Understanding statistical literacy: About knowledge of contexts and models

Comprensión de la cultura estadística (alfabetización estadística): Sobre el conocimiento de contextos y modelos

Iddo Gal

University of Haifa, Israel

Abstract

This paper examines challenges facing the promotion of statistical literacy, and better understanding of civic statistics. It focuses on one key notion of "context knowledge" ,which is part of the knowledge bases needed for statistical literacy. The paper examines the notion of "meaningful and important" contexts that are worthy of attention when teaching for statistical literacy, discusses how to bring such contexts into the classroom and make sure students understand them, and what questions or tasks about meaningful and important contexts are suitable or worthy to ask in the classroom. In addition, the paper briefly examines issues related to notions of "models" and "modeling," and how they relate to context knowledge. Based on the analysis offered, some implications and recommendations are discussed.

Keywords: Statistical literacy, social issues, critical interpretation, citizenship skills, numeracy, contextual knowledge.

Resumen

Este documento examina los desafíos de la cultura estadística (alfabetización estadística) y de una mejor comprensión de las estadísticas cívicas. Se centra en una de las bases de conocimiento necesarias para la alfabetización estadística. El trabajo examina la noción de contextos significativos e importantes, que merecen la atención cuando se enseña para conseguir la cultura estadística, discute cómo llevar estos contextos a la clase y asegurar que los estudiantes los comprenden y qué preguntas o tareas sobre contextos significativos e importantes se pueden considerar o merece la pena llevar a la clase. Además, el documento examina brevemente cuestiones relacionadas con las nociones de "modelo" y "modelización", y cómo se relacionan con el conocimiento del contexto. Con base en el análisis ofrecido, se discuten algunas implicaciones y recomendaciones.

Palabras clave: Alfabetización estadística, temas sociales, interpretación crítica, habilidades de ciudadanía, cultura aritmética, conocimiento contextual

1. Introduction

Within the domain of statistics education, teaching for the development of learners' statistical literacy has a special place, because it contributes to a critical and general educational outcome - the ability of graduates of education systems to engage with statistics in the real world. In this talk I want to problematize the notion of statistical literacy, challenge us to think deeper about some of its elements with a focus on contexts and models, and discuss some resulting implications. The talk is based in part on my own work, in part on insights gained through collaborative work with the ProCivicStat project[i], which is also mentioned in a companion keynote talk by Joachim Engel also given at CIVEEST 2019.

As a brief background, statistical literacy traditionally refers to comprehension of and engagement with statistical messages that may appear in the media, at work, and other

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real life contexts. Thus, it is a construct related to adult numeracy (Gal, 1997) and mathematical literacy (Kilpatrick, 2001), but goes beyond them, given differences between what is subsumed under "statistics" when broadly viewed, and the domain of mathematics.

Statistical literacy has been defined in several ways in the literature (Haack, 1979). My own definition (Gal, 2002) reads:

> The motivation and ability to access, understand, interpret, critically evaluate, and if relevant express opinions, regarding statistical messages, data-related arguments, or issues involving uncertainty and risk.

In a prior paper (Gal, 2002) I offered the first comprehensive conceptual model describing the building blocks of statistical literacy. (That framework has been based in part on earlier work I did on the conceptualization of adult numeracy). The original model is shown in Figure 1.

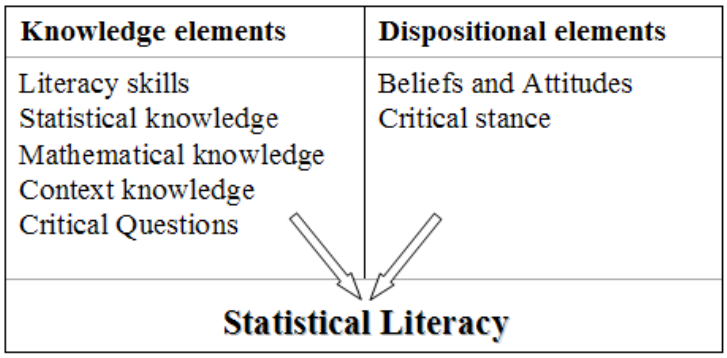

Figure 1. A model of statistical literacy (Gal, 2002)

Additional building blocks or revised facets that draw in part on this model have been described in recent work as part of the ProCivicStat project, as outlined in the Keynote talk by Joachim Engel in this CIVEEST 2019 conference, and explained in more detail in Ridgway, Nicholson and Gal (2018) and other publications by ProCivicStat. For an overview, and description of new developments, see the ProCivicStat *Call for Action and Recommendations* (ProCivicStat partners, 2018), at: http://iase-web.org/islp/pcs.

