Age-dependent Taxation, Retirement Behavior, and Work Hours Over the Life Cycle

Julian Díaz-Saavedra

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Julián Díaz-Saavedra
Universidad de Granada
julianalbertodiaz@ugr.es

Abstract

We use a computable overlapping generations model economy, which matches the stylized facts concerning retirement behavior, to analyze the consequences of three reforms designed to reduce tax rates on the labor supply of older workers. We find that these reforms increase the participation rates of the elderly and show that the gains, in terms of old age work hours, are non-trivial. However, we also find that the total labor supply response to the reforms is not so much an increase in total lifetime hours as it is a reallocation of hours over the life cycle. Finally, we show that these reforms, designed to increase the length of the working life of individuals, may not increase output.

Keywords: Computable general equilibrium, labor supply, retirement, age-dependent taxation
JEL classification: C68, J22, J26, H31

1 Introduction

Despite the reversal in recent years of the trend to early retirement of older workers, their participation rates continue to be low in most of developed countries, and these low rates generate serious concerns about the burden on public pension systems and the lost output. Early retirement reduces payroll tax revenues and increases pension payments. Consequently, it will exacerbate the financial imbalance of PAYG pension systems during the coming decades, when populations are expected to age. Additionally, the low participation rates of older workers may reduce output, and according to the OECD (1999), early retirement could cut growth rates in Europe and the United States by half a percentage point per year. To tackle these issues, the policy most commonly adopted to increase the participation rates of older workers, has been to increase the legal retirement ages for claiming pension benefits. Intuitively, this reduces the number of retired people and increases the total number of hours worked.

However, increasing retirement ages is costly from a political point of view, since such reforms have little support among voters. For instance, there is a widespread fear that those with physically demanding and often low-paid jobs are the main losers since they are not able to adjust their retirement behavior. Then, it can be argued that, rather than forcing workers to delay retirement, changes in tax and transfers programs...
could provide sufficient incentives to make the elderly voluntarily increase their working lifetime. Specifically, there is strong evidence that labor supply elasticity is not constant over the life cycle. For instance, French (2005), Goda et al. (2007) and French and Jones (2011) show that labor supply is particularly responsive at later stages in the life cycle. Consequently, and following the theory of optimal taxation, more elastic goods should be taxed less than inelastic goods so labor supply taxes at older ages should be lower than those at younger ages. Nevertheless, this is not the case in most developed countries, as labor supply taxes also increase with age. First, and because the progressivity of income tax codes, marginal tax rates increase with labor earnings. And since average earnings increase with age, a progressive tax code implies that marginal tax rates faced by an average worker also increase with age. And second, using data from national surveys, Gruber and Wise (2004) and Blondal and Scarpetta (1999) show that social security programs impose high implicit tax rates on continued work of elderly. Thus, reducing effective tax rates at certain points in the life cycle may therefore be a powerful tool for increasing working lifetimes.

This paper departs from the social security literature to focus on the labor supply effects of taxation. Specifically, we use a life-cycle model of labor supply and retirement to quantify the changes in participation rates, total work hours, aggregates and the pension system balance, in response to counterfactual experiments related to age-dependent taxation. Our quantitative experiments rely on the Spanish economy. Some dimensions of the Spanish both public pension system and income tax code differ from the US system, and tend to encourage retirement earlier than in the US. For instance, the Spanish system provides a minimum benefit level which is higher than in the US. Moreover, workers can receive this minimum amount at the first retirement age of 60 and thus, because delaying receipt of this minimum does not increase it, workers have an incentive to apply for this type of benefit at that age, especially low income workers. Another example is that the Spanish system raises the annual benefit by 3 per cent when a worker chooses to delay claiming benefits after the normal retirement age 65, this figure being 8 percent in the US. Since 3 percent is less than actuarially fair, it provides strong incentives to draw benefits by the age of 65. Finally, and as reported by Erosa, Fuster, and Kambourov (2011), Spanish high earners face higher marginal tax rates that their US counterparts. Consequently, and unsurprisingly, the participation rates of older workers are higher in the US than in Spain. For instance, in 2011, the participation rates for the age groups 60 to 64 and 65 to 69 were 54.5 and 32.1 percent in the US. In Spain, these rates were 37.7 and 5.2 percent that same year.

Our model economy of labor supply and retirement decisions builds on Díaz-Giménez and Díaz-Saavedra (2009), Erosa, Fuster, and Kambourov (2011), and İmrohoroğlu and Kitao (2012). As in Díaz-Giménez and Díaz-Saavedra (2009), our neoclassical model of labor endogenizes labor supply in both the extensive and intensive margins. That is to say, a worker chooses both the fraction of her life to devote to employment and what fraction of her period endowment to devote to work while employed. The social security and the taxation systems are modelled in great detail. In particular, our life-cycle model introduces the early and the normal age of retirement, minimum and maximum pensions, adjustment in pensions for early and late withdrawal, maximum covered earnings, and an exogenous replacement rate. As in Spain, our model also introduces proportional taxes on consumption and capital income, and a progressive tax code on personal income. We model three educational groups since retirement behavior differs significantly across educational groups. The benchmark model economy is calibrated to Spanish macro and micro data, and our calibration procedure follows the approach of Castañeda, Díaz-Giménez, and Rios-Rull (2003). First, we determine the value of a group of parameters directly from the data. And second, to determine the values of the remaining parameters, as it is the shock process of labor productivity, we construct a system of non-linear equations where these equations require that the aggregate and distributional statistics of our model economy replicate the values of the corresponding statistics in Spain in 2010. The procedure requires solving this system for many different initial values in order to find the best parameterization possible.

After the calibration procedure, we find that our model economy matches very well the main aggregates and ratios of the Spanish economy. In addition, its calibrated process of stochastic labor income allows the model to replicate the earnings and income distributions of the Spanish economy. Finally, it is particularly remarkable that our life-cycle model replicates reasonable well the statistics that characterise retirement...
behavior of older workers, since these statistics were not explicitly targeted during the calibration procedure. Specifically, our model economy is quantitative consistent with the age-dependent probabilities of exiting the labor force, and it also replicates the participation rates of older workers. This is because our retirement model introduces the key economic and institutional forces leading to retirement.

We analyze the quantitative consequences of three counterfactual experiments, where the motivation of these experiments rests on the literature of age-dependent taxation, which indicates efficiency gains from using age to target lower tax rates for those workers with higher labor supply elasticities. In the first experiment, we eliminate the labor income and payroll taxes for those workers aged 60 years old or over. In the second experiment, we increase from the current 3 percent to 8 percent, the annual increase in the retirement pension for each additional year worked beyond age 65. And the third reform eliminates the minimum retirement pension provided by the Spanish public pension system. The findings are that these reforms increase both the participation rates of older workers and the average retirement age, showing that the gains in terms of old-age work hours are non-trivial. For instance, our quantitative experiments predict that the hours worked for those aged 60+ increase by 13, 14, and 24 percent respectively, and that these changes in work hours differ across educational groups.

Differently from Laitner and Silverman (2011), we find that total hours worked are less responsive to an increase in old-age after tax wages. Laitner and Silverman (2011) analyze a US social security reform in which older individuals no longer face the OASI payroll tax and their subsequent earnings have no bearing on their benefits, and they find that this reform delays retirement ages by one year, and that it also brings significant efficiency gains. And this is because Laitner and Silverman (2011) present a life-cycle model and assume both that households make consumption and retirement decisions, and that households can not adjust their labor supply during the vesting period. This last assumption is important for generating a large response of total hours as discussed by Laitner and Silverman (2011). Our quantitative findings, however, show a significant reallocation of work hours over the life cycle, that is, workers work less when young knowing that they will work until an older age. Thus, we find that the reforms increase aggregate hours by 0.9 percent at most. Our results are consistent with the findings of McGrattan and Rogerson (1998), who report that in the last century, U.S. workers both shortened their working period and shifted work hours from older to younger ages, as social security coverage increased.

Also, our results, and contrary to those from Herbertsson and Orszag (2003), imply that very small efficiency gains could arise from these reforms. While Herbertsson and Orszag (2003) find that early retirement can be held responsible for a reduction of 5 to 7 per cent of the potential annual GDP in OECD countries, these numbers being higher for European countries, our findings imply that these reforms could increase output by less than one percent at most. Looking at the fine print, this is because Herbertsson and Orszag (2003) assume both that total labor input increases strongly with a higher employment rate of elderly, and that saving rates are not affected by the implied longer working lifetime. We already discussed that total labor supply could be much less responsive. But we also find that a shorter retirement period and higher retirement pensions may reduce saving rates, and consequently capital stock, as firstly suggested by Feldstein (1974). Thus, we think that when trying to quantify the output lost due to early retirement, it is important to consider the potential reallocation in work hours over the life-cycle and the change in saving rates.

Finally, we find that the higher participation rates of elderly does not help to cope with the pension burden. Specifically, we find that after these reforms, the social security budget remains roughly unchanged, and this is because what government saves on the payment of retirement benefits, it loses by having to pay higher benefits later on, due to the permanent increase in the benefit received in later years. Our results are in line with those from İmrohoroglu and Kitao (2012), who find that an increase in the earliest retirement age by two years, brings no significant variation on the U.S. Social Security budget. Thus, any policy aimed to increase the participation rate of elderly should be jointly implemented with a parametric change designed to scale down pension benefits.

4Under the current Spanish public pension system rules, workers aged 65 or over are exempt from paying payroll taxes
5The fact that younger cohorts respond in part by adjusting labor supply at earlier ages is also found by French and Jones (2011). These authors assume a 20 percent permanent increase in wages for workers aged 60, and find that workers shift hours from before the wage change to afterwards, so that aggregate hours change less.
Our paper is most closely related to two branches of the literature, where the first considers the possibility of age-adjusted tax rates⁶; see for example the papers of Fennell and Stark (2005), Kremer (2001), and Weinzierl (2011). To the best of our knowledge, however, most of the papers primarily consider age-based income tax reductions for young people, and only the papers by Goda et al. (2007) and Laitner and Silverman (2011) focused on earnings tax reductions for U.S. elderly. Goda et al. (2007) recommend eliminating the Medicare as a Secondary Payer requirement, since such provision imposes a tax rate ranging from 15 to 20 percent at age 65. The closest work to that reported here, nevertheless, is the paper by Laitner and Silverman (2011), who study a US social security reform in which older individuals no longer face the OASI payroll tax and their subsequent earnings have no bearing on their benefits. However, and as stated before, certain assumptions may contribute to amplify the efficiency gains that they obtain.

A second branch of the literature uses large scale, discrete time overlapping generations models, as pioneered by Auerbach and Kotlikoff (1987), to analyse social security reforms. Since then, such models have incorporated liquidity constraints, longevity risk and heterogeneity within cohorts, due to uninsurable shocks. Most of this literature, however, omits the extensive margin in labor supply. This is important, because endogenous retirement is precisely a quantitative significant margin through which changes in social security rules affect the economy. Stated another way, any social security reform which changes the marginal utility of working will affect average retirement age and the reported results. The papers by Díaz-Giménez and Díaz-Saavedra (2009), Imrohoroglu and Kitao (2012), and Erosa, Fuster, and Kambourov (2011) are three recent exceptions, although only the first two papers assume that households understand how the past cumulated labor earnings determine the Social Security benefits⁷.

