Randomized response estimation in multiple frame surveys

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ABSTRACT
Large scale surveys are increasingly delving into sensitive topics such as gambling, alcoholism, drug use, sexual behavior, domestic violence. Sensitive, stigmatizing or even incriminating themes are difficult to investigate by using standard data-collection techniques since respondents are generally reluctant to release information which concern their personal sphere. Further, such topics usually pertain elusive population (e.g., irregular immigrants and homeless, alcoholics, drug users, rape and sexual assault victims) which are difficult to sample since not adequately covered in a single sampling frame. On the other hand, researchers often utilize more than one data-collection mode (i.e., mixed-mode surveys) in order to increase response rates and/or improve coverage of the population of interest. Surveying sensitive and elusive populations and mixed-mode researches are strictly connected with multiple frame surveys which are becoming widely used to decrease bias due to undercoverage of the target population. In this work, we combine sensitive research and multiple frame surveys. In particular, we consider statistical techniques for handling sensitive data coming from multiple frame surveys using complex sampling designs. Our aim is to estimate the mean of a sensitive variable connected to undesirable behaviors when data are collected by using the randomized response theory. Some estimators are constructed and their properties theoretically investigated. Variance estimation is also discussed by means of the jackknife technique. Finally, a Monte Carlo simulation study is conducted to evaluate the performance of the proposed estimators and the accuracy of variance estimation..

KEYWORDS
Auxiliary information; Monte Carlo simulation, Multiple frame sampling; Privacy protection; Randomized response theory; Sensitive issues

1. Introduction

Mostly in socioeconomic and biomedical studies, very often the researcher has to gather information relating to highly sensitive, embarrassing or even threatening issues. When doing sensitive research, posing direct questions to the respondents by means of traditional data-collection methods (e.g., self-administered questionnaires with paper and pencil, computer-assisted telephone interviewing, computer-assisted self interviewing, audio computer-assisted self interviewing or by computer-assisted Web interviewing) may procure untruthful responses or even refuse to respond because of social stigma or fear about threat of disclosure. Such systematic nonsampling response errors lead to social desirability bias in the estimates of sensitive characteristics. Social desirability

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bias occurs when respondents tend to present themselves in a positive light, meaning that they overreport socially acceptable attitudes which conform to social norms (e.g., giving to charity, believing in God, church attendance, voting, healthy eating, doing voluntary work) and underreport socially disapproved, undesirable behaviours which deviate from social rules (e.g., xenophobia, anti-Semitism, gambling, consumption of alcohol, abortion, sexual violence, drug and enhancing substances, tax evasion). The effect is to flaw the quality of the collected data and produce unreliable analysis of the sensitive behavior under investigation.

To limit unacceptable rates of nonresponse and obtain more reliable data, indirect questioning methods (see, e.g, Chaudhuri and Cristofides, 2013), such as the randomized response (RR) theory (RRT), may be used. The RRT was originated by Warner (1965) who proposed a data-collection procedure that allows researchers to obtain more reliable sensitive information by increasing respondents' cooperation without jeopardizing privacy protection. To guarantee confidentiality to the respondents, a randomization device (decks of cards, coloured numbered balls, dice, coins, spinners, random number generators, etc.) is used to hide the answers in the sense that the respondents reply to one of two or more selected questions depending on the result of the device. Privacy is protected since respondents do not reveal to anyone the question that has been selected and nobody, except the respondent, knows the outcome generated by the randomization device. Since privacy is fully protected, the approach should, in principle, encourage greater cooperation from respondents and reduce their motivation to falsely report their attitudes. Hence, it is expected that survey participants are compliant with the rules prescribed by the adopted randomization mechanism and are completely honest in releasing their responses. The randomization device generates a probabilistic relationship between respondents' answers and the true sensitive status which is used to draw inferences about unknown sensitive population characteristics such as the prevalence of a stigmatizing attribute, the mean/total of a quantitative sensitive variable or its distribution function.

The most important claim made for the RRT is that it can yield valid estimates of sensitive behaviours. Many studies have assessed the validity of RR methods and shown that they can produce more reliable answers than other conventional data-collection methods based on direct questions in face-to-face interviews, self-administered questionnaires with paper and pencil, and computer assisted self interviews. On this, see, van der Heijden et al. (2000); Lara et al. (2004); Lensvelt-Mulders et al. (2005) just to name a few. On the other hand, using RRT incurs extra costs which can be outweighed only if the analyses are substantially better than those derived from straightforward question-and-answer designs.

The RRT has been applied in surveys covering a variety of sensitive topics including, for instance, racism (Ostapczuk et al., 2009; Krumpal, 2012), drug use (Kerkvliet, 1994; Striegel et al., 2010; Stubbe et al. 2013; Perri et al., 2017), abortion (Lara et al., 2004, 2006; Perri et al., 2016), sexual victimization (Krebs et al., 2011), academic cheating and plagiarism (Fox and Meijer, 2008; Jann et al., 2012), tax evasion (Houston and Tran, 2001; Korndörfer et al., 2014), HIV/AIDS infection and high-risk sexual behaviors (Arnab and Singh, 2010; Geng et al., 2016), animal diseases (Cross et. al., 2010; Gunarathne et al., 2016), illegal fishing and hunting (Nuno et al., 2013; Conteh et al., 2015).

Warner's study generated a rapidly-expanding body of research literature on alternative techniques for eliciting suitable RR schemes in order to estimate a population proportion (see, e.g., Arnab, 1996; Barabesi and Marcheselli, 2006; Barabesi, 2008; Gjestvang and Singh, 2006; Lee et al., 2013; Liu and Tian, 2013; Perri, 2008).

Standard RR methods have been basically conceived to be used in surveys which require a binary response (”yes” or ”no”) to a sensitive question, and seek to estimate the proportion of people presenting a given sensitive attribute. Nevertheless, empirical studies may address situations in which the response to a sensitive question results in a quantitative variable and the interest of the researcher relies, in the easiest case, on the estimation of the mean or the total of the sensitive variable under study. To deal with such situations, Warner’s idea has been promptly extended to sensitive quantitative variables by Greenberg et al. (1971), Eriksson (1973) and Pollock and Beck (1976). Since then a lot of mechanism have been proposed in the literature to scramble the response and, thus, protect respondents’ privacy (see, e.g., Eichhorn and Hayre, 1983; Bar-Lev et al., 2004; Saha, 2007a; Diana and Perri, 2010; Gupta et al., 2010; Odumade and Singh, 2010; Arcos et al., 2015, and the contributions collected in Chaudhuri et al., 2016). When dealing with quantitative sensitive variables, the idea is to ask respondents to not disclose the true value of the sensitive variable but rather to release a scrambled value obtained by algebraically perturb the true response making use of one or more scrambling random variables, independent each other and of the sensitive one, which distributions are completely known to the researcher.

