Versioning Goods and Joint Purchase: Substitution and Complementarity Strategies*

Francisco Martínez-Sánchez†
Universidad de Granada

April 1, 2008

Abstract

We analyze the monopolist's decision about how to design different versions of a good, i.e. whether to make them substitutes or complements, when consumers can buy them simultaneously. In this context, we find that versioning goods as substitutes or complements may be optimal for the monopolist, and the final result depends on the degree of concavity and convexity of the cost function.

JEL classification: L10; L12; L15

Keywords: Versioning Goods, Joint Purchase Option, Substitutes, Complementarity, Price Discrimination, Market Segmentation

*I thank Natalia Jiménez for her useful suggestions. I acknowledge financial support from Spanish MCYT under the Project SEJ2007-62081/ECON. Any remaing errors are, of course, mine alone.

†Depto. de Teoría e Historia Económica, Universidad de Granada, Campus La Cartuja, 18011 Granada, Spain. *E-mail address:* fms@ugr.es

1 Introduction

A common strategy for most firms is to create various versions of the same product (which is known as *versioning goods*) for price discriminating purposes and to increase profits, particularly in the industry of information goods (Belleflamme (2005)). Nevertheless, we believe that this strategy is not used to the full by some firms insofar as they only pay attention to introducing a new low-quality variant (or version) onto the market and fail to consider the possibility of making complementary or substitute variants so that consumers can buy them simultaneously. We refer to this possibility or strategy as *versioning substitute/complementary goods*.

To understand the strategy of versioning substitute/complementary goods better, we give three examples of the implementation of this strategy in the media, software and textile industries. We first consider a press market with two differentiated newspapers sold by a single publishing house (or multiproduct monopolist), where the source of differentiation is the size of the readership, as in Gabszewicz, Laussel and Sonnac (2001). Imaging that a general interest newspaper and an economic newspaper report on an economic news item. Then the publishing house designs both newspapers so that a consumer can read this news item in both newspapers or in only one (general or economic).

Secondly we consider a monopolist that develops two vertically differentiated versions of a software product, in which quality is denoted by the number of applications that they can run (Deneckere and McAfee (1996) and Gabszewicz and Wauthy (2003)). For instance, assume that the monopolist makes a low-quality variant a little more attractive at first sight than the high-end version, so that a consumer of the latter version must buy the low-end version if he wants a better appearance. Thus, he encourages consumers to buy both variants.

Finally we focus on women's decisions as to what quality-differentiated new dresses to buy, where they can be substitutes if they are similar in style and colours, or complementary if they are in different colours, for instance. It is possible that a woman may value two complementary dresses more because she can choose between them and combine each with different shoes, clothes and jewellery, so that she seems to have an extensive wardrobe (Gabszewicz, Sonnac and Wauthy (2001)). In this market, a monopolist can take this into account when it comes to designing a product line. As we can note in the last two examples, complementarity can be seen as a source of horizontal differentiation.

Versioning goods (or equivalently second degree price discrimination) has previously been analyzed from a theoretical viewpoint. In a seminal 1979 paper, Stokey provides conditions under which (second degree) price discrimination is not optimal. In a later study Salant (1989) shows that price discrimination is not optimal if the marginal cost function of improving quality is linear. However, under these conditions, Jing (2007) shows that the presence of network externalities restores the optimality of price discrimination (or versioning goods).

A special strategy of versioning goods consists of damaging a high-quality good because this is a cheaper way of producing a low-quality good than actually making a low-quality good. This particular strategy has been analyzed by Deneckere and McAfee (1996), who provide many examples of it in the chemicals, electronics and pharmaceutical industries, and show that it can be profitable for a firm and a Pareto improvement.

In a simple model of versioning information goods, Belleflamme (2005) shows that if the common valuation of different variants by consumers is zero and the constant marginal cost of producing one unit is the same for any variant, the monopolist will not prefer to version goods. In other words, it will offer the high-quality variant only.

In a recent paper Bhargava and Choudhary (in press) show that if the marginal cost of manufacturing is invariant in quality, versioning is optimal when the optimal market share of the lower-quality version offered alone is greater than the optimal market share of the high-quality version offered alone.

