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- *Computational Statistics & Data Analysis, Volume 50, Issue 10, 2006, Pages 2655-2667*
- DOI: https://doi.org/10.1016/j.csda.2005.04.015

Available online at www.sciencedirect.com
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Computational Statistics & Data Analysis 50 (2006) 2655 – 2667
COMPUTATIONAL STATISTICS & DATA ANALYSIS
www.elsevier.com/locate/csda

Modelling the mean of a doubly stochastic Poisson process by functional data analysis

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Received 15 June 2004; received in revised form 21 April 2005; accepted 21 April 2005
Available online 23 May 2005

Abstract

A new procedure for estimating the mean process of a doubly stochastic Poisson process is introduced. The proposed estimation is based on monotone piecewise cubic interpolation of the sample paths of the mean. In order to estimate the continuous time structure of the mean process functional principal component analysis is applied to its trajectories previously adapted to their functional form. A validation of the estimation method is presented by means of some simulations.
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Keywords: Doubly stochastic Poisson process; Monotone piecewise cubic interpolation; Functional principal component analysis; Functional data analysis

1. Introduction

The doubly stochastic Poisson process (DSPP) or Cox process is a generalization of the Poisson process whose intensity is also a stochastic process influenced by another external one, instead of being constant (homogeneous Poisson process) or a function of time (non-homogeneous Poisson process). The only restriction of the intensity process is that it is

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[1] This work was partially supported by projects MTM2004-05992 of Dirección General de Investigación, and MTM2004-04230 of Plan Nacional de I+D+I, Ministerio de Ciencia y Tecnología jointly by the FEDER.

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doi:10.1016/j.csda.2005.04.015

nonnegative. The parametric function of the DSPP is its mean. In case of being absolutely continuous the mean is the integral of the intensity process so that it is also stochastic. The mean process does not have very restrictive properties except that it has nonnegative and nondecreasing sample paths.

Many attempts of estimating the mean and the intensity processes can be found in literature. For instance, Boel and Beneš (1980) and Snyder and Miller (1991) formulate several approaches using filtering methodology where it is always necessary to impose a fixed model on its moments. Other important studies about point processes and in particular about DSPP are Daley and Vere-Jones (1988), Brèmaud (1981), Last and Brandt (1995), Rolski et al. (2001) and Benning and Korolev (2002).

An attempt of relaxing the statistical assumptions was done by Bouzas et al. (2002) where it was proposed a methodology to forecast each sample path of a DSPP in a future instant of time by Multivariate Principal Component Regression without any restrictive statistical hypothesis. The expression of the probability mass function of the DSPP in that future time point was also derived but then we assumed a truncated Normal distribution for the mean.

Bouzas et al. (2004) proposed an estimation of the intensity process of a DSPP from the functional data analysis (FDA) point of view, just from observed sample paths of the DSPP. They were divided into subtrajectories and the intensity process was estimated in a finite set of points using a point estimator. Then, the stochastic structure of the intensity process was estimated by functional principal component analysis (FPCA) approximated from the estimated values.

In this paper we present a new line of study of a DSPP based on developing a continuous time modelling of the mean process of a DSPP from discrete-time observations of the DSPP on the basis that the mean process has simple analytical properties as second-order moments, quadratic mean continuity and nondecreasing monotonicity but without any previous statistical knowledge. Taking into account that sample data associated to the mean process are functions only observed at a finite set of time points, FDA methodologies can be used to model this stochastic process. A general overview about FDA methods can be found in Ramsay and Silverman (1997) and Valderrama et al. (2000). In order to reconstruct the time functional form of the sample paths, it is usual to interpolate them from their discrete-time observations. The usual interpolation functions are linear piecewise functions, cubic splines or wavelets or sinusoidal functions. As the mean process is nondecreasing it is necessary to impose this property to the reconstructed functional sample paths. Therefore, as the usual interpolation functions do not preserve monotonicity, we propose to use monotone piecewise interpolation (Frisch and Carlson, 1980) which we will adapt to our framework. This type of interpolation will duplicate the dimension of the data space so that we propose to use FPCA to reduce it by reconstructing the sample paths in terms of set of uncorrelated variables.

Opposite to Bouzas et al. (2002, 2004), in this paper we introduce a new approximation to FPCA of nondecreasing processes based on monotone piecewise cubic interpolation that is applied to estimate the continuous time structure of the mean of a DSPP from discrete-time pointwise estimations.