Usually, RR methods, both for qualitative and quantitative variables, have been theoretically developed assuming that the observed responses are collected on sampled units selected according to simple random sampling. Indeed, most of the real studies are based on complex surveys involving, for instance, stratification, clustering and unequal probability sampling designs. Therefore, the RRT has been extended to more complex sampling design, as stratified sampling (Mahajan and Singh, 2005; Kim and Elam, 2007; Saha, 2007b; Singh and Tarray, 2015), or unequal probability sampling (Chaudhuri, 2001, 2004; Arnab and Dorffer, 2006; Saha, 2007a; Pal, 2008; Quatember, 2012). An interesting development of the RRT concerns the use of auxiliary information at the estimation stage to improve the performance of the randomization device without additional costs and without infringing respondents’ privacy. Recent contributions in this field have been proposed, among other, by Diana and Perri (2011, 2012); Gupta et al. (2012); Perri and Diana (2013); Koyuncu et al. (2014); Özgül and Cingi (2017) and Rueda et al. (2017a).

Readers interested in the RRT and other alternative indirect questioning techniques approaches are referred to the monographs by Fox and Tracy (1986), Fox (2016) Chaudhuri and Mukerjee (1988), Chaudhuri (2011), Chaudhury and Christofides (2013) and Chaudhuri et al. (2016).

Traditionally, surveys have been carried out using three main methods of data collection: face-to-face interviews, mail surveys and telephone interviews. Over the last 20 years, the picture has changed sharply. Telephone surveys have become a popular mode for collecting data, especially following the creation and development of computer-assisted telephone interviewing systems. Telephone interviews are often considered a less costly alternative to mail and face-to-face interviews and the population coverage reaches acceptable levels. However, telephone surveys also present some drawbacks with regard to coverage, due to the absence of a telephone in some households and the generalized use of mobile phone which are sometimes replacing fixed (land) lines entirely. The potential for coverage error as a result of the exponential growth of the cell phone-only population has led to the development of dual frame surveys. On the other hand, sensitive surveys usually meet elusive or hard-to-reach populations which are not listed in a unique sampling frame and required more than one frame to to be sufficiently covered.

Dual frame surveys were introduced as a device for reducing data-collection costs without affecting the accuracy of the results with respect to single frame surveys. Since then, the multiple frame sampling theory has experienced a noticeable development and several estimators for the total of a continuous variable have been proposed. Hartley (1962) proposed the first dual frame estimator which was improved by Lund (1968) and Fuller and Burmeister (1972). Bankier (1986), Kalton and Anderson (1986) and Skinner (1991) proposed dual frame estimators based on new techniques. Skinner and Rao (1996) and Rao and Wu (2010) applied likelihood methods to compute estimators that perform well in complex designs. More recently, Ranalli et al. (2016) used calibration techniques to derive estimators in the dual frame context. Although the copious literature on dual frame theory, there are very few studies that address the problem of estimating sensitive behaviors under this setting. Recently Rueda et al. (2015) proposed some dual frame estimators for proportions and means when data are obtained by using the RRT.

Surveys where data are collected from three sampling frames are also used in practice. Iachan and Dennis (1993) used a three frame survey to reach the homeless population of Washington D.C. metropolitan area. Frames in this survey were composed of homeless shelters, soup kitchens and street areas. On the other hand, the Canadian Community Health Survey conducted by Statistics Canada (2003) is based on a area frame, a list frame and a random digit dialing (RDD) frame. In the near future, importance of three frame surveys is expected to grow with the use of the internet as data-collection tool (Lohr, 2010). Indeed, it is very likely that dual frame surveys consisting of a cell and a landline frame evolve to three frame surveys incorporating a third frame of web users. A good revision of the problem of estimation in multiple frame surveys can be found in Lohr (2009). Recently Rueda et al. (2017b) proposed statistical techniques for handling ordinal data coming from a multiple frame survey in complex sampling designs.

To the best of our knowledge, theoretical or applied works do not exist under a multiple frame setting in the field of the RRT and, more in general, in the research area of indirect questioning techniques. This paper aims, thus, at proposing new estimators of the population mean of a sensitive variable when data are collected using the RRT under a multiple frame sampling approach.

This article proceeds as follows. In Sect. 2, we introduce the notation and propose a first multiple frame estimator of the unknown population mean in the RR setting. The unbiasedness of the multiplicity estimator is proved and its variance worked out (Sect. 2.1). Variance estimation is also discussed in Sect. 2.2 including the jackknife approach. In Sect. 3., other multiple frame estimators are extended to the RR setting. The performance of all the proposed estimators is investigated in Sect. 4 where the findings of a Monte Carlo simulation study are commented and graphically summarized. Finally, Sect. 5 concludes the article with some final remarks.

2. A multiple frame estimator for the RRT

Let $U = \{1, ..., k, ..., N\}$ be a finite population composed of N units labeled from 1 to N and let $A_1, \ldots, A_q, \ldots, A_Q$ be a collection of $Q \geq 2$ overlapping frames of sizes $N_1, \ldots, N_q, \ldots, N_Q$, all of them can be incomplete but it is assumed that overall they cover the entire target population U. With Q frames, there are $2^Q - 1$ possible disjointed domains. Let y be a sensitive variable to study and y_k its value for the k-th population unit. Let us suppose that population mean of y, say

$$\bar{Y} = \frac{1}{N} \sum_{k \in U} y_k,$$

is unknown and has to be estimated. This mean can be rewritten as follows

$$\bar{Y} = \frac{1}{N} \sum_{q=1}^{Q} \sum_{k \in A_q} \frac{y_k}{m_k}, \tag{1}$$

where m_k indicates the multiplicity of the k-th unit, i.e. the number of frames the unit k is included. Multiplicities m_k are needed in (**??**) to weight values y_k, otherwise, those units belonging to more than one frame would count more than once in the overall sum. Let s_q be a sample drawn from frame A_q under a particular sampling design, independently for $q = 1, \ldots, Q$ and let $\pi_k(q)$ and $\pi_{kj}(q)$ be the first and second-order inclusion probabilities under this sampling design, respectively. Let $d_k(q) = 1/\pi_k(q)$ be the sampling weight for unit k in frame A_q. Moreover, let n_q be the size of sample s_q and $s = \cup_q s_q$. We assume that duplicated units has a negligible chance to happen (Skinner 1991, Molina et al, 2015).

