ThE Papers 05/19

Departamento de Teoría e Historia Económica

Universidad de Granada

Old tittle: A coordination game to elicit social networks: New tittle: An experimental device to elicit social networks

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First version: Sept. 2005 THIS version: May 2006

*Pedro Rey revised this version of the paper. We also acknowledge to Coralio Ballester, Enrique Fatás, Antonio J. Morales & Pablo Guillén and participants at seminars in Granada (Flip-Flop) and Bilbao (Fae^2) and audience at Economic Science Association 2005 Meeting at Montreal. Pablo Brañas-Garza acknowledges finantial support from CICYT-SEJ 2004-07554/ECON and Fundación Ramón Areces.

An experimental device to elicit social networks*

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May 12, 2006

Abstract

This paper proposes an original mechanism to elicit latent social networks. Subjects are invited to reveal their friends' name and surname, together with a score measuring the strength of relationship. According

*We acknowledge comments from Pedro Rey, Coralio Ballester& Pablo Guillén and audience at seminars in Granada (Flip-Flop),Bilbao (Fae^2), Alicante and Montréal (ESA 2005).We also are very grateful to their helpful research assistance to Laura Barrientos, Jose A. Cobos, María Herrera, Eloy Hidalgo, Ana M^a Moreno, Julio Torres, Irene, F. Javier. Pablo Brañas-Garza acknowledges financial support from CICYT-SEJ 2004-07554/ECON. Natalia Jiménez acknowledges financial support by the Generalitat Valenciana under the project CTBPRB/2003/71. Pablo Brañas-Garza, Ramón Cobo-Reyes and Natalia Jiménez acknowledge financial support by Centro de Estudios Andaluces I+D 2006.

to the mechanism, subjects are rewarded of a fixed price either a) when they do not name anybody or b) when the scores of a randomly selected (bidirectional) link are sufficiently close. We test the mechanism's performance in the field. Our main results are: i) a very large percentage of links (75%) were corresponded. ii) the mechanism largely captures strong friendship relations and practically ignores weak relations. A simple model of friend–regarding preferences is developed to explain this evidence.

Keywords: friendship, networks, experiments, other–regarding preferences.

JEL Class.: C93, D85, Z13

1 Introduction

There is a growing literature[1] which highlights the importance of the **structure** of *social networks* in our social and economic life. These works explore (both theoretically and empirically) how the existence of social networks influence individuals' behavior in a wide variety of contexts, from job search to information transmission within a firm[2]. In this respect, being capable to properly map the structure of a network becomes crucial to understand how the network structure influences individuals' behavior and, vice versa, which is the impact of

[1] Take, for example Montgomery [20], Granovetter [14] or Calvó-Armengol and Jackson [5] which deal with job search through social contacts; Bloch [2], Goyal and Moraga [13] develop models related to industrial organization, specifically collusive alliances among corporations; Kranton and Minehart [18], and Wang and Watts [26] which analyze trade in non-centralized markets.

[2] Another different context can be found in Reuben & van Winden [22] where they evaluated reciprocity and emotions with an incomplete and inexact social network. They concluded that the complete social network is needed to better analyze this kind of problems.

individuals' decision on the social network's performance.

One of the reasons why, so far, the interest of the literature on these matters has been mainly theoretical, comes from the difficulty of measuring the structure and strength of social relationships in real-life contexts. By the same token, also the experimental literature works on environments in which the network is **exogenously** induced using monetary incentives.[3]

To the best of our knowledge, the seminal paper which proposes (and tests experimentally) a mechanism for network elicitation, is that of Mobius, Rosenblat & Quoc-Anh [19] (MRQ, hereafter)[4]. In their paper they develop the network elicitation mechanism[5] as follows: *i*) the experiment was conducted via internet, *ii*) the mechanism was a coordination game where subjects received 50 cents only if they named each other with 50 percent probability and zero otherwise, *iii*) subjects were asked how much time they spend on average per week together as a measure of link strength (if subjects agreed on this dimension of their friendship, with an error of half an hour, the probability of obtaining the money increased from 50% to 75%). Results obtained by MRQ mechanism were the following: *i*) 37% of links were symmetric links where both subjects had named each other *ii*) of those, 80% coincided in the time they spend to-

[3] Examples of the experimental research include COORDINATION networks (see Keser et al. [16], Berninghaus et al. [1], Corbae and Duffy [9], among others); COOPERATION networks (see Kirchkamp and Nagel [17], Cassar [6] or Riedl and Ule [24]); BUYER-SELLER networks (see Charness et al. [7]) network FORMATION (see Deck and Johnson [10], Callander and Plott [4], and Falk and Kosfeld [11] and Vanin [25]).

[4] There are some papers which also deal with social networks but their main goal is not the elicitation of the network but to use a network for a particular experiment. Reuben [23] and Brañas-Garza et al. [3] are some examples.

[5] The aim of their paper was to measure social capital in a real-world social networks, so they conduct a Dictator Game controlling for the variable "friendship".

gether ($\pm$ half an hour), and *iii*) the average number of friends elicited was 10 (most participants spent less than half an hour with their 10th friend).

MRQ mechanism motivates this paper. We consider that there are three potential problems in MRQ device *i*) as the experiment is conducted via internet, there exists the possibility of subjects speaking with each other about the game before answering *ii*) this game really gives subjects strong incentives to name a lot of people, given that it does not establish any kind of punishment if individuals do not coordinate in naming each other, *iii*) participants must have friends in order to earn money, moreover the earnings in expected terms are decreasing in the number of friends.

We develop a new mechanism which differs with MRQ mainly in the following dimensions: *i*) we set up a mechanism in which we reward the decision of abstaining to elicit any link, the reason is that people may not be willing to reveal private information –such as friends– since they might be aware of any negative consequence in the use of this information. So, although the analysis of the game would be much more difficult, for ethical reasons we consider that individuals should have an "exit" option (obtaining the **maximum payoff**). *ii*) In our mechanism, strength is measured on a not observable scale (as opposed to some observable measure, such as time spent together), we directly ask subjects about the level of the relationship, so we obtain the measure of this strength directly from subjects and we do not use a proxy for this variable and *iii*) Incentives are not only monetary, we use as well class grades, the main reason for this is that we consider that class grade is also a relevant payoff in the specific context we are analyzing: the classroom network.

Why a mechanism? Without incentives, some problems could arise. One of them would be that some subjects could be against revealing information about themselves to some unknown experimenters. Other potential problem would be that individuals are not going to take the task very seriously, so elicited links wouldn't reflect the real social network. Section 3 shows the results from a treatment without any kind of incentives, which highly support the necessity of a mechanism, given that only 5% of sent links were corresponded. Moreover, 13% of subjects didn't give us the permission to use their data.

