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Estudio de vigas metálicas en I cargadas excéntricamente respecto del plano del alma

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RESUMEN

La Tesis titulada "*ESTUDIO DE VIGAS METÁLICAS EN I SOLICITADAS POR CARGAS TRANSVERSALES EXCÉNTRICAS RESPECTO DEL PLANO DEL ALMA*" profundiza en el conocimiento de la respuesta de vigas metálicas en I solicitadas a cargas concentradas transversales que actúan excéntricamente respecto del plano del alma. En todo este estudio se ha considerado que la carga está aplicada a nivel de la cara exterior del ala superior.

El problema sobre el que trata esta tesis es bastante complejo de abordar matemáticamente ya que las vigas metálicas de débil espesor sometidas a cargas excéntricas muestran, aún para valores bajos de la carga aplicada, una respuesta claramente no lineal tanto geométrica como del material.

El estudio llevado a cabo en este trabajo ha sido pionero en este campo, que nunca anteriormente había sido objeto de una investigación exhaustiva.

Las escasas referencias existentes sobre el tema que versa esta tesis han puesto de manifiesto que el comportamiento, el modo de fallo y la carga última de una viga metálica en I sometida a cargas concentradas transversales al plano del alma y actuando con una excentricidad respecto de éste, dependen de múltiples factores, entre los que cabe destacar, por su importancia, los siguientes: parámetros geométricos (dimensiones de la viga y relaciones entre ellas), excentricidad a la que actúa la carga, longitud a través de la cual se aplica la carga y forma en la que ésta actúa (lineal o lateralmente distribuida).

Los ensayos que con anterioridad a esta tesis se llevaron a cabo sobre este tema pusieron de manifiesto que, en la mayoría de los casos, el modo de fallo excéntrico era sustancialmente diferente al modo de fallo centrado. En el caso de colapso excéntrico, se había observado que la carga de agotamiento de la viga se reducía a medida que aumentaba la excentricidad. Sin embargo, al ser muy escasa la base de datos existente, no había sido posible delimita los modos de fallo centrado y excéntrico que se presentan en vigas solicitadas por cargas transversales excéntricas.

Tampoco con anterioridad a la publicación de los resultados obtenidos de esta investigación se había formulado un procedimiento fiable –basado en resultados experimentales y resultados numéricos obtenidos a partir de modelos calibrados- para estimar la carga de agotamiento de vigas metálicas sometidas a cargas excéntricas respecto del plano del alma.

Para abordar el problema objeto de esta tesis ha sido necesario desarrollar una extensa campaña experimental: se han ensayado hasta rotura más de un centenar de vigas metálicas sometidas a cargas concentradas centradas y excéntricas. La campaña experimental se desarrolló en dos fases: una campaña inicial llevada a cabo en 2007, "Ekscentro 2007" (parte esencial de esta tesis puesto que estos ensayos fueron pioneros en el estudio de vigas metálicas en I cargas excéntricamente) y otra posterior que se planificó para complementar la campaña del 2007, en la que se abordaban los rangos de parámetros no ensayados anteriormente.

Para cubrir los casos que no han podido ser ensayados en laboratorio, por razones económicas, se ha recurrido a la modelización mediante Elementos Finitos. Estos modelos se han calibrado en base a los resultados experimentales.

Las normativas de estructuras metálicas recogen una formulación para obtener la carga última de vigas metálicas en I sometidas a cargas transversales en el plano del alma. Teniendo en cuenta la existencia de esta formulación, en esta tesis se ha definido un factor de reducción que relaciona la carga última de una viga cargada excéntricamente con la carga última correspondiente a la misma viga cargada en el plano del alma. La expresión propuesta para este factor de reducción se ha deducido aplicando la técnica de mínimos cuadrados ponderados.

Durante el desarrollo de esta investigación se ha comprobado que existe un parámetro fundamental que afecta al valor de la solicitación última de agotamiento: *la longitud sobre la carga es transmitida*. Este valor no había sido anteriormente introducido en las formulaciones desarrolladas para obtener la carga última de vigas metálicas cargadas excéntricamente. En esta tesis se ha obtenido un coeficiente de corrección, también a partir de mínimos cuadrados ponderados, que tiene en cuenta la influencia de la longitud de la carga en el valor de la carga de colapso de la viga.

Por último, se ha aplicado la técnica de Redes Neuronales Artificiales (Artificial Neural Networks) a este problema específico, con el objetivo de pronosticar los modos de colapso y la carga última de la viga.

Key words: *Estructuras metálicas,*
Vigas en I de débil espesor,
Cargas concentradas excéntricas ,
Influencia de la longtitud de la carga,
Modo de colapso
Pronóstico mediante redes neuronales

INTRODUCCIÓN

Los problemas relacionados con cargas concentradas se presentan con frecuencia en la práctica ingenieril. Las vigas metálicas de débil espesor a menudo están solicitadas por una carga concentrada, que actúa sobre una pequeña longitud o área del elemento estructural. El caso más habitual corresponde al caso de una compresión que actúa a nivel del ala superior; esta compresión comprime el alma de la viga en una región localizada debajo del punto de aplicación de la carga. Ver *Figura 1.1*.

En la práctica constructiva esta situación se presenta en numerosas situaciones, tales como:

- cargas transmitidas a elementos estructurales principales por elementos secundarios (p.e. vigas en una dirección que apoyan en vigas perpendiculares);
- uniones viga-columna metálicas no rigidizadas;
- extremos apoyados de vigas sin rigidizar;
- vigas carril para puentes grúa;
- vigas de puentes en la fase de lanzamiento;
- vigas transversales de puentes, sobre pilas y pilotes, durante la elevación del puente durante las tareas de revisión o mantenimiento, etc.

En las inmediaciones de la zona en la que la carga es aplicada aparecen tensiones y deformaciones. Estas tensiones y deformaciones no influyen en el análisis global de la estructura y no se consideran en la teoría de flexión lineal elástica. Sin embargo, estas tensiones y deformaciones pueden originar fenómenos de inestabilidad local en el elemento que pueden llevar al colapso total de la estructura.

La manera de evitar estos fenómenos de inestabilidad local es simple: colocar rigidizadores transversales en el alma de la viga en la localización en la que actúa la carga concentrada. Estos elementos auxiliares tienen como misión principal impedir el pandeo del alma, evitando así el colapso de la viga. Sin embargo, desde el punto de vista práctico, los rigidizadores transversales son elementos no deseados ya que suponen un obstáculo para la automatización de la producción de la estructura y ralentizan la construcción principalmente por el tratamiento anticorrosivo al que han de ser sometidos. En situaciones de carga estática, la disposición de rigidizadores transversales, aunque con los inconvenientes anteriormente señalados, es posible; sin embargo, la disposición de rigidizadores en el punto de aplicación de la carga no es posible en el caso de que la carga concentrada sea móvil, tal como sucede en la viga carril de puentes grúa y en las vigas de puentes, y en este caso es necesario hacer un estudio detallado del fenómeno de inestabilidad local objeto de esta tesis.

Aunque se han llevado a cabo algunas investigaciones teóricas y experimentales sobre el tema de las cargas concentradas en vigas metálicas de débil espesor, aún queda mucho por investigar en este campo. La determinación de la capacidad última de carga es particularmente interesante desde el punto de vista conceptual ya que se desarrollan tensiones y deformaciones elasto-plásticas y en este fenómeno la no linealidad geométrica se manifiesta incluso para valores muy bajos de la carga.

El comportamiento de vigas metálicas en I sometidas a cargas concentradas se puede dividir en los siguientes casos característicos, representados en la *Figura 1.1*:

- En función de la *posición de la carga*:
  - Carga centrada – que actúa en el plano del alma-, y
  - Carga excéntrica – que actúa con cierta excentricidad respecto del plano del alma-.

- En función del *carácter de la carga*:
  - Carga estática, y
  - Carga dinámica.

En la práctica resulta muy difícil cargar un elemento a nivel del eje del alma, particularmente cuando la carga es móvil. La situación más frecuente en la práctica constructiva corresponde a que la carga esté aplicada con cierta excentricidad respecto del plano del alma. Es de sumo interés determinar si esta inevitable excentricidad va a afectar al comportamiento, al modo de colapso y al valor de la carga última de la viga o si, por el contrario, esta excentricidad puede ser ignorada puesto que su influencia es poco importante en la respuesta estructural del elemento metálico en I.

Experimentalmente se ha comprobado que la respuesta del elemento no sólo va a depender del valor de la excentricidad, sino que otros factores tales como la geometría de la viga y la forma en la que la carga es aplicada van a afectar al comportamiento estructural de la viga. Por tanto, la combinación de todos estos factores ha de ser analizada.

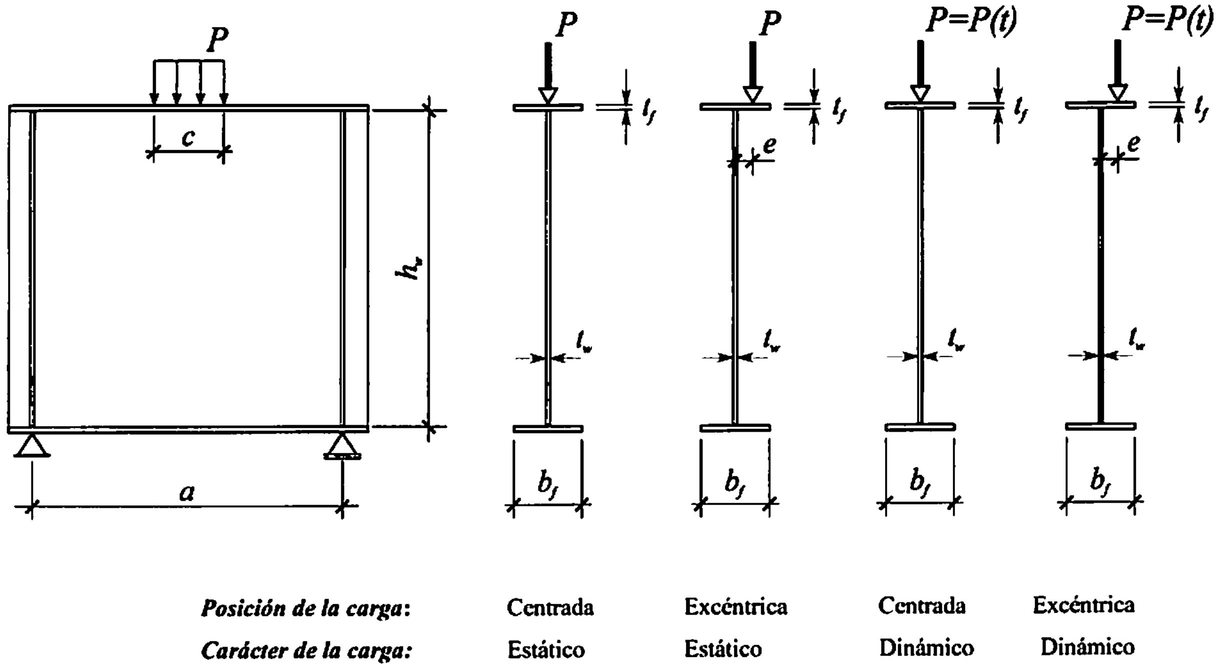

Figura 1.1 *- Casos característicos de cargas concentradas en vigas metálicas en I de débil espesor.*

La investigación sobre cargas concentradas en vigas metálicas de débil espesor (concretamente vigas en I, que son las más habituales en construcción) se inició en la segunda mitad del siglo XIX. Si embargo, no fue hasta los últimos cuarenta años del siglo XX cuando la investigación en este campo se intensificó, debido a la tendencia de racionalización de las estructuras de acero.

Mientras que algunos aspectos relativos a las cargas concentradas en vigas metálicas se han analizado a fondo otros, sin embargo, a penas han sido estudiados; en concreto, la situación de carga excéntrica respecto del plano del alma nunca anteriormente había sido investigada en profundidad hasta ahora.

Los primeros estudios de vigas metálicas en I excéntricamente cargadas se iniciaron a finales de 1980. Las escasas investigaciones que sobre el tema se han realizado en las últimas tres décadas han contribuido significativamente al conocimiento del problema pero aún quedan sin resolver cuestiones sobre el comportamiento, el modo de fallo y la carga de agotamiento de la viga.

La revisión del estado del arte llevada a cabo en este trabajo de investigación ha puesto de manifiesto la existencia de varias líneas de investigación, conectadas entre sí:

- *Investigación experimental:*

Los primeros ensayos de vigas metálicas en I se llevaron a cabo en Estados Unidos, a finales de la década de 1980 (Elgaaly and Nunan, 1989 [77] and Elgaaly and Salkar, 1990 [88]). Casi simultáneamente, en Czechoslovakia (Drdacky, 1989 [76]) se ensayaban en laboratorio vigas en I excéntricamente cargadas. Diez años después una nueva campaña experimental se inició en Montenegro, Yugoslavia (Lučić, 1998 [111] and Lučić, Šćepanović, 2001 [217]).

La base de datos completa correspondiente a las investigaciones experimentales anteriormente mencionadas, que consistía en 83 especimenes, ha sido considerada para el desarrollo de esta tesis. Obviamente, teniendo en cuenta la complejidad del problema y los numerosos parámetros que influyen en el mismo, estos 83 ensayos no eran suficientes para aclarar todas las cuestiones relativas al comportamiento de vigas en I bajo carga excéntrica.

Por este motivo, en la Facultad de Ingeniería Civil en Podgorica (Montenegro) se llevó a cabo otra extensa campaña experimental, "Ekscentro 2007". En esta ocasión se ensayaron 102 vigas metálicas. Esta campaña de ensayos ha sido parte esencial en esta investigación de doctorado. Tanto los resultados, como el desarrollo de la campaña experimental así como las conclusiones de la misma constituyen una parte fundamental de este trabajo de investigación.

- *Modelización con Elementos Finitos (FEM):*

Los elementos finitos han demostrado ser una herramienta válida para modelizar este tipo de problemas [180, 191] y obtener resultados fiables sobre deformaciones y cargas. La modelización permite obtener información sobre modelos no ensayados. Para calibrar el modelo algunas de las vigas ensayadas se modelizaron y los resultados obtenidos numéricamente se compararon con los experimentales para validarlos. Para los modelo estudiados se llevaron a cabo análisis no lineales. La modelización de la viga se realizó con elementos que permitían grandes deformaciones y grandes desplazamientos. La plasticidad se incluyó mediante una regla de endurecimiento cinemático y la ley tensión-deformación del acero que se introdujo en el modelo correspondía a la obtenida en laboratorio para el acero ensayado.

En Montenegro (2002) las modelizaciones de Elementos Finitos se llevaron a cabo mediante el software SAP2000 (Lučić and Šćepanović, 2002 [133]) mientras que en España (2005) se utilizó el programa ANSYS (Gil Martín et all. [181]).

- *Formulación para el cálculo de la carga última :*

Son dos las posibles maneras de determinar una formulación para la carga última de vigas metálicas sometidas a cargas transversales: ajustar una formulación basada en resultados experimentales o deducir una formulación teórico-matemática para el mecanismo de colapso observado en laboratorio.

En esta tesis, se ha optado por la primera opción y se ha ajustado la formulación a partir de los resultados experimentales, completados con datos obtenidos a partir de modelos numéricos de elementos finitos calibrados.

La primera expresión empírica publicada para la obtención de la carga última de vigas excéntricamente cargadas -la única que existía hasta ahora- estaba basada en los ensayos experimentales llevados a cabo en la década de los 80 [206]. Posteriores resultados experimentales obtenidos en 1998 [111] y 2001[217] pusieron de manifiesto la necesidad de revisión y modificación de esta expresión.

Una primera modificación de esta formulación se llevó a cabo en la Universidad de Granada, con la colaboración de los profesores Gil Martín y Hernández Montes, [178]. Experimentos posteriores en la campaña de " Ekscentro 2007 " hicieron necesaria una segunda revisión de las expresiones empíricas existentes, [191].

La segunda opción, consistente en una definición teórico-matemática del mecanismo de colapso en vigas de acero en I cargadas excéntricamente no se ha desarrollado en esta tesis puesto que este mecanismo no ha podido ser identificado con precisión en laboratorio.

- *Modelos de pronóstico con redes neuronales artificiales (ANN):*

El problema de definición del modo de colapso y la determinación de la carga última de vigas metálicas cargadas excéntricamente corresponden a situaciones muy específicas, en las que influyen numerosos parámetros que interactúan entre sí. En estas circunstancias, se ha comprobado que las redes neuronales artificiales constituyen una herramienta adecuada para la creación de modelos de pronóstico fiables.

La aplicación de las redes neuronales artificiales al problema sobre el que versa esta tesis se puso en práctica durante la preparación de la investigación experimental "Ekscentro 2007", [189, 190, 194, 218]. Posteriormente, cuando se obtuvieron los nuevos resultados experimentales, fue posible crear nuevas redes que mejoraron los modelos de predicción.

Todas las direcciones de investigación indicadas en los puntos anteriores, que se han presentado como independientes unas de otras, están conectadas entre sí. Los experimentos son la base tanto de la formulación de carga última de la viga como de los modelos analizados (FEM y RNA) y la validez y calidad de los modelos numéricos y de las expresiones matemáticas deducidas se deducen a partir de la comparación de sus resultados con los valores experimentales.

JUSTIFICACIÓN Y OBJETIVOS

Las cargas concentradas en estructuras metálicas se refieren a cargas que actúan sobre una longitud o área muy reducidas de un elemento estructural. Esta situación es muy frecuente en ingeniería y se suele presentar cuando el ala superior de la viga está sometida a cargas que comprimen el alma de la viga en las proximidades al punto de aplicación de la carga.

Las normativas vigentes de Estructuras Metálicas, tales como el Eurocódigo 3 o la norma española EAE, incluyen una formulación para verificar la estabilidad del alma de vigas en I solicitadas por cargas transversales en el plano del alma. La formulación, que se formula como una verificación de pandeo, es la envolvente de los tres mecanismos de fallo observados: aplastamiento, abolladura localizada y abolladura del alma. Sin embargo, esta formulación sólo es aplicable al caso de carga centrada y, por tanto, las normativas en vigor "obligan" a rigidizar la sección transversal cuando la carga está aplicada con alguna excentricidad respecto del plano del alma –puesto que no incluyen formulación para verificar la estabilidad del alma de la viga en esta situación-.

En la práctica, sin embargo, una pequeña excentricidad relativa al plano del alma es inevitable y, por tanto, es necesario abordar el estudio de la influencia de esta excentricidad en la respuesta estructural del elemento metálico.

Desde el punto de vista teórico resulta evidente que el modo o mecanismo de colapso de una viga cargada excéntricamente será diferente al caso centrado y que la carga última se reducirá a medida que aumenta la excentricidad debido a que los momentos torsores locales serán mayores. Resulta, pues, de gran interés práctico determinar el modo de colapso y la reducción de la carga última para el caso de vigas cargadas excéntricamente.

Para facilitar la aplicación técnica de los resultados de esta tesis, se ha definido un coeficiente de reducción, R, que relaciona las cargas últimas de una viga excéntricamente cargada con la correspondiente a la misma viga cuanto dicha excentricidad es nula. De esta manera, el ingeniero podrá determinar la carga máxima a partir de la normativa de estructuras metálicas y posteriormente aplicar los coeficientes aquí desarrollados para introducir el efecto de la excentricidad y de la longitud de la carga.

La única expresión existente para el factor de reducción con anterioridad a esta investigación, estaba basada en los resultados experimentales llevados a cabo en los años 80, y fue desarrollada por Galambos [206]. Esta expresión definía R en función de dos parámetros geométricos relevantes: e/b_f y t_f/t_w (siendo e la excentricidad de la carga, b_f el ancho del ala sobre la que se aplica la carga y t_f y t_w los espesores del ala y alma, respectivamente).

Sin embargo, ensayos posteriores llevados a cabo - que cubrían un rango más amplio de parámetros geométricos así como diferentes combinaciones de estos parámetros- pusieron de manifiesto la necesidad de revisión de esta expresión. Se comprobó, además, que existen otros factores - que no habían sido considerados en la formulación - tales como la distancia entre rigidizadores transversales, *a*, y la longitud a través de la cual la carga es aplicada, *c*, que influyen significativamente en el valor de la carga de agotamiento de vigas metálicas en I cargadas excéntricamente.

Las mayores discrepancias entre los ensayos y la formulación de Galambos aparecían para valores grandes de la excentricidad, por lo que en esta tesis se ha ampliado el rango de excentricidades estudiadas, habiéndose considerado valores de la excentricidad de hasta 30 mm.

En este trabajo de investigación se pretendía ajustar una formulación para el factor de reducción de la carga por efecto de la excentricidad, *R*, que tuviera en cuenta los resultados de todos los ensayos disponibles y que abordara, pues, un mayor rango de parámetros geométricos, valores de la excentricidad, modo de aplicación de la carga y longitud de ésta. Para ello ha sido necesario desarrollar una amplia campaña experimental, completada con modelos numéricos calibrados de elementos finitos.

Experimentalmente se ha comprobado que el mecanismo de fallo de vigas metálicas cargadas excéntricamente difiere de los tres mecanismos observados para cargas centradas. En esta tesis se ha pretendido definir y caracterizar los mecanismos de fallo observados para, en investigaciones posteriores, identificarlos teóricamente y ajustar una formulación matemática para los mismos.

Resumen De Los Resultados Obtenidos

Puesto que la campaña experimental llevada a cabo en Montenegro ha sido pionera en la investigación de cargas excéntricas concentradas en vigas en I, los resultados parciales que se iban obteniendo a medida que avanzaba la campaña, consistentes en: la descripción de los ensayos, de los modos de fallo observados y de los valores de las cargas últimas de agotamiento se han ido publicando en congresos específicos (ANEXO). Estas ponencias han resultado muy valiosas puesto que han permitido que otros investigadores sobre el tema aportaran ideas, que se han tenido en cuenta para la planificación de los ensayos posteriores.

Algunos resultados parciales obtenidos de aplicar la técnica de Redes Neuronales Artificiales también han sido recogidos en ponencias de congresos (ANEXO).

La descripción del grueso del ensayo así como una primera recopilación de la información obtenida hasta la fecha de los experimentos, fue publicada en la revista *Journal of Constructional Steel Research* (ANEXO). Esta revista es la única publicación específica de acero que existe en el SCI.

La campaña experimental ha permitido verificar que, aunque son muchos los parámetros que intervienen en la respuesta estructural de vigas metálicas sometidas a cargas transversales excéntricas, los parámetros relevantes son dos: el valor de la excentricidad, e, y el espesor del alma de la viga, t_w.

Una aportación fundamental de esta tesis ha sido la obtención de una expresión empírica para el factor R, coeficiente que considera el efecto de la excentricidad en el valor de la carga última. Esta expresión se ha obtenido a partir de un análisis de regresión de los resultados obtenidos de los ensayos llevados a cabo en Montenegro, más de un centenar de vigas, y de modelos numéricos numéricos calibrados que se han desarrollado. Esta nueva expresión se define, como la de Galambos, en función del parámetro e/b_f e incluye un nuevo parámetro $t_f \cdot a^B / t_w$, donde a es la separación entre rigidizadores transversales y B es una constante. Esta nueva expresión resulta mucho más aproximada que la propuesta por Galambos, especialmente para valores grandes de la excentricidad.

El desarrollo de la formulación para R así como la verificación de su validez en base a la comparación con los resultados existentes ha sido recogido en la publicación de *Engineering Structures* (ANEXO). Esta revista está dedicada a la ingeniería estructural y aparece en el SCI.

Hasta ahora, las expresiones para obtener el factor de reducción de resistencia asociado a la excentricidad, *R*, no tenían en cuenta el efecto de la longitud a través de la cual la carga es aplicada. En esta tesis se ha estudiado la influencia de la longitud de transmisión de la carga y se ha comprobado que ésta juega un papel muy importante en el valor de la carga última de agotamiento de la viga cuando el factor t_f/t_w –relación entre los espesores de ala y alma- es pequeño.

A partir de los ensayos y de los modelos numéricos de Elementos Finitos calibrados, se ha podido determinar que para valores de $t_f/t_w \leq 1.5$ el valor de la longitud de la carga, *c*, tiene un efecto importante en el valor del coeficiente de reducción de resistencia por efecto de la excentricidad. En esta tesis se han analizado valores de *c* de: 50, 100, 150 y 200 mm (los valores de *c* no ensayados en laboratorio se han modelizado mediante Elementos Finitos).

Se ha comprobado que el valor del coeficiente *R*, que considera el efecto de la excentricidad, aumenta con el valor de la longitud de la carga. Por tanto, cuando la carga se aplica a través de una longitud suficientemente grande, el valor obtenido para la solicitación de agotamiento será muy conservador - lo que resultará antieconómico- y es necesario corregirlo.

El desarrollo de la formulación para el coeficiente de corrección que tiene en cuenta el efecto de la longitud a través de la cual se transmite la carga también ha sido publicada en la revista *Journal of Constructional Steel Research* (ANEXO).

CONCLUSIONES Y PERSPECTIVAS

Todas las investigaciones previas a esta tesis sobre del comportamiento de vigas de acero en I cargadas excéntricamente han contribuido significativamente al avance del conocimiento sobre el tema, dado el vacío existente en este campo de investigación.

En esta tesis, se ha comprobado que la respuesta, el modo de colapso y el valor de la carga última de una viga cargada con una excentricidad relativa al plano del alma dependen de parámetros geométricos (dimensiones de las chapas de la viga y sus relaciones), del valor de la excentricidad, de la longitud a través de la cual la carga es transmitida y de la manera en la que la carga es aplicada (lineal o lateralmente distribuida). Se ha verificado que los factores anteriores no tienen una influencia aislada sino que la respuesta de la viga depende de cómo se combinen entre sí y se ha comprobado que las vigas cargadas excéntricamente se comportan de una manera distinta al caso de carga centrada, exhibiendo un modo de colapso diferente y una menor carga última.

En esta investigación no ha sido posible, sin embargo, delimitar una clara frontera entre el comportamiento centrado y el excéntrico ni definir el mecanismo de colapso para el modo excéntrico.

Las investigaciones previas publicadas sobre este tema no habían desarrollado una formulación para la obtención de la carga última de vigas asimétricamente cargadas; tampoco se habían acotado las situaciones para las que la excentricidad de la carga no influye en el modo de colapso ni en la carga última, pudiéndose asimilar la respuesta de la viga excéntrica al caso centrado y, por consiguiente, tampoco se habían identificado las circunstancias en las que la excentricidad tiene una influencia crucial en el comportamiento de la viga, con carga de colapso inferior al caso centrado.

Por lo tanto, los pocos estudios previos existentes sobre el tema no eran suficientes y la base de datos experimentales con la que se contaba antes de desarrollar esta tesis, que constaba sólo de 83 vigas ensayadas, era muy modesta para obtener resultados concluyentes.

Con el objetivo de abordar el tema en profundidad se propuso un trabajo de investigación amplío y de larga duración, dentro del cual se engloba esta tesis.

Las investigaciones de esta tesis han sido realizadas en varias direcciones:

- Se ha llevado a cabo una extensa investigación experimental "Ekscentro 2007" llevada a cabo en la Facultad de Ingeniería Civil en Podgorica (Universidad de Montenegro). Se han ensayado 102 vigas de prueba. Estos ensayos no sólo son esenciales para el desarrollo de la tesis sino que constituyen una gran aportación en este ámbito de investigación.

- La cooperación con la Universidad de Granada ha permitido, mediante modelos calibrados de Elementos Finitos, desarrollar una formulación empírica para el cálculo de la carga última de vigas metálicas en I cargadas excéntricamente y analizar la influencia de la longitud de carga en la respuesta de la viga. Concretamente, se ha estudiado en profundidad la influencia de la imperfección geométrica inicial y de la longitud de la carga en el comportamiento de la viga y se ha analizado cómo ésta última afecta a la carga de agotamiento de la viga.

- Mediante la aplicación de Redes Neuronales Artificiales (RNA) se han llevado a cabo modelos de diagnóstico para prever el modo de colapso y la carga última de vigas metálicas en I cargadas excéntricamente.

El análisis de la base de datos experimentales completa, que consta de los resultados de experimentaciones anteriores a esta tesis y de los resultados de "Ekscentro 2007", ha permitido dar respuesta a algunos interrogantes planteados sobre el modo de colapso y la carga última de vigas de acero en I cargadas excéntricamente. Exactamente estas dos cuestiones fueron las que motivaron la organización de "Ekscentro 2007".

A lo largo de la campaña experimental desarrollada se han observado, además de los modos de colapso centrado y excéntrico - ya detectados en investigaciones anteriores-, nuevos modos mixtos de colapso en las vigas cargadas excéntricamente. Todos los modos de colapso observados en laboratorio han sido descritos y definidos en detalle y también se ha establecido la influencia de ciertos parámetros, que afectan al valor de la carga de rotura, en los criterios para la identificación del modo de colapso. Sin embargo, se ha dejado para futuros trabajos de investigación la definición teórico-matemática de estos modos de colapso observados.

Teniendo en cuenta que todas las vigas ensayadas tienen una relación de aspecto del panel $a/h_w = 1$, este parámetro debe de ser variado en futuras investigaciones experimentales con objeto de evaluar su influencia.

Hasta ahora no se analizado experimentalmente la influencia del parámetro a, distancia entre rigidizadores transversales –que se suponen situados sólo en los apoyos- en la resistencia a torsión del ala y la manera en cómo influye en la respuesta de vigas cargadas excéntricamente. El valor de a/h_w, sin embargo, se ha variado en los modelos de Elementos Finitos pero no se han realizado estudios locales, que requieren de ensayos de laboratorio.

En futuras investigaciones, tanto experimentales como de modelización mediante Elementos Finitos, se deberá abordar la influencia de la forma de aplicación de la carga -y su simulación numérica- en la respuesta de la viga, con objeto de considerar situaciones habituales en la práctica constructiva. Los casos de auto-centrado de carga inicialmente excéntrica deben de ser aislados y analizados por separado.

A efectos prácticos, el resultado principal del trabajo de investigación ha sido el desarrollo de una formulación sencilla y fiable para el cálculo de la carga última de vigas cargadas excéntricamente respecto del plano del alma así como un factor corrector para introducir el efecto de la longitud de la carga.

Se ha podido comprobar que la carga última depende del modo de colapso y se han establecido los criterios fundamentales para la identificación modo de fallo. Asimismo, se ha verificado que, en el caso de "*modo de colapso centrado*" en vigas sometidas a carga excéntrica, la carga última no se reduce como consecuencia de la excentricidad, o sea, la carga de agotamiento ha resultado idéntica a la de una viga geométricamente idéntica cargada en el plano de simetría del alma. Por lo tanto, cualquiera de los modelos matemáticos existentes para la obtención de la carga de rotura en vigas cargas sin excentricidad se podrían emplear para determinar la carga de agotamiento cuando dicha excentricidad existe.

Por el contrario, para el *modo de colapso excéntrico* se ha comprobado que la carga última se reduce tanto más cuanto mayor es la excentricidad con la que se aplica la carga. El grado de reducción se ve afectado, además de por la excentricidad de la carga y por la longitud a través de la cual se aplica ésta, por parámetros geométricos de la viga.

Para facilitar el procedimiento de cálculo de la carga de agotamiento bajo solicitación excéntrica, y puesto que las normativas definen la carga última para el caso de carga centrada, se ha definido un factor de reducción *R*. Este coeficiente cuantifica el valor de la disminución de la resistencia de la viga metálica por efecto de la excentricidad. La expresión propuesta para *R* ha sido obtenida a partir de los datos experimentales y de modelos numéricos calibrados.

La aplicación de las expresiones de *R* se habrá de limitar al rango de los datos experimentales y FEM considerados. Esto supone una limitación en el empleo de la formulación desarrollada y deja el campo abierto para el estudio de vigas con otra geometría y la posterior verificación de la formulación y/o modificación de la misma en base a los nuevos resultados.

A pesar del avance que supone, este procedimiento de obtención de la carga última a partir del factor de reducción es artificial, ya que no puede explicar la naturaleza del colapso de la viga. Mientras los posibles mecanismos de colapso para el caso de carga centrada están definidos y formulados, no existe formulación alguna publicada al respecto de carga excéntricas. Futuras líneas de investigación habrán de definir una formulación matemática para determinar la carga última basada en el mecanismo de colapso de la viga.

Dado el elevado número de parámetros que intervienen en la respuesta de vigas metálicas excéntricas, se ha desarrollado un modelo de predicción o diagnóstico utilizando como herramienta las redes neuronales artificiales, tanto para la estimación de la carga última como del modo de colapso. Se ha comprobado que los resultados obtenidos a partir de los modelos de diagnóstico son aplicables aunque ha quedado patente la necesidad de realizar algunas modificaciones para lograr un mayor ajuste con los resultados experimentales y numéricos. Esta línea de investigación habrá de seguir desarrollándose puesto que los resultados serán interesantes no sólo para investigación sino también para la práctica constructiva.

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ANEXO

Publicaciones obtenidas del trabajo de tesis titulado:

"Estudio de vigas metálicas en I sometidas a cargas transversales excéntricas respecto del plano del alma"

Doctorando:Biljana Šćepanović, MSc, Dipl.Ing. in Struct.Eng.

Artículos en revistas internacionales recogidas en el SCI:

- Lučić D. , Šćepanović B.: *"Experimental investigation on locally pressed I-beams subjected to eccentric patch loading"*, Journal of Constructional Steel Research, Vol. 60, 2004, p.525-534.

- Šćepanović B, Gil-Martín L.M, Hernández-Montes E, Aschheim M, Lučić D: *"Ultimate Strength of I-Girders under Eccentric Patch Load: Derivation of a New Strength Reduction Coefficient"*, Engineering Structures, Vol.31, 2009, p.1403-1413.

- Gil-Martín L.M, Šćepanović B, Hernández-Montes E, Aschheim M, Lučić D: *"Eccentrically Patch Loaded Steel I-Girders: Influence of Patch Length on Ultimate Strength"*, Journal of Constructional Steel Research, Vol. 66, 2010, p.716-722.

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Experimental investigation on locally pressed I-beams subjected to eccentric patch loading

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Abstract

A review of experimental research performed on thin walled I girders subjected to patch loading is presented in this paper. Primarily, the aim of the research was to thoroughly investigate the behaviour of girders subjected to eccentric patch loading, i.e. loading out of the web plane. This experiment is connected to the previous experimental–theoretical research conducted at the Faculty of Civil Engineering, University of Montenegro, in the years of 1998 and 1999. Load was applied over the flange in increments until final girder collapse. Various web thicknesses were studied. Loading eccentricity was varied six times for each girder type. Altogether, there were 24 tests. Six of those tests were performed using strain gauges, so that some valuable information was gathered with respect to stress distribution and the formation of yielding lines over the girders. Previous experimental research showed that the collapse form of the girders under centric loading was different from the form of those loaded eccentrically. Now, it turns out that with small eccentricity, the collapse form of the girders is the same as that of those subjected to the centric loading. Web thickness proved to have considerable influence over the collapse form of the eccentrically loaded girders.

Keywords: Steel structures; Thin-walled girders; Stability; Ultimate load; Crippling; Eccentric patch loading; Experimental research

1. Introduction

This paper presents a review of experimental research performed on thin-walled I girders subjected to patch loading. What is understood by patch loading is loading with local effect over a small area or length of a structural element. Particularly

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intriguing is the problem of the load affecting upper I-profile flange so that the web, below loading, is locally pressed. This is a rather complex and challenging issue of extremely evident elasto-plastic stresses and deformations, and apart from that, there is a noticeable geometrical non-linearity even at the lowest extent of loading. The carrying capacity loss is of local character and is reflected by the local loss of girder stability below the loading.

So far, more than 30 experimental researches have been carried out worldwide, and more than 25 mathematical models or empirical expressions for calculating the failure load have been proposed [1]. In spite of such a large number of researches, there are still a number of problems to be looked into, and also, some of the parameters have not yet been thoroughly defined in terms of the extent to which they influence the ultimate load.

Patch loaded girders are widely used in engineering practice: crane girders loaded by crane wheels, bridge girders erected by launching, etc.

2. Reasons for the research

Primarily, the aim of the research was to thoroughly investigate the behaviour of girders subjected to eccentric patch loading, i.e. loading out of the web plane. This experiment is connected to the previous experimental–theoretical research conducted at the Faculty of Civil Engineering, University of Montenegro, in the years of 1998 and 1999 [2]. The previous research studied the behaviour of girders until collapse when girders were subjected to loading in the plane of the web and out of the web plane. Results of the research led to some quite new and useful conclusions. The behaviour of eccentrically loaded girders was particularly interesting: it was shown that the collapse form of these girders was quite different from that of the centrically loaded girders (Fig. 1). This phenomenon has not been sufficiently investigated so far. While on one hand (centrically loaded beams), we deal with a rather complex problem of elasto-plastic buckling and local stability loss, on the

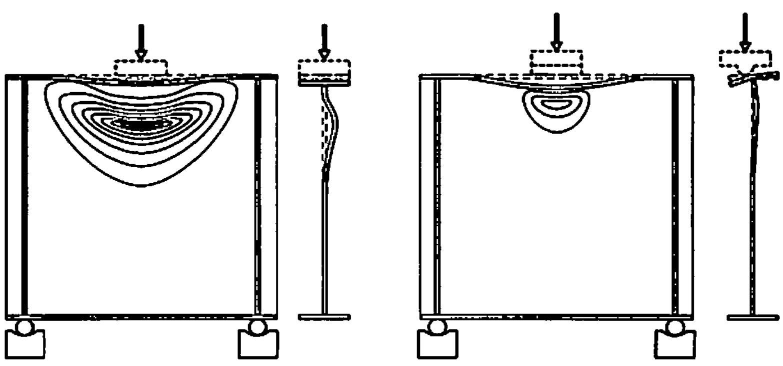

Fig. 1. Difference between collapse forms of the centrically and eccentrically loaded girders.

other (eccentrically loaded beams), we consider the problem of carrying capacity loss due to local elasto-plastic bending.

During the process of load increase, in the cases of eccentrically loaded girders, the most evident deformation is flange warping, accompanied by gentle web bending which follows the flange deformation. The web thickness and the flange stiffness influence the extent of flange warping. Flange warping and web bending increase until final collapse of the girder.

A plastification line appears on the flange and surrounds the load, forming an arch (Fig. 2). The length of the arch chord depends on the extent of eccentricity in the first place, and then on the relation t_f/t_w. As the eccentricity increases, the arch chord decreases almost linearly. Apart from the yielding line on the flange, there is a horizontal yielding line on the web. In cases of girders with thicker web this yielding line is far less evident, almost invisible. Loading eccentricity influences the depth of yielding line occurrence with respect to the loaded flange. The greater the eccentricity, the closer the yielding line gets to the loaded flange. Change of t_f/t_w does not affect the depth of line occurrence.

Apart from new findings about the phenomenon, a number of questions have not been answered after the research. A particularly intriguing question is the following: under what circumstances do the eccentrically loaded girders have the collapse form that is characteristic of centrically loaded girders, or to what extent of load eccentricity is the collapse form of the same character as that of centrically loaded beams? Is there anything that might be defined as an intermediate collapse form (a collapse form which is a combination of these two collapse forms)?

These were the reasons for initialising new experimental research, in an attempt to solve the open questions.

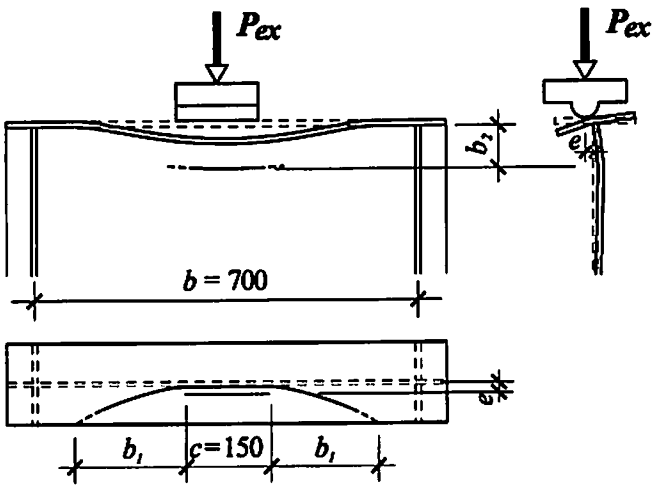

Fig. 2. Characteristic parameters of the girder collapse form.

3. Concept

According to the previous research, many parameters are supposed to influence the collapse forms. Naturally, a dominant parameter is the eccentricity value or ratio e/b_f (Fig. 3). Apart from this parameter, the influence of web thickness t_w should be studied, as well as influence of ratios b_f/t_f, t_f/t_w and d_w/t_w. The study of such a great number of parameters at the same time assumes a very extensive as well as expensive research. In the experimental research that was carried out, we studied only the influence of load eccentricity as well as web thickness over the collapse form.

Load eccentricity and web thickness were varied in a wide range, anticipating the occurrence of collapse forms that were to be studied.

The experimental research was divided into four series: *EB I*, *EB II*, *EB III* and *EB IV*. For all girders of one series web thickness was constant, whereas eccentricity was varied six times. Accordingly, there were 24 tests until girder collapse.

Series *EB II* and *EB III* had the same web thickness. Tests were repeated in series *EB III* so as that some additional measurements could be done.

The eccentricity and web thickness values are chosen in order to make a connection between this investigation and the previous one. The flange dimensions are the same for all girders. According to previous research experience, a somewhat greater flange stiffness has been chosen, since it turned out that for the greater ratio t_f/t_w, it was possible to have a collapse form similar to that of centrically loaded girders, even though the load had a certain eccentricity. Loading was applied linearly, by means of a half-cylinder, over the length of 50 mm. The length of the distributed load was shortened compared with the previous research because of the additional check of stress state in the loaded flange. There are some indications that more concentrated load might cause an earlier appearance of plastification zones in the flange.

The dimensions of the tested panels (web depth-d_w and girder span-b) are the same as in the previous experimental research: $b \times d_w = 700 \times 700$ mm.

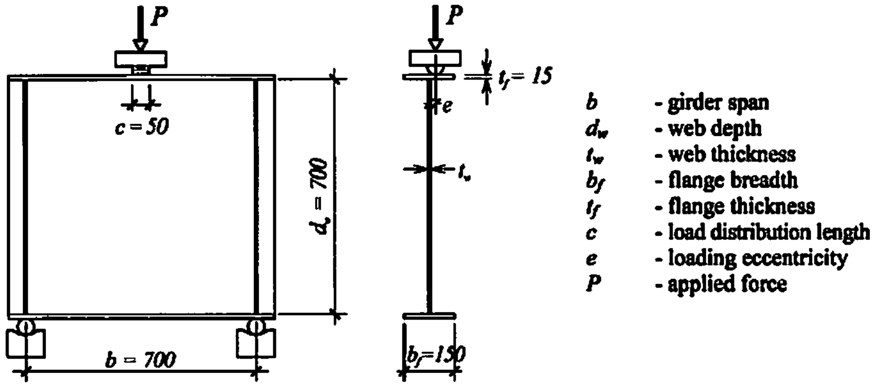

Fig. 3. Girder from the experiment.

Load was applied by means of hydraulic pump and press, over the load cell, and load distribution block with half-cylinder for linear load distribution.

4. Analysed parameters

A girder from the experiment, with symbols used in the paper, is illustrated in Fig. 3.

The following parameters are the same in all tests: $b = 700$ mm, $d_w = 700$ mm, $b_f = 150$ mm, $t_f = 15$ mm, $c = 50$ mm. The research consists of four series, each with six tests. The series are grouped together by web thickness, which is constant in each series. Load eccentricity varies six times in each series as follows: $e = 0$, 5, 10, 15, 20, 25 mm. An overview of parameters analysed in the experimental research is shown in Table 1.

Because of the collapse being very localised near the load distribution block, it is possible to test each girder twice. Load is applied over one flange once, and the next time, after turning the girder upside down, over the other flange. This enabled the usage of three girders of the same characteristics to perform six tests during one series.

5. Measurements, methods and course of the investigation

In series *EB I*, *EB II* and *EB IV*, the deflection of the loaded flange and the deflection outside the web plane were measured.

The deflections of the flange were measured at 13 points. A special arrangement, a horizontal traveller, was used in order to perform those measurements. By means of this mechanism, it was possible to record deflections of all 13 points with only one electrical displacement transducer which was, by means of a data logger, directly connected to a computer.

The appearance and development of web deformations were observed by measuring the deflections outside the web plane. The deflections were recorded in $3 \times 13 = 39$ measuring posts along three vertical lines. Only one electrical displacement transducer was used to record deflections of all 39 points. For guiding this transducer, the same frame-traveller as in the previous research was used.

In series *EB III*, apart from the loading, flange and web deformations were measured by means of strain gauges. In series *EB II* and *EB III*, the same girders were tested. However, in series *EB II*, flange and web deflections were measured, while in series *EB III* deformations in the flange plane and web plane were measured. The gauges were placed on each girder with respect to the collapse form of the same girder in series *EB II*. With both the flange and the web, strain gauges were placed on both sides of the plate (in the same direction). The purpose was to separate the membrane deformations from the bending deformations. In addition, two fixed electrical displacement transducers were used in series *EB III* to measure loaded flange deflections at two points that were symmetric with respect to the web.

The intensity of loading was recorded by a load cell.

Table 1
Overview of parameters analysed in experimental research

No.	Girder	b (mm)	d_w (mm)	t_w (mm)	b_f (mm)	t_f (mm)	c (mm)	e (mm)	b/d_w	d_w/t_w	c/b	t_f/t_w	b_f/t_f	e/b_f
1	*EB I*-1	700	700	3	150	15	50	0	1	233.3	0.07	5	10	0.00
2	*EB I*-2	700	700	3	150	15	50	5	1	233.3	0.07	5	10	0.03
3	*EB I*-3	700	700	3	150	15	50	10	1	233.3	0.07	5	10	0.07
4	*EB I*-4	700	700	3	150	15	50	15	1	233.3	0.07	5	10	0.10
5	*EB I*-5	700	700	3	150	15	50	20	1	233.3	0.07	5	10	0.13
6	*EB I*-6	700	700	3	150	15	50	25	1	233.3	0.07	5	10	0.17
7	*EB II*-1	700	700	6	150	15	50	0	1	116.7	0.07	2.5	10	0.00
8	*EB II*-2	700	700	6	150	15	50	5	1	116.7	0.07	2.5	10	0.03
9	*EB II*-3	700	700	6	150	15	50	10	1	116.7	0.07	2.5	10	0.07
10	*EB II*-4	700	700	6	150	15	50	15	1	116.7	0.07	2.5	10	0.10
11	*EB II*-5	700	700	6	150	15	50	20	1	116.7	0.07	2.5	10	0.13
12	*EB II*-6	700	700	6	150	15	50	25	1	116.7	0.07	2.5	10	0.17
13	*EB III*-1	700	700	6	150	15	50	0	1	116.7	0.07	2.5	10	0.00
14	*EB III*-2	700	700	6	150	15	50	5	1	116.7	0.07	2.5	10	0.03
15	*EB III*-3	700	700	6	150	15	50	10	1	116.7	0.07	2.5	10	0.07
16	*EB III*-4	700	700	6	150	15	50	15	1	116.7	0.07	2.5	10	0.10
17	*EB III*-5	700	700	6	150	15	50	20	1	116.7	0.07	2.5	10	0.13
18	*EB III*-6	700	700	6	150	15	50	25	1	116.7	0.07	2.5	10	0.17
19	*EB IV*-1	700	700	8	150	15	50	0	1	87.5	0.07	1.875	10	0.00
20	*EB IV*-2	700	700	8	150	15	50	5	1	87.5	0.07	1.875	10	0.03
21	*EB IV*-3	700	700	8	150	15	50	10	1	87.5	0.07	1.875	10	0.07
22	*EB IV*-4	700	700	8	150	15	50	15	1	87.5	0.07	1.875	10	0.10
23	*EB IV*-5	700	700	8	150	15	50	20	1	87.5	0.07	1.875	10	0.13
24	*EB IV*-6	700	700	8	150	15	50	25	1	87.5	0.07	1.875	10	0.17

Fig. 4 shows the measuring equipment at one of the tested girders.

There were five participants conducting the experiment in series *EB I*, *EB II* and *EB IV* (when deflections were measured): the leader of the investigation, a computer operator, a member operating the hydraulic pump and static data logger, a member pointing the displacement transducers to the measuring posts and a member working on the frame-traveller and horizontal traveller for guiding the displacement transducers. In series *EB III*, only four participants were needed to carry out the experiment: the leader of the investigation, a computer operator, a member operating the hydraulic pump and static data logger and a member controlling the position of the displacement transducers.

The loading was increased by a hydraulic pump and 800 kN press. The strain gauges, displacement transducers and load cell readings were taken by the static data logger. Data were transferred from the data logger to the computer by specialised software. That way, data monitoring (on the computer monitor) during the tests was enabled.

The loading was applied in increments. At the very beginning of each test, the intensity of the increments was approximately a fifth of the estimated failure load, then in later increments a tenth of the estimated failure load and even less near collapse. After each of the increments, deflection measurements were made in series *EB I*, *EB II* and *EB IV*, while in series *EB III* gauge readings were recorded after each 10 kN and more often near collapse. In later increments, the loading was kept constant for 2–5 min (before making the measurements), because of potential deformation stabilisation. The speed of load increase was approximately 0.5 kN/s.

Before the beginning, after the loading distribution apparatus had been initially pressed, measurements were made at the so-called zero (initial) increment. In cases of tests with strain gauges, zero measurement was performed before placing the

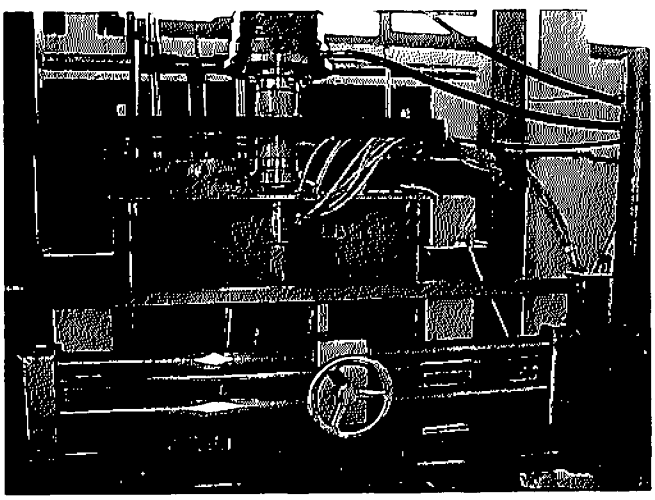

Fig. 4. Testing of girder *EB III*-1.

