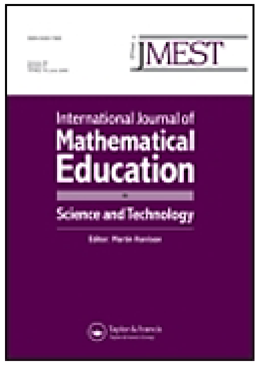

University Students' Engagement with Digital Mathematics Textbooks: A Case of Linear Algebra

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University Students' Engagement with Digital Mathematics Textbooks: A Case of Linear Algebra

We investigated university students' engagement with digital textbooks by analyzing their real-time viewing and use in a linear algebra course taught at four different universities. Viewing data over a semester was complemented with responses to bi-weekly open-ended surveys and other course artifacts, such as course syllabi. With our approach to ask students about their viewing data in surveys, we were able to relate student majors to textbook engagement and to document more and different uses of textbook elements than what has been reported so far. We used descriptive statistics and frequency graphs to analyse the viewing data and a grounded analytical approach to survey data. By using the didactical tetrahedron, we found that textbook engagement dropped when the algebraic language and content of the textbook was not institutionally legitimized and when the textbook did not directly satisfy interests associated with earning a degree. Textbook engagement was higher than what could have been anticipated by syllabi for students majoring in disciplines other than mathematics. We propose some areas for further research.

Keywords: engagement with textbooks; digital mathematics textbook; textbook use; linear algebra; university education

With the advances in new technologies, authors and publishers have been designing new kinds of mathematics textbooks in order to support their evolving nature from paper to digital; facilitating universal access and the addition of a range of interactive tools (Schacht, 2018). These new textbooks offer advantages over ordinary printed textbooks—lower costs, increased portability, access to multiple textbooks on one device—increasing their worldwide use, sometimes complementing, or replacing, paper textbooks. New questions arise about how these mathematics textbooks are being used by students and instructors and how users are integrating and appropriating these textbooks in and out of class (Gueudet et al., 2016). A major roadblock in answering these questions is methodological as it is difficult to capture data that can truthfully document how textbooks are used in real time with a large number of users; this roadblock limits the possibility of acquiring a well-grounded view of how users interact with these textbooks and their novel features (Gueudet, 2017).

In this paper, we report an investigation on undergraduate students' engagement with a linear algebra digital textbook (Beezer, 2017) that was written using a markup language called PreTeXt (https://pretextbook.org/). PreTeXt allows tracking real time viewing that can be used to understand students' engagement with the textbook. With this feature we gather data from multiple users simultaneously independently of their geographical location, facilitating a more comprehensive account from a large number of participants. We are primarily interested in operationalizing engagement using real-time data with the goal of contributing to an empirically grounded approach to the relationship between students and digital textbooks, an area that has, so far, limited research (Matić & Gracin, 2016).

Literature Review of University Students' Engagement with Textbooks

At the university level, various countries (USA, Canada, Australia) have launched national surveys of student engagement (e.g., The North American Survey of Student Engagement, piloted in 1999, and the Australasian Survey of Student Engagement, introduced in 2007). In that work engagement refers to "interactions" with faculty rather than to students' use of a learning tool. Research on university student engagement with digital textbooks are case studies at specific universities. These studies describe reading patterns (Abaci et al., 2017; Junco & Clem, 2015); use of interactive components (Abaci et al., 2017; Johnston & Ferguson, 2020); relation between student grades and time spent reading (Abaci et al., 2017; Allred & Murphy, 2019; Junco & Clem, 2015);

or skills, emotions, participation, and interaction (Bikowski & Casal, 2018). These researchers indicate that undergraduates see the e-textbook as a motivator and learning facilitator resource (Bikowski & Casal, 2018) but have documented different levels of reading per day (e.g., from 3 h or less per week, Berry et al., 2010, to 7 h and 20 min over the entire semester, Junco & Clem, 2015) or different uses of features (e.g., highlighting is reported as being widely used, Abaci et al., 2017; or not used Johnston & Ferguson, 2020). These differences can be the result of the poor conceptualization and inconsistent measurement of engagement across studies. A second problem is that the studies do not consider the potential impact that the discipline might have on resource use.

Other studies on student engagement investigate how much time and when students view digital textbooks in relation to the learning (e.g., Shepperd et al., 2008; Woody et al., 2010). The findings showed significant positive correlation between overall course grades and time spent reading (Abaci et al., 2017; Allred & Murphy, 2019; Junco & Clem, 2015), so digital textbook analytics are an effective early warning system to identify students at risk of academic failure (Junco & Clem, 2015). Comparing students who used an electronic versus a traditional paper textbook, results revealed that participants spent more time studying with a digital textbook than with a printed one but they prefer print texts for the learning process (Woody et al., 2010). Also, no differences between digital and printed groups in learning outcomes were evident (Shepperd et al., 2008).

In the case of university mathematics, this area of research is in its infancy. Berger (2019) conducted a small scale study with five students in one of her self-study calculus and precalculus courses, to identify typologies of reading styles derived from academic skills courses. She posits that there are five styles of reading, that depend on the connections made to other knowledge (some or strong) and how close the reading is done (close towards avoid). Some of these are interpreted in terms of textbook features, such as structure, voice, or genre. This study, however, did not address the mathematical content at stake. In a study with 330 university students in a pre-calculus course and 25 pre-service teachers who had a bachelor's degree in mathematics, Kondratieva (2018) investigated participants' experience using a digital interactive textbook on the topic of transformations of functions; the textbook included mathematical activities via software applications. Their analysis of surveys and interviews indicate that students found the textbook to be useful for studying and understanding transformation of functions, and that new features, such as interactive diagrams, helped to clarify ideas about the content. However, the experience was more engaging for pre-service teachers who used the text for reviewing the topic, than for pre-calculus students, who were learning the topic for the first time, which suggests that prior knowledge might influence students' perceptions of the usefulness, and therefore engagement, of a textbook. Gueudet and Pepin (2018) investigated four university students' use of digital resources and the linkage to the rules of the didactic contract. The analyses of the various types of data (interviews, observations of lectures, surveys, and resources used), showed different expectations about textbook use by lecturers and students. Lecturers expected students to use the textbook to learn and understand the concepts, and as a model for learning proof. The students instead searched for worked examples (as models for practices) in different kinds of resources such as their tutorial notes, textbooks and websites, and indicated that they focused on passing their examinations.

In summary, there is an interest in understanding affordances and limitations of digital mathematics textbooks for teaching undergraduate students and there is a wide range of approaches to collect data about student engagement with them. However, engagement has not been fully operationalized at the resource level—which is problematic for theory building—and there are few accounts of student engagement with textbooks in mathematics courses. Given that free digital textbooks are necessary to lower the costs of a college degree, questions about how students engage with them in courses that are needed for multiple STEM majors, such as linear algebra, become pressing.

We are interested in a particular type of open educational resource,[1] free digital textbooks that are open source, open access, and *interactive*. These textbooks are written with PreTeXt (https://pretextbook.org/), a purpose-built 'design neutral' markup language that allows for multiple renderings of the textbook in multiple formats, online and print (including Braille). These textbooks are distributed through a GNU Free Documentation License and thus, not subject to the traditional copyright agreements; as open-access and open-source, they can be accessed from any device connected to the Internet and modified for personal use. When updates are done, they are nearly immediately available. When using PreTeXt to write a textbook, authors must organize mathematical knowledge explicitly in a hierarchical fashion, incorporating simultaneously multimodal features of the various genres found in mathematics textbooks, such as "gross containers (e.g., 'chapter', 'theorem'), narrative content (paragraphs, titles, block quotations, table cells), mathematical symbolism (using LaTeX), and mathematical images" (O'Halloran et al., 2018, p. 868). The markup

[1] The OECD (2007) defines open educational resources (OER) as "digitized materials offered freely and openly for educators, students, and self-learners to use and reuse for teaching, learning, and research. OER includes learning content, software tools to develop, use, and distribute content, and implementation resources such as open licenses" (p. 10).

language has a precise designation for each of these elements of the structure which allows for such structure to be preserved no matter how the textbook is rendered. The textbooks include features that promote interaction: for example, automatic and immediate feedback for individual solutions or live computations in Python, via Sage cells, and links (called *knowls,* see Figure 1) that when clicked, open up boxes in the text so the reader gets access to specific content (e.g., definitions, theorems, examples, exercises, etc.). Additional functions of knowls include cross-referencing of the mathematical content, or SageMath calculations with the possibility to read the code.