In this paper, we start from having observed several sample paths of the DSPP. We propose to cut them into subtrajectories as in Bouzas et al. (2004) and after that, we obtain discrete estimations of some trajectories of the mean by means of an adequate point estimator. This procedure of estimation of the mean sample paths is explained in Section 2. Then, the functional forms are reconstructed by monotone piecewise interpolation in Section 3. FPCA is applied to the reconstructed sample paths so that the mean process is expressed as an orthogonal expansion in terms of its principal components. FPCA is adapted to the monotone interpolated sample paths of the mean in Section 4. Finally, Section 5 is dedicated to test the estimation methodology. This section presents how to apply the estimation methodology using a simulation of a DSPP whose intensity process has uniform distributed random variables as marginals. An estimated error is also defined and calculated in the explained example as in some other simulations.

2. Estimation of the mean process in a finite set of time points

Formally, a DSPP $\{N(t); t \geqslant t_0\}$ with intensity process $\{\lambda(t, x(t)); t \geqslant t_0\}$ is defined as a conditional Poisson process with intensity process $\{\lambda(t, x(t)); t \geqslant t_0\}$ given the information process $\{x(t); t \geqslant t_0\}$. The parametric function of the DSPP is its mean and it is denoted by $\Lambda(t, x(t))$, where as it is well known $\Lambda(t, x(t)) = \int_{t_0}^{t} \lambda(\sigma, x(\sigma))\, d\sigma$, so that it is also a stochastic process as mentioned in the introduction. The mean process will be denoted along the paper by $\Lambda(t)$ in order to simplify the notation. We will consider $N(t_0) = 0$ and $\lambda(t_0, x(t_0)) = 0$ as it is not restrictive.

This section explains how to obtain an estimation of the mean sample paths, $\Lambda_\omega(t)$, in a finite set of values from observed data of the DSPP.

Let us now start from the initial situation of having observed k independent trajectories of the DSPP $\{N(t); t \geqslant t_0\}$ with intensity process $\{\lambda(t, x(t)); t \geqslant t_0\}$ in the interval $[t_0, t_0 + rT]$ denoted by

$$\{N_\omega(t) : \omega = 1, \ldots, k\}.$$

We will derive from the observed trajectories of $N(t)$, estimated values of other new k trajectories of the mean process, $\Lambda_\omega(t)$, in a finite set of time points.

Firstly, making use of the idea of splitting up the trajectories proposed in Bouzas et al. (2004), we will cut each of the k observed sample paths of $N(t)$ into pieces of amplitude T, so we cut in the following time points

$$T_0 = t_0 < T_1 = t_0 + T < T_2 = t_0 + 2T < \cdots < T_r = t_0 + rT.$$

Cutting the time interval into these subintervals, we can also cut each observed $N_\omega(t)$ into r shorter sample paths in $[T_{i-1}, T_i)$ denoted by $N_{\omega i}(t)$. These new sample paths are independent due to the independence of increments of the DSPP, therefore they can be interpreted as observed in the same shorter interval $[T_0, T_1) \equiv [t_0, t_0 + T)$. These new

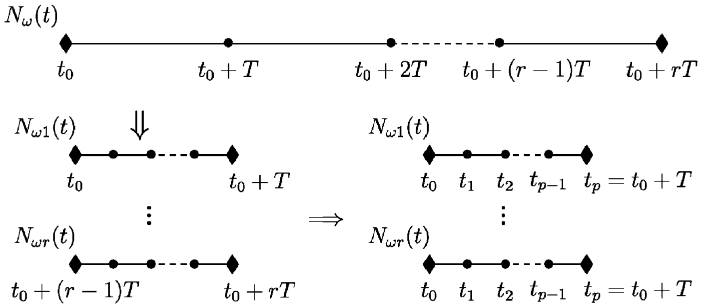

Fig. 1. Sketch of how to cut a sample path, $N_\omega(t)$, of the DSPP.

trajectories are defined as

$$N_{\omega i}(t) = N_\omega(t + (i-1)T) - N_\omega\left(t_0 + (i-1)T\right), \quad t \in [t_0, t_0 + T),$$

with $\omega = 1, \dots, k$ and $i = 1, \dots, r$.

We have split up each initial sample path of $N(t)$ in order to define a point estimator of the mean process at several time points of the interval $[t_0, t_0 + T)$ based on the sub-trajectories $N_{\omega i}(t)$, $i = 1, \dots, r$ of the initial ones. The point estimator for $\Lambda(t)$ is the natural estimator for the mean. Choosing a new partition in $[t_0, t_0 + T)$ defined by the knots

$$t_0 = t_0 < t_1 < \cdots < t_j < \cdots < t_p = t_0 + T, \tag{1}$$

the point estimator of the mean process in those time points is

$$\hat{\Lambda}_\omega\left(t_j\right) = \frac{\sum_{i=1}^{r} N_{\omega i}\left(t_j\right)}{r}, \tag{2}$$

for $j = 1, \dots, p$ and $\omega = 1, \dots, k$.

