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IMPROVEMENT THE SPATIOTEMPORAL RESOLUTION OF ATMOSPHERIC
AEROSOL OPTICAL PROPERTIES USING ELASTIC LIDAR MEASUREMENTS

Master's Thesis

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Abstract

Lidar technique is an active remote sensing technique that enables the retrieval of the optical properties of aerosol particles suspended in the atmosphere. This work presents an improvement in the spatiotemporal resolution of these properties using the ALHAMBRA lidar system, operated by Group of Physics of the Atmosphere (GFAT). Through the implementation of alternative inversion algorithms—specifically, the quasi-backscatter and iterative backscatter algorithms—the particle backscatter coefficient has been retrieved with high spatiotemporal resolution which allows obtaining a set of high resolution intensive aerosol properties.

These algorithms require a precise estimation of the lidar system's calibration factor as input. To this end, three calibration methods have been rigorously evaluated: the direct calibration method and two new approaches based on linear fits of the signal to attenuated backscatter and aerosol optical depth. These methods have been validated using synthetic lidar signals and have demonstrated robust performance when applied to real data under stable atmospheric conditions.

Additionally, overlap correction strategies have been studied to extend the range over which aerosol optical properties can be accurately visualized. In this context, a new overlap algorithm has been developed and validated, based on the assumption of a homogeneous atmosphere in the near-field region. This method showed excellent agreement with the traditional iterative algorithm, both in simulations and in real measurements.

The high-resolution visualization of intensive and extensive aerosol properties has enabled a detailed analysis of atmospheric events of interest. These representations facilitated the classification of atmospheric structures—like Saharan dust, anthropogenic pollution, and clouds—and the identification of interaction zones between them. Furthermore, they allowed for a more detailed study of particle dynamics within the boundary layer.

The results confirm the robustness and applicability of the proposed methodologies, establishing a solid foundation for future work. In particular, these advances lay the groundwork for more accurate and robust atmospheric monitoring.

Keywords: High resolution, calibration, backscatter, lidar, aerosol

List of Symbols and Acronyms

Symbols

z	Height [m]
λ	Wavelength [nm]
t	Time [s]
σ_{sca}	Scattering coefficient [Mm^{-1}]
σ_{sca}	Absorption coefficient [Mm^{-1}]
$\alpha, \alpha_p, \alpha_m$	Total, (p) Particle, (m) Molecular extinction coefficient [m^{-1}]
β, β_p, β_m	Total, (p) Particle, (m) Molecular backscatter coefficient [$\text{sr}^{-1}\text{m}^{-1}$]
$\beta^{\parallel}, \beta_p^{\parallel}, \beta_m^{\parallel}$	Parallel total, (p) particle, (m) molecular backscatter coefficient [$\text{sr}^{-1}\text{m}^{-1}$]
$\beta^{\perp}, \beta_p^{\perp}, \beta_m^{\perp}$	Perpendicular total, (p) particle, (m) molecular backscatter coefficient [$\text{sr}^{-1}\text{m}^{-1}$]
T, T_p, T_m	Total, (p) Particle, (m) Molecular transmittance [#]
AOD	Aerosol optical depth [#]
$AOD_{\text{lidar}}, AOD_{\text{photo}}$	(lidar) Lidar, (photo) Photometer aerosol optical depth [#]
C_{α}	Accumulated extinction coefficient [#]
z_m	Aerosol-free reference height [m]
z_{over}	Full-overlap height [m]
z_c	Arbitrary height [m]
LR, LR_p, LR_m	Total, (p) Particle, (m) Molecular lidar ratio [sr]
$\dot{a}$	Ångström Exponent [#]
R	Backscatter ratio [#]
$\delta_V, \delta_p, \delta_m$	(V) Volume, (p) Particle, (m) Molecular linear depolarization ratio [#]
λ_L, λ_R	(L) Elastic, (R) Raman wavelength [nm]
P, P_R	Elastic, (R) Raman lidar signal [a.u.]
RCS, RCS_R	Elastic, (R) Raman range-corrected lidar signal [a.u.]
O	Overlap function [#]
K	Lidar calibration factor [a.u.]
P_0	Lidar emitted power [W]
c	Speed of light [m/s]
τ	Lidar pulse duration [s]
A_T	Illuminated area [m]
g	Photomultiplier gain [V/W] or [Hz/W]
$\beta_{\text{att}}, \beta_{\text{att}}^{\text{mol}}$	Total, (mol) molecular attenuated backscatter coefficient [$\text{sr}^{-1}\text{m}^{-1}$]
Ω	Solid angle [sr]
n_j	Density of scatterers [Mm^{-1}]
G_T, H_T, G_R, H_R, η	Constants of lidar depolarization system [#]
$\text{sgn}(z)$	Sigmoidal function [#]
C	Amplitude of sigmoidal function[a.u.]
n	Steepness of sigmoidal function[a.u.]
z_0	Inflection point of sigmoidal function[a.u.]
z'_0	Initial height of simulated atmospheric boundary layer [m]
C_z	Growth of simulated atmospheric boundary layer [#]
n_z	Slope of the growth of the simulated atmospheric boundary layer [m^{-1}]
t_0	Onset time of the growth of the simulated atmospheric boundary layer [s]
A	Amplitude of the simulated entrainment zone [m]
T_i	Period of the simulated entrainment zone [s]
b	Intercept of a linear fit [a.u.]
R_f	Correlation coefficient of a linear fit [#]

Acronyms

LIDAR	L ight D etection A nd R anging
ABL	A tmospheric B oundary L ayer
ABLH	A tmospheric B oundary L ayer H eight
TOA	T op O f A tmosphere
FT	F ree T roposphere
EZ	E ntrainment Z one
IPCC	I ntergovernmental P anel on C limate C hange
BC	B lack C arbon
ECMW	E uropean C entre for M edium-range W eather forecasts
UGR	U niversidad de G ranada
GFAT	G ruppo de F ísica de la A tmosfera
IISTA	I nstituto I nteruniversitario de investigación del S istema T ierra en A ndalucía
CEAMA	C entro A ndaluz de M edio A mbiente
AGORA	A ndalusian G lobal O bservato R y of the A tmosphere
ACTRIS	A erosol, C louds and T race gases R esearch I nfra S tructure
AERONET	A erosol R obotic N ETwork
EARLINET	E uropean A erosol R esearch L idar N ETwork

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1 Introduction

Earth's atmosphere is a layer of gases that surrounds our planet. Within this mixture of gases, microscopic solid and liquid particles—known as atmospheric aerosols—are suspended. These aerosols can remain airborne for extended periods of time [Guerrero-Rascado, 2008].

These particles have gained increasing relevance as their impact on climate and air quality has become better understood [Horvath, 1998]. Atmospheric aerosols alter the Earth's energy balance by interacting with incoming radiation through scattering and absorption, and they play a key role in the hydrological cycle by acting as cloud condensation nuclei [Rejano et al., 2024], thereby influencing both precipitation patterns and the formation of ice crystals [Welti et al., 2009]. Numerous studies have been conducted to understand the mechanisms behind this balance, given its importance in the current context of climate change and global warming.

In the field of atmospheric physics, one of the most effective techniques for studying and characterizing aerosols is the LIDAR (Light Detection and Ranging) technique. This method involves emitting laser pulses into the atmosphere and analyzing the radiation backscattered by suspended particles. It allows for the retrieval of vertical profiles of various optical and physical aerosol properties with high spatial and temporal resolution (e.g., [Bravo-Aranda et al., 2015], [Granados-Muñoz et al., 2016], [Guerrero-Rascado et al., 2009]), providing essential data for their analysis and understanding.

The Group of Physics of the Atmosphere (GFAT) at the University of Granada operates an advanced lidar system called ALHAMBRA, located at the Andalusian Institute for Earth System Research (IISTA-CEAMA). Thanks to this system, GFAT is part of ACTRIS (Aerosol, Clouds and Trace Gases Research Infrastructure), a pan-European research infrastructure that provides high-quality data and information on atmospheric constituents. Additionally, since 2004, GFAT has been an active member of EARLINET (European Aerosol Research Lidar Network), a European network that coordinates atmospheric observations using lidar systems and offers an open-access database on aerosol distribution across Europe.

The ALHAMBRA lidar system is capable of generating measurement profiles using different wavelengths, polarization planes, and other parameters, providing a wealth of information about atmospheric aerosols. However, retrieving their optical and physical properties requires the use of inversion algorithms, such as the Klett algorithm (e.g., [Klett, 1981; Fernald, 1984]) or the Raman algorithm [Ansmann et al., 1992]. These algorithms have been implemented in an open-access Python module called GFATPY, designed to facilitate synergistic processing of the GFAT's multi-instrumental database. Since these algorithms require the assumption of specific parameters and involve complex implementation, the version implemented within GFATPY was successfully validated in [Villalba-Torres, 2024], including a sensitivity analysis of the assumed parameters [Villalba-Torres et al., 2024]. These algorithms are considerably sensitive to measurement noise, so the accuracy of the retrieved optical and physical properties is improved by applying spatiotemporal averaging, typically, 30-minute averages in time and from 15 to 250 meters in height, depending on the optical property and the algorithm.

Despite this spatiotemporal averaging has allowed advances in the atmospheric-aerosol research field, high resolution of optical and intensive atmospheric-aerosol properties (such as the backscatter-related Ångström exponent and the linear particle depolarization ratio) would help to deepen on:

1. **Atmospheric aerosol mixing processes:** a better spatiotemporal resolution allows for a more detailed analysis of aerosol transport, dispersion, and behavior, specially within the Atmospheric Boundary Layer (ABL). This contributes to a better understanding of its impact on air quality and regional climate, while also enhancing the modeling of atmospheric phenomena, such as cloud formation and pollutant diffusion. Consequently, it may provide valuable input for weather forecasting and environmental management.

2. **Atmospheric aerosol classification:** The ability to distinguish and classify different aerosol types (e.g., natural and anthropogenic sources) depends on their optical and intensive properties. A better resolution allows us to disentangle their origin, composition, and mixing level.

To do so, alternative inversion algorithms have been explored, such as the iterative method proposed by Di Girolamo et al. [1995], and the quasi particle backscatter coefficient approach described by Baars et al. [2017]. Despite these algorithms have already been tested in studies using spaceborne lidars (e.g., CALIPSO, [Omar et al., 2009]; EarthCARE, [Illingworth et al., 2015]; [Groß et al., 2015]), airborne High Spectral Resolution Lidar (HSRL) measurements ([Burton et al., 2012]; [Groß et al., 2013]), and small, sophisticated, and automatic ground-based lidars (e.g., Polly^{XT}_{IFT}, [Baars et al., 2017]), they have not yet been thoroughly explored for high-resolution ground-based lidar systems such as ALHAMBRA. **Alternative inversion algorithms have in common that require determining the lidar calibration factor.** Although there is no standardized method for its estimation, some works provide algorithms based on Klett inversion and particular atmospheric conditions (e.g., [Baars et al., 2017]). This Master's thesis also assesses how to estimate this factor, with the aim of achieving a robust lidar calibration.

1.1 Objectives

The main objective of this Master's thesis is to improve optical properties of atmospheric aerosols obtained from the ALHAMBRA lidar system by increasing their high spatio-temporal resolution and determining the ALHAMBRA's overlap function. This will enable a more accurate and detailed characterization of aerosol layers in the atmosphere. To achieve this goal, the following specific objectives (SO) are proposed:

- **SO1:** Literature review of alternative inversion algorithms. Analyzing the high-resolution products they generate, aimed at identifying the advantages and limitations of each algorithm.
- **SO2:** Design and implement an algorithm to generate synthetic two-dimensional lidar signals (height and time).
- **SO3:** Develop, implement, and evaluate algorithms to determine the lidar system calibration factor, using synthetic and real lidar signals.
- **SO4:** Apply high-resolution algorithms to retrieve extensive and intensive aerosol properties in a representative case study, such as a Saharan dust intrusion event.
- **SO5:** Design, implement, and validate a new method to determine the overlap function.

2 Theoretical basis

2.1 The Atmosphere

The Earth's atmosphere is a gaseous layer surrounding the planet, predominantly composed of N_2 (78%) and O_2 (21%) [Barry and Chorley, 1987]. The remaining 1% consists of trace gases, primarily argon, carbon dioxide, and water vapor, which play a crucial role in regulating the Earth's radiative balance. Other components present in lower proportions are the atmospheric aerosol particles, yet their impact on the radiative balance is also fundamental.

When analyzing atmospheric components, a distinction is typically made between particles and molecules due to their significantly different properties and characteristics. Molecular constituents refer to the gases that constitute the atmosphere. The concentration of molecules tends to be relatively uniform throughout the lower atmosphere, with notable exceptions in certain regions, such as the tropics or coastal areas, where water vapor levels are significantly higher [Randel et al., 1996]. However, horizontal variations are not as significant compared to vertical changes, where the concentrations of different molecules can vary rapidly.

Particle atmospheric components include atmospheric aerosols and cloud particles (suspended liquid water droplets and/or ice crystals). The atmospheric aerosols consist of small solid and/or liquid particles that remain suspended in the atmosphere for at least several minutes [Horvath, 1998]. Their characteristics are defined by their shape, size, and chemical composition. The vertical and horizontal distribution of aerosols strongly depends on geographic location. For example, marine aerosols are predominant over oceans, while mineral dust prevails over deserts.

Atmospheric aerosol particles influence the Earth-atmosphere system in various ways, either directly or indirectly, by exerting a radiative forcing—contributing positively (warming) or negatively (cooling) to the balance between incoming and outgoing radiation. Direct effects refer to the interaction of aerosols with radiation through absorption and scattering of shortwave (solar) and longwave (thermal) radiation, as well as longwave emission [Liou and T., 1975] [Liou, 1980]. Indirect effects involve processes by which aerosol particles alter the optical and microphysical properties of clouds [Twomey, 1977].

The influence of aerosol particles and other atmospheric components on climate is periodically assessed and analyzed by the Intergovernmental Panel on Climate Change (IPCC) through its Assessment Reports (AR). These reports compile the most relevant scientific information on climate change and are published on a regular basis. Figure 1 summarizes the contributions to radiative forcing from various atmospheric components [Myhre et al., 2014]. The results indicate that trace gases contribute to positive radiative forcing, leading to a warming effect on the planet. Among them, the radiative forcing associated with carbon dioxide (CO_2) is particularly significant, representing the largest contribution to anthropogenic warming.

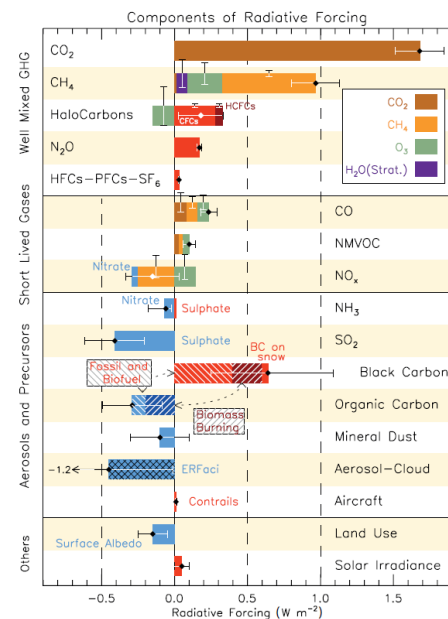


Figure 1: Radiative Forcing and Uncertainties of Different Atmospheric Components [Myhre et al., 2014].

Figure 1 shows that some aerosol particles, such as black carbon (BC), exert a positive forcing due to their strong absorption of solar radiation. Others particles, like mineral dust, have a negative forcing as they more effectively scatter radiation. The radiative forcing estimate for aerosol-cloud interactions is also presented, though it remains highly uncertain, making it a key research priority today. This highlights the importance of studying the properties of atmospheric aerosols to obtain the accurate information to identify and explain their effects on the radiative balance and climate.

2.2 Radiation-matter interaction

Radiation passing through the atmosphere interacts with various scattering and absorbing elements such as aerosol particles, molecules, raindrops, ice crystals, and other hydrometeors. The radiation-matter interaction consists of **absorption and scattering**, with their sum referred to as extinction. The ability to absorb and scatter radiation is quantified by the absorption coefficient (σ_{ab}) and the scattering coefficient (σ_{sca}), both having units of inverse length, typically Mm^{-1} [Guerrero-Rascado, 2008]. These coefficients represent the fraction of radiation that is absorbed or scattered by the particles.

Scattering is the process in which radiation is deflected into different directions in space, other than the incident direction, due to interaction with a particle or molecule. Scattering can be **elastic or inelastic (Raman)**. In elastic scattering, the incident light interacts with the scatterers without changing its energy, meaning that both the incident and scattered radiation have the same wavelength. In contrast, Raman scattering occurs when photons interact with molecules (primarily N_2 and O_2), causing the molecules to undergo a change in their quantum state (rotational or vibrational), resulting in a scattered photon with a different frequency than the incident one. **Molecules can scatter radiation both elastically and via Raman scattering, whereas aerosol particles can only scatter elastically.**

A fundamental quantity for the lidar technique is the **backscatter coefficient** (β). This coefficient measures the scattering in the backward direction, that is, the scattering that occurs at an angle (Ω) of 180° relative to the incident radiation. The backscatter coefficient is defined as the sum of the product between the number density of scatterers (n_j) and their effective cross-sections at 180° for the wavelength (λ), and its typical units are $[Mm^{-1}sr^{-1}]$ [Bravo-Aranda, 2014]:

$$\beta(z, \lambda) = \sum_j n_j \left. \frac{d\sigma_{j,sca}(z, \Omega, \lambda)}{d\Omega} \right|_{\Omega=\pi} \quad (1)$$

Both particles and molecules can absorb radiation. **Absorption** happens when an incident photon is taken up by a hydrometeor, adding to its internal energy and changing into other forms of energy. This process is selective, occurring only at specific wavelengths where absorption lines exist.

Extinction is the sum of the absorption and scattering processes and is quantified by the extinction coefficient (α) $[Mm^{-1}]$ [Guerrero-Rascado, 2008]:

$$\alpha(z, \lambda) = \sigma_{ab}(z, \lambda) + \sigma_{sca}(z, \lambda) \quad (2)$$

The total backscatter coefficient (β) can be decomposed into the sum of the particle backscatter coefficient (β_p) and the molecular backscatter coefficient (β_m) (Eq.3). The same applies to the total extinction coefficient (α) (Eq.4) [Weitkamp, 2005a].

$$\beta(z, \lambda) = \beta_p(z, \lambda) + \beta_m(z, \lambda) \quad (3)$$

$$\alpha(z, \lambda) = \alpha_p(z, \lambda) + \alpha_m(z, \lambda) \quad (4)$$

The values of α_m and β_m were theoretically derived by Lord Rayleigh in 1871 through his scattering laws. Rayleigh scattering assumes spherical scatterers and interactions where the wavelength of the

incident radiation is much larger than the size of the scatterer [Rayleigh, 1871]. Molecular scattering satisfies these conditions, allowing α_m and β_m to be calculated using Rayleigh theory, given the number of molecules in a given volume. Conversely, atmospheric aerosol particles, with sizes ranging from 0.1 nm to 10 μm , fall within the Mie scattering regime [Mie, 1908].