To test the robustness of the mechanism to changes in rewards, we have conducted a session with monetary incentives. The results obtained are very similar to the extra-credit point treatment and are explained in detail in Section 3.

Our main experimental results are: *i*) the number of corresponded links is extremely high (75% were corresponded, from which 80% are "exactly", according to our definition), *ii*) very few subjects choose not to name any friend, *iii*) all subjects have at least one link corresponded "exactly", *iv*) the average number of friends elicited is 4.5.

The remainder of the paper is arranged as follows. Section 2 describes the experimental design and procedures, while Section 3 reports the experimental results for the three treatments. Section 4 is devoted to develop a model which explains empirical results. Finally, Section 5 concludes.

2 Experimental design & procedures

2.1 Rewards

We conducted three treatments with different rewards, extra-credit points (TP), monetary (TM) and no incentives (TNI). All sessions were run as classroom experiments. We used classroom frame instead of voluntary participation because (*i*) to elicit a network first you need a real network, obviously the class is the closest network we have access and (*ii*) also, we supposed that (apart from monetary rewards) extra–credit points was one of the relevant payoff in this real situation: the class network. In the TP session, experimental subjects could receive either one extra-credit point or nothing (the grade system in Spain ranges from 0 to 10).

We have also used monetary rewards (5 Euros), to test the robustness of our mechanism to changes in the incentives. Instructions were identical in TP and TM except for the reward. In section 3 we show that main results remain in TM treatment.

The last treatment was conducted without any kind of incentives. There were neither game instructions nor rewards, so subjects simply were asked to reveal some information of their friends as in a mere questionnaire. In the Network Elicitation Mechanism subsection, we explain in detail which kind of information was requested from participants.

To clarify ideas, instructions in the appendix show the difference between the three treatments TP, TM and TNI.

2.2 Subjects

The experiments with the extra-credit point were conducted, in order to ensure the *maximum attendance* [6], in June 2004 during the exam of Microeconomics II, a first year course, at the University of Jaen. We included a “special question” as an additional item of the final exam. Students have very little exposure to game theory. We ran the experiment in three classes: Group 1 was made of students from the Degree in Business Studies (Group 1: morning and Group 2: evening groups) and Group 3 Degree in Law and Business (unique group) at the University of Jaen (Spain). These three groups consisted of 51, 53 and 31 students respectively.

The unique monetary incentives (TM) session was conducted in February 2006 at the University of Granada. The group was compounded of 39 students from Microeconomics I, a first year course in Economics Degree (they had no training in game theory)[7].

The TNI treatment was conducted also at the University of Granada in February 2006. The sample was 40 students from Microeconomics I, a first year course in Business Degree[8].

[6] To analyze the correspondence between links it was necessary to ensure the maximum attendance. If one individual who had been named did not play the game, it was impossible to verify if the link sent to this subject was corresponded. This situation would be problematic in order to study of the performance of our mechanism.

[7] As this treatment was not conducted during an exam, the maximum attendance was not guarantied. So, we have had to remove from our sample some links whose correspondence could not be verified.

[8] This group neither had training in game theory. As in TM session, in this treatment we had to remove some links which could not be checked.

2.3 Network Elicitation Mechanism (NEM)

The basic structure of the Network Elicitation Mechanism (NEM hereafter) is as follows. We asked the students to reveal the name and surname of their friends within the population and, in a scale from 1 to 4, the strength of each relationship.

Let s_{ij} define the "score" given by i to the ij relationship. This score ranges from 1 to 4 as follows[9]:

$s_{ij} = 1$: j is an acquaintance of i

$s_{ij} = 2$: j is a close acquaintance of i.

$s_{ij} = 3$: j is a friend of i.

$s_{ij} = 4$: j is a close friend of i.

Notice that we use the term ACQUAINTANCES to define "weak" social relationships ($score = 1, 2$), whereas we use the term FRIENDS to define "strong" social relationships ($score = 3, 4$). Finally, if individual i does not name individual j, we set $s_{ij} = 0$. Remark that ij or (i, j) represents a directed link from i to j[10].

NEM incentives for the TP treatment are described as follows. Subjects would receive a fixed prize (an extra point (out of 10) in their final exam)[11]

[9] Note that we have used a strength which ranges from 1 to 4 instead of only score 1 or 2 for acquaintances or friends respectively. The reason is that we wanted to increase the possibilities of players when they valued the relationship. The idea was to relax the classification of friendship. Increasing the space of strategies we reduced the transcendence of the decision and it would facilitate players' decisions about some partners.

[10] A directed graph G is an ordered pair $G := (V, A)$ with V, a set of vertices or nodes, and A, a set of ordered pairs of vertices, called directed edges or links (i, j). In other words, a graph or network in which relations among points or vertices are either unequal and reciprocal or non-reciprocal.

[11] For TM treatment, the fixed priced for the experiment was 5€.

under these two CASES:

- CASE 1: if they did not name anybody, or
- CASE 2: if they named at least one subject, if all of the following three rules apply.

rule 1 One out of the elicited links was chosen at random (each link selected with equal probability). Let j denoting the subject named in the randomly selected link;

rule 2 Subject i would receive the price only if also j has named her (i.e. only if $s_{ij} \neq 0$). That is, if Bill named Jimmy Carter and, Jimmy named Bill Clinton (both names and surnames).

rule 3 For obtaining the payoff, the friendship score should be also accurate, that is, if $D_{ij} = |s_{ij} - s_{ji}|$ □ 1, $s_{ij} \neq 0$, i.e. the difference in the scores given by i and j is not higher than 1.[12]

There is another feature of our mechanism which is worth to mention at this stage. According to CASE 1 subjects would secure the prize for themselves not naming anybody. As we have already mentioned, we state this rule to provide an “exit” option for subjects with no friends or reluctant to reveal private information. We were aware that this rule could be a potential problem to elicit the whole network. However, to ease this setback, we exploit the fact

[12] Note that rule 1 relaxes the mechanism, another possibility would have been to eliminate rule 1; so, all links for each individual would be checked. In this case individuals would perceive a high probability of losing the prize, so they would have incentives not to name anyone. As we consider the punishment would be extreme, we introduce rule 1.

that subject i may damage all subjects j who have named i if she doesn't name anybody. So we decided to highlight this possibility in the experimental instructions by explicitly warning subjects that their friends could be damaged by this decision, since "*those subject who named them could lose the prize*" (see instructions in the appendix).