The last two decades have seen increased attention to the teaching of statistics in general in school systems (Batanero, Burrill, & Reading, 2011) and at the college level, as well as more attention to statistical literacy (e.g., Budgett & Rose, 2017; Callingham & Watson, 2017; Watson, 2013). Yet, the construct of statistical literacy continues to remain elusive and at times with multiple meanings, and this challenges education systems and teachers. Several papers have referred to statistical literacy as understanding and using the basic language of statistics (see Ziegler & Garfield, 2018), without direct connection to the use of statistics in the outside world. I continue to hear presenters equating statistical literacy with entry-level content taught in the early phases of a statistics course, i.e., the simple or primitive aspects of regular statistical knowledge (i.e., "serious" statistics which are more formalized and advanced).

However, in my view statistical literacy is not a watered-down version or simple subset of formal statistics, but a stand-alone complex competency with many unique elements that are seldom addressed in regular statistics or mathematics instruction, although it of course requires some knowledge of statistics and mathematics, as shown in Figure 1.

With the above in mind, to advance professional practice and scholarly discussions regarding statistical literacy, it is important to continue and examine the core ideas that underlie what statistical literacy means and implies. This paper focuses on one key notion of "context knowledge" which is part of the knowledge bases noted in Figure 1. In addition the paper also briefly examines issues related to seemingly separate notions of "models" and "modeling," which receive increasing attention from statistics and mathematics educators, and highlights how they relate to context knowledge. Based on the analysis offered, some implications and recommendations are discussed.

2. The centrality of "context" for statistical literacy (and for statistics education in general)

"Context" seems to be a simple idea. After all, we all live our lives in context, not in a vacuum. And as educators, we hope that what we have taught our learners will help them to function effectively in multiple life contexts. However, within the classroom or lecture hall, "context" is far from being a simple notion. This is because "context" is not automatically present in the classroom - we need to bring it in. But on this, there is a difference between mathematics education (which is where much of statistics education happens in schools) and statistics education (which covers the school level as well as tertiary levels and many other teaching contexts, but at the tertiary level occurs mainly on its own, not under mathematics education).

In mathematics education (at the k-12 level) there are many discussions about contexts, authenticity, or realism of tasks and problems, and debates about the extent to which mathematics educators should worry about or relate to contexts as part of either teaching/learning or assessment processes. A succinct review of various views on contexts and contextualization can be found in Greatorex (2014). For example, as du Feu (2001) argues, there are five levels or types of contexts (or lack thereof) for tasks and activities used in the mathematics classroom: context-free, real, cleaned, parable (fictitious but possibly real), and contrived (invented to drive a mathematical problem, but usually not realistic). It follows that when learning mathematics, the role of context is debatable, and a real context is not always seen as needed, and sometimes is even perceived as a distraction that hurts the learning of the abstract ideas of mathematics.

In contrast, when teaching statistics, context is supposed to be a cornerstone of all activities. After all, the history of statistics shows that they were invented in large part to serve practical needs of states and enable administrators to understand the demographics and conditions of their citizens and economies. Moore (1990) has argued that statistics are "numbers in context", and that hence, in statistics, context is the source of meaning and basis for application of statistical procedures and for the interpretation of obtained results.

Let us now reconnect with the statistical literacy point of view, which emphasizes the importance of developing functional competencies. From this perspective, it can be argued that our overarching goal in teaching statistics is that when students leave the classroom and eventually graduate, they are both able *and* willing to engage effectively and make sense of the statistics and statistical messages that flow to them as citizens or consumers of statistical messages. If so, it is critical to ask - how are the *context*s in which real-world statistics, whether related to social, economic, environmental, or other life topics, chosen by teachers and brought into the statistics classroom?

