Centrist's Curse?
An Electoral Competition Model with Credibility Constraints*

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Abstract

I analyze a model of electoral competition in which a candidate's reputation and his need of credibility restricts his policy choice to a certain subset of the policy space, its ideology set. Candidates are office-motivated. They care about winning and also about the share of votes they get. I consider both two and three party systems. I describe the equilibrium outcomes assuming that plurality rule applies, and obtain for two party competition, in some cases, equilibrium outcomes different than what the median voter theorem suggests because of the restrictions on the ideology sets implied by the credibility constraints. I show that centrist parties are disadvantaged compared to leftist and rightist ones, since, in equilibrium, leftist and rightist parties choose policy points that are as close as possible to each other and obtain votes from the centrist parties' ideology set. A centrist candidate needs a higher concentration of voters in his credibility set compared to his opponents in order to have a chance to win. I also analyze a run-off system for three parties and show that centrist parties have more opportunities to win under this rule than under plurality rule.

KEYWORDS: Electoral competition, plurality, run-off, credibility, spatial models

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1 Introduction

In the last decades, it has been argued that the political view of society members (voters) has smoothened compared to the past and that most people would define themselves to be more centrist. A study conducted by CIS in Spain shows that the response to the question "In politics it is being talked about left and right. On a scale where 0 is left and 10 is right where would you put yourself?" had a mean of 4.56 in 1997 and 4.32 in 2005. Moreover, in 1997 51.3% of the people who answered the question located themselves between 4 and 6, that is, the very center of the political spectrum. Blais and Bodet (2006) report the results for a similar question for 21 countries and 31 elections and find that except for two countries (Germany and Israel) the median voter is located at the midpoint. Although this finding does not include the exact density around the center it can be argued that it shows the necessary condition to be able to talk about the importance of the center which is that the median voter should necessarily be located in the center. Considering this aspect, one should expect that centrist parties should have encountered an increasing advantage to win elections. Daalder (1984) states that the idea of defining themselves as centrist parties has been considered attractive by politicians. Duverger (1964) argues however, that "although the "Centre" is the main force of the parliamentary life", that is, although the majority of voters are located ideologically close to the very center of the political spectrum, the electoral system favors ideologically leftist or rightist parties. This paper intends to show the conditions under which one could think that Duverger's statement applies by analyzing a three candidate electoral competition model, and by arguing that plurality rule is an electoral system that favors leftist and rightist parties whereas a centrist candidate would have more chances of winning under a run-off rule than under plurality rule.

One of the main features of our analysis is the consideration of credibility constraints. In the real world electoral competition, it is a rather rare situation that candidates can choose their policy or campaigning points among all the hypothetically possible ones. Normally we see that candidates face restrictions over the sets of policies that they can propose as campaign promises. Furthermore, one can observe that the set of policies that are credible for one candidate usually does not coincide with the set of policies that are credible for a different candidate. In order to formalize this observation, in this paper I will assume that each candidate (or party) will be able to choose its policy point from a certain subset of the whole interval of policy points. In a one dimensional policy space, the strategy set of a candidate, that is, the subset of the policy space from which a candidate can choose a policy point that would be considered as credible by the voters would simply be an interval of the policy line. It can be argued that, the larger the interval of a candidate, the higher is the credibility that he has among the voters.

Think about parties or candidates that have been competing for a certain number of elections. A party having claimed or acted for a long period to be ideologically leftist would lose its credibility if it were to state that from now on it were to follow a rightist point of view. Alesina (1988) states that candidates obtain credibility through reputation in an infinitely repeated game. This reputation, for a party that wins the election, makes deviations from the announced policy point to another policy which is preferred by the party too costly. This result, I believe, could be considered as a justification for the assumption that I will follow in terms of the permissible intervals. It is assumed that competing candidates have a history of past competition, and the specified intervals define the limit points of policy announcements that would be considered as credible by voters.

I consider an electoral competition where candidates have a history of policy choices. This history defines them ideologically as leftist, rightist or centrist. In my framework, one could say that "Voters discipline candidates by believing some promises a candidate makes as long as"[1] these promises coincide with the set of promises that voters accept as credible in terms of the ideological statements of the candidates. That is, the intervals of acceptable policy points for each ideology are defined as the sets of policy points that voters consider credible given the party´s history of policy choices. Aragones et al. (2005) show that in a repeated electoral competition where voters can punish candidates, parties can credibly commit to policy points which belong to an interval around their ideal points. Taking this result as granted, I assume in my model that these intervals are exogenously determined considering that candidates already have a history of past choices.

Casamatta and De Donder (2005) also consider exogenously given credibility intervals, which are symmetric around the ideal points of the candidates. These intervals do not intersect and each candidate's interval has the same length. Different than my model, they assume that candidates care only about the policy implemented by the winner. They analyze equilibrium outcomes under plurality rule and proportional representation for two and four candidate electoral competitions. Under plurality rule they obtain that the introduction of extreme parties causes that the centrist parties choose either the same policy point under two parties or more extreme ones. Different than their finding, I obtain that the policy point decision of the centrist candidate depends only on the distribution of ideal points of voters. Samuelson (1984) constructs a model of two candidate electoral competition under probabilistic voting where candidates are restricted to choose their strategies close to their initial points, which are a product of previous political activity, in order not to loose credibility. Rather than taking the credibility sets as exogenously given he assumes that they are an endogenous function of candidates' initial points. He

[1] Aragones et al. (2005) p.4

obtains that, different than the median voter theorem suggests, in equilibrium it is not always the case that the two candidates tie.

Another important feature of the model presented here can be found in the definition of the pay-off function of the candidates. Stating that candidates care only about winning does not seem to be sufficient to explain real-world electoral competition phenomena. Neglecting the effect of the share of votes a candidate gets, would lead to unreasonable interpretations. It would mean that candidates with no chance of winning would never enter the competition. However, in many situations there exist candidates or parties who continue to compete despite the fact that they have no chance of winning. These observations seem to imply that a reasonable candidate utility function should include the share of votes as an argument. Therefore, I will include also the shares of votes they get into the utility function of the candidates. One can consider that the higher its share of votes, the higher is the funding that a candidate obtains from outside. Since outside funding is one of the vital sources that help parties to survive, this is an important aspect to be considered given that in real world parties compete during several elections. Moreover, as Osborne (1993) argues, entering the competition might be useful for a candidate even if he loses as it might increase their credibility. It can be argued that the higher the share of votes of a candidate the higher his credibility would be in further elections. In the following analysis I do not consider repeated elections. I analyze only one period electoral competition under plurality rule and run-off rule but this analysis helps to have an insight on real world electoral competition where parties compete for repeated elections.

I consider an electoral competition on a one dimensional policy space, where voters vote sincerely, with no abstention, and where each candidate can choose its policy point from a given subset of the whole policy space. Candidates possess full information about the distribution of ideal points of voters. I first consider a one-round electoral system and analyze the characteristics of the equilibrium outcomes for different credibility sets for two (leftist and rightist) and three candidate (leftist, rightist and centrist) elections. I assume that the winner is determined by plurality rule. I analyze the equilibrium outcomes, under different specifications for the intervals from which candidates can choose their policy points.

Cox (1987) shows that for a uniform distribution of voters, for candidates which are plurality maximizers, and for odd numbers of candidates (n) with $n \geq 3$, there exists no Nash Equilibrium in pure strategies. In fact, for three candidate competition he shows that there exist no equilibrium for a general distribution under plurality rule when candidates are either plurality maximizers or complete plurality maximizers or share maximizers. This result would also hold for a utility function of candidates defined as in my model, if candidates do not face any credibility constraints.

In order to ensure the existence of an equilibrium in all cases, I maintain two assumptions on voters' behavior. The first assumption, which I maintain for both two and three candidate electoral competition, characterizes the behavior of voters when two candidates choose the same policy point. It states that voters do not consider as identical, candidates who are located on the same policy point. This could be considered as a natural behavior since voters discipline candidates by believing only those policy announcements that belong to a certain subset of the whole policy space. So, if two candidates have different credible policy sets, voters would perceive them as different even if they announce the same policy point.