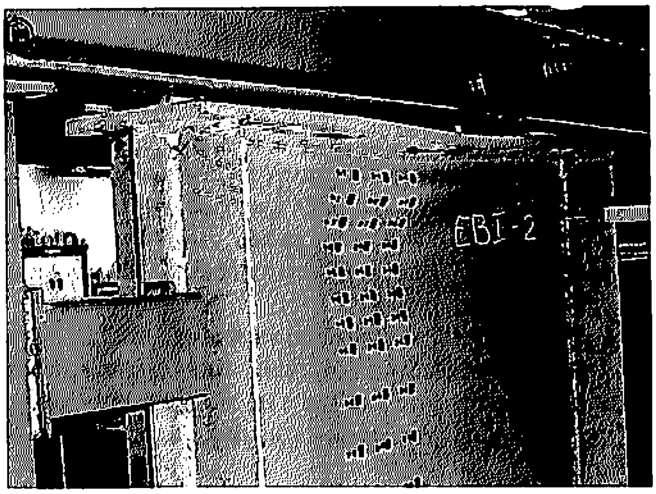

Fig. 5. Girder *EB I*-2 after collapse.

load distribution block, and the first increment followed the initial pressing of the apparatus. After reaching the failure load, residual deflections and residual strains were recorded.

6. The results of the experiment

As could be expected, the collapse form was typical of centrically loaded girders as well as of the eccentrically loaded ones (Fig. 1).

Fig. 6. Girder *EB II*-6 after collapse.

Table 2
Girders geometry, collapse form and flange deformations with failure loads

No.	Girder	b (mm)	d_w (mm)	t_w (mm)	b_f (mm)	t_f (mm)	c (mm)	e (mm)	Collapse form	Flange torsion	P_{cx} (kN)
1	*EB I*-1	700	700	3	150	15	50	0	Centric	None	132.9
2	*EB I*-2	700	700	3	150	15	50	5	Centric	None	128.3
3	*EB I*-3	700	700	3	150	15	50	10	Centric	Negligible	126.9
4	*EB I*-4	700	700	3	150	15	50	15	Centric	Negligible	135.3
5	*EB I*-5	700	700	3	150	15	50	20	Centric	Slightly warped	133.6
6	*EB I*-6	700	700	3	150	15	50	25	Centric	Slightly warped	123.9
7	*EB II*-1	700	700	6	150	15	50	0	Centric	None	340.2
8	*EB II*-2	700	700	6	150	15	50	5	Centric	Slightly warped	320.3
9	*EB II*-3	700	700	6	150	15	50	10	Centric	Slightly warped	325.6
10	*EB II*-4	700	700	6	150	15	50	15	Eccentric	Warped	295.6
11	*EB II*-5	700	700	6	150	15	50	20	Eccentric	Warped	242.9
12	*EB II*-6	700	700	6	150	15	50	25	Eccentric	Warped	196.9
13	*EB III*-1	700	700	6	150	15	50	0	Centric	None	342.2
14	*EB III*-2	700	700	6	150	15	50	5	Centric	Slightly warped	321.3
15	*EB III*-3	700	700	6	150	15	50	10	Centric	Slightly warped	300.6
16	*EB III*-4	700	700	6	150	15	50	15	Eccentric	Warped	267.3
17	*EB III*-5	700	700	6	150	15	50	20	Eccentric	Warped	227.6
18	*EB III*-6	700	700	6	150	15	50	25	Eccentric	Warped	186.9
19	*EB IV*-1	700	700	8	150	15	50	0	Centric	None	400.6
20	*EB IV*-2	700	700	8	150	15	50	5	Centric	Slightly warped	417.6
21	*EB IV*-3	700	700	8	150	15	50	10	Eccentric	Warped	393.9
22	*EB IV*-4	700	700	8	150	15	50	15	Eccentric	Warped	300.9
23	*EB IV*-5	700	700	8	150	15	50	20	Eccentric	Warped	245.3
24	*EB IV*-6	700	700	8	150	15	50	25	Eccentric	Warped	209.3

Web thickness greatly influenced the collapse form of the eccentrically loaded girders. A significant web buckling, i.e. the collapse form typical of centrically loaded girders, became evident with all tested girders of $t_w = 3$ mm (Fig. 5).

In cases of girders with $t_w = 6$ mm, the collapse form typical of the girders under centric loading became evident with $e = 0$, 5, and 10 mm, whereas in the cases with $e = 15$, 20 and 25 mm the collapse form was that of the eccentrically loaded girders (Fig. 6).

As far as the series with $t_w = 8$ mm is concerned, only the cases of $e = 0$ and 5 mm were characterised by the collapse form typical of centric loading, whereas in other cases ($e = 10$, 15, 20 and 25 mm) the collapse form was that of the eccentrically loaded girders.

The intensity of failure load decreases as the eccentricity increases in the cases of $t_w = 6$ and 8 mm, which is well known fact. But this is not a case with girders of $t_w = 3$ mm. This could be connected to the fact that collapse form typical of the eccentrically loaded girders failed to occur in either of the cases (series *EB I*).

It becomes evident that the collapse form of girders with slenderer web is typical of centrically loaded girders even for significant load eccentricities. Table 2 presents an overview of the tested girders with the descriptions of the collapse forms and the flange deformations together with the experimental failure loads.

7. Conclusions

Primarily, this research had the aim of studying girder behaviour, collapse form and intensity of ultimate load with beams that are not centrically loaded, but where the loading has a certain eccentricity with respect to the plane of the web.

This problem has not been investigated thoroughly, though it is quite often present in engineering practice, especially in cases of moving loads. Only few researches have been performed in the past [3–5].

This experimental work contributed to the development of new knowledge about the collapse form of eccentrically loaded girders. In future investigations, we need to introduce all the other parameters and study their influence on the failure load and the collapse form. This problem also requires to be presented in the form of a mathematical model in order to estimate the failure load.

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Ultimate strength of I-girders under eccentric patch loading: Derivation of a new strength reduction coefficient

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ABSTRACT

In structural applications I-girders are subjected to eccentric patch loading since even a small eccentricity of applied loads relative to the web plane is unavoidable in practice. Experimental research shows that the behaviour of eccentrically loaded girders differs from that of concentrically loaded girders. A reduction in ultimate strength due to eccentricity is evident, and some expressions for the reduction have been proposed. We show that these expressions overestimate the reduction in capacity for large eccentricities. In this paper, results of 102 tests on I-girders subjected to eccentric patch loading are presented and the variables involved in the phenomenon are evaluated based on a parametric study of experimental data and the results of several finite element models. The parametric study confirms that the parameters previously identified for evaluating the effect of small eccentricities are sufficient for addressing cases with larger eccentricities. New mathematical expressions are proposed herein to represent the reduction in strength across the full range of eccentricities considered.

1. Introduction

Patch loading is the loading applied over a small area or length of a structural element. A common situation in structural engineering occurs when a downward compressive load is applied to the flange of an I-section so that the web is compressed in the region below the applied load. Examples in practice are numerous and present in different types of structures, including crane and bridge girders.

Some eccentricity of load relative to the web plane is unavoidable in engineering practice (Fig. 1). When the ultimate strength of a girder is governed by patch loading, the ultimate strength of the girder depends on the degree of eccentricity and reduces as the eccentricity of the load increases. This decrease in the ultimate strength is expressed by the strength reduction coefficient R, which relates the ultimate strength of eccentrically loaded girders to the ultimate strength of concentrically loaded girders:

$$R = \frac{\text{ultimate strength of eccentrically loaded girder}}{\text{ultimate strength of concentrically loaded girder}}. \quad (1)$$

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Many parameters influence the behaviour and ultimate strength of girders prone to web crushing. Such girders often have thin webs and may be subjected to patch loading having non-zero eccentricities. The parameters include girder dimensions and their dimensionless ratios, load eccentricity and the conditions of load application. In regard to the conditions of load application, it was found that there is no reduction in the ultimate capacity due to eccentricity when load is applied through a thick "patch" plate placed eccentrically with respect to the plane of the web (see Fig. 1), due to prevention of rotation; however, reductions occurred when load was applied through a cylindrical roller (see Fig. 1), Galambos [1].

Until now, the experimental database available in the literature for eccentric patch loading consists of 35 tests conducted in the 1980s (22 specimens tested at the University of Maine, Elgaaly and Nunan [1]) and 13 additional specimens repoted by Elgaaly and Salkar [2] and 39 tests reported in 2001 (University of Montenegro, Lučić and Šćepanović [3]). The experimental database has been significantly improved by the addition of 102 tests conducted recently at the University of Montenegro summarized in Table 1. This last series fills in some gaps in parameter values remaining from the earlier tests and extends the range of existing test data. As in previous tests, the last series of girders had vertical stiffeners at both girder ends. In all of the new specimens, the distance between stiffeners, a, is 700 mm, the height of the web, h_w, is 700 mm, the width of the flanges, b_f, is 150 mm and the patch loading length, c, is 50 mm. Relative eccentricity e/b_f varies from

Table 1
Summary of the experimental data – Montenegro experiments 2007.

Series	Girder	a (mm)	h_w (mm)	t_w (mm)	b_f (mm)	t_f (mm)	c (mm)	e (mm)	t_f/t_w	e/b_f	$P_{u,exp}$ (kN)	R_{exp}
EB V	EB V - 1	700	700	5	150	10	50	0	2	0	229	1
	EB V - 2	700	700	5	150	10	50	5	2	0.033	212	0.93
	EB V - 3	700	700	5	150	10	50	10	2	0.067	197	0.86
	EB V - 4	700	700	5	150	10	50	15	2	0.100	175	0.76
	EB V - 5	700	700	5	150	10	50	20	2	0.133	153	0.67
	EB V - 6	700	700	5	150	10	50	25	2	0.167	129	0.56
EB VI	EB VI - 1	700	700	10	150	10	50	0	1	0	720	1
	EB VI - 2	700	700	10	150	10	50	5	1	0.033	575	0.80
	EB VI - 3	700	700	10	150	10	50	10	1	0.067	365	0.51
	EB VI - 4	700	700	10	150	10	50	15	1	0.100	313	0.43
	EB VI - 5	700	700	10	150	10	50	20	1	0.133	275	0.38
	EB VI - 6	700	700	10	150	10	50	25	1	0.167	220	0.31
EB VII	EB VII - 1	700	700	5	150	12	50	0	2.4	0	230	1
	EB VII - 2	700	700	5	150	12	50	5	2.4	0.033	225	0.98
	EB VII - 3	700	700	5	150	12	50	10	2.4	0.067	212	0.92
	EB VII - 4	700	700	5	150	12	50	15	2.4	0.100	180	0.78
	EB VII - 5	700	700	5	150	12	50	20	2.4	0.133	170	0.74
	EB VII - 6	700	700	5	150	12	50	25	2.4	0.167	149	0.65
EB VIII	EB VIII - 1	700	700	3	150	3	50	0	1	0	79	1
	EB VIII - 2	700	700	3	150	3	50	5	1	0.033	44	0.56
	EB VIII - 3	700	700	3	150	3	50	10	1	0.067	37	0.47
	EB VIII - 4	700	700	3	150	3	50	15	1	0.100	29	0.37
	EB VIII - 5	700	700	3	150	3	50	20	1	0.133	23	0.29
	EB VIII - 6	700	700	3	150	3	50	25	1	0.167	20	0.25
EB IX	EB IX - 1	700	700	3	150	6	50	0	2	0	95	1
	EB IX - 2	700	700	3	150	6	50	5	2	0.033	80	0.84
	EB IX - 3	700	700	3	150	6	50	10	2	0.067	69	0.73
	EB IX - 4	700	700	3	150	6	50	15	2	0.100	57	0.60
	EB IX - 5	700	700	3	150	6	50	20	2	0.133	47	0.49
	EB IX - 6	700	700	3	150	6	50	25	2	0.167	39	0.41
EB X	EB X - 1	700	700	3	150	9	50	0	3	0	102	1
	EB X - 2	700	700	3	150	9	50	5	3	0.033	105	1.03
	EB X - 3	700	700	3	150	9	50	10	3	0.067	107	1.05
	EB X - 4	700	700	3	150	9	50	15	3	0.100	90	0.88
	EB X - 5	700	700	3	150	9	50	20	3	0.133	85	0.83
	EB X - 6	700	700	3	150	9	50	25	3	0.167	70	0.69
EB XI	EB XI - 1	700	700	3	150	12	50	0	4	0	116	1
	EB XI - 2	700	700	3	150	12	50	5	4	0.033	113	0.97
	EB XI - 3	700	700	3	150	12	50	10	4	0.067	115	0.99
	EB XI - 4	700	700	3	150	12	50	15	4	0.100	110	0.95
	EB XI - 5	700	700	3	150	12	50	20	4	0.133	105	0.91
	EB XI - 6	700	700	3	150	12	50	25	4	0.167	115	0.99
EB XII	EB XII - 1	700	700	4	150	4	50	0	1	0	120	1
	EB XII - 2	700	700	4	150	4	50	5	1	0.033	70	0.58
	EB XII - 3	700	700	4	150	4	50	10	1	0.067	50	0.42
	EB XII - 4	700	700	4	150	4	50	15	1	0.100	45	0.38
	EB XII - 5	700	700	4	150	4	50	20	1	0.133	40	0.33
	EB XII - 6	700	700	4	150	4	50	25	1	0.167	35	0.29
EB XIII	EB XIII - 1	700	700	4	150	6	50	0	1.5	0	125	1
	EB XIII - 2	700	700	4	150	6	50	5	1.5	0.033	110	0.88
	EB XIII - 3	700	700	4	150	6	50	10	1.5	0.067	86	0.69
	EB XIII - 4	700	700	4	150	6	50	15	1.5	0.100	68	0.54
	EB XIII - 5	700	700	4	150	6	50	20	1.5	0.133	50	0.40
	EB XIII - 6	700	700	4	150	6	50	25	1.5	0.167	45	0.36
EB XIV	EB XIV - 1	700	700	4	150	8	50	0	2	0	140	1
	EB XIV - 2	700	700	4	150	8	50	5	2	0.033	129	0.92
	EB XIV - 3	700	700	4	150	8	50	10	2	0.067	130	0.93
	EB XIV - 4	700	700	4	150	8	50	15	2	0.100	100	0.71
	EB XIV - 5	700	700	4	150	8	50	20	2	0.133	86	0.61
	EB XIV - 6	700	700	4	150	8	50	25	2	0.167	75	0.54
EB XV	EB XV - 1	700	700	4	150	10	50	0	2.5	0	155	1
	EB XV - 2	700	700	4	150	10	50	5	2.5	0.033	148	0.95
	EB XV - 3	700	700	4	150	10	50	10	2.5	0.067	140	0.90
	EB XV - 4	700	700	4	150	10	50	15	2.5	0.100	138	0.89
	EB XV - 5	700	700	4	150	10	50	20	2.5	0.133	128	0.83
	EB XV - 6	700	700	4	150	10	50	25	2.5	0.167	115	0.74
EB XVI	EB XVI - 1	700	700	5	150	6	50	0	1.2	0	187	1
	EB XVI - 2	700	700	5	150	6	50	5	1.2	0.033	130	0.70
	EB XVI - 3	700	700	5	150	6	50	10	1.2	0.067	105	0.56
	EB XVI - 4	700	700	5	150	6	50	15	1.2	0.100	74	0.40

Table 1 (*continued*)

Series	Girder	a (mm)	h_w (mm)	t_w (mm)	b_f (mm)	t_f (mm)	c (mm)	e (mm)	t_f/t_w	e/b_f	$P_{u,exp}$ (kN)	R_{exp}
	EB XVI - 5	700	700	5	150	6	50	20	1.2	0.133	59	0.32
	EB XVI - 6	700	700	5	150	6	50	25	1.2	0.167	55	0.29
EB XVII	EB XVII - 1	700	700	5	150	8	50	0	1.6	0	209	1
	EB XVII - 2	700	700	5	150	8	50	5	1.6	0.033	200	0.96
	EB XVII - 3	700	700	5	150	8	50	10	1.6	0.067	145	0.69
	EB XVII - 4	700	700	5	150	8	50	15	1.6	0.100	130	0.62
	EB XVII - 5	700	700	5	150	8	50	20	1.6	0.133	98	0.47
	EB XVII - 6	700	700	5	150	8	50	25	1.6	0.167	83	0.40
EB XVIII	EB XVIII - 1	700	700	6	150	6	50	0	1	0	208	1
	EB XVIII - 2	700	700	6	150	6	50	5	1	0.033	170	0.82
	EB XVIII - 3	700	700	6	150	6	50	10	1	0.067	130	0.63
	EB XVIII - 4	700	700	6	150	6	50	15	1	0.100	104	0.50
	EB XVIII - 5	700	700	6	150	6	50	20	1	0.133	88	0.42
	EB XVIII - 6	700	700	6	150	6	50	25	1	0.167	69	0.33
EB XIX	EB XIX - 1	700	700	6	150	9	50	0	1.5	0	330	1
	EB XIX - 2	700	700	6	150	9	50	5	1.5	0.033	285	0.86
	EB XIX - 3	700	700	6	150	9	50	10	1.5	0.067	217	0.66
	EB XIX - 4	700	700	6	150	9	50	15	1.5	0.100	155	0.47
	EB XIX - 5	700	700	6	150	9	50	20	1.5	0.133	125	0.38
	EB XIX - 6	700	700	6	150	9	50	25	1.5	0.167	107	0.32
EB XX	EB XX - 1	700	700	6	150	12	50	0	2	0	300	1
	EB XX - 2	700	700	6	150	12	50	5	2	0.033	265	0.88
	EB XX - 3	700	700	6	150	12	50	10	2	0.067	311	1.04
	EB XX - 4	700	700	6	150	12	50	15	2	0.100	235	0.78
	EB XX - 5	700	700	6	150	12	50	20	2	0.133	202	0.67
	EB XX - 6	700	700	6	150	12	50	25	2	0.167	165	0.55
EB XXI	EB XXI - 1	700	700	5	150	10	150	10	2	0.067	240	0.93
	EB XXI - 2	700	700	5	150	10	150	5	2	0.033	250	0.97
	EB XXI - 3	700	700	10	150	10	150	10	1	0.067	640	0.73
	EB XXI - 4	700	700	10	150	10	150	5	1	0.033	790	0.90
	EB XXI - 5	700	700	5	150	12	150	10	2.4	0.067	255	0.96
	EB XXI - 6	700	700	5	150	12	150	5	2.4	0.033	255	0.96

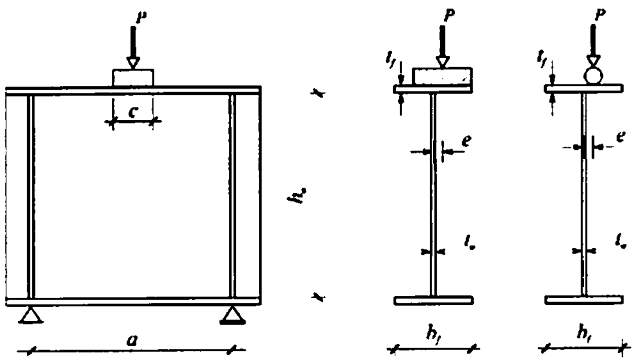


Fig. 1. I-girder under eccentric patch loading.

0 to 0.167; the thicknesses of the web and flanges are indicated in Table 1. Consequently, the enlarged database of test results allows a more detailed and comprehensive characterization of the influence of geometric parameters (e/b_f, t_f/t_w, a/h_w, ...) on the ultimate strength of eccentrically loaded girders.

The strength reduction coefficient, R, has been expressed by Galambos [4] in terms of two main parameters e/b_f and t_f/t_w. Graciano [5,6] stated that flange to web thickness (t_f/t_w) and flange to web yield strength ratios influence patch loading resistance in longitudinally stiffened webs. Galambos' expression for R (Eqs. (2) and (3)) was based on the experiments conducted in the 1980s by Elgaaly and Nunan [1] and Elgaaly and Salkar [2]. In [1,2] the flange to web yield strength ratio is 1. Based on these experiments, Galambos proposed an expression for R as a function of t_f/t_w and e/b_f:

$$R = m \cdot \frac{e}{b_f} + n < 1 \qquad (2)$$

$$\left.\begin{aligned} m &= -0.45 \cdot \left(\frac{t_f}{t_w}\right)^2 + 4.55 \cdot \left(\frac{t_f}{t_w}\right) - 12.75 \\ n &= 1.15 - 0.025 \cdot \left(\frac{t_f}{t_w}\right) \end{aligned}\right\} \qquad (3)$$

where e is the load eccentricity, b_f is the flange width, t_f is the flange thickness and t_w is the web thickness (Fig. 1). Because Eq. (2) with coefficients given by Eq. (3) is empirical, its use properly should be limited to the range of the test data, namely: $1 \le t_f/t_w \le 4$ and $e/b_f \le 1/6$. Data from more recent experimental tests at the University of Montenegro [3] indicate that for larger eccentricities, Eqs. (2) and (3) underestimate the strength reduction coefficient—Fig. 2 shows that Eq. (2) (using m and n from Eq. (3)) estimates well the Series III data by Elgaaly and Nunan [1] but provides a very conservative estimate of the strength reduction coefficient observed for most of the Series IV data from the same authors [1].

In this paper, results from the series of tests conducted at the University of Montenegro in 2007 are presented. Results from experiments are supplemented by finite element studies to discern the main parameters that influence R and a new expression for the strength reduction coefficient for girders subjected to eccentric patch loading is derived. In the experimental and finite element models, yield strength of the steel in the flange and yield strength of the steel in the web are equal.

2. Evaluation of the current expression for the reduction coefficient

The first experimental study on eccentric patch loading was published in 1989 by Elgaaly and Nunan [1]. In their study, geometric parameters were constrained for all girders as follows: $a/h_w = 1.1$, $b_f/t_f = 11.4$, $h_w/t_w = 45$, $t_f/t_w = 1.4$ and $c/a = 0.20$. Loads were applied at various eccentricities through a thick patch plate or a cylindrical bar. For loads applied by a cylindrical

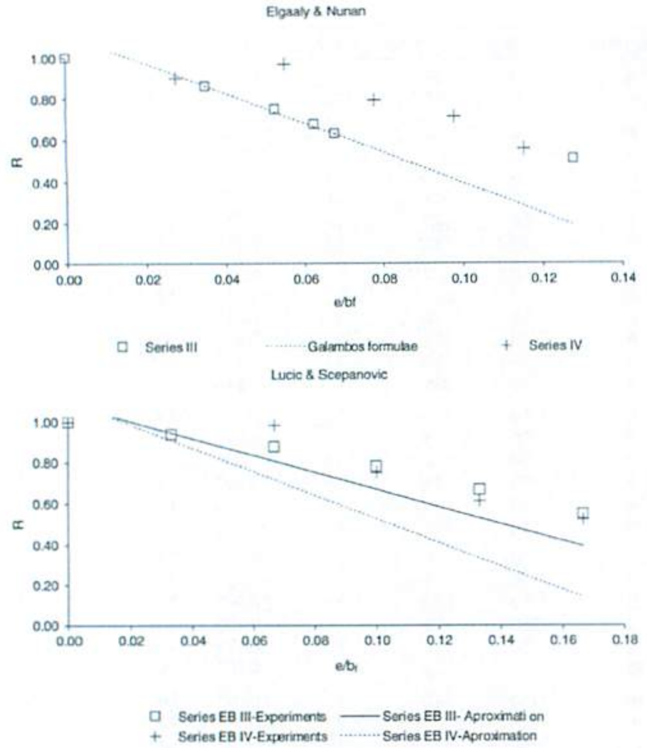


Fig. 2. Comparison of experimental and calculated values of R according to Eq. (2) with coefficients from Eq. (3).

bar, eccentricities varied from 0 to a maximum $e/b_f = 1/8$. For the series in which eccentric patch loading was applied through a thick "patch" plate, eccentricities (e/b_f) varied from 0 to 1/4.

Further experimental studies (Series J and S) were published by Elgaaly and Salkar in 1990 [2]. In Series J, geometric parameters of all girders were constrained to $a/h_w = 1.0$, $b_f/t_f = 24$, $h_w/t_w = 210$, $t_f/t_w = 2.1$ and $c/a = 0.2$. Eccentricities varied from 0 to a maximum $e/b_f = 1/8$. In Series S, girders had geometric parameters $a/h_w = 1.0$, $b_f/t_f = 12$ or 24, $h_w/t_w \approx 210$ (slender web), $t_f/t_w = 4.2$ or 2.1 (flange stiffness much greater than web stiffness) and $c/a = 0.2$ or 0.6. Specimens S7 and S8 had longitudinal stiffeners and are not considered in the present study.

In both studies ([1,2]), the ultimate strength clearly decreased as the load eccentricity increased. In Galambos' approximation (Eqs. (2) and (3)), the strength reduction coefficient decreases linearly with eccentricity, as may be seen in Eq. (2) and Fig. 2(a). The strength reduction coefficient represented by Galambos' equation increases for larger values of ratio t_f/t_w, so the ultimate strength would be projected to increase with t_f/t_w.

Eq. (2) does not consider the load length c. Markovic and Hajdin [7] concluded that the load length, c, has little influence for unstiffened webs. Šćepanović et al. [8] demonstrate that for the combination of small ratio t_f/t_w (<1.5) and small load eccentricity ($e/b_f < 0.1$) the load length c does influence the strength reduction coefficient R for transversal stiffened webs. It is clear that for $e/b_f < 0.1$ the load length has to be considered as a new parameter in any revised expression for R.

In experimental work by Lučić [9,10], load eccentricity, e, and flange and web thicknesses, t_f and t_w (i.e. ratios e/b_f, b_f/t_f, h_w/t_w and t_f/t_w) were varied. Held constant were parameters $a/h_w = 1$ and $c/a = c/h_w = 0.21$. Other parameters had the following values: $b_f/t_f = 12.5$ or 15, $h_w/t_w = 70$ or 140, $t_f/t_w = 1.0$, 2.0 or 2.4. The eccentricity ratio e/b_f varied from 1/10 to 1/5. As before, the ultimate strength decreased approximately linearly with an increase in eccentricity ratio. This experimental work also demonstrated that the reduction of ultimate strength is greater as t_f/t_w decreases, and appears to be inversely proportional to t_f/t_w. Thus, the original equation proposed by Galambos becomes increasingly conservative as t_f/t_w decreases (Fig. 2).

Experimental work by Lučić and Šćepanović [3] continued their previous studies. Load eccentricity, e, and web thickness, t_w, were varied. A larger range of eccentricity ratio e/b_f was studied, compared to the previous research [9,10], ranging from 1/30 to 1/6. Ranges of parameters h_w/t_w and t_f/t_w were extended to larger values: $h_w/t_w \approx 88$, 117 or 233, and $t_f/t_w \approx 1.9$, 2.5 or 5.0. Girder span, a, web depth, h_w, and flange width, b_f, were the same as in previous tests [9,10]. The flange was thicker ($t_f = 15$ mm, $b_f/t_f = 10$) than in the earlier tests by Lučić [9,10] and Elgaaly [1, 2]. Load length was shorter than before ($c/a = c/h_w = 0.07$). Ultimate strength reduced approximately linearly with increase in eccentricity, but the empirical strength reduction coefficient was greater than represented by Eqs. (2) and (3) for smaller ratios t_f/t_w.

Based on the results presented in [3,9,10], some samples of them are represented in Fig. 2(b), we conclude that the strength reduction coefficient of Eq. (2) (with m and n from (3)) is needlessly conservative for larger eccentricities as t_f/t_w decreases. Approximations of Galambos' equation for values of $e/b_f > 0.06$ gave conservative results, indicating that the strength reduction coefficient was underestimated in case of large eccentricities.

3. Experimental program

In the experimental program at the University of Montenegro, a total of 102 welded I-girders were tested (welded plate girders having an I-shaped cross section), in 17 series (Table 1). The tests were completed in 2007. The experiments were designed with the intention of obtaining an improved expression for the strength reduction coefficient, R, due to eccentric patch loading. The range of input parameters (girder dimensions and load eccentricity) was chosen to obtain experimental data over a larger range of parameter values than had previously been obtained, while also using girders having dimensions more typical of real engineering practice. The capacity of available laboratory equipment and space constraints also affected the test setup and girder dimensions. The load was applied at mid-span (Fig. 1), through a cylindrical roller placed on the upper flange. Transverse stiffeners were welded over the supports to prevent crippling and to insure localized failure beneath the load.

With regard to girder dimensions, only flange and web thicknesses, t_f and t_w, varied over a wide range, covering some existing gaps in previous experimental series..

The length of patch load, c, and eccentricity of the load, e, were the same as in previous tests in order to compare previous and new experimental results. The length of stiff bearing is $c = 50$ mm. Values of eccentricity used in the test series are $e = 0, 5, 10, 15, 20, 25$ mm.

Although the complete set of test results is summarized herein (Table 1) the two experimental tests in which the value of ultimate strength under eccentric loading was larger than for concentric loading were not considered in the derivation of the R coefficient by regression analysis.

4. Finite element model description and validation

Nonlinear finite element analysis (FEA) has been proved to be a useful tool to study the behaviour of plate girders under patch loading [11]. Because of the cost of experimental work, it is appropriate to use finite element simulations to understand parametric trends using validated models.

Nonlinear finite element analyses were conducted to better understand the effect of load eccentricity on ultimate strength

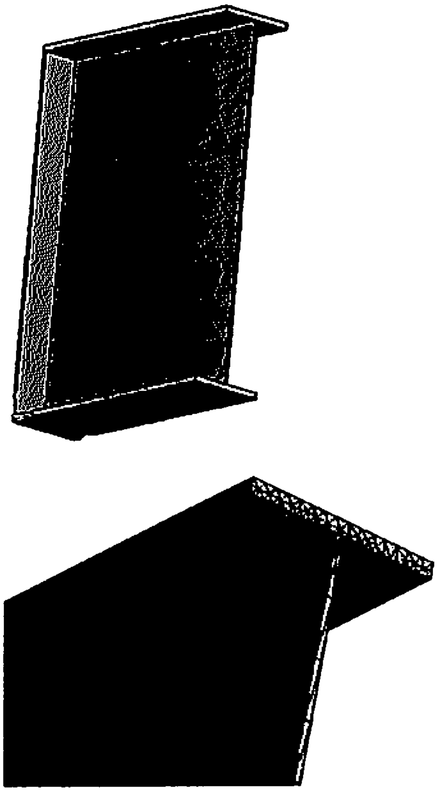


Fig. 3. Finite Element model for t_f = 12 mm, t_w = 6 mm, h_w = 700 mm, a = 1050 mm.

of patch loaded I-girders. As a starting point, the response of Specimens EB-I, EB-III and EB-IV [3] was simulated using the ANSYS program. The load was applied at midspan on the upper flange and vertical stiffeners were located at both girder ends, just as in the experiments. An initial out-of-plane unstressed configuration was established by deforming the geometry of the web by 5 mm (without inducing stress) to represent an initial imperfection. Graciano and Edlund [11] and Gil-Martín et al. [12] showed that, for small amplitudes, the amplitude of initial imperfection doesn't affect the load capacity. The SOLID92 (®) element was employed to model web, flanges and vertical stiffeners. This element is a ten node three-dimensional element having three degrees-of-freedom at each node, consisting of translations in the x, y, and z directions. Plasticity, large-deflection and large-strain capabilities were used. Residual stresses were not considered in the FE-models. Material nonlinearities were represented by using a nonlinear stress–strain relationship, while plasticity was represented using a kinematic hardening rule. Approximately 5000 elements were used for each 3D model of a test specimen.

Due to the symmetry in geometry, load conditions and expected deformation, just one half of each girder was modelled (Fig. 3). The patch load was transferred into the girder at the top of upper flange by controlling the displacement of the patch nodes in the FE analysis.

Table 2 compares some experimental results from Lučić and Šćepanović [3] with the FEA results of different beams. The numerical analysis was done with two steel models: an idealized version of European S235 steel (having an ideal yield strength of 235 MPa) and an idealized version of the behaviour of the actual steel used in the experiments (yield stress of 327 MPa). Material curves for both, experimental and FE models, are represented in Fig. 4. Validation of the FEM model was performed by comparing experimental and FEM results for several models.

Table 2
Validation of FEA models.

Model	Strength reduction coefficient, R		
	Experiment	ANSYS	
		f_y = 327 MPa	S235 (f_y = 235 MPa)
EBI e = 0 mm EBI e = 10 mm	0.95	0.98	0.99
EBIII e = 0 mm EBIII e = 15 mm	0.78	0.76	0.77
EBIV e = 0 mm EBIV e = 15 mm	0.75	0.72	0.75

The strength reduction coefficient, R, differed less than 3% from the experimentally determined values and the steel model had a negligible effect on ultimate strength.

5. Parametric finite element study

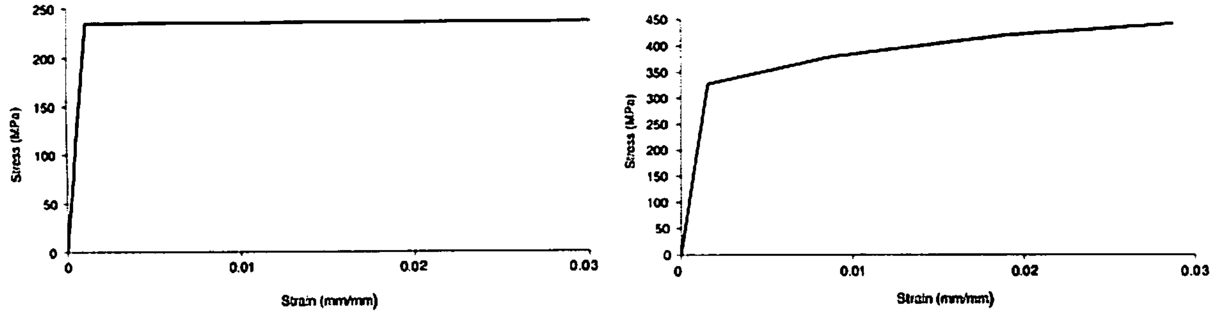
An expanded FEA study was done to evaluate the parametric dependency of the R coefficient. Some parameters were kept constant in the parametric study: h_w = 700 mm, t_w = 6 mm, b_f = 150 mm and c = 50 mm; other parameters were varied: t_f = 6, 12, 18 and 24 mm, a = 700, 910, 1050, 1260 and 1400 mm, e = 0, 6, 12, 18 and 24 mm. The full combination of parameters resulted in 100 cases. Table 3 shows the strength reduction coefficients obtained from the FEA.

Parameter values of the numerical tests compared with the parameter values of the experiments considered in this paper are represented in Fig. 5.

In Eqs. (2) and (3), the relative eccentricity was defined as e/b_f. This definition of relative eccentricity is retained presently, although other relative eccentricity definitions may be considered, including e/h_w, where h_w is the web depth. Values of the strength reduction coefficient for the two relative eccentricity ratios are shown in Fig. 6 for different values of t_f/t_w. In this figure, only FEA results for a = 700 mm are plotted. Because in the numerical analyses t_w is constant (t_w = 6 mm) different values of t_f indicate different values of t_f/t_w. The numerical data demonstrate the existence of a linear trend in R for large t_f/t_w (=4 and 3) that gradually changes to a quadratic trend for small t_f/t_w (=1) as a function of the relative eccentricity (e/b_f or e/h_w). A quadratic dependence of R on relative eccentricity (e/b_f) evidently would fit the data better than a linear dependence for larger values of relative eccentricity and for small values of t_f/t_w.

In order to evaluate the behaviour of different candidate $a - t_f - e$ relations, results of the FEA model for a = 910 mm are plotted in Fig. 7. Fig. 7 shows the response of the girder through the entire load–deformation response in terms of applied vertical load versus vertical deflection of the loading point. Two values of the thickness of the flange (t_f = 6 and 24 mm) are considered in Fig. 7. Figs. 6 and 7 show that the greater the value of t_f/t_w, the greater the value of R. In case of $t_f/t_w = 4$ (t_f = 24 mm, Fig. 7(b)), the ultimate load values for different eccentricities are very similar although the load–deflection curve shows a different behaviour with different eccentricities, i.e. for t_f = 24 mm and e = 0 and 6 mm maximum load happens for deflection of about 4 mm (Point A in Fig. 7(b)) while for e = 24 mm, the maximum load occurs at a deflection of about 8 mm (point B in Fig. 7(b)) and no peak appears in the load–deflection response.

Relations $R - a/h_w - t_f$ for e = 6 mm and e = 24 mm obtained from FEA are represented in Fig. 8. It can be observed that the parameter a/h_w has only a very small influence on the strength reduction coefficient, R, for the range of parameter values considered in the numerical simulation. Experimental results



(a) European steel -FEA-. (b) Serbian steel -experiments-.

Fig. 4. Material curves.

Table 3
Strength reduction coefficient obtained from FEA for European steel S235, considering maximal loads ($h_w = 700$ mm, $t_w = 6$ mm, $b_f = 150$ mm, $c = 50$ mm).

t_f (mm)	e (mm) 0	6	12	18	24	a (mm)
6	1	0.792	0.480	0.359	0.292	700
12	1	0.930	0.772	0.679	0.596	700
18	1	0.964	0.876	0.825	0.764	700
24	1	0.964	0.916	0.895	0.835	700
6	1	0.682	0.481	0.432	0.383	910
12	1	0.809	0.727	0.611	0.521	910
18	1	0.967	0.894	0.834	0.751	910
24	1	0.974	0.936	0.901	0.854	910
6	1	0.675	0.453	0.377	0.304	1050
12	1	0.870	0.694	0.623	0.539	1050
18	1	0.959	0.881	0.822	0.735	1050
24	1	0.968	0.955	0.929	0.877	1050
6	1	0.699	0.514	0.387	0.296	1260
12	1	0.878	0.705	0.626	0.547	1260
18	1	0.924	0.885	0.823	0.739	1260
24	1	0.989	0.947	0.926	0.906	1260
6	1	0.723	0.663	0.528	0.405	1400
12	1	0.921	0.814	0.735	0.672	1400
18	1	0.936	0.853	0.826	0.756	1400
24	1	0.985	0.967	0.936	0.931	1400

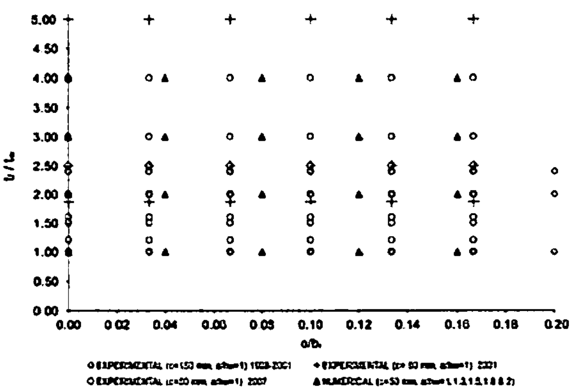


Fig. 5. Available models.

obtained by Bergfelt [13] for welded plate girders showed no influence of the panel aspect ratio, a/h_w, on longitudinally stiffened webs subjected to patch loading. However, it is clear that this parameter influences the behaviour of beams without longitudinal stiffeners. In Fig. 7(b) ($a = 910$ mm and $t_f = 24$ mm), post-buckling behaviour can be observed for small values of eccentricity ($e = 0$ or 6 mm) after the maximum strength is attained on the load–deflection curve. As the girder span a increases this behaviour is more evident. Figs. 9 and 10 illustrate this using

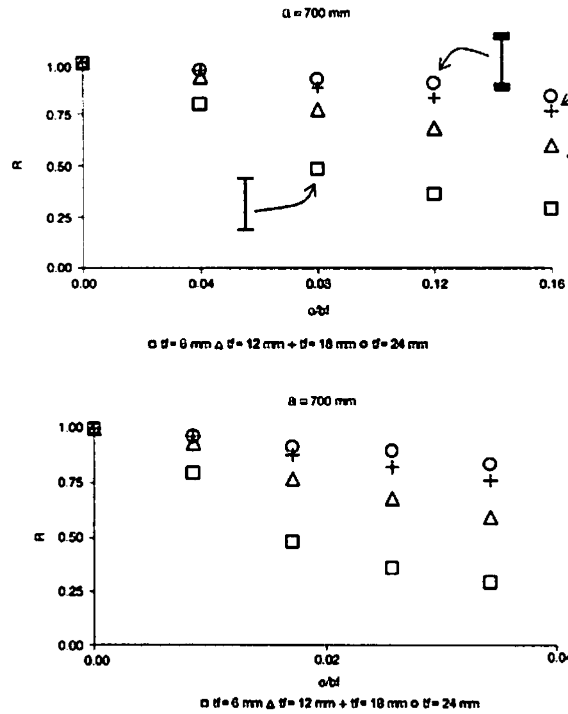
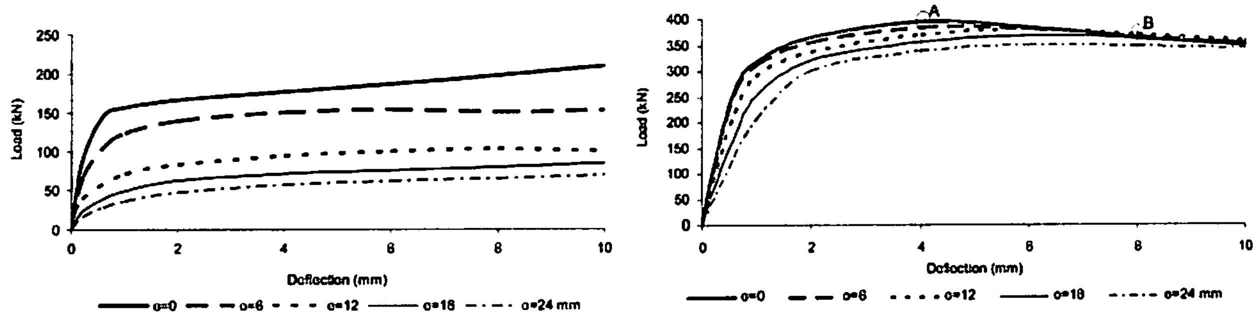


Fig. 6. Different relative eccentricities. FEM analysis results.

plots of load–deflection response for $a = 1260$ and 1400 mm respectively; computed response for values of $t_f = 12$ and 24 mm are shown in both figures. For $t_f = 12$ mm, no post-buckling reduction in load resistance is evident for $a = 1260$ mm (Fig. 9) but a post-buckling reduction in load resistance is evident for $a = 1400$ mm (Fig. 10). Similar results were obtained by Graciano and Edlund [11] who observed that the relationship t_f/t_w for a certain value of a affect the post peak behaviour of plate girders under patch loading.

Experiments conducted to date have not shown this post-buckling behaviour, because all test specimens have $a/h_w \approx 1$, for which post-buckling behavior would not be expected on the basis of FEA results. In the experimental work the maximum load observed in the load-deformation response is identified as the resistance load, so this reading should correspond to the value of maximum load in the load–deflection curve obtained by finite element analysis.

One reason for the increase of the strength reduction coefficient with t_f can be explained as follows. In Fig. 11, out-of-plane node displacements in the plane of symmetry of the web at a midspan section are represented for $a = 1260$ mm, $t_f = 24$ mm,



(a) (a) = 910/mm-t_f = 6 mm.

(b) a = 910/mm-t_f = 24 mm.

Fig. 7. Load–deflection graphs for different values of $t_f - e$, a = 910 mm.

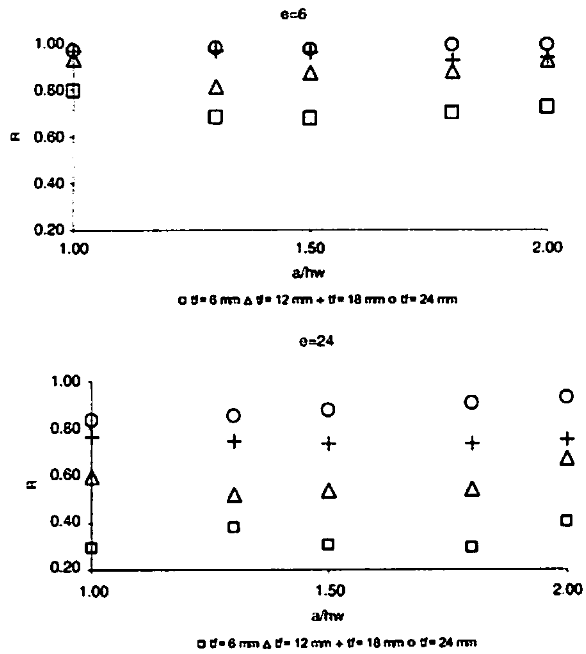


Fig. 8. Relations $R - a/h_w - t_f$ for e = 6 mm and e = 24 mm.

and $e = 0$ (Fig. 11(a)) and $e = 18$ mm (Fig. 11(b)). Out-of-plane nodal displacements are represented for different values of the vertical displacement of the loading point. Positive and negative signs denote data from different sides of undeformed web centerline. For concentrically loaded girders, similar behaviour was observed by Granath and Lagerqvist [14]. However, when load is applied eccentrically, portions of the web displace to either side of the undeformed web centerline, as shown in Fig. 11(b). Overall displacements associated with this mode of buckling of the web are reduced relative to the concentrically loaded case. The reduced displacements are associated with additional resistance in the web, relative to the concentrically loaded case.

6. Development of a revised expression for the strength reduction coefficient

To quantify the reduction in strength due to load eccentricity, a strength reduction coefficient, R, is defined (Eq. (1)). According to an earlier expression developed by Galambos [4], R is a function of t_f/t_w and varies linearly with e/b_f (Eqs. (2) and (3)). This expression was shown to give conservative estimates of R for large eccentricities ($e/b_f > 0.06$) in Fig. 2.

Several candidate expressions of R coefficient are evaluated in this paper. Even though the numerical studies indicated that the R coefficient should be a quadratic function of the relative eccentricity e/b_f, the first expression evaluated uses the form of the Galambos' expression, but with constants fit to the complete available data set, including all historical data:

$$m = A \cdot \left(\frac{t_f}{t_w}\right)^2 + B \cdot \left(\frac{t_f}{t_w}\right) + C$$
$$n = D + E \cdot \left(\frac{t_f}{t_w}\right). \tag{4}$$

Constants are fitting using the square least technique taking into account all experimental results ([1–3,9,10] and Table 1) and numerical values of R obtained from ANSYS (Table 3) for specimen dimensions not tested. Parameter values for the tests considered in developing the revised expression are represented in Fig. 5.

Values of constants for Eq. (4) and corresponding errors are shown in Table 4. The variability of the data respect to the model is measured through the residual sum of squares SS_{err}, expressed as:

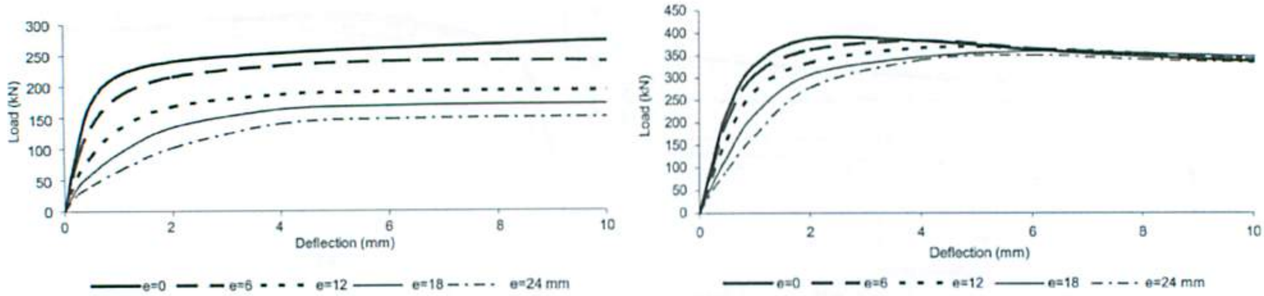
$$SS_{err} = \sum_{1}^{n} (R_M - R_F)^2 \tag{5}$$

where n is the number of data, R_M is the strength reduction coefficient obtained from experimental or numerical results and R_F is the strength reduction coefficient using the corresponding formulae for R.

The results of the revised Galambos' expression, obtained by changing coefficients A to E in Eq. (4) (Galambos' modified expression) and the original expression (Eq. (3)) are compared in Table 4. The error obtained using new coefficients is considerably reduced relative to that obtained with the original Galambos' expression. (Eqs. (2) and (3)).

Experimental and numerical results [1] show that the behaviour of the web under eccentric line loads is different from that under concentric line loads because under concentric loads the beam flange does not deform and only moves vertically downward into the web while under eccentric loads the flange deflected downward and twisted. Because the torsional and flexural stiffness of the flange and the value of the critical load for local buckling of the web under transverse forces (EC3-1-5. Art. 6.4 [15]) are a function of t_f^3 and t_w^3, respectively, a cubic relation between R and t_f/t_w is considered as another possible expression. A new formulation for the strength reduction coefficient is evaluated maintaining a linear relationship between R and e/b_f (Eq. (2)) but using cubic approximations for m and n:

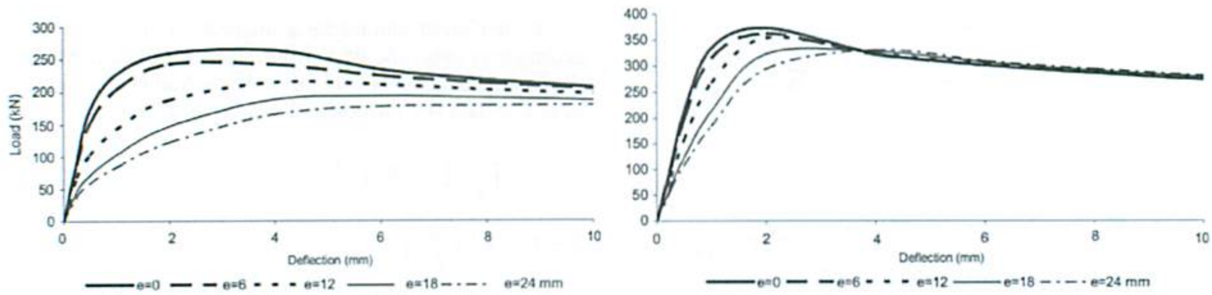
$$m = A \cdot \left(\frac{t_f}{t_w}\right)^3 + B \cdot \left(\frac{t_f}{t_w}\right)^2 + C \cdot \left(\frac{t_f}{t_w}\right) + D$$
$$n = E \cdot \left(\frac{t_f}{t_w}\right)^3 + F \cdot \left(\frac{t_f}{t_w}\right)^2 + G \cdot \left(\frac{t_f}{t_w}\right) + H. \tag{6}$$



(a) $a = 1260$ mm-$t_f = 12$ mm.

(b) $a = 1260$ mm-$t_f = 24$ mm.

Fig. 9. Load–deflection graphs for different values of $t_f - e$, $a = 1260$ mm.



(a) $a = 1400$/mm-$t_f = 24$ mm.

(b) $a = 910$/mm-$t_f = 24$ mm.

Fig. 10. Load–deflection graphs for different values of $t_f - e$, $a = 1400$ mm.

Table 4
Constant coefficients for Eq. (4).

