Asymptotic mass limit of large fully heavy compact multiquarks

M. C. Gordillo [1,2,*] and J. M. Alcaraz-Pelegrina [3,†]

[1] *Departamento de Sistemas Físicos, Químicos y Naturales, Universidad Pablo de Olavide, Carretera de Utrera km 1, E-41013 Sevilla, Spain*

[2] *Instituto Carlos I de Física Teórica y Computacional, Universidad de Granada, E-18071 Granada, Spain*

[3] *Departamento de Física, Universidad de Córdoba, Campus de Rabanales, Edif. C2 E-14071 Córdoba, Spain*

(Received 20 July 2023; accepted 31 August 2023; published 21 September 2023)

The properties of fully heavy arrangements including a number of quarks between 5 and 12 were calculated within the framework of a constituent quark model by using a diffusion Monte Carlo technique. We considered only clusters in which all the quarks had the same mass and whose number of particles and antiparticles were adequate to produce color singlets. All the multiquarks were in their lowest possible values of L^2 and S^2 operators. Thus, we considered only color-spin wave functions that were antisymmetric with respect to the interchange of *any* two quarks of the same type. We found that in both all-c and all-b multiquarks, the mass per particle levels off for arrangements with the number of quarks larger than or equal to six. The analysis of their structure implies that the fully heavy multiquarks are compact structures.

DOI: 10.1103/PhysRevD.108.054027

I. INTRODUCTION

Protons and neutrons are the basic constituents of atomic nuclei. Quantum chromodynamics (QCD) is the theory that describe them as a composite set of quarks and gluons interacting through a strong force. However, QCD does not consider only the light quarks (u, d) that make up the nucleons, but extends to other types of particles collectively called hadrons. Hadrons can include or be totally made of heavier quarks (s, c, b).

Unfortunately, as of today, it is impossible to analytically solve the QCD equations and deduce the hadron spectrum. Among the phenomenological QCD-inspired models designed to fill that gap, the so-called quark model stands out. It considers only the valence quarks and antiquarks within the hadrons and was independently proposed by Gell-Mann [1] and Zweig [2]. Even though it was designed to account for the properties of mesons (one quark and one antiquark) and baryons (three quarks), it also opens the door to larger associations of quarks such as tetra- and pentaquarks [3,4]. We can also include hexaquarks, such as the experimentally produced deuteron [5] and the well-established $d^*(2380)$ resonance [6–10]. The quark model does not impose, in principle, any limit to the upper size of those clusters of quarks, and in this work we use it to obtain the masses of all possible fully heavy multiquarks. Of all the possible compositions of those clusters, we focus on arrangements in which all the quarks have the same mass. Thus, we consider sets up to 12 c or b quarks and/or antiquarks. To do so, we have to solve the Schrödinger equation derived from the Hamiltonian [11]:

$$H = \sum_{i=1}^{N_q} \left(m_i + \frac{\vec{p}_i^{\,2}}{2m_i} \right) + \sum_{i<j}^{N_q} V(r_{ij}), \tag{1}$$

where N_q is the number of quarks, while m_i and $\vec{p}_i$ are the mass and momentum of the i quark. This is a nonrelativistic approximation, and it is expected to work best for the fully heavy ensembles considered in this work. In principle, it is possible to experimentally produce these multiquarks as the discovery of the X(6900) (thought to be a fully c-tetraquark) attests [12]. Note that $V(r_{ij})$ is a two-body potential that depends only on the distance between quarks, r_{ij}, and can be written as the sum of a one-gluon exchange term given by [13,14]

$$V_{\mathrm{OGE}}(r_{ij}) = \frac{1}{4}\alpha_s(\vec{\lambda}_i \cdot \vec{\lambda}_j)\left[\frac{1}{r_{ij}} - \frac{2\pi}{3m_i m_j}\delta^{(3)}(r_{ij})(\vec{\sigma}_i \cdot \vec{\sigma}_j)\right], \tag{2}$$

which includes both Coulomb and hyperfine terms, and the lineal confining potential

*cgorbar@upo.es
†fa1alpej@uco.es

$$V_{\mathrm{CON}}(\vec{r}_{ij}) = (b r_{ij} + \Delta)(\vec{\lambda}_i \cdot \vec{\lambda}_j), \tag{3}$$

which approximates the contribution of multigluon exchanges. Here, $\vec{\lambda}$ and $\vec{\sigma}$ are the Gell-Mann and Pauli matrices, respectively, and they account for the color and spin degrees of freedom. The Dirac delta function was regularized in the standard way [15–17] in order to make the calculations possible. The parameters needed to fully define the interaction were taken from Refs. [15,16] and are the same as those used in previous calculations for smaller clusters [17–20]. The masses of the hadrons computed with this potential were found to be in good agreement with experimental data, when available [17]. Since this nonrelativistic approximation applies best to heavy quarks, in order to describe light quarks (u, d, or s), we would need to include additional terms [21], something that we will not do in this work.