For three candidate competition, under this assumption, I can guarantee the existence of an equilibrium for the cases where the intervals of the three candidates intersect only at a single point. If the intervals overlap, then for some parameter values this assumption can not guarantee the existence of an equilibrium. Therefore, in this case I use a second assumption on voters' behavior. The second assumption I make characterizes the behavior of voters when the centrist candidate is located to the left of the leftist candidate or to the right of the rightist candidate. I assume that, in these two cases, the centrist candidate gets no votes. This behavior could be considered as a punishment applied by voters for a centrist candidate who chooses to be more leftist than the leftist candidate or to be more rightist than the rightist candidate.

The main result for two candidate electoral competition is that certain restrictions on intervals, such that the median voter's ideal point is located in the interval of at most one candidate, cause that candidates choose policy points different than the location of the median voter which might coincide or not depend on the exact characteristics of the intervals. Therefore, in these cases the outcome is not the same as it is suggested by the median voter theorem, which states that, in equilibrium, both candidates are located at the ideal point of the median voter. Otherwise, the results coincide with the predictions of the median voter theorem.

For the three candidate electoral competition, I show that in equilibrium the centrist candidate is disadvantaged compared to the other two candidates, that is, he is less likely to win. I describe the conditions under which a centrist candidate wins. I show also that since candidates are assumed to be office-motivated, leftist and rightist candidates prefer to converge to each other as much as their credibility constraints allow. The centrist candidate wins when his opponents face small credibility sets and when the share of voters with ideal points in his credibility set is fairly high. When the centrist candidate cannot win, in many cases his policy choice determines the winner.

Having shown that a centrist candidate faces disadvantages under plurality rule, the next step would be to argue whether this also holds for different election rules. I consider a run-off rule under the two assumptions mentioned above and show that the run-off rule can be more advantageous for centrist candidates. That is, the centrist candidate can win for larger credibility sets of his opponents compared to plurality rule. The run-off rule I analyze is similar to the system "used for presidential elections in France and several Latin American countries and in Israel between 1996 and 2001 to elect a prime minister" (Callander 2005).[2]

The outline of the paper is as follows. In the next section I build up the model. In Section 3, I analyze two-candidate electoral competition under the specifications of the model. In Section 4, I focus on the analysis of the three-candidate electoral competition. In Section 5, I analyze three-candidate electoral competition under a run-off rule. In the last section I conclude with a brief discussion of the results.

2 Model

The model that I set up follows the classical analysis of Hotelling (1929) and Downs (1957). I construct a spatial model of elections where the policy space is represented by the interval [0,1], where 0 could be considered as the most extreme leftist point and 1 as the most extreme rightist one.

2.1 Game

The game consists of a one period electoral competition. There is a fixed number of candidates[3] and a continuum of voters. Candidates declare their policy points simultaneously before the election and full-commitment is assumed. Each candidate can credibly commit to a policy as long as it belongs to a specified subset of the policy space. We assume that a non-credible strategy is always dominated by a credible one, and therefore, we consider only credible strategies as potential equilibrium strategies. Thus, the relevant strategy set for a candidate will be defined as a particular subset of the interval. The solution concept used is Nash equilibrium. The outcome of the game is the winner of the election who is determined by plurality rule, that is, the candidate who gets the highest amount of votes is the winner, or by a run-off rule. In the case of a tie, as a tie breaking rule, I assume that each candidate sharing the highest percentage of votes will have the same probability of winning.

[2] Callander (2005) gives also examples of other run-off rules.

[3] In Section 3 I analyze two candidate electoral competition and in Sections 4 and 5 I analyze three candidate electoral competition.

2.2 Candidates

I assume that candidates care only about winning with the highest share of votes possible, that is they are office-motivated. I consider first an environment in which two candidates j where $j \in J$ and $J = \{L, R\}$ are competing. Later on, I analyze an environment in which three candidates j where $j \in J$ and $J = \{L, C, R\}$ are competing. They could be considered as a leftist (L), a centrist (C) and a rightist (R) candidate. In the case with two candidates, I assume that the leftist candidate can choose a policy point from the interval $[0, a]$ and the rightist candidate from the interval $[1-a, 1]$ with $a \in (0, 1)$. For the case of three candidates, I assume that the leftist candidate can choose a point from $[0, a]$, the centrist candidate can choose from $[b, 1-b]$ and the rightist candidate from $[1-a, 1]$ with $a \in (0, 1)$ and $b \in (0, \frac{1}{2})$. Before the election, all candidates declare their policy choices. The policy point chosen by the leftist candidate is denoted as x_L, the policy point chosen by the centrist candidate as x_C, and the one of the rightist candidate is denoted as x_R. The utility function (U_j) of candidates has the following lexicographic form:

$$U_j = \begin{cases} \overline{U} + v(s_j) & \text{if } j \text{ wins the election} \\ \frac{\overline{U}}{n} + v(s_j) & \text{if } j \text{ gets the highest amount of votes with } n-1 \text{ more parties } (n = 2, 3) \\ v(s_j) & \text{if } j \text{ does not win the election} \end{cases}$$

for $j \in \{L, R\}$ if $n = 2$ and for $j \in \{L, C, R\}$ if $n = 3$ with $\frac{\overline{U}}{3} > v(1)$, $s_j \in [0, 1]$, $\sum_{j=1}^{n} s_j = 1$ and $n \leq 3$ where $v(s_j)$ is a strictly increasing function; $\overline{U}$ a fixed utility obtained from winning and s_j the share of votes candidate j gets. Candidates primarily care about winning and the share of votes they get. Since they are office-motivated the utility obtained from winning outweighs the utility obtained from the share of votes. So, their aim is to win with the highest share of votes possible. A candidate who cannot win, simply chooses the policy point that maximizes his share of votes.

2.3 Voters

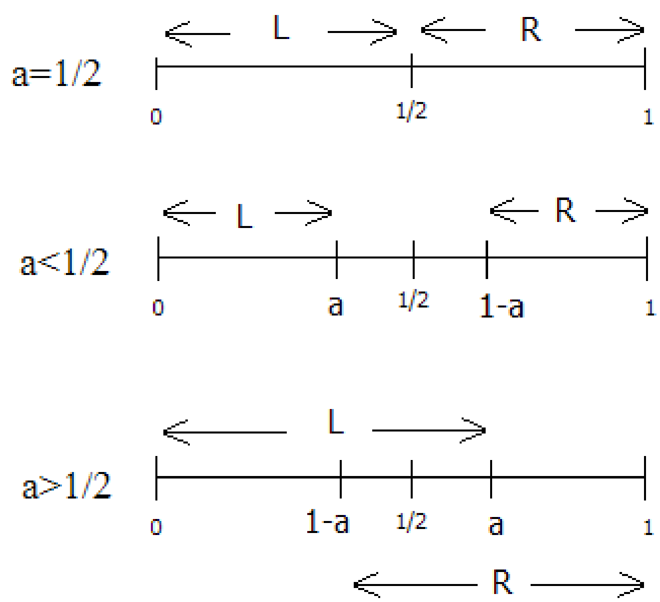
For the whole analysis, I assume that there is a continuum of voters who have single-peaked preferences over the policy space with $x_i \in [0, 1]$ referring to their ideal points for all $i \in I$. I assume that voters preferences are represented by a utility function of $U_i(x_j) = -|x_j - x_i|$ where $j \in \{L, C, R\}$ and where x_j's are the policy point announcements of the candidates. The distribution of voters' ideal points is given by a continuos distribution function $F(x)$ with $x \in [0, 1]$.

Moreover, I exclude the possibility of abstention by thinking of a law making voting obligatory where the penalty outweighs the cost of voting (or it could be assumed that voting has no cost). I assume that voters vote sincerely, which could also be thought as a situation in which voters do not posses any information regarding neither the distribution of voters nor of the median voter[4]. Voters are not strategic players of the game since sincere voting is assumed.

