



A theory of power laws in human reaction times: insights from an information-processing approach

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Human reaction time (RT) can be defined as the time elapsed from stimulus presentation until a reaction/response occurs (e.g., manual, verbal, saccadic, etc.). RT has been a fundamental measure of the sensory-motor latency at suprathreshold conditions for more than a century and is one of the hallmarks of human performance in everyday tasks (Luce, 1986; Meyer et al., 1988). Some examples are the measurement of RTs in sports science, driving safety or in aging. Under repeated experimental conditions the RT is not a constant value but fluctuates irregularly over time. Stochastic fluctuations of RTs are considered a benchmark for modeling neural latency mechanisms at a macroscopic scale (Luce, 1986; Smith and Ratcliff, 2004). Power-law behavior has been reported in at least three major types of experiments. (1) RT distributions exhibit extreme values. The probability density function (pdf) is often heavy-tailed and can lead to an asymptotic power-law distribution in the right tail (Holden et al., 2009; Moscoso del Prado Martín, 2009; Sigman et al., 2010). (2) RT variability (e.g., variance) is not bounded and usually shows a power relation with the mean, with an exponent β close to unity (Luce, 1986; Wagenmakers and Brown, 2007; Holden et al., 2009; Medina and Díaz, 2011, 2012). This relationship is a manifestation of Taylor's law (also called "fluctuation scaling") (Taylor, 1961; Eisler et al., 2008), although departures from power law have been reported (Eisler et al., 2008; Schmiedek et al., 2009). And (3), the mean RTs decay as the stimulus strength increases (Cattell, 1886), an issue that is

well-described by a truncated power function written in the form of Piéron's law (Piéron, 1914, 1920; Luce, 1986):

$$t_{n+1} = t_n + \frac{d}{S^p} \quad (1)$$

t_{n+1} indicates the mean RT, S is the stimulus strength (e.g., loudness intensity, odor concentration, etc.), t_n represents the asymptotic component of the mean RT reached at very high stimulus strength and d and p are two parameters (Luce, 1986). The sub-index n denotes the time step or order and it indicates a causal process: t_{n+1} grows from the previous stage t_n by an additive factor that depends on the stimulus strength S (Medina, 2009). The previous stage t_n contains those processes at the threshold at an earlier time and t_{n+1} in Equation (1) describes those processes at suprathreshold conditions at a later time (Norwich et al., 1989; Medina, 2009). The origin of power-law behavior in RTs has been a long-standing issue. Considerable effort has been dedicated in modeling each power relation separately. While it might be plausible that power laws in RTs could share a limited number of mechanisms, a successful theory remains unresolved. The ubiquity of power laws in many biological and physical systems has revealed the existence of multiple generative mechanisms (Mitzenmacher, 2004; Newman, 2005; Sornette, 2007; Frank, 2009). Research on a unifying framework that links power laws in RTs is an important issue for better understanding the emergent complex behavior of neural

activity in simple decisions and in dysfunctional states.

We propose that type (3) power laws govern the threshold for RT; and it follows consequently that power laws govern suprathreshold fluctuations in RT. Piéron's law is valid for each sensory modality (Chocholle, 1940; Banks, 1973; Luce, 1986; Overbosch et al., 1989; Pins and Bonnet, 1996; Bonnet et al., 1999), and in both simple and choice reaction times (Schweickert et al., 1988; Pins and Bonnet, 1996). Instead of diffusion models (Luce, 1986; Smith and Ratcliff, 2004), we use elements from information theory and statistical physics as the principal conceptual tools. We also discuss random multiplicative processes as an important approach to Piéron's law and power laws in RTs.

In our information-theoretic formalism, the information entropy function H always expresses a measure of uncertainty within a sensory neural network. High information entropy values indicates high uncertainty and vice versa. Information is related to the drop of uncertainty (measured, e.g., in bits). It is postulated that sensory perception is not an instantaneous act but it always takes time (Norwich, 1993). Initially, for a given external input signal, the sensory system encodes the stimulus efficiently and then, it adapts and transfers information over time. Therefore, the H -function depends explicitly on the time to represent a continuous process of sensory adaptation (Norwich, 1993). The human RT can be re-defined as the time needed to accumulate ΔH bits of information after

efficient encoding (Norwich et al., 1989; Norwich, 1993):

$$\Delta H = H\left(\frac{1}{t_0}\right) - H\left(\frac{1}{t_{n+1}}\right) > 0 \quad (2)$$