Atmospheric constituents modify the polarization state of light through scattering interactions. Accordingly, the backscatter coefficient β can be decomposed with respect to the polarization plane of the incident radiation. This allows for the definition of $\beta^{\parallel}$, which quantifies the component of backscattered radiation polarized parallel to the incident beam, and $\beta^{\perp}$, which quantifies the component polarized perpendicular to it. The sum of the polarized components yields the total backscatter coefficient [Bravo-Aranda, 2014]:

$$\beta(z, \lambda) = \beta^{\parallel}(z, \lambda) + \beta^{\perp}(z, \lambda) \quad (5)$$

The total extinction of solar radiation in the atmosphere for a perpendicular direction to the surface is evaluated using the **aerosol optical depth** (AOD). It is obtained by integrating α over the entire atmospheric column, from the surface ($z = 0$) to the aerosol-free reference height (z_m) [Bravo-Aranda, 2014]:

$$AOD(z_m, \lambda) = \int_0^{z_m} \alpha_p(z, \lambda) dz \quad (6)$$

However, if we are not interested in the extinction of the entire atmospheric column but rather in the accumulated extinction within a specific atmospheric thickness, it is useful to define the **accumulated extinction coefficient** (C_α) as the integral between the surface and the height z_c that we wish to consider:

$$C_\alpha(z_c, \lambda) = \int_0^{z_c} \alpha_p(z, \lambda) dz \quad (7)$$

Atmospheric transmittance is another quantity closely related to extinction. The Beer-Bouguer-Lambert law establishes a logarithmic relationship between the transmitted light in a medium and the distance it travels through that same material [Kneizys, 1980]:

$$T(z, \lambda) = \exp \left(- \int_0^{TOA} \alpha(z, \lambda) dz \right) \quad (8)$$

Equation 8 represents the total transmittance of the atmospheric column between the surface and the Top Of the Atmosphere (TOA). However, the integrated transmittance can also be calculated over a region of interest, such as from the surface up to a given height z_c .

Transmittance can be decomposed into its molecular (T_m) and particle (T_p) components as $T = T_p T_m$. The equation for T_p and T_m are:

$$T_p(z, \lambda) = \exp \left(- \int_0^{z_m} \alpha_p(z, \lambda) dz \right) = e^{-AOD(z_m, \lambda)} \quad (9)$$

$$T_m(z, \lambda) = \exp \left(- \int_0^{TOA} \alpha_m(z, \lambda) dz \right) \quad (10)$$

2.3 Intensive properties of atmospheric aerosol

The intensive properties of atmospheric aerosol are independent of its concentration, as they are linked to the intrinsic characteristics of the aerosol, such as its refractive index. Therefore, measuring these intensive properties enables the classification of atmospheric aerosols. In this subsection, two intensive properties relevant to this study are described:

- **Lidar Ratio (LR):** The lidar ratio is defined as the **ratio between the extinction and backscatter coefficients** ($LR = \alpha/\beta$ [sr]). It is highly sensitive to both the size of the scatterers and the refractive index [Ackermann, 1998]. The molecular lidar ratio can be approximately determined using Rayleigh scattering, since the values of α_m and β_m are known, leading to a constant value of $LR_m = 8\pi/3$ sr [Reddy and Kumar, 2013]. The values of LR_p range from 10 sr (for marine aerosols) to over 100 sr for smoke or urban aerosols. The following table summarizes the typical lidar ratio ranges for different types of atmospheric aerosols [Haarig et al., 2025]:

Type of aerosol	Lidar ratio (sr)		
	355 nm	532 nm	1064 nm
Marine aerosol	20-30	10-35	5-40
Ice clouds	26-36	31-41	33-43
Continental aerosol	45-70	40-65	15-60
Sulfate	57-77	30-70	15-45
Troposphere smoke	40-75	60-80	60-105
Stratosphere smoke	25-95	55-120	65-120
Mineral Dust	40-70	40-70	40-70

Table 1: Lidar ratio range for different aerosol particles at wavelengths of 355, 532, and 1064 nm. The intervals presented in the table are based on the literature review conducted in [Haarig et al., 2025]. The values for mineral dust are taken from [Gasteiger et al., 2011].

- **Ångström Exponent ($\hat{a}$):** The Ångström exponent is an intensive parameter that provides information about the predominant aerosol size in the studied atmosphere. A lower value of $\hat{a}$ indicates larger particle sizes. The Ångström exponent can be calculated from different optical coefficients measured at two wavelengths (λ_1 and λ_2) using the following equation [Ångström, 1929]:

$$\hat{a}(z, \lambda) = -\frac{\log(C_X(z, \lambda_1)/C_X(z, \lambda_2))}{\log(\lambda_1/\lambda_2)} \quad (11)$$

where C_X may be σ_{sp} , σ_{ab} , α , β or AOD.

- **Backscatter ratio (R):** Represents the ratio between the total backscatter coefficient and the molecular backscatter coefficient. This quantity allows for the quantification of the additional scattering caused by the presence of aerosol particles in the atmosphere [Weitkamp, 2005b]:

$$R(z, \lambda) = \frac{\beta(z, \lambda)}{\beta_m(z, \lambda)} = \frac{\beta_m(z, \lambda) + \beta_p(z, \lambda)}{\beta_m(z, \lambda)} \quad (12)$$

- **Volume linear depolarization ratio (δ_V):** This intensive property quantifies the sphericity of atmospheric constituents, including both aerosol particles and molecules. Lower depolarization values correspond to more spherical shapes, whereas higher values indicate greater morphological irregularity. It is computed as the ratio of the polarized components of the total backscatter coefficient [Bravo-Aranda, 2014]:

$$\delta_V(z, \lambda) = \frac{\beta^\perp(z, \lambda)}{\beta^\parallel(z, \lambda)} \quad (13)$$

- **Particle linear depolarization ratio (δ_p):** It is similar to δ_V , but exclusively considers the particle component of the backscatter coefficient, excluding molecular contributions [Bravo-Aranda, 2014]:

$$\delta_p(z, \lambda) = \frac{\beta_p^\perp(z, \lambda)}{\beta_p^\parallel(z, \lambda)} \quad (14)$$

2.4 Lidar technique

The lidar technique (**L**ight **D**etection **A**nd **R**anging) is an active remote sensing method that enables the retrieval of optical properties of atmospheric components with high spatial and temporal resolution. The measurements are performed under real atmospheric conditions and can cover a range of altitudes from the surface to over 100 km.

The lidar technique is based on a series of steps analogous to radar operation, but employing different wavelengths. Typically, lidar systems use wavelengths ranging from ultraviolet to infrared, most commonly **355, 532, and 1064 nm**. These wavelengths are chosen due to the ease of generating monochromatic laser light and the minimal molecular absorption of key atmospheric components, such as H_2O and O_3 , within this spectral range (atmospheric window) [Weitkamp, 2005c].

First, a monochromatic, coherent, and nearly collimated laser beam is emitted into the atmosphere. The backscattered light is then collected by the instrument's telescope and separated into its respective wavelengths. Subsequently, the optical signal is converted into an electrical signal using optical detectors and amplified through photomultiplier tubes (PMTs). Finally, the processed signal is transmitted to a high-frequency sampling system for storage. The following figure illustrates the main components of the lidar system:

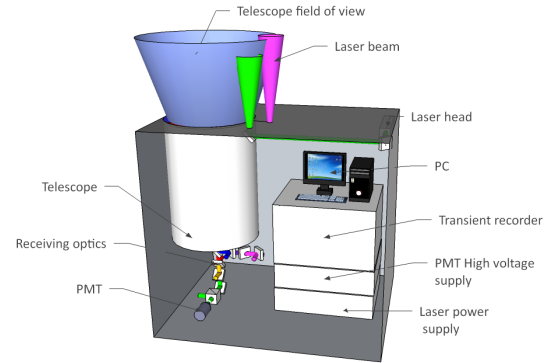


Figure 2: Schematic representation of the main parts of a lidar system [Bravo-Aranda, 2014].

The equation defining the signal P detected by the lidar system is given by [Weitkamp, 2005c]:

$$P(z, \lambda_L) = K(\lambda_L) \frac{O(z)}{z^2} \beta(z, \lambda_L) T^2(z, \lambda_L) \quad (15)$$

Where z represents the **range**, λ_L is the **laser wavelength** (elastic scattering), K is a **calibration factor**, $O(z)$ denotes the **overlap function**, $\beta(z, \lambda_L)$ is the **total backscatter coefficient**, and $T(z, \lambda_L)$ represents the **transmittance**. In the following, each variable is explained in detail.

The **lidar calibration factor** K gathers the emitted power P_0 , the speed of light c , the pulse duration τ , the illuminated area A_T , and the photomultiplier gain g , as described by the following equation [Weitkamp, 2005c]:

$$K(\lambda_L) = P_0(\lambda_L) \frac{c \tau A_T}{2} g \quad (16)$$

The function $O(z)$ describes the incomplete overlap between the emitted laser beam and the telescope's field of view at close range. The overlap function accounts for this effect by varying from 0, indicating no overlap, to 1, when the laser beam is fully within the telescope's field of view. Its effect influences the lidar signal generally in the first two kilometers [Wandinger and Ansmann, 2002].

Many lidar systems detect backscattered radiation at shifted wavelengths, when Raman scattering occurs. In such cases, the detected signal (P_R) depends on both the wavelength transmitted into the atmosphere (λ_L) and the wavelength of the inelastically scattered radiation (λ_R) [Weitkamp, 2005d]:

$$P_R(z, \lambda_L, \lambda_R) = K_R(\lambda_L) \frac{O(z)}{z^2} \beta_m^R(z, \lambda_R) T_m(z, \lambda_L) T_m(z, \lambda_R) \quad (17)$$

where R denote the Raman scattering.

Range Corrected Signal (RCS) is defined as the product of the received signal and the square of the distance to the scattering volume [Weitkamp, 2005a]:

$$RCS(z, \lambda_L) \equiv P(z, \lambda_L) z^2 = K(\lambda_L) O(z) \beta(z, \lambda_L) T^2(z, \lambda_L) \quad (18)$$

$$RCS_R(z, \lambda_L, \lambda_R) \equiv P_R(z, \lambda_L, \lambda_R) z^2 = K_R(\lambda_L) O(z) \beta_m^R(z, \lambda_R) T_m(z, \lambda_L) T_m(z, \lambda_R) \quad (19)$$

The **range corrected signal** is proportional to the **attenuated backscatter coefficient** $\beta_{att}(z, \lambda_L)$, which represents the product of the backscatter coefficient and the two-way transmittance (Eq. 20). This relationship enables RCS profiles to identify aerosol layers at a specific time and location in the atmosphere [Bravo-Aranda, 2014]:

$$RCS(z, \lambda_L) \propto \beta(z, \lambda_L) T^2(z, \lambda_L) \equiv \beta_{att}(z, \lambda_L) \quad (20)$$

It is worth noting that lidar signals are typically recorded using two complementary detection modes: **analog** and **photocounting**. In analog mode, the received optical signal is converted into a continuous voltage proportional to its intensity, making it suitable for strong returns. In contrast, photocounting mode registers individual photon events and is better suited for weak signals, such as those from high altitudes or Raman channels. The choice of detection mode directly influences how the signal is quantified and, consequently, how it is interpreted.

This distinction becomes particularly relevant when considering the units of the RCS. Although the theoretical lidar equation defines the received signal power $P(z, \lambda_L)$ as optical power, lidar systems do not measure this quantity directly. Instead, detectors convert the optical signal into electrical units: voltage [V] in analog mode or photon count rate [Hz] in photon-counting mode. These differences in measurement are accounted for in the system through the **photomultiplier gain** g , which is incorporated into the lidar calibration factor K (see Eq. 16).

Consequently, the units of both the RCS and K depend on the detection system: [V m²] for the RCS and [V sr m³] for K in analog mode, and [Hz m²] for the RCS and [Hz sr m³] for K in photon-counting mode. For this reason, both the RCS and K are commonly expressed in **arbitrary units** [a.u.], reflecting their dependence on the specific detection method employed.

3 Instrumentation

3.1 Lidar Raman Multi-spectral, Dual-LMRD (ALHAMBRA)

The ALHAMBRA dual multiwavelength Raman lidar system is operated at the urban station of the University of Granada (UGR), Spain, as part of the AGORA initiative (Andalusian Global Observatory of the Atmosphere). It is a member of the European Aerosol Research Lidar Network (EARLINET) and has recently been incorporated into the ACTRIS (Aerosol, Clouds and Trace Gases Research Infrastructure) network for aerosol remote sensing.

ALHAMBRA comprises two subsystems. The near-field subsystem is dedicated to monitoring the atmospheric boundary layer (ABL) and features elastic backscatter detection at 1064, 532, and 355 nm, as well as nitrogen vibrational Raman (VR) channels at 532 and 355 nm. The far-field subsystem extends measurements into the lower stratosphere, including elastic detection at the same three wavelengths and nitrogen rotational Raman (RR) detection channels at 1064, 532, and 355 nm. Furthermore, both subsystems are equipped with water vapor detection channels at 355 nm and the linear particle depolarization ratio (δ) can be retrieved at both 532 nm and 355 nm.

By incorporating the near-field subsystem, the total overlap height can be significantly reduced to approximately 200 meters above ground level, which enhances the system's ability to monitor the lowermost atmospheric layers with greater accuracy. This improved is particularly beneficial for studying aerosol dynamics within the atmospheric boundary layer (ABL), where processes such as emission, transport, and mixing are most active and variable.

Tables 2 and 3 list all the detection channels of the ALHAMBRA lidar system for the near-field and far-field subsystems, respectively. The channel names follow the notation $\lambda X_1 X_2 X_3$, where λ indicates the detection wavelength, X_1 specifies the subsystem: near-field (**n**) or far-field (**f**), X_2 denotes the polarization configuration: total (**t**), parallel (**p**), or perpendicular (**s**); and X_3 indicates the detection mode: analog (**a**) or photon-counting (**p**).

355				532				1064	
Total	Elastic Parallel	Perp	Raman	Total	Elastic Parallel	Perp	Raman	Elastic	Raman
355nta	355npa	355nsa	387nta	532nta	532npa	532nsa	607nta	1064nta	
355ntp	355npp	355nsp	387ntp	532ntp	532npp	532nsp	607ntp		X

Table 2: Channels of the near-field subsystem of ALHAMBRA. The Raman channels correspond to the vibrational channels of N_2 .

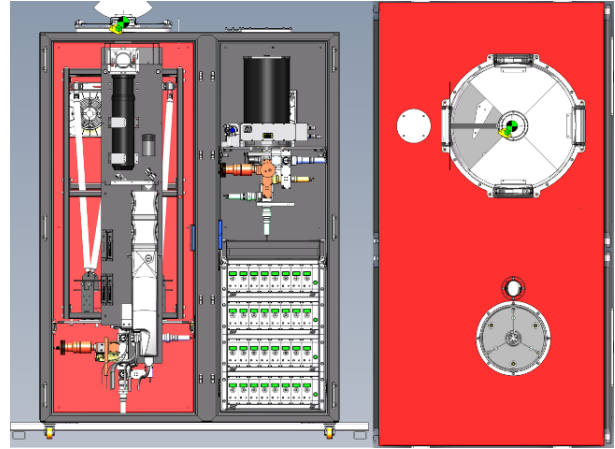


Figure 3: Scheme of the ALHAMBRA dual multiwavelength Raman lidar. The system front face features the far-field and the near-field subsystem. Also, the lidar system's top view show the co-axial far-field subsystem is situated at the top and the bi-axial near-field subsystem at the bottom [Fernández-Carvelo et al., 2024].

355				532				1064	
Total	Elastic Parallel	Perp	Raman	Total	Elastic Parallel	Perp	Raman	Elastic	Raman
355fta 355ftp	355fpa 355fpp	355fsa 355fsp	354fta 354ftp 408fta 408ftp	532fta 532ftp	X	X	531fta 531ftp	1064fta	1058fta

Table 3: Channels of the far-field subsystem of ALHAMBRA. The Raman channels correspond to the rotational channels of N_2 .

3.2 Sun and lunar photometer from AERONET

The AERONET (AErosol RObotic NETwork) program is a global network of ground-based sun photometers designed to provide long-term, quality-assured measurements of aerosol optical properties. Its core instrument, the Cimel CE-318, performs direct solar radiance measurements at multiple wavelengths, allowing for the retrieval of AOD, $\dot{a}$, and other key parameters. The network has been widely used to validate satellite data, constrain climate models, and support atmospheric monitoring efforts worldwide. Data products are generated using a well-established inversion algorithm that provides both spectral and microphysical aerosol properties under cloud-free conditions during daylight hours (e.g. [Holben et al., 1998], [Dubovik and King, 2000]).

To extend aerosol monitoring capabilities into nighttime periods, AERONET has incorporated lunar photometry by adapting the Cimel CE-318 to track and measure moonlight, operates between the first and last quarter, preferably near full moon. This advancement enables the retrieval of nocturnal AOD under favorable lunar phases and atmospheric conditions. Several studies have demonstrated the feasibility and reliability of these measurements, which significantly enhance the temporal coverage of aerosol observations and allow for the analysis of diurnal aerosol evolution [Barreto et al., 2016a] [Lopatin et al., 2021]. The inclusion of both sun and lunar photometry makes AERONET a proper reference for lidar system calibration and aerosol characterization.

4 Methodology

4.1 Processing lidar signals.

4.1.1 Preprocessing

Before applying any inversion algorithm to retrieve the optical properties of atmospheric aerosols, it is essential to perform a preprocessing step on the lidar data. This involves applying a series of treatments and filters to the received signal.

First, the background signal (**dark measurement**) is subtracted. This noise originates from both ambient sky light and the intrinsic electronic noise of the instrumentation. To remove it, a far-field region is selected where the useful signal is assumed to be negligible, and the mean value in this region is calculated and subtracted from the entire profile. For photocounting signals, the next step involves **dead-time correction**, which accounts for the time required by the detector to recover between successive photon events [Guerrero-Rascado, 2008].

After these preprocessing steps, the RCS is computed. However, the signal may still exhibit significant noise, particularly in regions with weak backscatter such as higher altitudes or Raman channels. Therefore, it is common practice to apply a **noise-reduction filter**, such as a moving average, Savitzky–Golay, or low-pass Gaussian filter. The choice of filter depends on the characteristics of the signal and the desired balance between noise suppression and signal preservation [Liu et al., 2020].

In this work, a **5-minute moving average filter** is employed. This technique smooths the data by replacing each data point with the average of its neighboring points within a specified temporal window—in this case, 5 minutes. The moving average reduces high-frequency fluctuations associated with random noise, thereby enhancing the signal-to-noise ratio while preserving the general structure and trends of the lidar profile.

