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Research Article

An ε-Constraint Method for Multiobjective Linear Programming in Intuitionistic Fuzzy Environment

Boris Pérez-Cañedo [iD],[1] **José Luis Verdegay** [iD],[2] **and Eduardo René Concepción-Morales** [iD][3]

[1]*Department of Mathematics, University of Cienfuegos, Cienfuegos 55100, Cuba*
[2]*Department of Computer Science and Artificial Intelligence, University of Granada, Granada 18071, Spain*
[3]*Department of Informatics, University of Cienfuegos, Cienfuegos 55100, Cuba*

Correspondence should be addressed to Eduardo René Concepción-Morales; econcepm@gmail.com

Effective decision-making requires well-founded optimization models and algorithms tolerant of real-world uncertainties. In the mid-1980s, intuitionistic fuzzy set theory emerged as another mathematical framework to deal with the uncertainty of subjective judgments and made it possible to represent hesitancy in a decision-making problem. Nowadays, intuitionistic fuzzy multi-objective linear programming (IFMOLP) problems are a topic of extensive research, for which a considerable number of solution approaches are being developed. Among the available solution approaches, ranking function-based approaches stand out for their simplicity to transform these problems into conventional ones. However, these approaches do not always guarantee Pareto optimal solutions. In this study, the concepts of dominance and Pareto optimality are extended to the intuitionistic fuzzy case by using lexicographic criteria for ranking triangular intuitionistic fuzzy numbers (TIFNs). Furthermore, an intuitionistic fuzzy ε-constraint method is proposed to solve IFMOLP problems with TIFNs. The proposed method is illustrated by solving two intuitionistic fuzzy transportation problems addressed in two studies (S. Mahajan and S. K. Gupta's, "On fully intuitionistic fuzzy multiobjective transportation problems using different membership functions," Ann Oper Res, vol. 296, no. 1, pp. 211–241, 2021, and Ghosh et al.'s, "Multi-objective fully intuitionistic fuzzy fixed-charge solid transportation problem," Complex Intell Syst, vol. 7, no. 2, pp. 1009–1023, 2021). Results show that, in contrast with Mahajan and Gupta's and Ghosh et al.'s methods, the proposed method guarantees Pareto optimality and also makes it possible to obtain multiple solutions to the problems.

1. Introduction

Modern societies depend every day to a greater extent on the use of intelligent automated or semiautomated technologies to, e.g., drive productivity in assembly lines, control communication systems, perform surveillance, and process large volumes of financial transactions [1]. With the advent of artificial intelligence paradigms, automated decision-making (ADM) systems began to undergo considerable development, and they are currently being deployed in public and private sectors. An ADM system uses computation to aid or replace organization decisions, judgments, and/or policy implementations that impact opportunities, access, liberties, rights and/or safety [2]. Important constituents of these systems are their underlying optimization models and algorithms. However, to act like human experts do when making decisions, to understand the context in which decisions are to be made, and to deal with real-world uncertainties, conventional optimization models and algorithms are not sufficient. In fact, ethical concerns about the use of ADM systems have been raised and the need for regulation has general consensus [3].

Zadeh [4] conceived the fuzzy set theory as a theory for modeling uncertainty due to subjective judgments, in which the concept of grade of membership of an element in a set

has a central role. Atanassov [5] introduced the intuitionistic fuzzy set (IFS) theory as a generalization of Zadeh's [4] theory and considered along with the concept of grade of membership to that of nonmembership. In this way, hesitancy can be quantified and included in a decision-making process. Both theories were soon applied to optimization under uncertainty and gave birth to fuzzy linear programming (FLP) and intuitionistic fuzzy linear programming (IFLP) (see [6–8]).

In recent years, FLP problems, in which the parameters and/or decision variables take on fuzzy numbers, have received a lot of attention and several solution approaches have been proposed [9]. A widespread approach for solving such problems is to transform them into conventional (nonfuzzy) ones by using linear ranking functions [9]. Those ranking functions, however simple to apply, have a major drawback, that is, they may transform different fuzzy numbers into the same real number. Two implications clearly arise. First, in FLP, such a transformation does not guarantee uniqueness of optimal objective values [10]. Second, in connection with the first implication, in the multiobjective case, using linear ranking functions does not always guarantee Pareto optimality [11]. To resolve these drawbacks, lexicographic ranking criteria have been suggested for the single-objective and multiobjective cases. These criteria use multiple indices hierarchically to compare fuzzy numbers and, under very few assumptions, can guarantee unique optimal objective values and Pareto optimality [11]. Hashemi et al. [12] proposed a two-phase lexicographic approach to solve FLP with symmetric LR-type fuzzy numbers. Kaur and Kumar [10] developed a lexicographic method for solving FLP problems, with trapezoidal fuzzy numbers, which have only fuzzy equality constraints. Ezzati et al. [13] introduced a lexicographic method for solving FLP problems with triangular fuzzy numbers, in which fuzzy inequality constraints are transformed into fuzzy equality constraints by means of fuzzy slack or surplus variables. However, although such an approach is valid in the nonfuzzy case, it may produce infeasible solutions in the fuzzy case. In [14], a method similar to that of Ezzati et al. [13] was proposed for solving FLP problems with LR-type fuzzy numbers and with only fuzzy equality constraints. Das et al. [15] used a lexicographic criterion for ranking trapezoidal fuzzy numbers and proposed a method for solving FLP problems with fuzzy equality and inequality constraints. However, Das et al.'s [15] method does not handle fuzzy inequality constraints correctly. All the previously mentioned methods were later generalized in [16], where a method to solve FLP problems with fuzzy equality and inequality constraints, LR-type fuzzy numbers, and arbitrary lexicographic ranking criteria was proposed. For the multiobjective case, Yang et al. [17] proposed a method for solving FLP problems with triangular fuzzy numbers and with only fuzzy equality constraints. Their method, however, does not produce multiple Pareto optimal solutions. In [11], an ε-constraint method was developed that guarantees Pareto optimality with respect to lexicographic ranking criteria and produces multiple Pareto optimal solutions.

Among the available approaches for solving IFLP problems, linear ranking function-based approaches also stand out for their simplicity, and are gaining popularity in the nonlinear case as well (see [18, 19]). Recently, in [20, 21], several methods were proposed to solve intuitionistic fuzzy multiobjective transportation (IFMOT) problems. However, the drawbacks of linear ranking functions were not addressed by the authors.

In this study, we take a step towards a future incorporation in ADM systems of optimization models and algorithms that effectively handle nonprobabilistic uncertainty. Specifically, we focus on solving intuitionistic fuzzy multiobjective linear programming (IFMOLP) problems, in which uncertainty in the data is represented by triangular intuitionistic fuzzy numbers (TIFNs). We do not thoroughly review the many available methods to solve such problems. Instead, since linear ranking functions are being frequently used to solve IFMOLP problems, so new methods based on their use are being proposed for the nonlinear case as well, and we believe that researchers should be made aware of the drawbacks of linear ranking function-based approaches. Consequently, our main purpose is to demonstrate that using linear ranking functions to solve IFMOLP problems does not always guarantee Pareto optimality. To this aim, we make the following contributions:

(i) The concepts of dominance and Pareto optimality are extended to the intuitionistic fuzzy case by using lexicographic criteria for ranking intuitionistic fuzzy numbers

(ii) An intuitionistic fuzzy ε-constraint method is developed to solve IFMOLP problems. This method generalizes previous results introduced in [11]

(iii) Two IFMOT problems previously addressed by Mahajan and Gupta [20] and Ghosh et al. [21] are solved to illustrate the advantages of the proposed method

The rest of the article is organized as follows. Section 2 presents some basic definitions concerning intuitionistic fuzzy numbers. Section 3 in brief describes a lexicographic method for solving IFLP problems that guarantees solutions with unique objective values. Section 4 presents the main results of this study (intuitionistic fuzzy dominance and Pareto optimality in terms of lexicographic ranking criteria, and the proposed intuitionistic fuzzy ε-constraint method). The proposed intuitionistic fuzzy ε-constraint method is demonstrated in Section 5 by solving two IFMOT problems. A comparison with Mahajan and Gupta's [20] and Ghosh et al.'s [21] methods is carried out in this section as well. Possible extensions of the proposed method to other uncertain environments are discussed in Section 6. Concluding remarks and future research lines are provided in Section 7.

2. Basic Definitions

This section presents basic definitions taken from Mahajan and Gupta [20] and Ghosh et al. [21].