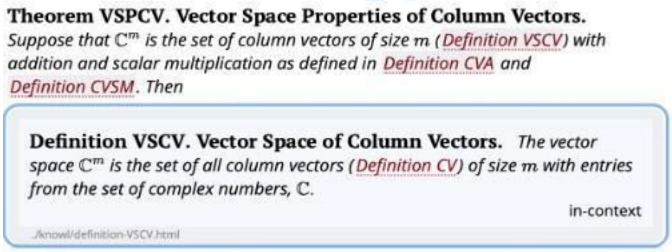


Figure 1. Example of a knowl for Definition VSCV, Vector Spaces of Column Vector. The slightly indented box opens up when the link in parenthesis is clicked (Beezer, 2017)

This precise designation allows tracking user activity when the textbook is accessed online via a browser. At any given moment it is possible to determine what is visible, and when users scroll down a section it is possible to detect when a new element is visible at the top of the viewing screen. Javascript records any changes in viewing, including going to a new page, scrolling down so that a new element enters the viewing window, or whenever users click on specific items (e.g., examples, proofs) to open or close it. “Tracking is recorded every second for the current item, and then the information is sent to the server every 10s (or when the user leaves the page) and recorded on the server log” (O’Halloran et al., 2018, p. 873). The recorded information includes also a unique user identification giving the total time each individual user spent viewing any given item.[2] Therefore, these textbooks open up the possibility to investigate a component of engagement at a high granular level. But tracking viewing is not sufficient for assessing engagement, unless users report what they do when they view their digital textbooks.

In this study we define engagement with digital textbooks as encompassing two aspects, viewing of the digital textbooks (that can be done with automatic tracking) and self-reported use of that textbook at the times of viewing. We work with *A First Course in Linear Algebra* (FCLA) (Beezer, 2017), a one-semester, introductory course in linear algebra (a gateway to STEM majors). The textbook has eight chapters (Systems of Linear Equations, Vectors, Matrices, Vector Spaces, Determinants, Eigenvalues, Linear Transformations, Representations, and Preliminaries). Each section follows a definition-theorem-proof presentation followed by a handful of exercises, some of which have worked out solutions. Differently from other studies that use a single class and a handful of observations, we include four groups of students at four different universities and collect data over a whole semester on engagement with elements that are specific to the mathematics textbook (definitions, theorems, proofs, examples, exercises, and solutions). These elements are labeled in the text (e.g., "Definition VS Vector Space") and are regularly referenced throughout the textbook, as *knowls*. To explore how university students with different majors engage with this digital linear algebra textbook, we sought answers to the following questions: what are students' viewing

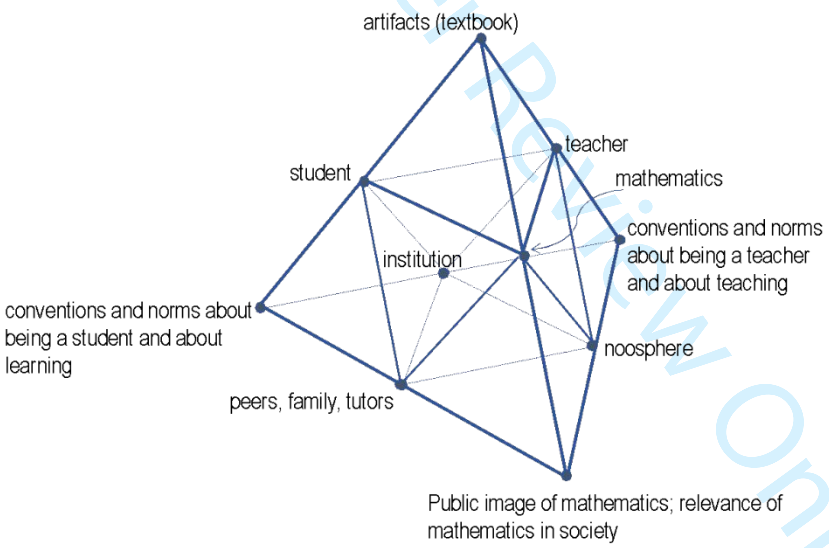
[2] If a user leaves their browser open for a long time, the script will record those seconds, suggesting a prolonged viewing. In our data, related to days close to the exams, we used a threshold of 300 minutes (5 hours). Only data from one student was removed, as their viewing exceeded 9 hours.

patterns of this digital textbook, what do students report about their textbook use, and what may explain such engagement?

Conceptual Orientation

Understanding student engagement with their resources is important because engagement shapes what occurs in the classroom (Rezat, 2013). Rezat and Straesser's (2012) didactical tetrahedron helps to conceptualize the role of resources in teaching (see Figure 2) allowing "for a situated understanding of the complex network of actions and their backings in the situation at hand" (Rezat, 2013, p. 660).

Figure 2. The socio-didactical tetrahedron (Rezat & Straesser, 2012, p. 648)



The four elements at the top indicate that textbooks mediate the interactions that occur between teacher, student, and mathematics: the work of students and teachers based on specific mathematics can be strongly determined by the textbook content and organization. The textbook can determine what is taught and when, providing a structure for any given course; in doing so, the textbook mediates the generation of other artifacts, such as a syllabus, lecture notes for a class, student homework, or exams. Students, in seeking to learn the content and instructors in seeking to teach it, rely on the textbook for these purposes. With this conceptualization researchers can model situations of mathematical teaching and learning with artifacts: each edge can be analyzed and then integrated to provide an account of how resources enter into, or influence, the interactions between the teacher, the student, and the content. The base of the tetrahedron fuses together two activity systems, one encompassing the work of teachers, and the other encompassing the work of the students, who gather together because they have a common goal, related to learning a particular piece of knowledge. The inner vertices of the base of the tetrahedron, represent various contexts and communities related to mathematics, the teacher, and students, the institution (where students study and instructors teach), the noosphere (the community of professionals and researchers who support teachers), peers, family, and tutors (the community that supports students). The other vertices represent the conventions and norms that regulate the work among people in each community and define particular roles related to mathematics (e.g., in society), for being a student and for being a teacher.

The tetrahedron is a powerful tool that provides an organizational frame for understanding engagement with the textbook, with teachers and students as the main users, showing the interplay between institutional and social contexts (Rezat 2013). It has been used in previous research on textbook use (e.g., Matić & Gracin, 2016; Nyman, 2017). Because the tetrahedron is a model, not a theory, in this work we expand on how we see mathematics textbooks, how we see students, and how we understand students' engagement with mathematics textbooks in the context of this investigation.

Pepin et al. (2016) remind us that "teachers often rely heavily on textbooks in their day-to-day teaching when they decide what to teach, how to teach it, and the kinds of tasks and exercises to assign to their students" and that the content and structure of the textbooks are closely tied to their design (p. 637). In the case of a digital or e-textbook, most significantly the interactions are expected to change because of the different interfaces. But this is also the case for students. There might be design features that create interfaces that facilitate communication between students and teachers, "for example in terms of feedback on (written) homework (which may be in or out of class)" (p. 638). At their core, mathematics texts still fulfill two pedagogical functions: curricular and conceptual (Love & Pimm, 1996; Valverde et al., 2000) and these are no different in a digital or an e-textbook. The curricular function pertains to the creation of a logical progression of mathematical knowledge based on building new ideas from old ones; the curricular function pertains to embodying "the development of cognitive structures in the learner" and attends to element such as "selection, ordering and pacing of the materials; consistency of language and symbolism; level of rigor; [attention to] students' assumed previous knowledge, attainment, and ability to process the text, and setting expectations for future readings" (p. 383). To accomplish these functions, mathematics textbooks use a number of devices to organize the reader's work: "exposition, explanation, questioning, exercises, and tests" (p. 386). As the name implies, exposition in mathematics texts expounds the subject matter discursively; it includes the definitions and theorems with their proofs; it may include questions and visual material, or questions for readers to ponder. Examples offer students "a model to be emulated in the exercises which follow" (p. 387). We consider the exposition text as constituting the mathematics theory-building blocks in the textbook. The examples may contain steps that students are to follow and are included with the expectation that students may generalize or transfer what is illustrated in the example into new situations—as given in the exercises. Exercises, "the principal means by which students are encouraged to be an active reader of the text" (p. 387) may mimic the examples or offer other types of problems. We consider the in-text examples and exercises as constituting the mathematics practicing blocks of the textbook. A main difference between a digital textbook and non-digital textbooks is how they are presented: the former can be viewed on a tablet, via browser, or through an application. Early generations of digital textbooks were "digitized versions of their paper counterparts" (Pepin et al, 2016, p. 639). Later generations included multimodal interactions, continuous content evolution, and personalization or customization, but these were mainly managed by publishing companies. The latest generation includes all open access and open source materials. E-textbooks have been classified as "integrative" when a digitized version of a paper bound textbook has other learning objects; evolving, when a community authors a textbook "that is permanently under development"; or interactive, "when it is a collection of learning objects that can be linked or combined" (Pepin et al., 2016, p. 640).