An important aspect to consider is that lidar profiling does not start exactly at ground ($z = 0$) but between tens and hundreds of meters above the instrument, depending on the system, known as the full-overlap height (z_{over}). If the whole profile is needed, the aerosol layer in the range $(0, z_{\text{over}})$ is assumed to be homogeneous and constant, taking its value as the profile value at z_{over} , an approach known as the **refill method** [Wandinger and Ansmann, 2002]. Alternatively, the system's overlap function can be explicitly determined using one of the techniques described in Section 4.6.

4.1.2 Reference height retrieval

Another critical requirement for applying inversion and overlap algorithms is the selection of an aerosol-free reference height (z_m). This height is determined through a **Rayleigh fit**, a key procedure that leverages the known properties of molecular scattering in clean atmospheres.

The method compares the experimentally attenuated backscatter with the attenuated molecular backscatter, defined as:

$$\beta_{\text{att}}^{\text{mol}}(z, \lambda_L) = \beta_m(z, \lambda_L) T_m^2(z, \lambda_L) \quad (21)$$

The algorithm is based on the sequential application of physical and statistical criteria to the signal, with the objective of identifying a set of height candidates to be used as the reference height. In this work, a combination of selection criteria has been employed — some presented in the Appendix of [Baars et al., 2016] — but the primary and most fundamental criterion is the one introduced in [Klett, 1981].

1. Signal normalization: This is done by dividing it by the mean signal within the height range where candidates for z_m are being evaluated.
2. The slopes (derivatives) of the β_{att} and $\beta_{\text{att}}^{\text{mol}}$ profiles are compared. Both derivatives are obtained by means of a moving linear regression. A point is considered a candidate if the relative difference between both derivatives is below the established tolerance (in our case, 1%) [Klett, 1981].

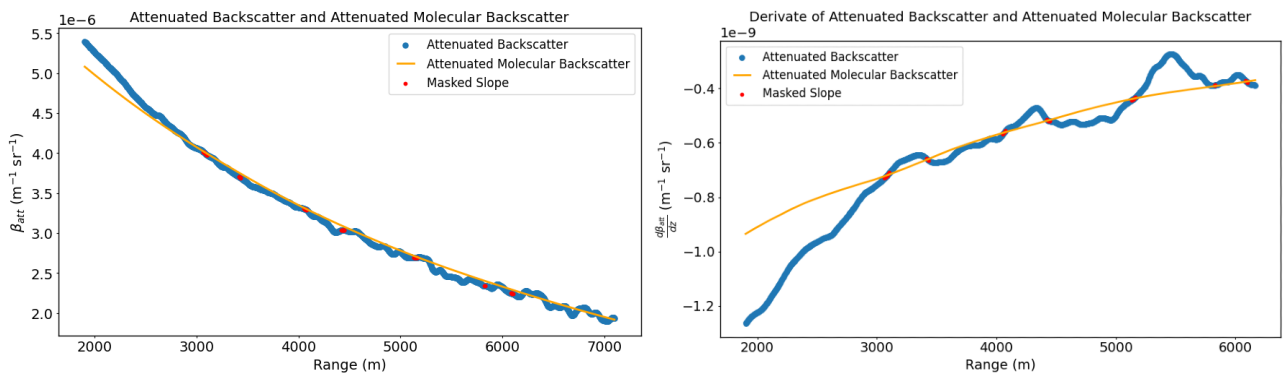


Figure 4: Representation of β_{att} (blue) and $\beta_{\text{att}}^{\text{mol}}$ (orange) [left plot], along with their derivatives [right plot], for a lidar signal detected in the 355 nm channel during the night of 2025/04/23. Candidate heights for z_m that satisfy the first criterion are marked with red dots.

3. β_{att} is computed in a window centred on each candidate identified in the previous step. A residual mask is applied to assess its proximity to a $\beta_{\text{att}}^{\text{mol}}$. Only those windows in which at least 80% of the points satisfy this criterion are accepted [Baars et al., 2016].
4. A linear regression between β_{att} and $\beta_{\text{att}}^{\text{mol}}$ is applied to the previous accepted windows, allowing for the estimation of the extinction coefficient. Both the extinction coefficient (slope) and its standard deviation are obtained. A candidate satisfies these criteria if the standard deviation of the extinction coefficient is less than or equal to its absolute value, thereby ensuring a robust estimate [Baars et al., 2016].
5. **Final Selection:** If multiple candidates satisfy all the previous criteria, a weighting function (w) is applied to each of them. This weighting is defined as [Baars et al., 2016]:

$$w = |\bar{x}| \cdot \sigma_x \cdot m$$

where $\bar{x}$ and σ_x are the mean and the standard deviation of β_{att} within the window and m is the slope of the linear fit (related to extinction).

4.2 Retrieval algorithms for particle backscatter coefficient

4.2.1 Klett-Fernald-Sasano algorithm

The algorithm presented by James D. Klett in 1981 allows the determination of the backscatter coefficient from an elastic lidar signal [Klett, 1981]. To apply the method, the following considerations must be taken into account:

- Only a single elastic backscattered signal is available.
- In each inversion of the lidar signal, calibration is performed at a reference height (z_m), so absolute system calibration (know K) is not required. This reduces the number of unknowns. The reference height is typically chosen in an aerosol-free region, where β_p is known (see Sec.4.1.2).
- It is necessary to assume either a column-averaged lidar ratio or a lidar ratio profile in the case of multiple aerosol layers.

Many discussions and studies have been conducted on the Klett algorithm (e.g. [Fernald, 1984] [Hughes et al., 1985] [Bissonnette, 1986]). In fact, the original Klett algorithm was reformulated following the work of Fernald [Fernald, 1984] and Sasano [Sasano et al., 1985]. The Klett–Fernald–Sasano algorithm explicitly separates the contribution of aerosol particles and atmospheric molecules.

The Klett method is based on solving a Bernoulli-type differential equation, which is derived by differentiating the natural logarithm of the range-corrected signal $RCS(z, \lambda_R)$ with respect to height. This operation is particularly sensitive to noise in the signal. The resulting differential equation is given by [Klett, 1981]:

$$\frac{d\beta(z)}{dz} - 2 LR_p(z) \beta^2(z) = \frac{d}{dz} \ln[RCS(z)] \quad (22)$$

Its solution provides a means to address the inversion problem and is expressed as follows [Klett, 1981]:

$$\beta_p(z) = \frac{RCS(z) \cdot \exp \left\{ -2 \int_{z_m}^z [LR_p(z') - LR_m] \beta_m(z') dz' \right\}}{\frac{RCS(z_m)}{\beta_p(z_m) + \beta_m(z_m)} - 2 \int_{z_m}^z LR_p(z') RCS(z') \cdot \exp \left\{ -2 \int_{z_m}^{z'} [LR_p(z'') - LR_m] \beta_m(z'') dz'' \right\} dz'} \quad (23)$$

The most critical aspect of this method is the selection of the particle lidar ratio and the reference height. On the one hand, the bibliography provides a wide range of tabulated lidar ratio values corresponding to different aerosol types, as summarized in table 1. On the other hand, the reference height is determined using the algorithm presented in section 4.1.2. In this context, it is worth noting that the choice of the reference height, which serves as the boundary condition in the differential inversion equation, can be interpreted as a form of relative calibration. That is, each time a profile is inverted, the solution of the equation is determined with respect to the value at z_m , allowing the retrieval to be consistent without requiring an absolute calibration of the signal.

4.2.2 Raman algorithm

The Raman algorithm consists of two stages. First, α_p is retrieved, and from it, β_p is computed. The particle extinction coefficient can be derived from the Raman lidar signal. After manipulating the Raman lidar equation, the following expression is obtained [Ansmann et al., 1990]:

$$\alpha_p(z, \lambda_L) + \alpha_p(z, \lambda_R) = \frac{d}{dz} \ln \left[\frac{N_R(z)}{RCS(z, \lambda_R)} \right] - \alpha_m(z, \lambda_L) - \alpha_m(z, \lambda_R) \quad (24)$$

where $N_R(z)$ represents the number of molecules that inelastically scatter the radiation. By isolating the particle extinction coefficient at the elastic wavelength, $\alpha_p(z, \lambda_L)$ and by assuming that the wavelength dependence of the aerosol extinction is $\alpha_p(\lambda_R)/\alpha_p(\lambda_L) = \lambda_L/\lambda_R$, the following expression is obtained [Ansmann et al., 1990]:

$$\alpha_p(z, \lambda_L) = \frac{\frac{d}{dz} \ln \left[\frac{N_R(z)}{RCS(z, \lambda_R)} \right] - \alpha_m(z, \lambda_L) - \alpha_m(z, \lambda_R)}{1 + \left(\frac{\lambda_L}{\lambda_R} \right)^{\hat{a}}} \quad (25)$$

Once α_p has been determined, β_p can be calculated from the ratio between the elastic and raman signals [Ansmann et al., 1992]:

$$\beta_p(z, \lambda_L) = -\beta_m(z, \lambda_L) + \beta(z_m, \lambda_L) \frac{P(z, \lambda_L) P(z_m, \lambda_R) N_R(z)}{P(z_m, \lambda_L) P(z, \lambda_R) N_R(z_m)} \frac{e^{-\int_{z_m}^z \alpha(x, \lambda_R) dx}}{e^{-\int_{z_m}^z \alpha(x, \lambda_L) dx}} \quad (26)$$

The Raman algorithm offers two major advantages over the Klett method. First, it does not require assuming a lidar ratio to obtain the solution, which significantly reduces uncertainty. Second, the effects of overlap are canceled out in this method, allowing the retrieval of β_p even in regions where overlap is incomplete.

However, it also presents some limitations. The main drawback is that the term $\frac{d}{dz} \ln \left[\frac{N_R(z)}{RCS(z, \lambda_R)} \right]$ is highly sensitive to signal noise. Additionally, $\hat{a}$ must be known to apply the algorithm.

4.3 Lidar calibration methods

4.3.1 Direct calibration

The absolute calibration is the estimation of the lidar calibration factor K . If the particle (T_p) and molecular (T_m) contributions to the transmittance are separated and full-overlap height is assumed in equation 18, K may be determined as [Baars et al., 2017]:

$$K(\lambda_L) = \frac{RCS(z, \lambda_L)}{[\beta_p(z, \lambda_L) + \beta_m(z, \lambda_L)] T_m^2(z, \lambda_L) T_p^2(z, \lambda_L)} \quad (27)$$

On the one hand, molecular properties are determined from temperature and pressure profiles. On the other hand, particle backscatter coefficient profiles are retrieved using the Klett inversion algorithm with a 10-minute resolution, assuming a lidar ratio and refilling β_p as described in Section 4.1.1. To this end, the lidar signal was averaged over 10-minute intervals, and the algorithm was applied to each averaged profile.

Lidar calibration factor profiles obtained using Eq. 27 are expected to remain constant, as the calibration factor is independent of height. The calibration factor is generally determined at high altitudes, in a particle-free region [Wiegner et al., 2014] [Cazorla et al., 2017], since substantial simplification of the calibration procedure is achieved:

$$K(\lambda_L, z_m) = \frac{RCS(z_m, \lambda_L)}{\beta_m(z_m, \lambda_L) T_m^2(z_m, \lambda_L) e^{-2AOD(z, \lambda_L)}} \quad (28)$$

Alternatively, the calibration height is set at the surface level where $T(z, \lambda_L) \approx 1$ [Kenzo-Yoshihara-Gómez, 2024], although an accurate knowledge of the overlap function is required:

$$K(\lambda_L, z_m) = \frac{RCS(z_m, \lambda_L)}{O(z_m) [\beta_p(z_m, \lambda_L) + \beta_m(z_m, \lambda_L)]} \quad (29)$$

Another important aspect to consider is that absolute lidar calibration is best performed at night. During nighttime, the signal is less affected by signal noise due to the absence of solar background radiation, and the atmospheric conditions tend to be more stable. These factors contribute to more reliable and consistent retrievals of aerosol optical properties [Baars et al., 2017].

The following sections (Sec.4.3.2 and Sec.4.3.3) introduces two novel methods based on the linear fitting of different quantities from the lidar equation, aimed at estimating the calibration factor. Both methods have been fully implemented and validated as part of this work.

4.3.2 Attenuated backscatter fitting

This method is based on determining the calibration factor from the slope of a linear fit between the RCS and the attenuated backscatter coefficient. For the sake of clarity, the elastic lidar equation is rewritten below:

$$RCS(z, \lambda_L) = K(\lambda_L) O(z) [\beta_p(z, \lambda_L) + \beta_m(z, \lambda_L)] T^2(z, \lambda_L)$$

And in terms of the attenuated backscatter coefficient:

$$RCS(z, \lambda_L) = K(\lambda_L) O(z) \beta_{att}(z, \lambda_L) \quad (30)$$

Above the full-overlap height ($O(z) = 1$), we can perform a linear fit of the RCS versus β_{att} , and obtain the calibration factor from the slope of the fit:

$$RCS(z, \lambda_L) = K(\lambda_L) \beta_{att}(z, \lambda_L) \quad (31)$$

This fitting can be performed in two ways: at a fixed height, by taking different time points and fitting the corresponding values of the signal and β_{att} at that height; or at a fixed time, by fitting a vertical profile, that is, using RCS and β_{att} values at different heights. In this study, the second approach is adopted. Specifically, data points of RCS and β_{att} between z_{over} and z_m are selected, as this is the region where the signal is less noisy and the overlap is complete.

To apply this method, it is also necessary to retrieve β_p using an inversion algorithm, such as Klett or Raman algorithm. As previously mentioned, the Klett algorithm was applied at 10 minutes resolution. Calibration was performed over entire nights, such that the linear fit between RCS and β_{att} was conducted every 10 minutes throughout the night, resulting in a time series of the calibration factor K . As in the explicit method, the representative value of K is taken as the mean of this time series.

4.3.3 Aerosol optical depth fitting

This method is based on determining the calibration factor from the intercept of a linear fit between the RCS signal and the AOD . Let us reconsider the elastic lidar equation (using simplified notation by omitting explicit functional dependencies) and full-overlap region ($z > z_{over}$):

$$RCS = K (\beta_p + \beta_m) T_m^2 T_p^2$$

Calibrate at an arbitrary height below the reference height (i.e., $z_c \in (z_{over}, z_m)$), the particle transmittance can be expressed in terms of the accumulated extinction coefficient C_α (see Eq. 7):

$$RCS = K (\beta_p + \beta_m) T_m^2 e^{-2C_\alpha}$$

By isolating the terms of K and C_α , the equation becomes:

$$\frac{RCS}{(\beta_p + \beta_m) T_m^2} = K e^{-2C_\alpha}$$

By taking the natural logarithm on both sides of the lidar equation, we obtain an expression that is linear with respect to C_α . This formulation allows for a linear fit, where the slope is -2 and the intercept corresponds to $\ln(K)$, from which the calibration factor K can be determined:

$$\ln \left(\frac{RCS}{(\beta_p + \beta_m) T_m^2} \right) = -2C_\alpha + \ln(K) \quad (32)$$

If calibration height is in the molecular region ($z > z_m$), $\beta_p(z, \lambda_L) = 0$ and $C_\alpha = AOD$. Thus, derive an expression that allows us to retrieve K through a linear fit with the AOD (Eq.33). This AOD can be computed either from the lidar signal or obtained from an independent reference instrument, such as a photometer. By using the reference AOD (AOD_{photo}), we can determine the lidar calibration factor without applying an inversion algorithm.

$$\ln \left(\frac{RCS}{\beta_m T_m^2} \right) = -2AOD + \ln(K) \quad (33)$$

As with the previous method, this linear fit may be carried out either at a fixed height or at a fixed time. In this case, the first approach is applied: the fit is performed at a fixed height using data points within a vertical window around the reference height. Within this region, the signal remains relatively free of noise and the aerosol layer is already fully developed, so $AOD = C_\alpha$.

4.4 Retrieval algorithms for high-resolution particle backscatter coefficient

As mentioned in Section 4.2, two algorithms—Klett and Raman—can be used to retrieve the particle backscatter coefficient. In this section, we present two alternative inversion algorithms that also retrieve β_p , but with high spatiotemporal resolution. These alternative algorithms require an absolute calibration of the lidar system (see section 4.3.1), unlike Klett and Raman algorithms that rely solely on relative calibration based on a reference height.

4.4.1 Iterative particle backscatter coefficient algorithm

This algorithm was first developed by Girolamo et al. [Di Girolamo et al., 1995], with the aim of estimating $\beta_p(z)$ using an iterative technique based on the relationship between the signal measured by the lidar (RCS profile) and the signal expected in a purely molecular atmosphere. From this comparison, the aerosol contribution in the atmosphere is progressively considered.

Initially, it is assumed that $\alpha_p = 0$ and $\beta_p = \beta_m$. A vertical resolution is also defined based on the average difference between points in the height profile.

The algorithm is repeated a maximum number of iteration i (typically 10) or until acceptable convergence is reached. Each iteration consists of the following steps:

1. **Simulation of the theoretical molecular signal:** The lidar signal is calculated assuming the atmosphere consists only of molecules (no aerosols), plus the aerosol estimate from the previous iteration, which is taken into account to compute the particle transmittance:

$$RCS_{mol}^i = K \beta_m T_m^2 \exp \left(-2 \int_0^{z_m} \alpha_p^{i-1} dz \right)$$

2. **Calculation of the Backscattering Ratio:** The ratio between the measured RCS and the molecular RCS is computed, which allows estimating the excess signal attributable to particles:

$$R^i = \frac{RCS}{RCS_{mol}^i}$$

Using this value, the particle backscatter profile is updated ($\beta_p = [R - 1] \beta_m$).

3. **Convergence evaluation:** To check whether the estimation has stabilized, the integral of the current β_p profile is compared with that of the previous iteration within the aerosol-affected region, i.e., from the surface up to the reference height. This is assessed by calculating the relative difference (Eq. 34); if it falls below a predefined tolerance threshold, the iterative process is terminated.

$$diff = \frac{|\int_0^{z_m} \beta_p^i dz - \int_0^{z_m} \beta_p^{i-1} dz|}{\int_0^{z_m} \beta_p^{i-1} dz} \quad (34)$$

4. **Profile update:** If convergence has not been reached, β_p is updated and α_p is recalculated using the lidar ratio ($\alpha_p = \beta_p \cdot LR_p$). This profile is corrected in the incomplete-overlap region using the refill technique (see section 4.1.1).

The process ends when convergence is achieved or the maximum number of iterations is reached. The final result is an estimated β_p profile with improved spatiotemporal resolution.

