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remote individuals**

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Abstract.

In assessing relative deprivation, the classical approach considers that individuals compare their income with each and every income of the distribution, and assign equal weight to these comparisons. In this paper we propose a more realistic alternative approach to obtain individual deprivation. We assume that the deprivation of the individual depends, to a greater extent, on the situation of those who are part of their social environment (neighbors, colleagues, family, or, in general, the individual's reference group) rather than on the situation of those in an unattainable situation from the individual's point of view. In developing their aspirations, individuals focus on the group to which they belong or at least, they feel they are likely to belong to. As a particular case, our proposal includes the classical approach, allowing us to explain some situations that do not fall under the assumptions of that approach.

*Keywords: Inequality, inter-individual comparisons, reference groups, weighting functions.*

*JEL codes: C10, D31, I38.*

# 1. Introduction

An individual feels deprived when he compares himself with other individuals who he considers to be better off. The economic literature on deprivation usually defines this concept in terms of income. Income is frequently considered an indicator of the individual's ability to own or consume commodities. Under this assumption, the interrelation between individual or social deprivation and income distribution inequality becomes evident. From an economic standpoint<sup>1</sup>, Runciman's (1966) approach has had the greatest impact. He defines that a person is relatively deprived of Y when (i) he does not have Y; (ii) he sees some other person or persons, which may include himself at some previous or expected time, as having Y; (iii) he wants Y; and (iv) he sees it as feasible that he should have Y.

Yitzhaki (1979) presented the first proposal to quantify deprivation from an individual and social level taking as a reference the income distribution in a population, while Hey and Lambert (1980) provide an alternative motivation to the same expression<sup>2</sup>. They identify the deprivation felt by an individual with respect to another individual with income  $y$ ,  $y > x$ , with the difference  $y - x$ . The interest of this approach is twofold. On the one hand, the results derived from this definition allow social welfare functions consistent with the Gini index (1914) to be obtained once it is proved that the mean social deprivation equals the absolute Gini index. On the other hand, this formulation provides a logical sequential order in its reasoning: (i) it explicitly indicates how to make inter-individual comparisons, something essential in this type of formulations; (ii) deprivation associated with a given level of income is then obtained; and (iii) mean social deprivation is finally determined.

Other references related to deprivation in terms of income are Yitzhaki (1982), Chakravarty and Chakraborty (1984), Berrebi and Silber (1985), Paul (1991), Chakravarty et al. (1995), Podder (1996), Chakravarty (1997), Chakravarty and Mukherjee (1998), Imedio et al. (1999), Ebert and Moyes (2000), Duclos (2000), Imedio and Bárcena (2003), Chakravarty (2007), Imedio and Bárcena (2008), Magdalou and Moyes (2009), Yitzhaki (2010), and Imedio et al. (2012). These studies propose alternative definitions of deprivation that allow different inequality measures such as the Bonferroni (1930) index, the De Vergottini (1940) index, and the generalized Gini indexes (Yitzhaki, 1983) to be interpreted as social deprivation measures.

When the notion of relative deprivation is invoked, one inevitably confronts difficult questions about how people actually evaluate their position in society (Pedersen, 2004). It is therefore necessary to take into account several aspects when formulating relative deprivation. First, an individual's deprivation, given his income level, depends on the group he belongs to or identifies with and on the group of individuals within which he confines his aspirations. Second, once we define the function that assigns deprivation to each income level, social deprivation can be assessed as the mean value of this function. Alternatively, it is possible to use weights that

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<sup>1</sup> This concept initially appeared in studies in the field of sociology to justify certain aspects of collective behavior. We can highlight those of Stouffer et al. (1949), Davis (1959), Runciman (1966), Gurr (1968) and Crosby (1976, 1979).

<sup>2</sup> This definition is motivated by Runciman (1966): "...relative deprivation is the extent of the difference between the desired situation and that of the person desiring it "

discriminate between the different parts of the distribution by assigning different importance to the deprivation associated with each income level. That is, in the same way that each index in the measurement of inequality incorporates its own criteria to add the information contained in the distribution according to the value judgments that underlie it, we can introduce different attitudes when assessing mean social deprivation through the use of weights. This is the case in Imedio et al. (2012), and it allows the indices of the  $\beta$  class (Imedio et al. 2009, 2010) – which includes, among others, the generalized Gini indices, the family introduced by Aaberge (2007) and the one by Imedio et al. (2011) – to be interpreted as social deprivation measures.