Definition 1. An intuitionistic fuzzy set $\widetilde{A}^I$ in X is defined as an object of the following form:

$$\widetilde{A}^I = \left\{ \left(x, \mu_{\widetilde{A}^I}(x), \nu_{\widetilde{A}^I}(x) \right) \mid x \in X \right\}, \tag{1}$$

where the functions $\mu_{\widetilde{A}^I}: X \longrightarrow [0,1]$ and $\nu_{\widetilde{A}^I}: X \longrightarrow [0,1]$ define the degree of membership and the degree of non-membership of the element $x \in X$, respectively, and for every $x \in X$, we have

$$0 \leq \mu_{\widetilde{A}^I}(x) + \nu_{\widetilde{A}^I}(x) \leq 1. \tag{2}$$

The degree of hesitation of $x \in X$ in the set $\widetilde{A}^I$ is given by

$$\pi_{\widetilde{A}^I}(x) = 1 - \mu_{\widetilde{A}^I}(x) - \nu_{\widetilde{A}^I}(x). \tag{3}$$

For the sake of simplicity and brevity in the exposition, we restrict the subsequent discussion to TIFNs, but our results can be easily extrapolated to the more general LR-type intuitionistic fuzzy numbers by simply adopting the definitions and arithmetic operations given by Singh and Yadav [22].

Definition 2. A TIFN $\widetilde{a}^I = (a_1, a, a_2; \overline{a}_1, a, \overline{a}_2)$, where $\overline{a}_1 \leq a_1 \leq a \leq a_2 \leq \overline{a}_2$, is an intuitionistic fuzzy set, whose membership and nonmembership functions are given as

$$\mu_{\widetilde{a}^I}(x) = \begin{cases} \dfrac{x - a_1}{a - a_1} & if\ a_1 \leq x \leq a, \\[2mm] \dfrac{a_2 - x}{a_2 - a} & if\ a < x \leq a_2, \\[2mm] 0 & otherwise, \end{cases}$$

$$\nu_{\widetilde{a}^I}(x) = \begin{cases} \dfrac{a - x}{a - \overline{a}_1} & if\ \overline{a}_1 \leq x \leq a, \\[2mm] \dfrac{x - a}{\overline{a}_2 - a} & if\ a < x \leq \overline{a}_2, \\[2mm] 1 & otherwise. \end{cases} \tag{4}$$

The set of all TIFNs is denoted by $\mathscr{T}(\mathbb{R})$. Figure 1 shows a graphical representation of a TIFN.

Definition 3. A TIFN $\widetilde{a}^I = (a_1, a, a_2; \overline{a}_1, a, \overline{a}_2)$ is non-negative, which is denoted by $\widetilde{a}^I \geq 0$, if $\overline{a}_1 \geq 0$.

Definition 4. Let $\widetilde{a}^I = (a_1, a, a_2; \overline{a}_1, a, \overline{a}_2)$ and $\widetilde{b}^I = (b_1, b, b_2; \overline{b}_1, b, \overline{b}_2)$ be the two TIFNs, and k be any real number. Then, the arithmetic operations are defined as follows:

(i) Addition: $\widetilde{a}^I \oplus \widetilde{b}^I = (a_1 + b_1, a + b, a_2 + b_2; \overline{a}_1 + \overline{b}_1, a + b, \overline{a}_2 + \overline{b}_2)$

(ii) Subtraction: $\widetilde{a}^I \ominus \widetilde{b}^I = (a_1 - b_2, a - b, a_2 + b_1; \overline{a}_1 - \overline{b}_2, a - b, \overline{a}_2 - \overline{b}_1)$

(iii) Multiplication: $\widetilde{a}^I \odot \widetilde{b}^I = \begin{pmatrix} \min\{a_1 b_1, a_1 b_2, a_2 b_1, a_2 b_2\}, ab, \max\{a_1 b_1, a_1 b_2, a_2 b_1, a_2 b_2\}; \\ \min\{\overline{a}_1 \overline{b}_1, \overline{a}_1 \overline{b}_2, \overline{a}_2 \overline{b}_1, \overline{a}_2 \overline{b}_2\}, ab, \max\{\overline{a}_1 \overline{b}_1, \overline{a}_1 \overline{b}_2, \overline{a}_2 \overline{b}_1, \overline{a}_2 \overline{b}_2\} \end{pmatrix}$

(iv) Scalar multiplication: $k \times \widetilde{a}^I = (ka_1, ka, ka_2; k\overline{a}_1, ka, k\overline{a}_2)$ if $k \geq 0$, and $k \times \widetilde{a}^I = (ka_2, ka, ka_1; k\overline{a}_2, ka, k\overline{a}_1)$ if $k < 0$.

Definition 5. Let $\widetilde{a}^I = (a_1, a, a_2; \overline{a}_1, a, \overline{a}_2)$ and $\widetilde{b}^I = (b_1, b, b_2; \overline{b}_1, b, \overline{b}_2)$ be the two TIFNs. We say $\widetilde{a}^I = \widetilde{b}^I$ if and only if $a_1 = b_1$, $a = b$, $a_2 = b_2$, $\overline{a}_1 = \overline{b}_1$, and $\overline{a}_2 = \overline{b}_2$.

Definition 6. Let $\widetilde{a}^I = (a_1, a, a_2; \overline{a}_1, a, \overline{a}_2)$ and $\widetilde{b}^I = (b_1, b, b_2; \overline{b}_1, b, \overline{b}_2)$ be the two TIFNs. We say $\widetilde{a}^I \leq \widetilde{b}^I$ if and only if $a_1 \leq b_1$, $a \leq b$, $a_2 \leq b_2$, $\overline{a}_1 \leq \overline{b}_1$, and $\overline{a}_2 \leq \overline{b}_2$.

Definition 7. Let $\widetilde{a}^I = (a_1, a, a_2; \overline{a}_1, a, \overline{a}_2) \in \mathscr{T}(\mathbb{R})$. Then, the accuracy function $f: \mathscr{T}(\mathbb{R}) \longrightarrow \mathbb{R}$ is defined in terms of the parameters of $\widetilde{a}^I$ as $f(\widetilde{a}^I) = (a_1 + a_2 + 4a + \overline{a}_1 + \overline{a}_2)/8$.

3. Method for Single-Objective Case

In this section, we briefly describe the lexicographic method proposed in [23] for solving IFLP problems. This method is necessary because the solution strategy consists in transforming the IFMOLP problem into a single-objective one. We begin with the definition of a lexicographic order on $\mathscr{T}(\mathbb{R})$ [23].

Definition 8. Let $f_t: \mathscr{T}(\mathbb{R}) \longrightarrow \mathbb{R}$ (for $t = 1, \ldots, 5$) be linear functions of the parameters of any TIFNs $\widetilde{a}^I$ and $\widetilde{b}^I$, with nonsingular coefficient matrix. Furthermore, let $\leq_{Lex}$ denote the lexicographic order relation on $\mathbb{R}^5$. We say that $\widetilde{a}^I$ is less than $\widetilde{b}^I$ (denoted by $\widetilde{a}^I \prec \widetilde{b}^I$) if $(f_t(\widetilde{a}^I))_{t=1,\ldots,5} <_{Lex} (f_t(\widetilde{b}^I))_{t=1,\ldots,5}$, and $\widetilde{a}^I$ is less than or equal to $\widetilde{b}^I$ (denoted by $\widetilde{a}^I \preceq \widetilde{b}^I$) if $(f_t(\widetilde{a}^I))_{t=1,\ldots,5} <_{Lex} (f_t(\widetilde{b}^I))_{t=1,\ldots,5}$ or $(f_t(\widetilde{a}^I))_{t=1,\ldots,5} = (f_t(\widetilde{b}^I))_{t=1,\ldots,5}$.

Remark 1. The functions $f_t: \mathscr{T}(\mathbb{R}) \longrightarrow \mathbb{R}$ (for $t = 1, \ldots, 5$) must be chosen such that they capture, as accurately as possible, the characteristics a decision-maker takes into account for comparing TIFNs.

Example 1. We consider the TIFNs $\widetilde{p}^I = (0, 1, 2; 0, 1, 2)$ and $\widetilde{q}^I = (0, 1.5, 2; -2, 1.5, 2)$, and the lexicographic ranking criterion is determined by $f_1(\widetilde{a}^I) = (a_1 + a_2 + 4a + \overline{a}_1 + \overline{a}_2)/8$, $f_2(\widetilde{a}^I) = a$, $f_3(\widetilde{a}^I) = a_1$, $f_4(\widetilde{a}^I) = a_2 - a_1$, and $f_5(\widetilde{a}^I) = \overline{a}_2$. By using Definition 8, we have $(f_t(0, 1, 2; 0, 1, 2))_{t=1,\ldots,5} = (1, 1, 0, 2, 2)$ and $(f_t(0, 1.5, 2; -2, 1.5, 2))_{t=1,\ldots,5} = (1, 1.5, 0, 2, 2)$. Since $(1, 1, 0, 2, 2) <_{Lex} (1, 1.5, 0, 2, 2)$, i.e., $(1, 1, 0, 2, 2)$ is lexicographically less than $(1, 1.5, 0, 2, 2)$; hence, we conclude that $\widetilde{p}^I \prec \widetilde{q}^I$.