4.4.2 Quasi particle backscatter coefficient algorithm

This algorithm developed by Holger Baars et al. in [Baars et al., 2017] aims to estimate a high resolution $\beta_p(z)$ (non-iteratively), using the lidar-measured signal and the known molecular properties of the atmosphere. This algorithm is based on the following steps:

1. **Computation of the attenuated backscatter coefficient** using the lidar calibration factor ($\beta_{att} = RCS/K$).
2. **Initial estimation of β_p** , assuming that attenuation is solely only due to molecular components, i.e., $T = T_m$:

$$^{quasi*}\beta_p(z, \lambda_L) = \frac{\beta_{att}(z, \lambda_L)}{T_m^2(z, \lambda_L)} - \beta_m(z, \lambda_L) \quad (35)$$

The lidar ratio (LR_p) is then used to calculate a first estimate of the extinction coefficient, $^{quasi*}\alpha_p(z, \lambda_L) = LR_p \cdot ^{quasi*}\beta_p(z, \lambda_L)$. Incomplete overlap region is corrected using the refill method.

3. **Computation of total transmittance** using $^{quasi*}\alpha_p(z, \lambda_L)$:

$$T(z, \lambda_L) = \exp \left(- \int \left[^{quasi*}\alpha_p(z, \lambda_L) + \alpha_m(z, \lambda_L) \right] dz \right) \quad (36)$$

4. **Refined estimation of the particle backscatter coefficient** $^{quasi}\beta_p(z, \lambda_L)$, using the total transmittance:

$$^{quasi}\beta_p(z, \lambda_L) = \frac{\beta_{att}(z, \lambda_L)}{T^2(z, \lambda_L)} - \beta_m(z, \lambda_L) \approx \beta_p(z, \lambda_L) \quad (37)$$

Both this algorithm and the iterative one presented in the previous section yield more reliable results at wavelengths of 532 nm and 1064 nm, as reported in studies such as [Baars et al., 2017]. This is mainly because, at 355 nm, molecular backscattering is significantly stronger than at 532 or 1064 nm. As a result, large uncertainties are introduced into the attenuation correction described in equations 35 and 37 when signals at 355 nm are considered, even if the lidar system parameters are accurately known.

On the other hand, both algorithms share common sources of uncertainty. The most significant is the assumption of a constant lidar ratio across the spatial and temporal domain under analysis. However, as indicated in Table 1, the lidar ratio can vary considerably between different aerosol layers. Another source of uncertainty arises from assuming that the calibration factor remains constant throughout the temporal domain. Nevertheless, as observed in the results of this study (see section 5.2.2), the calibration factors may exhibit slight temporal variations. Finally, the extinction profiles are corrected using the refill method, which relies on the assumption of a homogeneous atmosphere within the incomplete overlap region.

4.5 Retrieval for aerosol intensive properties

The particle backscatter coefficient enables the retrieval of aerosol intensive properties, as described in Section 2.3. Accurate determination of these properties with high spatial and temporal resolution allows for the characterization and classification of aerosol layers in the atmosphere, since parameters such as the depolarization ratio or the Ångström exponent are indicative of the particle shape and size. Consequently, they can be used to infer the aerosol origin (natural or anthropogenic) and assess its potential impact on climate and air quality.

There are two fundamental methods for determining the **lidar ratio** (LR_p):

- **Compare the AOD retrieved from the elastic lidar signal (AOD_{lidar}) with the AOD provided by an independent instrument, such as a photometer (AOD_{photo}).** The AOD values obtained from AERONET photometers are typically used as a reference. AERONET (*AErosol RObotic NETwork*) is a global network of ground-based remote sensing stations that provide high-quality and well-calibrated measurements [Holben et al., 1998].

$$AOD_{lidar} = \int_0^{z_m} (LR_p \cdot \beta_p^{klett}) dz \quad (38)$$

The method consists in tuning the lidar ratio when applying the Klett algorithm, so that AOD_{lidar} matches as closely as possible AOD_{photo} . In other words, the lidar ratio is iteratively adjusted until the best agreement between the two AOD estimates is achieved.

- **By combining elastic and raman signals,** it is possible to retrieve $\beta_p(\lambda_L)$ and $\alpha_p(\lambda_L)$ using the Raman algorithm. Once both coefficients are obtained, the lidar ratio can be calculated as their ratio [Ansmann et al., 1992].

The high-resolution **Ångström exponent** $\hat{a}$ can be computed from the high-resolution particle backscatter coefficient at two different wavelengths using equation 11. And the high-resolution **backscatter ratio** R also relies on high-resolution particle backscatter coefficient and is calculated using equation 12.

Freudenthaler [Freudenthaler, 2014] proposes an equation to obtain the **volume linear depolarization coefficient** δ_V from the ratio of the signals measured in the parallel and perpendicular polarization planes, together with a set of constants specific to the configuration of each lidar system (Eq. 39). These constants (G_T , G_R , H_T , H_R and η) are directly related to the instrumental characteristics of the lidar depolarization system. The spatiotemporal resolution of δ_V matches that of the measured signals, since its calculation essentially consists of a ratio between them.

$$\delta_V(z, \lambda_L) = \frac{\delta^*(z, \lambda_L)[G_T + H_T] - [G_R + H_R]}{[G_R - H_R] - \delta^*(z, \lambda_L)[G_T - H_T]} \quad (39)$$

The coefficient $\delta^*(z, \lambda_L)$ is calculated as follows, where $P^{\parallel}$ is the parallel signal and $P^{\perp}$ the perpendicular signal [Freudenthaler, 2014]:

$$\delta^*(z, \lambda_L) = \frac{1}{\eta} \frac{P^{\parallel}(z, \lambda_L)}{P^{\perp}(z, \lambda_L)} \quad (40)$$

The **particle linear depolarization ratio**, δ_p , at high resolution is determined using δ_V and β_p through the following equation [Baars et al., 2017]:

$$\delta_p(z, \lambda_L) = [\delta_V(z, \lambda_L) + 1] \left(\frac{\beta_m(z, \lambda_L) [\delta_m(z, \lambda_L) - \delta_V(z, \lambda_L)]}{\beta_p(z, \lambda_L) [\delta_m(z, \lambda_L) + 1]} + 1 \right)^{-1} - 1 \quad (41)$$

Equation 41 also makes use of the molecular linear depolarization ratio, which is defined as follows:

$$\delta_m(z, \lambda) = \frac{\beta_m^{\perp}(z, \lambda)}{\beta_m^{\parallel}(z, \lambda)} \quad (42)$$

4.6 Retrieval algorithms for the overlap function

4.6.1 Iterative overlap algorithm

The iterative algorithm [Wandinger and Ansmann, 2002] is based on the premise that the particle backscatter coefficient retrieved using the Raman algorithm (β_p^R) is unaffected by the overlap function, whereas the one obtained through the Klett algorithm (β_p^K) is. This method involves an iterative procedure to estimate the overlap function $O(z)$ from the difference between both backscatter coefficients:

$$\frac{\beta_p(z, \lambda_R) - \beta_p(z, \lambda_L)}{\beta_p(z, \lambda_R) + \beta_m(z, \lambda_L)} \propto \frac{RCS(z, \lambda_L)O(z)^{-1} - RCS(z, \lambda_L)}{RCS(z, \lambda_L)O(z)^{-1}} \propto 1 - O(z) \quad (43)$$

Based on this equivalence, an increment in the overlap function, $\Delta O(z)$, is computed at each iteration (i) using the following expression [Wandinger and Ansmann, 2002]:

$$\Delta O_i(z) = \frac{\beta_p(z, \lambda_R) - \beta_{p,i}(z, \lambda_L)}{\beta_p(z, \lambda_L) + \beta_m(z, \lambda_L)} \quad (44)$$

Using $\Delta O(z)$, the lidar signal is corrected to obtain a new estimate, which is then used as input for the Klett algorithm in the next iteration [Wandinger and Ansmann, 2002]:

$$P_{i+1}(z, \lambda_L) = P_i(z, \lambda_L) [1 + \Delta O_i(z)] \quad (45)$$

After about 9 iterations, β_p^K matches β_p^R , which allows us to estimate the overlap function $O(z)$. It's worth noting that this method requires assuming a lidar ratio profile, which adds some uncertainty.

4.6.2 Explicit overlap algorithm

The explicit algorithm provides an explicit formula to determine the overlap function. This expression requires the elastic and Raman signals, along with an assumed reference height z_m and a lidar ratio. The details leading to the explicit expression of the overlap function $O(z)$ are presented in the paper [Comerón et al., 2023]. The final result is given by:

$$\begin{aligned} O(z) = & \frac{\beta_m(z_m, \lambda_L) RCS(z, \lambda_R)}{\beta_m(z, \lambda_R) RCS(z_m, \lambda_R)} \cdot \exp \left\{ 2 \int_z^{z_m} [LR_p(x, \lambda_L) - LR_m(\lambda_L)] \beta_m(x, \lambda_L) dx \right\} \cdot \\ & \cdot \exp \left\{ \int_z^{z_m} [LR_m(\lambda_L) \beta_m(x, \lambda_L) - LR_m(\lambda_R) \beta_m(x, \lambda_R)] dx \right\} \cdot \exp \left\{ \int_z^{z_m} [\alpha_p(x, \lambda_L) - \alpha_p(x, \lambda_R)] dx \right\} \cdot \\ & \cdot \exp \left\{ 2 \frac{RCS(z_m, \lambda_R)}{RCS(z_m, \lambda_L)} \int_z^{z_m} \frac{LR_p(x, \lambda_L) \beta_m(x, \lambda_L) RCS(x, \lambda_L)}{RCS(x, \lambda_R)} e^{A(x)} dx \right\} \end{aligned} \quad (46)$$

where $A(x)$ is:

$$A(x) = - \int_x^{z_m} [LR_m(\lambda_L) \beta_m(x', \lambda_L) - LR_m(\lambda_R) \beta_m(x', \lambda_R)] dx' - \int_x^{z_m} [\alpha_p(x', \lambda_L) - \alpha_p(x', \lambda_R)] dx' \quad (47)$$

The explicit overlap algorithm does not perform well when the Raman and elastic wavelengths differ significantly. In such cases, the differential transmittance term $\int [\alpha_p(z, \lambda_L) - \alpha_p(z, \lambda_R)] dz$ becomes large, which may prevent the algorithm from converging. This issue is observed in the near-field subsystem channels of ALHAMBRA.

4.6.3 Homogeneous-backscatter overlap algorithm

This section presents an innovative method to determine the overlap function by means of an appropriate estimation of the calibration factor. This method is based on assuming a constant and homogeneous aerosol layer within the region of incomplete overlap, i.e., between the surface and the full-overlap height (z_{over}). Under this assumption, the refill method can be applied to β_p , filling in the coefficient values in that region with the value of $\beta_p(z_{\text{over}})$.

Denoting $\beta_p^{\text{re}}(z, \lambda_L)$ as refilled, Klett-derived particle backscatter coefficient, the overlap ($O_K(z)$) can be determined by (omitting functional dependencies):

$$O_K = \frac{RCS}{K [\beta_p^{\text{re}} + \beta_m] T_m^2 e^{-2 \int_0^{z_m} [LR_p \cdot \beta_p^{\text{re}}] dz}} \quad (48)$$

This formulation allows the estimation of the overlap function by means of a properly calibrated elastic signal and the refill-corrected particle backscatter coefficient within the incomplete overlap region.

The uncertainty associated with this method can be easily assessed using synthetic lidar signals. If we expand the RCS equation in the numerator, we observe that the calibration factor cancels out:

$$O_K = \frac{K O [\beta_p + \beta_m] T_m^2 e^{-2 \int_0^{z_m} [LR_p \cdot \beta_p] dz}}{K [\beta_p^{\text{re}} + \beta_m] T_m^2 e^{-2 \int_0^{z_m} [LR_p \cdot \beta_p^{\text{re}}] dz}}$$

Rewriting the expression in terms of the attenuated backscatter coefficient:

$$O_K(z) = O(z) \frac{\beta_{\text{att}}(z, \lambda_L)}{\beta_{\text{att}}^{\text{re}}(z, \lambda_L)} \quad (49)$$

If the atmosphere is homogeneous and stable in the first few meters above the ground, then $\beta_{\text{att}} \approx \beta_{\text{att}}^{\text{re}}$, and the homogeneous-backscatter overlap function —abbreviated as hom- β overlap function— coincides with the real overlap function.

4.7 Synthetic lidar signals

All algorithms and methods described in the previous methodology sections are complex to implement and rely on the assumption of various parameters, introducing sources of uncertainty. For this reason, synthetic lidar signals were generated. These signals offer a major advantage over real lidar measurements: the exact atmospheric properties are known a priori, allowing for a rigorous validation of the algorithms applied.

4.7.1 Generation of synthetic lidar signals with high spatiootemporal resolution

The generation of synthetic lidar signals requires the artificial simulation of the different variables involved in the lidar equation (Eq.15). Each of these quantities must be coherently modeled in order to reflect the physical behavior of the atmosphere and enable a realistic analysis of the simulated profiles [Villalba-Torres, 2024]. It follows how each term of the lidar equation is simulated:

- The **calibration factor** (K) is set to a fixed value.
- The **total backscatter** (β) and **extinction** (α) **coefficients** are calculated as the sum of their molecular and particle components (equations 3 and 4). On the one hand, the values of α_m and β_m

can be theoretically derived by assuming Rayleigh scattering, using atmospheric temperature and pressure profiles. These profiles can be obtained either from tabulated data of a standard atmosphere, such as the U.S. Standard Atmosphere (1976), from radiosonde measurements, or atmospheric models (such as, ECMWF). On the other hand, β_p and α_p have been simulated using a sigmoidal function (sgn), defined by the following equation:

$$sgn(z) = \frac{C}{1 + e^{-n(z-z_0)}} \quad (50)$$

where C is a parameter that scales the amplitude of the function, z_0 represents the inflection point—where the curve changes direction and the slope is maximal—and n controls the steepness of the curve. The greater the absolute value of n , the sharper the transition of the sigmoid function at z_0 is.

The parameter C is closely related to the concentration of particles within the simulated aerosol layer. The parameter z_0 corresponds to the top height of the aerosol layer. Finally, n determines how abrupt the transition between layers is.

- The **overlap function** ($O(z)$) is simulated using a sigmoidal function (Eq. 50). For the overlap function, the values of n and z_0 are variable, whereas $C = 1$ remain constant.
- The **transmittance** ($T(z, \lambda_L)$) is calculated by applying the Beer–Bouguer–Lambert law (Eq. 8) to the simulated α .

Figure 5 presents an example of a simulated signal that illustrates the different quantities involved in the elastic lidar return. The product of all the synthetic variables results in the lidar signals shown in subfigure 5d. Subfigure 5a shows the β_p and β_m profiles. The artificially generated sigmoidal shape of β_p is clearly visible, as well as the decrease of the molecular component with height. Subfigures 5b and 5c present the synthetic overlap function (with a sigmoidal shape) and the transmittance, respectively.

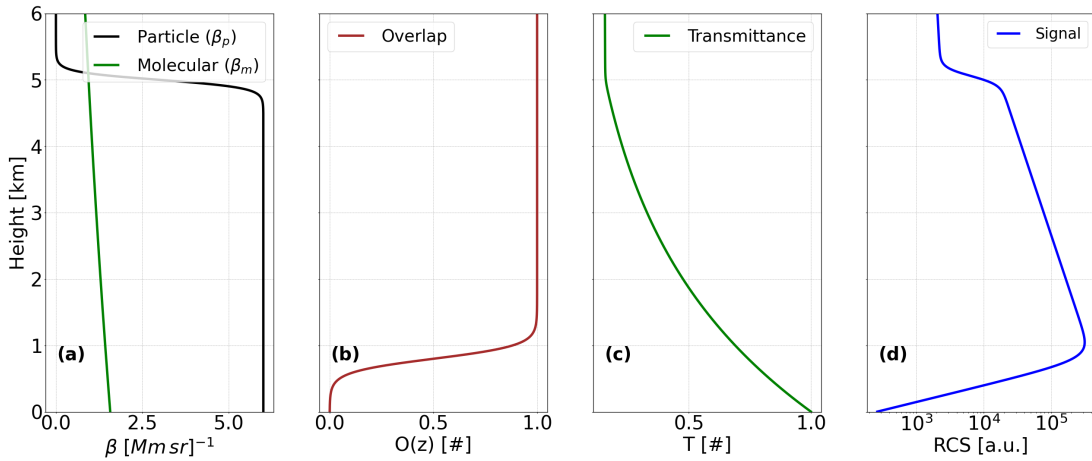


Figure 5: Synthetic lidar signals generated for $\lambda_L = 532$ nm, $LR_P = 60$ sr, and $K = 10^{11}$ a.u.. (a) Molecular and particle components of the total backscatter coefficient. The particle backscatter coefficient (β_p) was simulated using a sigmoidal function with parameters $C = 6 \cdot 10^{-6}$ ($Mm sr$)⁻¹, $n = 1/60$ m⁻¹ and $z_0 = 5000$ m. (b) Overlap function simulated using a sigmoid function with parameters $C = 1$, $n = 1/100$ m⁻¹ and $z_0 = 800$ m. (c) Atmospheric transmittance at the emitted wavelength. (d) Synthetic elastic range-corrected signal.

The next step involves generating time series of the lidar signal with vertical resolution, commonly known as quicklooks. To achieve this, it is necessary to simulate a temporal sequence of profiles with varying characteristics in order to reproduce the daily evolution of different aerosol layers in the atmosphere. An example of a full-day quicklook of a synthetic lidar signal is shown in figure 6.

The variation in aerosol concentration has been modeled by modifying the parameter C of the sigmoidal function as a function of time. In this way, any function $C(t)$ can be defined to describe how the aerosol particle concentration evolves throughout the day.

The variation in the vertical distribution of the aerosol layer(s) has been simulated by modifying the parameter z_0 of the sigmoidal function over time ($z_0(t)$). This approach allows for the representation of the progressive rise of the atmospheric boundary layer (ABL) after sunrise, a phenomenon associated with the solar heating of the Earth's surface. This heating induces thermal convection, which promotes vertical mixing of the lower atmosphere, causing the ABL to increase in height throughout the day. This behavior has been modeled using a sigmoidal function that describes its daily evolution (first term in the equation 51).

Meanwhile, the second term of the equation 51 represents the interaction between the free troposphere (FT) and the atmospheric boundary layer (ABL). The oscillations occurring at the top of the ABL, known as the entrainment zone (EZ), have been simulated using a cosine function.

$$z_0(ABLH) = z'_0 \cdot \left(\frac{C_z}{1 + e^{-n_z(t-t_0)}} + 1 \right) + A \cos \left(\frac{2\pi}{T_i} \right) \quad (51)$$

On the one hand, the parameter z'_0 represents the initial height of the aerosol layer, t_0 denotes the minute at which the layer begins to grow, C_z defines how much the layer grows, and n_z controls how abrupt the growth is. On the other hand, A and T_i represent the amplitude and period of the entrainment waves, respectively.