	Coefficients					Error $= \sum(R_M - R_F)^2$		
	A	B	C	D	E	Experiment	Numerical	All of them
Eq. (4) (Revised Galambos)	−0.32	3.60	−7.67	1.00	−0.0025	0.779	0.296	1.075
Eq. (3) (Original Galambos)	−0.45	4.55	−12.75	1.15	−0.025	5.413	3.699	9.113

Table 5
Constant coefficients for Eq. (6).

Coefficients						Error $= \sum(R_M - R_F)^2$		
A	B	C	D	G	H	Experiment	Numerical	All of them
−0.012	−0.22	2.85	−7.55	−0.0033	1.00	0.751	0.305	0.994
0	−0.22	2.85	−7.55	−0.0033	1.00	0.780	0.370	1.149

Constants are also fitted using the least squares technique. Values of constants and corresponding errors (in terms of the residual sum of squares (5)) are shown in Table 5, in which $E = F = 0$. Note that the total error increases only 15% if the coefficient A is set equal to zero in Eq. (6), indicating only slight dependence on the cubic power of t_f/t_w, Table 5.

Although Fig. 8 shows that the shape factor of the web plate, a/h_w, has a very small influence on R, a possible dependence on a/h_w is considered using Eq. (2) and m and n defined as:

$$m = A \cdot \left(\frac{t_f}{t_w}\alpha^G\right)^3 + B \cdot \left(\frac{t_f}{t_w}\alpha^G\right)^2 + C \cdot \left(\frac{t_f}{t_w}\alpha^G\right) + D$$
$$n = E + F \cdot \left(\frac{t_f}{t_w}\alpha^G\right) \tag{7}$$

where $\alpha = a/h_w$. Application of least squares fitting results in new constants (Table 6). In Table 6, the value of constant G is close to zero, confirming that the parameter α has very little influence on the strength reduction coefficient.

Following guidance developed in the numerical analysis, a quadratic function of e/b_f is now investigated:

$$R = m \cdot \left(\frac{e}{b_f}\right)^2 + n \cdot \left(\frac{e}{b_f}\right) + k \leq 1$$
$$m = A \cdot \left(\frac{t_f}{t_w}\right)^2 + B \cdot \left(\frac{t_f}{t_w}\right) + C \tag{8}$$
$$n = D + E \cdot \left(\frac{t_f}{t_w}\right).$$

Values of constants and errors are summarized in Table 7(a). The error is lower than that obtained for the previous candidate expressions.

Errors (SS_{err}) corresponding to the parabolic formulation of Eq. (8) are smaller than the previously proposed formulations (Table 4 to Table 7). The differences between different approximations of R are more important for the experimental data that for the numerical data. The scatter in the experiment versus numerical data is due to the imperfections inherent to all experimental conditions (residual stress, loading, initial out-of-straightness and variability in material properties).

In order to improve the formulation of R obtained from Eq. (8), numerical results have been "pondered" with a κ coefficient that

Table 6
Constant coefficients for Eq. (7).

Coefficients							Error $= \sum(R_M - R_F)^2$		
A	*B*	*C*	*D*	*E*	*F*	*G*	Experiment	Numerical	All of them
0.013	−0.42	3.28	−7.86	1.00	−0.0025	0.10	0.780	0.284	1.064

Table 7
Constant coefficients for Eq. (8).

(a) Coefficients (ponderation factor $\kappa = 2$)						Error $= \sum(R_M - R_F)^2$		
A	*B*	*C*	*D*	*E*	*k*	Experimental	Numerical	All of them
−0.864	−14.40	38.00	−12.30	4.22	1.01	0.530	0.258	0.788
(b) Coefficients (ponderation factor $\kappa = 1.7$)						Error $= \sum(R_M - R_F)^2$		
−1.03	−10.15	31.70	−11.30	3.65	1.00	0.587	0.215	0.802

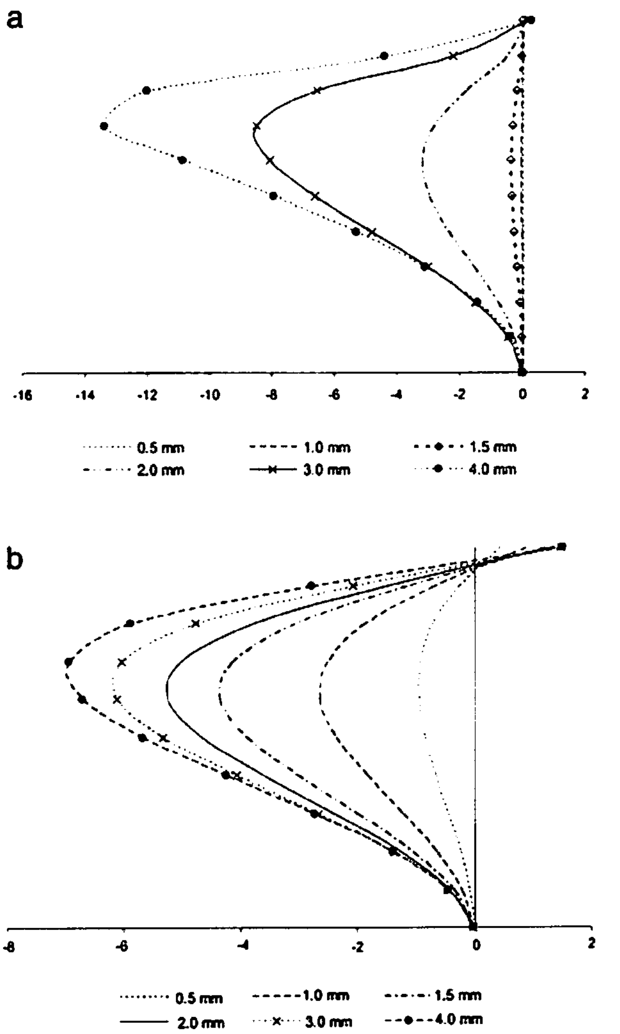


Fig. 11. Midspan out-of-plane displacements for different values of vertical deflection, $a = 1260$ mm, $t_f = 24$ mm. (a) $e = 0$ mm. (b) $e = 18$ mm.

varies from 1.2 to 2.7 using a technique called pondered least squares [16]. The function to minimize is:

$$PSS_{err} = \sum_{\text{numerical data}} (R_M - R_F)^{\kappa} + \sum_{\text{experimental data}} (R_M - R_F)^2. \quad (9)$$

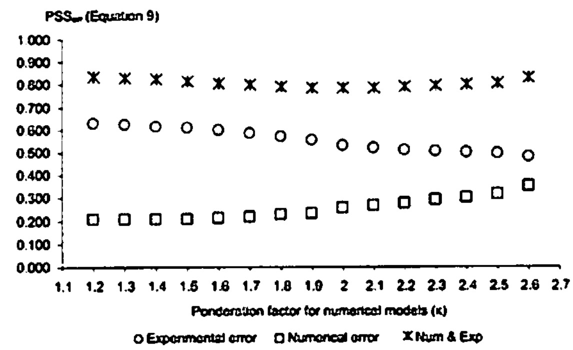


Fig. 12. Errors from pondered least squares technique.

Constants in Table 7(a) were obtained from least squares fitting, resulting in $\kappa = 2$. Fig. 12 shows values of PSS_{err} as function of the weighting factor κ. For $\kappa \leq 1.7$ errors for numerical and experimental data remain constant. Table 7(b) shows the constants for Eq. (8) in case of $\kappa = 1.7$ and the corresponding errors.

Experimental and numerical results are represented graphically in Fig. 13 with corresponding strength reduction coefficients according to Eq. (8). Fig. 13 shows clearly that the pondered least squares regression represented by Eq. (8) results in substantially less error than was obtained using the original equation given by Galambos (Eqs. (2) and (3)).

7. Practical design

Parts 1-5 of Eurocode 3 [15] includes a check of the buckling resistance of girder webs to patch loading at ultimate load limit state based on the research conducted by Lagerqvist and Johansson [17]. The methodology in [17] is based on the post-peak? strength of the plate girder, determined by a stability check using buckling curves for patch loading resistance. Carretero and Lebet [18] state that ultimate strength prediction using EC-3 Part 1-5 [15] gives conservative results, about 50% of experimental ultimate strengths.

Just as the way that other effects, such as the influence of longitudinal stiffeners on patch loading resistance [19] are considered, the influence on patch load eccentricity on strength is proposed herein based on regression analysis of experimental and finite element studies. The reduced strength is proposed to be determined as the product of the ultimate strength for concentric loading, obtained by EC-3 Part 1-5 [15], and the strength reduction coefficient, *R*, proposed herein. This procedure is possible because the failure mechanisms of the web are similar for both concentric and eccentric patch loading.

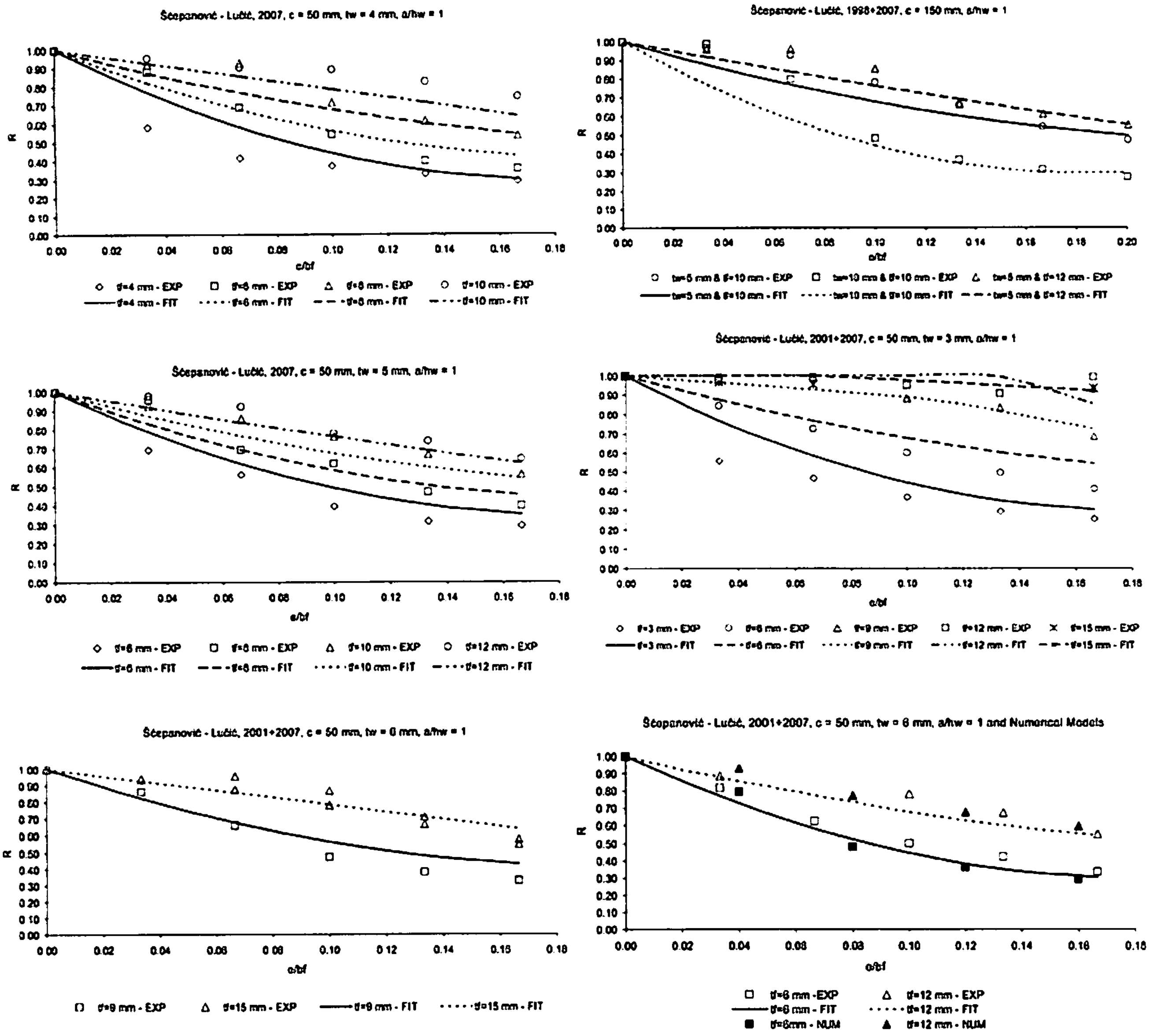


Fig. 13. Reduction coefficient approximation for Eq. (8) with $\kappa = 1.7$

The strength reduction coefficient R should be used only within the range of test data that is indicated in Fig. 5, for same steel material yield strength in flange and web and web panel aspect, a/h_w, ratio equal to 1.

8. Conclusion

The existing expressions for R were shown to be unnecessarily conservative for large eccentricities. A parametric study evaluated the variables that affect the ultimate strength of girders subjected to eccentric patch loading. For larger values of patch loading eccentricity, a parabolic dependence of R on relative eccentricity was shown to be more accurate than the linear expression currently available.

Finite element models indicate that as girder span a increases, the response of the girder through the entire load-deformation response shows a peak, followed by post-peak behaviour. However, the parameter a has a negligible influence on the strength reduction coefficient, R, which is concerned only with the ratio of maximum loads (under eccentric patch loading relative to concentric patch loading).

Results from a recent experimental program to characterize the strength reduction coefficient (R) for eccentric patch loadings were presented. A comprehensive dataset consisting of experimental data obtained in previous research [1–3,9,10], the recent experimental program, and finite element analyses described herein was developed. A regression analysis was used to evaluate several candidate expressions for the reduction coefficient. While the original linear equation and a re-calibrated linear equation both result in relatively high errors, a quadratic equation was found to significantly reduce the error. This new expression, given by Eq. (8), provides more accurate estimates of the R coefficient than the existing expression, particularly for large eccentricities. The proposed expression avoids the very conservative estimates of the R coefficient that can be obtained with the existing expression for large eccentricities.

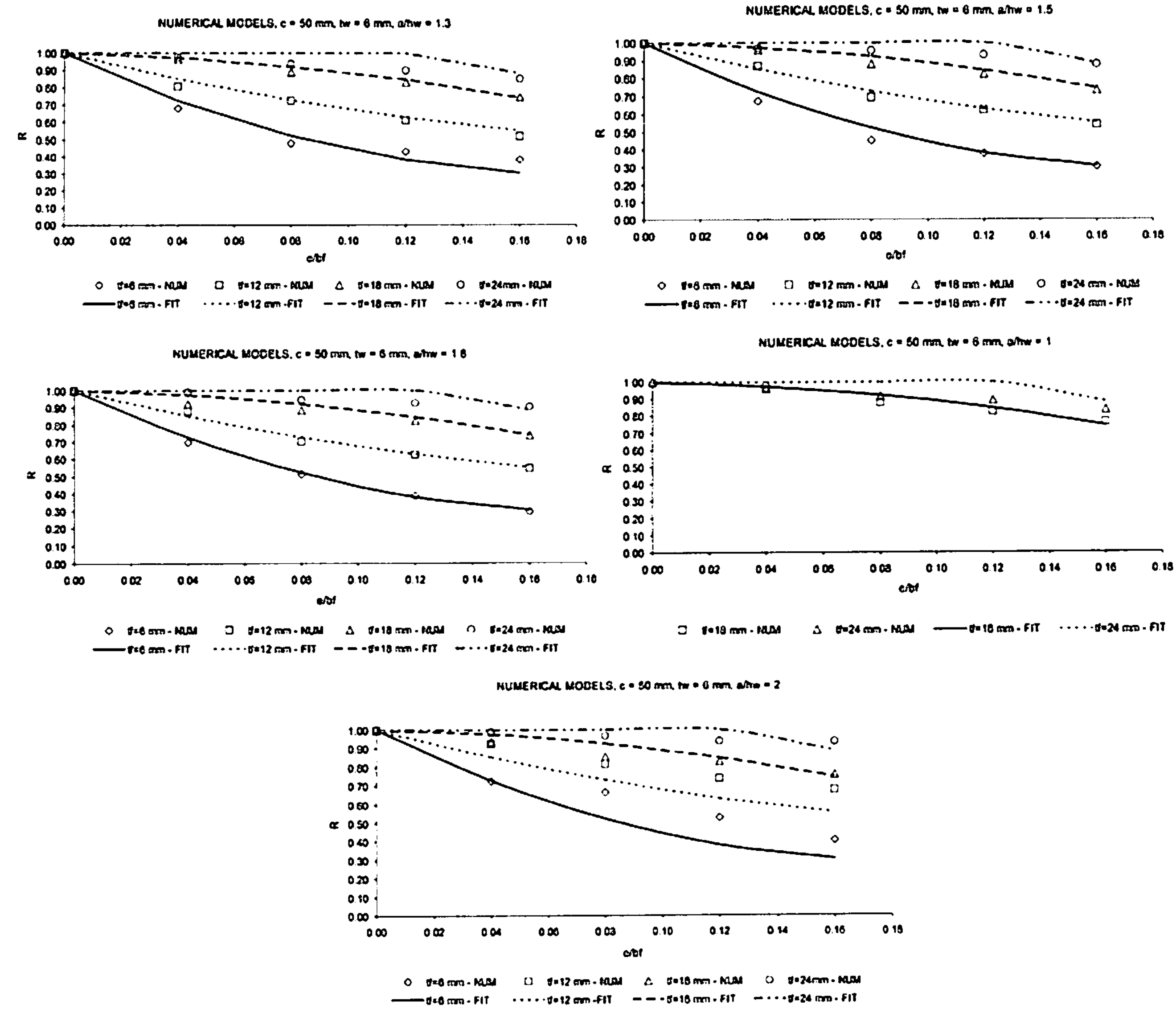


Fig. 13. (*continued*)

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Eccentrically patch-loaded steel I-girders: The influence of patch load length on the ultimate strength

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ABSTRACT

The ultimate strength of steel I-girders is reduced when the loads are applied at an eccentricity relative to the center of the section, compared with the centric case, for which the loads are applied in the plane of the web. Current expressions for the strength reduction coefficient for eccentrically patch-loaded steel I-girders do not account for the length of the patch loading. In this paper, the effect of patch load length is investigated using a substantially larger data set than was available in the past. The length of the patch load is found to have a significant influence on the strength reduction coefficient for small ratios of flange thickness to web thickness, t_f/t_w.

1. Introduction

The term patch loading describes loading applied over a small area or length of a structural element. A common situation in structural engineering occurs when a downward compressive load is applied to the flange of an I-section, so that the web is compressed in the region below the applied load. Examples are numerous and present in many different structures, including crane and bridge girders. Of course, some eccentricity of load relative to the web plane is unavoidable in engineering practice (Fig. 1).

Experimental and numerical analyses of eccentrically patch-loaded steel I-girders was begun by Elgaaly and Nunan [1] and Elgaaly and Salkar [2] at the University of Maine. Tests were also done by Drdacky [3] at the Institute of Theoretical and Applied Mechanics, Czech Academy of Sciences. A decade later, a new series of experiments was conducted at the University of Montenegro, as described by Lučić [4–6], Šćepanović [7], Lučić and Šćepanović [8] and Šćepanović et al. [9].

Experimental and numerical data show that the ultimate strength reduces as the eccentricity of patch loading increases. This decrease in the ultimate strength can be quantified by a strength reduction coefficient, R, which relates the ultimate strength of an eccentrically loaded girder to the ultimate strength of an otherwise similar centrically loaded girder:

$$R = \frac{\text{ultimate strength of eccentrically loaded girder}}{\text{ultimate strength of centrically loaded girder}}. \tag{1}$$

According to Galambos [10], R may be calculated by Eq. (2), which was developed on the basis of experiments reported by Elgaaly and Nunan [1] and Elgaaly and Salkar [2].

$$\left.\begin{aligned} R &= m \cdot \frac{e}{b_f} + n \leq 1 \\ m &= -0.45 \cdot \left(\frac{t_f}{t_w}\right)^2 + 4.55 \cdot \left(\frac{t_f}{t_w}\right) - 12.75 \\ n &= 1.15 - 0.025 \cdot \left(\frac{t_f}{t_w}\right) \end{aligned}\right\} \tag{2}$$

where e = load eccentricity, b_f = the width of the flange, t_f = the thickness of the flange, and t_w = the thickness of the web.

Experimental data from 1998, 2001 and 2007, described by Lučić [4–6], Šćepanović [7], Lučić and Šćepanović [8] and Šćepanović et al. [9] were used by Šćepanović et al. [9] to obtain an improved expression for the strength reduction coefficient:

$$\left.\begin{aligned} R &= m \cdot \left(\frac{e}{b_f}\right)^2 + n \cdot \left(\frac{e}{b_f}\right) + 1 \leq 1 \\ m &= -1.03 \cdot \left(\frac{t_f}{t_w}\right)^2 - 10.15 \cdot \left(\frac{t_f}{t_w}\right) + 31.70 \\ n &= -11.30 + 3.65 \cdot \left(\frac{t_f}{t_w}\right) \end{aligned}\right\}. \tag{3}$$

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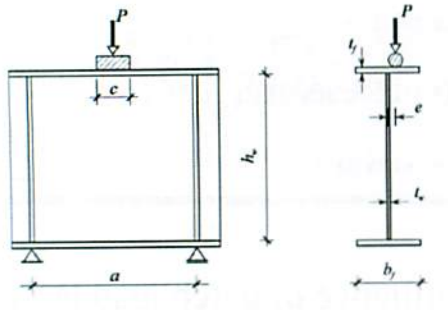


Fig. 1. I-girder under eccentric patch loading.

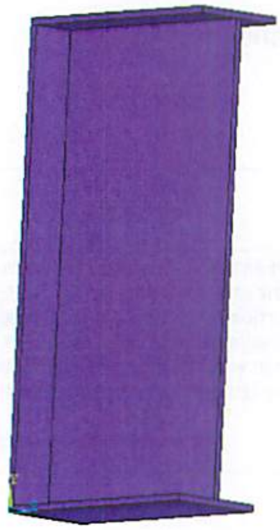


Fig. 2. Finite element model.

Eq. (3) was established considering both experimental and numerical finite element analysis results. The numerical and experimental models used to obtain Eq. (3) had a length of the patch load, c, equal to 50 mm.

Eqs. (2) and (3) identify the most important parameters that influence R. However, neither Eq. (2) nor Eq. (3) takes into account the length of the patch load, c, which is the subject of this article.

2. Available experimental data

In the first experimental study, by Elgaaly and Nunan [1], the same length of patch load, c, was used for all test specimens (c = 61 mm), so that the influence of this parameter was not evaluated. In subsequent studies at the University of Maine, by Elgaaly and Salkar [2], the parameter c was kept constant for most tests: c = 127 mm. These experimental tests were performed for different types of girder, and an evaluation of the influence of c was not possible.

The tests conducted at the University of Montenegro in 1998 and 2001, described by Lučić [4–6], Šćepanović [7] and Lučić and Šćepanović [8], used two values of c (50 and 150 mm), but the experimental campaign did not aim to study the influence of the patch load length, so tests with different values of c were not directly comparable.

In the 1998 experiments, described by Lučić [4–6], all girders, in all series, had the same values of span a, web depth h_w and flange width b_f ($a = h_w = 700$ mm, $b_f = 150$ mm). The series differed in the web and flange thicknesses. Of interest here are three girder geometries that were tested in three series: BIII-1, BIII-2 and BIII-3 (Table 1(a), in regular (non-bold) type). In series

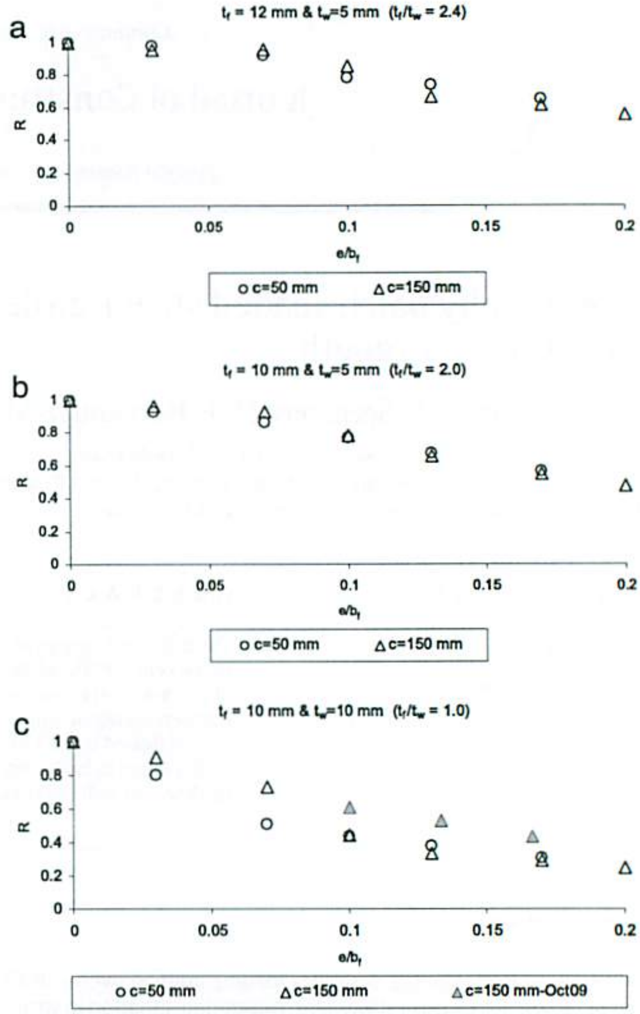


Fig. 3. Graphs of R versus e/b_f for tested load lengths c and different ratios t_f/t_w.

BIII-1: $t_w = 5$ mm, $t_f = 10$ mm (i.e. $t_f/t_w = 2$). In series BIII-2: $t_w = 10$ mm, $t_f = 10$ mm (i.e. $t_f/t_w = 1$). In series BIII-3: $t_w = 5$ mm, $t_f = 12$ mm (i.e. $t_f/t_w = 2.4$). In each series, girders were tested with 5 different load eccentricities $e = 0$ mm, 15 mm, 20 mm, 25 mm and 30 mm. Small eccentricities $e = 5$ mm and 10 mm were not considered. In all tests the load length was the same: $c = 150$ mm.

A new experimental series was conducted at the University of Montenegro in 2007, and this series provided data with which to evaluate the influence of the length of the load, c. In the 2007 tests, the girder span a, web depth h_w and flange width b_f were the same as in 1998 and 2001 ($a = h_w = 700$ mm, $b_f = 150$ mm). The web and flange thicknesses varied in a wide range ($t_w = 3$ mm, 4 mm, 5 mm, 6 mm and 10 mm, $t_f = 3$ mm, 4 mm, 6 mm, 8 mm, 9 mm, 10 mm and 12 mm) covering some gaps from previous tests and enlarging the range of existing test data. Line loads were applied along load lengths c = 50 mm or 150 mm, with eccentricity values $e = 0$ mm, 5 mm, 10 mm, 15 mm, 20 mm and 25 mm. The experimental data used to study the influence of the load length are grouped in series EB-XXI, EB-V, EB-VI and EB-VII, which are identified with bold type in Table 1(a).

3. Nonlinear finite element analysis

Graciano and Edlund [11] demonstrated that nonlinear finite element analysis can be a useful tool to study the behaviour of plate

Table 1
Experimental strength reduction coefficient R for different load lengths c ($a = h_w = 700$ mm, $b_f = 150$ mm).

(a) Specimens tested in Montenegro between 1998 and 2007

e (mm)	e/b_f	R_{exp} BIII-1 EB-XXI $c = 150$ mm	EB-V $c = 50$ mm	BIII-2 EB-XXI $c = 150$ mm	EB-VI $c = 50$ mm	BIII-3 EB-XXI $c = 150$ mm	EB-VII $c = 50$ mm
0	0.000	1.000	**1.000**	1.000	**1.000**	1.000	**1.000**
5	0.033	**0.965**	**0.926**	**0.904**	**0.799**	**0.959**	**0.978**
10	0.067	**0.927**	**0.860**	**0.732**	**0.507**	**0.959**	**0.922**
15	0.100	0.781	**0.764**	0.442	**0.435**	0.856	**0.783**
20	0.133	0.659	**0.668**	0.339	**0.382**	0.666	**0.739**
25	0.167	0.544	**0.563**	0.290	**0.306**	0.611	**0.648**
30	0.200	0.472		0.250		0.554	
	t_w (mm)	5		10		5	
	t_f (mm)	10		10		12	
	t_f/t_w	2		1		2.4	
	h_w/t_w	140		70		140	

(b) Specimens tested in Spain in 2009

e (mm)	e/b_f	R_{exp} $c = 150$ mm
0	0.000	1.000
15	0.100	0.604
20	0.133	0.525
25	0.167	0.439
	t_w (mm)	10
	t_f (mm)	10
	t_f/t_w	1
	h_w/t_w	70

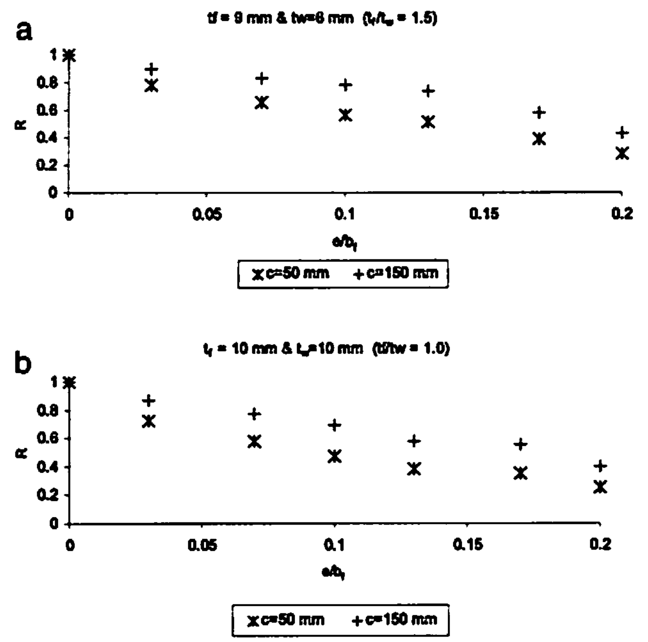


Fig. 4. Graphs of R versus e/b_f from finite element analysis for load lengths $c = 50$ mm and 150 mm and ratios $t_f/t_w = 1.5$ and 1.0.

girders under patch loading. Finite element analysis is used here to extend the parameter range for which the R values are established.

The available experimental data provide information on the influence of the patch load length for values of $t_f/t_w = 1, 2$ and 2.4. In order to obtain data for $t_f/t_w = 1.5$ and 1.0 a nonlinear finite element (FE) analysis was conducted using the ANSYS program. The load was applied at midspan on the upper flange and vertical stiffeners were located at both girder ends, just as in the experiments. An initial out-of-plane unstressed configuration was established by deforming the geometry of the web by 5 mm (without induced stress) to represent an initial imperfection. Raoul et al. [12], Graciano and Edlund [11], Granath [13] and Gil-Martín et al. [14] have shown that, for small amplitudes, the amplitude of initial imperfection does not affect the load capacity.

The element employed to model the web, flanges and vertical stiffeners is SOLID92. This element is a ten-node three-dimensional solid element having three degrees of freedom at each node, consisting of translations in the x, y, and z directions. Plasticity, large deflection and large strain capabilities are available and were used. Material nonlinearities were represented by a nonlinear elasto-plastic stress–strain relationship, while plasticity was represented using a kinematic hardening rule. Approximately 5000 elements were used in each 3D model of a test specimen. Due to the symmetry in geometry, load conditions and expected deformation, just one half of each girder was modelled (Fig. 2). The patch load was transferred into the girder at the top of the upper flange by controlling the displacement of the patch nodes in the FE analysis. Residual stresses were not considered in this work, and the weld fillet at the junction of web and flange was not modelled.

Validation of the FE model (FEM) is reported by Šćepanović et al. [9] by comparing experimental data with the FEM results. In those cases used to validate the model, the strength reduction coefficient R (see (1)) obtained from the finite element analysis differed less than 3% from the value of R determined empirically.

All parameters, except t_f and t_w, were kept constant in the numerical study in relation to the experimental tests. Table 2 shows the strength reduction coefficients obtained in the FEM numerical analyses.

4. Evaluation of results

In the 1998 tests reported by Lučić [4–6], with $c = 150$ mm, small eccentricities of 5 mm and 10 mm were not tested. Series EB-XXI conducted in subsequent experimental research in 2007 was undertaken to fill in these missing data. In series EB-XXI girders of the same dimensions as in 1998 (series BIII-1, BIII-2 and BIII-3) were tested with patch length $c = 150$ mm and with load eccentricities $e = 5$ mm and 10 mm (Table 1(a), bold data in columns 3, 5 and 7).

Fig. 5. Test setups used in Montenegro (left) and in Spain (right).

In series EB-V, VI and VII of the 2007 study, the tests mentioned in the previous paragraph were repeated, with the same girder geometries, but with $c = 50$ mm. The load eccentricity varied over the full range $e = 0$ mm, 5 mm, 10 mm, 15 mm, 20 mm and 25 mm. The girder dimensions in series EB-V corresponded to series BIII-1, EB-VI to BIII-2 and EB-VII to BIII-3 (Table 1(a), bold data in columns 4, 6 and 8).

In this way, a complete experimental dataset of strength reduction coefficient R values (Table 1(a)) was obtained for girders of three different geometry types (columns 3–4 for $t_f/t_w = 2$, columns 5–6 for $t_f/t_w = 1$ and columns 7–8 for $t_f/t_w = 2.4$ in Table 1(a)), each for $c = 50$ mm (columns 4, 6 and 8 in Table 1(a)) and for $c = 150$ mm (columns 3, 5 and 7 in Table 1(a)), over the full range of $e = 0$ mm, 5 mm, 10 mm, 15 mm, 20 mm and 25 mm.

A graphical comparison of experimental data for both values of c tested, i.e. $c = 50$ mm and 150 mm (Table 1(a)), is presented in Fig. 3, which shows the strength reduction coefficient R versus the dimensionless load eccentricity e/b_f. The comparison is made considering the ratio t_f/t_w as an important parameter for girders under eccentric patch loading. Fig. 3(a) is for $t_f/t_w = 2.4$, Fig. 3(b) is for $t_f/t_w = 2.0$ and Fig. 3(c) is for $t_f/t_w = 1.0$. Fig. 3(a) and (b) show that when $t_f/t_w \geq 2$ the patch length does not influence in the strength reduction coefficient value, R, for the entire range of values of the dimensionless load eccentricity, e/b_f, considered.

In Fig. 3(c), a noticeable reduction of the values of R appears for $e/b_f \geq 0.1$ and $c = 150$ mm (unfilled triangles) for specimens tested in 2007. In order to confirm this trend of the R versus e/b_f curve for $t_f/t_w = 1$, numerical values of the strength reduction coefficient R were obtained from FE analysis for this model and several values of the patch length, c (Table 2, Fig. 4(b)).

Figs. 3(c) and 4(b) correspond to the same model ($h_w = a = 700$ mm, $b_f = 150$ mm, $t_f = t_w = 10$ mm). Fig. 3(c) represents experimental values while Fig. 4(b) represents numerical values obtained from FE analysis. A comparison of Fig. 3(c) (unfilled triangles) and 4(b) shows that when $c = 150$ mm and $e/b_f \geq 0.1$, discrepancies exist between the experimental and numerical values of R. It is considered that these discrepancies may possibly be due to unintended errors in the setup or conduct or data acquisition associated with these experiments. In order to confirm the numerical results of Fig. 4(b), new tests were conducted in 2009 with $c = 150$ mm and values of eccentricity $e = 0$ (centric), 15 mm, 20 mm and 25 mm. The experimental program is summarized in the Appendix; the results obtained are summarized in Table 1(b). The 2009 experimentally determined values of R are presented in Fig. 3(c) using filled triangles, along with the results from the earlier tests in Montenegro. The new experimental results are seen to validate the numerical ones (Fig. 4(b)), described in the next section. The test setups used in Montenegro and Spain are shown in Fig. 5.

To obtain a relationship between R and e/b_f corresponding to $t_f/t_w = 1.5$, for which no experimental results exists, FE analyses were made using the program ANSYS as described in the next section. In Fig. 4(a), values of the strength reduction coefficient R versus the dimensionless load eccentricity e/b_f are presented for $t_f/t_w = 1.5$ and $c = 50$ mm and 150 mm, respectively.

A comparison of Fig. 3(a) and (b) with Figs. 3(c), 4(a) and (b) indicates that c has a less significant effect on the strength reduction coefficient R for the case of the ratio t_f/t_w greater or equal than 2.0. For a stocky flange and weak web, the behavior of the flange is close to that of a single supported plate; the flange can rotate freely for all values of c, making the patch length have negligible influence on R. In this case, where t_f/t_w is large enough, the bending stiffness of the flange has the largest influence on the behaviour of the girder subjected to patch loading; see [13].

Where the thickness of the flange is similar to thickness of the web, the flange is similar to a fixed-end plate due to the restraint against flange rotation provided by the web. In this case the influence of the patch length, c, is greater than in the previous case. Figs. 3(c), 4(a) and (b) imply that for small ratio t_f/t_w (≤ 1.5) the load length c has an influence on the strength reduction coefficient R.

5. Numerical study of the influence of patch length on the strength reduction coefficient

The existing expressions for the strength reduction coefficient R (Eqs. (2) and (3)) do not consider the patch length c, which is justified by Fig. 3(a) and (b). However, for ratios $t_f/t_w \leq 1.5$, the patch length c has a significant influence on the strength reduction coefficient R (Figs. 3(c), 4(a) and (b)).

In order to study the influence of c on the strength reduction coefficient, several numerical models with ratio t_f/t_w equal to 1.0 and 1.5 have been analyzed. Values of patch length equal to 50 mm, 100 mm, 150 mm and 200 mm have been analyzed. Values of R determined from the numerical study are summarized in Table 2 and presented in Fig. 6.

Fig. 6 shows the approximate expression corresponding to Eq. (3), proposed by Šćepanović et al. [9], represented by a grey color

Table 2
Strength reduction coefficient for the finite element model.

e (mm)	e/b_f	R			
		$c = 50$ mm	$c = 100$ mm	$c = 150$ mm	$c = 200$ mm
(a) ($a = h_w = 700$ mm, $b_f = 150$ mm, $t_f = 9$ mm and $t_w = 6$ mm)					
0	0.00	1.00	1.00	1.00	1.00
5	0.03	0.78	0.81	0.90	0.96
10	0.07	0.66	0.73	0.83	0.90
15	0.10	0.57	0.69	0.78	0.86
20	0.13	0.52	0.66	0.74	0.81
25	0.17	0.39	0.51	0.58	0.62
30	0.20	0.28	0.38	0.43	0.47
(b) ($a = h_w = 700$ mm, $b_f = 150$ mm, $t_f = 10$ mm and $t_w = 10$ mm)					
0	0.00	1.00	1.00	1.00	1.00
5	0.03	0.72	0.79	0.87	0.93
10	0.07	0.58	0.69	0.77	0.82
15	0.10	0.47	0.61	0.69	0.74
20	0.13	0.38	0.51	0.58	0.62
25	0.17	0.35	0.49	0.55	0.58
30	0.20	0.25	0.36	0.40	0.43

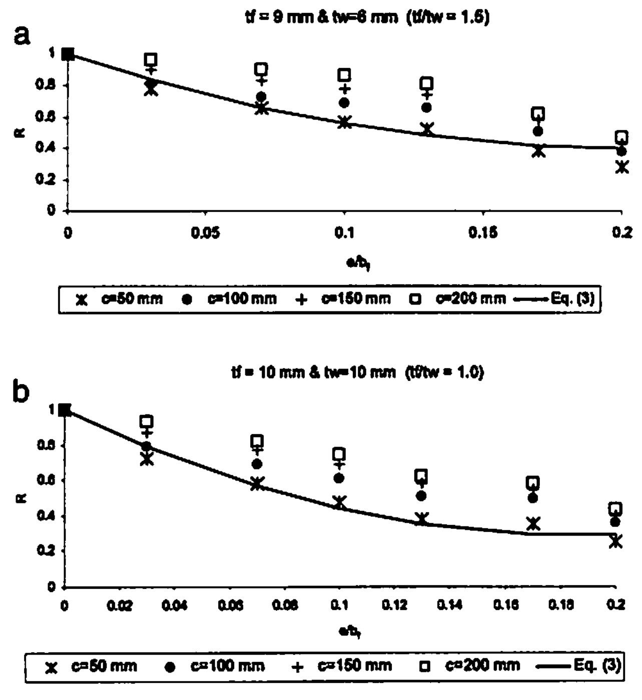


Fig. 6. Graphs of R versus e/b_f for load lengths $c = 50$ mm, 100 mm, 150 mm and 200 mm from FE analysis for ratios $t_f/t_w = 1.0$ and 1.5.

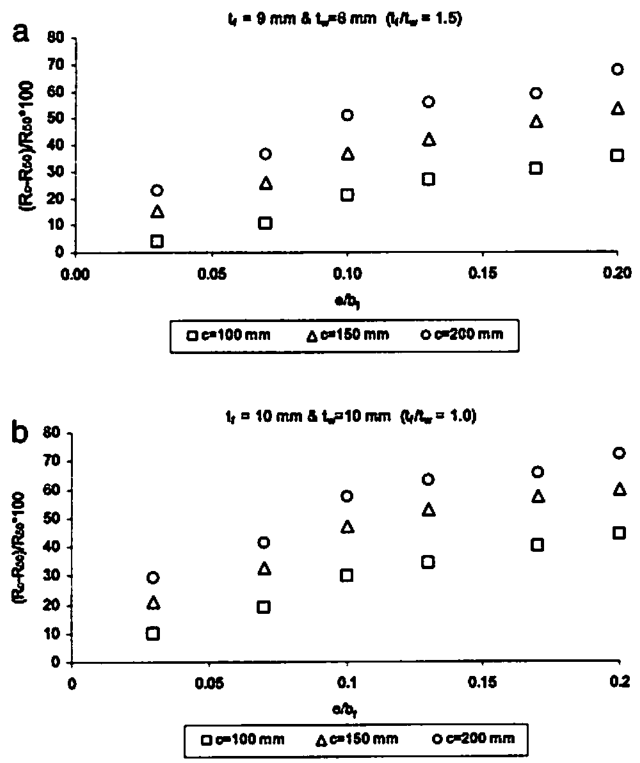


Fig. 7. Ponderation coefficient, k, for different load lengths c as a function of e/b_f.

curve. Because Eq. (3) was obtained from physical specimens and numerical models with $c = 50$ mm, this curve is very close to values of R for which $c = 50$ mm. Fig. 6 shows that the value of R increases with the value of the patch length, c, so values obtained from Eq. (3) are conservative for larger values of c.

To estimate the strength reduction coefficient R for values of c other than 50 mm, a ponderation coefficient k is defined. This coefficient has been obtained from numerical results corresponding to ratios t_f/t_w for which the ultimate strength of eccentric patch loading is affected by the length of patch, i.e. $t_f/t_w = 1.0$ and 1.5 (Fig. 4). The corrected reduction factor R for a length of patch load equal to c can be expressed as

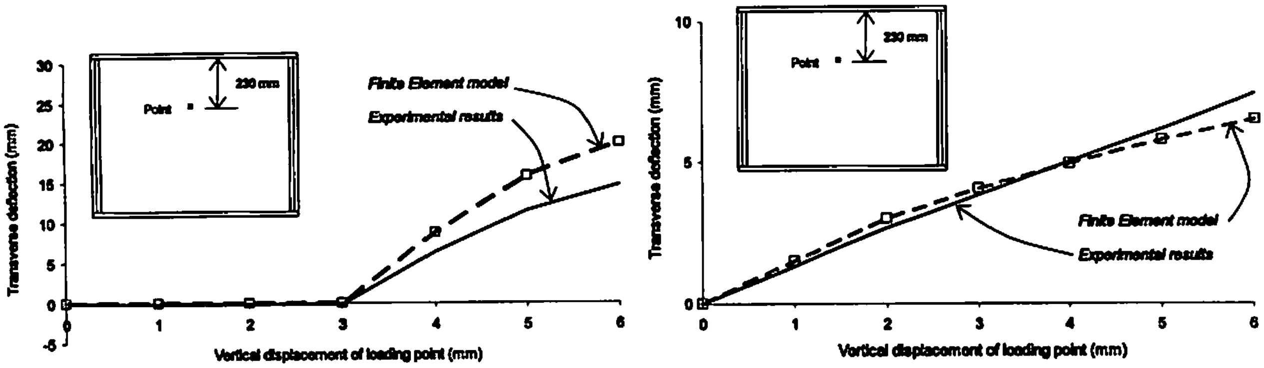
$$R_C = R_{50}(1 + k), \tag{4}$$

where R_{50} is the reduction factor coefficient, defined by Šćepanović et al. [15], obtained from Eq. (3), for $c = 50$ mm, and k is a ponderation coefficient, defined as

$$k = \frac{R_C - R_{50}}{R_{50}}. \tag{5}$$

Values of k were obtained from FE analysis for $t_f/t_w = 1.0$ and 1.5 and values of $c = 100$ mm, 150 mm and 200 mm. Values of k are represented in Fig. 7.

Fig. 7(a) and (b) plot values of k as a function of the dimensionless load eccentricity e/b_f for $c = 100$ mm, 150 mm and 200 mm for models with $t_f/t_w = 1.0$ and $t_f/t_w = 1.5$, respectively.



(a) Centrically loaded girder.

(b) Eccentrically loaded girder, $e = 25$ mm.

Fig. 8. Measured out-of-plane displacement at midspan and 230 mm under the loaded flange. (a) $e = 0$ mm (b) $e = 25$ mm. ($a = h_w = 700$ mm, $b_f = 150$ mm, $c = 150$ mm, $t_f = 10$ mm, $t_w = 10$ mm).

As provided in Fig. 7, k increases as the patch length c and the dimensionless load eccentricity e/b_f increase.

6. Conclusions

The ultimate strength of steel I-girders under eccentric patch loading was determined relative to the case of centric patch loading (loads in the plane of the web) by the use of a strength reduction coefficient, R. Current expressions for R do not account for the length of the patch loading, c.

In this paper, experimental results are used to observe that the length of patch loading has an effect on ultimate strength for ratios $t_f/t_w \leq 1.5$. Validated numerical models, using finite element analyses conducted with ANSYS, were used to obtain values of the strength reduction coefficient, R, for several values of patch length ($c = 50$ mm, 100 mm, 150 mm and 200 mm).

Values of R obtained from Eq. (3), corresponding to $c = 50$ mm, give conservative results as c and e/b_f increase. Improved estimates of ultimate strength are obtained through the pondered least squares method, which resulted in the definition of a ponderation coefficient, k (defined by Eq. (5), for $t_f/t_w \leq 1.5$) that results in improved estimates of R (Eq. (4)).

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Appendix. Description of 2009 experimental study

Tests of girders with $c = 150$ mm and values of $e = 0$ mm (centric), 15 mm, 20 mm and 25 mm were done in the University of Navarra under the direct supervision of Professor Hernández-Montes in 2009 to clarify the trends observed in Figs. 3(c) and 4(b). Specimens were loaded monotonically in an universal testing machine as illustrated in Fig. 5.

The development of web deformation was monitored by measuring deflections perpendicular to the web plane. The deflections were recorded at three measuring points along the vertical axis of the symmetry of the girder at midspan, at 130 mm, 240 mm and 350 mm under the loaded flange, respectively. A linear variable differential transducer (LVDT), with range of 100 mm and accuracy of 0.001 mm, was used to record the deflections at these three points. The tests were conducted in a stiff double servo-hydraulic steel frame (closed loop control) controlled by a Windows computer (IBERTEST PEV-2/ 400-W model). Each frame has a capacity of 400 kN (total capacity 800 kN). Fig. 5 right.

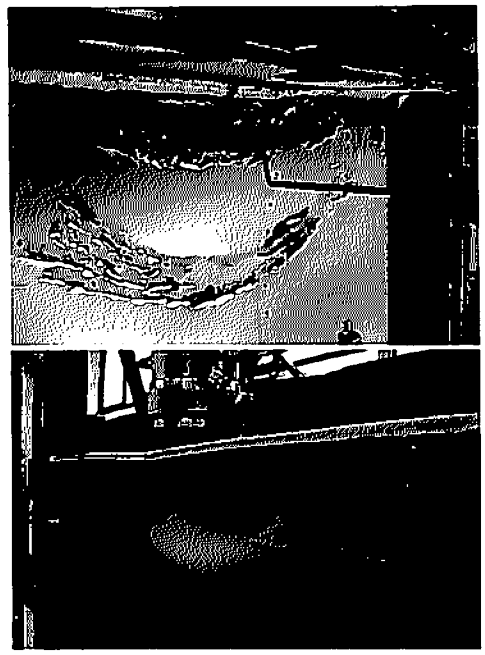

Fig. 9. Centric collapse mode observed in Montenegro corresponding to $a = h_w = 700$ mm, $b_f = 150$ mm, $c = 50$ mm, $t_f = 12$ mm, $t_w = 5$ mm and $e = 5$ mm.

The data recorded during the tests (read 6 times per second) were transferred to a spreadsheet. Comparison between FEM results and measured out-of-plane displacements of the point located at midspan and 230 mm under the loaded flange for the centrically loaded girder and for the eccentrically loaded girder, with $e = 25$ mm, are compared in Fig. 8(a) and (b), respectively. Good agreement between experimental and analytically determined displacements is apparent.

Because the collapse consists of localized deformation near the loaded flange, it was possible to test each girder twice. After the first test, the girder was turned upside down and retested.

For small values of eccentricity, a centric collapse mode was observed. In this case, the web has out-of-plane deflection associated with bending of the web to the loaded flange. Fig. 9 corresponds to a centric collapse mode observed in a specimen tested in Montenegro. When the eccentricity increases, the loaded flange rotates and out-of-plane deflections are induced in the web, under the loaded flange. Fig. 10 corresponds to an eccentric

Fig. 10. Eccentric collapse mode observed in Spain corresponding to $a = h_w = 700$ mm, $b_f = 150$ mm, $c = 150$ mm, $t_f = 10$ mm, $t_w = 10$ mm and $e = 15$ mm.

collapse mode observed in a specimen having eccentricity of 15 mm.

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- Šćepanović B: *"Patch Loading Researches at the University of Montenegro"*, invited presentation for the 9th Official Meeting of ECCS-TWG 8.3, London, October 2009.

PATCH LOADING RESEARCHES AT THE UNIVERSITY OF MONTENEGRO

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1. INTRODUCTION

Patch loading is loading applied over a small area or length of a structural element. A common situation in structural engineering is when local compressive patch load affects the flange of steel I-girder so that the web is compressed in the region bellow the applied load, Figure 1. Local stresses in web might cause local instability that may provoke element carrying capacity loss and, consequently, collapse of the whole structure. This is a rather complex and challenging issue of extremely evident elastic-plastic stresses and deformations. Apart from that, geometrical nonlinearity is noticeable even at the lowest loading level.