We assume that the variable under study y cannot be observed directly and that in each frame it is possible to use a different RR procedure to collect data on it. In order to consider a wide variety of RR devices, we consider the unified approach given by Arnab (1996) according to the individuals in the sample s_q use a generic RR model denoted by RR_q. For each $k \in s_q$ the RR_q induces a random variable Z_{qk} so that the revised randomized response R_{qk} is an unbiased estimation of y_k, the real value of the sensitive variable for the k-th unit in s_q. We consider RR_q, with $q = 1, \ldots, Q$, to be independent randomization devices such that the respective revised randomized responses R_{qk} satisfy the conditions (Arnab, 2004) $E_R(R_{qk}) = y_k$, $V_R(R_{qk}) = \sigma^2_{qk}$, $C_R(R_{qk}, R_{qj}) = 0$, where E_R, V_R and C_R denote the expectation, variance and covariance operators with respect to the RR mechanism.

Following Mecatti (2007), we propose the multiplicity estimator:

$$\hat{\bar{Y}}_M = \frac{1}{N} \sum_{q=1}^{Q} \sum_{k \in s_q} R_{qk} d_k^M(q), \tag{2}$$

with $d_k^M(q) = d_k(q)/m_k$.

2.1. Properties of the proposed estimator

In this section we describe the main properties of the proposed estimator. The results stated in the following theorem hold:

Theorem 2.1. *The multiplicity estimator $\hat{\bar{Y}}_M$ is an unbiased estimator of the population mean $\bar{Y}$ and has variance given by:*

$$V(\hat{\bar{Y}}_M) = \frac{1}{N^2} \left[\sum_{q=1}^{Q} \sum_{k \in A_q} \frac{\sigma^2_{qk}}{m_k} d_k^M(q) + \sum_{q=1}^{Q} \sum_{k \in A_q} \sum_{j \in A_q} \frac{y_k y_j}{m_k m_j} \left(\frac{\pi_{kj}(q)}{\pi_k(q)\pi_j(q)} - 1 \right) \right].$$

Proof. Let E_d and V_d denote the expectation and variance operators for any sampling design d. Moreover, let $I_k(q)$ the indicator function, with $E_d\left[I_k(q)\right] = \pi_k(q)$, which takes value 1 if the k-th unit in frame A_q is selected in the sample s_q, and 0 otherwise. Taking into account the two sources of variability induced by the sampling design and the randomization device, we have:

$$
\begin{aligned}
E(\hat{\hat{Y}}_M) &= \frac{1}{N} E_d E_R \left[\sum_{q=1}^{Q} \sum_{k\in s_q} R_{qk} d_k^M(q)\right] \\
&= \frac{1}{N} E_d E_R \left[\sum_{q=1}^{Q} \sum_{k\in A_q} R_{qk} d_k^M(q) I_k(q)\right] \\
&= \frac{1}{N} E_d \left[\sum_{q=1}^{Q} \sum_{k\in A_q} E_R(R_{qk}) d_k^M(q) I_k(q)\right] \\
&= \frac{1}{N} E_d \left[\sum_{q=1}^{Q} \sum_{k\in A_q} y_k d_k^M(q) I_k(q)\right] \\
&= \frac{1}{N} \sum_{q=1}^{Q} \sum_{k\in A_q} y_k d_k^M(q) E_d\left[I_k(q)\right] \\
&= \frac{1}{N} \sum_{q=1}^{Q} \sum_{k\in A_q} \frac{y_k}{m_k \pi_k(q)} \pi_k(q) \\
&= \frac{1}{N} \sum_{q=1}^{Q} \sum_{k\in A_q} \frac{y_k}{m_k} = \bar{Y}.
\end{aligned}
$$

Hence the proof of the unbiasedness of $\hat{\hat{Y}}_M$.

With regard to the variance of the estimator, it stems from the variance decomposition formula:

$$
V(\hat{\hat{Y}}_M) = E_d[V_R(\hat{\hat{Y}}_M)] + V_d[E_R(\hat{\hat{Y}}_M)].
$$

Hence, by omitting some steps, we have:

$$
\begin{aligned}
V(\hat{\hat{Y}}_M) &= \frac{1}{N^2} E_d \left[\sum_{q=1}^{Q} \sum_{k\in A_q} V_R(R_{qk})(d_k^M(q))^2 I_k(q)\right] + \frac{1}{N^2} V_d \left[\sum_{q=1}^{Q} \sum_{k\in s_q} y_k d_k^M(q)\right] \\
&= \frac{1}{N^2} \sum_{q=1}^{Q} \sum_{k\in A_q} \frac{\sigma_{qk}^2}{m_k} d_k^M(q) + \frac{1}{N^2} \sum_{q=1}^{Q} V_d\left(\sum_{k\in s_q} \frac{y_k^M}{\pi_k(q)}\right),
\end{aligned}
$$

where $y_k^M = y_k/m_k$. The quantity $\sum_{k\in s_q} y_k^M/\pi_k(q) = \hat{\bar{Y}}_{HT}(q)$ denotes the expression of the Horvitz-Thompson estimator (hereafter HT-estimator), computed on weighted values y_k^M, for which it is known that (see, e.g, Särndal et al., 1992):

$$V_d(\hat{\hat{Y}}_{HT}(q)) = \sum_{k \in A_q} \sum_{j \in A_q} \frac{y_k^M y_j^M}{\pi_k(q)\pi_j(q)} \left(\pi_{kj}(q) - \pi_k(q)\pi_j(q)\right].$$

Replacing y_k^M and y_j^M, and assembling the terms, expression (**??**) easily follows. Hence the proof. □

We observe that the variance of the multiplicity estimator $\hat{\hat{Y}}_M$ in the RR setting is composed of two terms. The second addendum is related to the variance of the HT-estimator which depends on the sampling designs and the y_k values in each frame. The first term depends on the sampling designs and also on the randomization mechanism used in each frame. It represents the cost to pay, in terms of efficiency, to increase respondents' confidentiality.

In the formulation of the proposed estimators it is assumed that the population size N is known and the HT-estimator is used as baseline. Alternatively, one can consider an Hájek-type estimator substituting N with $\hat{N}$, an unbiased design-based estimator of N. This is a special case of ratio estimator, and it can be more efficient than the HT-type estimator because the sample size in overlapping domains is not fixed. The estimators are thus approximately unbiased under certain conditions on the weights (see Lohr, 2009).