These papers fail to take into account that consumers may simultaneously buy low- and high-quality versions of a good. This has been analyzed in a duopoly model of vertical differentiation by Gabszewicz, Sonnac and Wauthy (2001) for complementary variants and by Gabszewicz and Wauthy (2003) for substitutes. These authors call this possibility a *joint purchase option*.

In the present paper, we develop a monopoly model of vertical product differentiation for analyzing the monopolist's decision about the possibility of versioning goods as substitutes or complements when the joint purchase option is available to consumers. In this context, we find that versioning goods as substitutes or complements is optimal for the monopolist, and the final result depends on the degree of concavity and convexity of the cost function.

The rest of the paper is organized as follows. Section 2 describes the model formally. The monopolist's decision about how to design different variants of a good is analyzed in Section 3. Section 4 analyzes equilibrium strategies and Section 5 concludes.

2 The Model

We consider a monopolist who decides whether to produce one or two variants of a product. The two variants are vertically differentiated as per Mussa and Rosen (1978), so there is a variant of high quality, indexed by u_h, and other of low quality, indexed by u_l. Thus, $u_h > u_l$. We assume that the monopolist produces at zero cost. He chooses the prices of the variants and the degree of substitution and complementarity between them so as to maximize profits.

Unlike previous models that analyze versioning strategies, in this paper we allow consumers to simultaneously buy the two variants of a product. This possibility can be viewed as consuming a third variant of quality u_2. Consumers are indexed by $\theta \in [0, 1]$ which represents consumers' tastes for the quality of a product. We assume that θ follows a uniform distribution. The utility of consumer θ is:

$$U(\theta, u_l, u_h, u_2, p_l, p_h) = \begin{cases} \theta u_h - p_h & \text{if he/she buys the high-quality variant} \\ \theta u_l - p_l & \text{if he/she buys the low-quality variant} \\ \theta u_2 - p_h - p_l & \text{if he/she buys both variants (or the "bundle")} \\ 0 & \text{if he/she does not buy} \end{cases} \quad (1)$$

where p_l and p_h are the price of the low- and high-quality variants, respectively. In order to find the monopolist's demand function, we consider two cases: (i) $u_2 \in]u_h, u_l + u_h[$ if the two variants are substitutes, and (ii) $u_2 \in [u_l + u_h, +\infty[$ if they are complementary.[1]

(i) Substitutes ($u_2 \in]u_h, u_l + u_h[$) Let θ_l be a consumer indifferent between buying the low-quality variant and not buying at all, from function (1), $\theta_l = p_l/u_l$. Let θ_h be a consumer indifferent between buying the high-quality variant and not buying at all, that is, $\theta_h = p_h/u_h$. Let θ_2 be a consumer indifferent between buying the bundle and not buying at all, that is, $\theta_2 = (p_l + p_h)/u_2$. Let θ_{hl} be a consumer indifferent between buying the high and low-quality variants, that is, $\theta_{hl} = (p_h - p_l)/(u_h - u_l)$. Let θ_{2l} be a consumer indifferent between buying the low-quality variant and one unit of both variants (bundle), that is, $\theta_{2l} = p_h/(u_2 - u_l)$. Let θ_{2h} be a consumer indifferent between buying the high-quality variant and one unit of both variants (bundle), that is, $\theta_{2h} = p_l/(u_2 - u_h)$.

From these definitions, we derive the monopolist's demand functions for each variant. But first we identify the critical regions of the domain of (p_l, p_h)-prices, which are shown in Figure 1(a) and coincide with those computed by Gabszewicz and Wauthy (2003). The region (or subdomain) P_1 is defined as

$$P_1 = \{(p_l, p_h) : p_l \geq u_2 - u_h\}$$

In this region the consumer who is most willing to pay to consume both variants ($\theta = 1$) prefers the high-quality variant alone to buying them at that price. Thus, the demand function of each variant is the same one that we obtain in a model of vertical differentiation in which buying the bundle is not possible, so the demand functions are as follows:

$$D_l(p_l, p_h) = \theta_{hl} - \theta_l; D_h(p_l, p_h) = 1 - \theta_{hl}. \quad (2)$$