Pedersen (2004) called for a sociologically inspired normative theory that takes seriously the possibility that not all comparisons within the larger society are equally relevant for social (self-)evaluation. As mentioned above, Imedio et al. (2012) answers the call corresponding to social evaluation by giving priority to deprivation associated to different parts of the income distribution. This paper answers the call corresponding to self evaluation by introducing an alternative approach that considers that the deprivation felt by an individual depends to a greater extent on the situation of those who are part of their immediate social environment (neighbors, colleagues, family or the reference group in general) rather than on the situation of those in an unattainable situation from the individual's point of view. Individuals develop their aspirations by focusing on those who belong to the group<sup>3</sup> into which they are integrated or at least the group they could potentially belong to. It therefore seems reasonable to assume that it is the individual, and not the social evaluator, who introduces a weighting scheme to add the deprivation he feels with respect to those with incomes greater than his own.

We assume that when the individual compares his income with similar or close ones he feels a more acute deprivation than when he compares it with remote incomes. We can even admit the possibility of ignoring incomes from a level deemed inaccessible. This is equivalent to assuming that individuals can consider only one range of the income distribution when evaluating their deprivation. This censoring of reference incomes can be justified by Runciman's view (1966, p.29) that "people often choose reference groups closer to their actual circumstances than those which might be forced on them if their opportunities were better than they are".

We believe that this paper brings more realism to the topic when obtaining the deprivation associated to each income level. The literature on this issue considers that each individual compares his income with each and every income of the distribution, assigning equal importance to all comparisons. That is, it is assumed that the reference group of each individual is the entire income distribution, and that all incomes have the same relevance. Relaxing these assumptions leads to some formal difficulties and the need to subjectively choose some elements involved in the analysis such as the length of the intervals where individuals make comparisons, the selection of weights used, etc. In this way, mean social deprivation is not, in general, a common measure of inequality. However, as it is a generalization of the classical approach, it will be included as a special case in which mean social deprivation is the absolute Gini index.

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<sup>3</sup> Although the reference groups can be formed in response to different variables, this paper considers that they are determined by income alone, as we discuss below.

In this approach, in which each individual only compares effectively with those of his immediate environment, social deprivation is lower than that obtained in the classical approach in which each individual compares with everyone else. This may help to explain why societies with high levels of inequality (according to the usual indexes) can show low levels of deprivation which do not always lead to social conflict. Therefore, the inequality perceived by individuals that belong to the distribution may be significantly lower than the inequality perceived by a social evaluator that simply evaluates it through a classic index. In an extreme case, if we consider perfect stratification where the population is divided into “classes” with the aspirations of members of each “class” limited to that class, or if we consider a policy designer that convinces each group to stick to their own folks with no cross-group comparison, society can tolerate large inequalities with a low level of deprivation. Popular images of feudal hierarchies or of social structures based on caste systems are examples of such deeply divided societies. In the most extreme case where the number of groups approaches the number of members in the society, high inequality can prevail with zero deprivation (Yitzhaki, 2010). In this sense, the relative deprivation theory could be applied to explain social conflict and struggle (Korpi, 1974; Chandra and Foster, 2005). As Runciman (1966) argued, there does not seem to be a strong correlation between the level of "class-political discontent" and objective indicators of inequality so this discontent is instead related to the gap which exists between one's economic and social conditions and the perceived conditions of some reference group.

The paper is organized as follows. The following section introduces the analytical framework and presents the classical approach in the study of deprivation. The third section develops our proposal, which consists of weighting deprivation among individuals depending on the degree of proximity between their respective situations (income levels). The fourth section includes an empirical illustration for two countries, considering different weighting functions. Finally, some brief conclusions are drawn.

## 2. Analytical framework: The classical definition of deprivation

Let us assume that the income distribution of a population is denoted by the random variable  $X$  whose domain is the semi-straight positive real,  $\mathbb{R}^+ = [0, \infty)$ , where  $F(\cdot)$  is its distribution function, and  $\mu = E(X) = \int_0^{\infty} x dF(x) < \infty$  its mean income.

The associated Lorenz curve,  $L(p)$ ,  $p = F(x)$ , is defined by:

$$L : [0, 1] \rightarrow [0, 1], \quad L(p) = \frac{1}{\mu} \int_0^x s dF(s), \quad 0 \leq p \leq 1. \quad [1]$$

For each  $p = F(x)$ ,  $L(p)$  is the proportion of total income accumulated by the set of units with an income less than or equal to  $x$ . It is clear that for  $0 \leq p \leq 1$ ,  $L(p) \leq p$ . In the case of perfect equality,  $L(p) = p$ , and  $L(p) = 0$  for  $0 \leq p < 1$ ,  $L(1) = 1$  if there is maximum concentration. For any distribution,  $X$ , the Lorenz curve is not decreasing and convex.