Questions about the place of the context in statistics education are important, if we want to remain true to the nature of statistics as "numbers in context". Yet, this is where I think we face a problem. There are many calls to improve conceptual understanding of "big ideas" in statistics and put less emphasis on computations and procedural knowledge (Chance & Rossman, 2001; Cobb, 2007; Malone et al., 2012; Ograjenšek & Gal, 2016). The *Guidelines for Assessment and Instruction in Statistics Education* (GAISE, 2016), which are receiving wide attention all over the world, emphasize in particular the need and importance of using "real data in the classroom", and are posited as leading to development of statistical literacy (Schield, 2016). However, there is a big difference between using "real" *data*, and linking instruction to *meaningful and important contexts.* This is because what matters is *not* the data, but the *answers and insights* we seek in the data. Thus, the *context* is the motivator for the questions which inform the instruction in statistics and the answers we generate based on given data. After all, regular data are collected *because* somebody out there in the real world has specific information needs and specific questions for which they seek answers.

The above leads to three key questions, listed in Figure 2:

1. What are "meaningful and important" contexts that are worthy of attention when teaching for statistical literacy?
2. How do we bring such "meaningful and important" contexts into the classroom and make sure students understand them?
3. What questions or tasks about meaningful and important contexts are suitable or worthy to ask in the classroom?

Figure 2. Three questions about contexts and tasks in statistical literacy

These questions are very broad, and each could be the focus of a separate paper, but given space constraints, each is only discussed briefly below. While the answers given below may be short and perhaps incomplete, it is hoped that when viewed together, they sketch an informative picture that can help further scholarly discussion about the practice of statistical literacy in the classroom. Note that in addition, a later section briefly reflects on how contexts are *modeled* in (official or civic) statistics; this can further inform some question-posing or task design for teaching statistical literacy.

2.1. About meaningful and important contexts

Two conditions describe what a meaningful and important context is. First, *the context should be authentic*, i.e. naturally occurring in the outside world, not contrived or fictitious. Thus, if teachers invent a local survey or make up fictitious data only so students have a survey to work on (e.g., about what are students' political opinions, body measurements, career plans, etc.), the data would be "real" in the sense that they describe some phenomena in the world which can be described statistically, but the context is made up and does not serve as a sensible source for a true "need to know" (see below).

The second condition is that *the context should invoke a genuine "need to know" and be of interest to an outside actor or stakeholder*, such as to politicians; policy-makers and managers in a government ministry (department), a public agency or a business

organization; to leaders or activists in a community-based or non-profit organization; or to concerned and empowered citizens. The notion of "Need to know" has been promoted by Ograjensek and Gal (2016), who argue that a critical challenge in statistics education involves issues of students' acceptance of *purpose* of the whole activity of generating data and interpreting a statistical message or data, and the value of good data within the broader societal contexts which statistics aims to inform. In combination, the two conditions also aim to maximize students' sense of relevance of the context to their current or future lives, and improve their motivation and interest.

The notion of a meaningful and important context has been examined in detail in the ProCivicStat project, under the dual terms "civic statistics" and "burning issues" (see *Call for Action and Recommendations* [ProCivicStat Partners, 2018], and the keynote by Engel for CIVEEST 2019). The assumption is that citizens need to be aware of and critically understand statistics and statistical messages regarding past trends, present situations, and possible future changes in key social and economic topics, which collectively are grouped under the heading *civic statistics*.

Civic statistics may be about topics such as demographics, employment, wages, migration, health, crime, poverty, access to services, education, human rights, public expenditures, pollution, production, and related areas. Such topics are seen as "burning" because they affect social and economic progress, impact well-being of citizens and communities. Hence, they are at the focus of public and political dialogue, of much interest to policy setting and to decision-makers at the national, regional, and local levels, and often discussed in the media. For these reasons, civic statistics contexts can be seen as both meaningful and important to a country or society, and hopefully to teachers and students.

ProCivicStat has argued (ProCivicStat Partners, 2018) that civic statistics have several general characteristics, related to *multivariate phenomena* that are analyzed and reported through *aggregation or disaggregation* and the *use of models* and in particular *indicators*, and involve *dynamic data* that change or spread over time. Such data may often be conveyed to the public via *rich texts* (printed or spoken), e.g., articles in the printed or digital media, TV news programs, social networks, and the like. Further, data and findings about trends in society are also communicated via *rich visualizations* that are broader in scope and at times more sophisticated compared with the (limited) range of ideas and representations included in introductory statistics or high-school classes.

2.2. How to bring contexts and statistics about them into the classroom

The first part of this question seems easy - but the second part is much more challenging. As explained above, there are many meaningful and important contexts; hence, teachers have much freedom to choose sources and topics in order to fit their and their students' interests. In his Keynote talk and paper, Joachim Engel has discussed some of the data sources from which raw data and datasets can be found, and illustrated the use of a tool developed by ProCivicStat, i.e., CivicStatMap (see: http://iase-web.org/islp/pcs), to coordinate datasets, background contextual information, and class activities about them.