2.2. Variance estimation

It can be proved that an analytical unbiased estimator of $V(\hat{\hat{Y}}_M)$ is given by:

$$\hat{v}(\hat{\hat{Y}}_M) = \frac{1}{N^2}\left[\sum_{q=1}^{Q}\sum_{k \in A_q} \frac{\sigma_{qk}^2}{m_k} d_k^M(q) + \sum_{q=1}^{Q}\sum_{k \in s_q}\sum_{j \in s_q} \frac{R_{qk}R_{qj}}{m_k m_j}\left(\frac{1}{\pi_k(q)\pi_j(q)} - \frac{1}{\pi_{kj}(q)}\right)\right]. \quad (3)$$

To compute the variance estimator $\hat{v}(\hat{\hat{Y}}_M)$, the second-order inclusion probabilities $\pi_{kj}(q)$ must be known for all units in each frame. In some common sampling designs (as cluster sampling with probability proportional to size) these probabilities are unknown or can be equal to 0 for some sampling units i, j. A simple alternative is the use of with replacement variance estimators or replicated sampling methods (for details on these techniques see, e.g., Särndal et al., 1992 and Wolter, 2007). Quenouille (1949) introduced the jackknife method to estimate the bias of an estimator by deleting one observation each time from the original data set and recalculating the estimator based on the rest of the data. In survey sampling, it is usual to use jackknife techniques due to their simplicity and because they are implemented in general purpose software packages, such as R.

If we consider a non-stratified design, the jackknife estimator for $V_d(\hat{\hat{Y}}_{HT}(q))$ is given by:

$$\hat{v}_J(\hat{\hat{Y}}_{HT}(q)) = \frac{n_q - 1}{n_q} \sum_{i \in s_q} \left(\hat{\hat{Y}}_{HT}^{(-i)}(q) - \tilde{Y}_{HT}(q)\right)^2,$$

where $\hat{\hat{Y}}_{HT}^{(-i)}(q)$ is the value taken by $\hat{\hat{Y}}_{HT}(q)$ after eliminating unit i from s_q and $\tilde{Y}_{HT}(q)$

is the average of $\hat{\hat{Y}}_M^{(-i)}(q)$ values. It is known that the jackknife variance estimator is asymptotically design unbiased for the variance of Horvitz-Thompson estimator (Wolter, 2007). So, for the large sample size n, and hence for n_q sufficiently large, we have $E[\hat{v}_J(\hat{\hat{Y}}_{HT}(q))] = V_d(\hat{\hat{Y}}_{HT})$. Thus, the adapted jackknife estimator

$$\hat{v}_J(\hat{\hat{Y}}_M) = \frac{1}{N^2}\left[\sum_{q=1}^{Q}\sum_{k\in A_q}\frac{\sigma_{qk}^2}{m_k}d_k^M(q) + \sum_{q=1}^{Q}\hat{v}_J(\hat{\hat{Y}}_{HT}(q))\right] \tag{4}$$

is an unbiased estimator for the variance of the proposed multiplicity estimator $\hat{\hat{Y}}_M$. Variance estimator (**??**) can be considered an extension to multiple frame setting of the adjusted variance estimator proposed by Arnab et al. (2015).

Variance estimators (**??**) and (**??**) rely on the assumption that σ_{qk}^2 is known. Indeed, real situations may be incurred where σ_{qk}^2 is unknown and has to be estimated. Hence, if $\hat{\sigma}_{qk}^2$ denote an RR-unbiased estimator of σ_{qk}^2, unbiased variance estimates may be achieved by modifying the first addendum in (**??**) and (**??**) and obtaining:

$$\hat{v}^*(\hat{\hat{Y}}_M) = \frac{1}{N^2}\left[\sum_{q=1}^{Q}\sum_{k\in s_q}\frac{\hat{\sigma}_{qk}^2}{m_k^2\pi_k^2(q)} + \sum_{q=1}^{Q}\sum_{k\in s_q}\sum_{j\in s_q}\frac{R_{qk}R_{qj}}{m_k m_j}\left(\frac{1}{\pi_k(q)\pi_j(q)} - \frac{1}{\pi_{kj}(q)}\right)\right] \tag{5}$$

and

$$\hat{v}_J^*(\hat{\hat{Y}}_M) = \frac{1}{N^2}\left[\sum_{q=1}^{Q}\sum_{k\in s_q}\frac{\hat{\sigma}_{qk}^2}{m_k^2\pi_k^2(q)} + \sum_{q=1}^{Q}\hat{v}_J(\hat{\hat{Y}}_{HT}(q))\right]. \tag{6}$$

3. Other multiple frame estimators

Lohr and Rao (2006) considered the so called single frame approach used by Kalton and Anderson (1986) to proposed a single frame estimator in a multiple frame context. Following their idea, we propose a new estimator in the form:

$$\hat{\hat{Y}}_{KA} = \frac{1}{N}\sum_{q=1}^{Q}\sum_{k\in s_q} R_{qk}d_k^{KA},$$

with $d_k^{KA} = \bar{\pi}_k^{-1}$ and $\bar{\pi}_k = \sum_{q:k\in A_q}\pi_k(q)$. To compute this estimator it is necessary to know not only the number of frames each unit belongs to but, also, the specific frames the unit is included in. This can be an important drawback particularly if misclassification issues are present. Note that the previous proposed estimator $\hat{\hat{Y}}_M$ only requires the knowledge of the multiplicity of each unit, no matter which frame unit belongs to.

We can combine $\hat{\hat{Y}}_M$ and $\hat{\hat{Y}}_{KA}$ and define the new composite multiplicity estimator:

$$\hat{\hat{Y}}_{CM} = \frac{1}{N}\sum_{q=1}^{Q}\sum_{k\in s_q} R_{qk}d_k^{CM}, \tag{7}$$

where

$$d_k^{CM} = \frac{\lambda_k d_k + (1 - \lambda_k) d_k^{KA}}{m_k},$$

being $0 < \lambda_k < 1$.

Usually, information at population level about auxiliary variables is available in surveys. Rao and Wu (2010) followed a single frame multiplicity based approach to extend the pseudo empirical likelihood estimator for the mean of a variable to the multiple frame setting. Calibration (Deville and Särndal, 1992) is also a well-known technique to deal with auxiliary information at the estimation stage. Some works link RR models and calibration techniques together (Tracy and Singh, 1999; Diana and Perri, 2012). Ranalli et al. (2016) proposed different calibration estimators for the dual frame case, which can be easily extended to the multiple frame context. Using this technique we define a calibration estimator for randomized responses in the multiple frame context.

