Delta\omega$ for ω_0, and the delay time t_{lag} to trigger the release of the golf ball once ω_0 is reached and after the sling is in parallel to the ground (reflected by activating the Hall sensor). There is a fourth control, a *fire* command which ensures that the shooting happens when expected. While one of us needs to operate the drill (see Fig. 1c), both ω_0 and t_{lag} are controlled remotely and in an automated way. Since the control board and the loaded sling must rotate at the same speed at the moment of the release, the drill operator needs to start the drill while tightly holding the sling in horizontal position and ensuring that the first rotation will be towards the ground. With this precaution, the constrained fall of the loaded sling and the rotation will be in the same direction, producing proper swinging after a transitory regime. Therefore, the departure speed v_0 and angle θ_0 respect to the ground are controlled indirectly through ω_0 and t_{lag}.

The experiments were recorded using a smartphone at 240 fps and the trajectories were analyzed by the open source software Tracker [8] (see Fig. 1c). Since Tracker interprets the video as 30 fps, 1 second corresponds to 8 seconds in Tracker, and this needs to be taken into account when calculating the speed. We calibrate the pixel size with two marks on the fixed pole separated by 50 cm, and we obtain the position, speed and angle of the speed vector of the golf ball over time.



t (s)	r (m)	v (m/s)	θ_v
1,367	0,738	1,369	64,1°
1,400	0,744	1,294	67,8°
1,433	0,754	1,270	66,6°
1,467	0,767	1,233	66,9°
1,500	0,779	1,233	69,6°
1,533	0,793	1,236	72,2°
1,567	0,805	1,155	72,2°
1,600	0,820	1,186	69,4°
1,633	0,840	1,124	71,0°
1,667	0,854	1,052	72,5°
1,700	0,873	1,091	70,5°

Fig. 1. a) Arduino control board showing the bluetooth module for wireless communication, the Hall effect sensor to act as a tachometer and position sensor, and a relay powering a solenoid that opens the span shackle releasing the projectile, a golf ball. **b)** Fixed pole where the variable speed drill is mounted, making the control board and sling rotate. **c)** Example of an automated release of the red golf ball, recorded via a smartphone and processed by Tracker, obtaining the position, speed and angle of the speed vector of the golf ball over time.

Automated sling vs sling shooting by hand

Our goals are two: i) to compare the reproducibility of the initial conditions for the automated sling machine against shooting by hand, and ii) characterize the automated sling shooting to identify strengths and weaknesses. The shooting by hand was performed trying to mimic the automated sling shooting to produce meaningful comparisons (see Fig. 2).

Fig. 2. Sling shooting by hand. Initial swinging circle (green), firing circle after extending the arm (red), and trajectory of the golf ball (yellow).

In the first experiment 5 automated shots were produced by fixing ω_0=170 rpm and t_{lag}=35 ms. Next, we shot 5 times aiming for the same v_0 and angle θ_0 as for the automated shots. The following table compares the results obtained after tracking the trajectories:

	v_0 (m/s)	θ_0 (°)
Automated	10.6±0.3	57±3
By hand	10.3±0.7	36±6

We see that the automation manages to reduce the dispersion of the speed by 57% and of the angle by 55%. But also it is remarkable our ability when shooting by hand in matching the range of an automated shot. We estimate through a complex process that it might be described as “aiming to reach a specific range”. We believe that this is probably due to the fact that it is easy to estimate and mimic the rotational motion from the trajectory of the sling and golf ball. However, it is much more difficult to match θ_0 because it depends on many more factors such as the way in which the golf ball is released. Moreover, in the case of the automated shot, the experimental $\omega_{0,exp}$=143±6 rpm is far from the commanded 170 rpm. To understand this discrepancy we must take into account that the commanded ω_0 is measured as an average in the previous lap at the instant of release, and the real instantaneous speed depends on how much the operator pulls the drill trigger at the last moment. This is still an uncontrolled factor in our setup. Next, we compared the ability of the automated machine to shooting by hand in making a progression of θ_0 for a fixed speed ω_0 (see Fig. 3). For the automated shooting in Fig. 3a, ω_0 was set to 170 rpm, and the progression in θ_0 was obtained by increasing t_{lag}, indicated by the legends and numbering of the trajectories from lower to higher t_{lag}. The angle θ_0 and v_0 were fitted for each trajectory and added in the legends. For the shooting by hand in Fig. 3b, 10 shots were produced trying to mimic the speed of the automated shooting. The legend corresponds to the order in which they were thrown, aiming for increasing θ_0. Taking into account that the aiming was to maintain a constant speed, and vary the angle progressively, we can see that this is not what we obtained. We see that despite the aim, the dispersion in θ_0 and v_0 is much higher than that of the automated sling. For the automated sling shooting, the following dependence of θ_0 with t_{lag} for fixed ω_0=170 rpm was found: $\theta_0(t_{lag}) = (0.92\pm0.14)\, t_{lag} + (19\pm3)\,^{\circ}$.

