ONTOLOGICAL BELIEFS AND THEIR IMPACT ON TEACHING ELEMENTARY GEOMETRY

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This paper proposes a conceptual framework to classify ontological beliefs on elementary geometry. As a first application, this framework is used to interpret nine interviews taken from secondary school teachers. The interpretation leads to the following results: (a) the ontological beliefs vary in a broad range, denying the assumption that a similar education provokes analogue opinions; and (b) ontological beliefs have a remarkable influence on the standards of proofs and on the epistemological status of theorems, and also on the role of drawing, constructions and their descriptions, media, and model building processes.

Keywords: Geometry; Ontology; Secondary school; Teachers' beliefs

Creencias Ontológicas y su Impacto en la Enseñanza de la Geometría Elemental

Este artículo propone un marco conceptual para clasificar las creencias ontológicas sobre la geometría elemental. Como primera aplicación, este marco se utiliza para interpretar nueve entrevistas realizadas a profesores de secundaria. La interpretación conduce a los siguientes resultados: (a) las creencias ontológicas varían en un amplio rango, negando la suposición de que una educación similar provoca opiniones análogas; y (b) las creencias ontológicas tienen una influencia notable en los estándares de las pruebas y en el estatus epistemológico de los teoremas, así como en la función del dibujo, las construcciones y sus descripciones, los medios y los procesos de construcción de modelos.

Términos clave: Creencias de los profesores; Educación secundaria; Geometría; Ontología

In recent years, teachers' beliefs have become a vivid exploratory focus of mathematics education (Calderhead, 1996). The main reason for this interest is the assumption that "what teachers believe is a significant determiner of what gets thought, how it gets thought, and what gets learned in the classroom" (Wilson &

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Cooney, 2002, p. 128). Following this idea, this article concerns the impact of ontological beliefs on teaching elementary geometry at secondary school. Especially, we consider their subtle influence on the modalities how geometrical issues are thought, presented, and managed.

A Classification of Ontological Beliefs on Geometry

The ontological background of a theory can be described as the answer to the following questions: To what kind of objects does the theory refer and what are the basic assumptions the theory claims upon these objects? Insofar, ontology is split into a referential and a theoretical aspect. This idea can be specified on the base of a particular kind of philosophy of science which is called the structuralist theory of science, primarily established by Sneed (1979) and elaborated by Stegmüller (1985). To establish our classification of ontological beliefs, we will combine this approach with an investigation of Struve (1990), who adopted this theory to mathematics education to analyse the influence of textbooks. As a further source, the concept of geometrical working spaces is used, which was developed to classify students' handling of geometrical problems (Houdement & Kuzniak, 2001).

Following the structuralist theory of science, we assume that a non-trivial (more or less scientific) theory can be described by two components, namely by its system of axioms and by a set of intended applications (Stegmüller, 1985). By the set of axioms, the conceptual and propositional content of a theory is given; and by the set of intended applications, the referential aspect of the theory is determined. In the case of elementary geometry, the set of axioms normally corresponds to an axiomatization of classical Euclidean geometry. Concerning the set of intended applications, already in history of mathematics, its content was controversial. We will distinguish between three influential opinions, which seem to cover the whole range of geometrical ontology (Kline, 1983):

- In a formalistic view, geometry is seen as an uninterpreted calculus without any reference, that is, the set of intended application is regarded as empty.
- In an idealistic view, geometry refers to a world of ideal objects which fulfils the Euclidean axioms without any approximation and which do not belong to the physical world.
- In an applied view, geometry refers to physical objects, typically with some approximation. At school, the paradigmatic real-world objects elementary geometry is applied to are drawing figures, figures produced by Interactive Geometry Software (IGS), and physical objects of middle dimension like balls, dice, chambers, ladders, bridges, and churches — especially the ornaments of their windows—.