The paper is organized as follows. Section 2 presents the model economy. Section 3 describes the calibration procedure. Section 4 presents the calibration results. Section 5 describes in great detail the counterfactual experiments. Section 6 presents the results, and Section 7 offers our conclusions.

2 The Model Economy

Our model economy, which resembles the model described in Díaz-Giménez and Díaz-Saavedra (2009), is an overlapping generations model economy⁸. We assume that it is populated by a continuum of heterogeneous households, a representative firm, and a government. We describe these three sectors below.

2.1 Population and Endowment Dynamics

We assume that households in our model economy differ in their age, \( j \in J \); in their education, \( h \in H \); in their employment status, \( e \in E \); in their assets, \( a \in A \); in their pension rights, \( b_t \in B_t \), and in their pensions \( p_t \in P_t \).⁹ Sets \( J, H, E, A, B_t, \) and \( P_t \) are all finite sets which we describe below. We use \( \mu_{j,h,e,a,b,p,t} \) to denote the measure of households of type \((j, h, e, a, b, p)\) at period \( t \). For convenience, whenever we integrate the measure of households over some dimension, we drop the corresponding subscript.

Age. Every household enters the economy when it is 20 years old and it is forced to exit the economy at age 100. Consequently, \( J = \{20, 21, \ldots, 100\} \). We also assume that each period every household faces a

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⁶See Banks and Diamond (2010) for an extensive review.
⁷Sánchez-Martín (2010) also presents a large scale overlapping generations model economy calibrated to the Spanish economy, with endogenous retirement, and where agents are heterogeneous regarding their education. However, his model economy does not introduce heterogeneity within educational groups. Unsurprisingly, therefore, his model economy fails to match the principal statistics characterizing the retirement behavior of the elderly in Spain.
⁸A distinguishing feature of this version is to model the progressivity of taxes on personal income. In addition, this latest version of our model economy is calibrated to a more recently data.
⁹To calibrate our model economy, we use data per person older than 20. Therefore our model economy households are really individual people.
conditional probability of surviving from age $j$ to age $j+1$, which we denote by $\psi_{jt}$. This probability depends on the age of the household and it varies with time, but it does not depend on the household’s education.

**Education.** We abstract from the education decision, and we assume that the education of every household is determined forever when they enter the economy. We consider three educational levels and, therefore, $H = \{1, 2, 3\}$. Educational level $h = 1$ denotes that the household has dropped out of high school; educational level $h = 2$ denotes that the household has completed high school but has not completed college; and educational level $h = 3$ denotes that the household has completed college.

**Population Dynamics.** In the real world the age distribution of the population changes because of changes in fertility, survival rates, and migratory flows. The population dynamics in our model economy are exogenous and we describe them in Appendix 1 below.

**Employment status.** Households in our economy are either workers, retirees, or disabled households. We denote workers by $\omega$, retirees by $\rho$, and disabled households by $d$. Consequently, $E = \{\omega, \rho, d\}$. Every household enters the economy as a worker. The workers face a positive probability of becoming disabled at the end of each period of their working lives. And they decide whether to retire at the beginning of each period once they have reached the first retirement age, which we denote by $R_0$. In our model economy, both the disability shock and the retirement decision are irreversible and there is no mandatory retirement age.

**Workers.** Workers receive an endowment of efficiency labor units every period. This endowment has two components: a deterministic component, which we denote by $\epsilon_{jh}$, and a stochastic idiosyncratic component, which we denote by $s$.

We use the deterministic component to characterize the life-cycle profile of earnings. This profile is different for each educational group, and we model it using quadratic functions on age of the form

$$\epsilon_{jh} = a_{1h} + a_{2h}j - a_{3h}j^2$$

We choose this functional form because it allows us to represent the life-cycle profiles of the productivity of workers in a very parsimonious way. We represent the calibrated versions of these functions in Panel A of Figure 1.

We use the stochastic component of the endowment shock, $s$, to generate earnings and wealth inequality within the age cohorts. We assume that $s$ is independent and identically distributed across the households, that it does not depend on the education level, and that it follows a first order, finite state Markov chain with conditional transition probabilities given by

$$\Gamma[s' | s] = \Pr\{s_{t+1} = s' | s_t = s\} = \{s_1, s_1, \ldots, s_n\}.$$  

We assume that the process on $s$ takes three values and, consequently, that $s \in \omega = \{s_1, s_2, s_3\}$. We make this assumption because it turns out that three states are sufficient to account for the Lorenz curves of the Spanish distributions of income and labor earnings in sufficient detail, and because we want to keep this process as parsimonious as possible.

**Retirees.** As we have already mentioned, workers who are $R_0$ years old or older decide whether remain in the labor force, or whether to retire and start collecting their retirement pension. They make this decision

\[\text{In this group we include every household that has not completed the compulsory education. Due to the changes in the Spanish educational laws, we define the compulsory studies to be either the Estudios Secundarios Obligatorios, the Graduado Escolar, the Certificado Escolar, or the Bachiller Elemental.}\]
after they observe their endowment of efficiency labor units for the period. In our model economy retirement pensions are incompatible with labor earnings and, consequently, retirees receive no endowment of efficiency labor units.

Disabled households. We assume that workers of education level $h$ and age $j$ face a probability $\varphi_{jh}$ of becoming disabled from age $j + 1$ onwards. The workers find out whether they have become disabled at the end of the period, once they have made their labor and consumption decisions. When a worker becomes disabled, she exits the labor market and it receives no further endowments of efficiency labor units, but she is entitled to receive a disability pension until she dies.

To determine the values of the probabilities of becoming disabled, we proceed in two stages. First we model the aggregate probability of becoming disabled. We denote it by $q_j$, and we assume that it is determined by the following function:

$$q_j = a_4 e^{(a_5 \times j)}$$

We choose this functional form because the number of disabled people in Spain increases more than proportionally with age, according to the Boletín de Estadísticas Laborales (2007).

Once we know the value of $q_j$ we solve the following system of equations:

$$\begin{align*}
q_{jh} \mu_{jh,2007} &= \sum_h \varphi_{jh} \mu_{jh,2007} \\
\varphi_{j2} &= a_6 \varphi_{j1} \\
\varphi_{j3} &= a_7 \varphi_{j1}
\end{align*}$$

This procedure allows us to make the disability process dependent on the educational level as is the case in Spain. We represent our calibrated values for $\varphi_{jh}$ in Panel B of Figure 1.\(^{11}\)

2.2 Preferences

We assume that households derive utility from consumption, $c_{jht} \geq 0$, and from non-market uses of their time and that their preferences can be described by the following standard Cobb-Douglas expected utility

\(^{11}\)The data on disability can be found at www.empleo.gob.es/es/estadisticas.
function:

\[
\max E \left\{ \sum_{j=20}^{100} \beta^{j-20} \psi_{jt} [c_{jht}^{\alpha} (1 - l_{jht})^{(1-\alpha)}]^{1-\sigma} / 1 - \sigma \right\}
\]  

where \(0 < \beta\) is the time-discount factor; 1 is the normalized endowment of productive time; and \(0 \leq l_{jht} \leq 1\) is labor. Consequently, \(1 - l_{jht}\) is the amount of time that the households allocate to non-market activities.

2.3 Technology

We assume that aggregate output, \(Y_t\), depends on aggregate capital, \(K_t\), and on the aggregate labor input, \(L_t\), through a constant returns to scale aggregate production function, \(Y_t = f(K_t, A_t L_t)\), where \(A_t\) denotes an exogenous labor-augmenting productivity factor whose law of motion is \(A_{t+1} = (1 + \gamma_t) A_t\), and where \(A_0 > 0\). We choose a standard Cobb-Douglas aggregate production function with capital share \(\theta\). Aggregate capital is obtained aggregating the capital stock owned by every household, and the aggregate labor input is obtained aggregating the efficiency labor units supplied by every household. We assume that capital depreciates geometrically at a constant rate, \(\delta\), and we use \(r\) and \(w\) to denote the prices of capital and of the efficiency units of labor before all taxes.

2.4 Government Policy

The government in our model economy taxes capital income, household income and consumption, and it confiscates unintentional bequests. It uses its revenues to consume, and to make transfers other than pensions. In addition, the government runs a pay-as-you-go pension system.

In this model economy the consolidated government and pension system budget constraint is

\[
G_t + P_t + Z_t = T_{at} + T_{st} + T_{yt} + T_{ct} + E_t + (F_t - F_{t+1})
\]  

where \(G_t\) denotes government consumption, \(P_t\) denotes pensions, \(Z_t\) denotes government transfers other than pensions, \(T_{at}, T_{st}, T_{yt}, T_{ct}\), denote the revenues collected by the asset income tax, the payroll tax, the household income tax, and the consumption tax, \(E_t\) denotes unintentional bequests, and \(F_t > 0\) denotes the value of the pension reserve fund at the beginning of period \(t\). Finally, \((F_t - F_{t+1})\) denotes the revenues that the government obtains from the pension reserve fund or deposits into it.

We assume that the pension reserve fund must be non-negative and that \(Z\) is thrown to the sea so that they create no distortions in the household decisions.

2.4.1 Taxes

Asset income taxes are described by the function

\[
\tau_a(y^a_t) = a_8 y^a_t
\]  

where \(y^a_t\) denotes the income that the households obtain from all their assets.

Household income taxes are described by the function

\[
\tau_y(y^b_t) = a_9 \left\{ y^b_t - \left[ a_{10} + (y^b_t)^{-a_{11}} \right]^{-1/a_{11}} \right\}
\]
where the tax base is
\[ y_t = y_t^b + y_t^t + p_t - \tau_a(y_t^g) - \tau_s(y_t^l) \] (9)
where \( y_t^b \) is labor income, before taxes, at period \( t \), and \( \tau_s(y_t^l) \) are payroll taxes that same period. Expression (8) is the function chosen by Gouveia and Strauss (1994) to model effective personal income taxes in the United States, and it is also the functional form chosen by Calonge and Conesa (2003) to model effective personal income taxes in Spain.

Consumption taxes are described by the function
\[ \tau_c(c_t) = a_{12} c_t. \] (10)

Finally, we assume that at the end of each period, once they have made their labor and consumption decisions, a share \((1 - \psi_{jt})\) of all households of age \( j \) die and that their assets are confiscated by the government.

### 2.4.2 The Pension System

**Payroll taxes.** In Spain the payroll tax is capped and it has a tax-exempt minimum. In our model economy the payroll tax function is the following:
\[ \tau_s(y_t^l) = \begin{cases} a_{13} \bar{y}_t - \left[ a_{13} \bar{y}_t \left( 1 + \frac{a_{14} y_t^l}{a_{13} \bar{y}_t} \right)^{-a_{13} \bar{y}_t} \right] & \text{if } j < R_1 \\ 0 & \text{otherwise} \end{cases} \] (11)
where parameter \( a_{13} \) is the cap of the payroll tax, and \( \bar{y}_t \) is per capita output at market prices at period \( t \).

This function allows us to replicate the Spanish payroll tax cap, but it does not allow us to replicate the tax exempt minimum. In Panel C of Figure 1 we represent the payroll tax function for our calibrated values of \( a_{13} \) and \( a_{14} \).