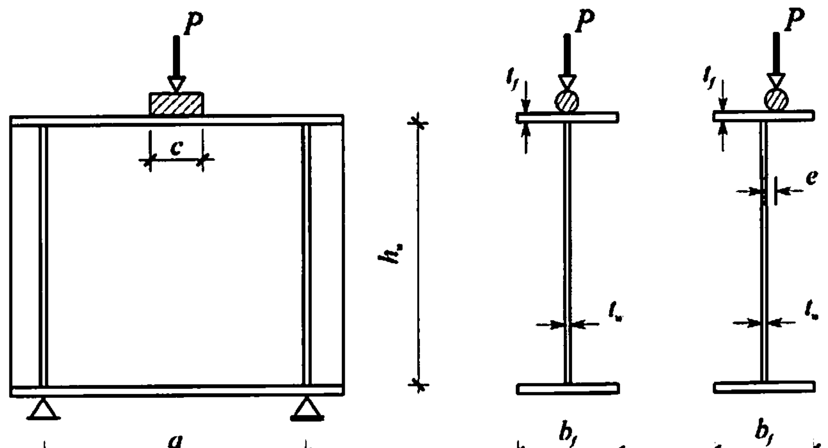


Figure 1. Patch loaded I-girder

Patch loaded girders are widely used in engineering practice. Examples are numerous and present in different structures, including crane girders loaded by crane wheels or bridge girders erected by launching.

More than 30 experimental researches had been carried out worldwide and more than 25 mathematical models or empirical expressions for failure load had been proposed until 1998. In spite of such a large number of researches, a number of questions have not yet been completely answered. Some parameters and their influence on behaviour, failure mode and ultimate load of patch loaded girders have not been thoroughly investigated and defined. The influence of load eccentricity relative to the web plane is particularly interesting and should be carefully analysed, having in mind the fact that some eccentricity is almost unavoidable in practice.

In attempt to help answering open questions and giving an explanation of certain patch loading issues, a series of patch loading researches started in 1998 at the Faculty of Civil Engineering in Podgorica, University of Montenegro, Montenegro.

2. EXPERIMENTAL RESEARCH BY *LUČIĆ, 1998*

The attention is paid to two problems having their own particularities, but also being mutually connected. The first one is carrying capacity loss of girders loaded in the web plane, Figure 2a. The second one is carrying capacity loss of girders under loading having a certain eccentricity relative to the web plane, Figure 2b. This research confirms the fact known from the previous research work of other authors (Elgaaly and associates; Drdacky; late 1980s). The two collapse modes, in centrically and eccentrically loaded girders, are quite different, Figure 2. Carrying capacity loss in the case of centric load is due to web buckling and local stability loss. In the case of eccentric load, girders lose carrying capacity due to local elastic-plastic bending and the ultimate load significantly reduces as the load eccentricity increases.

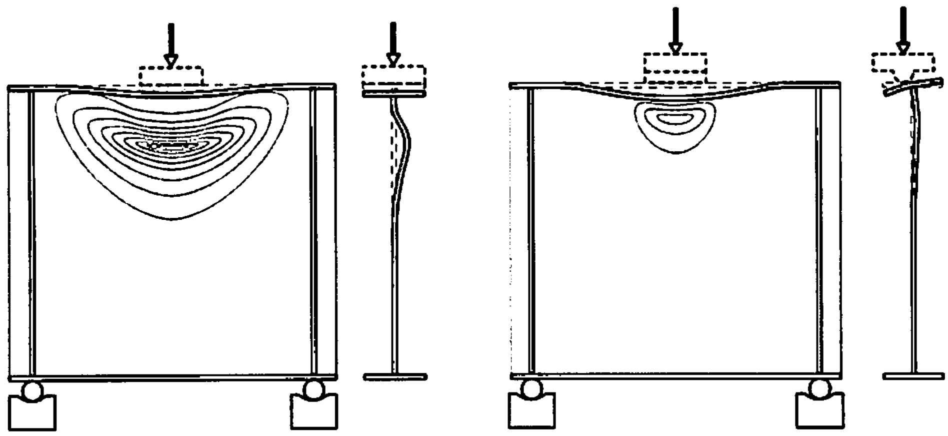
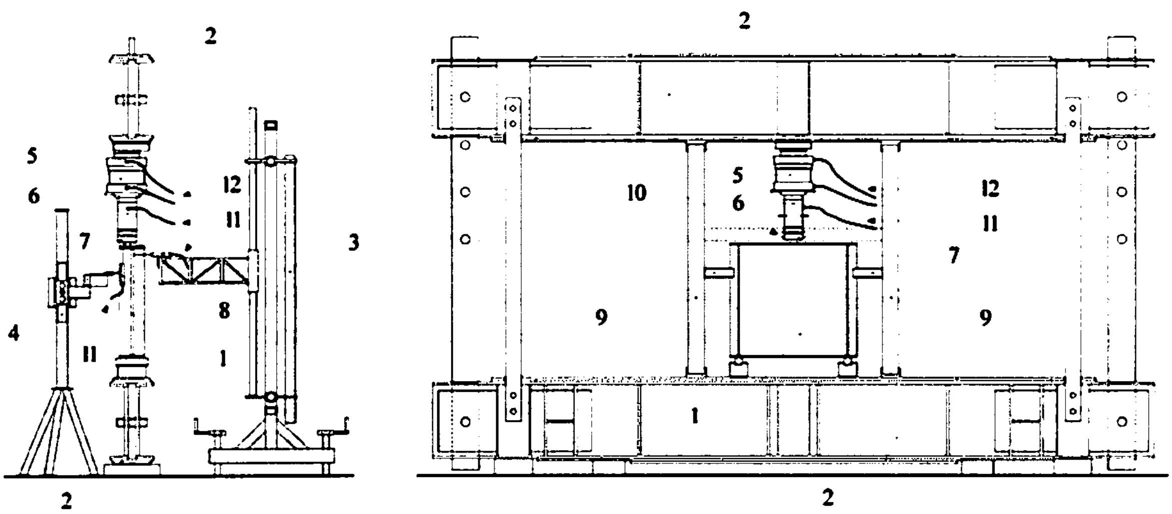


Figure 2. Collapse modes for centric and eccentric patch loading

For the need of our first patch loading experimental research, special arrangements for displacement transducers positioning as well as special load transferring blocks were particularly designed. This equipment was used in following experiments, too. All tests were carried out in a very stiff, closed steel frame, Figure 3.



1. *loaded girder*
2. *closed frame*
3. *frame-traveller for guiding the displacement transducer*
4. *horizontal traveller for guiding the displacement transducer*
5. *press (800 kN)*
6. *load cell*
7. *load distribution block*
8. *displacement transducer*
9. *stiffeners for preventing the lateral buckling*
10. *safety ring with holder*
11. *data logger cables*
12. *hydraulic pump cables*

Figure 3. Testing equipment

The research consisted of 3 series, with 12 tests in each. All together $3 \times 12 = 36$ tests were done on $36/2 = 18$ girders. Each girder was loaded up to the collapse twice, over both flanges. Deflection of loaded flange, deflection out of the web plane and the loading intensity were measured in the first series, on 12 centrically (i.e. in the web plane) loaded girders of different dimensions. Girder span, a, web depth, h_w, and flange width, b_f, were the same in all girders, Figure 4. Only web and flange thickness, t_w and t_f, varied. In the first series load was laterally distributed, i.e. applied over steel plate(s). The same girders were turned up side down and loaded over the other flange in the second series, by line load, i.e. over steel half-cylinder(s). Flange strains, web strains and load intensity were measured in the second series. Girders of same dimensions had approximately the same ultimate loads in cases of line load (second series) and laterally distributed load (first series). In the third series, 12 tests with eccentric loading were performed. Three girder types were tested, with four different load eccentricities. Each girder type had the twin-girders of the same dimensions in the first and second series, with zero eccentricity. Collapse mode of eccentrically loaded girders was completely different from the collapse mode of girders loaded in the web plane. Ultimate load decreased with the increase in load eccentricity. This reduction was approximately linear.

Tested girder is shown in Figure 4. Test results for the first and second series are summarised in Table 1, while results for the third series are in Table 2.

Table 1. Girders geometry, collapse form and failure loads of centrically loaded girders

No	a [mm]	h_w [mm]	t_w [mm]	b_f [mm]	t_f [mm]	c [mm]	$P_{u,exp}$ [kN] SERIES B I - laterally distributed load	$P_{u,exp}$ [kN] SERIES B II - line load
1	700	700	4	150	8	150	159	163
2	700	700	4	150	10	150	168	172
3	700	700	4	150	12	150	177	187
4	700	700	5	150	8	150	245	262
5	700	700	5	150	10	150	252	266
6	700	700	5	150	12	150	266	266
7	700	700	8	150	8	150	443	483
8	700	700	8	150	10	150	504	532
9	700	700	8	150	12	150	551	547
10	700	700	10	150	8	150	778	699
11	700	700	10	150	10	150	874	727 * (laterally distributed load)
12	700	700	10	150	12	150	815	812 * (laterally distributed load)

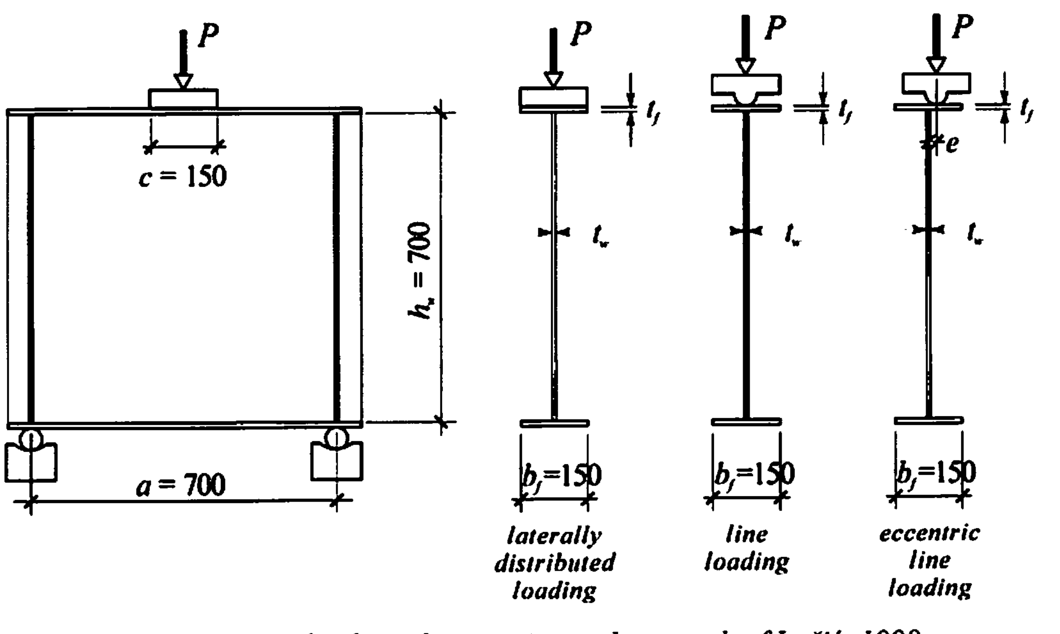


Figure 4. Girder from the experimental research of Lučić, 1998

3. MATHEMATICAL MODEL FOR CALCULATING ULTIMATE CENTRIC LOAD, *LUČIĆ, 1998*

Proposed mathematical model, based on experimental experience, defines ultimate load as a sum of two loads: P_{u1}, load at which collapse mechanism occurs in web, and P_{u2}, load that is spent for flange deformation in the moment of collapse.

Load P_{u1} is calculated by means of the strain energy concept applied to the failure mechanism in Figure 5. Girder failure happens by forming of two yielding lines in web. Lines are horizontal along the load length c. The distance between lines is h. In the moment when load reaches ultimate value, lines are developed along the length g. Yielding at the rest of lines (dashed line), as well as forming of plastic hinges in flange, happens latter, after the collapse mechanism happens in web at load P_{u1}. Accepted stress distributions along the yielding lines are shown in Figure 6.

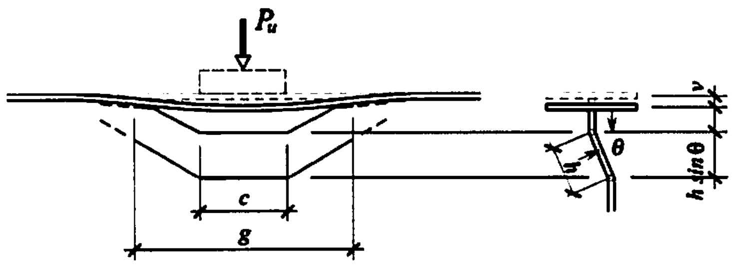


Figure 5. Collapse mechanism (Lučić, 1998)

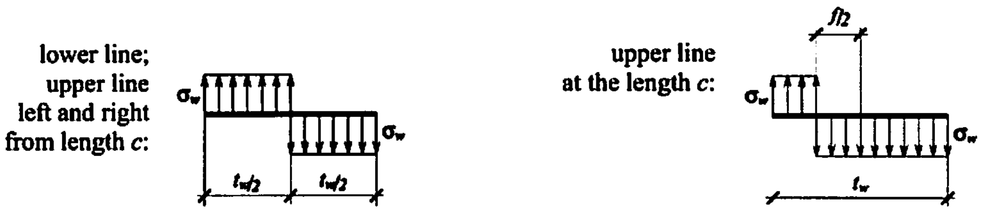


Figure 6. Stress distribution along the web yielding lines (Lučić, 1998)

Load P_{u2} is defined as a concentrated force that causes deflection v of simple beam (girder flange, herein) with span l. The span l and deflection v are obtained as experimental data. Deflection is measured. Span is the distance between zero-points at the bending moment diagram for flange.

Mathematical expression for ultimate load P_u is given by equations (1-3), where σ_w is yield stress of web, E_f and I_f are modulus of elasticity and moment of inertia for flange. Variables h, g, v, f, l are obtained by the calibration of model at the statistical sample consisting of 518 tests from 29 experimental researches.

$$P_{u1} = \sigma_w \cdot f \cdot c + \frac{\sigma_w \cdot t_w^2}{4} \cdot \left[1 - \left(\frac{f}{t_w}\right)^2\right] \cdot \frac{c}{h \cdot \cos\theta} + \frac{\sigma_w \cdot t_w^2}{4} \cdot \frac{2 \cdot g - c}{h \cdot \cos\theta} \quad (1)$$

$$P_{u2} = \frac{48 \cdot E_f \cdot I_f \cdot v}{l^3} \quad (2)$$

$$P_u = P_{u1} + P_{u2} \quad (3)$$

Theoretically, described failure mechanism is not a real mechanism and application of upper bound theorem of theory of plasticity is not completely correct. Nevertheless, this approach is taken as the most similar to the real failure in the web. The failure described with two yielding lines behaves like mechanism which propagates along the lines. This theoretical assumption together with experimental calibration of unknown variables gives semi-empirical character to this approach of calculating ultimate load.

4. MATHEMATICAL MODEL FOR CALCULATING ULTIMATE CENTRIC LOAD, *LUČIĆ & ALEKSIĆ, 2005*

Mathematical model from 1998 has been reconsidered and new, improved model is proposed. Generally, the concept is the same and ultimate load P_u is the sum of two loads, P_{u1} and P_{u2}. However, this model has better results, with better statistical indicators. Parameter $P_u/P_{u,exp}$ has mean value of 1.05 and variation coefficient of 15.96% at the statistical sample consisting of 729 tests from 33 experimental researches.

The load of collapse mechanism occurrence in web, P_{u1}, is again calculated by means of the strain energy concept. Collapse mechanism and stress distributions, slightly different from those in 1998, are shown in Figures 7, 8. A new variable, so called fictive load length c_1, which in general differs from the real load length c, is introduced.

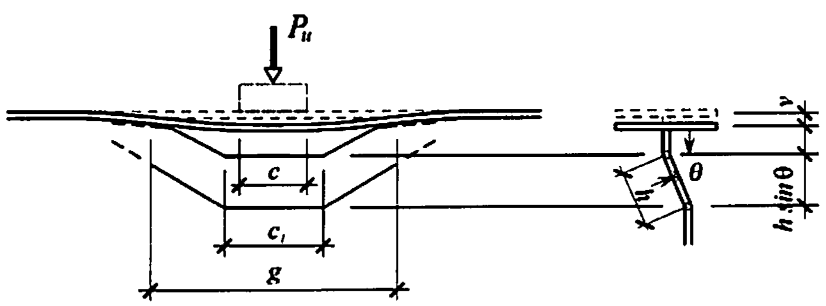


Figure 7. Collapse mechanism (Lučić & Aleksić, 2005)

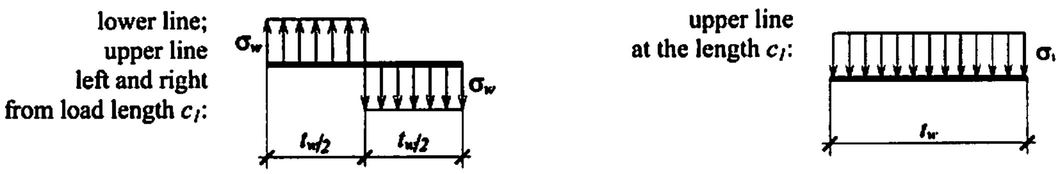


Figure 8. Stress distribution along the web yielding lines (Lučić & Aleksić, 2005)

Further increase in ultimate load, P_{u2}, after the collapse mechanism happens in web at load P_{u1}, is possible only on account of reserve in flange carrying capacity. If the limit carrying capacity of engaged flange part is less than P_{u1}, then $P_{u2} = 0$ and $P_u = P_{u1}$. Otherwise $P_u = P_{u1} + P_{u2}$, where P_{u2} is load spent for flange bending immediately after collapse mechanism have happened in web. The load P_{u2} is defined as a part of elastic carrying capacity, F_{u2}, of simple beam (girder flange, herein) with span l, loaded by concentrated force in mid-span. A part of this capacity has already been spent during the common deformation of flange and web before the occurrence of collapse mechanism in web. The rest, defined by coefficient k, makes P_{u2}.

Mathematical expression for ultimate load P_u is given by equations (4-6), where σ_w, σ_f are yield stresses of web and flange. Variables h, g, c_1, θ, l, k are obtained by the calibration of model at the statistical sample consisting of 729 tests from 33 experimental researches.

$$P_{u1} = \sigma_w \cdot t_w \cdot c_1 + \frac{\sigma_w \cdot t_w^2}{4} \cdot \frac{2 \cdot g - c_1}{h \cdot \cos\theta} \tag{4}$$

$$P_{u2} = F_{u2} \cdot k = \frac{2}{3} \cdot \sigma_f \cdot \frac{b_f \cdot t_f^2}{l} \cdot k \,; \qquad \begin{array}{l} t_w \le 3.5\,mm \;\Rightarrow\; k = 0.5 \\ t_w > 3.5\,mm \;\Rightarrow\; k = 0 \end{array} \tag{5}$$

$$P_u = P_{u1} + P_{u2} \tag{6}$$

5. EXPERIMENTAL RESEARCH "EKSCENTRO 2001", *LUČIĆ & ŠĆEPANOVIĆ, 2001*

"Ekscentro 2001" continued experimental research from 1998. In this experiment only eccentric patch loading was analysed. Both types of collapse mode were observed in eccentrically loaded girders: not only the one typical for eccentric load, but also the one typical for centric load. Under a certain circumstances, i.e. for the specific girder dimensions ratios, even for large eccentricity of load, girders collapsed the same way as if there was no eccentricity. Test results are summarised in Table 2.

Research was divided into 4 series, with 6 tests in each. All together $4 \times 6 = 24$ tests were done on $24/2 = 12$ girders. Same as in 1998, each girder was loaded up to the collapse twice, over both flanges. All girders in one series were of same dimensions, but load eccentricity varied six times. Series differed only by the web thickness. Three different web thicknesses were analysed. Other dimensions were the same in all 12 girders. Apart from load length, which is shorten herein ($c = 50$ mm), and flange thickness, which is constant for all the girders of "Ekscentro 2001", tested girders completely correspond to girders of the third series from previous experiment, Figure 4.

In the first, second and forth series, loaded flange deflection, web out-of-plane deflection and the loading intensity were measured. In the third series, with the same girders as in the second series, flange and web strains were measured, as well as load intensity.

6. EXPERIMENTAL RESEARCH "EKSCENTRO 2007", *ŠĆEPANOVIĆ & LUČIĆ, 2007*

"Ekscentro 2007" continued experimental research work from 1998 and 2001. Eccentric patch loading was studied. The influence of load eccentricity e, web and flange thicknesses, t_w and t_f, on collapse mode and ultimate load of eccentrically patch loaded steel I-girders were analysed. These three dimensions are chosen as parameters with the most significant influence, according to the previous research work. At the same time, important influential dimensionless parameters t_f/t_w and $a/t_w = h_w/t_w$ were analysed. Load eccentricity and plate thicknesses varied in wide range, with an expectation to obtain and study different collapse modes. Other dimensions of girder (span, a, web depth, h_w, and flange width, b_f) are the same as in our previous experiments, so that the tested girder corresponds to the eccentrically loaded girder in Figure 4. Line load is applied along the length $c = 50$ *mm* (as in 2001) or $c = 150$ *mm* (as in 1998).

Girder geometry, i.e. variable dimensions are chosen as follows:

- Web thickness, t_w, is chosen in order to:
  - enable connection of these tests with the existing ones from 1998 and 2001, i.e. to get several series with the same t_w and variable another parameter.
  - get girders with parameters $a/t_w = h_w/t_w$ which correspond with real practice.
  - get ultimate load which corresponds with available equipment in laboratory.
- Flange thickness, t_f, is chosen for each web thickness, t_w, separately, in order to have at least 4 different t_f for each t_w and to get, for each t_w, values of parameter t_f/t_w in arithmetic progression.

"Ekscentro 2007" is organised in 17 series, each with 6 tests. All together $17 \times 6 = 102$ tests were done on $102/2 = 51$ girders. Loaded flange deflection, web out-of-plane deflection and the loading intensity were measured. Results are summarised in Table 2.

As it was planned and expected, both collapse forms were observed in eccentrically loaded girders: the one typical of centrically loaded girders (*centric collapse mode*, Figure 9) as well as that one typical of the eccentrically loaded girders (*eccentric collapse mode*, Figure 10). However, the third type of collapse form, which might be defined as a combination of previously mentioned two types, so-cold *mixed collapse mode* was also observed.

Table 2. Girders geometry, collapse form and failure loads of eccentrically loaded girders

B III 1, B III 2, B III 3 – 1998 experiment
EB I, EB II, EB III, EB IV – "Ekscentro 2001"
EB V, EB VI ... EB XXI – "Ekscentro 2007"

No	SERIES	a [mm]	h_w [mm]	t_w [mm]	b_f [mm]	t_f [mm]	c [mm]	$P_{u,exp}$ [kN], TYPE OF COLLAPSE FORM: C – centric, E – eccentric, M – mixed $e = 0$ mm	$e = 5$ mm	$e = 10$ mm	$e = 15$ mm	$e = 20$ mm	$e = 25$ mm
1	**EB I**	700	700	3	150	15	50	133	128 C	127 C	135 C	134 C	124 C
2	**EB II**	700	700	6	150	15	50	340	320 C	326 C	296 E	243 E	197 E
3	**EB III**	700	700	6	150	15	50	342	321 C	301 C	267 E	228 E	187 E
4	**EB IV**	700	700	8	150	15	50	401	418 C	394 E	301 E	245 E	209 E
5	**EB V**	700	700	5	150	10	50	229	212 C/M	197 E	175 E	153 E	129 E
6	**EB VI**	700	700	10	150	10	50	720	575 E/M	365 E	313 E	275 E	220 E
7	**EB VII**	700	700	5	150	12	50	230	225 C	212 C/M	180 E/M	170 E	149 E
8	**EB VIII**	700	700	3	150	3	50	79 (E-M)	44 E	37 E	29 E	23 E	20 E
9	**EB IX**	700	700	3	150	6	50	95	80 E/M	69 E	57 E	47 E	39 E
10	**EB X**	700	700	3	150	9	50	102	105 C	107 C	90 C/M	85 E	70 E
11	**EB XI**	700	700	3	150	12	50	116	113 C	115 C	110 C	105 C/M	115 M
12	**EB XII**	700	700	4	150	4	50	120 (M)	70 E	50 E	45 E	40 E	35 E
13	**EB XIII**	700	700	4	150	6	50	125	110 E/M	86 E	68 E	50 E	45 E
14	**EB XIV**	700	700	4	150	8	50	140	129 C	130 C	100 E	86 E	75 E
15	**EB XV**	700	700	4	150	10	50	155	148 C	140 C	138 C/M	128 E/M	115 E/M
16	**EB XVI**	700	700	5	150	6	50	187	130 E	105 E	74 E	59 E	55 E
17	**EB XVII**	700	700	5	150	8	50	209	200 C/M	145 E	130 E	98 E	83 E
18	**EB XVIII**	700	700	6	150	6	50	208 (M)	170 E	130 E	104 E	88 E	69 E
19	**EB XIX**	700	700	6	150	9	50	330 (M)	285 E	217 E	155 E	125 E	107 E
20	**EB XX**	700	700	6	150	12	50	300	265 E/M	311 E	235 E	202 E	165 E
21	**B III** / **EB XXI**	700	700	5	150	10	150	252 / 266	250 C	240 C/M	202 E	171 E	141 E
22		700	700	10	150	10	150	874 / 727	790 E	640 E	386 E	297 E	254 E
23		700	700	5	150	12	150	266 / 266	255 C	255 C	228 E	177 E	162 E

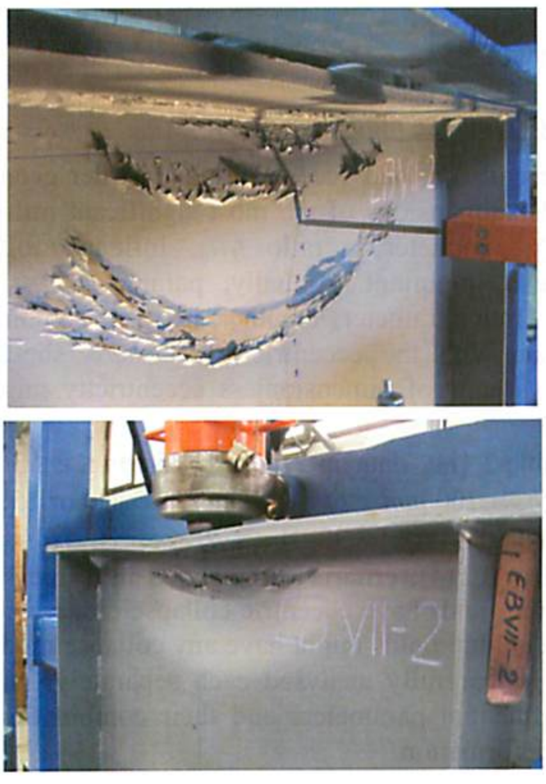


Figure 9. Centric collapse mode
(t_w = 5 mm, t_f = 12 mm, e = 5 mm)

Figure 10. Eccentric collapse mode
(t_w = 3 mm, t_f = 3 mm, e = 5 mm)

7. COLLAPSE MODE AND ULTIMATE LOAD OF ECCENTRICALLY PATCH LOADED I-GIRDERS – SUMMARY CONCLUSION OF EXPERIMENTAL TESTS

Three different collapse modes are observed in eccentrically patch loaded steel I-girders: eccentric, centric and mixed collapse mode. Mixed collapse mode, having characteristics of both, centric and eccentric collapse modes, may appear in two variants: as centric-mixed or as eccentric-mixed collapse mode, depending on dominant collapse mode characteristics.

Concerning engineering practice, the most important difference between collapse modes is in ultimate load. The reduction in ultimate load with an increase in load eccentricity is obvious in girders with eccentric collapse mode. For a certain girder geometry, even the smallest load eccentricity ($e = 5$ mm or $e/b_f = 1/30$) reduced ultimate load over 40% (e.g. series EB VIII, EB XII, Table 2). This decrease in ultimate load might be quantified by a reduction factor, R, which relates the ultimate load of eccentrically loaded girder to the ultimate load of centrically loaded girder and $R < 1$. In case of centric collapse mode in eccentrically loaded girders, ultimate load does not change significantly with an increase in load eccentricity, i.e. $R = 1$.

Hence, in order to estimate ultimate load, the first step is to determine collapse mode of eccentrically loaded I-girder, i.e. to recognise if $R = 1$ or $R < 1$. Having in mind ultimate load reduction ($R < 1$), the identification of eccentric collapse mode is the most important. In order to be able to estimate load reduction and its consequences, it is essential to precisely define the combination(s) of influential parameters which mean eccentric collapse mode.

When the collapse mode is recognised as an eccentric one, i.e. $R < 1$, ultimate load might be calculated in any of the following ways:

- by means of reduction factor (calculated from available empirical expressions which are obtained by numerical analysis of experimental and FEM modelling data) and ultimate load of centrically loaded girder (calculated by any of numerous available mathematical procedures),
- by means of mathematical model based on eccentric collapse mechanism (unfortunately, such model has not yet been suggested and available in literature),
- by means of artificial neural networks,
- any other reliable and applicable procedure.

Determination/recognition/identification of collapse mode

Occurrence of a certain collapse mode depends on load eccentricity and girder geometry. Web and flange thicknesses, t_w and t_f, are girder dimensions of the most significant influence. The most influential dimensionless geometry parameter is ratio t_f/t_w. Influence of web slenderness, i.e. parameter $a/t_w = h_w/t_w$ is also important. Actually, parameters t_f/t_w and $a/t_w = h_w/t_w$ should not be analysed separately. Both parameters together, i.e. their combination should be considered. Furthermore, correlation with the eccentricity ratio e/b_f should be established. Apart from ratio e/b_f, two more variants of dimensionless eccentricity might be considered: e/t_f and e/t_w.

Based on the experimental data from Table 2 [i.e. data in range: (0) $5\ mm \le e \le 30\ mm$, $3\ mm \le t_w \le 10\ mm$, $3\ mm \le t_f \le 15\ mm$, $a = h_w = 700\ mm$, $b_f = 150\ mm$, $c = 50$ or $150\ mm$; (0) $1/30 \le e/b_f \le 1/5$, (0) $0.33 \le e/t_f \le 8.33$, (0) $0.50 \le e/t_w \le 8.33$, $1 \le t_f/t_w \le 5$, $10 \le b_f/t_f \le 50$, $70 \le a/t_w = h_w/t_w \le 233$, $a/h_w = 1$, $c/a = 0.071$ or 0.214], criteria summarised in Table 3 have been established. Girders fulfilling criteria in grey fields have eccentric collapse mode, with the reduced ultimate load. Girders fulfilling criteria in blue fields might have any collapse mode and these criteria imply that such girders should be carefully analysed each separately, with its specific characteristics, considering several influential parameters and their combinations, as well as paying particular attention to the initial deformation.

Table 3. Dimensionless parameter criteria for collapse mode identification

dimensionless eccentricity	***criterion***	***collapse mode***
e/b_f	$h_w/t_w < 1050 \cdot e/b_f + 35$	E
	$h_w/t_w \geq 1050 \cdot e/b_f + 35$	E, C, M
	$t_f/t_w < 15 \cdot e/b_f + 0.5$	E
	$15 \cdot e/b_f + 0.5 \leq t_f/t_w \leq 15 \cdot e/b_f + 1.5$	E, C, M
	$t_f/t_w > 15 \cdot e/b_f + 1.5$	C
e/t_f	$h_w/t_w \leq 85 \cdot e/t_f + 60$	E
	$h_w/t_w \geq 85 \cdot e/t_f + 70$	C
	$85 \cdot e/t_f + 25 \leq h_w/t_w \leq 85 \cdot e/t_f + 105$	M
	$t_f/t_w < e/t_f + 0.5$	E
	$e/t_f + 0.5 \leq t_f/t_w \leq e/t_f + 1.7$	E, C, M
	$t_f/t_w > e/t_f + 1.7$	C
e/t_w	$t_f/t_w < 0.3 \cdot e/t_w + 0.8$	E
	$0.3 \cdot e/t_w + 0.8 \leq t_f/t_w \leq 0.3 \cdot e/t_w + 1.8$	E, C, M
	$t_f/t_w > 0.3 \cdot e/t_w + 1.8$	C

Criteria from Table 3 correlate dimensionless parameters t_f/t_w and $a/t_w = h_w/t_w$ with dimensionless eccentricity e/b_f, e/t_f or e/t_w. Such relations provide high level of reliability in collapse mode identification. Combining (checking) several criteria from Table 3 increases the reliability level.

Some girder dimensions or dimensionless parameters, analysed separately, without making relation(s) with the other parameter(s), may be used as a rough identifiers of collapse mode. Girders having thin flange ($t_f \leq 6$ mm) or thick web ($t_w \geq 8$ mm) or low web slenderness ($h_w/t_w < 100$) or ratio $t_f/t_w < 2$ are affected even with the lowest eccentricity. On the other side, girders with the ratio $t_f/t_w > 3$ have centric collapse mode even at the highest eccentricity.

Obviously, parameter t_f/t_w has the key-role nevertheless if analysed in correlation with the other parameters or separately, only by itself.

Ultimate load and its reduction

Eccentrically loaded girders with centric collapse mode behave as if loaded in the web plane, without significant change in ultimate load, P_u, due to load eccentricity.

In case of eccentric and mixed collapse mode, ultimate load, P_u, depends on load eccentricity, e, and reduces with an increase in eccentricity. The reduction is more emphasised for smaller ratio t_f/t_w and higher ratio $a/t_w = h_w/t_w$. For years this reduction has been considered approximately linear. However, the newest experimental data confirms this only for $t_f/t_w \geq 1.5$. In case of $t_f/t_w < 1.5$, more appropriate approximation of diagram $P_u - e$ would be bi-linear, parabolic or even by the line of higher order.

Ultimate load, P_u, increases with a decrease in web slenderness h_w/t_w, i.e. with an increase in web thickness, t_w. The same happens in both cases: at $t_f/t_w = const$, $t_f \neq const$, as well at $t_f/t_w \neq const$, $t_f = const$.

Ultimate load, P_u, increases with an increase in ratio t_f/t_w due to increase in flange thickness, t_f. However, if ratio t_f/t_w increases due to decrease in web thickness, t_w, ultimate load will also decrease.

Although the load length, c, influences ultimate load (longer the load length, higher the ultimate load), this parameter is generally not of a great influence for the reduction in ultimate load with the increase in load eccentricity. However, there are some implications that the load length might influence this reduction in ultimate load, for the combination of low ratio t_f/t_w and small eccentricity.

Small initial imperfection does not influence girder behaviour, collapse mode and ultimate load. Herein, "small" means "insignificant, negligible in comparison with the plate thicknesses". However, in case of thin flange and/or web, when initial deformation is of the same size order as plate thickness, it must not be neglected, since it will greatly influence girder behaviour and, consequently, its collapse mode and ultimate load.

8. EMPIRICAL EXPRESSION(S) FOR REDUCTION IN ULTIMATE LOAD OF ECCENTRICALLY PATCH LOADED I-GIRDERS

Experimental data shows that, in girders with eccentric collapse mode, ultimate load reduces as the load eccentricity increases. The reduction is quantified by a reduction factor:

$$R = \frac{\text{ultimate load of eccentrically loaded girder}}{\text{ultimate load of centrically loaded girder}} \tag{7}$$

Ultimate load of centrically patch loaded girder might be calculated by one of numerous and very accurate existing mathematical expressions. Ultimate load of eccentrically patch loaded girder then might be easily calculated if the reduction is evaluated correctly and confidently.

Our experiments, Table 2, with wider range of test data, implies that the original expression for reduction factor, published by Galambos in 1998, based on the 1980s experiments of Elgaaly and associates, should be modified.

In 2005, join work with the colleagues from Granada University, professors Gil Martín and Hernandez Montes, started through the CGHS programme. Among the others, this cooperation resulted in several modifications of original expression for reduction factor.

New, improved expressions are also empirical, obtained by regression analysis based on all available experimental data as well as on results of finite element modelling by ANSYS.

One of these expressions, which has very good match with the wide range of experimental and numerical data, is defined by (8). The reduction factor, R, is considered to be a quadratic function of the most relevant parameter e/b_f and, same as in the original expression, dependent on the most influential geometry parameter t_f/t_w:

$$\left.\begin{aligned} R &= m\cdot\left(\frac{e}{b_f}\right)^2 + n\cdot\left(\frac{e}{b_f}\right) + 1.01 \le 1 \\ m &= -0.864\cdot\left(\frac{t_f}{t_w}\right)^2 - 14.40\cdot\left(\frac{t_f}{t_w}\right) + 38.00 \\ n &= -12.30 + 4.22\cdot\left(\frac{t_f}{t_w}\right) \end{aligned}\right\} \tag{8}$$

Same as any other empirical expression, the (8) should be used only in the range of data used for regression analysis. Experimental data used for the derivation of (8) are in range as follows: $1 \le t_f/t_w \le 5$, $1/30 \le e/b_f \le 1/5$, $45 \le a/t_w \le 233$, $10 \le b_f/t_f \le 30$, $c/a = 0.071$ or 0.214, $a/h_w = 1$. ANSYS data used for the derivation of (8) are in range as follows: $1 \le t_f/t_w \le 4$, $1/25 \le e/b_f \le 1/6.25$, $a/t_w = 233$, $6.25 \le b_f/t_f \le 25$, $0.036 \le c/a \le 0.071$, $1 \le a/h_w \le 2$.

It has to be pointed out that every future experimental testing or FE modelling should be followed by new revision and adjusting of empirical expression for the ultimate load reduction factor in order to improve its accuracy.

9. ARTIFICIAL NEURAL NETWORK APPLICATION FOR THE FORECAST OF ULTIMATE LOAD IN ECCENTRICALLY PATCH LOADED I-GIRDERS

During the preparation of "Ekscentro 2007", modelling of subject issue was made by means of artificial neural networks. Depending on variable input parameters (girder geometry and load eccentricity) the ultimate load, as the only output, was forecasted. Artificial neural networks of different architecture, trained on experimental data from 1998 and 2001, were created.

By initiating trained neural network with some specific values of input variables, that have to be in range of training data, the forecast model of output variable (ultimate load) values is obtained. The most suitable presentation of model is graphical method, i.e. diagrams of output variable as a function of one input variable while other input variables have fixed values. Herein, it is interesting to get diagrams P_u-e for values of t_w that were not tested, or diagrams P_u-t_w for fixed values of e, Figure 11. Plenty of similar diagrams might be made and help to understand nature of relationship between output and one particular input variable. In addition, some conclusions about interaction between input parameters might be made, too.

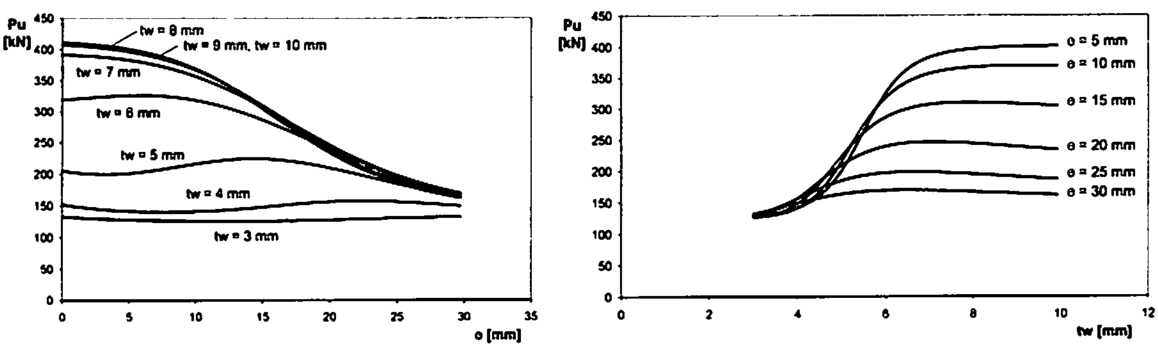


Figure 11. Forecast models $P_u(e)$ and $P_u(t_w)$

The results of "Ekscentro 2007" enlarged available training data base and enabled creation of more precise, more realistic neural networks and forecast models. The complete experimental database (containing data from 1998, 2001 and 2007, Table 2) is going to be used to forecast not only ultimate load, but also collapse mode of eccentrically patch loaded I-girders. This work is ongoing.

10. ONGOING AND FUTURE WORK

A new experimental research "Centro 2009", aiming analysis of centric patch loading in combination with global bending, has been recently finished. The presence of significant compression/tension stresses in the zone affected by patch loading is studied. Data analysis is ongoing.

At the same time, work on mathematical model for ultimate centric load calculation is continuing. The new model, which has just been constituted, has completely different approach and even better statistical indicators than the model by Lučić & Aleksić, 2005.

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ARTIFICIAL NEURAL NETWORK APPLICATION IN ECCENTRIC PATCH LOADING EXPERIMENTAL RESEARCH

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ABSTRACT: A series of experimental researches in the domain of patch loading in thin-walled steel I-girders has been carried out at the Faculty of Civil Engineering in Podgorica, University of Montenegro, since 1998. Patch loading is the loading acting locally, over a small area or length of a structural element. Particularly intriguing problem is eccentric patch loading - thin-walled steel I-girder loaded over flange by local load that has a certain eccentricity regarding the web plane. Neither collapse mechanism model nor generally valid empirical expression for failure load calculation has been proposed for this particular case of patch loading. During the preparation of new experimental research at the Faculty of Civil Engineering in Podgorica, University of Montenegro, planned for the summer 2006, the forecast models of this problem have been made, by means of artificial neural network. Depending on variable input parameters (girder geometry and load eccentricity), the failure load was predicted. Results of previous researches formed data base which provided training and validation data sets for neural network architecture definition and network training.

KEYWORDS: patch loading, load eccentricity, steel I-girder, failure load,
artificial neural network, back-propagation

1. INTRODUCTION

Patch loading is the loading that acts locally, over a small area or length of a structural element. Particularly intriguing problem is the case when the load affects the flange of a steel I-profile so that the web, below the loading, is locally pressed. Patch loaded girders are widely used in engineering practice: crane girders loaded by the crane wheels, bridge girders erected by launching etc.

So far about 35 experimental researches have been carried out worldwide, and about 30 mathematical models or empirical expressions for calculating the failure load, have been proposed [1]. However, only a few of those studies dealt with eccentric patch loading, i.e. loading with a certain eccentricity regarding the web plane, Figure 1.

It was shown that the collapse mode of most girders subjected to eccentric patch loading was quite different from the collapse mode of centrically loaded girders. While on the one hand (centrically loaded beams – load in the web plane), it is dealt with a rather complex problem of elasto-plastic buckling and local stability loss, on the other hand (eccentrically loaded beams – load with an eccentricity regarding the web plane), the problem of carrying capacity loss due to local elasto-plastic bending is considered.

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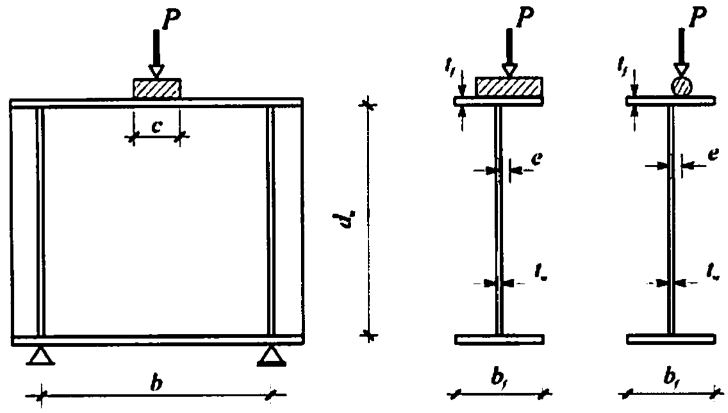

Figure 1. I-girder under eccentric patch loading

Many parameters influence the behaviour, collapse mode and ultimate load of thin-walled I-girder subjected to eccentric patch loading: geometric parameters (girder's dimensions and their ratios), load eccentricity and the manner of load application. The eccentricity value or ratio e/b_f is dominant parameter. However, web thickness t_w and ratios like b_f/t_f, t_f/t_w, d_w/t_w, c/d_w, as well as manner of load applying (line or laterally distributed load, Figure 1) are also of great importance.

Apart from new findings about the phenomenon of eccentrically locally loaded I-girders, numerous questions have not yet been answered. The most intriguing question is the following: under what circumstances do the eccentrically loaded girders have the same collapse mode as centrically loaded girders? Further, for the eccentric collapse mode neither collapse mechanism model nor generally valid empirical expression for failure load calculation has been proposed.

2. EXPERIMENTAL RESEARCHES

Experimental work in the domain of eccentric patch loading started at the University of Maine in 1988. At the same time some tests were done at the Institute of Theoretical and Applied Mechanics, Czech Academy of Sciences. Ten years later, a new series of experiments started at the University of Montenegro.

The first research in the patch loading domain, by Lučić, 1998, [1-3], beside centrically loaded included a number of eccentrically loaded I-girders. Load eccentricity, flange and web thicknesses (i.e. ratios b_f/t_f, d_w/t_w and t_f/t_w) were varied, Table 1. Panel aspect ratio $b/d_w = 1$ was the same for all girders ($b = d_w = 700$ mm), as well as the ratio $c/d_w = 0.21$ ($c = 150$ mm). Eccentricity varied in interval from $b_f/10$ to $b_f/5$ ($b_f = 150$ mm) and all eccentrically loaded girders had failure modes quite different from those of centrically loaded girders.

The next research, "Ekscentro 2001", by Lučić and Šćepanović, 2001, [4-5], was connected to the previous one. Variable parameters were eccentricity and web thickness, Table 2. In comparison with the previous research, eccentricity varied over a wider range, from $b_f/30$ to $b_f/6$, i.e. smaller eccentricities were also analysed. It might be said that this experimental study included analysis of thinner webs (regarding web depth and flange thickness). Girder span and web depth ($b = d_w = 700$ mm), i.e. panel aspect ratio ($b/d_w = 1$), as well as flange width ($b_f = 150$ mm) were the same as in previous study. The flange was thicker than earlier ($t_f = 15$ mm, $b_f/t_f = 10$). Load length was much shorter than before ($c/d_w = 0.07$, $c = 50$ mm). Collapse modes characteristic of both, centrically and eccentrically loaded girder, were observed. The smaller the eccentricity and the larger the ratio t_f/t_w (i.e. thinner web), the more possible the occurrence of the so-called "centric collapse mode" in eccentrically loaded girder.

Table 1. Test parameters and results for the research of Lučić, 1998

No	Girder	t_w mm	t_f mm	e mm	e/b_f	b_f/t_f	d_w/t_w	t_f/t_w	P_{exp} kN
1	B I 5	5	10	0*	0	15	140	2	251.7
2	B II 5	5	10	0**	0	15	140	2	266.3
3	B III 1/1	5	10	15	1/10	15	140	2	202.8
4	B III 1/2	5	10	20	1/7.5	15	140	2	170.7
5	B III 1/3	5	10	25	1/6	15	140	2	140.8
6	B III 1/4	5	10	30	1/5	15	140	2	122.2
7	B I 11	10	10	0*	0	15	70	1	874.1
8	B II 11	10	10	0*	0	15	70	1	727.3
9	B III 2/1	10	10	15	1/10	15	70	1	386.5
10	B III 2/2	10	10	20	1/7.5	15	70	1	296.6
11	B III 2/3	10	10	25	1/6	15	70	1	253.6
12	B III 2/4	10	10	30	1/5	15	70	1	218.8
13	B I 6	5	12	0*	0	12.5	140	2.4	265.5
14	B II 6	5	12	0**	0	12.5	140	2.4	266.4
15	B III 3/1	5	12	15	1/10	12.5	140	2.4	227.7
16	B III 3/2	5	12	20	1/7.5	12.5	140	2.4	177.2
17	B III 3/3	5	12	25	1/6	12.5	140	2.4	162.4
18	B III 3/4	5	12	30	1/5	12.5	140	2.4	147.3

* laterally distributed load; ** line load

Table 2. Test parameters and results for the research of Lučić and Šćepanović, 2001

No	Girder	t_w mm	e mm	e/b_f	d_w/t_w	t_f/t_w	P_{exp} kN
1	EB I-1	3	0	0	233.3	5	132.9
2	EB I-2	3	5	1/30	233.3	5	128.3
3	EB I-3	3	10	1/15	233.3	5	126.9
4	EB I-4	3	15	1/10	233.3	5	135.8
5	EB I-5	3	20	1/7.5	233.3	5	133.6
6	EB I-6	3	25	1/6	233.3	5	123.9
7	EB II-1	6	0	0	116.7	2.5	340.2
8	EB II-2	6	5	1/30	116.7	2.5	320.3
9	EB II-3	6	10	1/15	116.7	2.5	325.6
10	EB II-4	6	15	1/10	116.7	2.5	295.6
11	EB II-5	6	20	1/7.5	116.7	2.5	242.9
12	EB II-6	6	25	1/6	116.7	2.5	196.9
13	EB III-1	6	0	0	116.7	2.5	342.2
14	EB III-2	6	5	1/30	116.7	2.5	321.3
15	EB III-3	6	10	1/15	116.7	2.5	300.6
16	EB III-4	6	15	1/10	116.7	2.5	267.3
17	EB III-5	6	20	1/7.5	116.7	2.5	227.6
18	EB III-6	6	25	1/6	116.7	2.5	186.9
19	EB IV-1	8	0	0	87.5	1.875	400.6
20	EB IV-2	8	5	1/30	87.5	1.875	417.6
21	EB IV-3	8	10	1/15	87.5	1.875	393.9
22	EB IV-4	8	15	1/10	87.5	1.875	300.9
23	EB IV-5	8	20	1/7.5	87.5	1.875	245.3
24	EB IV-6	8	25	1/6	87.5	1.875	209.3

In an attempt to get closer to the answers for the above mentioned open questions concerning eccentric patch loading, a new experimental-theoretical research was initiated at the Faculty of Civil Engineering in Podgorica, in 2005. During the preparation of the experiment, planned for the summer 2006, in order to help us decide about number and geometrical parameters of test girders, the forecast models of this problem have been made, by means of artificial neural network.

3. ARTIFICIAL NEURAL NETWORK AND FORECAST MODEL

Two main steps in artificial neural network realisation are design of network architecture and network training. In order to have usable and reliable network, i.e. forecast model, it is important to have optimal proportion among the number of inputs, the number of outputs and the number of training and validation data [6].

Herein, the basic idea was to estimate the failure load depending on variable input parameters (girder geometry and load eccentricity). Several forecast models were made using different data from the Tables 1 and 2. Models comparison was done. In this paper only one model is presented. Through this example some specific issues of artificial neural network use will be pointed out.