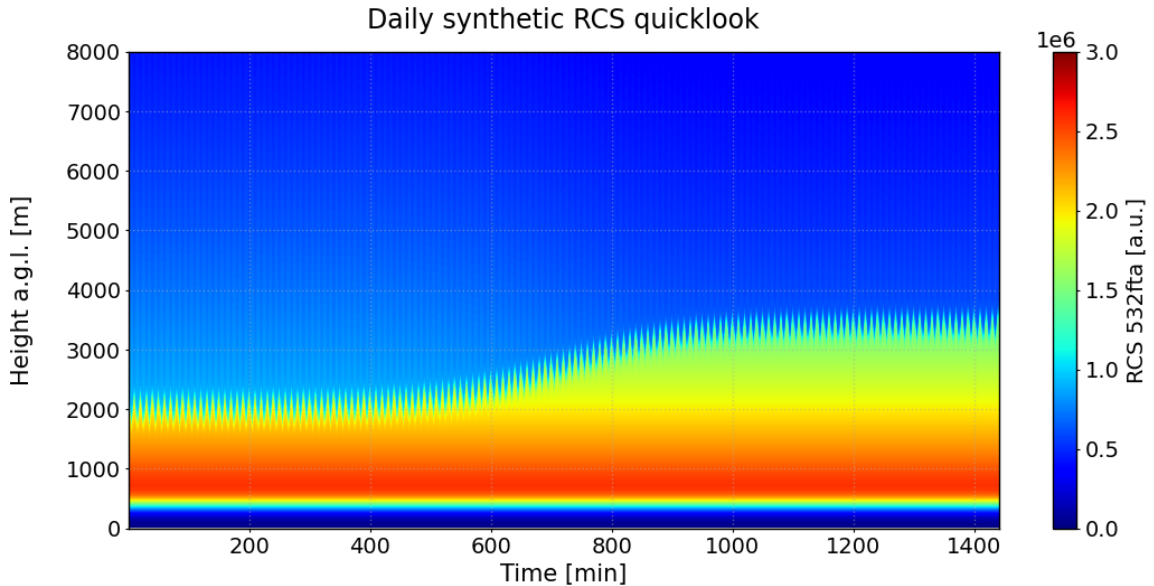


Figure 6: Daily quicklook of a synthetic range-corrected lidar signal generated for $\lambda_L = 532$ nm, $LR_P = 50$ sr, and $K = 10^{12}$ a.u. Particle backscatter coefficient (β_p) was simulated using $C = 1.5 \cdot 10^{-6} (\text{Mm sr})^{-1}$ and $n = 1/60 \text{ m}^{-1}$. Overlap function (O) was simulated with $z_0 = 400$ m and $n = 1/80 \text{ m}^{-1}$, remaining constant over time. Atmospheric Boundary Layer Height ($z_0(ABLH)$) was simulated with $z'_0 = 2000$ m, $C_z = 0.7$, $n_z = 1/100 \text{ m}^{-1}$, $t_0 = 720$ min (12:00 UTC), $A = 200$ m, and $T = 10$ min. Molecular properties were kept constant over time.

5 Results

5.1 Validation and assessment of the lidar calibration methods by means of synthetic lidar signals

5.1.1 2D lidar signal simulation

To validate the different algorithms to determine the lidar calibration factor described in section 4.3, a 12-hour synthetic signal at 532 nm has been generated with spatio-temporal resolution of 3.75 m and 60 s (figure 7). This simulation aims to represent an ALHAMBRA's measurement during stable conditions characterized by atmospheric stability and minimal boundary layer dynamics, since lidar system calibration is typically performed under these conditions.

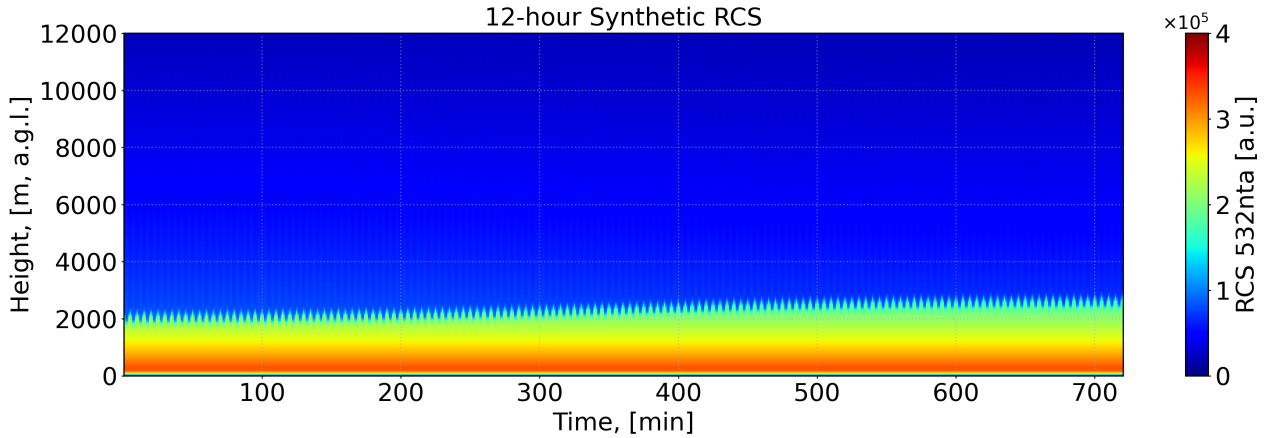


Figure 7: Quicklook of the synthetic range corrected signal generated for $\lambda_L = 532$ nm, $LR_p = 50$ sr, and $K = 10^{11}$ a.u.. Particle backscatter coefficient (β_p) was simulated with $C = 2 \cdot 10^{-6}$ ($Mm\ sr$) $^{-1}$ and $n = 1/60\ m^{-1}$. Overlap function (O) was simulated with $z_0 = 10$ m and $n = 1/50\ m^{-1}$, remaining constant over time. Atmospheric Boundary Layer Height ($z_0(ABLH)$) was simulated with a $z'_0 = 2000$ m, $C_z = 0.3$, $n_z = 1/100\ m^{-1}$, $t_0 = 350$ min, $A = 200$ m and $T = 5$ min (Eq.51). Molecular properties were kept constant over time.

It is important to note that overlap function is simulated with the full-overlap height at approximately 400 m. This choice aims to be as similar as possible to the one of the near-field subsystem of ALHAMBRA (i.e., 200 m), as the following sections will focus on near-field signals, taking into account limitations caused by the asymptotic trend to zero of the sigmoidal function in the lowermost region of the profile (Eq.50).

Since calibration algorithms require the particle backscatter coefficient (β_p), it is retrieved applying the Klett inversion algorithm to 10-minute averaged RCS profiles (Fig.8a). The resulting β_p profiles are subsequently refilled in the region $[0, z_{over}]$ m as described in section 4.1 (Fig.8b). The particle backscatter coefficient profiles retrieved using the Klett algorithm coincide with the synthetic β_p profiles [Villalba-Torres, 2024]. As it can be seen in Fig.8, the top height of the aerosol layer increases from approximately 2km to 3 km over the 12-hour simulation period, in agreement with the evolution displayed in the quicklook shown previously.

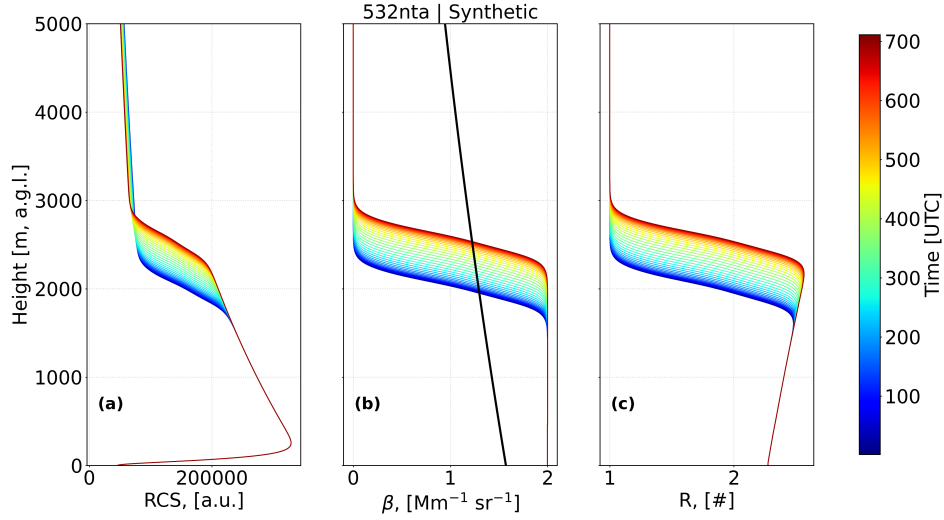


Figure 8: Synthetic lidar signals were generated with the characteristics described in figure 7: **(a)** Ten-minute averaged profiles of synthetic range-corrected signal. **(b)** Retrieved profiles of molecular (black line) and particle (colored lines) backscatter coefficient. **(c)** Profiles of backscatter ratio.

5.1.2 Validation of lidar calibration methods

The three lidar calibration methods are validated using the previous simulation case:

The direct calibration method (Sec. 4.3.1) is used to compute a time series of the calibration factor K with the 10-minute resolution (figure 9). Since this method retrieves vertically-resolved K , values at different heights (see legend) are displayed, although K remains constant at $(9.99877 \pm 0.00001) \cdot 10^{10}$ a.u. throughout the whole period, independently of the height. This is the expected behaviour since K is not range dependent. The retrieved value of K is very close to the synthetic one (10^{11} a.u.) (i.e., set during the generation of the simulated lidar signal) with a relative error of -0.012% . This result allows us to validate the implementation of the algorithm, where small differences are due to the refilling applied to β_p .

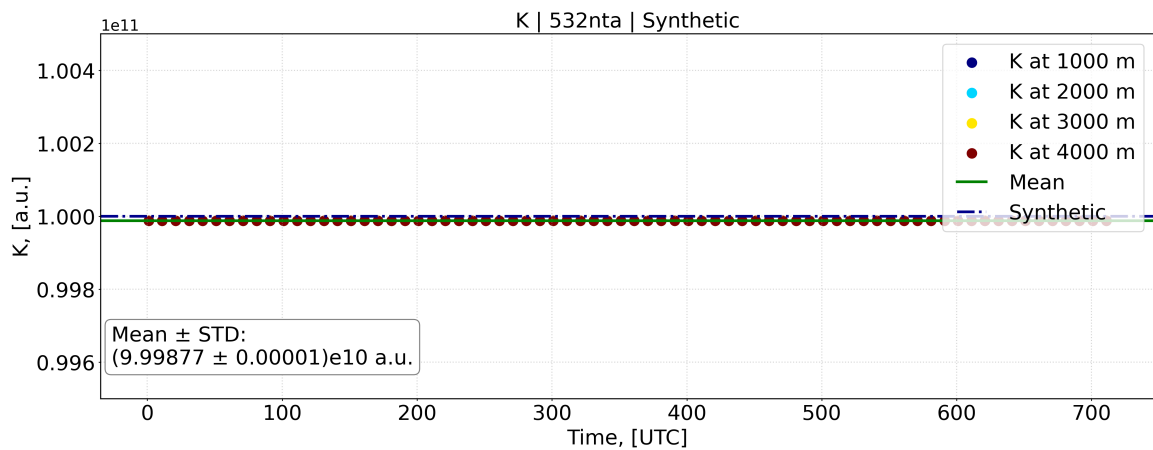


Figure 9: Time series of K , calculated by direct calibration method, are presented for calibrations performed at 1000, 2000, 3000, and 4000 meters. Each height is represented by colored markers (which overlap). The average of the time series is shown as a green line, while the synthetic reference value is indicated by a dashed blue line.

The attenuated backscatter fitting calibration method (Sec. 4.3.2) is performed by fitting 10-minutes average RCS and β_{att} at a chosen region (i.e., $z \in [z_{over}, z_m]$). The result is a time series of slopes (K) and intercepts (b), shown in figure 10. The error bars for K are very small and not appreciable at the scale of the figure. The value of K are identical to that retrieved using the direct method, with a relative error of -0.012% , confirming the validity of the β_{att} method.

According to equation 31, the intercept (b) is theoretically expected to be zero. Although this result is not exactly obtained, the b values are on the order of 10^{-3} , in contrast to the RCS values, which are on the order of 10^5 . Therefore, considering the orders of magnitude, the intercept behaves as expected. In terms of its temporal evolution, a slight increasing trend is observed, coinciding with the increasing AOD trend with time, that may be related with the computation of the transmittance which requires application of numerical methods to calculate integrals. Regarding the correlation coefficients, they are approximately equal to 1 for all the fits.

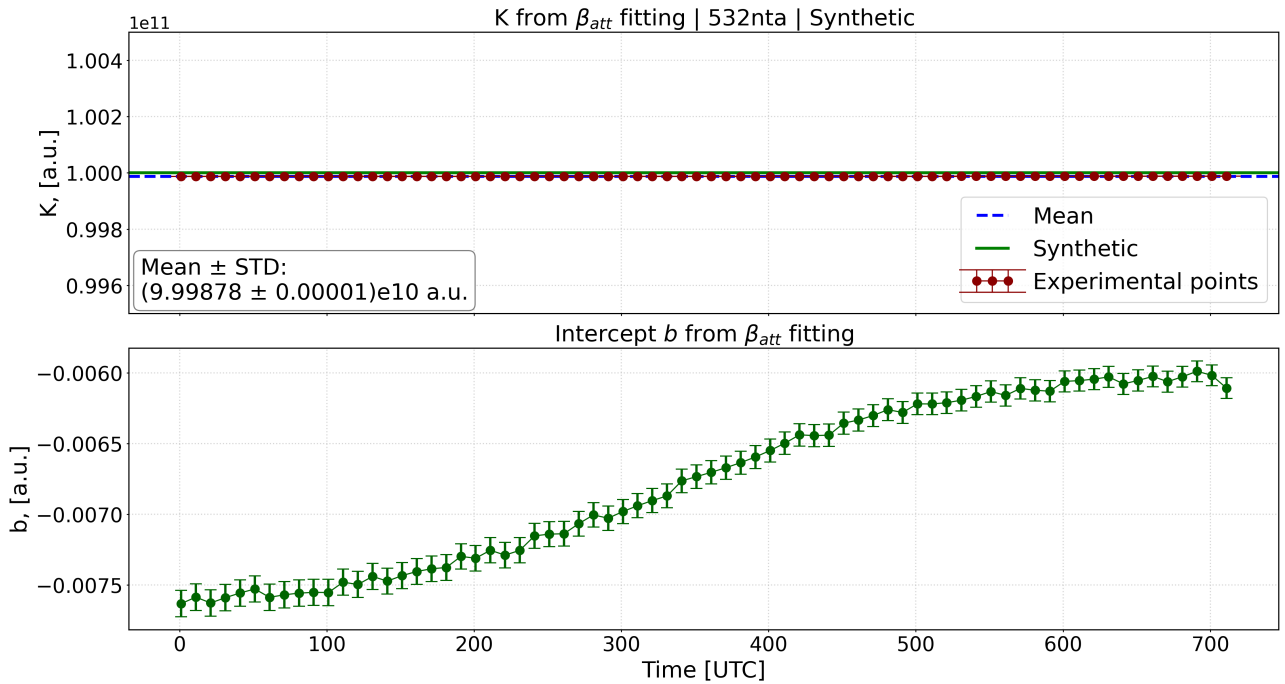


Figure 10: Time series of K and b represented with red and green error bars and calculated by the attenuated backscatter fitting calibration method. In the top panel, the average of the time series is shown as a green line, while the synthetic reference value is indicated by a dashed blue line.

The **aerosol optical depth fitting calibration method** (Sec. 4.3.3) is applied according to equation 33, using the time series of the AOD_{lidar} and the averaged RCS in the range $(z_m - 250m, z_m + 250m)$ as shown in figure 11. The correlation coefficient of $|R_f| = 1$ points to a perfect linear fit due to the use of synthetic data. From the intercept, a value of $K = (9.99869862 \pm 0.00000004) \cdot 10^{10}$ a.u. is obtained. A low relative error (-0.013% with respect to the synthetic reference value) and a slope of ~ 2 (agreeing with the expected linear fit equation) support the validity of the AOD method.

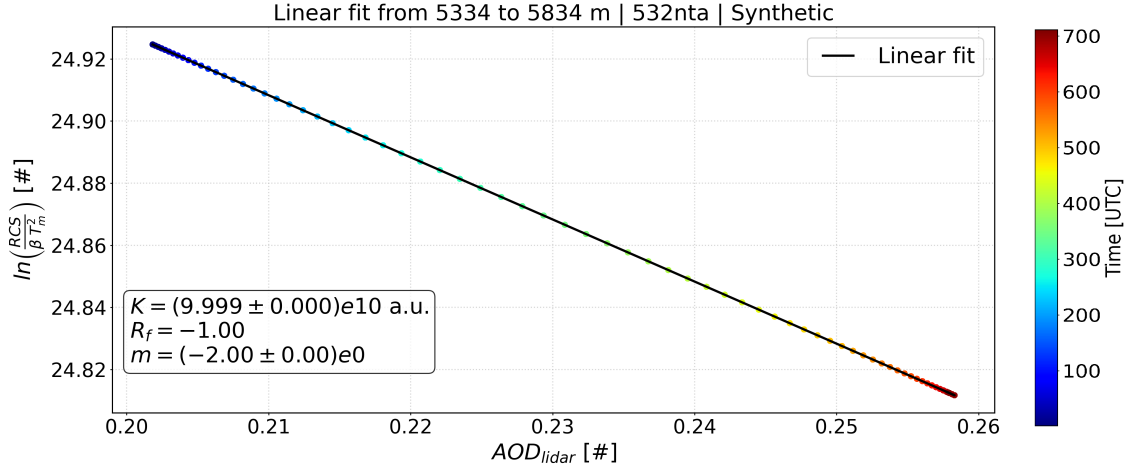


Figure 11: Aerosol optical depth fitting calibration method. Colored markers represent the temporal evolution of the experimental data points, and black line the linear fitting.

Therefore, the implementation and performance of the direct and β_{att} and AOD fitting methods are validated by means of synthetic lidar signals. This validation was also performed at 355 and 1064 nm, yielding analogous results (not shown).

5.1.3 Assessment of the lidar ratio and full-overlap height influence

The direct calibration method using real lidar signals requires the assumption of the particle lidar ratio (LR_p) and the full-overlap height (z_{over}) and thus, any different between the assumed and real values will affect the estimation of K . To do quantify the relation between the deviation between the assumed and real values of LR_p and z_{over} , the previous synthetic signal is used (7) where $LR_p = 50$ sr and $z_{over} \approx 400$ m.