2.4 Game-form and equilibria

Given CASES 1 and 2 the game-form of our mechanism is defined by $G = \{N, S_i, \pi_i\}$, where $N = \{1, 2, ..., n\}$ is the finite set of subjects, $S_i = \{s_{ij} \in \{0, 1, 2, 3, 4\}\}_{j \neq i}$ is the set of strategies of subject i and $\pi_i(\cdot)$ is the outcome function of subject i. The strategy vector of subject i over all relationships with all individuals in N is denoted as $\mathbf{s}_i = (s_{i1}, s_{i2}, ..., s_{ii-1}, s_{ii+1}...s_{in}) \in S_i \subset \Re^{n-1}$ and a strategy for each individual in N is denoted as a matrix containing all strategy vectors, $\mathbf{s} = (\mathbf{s}_1; \mathbf{s}_2; ...; \mathbf{s}_n) \in \prod_{i \in N} S_i \subset \Re^{(n-1)n}$. To define the outcome function $\pi_i(\mathbf{s})$, let $J_i = \{j \in N \setminus \{i\} \mid s_{ij} > 0\}$ be the set of subjects named by individual i, and $j_i = |J_i|$ its cardinality. Let also $\hat{\jmath}$ be the index indicating the subject randomly selected by the mechanism in case j_i >0, i.e. a random variable which can take any value within the range $\{1, 2, ..., j_i\}$ with probability $\frac{1}{j_i}$ (rule 1) only if $j_i > 0$. Then

$$\pi_i(\mathbf{s}) = \begin{cases} 1, if \begin{cases} D_{i\hat{\jmath}} \square\ 1 \; and \; j_i > 0 \\ j_i = 0 \end{cases} \\ 0, \; otherwise \end{cases}$$

To analyze the equilibria of this game we need to reduce its dimension, that is, we will show that this n-player game can be considered as $\begin{pmatrix} n \\ 2 \end{pmatrix}$ 2-player games. The intuition behind this result is as follows. When Player i has to decide which strategy to play with each Player j in the group, i will play this game as if this concrete Player j were the one randomly chosen for being checked (rule 1) given that each link is selected with the same probability. Thus, Player i will choose *independently* a strategy s_{ij}, maximizing her payoffs, for each of the players in the class ($\forall j \in N$). In sum, we state that n subjects playing NEM is equivalent to every pair of subjects (i, j) playing the 2-player game represented in next figure 1.

Figure 1: 2-PLAYER REDUCED NEM GAME

$s_{ij} \setminus s_{ji}$	0	1	2	3	4
0	1 1	0 1	0 1	0 1	0 1
1	1 0	1 1	1 1	0 0	0 0
2	1 0	1 1	1 1	1 1	0 0
3	1 0	0 0	1 1 2	1 1	1 1
4	1 0	0 0	0 0	1 1	1 1

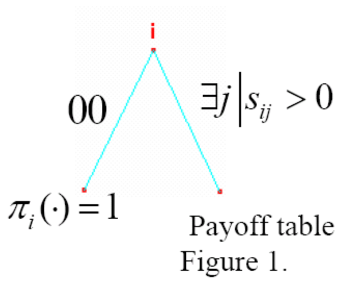
The following proposition 1 states formally the relationship between the NEM and the Reduced NEM game equilibria.

Proposition 1 *A strategy* $\mathbf{s}^* \in \prod_{i\in N} S_i$ *is a pure Nash equilibrium of the NEM if and only if* $\left(s_{ij}^*, s_{ji}^*\right)$ *is a pure Nash equilibrium of each of the* 2×2 REDUCED NEM GAME *for any pair of players* $(i, j) \in N$.

P roof. See appendix 2. ■

After this proposition we can illustrate the NEM game in next figure 2 in the extensive form:

Figure 2: NEM GAME EXTENSIVE FORM.



At the beginning of the game, player i has to decide between two options: i) not naming anybody ($s_{ij} = 00$): which means to play $s_{ij} = 0, \forall j \in N$ (assuring the extra credit point) and, *ii*) naming at least one individual of N ($\exists j | s_{ij} > 0$). If player i chooses the first option, the payoff will be 1. The second option leads player i to play with each individual j in N according to the payoff table described in figure 1. From figure 1 and figure 2, it is clear that $s_{ij} = 0$, $\forall j \in N$ is a weakly dominant strategy.

Note that the NEM can be considered as a coordination game in a certain sense, with some particular features: (i) there are two different possibilities of coordination, "negative coordination" –subjects do not name each other–, or "positive coordination" –subjects name each other–, (ii) "positive coordination" is only plausible if both subjects know the name and surname of each other and (iii) errors are permitted only in "positive coordination".

From figure 1 we can compute the set of Standard Nash equilibria in pure strategies for the 2-player Reduced NEM game[13]:

$$NE_2 = \left\{ \begin{array}{c} (0,0),\ (1,1),\ (1,2),\ (2,1),\ (2,2),\ (2,3), \\ (3,2),\ (3,3),\ (3,4),\ (4,3),\ (4,4) \end{array} \right\}$$

From proposition 1 we can compute the equilibria for the n-player NEM game by calculating the variations with repetition of those 11 equilibria (the total number of equilibria is $11^{\binom{n}{2}}$). Thus, the set of Standard Nash equilibria in pure strategies for the n-player NEM game is:

[13] As this is a one-shot game it isn't very useful to compute mixed strategies equilibria.

$$NE_n = \left\{ (\mathbf{s}_1^*; \mathbf{s}_2^*; ...; \mathbf{s}_n^*) \in \prod_{i \in N} S_i \mid \left(s_{ij}^*, s_{ji}^* \right) \in NE_2,\ \forall i, j \in N,\ i \neq j \right\}.$$

3 Results

After the brief overview of the NEM main properties, we evaluate the performance of the NEM device. To explore in depth NEM outcome, we analyze results in two ways: *i*) aggregate results to measure our NEM ability to obtain the latent network and *ii*) results per capita to complete the description of the obtained network. Remember that the whole experiment comprises 3 different groups ("networks") NET I, NET II, and NET III, with 51, 53 and 31 students respectively. Table 1 summarizes the verification of the experimental device for the three networks. Note that CORRESPONDED links mean that rule #2 is fulfilled (rule #3 can be fulfilled or not), whereas NON-CORRESPONDED referred to links which fail rule #2 and EXACT STRENGTH means that the referred link has been corresponded with the same score, i.e., rule #2 is satisfied and rule #3 holds with $D_{ij} = 0$ (see page 9).