By this threefold distinction, the first step of our classification is given. It is only defined by a difference in the set of intended applications, taken a complete Euclidean geometry as a theoretical background for granted. To analyze teachers' or students' beliefs, this assumption is inappropriate, since their geometrical propositions may differ from the standards of an axiomatic Euclidean geometry. For this reason, we introduce a second distinction on the theoretical level, insofar as we discriminate between an axiomatic Euclidean theory and an empirical one. In the first case, the individual theory follows the mathematical standards of an axiomatic elementary geometry —possibly except some minor mistakes due to human fallibility—; in the latter case, the individual theory lacks these standards significantly and consists of geometrical assumptions which substantially differ from a scientific view and which may be at most locally ordered, fulfilling the inferential standards of everyday discussions.

For our investigation, it is not necessary to describe the differences in the content of an individual empirical theory of geometry and a Euclidean one. We are rather interested in the question how the ontological difference influences the way of treating geometry on a meta-level, which we have initially circumscribed by keywords like standards of proving, presenting objects, or applying geometry. We claim that the differences on this meta-level are independent from the specific content of an empirical theory and only determined by its status as an empirical one. The main influence on these issues is already indicated by choosing the expression "empirical theory" for theories which do not fulfil axiomatic standards. Due to the lack of an elaborated axiomatic background, these theories cannot be treated in a formalistic or idealistic manner, since they afford neither a coherent calculus nor the conceptual strength to describe a world of idealistic objects sufficiently. Therefore, theories like these have to be regarded as empirical ones, which can only be denoted as geometrical, since they share the same set of intended applications with an applied Euclidean geometry and since they are used for similar purposes —for instance for measurement, for calculating lengths, angles, and areas or for formulating general theorems containing common geometrical concepts—. To distinguish between these two types, we will call an applied geometry which is intended to have a complete axiomatic Euclidean background a rationalistic geometry and an empirical geometry without such a background an empiristic geometry. This is the second distinction of our classification. Figure 1 gives a complete overview.