3.1. IFLP Model and a Lexicographic Solution Method. Let $\widetilde{c}_j^I$, $\widetilde{b}_i^I$ and $\widetilde{a}_{ij}^I \in \mathscr{T}(\mathbb{R})$ (for $i = 1, \ldots, m$ and $j = 1, \ldots, n$) denote the constant TIFNs and $\widetilde{x}^I = (\widetilde{x}_j^I)_{j=1,\ldots,n}$ variable TIFNs.
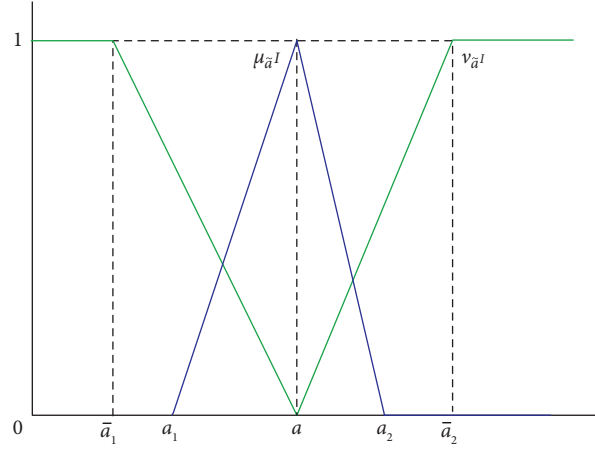
Figure 1: Graphical representation of a TIFN.

Then, by using Definitions 4, 5, and 8, the IFLP problem is formulated as

$$\min \widetilde{z}^I\left(\widetilde{x}^I\right) = \sum_j \widetilde{c}_j^I \odot \widetilde{x}_j^I,$$

$$s.t. \quad \widetilde{a}_i^I\left(\widetilde{x}^I\right) = \sum_j \widetilde{a}_{ij}^I \odot \widetilde{x}_j^I \{\preccurlyeq, =, \succcurlyeq\} \widetilde{b}_i^I \text{ for } i = 1, \ldots, m. \tag{5}$$

Let us denote by $I_{\preccurlyeq}$, $I_{\succcurlyeq}$, and $I_=$ the index sets of the $\preccurlyeq$ − type, $\succcurlyeq$ − type and equality constraints of problem (5), respectively. By using Definitions 5 and 8, IFLP (5) is transformed into the lexicographic optimization problem as shown by the following equation:

$$\text{lexmin}\left(f_t\left(\widetilde{z}^I\left(\widetilde{x}^I\right)\right)\right)_{t=1,\ldots,5}$$

$$\left(f_t\left(\widetilde{a}_i^I\left(\widetilde{x}^I\right)\right)\right)_{t=1,\ldots,5}\{\leq_{\text{Lex}}, \geq_{\text{Lex}}\}\left(f_t\left(\widetilde{b}_i^I\right)\right)_{t=1,\ldots,5} \text{ for } i \in I_{\preccurlyeq} \cup I_{\succcurlyeq},$$

$$s.t. \quad a_{i1}\left(\widetilde{x}^I\right) = b_{i1}, a_i\left(\widetilde{x}^I\right) = b_1, a_{i2}\left(\widetilde{x}^I\right) = b_{i2}, \tag{6}$$

$$\overline{a}_{i1}\left(\widetilde{x}^I\right) = \overline{b}_{i1}, \overline{a}_{i2}\left(\widetilde{x}^I\right) = \overline{b}_{i2} \text{ for } i \in I_=.$$

To handle the lexicographic constraints, the following transformation was proposed in [23] using binary variables, and the small and large positive constants ϵ and L, respectively:

$$\text{lexmin}\left(f_t\left(\widetilde{z}^I\left(\widetilde{x}^I\right)\right)\right)_{t=1,\ldots,5}$$

$$-L\sum_{k=1}^{t-1} y_{ik} + \epsilon y_{it} \leq f_t\left(\widetilde{b}_i^I\right) - f_t\left(\widetilde{a}_i^I\left(\widetilde{x}^I\right)\right) \leq Ly_{it} \text{ for } i \in I_{\preccurlyeq} \text{ and } t = 1, \ldots, 5,$$

$$-L\sum_{k=1}^{t-1} y_{ik} + \epsilon y_{it} \leq f_t\left(\widetilde{a}_i^I\left(\widetilde{x}^I\right)\right) - f_t\left(\widetilde{b}_i^I\right) \leq Ly_{it} \text{ for } i \in I_{\succcurlyeq} \text{ and } t = 1, \ldots, 5, \tag{7}$$

$$s.t. \quad y_{it} \in \{0, 1\} \text{ for } i \in I_{\preccurlyeq} \cup I_{\succcurlyeq} \text{ and } t = 1, \ldots, 5,$$

$$a_{i1}\left(\widetilde{x}^I\right) = b_{i1}, a_i\left(\widetilde{x}^I\right) = b_1, a_{i2}\left(\widetilde{x}^I\right) = b_{i2},$$

$$\overline{a}_{i1}\left(\widetilde{x}^I\right) = \overline{b}_{i1}, \overline{a}_{i2}\left(\widetilde{x}^I\right) = \overline{b}_{i2} \text{ for } i \in I_=.$$

Theorem 1. *Problems (5) and (7) are equivalent.*

Proof. See (16, 23).

Optimal solutions to problem (7) can be obtained by using the lexicographic method used for conventional multiobjective optimization [24]. □

4. Intuitionistic Fuzzy ε-constraint Method

In this section, a lexicographic method for solving IFMOLP problems is developed. This constitutes a generalization of the previous results from [11].

Let us consider the IFMOLP problem as

$$\min\left(\tilde{z}_r^I\left(\tilde{x}^I\right)\right)_{r=1,\ldots,p\geq 2} = \left(\sum_j \tilde{c}_{rj}^I \odot \tilde{x}_j^I\right)_{r=1,\ldots,p} \quad (8)$$

$$s.t. \quad \tilde{a}_i^I\left(\tilde{x}^I\right)\{\preccurlyeq,=,\succcurlyeq\}\tilde{b}_i^I \text{ for } i = 1,\ldots,m.$$

The solution strategy will be to scalarize problem (8), i.e., to transform it into an IFLP problem whose optimal solution is Pareto optimal for the multiobjective case, but, first, we need the following fundamental definitions:

Definition 9. Let $\preccurlyeq$ be given as in Definition 8 and $\tilde{X}^I$ denote the set of all feasible solutions to IFMOLP problem (8). A solution $\tilde{x}^I \in \tilde{X}^I$ is said to *dominate* $\tilde{y}^I \in \tilde{X}^I$ if $\tilde{z}_r^I(\tilde{x}^I) \preccurlyeq \tilde{z}_r^I(\tilde{y}^I)$ for $r = 1,\ldots,p$, with at least one strict inequality.

Example 2. We consider two feasible solutions to an intuitionistic fuzzy biobjective linear programming problem denoted by $\tilde{x}^I$ and $\tilde{y}^I$, with values in the objective functions $\tilde{z}_1^I(\tilde{x}^I) = (1,2,3;0,2,4)$, $\tilde{z}_2^I(\tilde{x}^I) = (0,1,2;0,1,2)$, $\tilde{z}_1^I(\tilde{y}^I) = (1,2,3;0,2,4)$, and $\tilde{z}_2^I(\tilde{y}^I) = (0,1.5,2;-2,1.5,2)$. We assume that the decision-maker ranks TIFNs according to the lexicographic criterion determined by $f_1(\tilde{a}^I) = (a_1 + a_2 + 4a + \overline{a}_1 + \overline{a}_2)/8$, $f_2(\tilde{a}^I) = a$, $f_3(\tilde{a}^I) = a_1$, $f_4(\tilde{a}^I) = a_2 - a_1$, and $f_5(\tilde{a}^I) = \overline{a}_2$. Then, by using Definition 9, we have, for the first objective function, $f_1(\tilde{z}_1^I(\tilde{x}^I)) = f_1(\tilde{z}_1^I(\tilde{y}^I)) = 2$, $f_2(\tilde{z}_1^I(\tilde{x}^I)) = f_2(\tilde{z}_1^I(\tilde{y}^I)) = 2$, $f_3(\tilde{z}_1^I(\tilde{x}^I)) = f_3(\tilde{z}_1^I(\tilde{y}^I)) = 1$, $f_4(\tilde{z}_1^I(\tilde{x}^I)) = f_4(\tilde{z}_1^I(\tilde{y}^I)) = 2$, and $f_5(\tilde{z}_1^I(\tilde{x}^I)) = f_5(\tilde{z}_1^I(\tilde{y}^I)) = 4$; and this clearly happens because $\tilde{z}_1^I(\tilde{x}^I) = \tilde{z}_1^I(\tilde{y}^I)$. However, for the second objective function, we have $f_1(\tilde{z}_2^I(\tilde{x}^I)) = f_1(\tilde{z}_2^I(\tilde{y}^I)) = 1$, and $f_2(\tilde{z}_2^I(\tilde{x}^I)) = 1 < f_2(\tilde{z}_2^I(\tilde{y}^I)) = 1.5$; hence, according to

Definition 8, $\tilde{z}_2^I(\tilde{x}^I)\prec\tilde{z}_2^I(\tilde{y}^I)$. So, we have $\tilde{z}_1^I(\tilde{x}^I) = \tilde{z}_1^I(\tilde{y}^I)$ and $\tilde{z}_2^I(\tilde{x}^I)\prec\tilde{z}_2^I(\tilde{y}^I)$; and consequently, $\tilde{x}^I$ dominates $\tilde{y}^I$.