We conceptualize the student as being in a mutual relationship with the textbook, with students being agents in determining their own engagement with the textbook in a social setting (Rezat, 2013). This conceptualization is aligned with prior research that documented that university instructors believe that university textbooks are written for students (Author, 2012) and presumes that students not only construct alternative meanings from the content of the textbook, but they also use it in ways that are not anticipated by either the authors or their teachers (Rezat, 2013). Students engage in various types of activities that involve their textbooks, such as practicing (Rezat, 2013), preparing for exams (Weinberg et al., 2012), or reading (Author, 2020; Berger, 2019), all presumably with the goal of learning the material or of demonstrating that they have learned sufficient material to pass the course. All these activities are predicated on a specific initial action, students viewing their textbooks, which is possible to track accurately in textbooks written in PreTeXt.

In order to fully speak about engagement, we need to describe what students do with their textbooks as well. Thus, for this reason we operationalize the mathematics textbook-student relationship as engagement and define it as being composed of two dual aspects: viewing of a textbook and its features and what students report about their use of those features. The combination of these two aspects allows us to fully speak about engagement with the textbook considering both objective and subjective information. Our research questions, as stated earlier, explore engagement from two perspectives, first from the actual viewing data that can be collected because the textbook is written in PreTeXt (e.g., timing and frequency of viewing, features viewed), and second, from student self-reports on their textbook use and other artifacts such as course syllabi. We use the socio-didactical tetrahedron to elucidate how the social context might shape students' engagement with their textbooks. In doing so, we give depth to the interaction represented by the artefact-student edge attending to possible influences on student engagement.

Method

The records used in this analysis were collected as part of a pilot study involving four instructors (and their students) who responded to a national call for participation and used the digital version of FCLA (Beezer, 2017). They taught the semester-long course (14-16 weeks) at four institutions, located in four different states. Their institutions varied in size and selectivity in admissions (none to high), and in their funding (private or public), which resulted in a diverse set of students (see Table 1). Each of the four instructors reported having over 10 years of full-time teaching experience.

Data Collection

The 107 participants in this study were students of instructors A, B, C, D using FCLA from January through June 2018; some characteristics of the groups are provided in Table 1.

Table 1. Participant Information

	Group A	Group B	Group C	Group D
No. of students	29	23	26	29
Number or percentage of female students as reported by the teachers	5	50%	40%	10*
Reported majors	Engineering, Computer and information science, Physics, Economics, Undecided	Mathematics, Computer Science	Mathematics, Computer science, Engineering, Physics, Chemistry, English	Secondary mathematics education, Computer Science, Undecided
Reported level of programme of students	1st-year, 2nd-year, 3rd-year, 4th-year	3rd-year, 4th-year	1st-year, 2nd-year	high-school, 2nd-year, 3rd-year, 4th-year, master
No. of class meetings per week	3	2	3	3
Class duration in minutes per meeting	50	75	50	60

Note. The instructor reported having at least one transgender student.

We collected different kinds of data: tracking data (automatically generated viewing data) that provide real-time information about textbook viewing and qualitative data including (1) student bi-weekly logs, (2) a student survey that was administered in the middle of the semester, and (3) course syllabi and an instructor survey. These are described next.

Tracking Data

We used tracking data to identify sections of the textbook that were viewed and for how long and analyzed two different representations of viewing this information: heatmap and element map.

Heatmap data. The heatmap represents cumulative textbook views of students since the beginning of the term. As seen in Figures 3, 4 and 5, the heatmap is a two-dimensional representation with two axes. The vertical axis corresponds to the chapters and sections of the textbook; the horizontal axis corresponds to days over a semester (Figure 3), hours over a day (Figure 4), and minutes per individual user (Figure 5). Inside the representation of Figures 3 and 4, various rectangles of different colors represent frequencies of textbook viewings, with darker colors representing heavier use (e.g., purple) and lighter colors representing less use (e.g., light pink). The color scale ranges from white, to pink, red, purple, gray, and black. The last three colors indicate that many students viewed the textbook chapters that day, with a maximum capacity of 50 viewings (the maximum number of students per group was 29) represented in black. In Figure 3, for example, there is a gray rectangle for the beginning of chapter System of Linear Equations, on January 29. These maps are interactive. When a user clicks on a rectangle, it is possible to zoom into the viewing of a particular day (Figure 4). By clicking again on a rectangle, we see viewing of individual users, each with their own ID; Figure 5 shows two users: one between 12:43-13:13pm with eight sections and a second between 13:34-13:52pm (7 sections).

Figure 3. Group B's Heat map (Full semester)

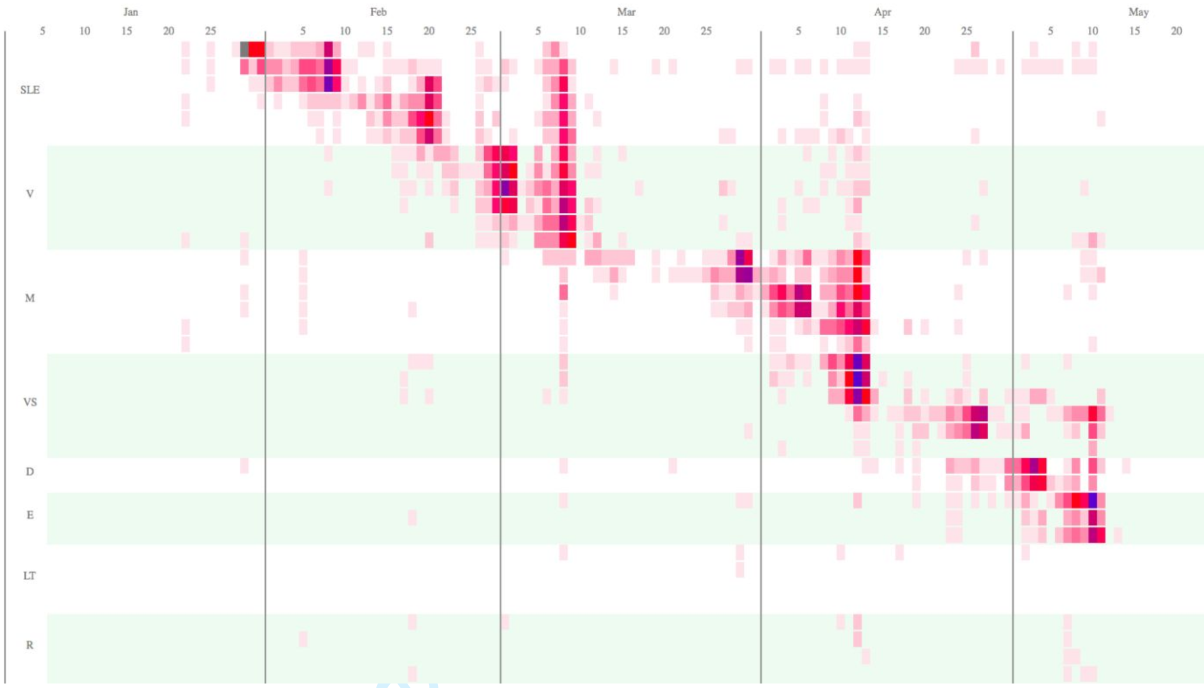


Figure 4. Group A's heat map (One day, March 23th)