Although datasets are essential for learning many statistical ideas, the citizen, or decision maker, seldom sees a raw dataset or needs to analyze it by herself or himself (even though in theory this could be possible, in the era of open data). Usually, citizens and decision makers alike are mostly consumers of statistical messages, not producers

or "number crunchers". The data are already analyzed by statistics producers or the media. Hence, the raw inputs that students should be able to learn to comprehend, analyze, and reason about, *cannot* be limited to data and datasets–but must include a healthy portion of text-based messages (or infographics, data displays, and dynamic visualizations) taken from the print and digital media, from statistics producers and in particular from official statistics agencies, TV programs and blogs, etc.

So, assume you found a suitable article, which conveys a set of statistical findings in a local or national newspaper or in a digital outlet. Typically, it will contain mainly or only text! Some graphs or charts may be included as well, but they are seldom the core of statistical messages that the public encounters. The context and the societal issues which motivate the data will not be clear to students without reading surrounding text or background information (e.g., on Wikipedia), which can save detailed explanation by the teacher. Thus, text comprehension is critical, since textual sources are the elements which need to be brought into the classroom to enable learners to directly engage with real-world tasks involving meaningful and important contexts.

2.3. Worthy questions or tasks for developing statistical literacy in context

What questions should you ask your students about the resource/article you found, in order to explore and develop their statistical literacy? And what questions would you like *them* (your students) to pose to the author/source of the article, or *about* the article or graphical display they were given? These are two different issues. The former relates to the approach to teaching about or for statistical literacy. The later refers to the many questions that together are called "critical questions" in the model in Figure 1, and "worry questions" in Gal (2002). Here I want to focus only on former, i.e., teaching-related questions, as they are a key for developing students' use of critical questions

As we all know, "question-posing" is part of the art of teaching. The term "question" should be seen broadly, to also cover the diverse tasks and assignments given to students, whether in class or as part of homework. Ainley, Gould and Pratt (2015) have argued for the use of tasks that have two characteristics, a clear purpose and present an engaging challenge for the learners, whether or not they refer to a real-world application. As Ograjensek and Gal (2016) argue, the questions posed to students should emerge from or relate to authentic contexts, because the context, not the data, dictates the "need to know", i.e., what matters to policy makers, public officials, managers, or researchers who are involved with social or economic issues. This implies that contexts used for statistics instruction should not be "contrived" or "fictitious" or "cleaned", as is very often the case.

A. ***Some background and a simple example.*** Question-posing when developing statistical literacy about meaningful and important societal issues raises dilemmas and requires instructional decisions. To illustrate some of them, I examine below a simple situation (based on Gal & Trostianitzer, 2016) – understanding information in a data table from an official statistics agency. (This in fact is seldom encountered by citizens "as is", but instead embedded in much text).

As a brief background, numerous scholars have examined issues associated with reading and interpreting information in tables and graphs, given their key role in statistics. Curcio's (1987) well known framework has discussed three stages, i.e., reading the data, reading between the data, and reading beyond the data. Friel, Curcio and Bright (2001) have outlined six related cognitive processes: reading, describing,

interpreting, analysing, predicting and extrapolating data. Building on these and other sources, Kemp and Kisanne (2010) offered a five-step framework to guide class discussions: *1: Getting started; 2: WHAT do the numbers mean? 3: HOW do they change or differ? 4: WHERE are the differences or relationships? and 5: WHY do they change?*

Later, Prodromou (2015) has demonstrated how Curcio's original conceptualization can be used in group-work on interpreting large 2-way tables comparing population trends (demographic changes, emigration and immigration, etc) in multiple countries.

Gal and Trostianitzer (2016) have argued that to address questions of purpose and motivation when using data or products of official statistics agencies that involve statistics about society, there is a need to extend the five-step framework introduced by Kemp and Kisanne (2010) by adding a sixth step that focuses on the *actual societal meaning or implications of the data*. This is illustrated in Figure 3.