Therefore, we have obtained estimated values of k sample paths of the mean process $\Lambda(t)$ at $p + 1$ time points of the interval $[t_0, t_0 + T)$.

A sketch of how to cut the time interval and the trajectories in order to obtain the point estimations of the mean process can be seen in Fig. 1.

3. Monotone piecewise cubic interpolation of the mean process

This section presents the main contribution of the paper. Section 2 was preparatory to obtain discrete estimated values of the mean, in case that they cannot be observed directly. It showed how to estimate the mean sample paths $\hat{\Lambda}_\omega(t)$, $\omega = 1, \dots, k$, only at a finite set of discrete time points $t_0, \dots, t_p$. Let us observe that the mean $\Lambda(t)$ is a continuous time stochastic process so that its sample paths are functions only observed at a finite set of time points as it is usual in the case of functional data. In order to model the mean process sample paths by using FPCA (see next section) we will first reconstruct the functional form of the mean process sample paths by approximating them in a finite space generated by a basis of functions. In literature about FDA, different methods have been considered for obtaining the basis coefficients of sample paths, as for example, least squares approximation on a space generated by trigonometric functions (Aguilera et al., 1995), wavelets (Ocaña et al., 1998) or cubic B-splines (Escabias et al., 2004a) and cubic spline interpolation (Aguilera et al., 1996; Escabias et al., 2004b). For a general review of how to reconstruct the true functional form of functional data, the interested lector can be referred to Ramsay and Silverman (1997).

In this paper, an interpolation method will be considered in order to reconstruct the functional form of the mean process sample paths that are supposed to be observed without error. As the mean of a DSPP is a nondecreasing process, its interpolation needs to conserve the monotonicity. Therefore, we propose its reconstruction by using the monotone piecewise cubic interpolation introduced by Fritsch and Carlson (1980). We will adapt this methodology to our context by expressing the interpolating function in terms of a set of linearly independent functions in the whole interval $[t_0, t_0 + T)$.

Let us denote by $\Lambda_\omega\left(t_j\right)$ the observed value of a general mean sample path $\Lambda_\omega(t)$ at the time points t_j $(j = 0, \ldots, p)$; we should work with the estimated values $\hat{\Lambda}_\omega\left(t_j\right)$ if they cannot be directly observed. Fritsch and Carlson (1980) constructed the following $\mathscr{C}^1$ monotone piecewise cubic interpolating function for a set of monotone data $\{(t_0, \Lambda_\omega(t_0)), \ldots, \left(t_p, \Lambda_\omega\left(t_p\right)\right)\}$

$$p_{\omega j}(t) = \Lambda_\omega\left(t_j\right) H_1(t) + \Lambda_\omega\left(t_{j+1}\right) H_2(t) + d_{\omega j} H_3(t) + d_{\omega j+1} H_4(t), \quad \begin{array}{l} t \in \left[t_j, t_{j+1}\right] \\ j = 0, \ldots, p-1, \end{array} \tag{3}$$

where

$$d_{\omega j} = \left.\frac{\mathrm{d}p_{\omega j}(t)}{\mathrm{d}t}\right|_{t=t_j}, \quad d_{\omega j+1} = \left.\frac{\mathrm{d}p_{\omega j}(t)}{\mathrm{d}t}\right|_{t=t_{j+1}}$$

and $H_s(t)$ are the usual Hermite basis functions for the interval $\left[t_j, t_{j+1}\right]$ given by

$$H_1(t) = \phi\left(\frac{t_{j+1} - t}{h_j}\right), \quad H_2(t) = \phi\left(\frac{t - t_j}{h_j}\right),$$
$$H_3(t) = -h_j \psi\left(\frac{t_{j+1} - t}{h_j}\right), \quad H_4(t) = h_j \psi\left(\frac{t - t_j}{h_j}\right), \tag{4}$$

with $h_j = t_{j+1} - t_j$, $\phi(x) = 3x^2 - 2x^3$ and $\psi(x) = x^3 - x^2$. For every subinterval $\left[t_j, t_{j+1}\right]$, the derivatives d_j and d_{j+1} are previously calculated by an algorithm initiated with the values given by the standard three-point difference formula to be satisfactory for preserving the monotonicity (Fritsch and Carlson, 1980).