The assessment of the lidar ratio influence is performed varying LR_p in range of ± 30 sr with steps of 5 sr for the wavelength 355, 532 and 1064 nm (i.e., ± 30 sr around the LR_p used to generate the lidar signals). Figure 12 shows the resulting K relative error, using the synthetic value (10^{11} a.u.) as the reference. The results show that 355 and 532 nm wavelengths are particularly sensitive to the lidar ratio. In contrast, the 1064 nm channel is the least affected by the LR_p , with maximum relative errors of $\pm 10\%$. Relative errors at 532 nm are -20% and +15% when the ratio is under- and overestimated by 30 sr, respectively. The 355 nm wavelength exhibits a more asymmetric response: underestimation leads to significantly larger errors (-20%), while overestimation results in a modest error of around +10%. Overall, these findings indicate that underestimating the lidar ratio causes more deviations from the real K toward higher values.

The influence of the assumed full-overlap height on the direct lidar calibration method was evaluated by varying its value between 100 and 400 m. Figure 13 illustrates that the relative error progressively decreases as we approach the reference value (i.e., the full-overlap height used to simulate the lidar signals). This highlights, first, the importance of correcting the backscatter profiles using the refill method (otherwise, the value of K may be underestimated), and, second, the overlap function or, instead, the full-overlap height is a key lidar configuration parameter to be properly identified (see section 4.6 for further details). Moreover, figure 13 that the influence of the full-overlap height becomes more pronounced at shorter wavelengths.

The same validation is performed for the other methods (β_{att} and AOD fitting), yielding similar results (not shown). Summarizing, the use of synthetic signals allowed us to quantify the influence of the estimation of the particle lidar ratio and the full-overlap height, being demonstrated to be crucial to keep the uncertainty of these parameter below 20% and 4%, respectively.

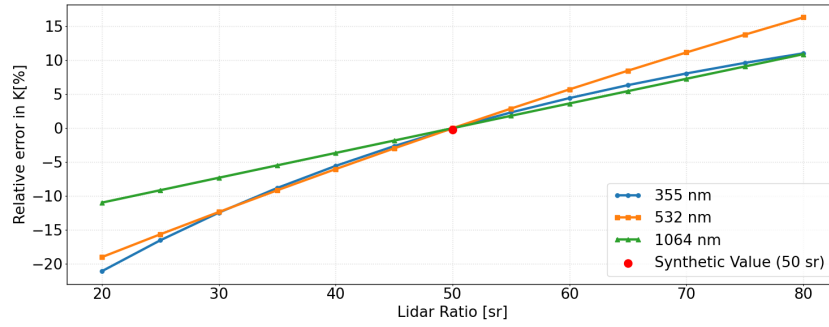


Figure 12: Assessment of the lidar ratio influence on the lidar calibration method by means of synthetic lidar signals.

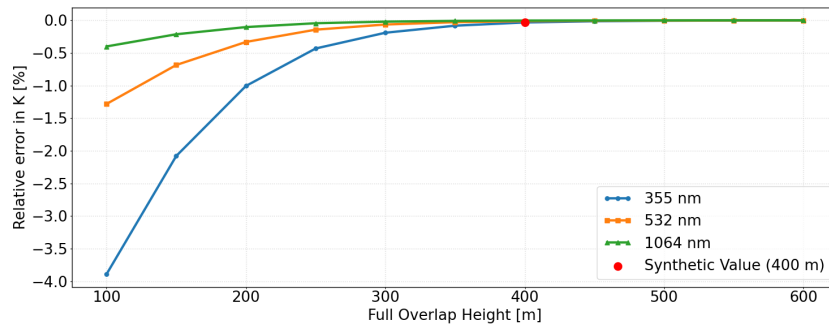


Figure 13: Assessment of the full-overlap height influence on the lidar calibration method by means of synthetic lidar signals.

5.2 Analysis of the lidar calibration methods by means of real lidar signals

5.2.1 Comparison in a real case of study

The three absolute calibration methods are applied to the ALHAMBRA's 532nta channel between 20:30 UTC on April 7, 2025, and 06:00 UTC on April 8, 2025 (Fig.14), to evaluate their performance under real atmospheric conditions. In the quicklook, an aerosol layer is observed with a decreasing thickness from 2 km at 19h00 UTC to 1 km from midnight. At the same time, RCS values within the aerosol layer slightly decreases, indicating a likely decrease in the aerosol load. This period is chosen as the atmospheric conditions are very stable.

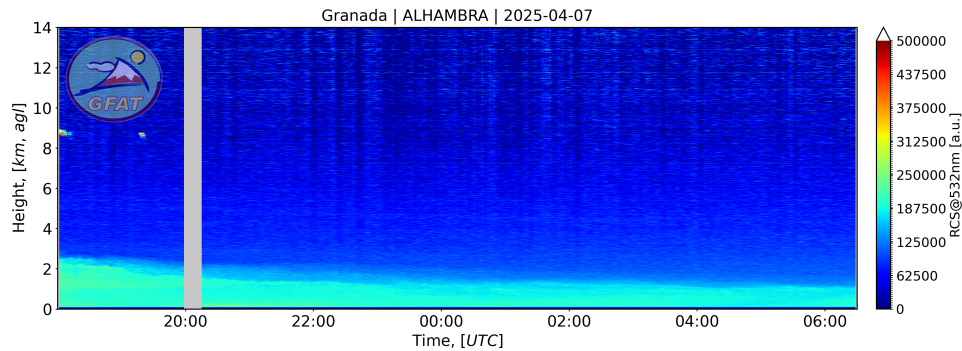


Figure 14: Time series of the lidar range-corrected signal with vertical resolution (quicklook) from ALHAMBRA (532nta channel) during the night of April 7-8, 2025. A 5-minute moving average is applied.

Between 20:30 UTC on April 7, 2025, and 06:00 UTC on April 8, 2025, β_p profiles are retrieved, applying the Klett algorithm to 10-minutes signals (Sec.4.2.1). The reference height is determined using the algorithm described in Section 4.1.2, resulting in a value of $z_m = 5605$ m. A particle lidar ratio of 60 sr was retrieved by the intercomparison of the Aod_{lidar} and the AOD_{photo} measured by a nearby lunar photometer (Lunar CIMEL CE-318 from UGR) as shown in figure 16 (see section 2.3). Despite AOD_{lidar} appears much more irregular —mainly due to the uncertainties associated with the inversion algorithms—, absolute differences remain below 0.02, which generally falls within the uncertainty range of the lunar photometer [Barreto et al., 2016b].

Figure 15 shows the RCS profiles (a), β_p and β_m profiles (b) and R profiles (c). This behavior is consistent with the retrieved particle backscatter profiles (figure 15b).

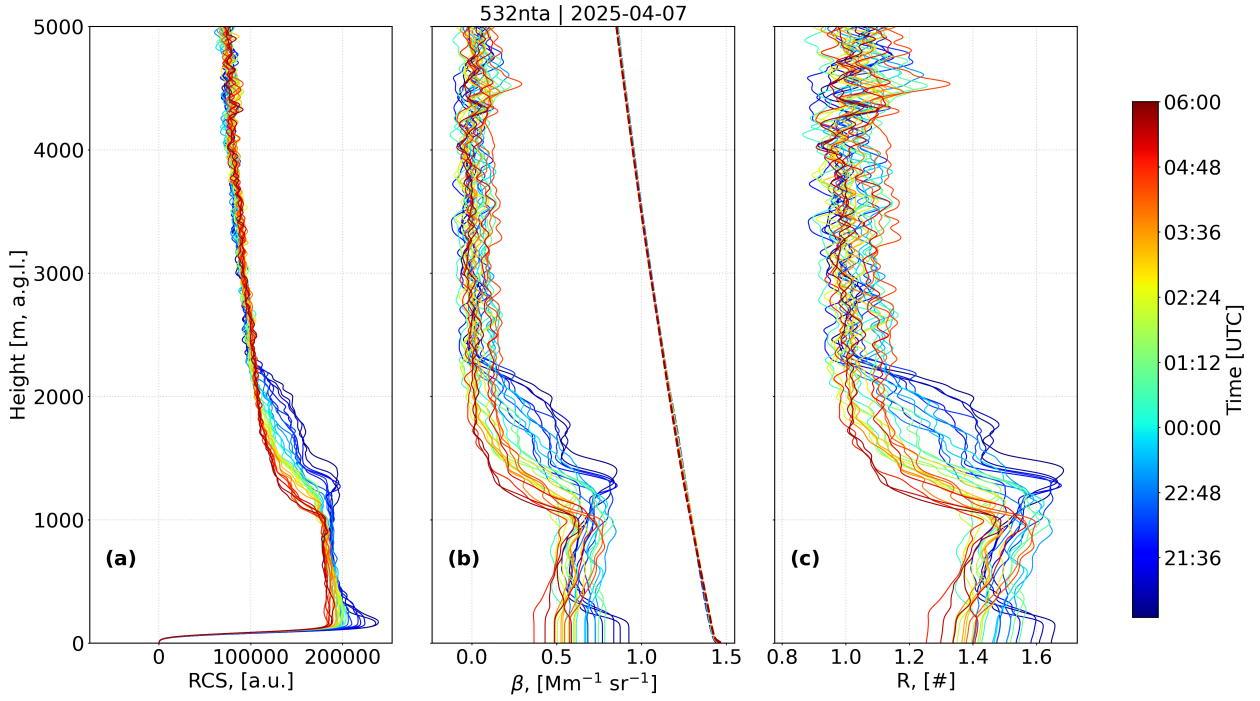


Figure 15: Signals from ALHAMBRA (532nta channel) during the night of April 7-8, 2025. (a) Ten-minute averaged profiles of range-corrected signal. (b) Retrieved profiles of molecular (dashed lines) and particle (continuous lines) backscatter coefficient refilled with $z_{over}=200$ m. (c) Profiles of backscatter ratio.

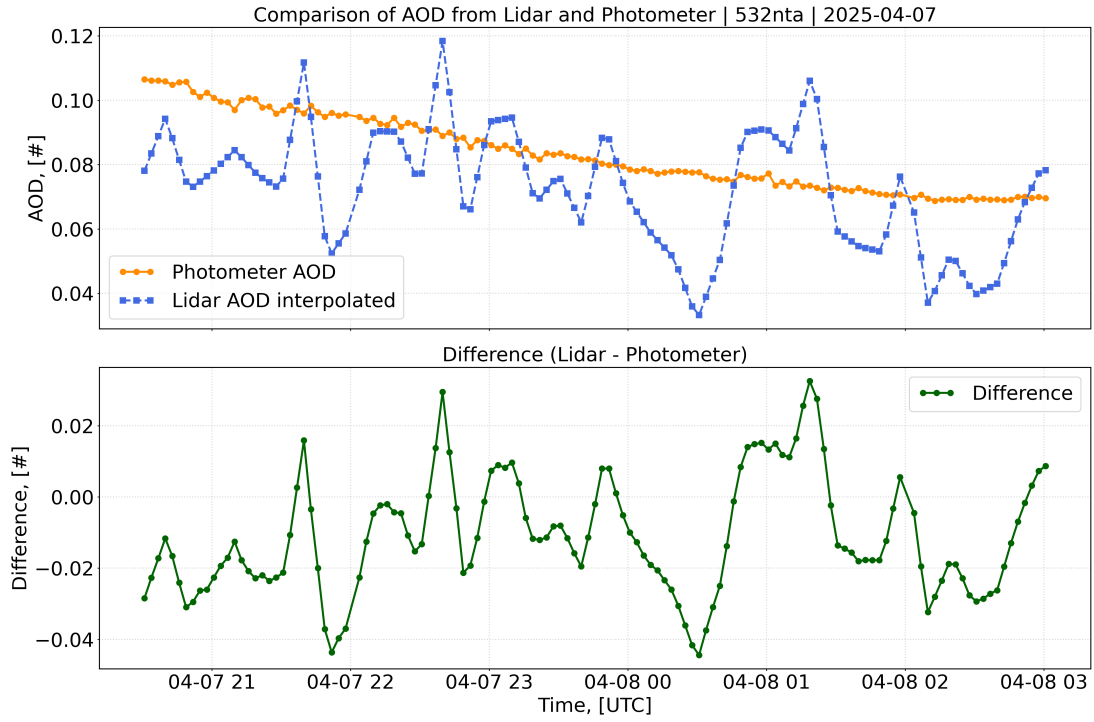


Figure 16: Comparison between AOD_{lidar} and AOD_{photo} at 532 nm for the night of April 7, 2025. The lunar photometer only measured until 3 a.m.

The **direct calibration method** is used to compute a time series of K shows in figure 17. The mean lidar calibration factor is $K = (1.02 \pm 0.03) \cdot 10^{11}$ a.u., with a coefficient of variation (std/mean) of 3.05%. This indicates a relatively stable calibration factor throughout the night. Additionally, the small uncertainty reflects the high robustness of the direct calibration method.

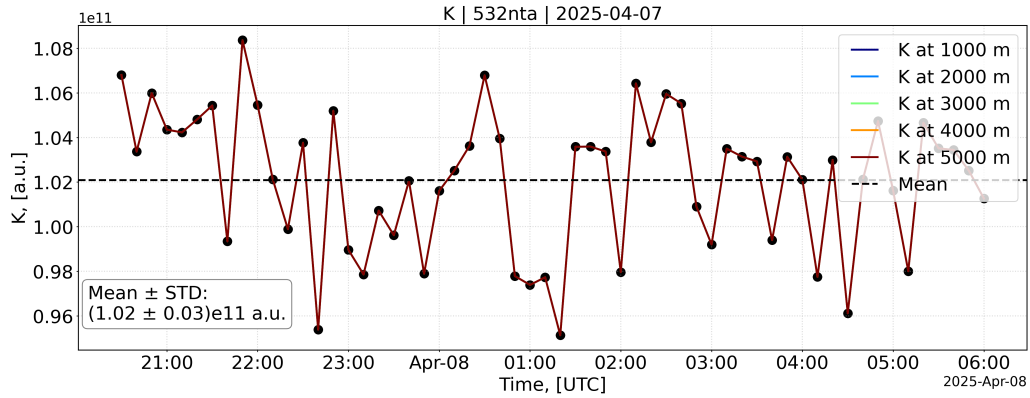


Figure 17: Time series of K at 532nta channel for the night of April 7-8, 2025, calculated by direct calibration method. Calibrations were performed at 1000, 2000, 3000, and 4000 meters, represented with different colored lines (which overlap). The average of the time series is shown as a dashed black line.

The **attenuated backscatter fitting calibration method** allows to obtain a calibration factor every 10 minutes, following the signal average. For illustrative purposes, Figure 18 shows the linear fits performed at 22:30. Gathering the linear fit coefficients from every time step, the time series of K and b shown in figure 19 are obtained. Regarding the correlation coefficients, they are nearly perfect, consistently close to 1 across all fits. This calibration method yields a lidar calibration factor of $K = (9.94 \pm 0.32) \cdot 10^{10}$ a.u. The coefficient of variation of K is 3.21%, demonstrating comparable stability

to the direct method. Overall, the results are consistent with those from the direct method, albeit with a very slight tendency to underestimate. Although the obtained intercepts are on the order of 10^2 —about three orders of magnitude lower than the RCS values (10^5)—they still deviate from the ideal intercept value of zero. This deviation may be related with the AOD since intercept correlates with AOD, a similar trend to the one observed with synthetic signals.

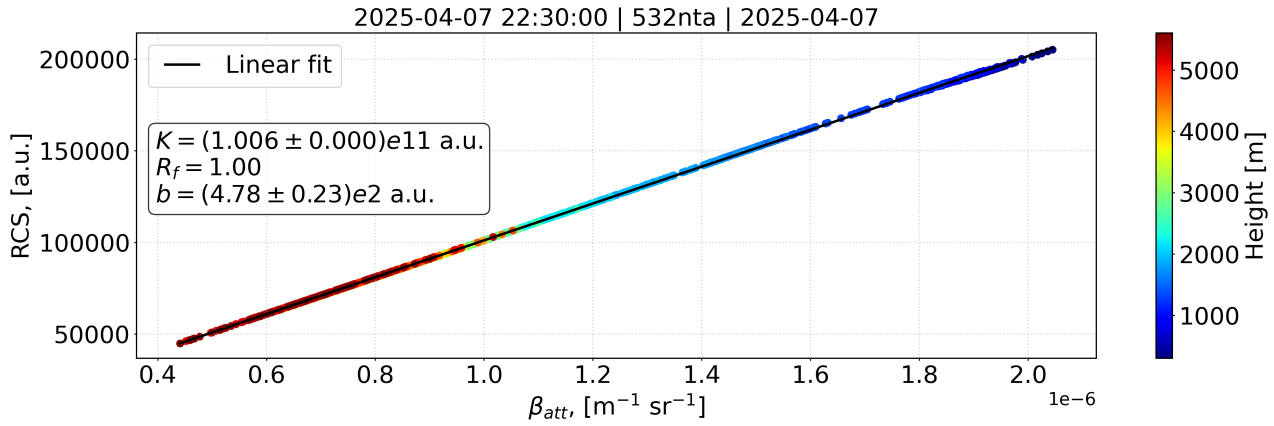


Figure 18: Attenuated backscatter fitting calibration method at 532 nm at 22:30 of April 7, 2025. Colored markers represent the temporal evolution of the experimental data points, and black line the linear fitting.

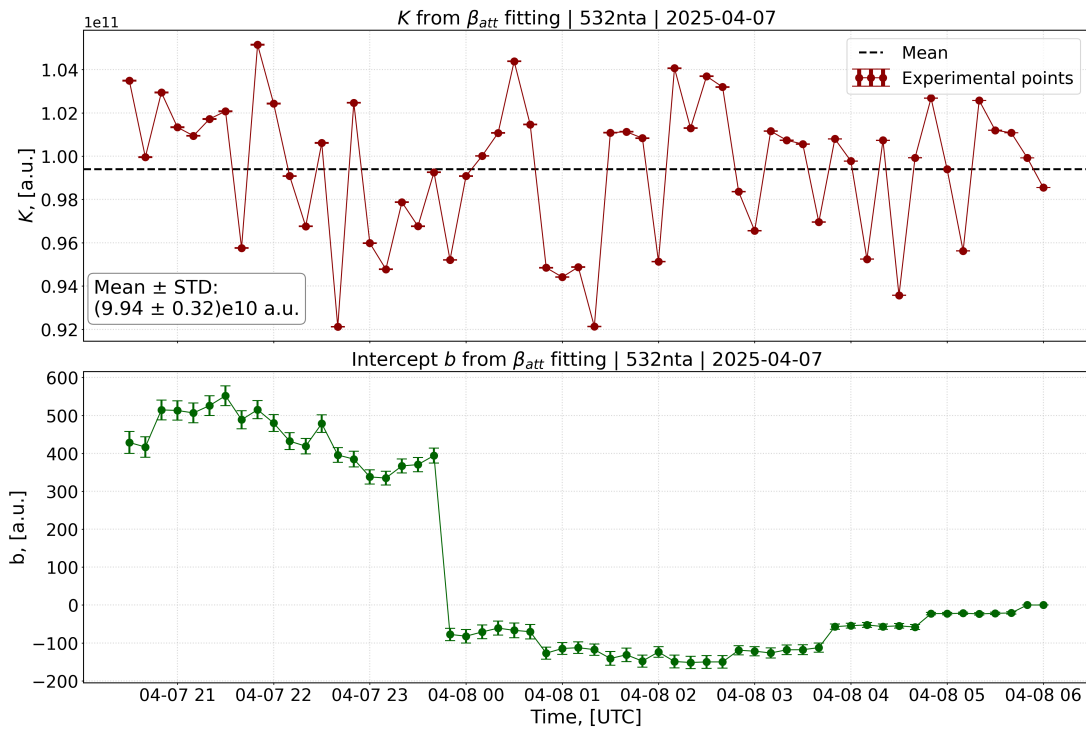


Figure 19: Time series of K (red error bars) and b (green error bars) at 532nta channel for the night of April 7-8, 2025, calculated by the attenuated backscatter fitting calibration method. In the top panel, the average of the time series is shown as a dashed black line.