II. METHOD

To solve the Schrödinger equation derived from the Hamiltonian in Eq. (1), we use a diffusion Monte Carlo (DMC) scheme [17,22–25]. This will provide us with the desired masses of the ground states of the different sets of quarks. This method requires an initial approximation to the real many-body wave function of the clusters, the trial function, which should include all the information known *a priori* about the different systems. We chose the expression [17]

$$\begin{aligned}\Psi(\mathbf{r_1}, \mathbf{r_2}, \ldots, \mathbf{r_n}, s_1, s_2, \ldots, s_n, c_1, c_2, \ldots, c_n)\\ = \Phi(\mathbf{r_1}, \mathbf{r_2}, \ldots, \mathbf{r_n})[\chi_s(s_1, s_2, \ldots, s_n) \otimes \chi_c(c_1, c_2, \ldots, c_n)],\end{aligned} \tag{4}$$

where $\mathbf{r_i}$, s_i, and c_i stand for the position, spin, and color of the particle i, which is inside a cluster of n quarks. In this work, we consider only multiquark states that are eigenvectors of the angular momentum operator, L^2 with eigenvalue $\ell = 0$. This means that Φ should depend on the distance between pairs of quarks and not on their absolute positions. Following Ref. [17], we have used

$$\Phi(\mathbf{r_1}, \mathbf{r_2}, \ldots, \mathbf{r_n}) = \prod_{i=1}^{N_q} \exp(-a_{ij} r_{ij}), \tag{5}$$

No other alternatives to the form of the radial part of the trial function were considered in this work since, in principle, the DMC algorithm should be able to correct its possible shortcomings and produce the exact masses of the arrangements [24]. The a_{ij} values were chosen in accordance with the boundary conditions of the problem [17]. Note that χ_s and χ_c are linear combinations of the eigenvectors of the spin and color operators defined by

$$F^2 = \left(\sum_{i=1}^{N_q} \frac{\lambda_i}{2}\right)^2 \tag{6}$$

and

$$S^2 = \left(\sum_{i=1}^{N_q} \frac{\sigma_i}{2}\right)^2, \tag{7}$$

with eigenvalues $F^2 = 0$ (colorless functions) and $S = 0$ or $1/2$, depending on whether the number of quarks in the multiquark is even or odd, respectively. These are the lowest possible eigenvalues for the spin operator and the only ones considered in this work. For instance, for the $cccc\bar{c}\,\bar{c}\,\bar{c}\,\bar{c}$ ($c^4\bar{c}^4$) octaquark, we have 23 color and 14 spin functions meeting those criteria. This means 322 $\chi_s \otimes \chi_c$ possible combinations. That said, we have to remember that since Eq. (5) is symmetric with respect to the exchange of any two identical quarks, we have to produce spin-color combinations that are antisymmetric with respect to those exchanges, as befits a set of fermions as quarks are. To do so, we apply the antisymmetry operator

$$\mathcal{A} = \frac{1}{N} \sum_{\alpha=1}^{N} (-1)^P \mathcal{P}_\alpha \tag{8}$$

to the color-spin set of functions. Here, N is the number of possible permutations of the set of quark indices, P is the order of the permutation, and $\mathcal{P}_\alpha$ represents the matrices that define those permutations. Once we construct the matrix derived from the operator in Eq. (8), we have to check if we can find any eigenvector with an eigenvalue equal to one. Then, these combinations will be the input of the DMC calculation [17]. For the octaquark, we have that, of all the 322 color-spin functions, only two are antisymmetric with respect to the interchange of *all* the pairs of quarks and, separately, of *all* the pairs of antiquarks. The analysis of the eigenvectors of the antisymmetry operator indicates that there are no antisymmetric color-spin functions for structures in which any of the quark or antiquark subsets contain more than six units. This means that the largest possible fully heavy multiquark is the $c^6\bar{c}^6$ dodecaquark. Moreover, neither the c^9 nonaquark nor the $c^7\bar{c}^4$ undecaquark, or their b counterparts, are viable structures. Independently, the $c^6\bar{c}^3$ nonaquark is also impossible since no antisymmetric color-spin combinations with respect to the interchange of any pair of c quarks were found.