For the following analysis it will be assumed that candidates possess complete information about the distribution of the voters. Throughout the whole analysis, I denote the ideal point of the median voter as m.

3 Restricted Electoral Competition With Two Candidates

In this section I analyze two candidate electoral competition where the credible policy sets are given such that the leftist candidate can choose a policy point from the interval $[0, a]$ and the rightist candidate from the interval $[1-a, 1]$ with $a \in (0, 1)$ for three different cases. First, I analyze the case where $a = \frac{1}{2}$ (the intervals of the two candidates intersect at a single point), then the case where $a < \frac{1}{2}$ (the intervals of the two candidates do not intersect) and finally the case where $a > \frac{1}{2}$ (the intervals of the two candidates overlap). (see figure)



[4]For two candidate competition, strategic voting and sincere voting would lead to the same equilibria. For three candidate competition given that there is a continuum of voters assuming that voters do not have any information about the distribution of ideal points, sincere voting would not lead to a non-rational behavior as voters could have any type of beliefs which would justify casting a vote to the most preferred candidate.

For the case of two candidates, a candidate has to get more votes than his opponent in order to win. Moreover, since the share of votes affects the utility of candidates, the candidates want to win with the highest share of votes possible. Since all voters vote, maximizing plurality is strategically equivalent to maximizing share of votes, satisfying the sufficient conditions of the equivalence theorem of Aranson et al. (1974).

As the intervals of both candidates may intersect or even overlap, it is possible that in equilibrium, they announce the same policy point. However, voters perceive the candidates as ideologically different and, therefore, when both candidates choose the same policy point, the following assumption on voters' behavior will be made:

Assumption 1: If $x_L = x_R$, then all voters with $x_i < x_L$ vote for the leftist candidate while all voters with $x_i > x_L$ vote for the rightist candidate.

This assumption could be considered as a tie-breaking rule for the voters. For some cases, without Assumption 1 there would not exist an equilibrium.[5] Throughout this section I assume that voters behave as Assumption 1 suggests.

For $a = \frac{1}{2}$, the results are as follows:

Proposition 1: If $a = \frac{1}{2}$, then for all m, in equilibrium, $x_L = x_R = \frac{1}{2}$ and:
i. L wins if $m \in [0, \frac{1}{2})$.
ii. R wins if $m \in (\frac{1}{2}, 1]$.
iii. L and R tie if $m = \frac{1}{2}$.

Proof: If $m \in [0, \frac{1}{2})$, L wins for sure by choosing x_L such that $\frac{1}{2} \geq x_L > m$ because he gets more than half of the votes, i.e. $s_L > s_R$. Similarly, if $m \in (\frac{1}{2}, 1]$, R wins for sure by choosing x_R such that $\frac{1}{2} \leq x_R < m$ because he can assure himself $s_R > s_L$. In both cases, under Assumption 1 the candidates maximize their share of votes by choosing $x_L = x_R = \frac{1}{2}$ because any $x_L \in [0, \frac{1}{2})$ is strictly dominated by $x_L = \frac{1}{2}$ and any $x_R \in (\frac{1}{2}, 1]$ is strictly dominated by $x_R = \frac{1}{2}$. The same holds if $m = \frac{1}{2}$. In this

[5] When the intervals of the two candidates intersect and the ideal point of the median voter is located on a point which is in the interval of only one of the two candidates, there exists no equilibrium. Consider for example the case where $a = \frac{1}{2}$ and $m < \frac{1}{2}$. Then $x_R = \frac{1}{2}$ is a strictly dominant strategy for R. Then, the best response of L would be $x_L = \frac{1}{2} - \epsilon$ with $\epsilon > 0$ and close to 0. So, there exists no equilibrium. Under Assumption 1, $x_L = \frac{1}{2}$ is a strictly dominant strategy for L and therefore, there exists an equilibrium. In general, Assumption 1 guarantees an equilibrium where the best response of a candidate would be to be located at an $\epsilon-$distance from his opponent without this assumption.

case, $s_R = s_L = \frac{1}{2}$ and they win with equal probability as a result of the tie breaking rule. If L deviates to $x_L < \frac{1}{2}$ he would decrease s_L which would lead to $s_R > s_L$ making R the winner. If R deviates to $x_R > \frac{1}{2}$ he would decrease s_R which would lead to $s_R < s_L$ making L the winner. Therefore, they have no incentives to deviate and $x_L = x_R = \frac{1}{2}$ constitutes the equilibrium. #

Proposition 1 states that in equilibrium both candidates will locate themselves at the same policy point meaning that there can not be any differentiation between leftist and rightist points of view. As opposed to the Hotelling model, candidates do not necessarily locate themselves at the ideal point of the median voter unless $m = \frac{1}{2}$ and they only win with equal probability if $m = \frac{1}{2}$. If $m \neq \frac{1}{2}$, the inclusion of share of votes to the utility function of candidates refines the existing equilibria. If the share of votes were not included, then, for instance, if $m \in [0, \frac{1}{2})$, the leftist candidate would be indifferent among choosing any $x_L \in (m, \frac{1}{2}]$ because he would be the winner for sure. The rightist candidate losing for sure, could locate himself at any point of his interval.

For $a < \frac{1}{2}$ the results are as follows:

Proposition 2: If $a < \frac{1}{2}$, then for all m, in equilibrium, $x_L = a$, $x_R = 1 - a$ and:
i. L wins if $m < \frac{1}{2}$.
ii. R wins if $m > \frac{1}{2}$.
iii. L and R tie if $m = \frac{1}{2}$.
Proof: $x_L = a$ strictly dominates any $x_L \in [0, a)$ and $x_R = 1-a$ strictly dominates any $x_R \in (1-a, 1]$. Therefore, in equilibrium $x_L = a$, $x_R = 1-a$. If $m < \frac{1}{2}$, then $s_L > s_R$ and L is the winner. If $m > \frac{1}{2}$ then $s_L < s_R$ and R is the winner. If $m = \frac{1}{2}$ then L and R win with equal probability since $s_L = s_R = \frac{1}{2}$.#

The conclusion that can be drawn from Proposition 2 is that if $a < \frac{1}{2}$ the candidates locate themselves as close to each other as possible but the credibility constraints do not allow them to choose the same policy point. As before, the inclusion of share of votes to the utility function of candidates refines the existing equilibria. If the share of votes were not included, then, for instance, if $m \in [0, a)$, the leftist candidate would be indifferent to locate himself at any $x_L \in (m, a]$ because he would be the winner for sure. The rightist candidate losing for sure, could locate himself at any point of his interval.

If $a > \frac{1}{2}$ then the results are as follows:

Proposition 3: If $a > \frac{1}{2}$, then in equilibrium:
i. $x_L = x_R = 1 - a$ and L wins if $m \in [0, 1 - a)$.
ii. $x_L = x_R = a$ and R wins if $m \in (a, 1]$.

iii. $x_L = x_R = m$ and R and L tie if $m \in [1-a, a]$.