Definition 10. A solution $\tilde{x}^I \in \tilde{X}^I$ is said to be Pareto optimal if there does not exist another $\tilde{y}^I \in \tilde{X}^I$ such that $\tilde{y}^I$ dominates $\tilde{x}^I$.

Without loss of generality, we may scalarize problem (8) as follows:

$$\min \tilde{w}^I,$$

$$\tilde{w}^I \oplus \sum_{r\neq 1} \lambda_r \times \tilde{s}_r^I = \tilde{z}_1^I(\tilde{x}^I) \oplus \sum_{r\neq 1} \lambda_r \times \tilde{p}_r^I \oplus \tilde{M}^I,$$

$$s.t. \quad \tilde{z}_r^I(\tilde{x}^I) \oplus \tilde{s}_r^I = \tilde{\epsilon}_r^I \oplus \tilde{p}_r^I \text{ for } r = 2,\ldots,p, \quad (9)$$

$$\tilde{s}_r^I \preccurlyeq \tilde{p}_r^I, \tilde{s}_r^I \geq 0, \tilde{p}_r^I \geq 0 \text{ for } r = 2,\ldots,p,$$

$$\tilde{a}_i^I(\tilde{x}^I)\{\preccurlyeq,=,\succcurlyeq\}\tilde{b}_i^I \text{ for } i = 1,\ldots,m.$$

In problem (9), $\tilde{w}^I$, $\tilde{s}_r^I$, and $\tilde{p}_r^I$ are the auxiliary variables, λ_r are the positive real constants, and $\tilde{\epsilon}_r^I$ are the TIFNs that bound the possible values of the objective functions $\tilde{z}_r^I(\tilde{x}^I)$ for $r = 2,\ldots,p$; this is similar to the conventional (non-fuzzy) ε-constraint method [24]. On the other hand, the TIFN $\tilde{M}^I = (-m/2,0,m/2;-m,0,m)$, where m is a big positive constant, has no implication in the optimization process because it is a constant, but it does serve a special purpose that is clarified in the following. Equation $\tilde{w}^I \oplus \sum_{r\neq 1}\lambda_r \times \tilde{s}_r^I = \tilde{z}_1^I(\tilde{x}^I) \oplus \sum_{r\neq 1}\lambda_r \times \tilde{p}_r^I$ is not always satisfied, i.e., there may not exist $\tilde{w}^I$ such that this equation is satisfied for optimal values of $\tilde{s}_r^I$, $\tilde{p}_r^I$, and $\tilde{x}^I$. Adding $\tilde{M}^I$ to the right-hand side in order to obtain $\tilde{w}^I \oplus \sum_{r\neq 1}\lambda_r \times \tilde{s}_r^I = \tilde{z}_1^I(\tilde{x}^I) \oplus \sum_{r\neq 1}\lambda_r \times \tilde{p}_r^I \oplus \tilde{M}^I$ resolves this issue.

Notice that, according to Definitions 2, 4, and 5, equation $\tilde{w}^I \oplus \tilde{a}^I = \tilde{b}^I$, with $\tilde{w}^I = (w_1,w,w_2;\overline{w}_1,w,\overline{w}_2)$, $\tilde{a}^I = (a_1,a,a_2;\overline{a}_1,a,\overline{a}_2)$, and $\tilde{b}^I = (b_1,b,b_2;\overline{b}_1,b,\overline{b}_2)$, is satisfied if and only if $\overline{w}_1 = \overline{b}_1 - \overline{a}_1 \leq w_1 = b_1 - a_1 \leq w = b - a \leq w_2 = b_2 - a_2 \leq \overline{w}_2 = \overline{b}_2 - \overline{a}_2$. As the following example shows, this is not always accomplished.

$$\tilde{w}^I \oplus (0,2,3;0,2,4) = (1,2,5;-1,2,7) \longrightarrow \tilde{w}^I = (1,0,2;-1,0,3). \quad (10)$$

We notice that $\tilde{w}^I$ does not satisfy, e.g., the condition $w_1 \leq w$ (the first component must be less than or equal to the second one); hence, $\tilde{w}^I$ is not a well-defined TIFN.

However, if we add $\tilde{M}^I = (-m/2,0,m/2;-m,0,m)$ to the right-hand side of $\tilde{w}^I \oplus \tilde{a}^I = \tilde{b}^I$, we get the inequalities $\overline{w}_1 = -m + \overline{b}_1 - \overline{a}_1 \leq w_1 = -m/2 + b_1 - a_1 \leq w = b - a \leq w_2 = m/2 + b_2 - a_2 \leq \overline{w}_2 = m + \overline{b}_2 - \overline{a}_2$. It can be seen that, by choosing m that is positive and big enough, all inequalities are satisfied regardless of the values of $\tilde{a}^I$ and $\tilde{b}^I$, and $\tilde{w}^I$ is now well-defined.

Theorem 2. *Let $(\tilde{x}^I,\tilde{s}_2^I,\ldots,\tilde{s}_p^I,\tilde{p}_2^I,\ldots,\tilde{p}_p^I,\tilde{w}^I)$ be an optimal solution to problem (9), with $\lambda_r > 0$ for $r = 2,\ldots,p$. Then, $\tilde{x}^I$ is a Pareto optimal solution to problem (8).*

Proof. (By contradiction) Let us suppose that $(\tilde{x}^I,\tilde{s}_2^I,\ldots,\tilde{s}_p^I,\tilde{p}_2^I,\ldots,\tilde{p}_p^I,\tilde{w}_x^I)$ is an optimal solution to problem (9), but $\tilde{x}^I$ is not a Pareto optimal solution to problem (8). Then, there exists $\tilde{y}^I \in \tilde{X}^I$ such that $\tilde{z}_r^I(\tilde{y}^I) \preccurlyeq \tilde{z}_r^I(\tilde{x}^I)$ for $r = 1,\ldots,p$, with at least one strict

TABLE 1: Data of the IFMOT problem of Mahajan and Gupta [20].

		D1	D2	D3	Supply
S1	Cost	(4, 6, 8; 2, 6, 10)	(5, 7, 9; 3, 7, 11)	(6, 8, 10; 4, 8, 12)	(20, 24, 28; 18, 24, 32)
	Delay time	(3, 6, 9; 0, 6, 12)	(7, 10, 13; 4, 10, 16)	(10, 15, 20; 5, 15, 25)	
S2	Cost	(7, 9, 11; 5, 9, 13)	(12, 14, 16; 10, 14, 18)	(10, 12, 14; 8, 12, 16)	(15, 18, 24; 12, 18, 30)
	Delay time	(8, 12, 16; 4, 12, 20)	(10, 14, 18; 6, 14, 20)	(12, 16, 20; 8, 16, 24)	
Demand		(16, 18, 22; 14, 18, 24)	(8, 12, 16; 6, 12, 20)	(11, 12, 14; 10, 12, 18)	

inequality. Let $Q = \{r: \tilde{z}_r^I(\tilde{y}^I) \prec \tilde{z}_r^I(\tilde{x}^I), r = 1, \ldots, p\}$ be the set of indices of the strict inequalities. We have, for all $r \in Q/\{1\}$, that $\tilde{z}_r^I(\tilde{y}^I) \oplus \tilde{s}_r^I \prec \tilde{z}_r^I(\tilde{x}^I) \oplus \tilde{s}_r^I = \tilde{\epsilon}_r^I \oplus \tilde{p}_r^I$; hence $\tilde{z}_r^I(\tilde{y}^I) \oplus \tilde{s}_r^I \prec \tilde{\epsilon}_r^I \oplus \tilde{p}_r^I$, and we can choose $\tilde{m}_r^I \prec \tilde{n}_r^I$ to obtain