Table 1: NEM VERIFICATION

	NET I		NET II		NET III		TOTAL	
	#links	%	*#links*	%	*#links*	%	*#links*	%
CORRESPONDED	220	76%	115	70%	114	75%	449	74%
(exact strength)	(180)	82%	(82)	71%	(98)	86%	360	80%
NON CORRESP.	69	24%	50	30%	38	25%	157	26%
TOTAL	289		165		152		606	

From Table 1 we state the following.

Result 1 (main): *On average, around 75% of our networks links are corresponded.*

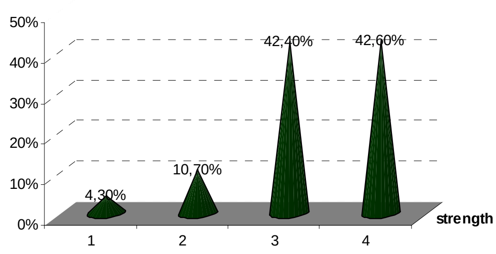
The later means that a remarkable percentage of links fulfill rule #2 (see page 9). Note that this result is remarkable since our accuracy rate doubles the previous experimental evidence (some 36,7% in Mobius *et al.* [19]). Also, our NEM provides a measurement of the strength of the relationship. The good performance of our NEM is also confirmed by

Result 2: *On average, around 80% of the corresponded links show an exact strength.*

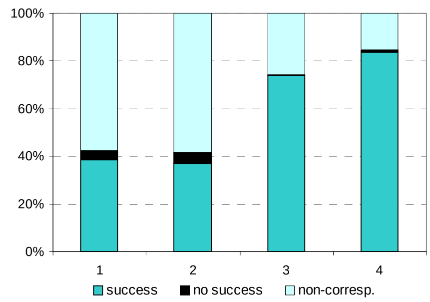
The above results show an overwhelming rate of correspondence between links (in spite of the fact that the probability of coordination without information is very low because individuals can play a lot of strategies).

We now focus on the *strength* of elicited links and their relative accuracy. Figure 3a) reports the relative frequency of each strength s_{ij} in the whole set of links[14].

Figure 3: STRENGTH AND ACCURACY OF ELICITED LINKS



a) Frequency of strength.



b) Percentage of corresponded links.

Thus, figure 3a) shows that the number of links associated to acquaintance relations is very small (15% over the total). Moreover, that the frequency of links associated to "friends" ($s_{ij} = 3$) and "close friends" ($s_{ij} = 4$) is very similar. This evidence is summarized in the following

Result 3: *Our NEM largely captures "friendship" relations (some 85%) and practically ignores "acquaintance" relations.*

Figure 3b) reports the percentage of SUCCESSFUL links –those links which fulfill rule #2 and rule #3 but not necessarily with $D_{ij} = 0$ (this is the difference with EXACT STRENGTH links), (see page 9)–, the NON-SUCCESSFUL links –those which fulfill rule #2 but fail rule #3- and NON-CORRESPONDED ones. Observe

[14] This frequency is an average of the three sessions conducted for the TM.

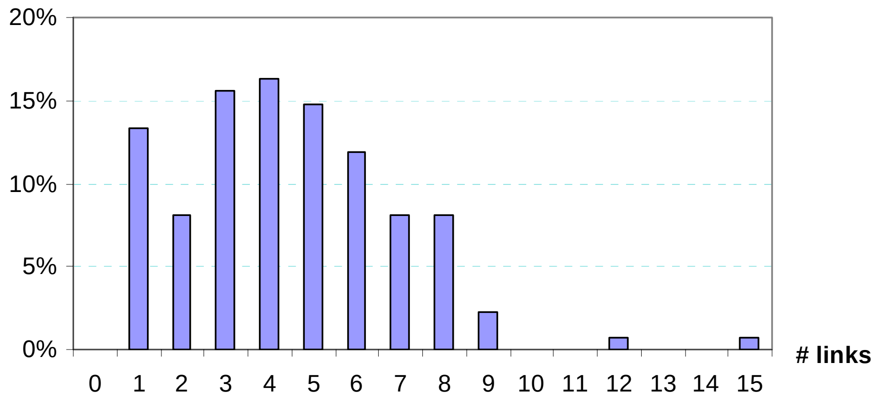
that coordination occurs much more frequently when subjects elicit friendship relationships rather than acquaintances:

Result 4: *Accuracy increases with the level of friendship.*

The above results clearly show that NEM mainly captures *friendship relations.*

Figure 4 reports the relative frequency of links *per capita* of our 135 participants.

Figure 4: Links per capita



Result 5: *The average number of links per capita is 4.49; with a range from 1 to 15 links. The median value is 4 and the mode is also 4.*

Result 5 shows that subjects name some friends and nobody decide to say that he has not any friend. Then, the following question arises: do subjects feel ashamed of saying they have no friends and then they always name someone? If the answer were positive then, there would exist some players who named partners randomly and they would not be corresponded at all[15]. The 135 participants were corresponded once at least; this statement, jointly to the fact that the probability of random coordination was close to zero, let us conjecture that subjects did not choose any partner randomly.

Recall that eliciting zero friends allowed subjects to get the prize for sure. As Figure 4 shows, *no subject opted for this option:*

Result 6: *All subjects revealed at least 1 link.*

The rest of this section explores first the strength of links that subjects sent **per capita** in the 3 networks and afterwards we study the probability of correspondence per capita.

Now, we will analyze the average strength per capita. Let $\widetilde{\ell_k}$ denote the average number of links sent per capita with strength k, where $k \in \{1,2,3,4\}$, that is:

$\widetilde{\ell_k} = \dfrac{\sum_R \left(\dfrac{\ell_R(k)}{n_R}\right)}{3}$, where $n_R = card\{N_R\}$ (# subjects in network R, $R \in \{I, II, III\}$) and $\ell_R(k) = card\{s_{ij} = k \mid i,j \in N_R\}$ (# total links sent with strength k in network R). Then, we have that $\widetilde{\ell_1} = 0.19$, $\widetilde{\ell_2} = 0.50$, $\widetilde{\ell_3} = 1.96$ and $\widetilde{\ell_4} = 1.88$. For instance, $\widetilde{\ell_2} = 0.50$ means that on average in each of the three networks, each subject sent 0.5 links with strength 2. Comparing these measures, we state:

[15] The probability of "positive coordination", given that the population is sufficiently large and strength must be "accurate", is close to zero even in the case a subject knows the name and surname of all people in his class.

Result 7: *The number of links sent to friends is four times larger that those sent to acquaintances. Also note that* $\widetilde{\ell}_3 > \widetilde{\ell}_4$ *and* $\widetilde{\ell}_2 > \widetilde{\ell}_1$.