Central Bureau of Statistics: Projections 2005 - 2030

Note: Figures are in Thousands

	Arabs		Jews		2005
Total (000)	%	Number (000)	%	Number (000)	Age
	59%	800	41%	2,284	0-24
	38%	512	48%	2,659	25-64
	3%	44	11%	629	+65
6,928 =	100%	1,356	100%	5,572	TOTAL

	Arabs		Jews		2030
Total (000)	%	Number (000)	%	Number (000)	Age
	47%	1,115	35%	2,737	0-24
	46%	1,082	51%	4,004	25-64
	7%	165	14%	1,129	+65
10,232=	100%	2,362	100%	7,870	TOTAL

Managerial simulation

You are a new, motivated manager at organization X.

You received from your senior management the following table, which is based on new data from the Central Bureau of Statistics.

The table shows projections regarding demographic trends among certain social and age groups.

Your manager asked to get your personal evaluation regarding the following key questions.

Q1. What key trends or changes you see between the social and age groups, for 2005?
Q2. What key trends or changes you see between the social and age groups, for 2030?
Q3. What key trends or changes do you see between 2005 and 2030?
Q4. What reasons or factors may explain the projected changes between 2005 and 2030, regarding these social and age groups? please list key ones.
Q5. Given these trends or changes, what are the implications or impact for long-range planning or needed managerial decisions in [organization]? Why?

Figure 3. Sample task - Table reading with opinion elicitation

Figure 3 shows a task given to students in a first-year course "Introduction to Human Services" that I am teaching in a Department focused on preparing students to manage service operations in organizations in the public and private sectors in Israel, which is a country with much ethnic, religious, and social diversity. The task was presented as a management simulation related to understanding client diversity, i.e., students had to operate in a managerial context in which they can be a real actor, not a passive or disinterested observer who just analyses data.

The 3-way data table in Figure 3 is based on real population projections published by the Israeli Bureau of Statistics and relates to anticipated changes in the demographic composition of the two largest subgroups in Israel, Jewish and Arab, hence is couched in a rich social reality known to all students. The data table in Figure 3 is comprised only of counts (in thousands) and percentages, which are typical in demographic tables, i.e., employs seemingly simple statistics that should be accessible to all students at the undergraduate or high-school level, even without learning any statistics. Yet, the table overall is richer than typical 2-way tables employed in learning cross-tabulation, as it has a 2x3x2 design (i.e., social group, age, year), which is needed to present a basic multivariate scenario.

B. More about question-posing. The five questions listed in Figure 3 are organized in a somewhat different way compared to the sequence proposed by Kemp and Kisanne (2010), which does not impose a structure on the actual process of [visual] analysis of the data. Q1 to Q3 cover the first four steps in the Kemp and Kisanne (2010) model described above, and aim to scaffold the thinking process by breaking down for students the table analysis into steps, i.e., first asking about trends or changes in 2005 (top half of the table), then in 2030 (bottom half) and finally across the two timeframes (whole table). This approach to question posing was employed in order to ensure that students pay attention to the key variables along which the data are organized, and can gradually build a picture of trends and changes within the data, from bivariate to multivariate.

Q4 and Q5 are the critical ones regarding *purpose* and *motivation*, and reflect our quest for a task design that directly connects with the context. Q4 addresses the same goal as Step 5 proposed by Kemp and Kisanne (WHY do they change) but uses a more structured phrasing that is tailored to the specific context. Q5 adds the new layer noted earlier and requires that students think about the actual societal or organizational *implications* of the data. Together, Q4 and Q5 take students from a mere analysis of how numbers change within the cells of a table, to engagement with a real-world context and to thinking about the social meaning and factors affecting a socially-loaded issue.

The example in Figure 3 is of course limited by design, being based on a single table and not having information about the social context and variables involved (because the basic demographics were known to all students), nor about richer statistical issues. In general, a much wider range of stimuli (i.e., articles, press releases, infographics, etc.) covering a wide range of civic topics should be used, such as where the students receive an article which describes a social phenomena of interest, and richer statistical information which can give rise to more diverse types of questions and tasks.