Proof: If $m \in [0, 1-a)$, the best reply for L is x_R if $x_R \leq a$ and a if $x_R > a$. The best reply for R is $x_L - \epsilon$ (ϵ sufficiently small) if $x_L > 1-a$ and $1-a$ if $x_L \leq 1-a$. Therefore, in equilibrium $x_L = x_R = 1-a$ and L wins for sure since $s_L > s_R$. If $m \in (a, 1]$, the best reply for L is $x_R + \epsilon$ (ϵ sufficiently small) if $x_R < a$ and a if $x_R \geq a$. The best reply for R is $1-a$ if $x_L \leq 1-a$ and a if $x_L > 1-a$. Therefore, in equilibrium $x_L = x_R = a$ and R wins for sure since $s_R > s_L$. If $m \in [1-a, a]$, then the best reply for L is $x_R + \epsilon$ (ϵ sufficiently small) if $x_R < m$ and x_R if $x_R \geq m$. The best reply for R is $x_L - \epsilon$ (ϵ sufficiently small) if $x_L > m$ and x_L if $x_L \leq m$. Therefore, in equilibrium $x_L = x_R = m$ and both candidates win with equal probability. #

If the intervals for the two candidates overlap, then, in equilibrium, both candidates locate themselves always at the same point. Moreover, both candidates have the same incentive of locating themselves at the median voter‘s ideal point. However, due to the restrictions put on the intervals, the policy point choice of both candidates coincides with the ideal point of the median voter only if the median voter‘s ideal point is in the interval of both candidates. As before, the inclusion of share of votes refines the existing equilibria. If the share of votes were not included, then, for instance, if $m \in [0, 1-a)$, the leftist candidate would be indifferent to locate himself at any $x_L \in (m, 1-a]$ because he would be the winner for sure. The rightist candidate losing for sure, could locate himself at any point of his interval.

4 Restricted Electoral Competition With Three Candidates

In this section I analyze the same type of electoral competition as above but now I consider three candidates, that is, a centrist candidate is introduced. The partition of the policy space is such that the leftist candidate can choose a policy point from the interval $[0, a]$, with $0 < a < 1$, the centrist candidate from $[b, 1-b]$ with $0 < b < \frac{1}{2}$ and the rightist candidate from $[1-a, 1]$. In order to cover all possibilities, I analyze four cases: $a = b$, $b > a$, $\frac{1}{2} > a > b$, and $a \geq \frac{1}{2}$. Similar to the two candidate electoral competition, there might occur situations where two candidates are located on the same policy point. For this cases, I assume that voters behave as the following assumption suggests.

Assumption 1.1: If $x_L = x_C$, then all voters with $x_i < x_L$ vote for the leftist candidate while all voters with $(x_R + x_C)/2 > x_i > x_L$ vote for the centrist candidate. Similarly, if $x_R = x_C$, then all voters with $x_i > x_R$ vote for the rightist candidate while all voters with $(x_L + x_C)/2 < x_i < x_R$ vote for the centrist candidate.

As in the previous section, this assumption could be considered as a tie-breaking rule for the voters.

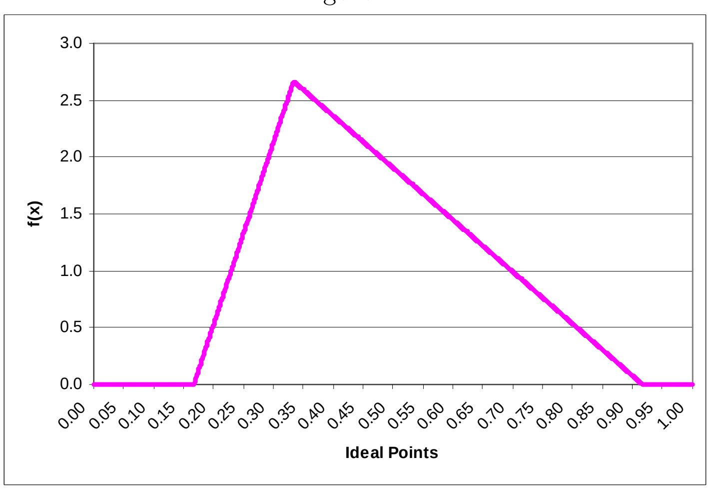
For some cases, without this assumption there does not exist an equilibrium.[6] Throughout the whole analysis that follows, I assume that voters behave as Assumption 1.1 suggests.

As opposed to the case for two candidates where maximizing the probability of winning is strategically equivalent to maximizing the share of votes, for three candidates as Osborne (1995) argues these two strategies that correspond to these two objectives do not need to coincide. That is, a candidate who would win may increase his share of votes by locating himself closer to one of his opponents and thereby increasing the share of votes of his other opponent that much so that he can not win any more. To visualize this situation, consider the following example:

Example 1: Consider the following density function (Figure 1) corresponding to a continuous and increasing distribution function:

$$f(x) = \begin{cases} 0 & \text{if } 0 \leq x < \frac{1}{6} \text{ or } \frac{11}{12} \leq x \leq 1 \\ \frac{48x-8}{3} & \text{if } \frac{1}{6} \leq x < \frac{1}{3} \\ \frac{88-96x}{21} & \text{if } \frac{1}{3} \leq x < \frac{11}{12} \end{cases}$$

Figure 1:



Suppose that the interval for the leftist, centrist and rightist candidates are $\left[0, \frac{1}{3}\right]$, $\left[\frac{1}{3}, \frac{2}{3}\right]$, and $\left[\frac{2}{3}, 1\right]$ respectively. During the following analysis, Assumption 1.1 will be maintained. Therefore, choosing

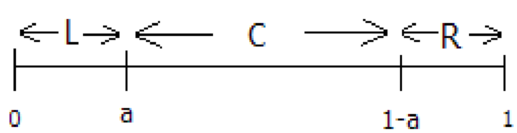
[6] Consider intervals with $a = b$ and a distribution of ideal points with $m < a$ whose density function is strictly decreasing for policy points greater than a. In that case, $x_C = a$ is a strictly dominant strategy for the centrist candidate. Then, the best response of the leftist candidate would be $x_L = a - \epsilon$ with $\epsilon > 0$ and close to 0. So, there exists no equilibrium. (Notice that x_R would be equal to $1 - a$)

$x_L = \frac{1}{3}$, $x_R = \frac{2}{3}$ is a strictly dominant strategy for L and R respectively. From the above density function it can easily be concluded that a share-maximizing centrist candidate would locate himself at $x_C = \frac{1}{3}$. Doing so, the share of votes each candidate gets will be $s_L = 0.222$, $s_C = 0.381$, $s_R = 0.397$ where the rightist candidate will be the winner. If, however, the centrist candidate locates himself at $x_C = \frac{5}{12}$ instead of $\frac{1}{3}$, then clearly the share of votes of the leftist candidate will increase while the share of votes of the rightist and centrist candidates will decrease. In fact, they take the following values: $s_L = 0.329$, $s_C = 0.35$, $s_R = 0.321$, meaning that the centrist candidate is the winner. So, the centrist candidate not being able to win the election with the highest share of votes possible, can achieve this aim with a lower share of votes by re-locating himself.

Therefore, the optimal strategy of a candidate that cares about winning under plurality does not need to coincide with the strategy that maximizes his share of votes. His optimal strategy is the one that makes him obtain the highest possible share of votes such that it is higher than the share of votes of all his opponents. Only a candidate that can never win, to maximize his utility, will simply choose the policy point which maximizes his share of votes.

4.1 Connected Intervals (a = b)

In this section, I analyze a situation where the intervals' intersection consists only of a single point, $(a = b)$, that is, the permissible interval for the leftist, centrist and rightist candidates are $[0, a]$, $[a, 1 - a]$ and $[1 - a, 1]$ respectively where $a \in (0, \frac{1}{2})$.



Which type of equilibria would we observe for an arbitrary distribution of ideal points of voters? If the ideal point of the median voter is located in the interval $[0, a)$, or in the interval $(1 - a, 1]$ then the result is as follows:

Proposition 4: In equilibrium: $x_L = a$ and $x_R = 1 - a$ and $x_C \in [a, 1 - a]$ that maximizes C's share of votes. The winner is the leftist candidate if $m \in [0, a)$ and the rightist candidate if $m \in (1 - a, 1]$.