$\tilde{z}_r^I(\tilde{y}^I) \oplus \tilde{s}_r^I \oplus \tilde{n}_r^I = \tilde{\epsilon}_r^I \oplus \tilde{p}_r^I \oplus \tilde{m}_r^I$ for $r \in Q/\{1\}$ and $\tilde{m}_r^I = \tilde{n}_r^I$ for the equalities. Thus, it follows that $(\tilde{y}^I, \tilde{s}_2^I \oplus \tilde{n}_2^I, \ldots, \tilde{s}_p^I \oplus \tilde{n}_p^I, \tilde{p}_2^I \oplus \tilde{m}_2^I, \ldots, \tilde{p}_p^I \oplus \tilde{m}_p^I, \tilde{w}_y^I)$ is a feasible solution to problem (9). Furthermore, we have that

$$\tilde{w}_x^I \oplus \sum_{r \neq 1} \lambda_r \times \tilde{s}_r^I = \tilde{z}_1^I(\tilde{x}^I) \oplus \sum_{r \neq 1} \lambda_r \times \tilde{p}_r^I \oplus \tilde{M}^I,$$

$$\tilde{w}_y^I \oplus \sum_{r \neq 1} \lambda_r \times \tilde{s}_r^I \oplus \sum_{r \neq 1} \lambda_r \times \tilde{n}_r^I = \tilde{z}_1^I(\tilde{y}^I) \oplus \sum_{r \neq 1} \lambda_r \times \tilde{p}_r^I \oplus \sum_{r \neq 1} \lambda_r \times \tilde{m}_r^I \oplus \tilde{M}^I. \tag{11}$$

Adding $\tilde{z}_1^I(\tilde{x}^I)$ to both sides of the second equation, we get

$$\tilde{z}_1^I(\tilde{x}^I) \oplus \tilde{w}_y^I \oplus \sum_{r \neq 1} \lambda_r \times \tilde{s}_r^I \oplus \sum_{r \neq 1} \lambda_r \times \tilde{n}_r^I = \tilde{z}_1^I(\tilde{y}^I) \oplus \tilde{z}_1^I(\tilde{x}^I) \oplus \sum_{r \neq 1} \lambda_r \times \tilde{p}_r^I \oplus \sum_{r \neq 1} \lambda_r \times \tilde{m}_r^I \oplus \tilde{M}^I. \tag{12}$$

Upon substitution, we get

$$\tilde{z}_1^I(\tilde{x}^I) \oplus \tilde{w}_y^I \oplus \sum_{r \neq 1} \lambda_r \times \tilde{s}_r^I \oplus \sum_{r \neq 1} \lambda_r \times \tilde{n}_r^I = \tilde{z}_1^I(\tilde{y}^I) \oplus \tilde{w}_x^I \oplus \sum_{r \neq 1} \lambda_r \times \tilde{s}_r^I \oplus \sum_{r \neq 1} \lambda_r \times \tilde{m}_r^I. \tag{13}$$

By simplifying, we get

$$\tilde{z}_1^I(\tilde{x}^I) \oplus \tilde{w}_y^I \oplus \sum_{r \neq 1} \lambda_r \times \tilde{n}_r^I = \tilde{z}_1^I(\tilde{y}^I) \oplus \tilde{w}_x^I \oplus \sum_{r \neq 1} \lambda_r \times \tilde{m}_r^I. \tag{14}$$

Since $\tilde{m}_r^I \prec \tilde{n}_r^I$ for all $r \in Q/\{1\}$, $\tilde{m}_r^I = \tilde{n}_r^I$ for the equalities, and $\lambda_r > 0$ for $r = 2, \ldots, p$, then we have that

$$\tilde{z}_1^I(\tilde{x}^I) \oplus \tilde{w}_y^I \prec \tilde{z}_1^I(\tilde{y}^I) \oplus \tilde{w}_x^I. \tag{15}$$

Now, if $1 \notin Q$ (meaning $\tilde{z}_1^I(\tilde{x}^I) = \tilde{z}_1^I(\tilde{y}^I)$), we get $\tilde{w}_y^I \prec \tilde{w}_x^I$; thus contradicting the fact that $(\tilde{x}^I, \tilde{s}_2^I, \ldots, \tilde{s}_p^I, \tilde{p}_2^I, \ldots, \tilde{p}_p^I, \tilde{w}_x^I)$ is an optimal solution to problem (9). On the other hand, if $1 \in Q$ (meaning $\tilde{z}_1^I(\tilde{y}^I) \prec \tilde{z}_1^I(\tilde{x}^I)$), we have $\tilde{z}_1^I(\tilde{y}^I) \oplus \tilde{w}_y^I \prec \tilde{z}_1^I(\tilde{x}^I) \oplus \tilde{w}_y^I \prec \tilde{z}_1^I(\tilde{y}^I) \oplus \tilde{w}_x^I$. Hence, $\tilde{z}_1^I(\tilde{y}^I) \oplus \tilde{w}_y^I \prec \tilde{z}_1^I(\tilde{y}^I) \oplus \tilde{w}_x^I$, which

also contradicts the fact that $(\tilde{x}^I, \tilde{s}_2^I, \ldots, \tilde{s}_p^I, \tilde{p}_2^I, \ldots, \tilde{p}_p^I, \tilde{w}_x^I)$ is an optimal solution to problem (9). □

5. Illustrative Examples

We present in this section two examples to illustrate the proposed method. The examples are taken from Mahajan and Gupta [20] and Ghosh et al. [21]. A comparison with their methods is carried out as well. Calculations were performed by using YALMIP toolbox [25] version 20180413 and Octave 5.2.0 on a computer with an Intel® Core™ i3-4005U @ 1.70 GHz × 4 and 4 GB RAM running Ubuntu 20.04.3 LTS. The lexicographic method from Section 3 was used with $\epsilon = 10^{-4}$ and $L = 10^4$, and m was set to 10^4 in the proposed ϵ-constraint method. Results are presented up to three decimal places.
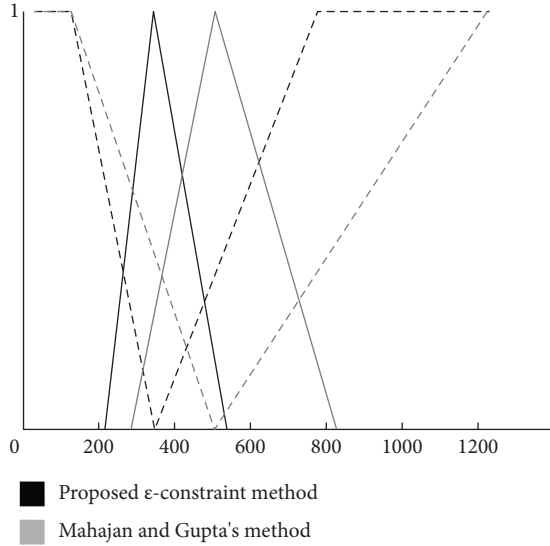
FIGURE 2: Intuitionistic fuzzy cost obtained with the proposed method and Mahajan and Gupta's method.
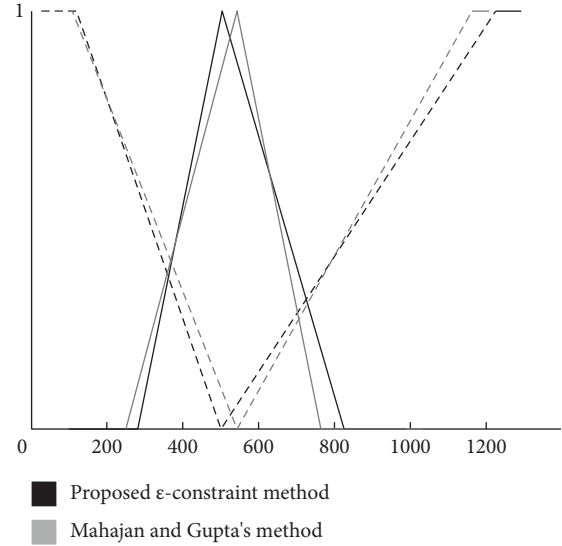

FIGURE 3: Intuitionistic fuzzy delay time obtained with the proposed method and Mahajan and Gupta's method.