In order to estimate FPCA later on, this interpolation has to be expressed as a function for the whole interval $\left[t_0, t_p\right)$, so we rewrite the Hermite functions as

$$H_{j,1}(t) = \begin{cases} \phi\left(\dfrac{t_{j+1}-t}{h_j}\right), & t \in \left[t_j, t_{j+1}\right], \\ 0 & \text{otherwise}, \end{cases}$$
$$H_{j,2}(t) = \begin{cases} \phi\left(\dfrac{t-t_j}{h_j}\right), & t \in \left[t_j, t_{j+1}\right], \\ 0 & \text{otherwise}, \end{cases}$$
$$H_{j,3}(t) = \begin{cases} -h_j\psi\left(\dfrac{t_{j+1}-t}{h_j}\right), & t \in \left[t_j, t_{j+1}\right], \\ 0 & \text{otherwise}, \end{cases}$$
$$H_{j,4}(t) = \begin{cases} h_j\psi\left(\dfrac{t-t_j}{h_j}\right), & t \in \left[t_j, t_{j+1}\right], \\ 0 & \text{otherwise}. \end{cases} \tag{5}$$

Then, from Eq. (3) and the new functions of Eq. (5), the interpolation of the trajectory $\Lambda_\omega(t)$ becomes

$$I\Lambda_\omega(t) = \sum_{j=0}^{p} \Lambda_\omega\left(t_j\right)\left[H_{j,1}(t) + H_{j-1,2}(t)\right] + \sum_{j=0}^{p} d_{\omega j}\left[H_{j,3}(t) + H_{j-1,4}(t)\right], \tag{6}$$

with $t \in [t_0, t_0 + T)$ and $H_{-1,2}(t) = H_{-1,4}(t) = H_{p,1}(t) = H_{p,3}(t) = 0$. In order to simplify the notation, we rewrite again the functions given in Eq. (5) as

$$\Phi_j(t) = \begin{cases} \phi\left(\dfrac{t-t_{j-1}}{h_{j-1}}\right), & t \in \left[t_{j-1}, t_j\right], \\ \phi\left(\dfrac{t_{j+1}-t}{h_j}\right), & t \in \left[t_j, t_{j+1}\right], \end{cases} \quad j \neq 0, p$$

$$\Psi_j(t) = \begin{cases} h_{j-1}\psi\left(\dfrac{t-t_{j-1}}{h_{j-1}}\right), & t \in \left[t_{j-1}, t_j\right], \\ -h_j\psi\left(\dfrac{t_{j+1}-t}{h_j}\right), & t \in \left[t_j, t_{j+1}\right], \end{cases} \quad j \neq 0, p$$

$$\Phi_0(t) = \phi\left(\frac{t_1-t}{h_0}\right), \quad t \in [t_0, t_1],$$

$$\Phi_p(t) = \phi\left(\frac{t-t_{p-1}}{h_{p-1}}\right), \quad t \in \left[t_{p-1}, t_p\right],$$

$$\Psi_0(t) = -h_0\psi\left(\frac{t_1-t}{h_0}\right), \quad t \in [t_0, t_1],$$

$$\Psi_p(t) = h_{p-1}\psi\left(\frac{t-t_{p-1}}{h_{p-1}}\right), \quad t \in \left[t_{p-1}, t_p\right], \tag{7}$$

so that with these new functions, Eq. (6) is finally expressed as

$$I\Lambda_{\omega}(t)=\sum_{j=0}^{p}\Lambda_{\omega}\left(t_j\right)\Phi_j(t)+\sum_{j=0}^{p}d_{\omega j}\Psi_j(t),\quad t\in[t_0,t_0+T),\ \omega=1,\ldots,k. \tag{8}$$

It is easy to show that the new functions defined by Eq. (7) are the usual Lagrange basis of cubic splines. The expression of Eq. (8) for all the sample paths can be unified this way

$$I\Lambda(t)=A\,B(t),\quad t\in[t_0,t_0+T), \tag{9}$$

where

$$I\Lambda(t)=\left(I\Lambda_1(t),\ldots,I\Lambda_k(t)\right)^{\mathrm{T}},$$
$$B(t)=\left(\Phi_0(t),\ldots,\Phi_p(t),\Psi_0(t),\ldots,\Psi_p(t)\right)^{\mathrm{T}}, \tag{10}$$

$$A=\begin{pmatrix}\Lambda_1\left(t_0\right) & \ldots & \Lambda_1\left(t_p\right) & d_{10} & \ldots & d_{1p}\\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots\\ \Lambda_k\left(t_0\right) & \ldots & \Lambda_k\left(t_p\right) & d_{k0} & \ldots & d_{kp}\end{pmatrix}, \tag{11}$$

where $^{\mathrm{T}}$ denotes transpose.