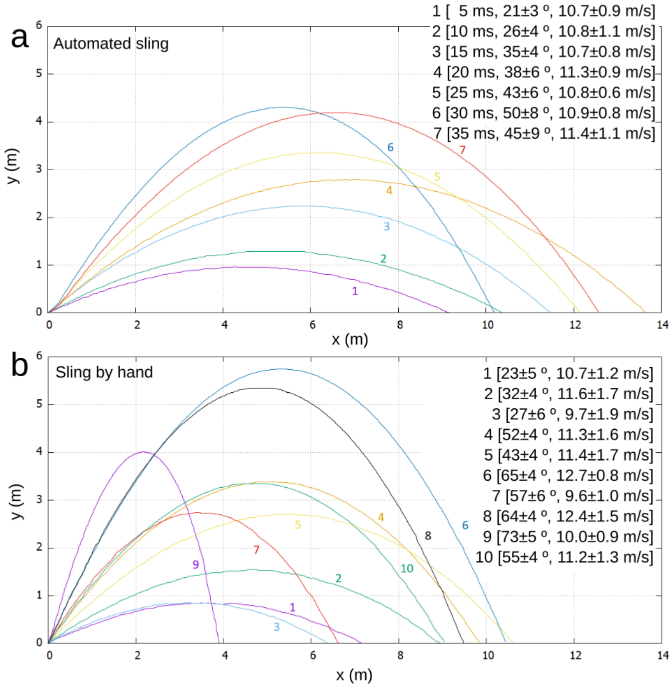


Fig. 3. a) Trajectories by automated sling shooting for 7 shots with fixed ω_0=170 rpm and increasing t_{lag} from 5 to 35 ms. **b)** 10 sling shots by hand where the shooter aimed for a progression of θ_0 at constant ω_0, but resulted in the numbered trajectories. θ_0 and v_0 fitted for each trajectory reported in the legends.

Additionally, in the case of the automated sling shooting, the opposite case respect to Fig. 3a can be studied, trajectories where t_{lag}=10 ms is fixed and ω_0 is increased (see Fig. 4). Since the control parameter is not θ_0, but t_{lag}, the angle increases with ω_0 as the speed increases. Given the results obtained by hand in Fig. 3b, we opted to not try to mimic this experiment which would be much harder. In this case, the range and maximum height h increase with ω_0. The 190 rpm value is the speed limit of our drill. While h is lower than in Fig. 3 because of the drill speed limitation in ω_0, a smoother progression was achieved in terms of h and range, which results from the control in the initial speed v_0: $v_0(\omega_0) = (0.06\pm0.01)\,\omega_0 + (1\pm2)\, m/s$.

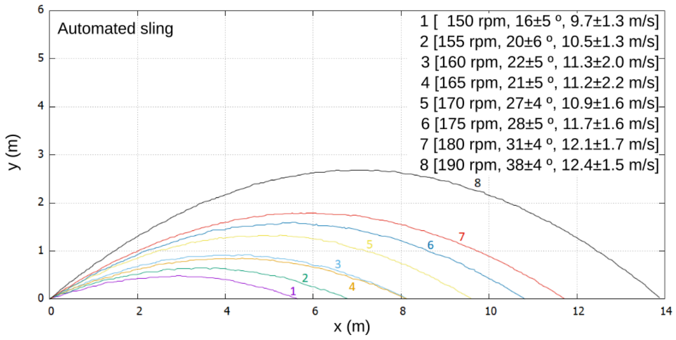


Fig. 4. Trajectories by automated sling shooting for 7 shots with fixed t_{lag}=10 ms and increasing ω_0 from 150 to 190 rpm. θ_0 and v_0 fitted for each trajectory reported in the legends.