Figure 1A represents the entropy function H in Equation (2). At least two stages can be differentiated. The H -function evolves from a previous state of maximum uncertainty reached at the encoding time t_0 , $H(1/t_0)$, to a final adapting stage with a lower uncertainty $H(1/t_{n+1})$ where a reaction occurs, ($t_{n+1} > t_0$). Maximum production of entropy and then, a reduction of uncertainty in ΔH as a function of time are concepts introduced from statistical physics, the latter as expressed by Boltzmann (Norwich, 1993). Based on

an analytical model of the H -function (Norwich, 1993), the gain of information ΔH is connected with the formation of an internal threshold in Equation (1) (Norwich et al., 1989; Medina, 2009):

$$d = t_n S_0^p \quad (3)$$

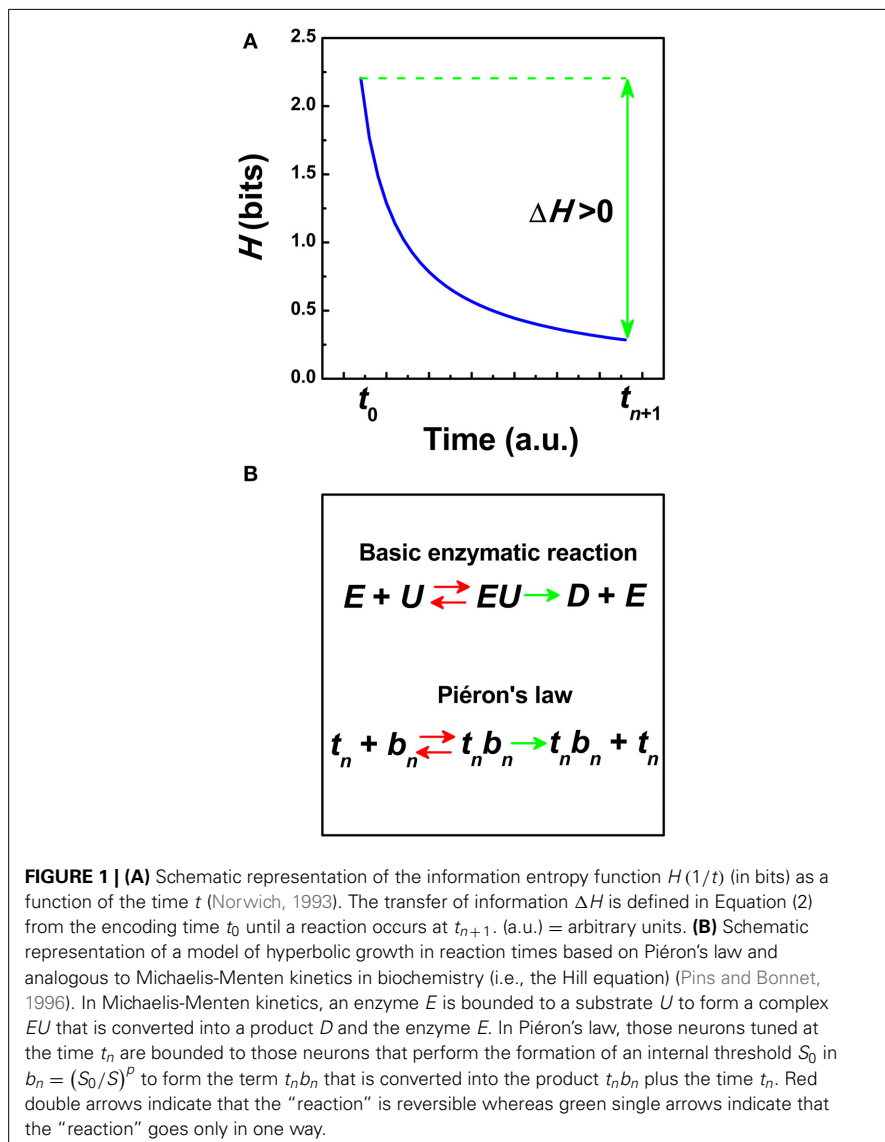
Piéron's law can be written as follows:

$$t_{n+1} = (b_n + 1) t_n, \quad (4)$$

where $b_n = (S_0/S)^p$. The parameter S_0 represents an estimation of the internal threshold that controls the RT: an external incoming signal S exceeding S_0 leads to a RT response (Norwich et al., 1989). Furthermore, S_0 varies based on several factors and provides the sensitivity