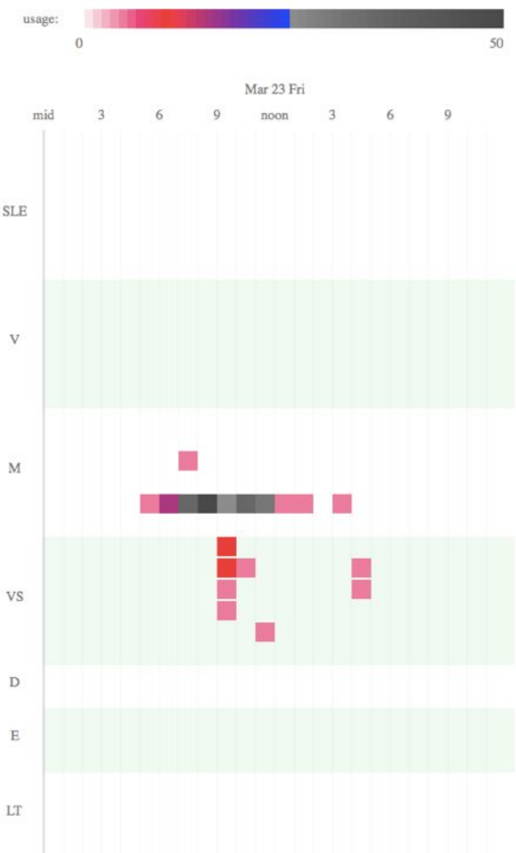
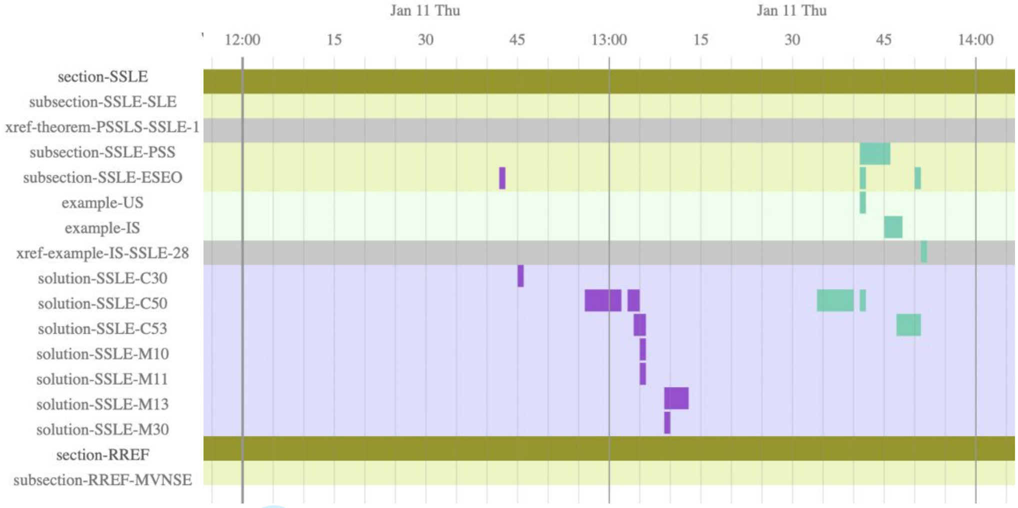
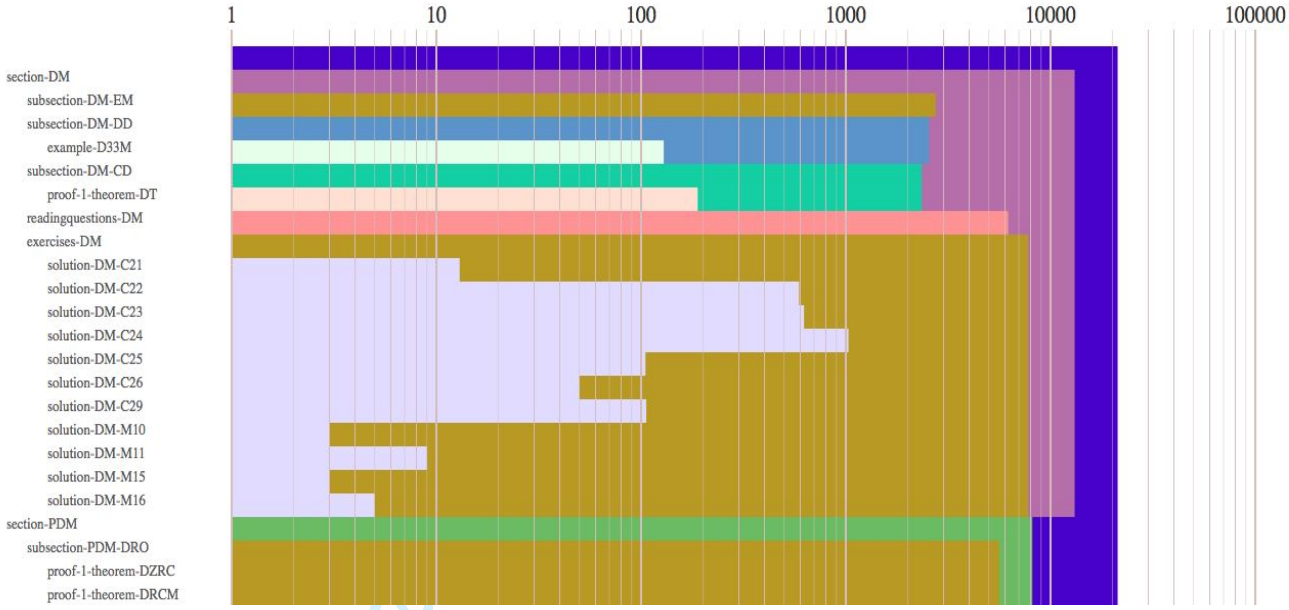


Figure 5. Individual users' map (One day, January 11th, 12:00-14:00 pm)



Element map. An element map is also a two-dimensional representation of cumulative viewings in seconds for each textbook element (definitions, theorems, proofs, examples, exercises, and solutions), structured by chapter and sections since the beginning of the term and using a logarithmic scale. The vertical axis of the element map corresponds to the sections, subsections, and elements of the textbook; the horizontal axis corresponds to the number of seconds that the sections, subsection or the element was viewed by all the participants of a group. Figure 6 shows an element map for the Determinant chapter for group B. Dark blue goes from the top to the bottom showing the time in seconds of all the viewings for all sections, subsections, and elements that this chapter includes, whereas purple in a smaller portion corresponds to the whole section Determinant of a Matrix, meaning that chapter Determinants was viewed for more than 20,000 seconds (see horizontally the end of the dark blue region), whereas section Determinant of a Matrix (with reference “section-DM”) was viewed for more than 10,000 seconds (see horizontally the end of purple region). Time for viewing textbook elements is counted per element; for example, the example of section Determinant of a Matrix, with reference “example-D33M” and light turquoise coloring, was viewed for more than 100 seconds. We collected 32 element maps (8 element maps per group) to obtain information about what textbook elements were viewed and how frequently.

Figure 6. Element map of chapter Determinants for Group B



Qualitative data

Student Bi-Weekly Logs. Six short surveys, hereafter logs, were distributed every other week to the students with the goal of gaining a deeper understanding of how students use their textbook and its elements both inside and outside of class. The logs could be accessed via a link from any mobile device or computer anytime. This method of data collection removes the interviewer, giving students more freedom in responding and eliminating the pressure to give answers that researchers may want to hear. The instructors offered students small incentives for completing the logs (e.g., extra credit in homework or quizzes). Each log had a different theme, and included open- and close-ended questions that addressed textbook use in many forms (e.g., "A representation (map) of how much students in your class used the textbook is here <link to a specific day in the term>. Please tell us what you were doing with a couple of sections represented by a couple of rectangles"). The questions were intended for students to provide a response that included the textbook element they used (e.g., definitions, theorems, proofs, examples), a description of what they were doing when they used that element, and their rationale. The questions targeted frequency of use ("how much do you use the following textbook elements?: A lot, A great deal, A moderate amount, A little, Not at all") and disposition ("indicate how much you like the following textbook elements?: Like a great deal, Like somewhat, Neither like nor dislike, Dislike somewhat, and Dislike a great deal"). Open-ended questions about the textbook (e.g., "In what ways could the textbook for the course be modified to better support you in learning the material of this course?") gave us students' affective statements about textbook elements.