Let $\mathbf{x}_q = (x_{q1}, x_{q2}, \ldots, x_{qp_q})'$ be a set of p_q auxiliary variables observed in the q-th frame, so that the vector $\mathbf{x}_{qk} = (x_{q1k}, x_{q2k}, \ldots, x_{qp_qk})'$ includes the values taken by the variables $\mathbf{x_q}$ on the k-th unit in frame A_q. In other words, we consider the case of complete auxiliary information. In addition, we examine the more general case in which auxiliary variables may differ in each frame, i.e. $\mathbf{x}_q \neq \mathbf{x}_q$, for $q, r = 1, \ldots, Q, q \neq r$. For the sample s_q selected from frame A_q, the values of the variables $\{y_k, \mathbf{x}_{qk}\}$ are observed.

A calibration estimator in the case of several sampling frames can be defined as:

$$\hat{\bar{Y}}_{CAL} = \frac{1}{N} \sum_{q=1}^{Q} \sum_{k \in s_q} R_{qk} d_k^{CAL}(q),$$

where $d_k^{CAL}(q)$ are such that they minimize $\sum_{q=1}^{Q} \sum_{k \in s_q} G\left(d_k^{CAL}(q), d_k^{M}(q)\right)$, where $G(\cdot, \cdot)$ is a particular distance function, subject to

$$\sum_{q=1}^{Q} \sum_{k \in s_q} d_k^{CAL}(q) \mathbf{x}_{qk} \delta_k(A_q) = \mathbf{t}_{xq}, \quad q = 1, ..., Q,$$

where $\delta_k(A_q)$ is the indicator variable that takes value 1 if the k-th unit is in frame A_q and 0 otherwise, and $\mathbf{t}_{xq}$ are the population totals of $\mathbf{x}_q$.

The proposed model calibration estimator eliminates overestimation issues by several means. We consider $d_k^M(q)$ as the starting weights for the calibration and, using the indicator variable $\delta_k(A_q)$, the calibration constraints ensure adjustment of the multiplicity issues by benchmarking all information on units from frame A_q included in the sample, irrespective of the frame they were originally selected from. Therefore, again, multiplicity is accounted for automatically by the constraints. The properties of this estimator can be derived from the properties of the calibrated estimators in multiple frames (see, e.g., Ranalli et al., 2016).

4. A Monte Carlo simulation study

In this section we run a Monte Carlo simulation study to compare empirically the performance of the proposed multiple frame estimators in an RR setting. In so doing, we assume that the taget variable has a normal distribution, $y \sim N(30, 3)$. We consider an artificial population size of $N = 10000$ units and establish $Q = 3$ frames: frame A has size $N_A = 5500$, frame B has size $N_B = 6500$ and frame C has size $N_C = 5000$. From these frames, we obtain the following $2^3 - 1$ domains: a, b and c, subsets of population units in frame A, B and C, respectively, with sizes $N_a = 1500$, $N_b = 1500$ and $N_c = 1000$; ab, ac and bc, subsets of population units in frames A and B, A and C, B and C, with sizes $N_{ab} = 2000, N_{ac} = 1000, N_{bc} = 2000$; abc domain, subset of population units in frames A, B and C, with size $N_{abc} = 1000$. Three scenarios with increasing sample sizes in each frame are included in the analysis. In the first scenario, we draw samples of size $n_A = 288$, $n_B = 371$ and $n_C = 582$ from each of the frames A, B and C. In the second scenario, we consider $n_A = 360$, $n_B = 464$ and $n_C = 728$ while for the third we set $n_A = 432$, $n_B = 557$ and $n_C = 874$.
Two experimental situations are investigated throughout the simulation:

ES1: Samples are selected according to simple random sampling without replacement (srswor) from all frames and in each sample we apply a different RR technique to produce data. Specifically, in frame A we use the Eichhorn and Hayre (1983) model: the i-th sampled units is asked to provide the randomized response $z_i = y_i w_i$ where w_i is a random value generated from the scrambling variable w whose distribution law is assumed to be known. We assume that w is Fisher distributed, $w \sim F(20, 20)$. In frame B, the Bar-Lev et al. (2004) device is adopted according to which the observed randomized response z_i is defined as:

$$z_i = \begin{cases} y_i & \text{with probability } p \\ y_i w_i & \text{with probability } 1 - p. \end{cases}$$

We assume $p = 0.6$ and w following the exponential distribution, $w \sim Exp(1)$. Finally, in frame C, we perform the Eriksson (1973) mechanism to scramble the true response. According to this mechanism, the randomized response released by the i-th respondent is:

$$z_i = \begin{cases} y_i & \text{with probability } p \\ w_i & \text{with probability } 1 - p, \end{cases}$$

where the value w_i is generated from the uniform discrete random variable $w = (27.86, 29.96, 32.33)$. The values of w are the quartiles of the y distribution. We assume $p = 0.7$.

ES2: The Bar-Lev et al. (2004) model is used to perturb the response in all frames and samples are selected according to srswor in frame A, stratified sampling in frame B and Midzuno sampling (see, e.g., Särndal et al., 1992) in frame C. For the stratified sampling we considered three strata depending of the value of y, while for implementing the Midzuno sampling design we used an auxiliary variable x highly correlated to y ($\rho_{yx} = 0.98$).

Under the two experimental settings (ES1 and ES2), two distinct simulation analyses are conducted. The first aims at assessing the performance of the proposed estimators in terms of bias and mean square error, the second refers to the accuracy of the jack-

Estimates	sample sizes	$\hat{\hat{Y}}_M$	$\hat{\hat{Y}}_{KA}$	$\hat{\hat{Y}}_{CM}$	$\hat{\hat{Y}}_{CAL}$
RB	$n_A = 288, n_B = 371, n_C = 582$	0.0137592	0.0130946	0.0174320	0.0137410
	$n_A = 360, n_B = 464, n_C = 728$	0.0122739	0.0115960	0.0154836	0.0122513
	$n_A = 432, n_B = 557, n_C = 874$	0.0117777	0.0111799	0.0144858	0.0117807
RMSE	$n_A = 288, n_B = 371, n_C = 582$	0.0002989	0.0002677	0.0004693	0.0002985
	$n_A = 360, n_B = 464, n_C = 728$	0.0002453	0.0002177	0.0003857	0.0002444
	$n_A = 432, n_B = 557, n_C = 874$	0.0002137	0.0001917	0.0003239	0.0002141

Table 1. ES1: Relative bias and relative mean square error for $\hat{\hat{Y}}_M$, $\hat{\hat{Y}}_{KA}$, $\hat{\hat{Y}}_{CM}$ and $\hat{\hat{Y}}_{CAL}$ under srswor in all frames.