Proof: L maximizes his share of votes at $x_L = a$ and R at $x_R = 1 - a$ being a strictly dominant strategy under Assumption 1.1. If $m \in [0, a)$, L is the winner because for $x_L = a$, $s_L \geq \frac{1}{2}$ which implies that $s_L > s_j \ j \in \{R, C\}$. If $m \in (1-a, 1]$, R is the winner because for $x_R = 1-a$, $s_R \geq \frac{1}{2}$ which implies that $s_R > s_j \ j \in \{L, C\}$. In both cases, C chooses $x_C \in [a, 1-a]$ such that he maximizes s_C. #

Notice that, if the share of votes would not affect the utility of candidates, for $m \in [0, a)$, the leftist candidate would be indifferent between any $x_L \in (m, a]$, since he would win the election for sure for any choice of x_L in this interval. Similarly, for $m \in (1-a, 1]$, the rightist candidate would be indifferent between any $x_R \in [1-a, m)$ since he would win the election for sure for any choice of x_R in this interval.

Now suppose that $m \in [a, 1-a]$ and suppose that the distribution of ideal points of voters is represented by a continuous and increasing cumulative distribution function $F(x)$ where $f(x)$ is the corresponding density function. Has the centrist candidate a chance to win? As above, $x_L = a$ and $x_R = 1-a$ are strictly dominant strategies for candidates L and R respectively under Assumption 1.1. Suppose that the centrist candidate chooses a point $x_c^{'}$. Then the share of votes each candidate gets will be as follows:

$$s_L = F\left(\frac{a + x_C^{'}}{2}\right), \ s_R = 1 - F\left(\frac{1 - a + x_C^{'}}{2}\right) \text{ and } s_C = F\left(\frac{1 - a + x_C^{'}}{2}\right) - F\left(\frac{a + x_C^{'}}{2}\right)$$

Therefore, in order to win the centrist candidate will choose x_C such that it satisfies:

$$x_C = \arg\max_{x_{c'}} \{s_C\}$$

$$s.t. \ s_C > s_j \ j \in \{L, R\}$$

$$x_C^{'} \in [a, 1-a]$$

That is, the centrist candidate knowing the equilibrium strategies of the rightist and leftist candidates will try to maximize his share of votes given that he wins. If the above maximization problem has no solution, he will try to win at least with some probability that is, he will face the above maximization problem with weak inequality(ies) for one of the two constraints or both. If it has no solution, then the centrist candidate will choose x_C such that it satisfies:

$$x_C = \arg\max_{x_{c'}} \{s_C\}$$

$$s.t. \ x_C^{'} \in [a, 1-a]$$

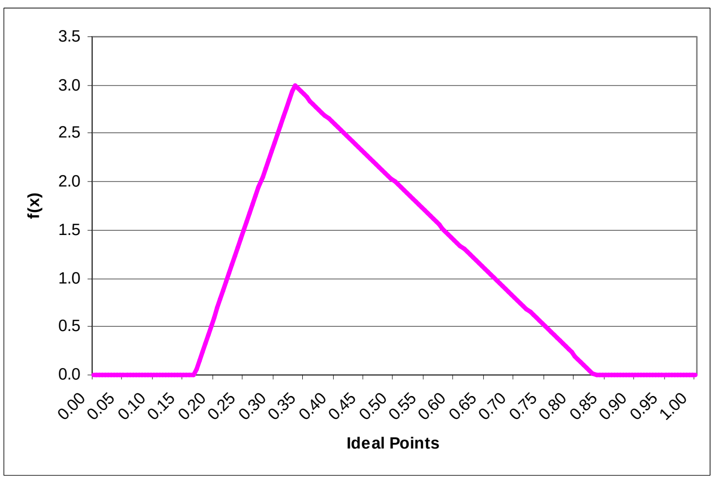
That is, if he cannot win, he will only try to maximize his share of votes.

Now, consider the following three examples with $a = \frac{1}{3}$ in which the median voter is located in the interval $\left[\frac{1}{3}, \frac{2}{3}\right]$. In the first example, I show a situation in which the centrist candidate wins for sure and in the second one a situation in which the centrist candidate can only tie with the other two candidates. The last example shows a situation in which the centrist candidate has no chance of winning.

Example 2: Consider the following density function (Figure 2) corresponding to a continuous and increasing distribution function:

$$f(x) = \begin{cases} 0 & \text{if } 0 \leq x < \frac{1}{6} \text{ or } \frac{5}{6} \leq x \leq 1 \\ 18x - 3 & \text{if } \frac{1}{6} \leq x < \frac{1}{3} \\ 5 - 6x & \text{if } \frac{1}{3} \leq x < \frac{5}{6} \end{cases}$$

Figure 2:

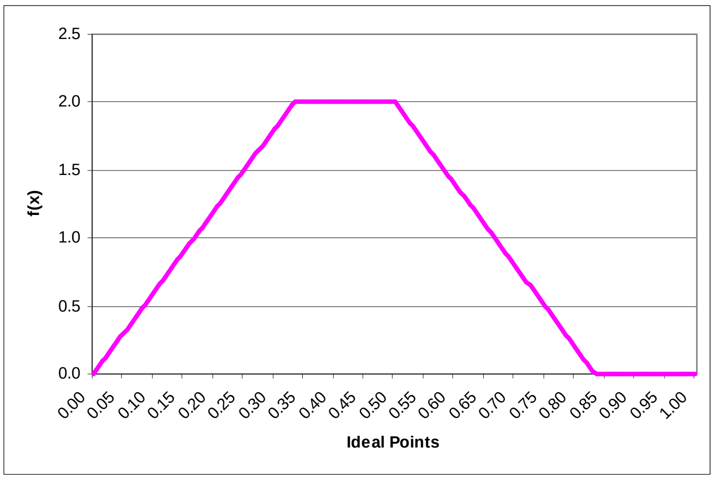


The best responses of the leftist and rightist candidates are $\frac{1}{3}$ and $\frac{2}{3}$ respectively, since they are strictly dominant strategies. Clearly, the centrist candidate maximizes his share of votes choosing $x_C = \frac{1}{3}$. If he wins for sure with this choice, it means that it is his best response. Indeed, in this case $s_L = \frac{3}{12}$, $s_R = \frac{4}{12}$ and $s_C = \frac{5}{12}$. So, in equilibrium, the centrist candidate wins for sure and maximizes his share of votes for $x_C = \frac{1}{3}$. If the shares of votes were not a part of the utility function of candidates, the centrist candidate would be indifferent among any x_C close enough to $\frac{1}{3}$ such that he still wins.

Example 3: Consider the following density function (Figure 3) corresponding to a continuous and increasing distribution function:

$$f(x) = \begin{cases} 6x & \text{if } 0 \leq x < \frac{1}{3} \\ 2 & \text{if } \frac{1}{3} \leq x < \frac{1}{2} \\ 5 - 6x & \text{if } \frac{1}{2} \leq x < \frac{5}{6} \\ 0 & \text{if } \frac{5}{6} \leq x \leq 1 \end{cases}$$

Figure 3:



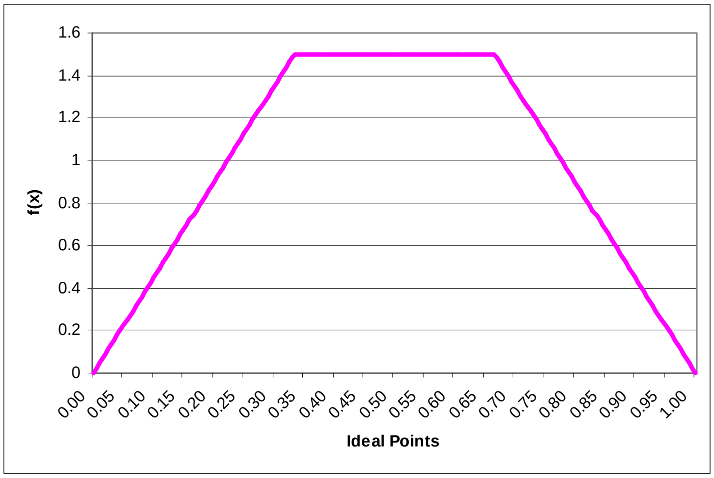
As before, the best responses of the leftist and rightist candidates are $\frac{1}{3}$ and $\frac{2}{3}$ respectively. Clearly, the centrist candidate maximizes his share of votes choosing $x_C = \frac{1}{3}$. For this choice, $s_L = s_R = s_C = \frac{1}{3}$. If he would choose any x_C different from $\frac{1}{3}$ his share of votes decrease while s_L would increase making L the winner. Therefore, in equilibrium all candidates have equal probability of winning.