However, the large percentage of subjects sending links with strength $s_{ij} = 4$ implies that *subjects do not play strategically with friends*[16], i.e., subjects do not name all friends with strength $s_{ij} = 3$.

It is also interesting to analyze the average percentage of non–corresponded links per strength and per capita, that is, to study when subjects fail naming other player.

Let $\widetilde{c}_k$ denote the average **percentage** of corresponded links per capita with strength $s_{ij} = k$, that is:

$$\widetilde{c}_k = \frac{\sum_r \left(\frac{\frac{c_R(k)}{n_R}}{\widetilde{\ell}_k} \right)}{3}, \text{ where } c_R(k) = card\{s_{ij} = k \mid D_{ij} \square 1, i \in N_R\}.$$

The obtained values are: $\widetilde{c}_1 = 0.31$; $\widetilde{c}_2 = 0.32$; $\widetilde{c}_3 = 0.74$; $\widetilde{c}_4 = 0.86$.

Result 8: *On average, the* ***percentage*** *of corresponded links is clearly larger for* friends *than for acquaintances. Also note that* $\widetilde{c}_4 > \widetilde{c}_3 > \widetilde{c}_2 > \widetilde{c}_1$.

In sum, previous results indicate that the number of friendship links ($s_{ij} > 2$) is larger than acquaintances. In sum,

Remark 1 *The NEG captures srong socila relations and nearly ignores weak relations.*

[16] To play strategically means that once a subject decides to name a friend (or acquaintance) and given that the difference in strength must be lower than 1 ($D_{ij} = |s_{ij} - s_{ji}| \leq 1$) to obtain the payoff, the optimal strength is 3 for friends (2 for acquaintances). See figure 1 for a detailed analysis of the strategies and equilibria.

3.1 Comparison with no incentive treatment (TNI)

This section highlights the importance of a mechanism to elicit in a more rigorous way a social network. Moreover, results obtained give evidence of some problems which can arise if there are no incentives. One of the potential problems is that some individuals could be reluctant to reveal private information. Another one, might be that subjects do not take the task very seriously, so elicited links would not reflect the real social network. Table 2 compares results between an unique session run with no incentives (TNI) and the average of the three sessions conducted with credit point reward (TP)[17].

Table 2: COMPARISON NO INCENTIVES (TNI) & CREDIT POINT(TP).

	N	CORRESP	EXACT	NO PERMISSION
TNI	40	4.85%	60%	13%
TP	45	74%	80%	0%

In table 2, N is the average number of subjects, *corresp* is the percentage of corresponded links, *exact* is the percentage of links with exact strength from the corresponded links, and *no permission* refers to the percentage of people who did not sign the authorization[18] to use their data of the experiment (obviously, they did not name anybody or give their own name).

Table 2 supports the above considerations about the potential problems

[17] As we have seen above, results in the three sessions of TP have very similar results, so the average is a good approximation.

[18] At the end of the experiment we asked subjects for signing a written authorization allowing us to show the results of their responses in this paper (of course, we assured subjects' anonymity in the process of showing the results).

which can emerge when using no incentives. On one hand, related to the problem that individuals maybe do not want to reveal private information, 13% of individuals did not allow us to use the information requested in the experiment. On the other hand, the second result shows the amazing high difference in the percentage of corresponded links, 4.85% in TNI as against 74% in TP (so, the network obtained in TNI is not a good approximation of the real network).

In sum, when incentives are not provided the obtained network seems to be unrealistic and less rigorous than if an appropriate mechanism is used.

3.2 Comparison with monetary rewards treatment (TM)

Now, we compare our treatment with points (TP) with those data generated with monetary rewards (TM). TP and TM share most of the features. Table 3 shows the main results for the two treatments.

Table 3: Comparison monetary (tm) and extra-credit point (tp).

	N	CORRESP	EXACT	SUCCESSFUL	NO NAME	%3,4 (corresp)	%1,2 (corresp)
TM	39	69%	52%	100%	7.7%	78%(79%)	22%(32%)
TP	45	74%	80%	98%	0%	85%(80%)	15%(38%)

where, *successful* is the percentage of corresponded links which fulfil rule 3, *no name* is the percentage of subjects who sent no links and *%3,4(corresp)* is the percentage of sent links with strength 3 or 4 (from those, the percentage of corresponded links).

Observe that the percentage of corresponded links in both treatments is very similar.

Although there is a considerable difference in the percentage of exact links, observe that results referring to *successful* variable are not so different for both treatments. Thus, the accuracy of the strength in corresponded links is not perfect in the TM, but very high, given that the difference in strength in all corresponded links is at most 1.

Monetary rewards have not a strong effect in the choice of the $(00, 00)$ equilibrium since the percentage of subject with 0 links in this treatment is only 7.7%.

Finally, table 3 shows that the percentage of friend and acquaintance relationships is very similar in both treatments (78% vs 85% for friends and the complementary for acquaintances), as well as the correspondence percentage (79% vs 80% for friends and 32% vs 38% for acquaintances). Hence, our mechanism captures mainly strong relationships. In sum,

i) previous results suggest that subjects are going to name other individuals and they are not going to assure their prize naming nobody and

ii) the NEM captures strong relations among subjects.

In section 4 we develop a model which explains subjects' behavior.

4 A simple model for the NEM

In this section, we develop a theoretical analysis with the aim of shedding light on the results obtained in previous section. In particular, we are interested in exploring the following result. Despite the fact that subjects earn the prize for sure if they say they have no friends, nobody played its weakly dominant strategy in TP[19]. Hence their preferences may depend not only on material payoffs but also on other considerations.

The induced game by NEM may be formally defined as a 3-tuple $\Gamma = \left\{N, \{s_i\}_{i\in N}, u_i\left(\pi_i\left(s_i, s_{-i}\right), \mu_i^{s_j}\right)\right\}$, where N is the set of participants in the experiment, $s_i = \{s_{ij}\}_{i\neq j}$ is the strategy (strength) of individual i respect to the ij relationship, $\mu_i^{s_i}$ is the probability assigned by player i to the first order beliefs of player j about the strategy s_i and, $u_i\left(\pi_i\left(s_i, s_{-i}\right)\right)$ is the utility associated to the outcome of individual i when he plays s_{ij} and the individual he named (j) plays s_{ji}.

Let us define r_{ij} as the real strength of the relationship between subjects i and j which is perceived by player i.