In order to unify the notation, we will rename the elements of $B(t)$ and A defined in (10) and (11) as

$$B(t)=\left(B_1(t),\ldots,B_{p+1}(t),B_{p+2}(t),\ldots,B_{2(p+1)}(t)\right)^{\mathrm{T}}, \tag{12}$$

$$A=\left(a_{\omega l}\right)_{\omega=1,\ldots,k;\ l=1,\ldots,2(p+1)}. \tag{13}$$

Therefore, we can rewrite (8) as follows

$$I\Lambda_{\omega}(t)=\sum_{l=1}^{2(p+1)}a_{\omega l}B_l(t);\quad t\in[t_0,t_0+T),\ \omega=1,\ldots,k. \tag{14}$$

4. Stochastic estimation of the mean process

This section estimates the continuous stochastic structure of the mean process. We have reconstructed the continuous-time functional form of the mean trajectories by monotone piecewise cubic interpolation in Section 3. Now, FPCA will be applied to reduce dimension and derive an orthogonal expansion for the mean in terms of uncorrelated random variables. Let us observe that in this case reducing dimension is very important because the number of sample observations $(p+1)$ has been duplicated by the monotone cubic spline interpolation approximation $(2(p+1))$ coefficients.

By analogy with the multivariate case, the functional principal components of a second-order and quadratic mean stochastic process with sample paths in the space $L^2\left[t_0,t_p\right]$ of square integrable functions are defined as uncorrelated generalized linear combinations of the process variables whose weight functions (principal factors) are obtained as the eigenfunctions of the sample covariance kernel (Ramsay and Silverman, 1997). In order to obtain the principal components (p.c.'s) of a stochastic process with sample paths in a finite dimension space generated by a set of linearly independent functions, Ocaña (1996) proved a theorem that shows the equivalence between FPCA with respect to the usual inner product in $L^2\left[t_0, t_p\right]$ and standard multivariate PCA in $\mathbb{R}^{2(p+1)}$. In this paper, we have adapted this result to the case of monotone interpolated trajectories obtaining the following equivalence between multivariate and functional PCA's.

Let us consider the centered interpolated mean process

$$\overline{I\Lambda}(t) = I\Lambda(t) - \mu_{I\Lambda}(t) = \left(A - \bar{A}\right)\, B(t),$$

where $\bar{A} = (\overline{a}_{\omega l})$ with elements $\overline{a}_{\omega l} = \frac{1}{k}\sum_{\omega=1}^{k} a_{\omega l}\,(l = 1, \ldots, 2(p+1), \omega = 1, \ldots, k)$ and the $\mu_{I\Lambda}(t) = \bar{A}\, B(t)$.

Let $\left(A - \bar{A}\right)$ be the matrix whose rows are the coefficients of each trajectory of the centered process with respect to the basis $\left\langle B_1(t), \ldots, B_{2(p+1)}(t)\right\rangle$ and $\mathbb{P}$ the matrix whose components are the usual inner products between basis functions given by $\left\langle B_i, B_j\right\rangle_u = \int_{t_0}^{t_p} B_i(t)B_j(t)\,\mathrm{d}t$. Then, FPCA of $\overline{I\Lambda}_\omega(t)$ in the space generated by the basis $\langle B_1(t), \ldots, B_{2(p+1)}(t)\rangle$ with respect to the usual metric in $L^2\left[t_0, t_p\right]$ is equivalent to multivariate PCA of the data matrix $\left(A - \bar{A}\right)\, \mathbb{P}^{1/2}$ with respect to the usual inner product in $\mathbb{R}^{2(p+1)}$. Let us observe that in the case of an orthonormal basis ($\mathbb{P} = \mathbb{I}$) FPCA is equivalent to multivariate PCA of the basis coefficients data matrix.

Once the eigenvectors g_j of the covariance matrix of $\left(A - \bar{A}\right)\, \mathbb{P}^{1/2}$ have been computed, the sample paths of the interpolated process $\overline{I\Lambda}(t)$ can be represented as follows in term of its p.c.'s

$$\overline{I\Lambda}_\omega(t) = \sum_{j=1}^{2(p+1)} \zeta_{\omega j}\, f_j(t), \quad \omega = 1, \ldots, k,$$

where $f_j(t)$ are eigenfunctions of the sample covariance function of the mean process given by

$$f_j(t) = \sum_{l=1}^{2(p+1)} f_{lj}\, B_l(t)$$

with the vector of coefficients being $f_j = \mathbb{P}^{-1/2}\, g_j$ and

$$\zeta_{\omega j} = \int_{t_0}^{t_p} \overline{I\Lambda}_\omega(t)\, f_j(t)\,\mathrm{d}t = \left(A_\omega - \bar{A}_\omega\right)\, \mathbb{P}^{1/2}\, g_j,$$

where $\left(A_\omega - \bar{A}_\omega\right)$ is the ωth row of $\left(A - \bar{A}\right)$.