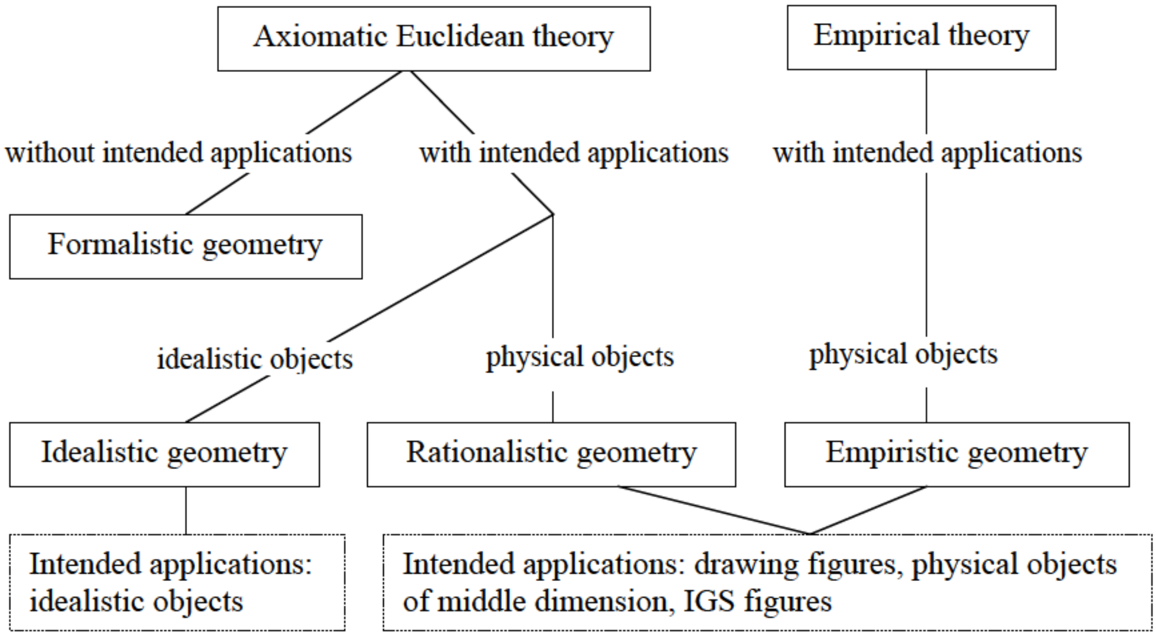

Figure 1. An ontological classification of elementary geometry

The expressions used in our classification obviously allude to $17^{th}/18^{th}$ century philosophy, not claiming to match the corresponding ideas of philosophers like Descartes, Locke, Hume, Leibniz, and Kant precisely, but claiming to follow their main intention that, from a rationalistic point of view, the basic assumptions of every empirical theory are independent from experience and, therefore, that they cannot be verified or falsified by experience, whereas an empiristic approach claims just the opposite: Basic assumptions are derived from experience and have to be verified or falsified by experience. This idea is elaborated within the theory of geometrical working spaces as a tool to analyse students' behaviour in the field of proving assumptions and solving problems, since it delimits the range of methods which is taken as allowed. We expand the ideas of Houdement and Kuzniak (2003) to our classification[1]. In Table 1 we summarize what roles typical aspects and methods of geometry play in respect of the ontological background.

[1] They only discriminate between three types of geometry, having no counterpart to our rationalistic one.

Table 1
Ontological Influences on Geometry

Formalistic geometry	Idealistic geometry	Rationalistic geometry	Empiristic geometry
Methods of proof and sources of knowledge			
Purely deductive, linked to axioms	Purely deductive, linked to axioms	Purely deductive, linked to axioms	Inferential arguments, experiments, intuitions
Role of experience, experiments, and measurement			
Heuristic	Heuristic	Heuristic and to identify geometrical objects	Basis of knowledge
Status of drawing			
Heuristic	Heuristic	An application of geometry	Objects of study and validation
Access to objects			
By relational or constructive descriptions	By relational or constructive descriptions	By experience and measurement	By experience and measurement
Experience			
Linked to a formal concept of space	Linked to an abstract Euclidean space	Linked to physical space, interpreted in Euclidean concepts	Linked to the measurable physical space without a predefined geometrical interpretation
Objects of intuition			
Internal to mathematics	Linked to idealistic figures	Linked to idealized real figures	Linked to perceptions

It is claimed that the content of Table 1 is a logical consequence of the different types of geometrical ontology; that is, the table is guided by the assumption that if someone possesses the ontological background mentioned in the sub-header of the table, it will be rational for him —and from an empirical point of view expectable— also to hold the statements in the below row. If this assumption is empirically traceable is one of our further tasks.

TEACHERS' ONTOLOGICAL BELIEFS ON GEOMETRY

Students' ontological beliefs on geometry are extensively investigated by two studies (Andelfinger, 1988; Struve, 1990). In our terms, they both end up in the result that students gain an empiristic view, assuming that the ontological background of teachers is a formalistic or idealistic one. In this article, the presupposition that teachers form a unified community of formalists and idealists is taken into question.

The empirical base of our investigation consists of semi-structured interviews taken from nine teachers of mathematics who are employed at German higher level secondary schools (so-called *Gymnasien*) and who teach mathematics from grade 7 to 13; that means that the age of their students ranges from 12 to 19 years. We refer to the teachers by the letters A to I. The aim of our whole investigation consists in the task of reconstructing the teachers' individual curricula of teaching geometry as subjective theories (Eichler, 2006). For this article, the results are restricted to ontological aspects. Subjective theories are defined as systems of cognitions containing a rationale which is, at least, implicit (Groeben, Wahl, Schlee, & Scheele, 1988). For this reason, the construct of subjective theories is a tool to reveal logical dependencies within the belief system of an individual. In our case, we are focussed on the dependencies between general ontological assumptions and the specific handling of geometry, guided by the following questions:

1. What types of ontological backgrounds occur according to our classification?
2. Do they lead to the consequences which are to be expected (see Table 1)?
3. Are there unexpected influences which do not seem to be accidental, but also implications of the ontological background?

Following our first question, we can conclude that every type of ontological background occur in our sampling. Our interpretation leads to the following classification (see Table 2).