5.1. Transportation Problem of Mahajan and Gupta. Let us consider the IFMOT problem presented as Example 2 by Mahajan and Gupta [20]. The problem consists of two sources (S1 and S2) and three destinations (D1, D2, and D3), in which the demand at each destination must be satisfied, and the total transportation costs and delay times must be minimized (see Table 1). The problem is formulated as the intuitionistic fuzzy biobjective linear programming problem as shown in the following equation:

$$\min\left(\widetilde{z}_1^I\left(\widetilde{x}^I\right),\widetilde{z}_2^I\left(\widetilde{x}^I\right)\right)=\left(\sum_{i,j}\widetilde{c}_{ij}^I\odot\widetilde{x}_{ij}^I,\sum_{i,j}\widetilde{t}_{ij}^I\odot\widetilde{x}_{ij}^I\right),$$

$$s.t.\quad\begin{aligned}&\sum_j\widetilde{x}_{ij}^I=\text{Supply}_i\text{ for }i=1,2,\\&\sum_i\widetilde{x}_{ij}^I=\text{Demand}_j\text{ for }j=1,2,3,\\&\widetilde{x}^I=\left(\widetilde{x}_{ij}^I\right)\geq0\text{ for }i=1,2\text{ and }j=1,2,3.\end{aligned}\quad(16)$$

In IFMOT problem (16), $\widetilde{c}_{ij}^I$ and $\widetilde{t}_{ij}^I$ denote the unitary cost and delay time of transporting $\widetilde{x}_{ij}^I$ units from source i to destination j, respectively.

By using Mahajan and Gupta's [20] method with the ranking function given in Definition 7 and linear membership functions, the solution to problem (16) is $\widetilde{x}_{11}^I=(12,12,13.062;12,12,13.062)$, $\widetilde{x}_{12}^I=(8,12,12.373;6,12,12.373)$, $\widetilde{x}_{13}^I=(0,0,2;0,0,6)$, $\widetilde{x}_{21}^I=(4,6,8.937;2,6,10.937)$, $\widetilde{x}_{22}^I=(0,0,3.062;0,0,7.062)$, and $\widetilde{x}_{23}^I=(11,12,12;10,12,12)$, with values in the objective functions $\widetilde{z}_1^I\left(\widetilde{x}^I\right)=(226,354,556.25;132,354,806.25)$ and $\widetilde{z}_2^I\left(\widetilde{x}^I\right)=(256,546,763.875;112,546,1161.75)$.

We will try to improve Mahajan and Gupta's [20] results by using the proposed method. To this aim, we formulate the IFLP problem as

$$\min\widetilde{w}^I,$$

$$\begin{aligned}&\widetilde{w}^I\oplus\lambda_2\times\widetilde{s}_2^I=\widetilde{z}_1^I\left(\widetilde{x}^I\right)\oplus\lambda_2\times\widetilde{p}_2^I\oplus\widetilde{M}^I,\\&\widetilde{z}_2^I\left(\widetilde{x}^I\right)\oplus\widetilde{s}_2^I=\widetilde{\epsilon}_2^I\oplus\widetilde{p}_2^I,\\&\widetilde{s}_2^I\preccurlyeq\widetilde{p}_2^I,\widetilde{s}_2^I\geq0,\widetilde{p}_2^I\geq0,\\s.t.\quad&\sum_j\widetilde{x}_{ij}^I=\text{Supply}_i\text{ for }i=1,2,\\&\sum_i\widetilde{x}_{ij}^I=\text{Demand}_j\text{ for }j=1,2,3,\\&\widetilde{x}^I=\left(\widetilde{x}_{ij}^I\right)\geq0\text{ for }i=1,2\text{ and }j=1,2,3,\end{aligned}\quad(17)$$

where $\lambda_2=0.01$ and $\widetilde{\epsilon}_2^I=(256,546,763.875;112,546,1161.75)$. To compare TIFNs, we resort to the lexicographic ranking criterion determined by the linear functions $f_1\left(\widetilde{a}^I\right)=(a_1+a_2+4a+\overline{a}_1+\overline{a}_2)/8$, $f_2\left(\widetilde{a}^I\right)=a$, $f_3\left(\widetilde{a}^I\right)=a_1$, $f_4\left(\widetilde{a}^I\right)=a_2-a_1$, and $f_5\left(\widetilde{a}^I\right)=\overline{a}_2$.

The first function is Mahajan and Gupta's [20] accuracy function, the second is the modal value of the TIFN $\widetilde{a}^I$, the third and fourth are the left endpoint and length of the support of $\mu_{\widetilde{a}^I}$, i.e., $cl\{x\mid x\in\mathbb{R},\mu_{\widetilde{a}^I}(x)>0\}=[a_1,a_2]$, respectively, and the last one is the right endpoint of the support of $1-\nu_{\widetilde{a}^I}$, i.e., $cl\{x\mid x\in\mathbb{R},\nu_{\widetilde{a}^I}(x)<1\}=[\overline{a}_1,\overline{a}_2]$.

On solving IFLP problem (17) with the lexicographic method from Section 3, we get $\widetilde{x}_{11}^I=(2.159,2.159,2.159;2.159,2.159,2.159)$, $\widetilde{x}_{12}^I=(8,12,16;6,12,20)$, $\widetilde{x}_{13}^I=(9.840,9.840,9.840;9.840,9.840,9.840)$, $\widetilde{x}_{21}^I=(13.840,15.840,19.840;11.840,15.840,21.840)$, and $\widetilde{x}_{23}^I=(1.159,2.159,4.159;0.159,2.159,8.159)$, with values in the objective functions $\widetilde{z}_1^I(\widetilde{x}^I)=(216.159,344.159,536.159;122.159,344.159,774.159)$ and $\widetilde{z}_2^I(\widetilde{x}^I)=(285.521,505.203,824.884;121.840,505.203,1224.565)$. Comparing both solutions, we have, for the first objective function,

Table 2: Pareto optimal solutions to IFMOT problem (16) obtained with the proposed method.

No			D1	D2	D3
1	Variables	S1	(12, 13.581, 17.581; 12, 13.581, 19.581)	(8, 10.418, 10.418; 6, 10.418, 12.418)	(0, 0, 0; 0, 0, 0)
		S2	(4, 4.418, 4.418; 2, 4.418, 4.418)	(0, 1.581, 5.581; 0, 1.581, 7.581)	(11, 12, 14; 10, 12, 18)
	1st objective			(226, 360.325, 568.325; 132, 360.325, 814.325)	
	2nd objective			(256, 452.838, 744.838; 112, 452.838, 1105.68)	
2	Variables	S1	(7.756, 7.756, 7.756; 7.756, 7.756, 7.756)	(8, 12, 16; 6, 12, 20)	(4.243, 4.243, 4.243; 4.243, 4.243, 4.243)
		S2	(8.243, 10.243, 14.243; 6.243, 10.243, 16.243)	(0, 0, 0; 0, 0, 0)	(6.756, 7.756, 9.756; 5.756, 7.756, 13.756)
	1st objective			(221.756, 349.756, 541.756; 127.756, 349.756, 779.756)	
	2nd objective			(268.731, 477.218, 785.705; 116.244, 477.218, 1174.19)	
3	Variables	S1	(4.398, 4.398, 4.398; 4.398, 4.398, 4.398)	(8, 12, 16; 6, 12, 20)	(7.601, 7.601, 7.601; 7.601, 7.601, 7.601)
		S2	(11.601, 13.601, 17.601; 9.601, 13.601, 19.601)	(0, 0, 0; 0, 0, 0)	(3.398, 4.398, 6.398; 2.398, 6.398, 10.398)
	1st objective			(218.398, 346.398, 538.398; 124.398, 346.398, 776.398)	
	2nd objective			(278.805, 494.009, 809.213; 119.602, 494.009, 1204.42)	

Table 3: Data of the IFMOT problem of Ghosh et al. [21].

		D1		D2		Supply
		C1	C2	C1	C2	
S1	Cost	(2, 5, 8; 1, 5, 9)	(9, 10, 13; 8, 10, 14)	(2, 4, 5; 1, 4, 6)	(3, 6, 8; 2, 6, 9)	(150, 180, 200; 130, 180, 220)
	Fixed-charge	(100, 150, 250; 80, 150, 270)	(200, 260, 320; 180, 260, 340)	(150, 200, 250; 130, 200, 270)	(180, 240, 300; 160, 240, 320)	
	Deterioration	(0.7, 1.0, 1.1; 0.6, 1.0, 1.2)	(0.9, 1.0, 1.3; 0.8, 1.0, 1.4)	(1.2, 1.3, 1.5; 1.1, 1.3, 1.6)	(1.4, 1.6, 1.7; 1.3, 1.6, 1.8)	
	Time	(11, 12, 14; 10, 12, 15)	(14, 15, 17; 13, 15, 18)	(6, 8, 10; 5, 8, 11)	(8, 9, 11; 7, 9, 12)	
S2	Cost	(4, 6, 9; 3, 6, 10)	(2, 4, 7; 1, 4, 8)	(5, 6, 9; 4, 6, 10)	(3, 4, 6; 2, 4, 7)	(250, 290, 310; 230, 290, 330)
	Fixed-charge	(100, 200, 250; 80, 200, 270)	(150, 200, 300; 120, 200, 330)	(260, 300, 350; 240, 300, 370)	(100, 140, 180; 70, 140, 210)	
	Deterioration	(0.3, 0.4, 0.7; 0.2, 0.4, 0.8)	(0.4, 0.5, 0.7; 0.3, 0.5, 0.8)	(0.8, 1.0, 1.3; 0.7, 1.0, 1.4)	(0.8, 1.0, 1.2; 0.7, 1.0, 1.3)	
	Time	(3, 6, 8; 2, 6, 9)	(5, 9, 11; 4, 9, 12)	(4, 6, 10; 3, 6, 11)	(9, 11, 14; 8, 11, 15)	
Demand		(240, 270, 300; 220, 270, 320)		(160, 200, 210; 140, 200, 230)		