Syllabi and Instructor Survey. All instructors were asked to provide their syllabus (content to be covered with approximate times and due dates for various assessments) for the course which was important for us to understand how the instructors planned to use their textbook throughout the semester. The instructors also answered a survey before their course started in which they described, among other things, the goals for the course, how they used their textbooks for planning, and the profile of the enrolled students.

Data analysis

The analysis was performed in four phases. In the first phase, we focused on the heat maps. First, a group was selected and then their syllabus was searched for the following information: daily schedule; textbook chapters covered; days dedicated to each chapter; vacation days; dates and time for class meetings, office hours, exams, quizzes, and homework. Second, we marked on the corresponding heatmap the dates preceding an exam, a quiz, or a homework deadline in order to establish a connection between these dates and the use represented in the heat map.

In the second phase, we attended to the horizontal axes of both the heat maps and elements maps, as they account for the time when the textbook was viewed, to identify and count for how long the textbook chapters, sections, and textbook elements were viewed by users. This quantitative information was organized in tables and represented in frequency graphs.

In the third phase and with the goal of complementing and adding to the information collected from the first two phases, we analyzed the log questions to obtain more detailed explanations about the use of the textbook and its elements by students. To do the analysis we open coded the student responses to all logs (n = 484), and created a grounded categorization of the textbook elements more frequently used and the justifications for such use (need to provide an example here? or perhaps a table?). Then we searched the literature for terms that could capture the essence of the open codes. The justifications of use per textbook element were used in multiple-choice questions in the last log for confirmation from the full sample (e.g., "What best describes your use of the definitions?: choose all that apply: Read to understand concepts, read to increase vocabulary, be informed about different articulation from class, use their notation to solve examples/homework problems, read to figure out how to solve homework/other problems, highlight, copy them into notes/note cards, memorize, don't use, other (please specify)). We also used descriptive statistics to analyze the student responses to the questions on textbook use choices, frequency of use and disposition. In the fourth phase, we repeated this process for each group and then created summary tables across the four cases.

Results

We present the findings in two sections. The first section describes first when and for what purpose the textbook was viewed by the students and next the patterns of element viewing of three of the most viewed chapters. The second section presents the students' reported use of specific elements of FCLA, how often they used them and why.

Students' Viewing of the Textbook

According to the heat maps, textbook viewing was more frequent during the days when the classes were offered, around exam days, and when homework assignments were due. Viewing during class sessions corresponded to the length of the class sessions, which indicates that the instructors required some textbook use during class. The days when the heat maps appeared white—indicating no textbook use—corresponded to school breaks, holidays, or weekends. In terms of hours of the day, we saw viewing data at all hours of the day, even at night. Outside of class, students spent time viewing the textbook, sometimes more than six hours per week, during class days, the day before class, or the last day of their spring break. We analyzed the days of the exams or quizzes and three days prior because, for all the groups, those were the days with the highest frequency of viewing.

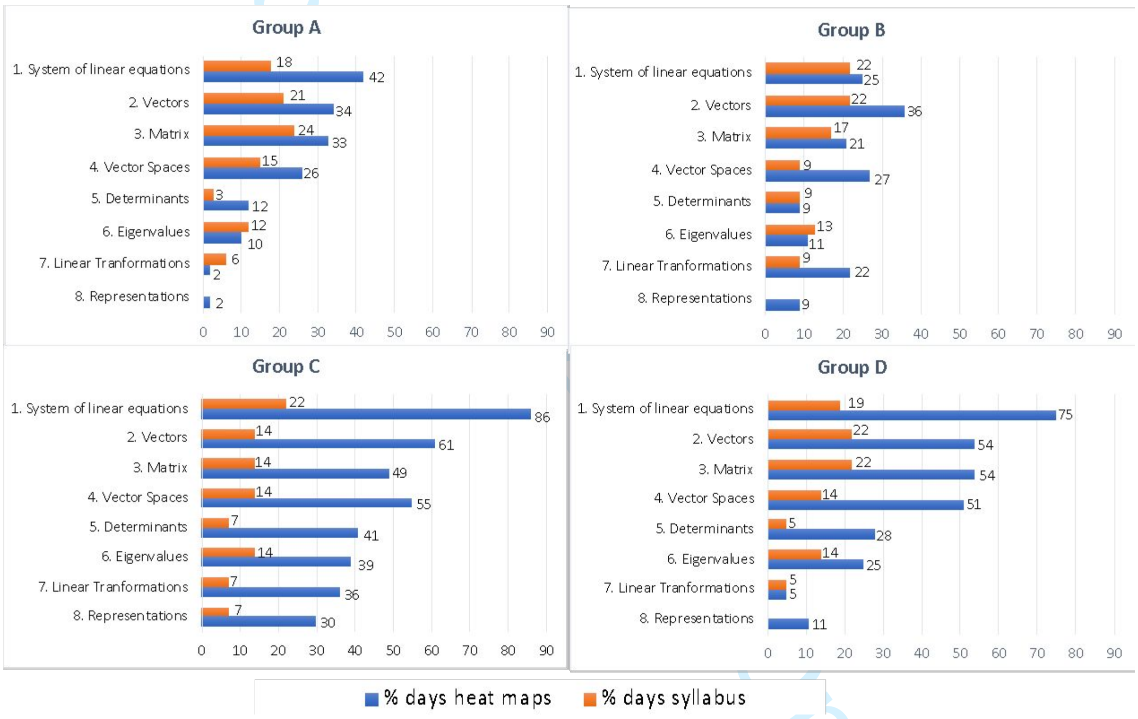
The tracking data reveal that students viewed the textbook the day of the exam, the day before, and, and two and three days before. The number of students per group who consulted the textbook within a three-day period before the exam varied, with a maximum of 17 (58%) students in Group C, one day before the last exam. We noticed that some students viewed the textbook for periods of time of various lengths (between 4 min and 5 hours) suggesting that students viewed FCLA for various goals, while others viewed it more frequently on the days prior to the exam. Across the 11 examinations, the average length of viewing was 86 minutes per day, almost an hour and a half, a high average with a high standard deviation (SD=100 min), suggesting wide variation in these times of viewing between participants.

The logs provide an indication about what students did during and outside of class when viewing their textbooks. During class, they said they used their textbooks to solve problems or exercises when their instructor asked them to, by going through the textbook and extracting key ideas that were used in the exercises (e.g., "I usually do the examples without looking at the solution. Then I go through the detailed solution and try to understand the concepts. I then move to the assigned problems and try to do them one by one."). Furthermore, when students were confused with the solving process and did not know how to work out the problems, they said that their instructor let them discuss and review some sections together, looking back at some theorems and examples (e.g., "We mainly went over the theorems that the book includes, and did some examples on vector operations.").

Outside of class, students said that they use their textbooks to prepare for the upcoming class (e.g., "I looked over the introduction and took a look at some problems. I read what the chapter gave me and just glanced at the problems. I did not solve anything."), study for exams or quizzes (e.g., "Most recently I used the section to study for my upcoming quiz. Going through and doing the problems provided and being able to look at the solutions really helps."), refresh their memory about how to solve their homework assignments (e.g., "I skimmed the chapter for the notation of a matrix A^2"), check problems or exercises similar to those proposed in the homework (e.g., "I looked at and worked out the problems to learn how to do them."), catch up if they had missed a class (e.g., "due to my missing of a specific lesson (*MISLE - Matrix Inverses and Systems of Linear Equations*), I actually read through a section of the book, reading the theorems, proofs, examples, etc."), or seek help when they did not fully understand their instructor's explanation (e.g., "I looked at definitions and examples to reunderstand (*sic*) how the problems are solved.").