3.1 NEURAL NETWORK ARCHITECTURE

The chosen neural network model is four-level neural network with four inputs, forty neurons in hidden levels (2 levels x 20 neurons) and one output. The only output value is ultimate load P_u. The input variables are e, t_w, t_f and c. The other models, that are not presented in the paper, differed from this one in the number and type of input data, as well as in structure of hidden levels. While inputs are dimensional herein, models with dimensionless inputs (ratios e/b_f, b_f/t_f, d_w/t_w, t_f/t_w, c/d_w) are also considered. Nevertheless some advantages of dealing with dimensionless parameters, the model with dimensional input parameters is chosen for the presentation as visually more suitable one.

3.2 NEURAL NETWORK TRAINING

The data base for neural network training, i.e. model fitting, consisted of 32 training data from Tables 1 and 2 (shaded rows). The partition of data base into training and validation set was done. 14 of 32 data (dark shaded rows) were chosen for validation set. The rest of the data (pale shaded rows) formed training set. The training of feed-forward neural network was done by means of improved back-propagation algorithm. Logistic sigmoidal function $f(x) = 1/(1+e^{-x})$ was used as activation function. The number of training cycles in one iteration was 500. Iterative process of model fitting was stopped when error of validation set showed intention to grow after lessening trend. The error of training set constantly decreased. The training was done in MS Excel, by means of Visual Basic.

3.3 RESULTS – NEURAL NETWORK MODEL APPLICATION

By initiating trained neural network with some specific input values, that have to be in range of training data, it is possible to get forecast model for output value. The most suitable presentation of the model is graphical method, i.e. diagrams of output variable as a function of one input variable while other input variables have fixed values.

For the subject problem, it is interesting to get diagrams P_u-e for values of t_w that were not tested. In Figure 2 these diagrams are shown together with the diagrams for tested t_w values. Experimental data P_u^{exp} for tested t_w values are not in these diagrams, but also neural network values of P_u. Similarly, diagrams P_u-t_w for fixed values of e might be drawn, Figure 3. All of these diagrams assume fixed values of other inputs (t_f and c), as stated in captions of Figures 2 and 3.

Plenty of similar diagrams might be made and help to understand nature of relationship between output and particular input variable. In addition, some conclusion about interaction between input parameters might be made, too.

It is important to emphasize that these forecast models might be used and might give reliable results only for input parameters in range of experimental (training) data, i.e. data used for model fitting (network training).

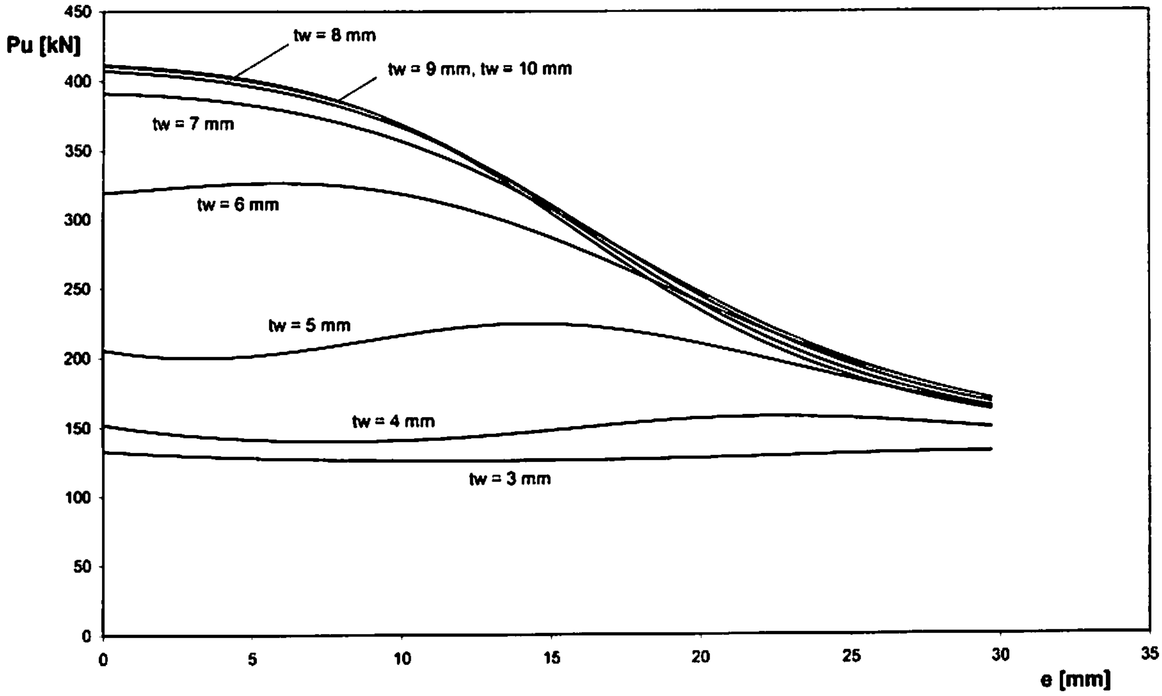

Figure 2. Forecast model $P_u(e)$ for $t_f = 15$ *mm* and $c = 50$ *mm*

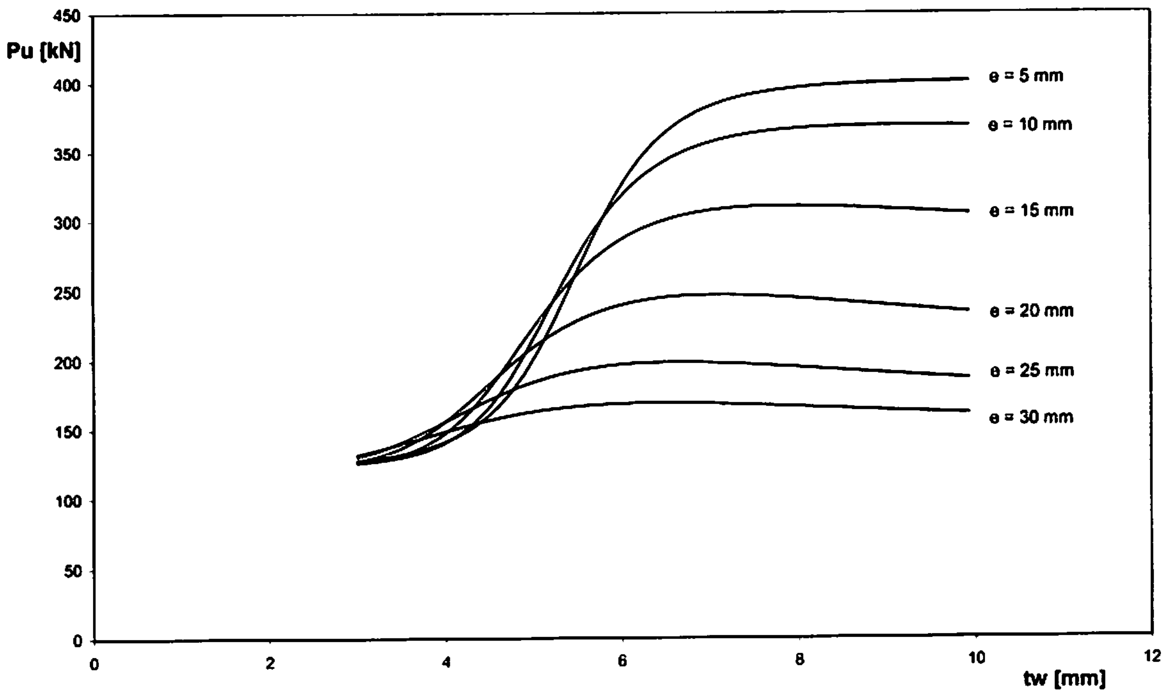

Figure 3. Forecast model $P_u(t_w)$ for $t_f = 15$ *mm* and $c = 50$ *mm*

As it might be seen in Figures 2 and 3, curves P_u-e and P_u-t_w are rather realistic according to the available experimental data. Decrease of failure load P_u with the increase of load eccentricity e as well as ultimate load P_u increase with web thickness t_w increase are obvious. At first glance, curves P_u-e for t_w = 3,4 and 5 *mm*, Figure 2, seem to be non-regular because of trend direction changing. It might be explained by the experimentally proved fact that for large ratio t_f/t_w, as in case of these curves, eccentrically loaded girders behave the same way as centrically loaded ones, nevertheless the eccentricity level. Therefore these data can not fit the model completely. However, general trend-lines of these curves are correct, i.e. ultimate load is approximately constant, does not or only slightly change as eccentricity grows. It is expected the new experimental researches to confirm this forecast.

4. CONCLUSION

The experimental data base for the eccentric patch loading in steel I-girders is still rather modest, having in mind complexity of the problem and large number of influential parameters. For now on, it is interesting to use artificial neural network as a method to estimate failure load for different combinations of input parameters (load eccentricity and girder geometry) that have not been covered by experimental research. The purpose of this prediction is to plan new experiment that will, among the other benefits, confirm or refute the validity of such forecast modelling.

The new experimental results will be used for work on definition of collapse mechanism and calibration of empirical expression for failure load. Furthermore, these data are going to enlarge available training data base and enable creation of more precise, more realistic neural network models. Such fitted models might be used not only in the purpose of research, but also in engineering practice.

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Reduction in Failure Load of I-Girders Under Eccentric Patch Loading

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Abstract
Experimental research shows that the behaviour and failure mode of most eccentrically patch loaded girders differ from those of centrically patch loaded girders. A reduction in the ultimate load due to eccentricity is evident, and has been quantified by a reduction factor that relates the ultimate load of the eccentrically loaded girders to that of the centrically loaded girders. An existing expression for the reduction factor is based on results from experiments conducted in 1980s. It reflects an experimentally proved fact that the ultimate load decreases linearly as the eccentricity increases. The reduction factor, R, is a function of two main geometric parameters e/b_f and t_f/t_w. New test data indicates that the distance between vertical web stiffeners, a, is also important. Recent experiments covering a wider range of geometric parameters and different combinations of parameter values imply that the existing expression for reduction factor should be revised. The improved empirical expression proposed in the paper was shown to match experimental data better than the existing one, especially for large eccentricities.

Introduction
Patch loading is loading applied over a small area or length of a structural element. A common situation in structural engineering occurs when a downward compressive load is applied to the flange of I-section so that the web is compressed in the region below the applied load. Examples are numerous and present in different structures, including crane and bridge girders.

Some eccentricity of load relative to the web plane is unavoidable in engineering practice (Figure 1). It is evident that most eccentrically loaded girders have a collapse mode quite different from that of centrically loaded girders. The ultimate load reduces as the load eccentricity increases. This decrease in the ultimate load is expressed by a reduction factor, R, that relates the ultimate load of eccentrically loaded girders to the ultimate load of centrically loaded girders.

Many parameters influence the behaviour, collapse mode and ultimate load of thin-walled I-girders subjected to eccentric patch loading: geometric parameters (girder's dimensions and their dimensionless ratios), load eccentricity and the manner of load application. The most relevant parameters are load eccentricity, e, and web thickness, t_w.

An existing expression for the ultimate load reduction factor, R, is based on experimental studies conducted in 1980s. Reduction factor is expressed in terms of two main geometric parameters: e/b_f and t_f/t_w. However, recent experimental work indicates that other parameters, such as the distance between vertical web stiffeners, a, appear to be relevant as well.

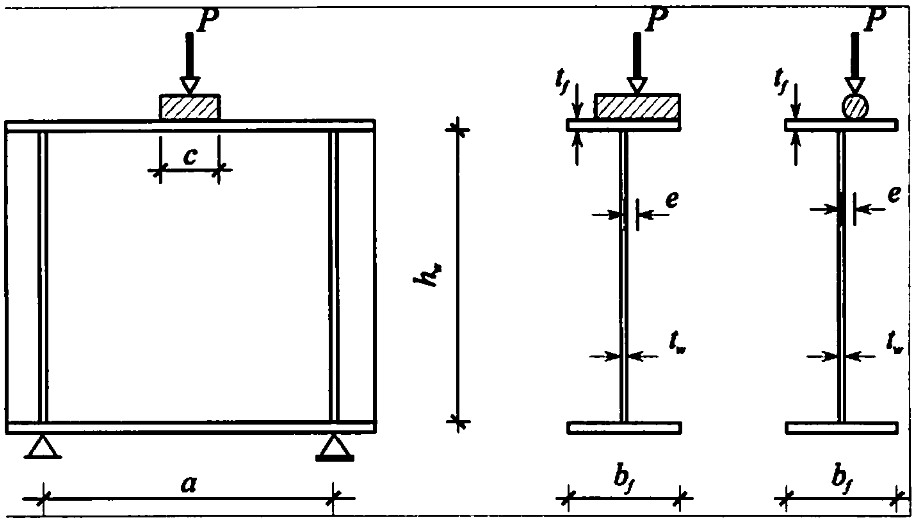

Figure 1. I-girder under eccentric patch loading

Experimental Studies

A large amount of experimental research on patch loaded girders has been carried out worldwide. However, only a few investigations have dealt with eccentric patch loading.

Experimental work started at the University of Maine in late 1980s [1-2]. At the same time some tests were done at the Institute of Theoretical and Applied Mechanics, Czech Academy of Sciences [3]. Ten years later a new series of experiments were initiated at the University of Montenegro [4-7].

The first experimental study on eccentric patch loading was published in 1989 by Elgaaly and Nunan, University of Maine [1]. The influence of load eccentricity was studied. Loads were applied at various eccentricities through a thick patch plate (Series I, II) or a cylindrical bar (Series III, IV). For the series in which laterally distributed eccentric patch loading was applied through a thick plate, the collapse mode was very similar to that obtained for the centrically loaded specimens. No significant reduction in ultimate load was observed as the ratio e/b_f increased to a maximum of 1/8. However, for the series in which a line load was applied by a cylindrical bar, the girder behaviour clearly differed from the behaviour of centrically loaded girders. The failure load clearly decreased as the load eccentricity increased. The reduction was found to be approximately linear with eccentricity. The maximum eccentricity ratio was $e/b_f = 1/8$. Geometric parameters of all tested girders were: $a/h_w \approx 1$, $b_f/t_f \approx 11$, $h_w/t_w \approx 45$ (low web slenderness), $t_f/t_w = 1.4$ and $c/a = 0.20$.

The second experimental study, by Elgaaly and Sturgis, University of Maine [2], used girders having geometric parameters $a/h_w = 1$, $b_f/t_f \approx 24$, $h_w/t_w \approx 210$ (slender web), $t_f/t_w = 2.1$ and $c/a = 0.2$. The maximum eccentricity ratio was $e/b_f = 1/8$. It was concluded that the ultimate load increased with an increase in the ratio t_f/t_w. The ultimate load clearly reduced as the eccentricity ratio e/b_f increased. The reduction was approximately linear with the increase in eccentricity ratio.

In the experiments of Elgaaly and Salkar, University of Maine [2], ratios b_f/t_f, t_f/t_w, c/a were varied along with the eccentricity ratio and some specimens had longitudinal stiffeners. Only results for girders without stiffeners and having non-dimensional ratios of $a/h_w = 1$, $b_f/t_f \approx 12$, $h_w/t_w \approx 210$ (slender web), $t_f/t_w \approx 4.2$, $c/a = 0.2$, were considered herein. The maximum eccentricity ratio was $e/b_f = 1/6$. This research confirmed that the failure load decreases, approximately linearly, with increase in ratio e/b_f. Of note is that this study showed that the reduction is smaller for larger values of ratio t_f/t_w. In particular, when the flange is much thicker than the web, the ultimate load does not reduce significantly with increase in eccentricity ratio, even for very slender webs ($h_w/t_w > 200$).

While in the experiments of Elgaaly et al. [1-2] load eccentricity ratio varied up to a maximum 1/6 ("small eccentricity"), the research of Drdacky [3] treated girders with eccentricity ratio e/b_f up to 5/3. Web deflections, stresses and strains proved to be linearly dependent on the eccentricity, but the girders were not loaded to failure. Therefore, these tests are not considered further in the paper, as they are not relevant to the development of a reduction factor for ultimate load.

In the research of Lučić, University of Montenegro [4-5], load eccentricity and flange and web thicknesses (i.e. ratios e/b_f, b_f/t_f, h_w/t_w and t_f/t_w) were varied. Held constant were parameters $a/h_w = 1$ and $c/a = 0.21$. The eccentricity ratio, e/b_f, varied from 1/10 to 1/5. All eccentrically loaded girders had collapse modes quite different from those of centrically loaded girders. Compared with Elgaaly's tests, the set of specimens in Lučić's tests exhibited more prominently the eccentric collapse mode characteristics. This may be attributed to a thicker web (Elgaaly: $t_w = 3 - 6.4$ mm; Lučić: $t_w = 5 - 10$ mm) and greater eccentricity ratio (Elgaaly: max $e/b_f = 8$; Lučić: min $e/b_f = 10$) in Lučić's tests relative to Elgaaly's tests. As before, ultimate load decreased approximately linearly with an increase in eccentricity ratio. This experimental study demonstrated that the reduction is greater for smaller t_f/t_w ratios. Herein the reduction is inversely proportional to the t_f/t_w.

The research of Lučić and Šćepanović, University of Montenegro [6-7], continued previous studies. Load eccentricity, e, and web thickness, t_w, were varied. In comparison with the previous research [4-5], eccentricity ratio varied over a larger range, from 1/30 to 1/6, i.e. smaller eccentricities were also analysed. Ranges of parameters h_w/t_w and t_f/t_w were also larger, i.e. greater values of these parameters were analysed. It might be said that this experimental study included analysis of thinner webs, regarding web depth and flange thickness. Girder span, a, and web depth, h_w, i.e. panel aspect ratio ($a/h_w = 1$), as well as flange width, b_f, were the same as in previous tests [4-5]. The flange was thicker than earlier (herein: $b_f/t_f = 10$; earlier: $b_f/t_f = 12.5\text{-}15$). Load length was shorter than before ($c/a = 0.07$). Collapse modes characteristic of both eccentrically and centrically loaded girders were observed. The smaller the eccentricity ratio and the larger the t_f/t_w ratio (i.e. thinner web relative to flange) the more likely a centric collapse mode would develop in girders having eccentric patch loading. The behaviour and collapse mode typical of the centrically loaded girders became evident in all tested girders of Series EB I. Ultimate load did not change significantly with eccentricity increase in Series EB I. In Series EB II, III and IV ultimate load depended on eccentricity in the same way as in previous studies [1-5]. It reduced linearly with increase in eccentricity, and the reduction was more emphasised for smaller ratios t_f/t_w.

Comparison of Existing Ultimate Load Reduction Factor Expression with Test Data

Experimental data shows a reduction in the ultimate load due to load eccentricity. This reduction is quantified by a reduction factor:

$$R = \frac{\textit{ultimate load of eccentrically loaded girder}}{\textit{ultimate load of centrically loaded girder}} \tag{1}$$

According to [8], R is a function of t_f/t_w and varies linearly with e/b_f:

$$\left.\begin{aligned} & R = m\cdot\frac{e}{b_f}+n<1 \\ & \quad m=-0.45\cdot\left(\frac{t_f}{t_w}\right)^2+4.55\cdot\left(\frac{t_f}{t_w}\right)-12.75 \\ & \quad n=1.15-0.025\cdot\left(\frac{t_f}{t_w}\right) \end{aligned}\right\} \tag{2}$$

Equation (2) is empirical, based on the experiments of Elgaaly at al [1-2]. Therefore it is applicable only for $1 \le t_f/t_w \le 4$ and $e/b_f \le 1/6$, which reflects the ranges of test data [8].

Equation (2) has been applied to a larger experimental data set that includes new experimental data from the University of Montenegro [4-7], as well as the original data from the University of Maine [1-2]. In Figures 2-4 values R_{eq} calculated from the equation (2) have been compared with experimental values R_{exp}, by means of diagrams R_{exp}/R_{eq} versus e/b_f.

Excluded from the analysed experimental data set are results from Series I and II from Elgaaly and Nunan's tests [1]. The girders of these series behaved as centrically loaded, without significant reduction in ultimate load due to eccentricity. Such behaviour is a consequence of the manner of load application (laterally distributed load).

Data Series III from Elgaaly and Nunan's tests [1], Series J from Elgaaly and Sturgis tests [2] and Series S from Elgaaly and Salkar tests [2] match well the equation (2) for eccentricities $e/b_f < 1/12$, i.e. $R_{exp}/R_{eq} \approx 1$ (Figure 2).

In Series IV of Elgaaly and Nunan's tests [1] there is no zero-eccentricity girder (centrically loaded girder). The missing ultimate load for $e = 0$ is therefore obtained by extrapolation of existing experimental data. A certain discrepancy between the experimental and calculated data for R is evident, i.e. $R_{exp}/R_{eq} > 1$ even at low eccentricities $e/b_f < 1/12 \approx 0.08$ (Figure 2).

Experimental data from the University of Montenegro does not match the equation (2). The discrepancy ($R_{exp}/R_{eq} \neq 1$) is evident even at low eccentricities and increases significantly with the increase in load eccentricity (Figures 3-4). This fact implies that equation (2) does not take some physical behaviour into account. Clearly, the reduction factor does not depend on e/b_f and t_f/t_w only, but also on other geometric parameters. Experimental work indicates that for larger eccentricities ($e/b_f > 1/12 \approx 0.08$), the distance between adjacent transverse stiffeners, a, is a relevant parameter that has to be considered.

Some Montenegro tests have parameters t_f/t_w and e/b_f in the range of equation (2), but other parameters, such as c/a or a/t_w, are different than in the Maine tests. In the tests of Lučić and Šćepanović [6-7], Series EB II, EB III, EB IV, parameters t_f/t_w and e/b_f are in range recommended for equation (2). However, while in the Maine tests $c/a \approx 0.20$, in the Montenegro tests $c/a \approx 0.07$. In the tests of Lučić [4-5], Series B III 1, B III 3, parameters t_f/t_w and e/b_f are in the range recommended for equation (2) and of the same order as in tests of Elgaaly and Nunan [1]. However, while in the Maine tests $a/t_w \approx 45$, in the Montenegro tests $a/t_w = 140$.

Apart from that, some specimens in the Montenegro tests are out of range for t_f/t_w in equation (2). In Series EB I of Lučić and Šćepanović [6-7] $t_f/t_w = 5 > 4$, so that equation (2) cannot be applied to these girders.

Equation (2) limits the reduction factor as: $R < 1$. Mathematically, due to the expression for m, negative values of R might be obtained from equation (2). That occurred in Series B III 2 of Lučić's tests, Figure 3, because of the ratio $t_f/t_w = 1$. Other parameters were approximately as in Elgaaly and Nunan's tests. To maintain $R > 0$, the range of t_f/t_w in which equation (2) is applicable should therefore be corrected to $1.4 \leq t_f/t_w \leq 4$, since the data upon which the equation (2) is based (tests of Elgaaly et al.) are in that range. Furthermore, equation (2) does not include the case of centrically loaded girders in a proper way. For $e = 0$ reduction factor R should be equal to 1 for any value t_f/t_w. However, $R = 1$ is obtained from equation (2) only for $t_f/t_w = 6$, which is not realistic. Logical constraints for R should be: $0 < R \leq 1$, and $R = 1$ for $e/b_f = 0$.

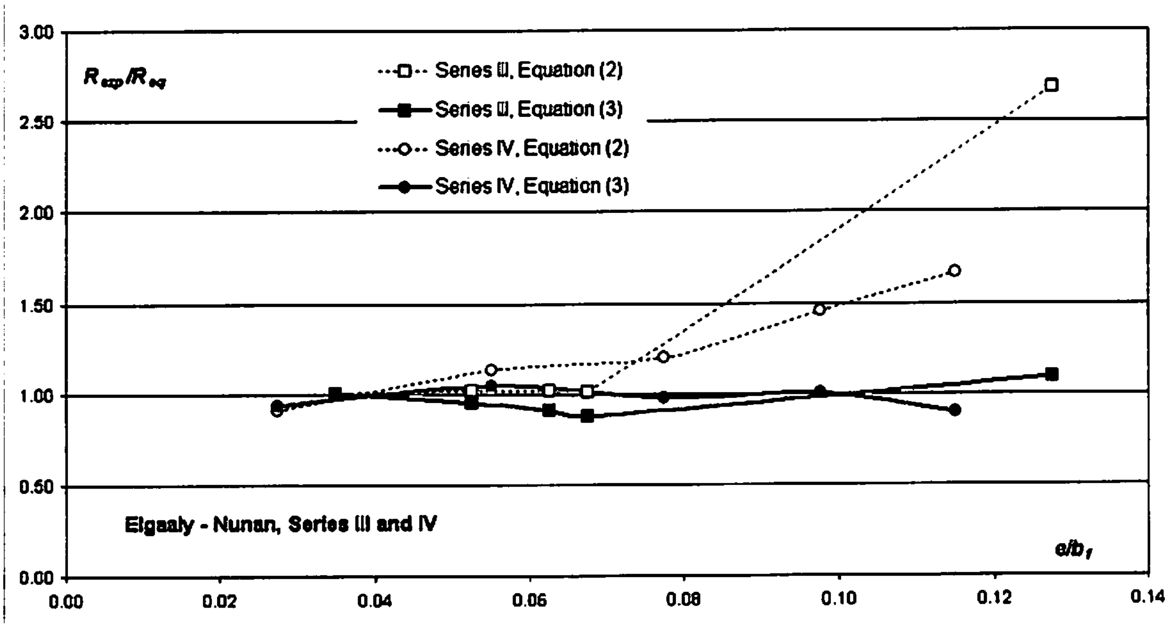

Figure 2. Elgaaly and Nunan's tests.
Comparison of experimental and calculated values of R

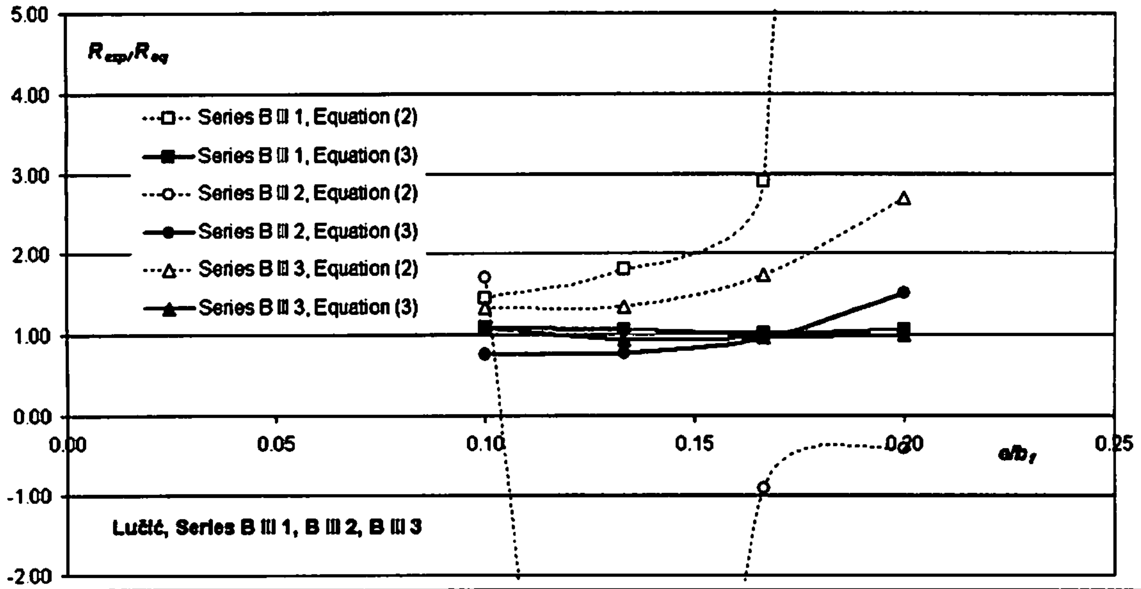

Figure 3. Lučić's tests.
Comparison of experimental and calculated values of *R*

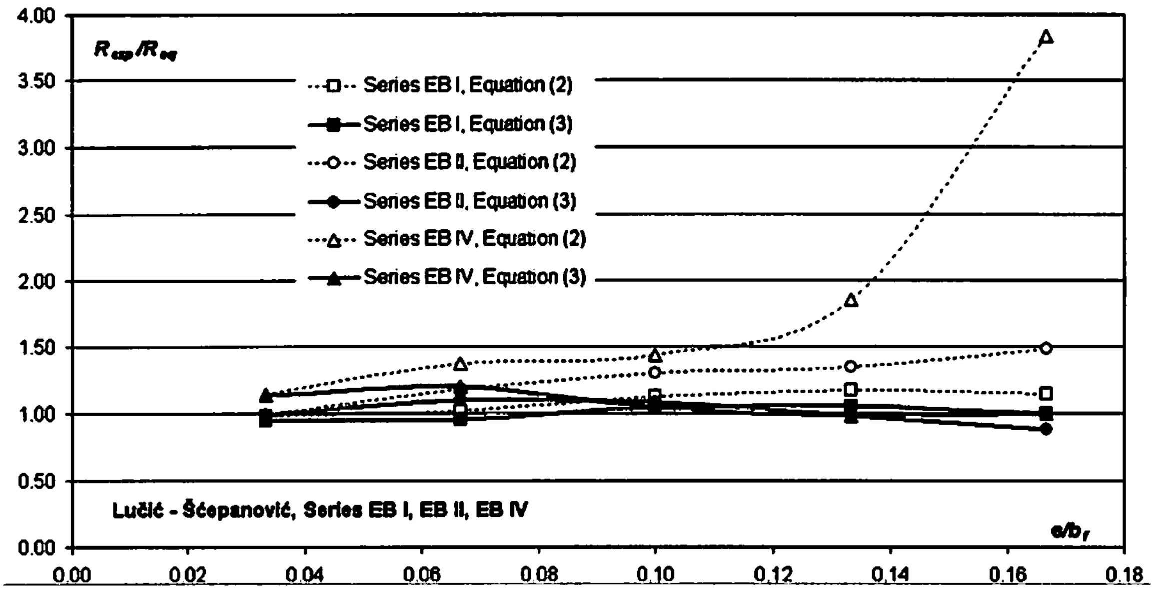

Figure 4. Lučić and Šćepanović's tests.
Comparison of experimental and calculated values of *R*

It might be said that equation (2), based on the 1980s experimental work, is good for the range $1.4 \leq t_f/t_w \leq 4$, $e/b_f < 1/12$ and the other parameters (b_f/t_f, a/t_w, c/a) as in corresponding tests. However, recent experiments, with a different range of parameters ($1 \leq t_f/t_w \leq 5$, $e/b_f \leq 1/5$, $70 \leq a/t_w \leq 230$, $10 \leq b_f/t_f \leq 15$, $c/a = 0.07$ or 0.20) and different combinations of parameter values, indicate a revision of equation (2) may be required to improve the accuracy of the reduction factor for this broader set of parameters and to comply with the logical constraints described previously.

Improved Expression for Reduction Factor

An objective of this study was to adapt the existing mathematical formulation of the reduction factor in order to improve the accuracy of the estimates of R relative to the available experimental data. The resulting expression for R is expected to be applicable to a wider range of geometric parameters (e/b_f, t_f/t_w, b_f/t_f, a/t_w, c/a).

The basic idea is to keep the linear form of R from equation (2), considering other geometrical parameters, apart from t_f/t_w, in coefficients m and n. A new parameter $t_f \cdot a^B/t_w$ is introduced.

Proposed is a new expression for the reduction factor R:

$$\left.\begin{array}{l} R = m \cdot \dfrac{e}{b_f} + n \\ \quad\quad \textit{where:} \quad 0 < R \leq 1 \\ \quad\quad\quad m = A \cdot \left(\dfrac{t_f \cdot a^B}{t_w}\right)^2 + C \cdot \left(\dfrac{t_f \cdot a^B}{t_w}\right) + D \\ \quad\quad\quad n = E + F \cdot \left(\dfrac{t_f \cdot a^B}{t_w}\right) \\ \quad\quad\quad a \rightarrow [mm] \end{array}\right\} \quad (3)$$

Coefficients A, B, C, D, E and F, Table 1, are determined by fitting expression (3) to the experimental data, by the pondered least square technique [9]. In this data regression analysis the following experimental data was used:

- Series III and IV of Elgaaly and Nunan's tests [1],
- Series B III 1, B III 2 and B III 3 of Lučić's tests [4-5],
- Series EB I, EB II, EB III and EB IV of Lučić and Šćepanović's tests [6-7].

Series J of Elgaaly and Sturgis [2] and Series S of Elgaaly and Salkar [2] were not used in the data regression analysis since not being appropriate for this kind of numerical analysis. They consist of a small number of samples, with repeated parameters, what is not suitable for pondered least square technique applied in expression fitting.

Table 1. Coefficients A, B, C, D, E and F in equation (3)

A	B	C	D	E	F
$-5.6 \cdot 10^{-4}$	0.44	0.11	-6	1	$4.6 \cdot 10^{-4}$

In Figures 2-4, both equations (2) and (3) are compared with the experimental data. Equation (2) has a good match with all tests ($R_{exp}/R_{eq} \approx 1$) in the domain of small eccentricities ($e/b_f < 1/12 \approx 0.08$), except for slight discrepancy in Series IV of Elgaaly and Nunan. For larger eccentricities, the values of R obtained from equation (2) are smaller than the experimental values ($R_{exp}/R_{eq} > 1$), in a safe way. Equation (3) matches the experiments much better, covering a larger range of eccentricity values.

In equation (2) a lower limit for R has not been considered at all. As already explained, negative values of R, that are not realistic, occur. Neither mathematical expression (2) nor (3) itself respect the logical range for R: $0 < R \leq 1$. Therefore, $0 < R \leq 1$ should be prescribed as an additional compulsory condition for both equations.

Like equation (2), equation (3) is empirical too, and should be used only within the range of the corresponding test data. Equation (3) is based upon the experiments with the following parameters: $1 \leq t_f/t_w \leq 5$, $1/30 \leq e/b_f \leq 1/5$, $45 \leq a/t_w \leq 233$, $10 \leq b_f/t_f \leq 15$, $c/a = 0.071$ or 0.214. It has to be mentioned that in all analysed tests the panel aspect ratio was $a/h_w = 1$.

Conclusion

Many parameters influence the behaviour, collapse mode and ultimate load of eccentrically patch loaded I-girders. The apparent reduction of the ultimate load due to load eccentricity may be quantified by a reduction factor, R, that is a function of geometric parameters. Recent experimental tests suggest the existing expression for the reduction factor should be modified, particularly in the case of large load eccentricities. The empirical expression presented in the paper, obtained by regression analysis based upon a larger number of experimental tests, considers the reduction factor, R, to be a function of e/b_f and a new parameter $t_f \cdot a^B / t_w$, where B is a constant. In the previous expression, R was a function of parameters e/b_f and t_f/t_w. The new expression was shown to match experimental data better than the previous one, especially for large eccentricities.

It has to be pointed out that every future experimental testing should be followed by new revision and adjusting of empirical expression for the ultimate load reduction factor in order to improve its accuracy. The reduction factor will also be analysed in the current experimental research at the University of Montenegro, completely devoted to eccentric patch loading, having 102 girders loaded up to the collapse.

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ГОДИШНИК НА УНИВЕРСИТЕТА ПО АРХИТЕКТУРА, СТРОИТЕЛСТВО И ГЕОДЕЗИЯ – СОФИЯ

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PATCH LOADED STEEL I-GIRDERS – RESEARCHES AT THE FACULTY OF CIVIL ENGINEERING IN PODGORICA, MONTENEGRO –

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ABSTRACT:

Patch loading is loading applied over a small area or length of a structural element. A common situation in structural engineering occurs when a downward compressive patch load is applied to the flange of steel I-girder so that the web is compressed in the region bellow the applied load. Numerous examples are present in different structures, including crane and bridge girders. Two issues having their own particularities, but also being mutually connected, should be analysed. The first one is carrying capacity loss of girders loaded in the web plane. The second one is carrying capacity loss of girders under loading having a certain eccentricity relative to the web plane.

The paper presents a review of patch loading experimental and theoretical researches carried out at the Faculty of Civil Engineering in Podgorica, Montenegro. Experimental work was initiated in 1998. Girders loaded in the web plane, as well as girders with the eccentric load were analysed. Experimental analysis was continued in 2001. Only eccentric loading was treated. A new, extensive experimental research, completely devoted to eccentrically patch loaded steel I-girders is ongoing. Two mathematical models for ultimate load of centrically patch loaded I-girders were proposed, in 1998 and 2005. In both models, based on experimental experience, ultimate load is calculated by means of the strain energy concept applied to the failure mechanism. Finite element modelling by means of computer software SAP 2000, NonLinear Version 6.11, followed experimental research from 2001. Only linear analysis was done. During the preparation of current experimental research, ultimate load forecast models were made by means of artificial neural networks. Networks of different architecture were trained on experimental data from 1998 and 2001.

Key words: patch loading, load eccentricity, steel I-girder, ultimate load, experimental research, mathematical model, collapse mechanism, finite element modelling, artificial neural network

1. Introduction

Patch loading is loading applied over a small area or length of a structural element. A common situation in structural engineering occurs when a downward compressive patch load is applied to the flange of steel I-girder so that the web is compressed in the region bellow the applied load, Figure 1. Local stresses in web might cause local instability that may provoke element carrying capacity loss and, consequently, collapse of the whole structure. This is a rather complex and challenging issue of extremely evident elastic-plastic stresses and deformations. Apart from that, geometrical nonlinearity is noticeable even at low loading.

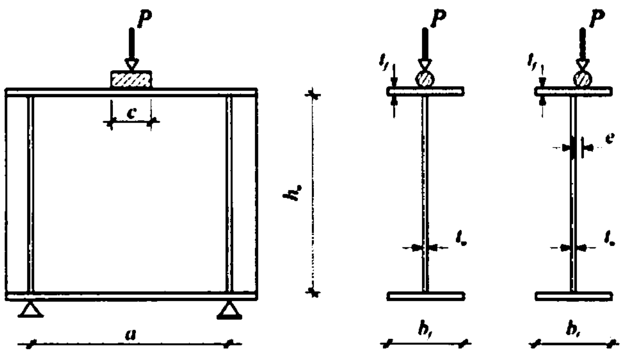


Figure 1. Patch loaded I-girder

Patch loaded girders are widely used in engineering practice. Examples are numerous and present in different structures, including crane and bridge girders.

More than 30 experimental researches had been carried out worldwide and more than 25 mathematical models or empirical expressions for failure load had been proposed until 1998. In spite of such a large amount of researches, a large number of questions have not yet been completely answered. Some parameters and their influence on behaviour, failure mode and ultimate load of patch loaded girders have not been thoroughly investigated and defined. The influence of load eccentricity relative to the web plane is particularly interesting and should be carefully analysed, having in mind the fact that some eccentricity is almost unavoidable in practice.

In attempt to help answering open questions and giving an explanation of certain patch loading issues, a series of patch loading researches started in 1998 at the Faculty of Civil Engineering in Podgorica, Montenegro.

2. Experimental research, 1998

The attention is paid to two problems having their own particularities, but also being mutually connected. The first one is carrying capacity loss of girders loaded in the web plane, Figure 2a. The second one is carrying capacity loss of girders under loading having a certain eccentricity relative to the web plane, Figure 2b. This research confirms the fact known from the previous research work of other authors. The two collapse modes, in centrically and eccentrically loaded girders, are quite different, Figure 2. Carrying capacity loss in the case of centric load is due to web buckling and local stability loss. In the case of eccentric load, girders lose carrying capacity due to local elastic-plastic bending.

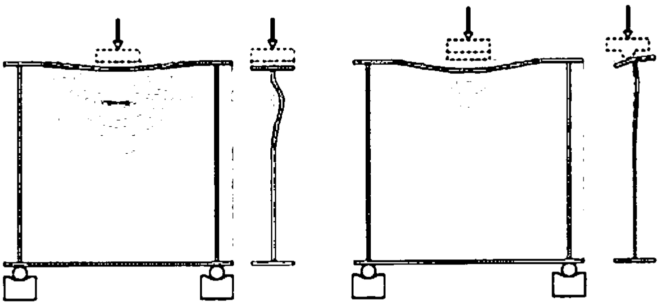
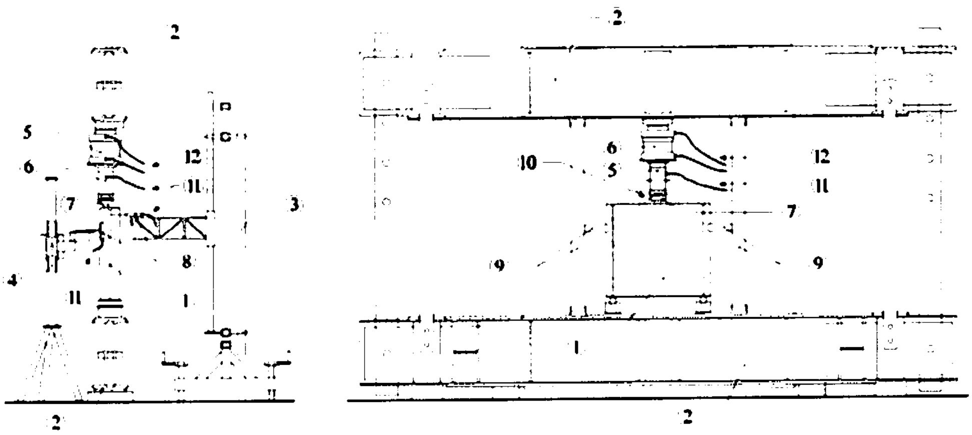


Figure 2. Collapse modes for centric and eccentric patch loading

Before the main research, a preliminary investigation was carried out in order to optimise the main investigation, to define optimum girder dimensions, to test measuring instruments and laboratory equipment, as well as to make the members of working team accustomed to each other and to testing process. Four tests of centrically loaded girders and four tests of eccentrically loaded girders were performed in the preliminary investigation. Girder dimensions and their ratios varied in a wide range, as well as load eccentricity. Different manners of load applying were used: by steel bar, ball and thick plate.



1. loaded girder
2. closed frame
3. frame-traveller for guiding the displacement transducer
4. horizontal traveller for guiding the displacement transducer
5. press (800 kN)
6. load cell
7. load distribution block
8. displacement transducer
9. stiffeners for preventing the lateral buckling
10. safety ring with holder
11. data logger cables
12. hydraulic pump cables

Figure 3. Testing equipment

The main research consisted of 3 series, with 12 tests in each. All together $3 \times 12 = 36$ tests were done on $36/2 = 18$ girders, during 51 working day. Each girder was loaded up to the collapse twice, over both flanges. Deflection of loaded flange, deflection out of the web plane and the loading intensity were measured in the first series, on 12 centrically loaded girders of different dimensions. Girder span ($a = 700$ mm), web depth ($h_w = 700$ mm) and flange width ($b_f = 150$ mm) were the same in all girders. Only web and flange thickness varied ($t_w = 4, 5, 8, 10$ mm; $t_f = 8, 10, 12$ mm). The same girders were turned up side down and loaded over the other flange in the second series. Flange strains, web strains and load intensity were measured this time. In the third series, 12 tests with eccentric loading were performed. Three girder types were tested, with four different load eccentricities

(e/b_f = 1/10, 2/15, 1/6, 1/5). Each girder type had the twin-girders of the same dimensions in the first and second series, with zero eccentricity. Girder types differed by the flange and web thickness (t_w = 5, 10 mm; t_f = 10, 12 mm), as in the first two series.

For the need of patch loading research, in addition to common and standard laboratory equipment, specially designed arrangements for electrical displacement transducers positioning, as well as particularly designed load transferring blocks were used. All tests were carried out in a very stiff, closed steel frame, Figure 3.

3. Mathematical model for calculating ultimate centric load, 1998

Proposed mathematical model, based on experimental experience, defines ultimate load as a sum of two loads: P_{u1}, load at which collapse mechanism occurs in web, and P_{u2}, load that is spent for flange deformation in the moment of collapse.

Load P_{u1} is calculated by means of the strain energy concept applied to the failure mechanism in Figure 4. Girder failure happens by forming of two yielding lines in web. Lines are horizontal along the load length c. The distance between lines is h. In the moment when load reaches ultimate value, lines are developed along the length g. Yielding at the rest of lines (dashed line), as well as forming of plastic hinges in flange, happens latter, in so-called secondary mechanism. Accepted stress distributions along the yielding lines are shown in Figure 5.

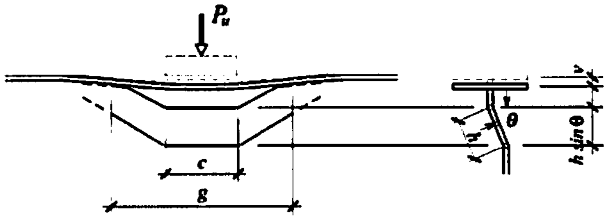
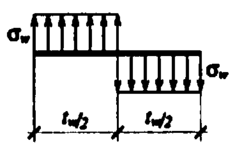


Figure 4. Collapse mechanism

lower line;
upper line
left and right
from length c:



upper line
at the length c:

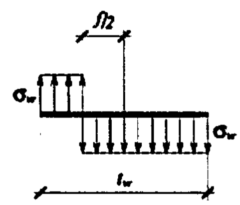


Figure 5. Stress distribution along the web yielding lines

Load P_{u2} is defined as a concentrated force that causes deflection v of simple beam (girder flange, herein) with span l. The span l and deflection v are obtained as experimental data. Deflection is measured. Span is the distance between zero-points at the bending moment diagram for flange.

Mathematical expression for ultimate load P_u is given by equations (1-3), where σ_w is yield stress of web, E_f and I_f are modulus of elasticity and moment of inertia for flange. Variables h, g, v, f, l are obtained by the calibration of model at the statistical sample consisting of 518 tests from 29 experimental researches.

$$P_{u1} = \sigma_w \cdot f \cdot c + \frac{\sigma_w \cdot t_w^2}{4} \cdot \left[1 - \left(\frac{f}{t_w}\right)^2\right] \cdot \frac{c}{h \cdot \cos\theta} + \frac{\sigma_w \cdot t_w^2}{4} \cdot \frac{2 \cdot g - c}{h \cdot \cos\theta} \tag{1}$$

$$P_{u2} = \frac{48 \cdot E_f \cdot I_f \cdot v}{l^3} \tag{2}$$

$$P_u = P_{u1} + P_{u2} \tag{3}$$

4. Experimental research "Ekscentro 2001", 2001

"Ekscentro 2001" continued experimental research from 1998. In this experiment only eccentric patch loading was analysed. Both types of collapse mode were observed in eccentrically loaded girders: not only the one typical for eccentric load, but also the one typical for centric load. Under a certain circumstances, i.e. for the specific girder dimensions ratios, even for large eccentricity of load, girders collapsed the same way as if there was no eccentricity.

Research was divided into 4 series, with 6 tests in each. All together $4 \times 6 = 24$ tests were done on $24/2 = 12$ girders, during 11 working day, after several-month preparation. Same as in 1998, each girder was loaded up to the collapse twice, over both flanges. All girders in one series were of same dimensions, but load eccentricity varied six times (e/b_f = 0, 1/30, 1/15, 1/10, 2/15, 1/6). Series differed only by the web thickness. Three different web thickness were analysed (t_w = 3, 6, 8 mm). Other dimensions were the same in all 12 girders ($a = h_w$ = 700 mm, b_f = 150 mm, t_f = 15 mm).

In the first, second and forth series, loaded flange deflection, web out-of-plane deflection and the loading intensity were measured. In the third series, with the same girders as in the second series, flange and web strains were measured, as well as load intensity.

Beside the common laboratory equipment and equipment specially designed for the research in 1998, a new load transferring block, as well as a new arrangement for electrical displacement transducers positioning were also constructed for the need of "Ekscentro 2001". All tests were carried out in the same, very stiff, closed steel frame, Figure 3.

5. Finite element modelling, 2002

"Ekscentro 2001" was followed by the finite element modelling, by means of computer software SAP 2000 (NonLinear Version 6.11). The idea was to compare experimental end theoretical results in order to obtain more detailed picture of stress-strain state in girder and to asses the capability of SAP 2000, commercial software widely used in engineering practice, to be used for modelling of such problem. Only linear analysis was done, a basic step for a future analysis in non-linear domain.

Several 3D models (flanges + web + stiffeners + load positioned where it really acts) were made. Two models were chosen for results comparison: one model with shell elements (approx 94000 dof) and one model with solid elements (two layers of solids in flange, one layer in web, approx 103000 dof), Figure 6. Comparative analysis of results for stresses in central cross section of girder was done. At first, comparison of different SAP models was done. Afterwards, SAP results were compared with the experimental results.

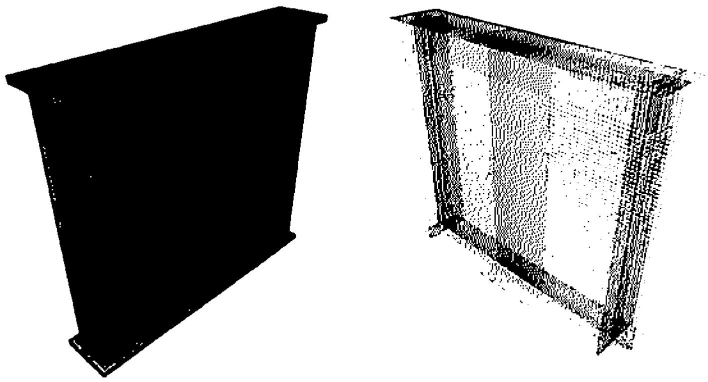


Figure 6. SAP 2000 models with solid and shell finite element

6. Mathematical model for calculating ultimate centric load, 2005

Mathematical model from 1998 has been reconsidered and new, improved model is proposed. Generally, the concept is the same and ultimate load P_u is the sum of two loads, P_{u1} and P_{u2}.

The load of collapse mechanism occurrence in web, P_{u1}, is again calculated by means of the strain energy concept. Collapse mechanism and stress distributions, slightly different from those in 1998, are shown in Figures 7, 8. A new variable, so called fictive load length c_1, which in general differs from the real load length c, is introduced.

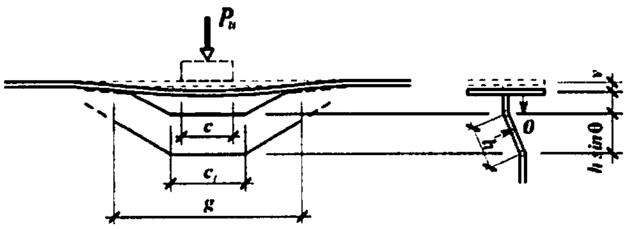
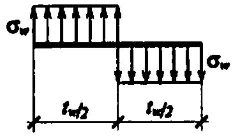


Figure 7. Collapse mechanism

lower line; upper line left and right from load length c_1:



upper line at the length c_1:

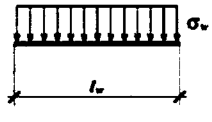


Figure 8. Stress distribution along the web yielding lines

Further increase in ultimate load, P_{u2}, after the collapse mechanism happens in web at load P_{u1}, is possible only on account of reserve in flange carrying capacity. If the limit carrying capacity of engaged flange part is less than P_{u1}, then $P_{u2} = 0$ and $P_u = P_{u1}$. Otherwise $P_u = P_{u1} + P_{u2}$, where P_{u2} is load spent for flange bending immediately after collapse mechanism have happened in web. The load P_{u2} is defined as a part of elastic carrying capacity, F_{u2}, of simple beam (girder flange, herein) with span l, loaded by concentrated force in mid-span. A part of this capacity has already been spent during the common deformation of flange and web before the occurrence of collapse mechanism in web. The rest, defined by coefficient k, makes P_{u2}.