The Gini (1914) index,  $G$ , is defined from the Lorenz curve,  $L(\cdot)$ :

$$G = 2 \int_0^1 (p - L(p)) dp = 1 - 2 \int_0^1 L(p) dp. \quad [2]$$

Its value is twice the area between the Lorenz curve and the line of perfect equality.

Another way of expressing  $G$  from the differences between incomes of the distribution is:

$$G = \frac{1}{\mu} \int_0^{\infty} \int_x^{\infty} (y - x) dF(y) dF(x) = \frac{1}{\mu} \int_0^{\infty} \left( \int_0^x (x - y) dF(y) \right) dF(x). \quad [3]$$

$G \in [0, 1]$ ,  $G=0$  if there is perfect equality, and  $G=1$  in the case of maximum concentration. This is a compromise index, that is, a relative measure (invariant under changes of scale) which becomes an absolute index (invariant under changes of origin) when multiplied by the average income, thus leading to the absolute Gini index,  $\mu G$ .

If income is a discrete variable represented by  $\vec{x} = (x_1, x_2, \dots, x_N) \in \mathbb{R}_+^N$ , with

$x_1 \leq x_2 \leq \dots \leq x_N$ ,  $\mu = \frac{1}{N} \sum_{i=1}^N x_i$  is the population mean. When considering the cumulative shares of population  $\{i/N\}_{1 \leq i \leq N}$ , the Lorenz curve provides their respective shares in total income  $\{L(i/N)\}_{1 \leq i \leq N}$ , with

$$L(p) = L(i/N) = \frac{\sum_{1 \leq j \leq i} x_j}{N\mu}, 1 \leq i \leq N. \quad [4]$$

The graph of  $L(\cdot)$  is given by the points  $(0, 0)$ ,  $\{(i/N, L(i/N))\}_{1 \leq i \leq N}$  and the polygonal connecting every two consecutive points. In this case, the Gini index can be expressed as:

$$G = \frac{\sum_{i=1}^n \left( \sum_{j=1}^n |x_i - x_j| \right)}{2N^2\mu} = \frac{\sum_{i=1}^n \left( \sum_{j=i+1}^n (x_j - x_i) \right)}{N^2\mu} = \frac{\sum_{i=1}^n \left( \sum_{j=1}^{i-1} (x_i - x_j) \right)}{N^2\mu} \quad [5]$$

The following definition is the starting point of the classical approach in the analysis of deprivation (Yitzhaki, 1979; Hey and Lambert, 1980).

**Definition.** The deprivation felt by an individual with income  $x$  with respect to an individual with income  $y$ ,  $D(x, y)$ , is given by:

$$D(x, y) = \begin{cases} y - x, & \text{if } y > x \\ 0, & \text{if } y \leq x. \end{cases} \quad [6]$$

Hence, the deprivation felt by an individual with a given income level is given by the income difference with respect to those richer than him, and it is zero with respect to those that have less income.

When comparing a given income level  $x$  with each and every one of the income distribution and averaging the resulting differences according to their density<sup>4</sup>, we obtain the mean

<sup>4</sup> Runciman defines the degree of deprivation inherent in not having  $Z$  when others have it as an increasing function of the proportion of persons in the reference group who have  $Z$ . Using promotions as an example,

value of the deprivation associated with income level  $x$ ,  $D(x)$ . This function has the following expression<sup>5</sup>:

$$D(x) = \int_0^{\infty} D(x,y)dF(y) = \int_x^{\infty} (y-x)dF(y) = \mu(1-L(F(x))) - x(1-F(x)) = (1-F(x))(M[x^+] - x), \quad [7]$$

where

$$M[x^+] \equiv M[x, \infty) = \mu(1-L(F(x)))/(1-F(x)) \quad [8]$$

is the mean income of individuals with income greater than or equal to  $x$ .

Therefore,  $D(x)$  is the product of the proportion of individuals with income greater than  $x$ ,  $1-F(x)$ , and the difference between the mean income of such a group and  $x$ .

$D(\cdot)$  satisfies the following properties:

- (i)  $D(\cdot)$  is a strictly decreasing function of the level of income
- (ii)  $D(0) = \mu$
- (iii)  $D(x) \rightarrow 0$ , if  $x \rightarrow \infty$ .