TABLE 4: Pareto optimal solutions to IFMOT problem (21) obtained with the proposed method.

| No		D1		D2	
		C1	C2	C1	C2
1	Variables S1	(0, 0, 10; 0, 0, 10)	(0, 0, 0; 0, 0, 0)	(150, 180, 190; 130, 180, 210)	(0, 0, 0; 0, 0, 0)
	Variables S2	(40, 40, 40; 40, 40, 40)	(200, 230, 250; 180, 230, 270)	(10, 20, 20; 10, 20, 20)	(0, 0, 0; 0, 0, 0)
	1st objective		(1570, 2900, 4720; 1040, 2900, 5620)		
	2nd objective		(280, 385, 525; 212, 385, 624)		
	3rd objective		(18, 29, 53; 14, 29, 58)		
2	Variables S1	(0, 0, 10; 0, 0, 10)	(0, 0, 0; 0, 0, 0)	(150, 180, 190; 130, 180, 210)	(0, 0, 0; 0, 0, 0)
	Variables S2	(50, 60, 60; 50, 60, 60)	(190, 210, 230; 170, 210, 250)	(0, 0, 0; 0, 0, 0)	(10, 20, 20; 10, 20, 20)
	1st objective		(1410, 2740, 4530; 870, 2740, 5440)		
	2nd objective		(279, 383, 523; 211, 383, 622)		
	3rd objective		(23, 34, 57; 19, 34, 62)		
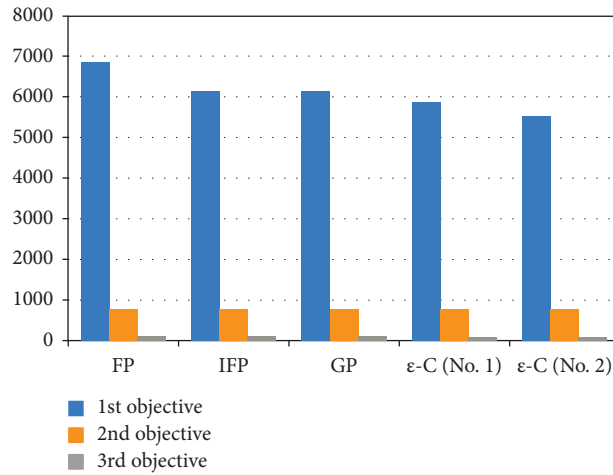
Figure 4: Values of the objective functions (in terms of the first comparison index) obtained with Ghosh et al.'s [21] methods and the proposed method.

$$f_1(216.159, 344.159, 536.159; 122.159, 344.159, 774.159) = 378.159$$
$$< f_1(226, 354, 556.25; 132, 354, 806.25) = 392.062, \tag{18}$$

and, for the second objective function, we have

$$f_1(285.521, 505.203, 824.884; 121.840, 505.203, 1224.565) = 559.703$$
$$= f_1(256, 546, 763.875; 112, 546, 1161.75) = 559.703, \tag{19}$$

but

$$f_2(285.521, 505.203, 824.884; 121.840, 505.203, 1224.565)$$
$$= 505.203 \tag{20}$$
$$< f_2(256, 546, 763.875; 112, 546, 1161.75) = 546.$$

This means that the solution obtained with the proposed method dominates the solution obtained with Mahajan and Gupta's [20] method. Consequently, Mahajan and Gupta's [20] method does not guarantee Pareto optimal solutions. Figures 2 and 3 depict the values of the objective functions, and it is noticeable in the fact that the proposed method produced results with lower total cost and delay time.

Furthermore, as Table 2 shows, the proposed method can produce multiple Pareto optimal solutions. This is also an advantage over Mahajan and Gupta's [20] method.

5.2. Transportation Problem of Ghosh et al. Let us consider the intuitionistic fuzzy three-objective fixed-charge solid transportation problem presented by Ghosh et al. [21]. The problem consists of two sources (S1 and S2) and two destinations (D1 and D2), in which the demand of fruits at each destination must be satisfied using the two types of conveyances (C1 and C2), and the total transportation costs, deterioration and the transportation times must be minimized (see Table 3). The problem is formulated as the intuitionistic fuzzy three-objective linear programming problem which is represented as

$$\min\left(\widetilde{z}_1^I\left(\widetilde{x}^I\right), \widetilde{z}_2^I\left(\widetilde{x}^I\right), \widetilde{z}_3^I\left(\widetilde{x}^I\right)\right) = \left(\sum_{i,j,k}\left(\widetilde{c}_{ijk}^I \odot \widetilde{x}_{ijk}^I \oplus \widetilde{f}_{ijk}^I \odot \left(y_{1ijk}, y_{ijk}, y_{2ijk}; \overline{y}_{1ijk}, y_{ijk}, \overline{y}_{2ijk}\right)\right)\right)|,$$

$$\sum_{i,j,k}\widetilde{d}_{ijk}^I \odot \widetilde{x}_{ijk}^I, \sum_{i,j,k}\widetilde{t}_{ijk}^I \odot \left(y_{1ijk}, y_{ijk}, y_{2ijk}; \overline{y}_{1ijk}, y_{ijk}, \overline{y}_{2ijk}\right)\right),$$

$$\sum_{j,k}\widetilde{x}_{ijk}^I \le \text{Supply}_i \text{ for } i = 1, 2,$$

$$\sum_{i,k}\widetilde{x}_{ijk}^I \ge \text{Demand}_j \text{ for } j = 1, 2, \tag{21}$$

$$\text{s.t.} \quad \sum_{i,j}\widetilde{x}_{ijk}^I \le \text{Capacity}_k \text{ for } k = 1, 2,$$

$$y_{1ijk} = \begin{cases} 1 & \text{if } x_{1ijk} > 0, \\ 0 & \text{otherwise}, \end{cases} \quad y_{ijk} = \begin{cases} 1 & \text{if } x_{ijk} > 0, \\ 0 & \text{otherwise}, \end{cases} \quad y_{2ijk} = \begin{cases} 1 & \text{if } x_{2ijk} > 0, \\ 0 & \text{otherwise}, \end{cases}$$

$$\overline{y}_{1ijk} = \begin{cases} 1 & \text{if } \overline{x}_{1ijk} > 0, \\ 0 & \text{otherwise}, \end{cases} \quad \overline{y}_{2ijk} = \begin{cases} 1 & \text{if } \overline{x}_{2ijk} > 0, \\ 0 & \text{otherwise}, \end{cases}$$

$$\widetilde{x}^I = \left(\widetilde{x}_{ijk}^I = \left(x_{1ijk}, x_{ijk}, x_{2ijk}; \overline{x}_{1ijk}, x_{ijk}, \overline{x}_{2ijk}\right)\right) \ge 0 \text{ for } i = 1, 2, j = 1, 2 \text{ and } k = 1, 2.$$

In IFMOT problem (21), $\widetilde{c}_{ijk}^I$, $\widetilde{f}_{ijk}^I$, $\widetilde{d}_{ijk}^I$, and $\widetilde{t}_{ijk}^I$ denote the unitary cost, fixed-charge, deterioration rate, and time of transporting $\widetilde{x}_{ijk}^I$ units of fruits from source i to destination j using the conveyance k, respectively. Two conveyances are used, one of type C1 with capacity $(200, 240, 260; 180, 240, 280)$ and the other of type C2 with capacity $(200, 230, 250; 180, 230, 270)$.

Ghosh et al. [21] used the linear ranking function $f(\widetilde{a}^I) = (2a_1 + 2a_2 + 34a + \overline{a}_1 + \overline{a}_2)/20$ to transform the problem into a conventional multiobjective one, which was

solved by using three different methods: fuzzy linear programming (via Zimmermann's [26] approach), intuitionistic fuzzy linear programming (via Angelov's [8] approach), and goal programming. In what follows, we present the results obtained when the proposed method is used.