**Retirement pensions.** A household of age \( j \geq R_0 \), who chooses to retire, receives a retirement pension which is calculated according to the following formula, which replicates the main features of Spanish retirement pensions:
\[ p_t = \phi (1.03)^{12} (1 - \lambda_j) \left[ \frac{1}{N_b} \sum_{t=j-N_b}^{j-1} \min \{a_{15} \bar{y}_t, y_t^l\} \right] \] (12)
where the last expression on the right hand side is called *Regulatory Base*. In this expression 12, parameter \( N_b \) denotes the number of consecutive years immediately before retirement that are used to compute the retirement pensions; parameter \( 0 < \phi \leq 1 \) denotes the pension system replacement rate; variable \( v \) denotes the number of years that the worker remains in the labor force after reaching the normal retirement age;\(^{12}\) function \( 0 \leq \lambda_j < 1 \) is the penalty paid for early retirement; and \( a_{15} \bar{y}_t \) is the maximum covered earnings.

Pensions in our model economy are computed upon retirement and their real value remains unchanged. We also model minimum and maximum retirement pensions. Formally, we require that \( p_{0t} \leq p_t \leq p_{mt} \), where \( p_{0t} \) denotes the minimum pension and \( p_{mt} \) denotes the maximum pension. We update the minimum pension so that it remains a constant proportion of output per capita.\(^{13}\)

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12 This late retirement premium was introduced in the 2002 reform of the Spanish public pension system.
13 In Spain normal and maximum pensions are adjusted using the inflation rate and minimum pensions are increased discretionally. This has implied that over the last decade or so the Spanish minimum pension has roughly kept up with per capita GDP, and that the maximum pension and normal pensions have decreased as a share of per capita GDP. This little known fact is known as the *silent reform*. 
The Spanish Régimen General de la Seguridad Social\textsuperscript{14} establishes that the penalties for early retirement are a linear function of the retirement age. To replicate this rule, our choice for the early retirement penalty function is the following

\[
\lambda_j = \begin{cases} 
an_{16} - a_{17}(j - R_0) & \text{if } j < R_1 \\
0 & \text{if } j \geq R_1 
\end{cases}
\] (13)

Finally, the Spanish pension replacement rate is a function of the number of years of contributions. In our model economy we abstract from this feature because it requires an additional state variable. It turns out that this last assumption is not very important because, in our model economy, 99.99 of all workers aged 20-60 in our benchmark model economy choose to work in our calibration year. This suggests that the number of workers who would have been penalized for having short working histories in our model economy is very small.

\textit{Disability pensions}. We model disability pensions explicitly for two reasons: because they represent a large share of all Spanish pensions (10.7 percent of all pensions in 2010), and because, in many cases, disability pensions are used as an alternative route to early retirement.\textsuperscript{15} To replicate the current Spanish rules, we assume that there is a minimum disability pension which coincides with the minimum retirement pension. And that the disability pensions are 75 percent of the households’ retirement claims. Formally, we compute the disability pensions as follows:

\[
p_t = \max\{p_0t, 0.75b_t\}.
\] (14)

\textit{The pension reserve fund}. We assume that pension system surpluses, \((T_{st} - P_t)\), are deposited into a non-negative pension reserve fund which evolves according to

\[
F_{t+1} = (1 + r^*)F_t + T_{st} - P_t
\] (15)

where parameter \(r^*\) is the exogenous rate of return of the fund’s assets. We assume that, when the pension reserve fund runs out, the government changes the consumption tax rate as needed in order to finance the pensions.

2.5 Market Arrangements

\textit{Insurance Markets}. We assume that there are no insurance markets for the stochastic component of the endowment shock. This is a key feature of our model economy. When insurance markets are allowed to operate, every household of the same age and education level is identical, and the earnings and wealth inequality disappears almost completely.

\textit{Assets}. We assume that the households in our model economy cannot borrow. Since leisure is an argument of their utility function, this borrowing constraint can be interpreted as a solvency constraint that prevents the households from going bankrupt in every state of the world. These restrictions give the households a precautionary motive to save. They do so accumulating real assets, which we denote by \(a_t\), and which take the form of productive capital. For computational reasons we restrict the asset holdings to belong to the

\textsuperscript{14}The Spanish Régimen General de la Seguridad Social is the most important pension program in the Spanish Social Security System. For instance, 82.1 percent of the affiliated workers and 54.9 percent of existing pensions belonged to this program in 2010.

\textsuperscript{15}See Boldrin and Jiménez-Martín (2003) for an elaboration of this argument.
discrete set $\mathcal{A} = \{a_0, a_1, \ldots, a_n\}$. We choose $n = 99$, and assume that $a_0 = 0$, that $a_{99} = 75$, and that the spacing between points in set $\mathcal{A}$ is increasing.\footnote{In overlapping generation models with finite lives and no altruism there is no need to impose an upper bound for set $\mathcal{A}$ since households who reach the maximum age will optimally consume all their assets. Imrohoroglu, Imrohoroglu, and Joines (1995) make a similar point.}

**Pension Rights.** We assume that the workers’ pension rights belong to the discrete set $B_t = \{b_{0t}, b_{1t}, \ldots, b_{mt}\}$.\footnote{Set $B_t$ changes with time because its upper bound is the maximum covered earnings which are proportional to per capita output.} Let parameter $N_b$ denote the number of years of contributions that are taken into account to calculate the pension. Then, when a worker’s age is $R_0 - N_b < j < R_0$, the $b_{jt}$ record the average labor income earned by that worker since age $R_0 - N_b$. And when a worker is older than $R_0$, the $b_{jt}$ record the average labor income earned by that worker during the previous $N_b$ years. We assume that $b_{0t} = 0$, and that $b_{mt} = a_{15} \bar{y}_t$, where $a_{15} \bar{y}_t$, and as we said before, denotes the maximum earnings covered by the pension system. We also assume that $m = 9$ and that the spacing between points on $B_t$ is increasing.

**Pensions.** We assume that both the disability and retirement pensions belong to set $P_t = \{p_{0t}, p_{1t}, \ldots, p_{nt}\}$. The rules of the pension system determine the mapping from pension rights into pensions, and workers take into account this mapping when they decide how much to work and when to retire. Since this mapping is single valued, and cardinality of the set of pension rights, $B_t$, was 10, $m = 9$ also for $P_t$. Finally, we assume that the distances between any two consecutive points in the pensions set is increasing.

### 2.6 The Households’ Decision Problem

We assume that the households in our model economy solve the following decision problem:

$$
\max_{E} \left\{ \sum_{j=20}^{100} \beta^{j-20} \psi_{jt} [c_{jht}^a (1 - l_{jht})(1-\alpha)^{(1-\sigma)/1-\sigma}] \right\}
$$

subject to

$$
c_{jht} + a_{jht+1} + \tau_{jht} = y_{jht} + a_{jht}
$$

and where

$$
\tau_{jht} = \tau_a(y_{jht}^a) + \tau_s(y_{jht}^b) + \tau_y(y_{jht}^l) + \tau_c(c_{jht})
$$

$$
y_{jht} = y_{jht}^a + y_{jht}^b + p_t
$$

$$
y_{jht}^a = a_{jht} r_t
$$

$$
y_{jht}^b = \epsilon_{sht} t_{jht} w_t
$$

$$
y_{jht}^l = y_{jht}^a + y_{jht}^l + p_t - \tau_a(y_{jht}^a) - \tau_s(y_{jht}^l)
$$

where $a_{jht} \in \mathcal{A}$, $p_t \in P_t$, $s_t \in \omega$ for all $t$, and $a_{jht0}$ is given. Notice that every household can earn capital income, only workers can earn labor income, and only retirees and disabled households receive pensions.

### 2.7 Definition of Equilibrium

Let $j \in J$, $h \in H$, $e \in \mathcal{E}$, $a \in \mathcal{A}$, $b_t \in B_t$, and $p_t \in P_t$, and let $\mu_{j,h,e,a,b,p,t}$ be a probability measure defined on $\mathcal{R} = J \times H \times \mathcal{E} \times \mathcal{A} \times B_t \times P_t$.\footnote{Recall that, for convenience, whenever we integrate the measure of households over some dimension, we drop the corresponding subscript.} Then, given initial conditions $\mu_0$, $A_0$, $E_0$, $F_0$, and $K_0$, a competitive equilibrium
for this economy is a government policy, \( \{ G_t, P_t, Z_t, T_{st}, T_{yt}, T_{ct}, E_{t+1}, F_{t+1} \}_{t=0}^{\infty} \), a household policy, \( \{ c_t(j, h, e, a, b, p), h_t(j, h, e, a, b, p), a_{t+1}(j, h, e, a, b, p) \}_{t=0}^{\infty} \), a sequence of measures, \( \{ \mu_t \}_{t=0}^{\infty} \), a sequence of factor prices, \( \{ r_t, w_t \}_{t=0}^{\infty} \), a sequence of macroeconomic aggregates, \( \{ C_t, I_t, Y_{t+1}, K_t \}_{t=0}^{\infty} \), a function, \( Q \), and a number, \( r^* \), such that:

(i) The government policy and \( r^* \) satisfy the consolidated government and pension system budget constraint described in Expression (6) and the law of motion of the pension system fund described in Expression (15).

(ii) Firms behave as competitive maximizers. That is, their decisions imply that factor prices are factor marginal productivities \( r_t = f_1(K_t, A_t L_t) - \delta \) and \( w_t = f_2(K_t, A_t L_t) \).

(iii) Given the initial conditions, the government policy, and factor prices, the household policy solves the households’ decision problem defined in Expressions (16), through (22).

(iv) Gross savings, consumption, factor inputs, pension payments, tax revenues, and accidental bequests are obtained aggregating over the model economy households as follows:

\[
K_t = \int a_{jht} d\mu_t \quad (23)
\]
\[
C_t = \int c_{jht} d\mu_t \quad (24)
\]
\[
L_t = \int \epsilon_{jh} s_{jht} d\mu_t \quad (25)
\]
\[
P_t = \int p_t d\mu_t \quad (26)
\]
\[
T_{ct} = \int \tau_{ct}(c_{jht}) d\mu_t \quad (27)
\]
\[
T_{at} = \int \tau_{a}(y^a_{jht}) d\mu_t \quad (28)
\]
\[
T_{st} = \int \tau_{s}(y^s_{jht}) d\mu_t \quad (29)
\]
\[
T_{yt} = \int \tau_{y}(y^y_{jht}) d\mu_t \quad (30)
\]
\[
E_t = \int (1 - \psi_{jt}) a_{jht+1} d\mu_t \quad (31)
\]

where \( y^a_{jht} = a_{jht} r_t, y^s_{jht} = \epsilon_{jh} s_{jht} w_t \), and \( y^y_{jht} = y^a_{jht} + y^s_{jht} + p_t - \tau_a(y^a_{jht}) - \tau_s(y^s_{jht}) \), and all the integrals are defined over the state space \( \Re \).