Another way of expressing  $D(x)$  in terms of  $p = F(x)$  using the first derivative of the Lorenz curve,  $L'(p) = dL(p)/dp = x/\mu$ , is:

$$D(x) = \mu(1-L(p)) - \mu L'(p)(1-p). \quad [9]$$

The expected value of the function  $D(\cdot)$  is the mean social deprivation. This value, following [3], is the absolute Gini index of the income distribution,  $\mu G$

$$E(D(X)) = \int_0^{\infty} D(x)dF(x) = \int_0^{\infty} \left( \int_0^{\infty} (y-x)dF(y) \right) dF(x) = \mu G. \quad [10]$$

In the previous approach each individual compares his income with all others in the distribution and the same treatment is applied to all the values that result from these comparisons when computing the mean deprivation associated with each income level. No other circumstances are taken into account. Thus, the poorest individuals of the population experience deprivation with respect to the situation of the richest. In doing so, each individual weights his deprivation with respect to those with a higher income with the same intensity, including the deprivation resulting from comparisons with income levels considered inaccessible. A more realistic approach in assessing the deprivation of an individual involves the possibility of discriminating between those with higher incomes according to his own perceptions or aspirations.

In the next section we propose a procedure to take into account, at least partially, this new point of view when obtaining the deprivation of every income level and, from it, the social deprivation.

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Runciman (1966, p.19) writes: "The more people a man sees promoted when he is not promoted himself, the more people he may compare himself with in a situation where the comparison will make him feel relatively deprived".

<sup>5</sup> Proof of the results of this work are available from the authors upon request.

### 3. Weighted deprivation depending on the proximity of individuals

We continue assuming that the deprivation experienced by an individual with income  $x$  with respect to another individual with income  $y$ ,  $D(x, y)$  is given by the previous definition. Now, given income level  $x$ , rather than proceeding as in [7] to assess the deprivation attached to income  $x$ , the differences  $y - x$ ,  $y > x$ , are weighted by a function,  $\omega_{x,h}(\cdot): \mathbb{R}_0^+ \rightarrow \mathbb{R}_0^+$ , where  $\mathbb{R}_0^+ = [0, \infty)$ , whose support is the interval  $[x, x + h]$ ,  $h > 0$ , therefore,  $\omega_{x,h}(y) = 0$ ,  $y \notin [x, x + h]$ . Let us assume that  $\omega_{x,h}(\cdot)$  is nonnegative, not increasing, and continuous such that:

$$\int_x^\infty \omega_{x,h}(y) dF_{x,h}(y) = \int_x^{x+h} \omega_{x,h}(y) dF_{x,h}(y) = 1. \quad [11]$$

$F_{x,h}(\cdot)$  is the distribution function of the income variable,  $X$ , restricted to the interval  $[x, x + h]$ .

It is:

$$F_{x,h}(y) = \begin{cases} 0, & y < x, \\ \frac{F(y) - F(x)}{F(x+h) - F(x)}, & x \leq y \leq x+h, \\ 1, & y > x+h. \end{cases} \quad [12]$$

The above conditions ensure that: (a) the individual with income  $x$  does not, in fact, feel deprivation in relation to incomes greater than  $x + h$ ,  $h > 0$ ; (b) the differences  $y - x$ ,  $y > x$ , are assigned a decreasing or constant weight; and (c)  $\omega_{x,h}(\cdot)$  is actually a weighting function.

We also assume that the kind of function to which  $\omega_{x,h}(\cdot)$  belongs is the same for all  $x \in \mathbb{R}_0^+$  and that the value of  $h > 0$  remains constant along the income scale. That is, the width of the intervals that individuals consider to be relevant in assessing deprivation is identical for all of them and the weighting criteria also coincides. These conditions, which are certainly restrictive, make the analytical treatment of the problem tractable.

Under the previous assumptions, if  $D_\omega(x)$  is the deprivation associated to income  $x$  when using the weighting function  $\omega_{x,h}(\cdot)$ , it is verified that:

$$\begin{aligned} D_\omega(x) &= \int_0^\infty D(x,y) \omega_{x,h}(y) dF(y) = \int_x^\infty (y-x) \omega_{x,h}(y) dF(y) = \\ &= \int_x^{x+h} (y-x) \omega_{x,h}(y) dF(y) = (F(x+h) - F(x)) \int_x^{x+h} (y-x) \omega_{x,h}(y) dF_{x,h}(y) = \\ &= (F(x+h) - F(x))(M_\omega[x, x+h] - x), \end{aligned} \quad [13]$$

where

$$M_\omega[x, x+h] = \int_x^{x+h} y \omega_{x,h}(y) dF_{x,h}(y) \quad [14]$$



is the weighted mean income of the interval  $[x, x + h]$ , using  $\omega_{x,h}(\cdot)$  as the weighting function.