Then, the mean process $\Lambda(t)$ can be approximated by the following truncated orthogonal p.c.'s decomposition of its interpolation denoted by $\Lambda^q(t)$

$$\Lambda^q(t) \simeq I\Lambda^q(t) = \mu_{I\Lambda}(t) + \sum_{j=1}^{q} \zeta_j\, f_j(t). \tag{15}$$

This way, the dimension $2(p+1)$ is reduced to q so that we get to explain a high proportion of the total variance of the mean process (as next to 1 as possible) given by $\sum_{j=1}^{q} \lambda_j / \sum_{j=1}^{2(p+1)} \lambda_j$ where λ_j is the variance of the jth p.c. ζ_j given by the jth eigenvalue of the covariance matrix of $\left(A - \bar{A}\right) \mathbb{P}^{1/2}$ associated to the jth eigenvector g_j.

5. Simulation

This section presents how to apply the methodology of estimation of the mean to a simulation of a DSPP. It also provides an approximation to the mean-squared error of the estimation in order to test the presented procedure.

We have simulated 150 sample paths $(N_1(t), \ldots, N_{150}(t))$ of a Poisson process, $N(t)$, whose random intensity is a random variable with uniform distribution in [0, 1] each time of the interval [0, 300] (see Grigoriu (1995) for further details). The sample paths of $\Lambda(t)$ have been estimated as explained in Section 2. Each long trajectory of $N(t)$ in [0, 300] has been split up into 30 shorter ones on the interval [0, 10]. Therefore, from each original sample path $N_\omega(t)$ we obtain 30 subtrajectories $\{N_{\omega i}(t) : t \in [0, 10), i = 1, \ldots, 30, \omega = 1, \ldots, 150\}$. Therefore, we have fixed $k = 150$, $t_0 = 0$, $r = 30$ and $T = 10$ (see Section 2).

For each ω, the mean process has been estimated at a finite set of time points $t_j = j \times \frac{1}{2}$ $(j = 0, \ldots, 20)$ by means of the point estimator given by Eq. (2). That is, for each ω, a sample path of the mean, $\Lambda_\omega(t)$, has been estimated at 21 equally spaced time points on the interval [0, 10).

Firstly, taking into account that the mean is not directly observed (simulated) as in most real observed processes, the functional form of each of its 150 sample paths are reconstructed by monotone piece wise cubic interpolation as explained in Section 3 from the point estimations $\hat{\Lambda}_\omega\left(t_j\right)$. These interpolated functions, $I\hat{\Lambda}_\omega(t)$ $(\omega = 1, \ldots, 150)$ are linear combinations of the basis functions $\left\langle B_1(t), \ldots, B_{2(p+1)}(t)\right\rangle$ and their coefficients form the matrix A (see Eqs. (9)–(11)). Two examples of this type of monotone interpolation (sample paths number 75 and 150) can be seen in Figs. 2 and 3. As an example, some elements of the interpolation matrix $A_{150\times 42}$ for this case are presented

$$
\begin{array}{cccccccccc}
\Lambda_\omega(t_0) & \Lambda_\omega(t_1) & \Lambda_\omega(t_2) & \ldots & \Lambda_\omega(t_{21}) & d_{\omega 0} & d_{\omega 1} & d_{\omega 2} & \ldots & d_{\omega 21}
\end{array}
$$

$$
\begin{pmatrix}
0.200 & 0.366 & 0.600 & \ldots & 5.200 & 0.266 & 0.388 & 0.622 & \ldots & 0.800 \\
0.233 & 0.533 & 0.733 & \ldots & 5.770 & 0.700 & 0.480 & 0.560 & \ldots & 0.700 \\
0.200 & 0.566 & 0.900 & \ldots & 5.700 & 0.766 & 0.698 & 0.592 & \ldots & 0.133 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
0.266 & 0.433 & 0.666 & \ldots & 5.870 & 0.266 & 0.388 & 0.388 & \ldots & 0.103 \\
0.200 & 0.500 & 0.700 & \ldots & 5.030 & 0.700 & 0.480 & 0.457 & \ldots & 0.267 \\
0.300 & 0.600 & 0.866 & \ldots & 5.030 & 0.633 & 0.564 & 0.564 & \ldots & 0.467 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
0.367 & 0.633 & 0.767 & \ldots & 5.230 & 0.667 & 0.356 & 0.356 & \ldots & 0.400
\end{pmatrix}.
$$