Mathematical expression for ultimate load P_u is given by equations (4-6), where σ_w, σ_f are yield stresses of web and flange. Variables h, g, c_1, θ, l, k are obtained by the calibration of model at the statistical sample consisting of 729 tests from 33 experimental researches.

$$P_{u1} = \sigma_w \cdot t_w \cdot c_1 + \frac{\sigma_w \cdot t_w^2}{4} \cdot \frac{2 \cdot g - c_1}{h \cdot \cos\theta} \tag{4}$$

$$P_{u2} = F_{u2} \cdot k = \frac{2}{3} \cdot \sigma_f \cdot \frac{b_f \cdot t_f^2}{l} \cdot k; \qquad \begin{aligned} t_w &\leq 3.5\ mm \Rightarrow k = 0.5 \\ t_w &> 3.5\ mm \Rightarrow k = 0 \end{aligned} \tag{5}$$

$$P_u = P_{u1} + P_{u2} \tag{6}$$

7. Artificial neural network application for ultimate load forecast

During the preparation of new experimental research at the University of Montenegro, planned for 2007, modelling of subject issue was made by means of artificial neural networks. Depending on variable input parameters (girder geometry and load eccentricity) the ultimate load, as the only output, was forecasted. Artificial neural networks of different architecture, trained on experimental data from 1998 and 2001, were created.

By initiating trained neural network with some specific values of input variables, that have to be in range of training data, the forecast model of output variable (ultimate load) values is obtained. The most suitable presentation of model is graphical method, i.e. diagrams of output variable as a function of one input variable while other input variables have fixed values. Herein, it is interesting to get diagrams P_u-e for values of t_w that were not tested, or diagrams P_u-t_w for fixed values of e. Plenty of similar diagrams might be made and help to understand nature of relationship between output and one particular input variable. In addition, some conclusions about interaction between input parameters might be made, too.

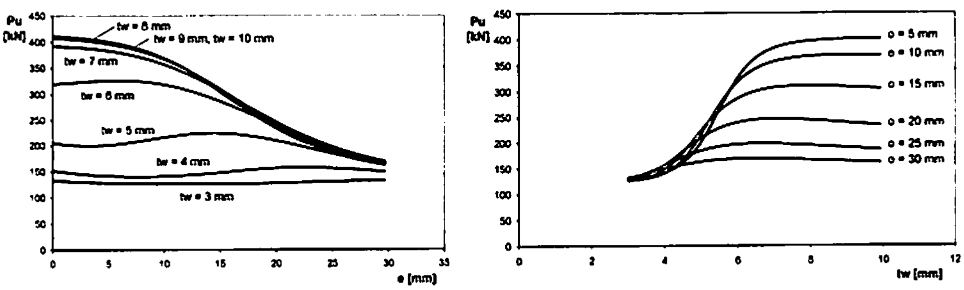


Figure 9. Forecast models $P_u(e)$ and $P_u(t_w)$

The new experimental results are going to enlarge available training data base and enable creation of more precise, more realistic neural network models. Such fitted models might be used not only for the purpose of research, but also in engineering practice.

8. Instead of conclusions

Despite the fact that a large amount of patch loading researches have been carried out worldwide, a large number of questions in a rather wide and complex domain of patch loading are still open.

Researches carried out at the Faculty of Civil Engineering in Podgorica, University of Montenegro, partially answered some of those questions. However, that is not enough and further research work is already ongoing. A new experimental research is planned for 2007, as well as further work on mathematical models. Some other approaches to the ultimate load calculation are also being considered, like empirical expression formulation, that is particularly interesting for eccentrically loaded girders.

Several MSc and PhD theses are based on presented research work. Two new PhD theses are planned for the academic year 2007/08. Results of these researches have been published in journals and presented at international conferences [1-28].

The laboratory of the Faculty of Civil Engineering in Podgorica also hosts scientists from other universities and enable them to perform their own investigations. Experimental research that was part of PhD thesis of dr Nenad Marković, "Buckling of plate girders under the action of patch loading", Faculty of Civil Engineering, University of Belgrade, Serbia, was carried out in our laboratory. The two faculties have qualitative and long lasting cooperation.

The cooperation with several European universities and institutes has been established. Members of our patch loading research team realised a number of visits and few-month stays at the Institute of Theoretical and Applied Mechanics in Prague, Academy of Sciences of Czech Republic, University of Birmingham, Cardiff University, RWTH Aachen, TU Wien, TU Graz, University of Granada, Technical University of Bratislava, University of Architecture, Building and Geodesy in Sofia.

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PATCH LOADING RESEARCHES AT THE UNIVERSITY OF MONTENEGRO

SUMMARY

Patch loading is loading applied over a small area or length of a structural element. Examples of patch loaded steel I-girders are numerous and present in different structures, including crane and bridge girders. The paper presents a review of patch loading experimental and theoretical researches carried out at the Faculty of Civil Engineering in Podgorica, University of Montenegro. Experimental work was initiated in 1998 and continued in 2001. In 2007 one extensive experimental research has already been organised and one more is planned. Two mathematical models for ultimate load of centrically patch loaded I-girders were proposed, in 1998 and 2005. Finite element modelling, as well as artificial neural networks modelling were also considered.

Key words: patch loading, steel I-girder, ultimate load, mathematical model, experimental research

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1. INTRODUCTION

Patch loading is loading applied over a small area or length of a structural element. A common situation in structural engineering occurs when a downward compressive patch load is applied to the flange of steel I-girder so that the web is compressed in the region bellow the applied load, Figure 1. Local stresses in web might cause local instability that may provoke element carrying capacity loss and, consequently, collapse of the whole structure. This is a rather complex and challenging issue of extremely evident elastic-plastic stresses and deformations. Apart from that, geometrical nonlinearity is noticeable even at the lowest loading.

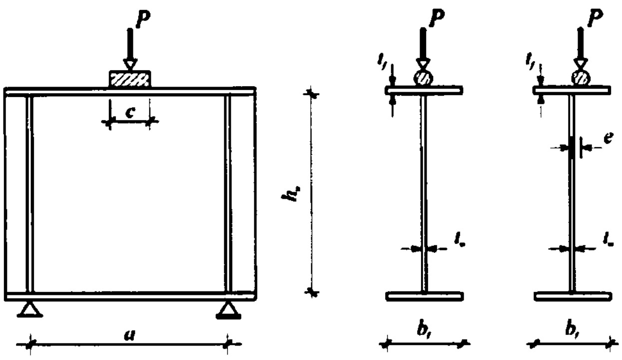

Figure 1: Patch loaded I-girder

Patch loaded girders are widely used in engineering practice. Examples are numerous and present in different structures, including crane and bridge girders.

More than 30 experimental researches had been carried out worldwide and more than 25 mathematical models or empirical expressions for failure load had been proposed until 1998. In spite of such a large amount of researches, a large number of questions have not yet been completely answered. Some parameters and their influence on behaviour, failure mode and ultimate load of patch loaded girders have not been thoroughly investigated and defined.

In attempt to help answering open questions and giving an explanation of certain patch loading issues, a series of patch loading researches was initiated in 1998 at the University of Montenegro, Faculty of Civil Engineering in Podgorica, Montenegro.

2. EXPERIMENTAL RESEARCH, 1998

The attention is paid to two problems having their own particularities, but also being mutually connected. The first one is carrying capacity loss of girders loaded in the web plane. The second one is carrying capacity loss of girders under loading having a certain eccentricity relative to the web plane.

This research confirms the fact known from the previous research work of other authors. The two collapse modes, in centrically and eccentrically loaded girders, are quite different. Carrying capacity loss in the case of centric load is due to web buckling and local stability loss. In the case of eccentric load, girders lose carrying capacity due to local elastic-plastic bending.

Before the main research, a preliminary investigation was carried out in order to optimise the main investigation, to define optimum girder dimensions, to test measuring instruments and laboratory equipment, as well as with the intention to make the members of working team accustomed to each other and to testing process. Four tests of centrically loaded girders and four tests of eccentrically loaded girders were performed in the preliminary investigation. Girder dimensions and their ratios varied in a wide range, as well as load eccentricity. Different manners of load applying were used: by steel bar, ball and thick plate.

The main research consisted of 3 series, with 12 tests in each. All together $3 \times 12 = 36$ tests were done during 51 working day. Deflection of loaded flange, deflection out of the web plane and the loading intensity were measured in the first series, on 12 centrically loaded girders of different dimensions.

Girder span (a = 700 mm), web depth (h_w = 700 mm) and flange width (b_f = 150 mm) were the same in all girders. Only web and flange thicknesses varied (t_w = 4, 5, 8, 10 mm; t_f = 8, 10, 12 mm). The same girders were turned up side down and loaded over the other flange in the second series. Flange strains, web strains and load intensity were measured this time. In the third series, 12 tests with eccentric loading were performed. Three girder types were tested, with four different load eccentricities (e/b_f = 1/10, 2/15, 1/6, 1/5). Each girder type had the twin-girders of the same dimensions in the first and second series, with zero eccentricity. Girder types differed by the flange and web thickness (t_w = 5, 10 mm; t_f = 10, 12 mm), as in the first two series.

For the need of patch loading research, in addition to standard laboratory equipment, specially designed arrangements for displacement transducers guiding, as well as particularly designed load distribution blocks were used. All tests were carried out in a very stiff, closed steel frame, Figure 2.

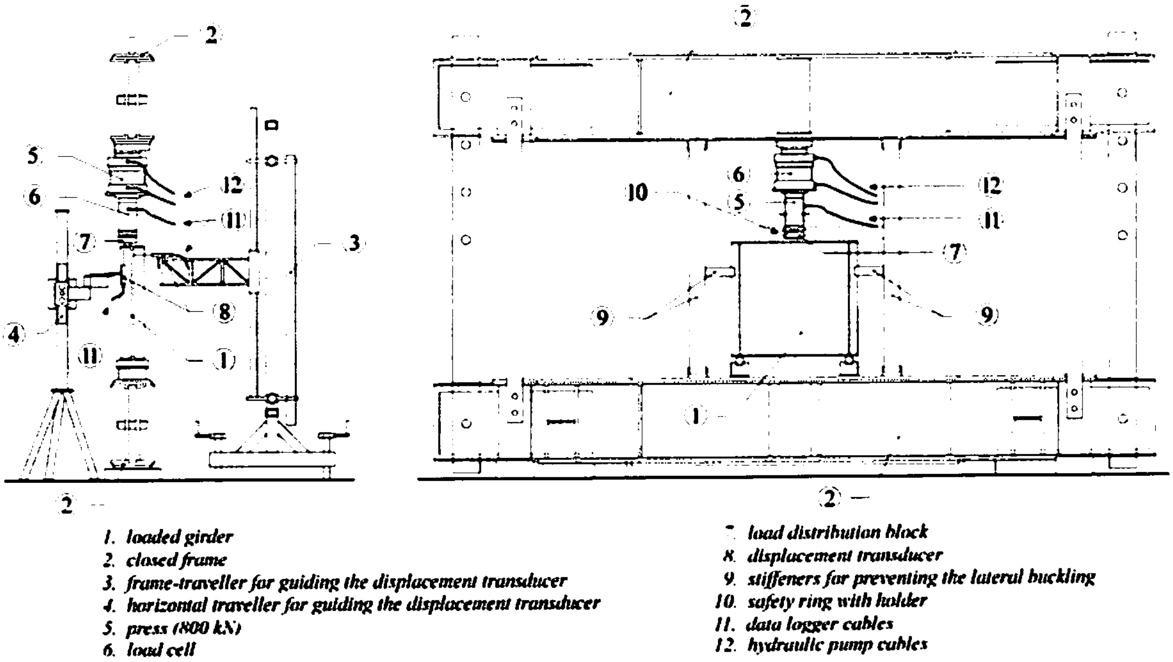

Figure 2: Testing equipment

3. MATHEMATICAL MODEL FOR ULTIMATE LOAD OF CENTRICALLY PATCH LOADED I-GIRDERS, 1998

Proposed mathematical model, based on experimental experience, defines ultimate load $P_u = P_{u1} + P_{u2}$, as a sum of two loads: P_{u1}, load at which collapse mechanism occurs in web, and P_{u2}, load that is spent for flange deformation in the moment of collapse.

Load P_{u1} is calculated by means of the strain energy concept applied to the failure mechanism in Figure 3a. Girder failure happens by forming of two yielding lines in web. Lines are horizontal along the load length c. The distance between lines is h. In the moment when load reaches ultimate value, lines are developed along the length g. Yielding at the rest of lines (dashed line), as well as forming of plastic hinges in flange, happens latter, in so-called secondary mechanism.

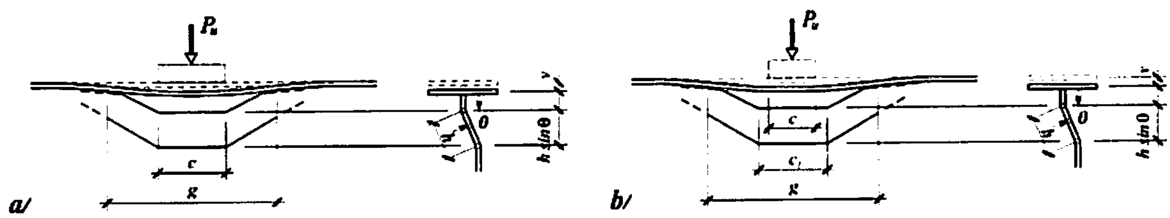

Figure 3: Collapse mechanisms: a/ 1998 model, b/ 2005 model

Load P_{u2} is defined as a concentrated force that causes deflection v of simple beam (girder flange, herein) with span l. The span l and deflection v are obtained as experimental data. Deflection is measured. Span is the distance between zero-points at the bending moment diagram for flange.

Mathematical expressions for ultimate load components P_{u1}, P_{u2} are given by equations (1-2), where σ_w is yield stress of web, E_f and I_f are modulus of elasticity and moment of inertia for flange. Variables h, g, v, f, l are obtained by the calibration of model at the statistical sample consisting of 518 tests from 29 experimental researches.

$$P_{u1} = \sigma_w \cdot f \cdot c + \frac{\sigma_w \cdot t_w^2}{4} \cdot \left[1 - \left(\frac{f}{t_w}\right)^2\right] \cdot \frac{c}{h \cdot \cos\theta} + \frac{\sigma_w \cdot t_w^2}{4} \cdot \frac{2 \cdot g - c}{h \cdot \cos\theta} \tag{1}$$

$$P_{u2} = \frac{48 \cdot E_f \cdot I_f \cdot v}{l^3} \tag{2}$$

4. EXPERIMENTAL RESEARCH "EKSCENTRO 2001", 2001

"Ekscentro 2001" continued experimental research from 1998. Only eccentrically patch loaded girders were analysed. Under a certain circumstances, i.e. for the specific girder dimensions ratios, even for large eccentricity of load, girders collapsed the same way as if there was no eccentricity.

Research was divided into 4 series, with 6 tests in each. All together $4 \times 6 = 24$ tests were done during 11 working days, after several-month preparation. All girders in one series were of same dimensions, but load eccentricity varied six times (e/b_f = 0, 1/30, 1/15, 1/10, 2/15, 1/6). Series differed only by the web thickness (t_w = 3, 6, 8 mm). Other dimensions were the same in all girders ($a = h_w = 700$ mm, $b_f = 150$ mm, $t_f = 15$ mm). In the first, second and forth series, loaded flange deflection, out-of-plane web deflection and the loading intensity were measured. In the third series, with the same girders as in the second series, flange and web strains were measured, as well as load intensity.

Beside the common laboratory equipment and equipment specially designed for the research in 1998, a new load distribution block, as well as a new arrangement for electrical displacement transducers guiding were also constructed for the need of "Ekscentro 2001". All tests were carried out in the same, very stiff, closed steel frame, Figure 2.

5. MATHEMATICAL MODEL FOR ULTIMATE LOAD OF CENTRICALLY PATCH LOADED I-GIRDERS, 2005

Mathematical model from 1998 has been reconsidered and new, improved model is proposed. Generally, the concept is the same and ultimate load P_u is the sum of two loads, P_{u1} and P_{u2}.

The load of collapse mechanism occurrence in web, P_{u1}, is again calculated by means of the strain energy concept. Collapse mechanism is shown in Figure 3b and it is slightly different from the one in 1998. A new variable, so called fictive length of load c_1, which in general differs from the real length of load c, is introduced.

Further increase in ultimate load, P_{u2}, after the collapse mechanism happens in web at load P_{u1}, is possible only on account of reserve in flange carrying capacity. If the limit carrying capacity of engaged flange part is less than P_{u1}, then $P_{u2} = 0$ and $P_u = P_{u1}$. Otherwise $P_u = P_{u1} + P_{u2}$, where P_{u2} is load spent for flange bending immediately after collapse mechanism have happened in web. The load P_{u2} is defined as a part of elastic carrying capacity, F_{u2}, of simple beam (girder flange, herein) with span l, loaded by concentrated force in mid-span. A part of this capacity has already been spent during the common deformation of flange and web before the occurrence of collapse mechanism in web. The rest, defined by coefficient k, makes P_{u2}.

Mathematical expressions for ultimate load components P_{u1}, P_{u2} are given by equations (3-4), where σ_w, σ_f are yield stresses of web and flange. Variables h, g, c_1, θ, l, k are obtained by the calibration of model at the statistical sample consisting of 729 tests from 33 experimental researches.

$$P_{u1} = \sigma_w \cdot t_w \cdot c_1 + \frac{\sigma_w \cdot t_w^2}{4} \cdot \frac{2 \cdot g - c_1}{h \cdot \cos\theta}; \tag{3}$$

$$P_{u2} = F_{u2} \cdot k = \frac{2}{3} \cdot \sigma_f \cdot \frac{b_f \cdot t_f^2}{l} \cdot k; \qquad \begin{array}{lll} t_w \le 3.5\, mm & \Rightarrow & k = 0.5 \\ t_w > 3.5\, mm & \Rightarrow & k = 0 \end{array} \tag{4}$$

6. EXPERIMENTAL RESEARCH "EKSCENTRO 2007", 2007

"Ekscentro 2007" continued experiments from 1998 and 2001. Only eccentric patch loading was analysed. As expected, both collapse modes, the one typical for centric loading, as well as the one typical for eccentric loading, were observed in eccentrically loaded girders.

Being the most extensive experiment so far, concerning eccentric patch loading, research was divided into 17 series, with 6 tests in each. All together 17 × 6 = 102 tests were done during 36 working days, after several-month preparation. All girders in one series were of same dimensions, but load eccentricity varied six times (e/b_f = 0, 1/30, 1/15, 1/10, 2/15, 1/6). Series differed by the web and flange thicknesses, which varied in a wide range, covering some "gaps" from previous tests and enlarging range of existing test data (t_w = 3, 4, 5, 6 and 10 mm, t_f = 3, 4, 6, 8, 9, 10 and 12 mm). Other dimensions were the same in all girders ($a = h_w$ = 700 mm, b_f = 150 mm), the same as in 1998 and 2001 experiments. The loaded flange deflection, out-of-plane web deflection and the loading intensity were measured.

Beside the common and standard laboratory equipment and equipment specially designed for the 1998 and 2001 researches, some new arrangements for electrical displacement transducers guiding were also constructed for the need of "Ekscentro 2007". Also some new safety elements and measures were applied, having in mind very high level of loading. All tests were carried out in the same, very stiff, closed steel frame, Figure 2.

7. INSTEAD OF CONCLUSION

Despite the fact that numerous patch loading researches have been carried out worldwide, a large number of questions in a rather wide domain of patch loading are still open.

Researches carried out at the Faculty of Civil Engineering in Podgorica, University of Montenegro, partially answered some of those questions. However, that is not enough and further research work is already ongoing. A new experimental research is planned for the end of 2007, as well as further work on mathematical models. Some other approaches to the ultimate load calculation are also being considered, like artificial neural networks application or empirical expressions formulations. This is particularly interesting for eccentrically loaded girders. "Ekscentro 2001" was followed by the finite element modelling, by means of computer software SAP 2000 (NonLinear Version 6.11). Only linear analysis was done, a basic step for a future analysis in non-linear domain.

Several MSc and PhD theses are based on presented research work. Two new PhD theses are planned for the academic year 2007/08. Results of these researches have been published in journals and presented at international conferences [1-31].

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RESEARCH WORK CONCERNING ECCENTRICALY PATCH LOADED STEEL I-GIRDERS

Summary

Patch loading acts locally, over a small area or length of a structural element. It is a common situation in structural engineering that local compressive load affects the flange of steel I-profile so that the web is compressed in the region below the applied load. Some eccentricity of load relative to the web plane is unavoidable in engineering practice. Numerous examples are present in different structures, including crane and bridge girders. The paper summarise investigations and knowledge on eccentrically patch loaded steel I-girders and suggests some directions of future research work in the subject domain.

Key words

patch load, load eccentricity, steel I-girder, ultimate load, experiment

ISTRAŽIVAČKI RAD U OBLASTI EKSCENTRIČNO LOKALNO OPTEREĆENIH ČELIČNIH I-NOSAČA

Rezime

Patch loading podrazumijeva usko podijeljeno, lokalno opetrećenje, koje djeluje na maloj dužini/površini konstruktivnog elementa. U građevinskom konstrukterstvu je uobičajeno da ovakvo opterećenje djeluje na nožici čeličnog I-nosača, tako da je rebro lokalno pritisnuto neposredno ispod opterećenja. Izvjestan ekscentricitet opterećenja u odnosu na ravan rebra gotovo je neizbježan. Primjeri u inžinjerskoj praksi su brojni, uključujući kranske i mostovske nosače. U radu su ukratko predstavljena dosadašnja istraživanja i saznanja, sa nekim smjernicama za dalji istraživački rad u predmetnoj oblasti.

Ključne riječi

lokalno opterećenje, ekscentricitet, čelični I-nosač, granično opterećenje, eksperiment

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1. INTRODUCTION

Patch loading acts locally, over a small area or length of a structural element. It is a common situation in structural engineering that local compressive load affects the flange of steel I-profile so that the web is compressed in the region below the applied load. Numerous examples are present in different structures, including crane and bridge girders.

Although some eccentricity of load relative to the web plane is unavoidable in engineering practice, Figure 1, rather modest amount of worldwide research work has treated this issue. While over 30 experimental researches dealt with I-girders patch loaded in the web plane, influence of load eccentricity was analysed in only 7 experimental studies [1-3,5-7]. Experimental work was followed by finite element method modelling, by means of various computer software, mostly in linear domain [1,2,6]. While over 25 mathematical expressions for centric ultimate load have been proposed, only 2 empirical expressions for eccentric ultimate load might be found in literature [4,9]. Artificial neural networks are also used nowadays for failure load estimation [8].

It has been shown that many parameters influence the behaviour, collapse mode and ultimate load of eccentrically patch loaded I-girders: geometric parameters (girder's dimensions and their dimensionless ratios), load eccentricity and the manner of load application. However, the exact influence neither of these parameters separately nor of their combinations are completely defined.

It is evident that most eccentrically loaded girders have a collapse mode quite different from that of centrically loaded girders. Carrying capacity loss in the case of centric load is due to web buckling and local stability loss. In the case of eccentric load, girders lose carrying capacity due to local elastic-plastic bending. However, it is not defined for what combination of influential parameters the eccentrically loaded girders have the same collapse form as the one of centrically loaded girders.

The reduction in ultimate load with an increase in load eccentricity is obvious in girders with eccentric collapse mode. It is quantified by a reduction factor, R, that relates the ultimate load of eccentrically loaded girders to the ultimate load of centrically loaded girders. The reduction factor might have been calculated from an original empirical expression based on 1980s tests. Although improved empirical expression has been proposed recently, it is still not generally valid, does not give completely confident results in all cases. Both expressions are obtained by numerical analysis of experimental data. Consequently, their application is limited by the range of test data. Any mathematical model for ultimate load calculation based on collapse mechanism has not yet been proposed.

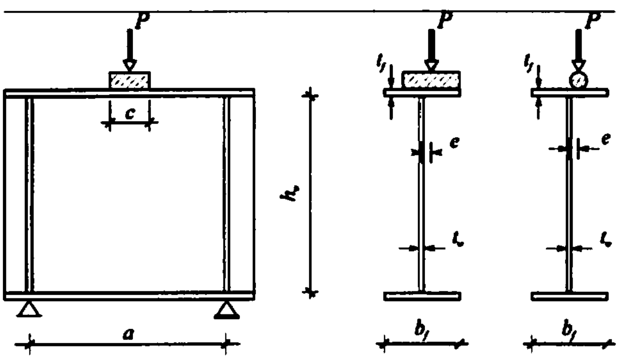

Figure 1. Eccentrically patch loaded I-girder

2. EXPERIMENTAL RESEARCH WORK

Experimental work started at the University of Maine in late 1980s [1,2]. At the same time some tests were done at the Institute of Theoretical and Applied Mechanics, Czech Academy of Sciences [3]. Ten years later a new series of experiments were initiated at the University of Montenegro [5-7].

In the experimental studies at the University of Maine, by Elgaaly and associates [1,2], loads were applied to around 35 rolled and built-up I-girders at various eccentricities, as a laterally distributed load, Figure 1a, or as a line load, Figure 1b. In the case of laterally distributed eccentric patch loading, the collapse mode was almost the same as in centrically loaded specimens. No significant reduction in ultimate load was observed as the eccentricity ratio e/b_f increased to the maximum of 1/6. However, in the case of line load the girder behaviour obviously differed from the behaviour of centrically loaded girders. Web bending, as well as flange bending and twisting were evident even at low load, and became more pronounced near the failure. The failure load clearly decreased as the load eccentricity increased. The reduction was approximately linear with the increase in eccentricity. It was also shown that the reduction was smaller for larger values of ratio t_f/t_w. In particular, when the flange is much thicker than the web, the ultimate load does not reduce significantly with the increase in eccentricity ratio, even for very slender webs ($h_w/t_w > 200$). It was concluded that the ultimate load did not depend significantly on panel aspect ratio a/h_w.

The research of Drdacky, ITAM, Czech Academy of Sciences, 1989, [3], treated girders with eccentricity ratio e/b_f up to 5 /3. Girders were not loaded to failure. Web deflections, stresses and strains proved to be linearly dependent on the eccentricity. Contrary to the load eccentricity, small initial geometric imperfection did not influence web bending significantly.

In the first research of Lučić and associates, University of Montenegro, 1998, [5], load eccentricity and flange and web thickness (i.e. ratios e/b_f, b_f/t_f, h_w/t_w and t_f/t_w) were varied in 18 tests (12 eccentrically + 6 comparative centrically loaded girders). The eccentricity ratio, e/b_f, varied from 1/10 to 1/5. All eccentrically loaded girders had collapse modes quite different from those of centrically loaded girders. Compared with Elgaaly's tests, the set of specimens in Lučić's tests exhibited more prominently the eccentric collapse mode characteristics. During the process of load increase, the most evident deformation is the flange warping accompanied by gentle web bending which follows the flange deformation. It seems that initial imperfection does not influence girder behaviour. As before, ultimate load decreases approximately linearly with an increase in eccentricity ratio. This experimental study demonstrated that the reduction of ultimate load was greater for smaller t_f/t_w ratios. Herein the reduction is inversely proportional to the t_f/t_w.

The research named "Ekscentro 2001", University of Montenegro, 2001, [6,7], continued previous studies. Load eccentricity, e, and web thickness, t_w, were varied in 24 tests (20 eccentrically + 4 comparative centrically loaded girders). In comparison with the previous research [5], eccentricity ratio varied over a larger range, from 1/30 to 1/6, i.e. smaller eccentricities were also analysed. Ranges of parameters h_w/t_w and t_f/t_w were also larger, i.e. greater values of these parameters were analysed. Collapse modes characteristic of both eccentrically and centrically loaded girders were observed in eccentrically loaded girders. The smaller the eccentricity ratio and the larger the t_f/t_w ratio (i.e. thinner web relative to flange), the more likely a centric collapse mode would develop in girders having eccentric patch loading.

A quite extensive experimental research "Ekscentro 2007" was organised in 2007, at the University of Montenegro, continuing previous research work [5-7]. It was completely devoted to eccentric patch loading, having 102 girders loaded up to the collapse (86 girders with eccentric load + 16 comparative girders with centric load). Wider ranges of influential parameters were analysed. The particular attention was paid to the following: e/b_f, h_w/t_w, t_f/t_w and c/a. Having in mind complexity of subject issue, large number of influential parameters and their mutual connectivity, existing experimental data base was rather modest. Hence, the importance of this experiment, its valuable data and conclusions are emphasised. Experimental data are still being processed.

3. FINITE ELEMENT METHOD (FEM) MODELLING

Parallel with the experimental researches, FEM modelling of eccentrically patch loaded steel I-girders has been developed. Various softwares were used at the universities in Maine, Montenegro and Spain.

The same as experimental work, FEM modelling was launched at the University of Maine, in late 1980s. Initially linear analysis [1] was continued as non-linear, by means of software developed by Elgaaly, Caccese and Du [2]. Only web panel was modelled. Flanges and web stiffeners were substituted with appropriate support conditions. Eccentric load was applied as a centric one in combination with a proper bending moment.

Experiment "Ekscentro 2001", at the University of Montenegro, was followed by problem modelling with finite element method, by means of computer software SAP 2000 (NonLinear Version 6.11) [6]. Only linear analysis was done. Full 3D model of loaded I-girder was made in few variations, with shell or solid elements.

Nonlinear FEM modelling is currently ongoing at the University of Granada, Spain, by means of ANSYS software.

4. EMPIRICAL EXPRESSION FOR ULTIMATE LOAD CALCULATION

Experimental data shows that, in girders with eccentric collapse mode, ultimate load reduces as the load eccentricity increases. The reduction is quantified by a reduction factor:

$$R = \frac{\text{ultimate load of eccentrically loaded girder}}{\text{ultimate load of centrically loaded girder}} \tag{1}$$

According to [4], R is a function of t_f/t_w and varies linearly with e/b_f:

$$\begin{gathered} R = m \cdot \frac{e}{b_f} + n < 1; \\ m = -0.45 \cdot \left(\frac{t_f}{t_w}\right)^2 + 4.55 \cdot \left(\frac{t_f}{t_w}\right) - 12.75; \quad n = 1.15 - 0.025 \cdot \left(\frac{t_f}{t_w}\right) \end{gathered} \tag{2}$$

Equation (2) is empirical, based on the experiments of Elgaaly at al [1,2]. Therefore it is applicable only for $1 \leq t_f/t_w \leq 4$ and $e/b_f \leq 1/6$, which reflects the ranges of test data [4].

Recent experimental work implies that the original expression (2) should be modified. According to [9], the improved expression is obtained by regression analysis, based upon a large number of data from the experiments presented in this paper, finishing with "Ekscentro 2001". The reduction factor is considered to be a function of the most relevant parameter e/b_f and a new parameter $t_f \cdot a^{0.44}/t_w$:

$$R = m \cdot \frac{e}{b_f} + n < 1; \quad 0 < R \leq 1; \quad a \rightarrow [mm]$$

$$m = -5.6 \cdot 10^{-4} \cdot \left(\frac{t_f \cdot a^{0.44}}{t_w}\right)^2 + 0.11 \cdot \left(\frac{t_f \cdot a^{0.44}}{t_w}\right) - 6; \quad n = 1 + 4.6 \cdot 10^{-4} \cdot \left(\frac{t_f \cdot a^{0.44}}{t_w}\right) \tag{3}$$

The new expression (3) was shown to match experimental data better than the previous one (2), especially for large eccentricities. Besides, the range of used test data is larger, so that the expression (3) has a wider domain of application [9]: $1 \leq t_f/t_w \leq 5$, $1/30 \leq e/b_f \leq 1/5$, $45 \leq a/t_w \leq 233$, $10 \leq b_f/t_f \leq 15$, $c/a = 0.071$ or 0.214, $a/h_w = 1$.

It has to be pointed out that every future experimental testing should be followed by new revision and adjusting of empirical expression for the ultimate load reduction factor in order to improve its accuracy. The experimental data from "Ekscentro 2007" are being processed in order to propose new modification of R expression.

Ultimate load of centrically patch loaded girder might be calculated by one of numerous and very accurate existing mathematical expressions. Ultimate load of eccentrically patch loaded girder then might be easily calculated if the reduction is evaluated correctly and confidently. Still, the application of this too simplified procedure has a lot of limits and it is recommendable to define mathematical model for ultimate load that is based on collapse mechanism.

5. ARTIFICIAL NEURAL NETWORK APPLICATION FOR ULTIMATE LOAD FORECAST

During the preparation of experimental research "Ekscentro 2007", modelling of subject issue was made by means of artificial neural networks [8]. Depending on variable input parameters (girder geometry and load eccentricity) the ultimate load (output) was forecasted. Artificial neural networks of different architecture, trained on experimental data from 1998 and 2001, were created.

By initiating trained neural network with some specific values of input variables, that have to be in range of training data, the forecast model of output variable (ultimate load) values is obtained. Herein, it was interesting to get diagrams P_u-e for values of t_w that were not tested, or diagrams P_u-t_w for fixed values of e. Plenty of similar diagrams might be made and help to understand nature of relationship between output and one particular input variable. Some conclusions about interaction between input parameters might be made, too.

It is important to emphasize that these forecast models might be used and might give reliable results only for input parameters in range of experimental (training) data, i.e. data used for model fitting (network training). The new experimental results are going to enlarge available training data base and enable creation of more precise, more realistic neural network models. Such fitted models might be used not only for the purpose of research, but also in engineering practice.

6. CONCLUSIONS

Presented research work and gathered knowledge on eccentrically patch loaded steel I-girders make a valuable contribution to the development of structural engineering science and practice. However, quite a lot more should be done in this research area.

Main problems that are still awaiting for the complete solution are separation of centric and eccentric failure mode, as well as estimation of ultimate load in eccentrically loaded girders.

Having in mind complexity of subject issue and large number of influential parameters and their mutual connectivity, the experimental data base existing until 2007 was rather modest, but notably enlarged by "Ekscentro 2007". It is now qualitative basis for all other research directions and means (finite element modelling, artificial neural network modelling, formulation of the expression for ultimate load calculation). Although "Ekscentro 2007" provides new and valuable knowledge about subject topic, experimental research should not stop here.

Regarding the estimation of ultimate load, the most proper way would be the proposition of mathematical model based on eccentric collapse mechanism. For the purpose of eccentric collapse mechanism recognition, as well as for mathematical model proposal and calibration, experimental data are basics. FEM and ANN models might also be useful.

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AN EXPERIMENTAL RESEARCH "EKSCENTRO 2007"

Experimental testing of eccentrically patch loaded steel I-girders

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INTRODUCTION

Patch loading acts locally, over a small area or length of a structural element. It is a common situation in structural engineering that local compressive load affects the flange of steel I-girder so that the web is compressed in the region below the applied load. Numerous examples are present in different structures, including crane and bridge girders.

Although some eccentricity of load relative to the web plane is unavoidable in engineering practice (*Fig. 1*), rather modest amount of worldwide research work has treated this issue. While over 33 experimental researches dealt with I-girders patch loaded in the web plane, influence of load eccentricity was analysed in only 6 experimental studies [1-3,5-7]. Experimental work started at the University of Maine in late 1980s [1,2]. At the same time some tests were done at the Institute of Theoretical and Applied Mechanics, Czech Academy of Sciences [3]. Ten years later a new series of experiments were initiated at the University of Montenegro [5-7]. Experimental work was followed by finite element method modelling, by means of various computer software, mostly in linear domain [1,2,6]. While over 28 mathematical expressions for centric ultimate load have been proposed, only 2 empirical expressions for eccentric ultimate load might be found in literature [4,10]. Mathematical model for ultimate load calculation based on collapse mechanism, or some other theoretical approach, has not yet been proposed. Artificial neural networks are also used nowadays for failure load estimation [9].

It has been shown that many parameters influence collapse mode and ultimate load of eccentrically patch loaded I-girders: geometric parameters (girder's dimensions and their dimensionless ratios), load eccentricity and load application (line or laterally distributed load). However, the exact influence none of these parameters separately nor of their combinations are completely defined.

It is evident that most eccentrically loaded girders have a collapse mode quite different from that of centrically loaded girders. Carrying capacity loss in the case of centric load is due to web buckling and local stability loss. In the case of eccentric load, girders lose carrying capacity due to local elastic-plastic bending. However, it is not defined for what combination of influential parameters the eccentrically loaded girders have the same collapse form as centrically loaded girders.

The reduction in ultimate load with an increase in load eccentricity is obvious in girders with eccentric collapse mode. However, there is still no confident mathematical expression, for calculation of this reduction in ultimate load.

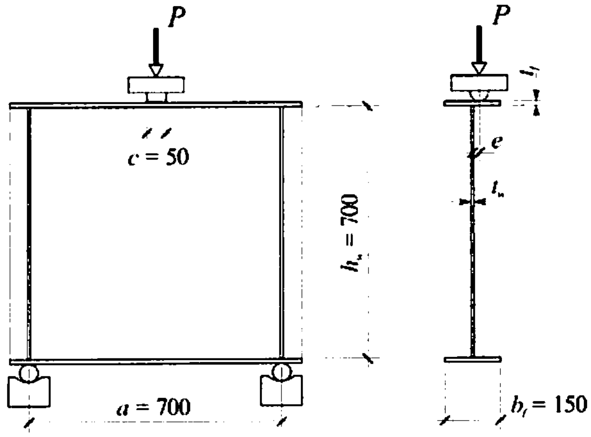


Figure 1. Eccentrically patch loaded I-girder – specimen of "Ekscentro 2007"

1 CONCEPT OF "EKSCENTRO 2007"

An existing experimental database [1-3,5-7] was not large enough to describe completely behaviour, collapse mode and ultimate load of eccentrically patch loaded steel I-girders. New, extensive experimental research "Ekscentro 2007" was organised in 2007 at the University of Montenegro, Faculty of Civil Engineering in Podgorica.

"Ekscentro 2007" continues series of patch loading experimental research at the University of Montenegro which started in 1998 [5] and proceeded through "Ekscentro 2001" [6,7].

According to the previous researches, numerous parameters influence behaviour, collapse mode and ultimate load of eccentrically patch loaded steel I-girders. Dominant parameter is the load eccentricity e or ratio e/b_f (*Fig. 1*). Apart from this parameter, the influence of geometry parameters should be studied. Girder dimensions, primarily web and flange thicknesses t_w and t_f, as well as ratio t_f/t_w are of important influence. Other ratios, like b_f/t_f, a/t_w, h_w/t_w should also be considered. Attention should also be paid to the load length c or ratio c/a and to the load application manner (linear or laterally distributed load). The study of such a great number of parameters at the same time assumes a very extensive as well as expensive research. On the other side, it is not possible to analyse influence of only one geometry parameter, separately, since it is not possible to vary only one of these parameters while all the others are constant. Variation of one girder dimension (e.g. t_w) automatically means variation of several dimensionless parameters (t_f/t_w, a/t_w, h_w/t_w). Therefore, formulating appropriate concept of the experiment (choice of input variable parameters and their variation) demands special attention.

Basic premises of "Ekscentro 2007" concept are:

- New experimental results should upgrade an existing database, fill in "the gaps" i.e. missing values of certain parameters in order that new data make one whole with the existing data.
- Variation of input parameters should be regular, with regular increment, as much as possible, in order to help easier defining their influence.
- Girder dimensions, load application manner and course of testing should correspond to the available equipment in the laboratory of the Faculty of Civil Engineering in Podgorica, in order to enable simple and successful realisation of the experiment.

Having in mind all above stated, it is decided for "Ekscentro 2007" to analyse the influence of load eccentricity e, web and flange thicknesses, t_w and t_f, on collapse mode and ultimate load of eccentrically patch loaded steel I-girders. These three dimensions are chosen as parameters with the most significant influence. Load eccentricity and plate thicknesses will vary in wide range, with an expectation to obtain different collapse modes that should be studied. Other dimensions of girder (span a, web depth h_w and flange width b_f) are the same as in previous Montenegrian experiments ($a = 700$ *mm*, $h_w = 700$ *mm*, $b_f = 150$ *mm*). Linear load is applied along the length c of 50 *mm* (as in 2001) or 150 *mm* (as in 1998). Test specimen is shown in *Figure 1*.

"Ekscentro 2007" is organised in 17 series (*EB V, EB VI, EB VII ... EB XXI*), each with 6 girders. All together $17 \times 6 = 102$ girders were tested. Their parameters are summarised in *Table 1*. In series *EB V – EB XX* all six girders of one series have the same geometry. Only load eccentricity varies in one series: $e = 0$, 5, 10, 15, 20 and 25 *mm* (as in "Ekscentro 2001"). Series *EB XXI* differs from this rule. It has 3 different types of girder (as in 1998 experiment), 2 pieces of each type, tested with $e = 5$ and 10 *mm*. This series complements 1998 experiment with missing load eccentricities.

In 1998 tests and series *EB XXI* of "Ekscentro 2007", load length is $c = 150$ *mm*. In series *EB V*, *EB VI* and *EB VII*, the girders of the same geometry are tested with $c = 50$ *mm*. This enables analysis of one more parameter – load length c – although it is not parameter of dominant influence.

For series *EB VIII, EB IX ... EB XX*, girder geometry is chosen as follows:

- Web thickness is $t_w = 3$, 4, 5 or 6 *mm*, in order to
  - enable connection of these series with the existing ones, i.e. to get several series with the same t_w and variable another parameter.
  - get girders with parameters a/t_w, h_w/t_w which correspond with real practice.
  - get ultimate load which corresponds with available equipment in our laboratory.

- Flange thickness t_f is chosen for each web thickness t_w, separately, in order to have at least 4 different t_f for each t_w and to get, for each t_w, values of parameter t_f/t_w in arithmetic progression.

The dimensionless parameter t_f/t_w is chosen as the most influential one, according to the previous researches. Apart from that, ratio $a/t_w = h_w/t_w$ will also be analysed, while ratio b_f/t_f, as a less important will not be treated.

2 MEASUREMENTS, METHOD AND COURSE OF THE INVESTIGATION

In all girders of all series the same measurements were done: the deflection of loaded flange and the deflection outside the web plane. All measurements were done in only one half of the girder, having in mind symmetry of girder, load and support conditions. Previous research experience approves this measurement concept.
The deflections of the flange were measured at 7 + 1 = 8 measuring points (*Fig. 2*). Seven points are on the same side of web as the load eccentricity. The eighth measuring point, that is intended to help evaluation of flange torsion, is on the opposite side of the web, in the central cross section of girder. A special arrangement, horizontal traveller designed for "Ekscentro 2001" (*Fig. 3b*), was used to perform measurings in 7 points on the loaded side of the flange. By means of this equipment it was possible to record deflections of all 7 points with only one electrical displacement transducer. The deflection of the eighth measuring point was measured by another electrical displacement transducer, which was fixed to the vertical holder, specially designed for this purpose (*Fig. 3c*).
The appearance and development of web deformations were observed by measuring the deflections outside the web plane. The deflections were recorded in 13 measuring posts along the vertical axis of symmetry in the girder midspan (*Fig. 2*). Only one electrical displacement transducer was used to record deflections of all 13 points. For guiding this transducer, the same frame-traveller as in the previous researches (in 1998 and 2001) was used (*Fig. 3d*).

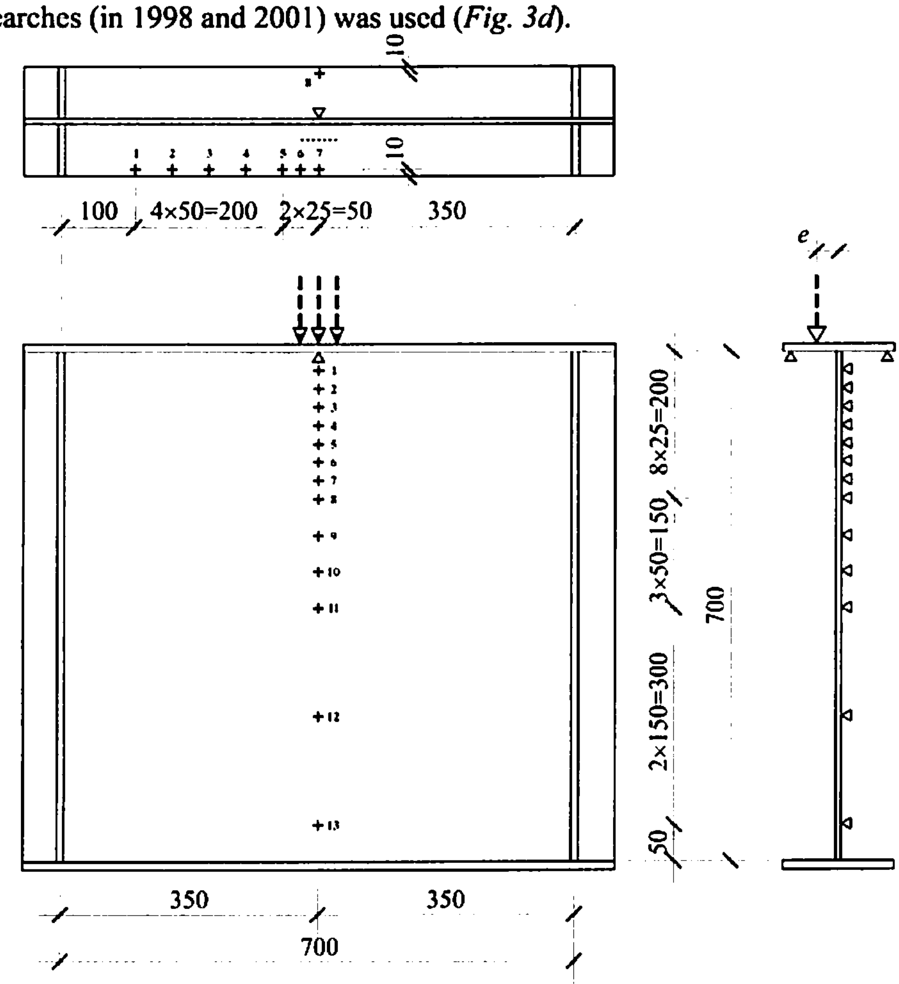


Figure 2. Deflection measuring points

Three digital displacement transducers, with range of 100 *mm* and accuracy of 1/50 *mm* were used. The intensity of loading was recorded by digital load cell with capacity of 1000 *kN* and accuracy of 0.3 *kN*. Connection of displacement transducers and load cell with data logger enables direct electronic record as well as hard copy of measuring data. The loading was increased by hydraulic pump and press with capacity of 800 *kN*.

All tests were done in a very stiff, closed steel frame (*Fig. 3a*). The loading was applied in increments: 20%, 40%, 60%, 70%, 80%, 90% and ≈100% of the estimated failure load. The failure load was estimated by means of mathematical model for failure load of centrically loaded girder [8], based on collapse mechanism, and by means of empirical expression for reduction factor of failure load in eccentrically loaded girder [10]. After each of the increments the deflection measuring were made while load was kept constant. In later increments, closer to collapse, the loading was kept constant for 2 to 5 minutes before reading the measuring, because of the possible deformation stabilisation. At the beginning, after the loading distribution apparatus had been initially pressed, measuring was made at the so-called zero (initial) increment. After reaching the failure load, residual deflections were recorded.

Because of the collapse being very localised near the load distribution block, it is possible to test each girder twice. Load is applied over one flange once, and the next time, after turning the girder up side down, over the other flange. This enabled usage of 51 girders to perform 102 tests.

Mechanical characteristics of steel were determined by tensile test. Three samples of each plate thickness were tested.

3 RESULTS OF "EKSCENTRO 2007"

As it was planned and expected, both collapse forms were observed in eccentrically loaded girders: the one typical of centrically loaded girders (*Fig. 4*) as well as that one typical of the eccentrically loaded girders (*Fig. 5*). However, the third type of collapse form, that might be defined as a combination of previously mentioned two types, was also observed. *Table 1* presents an overview of the tested girders with the experimental ultimate loads and the types of the collapse forms.

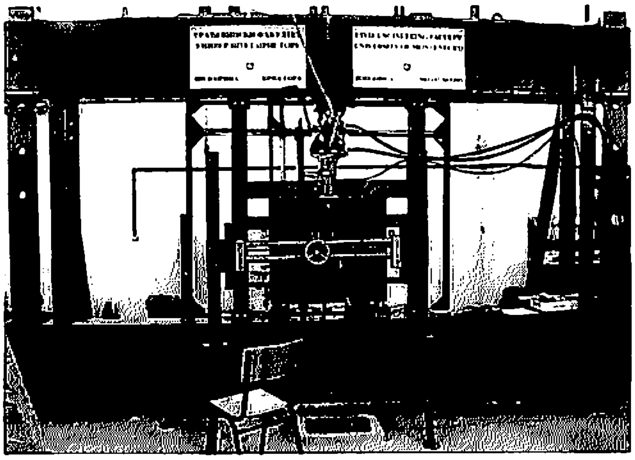
Apart from the load eccentricity, e, both dimensions, web thickness, t_w, and flange thickness, t_f, i.e. dimensionless parameter t_f/t_w greatly affects behaviour, collapse form and ultimate load of the eccentrically loaded girders. Influence of web slenderness, i.e. dimensionless parameter $a/t_w = h_w/t_w$ is also important. Actually, parameters t_f/t_w and $a/t_w = h_w/t_w$ should not be analysed separately. Both parameters together, i.e. their combination should be considered.

With an increase in t_f/t_w the possibility of centric collapse mode in eccentrically loaded girder also increases. This is obvious when series with the same web thickness, t_w, are compared. Only for $t_f/t_w = 1$ eccentric collapse mode happens at smallest eccentricity. As soon as $t_f/t_w > 1$, eccentrically loaded girders, until a certain level of load eccentricity, behave as if the load is centric. For higher values of ratio $a/t_w = h_w/t_w$, such behaviour is evident even for significant load eccentricity.

In case of centric collapse mode, ultimate load does not change as the load eccentricity rises. The failure load decreases as the load eccentricity increases in the cases of eccentric collapse form. Higher the ratios t_f/t_w and $a/t_w = h_w/t_w$, more rapid the decrease in ultimate load.

Although the load length, c, influences ultimate load (longer the load length, higher the ultimate load), this parameter is generally not of a great influence for the reduction of ultimate load with the increase of load eccentricity. However, there are some implications that the load length, c, might influence this reduction of ultimate load, for the combination of small ratio t_f/t_w and small load eccentricity, e.