A closer analysis of the viewing by chapter, reveals that once the viewing of any chapter had begun, participants continued to do so throughout the semester. For example, the first chapter, Systems of Linear Equations, was viewed by Group B from mid-January to mid-May (see Figure 3). We contrasted these data with the days allocated to each chapter in the syllabus of each group. Figure 7 presents the percentage average of days that students of each group viewed the chapters according to the heat maps and the percentage average of days planned according to their syllabus with respect to the total number of days in a semester.

Figure 7. Percentage of days that groups viewed each chapter and percentage of days dedicated in the course to each chapter according to the syllabus



Note. The blue bars do not add up to 100% because students viewed more than one chapter on the same day. The orange bars do add up to 100% because in the syllabus each chapter is assigned to an individual day.

Except for group B—the only group composed of mathematics or computer science majors—the number of days in which chapters were viewed decreased as the semester went on. The first four chapters—Systems of Linear Equations, Vectors, Matrices, and Vector Spaces—had the greatest number of days viewed, representing approximately 70% of the days planned by syllabi.

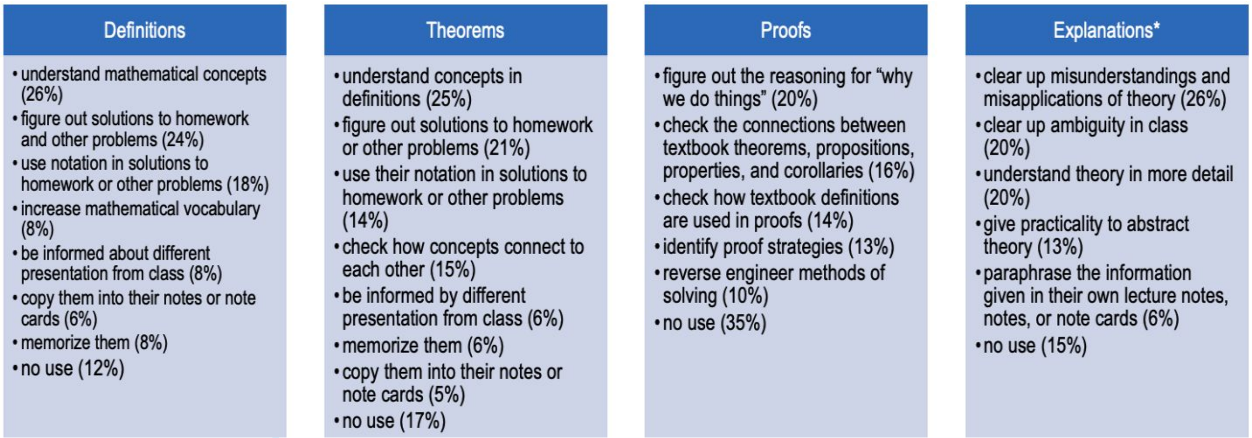
Comparing the percentage average of days that students viewed each chapter with the percentage average of days which instructors dedicated to those chapters according to the syllabus, five points stand out. First, chapters with a high viewing and high numbers of days in the syllabus: System of Linear Equations, Vectors, Matrices, and Vector Spaces. Specifically, the chapter System of Linear Equations has a similar percentage of days assigned across all the syllabi (18%-22%) and it was the most viewed chapter in three groups (A, C, & D). Second, the Determinants chapter stands out for having little presence in the syllabi (3%-9% of days), but a considerable percentage of days in which this chapter was viewed by students of three groups (A, C, & D). Third, and contrary to what happened with Determinants, Eigenvalues had a high percentage of days assigned in the syllabi (12%-14%), but a low number of days in which participants viewed it, relative to the rest of the chapters. Fourth, Linear Transformations had a low percentage of days both according to the syllabi (5%-9%) and to the days that students of the groups viewed this chapter. Fifth, the Representations chapter is only considered in the syllabus of Group C, but students from all groups viewed this chapter between 3% and 30% of days. The variations in the number of days in which the textbook is viewed and the plans espoused in the syllabus suggest that syllabus can’t be used as a proxy for estimating students' viewings of the textbook.

It appears that participants in Groups A and B adhered more to their instructors’ intended viewing of the textbook than participants in Groups C and D, as expressed by the time allocation of different textbook chapters in syllabi; we believe that these patterns are related to the characteristics of the groups. Participants in Groups A and B were at large state universities that offered multiple sections for the course; the majority of students were engineering majors in Group A and mathematics and computer science majors in Group B. Participants in Groups C and D were at smaller universities and had more diversity of student majors (e.g., secondary mathematics teaching, economics, chemistry); in smaller universities departments may need to be more inclusive of all majors to be able to offer mathematics courses.

Students' Reported Use of the Textbook

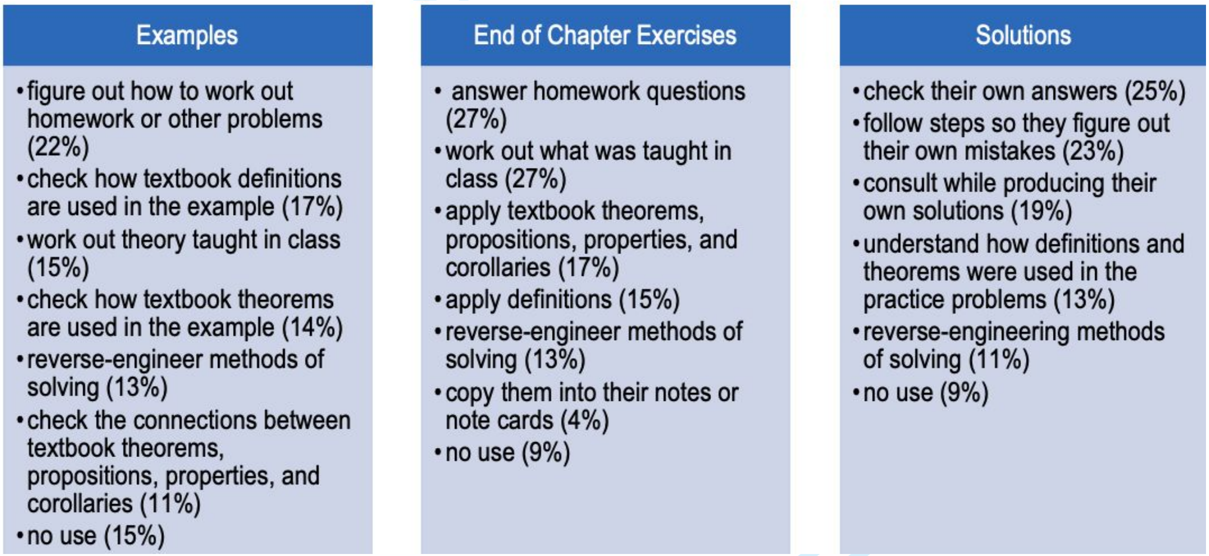
FCLA has 1,560 different elements distributed into 101 definitions (6%), 222 theorems (14%), 222 proofs (14%), 515 exercises (33%), 258 solutions to exercises (17%), and 242 examples (16%). Thus, relative to the other elements, exercises are quite prominent, almost double of the next most frequent features, solutions, examples, and theorems and their proofs. Exercises and solutions account for 50% of all elements in the textbook. It is surprising, however, that relative to this number of exercises and solutions the number of viewings and the time that participants spent in these elements is much higher. According to the element maps, in the 17,405 viewings, 81% of the viewing time was for exercises and their solutions (corresponding to approximately 54% of the viewings), 15% of time was for examples (31% of viewings), and 4% of time was for theorems, proofs and definitions (15% of viewings). Students' viewings of the mathematics practicing block elements (85% of viewings in the elements map) were more than those of elements related to theory-building blocks in mathematics (25% of viewings in the element maps). Figure 8 and 9 presents our analysis of students' reported use of these elements, according to whether they are related to theory-building blocks (definitions, theorems, proofs, and explanations, Figure 8) or to practicing with mathematics (examples, end-of-chapter exercises, and available solutions, Figure 9).