C. More general thoughts about critical interpretation and "opinion" questions. A key idea presented earlier is that students should engage directly with societal meaning or implications of the statistics and data they encounter. This implies that students should be asked to generate *opinions*, which by design cannot always be classified as right or wrong, and may have various shades of gray. On this, Gal (1998) has discussed the design of tasks that can elicit open-ended opinions of statistical messages, and offers four general heuristics:

1. A question or task should not provide specific hints as to where in the text or table or graph the student should look. The idea is that the student will have to decide on her own what parts of the text and inherent displays to examine, and how different aspects of the text, or data points in a graph or display, relate to each

other; the student's ideas and thinking process about text interpretation, data analysis and interpretation can be thus revealed through follow-up questioning;

2. A question or task should suggest to the student that a judgment (opinion) is called for, rather than a precise "mathematical" response in the form of a specific number. We want students to think about the meaning and interpretation, not look right away for some statistical terms or numbers to crunch; also, we want students to be sensitive and open to ambiguities or inconsistencies in what the text or statistics portray, and not assume that there is a single right answer;
3. An "opinion" question or task should inform students that they will be asked to explain and justify any opinion they introduce, and alert them to the need to examine all the information presented and reflect on it before responding. This may seem obvious when answering a mathematical task, but is not that obvious when students have to examine a text with embedded statistics, since the text may contain much information.
4. A question or task should seek an answer that can be of some service to an actor or stakeholder, i.e., a citizen or manager or decision-maker operating within the given context, i.e., the question should be an outgrowth of a functional "need to know." Unless students are presented with a reasonably realistic context, they may not use their "real head" and may not bring into the task the full power of their knowledge and reasoning skills; that said, the same situation may not be equally familiar to all students, and they may have different types of world knowledge or make different assumptions about any given context; we discuss this perplexing issue later on).

D. ***Where to find more ideas for tasks, questions, and class assignments related to statistical literacy.*** The four general heuristics above offer generic ideas that have to be adapted to local circumstances, i.e., the class goals, type of stimulus or resource (e.g., article, press release, infographic), the level and statistical background of the students, time constraints teaching context, etc. Many more examples for tasks and questions in this regard are offered inside lesson plans and activities developed by the ProCivicStat project (see the CivicStatMap tool here: http://iase-web.org/islp/pcs and play with its internal options). You can also explore other resources on the website of the International Statistical Literacy Project at the same URL, or see the rich collections developed by Milo Schield and his colleagues at: www.statlit.org.

3. Models and contexts

In this final section I want to present some observations on the connection between the ideas about contexts (and context knowledge) discussed in prior sections, and notions of models and modeling. Analyzing the connection between models and contexts is essential for better understanding of statistical literacy, and I relate to this here because I believe this connection seems to be misunderstood or seen in a narrow way by most of the literature on modeling, even in statistics education.

Modeling and the use of models are getting increased attention in mathematics education, as well as in statistics education both at the K-12 and more advanced levels. This trend is reflected in over 25 papers on modeling and statistics education in two recent special issues, i.e., the Nov 2017, Vol 16(2), of the *Statistics Education Research Journal*, and the December 2018 Vol 59)7) of *ZDM Mathematics Education*. The relevant literature is important but too large to summarize here, given space constraints.

With the risk of being simplistic, we can say that a model is a representation (but also simplification) of reality, which aims to capture and reflect key building blocks or elements in that reality, as well as the relationships or influences between these elements. (Broader summaries about models can be found in many sources, e.g., Doerr, Delmas, & Makar, 2017, and others in the two special issues mentioned above). In statistics, we all know that models are of prime importance. The literature views as "models" both simple statistics (e.g., average, standard deviation) and more advanced ones (e.g., correlation, regression, other methods subsumed under 'general linear models', and many more) since they serve as tools for summarizing distributions and multivariate data, and enable us to describe relationships between variables, make predictions and are the basis for inference from samples to populations, and more. Further, given their mathematical properties, such models also offer ways to estimate the goodness of fit, or precision, or degree or error, in predictions and inferences.

However, given that they are based mainly or only on mathematical expressions, such models are not directly connected to contexts. They can be studied or taught in class in the abstract, or using contrived contexts - as is often the case. So, where is the linkage to the real and authentic context? How does it relate to modeling–and to statistical literacy? There are several possible answers. Below I sketch, briefly, one possible answer. The gist of my current thinking on this is that while we want students to create a "model of a situation", *in fact there are multiple (and sometimes competing) families of models* which students and (future) citizens need to know about. Given time constraints, below I only highlight the crucial importance of *conceptual models,* which are mostly ignored in the literature, and are very different than other kinds of models (one of which, but not the only one, is a statistical model). The easiest way to explain what is a "conceptual model" is to examine the area of *statistical indicators*.