The detailed processing of experimental data, considering not only results of "Ekscentro 2007", but also results of previous experimental work that make a whole with "Ekscentro 2007", is ongoing. It is expected to lead to other important and interesting conclusions concerning behaviour of eccentrically patch loaded steel I-girders, primarily occurrence of centric or eccentric collapse mode in eccentrically loaded girder. Endeavours will be made to give conclusions not only in descriptive form, but also in quantitative form. This refers particularly to the confident expression for valid calculation of ultimate load in eccentrically loaded girders.



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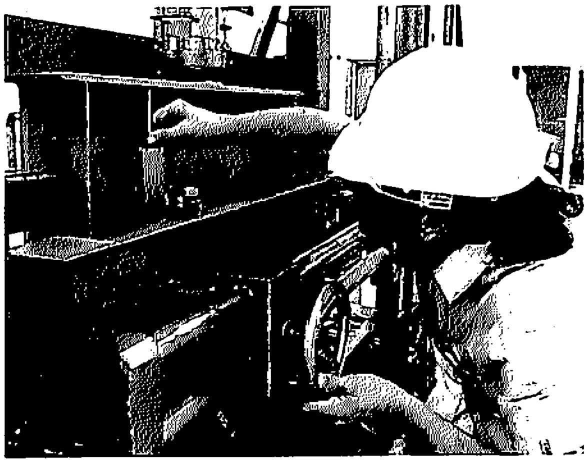

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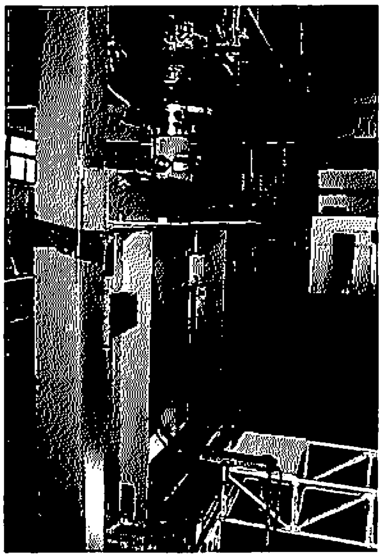

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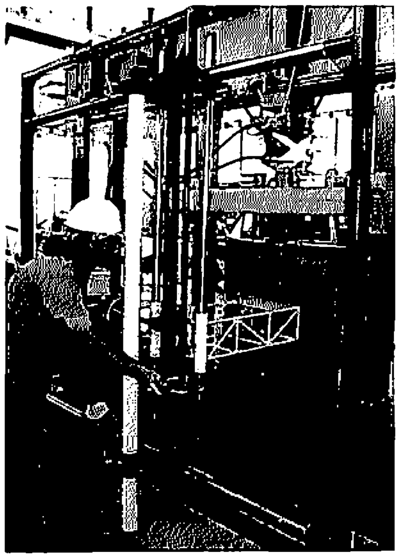

d/

Figure 3. Details of testing:

a/ General disposition (loaded test specimen in the stiff closed steel frame)

b/ Flange deflection measuring, points 1-7 (horizontal traveller)

c/ Flange deflection measuring, point 8 (vertical holder)

d/ Web deflection measuring (frame-traveller)

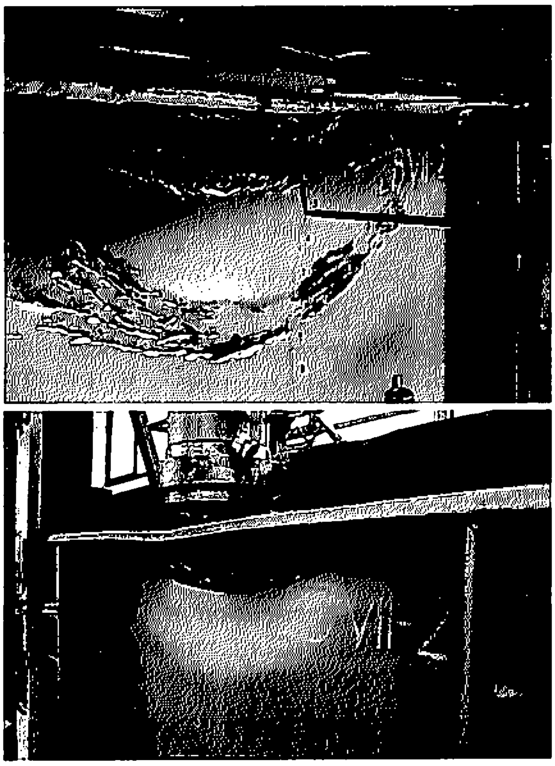

Figure 4. Centric collapse mode ($t_w = 5\ mm$, $t_f = 12\ mm$, $e = 5\ mm$)

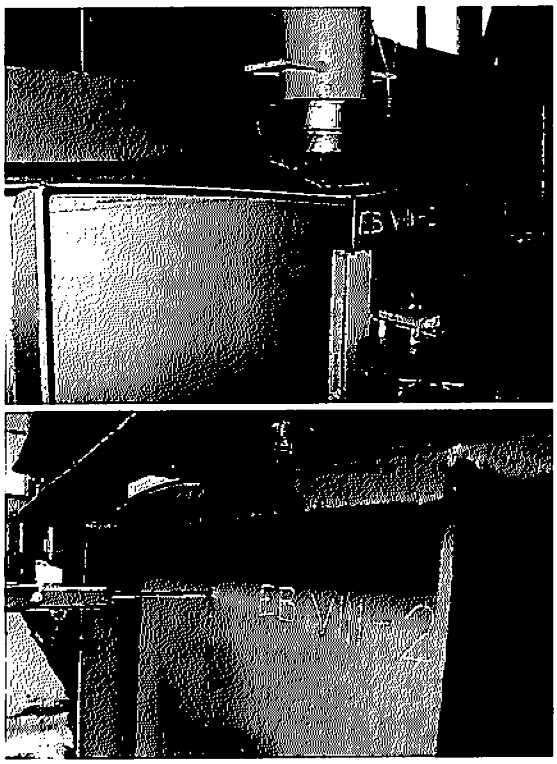

Figure 5. Eccentric collapse mode ($t_w = 3\ mm$, $t_f = 3\ mm$, $e = 5\ mm$)

Table 1. Girders geometry, collapse form and failure loads

No	SERIES	a	h_w	t_w	b_f	t_f	c	$P_{u,ex}$ [kN], TYPE OF COLLAPSE FORM: C – centric, E – eccentric, M – mixed					
		[mm]	[mm]	[mm]	[mm]	[mm]	[mm]	$e = 0$ mm	$e = 5$ mm	$e = 10$ mm	$e = 15$ mm	$e = 20$ mm	$e = 25$ mm
1	EB V	700	700	5	150	10	50	229	212 C/M	197 E	175 E	153 E	129 E
2	EB VI	700	700	10	150	10	50	720	575 E/M	365 E	313 E	275 E	220 E
3	EB VII	700	700	5	150	12	50	230	225 C	212 C/M	180 M	170 E	149 E
4	EB VIII	700	700	3	150	3	50	79	44 E	37 E	29 E	23 E	20 E
5	EB IX	700	700	3	150	6	50	95	80 C/M	69 E	57 E	47 E	39 E
6	EB X	700	700	3	150	9	50	102	105 C	107 C	90 C/M	85 E	70 E
7	EB XI	700	700	3	150	12	50	116	113 C	115 C	110 C	105 C/M	115 M
8	EB XII	700	700	4	150	4	50	120	70 E	50 E	45 E	40 E	35 E
9	EB XIII	700	700	4	150	6	50	125	110 E/M	86 E	68 E	50 E	45 E
10	EB XIV	700	700	4	150	8	50	140	129 C	130 C	100 E	86 E	75 E
11	EB XV	700	700	4	150	10	50	155	148 C	140 C	138 C/M	128 M	115 E
12	EB XVI	700	700	5	150	6	50	187	130 E	105 E	74 E	59 E	55 E
13	EB XVII	700	700	5	150	8	50	209	200 C/M	145 E	130 E	98 E	83 E
14	EB XVIII	700	700	6	150	6	50	208	170 E	130 E	104 E	88 E	69 E
15	EB XIX	700	700	6	150	9	50	330	285 E	217 E	155 E	125 E	107 E
16	EB XX	700	700	6	150	12	50	300	265 M	311 E	235 E	202 E	165 E
	EB XXI	700	700	5	150	10	150		250 C	240 C			
17	EB XXI	700	700	10	150	10	150		790 E	640 E			
	EB XXI	700	700	5	150	12	150		255 C	255 C			

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FEM MODELLING OF ECCENTRICALY PATCH LOADED STEEL I-GIRDERS

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Abstract: Patch loading acts locally, over a small area or length of a structural element. It is a common situation in structural engineering that local compressive load affects the flange of steel I-profile so that the web is compressed in the region below the applied load. Although some eccentricity of load relative to the web plane is unavoidable in engineering practice, rather modest amount of worldwide research work has treated this issue. While over 33 experimental researches dealt with I-girders patch loaded in the web plane, influence of load eccentricity was analysed in only 6 experimental studies. Experimental research shows that the behaviour and failure mode of the most of eccentrically loaded girders differ from those of centrically loaded girders. Parallel with the experimental researches, FEM modelling of eccentrically patch loaded steel I-girders has been developed. Various software was used at the universities in Maine, Montenegro and Spain.

Key words: patch loading, load eccentricity, steel I-girder, FEM modelling

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1. Introduction

Patch loading acts locally, over a small area or length of a structural element. It is a common situation in structural engineering that local compressive load affects the flange of steel I-profile so that the web is compressed in the region below the applied load. Examples are numerous and present in different structures, including crane and bridge girders.

Although some eccentricity of load relative to the web plane is unavoidable in engineering practice (Figure 1), rather modest amount of worldwide research work has treated this issue. While over 33 experimental researches dealt with I-girders patch loaded in the web plane, influence of load eccentricity was analysed in only 6 experimental studies. Experimental work started at the University of Maine in late 1980s [Elgaaly & Nunan 1989; Elgaaly & Salkar 1990]. At the same time some tests were done at the Institute of Theoretical and Applied Mechanics, Czech Academy of Sciences [Drdacky 1990]. Ten years later a new series of experiments were initiated at the University of Montenegro [Lučić 1999, 2001, 2003; Lučić & Šćepanović 2002, 2004, 2005; Šćepanović 2002; Šćepanović at all 2008]. Experimental research shows that the behaviour and failure mode of the most of eccentrically loaded girders differ from those of centrically loaded girders.

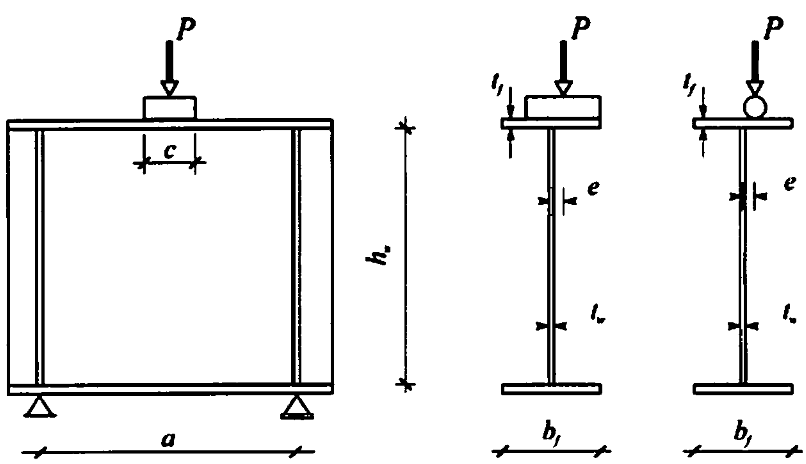

Figure 1: Eccentrically patch loaded I-girder

Parallel with the experimental researches, FEM modelling of eccentrically patch loaded steel I-girders has been developed. Various software was used at the universities in Maine, Montenegro and Spain.

2. FEM modelling at the University of Maine

The same as experimental work, FEM modelling was launched at the University of Maine, in late 1980s. Initially linear analysis was continued as non-linear, by means of software developed by Elgaaly, Caccese and Du. Only web panel was modelled. Flanges and web stiffeners were substituted with appropriate support conditions. Eccentric load was applied as a centric one in combination with a proper bending moment.

As a first step, linear finite element model was developed for comparison with the experimental results of Elgaaly and Nunan tests [Elgaaly & Nunan 1989]. One half of web panel model consisted of 150 rectangular flat plate finite elements with five degrees of freedom per node (drilling rotation omitted). Web panel was assumed to be simply supported perpendicular to its plane all around its edges. In-plane displacements were restrained in the vertical direction only at the two supported nodes, corresponding to the girder support points, i.e. web vertical stiffeners. Nodal loads and moments were applied along a finite length of the upper edge to simulate the eccentric patch loading. Two cases were considered: eccentric (nodal loads and moments) and corresponding centric load (only nodal loads). The FEM analysis results were compared with the test results obtained at the initial stages of loading, well before any nonlinearities, before large deflections or local yielding occurred.

Out-of-plane deflections at the vertical centreline of the web panel were compared. The measured deflections were consistently smaller than the corresponding calculated deflections. This is due to the assumption made in the FEM analysis that the flanges do not restrain the web from the rotation. In reality the flanges partially restrain the web from rotation. The measured deflections were consistently about 2/3 the calculated deflections.

In-plane deformations of FEM model were the same in cases of centric and eccentric load. Out-of-plane deflections appeared only in case of eccentric load. Calculated deflection shape was observed during the testing.

Above described linear FEM analysis by Elgaaly and Nunan was continued in more depth by Elgaaly and Sturgis [Sturgis 1989]. Both analysis, being linear in nature, were useful in studying web behaviour only before nonlinearities, i.e. well before girder collapse. Elgaaly and Salkar [Elgaaly & Salkar 1990] used a non-linear program developed by Elgaaly, Caccese and Du [Elgaaly at all 1989]. It uses a three-dimensional isoparametric doubly curved shell element, which is a degeneration of isoparametric hexahedron H20. The analysis combined the effects of large displacements and material nonlinearities. An updated Lagrangian method was used. The inelastic material behaviour was modelled based on Prandtl-Reuss flow theory of plasticity and the Von Mises yield criterion. Initially, the stability of the element and software was examined with respect to the load increments and the number of nodes in a model. Results of a model with 475 nodes and smallest load increment of 500 pounds (226.8 *kg*) were considered. The validity of FEM analysis was established by comparing analytically calculated and experimentally obtained values of collapse load. Experimental collapse loads were consistently higher than calculated ones. The difference was up to 10 %. Further, a detailed comparison was made between the strains and displacements calculated by FEM model and those observed from the experiment. Out-of-plane web displacements from tests were higher than calculated displacements.

3. FEM modelling at the University of Montenegro

Experimental research "Ekscentro 2001", organised in 2001, as the second one in series of patch loading researches at the University of Montenegro, was followed by problem modelling with finite element method, by means of computer software SAP 2000, NonLinear Version 6.11 [Lučić & Šćepanović 2005; Šćepanović 2002]. The idea was to compare experimental and theoretical results in order to obtain more detailed picture of stress-strain state in girder as well as to assess the capability of this commercial software, widely used in engineering practice, to be used for modelling such problem. Only linear analysis was done.

Full 3D model (flanges + web + stiffeners + load positioned where it really acts = copy of real girder) was made in few variations. For the comparative analysis one model with shell elements ("full shell behaviour", 6 degrees of freedom per node, approx. 94000 d.o.f.) and one model with solid elements (2 layers of solid elements in flange, only one layer in web; "incompatible bending modes", 3 degrees of freedom per node, approx. 103000 d.o.f.) were chosen, Figure 2.

Comparative analysis of results for stresses in central cross section of girder was done. At first, comparison of different SAP models was done. Afterwards, SAP results were compared with experimental results. Figure 3 shows strain gauges whose readings were used. All comparisons were done for web stress component in direction 2, flange stress component in direction 1, membrane and bending stresses in flange and web. Stresses were compared for different load levels, until the appearance of (theoretical) yielding in the girder. As an example, Figures 4-7 show graphical comparison of results for one tested girder.

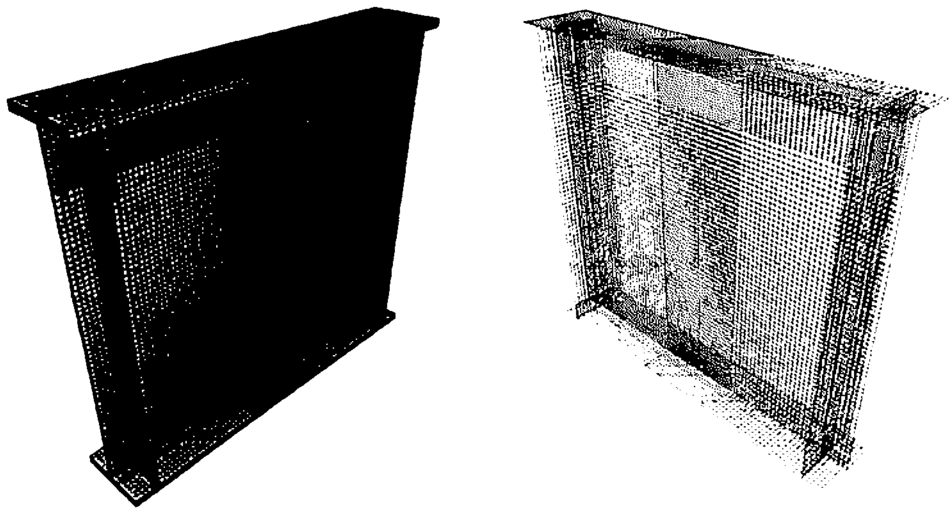

Figure 2: SAP 2000 models with solid and shell finite elements

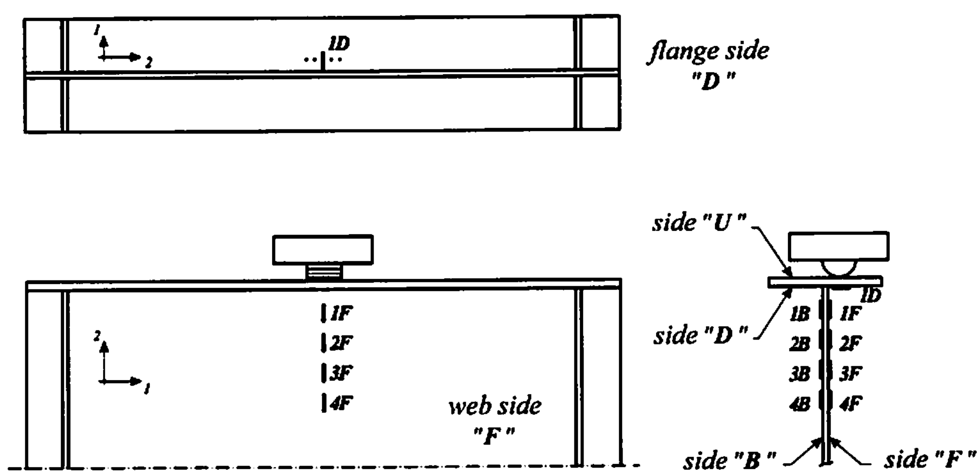

Figure 3: Strain gauges in central cross section

Shell and solid elements gave almost the same results for web stresses. It was not the case with flange stresses. For more precise, more real stress state, flange should be modelled with more than two layers of solid elements.

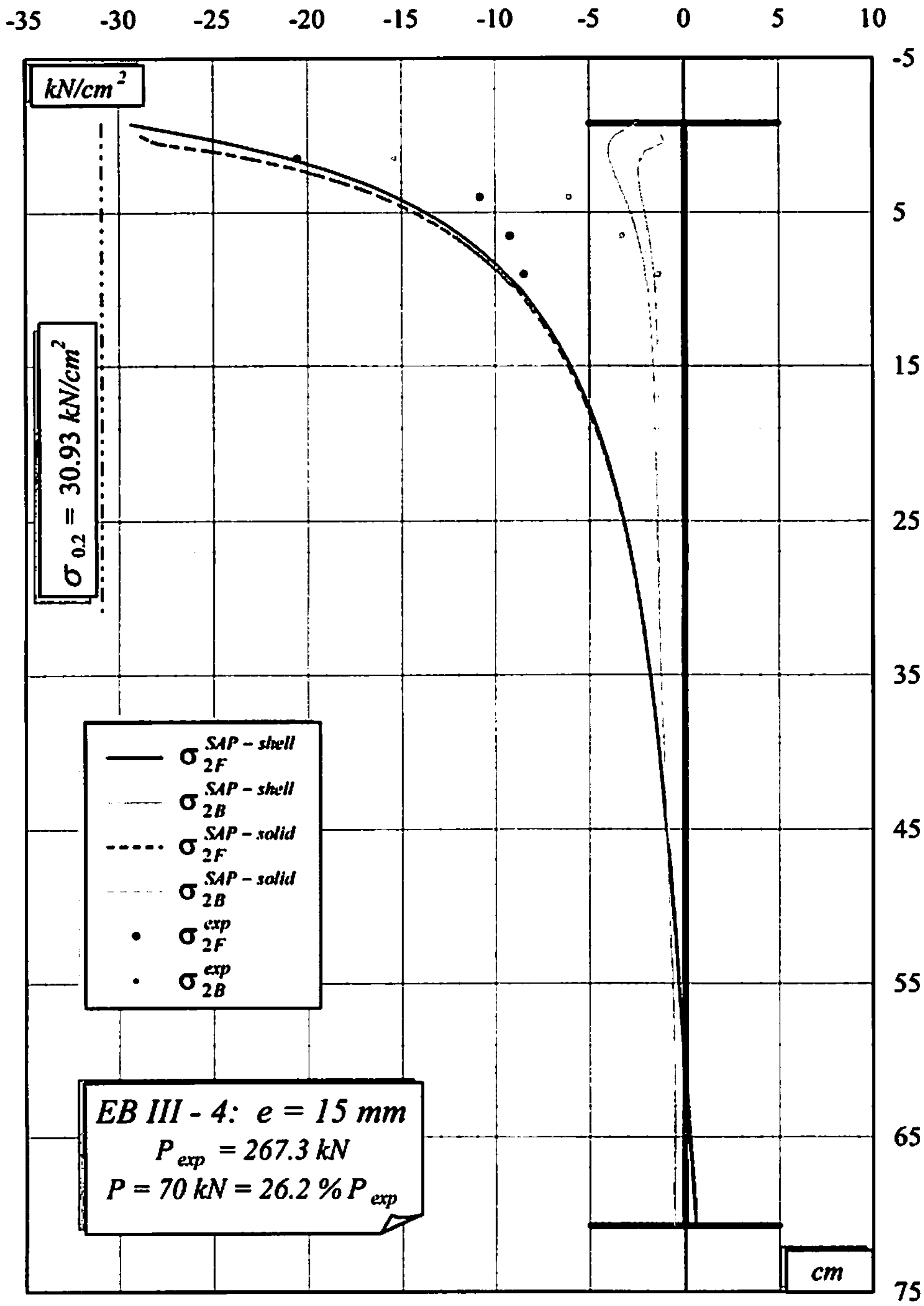

Figure 4: Comparison of results for girder EBIII - 4: web stress component σ_2

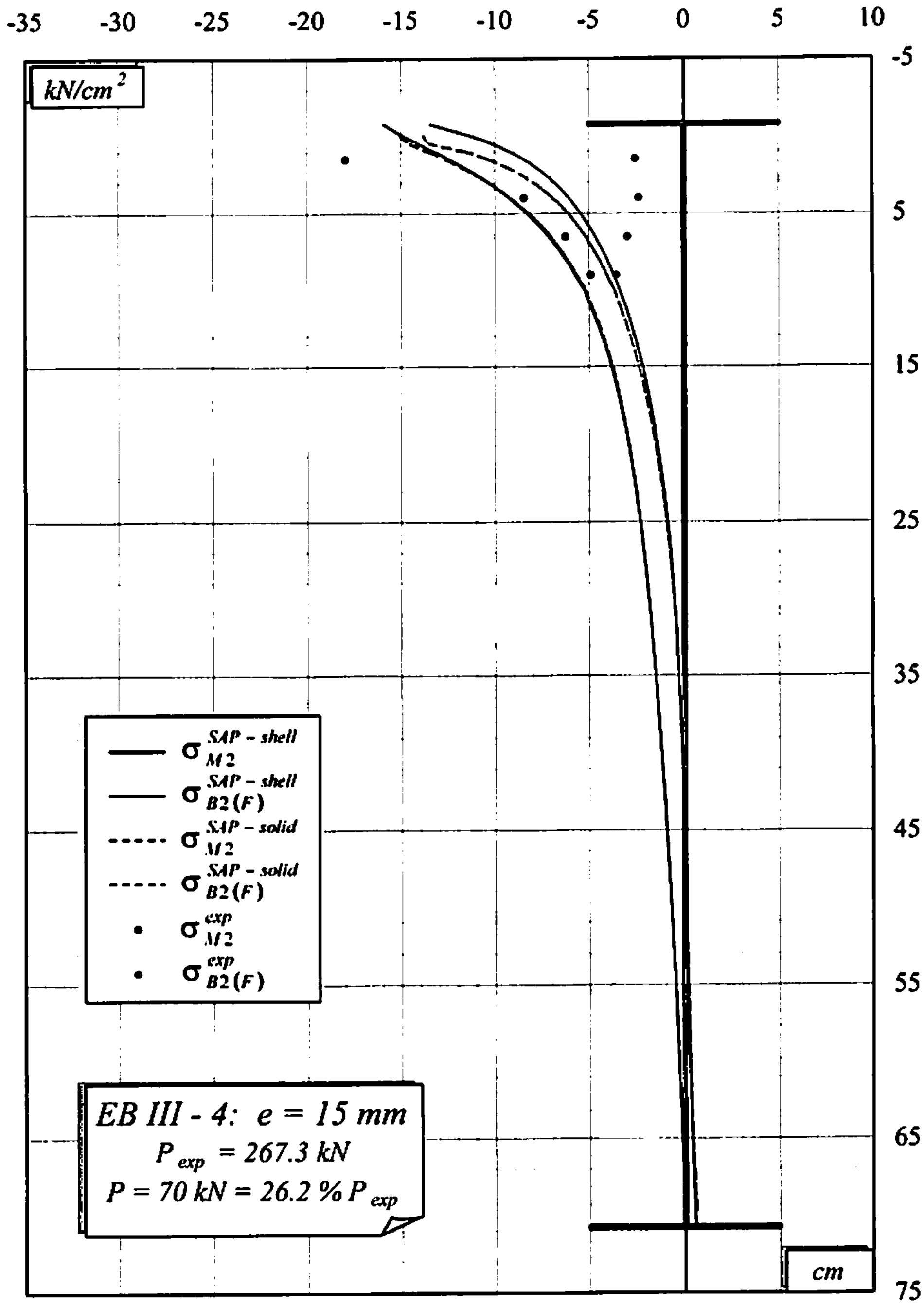

Figure 5: Comparison of results for girder EBIII - 4: membrane and bending web stresses

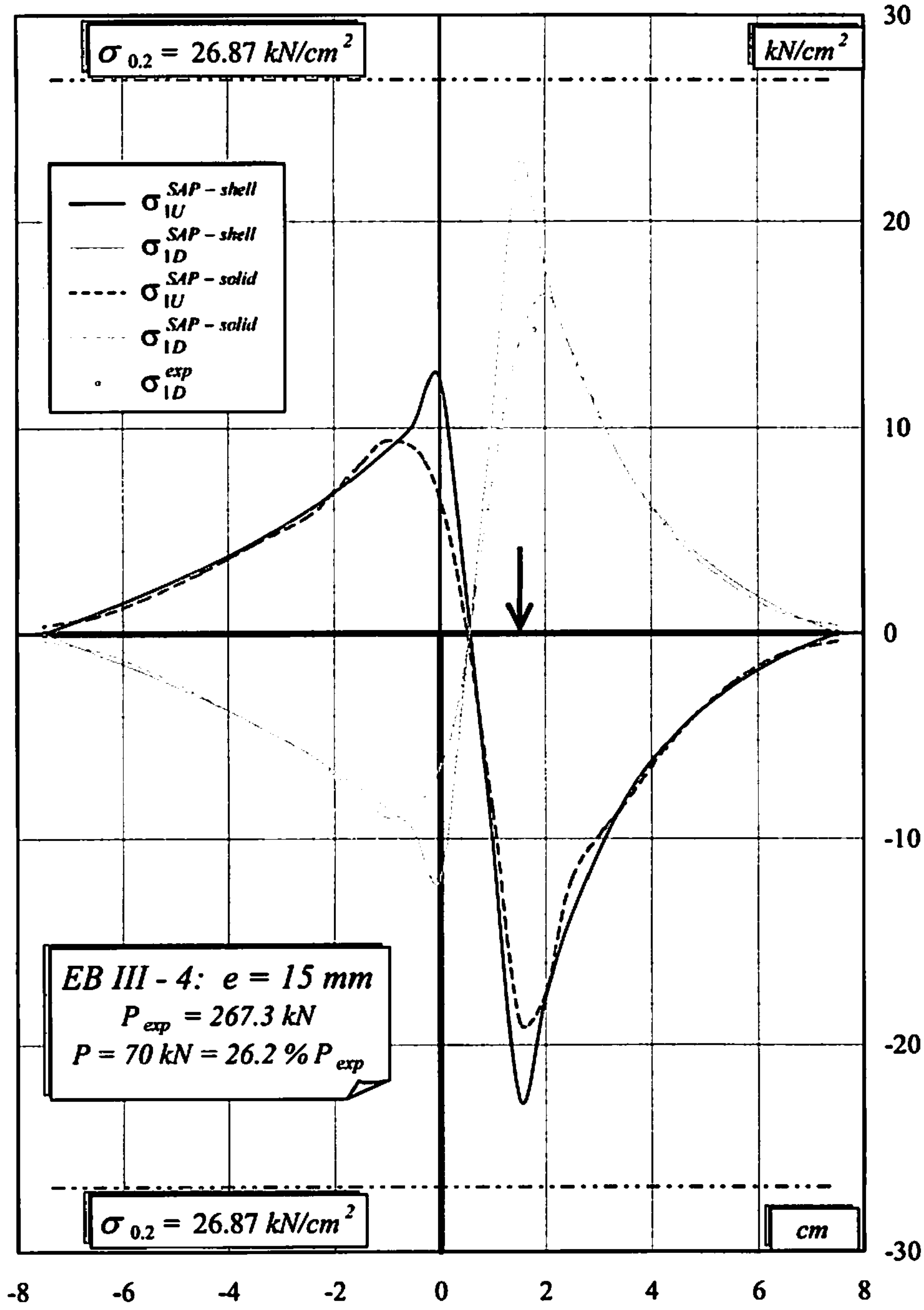

Figure 6: Comparison of results for girder EBIII - 4: flange stress component σ_l

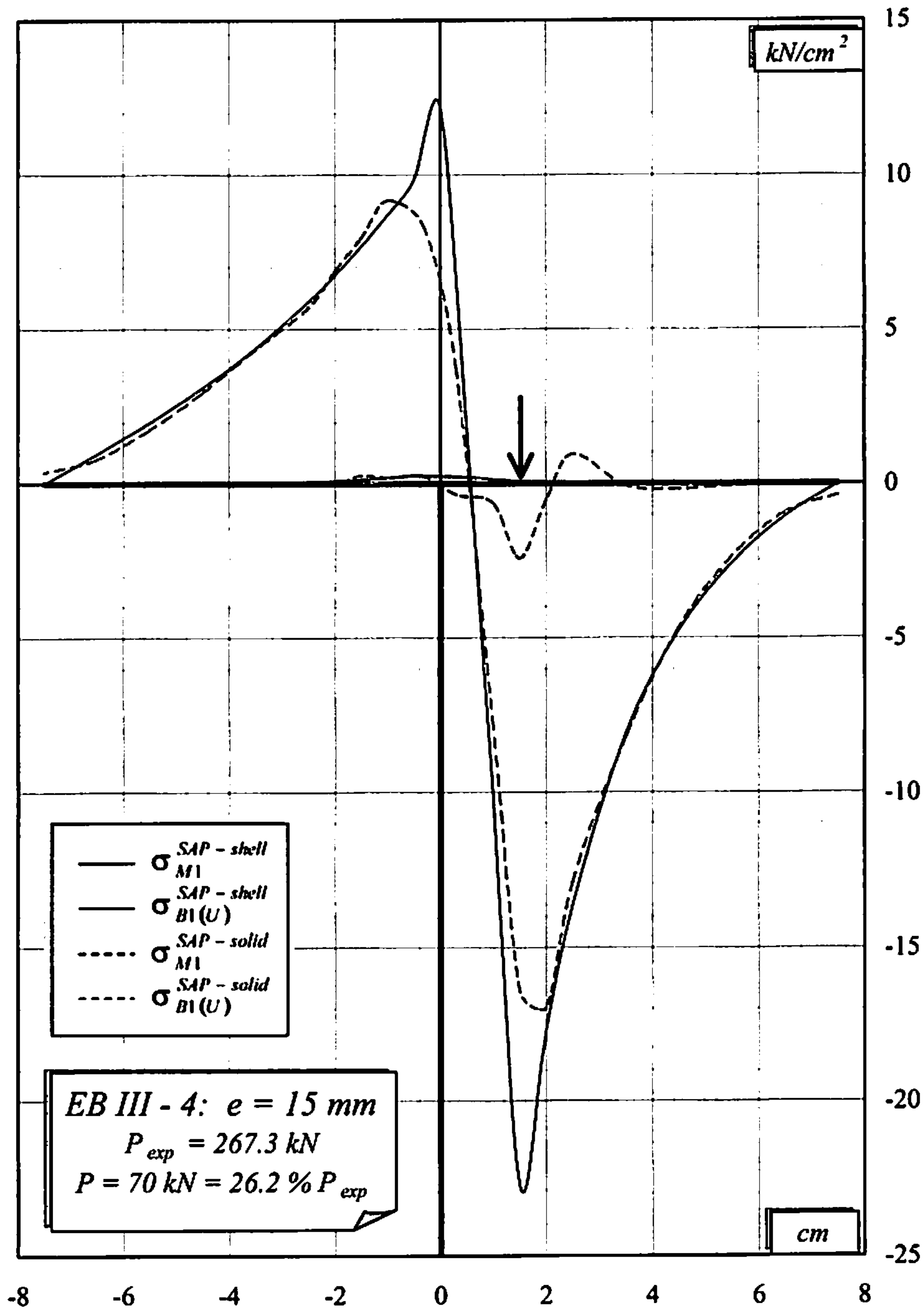

Figure 7: Comparison of results for girder EBIII - 4: membrane and bending flange stress

SAP results shows that for $e > 10$ *mm* flange does not behave like cantilever beam (Figure 8c), but more like systems in Figures 8a, 8b. This is not quite valid for $e = 5$ *mm*.

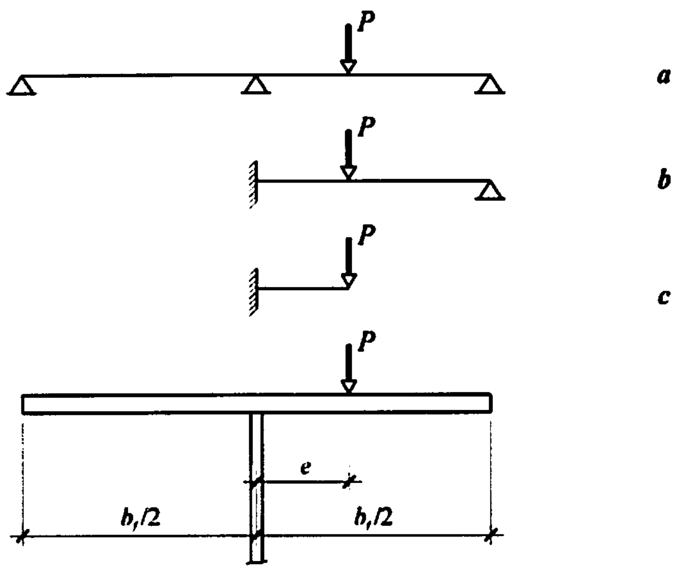

Figure 8: Different static systems of loaded flange

Analysis of SAP web stress σ_2, in the case of centrically loaded girder, shows that the angle of load distribution is not 45°, but smaller (≈ 35°). The load distribution angle slightly grows as the distance from loaded flange grows (Figure 9).

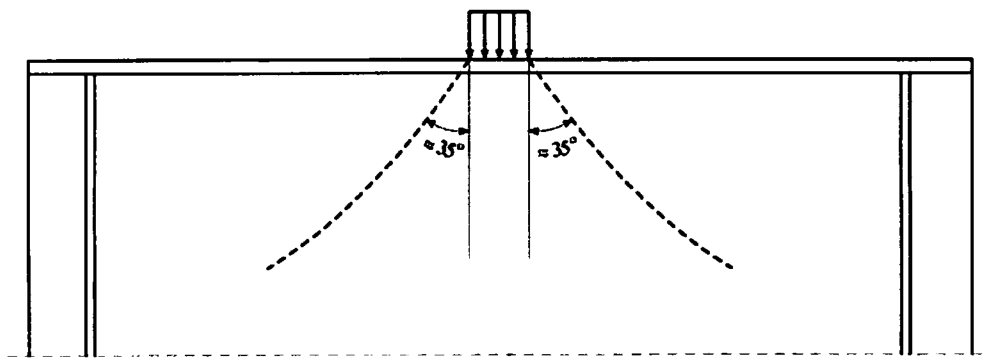

Figure 9: Load distribution in web of centrically loaded girder

SAP stresses σ_1 in flange are constantly higher than experimental values. This discrepancy rises as load eccentricity rises.

In general, experimental and SAP values of stresses σ_2 in web are almost the same. A certain discrepancy appears only locally, near the loaded flange.

4. FEM modelling at the University of Granada

Patch loading research teams from the University of Montenegro and the University of Granada have established a cooperation concerning some issues of the eccentric patch loading research. Among the other, joint work aims expression for ultimate strength reduction coefficient, which relates collapse loads of eccentrically patch loaded girder and corresponding centrically loaded girder [Šćepanović at all 2008]. Nonlinear FEM modelling is currently ongoing at the University of Granada, Spain, by means of ANSYS software.

As a starting point, the response of test specimens from the experimental research "Ekscentro 2001" (University of Montenegro) was simulated using the ANSYS program. Full 3D models (flanges + web + stiffeners) were made. An initial out-of-plane unstressed configuration was established by deforming the geometry of the web by 5 *mm* (without inducing stress) to represent an initial imperfection. The SOLID92 (®) element was employed to model web, flanges and vertical stiffeners. This element is a ten-node three-dimensional element having three degrees of freedom at each node (translations in the x, y and z directions). Plasticity, large-deflection and large-strain capabilities were used. Residual stresses were not considered in the FE-models. Material nonlinearities were represented by using a nonlinear stress-strain relationship, while plasticity was represented using a kinematic hardening rule. Approximately 5000 elements were used for each 3D model of a test specimen. Due to the symmetry in geometry, load conditions and expected deformation, just one half of each girder was modelled (Figure 10). The patch load was transferred into the girder at the top of upper flange by controlling the displacements of the patch nodes in the FE analysis.

The numerical analysis was done with two steel models: an idealized version of European S235 steel (having an ideal yield stress of 235 *MPa*) and an idealized version of the behaviour of the actual steel used in the experiment (yield stress of 327 *MPa*).

Validation of FEM models was established by comparison with experimental results. The FEM determined ultimate strength reduction coefficient differed less than 3 % from the experimentally obtained values of reduction coefficient. The steel model had a negligible effect on ultimate strength reduction coefficient.

An expanded FEM study is going to be done in order to evaluate the parametric dependency of the ultimate strength reduction coefficient.

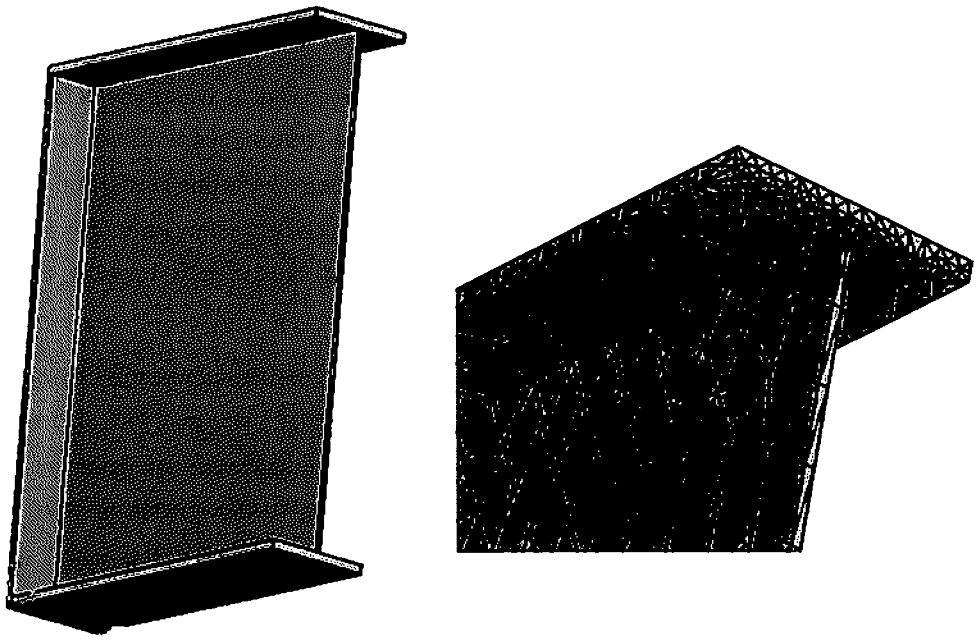

Figure 10: ANSYS finite element model

5. Conclusion

Parallel with the experimental researches, FEM modelling of eccentrically patch loaded steel I-girders has been developed. Various software was used at the universities in Maine, Montenegro and Spain. Both, linear and non-linear analysis were performed.

Linear FEM modelling might describe girder's behaviour successfully, but only at the initial stages of loading, well before appearance of any nonlinearity. The fact is that most of girders in engineering practice remain in that domain. However, linear analysis does not provide any answer regarding girder's collapse.

Eccentrically patch loaded girder's collapse mode and collapse load might be analysed only by means of non-linear FEM modelling. Recent results of modelling by means of ANSYS software imply successful implementation of non-linear FEM modelling in the analysis of eccentrically patch loaded steel I-girders.

Further work in domain of non-linear FEM analysis is ongoing.

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Ultimate capacity of eccentrically patch loaded steel I-girders

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ABSTRACT: Behaviour and collapse mode of most eccentrically patch loaded steel I-girders differ from case of centric patch load. Reduction in ultimate capacity with an increase in load eccentricity is obvious in case of eccentric collapse mode. Still, some eccentrically loaded girders, with certain girder geometry, behave as if there is no load eccentricity, having centric collapse mode, with no reduction in ultimate capacity. The paper presents mathematical models, i.e. calculation procedures for ultimate load capacity of eccentrically patch loaded steel I-girders, based on experimental research. Both cases are considered: eccentrically loaded girders with eccentric, but also with centric collapse mode. Suggested calculation procedures are intended to improve current standards and Eurocodes.

1 INTRODUCTION

Patch loading acts locally, over a small area or length of a structural element. It is a common situation in structural engineering that local compressive load affects the flange of steel I-profile so that the web is compressed below the applied load, Figure 1. Local stresses in web might cause local instability that may provoke element carrying capacity loss and, consequently, collapse of the whole structure.

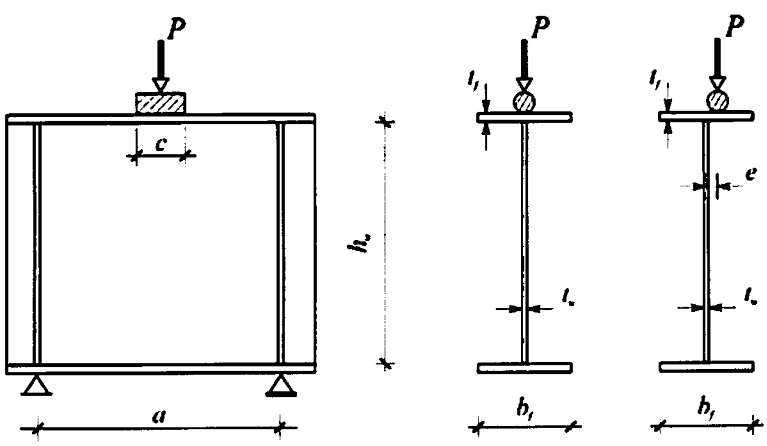


Figure 1. Patch loaded I-girder

This is a rather complex and challenging issue of extremely evident elastic-plastic stresses and deformations. Apart from that, geometrical non-linearity is noticeable even at the lowest loading level. Patch loading examples are numerous and present in different structures, including crane girders loaded by crane wheels or bridge girders erected by launching.

In spite of a large number of worldwide researches in patch loading domain, there are still a number of problems to be looked into. Particularly intriguing is issue of eccentric patch loading.

Although some eccentricity of load relative to the web plane is unavoidable in engineering practice, rather modest amount of worldwide research work has treated this issue. While over 35 experimental researches dealt with I-girders patch loaded in the web plane, influence of load eccentricity was analysed in only 7 experimental studies. Experimental work started at the University of Maine in late 1980s (Elgaaly & Nunan 1989, Elgaaly & Salkar 1990). At the same time some tests were done at the Institute of Theoretical and Applied Mechanics, Czech Academy of Sciences (Drdacky 1989). Ten years later a new series of experiments were initiated at the University of Montenegro (Lučić 2001, Lučić & Šćepanović 2002/2004, Šćepanović 2002, Šćepanović et al. 2008). Experimental work was followed by finite element method modelling, by means of various computer software (Elgaaly & Nunan 1989, Elgaaly & Salkar 1990, Šćepanović 2002, Šćepanović et al. 2009). While over 28 mathematical expressions for centric ultimate load have been proposed, only few empirical expressions for eccentric ultimate load might be found in literature: the original one, based on the earliest experiments from 1980s (Galambos 1998) and its modifications, based on the recent experiments from last decade (Šćepanović et al. 2009). Artificial neural networks are also used nowadays for failure load estimation (Knežević et al. 2006).

According to the experimental researches, numerous parameters influence the behaviour, collapse mode and ultimate load of eccentrically patch loaded steel I-girders: geometric parameters (girder's dimensions and their dimensionless ratios), load eccentricity and the manner of load application (line or laterally distributed load). Dominant parameter is the load eccentricity, e, or ratio e/b_f. Apart from this parameter, the influence of girder geometry parameters should be studied. Girder dimensions, primarily web and flange thicknesses, t_w and t_f, as well as ratio t_f/t_w are of important influence. Other ratios, like b_f/t_f, a/t_w, h_w/t_w should be considered as well. Attention should also be paid to the load length, c, or ratio c/a and to the load application manner. Influential parameters should not be considered separately. Combinations of certain parameters should be carefully analysed. However, the exact influence neither of these parameters separately nor of their combinations is completely defined.

It is evident that most (but not all!) eccentrically loaded girders have a collapse mode quite different from that of centrically loaded girders, Figure 2. Carrying capacity loss in the case of centric load is due to web buckling and local stability loss. In the case of eccentric load, girders mostly lose carrying capacity due to local elastic-plastic bending. However, eccentrically loaded girders under certain circumstances behave as if loaded in the web plane. It has not yet been completely defined for what combination of influential parameters the eccentrically loaded girders have the same collapse form as the one of centrically loaded girders.

The reduction in ultimate load with an increase in load eccentricity is obvious in girders with eccentric collapse mode. This decrease in the ultimate load is quantified by a reduction factor, R, which relates the ultimate load of eccentrically loaded girders to the ultimate load of centrically loaded girders. The reduction factor might be calculated from empirical expressions which are obtained by numerical analysis of experimental and FEM modelling data. Consequently, the application of empirical expressions is limited by the range of test data.

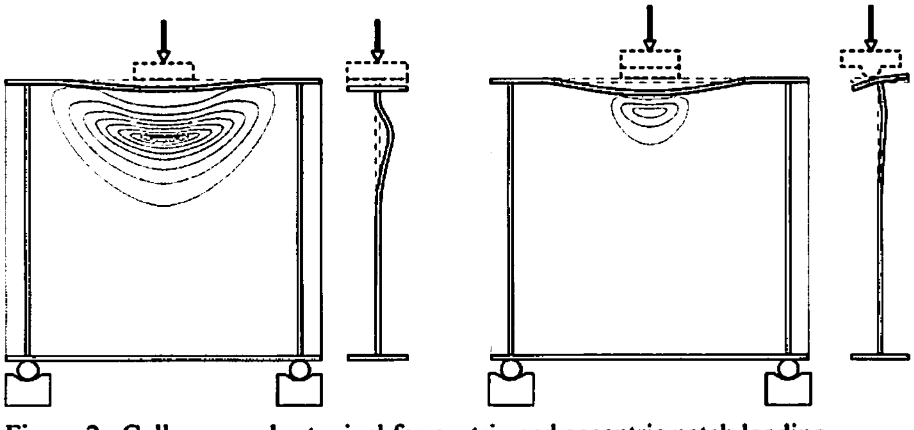


Figure 2. Collapse modes typical for centric and eccentric patch loading

2 COLLAPSE MODES

Experimental researches from the last decade (Lučić 2001, Lučić & Šćepanović 2002/2004, Šćepanović 2002, Šćepanović et al. 2008) demonstrate that both collapse modes occur in eccentrically loaded girders: not only the collapse mode typical for eccentric load (so-called "eccentric collapse mode"), but also the collapse mode typical for centric load, i.e. for load in the web plane (so-called "centric collapse mode"), Figure 2. Occurrence of a certain collapse mode depends on load eccentricity and girder geometry. The most influential geometry parameter is ratio t_f/t_w. Influence of web slenderness, i.e. dimensionless parameter $a/t_w = h_w/t_w$ is also important. Actually, parameters t_f/t_w and $a/t_w = h_w/t_w$ should not be analysed separately. Both parameters together, i.e. their combination should be considered. Furthermore, correlation with the eccentricity ratio e/b_f should be established. In the last experimental testing, "Ekscentro 2007" (Šćepanović et al. 2008), the third type of collapse mode, that might be defined as a combination of previously mentioned centric and eccentric modes (so-called "mixed collapse mode"), was also observed. However, it has been still studied and therefore is not a subject of this paper.
Table 1 summarises the experimental data of tests carried out at the University of Montenegro (Lučić 2001, Lučić & Šćepanović 2002/2004, Šćepanović 2002, Šćepanović et al. 2008), in last decade. Girders geometry, load eccentricities, ultimate loads and collapse modes are presented.