(v) Net investment \( I_t \) is

\[
I_t = K_{t+1} - (1 - \delta) K_t \quad (32)
\]

(vi) The goods market clears:

\[
C_t + \int (a_{jht+1} - a_{jht}) d\mu_t + G_t + [Z_t + (F_{t+1} - F_t)] = F(K_t, A_t L_t). \quad (33)
\]

The last term of the left-hand side of this expression is not standard. Transfers other than pensions, \( Z_t \), show up in this expression because we assume that the government throws them to the sea. And the change in the value of the pension reserve fund, \( (F_{t+1} - F_t) \) shows up because pension system surpluses are invested in the pension fund and pension system deficits are financed with the fund until it is depleted.\(^{19}\)

\(^{19}\)The last term of the left-hand side of Expression (33) would show up as net exports in the standard national income and product accounts.
(vii) The law of motion for $\mu_t$ is:

$$\mu_{t+1} = \int_{\mathbb{R}} Q_t d\mu_t.$$  

(34)

Describing function $Q$ formally is complicated because it specifies the transitions of the measure of households along its six dimensions: age, education level, employment status, assets holdings, pension rights, and pensions. An informal description of this function is the following:

We assume that new-entrants, who are 20 years old, enter the economy as able-bodied workers, that they draw the stochastic component of their endowment of efficiency labor units from its invariant distribution, and that they own zero assets and zero pension rights. Their educational shares are exogenous and they determine the evolution of $\mu_{ht}$. We also assume that new-entrants who are older than 20 replicate the age, education, employment status, wealth, pension rights, and pensions share distribution of the existing population.

The evolution of $\mu_{jht}$ is exogenous, it replicates the Spanish demographic projections, and we compute it following a procedure that we describe in Appendix 1 below. The evolution of $\mu_{et}$ is governed by the conditional transition probability matrix of its stochastic component, by the probability of becoming disabled, and by the optimal decision to retire. The evolution of $\mu_{at}$ is determined by the optimal savings decision, the unintentional bequests, and the age-dependent net migration flows estimated by the Spanish Instituto Nacional de Estadística (INE). The evolution of $\mu_{bt}$ is determined by the rules of the Spanish public pension system which we have described in Section 2.1. Finally, we assume that once a household retires or becomes disabled its retirement or disability pensions never change.

3 Calibration

To calibrate our model economy we do the following: First, we choose a calibration target country —Spain in this article— and a calibration target year —2010 in this article. Then we choose the initial conditions and the parameter values that allow our model economy to replicate as closely as possible selected macroeconomic aggregates and ratios, distributional statistics, and the institutional details of our chosen country in our target year.

3.1 Initial conditions

To determine the initial conditions, first we choose an initial distribution of households, $\mu_0$. In Appendix 1 we provide a detailed description about how we obtain that distribution. The initial distribution of households implies an initial value for the capital stock. This value is $K_{2010} = 12.1034$. The initial distribution of households and the initial survival probabilities determine the initial value of unintentional bequests, $E_{2010}$. We must also specify the initial values for the productivity process, $A_{2010}$, and for the pension reserve fund $F_{2010}$. Since $A_{2010}$ determines the units which we use to measure output and does nothing else, we choose $A_{2010} = 1.0$. Finally, our choice for the initial value of the pension reserve fund is $F_{2010} = 0.0612 Y^*_t$, where $Y^*_t$ denotes output at market prices, which we define as $Y^*_t = Y_t + T_{ct}$. This number corresponds to the value of the Spanish pension fund at the end of 2010.

3.2 Parameters

When all is told and once the initial conditions are specified, to characterize our model economy fully, we must choose the values of a total of 50 parameters. Of these 50 parameters, 3 describe the household preferences, 21 the process on the endowment of efficiency labor units, 4 the disability risk, 3 the production technology,
12 the pension system rules, and 7 the remaining components of the government policy. To choose the values of these 50 parameters we need 50 equations or calibration targets which we describe below.

### 3.3 Equations

To determine the values of the 50 parameters that identify our model economy, we do the following. First, we determine the values of a group of 31 parameters directly using equations that involve one parameter only. To determine the values of the remaining 19 parameters we construct a system of 19 non-linear equations. Most of these equations require that various descriptive statistics of our model economy replicate the values of the corresponding Spanish statistics in 2010. We describe the determination of both sets of parameters in the subsections below.

#### 3.3.1 Parameters determined using single equations

*The life-cycle profile of earnings.* We measure the deterministic component of the process on the endowment of efficiency labor units independently of the rest of the model. We estimate the values of parameters of the three quadratic functions that we describe in Expression (1), using the age and educational distributions of hourly wages reported by the *Instituto Nacional de Estadística* (INE) in the *Encuesta de Estructura Salarial* (2010) for Spain. This procedure allows us to identify the values of 9 parameters.

*The disability risk.* We want the probability of becoming disabled to approximate the data reported by the *Boletín de Estadísticas Laborales* (2007) for the Spanish economy. We use this dataset to estimate the values of parameters $a_4$ and $a_5$ of Expression (3) using an ordinary least squares regression of $q_j$ on $j$. According to the *Instituto de Mayores y Servicios Sociales*, in 2008 in Spain 62.6 percent of the total number of disabled people aged 25 to 44 years old had not completed high school, 26.9 percent had completed high school, and the remaining 10.5 percent had completed college. We use these shares to determine the values of parameters $a_6$ and $a_7$ of Equation (4). Specifically, we choose $a_6 = 0.269/0.626 = 0.4297$ and $a_7 = 0.105/0.626 = 0.1677$. This procedure allows us determine the values of 4 parameters.

*The pension system.* In 2010 in Spain, the payroll tax rate paid by households was 28.3 percent and it was levied only on the first 44,772 euros of annual gross labor income. Hence, the maximum contribution was 12,670 euros which correspond to 45.53 percent of the Spanish GDP per person who was 20 or older. To replicate this feature of the Spanish pension system we choose the value of parameter $a_{13}$ of our payroll tax function to be $a_{13} = 0.4553$.

Our choice for the number of years used to compute the retirement pensions in our benchmark model economy is $N_b = 15$. This is because the Spanish *Régimen General de la Seguridad Social* considers the last 15 years of contributions prior to retirement to compute the pension.

We assume that the minimum pension, the maximum pension, and the maximum covered earnings are directly proportional to per capita income. Our targets for the proportionality coefficients are $b_0 = 0.1731$, $b_{mt} = 1.2567$, and $a_{15} = 1.6089$. These numbers correspond to their values in 2010 in Spain for workers included in the *Régimen General*.\(^{20}\)

We choose our first and normal retirement ages to be $R_0 = 60$ and $R_1 = 65$. In Spain the first retirement age was 60 until 2002. This rule was changed in 2002 when the first retirement age was changed to 61, with

\(^{20}\)Specifically, in 2010 the minimum retirement pension in Spain was 4,817 euros, the maximum pension was 34,970 euros, the maximum covered earnings were 44,772 euros, and GDP per person who was 20 or older was 27,827 euros.
Table 1: The values of 38 of the model economy parameters

<table>
<thead>
<tr>
<th>Parameter obtained directly</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Earnings Life-Cycle</td>
<td>$a_{1,1}$</td>
<td>0.9189</td>
</tr>
<tr>
<td></td>
<td>$a_{1,2}$</td>
<td>0.8826</td>
</tr>
<tr>
<td></td>
<td>$a_{1,3}$</td>
<td>0.5064</td>
</tr>
<tr>
<td></td>
<td>$a_{2,1}$</td>
<td>0.0419</td>
</tr>
<tr>
<td></td>
<td>$a_{2,2}$</td>
<td>0.0674</td>
</tr>
<tr>
<td></td>
<td>$a_{2,3}$</td>
<td>0.1648</td>
</tr>
<tr>
<td></td>
<td>$a_{3,1}$</td>
<td>0.0006</td>
</tr>
<tr>
<td></td>
<td>$a_{3,2}$</td>
<td>0.0008</td>
</tr>
<tr>
<td></td>
<td>$a_{3,3}$</td>
<td>0.0021</td>
</tr>
<tr>
<td>Disability Risk</td>
<td>$a_4$</td>
<td>0.000449</td>
</tr>
<tr>
<td></td>
<td>$a_5$</td>
<td>0.0924</td>
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<tr>
<td></td>
<td>$a_6$</td>
<td>0.4291</td>
</tr>
<tr>
<td></td>
<td>$a_7$</td>
<td>0.1677</td>
</tr>
<tr>
<td>Preferences</td>
<td>$\sigma$</td>
<td>4.0000</td>
</tr>
<tr>
<td>Technology</td>
<td>$\theta$</td>
<td>0.3669</td>
</tr>
<tr>
<td></td>
<td>$\gamma$</td>
<td>0.0000</td>
</tr>
<tr>
<td>Public Pension System</td>
<td>$a_{16}$</td>
<td>0.4000</td>
</tr>
<tr>
<td></td>
<td>$a_{17}$</td>
<td>0.0800</td>
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<tr>
<td></td>
<td>$N_6$</td>
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</tr>
<tr>
<td></td>
<td>$R_0$</td>
<td>60</td>
</tr>
<tr>
<td></td>
<td>$R_1$</td>
<td>65</td>
</tr>
<tr>
<td></td>
<td>$r^*$</td>
<td>0.0200</td>
</tr>
<tr>
<td>Government Policy</td>
<td>$a_9$</td>
<td>0.4500</td>
</tr>
<tr>
<td></td>
<td>$a_{11}$</td>
<td>1.0710</td>
</tr>
<tr>
<td>Parameters determined by guesses for $(K, L)$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Public Pension System</td>
<td>$a_{13}$</td>
<td>0.4553</td>
</tr>
<tr>
<td></td>
<td>$a_{15}$</td>
<td>1.6089</td>
</tr>
<tr>
<td></td>
<td>$b_{0t}$</td>
<td>0.6639</td>
</tr>
<tr>
<td></td>
<td>$b_{mt}$</td>
<td>4.6021</td>
</tr>
<tr>
<td>Government Policy</td>
<td>$G$</td>
<td>0.7562</td>
</tr>
<tr>
<td></td>
<td>$a_8$</td>
<td>0.1907</td>
</tr>
<tr>
<td></td>
<td>$a_{12}$</td>
<td>0.2113</td>
</tr>
<tr>
<td>Parameters determined solving the system of equations</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Preferences</td>
<td>$\alpha$</td>
<td>0.2979</td>
</tr>
<tr>
<td></td>
<td>$\beta$</td>
<td>1.0460</td>
</tr>
<tr>
<td>Technology</td>
<td>$\delta$</td>
<td>0.0724</td>
</tr>
<tr>
<td>Public Pension System</td>
<td>$a_{14}$</td>
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</tr>
<tr>
<td></td>
<td>$\phi$</td>
<td>0.8279</td>
</tr>
<tr>
<td>Government Policy</td>
<td>$a_{10}$</td>
<td>0.0672</td>
</tr>
<tr>
<td></td>
<td>$Z$</td>
<td>-0.0807</td>
</tr>
</tbody>
</table>
some exceptions. We choose \( R_0 = 60 \) because in 2010 a large number of workers were still retiring at that age.\(^{21}\)

To identify the early retirement penalty function, we choose \( a_{16} = 0.4 \), and \( a_{17} = 0.08 \). This is because we have chosen \( R_0 = 60 \), and because in Spain the penalties for early retirement are 8 percent for every year before age 65. Finally, for the rate of return on the pension reserve fund’s assets we choose \( r^* = 0.02.\(^{22}\)

These choices allow us to determine directly the values of 10 parameters.

**Government policy.** We choose directly the values of government consumption, \( G_t \), of the tax rate on capital income, \( a_8 \), of parameters \( a_9 \) and \( a_{11} \) of the household income tax function, and of the tax rate on consumption, \( a_{12} \). We describe our procedure to choose the value of these five parameters in Appendix 2.