Region P_2 is defined by

$$P_2 = \left\{(p_l, p_h) : p_l < u_2 - u_h; p_l \geq p_h \frac{u_l}{u_h}\right\},$$

so

$$\theta_{hl} \leq \theta_h \leq \min\{\theta_l, \theta_{2l}\} \leq \max\{\theta_l, \theta_{2l}\} < \theta_{2h}.$$

Thus, the demands are as follows[2]

$$D_l(p_l, p_h) = 1 - \theta_{2h}; D_h(p_l, p_h) = 1 - \theta_h. \quad (3)$$

[1] See Gabszewicz, Sonnac and Wauthy (2001) and Gabszewicz and Wauthy (2003) for a detailed analysis of demand functions of complementary and substitute variants of the same product in a duopoly model of vertical differentiation, respectively.

[2] Notice that $\theta_2 > \theta_h$.

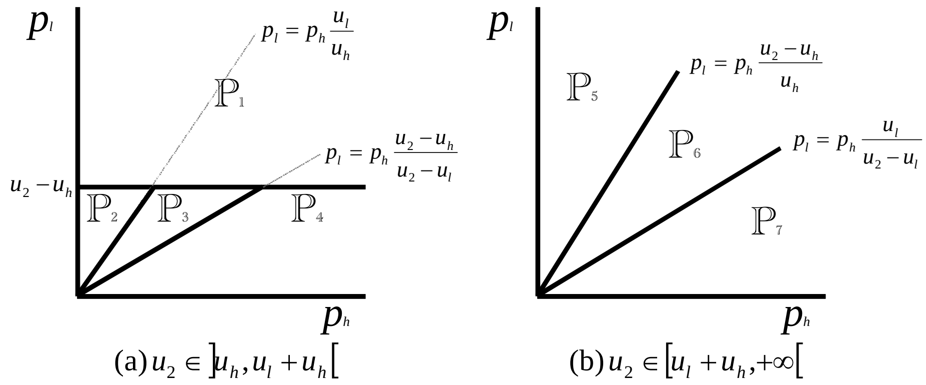

Figure 1: Partition of the price region

Notice that in P_2 all consumers who buy low-quality variant also buy the high-quality variant, so there are some consumers who only buy variant h. Thus the demand for variant h coincides with the monopoly's demand when it is offered alone. Region P_3 is defined as follows

$$P_3 = \left\{ (p_l, p_h) : p_l < u_2 - u_h; p_h \frac{u_2 - u_h}{u_2 - u_l} \leq p_l \leq p_h \frac{u_l}{u_h} \right\},$$

so

$$\theta_l \leq \theta_h \leq \theta_{hl} \leq \theta_{2l} \leq \theta_{2h}. \tag{4}$$

Thus, demands are as follows[3]

$$D_l (p_l, p_h) = 1 - \theta_{2h} + \theta_{hl} - \theta_l = 1 + \frac{p_h}{u_h - u_l} - p_l K; D_h (p_l, p_h) = 1 - \theta_{hl}, \tag{5}$$

where

$$K = \frac{u_l (u_2 - u_l) + (u_h - u_l)(u_2 - u_h)}{u_l (u_h - u_l)(u_2 - u_h)}.$$

In this region, there are consumers who only buy the low- or high-quality variant. Finally, region P_4, which is symmetric to P_2, is defined as follows

$$P_4 = \left\{ (p_l, p_h) : p_l < u_2 - u_h; p_l \leq p_h \frac{u_2 - u_h}{u_2 - u_l} \right\},$$

so

$$\theta_l < \min \{\theta_{2h}, \theta_h\} \leq \max \{\theta_{2h}, \theta_h\} \leq \theta_{2l} \leq \theta_{hl}.$$

Thus, the demands are as follows[4]

$$D_l (p_l, p_h) = 1 - \theta_l; D_h (p_l, p_h) = 1 - \theta_{2l}. \tag{6}$$

[3] Notice that $\theta_2 > \theta_l$.
[4] Notice that $\theta_2 > \theta_l$.