Table 1. Experimental data of eccentrically patch loaded I-girders (University of Montenegro experiments)

series	*a*	h_w	t_w	b_f	t_f	*c*	$P_{u,ex}$ *[kN]; type of collapse mode: C – centric, E – eccentric, M – mixed*					
	[mm]	*[mm]*	*[mm]*	*[mm]*	*[mm]*	*[mm]*	*e = 0 mm*	*e = 5 mm*	*e = 10 mm*	*e = 15 mm*	*e = 20 mm*	*e = 25 mm*
EB I	700	700	3	150	15	50	133	128 C	127 C	135 C	134 C	124 C
EB II	700	700	6	150	15	50	340	320 C	326 C	296 E	243 E	197 E
EB III	700	700	6	150	15	50	342	321 C	301 C	267 E	228 E	187 E
EB IV	700	700	8	150	15	50	401	418 C	394 E	301 E	245 E	209 E
EB V	700	700	5	150	10	50	229	212 C/M	197 E	175 E	153 E	129 E
EB VI	700	700	10	150	10	50	720	575 E/M	365 E	313 E	275 E	220 E
EB VII	700	700	5	150	12	50	230	225 C	212 C/M	180 M	170 E	149 E
EB VIII	700	700	3	150	3	50	79	44 E	37 E	29 E	23 E	20 E
EB IX	700	700	3	150	6	50	95	80 C/M	69 E	57 E	47 E	39 E
EB X	700	700	3	150	9	50	102	105 C	107 C	90 C/M	85 E	70 E
EB XI	700	700	3	150	12	50	116	113 C	115 C	110 C	105 C/M	115 M
EB XII	700	700	4	150	4	50	120	70 E	50 E	45 E	40 E	35 E
EB XIII	700	700	4	150	6	50	125	110 E/M	86 E	68 E	50 E	45 E
EB XIV	700	700	4	150	8	50	140	129 C	130 C	100 E	86 E	75 E
EB XV	700	700	4	150	10	50	155	148 C	140 C	138 C/M	128 M	115 E
EB XVI	700	700	5	150	6	50	187	130 E	105 E	74 E	59 E	55 E
EB XVII	700	700	5	150	8	50	209	200 C/M	145 E	130 E	98 E	83 E
EB XVIII	700	700	6	150	6	50	208	170 E	130 E	104 E	88 E	69 E
EB XIX	700	700	6	150	9	50	330	285 E	217 E	155 E	125 E	107 E
EB XX	700	700	6	150	12	50	300	265 M	311 E	235 E	202 E	165 E
EB XXI + B III 1	700	700	5	150	10	150	260	250 C	240 C	202 E	171 E	141 E
EB XXI + B III 2	700	700	10	150	10	150	874	790 E	640 E	387 E	297 E	254 E
EB XXI + B III 3	700	700	5	150	12	150	266	255 C	255 C	228 E	177 E	162 E

The smaller the eccentricity ratio e/b_f and the larger the t_f/t_w ratio (i.e. thinner web relative to flange), the more likely a centric collapse mode would develop in girders having eccentric patch loading. This is obvious when series with the same web thickness, t_w, are compared (e.g. $t_w = 3$ mm: EB VIII, EB IX, EB X, EB XI & EB I; $t_w = 4$ mm: EB XII, EB XIII, EB XIV & EB XV; $t_w = 5$ mm: EB XVI, EB XVII, EB V & EB VII; $t_w = 6$ mm: EB XVIII, EB XIX, EB XX, EB II & EB III). Only for $t_f/t_w = 1(1.2)$ eccentric collapse mode happens at smallest eccentricity. As soon as $t_f/t_w > 1(1.2)$, eccentrically loaded girders, until a certain level of load eccentricity, behave as if the load is centric. For higher values of ratio $a/t_w = h_w/t_w$, such behaviour is evident even for significant load eccentricity. The centric collapse mode occurs with eccentricities: $e/b_f \leq 1/30$ at $t_f/t_w \leq 2$; $e/b_f \leq 1/15$ at $t_f/t_w = 2 \div 2.5$; $e/b_f \leq 1/10$ at $t_f/t_w = 3$; $e/b_f \leq 1/6$ at $t_f/t_w = 4 \div 5$.
Having in mind common values of ratio $t_f/t_w \leq 2$ in engineering practice, it might be said that even small eccentricities, from $e/b_f \geq 1/30$, will lead to eccentric collapse mode, in general, which means reduction of ultimate load.

2.1 *Centric collapse mode*

During the process of load increase, both flange and web deformations are insignificant until the collapse, which happens suddenly. The loaded flange remains almost completely horizontal until the moment of collapse, when it moves downwards (below the loading), due to web crippling and/or buckling. Flange simply sinks, as it follows collapse deformation of web. A considerable web squashing is obvious in girders with thicker webs (t_w = 8 and 10 *mm*). The thicker the web, the more evident the squashing (yielding) becomes, before reaching failure load. However, apart from the obvious web squashing under the loading, the girder collapse is characterised by the occurrence of a crippling in the web. The noticeable web buckle is characterised by two yielding lines, Figures 3, 4. The upper line slightly arches the load distribution block. This line divides the upper part of the web, which remains flat and perpendicular to the flange, from the lower part where the buckling is evident. The lower yielding line outlines the buckling form. Both web thickness and flange stiffness influence the web buckling form. The stiffer the flange is, the closer (to the loaded flange) the upper yielding line gets, whereas the lower yielding line goes deeper into the web. The thicker the web is, the smaller the crippling area becomes (including both the width and the depth).

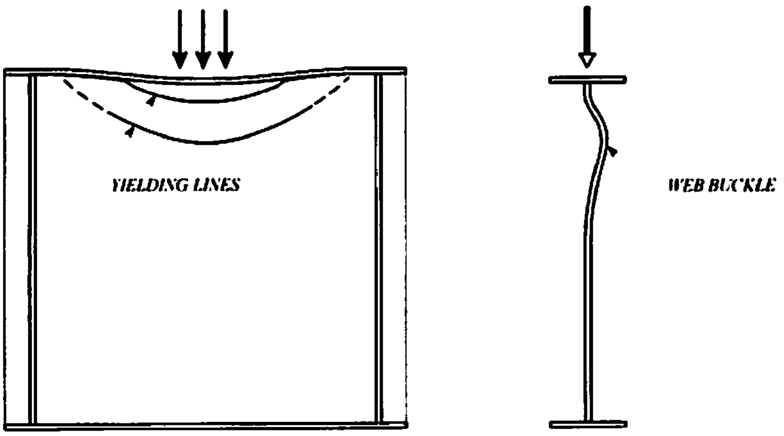


Figure 3. Centric collapse mode

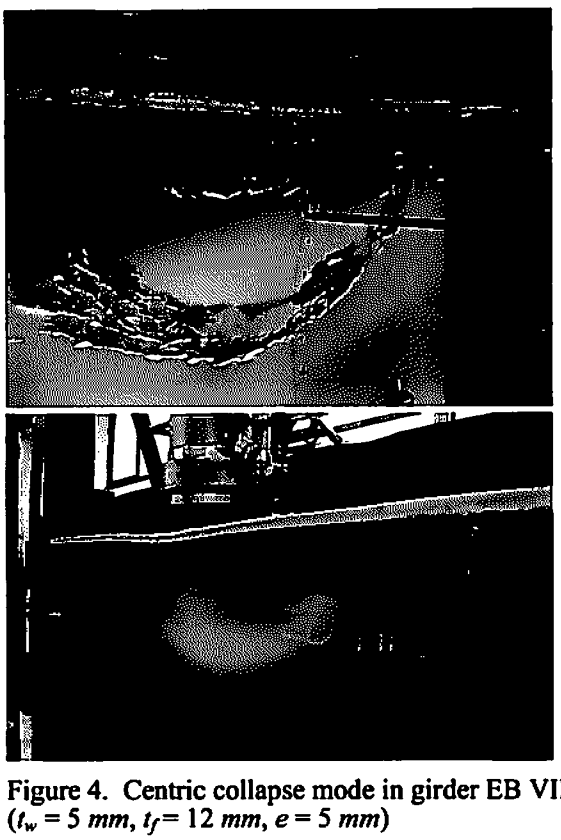


Figure 4. Centric collapse mode in girder EB VII-2 ($t_w = 5$ *mm*, $t_f = 12$ *mm*, $e = 5$ *mm*)

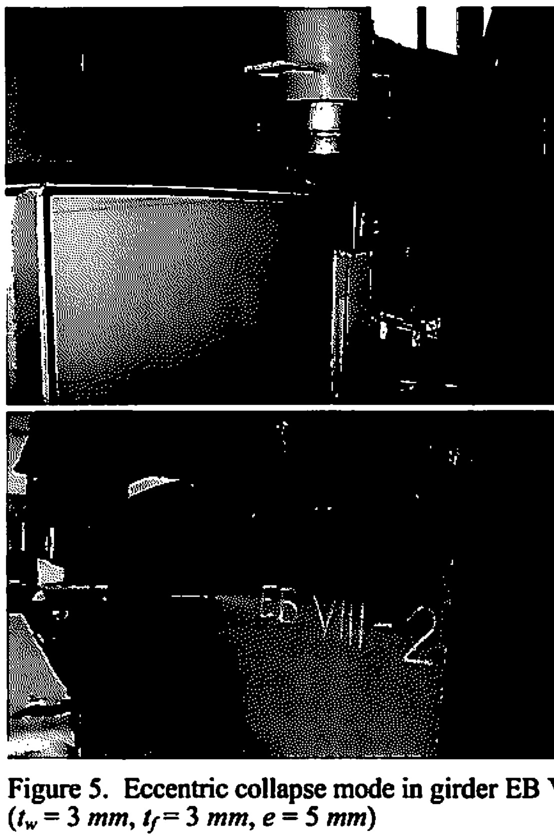
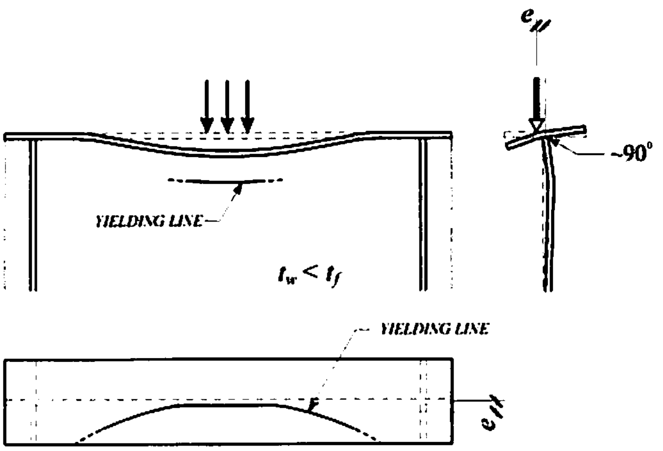


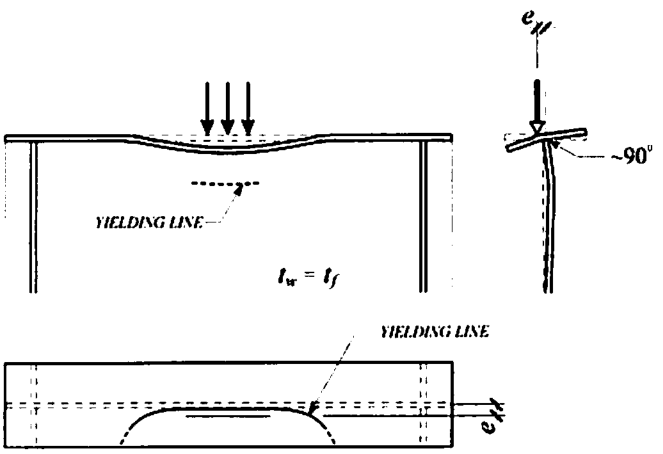
Figure 5. Eccentric collapse mode in girder EB VIII-2 ($t_w = 3$ *mm*, $t_f = 3$ *mm*, $e = 5$ *mm*)

2.2 *Eccentric collapse mode*

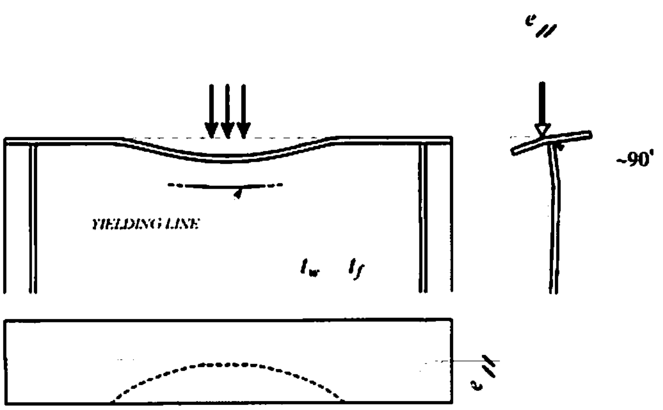
During the process of load increase, the most evident deformation is flange warping accompanied by gentle web bending that follows the flange deformation, due to flange-web constraining. The web thickness and the flange stiffness influence the extent of flange warping. The larger the eccentricity, the more pronounced the deformation is, especially in flange. Flange warping and web bending grow until the final collapse of girder. A yielding line appears in the flange and surrounds the load, forming an arch, Figures 5, 6. The length of arch chord primarily depends on the extent of eccentricity, and secondarily on the ratio t_f/t_w. As the eccentricity increases, the arch chord decreases almost linearly. The smaller the ratio t_f/t_w, the "shorter" the chord, i.e. the "shaper" the arch, Figures 6 a, b. In girders with rather thick flange, yielding line in flange is not clearly visible, i.e. it is not totally developed, Figure 6 c. Apart from yielding line in the flange, there is a horizontal yielding line in the web. In cases of girders with thicker web this yielding line is far less evident, almost invisible, Figure 6 b. Loading eccentricity influences the depth of yielding line in web, i.e. its distance from the loaded flange. The greater the eccentricity, the closer the yielding line gets to the loaded flange. The change of t_f/t_w ratio does not affect the depth of web yielding line appearance.



a/ In girders with $t_w < t_f$ and $b_f/t_f \geq 12.5$, both yielding lines, in flange and in web, are clearly visible.



b/ In girders with $t_w = t_f$ and $b_f/t_f \geq 12.5$, yielding line in web is almost invisible.



c/ In girders with thick flange ($b_f/t_f = 10$), yielding line in flange is not clearly visible, i.e. it is not totally developed.

Figure 6. Eccentric collapse mode

3 ULTIMATE LOAD

In case of centric collapse mode in eccentrically loaded girders, ultimate load, P_u, does not change significantly with an increase in the load eccentricity, *e*. In case of eccentric collapse mode, ultimate load depends on eccentricity and reduces approximately linearly with an increase in eccentricity. The reduction is more emphasised for smaller ratio t_f/t_w and higher ratio $a/t_w = h_w/t_w$.
Although the load length, *c*, influences ultimate load (longer the load length, higher the ultimate load), this parameter is generally not of a great influence for the reduction of ultimate load with the increase of load eccentricity. However, there are some implications that the load length might influence this reduction of ultimate load, for the combination of low ratio t_f/t_w and small eccentricity.

3.1 *Ultimate load in case of centric collapse mode*

Ultimate load of eccentrically patch loaded girder in case of centric collapse mode almost does not differ from ultimate load of corresponding girder loaded in the web plane. Hence, any procedure for calculation of ultimate load of I-girder under centric patch loading might be used. Over 28 mathematical models might be found in literature. Herein, the model proposed by the paper authors (Aleksić 2005, Lučić et al. 2007), is presented.
Proposed mathematical model, based on experimental experience, defines ultimate load $P_u = P_{u1} + P_{u2}$, as a sum of two loads: P_{u1}, load at which collapse mechanism occurs in web, and P_{u2}, load that is spent for flange deformation after reaching force P_{u1}.
Load P_{u1} is calculated by means of the strain energy concept applied to the failure mechanism in Figure 7. Girder failure happens by forming of two yielding lines in web. Lines are horizontal along the fictive load length, c_1, which in general differs from the real length of load, *c*. The distance between lines is *h*. In the moment when load reaches ultimate value P_{u1}, lines are developed along the length *g*. Yielding at the rest of lines (dashed line), as well as forming of plastic hinges in flange, happens latter, after the ultimate load P_{u1} is reached, during the collapse mechanism development.

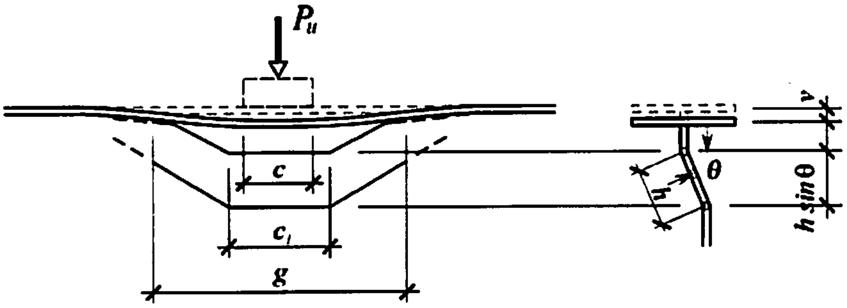


Figure 7. Collapse mechanism

Further increase in ultimate load, P_{u2}, after the collapse mechanism happens in web at load P_{u1}, is possible only on account of reserve in flange carrying capacity. If the limit carrying capacity of engaged flange part is less than P_{u1}, then $P_{u2} = 0$ and $P_u = P_{u1}$. Otherwise $P_u = P_{u1} + P_{u2}$, where P_{u2} is load spent for flange bending immediately after collapse mechanism have happened in web. The load P_{u2} is defined as a part of elastic carrying capacity, F_{u2}, of simple beam (girder flange, herein) with span *l*, loaded by concentrated force in mid-span. A part of this capacity has already been spent during the deformation of flange along with web, before the occurrence of collapse mechanism in web. The rest, defined by coefficient *k*, makes P_{u2}.
Mathematical expressions for ultimate load components P_{u1}, P_{u2} are given by Equations 1 and 2, where σ_w, σ_f are yield stresses of web and flange and t_w, t_f, b_f are girder dimensions, Figure 1. Variables h, g, c_1, θ, l, k are obtained by the calibration of model at the statistical sample consisting of 729 tests from 33 experimental researches.

$$P_{u1} = \sigma_w \cdot t_w \cdot c_1 + \frac{\sigma_w \cdot t_w^2}{4} \cdot \frac{2 \cdot g - c_1}{h \cdot \cos\theta}; \tag{1}$$

$$P_{u2} = F_{u2} \cdot k = \frac{2}{3} \cdot \sigma_f \cdot \frac{b_f \cdot t_f^2}{l} \cdot k; \qquad \begin{array}{l} t_w \le 3.5\,mm \Rightarrow k = 0.5 \\ t_w > 3.5\,mm \Rightarrow k = 0 \end{array} \tag{2}$$

Statistical analysis of proposed procedure is done not only for the total statistical sample consisting of 729 tests, but also separately, for each particular research (33 experimental researches). Following statistical parameters are calculated: average value, standard deviation and variation coefficient of the ratio P_{ex}/P_u, where P_{ex} and P_u are experimental and calculated values of ultimate load. Normal (Gauss) distribution is used. Statistical parameters confirm that proposed procedure matches experimental data very well. For the total sample (729 tests) the average of ratio P_{ex}/P_u is 1.05 with variation coefficient 15.96%.
In order to verify the proposed model, a comparison with 23 previously proposed models is done. The new model has the lowest coefficient of variation.

3.2 *Ultimate load in case of eccentric collapse mode*

Experimental data shows that, in girders with eccentric collapse mode, ultimate load reduces as the load eccentricity increases. The reduction might be quantified by a reduction factor:

$$R = \frac{\textit{ultimate load of eccentrically loaded girder}}{\textit{ultimate load of centrically loaded girder}} \tag{3}$$

Ultimate load of centrically patch loaded girder might be calculated by one of numerous and very accurate existing mathematical expressions (e.g. Equation 1-2). Ultimate load of eccentrically patch loaded girder then might be easily calculated if the reduction is evaluated correctly and confidently.
According to Galambos (1998), reduction factor R is a function of t_f/t_w and varies linearly with e/b_f:

$$\left.\begin{aligned} & R = m\cdot\frac{e}{b_f}+n<1 \\ & \quad m=-0.45\cdot\left(\frac{t_f}{t_w}\right)^2+4.55\cdot\left(\frac{t_f}{t_w}\right)-12.75 \\ & \quad n=1.15-0.025\cdot\left(\frac{t_f}{t_w}\right) \end{aligned}\right\} \tag{4}$$

Equation 4 is empirical, based on the 1980s experiments of Elgaaly et al. (1989, 1990). Therefore it is applicable only for $1 \le t_f/t_w \le 4$ and $e/b_f \le 1/6$, which reflects the ranges of test data.
Recent experimental work implies that the original expression (Equation 4) should be modified. According to Šćepanović et al. (2009), the improved expression is obtained by regression analysis, based upon a large number of experimental data, as well as upon results of FEM modelling by ANSYS. The reduction factor, R, is considered to be a quadratic function of the most relevant parameter e/b_f and, same as earlier, dependent on the most influential geometry parameter t_f/t_w:

$$\left.\begin{aligned} & R = m\cdot\left(\frac{e}{b_f}\right)^2+n\cdot\left(\frac{e}{b_f}\right)+1.01\le 1 \\ & \quad m=-0.864\cdot\left(\frac{t_f}{t_w}\right)^2-14.40\cdot\left(\frac{t_f}{t_w}\right)+38.00 \\ & \quad n=-12.30+4.22\cdot\left(\frac{t_f}{t_w}\right) \end{aligned}\right\} \tag{5}$$

The new expression (Equation 5) was shown to match experimental data better than the previous one (Equation 4), especially for large eccentricities. Besides, the range of data used for regression analysis is larger, so that the modified expression (Equation 5) has a wider domain of application. Experimental data are in range as follows: $1 \le t_f/t_w \le 5$, $1/30 \le e/b_f \le 1/5$, $45 \le a/t_w \le 233$, $10 \le b_f/t_f \le 30$, $c/a = 0.071$ or 0.214, $a/h_w = 1$. ANSYS data are in range as follows: $1 \le t_f/t_w \le 4$, $1/25 \le e/b_f \le 1/6.25$, $a/t_w = 233$, $6.25 \le b_f/t_f \le 25$, $0.036 \le c/a \le 0.071$, $1 \le a/h_w \le 2$.
It has to be pointed out that every future experimental testing or FEM modelling should be followed by new revision and adjusting of empirical expression for the ultimate load reduction factor in order to improve its accuracy.

4 CONCLUSION

Patch loaded steel I-girders are widely present in engineering practice. Current design standards and norms do not account with possible load eccentricity, i.e. patch loading out of web plane, which is almost unavoidable in practice. Experimental research shows that even small eccentricity of load may cause reduction of girder ultimate capacity in comparison with carrying capacity of girder loaded in the web plane. The paper presents mathematical models, i.e. calculation procedures for ultimate capacity of eccentrically patch loaded steel I-girders, based on experimental research, FEM and numerical modelling. Both cases are considered: eccentrically loaded girders with eccentric, but also with centric collapse mode. Suggested calculation procedures are intended to improve current standards and Eurocodes.

Determination of ultimate load of eccentrically patch loaded I-girders by means of reduction factor applied to the ultimate load of centrically loaded girders is one possible approach to problem solution. However, question – when eccentrically loaded girders have centric collapse mode, with ultimate load reduction factor $R = 1$ – is still without complete answer. Further work in this direction should result in universal mathematical procedure, applicable and valid for any type of collapse mode, with the following steps: 1/ recognition of collapse mode (centric or eccentric, depending on girder geometry and load eccentricity), which will lead to the determination of corresponding reduction factor ($R = 1$ or $R < 1$); 2/ ultimate load calculation.

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STEEL I-GIRDERS UNDER ECCENTRIC PATCH LOADING

Biljana Šćepanović*, Duško Lučić, Srđa Aleksić*****

Abstract: Patch loading acts locally, over a small area or length of a structural element. It is a common situation in structural engineering that local compressive load affects the flange of steel I-profile so that the web is compressed in the region below the applied load. Numerous examples are present in different structures, including crane and bridge girders. Some eccentricity of load relative to the web plane is unavoidable in engineering practice. It has been shown that many parameters influence the behaviour, collapse mode and ultimate load of eccentrically patch loaded I-girders: geometric parameters (girder's dimensions and their dimensionless ratios), load eccentricity and the manner of load application. It is evident that most eccentrically loaded girders have a collapse mode quite different from that of centrically loaded girders. The paper summarise investigations on eccentrically patch loaded steel I-girders and suggests some directions of future research work in the subject domain.

Introduction

Patch loading acts locally, over a small area or length of a structural element. It is a common situation in structural engineering that local compressive load affects the flange of steel I-profile so that the web is compressed in the region below the applied load. Numerous examples are present in different structures, including crane and bridge girders.

Although some eccentricity of load relative to the web plane is unavoidable in engineering practice, Figure 1, rather modest amount of worldwide research work has treated this issue. While over 30 experimental researches dealt with I-girders patch loaded in the web plane, influence of load eccentricity was analysed in only six experimental studies [1-7]. Experimental work was followed by finite element method modelling, by means of various computer software, mostly in linear domain [1,2,6]. While over 25 mathematical expressions for centric ultimate load have been proposed, only one empirical expression for eccentric ultimate load, based on the earliest experiments from 1980s, might be found in literature [4]. Its modification has been proposed recently [9], taking recent experiments into consideration. Artificial neural networks are also used nowadays for failure load estimation [8].

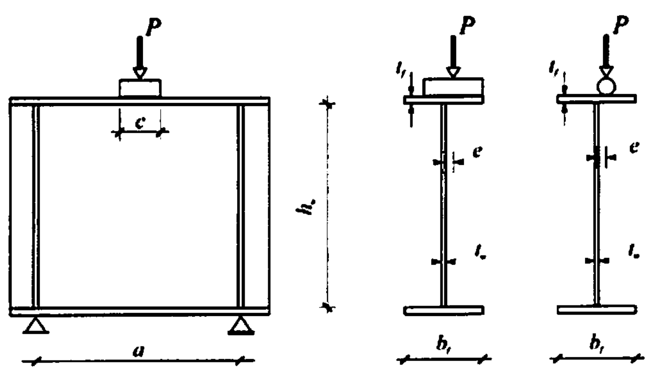

Fig. 1 Eccentrically patch loaded I-girder

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It has been shown that many parameters influence the behaviour, collapse mode and ultimate load of eccentrically patch loaded I-girders: geometric parameters (girder's dimensions and their dimensionless ratios), load eccentricity and the manner of load application. However, the exact influence neither of these parameters separately nor of their combinations is completely defined.

It is evident that most eccentrically loaded girders have a collapse mode quite different from that of centrically loaded girders. Carrying capacity loss in the case of centric load is due to web buckling and local stability loss. In the case of eccentric load, girders lose carrying capacity due to local elastic-plastic bending. The reduction in ultimate load with an increase in load eccentricity is obvious in girders with eccentric collapse mode. However, it is not defined for what combination of influential parameters the eccentrically loaded girders have the same collapse form as the one of centrically loaded girders.

Experimental research work

The first experimental study on eccentric patch loading was published in 1989 by Elgaaly and Nunan, University of Maine [1]. Loads were applied to 22 rolled I-girders of the same dimensions, at various eccentricities, through a thick patch plate (laterally distributed load, Figure 1a) or a cylindrical bar (line load, Figure 1b). In the case of laterally distributed eccentric patch loading, the collapse mode was almost the same as in centrically loaded specimens. No significant reduction in ultimate load was observed as the ratio e/b_f increased to a maximum of 1/8. However, in the case of line load, the girder behaviour obviously differed from the behaviour of centrically loaded girders. Web bending, as well as flange bending and twisting were evident even at low load, and became more pronounced near the failure. The failure load clearly decreased as the load eccentricity increased. The reduction was found to be approximately linear with eccentricity.

The second experimental study, by Elgaaly and Sturgis, University of Maine, 1989, [2], used several rolled and built-up I-girders. The maximum eccentricity ratio was $e/b_f = 1/8$. Ultimate load increased with an increase in the ratio t_f/t_w, but did not depend significantly on panel aspect ratio a/h_w. The ultimate load clearly (and approx. linearly) reduced as the eccentricity ratio e/b_f increased.

In the experiments of Elgaaly and Salkar, University of Maine, 1989, [2], ratios b_f/t_f, t_f/t_w, c/a were varied along with the eccentricity ratio. The maximum eccentricity ratio was $e/b_f = 1/6$. This research confirmed that the failure load decreased, approximately linearly, with increase in ratio e/b_f. This study also showed that the reduction was smaller for larger values of ratio t_f/t_w. In particular, when the flange is much thicker than the web, the ultimate load does not reduce significantly with the increase in eccentricity ratio, even for very slender webs ($h_w/t_w > 200$).

While in the experiments of Elgaaly et al. [1-2] load eccentricity ratio varied up to a maximum 1/6 ("small eccentricity"), the research of Drdacky, ITAM, Czech Academy of Sciences, 1989, [3], treated girders with eccentricity ratio e/b_f up to 5 /3. Web deflections, stresses and strains proved to be linearly dependent on the eccentricity. Girders were not loaded to failure. The influence of load eccentricity relative to initial web imperfection was analysed. It has been shown that, contrary to the load eccentricity, small initial geometric imperfection did not influence web bending significantly.

In the research of Lučić, University of Montenegro, 1998, [5], load eccentricity and flange and web thickness (i.e. ratios e/b_f, b_f/t_f, h_w/t_w and t_f/t_w) were varied in 18 tests (12 eccentrically + 6 comparative centrically loaded girders). The eccentricity ratio, e/b_f, varied from 1/10 to 1/5. All eccentrically loaded girders had collapse modes quite different from those of centrically loaded girders. Compared with Elgaaly's tests, the set of specimens in Lučić's tests exhibited more prominently the eccentric collapse mode characteristics. During the process of load increase, the most evident deformation is the flange warping accompanied by gentle web bending which follows the flange deformation. The web thickness and the flange stiffness influence the extent of flange warping. Flange warping and web bending grow until the final collapse of girder. It seems that initial imperfection does not influence girder behaviour. As before, ultimate load decreases approximately linearly with an increase in eccentricity ratio. This experimental study demonstrated that the reduction was greater for smaller t_f/t_w ratios. Herein the reduction is inversely proportional to the t_f/t_w.

The research named "Ekscentro 2001", by Lučić and Šćepanović, University of Montenegro, 2001, [6-7], continued previous studies. Load eccentricity, e, and web thickness, t_w, were varied in 24 tests. In comparison with the previous research [5], eccentricity ratio varied over a larger range, from 1/30 to 1/6. Collapse modes characteristic of both eccentrically and centrically loaded girders were observed in eccentrically loaded girders. The smaller the eccentricity ratio and the larger the t_f/t_w ratio (i.e. thinner web relative to flange), the more likely a centric collapse mode would develop in girders having eccentric patch loading. The behaviour and collapse mode typical of the centrically loaded girders became evident, even for maximum e/b_f, in all tested girders with ratio $t_f/t_w = 5$. Ultimate load did not change significantly with eccentricity increase in these girders. In girders with ratio $t_f/t_w = 2.5$ or 1.875 ultimate load depended on eccentricity in the same way as in previous studies [1,2,3,5]. It reduced linearly with an increase in eccentricity, and the reduction was more emphasised for smaller ratios t_f/t_w. The centric collapse mode occurred only with small eccentricities ($e/b_f \leq 1/15$ at $t_f/t_w = 2.5$; $e/b_f \leq 1/30$ at $t_f/t_w = 1.875$), whereas in the cases with larger eccentricity collapse mode was the one typical for eccentrically loaded girders, characterised with web bending and flange bending and torsion. The bigger the eccentricity, the more pronounced the deformation is, especially in flange.

Finite element method (FEM) modelling

Parallel with the experimental researches, FEM modelling of eccentrically patch loaded steel I-girders has been developed. The same as experimental work, FEM modelling was launched at the University of Maine, in late 1980s. Initially linear analysis [1] was continued as non-linear, by means of software developed by Elgaaly, Caccese and Du [2]. Experiment "Ekscentro 2001", at the University of Montenegro, was followed by problem modelling with finite element method, by means of computer software SAP 2000 (NonLinear Version 6.11) [6]. Only linear analysis was done. Nonlinear FEM modelling is currently ongoing at the University of Granada, Spain, by means of ANSYS software.

Empirical expression for ultimate load calculation

Experimental data shows that, in girders with eccentric collapse mode, ultimate load reduces as the load eccentricity increases. The reduction might be quantified by a reduction factor:

$$R = \frac{\textit{ultimate load of eccentrically loaded girder}}{\textit{ultimate load of centrically loaded girder}} \tag{1}$$

According to [4], R is a function of t_f/t_w and varies linearly with e/b_f. Expression for R [4] is empirical, based on the experiments of Elgaaly at al [1,2]. Therefore it is applicable only for $1 \leq t_f/t_w \leq 4$ and $e/b_f \leq 1/6$, which reflects the ranges of test data [4].

Recent experimental work implies that the original expression should be modified. According to [9], the improved expression is obtained by regression analysis, based upon a large number of data from the experiments briefly presented in this paper. The reduction factor R is considered to be a function of the most relevant parameter e/b_f and a new parameter $t_f \cdot a^{0.44}/t_w$. The new expression for R [9] matches experimental data better than the original one, especially for large eccentricities. Besides, the range of used test data is larger, so that the modified expression has a wider domain of application [9].

It has to be pointed out that every future experimental testing should be followed by new revision and adjusting of empirical expression for the ultimate load reduction factor in order to improve its accuracy.

Artificial neural network application for ultimate load forecast

During the preparation of new experimental research at the University of Montenegro, modelling of subject issue was made by means of artificial neural networks [8]. Depending on variable input parameters (girder geometry and load eccentricity) the ultimate load was forecasted. Artificial neural networks of different architecture, trained on experimental data from 1998 and 2001, were created. It is important to emphasize that these forecast models might be used and might give reliable results only for input parameters in range of experimental (training) data, i.e. data used for model fitting (network training). The new experimental results are going to enlarge available training data base and enable creation of more precise, more realistic neural network models. Such fitted models might be used not only for the purpose of research, but also in engineering practice.

Conclusions

Presented research work and gathered knowledge on eccentrically patch loaded steel I-girders make a valuable contribution to the development of structural engineering science and practice. However, quite a lot more should be done in this research area.

Main problems that are still waiting for the complete solution are separation of centric and eccentric failure mode, as well as estimation of ultimate load in eccentrically loaded girders.

For the beginning, future research should be concentrated on the experimental work. Having in mind complexity of subject issue and large number of influential parameters, existing experimental data base is rather modest. It should necessarily be enlarged in order to make qualitative basis for all other research directions and means (finite element modelling, artificial neural network modelling, formulation of the expression for ultimate load calculation).

Regarding the estimation of ultimate load, the most proper way would be the proposition of mathematical model based on eccentric collapse mechanism. For the purpose of eccentric collapse mechanism recognition, as well as for mathematical model proposal and calibration, more experiments are needed.

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INTERNACIONALNI NAUČNO-STRUČNI SKUP
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ECCENTRICALLY PATCH LOADED STEEL I-GIRDERS - COLLAPSE MODE AND ULTIMATE LOAD -

Summary

It is evident that most eccentrically patch loaded steel I-girders have a collapse mode quite different from that of centrically loaded girders. Reduction of ultimate load with the increase in load eccentricity is obvious in case of eccentric collapse mode. Numerous parameters influence the behaviour, collapse mode and ultimate load of eccentrically patch loaded steel I-girders. After the last, rather extensive experiment "Ekscentro 2007", the complete experimental database is finally sufficient to enable enough precise definition of the influence of certain parameters to the collapse mode and ultimate load of eccentrically patch loaded steel I-girders.

Key words

Patch load, load eccentricity, steel I-girder, collapse mode, ultimate load, experiment

EKSCENTRIČNO LOKALNO OPTEREĆENI ČELIČNI I-NOSAČI - OBLIK I SILA LOMA -

Rezime

Evidentno je da većina ekscentrično lokalno opterećenih čeličnih I-nosača ima oblik loma potpuno različit od oblika loma centrično opterećenih nosača. U slučaju ekscentričnog loma, očigledno je smanjenje sile loma pri povećanju ekscentriciteta opterećenja. Na ponašanje, oblik i silu loma ekscentrično lokalno opterećenih čeličnih I-nosača utiču brojni parametri. Nakon poslednjeg, veoma obimnog eksperimenta "Ekscentro 2007", kompletna baza eksperimentalnih podataka konačno omogućava dovoljno precizno definisanje uticaja pojedinih parametara na oblik i silu loma ekscentrično lokalno opterećenih čeličnih I-nosača.

Ključne riječi

Lokalno opterećenje, ekscentricitet, čelični I-nosač, oblik loma, sila loma, eksperiment

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1. INTRODUCTION

Patch loading is loading applied over a small area or length of a structural element. A common situation in structural engineering is when local compressive patch load affects the flange of steel I-girder so that the web is compressed in the region bellow the applied load. Local stresses in web might cause local instability that may provoke element carrying capacity loss and, consequently, collapse of the whole structure. This is a rather complex and challenging issue of extremely evident elastic-plastic stresses and deformations. Apart from that, geometrical nonlinearity is noticeable even at the lowest loading level.

Patch loaded girders are widely used and present in different structures, including crane girders loaded by crane wheels or bridge girders erected by launching.

Although some eccentricity of load relative to the web plane is unavoidable in engineering practice, *Figure 1*, rather modest amount of worldwide research work has treated this issue in comparison with the amount of worldwide research work that treats centric patch loading.

It has been shown that many parameters influence the behaviour, collapse mode and ultimate load of eccentrically patch loaded I-girders: geometric parameters (girder's dimensions and their dimensionless ratios), load eccentricity and the manner of load application. However, the exact influence neither of these parameters separately nor of their combinations were completely defined until recently.

It is evident that most eccentrically loaded girders have a collapse mode quite different from that of centrically loaded girders. Carrying capacity loss in the case of centric load is due to web buckling and local stability loss. In the case of eccentric load, girders lose carrying capacity due to local elastic-plastic bending. However, until recently, it was not defined for what combination of influential parameters the eccentrically loaded girders have the same collapse form as the one of centrically loaded girders.

The reduction in ultimate load with an increase in load eccentricity is obvious in girders with eccentric collapse mode. It is quantified by a reduction factor, R, that relates the ultimate load of eccentrically loaded girders to the ultimate load of centrically loaded girders. However, until recently, there was no confident and widely valid expression for R.

Series of experimental researches at the Faculty of Civil Engineering, University in Montenegro, during the last decade, finally enable answers to the above open questions.

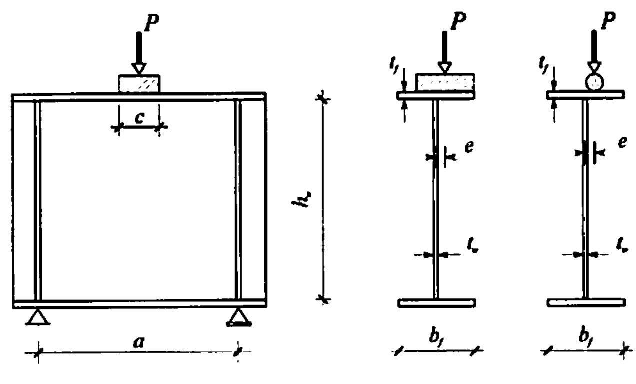

Figure 1. Eccentrically patch loaded I-girder

2. EXPERIMENTAL RESEARCH WORK

Experimental work started at the University of Maine in late 1980s, by Elgally and associates [1,2]. All together, around 35 rolled and built-up I-girders were tested to the collapse. At the same time, some tests were done at the Institute of Theoretical and Applied Mechanics, Czech Academy of Sciences, by Drdacky [3]. Girders were not loaded to the failure. Ten years later, a new series of experiments were initiated at the University of Montenegro, by Lučić and associates [4-7]. *Table 1* summarises results of experiments from 1998, 2001 and 2007, with all together 144 tests regarding eccentric patch loading.

Table 2. Girders geometry, collapse form and ultimate loads in Montenegrin experiments

B III 1, B III 2, B III 3* – *1998 experiment, 18 tests
EB I, EB II, EB III, EB IV* – *"Ekscentro 2001", 24 tests
EB V, EB VI ... EB XXI* – *"Ekscentro 2007", 102 tests

SERIES	a [mm]	h_w [mm]	t_w [mm]	b_f [mm]	t_f [mm]	c [mm]	$P_{u,exp}$ [kN], TYPE OF COLLAPSE FORM: C – centric, E – eccentric, M – mixed $e = 0$ mm	$e = 5$ mm	$e = 10$ mm	$e = 15$ mm	$e = 20$ mm	$e = 25$ mm
EB I	700	700	3	150	15	50	133	128 C	127 C	135 C	134 C	124 C
EB II	700	700	6	150	15	50	340	320 C	326 C	296 E	243 E	197 E
EB III	700	700	6	150	15	50	342	321 C	301 C	267 E	228 E	187 E
EB IV	700	700	8	150	15	50	401	418 C	394 E	301 E	245 E	209 E
EB V	700	700	5	150	10	50	229	212 C/M	197 E	175 E	153 E	129 E
EB VI	700	700	10	150	10	50	720	575 E/M	365 E	313 E	275 E	220 E
EB VII	700	700	5	150	12	50	230	225 C	212 C/M	180 E/M	170 E	149 E
EB VIII	700	700	3	150	3	50	79 (E-M)	44 E	37 E	29 E	23 E	20 E
EB IX	700	700	3	150	6	50	95	80 E/M	69 E	57 E	47 E	39 E
EB X	700	700	3	150	9	50	102	105 C	107 C	90 C/M	85 E	70 E
EB XI	700	700	3	150	12	50	116	113 C	115 C	110 C	105 C/M	115 M
EB XII	700	700	4	150	4	50	120 (M)	70 E	50 E	45 E	40 E	35 E
EB XIII	700	700	4	150	6	50	125	110 E/M	86 E	68 E	50 E	45 E
EB XIV	700	700	4	150	8	50	140	129 C	130 C	100 E	86 E	75 E
EB XV	700	700	4	150	10	50	155	148 C	140 C	138 C/M	128 E/M	115 E/M
EB XVI	700	700	5	150	6	50	187	130 E	105 E	74 E	59 E	55 E
EB XVII	700	700	5	150	8	50	209	200 C/M	145 E	130 E	98 E	83 E
EB XVIII	700	700	6	150	6	50	208 (M)	170 E	130 E	104 E	88 E	69 E
EB XIX	700	700	6	150	9	50	330 (M)	285 E	217 E	155 E	125 E	107 E
EB XX	700	700	6	150	12	50	300	265 E/M	311 E	235 E	202 E	165 E
B III EB XXI	700	700	5	150	10	150	**252 / 266**	250 C	240 C/M	**202 E**	**171 E**	**141 E**
	700	700	10	150	10	150	**874 / 727**	790 E	640 E	**386 E**	**297 E**	**254 E**
	700	700	5	150	12	150	**266 / 266**	255 C	255 C	**228 E**	**177 E**	**162 E**

Experimental studies from the University of Maine [1,2], show that in the case of laterally distributed eccentric patch loading (*Figure 1a*), the collapse mode was almost the same as in centrically loaded specimens. No significant reduction in ultimate load was observed as the eccentricity ratio e/b_f increased to the maximum of 1/6. However, in the case of line load (*Figure 1b*) the girder behaviour obviously differed from the behaviour of centrically loaded girders. The failure load clearly decreased as the load eccentricity increased. The reduction was approximately linear with the increase in eccentricity.

Researches at the University of Montenegro, from 1998 [4] and 2001 [5,6], generally proved findings from 1980s tests. Also some new facts appeared. Collapse modes characteristic of both eccentrically and centrically loaded girders were observed in eccentrically loaded girders. The smaller the eccentricity ratio e/b_f and the larger the t_f/t_w ratio (i.e. thinner web relative to flange), the more likely a centric collapse mode would develop in girders having eccentric patch loading. Only quite extensive experimental research from 2007 [7], revealing more new happenings, like mixed collapse modes, being analysed in conjunction with the previous researches, enabled more precise definition of the influence of certain parameters to the collapse mode and ultimate load of eccentrically patch loaded I-girders.

3. SUMMARY CONCLUSION OF EXPERIMENTAL TESTS

Three different collapse modes are observed in eccentrically patch loaded steel I-girders: eccentric, centric and mixed collapse mode. Mixed collapse mode, having characteristics of both, centric and eccentric collapse modes, may appear in two variants: as centric-mixed or as eccentric-mixed collapse mode, depending on dominant collapse mode characteristics.

Concerning engineering practice, the most important difference between collapse modes is in ultimate load. The reduction in ultimate load with an increase in load eccentricity is obvious in girders with eccentric collapse mode. For a certain girder geometry, even the smallest load eccentricity ($e = 5$ mm or $e/b_f = 1/30$) reduced ultimate load over 40% (e.g. series EB VIII, EB XII, *Table 1*). This decrease in ultimate load might be quantified by a reduction factor, R, which relates the ultimate load of eccentrically loaded girder to the ultimate load of centrically loaded girder and $R < 1$. In case of centric collapse mode in eccentrically loaded girders, ultimate load does not change significantly with an increase in load eccentricity, i.e. $R = 1$.

Hence, in order to estimate ultimate load, the first step is to determine collapse mode of eccentrically loaded I-girder, i.e. to recognise if $R = 1$ or $R < 1$. Having in mind ultimate load reduction ($R < 1$), the identification of eccentric collapse mode is the most important. In order to be able to estimate load reduction and its consequences, it is essential to precisely define the combination(s) of influential parameters which mean eccentric collapse mode. When the collapse mode is recognised as an eccentric one, i.e. $R < 1$, ultimate load might be calculated in any of the following ways:

- by means of reduction factor (calculated from available empirical expressions) and ultimate load of centrically loaded girder (calculated by any of numerous available mathematical procedures),
- by means of mathematical model based on eccentric collapse mechanism (unfortunately, such model has not yet been suggested and available in literature),
- by any other reliable and applicable procedure, e.g. by artificial neural networks.

3.1. DETERMINATION/IDENTIFICATION OF COLLAPSE MODE

Occurrence of a certain collapse mode depends on load eccentricity and girder geometry. Web and flange thicknesses, t_w and t_f, are girder dimensions of the most significant influence. The most influential dimensionless geometry parameter is ratio t_f/t_w. Influence of web slenderness, i.e. parameter $a/t_w = h_w/t_w$ is also important. Actually, parameters t_f/t_w and $a/t_w = h_w/t_w$ should not be analysed separately. Both parameters together, i.e. their combination should be considered. Furthermore, correlation with the eccentricity ratio e/b_f should be established. Apart from ratio e/b_f, two more variants of dimensionless eccentricity might be considered: e/t_f and e/t_w.

Based on the experimental data from *Table 1*, criteria summarised in *Table 2* have been established. Girders fulfilling criteria in dark grey fields have eccentric collapse mode, with the reduced ultimate load. Girders fulfilling criteria in light gray fields might have any collapse mode and these criteria imply that such girders should be carefully analysed each separately, with its specific characteristics, considering several influential parameters and their combinations, as well as paying particular attention to the initial deformation.

Table 2. Dimensionless parameter criteria for collapse mode identification

dimensionless eccentricity	***criterion***	***collapse mode***
e/b_f	$h_w/t_w \leq 1050 \cdot e/b_f + 35$	E
	$h_w/t_w \geq 1050 \cdot e/b_f + 35$	E, C, M
	$t_f/t_w < 15 \cdot e/b_f + 0.5$	E
	$15 \cdot e/b_f + 0.5 \leq t_f/t_w \leq 15 \cdot e/b_f + 1.5$	E, C, M
	$t_f/t_w > 15 \cdot e/b_f + 1.5$	C
e/t_f	$h_w/t_w \leq 85 \cdot e/t_f + 60$	E
	$h_w/t_w \geq 85 \cdot e/t_f + 70$	C
	$85 \cdot e/t_f + 25 \leq h_w/t_w \leq 85 \cdot e/t_f + 105$	M
	$t_f/t_w < e/t_f + 0.5$	E
	$e/t_f + 0.5 \leq t_f/t_w \leq e/t_f + 1.7$	E, C, M
	$t_f/t_w > e/t_f + 1.7$	C
e/t_w	$t_f/t_w < 0.3 \cdot e/t_w + 0.8$	E
	$0.3 \cdot e/t_w + 0.8 \leq t_f/t_w \leq 0.3 \cdot e/t_w + 1.8$	E, C, M
	$t_f/t_w > 0.3 \cdot e/t_w + 1.8$	C

Criteria from *Table 2* correlate dimensionless parameters t_f/t_w and $a/t_w = h_w/t_w$ with dimensionless eccentricity e/b_f, e/t_f or e/t_w. Such relations provide high level of reliability in collapse mode identification. Combining (checking) several criteria from *Table* 2 increases the reliability level.

Some girder dimensions or dimensionless parameters, analysed separately, without making relation(s) with the other parameter(s), may be used as a rough identifiers of collapse mode. Girders having thin flange ($t_f \leq 6$ mm) or thick web ($t_w \geq 8$ mm) or low web slenderness ($h_w/t_w < 100$) or ratio $t_f/t_w < 2$ are affected even with the lowest eccentricity. On the other side, girders with the ratio $t_f/t_w > 3$ have centric collapse mode even at the highest eccentricity.

Obviously, parameter t_f/t_w has the key-role nevertheless if analysed in correlation with the other parameters or separately, only by itself.

3.2. ULTIMATE LOAD AND ITS REDUCTION

Eccentrically loaded girders with centric collapse mode behave as if loaded in the web plane, without significant change in ultimate load, P_u, due to load eccentricity.

In case of eccentric and mixed collapse mode, ultimate load, P_u, depends on load eccentricity, e, and reduces with an increase in eccentricity. The reduction is more emphasised for smaller ratio t_f/t_w and higher ratio $a/t_w = h_w/t_w$. For years this reduction has been considered approximately linear. However, the newest experimental data confirms this only for $t_f/t_w \geq 1.5$. In case of $t_f/t_w < 1.5$, more appropriate approximation of diagram $P_u - e$ would be bi-linear, parabolic or even by the line of higher order.

Ultimate load, P_u, increases with a decrease in web slenderness h_w/t_w, i.e. with an increase in web thickness, t_w. The same happens in both cases: at $t_f/t_w = const$, $t_f \neq const$, as well at $t_f/t_w \neq const$, $t_f = const$.

Ultimate load, P_u, increases with an increase in ratio t_f/t_w due to increase in flange thickness, t_f. However, if ratio t_f/t_w increases due to decrease in web thickness, t_w, ultimate load will also decrease.

Although the load length, c, influences ultimate load (longer the load length, higher the ultimate load), this parameter is generally not of a great influence for the reduction in ultimate load with the increase in load eccentricity. However, there are some implications that the load length might influence this reduction in ultimate load, for the combination of low ratio t_f/t_w and small eccentricity.

Small initial imperfection does not influence girder behaviour, collapse mode and ultimate load. Herein, "small" means "insignificant, negligible in comparison with the plate thicknesses". However, in case of thin flange and/or web, when initial deformation is of the same size order as plate thickness, it must not be neglected, since it will greatly influence girder behaviour and, consequently, its collapse mode and ultimate load.

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