**Preferences.** Of the four parameters in the utility function, we choose the value of \( \sigma \) directly. Specifically, we choose \( \sigma = 4.0 \). This choice and the value of the share of consumption in the utility function, imply that the relative risk aversion in consumption is 1.8937, which falls within the 1.5-3 range which is standard in the literature.

**Technology.** According to the OECD data, the capital income share in Spanish GDP was 0.3669 in 2008. Consequently, we choose \( \theta = 0.3669 \) directly. We also choose the growth rate of total factor productivity directly. We discuss this choice in Appendix 3 below.

Specifically, we assume that the value of the growth rate of the labor-augmenting productivity process is \( \gamma = 0 \). The rationale for this choice is as follows. According to Balmaseda, Melguizo, and Taguas (2006), between 1988 and 2004, the average annual productivity growth rate, measured as output per employee, was only 0.6 percent. Moreover, Boldrin, Conde-Ruiz, and Díaz-Giménez (2010) show that for the period 1999-2006, the growth rate of labor productivity has been negative. Consequently, our choice resembles the average behavior of Spanish labor productivity during the last few years.

**Adding up.** So far we have determined the values of 31 parameters directly. We report their values in the first two blocks of Table 1.

### 3.3.2 Parameters determined using a system of equations

We still have to determine the values of 19 parameters. To find the values of those 19 parameters we need 19 equations. Of those equations, 14 require that model economy statistics replicate the value of the corresponding statistics for the Spanish economy in 2010. The government budget constraint allows us to determine the value of \( Z/Y^* \) residually. And the 4 remaining equations are normalization conditions.

**Aggregate Targets.** We report the values of the 6 Spanish macroeconomic aggregates and ratios that we target in Table 2. According to the Spanish Encuesta de Empleo del Tiempo (2010), the average number of hours worked per worker was 36.79 per week. If we consider the endowment of disposable time to be 14 hours per day, the total amount of disposable time is 96 hours per week. Dividing 36.79 by 96 we obtain 37.5 percent which is the share of disposable time allocated to working in the market that we target. Consequently,

\(^{21}\)In 2010 in Spain 22.4 percent of the people who opted for early retirement were 60 years old or younger. And 5.78 percent of the total number of retirees were 60 or younger. See Ministerio de Trabajo e Inmigración (MTIN), Anuario de Estadísticas 2010 (http://www.empleo.gob.es/estadisticas/ANUARIO2010/PEN/index.htm).

\(^{22}\)In Díaz-Giménez and Díaz-Saavedra (2009) we also run simulations \( r^* = 0.01, r^* = 0.03, \) and \( r^* = 0.04 \). We found that the changes implied by the various values of \( r^* \) were small and that they did not modify the qualitative conclusions of that article.
the Frisch elasticity of labour supply implied in our model is 0.77, which is in the middle of the range of econometric estimates. We describe how we obtain the remaining targets in Appendix 2.

**Distributional Targets.** We target the 3 Gini indexes and 5 points of the Lorenz curves of the Spanish distributions of earnings, income and wealth for 2004. We have taken these statistics from Budría and Díaz-Giménez (2006), and we report them in bold face in Table 6. Castañeda Díaz-Giménez and Rios-Rull (2003) argue in favor of this calibration procedure to replicate the inequality reported in the data. These targets give us a total of 8 additional equations.

**The Government Budget.** The government budget is an additional equation that allows us to obtain residually the government transfers to output ratio, $Z_t/Y_t^*$. 

**Normalization conditions.** Finally, in our model economy there are 4 normalization conditions. The transition probability matrix on the stochastic component of the endowment of efficiency labor units process is a Markov matrix and therefore its rows must add up to one. This gives us three normalization conditions. We also normalize the first realization of this process to be $s(1) = 1$.

### Table 3: The Stochastic Component of the Endowment Process

<table>
<thead>
<tr>
<th>Transition Probabilities</th>
<th>$\pi^*(s)$%</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s = s_1$</td>
<td>1.0000</td>
</tr>
<tr>
<td>$s = s_2$</td>
<td>2.0856</td>
</tr>
<tr>
<td>$s = s_3$</td>
<td>11.2892</td>
</tr>
</tbody>
</table>

*a* $\pi^*(s)$% denotes the invariant distribution of $s$.

**Computation.** To determine the values of these 19 parameters first we solve the system of 14 non-linear equations in 14 unknowns that we obtain when we equate the relevant statistics of the model economy to their corresponding Spanish targets. Once we had chosen the best solution to this system, we obtained the values of the remaining 5 parameters from the government budget and from our normalization conditions. In the third block of Table 1 and in the first two blocks of Table 3, we report the values of the 19 unknowns.

### 4 Calibration Results: The Benchmark Model

We check that our theoretical framework is consistent with Spanish data. The single most important feature of the Spanish economy that our model economy should approximate is the retirement behavior of Spanish households if we want to consider seriously our quantitative findings. Consequently, we begin this section by analyzing in great detail the statistics characterizing retirement behavior, both in Spain and in our benchmark model economy. Subsequently, we consider the main aggregates and ratios, and finally we examine the distributions on earnings, income, pensions and wealth.
4.1 Retirement behavior

An initial overview In Table 4 we report the average retirement ages and the participation rates of those aged 60 to 64. The table shows that the model predicts an average retirement age of 63.5 years, and that this number is 1.2 years higher than its empirical counterpart. The model also predicts increasing average retirement ages in proportion to the number of years of education. Unfortunately, the actual statistics are not available, but this relationship is highly plausible, since participation rates in Spain also increase with education (see column 3 of Table 4).

Table 4: Retirement Ages And Participation Rates

<table>
<thead>
<tr>
<th></th>
<th>Avg Ret Ages</th>
<th>Part rates at 60-64 (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Spain¹</td>
<td>Model</td>
</tr>
<tr>
<td>All</td>
<td>62.3</td>
<td>63.5</td>
</tr>
<tr>
<td>Dropouts</td>
<td>n.a.</td>
<td>63.1</td>
</tr>
<tr>
<td>High School</td>
<td>n.a.</td>
<td>63.8</td>
</tr>
<tr>
<td>College</td>
<td>n.a.</td>
<td>64.4</td>
</tr>
</tbody>
</table>

¹The Spanish data is for both males and females in 2010 (Source: Eurostat).
²The Spanish data is from both the Encuesta de la Población Activa, and the Encuesta de Empleo del Tiempo 2010, excluding the unemployed and non-participants who do not collect either retirement or disability pensions.

The total participation rate of those households aged 60 to 64 is 53.9 percent in our model economy, and 56.6 percent in Spain. The table also shows that participation rates in Spain increase with education. This is because even though all educational types value leisure equally, the foregone labor income (which is the opportunity cost of leisure) is lower for less educated workers. A second reason is that low educated workers are those who most take advantage of early retirement provisions, such as the minimum retirement pension provided by the Spanish pension system. The model successfully reproduces this tendency, and also does a good job in replicating the participation rates for all educational types.²³

Further details. An examination of other statistics on retirement behavior increases our confidence in our model economy as a tool for policy analysis. Panel A of Figure 2 shows the age-dependent empirical profile for claiming retirement benefits in Spain. This profile, which displays peaks at the first and the normal retirement age, is a common stylized fact across those countries operating a defined benefit pension system (see Gruber and Wise, 1999). Our model economy successfully matches the empirical profile, as the claiming of benefits is also concentrated at the first and normal retirement ages. Close scrutiny reveals that hazard is higher at age 65 in the model economy, reaching 81.9 percent. For Spain, this figure is 71.8.

Our model economy also predicts a much higher probability of low-educated workers leaving the labor force at the age of 60 (see Panel B of Figure 2). In fact, 80 percent of those who retire at this age are dropout workers. This is consistent with the findings of Sánchez-Martín (2010), who report that at age 60 low income workers have a much higher probability of retiring than high income workers. In both Spain and our model economy, the minimum retirement pension provided by the pension system is mainly behind this fact, since this type of pension strongly affects retirement behavior, and it is currently received by 27 percent of all retirees in Spain, this number reaching 31 percent in our model economy. Workers can receive this type of pension since the first retirement age of 60, and are also aware that delaying the receipt of this minimum amount does not increase it. In other words, a worker entitled to this amount faces a significant implicit tax on continued work. Consequently, workers, and especially low income workers, have the incentive to apply for this benefit at age 60. In our model economy, 97 percent of those who leave the labor force at age 60 receive this minimum pension, while Jiménez-Martín and Sánchez-Martín (2006) find that this figure in

²³When making this comparison it must be remembered that there exist certain fundamental differences between Spain and our model economy. In Spain, people of working age fall into one of five categories: employed, unemployed, retired, disabled, and other non-participants. In our model economy we only have three: employed, disabled, and retired. This is why we present the Spanish data in these three categories.
Spain is 67 percent. Retirement hazards fall after age 60 in both Spain and in our model economy, due to the same key economic force. Those workers with pension entitlements higher than the minimum pension, by working one further year can reduce by up to 8 percent the annual early retirement penalty applied to his or her pension. This means that after age 60, many workers face an implicit subsidy to continuing in work, which may amount to 25 percent of their net salary level in the relevant year, as shown by Boldrin et al. (1997). Expressed another way, these workers can increase their Social Security Wealth (SSW) if they choose to work at least one more year. Notice also that this implicit subsidy is reduced as age approaches to 65, because the interaction between labor income dynamics and the Regulatory Base. Consequently, retirement hazards increases after age 61 in both Spain and our model economy.

The picture is different at the age of 65. Because the Spanish pension system provides no economic incentives to delay retirement beyond this age, and also because of the drop in the Regulatory Base resulting from the worker’s labor income dynamics, SSW continues to be reduced for most workers who would remain in the labor force. In addition, the marginal tax rate on labor income may turn out to be higher than the marginal tax rate on pension income, due to the high progressivity of the Spanish income tax schedule. Consequently, these workers choose to leave the labor force to avoid the high implicit tax on continuing to work. Boldrin et al. (1997), Argimón et al. (2009), and Sánchez-Martín (2010) find that the probability of retirement at age 65 is independent of salary level, and our model economy replicates this stylized fact reasonably well. For instance, at age 65, retirement hazards are similar for all educational groups, and are over 75 percent (see Panel B of Figure 2).

Finally, Panel C of Figure 2 compares the age-dependent aggregate participation rates in the data and in our model economy. The data are based on the Encuesta de Empleo del Tiempo (2010), reported by the INE. This panel shows that our model economy is successful in matching quantitatively the decline in the participation rate starting at age 50, more sharply after age 60, in Spain.

Overall assessment. An accurate assessment of the questions we pose in this paper requires a model economy.
that captures the key institutional and economic forces leading to retirement. Our model economy describes in great detail both the Spanish tax system and the rules of the Spanish Public Pension System. It also incorporates a calibration procedure for the earnings process which is consistent with earnings inequality in Spain (see below). Thus, our model economy successfully matches distribution of retirement and other key features of retirement behavior found in Spanish data. This is particularly remarkable since the calibration procedure did not explicitly target the various facts on retirement behavior.

4.2 Aggregates and Ratios

*Macroeconomic Aggregates and Ratios.* In Table 5 we report the macroeconomic aggregates and ratios in Spain and in our benchmark model economy for 2010. We find that our benchmark model economy does a good job in replicating most of the values for the chosen targets.