Figure 8. Reported uses of theory building blocks. N=103



Note. *In the multiple-choice questions, we added "explanation" which was repeatedly mentioned by students in open-ended responses. We assumed "explanation" to be part of exposition, and thus belonging to the theory-building block.

Figure 9. Reported uses of practicing blocks. N=103



In the logs, the students reported on a Likert scale about their like-dislike for the FCLA elements. Except for proofs, all the elements appear to be well liked. The highest ranked elements were exercises, solutions, examples, and definitions with more than 75% of the participants stating that they liked them. Theorems appear towards the middle, with 56% of participants ranking this element in the category of like. Finally, proofs were the lowest ranked with 36% of the participants reporting that they liked them. In Figure 8 we see that proofs were also the least used element with 35% of the participants reported that they didn't use them. Students reported having difficulties with algebraic language and content when referring to definitions, theorems, and proofs, and that these obscured the connection between theory-building blocks and practice blocks (examples, exercises, solutions). Theorems and proofs received negative affective statements (e.g., "Hate having to memorize [theorems]", "The proofs are confusing. I do not know how to make sense of proofs"). Students mentioned difficulties when they encountered different textbook notation from what was done in class and said that they struggled with the level of abstraction.

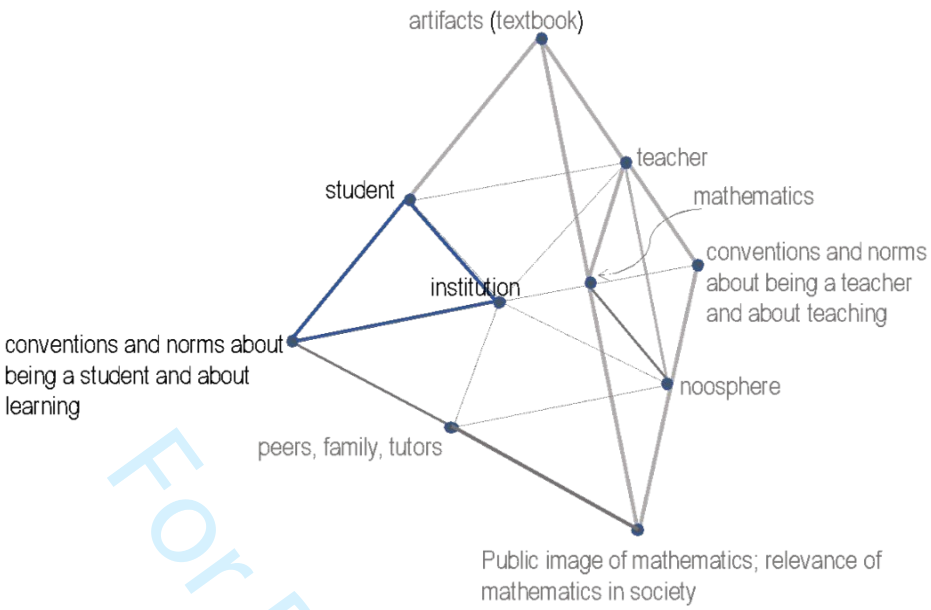
Discussion

The main goal of this study was to investigate student engagement with digital textbooks by using real-time viewing data to identify patterns of textbook viewing, and student self-reports on these data to gain insights into their textbook use. We interpreted such engagement in terms of the socio-didactical tetrahedron. We saw the digital textbook written in PreText as the key that gave us access to a large volume of viewing data. Coinciding with other studies (e.g., Kondratieva, 2018), we found that students considered the new features of the textbook useful. Unlike other research reporting that students did not take advantage of the new elements of the dynamic textbook because they preferred to use other resources (Rezat, 2017), our participants were clicking on the *knowls* to view various textbook elements (e.g., definitions, solutions) either as cross-references or as new content.

Our analyses linked the viewings to class sessions, exam days, or when homework assignments were due, indicating that students use the textbook for many more uses than ever before. Many of those uses support their understanding and clarification of the material from class sessions, in some ways contradicting other studies, and current wisdom, that indicate that students just use their practice-block elements in the textbook when doing homework or studying for exams (Gueudet & Pepin, 2018; Weinberg et al., 2012). These discrepancies between the findings of previous research and our findings might be due to the means of data collection (tracking data and self-reports on these data versus self-report alone).

In this study, we found that students use their textbooks more for homework and exams. This is not surprising and can be described by the relations depicted in the triangle institution-conventions and norms about being a student-student (see Figure 10) in the socio-didactical tetrahedron, which brings to the fore institutional features that influence the relation between students and textbook. Homework and exams are forms of summative assessments specific to the students' institution, whose syllabi in the US are built based on the textbook. As such, homework and exams play a major role in students' viewings of the textbook because, for students, passing grades per se can ensure the eventual earning of a degree. That being said, the importance of assessment that results in passing grades for the students, reveals the students' collective understanding of what is expected of them as students, and thus their perspectives of conventions and norms about being a student (i.e., to pass required courses). Such a situation also hints that some students might see their institution through the lens of a degree-awarding body rather than a space in which it is expected that the subject-matter will be accessible.

Figure 10. Institutional features underlying students' viewing of textbooks.



Our findings not only confirmed students' textbook use for homework or prior to exams, but also is the first to relate student majors to textbook engagement and each textbook element to many more and different uses than what has been reported so far. We found that FCLA was approached in a practical way by participants as they viewed exercises, solutions, and examples more than definitions, theorems, and proofs. This finding could be explained by the composition of the classes because about 63% of the participants were pursuing science and engineering majors, disciplines that may approach mathematics with a more utilitarian orientation. But because prior research with printed mathematics textbooks exclusively used by mathematics majors (e.g., Gueudet & Pepin, 2018) and by students in other majors (Weinberg et al., 2012) found similar trends, we confirm these findings as well.

Why are proofs in particular so disliked and unused, when it is a key element in the theory building block? FCLA attempts to build a rigorous mathematical approach to linear algebra but it does not explicitly emphasize the use-value of proofs:

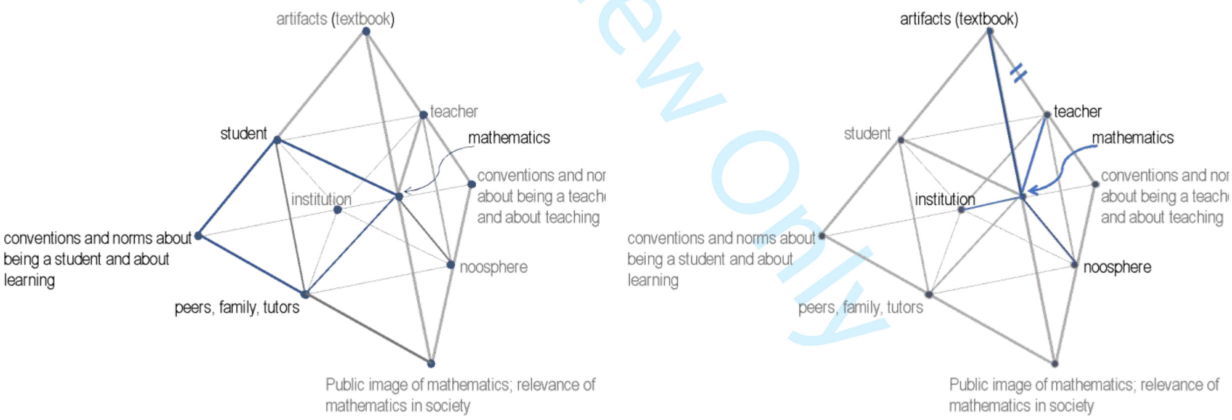
> Linear algebra is an ideal subject for the novice mathematics student to learn how to develop a subject precisely, with all the rigor mathematics requires.