Our surprising results (with particular reference to the absence of subjects not naming anybody in TP) suggests that subjects preferences regard not only for their own material payoffs but also for their friends payoff. As stated in Geanokoplos et al. [12]:

> "The traditional theory of games is not well suited to the analy-

[19] In TM only 7.7% of the individuals played this weakly dominant strategy.

sis of belief dependent psychological considerations as surprise, confidence, gratitude, disappointment, embarrassment and so on".

A behavioral model which introduces these considerations (defined in the literature as *belief-dependent motivations*) is the Fairness Theory of Rabin [21]. A modified version setting applied to the NEM could be useful to analyze theoretically the reasons for subjects hardly never playing a weakly dominant strategy in traditional game theory. In particularly, it can be shown that applying this model and the "guilt aversion" concept defined by Charness and Dufwenberg [8], the only efficient equilibria coincide with the ones more frequently elicited by the NEM, whenever the weight of belief-dependent motivations in subjects' utility function is sufficiently large. Although no naming any friend is still an equilibrium, under certain conditions is not efficient.

Let us define the utility function of individuals as follows:

$$u_i\left(\bar{s}_{ij}, \bar{s}_{ji}, \mu_i^{s_i}\right) = \pi_i\left(\bar{s}_{ij}, \bar{s}_{ji}\right) + \theta_i^{r_{ij}}\Psi_i\left(\bar{s}_{ij}, \bar{s}_{ji}, \mu_i^{s_i}\right)$$

, where $\pi_i\left(\bar{s}_{ij}, \bar{s}_{ji}\right)$ are the material payoffs corresponded to the payoff table described in figure 1, and $\Psi_i\left(\bar{s}_{ij}, \bar{s}_{ji}, \mu_i^{s_i}\right)$ represents the psychological payoffs which are weighted with a parameter $\theta_i^{r_{ij}}$ which depends on the real relationship between i and j perceived by i; in fact, it may be assumed to be increasing in r_{ij}. Ψ_i can be decomposed into two terms:

$$\Psi_i\left(\bar{s}_{ij},\bar{s}_{ji},\mu_i^{s_i}\right)=k_i\left(\bar{s}_{ij},\bar{s}_{ji}\right)-g_i\left(\bar{s}_{ij},\bar{s}_{ji},\mu_i^{s_i}\right)$$

, where $k_i\left(\bar{s}_{ij},\bar{s}_{ji}\right)$ represents a modified "kindness" function of the one developed by Rabin (1993), and $g_i\left(\bar{s}_{ij},\bar{s}_{ji},\mu_i^{s_i}\right)$ represents the guilt aversion[20] of subject i. Those functions are defined as follows:

$$\begin{aligned}
k_i\left(\bar{s}_{ij},\bar{s}_{ji}\right) &= \left[1+\left(\pi_i\left(\tilde{s}_{ij},\bar{s}_{ji}\right)-\pi_i\left(\bar{s}_{ij},\bar{s}_{ji}\right)\right)\right]\left[\pi_j\left(\bar{s}_{ij},\bar{s}_{ji}\right)-\pi_j\left(\tilde{s}_{ij},\bar{s}_{ji}\right)\right] \\
\tilde{s}_{ij} &\in \begin{array}{ll} \underset{s_{ij}}{\arg\min} & \pi_j\left(s_{ij},\bar{s}_{ji}\right) \\ s.a. & \pi_i\left(\bar{s}_{ij},\bar{s}_{ji}\right) \Box \pi_i\left(s_{ij},\bar{s}_{ji}\right) \end{array}
\end{aligned}$$

$$\begin{aligned}
g_i\left(\bar{s}_{ij},\bar{s}_{ji},\mu_i^{s_i}\right) &= \sum_{s_i\in\check{S}_{ij}}\left(\mu_i^{s_i}\left[\pi_j\left(s_{ij},\bar{s}_{ji}\right)-\pi_j\left(\bar{s}_{ij},\bar{s}_{ji}\right)\right]\right) \\
\check{S}_{ij} &= \left\{ s_{ij} \;\middle|\; s_{ij}\in \begin{array}{ll} \underset{s_i}{\arg\max} & \pi_j\left(s_{ij},\bar{s}_{ji}\right) \\ s.a. & \pi_i\left(\bar{s}_{ij},\bar{s}_{ji}\right) \Box \pi_i\left(s_{ij},\bar{s}_{ji}\right) \end{array} \right\}
\end{aligned}$$

The kindness function of subject i is composed by two terms. The second term compares: *i)* j's payoffs with i's current strategy $\left(\pi_j\left(\bar{s}_{ij},\bar{s}_{ji}\right)\right)$ to *ii)* j's payoffs when i tries to minimize them $\left(\pi_j\left(\tilde{s}_{ij},\bar{s}_{ji}\right)\right)$, whenever i maintains or increases his current payoffs.

[20] Recall that this term is taken from Charness and Dufwenberg [8] and we adapt it to our framework. Charness et al. concept is based on the idea that "a decision-maker suffers from guilt to the extent he believes he hurts others relative to what others believe he will do, and he tends to avoid such choices".

The first term takes into account how much payoff is sacrificing player i. This term will be 1 when subject i does not sacrifice her own payoffs in order to not decrease j's payoffs. This term is strictly higher than 1 only in case subject i must sacrifice his current payoff for not reducing j's payoffs.

To sum up, an individual i will feel that she is being "kind" to subject j if she is not reducing j's payoffs maintaining her own payoffs. Player i's sense of kindness will be higher when she is also sacrificing her own payoffs trying to avoid reducing j's payoffs[21].

The guilt aversion function tries to capture a situation where a subject feels guilty because he decreases another subject's payoffs. Here, we consider the guilt in a strong way given that to compute it, subject i compares what subject j obtains with i 's current strategy, $\bar{s}_i$ and what it would be the maximum payoff of j if i would favor j utmost. That is, the guilt is given by the difference between the payoffs player j could obtain if player i tried to maximize them and the payoffs player j obtain with the current player i's strategy. This difference is pondered by player i second order beliefs, i.e, the probability that i thinks that player j assigns to player i playing a strategy which maximizes j's payoffs.

These functions should be normalized but in our setting this is not necessary given that payoffs are always 0 or 1.

In order to simplify the analysis we formulate the following assumptions:

Assumption 1: *There is Common Knowledge between any subjects i and j,*

[21] This function can be defined different depending on the reference point, but in our setting as material payoffs are or 1 or 0, most of them are analogous. This is also true for the guilty aversion function.

on the game in which they are enrolled, a reduced form of the game is represented in figure 1 (page 11).