(ii) Complementary variants ($u_2 \in [u_l + u_h, +\infty[$) We now describe the monopolist's demand function when the two variants are complementary. Thus, we take into account only the indifferent consumers θ_l, θ_h and θ_2, described above. Three critical regions of the domain of (p_l, p_h)-prices are identified, which are shown in Figure 1(b) and coincide with those computed by Gabszewicz, Sonnac and Wauthy (2001). The region (or subdomain) P_5 is defined as follows

$$P_5 = \left\{ (p_l, p_h) : p_l \geq p_h \frac{u_2 - u_h}{u_h} \right\},$$

so

$$\theta_h \leq \theta_2 \leq \theta_l. \tag{7}$$

Thus, the demand for each variant is as follows

$$D_l (p_l, p_h) = 1 - \theta_2; D_h (p_l, p_h) = 1 - \theta_h. \tag{8}$$

The region P_6 is as follows

$$P_6 = \left\{ (p_l, p_h) : p_h \frac{u_l}{u_2 - u_l} \leq p_l \leq p_h \frac{u_2 - u_h}{u_h} \right\},$$

so

$$\theta_2 \leq \min \{\theta_l, \theta_h\}.$$

Thus, the demand for each variant is as follows

$$D_l (p_l, p_h) = D_h (p_l, p_h) = 1 - \theta_2. \tag{9}$$

In this region, firms' demand coincides with the demand for the bundle (the purchase of the two variants). Finally, region P_7 is symmetric to region P_5, so it is described as follows

$$P_7 = \left\{ (p_l, p_h) : p_l \leq p_h \frac{u_l}{u_2 - u_l} \right\},$$

so

$$\theta_l \leq \theta_2 \leq \theta_h.$$

Thus, the demand for each variant is as follows

$$D_l (p_l, p_h) = 1 - \theta_l; D_h (p_l, p_h) = 1 - \theta_2. \tag{10}$$

The monopolist's profit is as follows

$$\pi (u_l, u_h, u_2, p_l, p_h) = p_l D_l (p_l, p_h) + p_h D_h (p_l, p_h) - C (u_2) \tag{11}$$

where $D_l (.)$ and $D_h (.)$ represent the demand for the low- and high-quality variants, respectively, and $C (u_2)$ represents the fixed cost of making variants less close substitutes or more complementary, so that $C' (u_2) > 0$.[5] Notice that a lower (higher) u_2 implies that variants are closer substitutes (more complementary).

[5] This cost function is in line with the cost function of quality-improvement considered by Motta (1993).

The timing of the complete information game is as follows. First, the monopolist decides the relationship between the high- and low-quality variants, i.e. it decides the values of u_2. Next it sets the price of each variant. Finally, consumers decide to buy the high-quality variant, the low-quality variant, the bundle or neither after they have observed the prices of the variants.

In the next section, we look for the subgame perfect equilibrium (SPE).

3 Equilibrium

In this section, we seek to find the monopolist's equilibrium strategy, which consists of a choice about the degree of relationship between the high- and low-quality variants (substitutes or complements) and about the prices of the variants. To that end, in the next two subsections we look for the optimal price strategy when the two variants are substitutes and complements, respectively. Then we seek the optimal degree of relationship between them.

3.1 Substitution

We now obtain the optimal price strategy for the regions P_1, P_2, P_3 and P_4, as described above. The monopolist, in region P_3, faces the demands (5), so that by maximizing the monopolist's profit function (11), we have that the optimal prices, demands and profit are:

$$\overline{p}_3^* = (p_l, p_h) = \left(\frac{u_l(u_2-u_h)}{u_2+u_l-u_h}, \frac{u_l(u_2-u_l)+u_h(u_2-u_h)}{2(u_2+u_l-u_h)}\right); \quad D_l^{*3} = D_h^{*3} = \frac{1}{2}; \quad \pi^{*3} = \frac{u_2(u_h+3u_l)-(u_l+u_h)^2}{4(u_2+u_l-u_h)}. \tag{12}$$

Notice that $\overline{p}_3^* \in P_3$ and the market is not completely covered. The demand functions that the monopolist faces in region P_2 are (3), so that the optimal prices are $\overline{p}_2^* = (p_l, p_h) = ((u_2 - u_h)/2, u_h/2)$. Notice that $\overline{p}_2^* \notin P_2$, so the best strategy lies at the frontier with P_3, which by continuity is itself dominated by the best strategy in the interior of P_3. Thus, the optimal strategy is not in region P_2. Given that region P_4 is symmetric to region P_2, we obtain that nor is the optimal strategy in region P_4.