Once we establish the characteristics of the trial function including all the proper constraints (spin, color, and angular momentum), the DMC algorithm proceeds to create a series of quark configurations compatible with those constraints in the trial by applying the recipe described in Ref [17] to a set of initial random quark coordinates. If the trial function is reasonably close to the ground state, the procedure will give us the state with the minimum mass. However, the

DMC scheme is able to converge to an excited state if that initial approximation is known to be orthogonal to the one with the minimum energy [24]. To be sure that we have reached the equilibrium state, the first part of the Markov chain generated by the DMC algorithm is neglected in the calculation of the observables of interest. This means a set of 10^5 configurations out of 6×10^5 of a typical Monte Carlo run. We obtained the multiquark masses as the average of those last configurations, with error bars (1 standard deviation of the mass values) coming from the statistical nature of the procedure. However, to avoid spurious correlations, only data from Monte Carlo configurations located 1000 steps apart were considered; i.e., the masses and the error bars were calculated using 500 different values. Unfortunately, the DMC technique only produces the masses, not the possible experimental widths.

III. RESULTS

The masses of the multiquarks obtained by the DMC method are given in Table I. As indicated above, they are all colorless clusters with $S = 0$ or $1/2$ depending on whether the total number of quarks is even or odd, respectively. We have to stress that the color-spin functions used in the calculations are the eigenvalues of the antisymmetry operator given in Eq. (8), with no quark groupings that are different from those that put together identical particles. For instance, in pentaquarks, we do not take into account baryon + meson or diquark + diquark + antiquark arrangements [26,27] but a function that is antisymmetric with respect to the exchange of *any* pair of the four quarks considered to be undistinguishable. In any case, the results for that particular multiquark are virtually identical to those of Ref. [28], in which the same function is used. Those results validate our approach, which allows us to dispense with Young-tableaux diagrams to calculate larger clusters.

To better visualize the results in Table I, we display the mass per particle as a function of the number of particles in the clusters in Figs. 1 and 2. The data not given in Table I are taken from Refs. [17,18]. It is immediately apparent that, from the open-charm hexaquark up, the mass per particle of the clusters reaches a plateau for both c and b multiquarks. This basically means that to modify the number of quarks beyond six, we have to increase the mass of the system by constant values of 1649 and 4854 MeV per particle for c and b multiquarks, respectively.

The structure of the clusters can be deduced from the radial distribution functions, depicted in Figs. 3 and 4. These give us the probability of having another particle at a particular distance from a given one. We show only the more representative structures, the remaining ones being similar to those displayed. First, we can see that all the clusters are compact structures; i.e., the probability of finding another

TABLE I. Masses of the mutiquarks considered in this work in MeV. The error bars of the DMC procedure are given in parentheses.

	$c^4\bar{c}$	$c^3\bar{c}^3$	$c^5\bar{c}^2$
Mass	8195(2)	9614(2)	11543(4)
	$c^4\bar{c}^4$	$c^5\bar{c}^5$	$c^6\bar{c}^6$
Mass	13133(4)	16539(4)	19808(4)
	$b^4\bar{b}$	$b^3\bar{b}^3$	$b^5\bar{b}^2$
Mass	24211(2)	28822(2)	33970(4)
	$b^4\bar{b}^4$	$b^5\bar{b}^5$	$b^6\bar{b}^6$
Mass	38815(4)	48599(4)	58232(4)

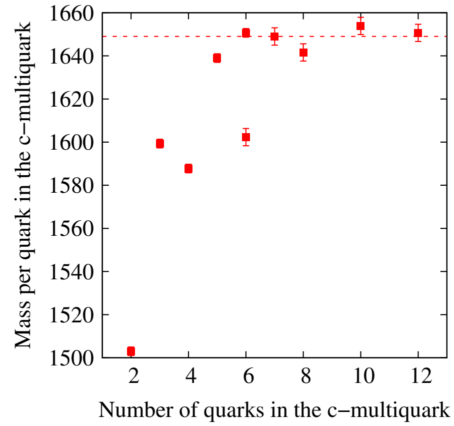

FIG. 1. Mass per particle in all the c multiquarks considered in this work. Where not shown, error bars are of the size of the symbols. The data for the meson, baryon, and tetraquark are taken from Ref. [17], while the upper symbol for the hexaquark corresponds to the mass of the open charm hexaquark given in Ref. [20]. The dashed line represents the average mass for the open charm hexaquark, and the hepta-, octa-, nona-, deca-, and docedaquarks.