($1/S_0$) of the sensory system (e.g., in vision the human contrast sensitivity function) (Felipe et al., 1993; Murray and Plainis, 2003). The model of Piéron's law in Equation (4) sets a number of important properties. First property, Equation (4) indicates the existence of multiplicative interactions in a cascade between different time scales: the mean RT is expressed in terms of the asymptotic time, t_n , and Piéron's law is written in multiples of threshold S_0 . That is, we work with dimensionless ratios of S_0/S (Norwich, 1993). Different interpretations of the exponent p have been reported. S_0^p could be interpreted as the transfer or transducer function between neurons (Copelli et al., 2002; Billock and Tsou, 2011) at the threshold. The exponent p usually takes non-integer values and could indicate a signature of self-organized criticality in a phase transition (Kinouchi and Copelli, 2006). Here the concept of phase transition does not deal with the classical view of different states of matter in thermodynamics (e.g., liquid vs. gas), but with different states of connectivity between neurons as modeled by branching processes (Kinouchi and Copelli, 2006). Alternatively, power functions S_0^p can be derived from Mackay transforms (Mackay, 1963) and the exponent p could represent oscillatory synchronization states between neurons (Billock and Tsou, 2005, 2011). The model of Piéron's law in Equation (4) is a useful alternative approach and optimal information transfer is related with the entropy function H (Norwich, 1993). Low values of p will promote a minimum in ΔH after efficient encoding, i.e., an *Infomin* principle at the macroscopic scale (Medina, 2011, 2012).

Second property, the threshold barrier S_0 is not a fixed static value but unstable and fluctuates over time due to the presence of endogenous or internal noise (Faisal et al., 2008). Consequently, RTs are influenced and modified by neural noise. Therefore, Equation (4) is not deterministic and is included in a general class of discrete-time stochastic equations that has been used in many applications such as in epidemics, finance, etc. (Levy and Solomon, 1996; Sornette and Cont, 1997; Takayasu et al., 1997; Newman, 2005; Sornette, 2006). The term b_n is a random and positive multiplicative factor that depends on the temporal fluctuations



of S_0 and thus, on ΔH . It has been demonstrated that the model of Piéron's law in Equation (4) produce type (1) power laws. RT pdfs obey a transition from a log-normal distribution into a power law in the right tail (Medina, 2012). If RTs are longer than the asymptotic term, t_n , the RT pdf is distributed as a power law with an exponent γ that depends on the exponent p of Piéron's law (Medina, 2012): $\gamma = 1 + (c/p)$, c being a constant. Two different regimes are observed: for those values $p > 0.6$ the central moments diverge and if $p \leq 0.6$ they are finite (Medina, 2012). Therefore, long RTs compared to the asymptotic term t_n are considered intermittent events over time. Their distribution is characterized by power law pdfs that might have finite or infinite variance. A cautionary note should be mentioned here. The magnitude of p could also depend on the metric of the stimulus strength S selected and values different from the boundary $p \cong 0.6$ might be possible. For instance, this is important when testing power law RT pdfs in color vision because an appropriate color contrast metric has not been established (Medina and Diaz, 2010).

Third property, the reciprocal of Piéron's law is invariant under rescaling (Chater and Brown, 1999; Medina, 2009). Taking the reciprocal of the mean RT, $R = 1/t_{n+1}$, and the reciprocal of the irreducible asymptotic term, $R_{\max} = 1/t_n$ in Equation (4), then, $R = R_{\max} [1 + (S_0/S)^p]$. Therefore, the reciprocal of the Equation (4) defines an affine transformation over multiple time scales that can be mapped into the Naka-Rushton equation at the cellular level (Naka and Rushton, 1966) and the Michaelis-Menten equation in enzyme reactions at the sub-cellular level (Michaelis and Menten, 1913; Pins and Bonnet, 1996). This suggests that some general properties of RT patterns governed by Piéron's law could be mirrored in part into the dynamics of the Naka-Rushton equation and/or the Michaelis kinetics (Medina, 2009, 2012). The Naka-Rushton equation represents a canonical form of non-linear gain control in neural responses before saturation (Albrecht and Hamilton, 1982; Billock and Tsou, 2011; Carandini and Heeger, 2012). Threshold normalization in the Naka-Rushton

equation is often modeled as a pool of many neurons tuned to different stimulus properties (Heeger, 1992; Carandini and Heeger, 2012). In the Michaelis-Menten equation, the normalization factor is the Michaelis constant and indicates the substrate concentration at a reference value. The Michaelis constant is related with the substrate's affinity for the enzyme and depends on many factors (Murray, 2002). **Figure 1B** represents a schematic model of RT growth based on Piéron's law and an analogy with enzyme kinetics.