Estimates	sample sizes	$\hat{\hat{Y}}_M$	$\hat{\hat{Y}}_{KA}$	$\hat{\hat{Y}}_{CM}$	$\hat{\hat{Y}}_{CAL}$
RB	$n_A = 288, n_B = 371, n_C = 582$	0.0177636	0.0172636	0.0220091	0.0175873
	$n_A = 360, n_B = 464, n_C = 728$	0.0157843	0.0153962	0.0194660	0.0155997
	$n_A = 432, n_B = 557, n_C = 874$	0.0149888	0.0146821	0.0186161	0.0146604
RMSE	$n_A = 288, n_B = 371, n_C = 582$	0.0004872	0.0004690	0.0007593	0.0004733
	$n_A = 360, n_B = 464, n_C = 728$	0.0003906	0.0003689	0.0006047	0.0003793
	$n_A = 432, n_B = 557, n_C = 874$	0.0003472	0.0003327	0.0005292	0.0003334

Table 2. ES2: Relative bias and relative mean square error for $\hat{\hat{Y}}_M$, $\hat{\hat{Y}}_{KA}$, $\hat{\hat{Y}}_{CM}$ and $\hat{\hat{Y}}_{CAL}$ considering srswor in frame A, stratified sampling in frame B and Midzuno sampling in frame C.

knife variance estimates. For each simulation setting, we consider $B = 1000$ simulation runs.

4.1. Efficiency of the estimates

The performance of the estimator $\hat{\hat{Y}}_. = \hat{\hat{Y}}_M, \hat{\hat{Y}}_{KA}, \hat{\hat{Y}}_{CM}, \hat{\hat{Y}}_{CAL}$ is investigated by means of the relative bias (RB) and the relative mean square error (RMSE):

$$\text{RB}(\hat{\hat{Y}}_.) = \frac{\sum_{k=1}^{B} |\hat{\hat{Y}}_.^{(k)} - \bar{Y}|}{B\bar{Y}}$$

and

$$\text{RMSE}(\hat{\hat{Y}}_.) = \frac{\sum_{k=1}^{B} (\hat{\hat{Y}}_.^{(k)} - \bar{Y})^2}{B\bar{Y}^2},$$

where $\hat{\hat{Y}}_.^{(k)}$ denotes the estimate of $\bar{Y}$ computed on the k-th simulation run. The composite multiplicity estimator $\hat{\hat{Y}}_{CM}$ is computed with λ_k as given in Singh and Mecatti (2011), while the calibration estimator $\hat{\hat{Y}}_{CAL}$ employes as auxiliary information the population frame size ($\mathbf{x}_{qk} = \delta_k(A_q)$ and $\mathbf{t}_{xq} = N_q$). The results of the simulation study for the two experimental situations are graphically depicted in Figures **??** and **??**, while Tables **??** and **??** give the values of RB and RMSE for all estimators in the different sample size scenarios.

From results in Tables **??** and **??** we observe that the bias for all the estimators considered is negligible. With respect to the efficiency, no significative differences can be ascertained between the estimators $\hat{\hat{Y}}_M$, $\hat{\hat{Y}}_{KA}$ and $\hat{\hat{Y}}_{CAL}$. In general, the best performance in terms of RMSE is achieved by $\hat{\hat{Y}}_{KA}$, but this estimator needs full information at frame level. The composite multiplicity estimator $\hat{\hat{Y}}_{CM}$ shows the worst behavior. This is probably due to the fact that the value of λ_k, used to compute the estimator, is obtained by minimizing the variance of the final weights d_k^{CM} and not of the variance of the estimator. This is a suboptimal solution which does not require estimation of variances and covariances.

As might be expected, we also observe that both the RB and the RMSE decrease as the frame sample sizes increase, a clear indication this of the consistency of the estimates.

4.2. Jackknife variance estimation

We now investigate the performance of the variance estimation for all the proposed estimators. In particular, we focus on the jackknife variance given in (**??**) for $\hat{\hat{Y}}_M$. The extension to the other estimators introduced in Section 3 readily follows. Similarly to the previous simulation study, the accuracy of the jackknife variance estimates is assessed by means of RB and RMSE:

$$\mathrm{RB}(\hat{v}_J^*(\hat{\hat{Y}}_.)) = \frac{\sum_{k=1}^B \left|\hat{v}_J^{*(k)}(\hat{\hat{Y}}_.) - V(\hat{\hat{Y}}_.)\right|}{BV(\hat{\hat{Y}}_.)}$$

and

$$\mathrm{RMSE}(\hat{v}_J^*(\hat{\hat{Y}}_.)) = \frac{\sum_{k=1}^B \left(\hat{v}_J^{*(k)}(\hat{\hat{Y}}_.) - V(\hat{\hat{Y}}_.)\right)^2}{BV(\hat{\hat{Y}}_.)^2},$$

where $\hat{v}_J^{*(k)}(\hat{\hat{Y}}_.)$ denotes the jackknife variance estimate computed on the k-th simulation run and

$$V(\hat{\hat{Y}}_.) = \frac{1}{B-1}\sum_{k=1}^B (\hat{\hat{Y}}_.^{(k)} - \bar{Y}_.)^2,$$

with $\bar{Y}_. = \sum_{k=1}^B \hat{\hat{Y}}_.^{(k)}/B$, $\hat{\hat{Y}}_. = \hat{\hat{Y}}_M, \hat{\hat{Y}}_{KA}, \hat{\hat{Y}}_{CM}, \hat{\hat{Y}}_{CAL}$.

For the two experimental settings (ES1 and ES2), the outcomes of this study are graphically summarized in Figures **??** and **??**, while Tables **??** and **??** report the values of the RB and RMSE for all estimators in the different sample size scenarios.

Jackknife variance	sample sizes	$\hat{\hat{Y}}_M$	$\hat{\hat{Y}}_{KA}$	$\hat{\hat{Y}}_{CM}$	$\hat{\hat{Y}}_{CAL}$
	$n_A = 288, n_B = 371, n_c = 582$	1.414224	1.574549	2.148209	1.423234
RB	$n_A = 360, n_B = 464, n_c = 728$	1.328427	1.506447	2.031942	1.341068
	$n_A = 432, n_B = 557, n_c = 874$	1.192673	1.334032	1.954348	1.192478
	$n_A = 288, n_B = 371, n_c = 582$	2.236686	2.767015	5.129108	2.263754
RMSE	$n_A = 360, n_B = 464, n_c = 728$	1.932149	2.477034	4.503975	1.967957
	$n_A = 432, n_B = 557, n_c = 874$	1.540854	1.923288	4.095583	1.540494

Table 3. ES1: Relative bias and relative mean square error for jackknife variance estimates under srswor in all frames.