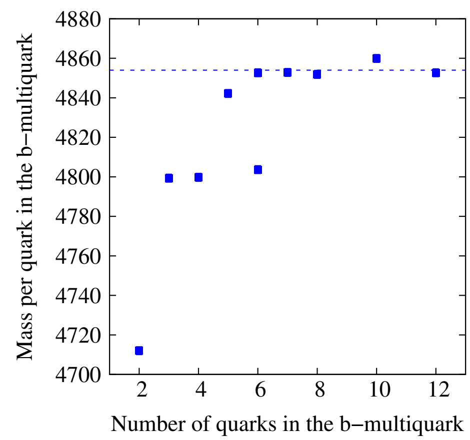

FIG. 2. Same as in the previous figure but for b multiquarks. Error bars are of the size of the symbols and are not shown for simplicity. The dashed line has the same meaning as in Fig. 1 but for the b multiquarks. The source of the data is the same as for their c counterparts.

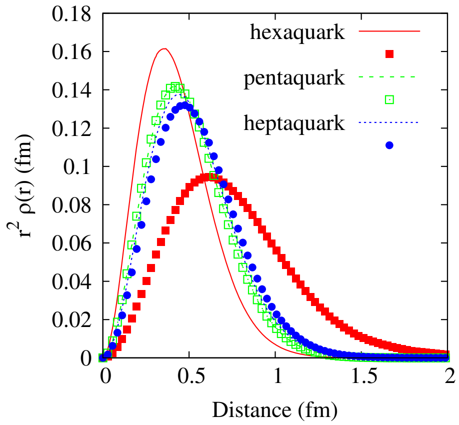

FIG. 3. Radial distribution functions for different particle pairs in c multiquarks. The solid lines are $c-c$ pairs (equivalent to $\bar{c}-\bar{c}$), and the symbols are $c-\bar{c}$ pairs. The hexaquark displayed corresponds to the $c^3\bar{c}^3$ hidden charm structure. All distributions are normalized to one.

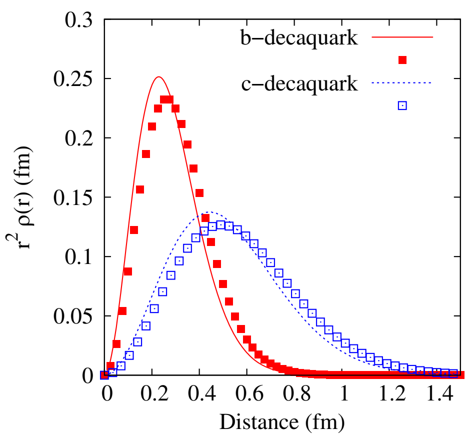

FIG. 4. Comparison between the pair distribution functions for a c-decaquark and a b-decaquark. Lines, $q-q$ pairs; symbols, $q-\bar{q}$ pairs.

particle at distances beyond a maximum of 2 fm goes rapidly to zero. In addition, in the majority of cases there is very little difference between the probability of finding another quark (solid lines) or an antiquark (symbols) for any particle at a given distance. This is similar to what happens for smaller multiquarks [17,18]. The only exception is the hidden-charm hexaquark, in which the $c-c$ and $c-\bar{c}$ are noticeably different, and in which the first of them is virtually identical to that corresponding to the ccc baryon. This is because, in that system, the quarks and antiquarks group together to produce a baryon and an antibaryon glued together. The same happens with the $b^3\bar{b}^3$ system.