The exponent p of Piéron's law could be related to the scaling exponent β of the variance-mean relationship in type (2) power law. A power law relationship between variance and mean of the stimulus population has been proposed in the H -function (Norwich, 1993) and this relationship could be compatible with the RT variance-mean relationship in the regime around $p > 0.6$ (Medina, 2011, 2012). Alternative approaches have explored α -stable processes to relate type (1) power laws and long-range correlations (Ihlen, 2013). Tweedie exponential dispersion models are also able to describe type (2) power laws in many biological and physical processes (Eisler et al., 2008; Kendal and Jørgensen, 2011; Moshitch and Nelken, 2014). However, a connection between Piéron's law and α -stable and Tweedie models remains unknown.

In summary, maximum entropy H and then, adaptation over time in Equation (2) leads to a type (3) power law, Piéron's law Equation (1). The H -function also explains many empirical relations of sensory perception (Norwich, 1993). An important message of the entropy function H is that the term d in Piéron's law depends explicitly on a sensory threshold S_0 by the power law Equation (3). There is also experimental evidence that RTs and threshold-based sensitivities are mediated by common sensory processes (Felipe et al., 1993; Murray and Plainis, 2003). Therefore, temporal fluctuations at the sensory threshold S_0 affect RT fluctuations at suprathreshold conditions and this can be described by means of a simple random multiplicative process in Equation (4). The same multiplicative process produces non-Gaussian RT distributions and type (1) power law RT pdfs. The model of Piéron's law in Equation (4) also

generates fractal-like behavior that extends to smaller time scales. The reciprocal of Equation (4) provides a direct link with neural gain control in single neurons as exemplified by the Naka-Rushton equation and a possible analogy with enzyme kinetics within neurons as exemplified by the Michaelis-Menten kinetics.

REFERENCES

- Albrecht, D. G., and Hamilton, D. B. (1982). Striate cortex of monkey and cat: contrast response function. *J. Neurophysiol.* 48, 217–237.
- Banks, W. P. (1973). Reaction time as a measure of summation warmth. *Percept. Psychophys.* 13, 321–327. doi: 10.3758/BF03214147
- Billock, V. A., and Tsou, B. H. (2005). Sensory recoding via neural synchronization: integrating hue and luminance into chromatic brightness and saturation. *J. Opt. Soc. Am. A Opt. Image Sci. Vis.* 22, 2289–2298. doi: 10.1364/JOSAA.22.002289
- Billock, V. A., and Tsou, B. H. (2011). To honor fechner and obey stevens: relationships between psychophysical and neural nonlinearities. *Psychol. Bull.* 137, 1–18. doi: 10.1037/a0021394
- Bonnet, C., Zamora, M. C., Buratti, F., and Guirao, M. (1999). Group and individual gustatory reaction times and Piéron's law. *Physiol. Behav.* 66, 549–558. doi: 10.1016/S0031-9384(98)00209-1
- Carandini, M., and Heeger, D. J. (2012). Normalization as a canonical neural computation. *Nat. Rev. Neurosci.* 13, 51–62. doi: 10.1038/nrn3136
- Cattell, J. M. (1886). The influence of the intensity of the stimulus on the length of the reaction time. *Brain* 8, 512–515. doi: 10.1093/brain/8.4.512
- Chater, N., and Brown, G. D. A. (1999). Scale-invariance as a unifying psychological principle. *Cognition* 69, B17–B24. doi: 10.1016/S0010-0277(98)00066-3
- Chochole, R. (1940). Variations des temps de réaction auditifs en fonction de l'intensité à diverses fréquences. *Année Psychol.* 41, 64–124. doi: 10.3406/psy.1940.5877
- Copelli, M., Roque, A. C., Oliveira, R. F., and Kinouchi, O. (2002). Physics of psychophysics: Stevens and Weber-Fechner laws are transfer functions of excitable media. *Phys. Rev. E* 65:060901. doi: 10.1103/PhysRevE.65.060901
- Eisler, Z., Bartos, I., and Kertész, J. (2008). Fluctuation scaling in complex systems: Taylor's law and beyond. *Adv. Phys.* 57, 89–142. doi: 10.1080/00018730801893043
- Faisal, A. A., Selen, L. P. J., and Wolpert, D. M. (2008). Noise in the nervous system. *Nat. Rev. Neurosci.* 9, 292–303. doi: 10.1038/nrn2258
- Felipe, A., Buades, M. J., and Artigas, J. M. (1993). Influence of the contrast sensitivity function on the reaction-time. *Vision Res.* 33, 2461–2466. doi: 10.1016/0042-6989(93)90126-H
- Frank, S. A. (2009). The common patterns of nature. *J. Evol. Biol.* 22, 1563–1585. doi: 10.1111/j.1420-9101.2009.01775.x
- Heeger, D. J. (1992). Normalization of cell responses in cat striate cortex. *Visual Neurosci.* 9, 181–197. doi: 10.1017/S0952523800009640