<table>
<thead>
<tr>
<th></th>
<th>C/Y</th>
<th>K/Y</th>
<th>h</th>
<th>Tp/Y*</th>
<th>Tp/Y+</th>
<th>P/Y*</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spain</td>
<td>51.5</td>
<td>3.28</td>
<td>37.5</td>
<td>7.4</td>
<td>10.1</td>
<td>10.3</td>
</tr>
<tr>
<td>Model</td>
<td>51.4</td>
<td>3.28</td>
<td>38.0</td>
<td>7.7</td>
<td>10.1</td>
<td>10.2</td>
</tr>
</tbody>
</table>

4.3 Inequality

*Distributional statistics* In Table 6 we report the Gini indices and selected points of the Lorenz curves for earnings, income, pensions and wealth in Spain and in our model economy. The statistics reported in bold are our eight calibration targets. The source for the Spanish data on earnings, income and wealth is the 2004 Financial Survey of Spanish Families, as reported in Budría and Díaz-Giménez (2006). We take the Gini index of pensions from Conde-Ruiz and Profeta (2007). The model economy statistics correspond to 2010. In Figure 3 we plot the Lorenz curves of these distributions in our model economy.

We find that our heterogeneous household model economy replicates all the Spanish Gini indices reasonably well. When we compare the earnings and income shares of the quintiles in the model economy, we find that the top quintiles of these two distributions earn more than in Spain. The fact that the model economy can account reasonably well for both the Lorenz income curve and the Gini pension index is particularly remarkable, since we have not used any of its points as our calibration targets. We also find that wealth is similarly concentrated in our model economy and in Spain. Despite this, the greatest differences between our heterogeneous household model economy and the Spanish data lie in the top 1 percent of wealth distribution, since wealth is considerably more concentrated in Spain. This disparity was expected, because in general overlapping generations economies fail to account for the large shares of wealth owned by the richest households in the data.²⁵

5 The reforms

This paper studies the consequences of three counterfactual experiments. The logic of these experiments, or reforms, is supported by the literature on age-dependent taxation, which points to efficiency gains from

²⁵See Castañeda et al. (2003) for an elaboration of this argument.
Table 6: The Distributions of Earnings, Income, Pensions, and Wealth∗

<table>
<thead>
<tr>
<th></th>
<th>Bottom Tail</th>
<th>Quintiles</th>
<th>Top Tail</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Gini</td>
<td>1–5</td>
<td>5–10</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Spain</td>
<td>0.49</td>
<td>0.0 0.7</td>
<td>1.2</td>
</tr>
<tr>
<td>Model</td>
<td>0.48</td>
<td>0.1 0.8</td>
<td>1.3</td>
</tr>
<tr>
<td>Spain</td>
<td>0.42</td>
<td>0.0 0.7</td>
<td>1.1</td>
</tr>
<tr>
<td>Model</td>
<td>0.44</td>
<td>0.1 0.9</td>
<td>1.5</td>
</tr>
<tr>
<td>Spain</td>
<td>0.32</td>
<td>n.a. n.a.</td>
<td>n.a.</td>
</tr>
<tr>
<td>Model</td>
<td>0.36</td>
<td>0.4 1.7</td>
<td>2.2</td>
</tr>
<tr>
<td>Spain</td>
<td>0.57</td>
<td>−0.1 0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Model</td>
<td>0.57</td>
<td>0.0 0.0</td>
<td>0.0</td>
</tr>
</tbody>
</table>

∗The source for the Spanish data of earnings, income, and wealth is the 2004 Encuesta Financiera de las Familias Españolas as reported in Budría and Díaz-Giménez (2006). We take the Pensions Gini index in Spain from Conde-Ruiz and Profeta (2007). The model economy statistics correspond to 2008. The statistics in bold face have been targeted in our calibration procedure.

using age to target lower tax rates at households with greater elasticities of labor supply. The current section describes these reforms in detail.

Reform 1. This reform eliminates all labor income taxes for those workers aged 60 or over. Consequently, such workers are exempt from paying personal income tax imputed to labor activities, and workers in the 60-64 age range are exempt from paying payroll taxes. It must be noted that this reform affects the revenue side of both the pension system budget (due to lower payroll tax collection) and the government budget (due to lower income tax collection).

Reform 2. This reform has two objectives. Firstly, to resemble the annual pension increase for every year worked after age 60. Secondly, and more importantly, to eliminate the current implicit tax on continued work from age 65 onwards. The reform fixes at 8 percent per year the annual increase in pension for each year worked beyond age 65. Under the current Spanish public pension system, this figure is 6 percent for every year worked between ages 60 and 64, but only 3 percent for every year worked beyond age 65. Thus, since the 3 percent annual premium is actuarially less than fair, the pension system provides strong incentives to claim benefits by age 65. To give an example, let us define $SSW_{jt}$ as the Social Security Wealth of an individual of age $j$ at period $t$. Then, and considering only this dimension, a worker would be indifferent between retiring at age $j$ or at age $j+1$, if $SSW_{jt}$ and $SSW_{j+1t}$ are equal. In this case, the pension system is said to be actuarially neutral.

Figure 4 shows the age-dependent expected lifetime for Spanish households, based on the current age-dependent survival probabilities computed by the Spanish National Statistics Institute (INE). Thus, expected lifetime at the age of 65 is 17.4 years and falls to 16.5 years 1 year later. Consequently, for a worker to be indifferent between retiring at age 65 or 66, the annual premium in his or her pension must be approximately 7.5 percent. As this is not the case, the Spanish pension system imposes an implicit tax on continued work.

26 It must be remembered that workers aged 65 or over were exempt from paying payroll taxes in the benchmark model economy, following the Spanish public pension system rules.

27 We could also have chosen a revenue-neutral reform. That is to say, the missing revenues from the lower income and payroll tax collections could be recovered by a higher tax rate on young workers income, for instance. However, the increase in income taxes early in life might encourage the reallocation of work hours over the life cycle (see later on).

28 Social Security Wealth at age $j$ in period $t$ is defined here, as the present expected value at period $t$ of the future stream of pension payments from age $j$ onwards, to which a household is entitled to over his or her remaining lifetime.
beyond the age of 65, since working one additional year after age 65 reduces a household’s Social Security Wealth. For instance, Boldrin et al. (1997) find that this implicit tax may reach 80 percent of the age 65 net salary level, while Duval (2003) presents an even higher figure, 90 percent.

Reform 3. All public pension schemes have an insurance aspect, which implies redistribution among individuals. Public pensions can thus increase a household’s lifetime wealth, allowing it to finance its retirement with less work effort. In the case of the Spanish public pension system, this redistribution is in part due to the minimum retirement pension. Specifically, a worker who has paid payroll taxes for at least 15 years, is entitled to receive a retirement pension after the first legal retirement age of 60. Additionally, this pension should never be lower than the minimum retirement pension, a guaranteed minimum fixed on a year by year basis by the Spanish government. Boldrin et al. (1997) show that such pension type imposes an implicit tax on continued work, because it neutralizes the strong incentives to work associated with early retirement penalties. As working an additional year does not increase the minimum pension, the best strategy for a

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*Sánchez-Martín (2010) shows that most low income workers are beneficiaries of this type of pension when they decide to retire.
worker entitled to this guaranteed minimum is to leave the labor force as soon as the pension is available, at
the first retirement age of 60. This reform eliminates the minimum retirement pension, and consequently,
its associated implicit tax on continued work, especially for low-income workers.

6 Results

We compute the equilibrium of our benchmark economy and then compute three additional equilibria, one
for each reform. To keep the distortions caused by the reforms as low as possible, and to make the
comparisons meaningful, we assume that government expenditure, transfers, and consumption, income and
payroll tax rates are identical in all four model economies. The benchmark and the reformed model economies
differ in their income tax and payroll tax collections, in their pension payments, and in their unintentional
bequests, which are endogenous. In Appendix 3, we show our results when we compute the transition from
the calibrated benchmark economy to a new economy under an alternative policy.

6.1 Retirement and hours of work over the life cycle.

Tables 7 and 8 present the results regarding differences in participation rates and hours of work between the
benchmark and the reformed economies. Figures 5 and 6 compare the age dependent retirement probabilities
from the labor force, and participation rates of elderly. Our principal findings are given immediately below.

Retirement behavior. All the reforms increase retirement ages, and consequently, the participation rates of
the elderly (see Figure 6). Eliminating all labor income taxes for those aged 60 or over increases the average
retirement age by almost 4 months. It also increases by almost 4 points the participation rates for those
workers aged 60 to 64 (see the second block in Table 7). We also find that highly educated workers delay their
retirement by more than their lesser educated counterparts, as part of this reform eliminates a progressive
tax (i.e. the personal income tax imputed to labor activities) which taxes high earners heavily. Note that
the longer working lifetime modifies the age-dependent profile of retirement hazards from the labor force
(see the left hand panel of Figure 5), since these probabilities decrease.

Table 7: Average Retirement Ages and Participation Rates for Workers Age 60-64

<table>
<thead>
<tr>
<th></th>
<th>Benchmark</th>
<th>Reform 1(^a)</th>
<th>Reform 2(^b)</th>
<th>Reform 3(^c)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Ret. Ages</td>
<td>Part. Rates</td>
<td>Diff. in R. Ages</td>
<td>Diff. in P. Rates</td>
</tr>
<tr>
<td>All</td>
<td>63.51</td>
<td>53.89</td>
<td>0.32</td>
<td>3.80</td>
</tr>
<tr>
<td>Dropouts</td>
<td>63.06</td>
<td>40.55</td>
<td>0.20</td>
<td>2.30</td>
</tr>
<tr>
<td>High School</td>
<td>63.82</td>
<td>65.22</td>
<td>0.43</td>
<td>6.58</td>
</tr>
<tr>
<td>College</td>
<td>64.38</td>
<td>79.47</td>
<td>0.70</td>
<td>4.00</td>
</tr>
</tbody>
</table>

\(^a\) Reform 1 eliminates all labor income taxes for those aged 60 or more.
\(^b\) Reform 2 sets in 8 percent per year the increase in pension for each additional worked year
after age 65.
\(^c\) Reform 3 eliminates the minimum retirement pension.
\(^d\) Difference in participation rates for workers aged 65 to 69 years.

The second reform, which increases the annual pension premium for every year worked beyond age 65,
increases by 0.6 years the average retirement age, and by almost 2 points the participation rate of workers

\(^30\) Recall that our benchmark model economy is the model economy where the tax and transfers programs remain unchanged.
\(^31\) There is a small difference in the retirement hazard at age 60, because retirement probability at this age depends heavily
on the minimum retirement pension, a guaranteed amount this first reform does not modify.
aged 60 to 64. However, and more important, this reform increases by almost 10 points the participation rate of those aged 65 to 69 (see the third block in Table 7). Again, high school and college workers are those who delay their retirement most, but this time because these educational groups had higher ex-ante retirement ages, and thus benefit more from this reform. Consequently, there is a 17 a 16 point increases in their participation rates. As a result, the consequences for retirement hazards are straightforward: as some workers decide, following the reform, to work beyond age 65, the retirement hazard at this age falls from 71 to 31 percent (see central panel of Figure 5).