IV. CONCLUSIONS

In this work we have calculated the color-spin functions with an algorithm that dispenses with the need to use Clebsch-Gordan coefficients. This is necessary since the increase in the number of color-spin functions with the number of quarks makes that approximation impossible. For instance, for a heptaquark, we have 11 color and 14 spin functions that make a total of 154 combinations. This is to be compared with the 15 color-spin possibilities for a pentaquark [28] or the 25 for an open-charm hexaquark [29,30]. The use of this technique in combination with a DMC algorithm, originally developed to deal with many-body systems, allowed us to obtain the masses of all possible fully heavy s-wave multiquarks, considering clusters larger than the fully heavy hexaquarks in the previous literature [31]. What we found is that, from a number of quarks beyond six, the mass of those systems is linearly proportional to the number of particles in the arrangements; i.e., in relative terms, there is no mass penalty in producing progressively larger multiquarks, as in going from a meson to a tetraquark. This means that, in mass terms, it is equally probable to have an open-charm hexaquark as to produce a heptaquark or an octoquark.

To our knowledge, the only arrangements containing exclusively c and/or $\bar{c}$ quarks that have been experimentally obtained were the η_c, J/ψ [32], and X(6900) structures [12,33], the last one being detected in 2020 and confirmed in 2022. This means that the multiquarks whose masses are given in Table I are beyond today's experimental capabilities. We are then in a similar situation with respect to those clusters as in the case of fully heavy charmed tetraquarks in the 1980s, when their masses were remarkably well approximated by theoretical calculations [34,35] but there was no way to produce them experimentally. This has not been a deterrent to many theoretical groups, which have produced values for the masses for all the heavy tetraquarks with different compositions (see, for instance, Refs. [17,36] and references therein). The same can be said of the pentaquark masses calculated in Refs. [26–28]. In any case, there is no physical reason the clusters in Table I could not be produced in the future, and our results could be a guide to their possible masses and arrangements. In particular, these could be resonances that could (or not) decay into more fundamental structures and still be detected, as in the case of X(6900), thought to be an excited p-wave tetraquark whose mass exceeds by approximately 700 MeV that of a pair of J/ψ mesons.

ACKNOWLEDGMENTS

We acknowledge financial support from Ministerio de Ciencia e Innovación MCIN/AEI/10.13039/501100011033 (Spain) under Grant No. PID2020–113565GB-C22 and from Junta de Andalucía group PAIDI-205. We also acknowledge the use of the C3UPO computer facilities at the Universidad Pablo de Olavide.

[1] M. Gell-Mann, A schematic model of baryons and mesons, Phys. Lett. **8**, 214 (1964).

[2] G. Zweig, Developments in the quark theory of hadrons, Report No. CERN-TH-412, NP-14146, 1964.

[3] H. X. Chen, W. Chen, X. Liu, and S.-L. Zhu, The hidden-charm pentaquark and tetraquark states, Phys. Rep. **639**, 1 (2016).

[4] H. X. Chen, W. Chen, X. Liu, Y. R. Liu, and S.-L. Zhu, A review of the open charm and open bottom systems, Rep. Prog. Phys. **80**, 076201 (2017).

[5] H. C. Urey, F. G. Brickwedde, and G. M. Murphy, A hydrogen isotope of mass 2, Phys. Rev. **39**, 164 (1932).

[6] M. Bashkanov, C. Bargholtz, M. Berlowski, D. Bogoslawsky, H. Calen, H. Clement *et al.* (CELSIUS-WASA Collaboration), Double-Pionic Fusion of Nuclear Systems and the "ABC" Effect: Approaching a Puzzle by Exclusive and Kinematically Complete Measurements, Phys. Rev. Lett. **102**, 052301 (2009).

[7] P. Adlarson *et al.* (WASA-at-COSY Collaboration), Abashian-Booth-Crowe Effect in Basic Double-Pionic Fusion: A New Resonance?, Phys. Rev. Lett. **106**, 242302 (2011).

[8] P. Adlarson *et al.* (WASA-at-COSY Collaboration), Isospin decomposition of the basic double-pionic fusion in the region of the ABC effect, Phys. Lett. B **721**, 229 (2013).

[9] P. Adlarson *et al.* (WASA-at-COSY Collaboration), Measurement of the $pn \to pp\pi^0\pi^-$ reaction in search for the recently observed resonance structure in $d\pi^0\pi^0$ and $d\pi^+\pi^-$ systems, Phys. Rev. C **88**, 055208 (2013).

[10] P. Adlarson *et al.* (WASA-at COSY Collaboration and SAID Data Analysis Center), Evidence for a New Resonance from Polarized Neutron-Proton Scattering, Phys. Rev. Lett. **112**, 202301 (2014).

[11] A. Valcarce, H. Garcilazo, F Fernández, and P. González, Quark model study of few-baryon systems, Rep. Prog. Phys. **68**, 965 (2005).