In region P_1, the monopolist faces the demand functions (2), so we have that if $u_2 \leq u_h + u_l/2$, optimal prices, demands and profit are:

$$\overline{p}_1^* = (p_l, p_h) = \left(\frac{u_l}{2}, \frac{u_h}{2}\right); \quad D_l^{*1} = 0; \quad D_h^{*1} = \frac{1}{2}; \quad \pi^{*1} = \frac{u_h}{4}; \tag{13}$$

Notice that $\overline{p}_1^* \in P_1$ if $u_2 \leq u_h + u_l/2$, otherwise $\overline{p}_1^* \notin P_1$. So in the last case the best strategy lies at the frontier with P_2, P_3 and P_4, which by continuity and the result obtained before is itself dominated by the best strategy in the interior of P_3. Thus, in this region we obtain that the optimal strategy for the monopolist is to sell the high-quality variant only if u_2 is low enough. This is the standard result obtained by the previous studies of versioning goods or second-degree price discrimination (Stokey (1979), Salant (1989) and Belleflamme (2005)). Therefore, the optimal price strategy is in region P_3 if $u_2 \geq u_h + u_l/2$.

Nevertheless, when $u_2 \leq u_h + u_l/2$, the optimal price strategy is in regions P_1 or P_3. By comparing π^{*1} and π^{*3} we get the optimal strategy for the monopolist, which is $\overline{p}_1^*$ if $u_2 \leq u_h + u_l/3$, otherwise it is $\overline{p}_3^*$. This means that versioning products as substitutes is not optimal when they are very close substitutes, i.e. when the utility associated with the joint purchase option is low enough. These results are summarized in the following proposition.

Proposition 1 *If the two variants are substitutes and consumers can buy them simultaneously, the monopolist decides to sell both at prices*

$$p_s^* = \begin{cases} \overline{p}_1^* & \text{if } u_2 \leq u_h + u_l/3, \\ \overline{p}_3^* & \text{if } u_2 \geq u_h + u_l/3, \end{cases}$$

From Proposition 1, we observe that the monopolist versions a product if u_2 is high enough. Otherwise it does not offer consumers a low-quality variant.

3.2 Complementarity

In this subsection, we consider that variants are complementary, so the partition of the domain of prices is as drawn in Figure 1(b). In region P_5, the monopolist's demand functions are (8), so the optimal prices, demands and profit are:

$$\overline{p}_5^* = \left(\tfrac{u_2(2u_2-u_h)}{4u_2-u_h}, \tfrac{u_2u_h}{4u_2-u_h}\right); \quad D_l^{*5} = \tfrac{2u_2-u_h}{4u_2-u_h}; \quad D_h^{*5} = \tfrac{3u_2-u_h}{4u_2-u_h}; \quad \pi^{*5} = \tfrac{u_2^2}{4u_2-u_h}. \tag{14}$$

Notice that $\overline{p}_5^* \in P_5$. The demands that the monopolist faces in region P_6 are (9). Since the bundle is considered as a third good, the firm behaves as a monopolist that only sells that good, so that the optimal prices $\overline{p}_6^*$ are those such that $p_l^* + p_h^* = p_2^* = \frac{u_2}{2}$.[6] Thus, the demand and profit correspond to those in a monopoly, i.e. $D_2^{*6} = 1/2$ and $\pi^{*6} = u_2/4$. Finally, we consider region P_7, in which the monopolist's demands are (10), so the optimal prices, demands and profit are:

$$\overline{p}_7^* = \left(\tfrac{u_2u_l}{4u_2-u_l}, \tfrac{u_2(2u_2-u_l)}{4u_2-u_l}\right); \quad D_l^{*7} = \tfrac{3u_2-u_l}{4u_2-u_l}; \quad D_h^{*7} = \tfrac{2u_2-u_l}{4u_2-u_l}; \quad \pi^{*7} = \tfrac{u_2^2}{4u_2-u_l}. \tag{15}$$

Notice that $\overline{p}_7^* \in P_7$. By comparing the profits obtained in regions P_5, P_6 and P_7, we have $\pi^{*6} < \pi^{*7} < \pi^{*5}$. Thus, when variants are complementary and consumers can simultaneously buy both variants, the monopolist's optimal price is p^{*5}, which means that versioning products is also optimal in this case. This result is shown in Proposition 2.