Assumption 2: *Each subject has only psychological considerations over other individuals payoffs on the part of material payoffs which directly depends on himself.*

This implies that at the moment of computing the psychological payoffs of individual i when naming subject j, he considers that the random selected link is the link ij. That is, player i is not going to introduce in his psychological payoffs (respect to individual j) considerations about the strategies that other players are playing with individual j.

In figure 5 we compute payoffs according to utility function [4] of any two subjects i and j in N.

The Nash equilibria in pure strategies in the 2-player reduced NEM game remain:

$$NE_2^{'} = \{(0,0),\ (1,1),\ (1,2),\ (2,1),\ (2,2),\ (2,3),\ (3,2),\ (3,3),\ (3,4),\ (4,3),\ (4,4)\}.$$

However, the main difference within this model is that if the condition $1 < 1 + \theta_k^{r_{kh}}$ holds $\forall k \neq h$, then $(0,0)$ is not an efficient equilibrium. And therefore, the equilibrium in which every subject doesn't name anybody is not an efficient equilibrium in the n-player NEM game. This could be a possible explanation why no subject play this equilibrium in TP treatment and very few people in the TM treatment.

5 Conclusions

Recent literature highlights the importance of obtaining the architecture of social interactions underlying subjects. This paper provides an innovative mechanism to elicit social networks.

In this mechanism friends and acquaintances are costly in the sense that subjects have the probability of losing the payoff when they name a friend or acquaintance under some preferences.

We have conducted three different treatments which differ in the type of incentive. The first two were based on credit points and monetary awards. They display very similar results, which indicates that the NEM is robust to changes in awards. The last one was the baseline and was run with no incentives at all. Its results decidedly support the necessity of a mechanism to elicit social networks, given the pretty reduced percentage of correspondence of 5%.

The main difference between our mechanism and the previous ones (MRQ) is that the NEM provides very low incentives to name a lot of people given that if subjects do not name anybody, they assure the maximum payoff (note that we introduced this rule in the mechanism in order to provide an "exit" option for those subjects with no friends or reluctant to reveal their private information). In each decision subjects take respect to a friend or an acquaintance, they are aware about the risk of loosing a sure payoff. Therefore, all relationships captured by this mechanism are true friends (recall that the probability of a random coordination is close to zero and the percentage of corresponded links is 70% and 75% for TM and TP respectively) but it might be friends that are not elicited by our device. Even if this is the case, we are achieving our goal: assuring true friendship relations by penalizing mistakes in coordination when naming friends. In future research, we want to study other experimental problems where the friendship relations are relevant , so we can extract a sample of true friends from our network and control the “friend” variable, for a more accurate analysis of the problem.

The most surprising result is that there is no subject (in the treatment where rewards were credit points) or very few subjects (7.7% in monetary rewards treatment) who reveal no link despite of this being a weakly dominant strategy. It is important to note that this result is not due to the fact that individuals feel ashamed to say they have no friends. The reason is that all subjects are corresponded “exactly” (with no difference in strength) by at least one subject and we have already explained that the probability that this coordination happens at random is negligible.

The latter results suggest that subjects preferences regard not only for their own material payoffs but also for their friends payoff. Thus, in an attempt to explain those results, we develop a behavioral model which introduces other considerations denominated *belief-dependent motivations* in the literature. We combine the concept of “kindness” from the Fairness Theory of Rabin [21] and the notion of “guilt aversion” from Charness et al. [8]. This setting is adapted to our NEM to analyze theoretically the reasons for subjects never playing a weakly dominant strategy in traditional game theory. In particularly, it can be shown that the only efficient equilibria coincides with the ones more frequently elicited by the NEM, whenever the weight of belief-dependent motivations in subjects' utility function is sufficiently large. Although, no revealing any link is still an equilibrium, under this setting it is not efficient.

Finally, remark that the main result of our mechanism is that a significant percentage of $70\% - 75\%$ of the links were corresponded (names and surnames) and, from those nearly 100% display a quite accurate strength (difference in strength 1 or lower). The correspondence obtained by NEM doubles previous experimental evidence (MQR). These results let us think that our network captures most of the relationships among individuals.

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6 Appendix 1

INSTRUCTIONS[22]

Hello, now you're going to take part in an Economic Experiment. We thank you in advance for your collaboration. This is part of a project coordinated by a teacher from the University of Alicante and he asks you for your collaboration to carry it out. The aim of this Experiment is studying how individuals take their decisions in certain environments. The instructions are simple.

If you follow them carefully, you will receive an additional POINT IN THE FINAL MARK OF MICROECONOMICS II [AMOUNT OF MONEY] confidentially at the end of the experiment.

You can ask the queries you may have at any time, raising your hand but without speaking. Except for these questions, any kind of communication between you is forbidden and subject to your expulsion from the Experiment.

Please, write a list with the name and surname of all you friends from the class. After their names, you have to write a number:

1 if you hardly know him/her; 2 He/she is only someone you know; 3 if he/she is your friend; 4 if he/she is a very close friend.

¿How do I GET THE POINT [RECEIVE THE MONEY]? We take your list and take out randomly the name of one (only one) of your friends (the ones you have mentioned); then, we look at your friend's list and see whether:

i) he/she has mentioned you and

ii) he/she has scored you with a similar number to the one you have rated

[22] In CAPITAL are highlighted differences between TP and TM (TM in brackets).

him/her (this means a maximum difference of one point).

If i) and ii) are affirmative you win THE POINT [5€]. If i) or ii) fails, then you win nothing (0 POINT [0€]).

Example. My List is:

Jose Pérez with a 3.

Juan Martínez with a 4.

Emilio López with a 1.

Jose Antonio Rodríguez with a 2.

Randomly, José Pérez was chosen from my list. They then looked at his list and he had rated me with a 4. As the difference in the scoring was just one point, I win THE POINT FOR MICROECONOMICS II [5€]. If I had rated him with 2 points, I would have won nothing.

NOTICE 1. If you mention no-one, you also receive THE POINT FOR MICROECONOMICS II [5€].

NOTICE 2. (about the notice above). Be aware that if you mention no-one but someone mentions you, you will be prejudicing him or her. In other words, a friend who mentions you would not receive THE POINT FOR MICROECONOMICS II [5€] because you don't include him/her in your friends' list[23].

[23] For the TNI treatment, instructions were as follows:
Hello, now you're going to take part in an Economic Experiment. We thank you in advance for your collaboration. This is part of a project coordinated by a teacher from the University of Alicante and he asks you for your collaboration to carry it out. The aim of this Experiment is studying how individuals take their decisions in certain environments. The instructions are simple.
You can ask the queries you may have at any time, raising your hand but without speaking. Except for these questions, any kind of communication between you is forbidden and subject to your expulsion from the Experiment.