Jackknife variance	sample sizes	$\hat{\hat{Y}}_M$	$\hat{\hat{Y}}_{KA}$	$\hat{\hat{Y}}_{CM}$	$\hat{\hat{Y}}_{CAL}$
	$n_A = 288, n_B = 371, n_c = 582$	1.417381	1.526632	2.086405	1.961645
RB	$n_A = 360, n_B = 464, n_c = 728$	1.382939	1.544858	2.075784	1.920892
	$n_A = 432, n_B = 557, n_c = 874$	1.313325	1.404051	1.935690	1.847420
	$n_A = 288, n_B = 371, n_c = 582$	2.248999	2.601825	4.858967	4.179665
RMSE	$n_A = 360, n_B = 464, n_c = 728$	2.104424	2.615741	4.729312	3.957605
	$n_A = 432, n_B = 557, n_c = 874$	1.868052	2.133943	4.052171	3.619694

Table 4. ES2: Relative bias and relative mean square error for jackknife variance estimates under srswor in frame A, stratified sampling in frame B and Midzuno sampling in frame C.

The results summarized in Tables **??** and **??** point out that the jackknife variance estimation for $\hat{\hat{Y}}_M$ produces RB and RMSE smaller than the other estimators, expect for the third sample sizes scenario in the first experimental situation where the best performance is achieved by $\hat{\hat{Y}}_{CAL}$ even if the efficiency gain upon $\hat{\hat{Y}}_M$ is nearly negligible. As in the previous study, the highest values for the RB and RMSE are ascribable to $\hat{\hat{Y}}_{CM}$.

We also observe that the accuracy of the variance estimation improves as the frame sample sizes increase.

5. Conclusions

In this article, we presented new estimators to determine the mean of a sensitive variable when data are obtained from several frames using some scrambled response models. We introduced a way to combine estimates from the different frames and considered different estimators based on different level of information.

In the absence of auxiliary information, the first proposed estimator $\hat{\hat{Y}}_M$, based on the idea of multiplicity due to Mecatti (2007), is applicable only if basic frame level information is available for all sampled units. This information pertain to the selection probability from the sampled frame and the number of frames from which the unit could have been selected but without the frame identification. The second proposed estimator $\hat{\hat{Y}}_{KA}$ needs full frame level information: the identification of frame membership for every sampled unit and the knowledge of inclusion probability for every frame in which the unit belongs to. The third proposed estimator $\hat{\hat{Y}}_{CM}$ combines the two previous estimators. Finally, in the presence of auxiliary variables, we used the calibration weighting method to define the fourth estimator $\hat{\hat{Y}}_{CAL}$. The calibration approach is very flexible and allows auxiliary information to be introduced at several different levels.

In practice, a different sampling procedure might feasibly be applied for each frame, or even no randomization at all (i.e., direct response) for a particular frame. The use of the RR approach has certain advantages (privacy protection is assured to respondents and measurement errors are reduced) but also drawbacks (the variance of the estimates is increased by the randomization mechanism and individual response patterns cannot be interpreted directly, due to the observation of randomized responses, nor can individuals or groups of individuals be compared). That said, an interesting idea that may be worthwhile pursuing for ongoing research may be combining randomized responses end direct questions in multiple frame surveys in order to realize a satisfactory trade-off between privacy protection, reliable data and efficiency in the estimates.

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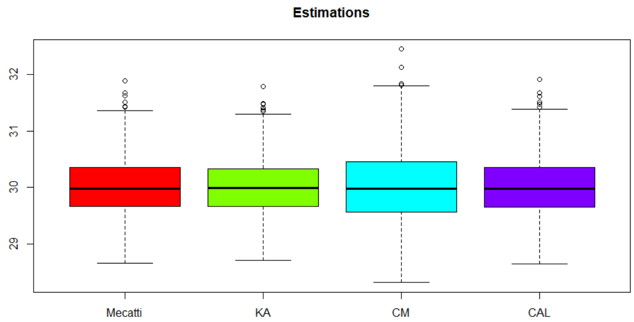

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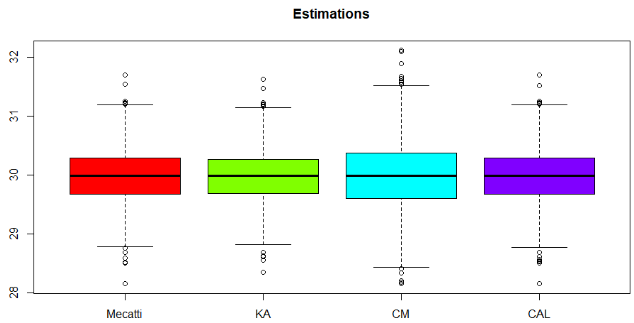

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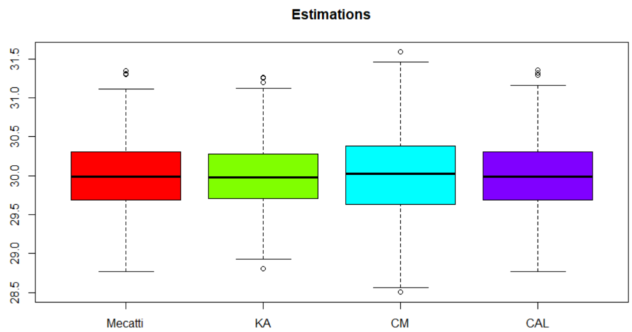
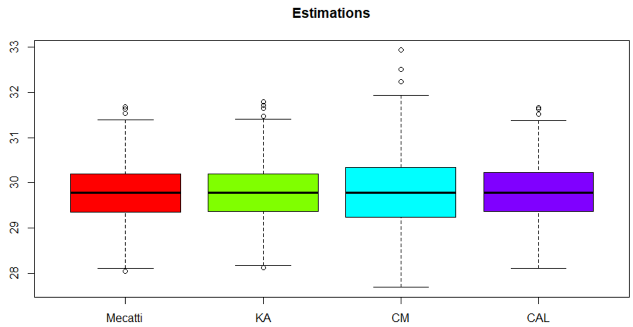


(c) $n_A = 432, n_B = 557, n_C = 874$

Figure 1. ES1: Estimates for $\hat{\hat{Y}}_M$, $\hat{\hat{Y}}_{KA}$, $\hat{\hat{Y}}_{CM}$ and $\hat{\hat{Y}}_{CAL}$ under srswor in all frames.