Proposition 2 *If the two variants are complementary and consumers can buy them simultaneously, the monopolist sells both at prices* $p_c^* = \overline{p}_5^*$.

3.3 Substitute or Complementary Variants

In this subsection, taking into account the previous results, we seek to establish the monopolist's optimal choice about the degree of substitution or complementarity. The monopolist faces the following profit

[6] We consider that the prices of the variants are such that $\overline{p}_6^* \in P_6$.

function:

$$\pi(p_l, p_h) = \begin{cases} \frac{u_h}{4} - C(u_2) & \text{if } u_h < u_2 \leq u_h + u_l/3; \\ \frac{u_2(u_h+3u_l)-(u_h+u_l)^2}{4(u_2+u_l-u_h)} - C(u_2) & \text{if } u_h + u_l/3 \leq u_2 < u_l + u_h; \text{ and} \\ \frac{u_2^2}{4u_2-u_h} - C(u_2) & \text{if } u_2 \geq u_l + u_h. \end{cases} \tag{16}$$

By maximizing this profit function on u_2, we find the equilibrium degree of substitution or complementarity. In particular, we find two equilibrium candidates: one where the two variants are substitutes (u_2^s) and another where they are complementary (u_2^c). Given the assumptions in the model developed here, we cannot obtain the explicit expression of u_2, but we can provide conditions that allow us to select the equilibrium. Namely, as we show in Proposition 3, when the cost function of making variants complementary is slightly concave or convex, the only equilibrium possible is for the two variants to be substitutes; and if the cost function is convex enough, we cannot select between the two candidates.

Proposition 3 *When a monopolist can version substitute/complementary goods and the joint purchase option is available to consumers, we have that*

(a) if $C''(u_2) \in \left[-\infty, -\frac{2u_l^2}{(u_2+u_l-u_h)^3}\right[$*, there is no equilibrium;*

(b) if $C''(u_2) \in \left[-\frac{2u_l^2}{(u_2+u_l-u_h)^3}, \frac{2u_h^2}{(4u_2-u_h)^3}\right[$*, the only equilibrium is* $u_2^* = u_2^s$*;*

(c) otherwise, u_2^s *and* u_2^c *can be an equilibrium.*

Proof of Proposition 3. *We maximize the profit function (16), so the first order condition (FOC) is:*

$$\frac{\partial \pi(p_l, p_h)}{\partial u_2} = \begin{cases} -C'(u_2) = 0 & \text{if } u_h < u_2 \leq u_h + u_l/3; \\ \frac{u_l^2}{(u_2+u_l-u_h)^2} - C'(u_2) = 0 & \text{if } u_h + u_l/3 \leq u_2 < u_l + u_h; \text{ and} \\ \frac{2u_2(2u_2-u_h)}{(4u_2-u_h)^2} - C'(u_2) = 0 & \text{if } u_2 \geq u_l + u_h. \end{cases} \tag{17}$$

From the FOC, given that $C'(u_2) > 0$*, we find the implicit expressions that define the equilibrium degrees of substitution and complementarity and that* $u_2^s \in [u_h + u_l/3, u_l + u_h[$*. The second order condition (SOC) is:*

$$\frac{\partial \pi^2(p_l, p_h)}{\partial^2 u_2} = \begin{cases} -C''(u_2) \leq 0 & \text{if } u_h < u_2 \leq u_h + u_l/3; \\ -\frac{2u_l^2}{(u_2+u_l-u_h)^3} - C''(u_2) \leq 0 & \text{if } u_h + u_l/3 \leq u_2 < u_l + u_h; \text{ and} \\ \frac{2u_h^2}{(4u_2-u_h)^3} - C''(u_2) \leq 0 & \text{if } u_2 \geq u_l + u_h. \end{cases} \tag{18}$$