Table 2
Ontological Classification of Teacher A to I

	Ontological background			
	Formalist	Idealist	Rationalist	Empirist
Teacher	A, I	B, D, F	C, G	E, H

We present here a single case study per category, and we restrict the empirical base to one significant phrase —all transcripts are translated by the author—. Mr. A's ontological background is a clearly formalistic one. He regrets that time limits him to implement it extensively:

Interviewer: What do you think of formalism in mathematics?

Mr. A: I loved it at university. It is pure reasoning… I would like to do such a thing [at school], but that is difficult, since I only teach four lessons a week… Five years back, when I had five lessons to teach, I did it and I did it gladly.

Mrs. D's answer provides an example of an idealistic view:

Mrs. D: The beauty of mathematics is the fact that everything there is logical and dignified… Everywhere else, there are mistakes and approximations, but not in mathematics. There is everything in a status in which it ideally has to be. [It is important] to recognize that there are ideal things and objects in mathematics and that, in reality, they are similar, but not equal.

It is interesting to note the subtle difference between Mrs. D and Mr. C below. Whereas Mrs. D stresses that mathematical objects are ideal and do not occur in reality, Mr. C refers to physical objects by geometrical terms without doubts, but emphasizes that some kind of abstraction is necessary, which indicates that he holds a rationalistic view of geometry, and not a empiristic one:

Mr. C: I make them [the students] search for shapes in reality and to prescind from them. Then this cone is a steeple or an ice-cream cornet… There are some basic shapes which are consistently occurring in life.

Since the difference between a rationalist and an empirist does not arise from a referential disagreement —they both refer to physical objects—, we omit a quotation concerning this issue and present two key phrases which show that this difference depends on the status of the geometrical theory:

Interviewer: What do you say if a student claims that he can see that something is as it is? Do you insist on a proof?

Mr. C: As far as classical proofs are concerned, I think: Yes, I do. If someone asserted in case of the Pythagorean theorem "By measuring, the theorem holds", then something of value would disappear… something which is genuinely mathematical… If geometry just consisted of measurement, calculation, drawing, and constructing, then I would regard it as meagre.

In the context of IGS and congruence, Mr. H refers to proofs. It is obvious that he allows experience and measurement to be bearers of knowledge. Insofar, he holds an empiristic view of geometry.

Interviewer: What is your experience with interactive geometry software?

Mr. H: It is possible to demonstrate and to prove many things by such software, for example Thales' theorem. We move the third point of the triangle on the arc of the circle and observe that it [the angle] always equals 90°, and we take this as a proof… All triangles are cut out and laid on top of another, and we observe that they are all equal… and we achieve the insight that three attributes are sufficient to construct the same triangle… Thereby, the concept of congruence is given. What does congruence mean? That means that something can be laid on something different without overlaps… We introduce π by measuring the circumferences of circles… That is more exact and more concrete for the students as if we went from a quadrangle to a pentagon, to a hexagon…, and sometime, we get an infinitygon, which we call a circle. [Using the latter method,] the aberrations are significant at the beginning, and it is difficult to draw a triacontagon… So, it is worth to ask if this method makes sense, since for students, it will be important to solve specific things. That won't have to be exact.

At a first result, we can conclude that the ontological beliefs of teachers are more divers than assumed by Struve (1990) and Andelfinger (1988). Especially, even the empiristic type which is supposed to be limited to students occurs twice in a sampling of nine individuals. It would be interesting, if a quantitative investigation could confirm this remarkable percentage. The claims in Table 1 are empirically detectable. Here we tried to choose quotations which make our assumption plausible and which should have shown that the ontological background is the crucial influence on the epistemological status of geometric theorems and, therefore, on the role of experience and measurement.