7 Appendix 2

Proposition 1 A strategy $\mathbf{s}^* = (\mathbf{s}_1^*; \mathbf{s}_2^*; ...; \mathbf{s}_n^*) \in \prod_{i \in N} S_i$ is a pure Nash equilibrium of the NEM game if and only if $\left(s_{ij}^*, s_{ji}^*\right)$ is a pure Nash equilibrium of each of the 2-player Reduced NEM games for any pairs of players (i, j) in N.

Proof of Proposition 1:.

For the if part, first suppose that $\mathbf{s}^* = (\mathbf{s}_1^*; \mathbf{s}_2^*; ...; \mathbf{s}_n^*)$ is a pure Nash equilibrium for the NEM game. Then, it satisfies:

i) $\mathbf{s}^* = (\mathbf{s}_1^*; \mathbf{s}_2^*; ...; \mathbf{s}_n^*) \in \prod_{i \in N} S_i$

ii) $\pi_i\left(\mathbf{s}_i^*; \mathbf{s}_{-i}^*\right) \geq \pi_i\left(\mathbf{s}_i; \mathbf{s}_{-i}^*\right)$, $\forall \mathbf{s}_i \in S_i$, and $\forall i \in N$.

For the structure of the game (only one link, $s_{ij} > 0 \leftrightarrow j \in J_i$, is randomly checked for each subject in N), payoffs can be considered in expected terms. So condition *ii)* becomes:

ii)' $\pi_i^e\left(\mathbf{s}_i^*; \mathbf{s}_{-i}^*\right) \geq \pi_i^e\left(\mathbf{s}_i; \mathbf{s}_{-i}^*\right)$, $\forall \mathbf{s}_i \in S_i$, and $\forall i \in N$.

According to the rules explained in section NEM (see page 9), *ii)'* can be developed as follows.

Case 1 $\exists j \in N \left| s_{ij}^* > 0\right.$

Please, write a list with the name and surname of all you friends from the class. After their names, you have to write a number:

1 if you hardly know him/her; 2 He/she is only someone you know; 3 if he/she is your friend; 4 if he/she is a very close friend.

Thank you very much.

iii) $\sum_{j\in J_i^*} \frac{1}{j_i^*}\pi_i\left(s_{ij}^*, s_{ji}^*\right) \geq \sum_{j\in J_i} \frac{1}{j_i}\pi_i\left(s_{ij}, s_{ji}^*\right)$, $\forall s_{ij} \in \{1,2,3,4\}$, $\forall j \in J_i$ and $\forall i \neq j \in N$.

Note that we have previously denoted $J_i = \{j \in N \setminus \{i\} \mid s_{ij} > 0\}$, $j_i = |J_i|$, $J_i^* = \{j \in N \setminus \{i\} \mid s_{ij}^* > 0\}$ and $j_i^* = |J_i^*|$.

Another feature which can be deduced from the particular structure of the NEM n-player game is that in all pure equilibria all subjects must obtain payoffs equal to 1 (if not, it is because they have obtained 0 payoff, so the have incentives to deviate). Hence, it is satisfied that:

iv) $\pi_i\left(s_{ij}^*, s_{ji}^*\right) = 1, \forall s_{ij}^* \in \{1,2,3,4\}, \forall s_{ji}^* \in \{0,1,2,3,4\}\ \forall j \in J_i$ and $\forall\{i,j\} \in N$, $i \neq j$.

In addition, it can be considered that when $s_{ij}^* = 0$, $\pi_i\left(s_{ij}^*, s_{ji}^*\right) = 1$, given that if a subject doesn't name anybody she obtained 1 for sure.

Finally, it is directly from *iv)* and the previous consideration that the following conditions hold:

v) $\pi_i\left(s_{ij}^*, s_{ji}^*\right) \geq \pi_i\left(s_{ij}, s_{ji}^*\right), \forall s_{ij} \in \{0,1,2,3,4\}$ and $\forall\{i,j\} \in N$, $i \neq j$.

Thus, it have been proved that $\left(s_{ij}^*, s_{ji}^*\right)$ is also an equilibrium in pure strategies for any pair (i,j) of subjects in N of the 2-player game represented in figure 1 (see page 11).

Case 2 $s_{ij}^* = 0$, $\forall j \in N$, $j \neq i$.

In this case it is trivial that if each subject i in N doesn't name anybody, those strategies also constitute an equilibrium for the 2-player Reduced NEM game.

For the only if part, suppose that $\left(s_{ij}^*, s_{ji}^*\right)$ is an equilibrium in pure strategies for any pair (i, j) of subjects in N of the 2-player Reduced NEM game (figure 1). Then, by definition:

i) $\pi_i\left(s_{ij}^*, s_{ji}^*\right) \geq \pi_i\left(s_{ij}, s_{ji}^*\right), \forall s_{ij} \in \{0,1,2,3,4\}$ and $\forall \{i,j\} \in N$, $i \neq j$.

With an analogous reasoning as in the if part, it can be deduced that:

ii) $\pi_i\left(s_{ij}^*, s_{ji}^*\right) = 1$, $\forall s_{ij}^* \in \{0,1,2,3,4\}, \forall j \in J_i$ and $\forall \{i,j\} \in N$, $i \neq j$.

Case 1 $\exists j \in N \left| s_{ij}^* > 0 \right.$

From *ii)* it can be computed the payoff for the NEM n-player game in equilibrium:

iii) $\sum_{j \in J_i^*} \frac{1}{j_i^*} \pi_i\left(s_{ij}^*, s_{ji}^*\right) = 1,\ \forall s_{ij} \in \{1,2,3,4\}, \forall j \in J_i$ and $\forall i \neq j \in N$.

Finally, given that payoffs in the NEM n-player game can take only two values: 0 or 1, it is clear that a convex combination of payoffs is always lower or equal to 1, and hence the equilibrium conditions hold:

iv) $\sum_{j \in J_i^*} \frac{1}{j_i^*} \pi_i\left(s_{ij}^*, s_{ji}^*\right) \geq \sum_{j \in J_i} \frac{1}{j_i} \pi_i\left(s_{ij}, s_{ji}^*\right),\ \forall s_{ij} \in \{1,2,3,4\}, \forall j \in J_i$ and $\forall i \neq j \in N$.

Case 2 $s_{ij}^* = 0$, $\forall j \in N$, $j \neq i$.

In this case it is trivial that if each subject i in N doesn't name anybody in each of the 2-player Reduced NEM games, those strategies also constitute an equilibrium for the n-player NEM game. ■