When each individual compares his income effectively to all those with a greater income, we are assuming that  $h \rightarrow \infty$ , that is, the support of the weighting function, now denoted as  $\omega_x(\cdot)$ , is  $[x, \infty)$ . In that case equations [11] and [12] would be given by:

$$\int_x^{\infty} \omega_x(y) dF_x(y) = 1 \quad \text{and} \quad F_x(y) = \begin{cases} 0, & y \leq x, \\ \frac{F(y) - F(x)}{1 - F(x)}, & y > x. \end{cases} \quad [15]$$

The deprivation associated to income  $x$ ,  $D_{\omega}(x)$ , is:

$$\begin{aligned} D_{\omega}(x) &= \int_0^{\infty} D(x, y) \omega_x(y) dF(y) = \int_x^{\infty} (y - x) \omega_x(y) dF(y) = \\ &= (1 - F(x)) \int_x^{\infty} (y - x) \omega_x(y) dF_x(y) = (1 - F(x))(M_{\omega}[x^+] - x), \end{aligned} \quad [16]$$

where  $M_{\omega}[x^+)$  is the weighted mean income of those individuals with an income greater than  $x$ .

Expressions [13] and [16] present a clear analogy with [7].  $D_{\omega}(x)$  is the product of the proportion of individuals whose income belongs to the interval  $[x, x + h]$  (or the half line  $[x, \infty)$ ) and the difference between the weighted mean income of the interval (or half line) and income  $x$ .

The characteristics of function  $D_{\omega}(\cdot)$  depend on:

- The weighting function,  $\omega_{x,h}(\cdot)$
- The width of the interval in which the individual makes his comparisons
- The shape of the underlying income distribution function,  $F(\cdot)$

It is immediate that when  $x \rightarrow \infty$ ,  $D_{\omega}(x) \rightarrow 0$ , that is, property (iii) of  $D(\cdot)$  in the classical approach is still satisfied. However,  $D_{\omega}(\cdot)$ , in general, does not satisfy properties (i) and (ii). We cannot assure that  $D_{\omega}(\cdot)$  is a strictly decreasing function of income. As we shall see below, in specific cases,  $D_{\omega}(\cdot)$  may be increasing in some parts of the distribution.

By averaging the deprivation associated with each income level  $D_{\omega}(\cdot)$  along the entire distribution, we obtain the mean social deprivation:

$$E(D_{\omega}(X)) = \int_0^{\infty} \left( \int_x^{\infty} (y - x) \omega_{x,h}(y) dF(y) \right) dF(x), \quad [17]$$

or

$$E(D_{\omega}(X)) = \int_0^{\infty} \left( \int_x^{\infty} (y - x) \omega_x(y) dF(y) \right) dF(x). \quad [18]$$

The above expressions are similar to [10] that provides the absolute Gini index,  $\mu G$ , when averaging all income differences  $y - x$ ,  $y > x$ , across the distribution, without discriminating between them. Therefore, we can say that [17] and [18] are generalizations of the absolute Gini index when the above differences are weighted with the functions  $\omega_{x,h}(\cdot)$  or  $\omega_x(\cdot)$ , with both

expressions depending on the type of weighting function and, in the first case, also on  $h$ , the width of the interval in which the individual makes comparisons. In particular,  $\mu G$  is obtained when  $h \rightarrow \infty$  and  $\omega_x(y) = 1, y > x, x \in \mathbb{R}_0^+$ .

It is clear that the mean social deprivation obtained from expressions [17] or [18] is zero if there is an egalitarian distribution of income. However, the mean social deprivation will also be zero when there is an egalitarian distribution in the reference group of each and every one of the individuals. Again, the perception of the individual takes precedence over the general view that an outside observer may have of the distribution, thus justifying that situations of high inequality without social conflict can occur.

### Particular cases for different weights

1. **Constant weight.** If  $\omega_{x,h}(\cdot)$  is a constant weight function in  $[x, x+h]$ , by imposing [11], we get:

$$\omega_{x,h}(y) = \begin{cases} 0, & y < x, \\ 1, & x \leq y \leq x+h, \\ 0, & y > x+h. \end{cases} \quad [19]$$

In this case the individual follows a uniform criterion in his reference group when assessing deprivation.

The mean deprivation associated with income level  $x$  is given by:

$$\begin{aligned} D_\omega(x) &= \int_x^{x+h} (y-x) dF(y) = (F(x+h) - F(x)) \int_x^{x+h} (y-x) dF_{x,h}(y) = \\ &= (F(x+h) - F(x))(M[x, x+h] - x), \end{aligned} \quad [20]$$

where  $M[x, x+h]$  is the mean income in the interval  $[x, x+h]$ .

The mean social deprivation is:

$$E(D_\omega(X)) = \int_0^\infty \left( \int_x^{x+h} (y-x) dF(y) \right) dF(x)$$

whose value depends on  $h$ .