We can easily check that,

(a) if $C''(u_2) \in \left[-\infty, -\frac{2u_l^2}{(u_2+u_l-u_h)^3}\right[$*, there is no equilibrium;*

(b) if $C''(u_2) \in \left[-\frac{2u_l^2}{(u_2+u_l-u_h)^3}, \frac{2u_h^2}{(4u_2-u_h)^3}\right[$*, the only equilibrium candidate that satisfies SOC is* u_2^s*;*

(c) otherwise, the two candidates satisfy SOC, so that u_2^s *and* u_2^c *can be an equilibrium.*

■

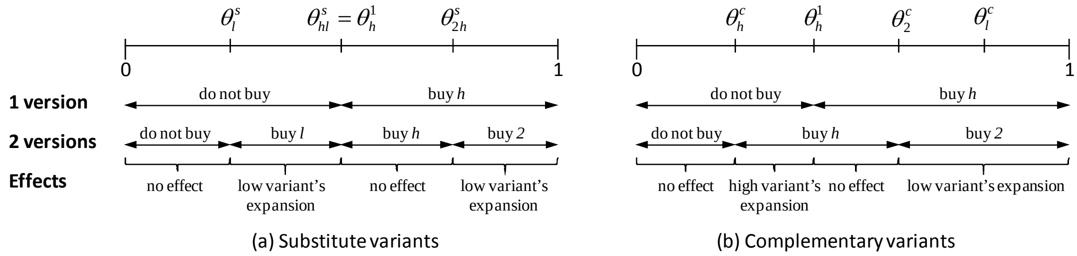

Figure 2: Effect of versioning substitute-complementary goods

4 Analysis of Equilibrium

By comparing the equilibrium demands when the two variants are substitutes with those when they are complementary, we have

$$D_l^c < D_l^s = D_h^s < D_h^c, \tag{19}$$

where superscripts s and c represent the equilibrium when the variants are substitutes and complements, respectively. From (19), we observe that the monopolist sells the low-quality variant more when it is a substitute for the high-quality variant.

Let $\theta_h^1 = 1/2$ be a the consumer indifferent between buying the high-quality variant and not buying at all when the monopolist decides to offer the high-quality variant only. From relationship (4) and the equilibrium results when variants are substitutes on (12), we have

$$\theta_l^s < \theta_h^1 = \theta_{hl}^s < \theta_{2h}^s. \tag{20}$$

In the same way, from relationship (7) and the equilibrium results when variants are complementary on (14), we have

$$\theta_h^c < \theta_h^1 < \theta_2^c < \theta_l^s. \tag{21}$$

Therefore, from relationships (20) and (21), we show that there is no cannibalization effect.[7] This explains why versioning substitute/complementary goods is optimal when the joint purchase option is available to consumers. This last result is summarized in Proposition 4 and illustrated in Figure 2.

Proposition 4 *If a monopolist can version substitute/complementary goods and consumers can simultaneously buy the two versions, there is no cannibalization effect.*

As can be seen from Figure 2, the expansion effect is felt only in variant l when the two variants are substitutes, but if they are complementary, the expansion effect is felt in both the low- and high-quality variants.

Let u_2^{s*} and u_2^{c*} be the equilibrium degree of substitution and complementarity, respectively. Since we do not know the explicit expression of u_2^{s*} and u_2^{c*}, we cannot make a complete analysis of equilibrium,

[7] The cannabalization effect represents the fact that there are consumers who would buy the high-quality variant if it were the only one available, but who would buy the low-quality variant otherwise.

but we can find how these equilibria change as the two variants differ more in quality. Notice that a higher u_2^{s*} implies that variants are less close substitutes, and a higher u_2^{c*} implies that they are more complementary. Through the theorem of implicit function, we obtain that a higher quality of the high-quality variant implies, ceteris paribus (which is equivalent to a higher differentiation), that variants are less close substitutes and less complementary. Intuitively, this means that when the differentiation between variants increases, the monopolist will seek to boost joint purchase if variants are substitutes, and seek to save costs if variants are complementary. The result is shown in the following proposition.