Finally, the last reform principally affects the retirement behavior of low-educated workers, since these workers increase their participation rate by 13 points (see the fourth block in Table 7), and on average they delay their retirement age by 1.5 years. By eliminating the minimum retirement pension, this reform brings the incentive effects stemming from penalizations for early retirement as working an additional year after age 60, increases all pension entitlements by 8 percent. Consequently, the age-dependent probabilities of exiting the labor force are lower in this reformed economy, especially at the age of 60, where the figure falls from 15 to 2 percent (see the right panel of Figure 5). This results is because at the age of 60, most of workers who chose to retire at that age in the benchmark economy, were low income workers entitled with the minimum pension.

Thus, we find that changes in tax and transfers programs induce a major shift in retirement behavior of older workers. Also, our findings are consistent with those from Erosa et al. (2011), who find that these programs may account for a large decline in the aggregate labor supply late in the life cycle.

Work hours over the life cycle. Since all the reforms increase the participation rates of the elderly, they also increase the total number of work hours of those aged 60 or over (see Table 8). In all cases, we find that the additional number of work hours by this age group in the reformed economies is striking. The percentage increase ranges from 14 percent in the first reform to 24 percent in the third reform, showing that the gains in terms of old age work hours are non trivial. It should be noted that these rises, like the changes in participation rates, differ notably across educational groups. If in the first reform more educated workers are those who most increase the total number of hours, low-educated workers increase their work hours in the third reformed economy by a dramatic 56 percent.

Differently from Laitner and Silverman (2011), we find that total hours worked are less responsive to an increase in old-age after tax wages (see the first block of Table 8). Laitner and Silverman (2011) analyze a US social security reform in which older individuals no longer face the OASI payroll tax and their subsequent
Figure 6: Participation Rates (%)

Table 8: Difference in Hours Worked With The Benchmark Economy (%)

<table>
<thead>
<tr>
<th></th>
<th>Reform 1</th>
<th>Reform 2</th>
<th>Reform 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ages</td>
<td>Ages</td>
<td>Ages</td>
<td>Ages</td>
</tr>
<tr>
<td>20-39</td>
<td>-0.70</td>
<td>-0.37</td>
<td>-0.54</td>
</tr>
<tr>
<td>40-59</td>
<td>-1.32</td>
<td>-0.90</td>
<td>-0.72</td>
</tr>
<tr>
<td>60+</td>
<td>13.59</td>
<td>14.42</td>
<td>24.12</td>
</tr>
<tr>
<td>All</td>
<td>-0.41</td>
<td>-0.03</td>
<td>0.91</td>
</tr>
<tr>
<td>Dropouts</td>
<td>-0.28</td>
<td>-0.04</td>
<td>0.14</td>
</tr>
<tr>
<td>High School</td>
<td>-1.05</td>
<td>-0.76</td>
<td>3.08</td>
</tr>
<tr>
<td>College</td>
<td>-1.22</td>
<td>-0.95</td>
<td>52.60</td>
</tr>
<tr>
<td></td>
<td>12.25</td>
<td>15.72</td>
<td>12.37</td>
</tr>
<tr>
<td></td>
<td>0.58</td>
<td>0.26</td>
<td>-0.06</td>
</tr>
<tr>
<td></td>
<td>-0.60</td>
<td>0.03</td>
<td>-0.01</td>
</tr>
<tr>
<td></td>
<td>-0.47</td>
<td>-0.02</td>
<td>0.01</td>
</tr>
</tbody>
</table>

- Reform 1 eliminates all labor income taxes for those aged 60 or more.
- Reform 2 sets in 8 percent per year the increase in pension for each additional worked year after age 65.
- Reform 3 eliminates the minimum retirement pension.

In summary, our results suggest that for the cohort of workers nearing retirement, changes in the tax rates on their labor supply could lead to significant changes in that supply. Younger cohorts, however, will respond in part by adjusting their labor supply at earlier ages, so that lifetime and aggregate labor change less. This is consistent with the results of McGrattan and Rogerson (1998), who find that in the last century, US workers both shortened their working period and shifted work hours from older to younger ages, as social security earnings have no bearing on their benefits, and they find that this reform delays retirement ages by one year, and that it also brings significant efficiency gains. And this is because Laitner and Silverman (2011) present a life-cycle model and assume both that households make consumption and retirement decisions, and that households can not adjust their labor supply during the vesting period. This last assumption is crucial for generating a large response of total hours as discussed by Laitner and Silverman (2011). Our quantitative findings show a significant reallocation of work hours over the life cycle, that is, workers work less when young knowing that they will work until an older age, and will enjoy higher retirement pensions thereafter. Thus, we find that the reforms increase aggregate hours by 0.9 percent at most. 

This reallocation of hours is somewhat different following the third reform, as low-educated workers reduce their working hours when youngest. Subsequently, and because labor income during the 15 years prior to retirement is used to compute the retirement pension, this group works harder at middle age.
coverage increased. Furthermore, our results are in line with French and Jones (2011), who find that a higher net wage rate at older ages makes these workers work harder when old, and work less when they are young.

6.2 Aggregates and the Pension System.

Less (more) work hours tend to decrease (increase) effective labor (see Table 9). Moreover, these reforms may also reduce saving rates, and consequently the capital stock, since a shorter retirement period together with a higher retirement pension reduce the optimal level of assets needed to support consumption when retired, as firstly suggested by Feldstein (1974). Our results are also consistent with those of ˙Imrohoroglu and S. Kitao (2012), who find that raising the earliest retirement age by two years in U.S., could reduce capital by 1.5 percent. Thus, output decreases by 0.36, and 0.52 percent after the two first reforms, and it only increases by 0.05 percent in the third reformed economy. Finally, and regarding the balance of the pension system, we find that the first reform increases the initial pension system deficit by almost 0.5 percent of output (see also Table 9), because the higher retirement age cannot compensate for the lost payroll tax revenue. The second reform brings with it no significant variation, while the third reform brings a pension surplus of 0.1 points of output, because the significant increase in retirement ages more than compensates for the higher pensions.

<table>
<thead>
<tr>
<th></th>
<th>Output ($Y$)</th>
<th>Capital ($K$)</th>
<th>Labor ($N$)</th>
<th>Pension Surplus*($T_s - P$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Benchmark</td>
<td>3.3369</td>
<td>12.0171</td>
<td>1.5876</td>
<td>-0.073</td>
</tr>
<tr>
<td>Reform 1**</td>
<td>-0.36</td>
<td>-0.65</td>
<td>-0.21</td>
<td>-0.505</td>
</tr>
<tr>
<td>Reform 2**</td>
<td>-0.52</td>
<td>-1.16</td>
<td>-0.16</td>
<td>-0.082</td>
</tr>
<tr>
<td>Reform 3**</td>
<td>0.05</td>
<td>-0.14</td>
<td>0.15</td>
<td>0.102</td>
</tr>
</tbody>
</table>

* As a percentage of output.
** Difference, in percentual points, with the Benchmark economy.

In conclusion, increasing the participation rates of the elderly may not increase output. If this occurs, the efficiency gains could be small and distanced from previous estimates. Herbertsson and Orszag (2003) find that early retirement can be held responsible for a reduction of 5 to 7 per cent of the potential annual GDP in OECD countries, these numbers being higher for European countries. Looking at the fine print, this is because Herbertsson and Orszag (2003) assume both that total labor input increases strongly with a higher employment rate of elderly, and that physical capital is not affected by the implied longer working lifetime. We already discussed that total labor supply could be much less responsive. But we also find that a shorter retirement period may reduce saving rates, and consequently capital stock. Thus, we think that when trying to quantify the output lost due to early retirement, it is important to consider the potential shift in hours over the life-cycle and the change in saving rates. Furthermore, a longer working lifetime may not improve pension sustainability, since what the government saves, by avoiding payments of retirement benefits to a number of individuals, it could lose by having to pay higher benefits later on, due to the permanent increase in the benefits received in later years.

7 Conclusions

This paper presents a multiperiod general equilibrium overlapping generations model economy, populated by ex-ante heterogeneous households. Our model economy introduces both extensive and intensive margins
of labor supply. We calibrate our model economy to the Spanish economy and find that it successfully replicates many important stylized facts regarding the retirement behavior of older workers in Spain. The model economy is used to study the consequences of three counterfactual experiments aimed at eliminating labor supply taxes for older workers. The rationale for these exercises rests on the literature on age-dependent taxation, which points to efficiency gains from using age to target lower tax rates at households with a greater elasticity of labor supply.

We find four main results. Firstly, reducing the labor supply taxation of the elderly significantly increases their participation rates, and consequently their retirement ages; the gains, in terms of old age work hours, are thus non-trivial. Secondly, we find that the labor supply response could not be so much an increase in total lifetime hours as a reallocation of hours over the life cycle. If this is the case, this is because younger workers will need to work less when young, knowing that they will work until an older age, and will enjoy a higher retirement pension thereafter. Thirdly, higher participation rates of the elderly might not increase output, since a shorter retirement period may reduce saving rates, and consequently physical capital. Thus, when trying to quantify the output lost due to early retirement, we think that it is important to consider both the potential shift in hours over the life cycle and the change in saving rates. Fourthly, the implied lower dependency rate, due to a higher participation rate, might not help to contain the social security deficit generated by population ageing. This is because it may be that what government saves by avoiding payments to certain retirees, it loses by having to pay higher pensions later.
References


Appendix 1: The Initial Distribution of Households

Recall that $\mu_{j,h,e,a,b,p,t}$ denotes the measure of households of type $(j, h, e, a, b, p)$ at period $t$ and that, whenever we integrate the measure of households over some dimension, we drop the corresponding subscript. To obtain $\mu_{2010}$, we proceed as follows:

1. We take the measure $\mu_{j,2010}$ for all $j = \{20, 21, ..., 100\}$ directly from the Spanish economy published by the National Institute of Statistics (INE). However, to solve the households’ decision problem we use the survival probabilities only.\(^{33}\)

2. We also take the measure $\mu_{j,h,2010}$ directly from the Spanish economy published by the National Institute of Statistics (INE). Specifically, we take from the Encuesta de Población Activa the population aged 20 and over by level of education attained and age group, for both males and females.\(^{34}\)

3. Next, we solve the decision problem of the model economy households. We obtain $\mu_{20,j,e,2010}$ from $\mu_{20,j,2010}$ and the invariant distribution of the stochastic component of the endowment of efficiency labor units process.\(^{35}\)

   To compute $\mu_{j,h,e,2010}$ for $j = \{21, 22, ..., 100\}$, we use the conditional transition probability matrix of the stochastic component of the endowment of efficiency labor units process, the probability of becoming disabled, and the optimal decision to retire.

4. To obtain $\mu_{20,h,e,a,b,p,2010}$, we assume that new-entrants own zero assets and have zero pension claims. For $j = 21, 22, ..., 44$, we use the household’s optimal saving decisions at age $j - 1$ and the pension system rules. From age $R_0 - N_b$ onwards, we accumulate the labor income to determine the pension claims and the optimal labor supply decisions.

5. Finally, to obtain $\mu_{j,h,e,a,b,p,2010}$, we use the optimal retirement decisions and the pension system rules.

Notice that steps 3, 4 and 5 must be computed simultaneously in the same loop.

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\(^{33}\)The latest demographic and survival probabilities can be found at http://www.ine.es/jaxiBD/tabla.do?per=01&type=db&divi=EPOB&kidtab=2.

\(^{34}\)The data can be found at http://www.ine.es/jaxiBD/menu.do?L=0&divi=EPA&his=1&type=db.