FURTHER INFLUENCES

The first part of our investigation was guided by a pre-defined hypothesis. Already in the quotations above, it is noticeable that ontological beliefs have an unexpected impact on further aspects of teaching geometry. For instance, Mr. H's students would presumably gain a physically defined notion of congruence and approximation and no elaborated concept of limits and irrational numbers. Unexpected impacts leads to theory construction. We will present our results in Table 3, not being able to establish our claims in detail. Instead, we will quote some unconnected episodes taken from different positions of our interviews to make our deliberations plausible and to consider the differences between a formalistic and idealistic view, which was of minor interest until now —arguing for the assumption that a community of idealists and formalist is a fiction—.

Table 3
Assumptions on the Ontological Impact on Aspects of Teaching Geometry

Formalistic geometry	Idealistic geometry	Rationalistic geometry	Empiristic geometry
	Purpose of proofs		
Verify the truth, reveal logical dependencies	Verify the truth, tools to remember content	Verify the truth	Make the truth of a sentence plausible
	Objects to prove		
General theorems	General theorems, attributes of objects	General theorems	Unclear
	Didactic aims of proving		
Argumentative abilities, insights in the nature of mathematics	Argumentative abilities, insights in the nature of mathematics	Argumentative abilities, insights in the nature of mathematics	Of minor interest
	Content of school mathematics		
Of minor importance, exchangeable in principle	Important entity to learn, large amount desirable	Important entity to learn, medium amount desirable	Is to restrict to practically useful topics
	Object studies		
Of minor, only didactic interest	Important task, no physical objects allowed	To learn the approximative use of geometrical concepts in real world situations	To achieve knowledge by experience
	Type of definitions		
According logical standards	According logical standards	According logical standards	Derived from experience

Table 3
Assumptions on the Ontological Impact on Aspects of Teaching Geometry

Formalistic geometry	Idealistic geometry	Rationalistic geometry	Empiristic geometry
Purpose of theories and axiomatization			
Objects of study and objects to achieve deductive abilities	Tools to describe mathematical objects, different approaches desirable	Tools to describe mathematical objects, of medium interest	Of minor interest, possibly as a tool to solve practical problems
Influence of IGS			
Decreases the insight in the necessity of proving	Allows complex constructions, identifies (in)adequate constructions	Identifies (in)adequate constructions	Additional source of mathematical knowledge, introduces motional aspects
Role of construction descriptions			
Of minor interest	Most important way to access objects	Of minor interest	Obsolete
Model building processes			
Motivation, occasions to learn further argumentative abilities	Contains an "unmathematical" way of thinking, didactical tool	Contains an "unmathematical" way of thinking	Important justification of teaching mathematics
Problem solving			
Train argumentative abilities	To train argumentative abilities	To train argumentative abilities	To link to real-world problems
Role of algorithms			
Tools and objects to justify	Tools and objects to justify	Tools and objects to justify	Tools

We observe remarkable differences between a typical formalist and a typical idealist in matters of content, axiomatization, constructions, model building, and IGS.

Mr. A: It doesn't matter what content we teach. The most important thing is that it is mathematics. The essence of mathematics can be found in every part of it: this consistency… The necessity of proof is reduced by IGS, since there are always 90° [in case of Thales' theorem…] I want that students solve complex problems in larger contexts… and that they justify algorithms… Concerning analysis and probability theory, there are many things which cannot be proved [at school], and in geometry, I don't see this at all… Arguing, thinking in conceptual hierarchies, problem solving, and model building —these are the higher goals in my view—.

Mr. B: On the way from a real situation to a mathematical model,… an argumentation arises which was untypical for teaching mathematics until now… I regard problem solving as a very important part of geometry,… whereas describing the real world is not in the first place… There are some very challenging constructions, but with IGS, there is no problem… In an optimistic view, I expect that, after school, a student copes with the complete mathematical contents and methods of secondary school.

This summarization of short episodes may illustrate why we have chosen the topics and assumptions mentioned in Table 3. From a meta-level, the differences between formalists and idealists seem to arise from the ontological attitude that an idealist is more interested in (idealistic) objects and their properties and constructions, whereas a formalist stresses theories, conception, and deductions, which opens an access to general abilities in the field of argumentations, problem solving, and model building.

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