> Unfortunately, much of this rigor seems to have escaped the standard calculus curriculum, so for many university students this is their first exposure to careful definitions and theorems, and the expectation that they fully understand them, to say nothing of the expectation that they become proficient in formulating their own proofs. We have tried to make this text as helpful as possible with this transition. Every definition is stated carefully, set apart from the text. Likewise, every theorem is carefully stated, and almost every one has a complete proof. (Preface, FCLA)

We think that because proof is of use-value mostly to mathematicians, the 63% of participating students majoring in another discipline may not see the relevance of proofs to the learning of mathematics. Social influences (e.g., by peers and support community), which may underlie students' perception of proofs, can be described by the quadrilateral student-mathematics-peers&family-conventions and norms about learning (see Figure 11a). These features also reveal a lens with which some students view mathematics: that of the use-value. Also, the conventions and norms about being a student relate to the learning outcomes of the course. As these are introductory courses to linear algebra, only the syllabi of Groups C and D—situated in smaller universities that included more diversity of student majors—explicitly included "proof", probably in an attempt to make students aware of the expectation of using them.

As other studies have suggested, algebraic symbols influence the comprehension of the content (e.g., Österholm, 2006) so the language and content of FCLA might have affected its use. To explain the role played by students' difficulties with definitions, theorems, and proofs in reducing use of those textbook elements, we drew on the mathematics vertex of the socio-didactical tetrahedron. FCLA's author, a member of the noosphere, included algebraic language and content that may have been rooted in the institutions he belonged to while designing it. This situation can be expressed by the path noosphere-mathematics-artefact-student (see Figure 11b); the students acquire the end-product of this process in the form of the textbook. However, their instructor produced teaching materials for linear algebra by using algebraic language and content, legitimized by the institution where the students studied and different from that of the textbook. So, the path for the production of the teaching materials becomes institution-mathematics-teacher-student (i.e., instructor, see Figure 11b). Focusing on the upper pyramid of the socio-didactical tetrahedron, the teaching materials (vertex teacher) conflict with the textbook (artifacts vertex) in terms of the algebraic language and content (mathematics vertex), so the relation Teacher-Textbook breaks (represented by the hash marks on the line on Figure 11b) and student difficulties arise. In such a situation, the students may choose the institutionalized materials over the textbook as those concur with conventions and norms about being a student (i.e., use of legitimized language and content by the institution).

Figure 11. (a) Social features underlying students' use of proofs. (b) A break in the relationship Teacher-Textbook.

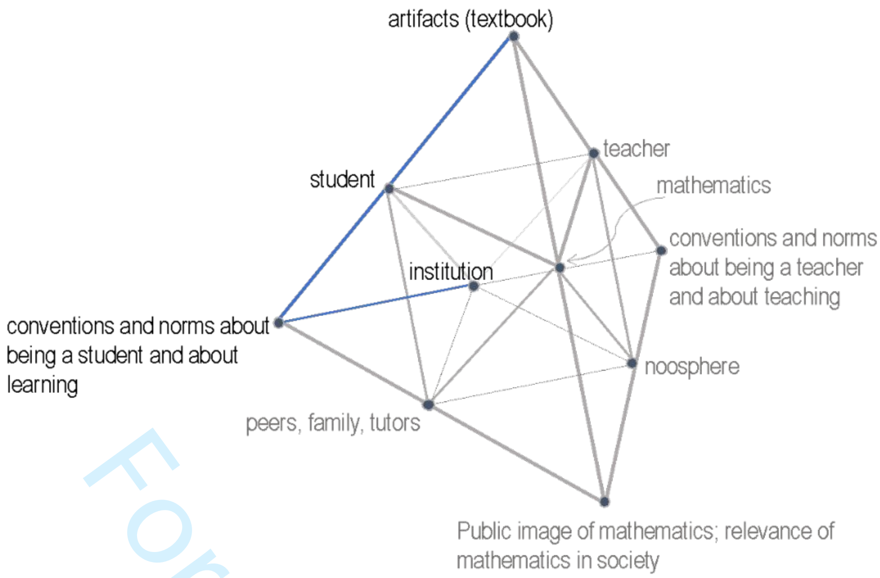


(a) (b)

We also found that participants spent more time on the first four chapters of FCLA, independently of the time the instructor assigned to those in the syllabus; possibly students need to go back to the first chapter to review the content or to study for an exam, because it provides foundational definitions that are used throughout the course. Furthermore, we found that students who study in a mathematics, computer science or engineering programme adhere more to their lecturers' intended viewing of the textbook than students who study in other programmes (chemistry, biology, etc.). Also, for the last students, the number of days in which chapters were viewed decreased as the semester went on.

The relations depicted in the socio-didactical tetrahedron can explain the difference of viewing time between groups; we considered the institution vertex of the socio-didactical tetrahedron and the path institution-conventions and norms about being a student-student-textbook (see Figure 12). The institution (through accreditation processes) regulates majors and subsequently the mathematics content (through departmental negotiations) that ought to be covered in order to satisfy degree requirements. Students' awareness of the expectations of required courses for their major might regulate the conventions and norms about being a student in a particular program (i.e., the need to pass their linear algebra course). Because linear algebra is a fundamental course for engineering, computer science, and mathematics disciplines, we presume that the students of these majors (namely Groups A and B) will want to be familiar with its contents and language so their viewing time of the textbook would be closer to the intended viewing of the textbook by the instructor than students in the other two groups (C and D).

Figure 12. Institutional features underlying students' viewing of the textbook.



Although we obtained meaningful results for research on the use of this "latest generation of digital mathematics textbooks" (O'Halloran et al., 2018) and the findings are supported through triangulation of multiple data sources, the present study has some limitations. On the one hand, the tracking data underestimates viewing because some students reported printing the textbook as they preferred to read it on paper instead of on an electronic device. On the other hand, we had no instance of a student staying on a page of the digital textbook for such a prolonged time that may have indicated tracking of unused viewing time so we are confident that the information allowed us to describe actual viewing of the textbook across the four groups. Our analysis did not include instructors' perspective on how the textbook is incorporated into the course, what the relative importance of chapters and elements is, or how they assess their students. As what instructors do affects the ways students use their textbook, we plan to explore this element in the near future. Another limitation of our research is related to the generalizability of findings: 107 participants and one digital textbook in four courses is a sample that limits our ability to study other factors that play a role in these results (e.g., year of study). However, our findings contribute to this research area, because of our operationalization of engagement as a combination of real-time viewing with self-report use.

Conclusion

The high level of connectivity of the mathematics content of FCLA seems to provide benefits to students related to mathematics connections, availability of examples and problems, portability and easiness of access, and cost savings. However, similar to previous findings with mathematics printed textbooks (Österholm, 2006), students' difficulties with the algebraic language and content led them to avoid a key piece of theory-building: proofs. In this sense, it is essential that textbook authors present the proofs in a manner that makes them useful and accessible elements. Instructors could also motivate students to work on proofs from the outset of the course, by explicitly commenting on their value for knowledge building in mathematics and include valuable reasoning techniques that might be useful in other subjects.

In this study, we identified how the institution regulates the Student-Textbook relationship. This is a dynamic relationship as it depends on institutional and social features that shape students' views of mathematics (relevant to the use-value) and of being a student (thus acquiring a degree). Our study suggests that students' textbook use can consistently vary, and as Weinberg et al. (2012) note, students use the text in ways that are not consistent with text structure. Textbook use seems to drop when the textbook language and content is not legitimized by the institution, when the textbook does not satisfy the interests of degree-earning, and when the narrative of the textbook does not showcase the use-value of the mathematics (and the students' support community may not recognize its relevance). Also, students' textbook use rises far beyond instructor's expectations when the students are unfamiliar with the nature of mathematics as they major in different disciplines.

Textbook production is expensive, and thus, research documenting how open access textbooks can be made widely available is important. This study empirically documents information that we had experienced but that is considered anecdotal. Our findings documenting naturalistic practice are uncommon, and potentially providing a baseline for future studies that may seek to contrast specific interventions. Particularly, these results raise new questions about student engagement with textbooks in undergraduate mathematics classes, such as, how do the patterns of engagement with digital textbooks change with other mathematics courses? How do instructors influence student engagement with their digital textbooks? What do we learn from student engagement with specific mathematical tasks? Do students of various groups (age, race/ethnicity, gender, SES) engage differently with their textbooks? Research in this area advances this knowledge and supports students in becoming aware of how to best use their textbooks for learning; it may also enable students' views of the institution to change direction towards a space where mathematics is learnable.

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Declaration of interest

The authors declare that there is no conflict of interest.

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