\(^{35}\)Note that we have assumed that there are no disabled households of age 20.
Appendix 2: Calibration of the Model Economy Ratios

A2.1 Calibration of the Macroeconomic Ratios

- The Spanish National Income and Product Data reported by the Instituto Nacional de Estadística (INE) for 2010 are the following:

<table>
<thead>
<tr>
<th>Shares of GDP (%)</th>
<th>Million Euros</th>
</tr>
</thead>
<tbody>
<tr>
<td>Private Consumption</td>
<td>56.72</td>
</tr>
<tr>
<td>Public Consumption</td>
<td>21.08</td>
</tr>
<tr>
<td>Consumption of Non-Profits</td>
<td>1.00</td>
</tr>
<tr>
<td>Gross Capital Formation</td>
<td>23.30</td>
</tr>
<tr>
<td>Exports</td>
<td>27.00</td>
</tr>
<tr>
<td>Imports</td>
<td>29.12</td>
</tr>
<tr>
<td>Total (GDP)</td>
<td>100.00</td>
</tr>
</tbody>
</table>

We adjust the amounts reported in Table 10 according to Cooley and Prescott (1995) and we obtain the following numbers:

- Adjusted Public Consumption: Public Consumption = 221,715 million euros.

The next adjustment is to allocate Net Exports to our measures of C, I, and G. To that purpose, we compute the shares of each of those three variables in the sum of the three and we allocate Net Exports according to those shares. The sum of the three variables is 1,073,613 million euros and the shares of C, I, and G are 51.49, 27.86, and 20.65 percent.

Next we redefine the model economy’s output and consumption from factor cost to market prices as follows: $Y^* = Y + T_c$, where $Y^*$ is the model economy’s output at market prices and $T_c$ is the consumption tax collections, and $C^* = C + T_c$, where $C^*$ is the model economy’s consumption at market prices.

Finally we use $C^*/Y^* = 51.49$ and $G/Y^* = 20.65$ as targets.

A2.2 Calibration of the Government Policy Ratios

- In Table 11 we report the 2010 revenue and expenditure items of the consolidated Spanish public sector. Notice that the GDP share of Government consumption differs from the one that we have computed in Section A2.1 because here we use its unadjusted value.

If we ignore the public pension system, the government budget in the model economy in 2010 is

$$G_{2010} + Z_{2010} = T_{c,2010} + T_{k,2010} + T_{w,2010} + E_{2010}$$

(35)
Table 11: Spanish Public Sector Expenditures and Revenues in 2010*

<table>
<thead>
<tr>
<th>Expenditures</th>
<th>Millions of euros</th>
<th>Percentage of GDP</th>
<th>Revenues</th>
<th>Millions of euros</th>
<th>Percentage of GDP</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumption</td>
<td>221,715</td>
<td>21.08</td>
<td>Sales and gross receipts taxes(^a)</td>
<td>94,234</td>
<td>8.96</td>
</tr>
<tr>
<td>Investment</td>
<td>40,091</td>
<td>3.81</td>
<td>Payroll taxes(^b)</td>
<td>106,599</td>
<td>10.13</td>
</tr>
<tr>
<td>Pensions(^c)</td>
<td>109,000</td>
<td>10.36</td>
<td>Individual income taxes</td>
<td>77,542</td>
<td>7.37</td>
</tr>
<tr>
<td>Other</td>
<td>108,839</td>
<td>10.35</td>
<td>Corporate profit taxes</td>
<td>19,425</td>
<td>1.84</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Other revenues</td>
<td>83,626</td>
<td>9.96</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Deficit</td>
<td>98,218</td>
<td>9.33</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Total</td>
<td>479,645</td>
<td>45.62</td>
</tr>
</tbody>
</table>


*Shares of nominal GDP at market prices.

\(^a\)It includes the tax collections from the Value Added Tax and other taxes on products.

\(^b\)Total revenues from the Spanish Social Security.

\(^c\)Total expenditure from the Spanish Social Security.

- Unintentional bequests, \(E_{2010}\), are exogenous.
- We target the output shares of \(T_{c,2010}\), \(T_{k,2010}\), and \(T_{y,2010}\), so that they replicate the GDP shares of Sales and Gross Receipt Taxes, Corporate Profit Taxes, and Individual Income taxes.
- We have already targeted the output ratio of government consumption and we have already accounted for government investment.
- We define the output share of transfers other than pensions, \(Z_{2010}\), residually to satisfy the budget.
- We report the model economy government budget items in Table 12 below.

Table 12: Model Economy Public Sector Expenditures and Revenues in 2010 (%\(Y^*\) Shares)

<table>
<thead>
<tr>
<th>Expenditures</th>
<th>Revenues</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumption and Investment ((G))</td>
<td>20.65</td>
</tr>
<tr>
<td>Pensions ((P))</td>
<td>10.35</td>
</tr>
<tr>
<td>Other Transfers ((Z))</td>
<td>0.83</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Total</td>
</tr>
</tbody>
</table>
Appendix 3: The Transitions

We also report our findings when we compute the transition dynamics from the calibrated benchmark economy to a new economy under an alternative policy. To this end, we assume that the government unexpectedly announces and implements these reforms at the beginning of year 2012. Then, the benchmark model economy and the reformed model economies share the initial conditions described above, and the demographic and the educational transitions, and the growth and fiscal policy scenarios that we now describe.

**Demographic Transition.** We use the demographic projections of the *Instituto Nacional de Estadística* (INE). The INE reports and projects the age distribution of Spanish residents from 2011 to 2052 for people aged from zero to 100 and more. Call those age cohorts $N_{jt}$ and let $N_{t} = \sum_{j=20}^{100+}N_{jt}$. Then, the age distribution of the households in our model economy is $\mu_{jt} = N_{jt}/N_{t}$ for $j = 20, 21, ..., 99, 100+$ and for $t = 2011, 2012, ..., 2051, 2052$. Note that according to these projections, the share of those aged 65+ over those aged 20-64 years will increase from 26 percent in 2010, to 77 percent in 2050 (see Panel A of Figure 7).

**Educational Transition.** We obtain $\mu_{jht}$, for $t = \{2011, 2012, ..., 2050\}$ from $\mu_{j,h,2010}$, from $\mu_{jt}$ for $t = \{2011, 2012, ..., 2050\}$, and from the following assumption: from 2011, newborns aged 20 enter to the economy with the same educational levels of the most educated age group so far, which is the one born between 1980 and 1984. Consequently, the shares of dropouts, high school, and college workers aged 20 to 64 vary from 19, 60, and 21 percent in 2010 to 9, 65, and 26 percent in 2050 (see Panel B of Figure 7).

**Growth Scenario.** We assume that the value of the growth rate of the labor-augmenting productivity process is $\gamma = 0$. The rationale for this choice is as follows. According to Balmaseda, Melguizo, and Taguas (2006), between 1988 and 2004, the average annual productivity growth rate, measured as output per employee, was only 0.6 percent. Moreover, Boldrin, Conde-Ruiz, and Díaz-Giménez (2010) show that for the period 1999-2006, the growth rate of labor productivity has been negative. Consequently, our choice resembles the average behavior of Spanish labor productivity during the last few years.

**Fiscal Policy Scenario.** Recall that the consolidated government and pension system budget constraint in

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36 This data can be found at http://www.ine.es/jaxi/menu.do?type=pcaxis&path=%2Ft20%2Fp251&file=inebase&L=0.
37 Note however that in our model economies there are two other sources of output growth: demographic and educational changes. The benchmark model economy and the three reformed economies share the educational and demographic transitions and the value of $\gamma$. But output grows at different rates in the four model economies because of the endogenous changes brought about by the reforms.

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our model economy is

\[
G_t + P_t + Z_t = T_{at} + T_{st} + T_{gt} + T_{ct} + (F_t - F_{t+1}) + E_t
\]  

(36)

In this expression, \(G_t\) is exogenous, and in our benchmark model economy we require \(G_t\) to be 20.65 percent of output, at market prices, every period \(t\). The remaining variables are endogenous. In our four model economies the consumption, capital income and payroll tax rates, the parameters that determine the household income tax function, and the values of government expenditures are identical.

**Factor Prices.** We also simulate the reforms under the assumption of fixed prices. Specifically, we assume as factor prices those obtained in the initial equilibrium of the benchmark model economy, and the rationale of this choice is twofold. First, because it can be arguable that Spain is a small open economy, and so the world interest rate is considered as a given, becoming an exogenous wage rate. And second, to eliminate the general equilibrium adjustment in prices because the demographic and educational transitions.

![Figure 8: Difference in participation rates 60-64 (%)](image)

The results from this second exercise are in line with our previous findings. That is to say, the reforms increase the participation rates of the elderly, and households continue to reallocate work hours across the life cycle. As before, these results differ across both the reforms and educational groups (see Figures 8, and 9). Note also that the educational transition, which implies higher percentages of skilled workers, makes the increase in hours worked by older workers, grow after the second reform. For the same reason, however, this increase tends to fall in the third reformed economy.

Two results, however, are worthy of mention. Firstly, we find that the expected aging of the Spanish population will lead to a sustained increase in pension payments over the coming decades. According to our results, and under the current pension system’s rules, the pension fund depletion would be in 2018, and the pension deficit in 2050 would amount 20.7 percent of output. Overall, the accumulated pension deficits over the next 40 years would be to more than 500 percent of output in 2050 (see Table 13). As expected due to our previous results, we find that there is not a significant effect of these reforms on pension system sustainability.

Secondly, we find that all the reforms increase hours worked and output in the long run (see Figure 10). This

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38Note that we assume in all the economies that whenever there is a surplus in the pension system, it is invested in the pension fund, and whenever there is a deficit, it is financed using the fund. And that the government borrows as much as necessary to finance any further pension system deficits after the fund is exhausted, paying the same exogenous rate \(r^*\) on this loans. We make this choice to minimize the large distortions that the growing pension system deficits would create if they were
Table 13: The Pension System Deficit and Fund

<table>
<thead>
<tr>
<th></th>
<th>Pension Fund Depletion</th>
<th>Pension Fund in 2050*</th>
<th>Pension Deficit in 2050*</th>
</tr>
</thead>
<tbody>
<tr>
<td>Benchmark</td>
<td>2018</td>
<td>−505.78</td>
<td>20.74</td>
</tr>
<tr>
<td>Reform 1</td>
<td>2016</td>
<td>−544.18</td>
<td>20.83</td>
</tr>
<tr>
<td>Reform 2</td>
<td>2018</td>
<td>−490.61</td>
<td>21.17</td>
</tr>
<tr>
<td>Reform 3</td>
<td>2019</td>
<td>−491.76</td>
<td>20.73</td>
</tr>
</tbody>
</table>

* As a percentage of output.

is because of the increase in the share of the elderly as population ages. As a result, we find that output increases 1.4, 1.8, and 0.2 percent in 2050\(^{40}\).

\(^{39}\)In the above computations, we do not introduce the latest reform to the Spanish pension system. This reform, approved by the Spanish government in 2011, was aimed at reducing the long run pension imbalance. However, it is unable to deal with the sharp aging of the Spanish population. See Díaz-Giménez and Díaz-Saavedra (2012) for a quantitative analysis of this issue.

\(^{40}\)After the third reform, the higher retirement age of dropouts is partly compensated by the small share of this group within the labor force during the next decades.
Figure 10: Difference in Hours Worked and Output (%)