Proposition 5 *In any SPE, a higher differentiation implies lower substitution if variants are substitutes, and lower complementarity if they are complementary.*

Proof of Proposition 5. *Let*

$$\begin{aligned} F(u_l, u_h, u_2) &= \frac{u_l^2}{(u_2+u_l-u_h)^2} - C'(u_2)\,, \text{ and} \\ G(u_l, u_h, u_2) &= \frac{2u_2(2u_2-u_h)}{(4u_2-u_h)^2} - C'(u_2)\,. \end{aligned} \tag{22}$$

From the theorem of implicit function and the results obtained from the Proof of Proposition 3, we have

$$\begin{aligned} \frac{du_2^{s*}}{du_h} &= -\frac{F_{u_h}}{F_{u_2}} = \frac{\frac{2u_l^2}{(u_2+u_l-u_h)^3}}{\frac{2u_l^2}{(u_2+u_l-u_h)^3} + C''(u_2)} > 0, \text{ and} \\ \frac{du_2^{c*}}{du_h} &= -\frac{G_{u_h}}{G_{u_2}} = \frac{\frac{2u_2u_h}{(4u_2-u_h)^3}}{\frac{2u_h^2}{(4u_2-u_h)^3} - C''(u_2)} < 0. \end{aligned} \tag{23}$$

■

Notice that when the monopolist decides on the utility level of the joint purchase option, it compares the profits and costs from encouraging joint purchase and decreasing competition (a higher u_2) with those obtained by saving costs and increasing competition (a lower u_2).

5 Conclusions

We analyze the monopolist's decision about how to design different versions of a good, i.e. whether it decides to make them substitutes or complements, when consumers can buy them simultaneously. The framework of analysis used is a monopoly model with vertical differentiation, where the monopolist also sets prices.

In this context, we find that versioning goods as substitutes or complements is optimal for the monopolist because this strategy eliminates the cannibalization effect. Moreover, when the cost function of making variants complementary is slightly concave or convex, the only equilibrium is when the two variants are substitutes; and if the cost function is convex enough, we cannot identify whether the monopolist makes variants substitutes or complements in equilibrium.

Another result is that if variants are very close substitutes, the monopolist offers the high-quality variant only. In other words, he decides to not create a low-quality variant of an existing good if it is a close substitute for the high-quality variant. Otherwise, versioning goods is optimal, independently of whether variants are substitutes or complements.

We also show that when the differentiation between variants increases, the monopolist seeks to boost joint purchase if variants are substitutes, and to save costs if variants are complementary.

References

Belleflamme, P., 2005, Versioning in the Information Economy: Theory and Applications, *CESifo Economic Studies*, 51 (2-3), 329-358.

Bhargava, H. K. and V. Choudhary, in press, Research Note: When is Versioning Optimal for Information Goods?, *Management Science*.

Deneckere, R. J. and R. P. McAfee, 1996, Damaged Goods, *Journal of Economics and Management Strategy*, 5 (2), 149-174.

Gabszewicz, J.J., Laussel, D. and N. Sonnac, 2001, Press advertising and the ascent of the "Pensée Unique," *European Economic Review*, 45, 641-651.

Gabszewicz, J. J., Sonnac, N. and X. Wauthy, 2001, On price competition with complementary goods, *Economics Letters*, 70, 431-437.

Gabszewicz, J. J. and X. Wauthy, 2003, The option of joint purchase in vertically differentiated markets, *Economic Theory*, 22, 817-829.

Jing, B., 2007, Network externality and market segmentation in a monopoly, *Economics Letter*, 95, 7-13.

Motta, M., 1993, Endogenous quality choice: price vs. quantity competition, *The Journal of Industrial Economics*, 41, 113-131.

Mussa, M. and S. Rosen, 1978, Monopoly and Product Quality, *Journal of Economic Theory*, 18, 301-317.

Salant, S.W., 1989, When is inducing self-selection suboptimal for a monopolist?, *Quaterly Journal of Economics*, 104, 391-397.

Stokey, N., 1979, Intertemporal price discrimination, *Quaterly Journal of Economics*, 93, 355-371.