- Holden, J. G., Van Orden, G. C., and Turvey, M. T. (2009). Dispersion of response times reveals cognitive dynamics. *Psychol. Rev.* 116, 318–342. doi: 10.1037/a0014849
- Ihlen, E. A. F. (2013). The influence of power law distributions on long-range trial dependency of response times. *J. Math. Psychol.* 57, 215–224. doi: 10.1016/j.jmp.2013.07.001
- Kendal, W. S., and Jørgensen, B. (2011). Tweedie convergence: a mathematical basis for Taylor's power law, $1/f$ noise, and multifractality. *Phys. Rev. E* 84:066120. doi: 10.1103/PhysRevE.84.066120
- Kinouchi, O., and Copelli, M. (2006). Optimal dynamical range of excitable networks at criticality. *Nat. Phys.* 2, 348–352. doi: 10.1038/nphys289
- Levy, M., and Solomon, S. (1996). Power laws are logarithmic Boltzmann laws. *Int. J. Mod. Phys. C-Phys. Comput.* 7, 595–601. doi: 10.1142/S0129183196000491
- Luce, R. D. (1986). *Response Times*. New York, NY: Oxford University Press.
- Mackay, D. M. (1963). Psychophysics of perceived intensity: a theoretical basis for Fechner's and Stevens' laws. *Science* 139, 1213–1216. doi: 10.1126/science.139.3560.1213-a
- Medina, J. M. (2009). $1/f$ (alpha) noise in reaction times: a proposed model based on Piéron's law and information processing. *Phys. Rev. E* 79:011902. doi: 10.1103/PhysRevE.79.011902
- Medina, J. M. (2011). Effects of multiplicative power law neural noise in visual information processing. *Neural Comput.* 23, 1015–1046. doi: 10.1162/NECO_a_00102
- Medina, J. M. (2012). Multiplicative processes and power laws in human reaction times derived from hyperbolic functions. *Phys. Lett. A* 376, 1617–1623. doi: 10.1016/j.physleta.2012.03.025
- Medina, J. M., and Diaz, J. A. (2010). S-cone excitation ratios for reaction times to blue-yellow suprathreshold changes at isoluminance. *Ophthalmic Physiol. Opt.* 30, 511–517. doi: 10.1111/j.1475-1313.2010.00745.x
- Medina, J. M., and Diaz, J. A. (2011). "Response variability of the red-green color vision system using reaction times," in: *Applications of Optics and Photonics*, ed M. F. M. Costa (Braga: Proc. SPIE), 800131–800139.
- Medina, J. M., and Diaz, J. A. (2012). $1/f$ noise in human color vision: the role of S-cone signals. *J. Opt. Soc. Am. A-Opt. Image Sci. Vis.* 29, A82–A95. doi: 10.1364/JOSAA.29.000A82
- Meyer, D. E., Osman, A. M., Irwin, D. E., and Yantis, S. (1988). Modern mental chronometry. *Biol. Psychol.* 26, 3–67. doi: 10.1016/0301-0511(88)90013-0
- Michaelis, L., and Menten, M. L. (1913). Die kinetik der invertinwirkung. *Biochem. Z* 49, 333–369.
- Mitzenmacher, M. (2004). A brief history of generative models for power law and lognormal distributions. *Internet Math.* 1, 226–251. doi: 10.1080/15427951.2004.10129088
- Moscoso del Prado Martín, F. (2009). "Scale invariance of human latencies," in: *31st Annual Conference of the Cognitive Science Society*, eds N. A. Taatgen and V. Rijn (Austin, TX: Proc. Cognitive Science Society), 1270–1275.
- Moshitch, D., and Nelken, I. (2014). Using Tweedie distributions for fitting spike count data. *J. Neurosci. Meth.* 225, 13–28. doi: 10.1016/j.jneumeth.2014.01.004
- Murray, I. J., and Plainis, S. (2003). Contrast coding and magno/parvo segregation revealed in reaction time studies. *Vision Res.* 43, 2707–2719. doi: 10.1016/S0042-6989(03)00408-5