Example 4: Consider the following density function (Figure 4) corresponding to a continuous and increasing distribution function:

$$f(x) = \begin{cases} \frac{9x}{2} & \text{if } 0 \leq x < \frac{1}{3} \\ \frac{3}{2} & \text{if } \frac{1}{3} \leq x < \frac{2}{3} \\ \frac{9-9x}{2} & \text{if } \frac{2}{3} \leq x \leq 1 \end{cases}$$

As before, the best responses of the leftist and rightist candidates are $\frac{1}{3}$ and $\frac{2}{3}$ respectively. Therefore, both L and R can guarantee themselves a share of vote of at least $\frac{1}{4}$. Therefore, for any choice of C,

Figure 4:



at least one of these two candidates obtains a share of votes larger than $\frac{1}{4}$. Furthermore, the centrist candidate gets the same share of votes for any choice of x_C which is equal to $\frac{1}{4}$. So, C can never win. Therefore we have the following type of equilibria. If $x_C = \frac{1}{2}$, L and R tie in equilibrium. If $x_C \in [\frac{1}{3}, \frac{1}{2})$, R wins for sure. If $x_C \in (\frac{1}{2}, \frac{2}{3}]$, L wins for sure.

The conclusion that can be drawn from the above examples is that under an arbitrary distribution of ideal points of voters, in many cases the centrist candidate has no chance of winning even if the ideal point of the median voter is located in his interval.

In order to analyze the characteristics of the equilibrium outcome when the ideal point of the median voter is located in the interval $[a, 1-a]$ consider a uniform distribution of ideal points of voters. The following proposition states the conditions for the existence of an equilibrium in which the centrist candidate wins and also describes the equilibria where the centrist candidate can not win. The result is as follows:

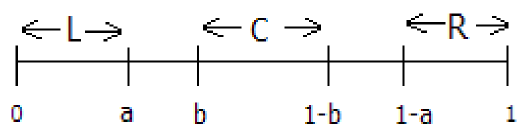
Proposition 5: In equilibrium: $x_L = a$, $x_R = 1-a$; if $a < \frac{1}{6}$, $x_C \in (3a, 1-3a)$ and C wins for sure; if $a = \frac{1}{6}$, $x_C = \frac{1}{2}$ and all three candidates win with equal probability; if $a > \frac{1}{6}$ the winner is L if $x_C \in (\frac{1}{2}, 1-a]$, R if $x_C \in [a, \frac{1}{2})$ and R and L with equal probability if $x_C = \frac{1}{2}$.

Proof: As before, choosing $x_L = a$, $x_R = 1-a$ is a strictly dominant strategy for L and R respectively. For any choice of policy point of C, $s_C = \frac{1-2a}{2}$. If $x_C > \frac{1}{2}$, $s_L > s_R$. Therefore, C wins if $s_C = \frac{1-2a}{2} > s_L = \frac{x_C+a}{2}$ i.e. if $x_C < 1-3a$. If $x_C < \frac{1}{2}$, $s_L < s_R$. Therefore, C wins if $s_C = \frac{1-2a}{2} > s_R = a + \frac{1-a-x_C}{2}$ i.e. if $x_C > 3a$. So, C wins for sure choosing $x_C \in (3a, 1-3a)$ which can only be a valid interval if $a < \frac{1}{6}$. So, for $a > \frac{1}{6}$, C has no chance of winning. Clearly, for $a = \frac{1}{6}$, the above conditions hold only for the case of equality leaving $x_C = \frac{1}{2}$ the only choice for C, resulting in equal shares of votes for all three candidates. If $a > \frac{1}{6}$, $s_C = \frac{1-2a}{2} < \frac{1}{3}$, $\forall x_C \in [a, 1-a]$. So, C has no possibility of winning. So, any $x_C \in [a, 1-a]$ could be an equilibrium strategy for C. If $x_C = \frac{1}{2}$, then $s_L = s_R$. From the tie-breaking rule, L and R win with equal probability. If $x_C \in (\frac{1}{2}, 1-a]$, $s_L > s_R$. So, L wins. If $x_C \in [a, \frac{1}{2})$ then $s_L < s_R$. So R wins. #

The conclusion that can be drawn is that, with intervals intersecting at a single point the centrist candidate can only win if the leftist and rightist candidates are forced to be polarized.

4.2 Disconnected Intervals (b > a)

Now I consider credible policy sets with $b > a$, that is, the intervals of the candidates do not intersect.



For this specification of intervals, for an arbitrary distribution of ideal points of voters, where the ideal point of the median voter is located in the interval $[0, \frac{a+b}{2})$ or in the interval $(1 - \frac{a+b}{2}, 1]$ the result is as follows:

Proposition 6: Under an arbitrary distribution of ideal points, in equilibrium: $x_L = a$ and $x_R = 1 - a$.

i. If $m \in [0, \frac{a+b}{2})$, L wins and C chooses the policy point that maximizes his share of votes.

ii. If $m \in (1 - \frac{a+b}{2}, 1]$, R wins and C chooses the policy point that maximizes his share of votes.

Proof: If $m \in [0, \frac{a+b}{2})$, L wins because for $x_L = a$, $s_L \geq \frac{1}{2}$ which implies that $s_L > s_j$ $j \in \{R, C\}$. Notice that $\forall m \in [0, \frac{a+b}{2})$, $x_L = a$ is a strictly dominant strategy for L. $x_R = 1-a$ is a strictly dominant for R under Assumption 1.1. If $m \in (1 - \frac{a+b}{2}, 1]$, R is the winner because for $x_R = 1 - a$, $s_R \geq \frac{1}{2}$ which implies that $s_R > s_j$ $j \in \{L, C\}$. Notice that $\forall m \in (1 - \frac{a+b}{2}, 1]$, $x_R = 1 - a$ is a strictly dominant strategy for R. $x_L = a$ is a strictly dominant strategy for R under Assumption 1.1. #

Different from Section 4.1, in this case for the same value of a the intervals in which the winner is the leftist or rightist candidate for sure for any distribution of ideal points of voters are larger since the centrist candidate is forced to choose from a more restricted interval than before. If m is located in the interval $[\frac{a+b}{2}, 1-\frac{a+b}{2}]$ then as in Section 4.1 depending on the distribution of ideal points of voters the winner can be the leftist, centrist or rightist candidate.

Now in order to describe the equilibrium characteristics when the ideal point of the median voter is located in the interval $[\frac{a+b}{2}, 1-\frac{a+b}{2}]$ consider a uniform distribution. For a uniform distribution of ideal points of voters the results are as follows:

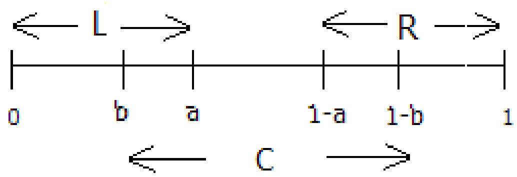
Proposition 7: Under a uniform distribution of ideal points, with intervals such that $b > a$, in equilibrium, $x_L = a$ and $x_R = 1-a$. If $a < \frac{1}{6}$, and $a > \frac{b}{3}$ then, in equilibrium, $x_C \in (3a, 1-3a)$ and he wins. If $a < \frac{1}{6}$ and $a \leq \frac{b}{3}$ then, in equilibrium, $x_C \in [b, 1-b]$ and he wins for sure. If, $a > \frac{1}{6}$, and $x_C \in [b, \frac{1}{2})$, in equilibrium, the winner is the leftist candidate; if $a > \frac{1}{6}$, and $x_C \in (\frac{1}{2}, 1-b]$, the winner is the rightist candidate. If $a > \frac{1}{6}$, and $x_C = \frac{1}{2}$, in equilibrium, the leftist and rightist candidates win with equal probability. If $a = \frac{1}{6}$, in equilibrium, $x_C = \frac{1}{2}$, and all three candidates win with equal probability.