A. About indicators and their underlying models. As clarified in the introduction, we discuss statistical literacy, because we want to make sure graduates can interpret and critically understand and discuss the statistics in and about society (broadly viewed). But - what kind of official statistics are conveyed to the public (and to policy-makers) via media channels? As Gal and Ograjensek argue (2017), the answer is complex, of course, as many types of statistical findings and insights are shared, and their flavor may change across topics or countries. Yet in all countries, official statistics providers create messages and constantly update the public about the levels or changes and trends in hundreds of *indicators*, such as employment levels, hospital usage or mortality, school performance, consumer spending, crime, income inequality (e.g., Gini coefficient), and many dozens of other topics that reflect the state of some aspect of our society, economy, or well-being of citizens and organizations.

Indicators are of interest to us because they are a key product category of official statistics that usually influences policy-makers, politicians and hence public discourse. Yet, surprisingly, despite their centrality in society and their prevalence in public and political discourse, indicators are seldom described or analyzed in textbooks and in statistics curricula (But see Haack, 1979)

From a technical (statistical) perspective, indicators such as those listed earlier, and most others in use by official statistics providers, are often not raw variables, such as those used in introductory statistics, but rather *combinations* of data elements that may be expressed as percentages, ratios, or numbers on arbitrary scales. Thus, there are both *computational models* and *assessment models* that underlie each indicator, since the indicators may be computed or derived, from simple rates to complex aggregates of

weighted elements. They may be based either on "objective" sources (e.g., administrative data that underlie an indicator on "consumer spending") or survey -based subjective data which are the basis for the important indicator on "consumer confidence". However they are defined, indicators are widely used by official statistics providers to report on a wide range of issues, hence their understanding is essential for all citizens.

B. Examples: conceptual models that underlie the definition of indicators. Computational and assessment models illustrated above are important - but they are only part of the story, and still not directly connected to context. Now we come to conceptual models, which pertain to *how the social or economic phenomena of interest is defined*. Let's start with a seemingly simple example regarding a social phenomena such as "equality"– specifically the equality of income of men and women. This is a hotly debated topic, and perhaps the most well-known indicator in this regard is "gender pay gap". See Table 4, which shows selected parts from the definitions in Wikipedia when searching in Spanish and English (texts differ somewhat because of how each Wikipedia entry was created and possible variations between authors).

Wikipedia Definition: "Brecha salarial de género", Spanish *(excerpts)*

La **diferencia salarial de género**...es la diferencia existente entre las retribuciones salariales de los hombres y de las mujeres expresada como un porcentaje del salario masculino, de acuerdo con la OCDE. La Comisión Europea define la brecha salarial de género como «la diferencia relativa en el ingreso bruto promedio de mujeres y hombres dentro de la economía en su conjunto».....

... Es importante diferenciar entre la diferencia salarial ajustada y la no ajustada. El cálculo no ajustado de la diferencia salarial de género no tiene en cuenta las diferencias personales a la hora de establecer la comparación, como por ejemplo, edad o educación, ni tampoco las características del puesto de trabajo, como por ejemplo, sector de actividad....

Wikipedia Definition: "Gender Pay Gap" English *(excerpts)*

G**ender pay gap** .....is the average difference between the remuneration for men and women who are working. Women are generally paid less than men.

....There are two distinct numbers regarding the pay gap: ***unadjusted*** versus ***adjusted*** pay gap. The latter takes into account differences in hours worked, occupations chosen, education and job experience.

....Factors like this contribute to lower yearly earnings for women; while the pay gap has narrowed over time, a gender pay gap still exists, even when controlling for these external factors. *Unadjusted* pay gaps are much higher. In the United States, for example the unadjusted average female's annual salary has commonly been cited as being 78% of the average male salary, compared to 80-98% for the adjusted average salary.

Figure 4. Definitions of "Gender Pay gap" in English and Spanish Wikipedia entries

As can be seen, the definition of the Gender Pay Gap indicator combines or implies the need to understand many statistical ideas, some are seemingly simple (average, distribution, rate), but some more complex (e.g., adjusted and unadjusted rates) which are seldom discussed in introductory courses. However, what is important about the definition is that it is not a statistical (computational) model, but first of all a *conceptual*

model - a view of the entities that matter in the real world ("men" and "women") and what about them needs to be known and taken into account (""income" [in the sense of "salary"] and other factors that need to be controlled for. There are of course many additional examples for indicators that have more complex conceptual definitions, such as regarding "violence" or "poverty". Some will be shown during the talk.