Afterwards, FPCA has been applied to the interpolated trajectories (see Section 4). Once the p.c.'s ζ_j, the eigenfunctions $f_j(t)$ and the eigenvalues λ_j have been computed, it was found out that the first p.c. accumulates 79.81% of the total variance, the first two 91.01, the

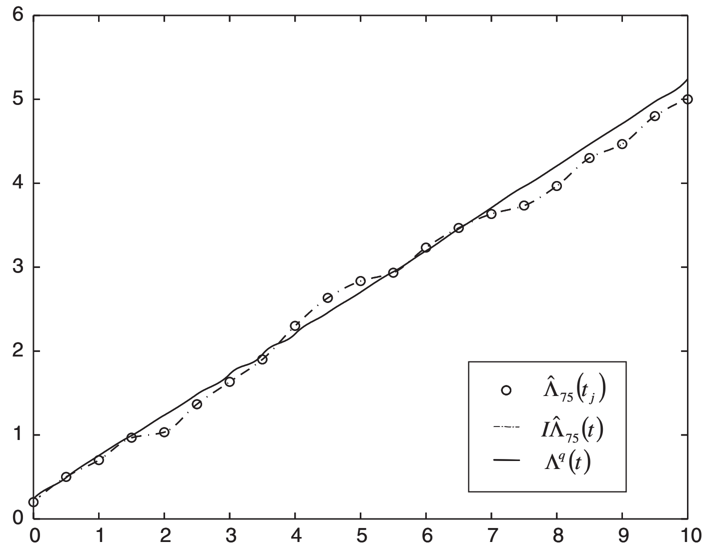

Fig. 2. Point estimations, monotone interpolation and FPC expansion with 4 p.c.'s of the mean process associated to sample path number 75.

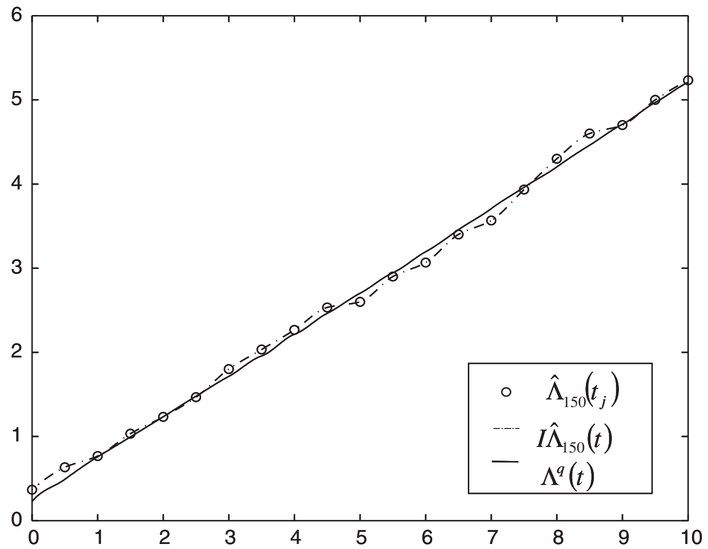

Fig. 3. Point estimations, monotone interpolation and FPC expansion with 4 p.c.'s of the mean process associated to sample path number 150.

first three 94.52 and the first four p.c.'s 96.06. Then, the mean process has been modelled in terms of the first four p.c.'s by means of the orthogonal expansion given by Eq. (15) with $q = 4$. The reconstruction of some of the mean process sample paths (number 75 and 150) next to their discrete time estimations and interpolations are also displayed in Figs. 2 and 3.