Proof: $x_L = a$, $x_R = 1-a$ are strictly dominant strategies for L and R respectively implying that for any choice of x_C, $s_C = \frac{1-2a}{2}$. If C locates himself to the right of $\frac{1}{2}$, $s_L > s_R$. Therefore, C wins if $s_C = \frac{1-2a}{2} > s_L = a + \frac{x_C - a}{2}$ i.e. if $x_C < 1-3a$. If he locates himself to the left of $\frac{1}{2}$, $s_R > s_L$. Therefore, C wins if $s_C = \frac{1-2a}{2} > s_R = a + \frac{1-a-x_C}{2}$ i.e. if $x_C > 3a$. $(3a, 1-3a)$ is a valid interval only if $a < \frac{1}{6}$. Notice that if $1-3a > 1-b$ i.e. $\frac{b}{3} > a$, then $1-3a$ and $3a$ lie outside the interval of C. So,if $a < \frac{1}{6}$ and $a > \frac{b}{3}$, C wins for sure choosing $x_C \in (3a, 1-3a)$. If $a < \frac{1}{6}$ and $\frac{b}{3} \geq a$, C wins for sure choosing $x_C \in [b, 1-b]$. Clearly, for $a = \frac{1}{6}$, the above inequalities hold only for the case of equality leaving $x_C = \frac{1}{2}$ as the only choice for C, resulting in equal shares of votes for all candidates. If $a > \frac{1}{6}$, and $x_C = \frac{1}{2}$, then $s_L = s_R = \frac{2a+1}{4} > \frac{1-2a}{2} = s_C$. From the tie-breaking rule, L and R win with equal probability. If $x_C \in (\frac{1}{2}, 1-b]$, $s_L > s_R$. So, L is the winner. If $x_C \in [b, \frac{1}{2})$ then $s_R > s_L$. So, R is the winner. #

The results are very similar to the ones of Proposition 5. The critical value, that makes the centrist candidate the winner is the same, namely $\frac{1}{6}$. The only difference is that, if we consider the same value of a for both cases, since in the case at hand $b > a$, whether he wins or not, the interval in which the centrist candidate can locate himself is smaller than it is in Section 5.1 leaving him with a smaller set of choice.

4.3 Intervals Overlapping with Neighbors ($\mathbf{b < a <} \frac{1}{2}$)

In the above analysis the intervals for the leftist and centrist candidates and the intervals for the rightist and centrist candidates were intersecting only at a single point or not at all. Now, I analyze the case where the intervals for the rightist and centrist candidates and the intervals for the leftist and centrist candidate overlap whereas the intervals of the leftist and rightist candidate do not intersect. That is, $\frac{1}{2} > a > b$.



In the previous two cases the credible policy sets were such that the centrist candidate would choose for sure a policy point to the right of the leftist candidate and to the left of the rightist candidate. In the case at hand, however, the centrist candidate can locate himself to the left of the leftist candidate or to the right of the rightist candidate. For such a choice, he could not be considered as the centrist candidate anymore being more leftist than the leftist or more rightist than the rightist candidate. Therefore, the following assumption will be made:

Assumption 2: If $x_C < x_L$ or $x_C > x_R$, then the centrist candidate gets no votes.

This assumption states that voters punish the centrist candidate by giving him no votes if he chooses to be more leftist or more rightist than the corresponding candidate. That is, although voters consider some policy choices as credible claims proposed by two different candidates, they still force candidates to locate themselves according their ideological labels. I assume for the whole section that voters behave as Assumption 2 suggests. Without this assumption, there would exist no equilibrium for a uniform distribution of ideal points of voters if $a > \frac{1}{4}$. [7]

For an arbitrary distribution of ideal points of voters, if the ideal point of the median voter is located in the interval $[0, a)$, or in the interval $(1 - a, 1]$, the following result is obtained:

Proposition 8: Under an arbitrary distribution of ideal points, in equilibrium, $x_L = a$, $x_R = 1 - a$;
i. If $m \in [0, a)$, L wins and C chooses the policy point that maximizes his share of votes.
ii. If $m \in (1 - a, 1]$, R wins and C chooses the policy point that maximizes his share of votes.

Proof: For both cases $x_L = a$ is a strictly dominant strategy for L giving him the highest share of votes possible. Similarly, $x_R = 1 - a$ is a strictly dominant strategy for R. So, if $m \in [0, a)$, L wins since for $x_L = a$, $s_L \geq \frac{1}{2}$. If $m \in (1 - a, 1]$, R wins since for $x_R = 1 - a$, $s_R \geq \frac{1}{2}$. In both cases, C having no chance of winning chooses $x_C \in [a, 1 - a]$ that maximizes s_C. #

[7] See the appendix for a formal anaysis

If the location of the median voter is in the interval $[a, 1-a]$, then the winner would depend on the distribution of the ideal points and the length of the intervals of the leftist and rightist candidates. Now to be able to say more about what happens if m is in this interval, consider a uniform distribution of ideal points. In this case, we get the following result:

Proposition 9: For intervals where $\frac{1}{2} > a > b$, and a uniform distribution of ideal points, in equilibrium, $x_L = a$, and $x_R = 1-a$ and:

i. If $a < \frac{1}{6}$, $x_C \in (3a, 1-3a)$ and the centrist candidate is the winner.

ii. If $a = \frac{1}{6}$, then $x_C = \frac{1}{2}$, and all three candidates win with equal probabilities.

iii. If $a > \frac{1}{6}$ and $x_C \in [a, \frac{1}{2})$, the winner is the rightist candidate.

iv. If $a > \frac{1}{6}$ and $x_C \in (\frac{1}{2}, 1-a]$, the winner is the leftist candidate.

v. If $a > \frac{1}{6}$ and $x_C = \frac{1}{2}$ the leftist and rightist candidates win with equal probability.

Proof: In equilibrium, $x_L = a$, and $x_R = 1-a$ as they are strictly dominant strategies. $s_C = 0$ if $x_C < a$ or $x_C > 1-a$. So, in equilibrium, $x_C \in [a, 1-a]$. Therefore, $s_C = \frac{1-2a}{2}$.

i. If $x_C \in (\frac{1}{2}, 1-a]$, $s_L > s_R$. Therefore, C wins if $s_C = \frac{1-2a}{2} > s_L = \frac{x_C+a}{2}$ i.e. if $x_C < 1-3a$. If $x_C \in [a, \frac{1}{2})$, $s_R > s_L$. Therefore, C wins if $s_C = \frac{1-2a}{2} > s_R = a + \frac{1-a-x_C}{2}$ i.e. if $x_C > 3a$. So, C wins for sure choosing $x_C \in (3a, 1-3a)$ which can only be a valid interval if $a < \frac{1}{6}$ meaning that C wins for sure if $a < \frac{1}{6}$.

ii. For $a = \frac{1}{6}$, the above inequalities hold only for the case of equality leaving $x_C = \frac{1}{2}$ the only choice for C, resulting in equal shares of votes for all three candidates. Notice that if $a > \frac{1}{6}$, C can never win. So, for $a > \frac{1}{6}$, any $x_C \in [a, 1-a]$ could be included as an equilibrium strategy.

iii. If $x_C \in [a, \frac{1}{2})$ then $s_R = 1 - \frac{(1-a+x_C)}{2} > \frac{(a+x_C)}{2} = s_L$. So, R is the winner.