The key point which I aim to highlight (based both on the preliminary example in Figure 4 and others to be shown during the talk) is that every important indicator has a conceptual definition which makes a statement about *what* are the things (constructs, variables) that matter in the world and need to be quantified─whereas h*ow* they are quantified is another matter and is described in separate models, i.e., a statistical model, an assessment model, etc.

The conceptual definition in fact makes a statement about what *is* the phenomenon of interest, through listing the things or factors that one must consider when trying to understand it and quantify it with a single indicator (i.e., a single number). The conceptual definition also may describe what other issues one must consider when discussing this phenomenon (e.g., the things to be "adjusted" regarding 'gender pay gap'). Without understanding *what* is the meaning of all the real-world entities included or implied by the definition, and their *correlates* (e.g., salary, position, experience, part-time vs. full time, etc) and *why* they matter, one cannot interpret the data and give it meaning within a specific social context, nor understand the social or economic *implications* of the measured values of the indicator.

Lastly, it should be noted that conceptual models (and of course statistical and assessment models), can be challenged and are often criticized. Quite often we hear of different stakeholders who argue that the conceptual definition of some topic (e.g., "poverty" and what is a "poverty line") ignores some important variables, or need to be computed in a different way, and this gives rise to competing conceptual definitions of the same social or economic phenomenon. It is thus no wonder that conceptual definitions may develop and change over time, to reflect society's needs for information about itself, and the points of view of different stakeholders.

4. Summary and implications

The development of statistical literacy as a core competency of school and university graduates is an essential outcome of educational processes at the school and university levels alike, if they are expected to function effectively in a statistics-filled world. This explains why the first of the six key recommendation emerging from the ProCivicStat *project* states that "*Statistics education activities should promote engagement with social issues and develop learners' critical understanding of statistics about key civic phenomena*" *(*See: *Call for Action and Recommendations,* ProCivicStat partners, 2018)

While the general importance of developing statistical literacy of graduates seems to be accepted to all actors involved in statistics education, the actual routes towards this goal are filled with multiple challenges. This paper has focused on some of the challenges and dilemmas that teaching for statistical literacy may entail. The paper aimed to highlight that understanding of the *context* within which statistics are desired and generated is an essential component of teaching statistics in general, and statistical literacy in particular.

The need to attend to contexts has many advantages–but is far from being simple or easy for educators who are focused on developing their students' technical mastery of statistical constructs and methods. For this reason, the paper posited three questions which every educator involved in teaching statistics should consider, regarding what contexts are valuable (meaningful and important), how contexts can be engaged in the classroom, and what tasks or questions to pose about statistics in contexts, and the importance of attending to the need to develop students' ability to express *opinions* about civic statistics and about their social implications.

The last section, regarding the connection between models (and modeling) and context, focused on official statistical indicators, which are arguably one of the most important types of products of official statistics agencies, from the point of view of statistical literacy. The sections tried to convince readers that it is not possible to fully understand any statistical indicator without engaging with the conceptual models that underlie that indicator, and reflect on the actual societal and economic contexts within which the indicator is generated and to which its implications do apply.

Of course, the messages in this paper need to be placed in, well, context! The arena of statistical literacy is much wider (See Gal, 2002; Watson, 2013), and many more issues need to be addressed when teaching for statistical literacy. The paper focused on one issue, i.e., context and contextualization, which I believe is misunderstood or neglected in statistics education. Further, the importance of contexts was discussed here mainly in connection with data and findings that are based on datasets and surveys that are in typical use by statistics producers and relate to outcomes from what is commonly referred to as "data analysis" in statistics education. That said, contexts are of equal importance for critical understanding of messages and public communications related to chance, risk, and probability, which fit more under *probability literacy* (Gal, 2004).

If the ideas in this paper are accepted, there are many implications for the design of statistics courses. More attention is also needed to statistical indicators, which in fact cover a wide range of both basic and advanced statistics which every citizen should know. The ideas in this paper (which have informed but also build in part on ideas and work generated as part of the *ProCivicStat* project) require changes in the level of contextualization of assessment tasks (as opposed to using "clean" problems with little or no context). All this in turn requires changes in how teachers are trained to teach statistics, and in how they approach question-posing in the classroom, and the amount of time and effort to be devoted to core statistical ideas and the contexts to which they apply. I hope that these ideas, and the dilemmas they involve, will continue to be discussed and debated in coming years.

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