- Murray, J. D. (2002). *Mathematical Biology*. Berlin, Germany: Springer-Verlag.
- Naka, K. I., and Rushton, W. A. H. (1966). S-potentials from luminosity units in retina of fish (cyprinidae). *J. Physiol. Lond.* 185, 587–599.
- Newman, M. E. J. (2005). Power laws, Pareto distributions and Zipf's law. *Contemp. Phys.* 46, 323–351. doi: 10.1080/00107510500052444
- Norwich, K. H. (1993). *Information, Sensation, and Perception*. San Diego, CA: Academic Press.
- Norwich, K. H., Seburn, C. N. L., and Axelrad, E. (1989). An informational approach to reaction times. *Bull. Math. Biol.* 51, 347–358. doi: 10.1007/BF02460113
- Overbosch, P., Dewijk, R., Dejonge, T. J. R., and Koster, E. P. (1989). Temporal integration and reaction times in human smell. *Physiol. Behav.* 45, 615–626. doi: 10.1016/0031-9384(89)90082-6
- Piéron, H. (1914). Recherches sur les lois de variation des temps de latence sensorielle en fonction des intensités excitatrices. *Année Psychol.* 20, 17–96. doi: 10.3406/psy.1913.4294
- Piéron, H. (1920). Nouvelles recherches sur l'analyse du temps de latence sensorielle et sur la loi qui relie ce temps à l'intensité de l'excitation. *Année Psychol.* 22, 58–60. doi: 10.3406/psy.1920.4404
- Pins, D., and Bonnet, C. (1996). On the relation between stimulus intensity and processing time: Piéron's law and choice reaction time. *Percept. Psychophys.* 58, 390–400. doi: 10.3758/bf03206815
- Schmiedek, F., Lövdén, M., and Lindenberger, U. (2009). On the relation of mean reaction time and intraindividual reaction time variability. *Psychol. Aging* 24, 841–857. doi: 10.1037/a0017799
- Schweickert, R., Dahn, C., and McGuigan, K. (1988). Intensity and number of alternatives in hue identification: Piéron's law and choice reaction time. *Percept. Psychophys.* 44, 383–389. doi: 10.3758/bf03210422
- Sigman, M., Etchemendy, P., Fernandez Slezak, D., and Cecchi, G. A. (2010). Response time distributions in rapid chess: a large-scale decision making experiment. *Front. Neurosci.* 4:60. doi: 10.3389/fnins.2010.00060
- Smith, P. L., and Ratcliff, R. (2004). Psychology and neurobiology of simple decisions. *Trends Neurosci.* 27, 161–168. doi: 10.1016/j.tins.2004.01.006
- Sornette, D. (2006). *Critical Phenomena in Natural Sciences*. Berlin, Heidelberg: Springer-Verlag.
- Sornette, D. (2007). "Probability distributions in complex systems," in *Encyclopedia of Complexity and System Science*, ed R.A. Meyers (New York, NY: Springer), 7009–7024.
- Sornette, D., and Cont, R. (1997). Convergent multiplicative processes repelled from zero: power laws and truncated power laws. *J. Phys. I* 7, 431–444. doi: 10.1051/jp1:1997169
- Takayasu, H., Sato, A. H., and Takayasu, M. (1997). Stable infinite variance fluctuations in randomly amplified Langevin systems. *Phys. Rev. Lett.* 79, 966–969. doi: 10.1103/PhysRevLett.79.966
- Taylor, L. R. (1961). Aggregation, variance and the mean. *Nature* 189, 732–735. doi: 10.1038/189732a0
- Wagenmakers, E. J., and Brown, S. (2007). On the linear relation between the mean and the standard deviation of a response time distribution. *Psychol. Rev.* 114, 830–841. doi: 10.1037/0033-295X.114.3.830

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