The **lidar AOD fitting calibration method** is used to compute K as shows in figure 20. Using this method, a value of $K = (1.081 \pm 0.001) \cdot 10^{11}$ a.u. is obtained, representing a slight overestimation compared to the values from the previous methods. The uncertainty associated with K is smaller since this result comes from a single fit rather than the average of a time series. Additionally, a slope of $m = (-2.85 \pm 0.01)$ is obtained, near to the theoretical value of -2, confirming the consistency of the results. This method shows lower correlation coefficient, with $|R_f| = 0.92$, likely because this method uses data from a given period where K may vary due to instability of the emitted laser power.

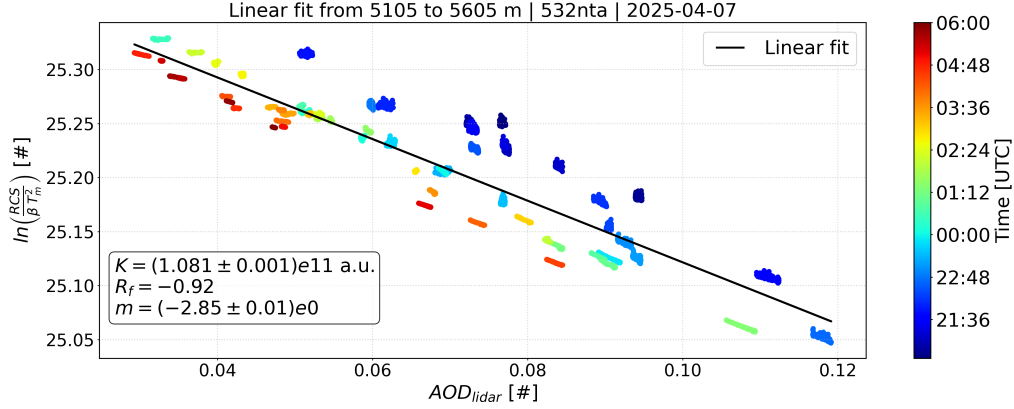


Figure 20: Lidar aerosol optical depth fitting calibration method at 532 nm for the night of April 7, 2025. Colored markers represent the temporal evolution of the experimental data points, and black line the linear fitting.

Finally, since we have lunar photometer data for this day, we can replicate the previous method **using the photometer AOD** instead of the lidar AOD (figure 21). This approach holds great potential because it allows us to determine the lidar calibration factor without performing profile inversion (thus eliminating the need to use Klett) and by utilizing an independent instrument. This not only simplifies the calibration process but also provides a valuable cross-validation between different measurement systems, increasing confidence in the retrieved calibration factor.

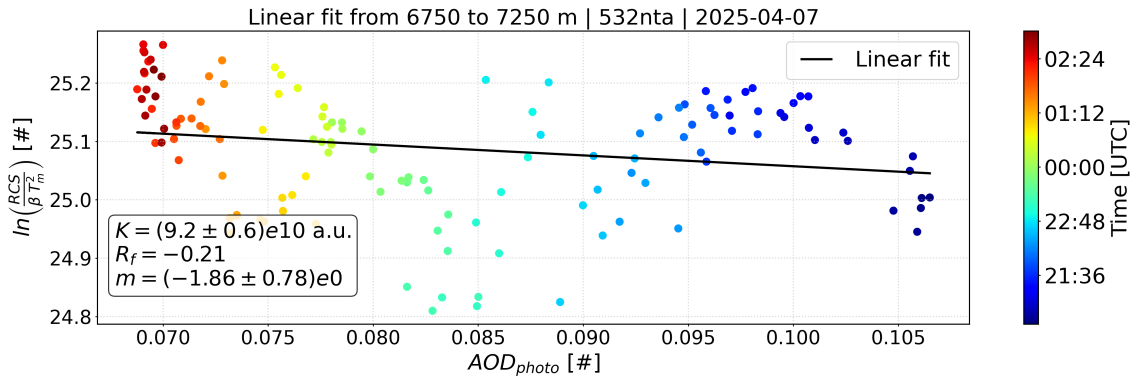


Figure 21: Lunar photometer aerosol optical depth fitting calibration method at 532 nm for the night of April 7, 2025. Colored markers represent the temporal evolution of the experimental data points, and black line the linear fitting.

Using the photometer, results consistent with those from the other methods are obtained: $K = (9.2 \pm 0.6) \cdot 10^{10}$ a.u., which represents a 5.88% underestimation compared to the value obtained using the direct method. This result indicates that, although the correlation is lower, the method successfully provides a reliable estimate of the lidar calibration factor without requiring signal inversion, offering a significant advantage in terms of simplicity and robustness.

These calibration methods are also applied to 355nta and 1064nta channels. The results are summarized in Table 4, showing good agreement among the different calibration methods, except for the photometer-based method at 1064 nm, which deviates more significantly from the others. As discussed in the following section, the 1064 nm channel is the one exhibiting the largest discrepancies between methods.

Methods	355 nm (a.u.)	532 nm (a.u.)	1064 nm (a.u.)
Direct	$(5.38 \pm 0.02) \cdot 10^{10}$	$(1.02 \pm 0.03) \cdot 10^{11}$	$(6.9 \pm 0.5) \cdot 10^{12}$
β_{att} fitting	$(5.03 \pm 0.02) \cdot 10^{10}$	$(9.9 \pm 0.3) \cdot 10^{10}$	$(6.8 \pm 0.5) \cdot 10^{12}$
AOD_{lidar} fitting	$(5.365 \pm 0.001) \cdot 10^{10}$	$(1.081 \pm 0.001) \cdot 10^{11}$	$(6.83 \pm 0.03) \cdot 10^{12}$
AOD_{photo} fitting	$(5.1 \pm 0.1) \cdot 10^{10}$	$(9.2 \pm 0.6) \cdot 10^{10}$	$(5.4 \pm 0.4) \cdot 10^{12}$

Table 4: Lidar calibration factor obtained using different methods for the 355nta, 532nta, and 1064nta channels of ALHAMBRA during the night of April 7, 2025.

Summarizing, similar results are obtained across three wavelengths, confirming the validity of the newly implemented methods and the robustness of the retrieved lidar calibration factor values.

5.2.2 Temporal stability assessment

This section aims to assess the temporal stability of the lidar calibration factor over an entire month. To this end, calibrations were performed on nine clear-sky nights throughout April 2025 using the three calibration methods across the three ALHAMBRA near-field subsystem channels: 355nta, 532nta, and 1064nta. This particular month was selected because a Saharan dust intrusion occurs toward its end, which will be examined in greater detail in the following section.

Figure 22 shows the temporal evolution of the calibration factor throughout April 2025. Tables 5 and 6 present the average calibration factors and their corresponding coefficients of variation for each channel and method. The 355nta channel shows the lowest variability, with a maximum difference of 3.89% using the AOD fit method. In contrast, the 1064nta channel exhibits the highest variability, reaching up to 21.11% with the β_{att} fit method.

Additionally, when comparing the different methods for a given channel, the AOD fit method consistently shows the largest deviations between measurements and tends to overestimate the calibration factor relative to the other methods —particularly for the 1064nta channel.

Finally, a common trend across all time series is observed: calibration factors tend to increase toward the end of the month. This is attributed to the unstable conditions caused by a Saharan dust outbreak.

Methods	355 nm (a.u.)	532 nm (a.u.)	1064 nm (a.u.)
Direct	$(5.32 \pm 0.09) \cdot 10^{10}$	$(1.07 \pm 0.08) \cdot 10^{11}$	$(5.7 \pm 1.2) \cdot 10^{12}$
β_{att} fitting	$(5.07 \pm 0.05) \cdot 10^{10}$	$(1.05 \pm 0.07) \cdot 10^{11}$	$(5.6 \pm 1.2) \cdot 10^{12}$
AOD_{lidar} fitting	$(5.4 \pm 0.2) \cdot 10^{10}$	$(1.2 \pm 0.1) \cdot 10^{11}$	$(7.0 \pm 1.2) \cdot 10^{12}$

Table 5: Mean of the lidar calibration factor K for 355nta, 532nta, and 1064nta, calculated using the three calibration methods throughout April 2025.

Methods	355 nm	532 nm	1064 nm
Direct	1.65%	7.42%	21.04%
β_{att} fitting	1.08%	7.04%	21.11%
AOD_{lidar} fitting	3.89%	10.24%	17.43%

Table 6: Coefficient variation of the lidar calibration factor K for 355nta, 532nta, and 1064nta, calculated using the three calibration methods throughout April 2025.

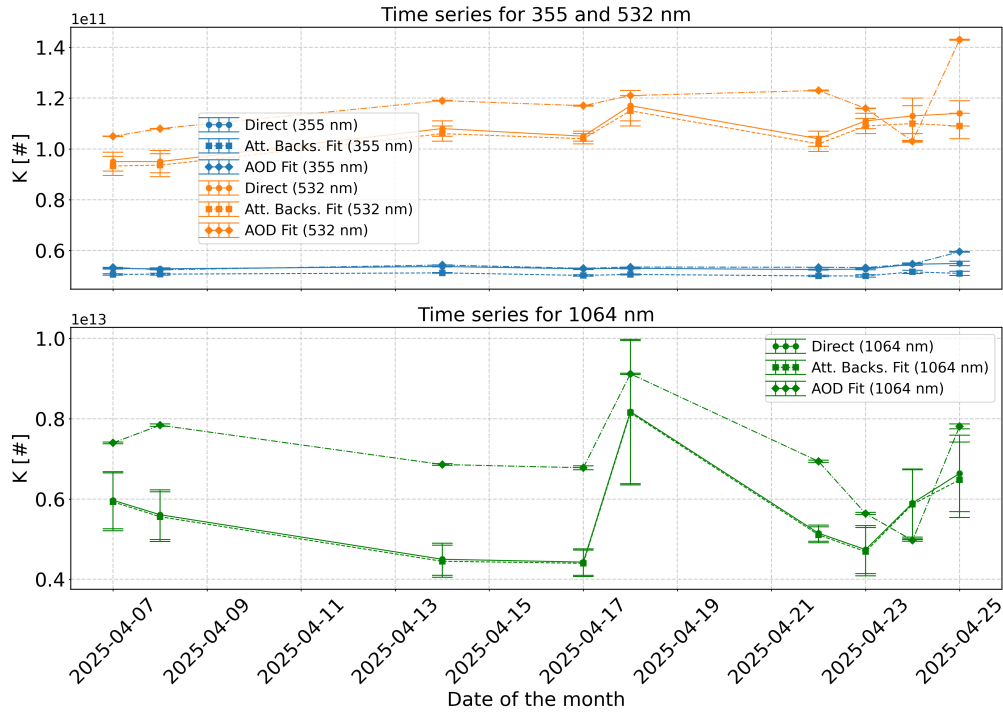


Figure 22: Evolution of the lidar calibration factor K for 355nta, 532nta, and 1064nta, calculated using the three calibration methods throughout April 2025.

Therefore, the lidar calibration constant (K) stability is generally high, although the direct and the attenuated backscatter fitting method provides better results. It is worthy to note that regular calibration is still necessary to account for potential variations due to changes in the lidar system configuration and the gradual degradation of laser power and photomultiplier gains.

5.3 High-resolution analysis of a Saharan dust intrusion event

Once the calibration factor is obtained for 355, 532 and 1064 nm; high-resolution products can be obtained. As case study of a dust outbreak, occurred on April 24 and 25, 2025, is presented (Fig. 23). Around 00:30 UTC, a Saharan layer is detected over Granada around 2 km, clearly decoupled from the lowermost layer [0, 1.75] km. This period is especially interesting because, in addition to the incoming dust layer, the other layer is linked to local aerosol (e.g., urban pollution). From 04:00 UTC, aerosol layers merges and the aerosol structure becomes more complex with time (several layers and internal dynamics), being worthy to identify a strong dust layer around 2 km between 18:00 and 6:00 UTC (night from April 24 to 25). Additionally, extremely high RCS values between 6 and 11 km and some spots at around 4 km are lined to cirrus and cumulus clouds, respectively. Such Saharan dust intrusion events are typical in Granada, particularly during the summer months, due to the city's geographical proximity to the Sahara Desert. This knowledge together with satellite and ancillary

information (not shown) was used to identify this event with Saharan dust. This combination of multilayering structure and cloud presence allows for a convenient analysis in real conditions.

Using the algorithms described in Section 4.4, high-resolution particle backscatter coefficients (β_p) at 532 and 1064 nm were retrieved. The results obtained with the iterative and quasi-backscatter algorithms are analogous; therefore, only the results from the iterative method are shown in Fig. 24 (top). As inputs to these algorithms, the averaged calibration factors at 532 and 1064 nm for April were used, along with a constant lidar ratio (LR_p) of 50 sr for both wavelengths. This value was selected based on previous studies of similar dust events in Granada (e.g., [Soupiona et al., 2019], [Fernández et al., 2019])

The 2D β_p representation may be used as proxy of the concentration of aerosol particles in the atmosphere. The highest values of β_p —exceeding $2Mm^{-1}sr^{-1}$ —are associated with both the Saharan dust layer and clouds. In the quicklook, a gradual downward movement of the Saharan dust layer becomes clearly visible starting at 15:00 UTC on April 25, eventually merging with a lower aerosol layer attributed to urban pollution.

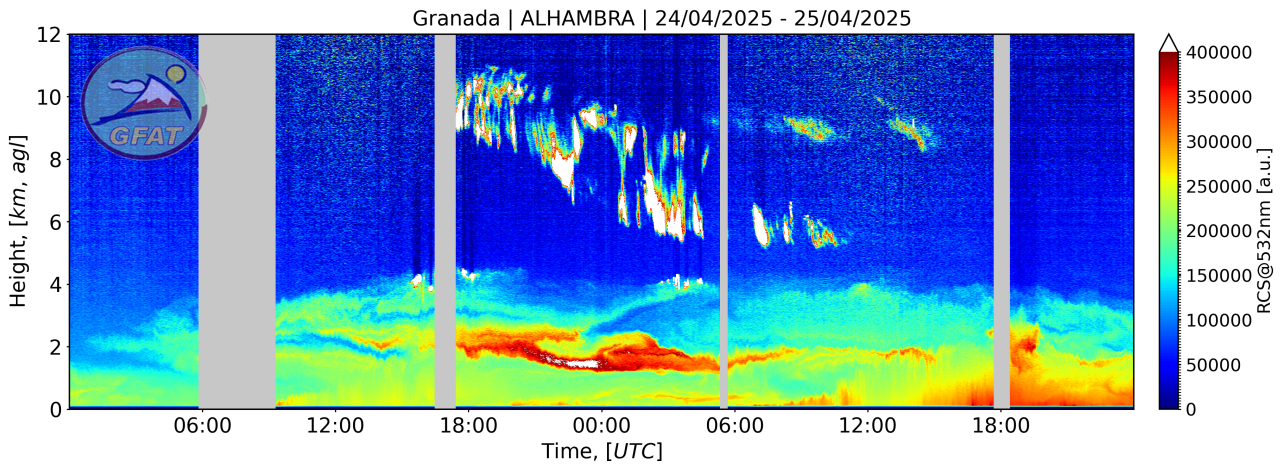


Figure 23: Time series of the lidar range-corrected signal (quicklook) with vertical resolution from ALHAMBRA (532nta channel) during April 24–25, 2025. A 5-minute moving average has been applied.

For a better identification of aerosol types, intensive properties are useful (see section 2.3), and thus, high spatio-temporal resolution backscatter ratio at 532 nm, Ångström exponent at 532-1064 nm and linear particle depolarization at 532 nm are shown in figure 24. It is worthy to note that it is the first time that this high-resolution retrieval is performed with lidar data measured in Granada.

The backscatter ratio, which quantifies the dominance of aerosol particles over a molecular atmosphere ($R - 1$), reveals patterns consistent with those seen in the β_p quicklook, further supporting the interpretation of aerosol layering and temporal evolution.

In the **Ångström exponent** ($\hat{a}$) quicklook, values were filtered to display only regions where $1.4 < R < 60$, thereby excluding both the purely molecular region ($R < 1.4$) and areas where the signal becomes attenuated ($R > 60$). The backscatter-related Ångström exponent below one indicates coarse-mode predominance. Additionally, the **linear particle depolarization ratio** (δ_p) is shown. This intensive property provides information about particle shape: low values indicate spherical particles, while high values reflect more irregular shapes. The combined analysis of both intensive optical properties allows for a more accurate classification of the various aerosol layers present in the atmosphere.

In the lowermost layer [0, 1.75] km, high $\dot{a}_{532,1064}$ (around 1.5) and low δ_p (<0.1) are observed. This indicates the presence of fine and spherical particles, which are typical of anthropogenic pollution aerosols, as reported in the literature (e.g., [Yang et al., 2009]).

In the Saharan dust layer, the Ångström exponent decreases to values between 1.1 and 1.25, indicating a predominance of coarse-mode particles typically associated with desert-origin intrusions. In this case, depolarization ratios increase significantly, with values of $\delta_p \approx 0.3$, which confirms the irregular and non-spherical nature of mineral dust particles suspended in the atmosphere.

However, a more detailed analysis reveals spatial heterogeneity within the layer. Specifically, regions with $\delta_p > 0.25$ can be interpreted as areas dominated by *pure mineral dust*, where the Ångström exponent is around 1.1, consistent with a strong presence of coarse particles. These areas likely correspond to the core of the dust plume, located between 2 and 4.5 km in altitude, and observed between approximately 18:00 UTC on April 24 and 06:00 UTC on April 25.

In contrast, regions where $\delta_p < 0.25$ suggest the presence of *mixed aerosol layers*, where Saharan dust is likely interacting with other fine-mode particles such as urban aerosols [Yang et al., 2013]. In these mixed regions, the Ångström exponent tends to increase slightly, typically showing values close to 1.25, indicating a more balanced contribution from both fine and coarse aerosol modes. This mixing can occur as the dust layer descends and interacts with the polluted planetary boundary layer, particularly during the late hours of April 25, from around 15:00 UTC and continuing into the night of April 25–26.