2. **Constant weight with  $h \rightarrow \infty$ .** If in the previous case we assume  $h \rightarrow \infty$ , the individual does not discriminate between higher incomes depending on the proximity to his own income, that is, if the incomes are close or remote. This reasoning coincides exactly with the classical approach so that:

$$\begin{aligned} D_\omega(x) &= \int_x^\infty (y-x) dF(y) = (1 - F(x))(M(x^+) - x), \\ E(D_\omega(X)) &= \int_0^\infty \left( \int_x^\infty (y-x) dF(y) \right) dF(x) = \mu G. \end{aligned}$$

For the constant weight referred to in the first case, the individual with income  $x$  assigns the same importance to the deprivation he feels with respect to higher incomes up to  $x+h$ , but

attaches a zero weight to deprivation felt with respect to  $x + h + \varepsilon$ ,  $\varepsilon$  positive and arbitrarily small. There is, therefore, an abrupt jump in assigning weights. This circumstance can be avoided by considering weights with a more or less marked decreasing rate.

3. **Linear weight.** If  $\omega_{x,h}(\cdot)$  is a linear function (piecewise) as:

$$\omega_{x,h}(y) = \begin{cases} 0, & y < x, \\ ay + b, & x \leq y \leq x + h, \\ 0, & y > x + h. \end{cases} \quad [21]$$

parameters  $a$  and  $b$  are determined by imposing that  $\omega_{x,h}(\cdot)$  satisfies the condition [11] and that its graph passes through the point  $(x + h, 0)$ , so that from  $x + h$  the weight is zero. By imposing these two conditions:

$$a = 1/(M[x, x + h] - (x + h)) < 0, \quad b = -a(x + h) > 0.$$

In applying [13] and [17], we obtain the deprivation associated to each income level and the mean social deprivation.

The previous examples contain only the simplest types of weighting function, particularly the one giving rise to the classical approach. Of course there are many possibilities for selecting weighting functions with different rates of decline and supports.

## 4. Illustration

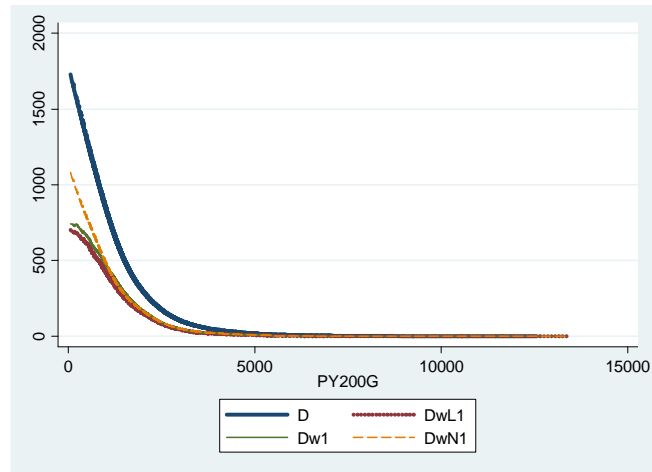
As noted above, weighted deprivation associated with an income level depends on the weighting function chosen, on the width of the interval in which the individual performs his comparisons, and on the shape of the underlying income distribution. In this section we illustrate these properties using the European Statistics on Income and Living Conditions<sup>6</sup> (EU-SILC) for the year 2009. We define deprivation in terms of the monthly gross income of employees.

We first check the effect of the weighting function employed. To do so, we use the Spanish income distribution considering four weights: (i) the constant weight considering an interval with a width equal to the mean income; (ii) the decreasing linear weighting function with the same interval as in the previous case; (iii) the normal weighting function truncated from the mode, in this case individuals compare with all higher incomes in the distribution; and (iv) the constant weighting function when the width of the interval tends to infinity, which is equivalent, as mentioned above, to the classical approach of Hey and Lambert (1980). The corresponding deprivation functions are represented in Figure 1.

Figure 1. Weighting deprivation functions for Spain 2009

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<sup>6</sup> This is an instrument that aims to collect timely and comparable cross-sectional and longitudinal multidimensional microdata on income, poverty, social exclusion and living conditions. This instrument is anchored in the European Statistical System (ESS).



Note:  $D$  denotes Hey and Lambert's (1980) definition of deprivation or constant deprivation with  $h \rightarrow \infty$ .  $Dw1$  is the constant deprivation with an interval of width equal to mean income.  $DwL1$  is the decreasing linear weighting function with an interval of width equal to mean income.  $DwN1$  is the normal weighting function truncated from the mode.

Figure 1 above shows that, for each income level, the lowest deprivation is the one that considers the decreasing linear weighting where the deprivation associated to the constant weighting function is slightly higher. Both are dominated by the deprivation function obtained for a normal weight, whose value depends on its variance. The highest levels of deprivation result from applying the classical approach (Hey and Lambert, 1980).