[12] R. Aaij *et al.* (LHCb Collaboration), Observation of structure in the J/Ψ-pair mass spectrum, Sci. Bull. **65**, 1983 (2020).

[13] A. De Rújula, H. Georgi, and S. L. Glashow, Hadron masses in a gauge theory, Phys. Rev. D **12**, 147 (1975).

[14] R. K. Bhaduri, L. E. Cohler, and Y. Nogami, A unified potential for mesons and baryons, Nuovo Cimento Soc. Ital. Fis. **65A**, 376 (1981).

[15] C. Semay and B. Silvestre-Brac, Diquonia and potential models, Z. Phys. C **61**, 271 (1994).

[16] B. Silvestre-Brac, Spectrum and static properties of heavy baryons, Few-Body Syst. **20**, 1 (1996).

[17] M. C. Gordillo, F. De Soto, and J. Segovia, Diffusion Monte Carlo calculations of fully-heavy multiquark bound states, Phys. Rev. D **102**, 114007 (2020).

[18] M. C. Gordillo, F. De Soto, and J. Segovia, Structure of the X(3872) as explained by a diffusion Monte Carlo calculation, Phys. Rev. D **104**, 054036 (2021).

[19] M. C. Gordillo, F. De Soto, and J. Segovia, $x(3872)$'s excitation and its connection with production at hadron colliders, Phys. Rev. D **106**, 094004 (2022).

[20] J. M. Alcaraz-Pelegrina and M. C. Gordillo, Diffusion Monte Carlo calculations of fully heavy compact hexaquarks, Phys. Rev. D **106**, 114028 (2022).

[21] J. Vijande, F. Fernández, and A. Valcarce, Constituent quark model study of the meson spectra, J. Phys. G **31**, 481 (2005).

[22] M. H. Kalos, Energy of a boson fluid with Lennard-Jones potentials, Phys. Rev. A **2**, 250 (1970).

[23] J. Boronat and J. Casulleras, Monte Carlo analysis of an interatomic potential for He, Phys. Rev. B **49**, 8920 (1994).

[24] B. Hammond, W. Lester, and P. Reynolds, *Monte Carlo Methods in Ab Initio Quantum Chemistry* (World Scientific, Singapore, 1994).

[25] J. Sánchez-Baena, J. Boronat, and F. Mazzanti, Diffusion Monte Carlo methods for spin-orbit-coupled ultracold Bose gases, Phys. Rev. A **98**, 053632 (2018).

[26] G. Yang, J. Ping, and J. Segovia, Fully charm and bottom pentaquarks in a lattice-QCD inspired quark model, Phys. Rev. D **106**, 014005 (2022).

[27] Y. Yan, Y. Wu, X. Hu, H. Huang, and J. Ping, Fully heavy pentaquarks in quark models, Phys. Rev. D **105**, 014027 (2022).

[28] H.-T. An, S.-Q. Luo, Z.-W. Liu, and X. Liu, Fully heavy pentaquark states in constituent quark model, Phys. Rev. D **105**, 074032 (2022).

[29] H. Kim, K. S. Kim, and M. Oka, Hexaquark picture for $d^*(2380)$, Phys. Rev. D **102**, 074023 (2020).

[30] Q. Lü, D. Y. Chen, and Y. Dong, Fully-heavy hexaquarks in a constituent quark model, arXiv:2208.03041.

[31] J. M. Richard, A. Valcarce, and J. Vijande, Very Heavy Flavored Dibaryons, Phys. Rev. Lett. **124**, 212001 (2020).

[32] M. Tanabashi *et al.* (Particle Data Group), Review of particle physics, Phys. Rev. D **98**, 030001 (2018).

[33] H.-X. Chen, Wei Chen, Xiang Liu, Yan-Rui Liu, and S.-L. Zhu, An updated review of the new hadron states, Rep. Prog. Phys. **86**, 026201 (2023).

[34] J. P. Ader, J. M. Richard, and P. Taxil, Do narrow heavy multiquark states exist?, Phys. Rev. D **25**, 2370 (1982).

[35] A. Badalyan, B. Ioffe, and A. Smilga, Four-quark states in heavy quark systems, Nucl. Phys. **B281**, 85 (1987).

[36] H. An, S. Q. Luo, Z. W. Liu, and X. Liu, Spectroscopic behavior of fully heavy tetraquarks, Eur. Phys. J. C **83**, 740 (2023).