Theory

Once the golf ball is released and the initial speed v_0 and angle θ_0 is tracked from the movies, the automated sling shot can be modeled as a parabolic motion with a friction coefficient β with air:

$$\beta = \frac{C_D\,\rho\,\pi\,R^2 v}{2\,m} = 0.087\pm0.007\ \mathrm{s}^{-1}$$
$$C_D \simeq \frac{24}{Re} + \frac{4}{\sqrt{Re}} + 0.4 = 0.424\pm0.001$$
$$Re = \frac{\rho air v D}{\mu} = 0.424\pm0.001 \qquad (1)$$

where C_D is the dimensionless drag coefficient for a golf ball for a low Reynolds number Re < $2\cdot10^5$, $\rho=1164\pm5\ kg/m^3$ is the air density, and $R=21.45\pm0.05$ mm, $v=10.9\pm0.8$ m/s, and $m=45\pm1$ g, are the radius, average speed and mass of the golf ball, respectively. Therefore, the acceleration in the x and y planes can be described as in Eq. 2:

$$a_x(t) = -\beta v_{0x}$$
$$a_y(t) = -g - \beta v_{0y} \qquad (2)$$

where v_{0x} and v_{0y} are the horizontal and vertical components of v_0, and $g=9.7964\pm0.0001$ m/s^2 is the gravity in Granada (Spain). After integration of Eq. 2, we obtain the position as a function of time in Eq. 3:

$$x(t) = v_{0x}\, e^{-\beta t}$$

$$y(t) = (v_{0y} + \frac{g}{\beta})\, e^{-\beta t} - \frac{g}{\beta} \qquad (3)$$

The parameters that we can compare easily between experiments and theory are three: i) the range R, the horizontal range of the shot, ii) the flight time T, the time it takes for the projectile to return to the ground, and iii) h, the max value of y, the transition between the ascending and descending flights. For the sake of keeping the manuscript short we will exemplify the agreement with the later parameter, but the agreement is similar for all parameters. In Fig. 5, we represented the maximum height h_{theory} calculated from Eq. 2 for the initial conditions θ_0 and v_0 against the $h_{experiments}$, all of them extracted from the experimental trajectories. A perfect agreement would be the line of slope 1. We plot points corresponding to all the trajectories mentioned in this work. The agreement is quite significant, regardless of the shot being performed automatically or by hand. This is expected as the parabolic motion is univocal for the initial experimental conditions θ_0 and v_0. Therefore, more than comparing theory and experiments, a rather expected result, this plot serves as a double check to ensure that we properly extracted the initial conditions.

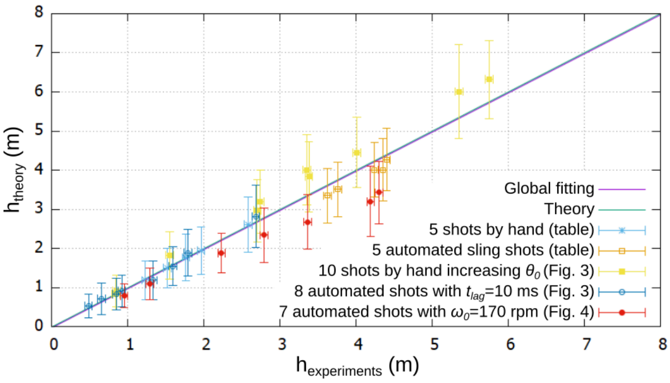


Fig. 5. Maximum height *h*. Comparison between theory and experiments for all the automated and shots by hand reported in this work, including a global fitting of all points and the expected results from theory.

Final remarks

The physics of the sling is as amazing as it is complex to model, due to many parameters affecting the so-called skill of the shooter. In this work, we automated the sling shooting with a machine based on an Arduino control board. It runs a software that includes real-time communication via bluetooth, a tachometer, and control of the swinging speed and firing delay with respect to a fixed reference point. With this setup we gain both reproducibility and absolute control over the shootings. One of the main limitations is the low range of 14 meters that we can obtain, limited by the drill speed and for a golf ball as projectile. This range is far from >100 m reached by conventional sling shooting of stones by hand [6]. Given the initial set of measurements performed, a possible improvement for the automation would be to calibrate θ_0 and v_0 as a function of ω_0 and t_{lag}, the parameters that we can control directly now, and implement such calibration in the Arduino software. In this way, we would be able to shoot by choosing θ_0 and v_0, more natural parameters of the shooting. Nevertheless, ω_0 and t_{lag} are the parameters that we can control directly with the tachometer implemented in the machine. It would also be interesting to think of ways in which the machine can be modified to include mimicking the extension of the shooter arm and possible changes in the plane of swinging during shooting, as it usually happens during a sling shooting by hand. Finally, we think that this is a fun and rather easy way to systematically study such an ancient art as it is the sling, and why not, we encourage the readers to invent new tools in the process.

Acknowledgments

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Supplementary material

Schematics of the Arduino control board, the software and control flowchart for Arduino, and movies showing the automated sling shooting are available online.

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