To verify the effect of interval width given the income distribution and the weighting function, we compare the deprivation functions in Spain obtained with the constant weight by considering two different amplitude ranges: mean income and 50% of the mean income (Figure 2).

Figure 2. Deprivation function with constant weight function considering mean income as the width of the interval ( $Dw1$ ) and 50% of the mean income ( $Dw05$ ) in Spain for 2009

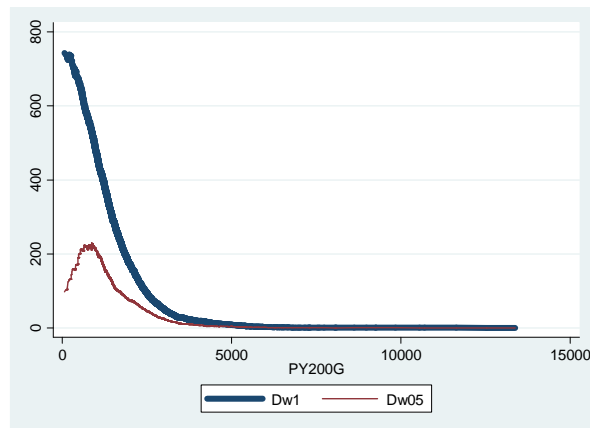
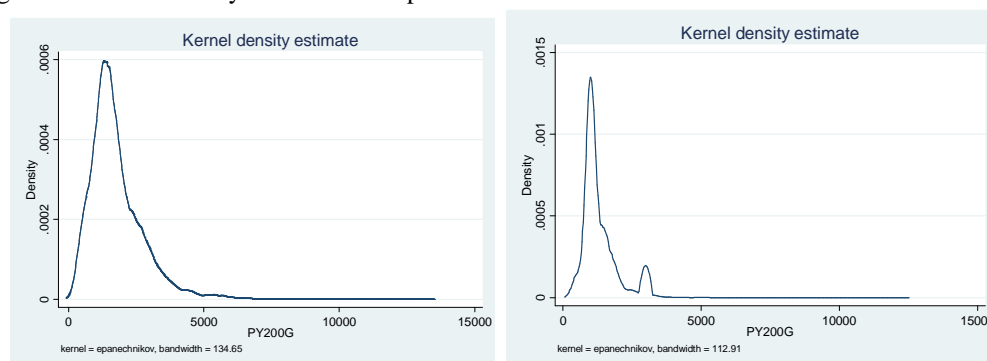


Figure 2 shows that, for the given distribution and the given weighting function, by increasing the width of the interval in which individuals compare with others, the deprivation associated with each income level also increases. Moreover, the  $Dw05$  curve shows that the weighted deprivation function is not necessarily decreasing or convex.

In order to illustrate that the weighted deprivation also depends on the underlying distribution function, we consider the income distribution in Greece for 2009, whose density

function is significantly different from that of Spain as shown in Figure 3. The Greek income distribution displays a local mode.

Figure 3. Income density functions for Spain and Greece

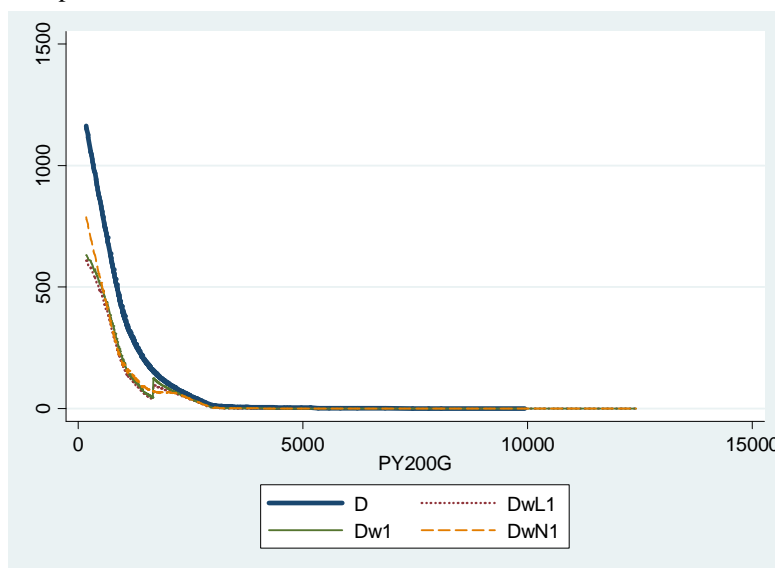


a) Spain

b) Greece

The weighted deprivation functions for Greece are represented in Figure 4 using the same assumptions as for Spain in Figure 1. Comparing the two graphs, we find that the deprivation functions for Greece are influenced by the underlying bimodal distribution. Abrupt changes occur in the weighted deprivation functions, specifically about 2000 euros in income. Around this level of income, the interval where the individual make comparisons begins to include the local mode shown in Figure 3.b.