This distinction between pure and mixed dust regions is crucial for accurately characterizing the aerosol population and its optical properties, as well as for understanding the microphysical processes driving the evolution of the layer.

For cirrus clouds, the lowest Ångström exponent values are observed, ranging from 0.5 to 0.75, indicating the presence of large particles, likely ice crystals, as also suggested by the highest depolarization values ($\delta_p \approx 0.45$), which confirm their irregular shape. These results are consistent with previous findings reported in the literature [Chen et al., 2002]. However, Ångström exponent analysis within clouds is limited due to the likelihood of lidar signal attenuation. Since this attenuation is wavelength-dependent, it may lead to unrealistic Ångström exponent values.

A particularly noteworthy phenomenon is observed during the progressive descent of cirrus clouds throughout April 25: the Ångström exponent increases, and the depolarization ratios decrease as the cloud descends. This suggests a change in cloud microphysics, likely associated with the melting of ice crystals and the formation of smaller, more spherical water droplets, consistent with the transition from cirrus to mixed-phase or liquid clouds in the lower troposphere.

Therefore, the use of extensive and intensive aerosol optical properties with high spatiotemporal resolution offers significant advantages for the study of atmospheric particle dynamics and their accurate classification. By simultaneously analyzing parameters related to particle concentration (particle backscatter coefficient and backscatter ratio), size (Ångström exponent) and shape (linear particle depolarization ratio), it becomes possible to distinguish between diverse aerosol layers—such as anthropogenic pollution, mineral dust intrusions, and cloud formations—with greater confidence. This detailed insight enhances our understanding of aerosol sources, transport mechanisms, and their interactions with meteorological phenomena, which is crucial for improving atmospheric modeling and air quality assessments. Ultimately, high-resolution intensive properties would enable a more detailed discussion on entrainment processes, coupled and decoupled layer structures, and aerosol mixing within the case study. However, a thorough analysis of these processes lies beyond the scope of the present work.

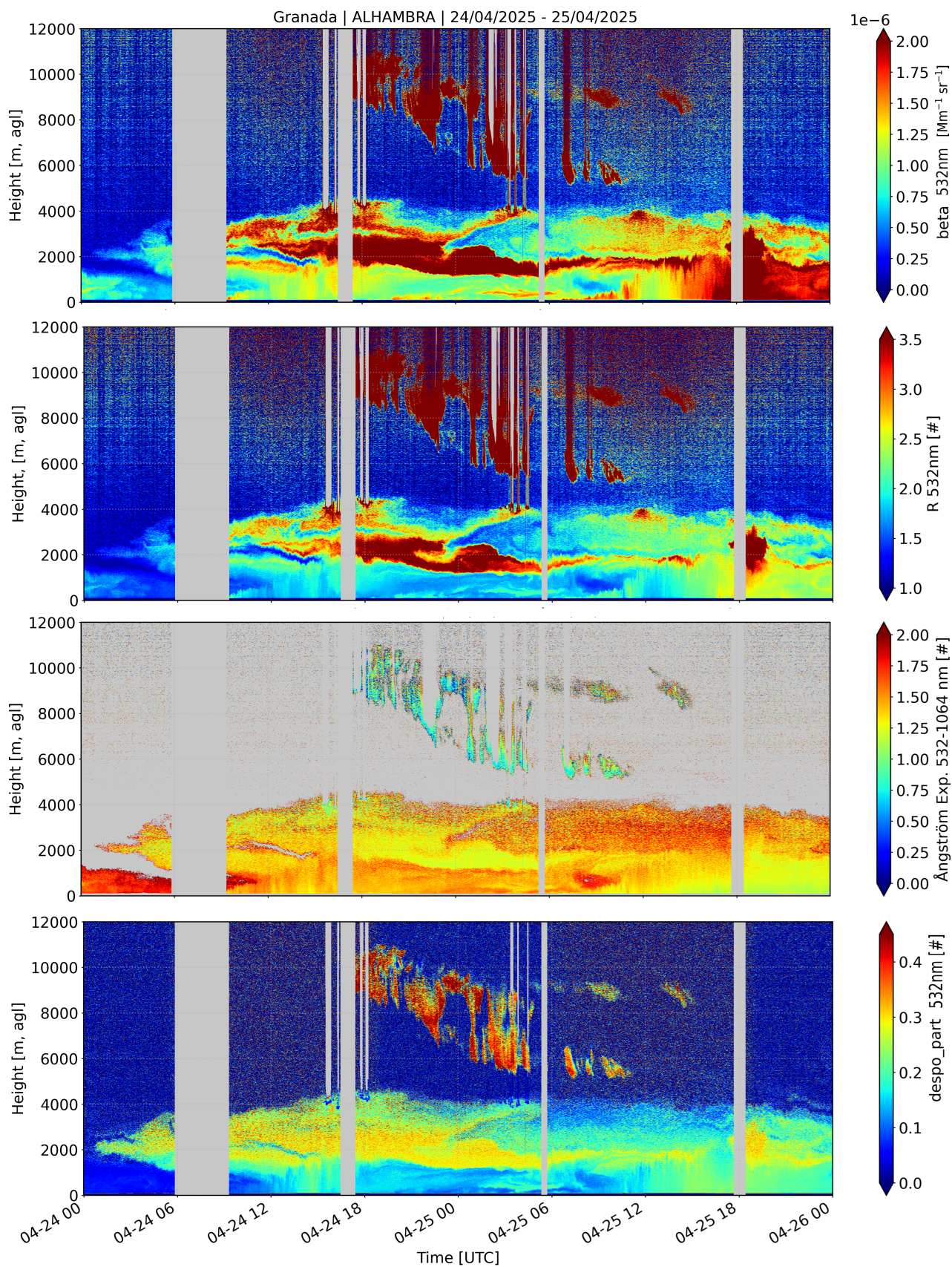


Figure 24: Quicklook of the particle backscatter coefficient at 532 nm and the aerosol intensive properties: Backscatter ratio at 532 nm, Ångström exponent at 532-1064 nm and linear particle depolarization at 532 nm with high spatiotemporal resolution during April 24–25, 2025.

5.4 Validation and assessment of the homogeneous-backscatter overlap algorithm

The objective of this section is twofold: to validate the new method for determining the overlap function, the homogeneous-backscatter overlap, described in Section 4.6.3 using synthetic signals, and to intercompare this function with other well-established methods (iterative, see Section 4.6.1; and explicit, see Section 4.6.2) in the context of a real case study.

As previously discussed, the homogeneous-backscatter overlap algorithm works properly when the atmosphere is stable and homogeneous. This assumption is generally fulfilled for the near-field subsystem, since no significant changes are typically observed in the aerosol layer within the first few hundred meters. Therefore, to validate the algorithm, the simulated example shown in figure 7 is used, where the overlap function is designed to resemble that of the near-field subsystem of AL-HAMBRA.

Figures 25(a) and 26(a) show the time-averaged overlap functions obtained with 10-minute resolution for the three methods applied to the synthetic example at 355 nm and 532 nm wavelengths, respectively, along with the synthetic reference overlap function. The absolute deviations from the reference value are also presented in figures 25(b) and 26(b).

The largest deviations for the homogeneous-backscatter overlap method are observed within the first 70 meters, where absolute differences of up to -0.08 from the synthetic reference value are reached at both wavelengths. However, these discrepancies can be attributed to limitations in generating synthetic overlap functions at very low altitudes using the sigmoidal model (Eq. 50), as well as to the selection of the full-overlap height used in the refill method. Thus, these initial deviations do not significantly compromise the overall accuracy of the homogeneous-backscatter overlap correction at higher altitudes.

In contrast, the iterative algorithm accurately retrieves the overlap function with very low errors. The explicit method shows the most significant deviations. As discussed in Section 4.6.2, this algorithm fails when the elastic and Raman wavelengths differ substantially, such as in the 355–387 nm and 532–607 nm channel pairs.

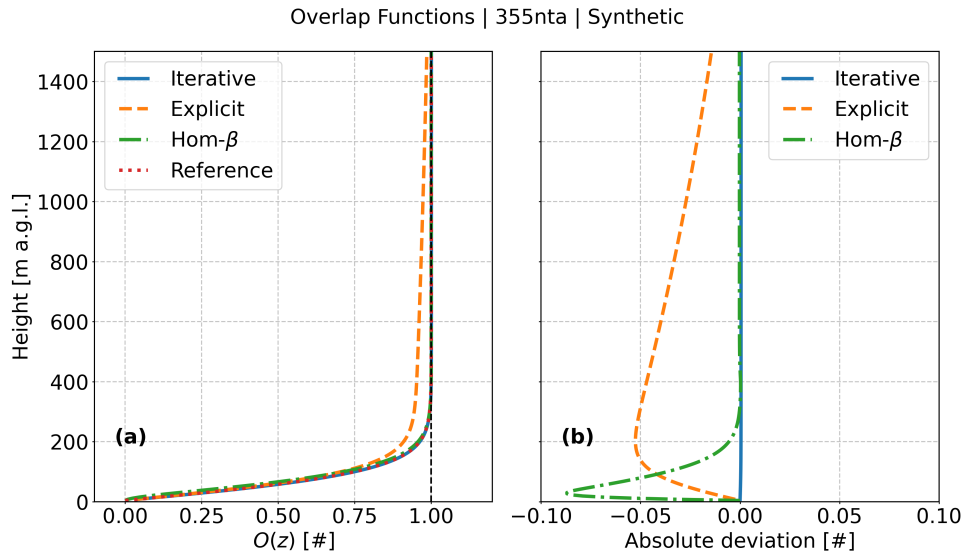


Figure 25: (a) Time-averaged overlap functions at 355 nm obtained using the three methods applied to the synthetic case shown in figure 7: iterative (blue), explicit (orange), and homogeneous-backscatter, denoted as Hom- β (green). The synthetic reference overlap is shown as a dashed red line. (b) Absolute deviations of the synthetic reference for each method, using the same color scheme as in a.

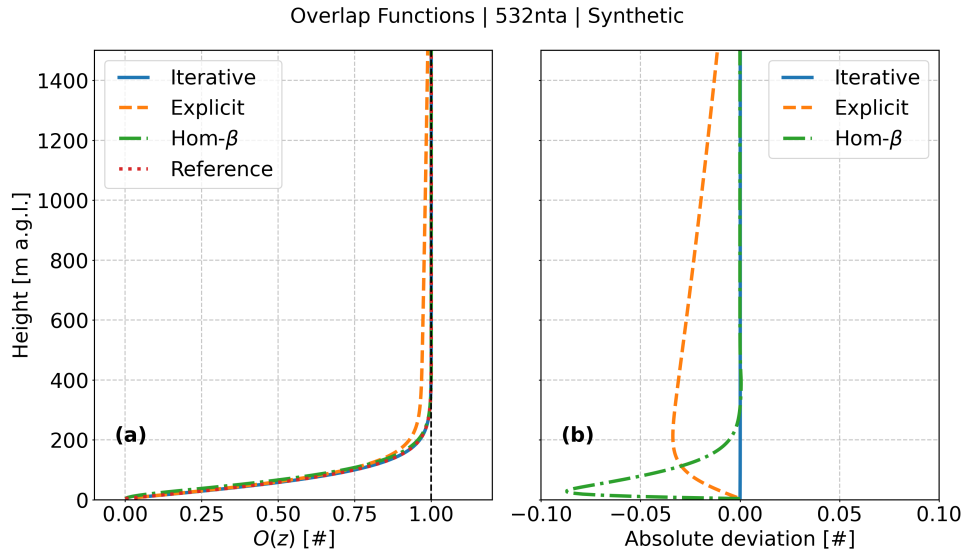


Figure 26: Same as figure 26 but for 532 nm

Figure 27 shows the time-averaged overlap functions obtained with 10-minute resolution using the three methods applied to the 355nta and 532nta channels of ALHAMBRA during the night of March 29, 2025. A good agreement is observed between the iterative and homogeneous-backscatter methods in both cases, particularly for the 532nta channel, which reinforces the validity of the homogeneous-backscatter method under real atmospheric conditions. In contrast, the explicit method exhibits larger deviations due to differences in spectral transmittance (see 4.6.2).

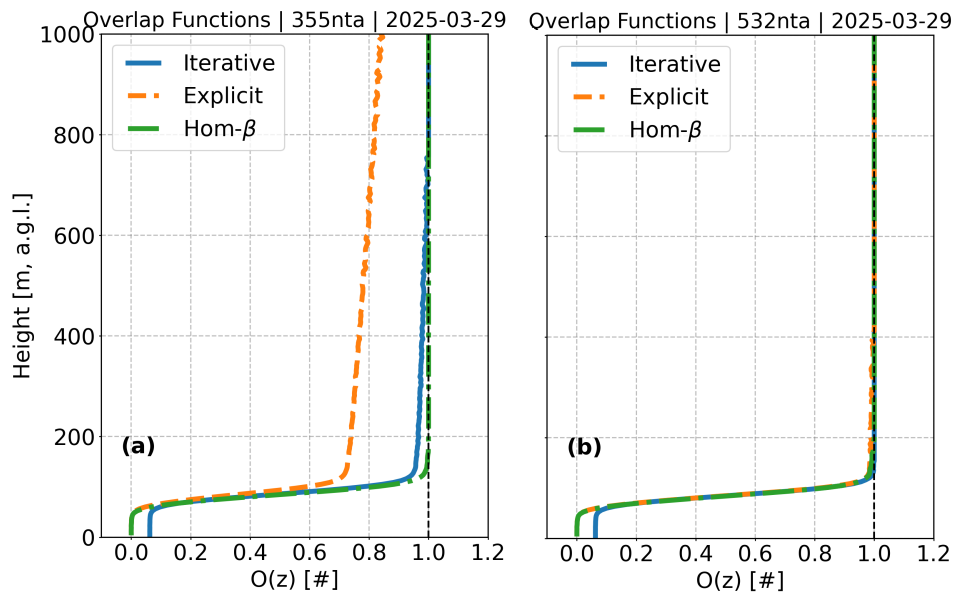


Figure 27: Time-averaged overlap functions obtained with 10-minute resolution using the three methods applied to the 355nta (a) and 532nta (b) channels of ALHAMBRA during the night of March 29, 2025: iterative (blue), explicit (orange), and homogeneous-backscatter, denoted as Hom- β (green)

Therefore, the correct implementation and validity of the homogeneous-backscatter algorithm are confirmed, as it yields results consistent with those obtained using the iterative algorithm for both synthetic and real lidar signals.

6 Conclusions

This work has successfully tackled the enhancement of the retrieval of optical properties of atmospheric aerosols from lidar signals by increasing their high spatio-temporal resolution and correcting the incomplete overlap. To achieve this, **two-dimensional synthetic lidar signals were generated to validate all the algorithms implemented in this work and to assess the input parameters such as particle lidar ratio and full-overlap height.** In light of the results, synthetic lidar signals can be highlighted as a fundamental tool for assessing the accuracy, robustness, and limitations of retrieval algorithms.

Three calibration methods (direct, attenuated backscatter fitting, and aerosol optical depth fitting) were rigorously validated using synthetic lidar signals. All methods presented high accuracy, with relative errors below 0.015%, confirming their robustness. The dependence with lidar ratio and full-overlap height of the calibration methods was evaluated, being identified that a negative bias of 30 sr from the reference LR_p results in relative errors of K of up to -20% at 355 and 532 nm, whereas a positive bias of 30 sr, caused a deviations of approximately +10% and +15% at 355 and 532 nm, respectively. K at 1064 nm was the least affected, with a maximum relative error of $\pm 10\%$ for $LR_p \pm 30$ sr. The analysis of full-overlap height influence indicates that the K is underestimated when the full-overlap height is underestimated, particularly at shorter wavelengths.

Once the methods were validated and evaluated, lidar data of ALHAMBRA measured on April 7, 2025 were used to obtain K values for the 355 nta, 532 nta, and 1064 nta channels. All methods produced consistent results with low coefficient of variations ($< 3.3\%$). It is worthy to mention that the AOD fitting method using photometer-derived AOD is a promising approach since no inversion is required and it relies on independent measurements. However, this method should be carefully applied since significant differences may appear between the lidar- and photometer-derived AOD.

The temporal stability of the lidar calibration factor was also assessed. To this end, K of 355 nta, 532 nta, and 1064 nta channels were obtained with the three calibration methods on nine clear-sky nights throughout April 2025. The K of the 355 nta channel showed low variability ($< 3.9\%$), whereas the K of 1064 nta channel exhibited the largest fluctuations ($> 21\%$).

The determination of K values for the 355 nta, 532 nta, and 1064 nta channels allowed to retrieve the particle backscatter coefficient with high resolution using the **alternative inversion algorithms**—i.e., the quasi-backscatter and iterative backscatter methods—and the aerosol-particle intensive properties. The analysis of high-resolution extensive and intensive properties allowed for a detailed characterization of **scatterer classification (e.g., anthropogenic and dust particles, as well as clouds)** and atmospheric structures. Furthermore, these properties proved essential for identifying regions where aerosol-cloud interactions occur. They also provided valuable insight into the **dynamics of aerosol particles within the atmospheric boundary layer.**

To extend lidar signal toward the surface, iterative and explicit methods for determining the overlap function were validated. The iterative-method overlap function almost matches the synthetic overlap function, whereas the explicit method exhibited larger discrepancies due to differences in the spectral transmittance (e.g., 355–387 nm). Additionally, a new method, named homogeneous-backscatter overlap algorithm, for estimating the overlap function was developed based on the lidar calibration factor and assuming a homogeneous and stable atmosphere in the lowermost region of the atmosphere. This algorithm was validated using synthetic lidar signals, showing maximum absolute deviations of -0.08 within the first 70 meters. These discrepancies are attributed to limitations of the sigmoidal model and uncertainties related to the refill method. When applied to real ALHAMBRA measurements under stable atmospheric conditions, the algorithm closely matched the results of the iterative method—considered the reference method in lidar community—at both 355 nm and 532 nm. These results confirm the correct implementation and reliability of the homogeneous-backscatter method under stable atmospheric conditions.

This work has demonstrated the potential of the high-resolution ground-based lidar systems, as ALHAMBRA, for the detailed characterization of atmospheric aerosols through the application of alternative inversion methods. The validation of new strategies —both for estimating the calibration factor and determining the overlap function— has significantly improved the spatiotemporal resolution of aerosol optical properties, while also extending current knowledge of the lidar technique. These advances lay the groundwork for more accurate and robust atmospheric monitoring.

In the future, the Monte Carlo techniques may help with the quantification of the uncertainties of extensive and intensive properties, associated with lidar calibration methods, allowing for a more robust analysis of the retrieved results. Moreover, the availability of high-resolution optical aerosol properties will increase the lidar database quality, serving as a valuable input for aerosol classification methods based on artificial intelligence. The spatiotemporal representation of these intensive and extensive properties will also facilitate more rigorous studies of aerosol dynamics within the atmospheric boundary layer, contributing to a deeper understanding of processes such as mixing, layering, and transport of particulate matter under different meteorological conditions.

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