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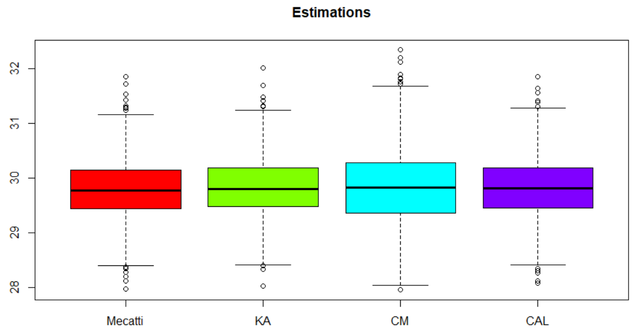

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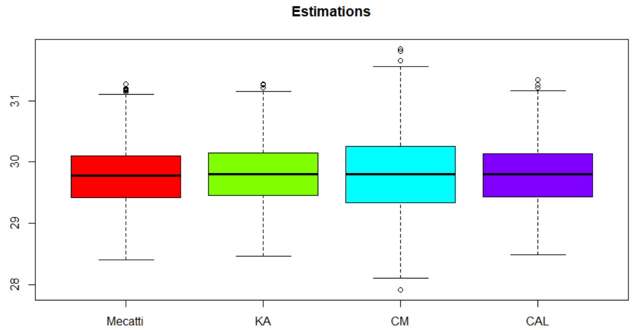
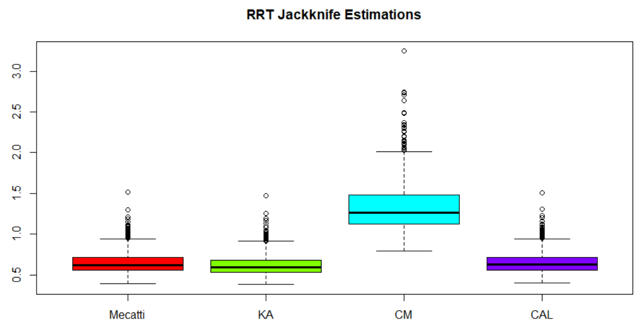
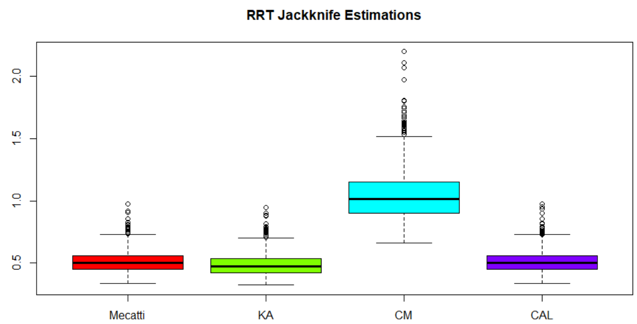


(c) $n_A = 432, n_B = 557, n_C = 874$

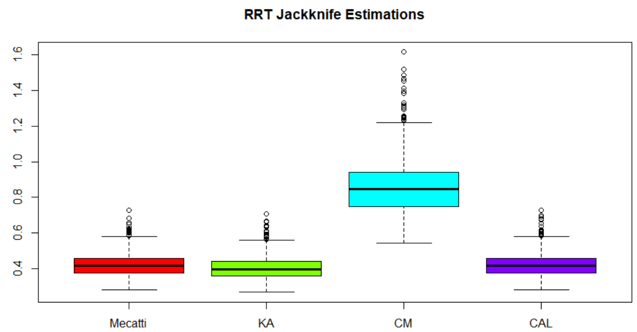
Figure 2. ES2: Estimates for $\hat{\hat{Y}}_M$, $\hat{\hat{Y}}_{KA}$, $\hat{\hat{Y}}_{CM}$ and $\hat{\hat{Y}}_{CAL}$ under srswor in frame A, stratified sampling in frame B and midzuno sampling in frame C.



(a) $n_A = 288, n_B = 371, n_C = 582$

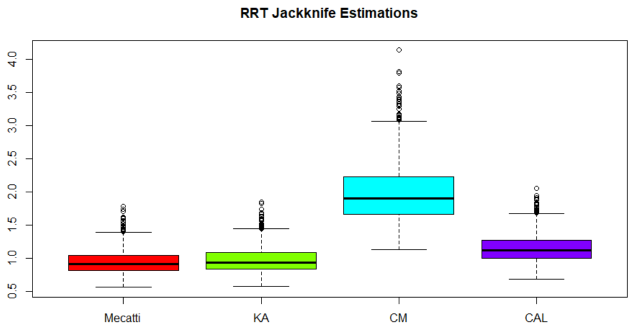


(b) $n_A = 360, n_B = 464, n_C = 728$

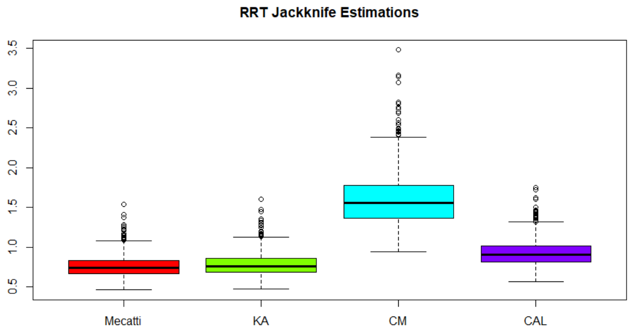


(c) $n_A = 432, n_B = 557, n_C = 874$

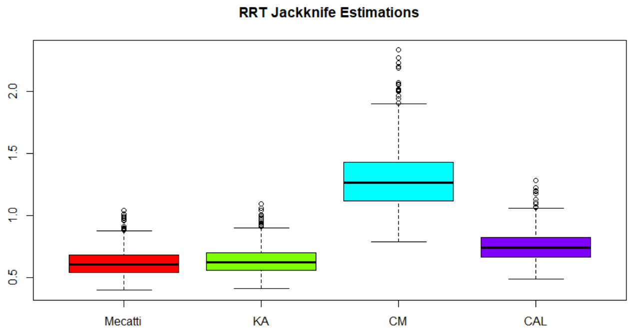
Figure 3. ES1: Jackknife variance estimates for $\hat{\hat{Y}}_M$, $\hat{\hat{Y}}_{KA}$, $\hat{\hat{Y}}_{CM}$ and $\hat{\hat{Y}}_{CAL}$ under srswor in all frames.



(a) $n_A = 288, n_B = 371, n_C = 582$



(b) $n_A = 360, n_B = 464, n_C = 728$



(c) $n_A = 432, n_B = 557, n_C = 874$

Figure 4. ES2: Jackknife variance estimates for $\hat{\hat{Y}}_M$, $\hat{\hat{Y}}_{KA}$, $\hat{\hat{Y}}_{CM}$ and $\hat{\hat{Y}}_{CAL}$ under srswor in frame A, stratified sampling in frame B and Midzuno sampling in frame C.

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