Figure 4. Weighted deprivation function for Greece 2009



Note: D denotes Hey and Lambert's (1980) definition of deprivation or constant deprivation with  $h \rightarrow \infty$ . Dw1 is the constant deprivation with an interval of width equal to mean income. DwL1 is the decreasing linear weighting function with an interval of width equal to mean income. DwN1 is the normal weighting function truncated from the mode.

In this case, unlike in the classical approach, it becomes clear that deprivation may increase in certain segments of the distribution. In the income distribution for Greece there is a segment of incomes with low density (incomes lower than the local mode) and subsequent segments with higher density. Therefore, when considering the constant weight or the linear decreasing weight, an individual that compares his income with higher incomes in a low density

interval will experience less deprivation than another individual with a higher income, but who makes comparisons on an interval with higher density. This explains the possibility that individuals will experience a lower deprivation than others with higher incomes.

Of course, the influence of the underlying distribution will be greater the smaller the width of the interval in which the individual makes comparisons or the greater the decreasing rate of the weighting function.

The following table shows the values of the mean social weighted deprivation for the various weighting functions considered in Greece and Spain.

Table 1. Mean social weighted deprivation considering different weighting functions (euros)

	Spain	Greece
D	522.7	329.3
Dw1	288.1	170.9
DwL1	256.7	151.3
DwN1	297.1	163.1
$\mu$ (mean)	1781.1	1335.3

Note: D denotes Hey and Lambert's (1980) definition of deprivation or constant deprivation with  $h \rightarrow \infty$ . Dw1 is the constant deprivation with an interval of width equal to mean income. DwL1 is the decreasing linear weighting function with an interval of width equal to mean income. DwN1 is the normal weighting function truncated from the mode.

The results shown in the previous table are consistent with the relationship between the graphs of the respective deprivation functions. The highest mean social deprivation is the one corresponding to the classical approach. Given the width of interval,  $h$ , the mean social deprivation obtained by weighting with a constant weighting function is greater than that obtained with a decreasing linear weighting function. The relationship of these values with those resulting from applying a normal weight depends on the shape of the normal function, which is determined by its variance. Moreover, for each of the cases considered, mean social deprivation in Spain is higher than in Greece as a consequence not only of the characteristics of their distributions, but, above all, of the difference between mean incomes.

## 5. Conclusions

Any formulation of relative deprivation has an underlying set of value judgments and normative aspects are therefore present. The characteristics of the income distribution influence the assessment of deprivation at both individual and social levels, but the specification of a set of subjective criteria to try to take into account the behavior of individuals is also required.

The main difference between the approach of this work and the classical approach is that we assume that, in assessing their deprivation, individuals discriminate between those closer to them in their reference group (because they have a similar situation or at least a situation considered to be attainable) and those who are in situations that they perceive as inaccessible. This distinction between "close" and "remote" is modulated by the use of weights. When the reference group of each individual is the total population and equal importance is given to each comparison, we obtain the results of the classical approach and the mean social deprivation coincides with the absolute Gini index.

Our proposal introduces a more realistic assessment of deprivation. It allows justifying that deprivation is not necessarily a decreasing function of income, that is, higher income does not always mean less deprivation. As Runciman (1966) suggested, if groups that are fairly well off tend to compare themselves with even more privileged strata, this could explain why they are sometimes more discontent than genuinely (read: objectively) poor people as the latter presumably tend to be much more modest in their choice of reference group. Moreover, an outside observer may evaluate inequality with the usual index although this assessment may not be shared by the members of that society. For example, a sharply polarized income distribution (i.e. poor and rich with a small middle class) presents a high inequality index by applying a classic index. At the same time, the mean weighted social deprivation could be small if non-overlapping reference groups of individuals are placed at different poles. This view may explain the lack of social protest in situations of high inequality.

Finally, the formation of each individual's reference group is key to assessing individual and social deprivation. In this work we have assumed that the reference groups are determined solely by level of income, and have the same width for all individuals. Of course, the specification of the relevant reference population is debatable, and as we have seen in the illustration, it is likely to affect the results of social evaluation. Future research is needed to ensure that the formation of such groups is an endogenous decision made by the individual. In this respect we would need to introduce a wide spectrum of dimensions correlated to the notion of within-group identity that aid in assessing the formation of the reference group.

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