By using the proposed intuitionistic fuzzy ε-constraint method, the IFMOT problem (21) is transformed into a single-objective problem as represented by the following equation:

$$\min \widetilde{w}^I,$$

$$\widetilde{w}^I \oplus \lambda_1 \times \widetilde{s}_1^I \oplus \lambda_2 \times \widetilde{s}_2^I = \widetilde{z}_3^I\left(\widetilde{x}^I\right) \oplus \lambda_1 \times \widetilde{p}_1^I \oplus \lambda_2 \times \widetilde{p}_2^I \oplus \widetilde{M}^I,$$

$$\widetilde{z}_1^I\left(\widetilde{x}^I\right) \oplus \widetilde{s}_1^I = \widetilde{\epsilon}_1^I \oplus \widetilde{p}_1^I,$$

$$\widetilde{z}_2^I\left(\widetilde{x}^I\right) \oplus \widetilde{s}_2^I = \widetilde{\epsilon}_2^I \oplus \widetilde{p}_2^I,$$

$$\widetilde{s}_1^I \preccurlyeq \widetilde{p}_1^I, \widetilde{s}_2^I \preccurlyeq \widetilde{p}_2^I, \widetilde{s}_1^I \ge 0, \widetilde{p}_1^I \ge 0, \widetilde{s}_2^I \ge 0, \widetilde{p}_2^I \ge 0,$$

$$\sum_{j,k}\widetilde{x}_{ijk}^I \le \text{Supply}_i \text{ for } i = 1, 2,$$

$$\sum_{i,k}\widetilde{x}_{ijk}^I \ge \text{Demand}_j \text{ for } j = 1, 2, \tag{22}$$

$$\text{s.t.} \quad \sum_{i,j}\widetilde{x}_{ijk}^I \le \text{Capacity}_k \text{ for } k = 1, 2,$$

$$y_{1ijk} = \begin{cases} 1 & \text{if } x_{1ijk} > 0, \\ 0 & \text{otherwise}, \end{cases} \quad y_{ijk} = \begin{cases} 1 & \text{if } x_{ijk} > 0, \\ 0 & \text{otherwise}, \end{cases} \quad y_{2ijk} = \begin{cases} 1 & \text{if } x_{2ijk} > 0, \\ 0 & \text{otherwise}, \end{cases}$$

$$\overline{y}_{1ijk} = \begin{cases} 1 & \text{if } \overline{x}_{1ijk} > 0, \\ 0 & \text{otherwise}, \end{cases} \quad \overline{y}_{2ijk} = \begin{cases} 1 & \text{if } \overline{x}_{2ijk} > 0, \\ 0 & \text{otherwise}, \end{cases}$$

$$\widetilde{x}^I = \left(\widetilde{x}_{ijk}^I = \left(x_{1ijk}, x_{ijk}, x_{2ijk}; \overline{x}_{1ijk}, x_{ijk}, \overline{x}_{2ijk}\right)\right) \ge 0 \text{ for } i = 1, 2, j = 1, 2$$

$$\text{and } k = 1, 2.$$

Table 4 shows two Pareto optimal solutions to IFMOT problem (21) obtained by solving its scalarized version IFLP problem (22) with the lexicographic ranking criterion

determined by $f_1(\widetilde{a}^I) = (2a_1 + 2a_2 + 34a + \overline{a}_1 + \overline{a}_2)/20$ (this is the same linear ranking function used by Ghosh et al. [21]), $f_2(\widetilde{a}^I) = a$, $f_3(\widetilde{a}^I) = a_1$, $f_4(\widetilde{a}^I) = a_2 - a_1$, and

$f_5(\tilde{a}^I) = \overline{a}_2$, with $\lambda_1 = \lambda_2 = 0.01$, $\tilde{\epsilon}_1^I = (1858, 3218, 5122; 1262, 3218, 6084)$ and $\tilde{\epsilon}_2^I = (280.3, 392.8, 531.8; 212.1, 392.8, 632.4)$ (solution no. 1), and $\tilde{\epsilon}_1^I = (1410, 2740, 4530; 870, 2740, 5440)$ and $\tilde{\epsilon}_2^I = (279, 383, 523; 211, 383, 622)$ (solution no. 2). Figure 4 depicts the values of the objective functions in terms of the first comparison index. It can be noticed that the proposed method improves in both solutions almost all values of the objective functions obtained with Ghosh et al.'s [21] methods. In fact, solutions nos. 1 and 2 are not dominated. In contrast, solution no. 2 dominates those obtained with Ghosh et al.'s [21] methods. The reader is encouraged to verify these claims by using the ranking criteria and Ghosh et al.'s [21] data. For example, the solution obtained by Ghosh et al. [21] via Zimmermann's [26] approach with linear membership functions (FP in Figure 4) is compared to solution no. 2 as shown in the following equation:

$$f_1\left(\tilde{z}_{1_{\epsilon-C}}^I\right) = 5567.5 < f_1\left(\tilde{z}_{1_{FP}}^I\right) = 6860.5,$$

$$f_1\left(\tilde{z}_{2_{\epsilon-C}}^I\right) = 772.95 < f_1\left(\tilde{z}_{2_{FP}}^I\right) = 789.05, \qquad (23)$$

$$f_1\left(\tilde{z}_{3_{\epsilon-C}}^I\right) = 69.85 < f_1\left(\tilde{z}_{3_{FP}}^I\right) = 87.3.$$

Therefore, according to Definition 9, we conclude that the solution obtained with the proposed method dominates the one obtained with Ghosh et al.'s [21] method. Similarly, we would conclude the same about the other solutions.

6. On the Extension of the Proposed Method to Other Uncertain Environments

Several authors have solved optimization problems in which uncertainty in the data is modeled by using different extensions of Zadeh's [4] fuzzy sets (see, e.g., [22, 27–29], and [30]). To extend the proposed ϵ-constraint method to those uncertain environments, we must use the appropriate arithmetic operations in each case, choose a lexicographic ranking criterion that adequately captures the decision-maker's preferences, and derive the appropriate value for the constant $\tilde{M}^I$ in problem (9). For example, if the uncertainty were to be modeled with LR-type intuitionistic fuzzy numbers, then Singh and Yadav's [22] arithmetic operations could be used along with the lexicographic ranking criterion proposed in [23] and $\tilde{M}^I = (0, m/2, m/2, m, m)_{LR}$. If, on the other hand, the uncertainty were best represented by trapezoidal intuitionistic fuzzy numbers, then the arithmetic operations for those numbers [27], the lexicographic ranking criterion proposed by Lakshmana Gomathi Nayagam et al. [31], and $\tilde{M}^I = (-m/2, -m/4, m/4, m/2; -m, -m/3, m/3, m)$ could be used with the proposed ϵ-constraint method. A similar approach would be followed with Pythagorean fuzzy numbers [30].

7. Conclusions and Future Work

In this article, we proposed an intuitionistic fuzzy ϵ-constraint method for solving IFMOLP problems, in which uncertainty in the data is represented by TIFNs. In doing so, we extended recent results from [11] to the intuitionistic fuzzy environment. The proposed method was illustrated by solving two IFMOT problems addressed in the recent literature. On the basis of the present study, we conclude the following:

(i) Linear ranking function-based methods do not always guarantee Pareto optimal solutions to IFMOLP problems

(ii) The proposed intuitionistic fuzzy ϵ-constraint method guarantees Pareto optimality in terms of lexicographic ranking criteria

(iii) Mahajan and Gupta's [20] and Ghosh et al.'s, [21] methods cannot produce multiple solutions

(iv) The proposed method can produce multiple Pareto optimal solutions, which is a valuable feature for analyzing alternative solutions in practical decision-making

A limitation of the proposed method is that it uses binary variables to handle lexicographic constraints. This feature certainly increases the computational burden and is a disadvantage with respect to Mahajan and Gupta's [20] and Ghosh et al.'s' [21] methods. Therefore, research studies must continue to develop more efficient approaches for handling lexicographic constraints, and testing is needed on large-scale problems to gain insights on the true efficiency of the proposed method. In this regard, hybridization with metaheuristic algorithms seems to be a promising research line to explore.

We hope that, along with the numerical demonstrations, the methodological aspects for dealing with IFMOLP problems using lexicographic ranking criteria discussed in this study will motivate researchers to extend our results to other uncertain environments, such as those modeled with interval-valued fuzzy sets [28], interval-valued intuitionistic fuzzy sets [29, 32], and Pythagorean fuzzy sets [30]. This topic and intuitionistic fuzzy nonlinear optimization are lines of work that we may explore in the future. We will definitely focus on developing a library in a programming language with wide support for scientific calculations, such as Python or R, to reach more application areas, ease comparison with alternative methods from the literature, carry out efficiency analysis in large-scale problems, and to develop ADM systems in areas of economic interest, such as transportation and tourism.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have are no conflicts of interest.

Acknowledgments

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