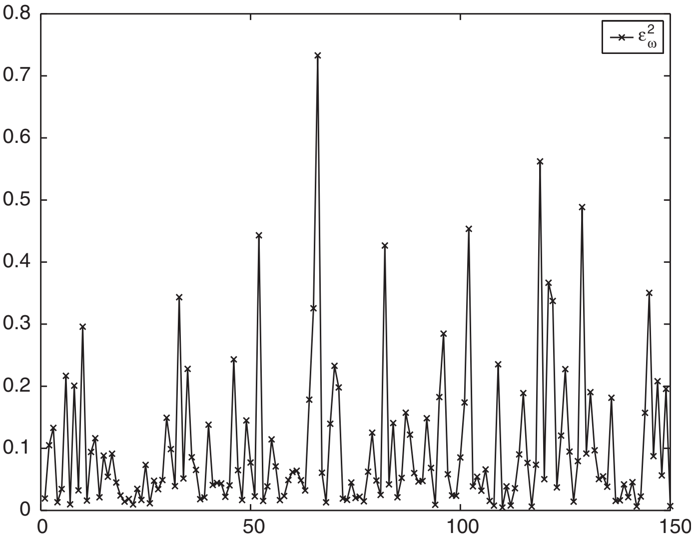

Fig. 4. Representation of mean squared errors of the FPC estimation.

In order to validate this methodology, we have evaluated the accuracy of the mean estimation provided by it. The usual way of doing it is to compare the estimated parameter with the real one used to obtain the simulation. The DSPP simulation is based on the intensity not on the mean process so we do not have the real mean to compare the estimation with it. Instead, we have defined the following approximated mean-squared error of estimation of $\Lambda(t)$ on the observed time points:

$$\varepsilon^2 = \frac{1}{k} \sum_{\omega=1}^{k} \varepsilon^2_\omega, \quad \text{where } \varepsilon^2_\omega = \frac{1}{p} \sum_{j=0}^{p} \left[\Lambda^q_\omega \left(t_j\right) - \hat{\Lambda}_\omega \left(t_j\right) \right]^2 \quad (\omega = 1, \ldots, k)$$

and $\hat{\Lambda}_\omega \left(t_j\right)$ is given by Eq. (2). The use of $\hat{\Lambda}_\omega \left(t_j\right)$ is justified because, as it is well known, this point estimator has optimal properties so that once it has been calculated over a big number of subtrajectories it can be supposed to be very close to the real value of $\Lambda_\omega \left(t_j\right)$. The errors ε^2_ω can be seen in Fig. 4. The global error ε^2 has become 0.1006 which is quite small, so we can conclude the goodness of the estimation.

We have chosen the intensity process of the simulation presented as an example to show the methodology. It could be thought that choosing other distributions for the intensity, the goodness of estimation might not be acceptable. In order to test the sensitivity of the proposed method with respect to the intensity distribution, we have considered other examples of intensity processes and we have come to the conclusion that the corresponding errors are also small. In order to have a similar situation except the type of intensity process so that the errors can be compared, we have simulated 150 sample paths of the DSPP in [0, 300] and considered $t_0=0$, $r=30$ and $T=10$ in every example and we have chosen FPC representation that accumulates at least 95% of the total variability. As in the case developed above, the mean discrete points are estimated, the mean sample paths interpolated and the FPCA calculated. The examples are the following.

Choosing the intensity process having marginals distributed as a gamma (0.5,1), the 150 nondecreasing mean sample paths grow from $I\hat{\Lambda}(t_0) = 0$ to $I\hat{\Lambda}\left(t_p\right) \in [4.0667, 6.7]$, the first 4 p.c.'s explain 95.96% of the total variance and the error ε^2 is 0.1388.

Having marginals distributed as a gamma (0.5,3), the 150 nondecreasing mean sample paths grow from $I\hat{\Lambda}(t_0) = 0$ to $I\hat{\Lambda}\left(t_p\right) \in [12.6, 18.6667]$, the first 4 p.c.'s explain 95.39% of the total variance and ε^2 is 0.6393.

Having marginals distributed as a gamma (3,1), the 150 nondecreasing mean sample paths grow from $I\hat{\Lambda}(t_0) = 0$ to $I\hat{\Lambda}\left(t_p\right) \in [26.7, 35.1333]$, the first 4 p.c.'s explain 96.12% of the total variance and ε^2 is 0.9483.

Having marginals distributed as a gamma (3,3), the 150 nondecreasing mean sample paths grow from $I\hat{\Lambda}(t_0) = 0$ to $I\hat{\Lambda}\left(t_p\right) \in [85.83333, 100.5333]$, the first 4 p.c.'s explain 95.34% of the total variance and ε^2 is 4.0886.

In the case that it is considered the time-dependent intensity $\lambda(t) = U\left(\sin^2 t + 0.5\right)$, where U is a random variable with uniform distribution in [0, 1] then the 150 nondecreasing mean sample paths grow from $I\hat{\Lambda}(t_0) = 0$ to $I\hat{\Lambda}\left(t_p\right) \in [3.7667, 6.5]$, the first 4 p.c.'s explain 95.89% of the total variance and ε^2 is 0.1068.

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