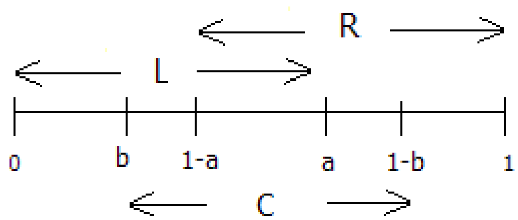
iv. If $x_C \in (\frac{1}{2}, 1-a]$, $1 - \frac{(1-a+x_C)}{2} < \frac{(a+x_C)}{2}$ so that L is the winner.

v. If $x_C = \frac{1}{2}$, $s_L = s_R = \frac{1-2a}{2}$. From the tie-breaking rule, L and R win with equal probability. #

The conclusion that can be drawn from the above result is that if the centrist candidate is required to locate himself to the right of the leftist candidate and to the left of the rightist candidate, the results are the same whether the intervals overlap or only intersect at a single point. Moreover, the interval of the centrist candidate does not play a role. The only parameter that affects the outcome is, a, the one that specifies the intervals of the leftist and rightist candidates.

4.4 Intervals Overlapping with all Opponents ($\frac{1}{2} \leq a$)

Finally, I consider the case where $a \geq \frac{1}{2}$.



As in the previous section, I assume that voters behave according to Assumption 2. Without this assumption, given that any $x_L \in [0, b)$ and any $x_R \in (1-b, 1]$ are strictly dominated we would arrive at the classical three candidate competition on the interval $[b, 1-b]$. It is well known that an equilibrium fails to exist. Moreover, now a leftist candidate could locate himself to the right of a rightist candidate. Therefore, in addition to Assumption 1.1, I will use Assumption 1, which states how voters behave if the leftist and rightist candidates are located on the same place. Notice that, this Assumption is applicable without any modification because of the implications of Assumption 2.

Therefore, for an arbitrary distribution of ideal points a similar argument to Proposition 9 would also hold for this case. Since $a \geq \frac{1}{2}$, we can define the winner for any type of distributions of ideal points of voters. The leftist and rightist candidates would win with equal probability locating themselves at m. If $m \in (1-a, a)$; L would win if $m \in [0, 1-a]$ with $x_L = x_R = 1-a$ and R would win if $m \in [a, 1]$ with $x_L = x_R = a$.

As a next step, I consider a uniform distribution of ideal points of voters. The result is as follows:

Proposition 10: Under a uniform distribution of ideal points of voters, the strategies $x_L = x_R = \frac{1}{2}$ and $x_C \in [b, 1-b]$ constitute the only Nash Equilibrium for intervals being such that $a \geq \frac{1}{2}$, in which the leftist and rightist candidate win with equal probabilities.

Proof: First check whether the above stated strategies constitute indeed an equilibrium. C is indifferent for any $x_C \in [b, 1-b]$ because from Assumptions 1.1 and 2, $s_C = 0$. If L deviates R would win for sure. So, he has no incentives to deviate. Similarly, if R deviates, L would win for sure. So, he has no incentives to deviate neither. Now, suppose that there exists an equilibrium in which L and R are located on a point different from $\frac{1}{2}$. If $x_L = x_R \neq \frac{1}{2}$, they tie. So, both would have incentives to deviate ε to the right if $x_L = x_R < \frac{1}{2}$ or ε to the left if $x_L = x_R > \frac{1}{2}$ to win for sure. If $x_R = \frac{1}{2}$ and $x_L \neq \frac{1}{2}$ R would win. So, L has incentive to deviate to $\frac{1}{2}$ for any choice of C where R and L would tie. A similar argument holds if $x_R \neq \frac{1}{2}$ and $x_L = \frac{1}{2}$. If $x_L \neq \frac{1}{2}$ and $x_R \neq \frac{1}{2}$ then depending on the choice of C anyone of the three could be the winner. Then, L or R would have the incentive to move closer to each other to increase at least their share of votes or even their probability of winning. So, having analyzed all other possible cases, it can be said that there exists no other type of equilibrium than the one stated above. #

The above result shows that in equilibrium there is only place for two parties, namely the leftist and rightist one. It can be said that in a society where the label of being "leftist" or "rightist" plays an important role "we arrive at the paradoxical situation that the Centre influences the whole of parliamentary life in the very country in which the electoral system prevents the formation of a Centre party."[8]

4.5 Discussion of the Results

In the analysis above the equilibrium outcomes are described for different credibility sets of candidates, which cover all the possibilities. What do these outcomes have in common? In all cases, for leftist and rightist candidates, it is a strictly dominant strategy to choose the policy point that is the most centrist one of their credibility sets. Therefore, the choice of the centrist candidate is the result of the maximization of his utility taking these strictly dominant strategies into account.

The winner depends heavily on the distribution of voters' ideal points. The leftist or rightist candidates are the winners if the median voter's ideal point is located in their credibility set and they apply the policy corresponding to the most centrist policy point allowed by their credibility sets. The optimal choice of the centrist candidate depends on the distribution of voters' ideal points. Under a uniform distribution of ideal points, for instance, where the median voter's ideal point is at the midpoint (C's interval), the centrist candidate can only win if his interval is four times as large as his opponents'. In this case, for a policy choice that is not too far from his two opponents' choices, that is, for a policy choice around the middle point, the centrist candidate is the winner. A necessary condition such that the centrist candidate wins is that the median voter's ideal point belongs to the interval of the centrist candidate.

For an arbitrary distribution of ideal points, when the ideal point of the median voter is in the interval of the centrist candidate, as it can be seen from the three examples given in Section 4.1 the winner can be any of the three candidates depending on the distribution of ideal points of voters. As the examples illustrate it might well happen that the centrist does not win even though his interval includes more than half of the voters..

First, the centrist candidate needs heavily concentrated ideal points of voters around the middle of the space. That is, a centrist candidate can only win if more than half of the voters' ideal points are

[8] Duverger, 1964 p. 387

located only in his credibility set. For some distributions he even needs 60% or more of support in his credibility set. Secondly, some asymmetry in the distribution of ideal points gives the centrist candidate higher chances of winning. That is, for a skewed distribution where the median voter is located at a point to the left or right of the middle point but still only in the interval of the centrist candidate, a centrist candidate can win with a less concentrated distribution in his credibility set compared to a symmetric distribution[9]. In short, the centrist candidate needs a much higher amount of voters in his credibility set compared to his opponents in order to have a chance to win the election.

One point that should be taken into account is that I only analyzed symmetric intervals. It should not be difficult to argue that, the conclusions would be almost identical under asymmetric intervals.

Of course, it should not be forgotten that this results are implied by plurality rule. Therefore, in order to see whether this disadvantage of a centrist candidate would also hold under another election rule, in the next section the same electoral competition as before will be analyzed under a run-off rule.

5 Restricted Electoral Competition With Three Candidates under a Run-Off Rule

Until now, I analyzed electoral competition of a single round under plurality rule. How would the outcome change if candidates would run an electoral competition under a run-off rule? Under a run-off rule, if in the first round none of the candidates reaches an absolute majority the candidates who got the first and second highest amount of votes run for a second time whereas the other candidates are eliminated. The winner is the candidate who gets the highest amount of votes in the second round, where in each round voters are required to cast a new ballot. I assume that candidates cannot change their policy points after the first round and that only the share of votes obtained in the first round affects candidates' utilities. That is because the first round shows the true preferences of voters, i.e. the true support of parties, whereas in the second round some voters are forced to vote for their second best choice as their first choice might have been eliminated in the first round.

We have seen that under an electoral competition where plurality rule is applied, the centrist candidate is disadvantaged compared to his opponents. Does a run-off rule change this result? I analyze the results for voters which behave according to Assumption 1 and Assumption 2 (where it is applicable). Clearly, for a general distribution where the median voter´s ideal point is located in the interval of the leftist or

[9] These results were obtained by considering different uni-modal Beta distributions.

rightist candidate, these candidates would win for sure in the first round with an absolute majority. Thus in run-off there is no need for the second round.

If m is located at a point which belongs only to the centrist candidate's interval, the winner would depend on the characteristics of the distribution.[10] As in the previous sections, I consider a uniform distribution of ideal points.

I first analyze the case where $a = b$ and assume a uniform distribution of ideal points of voters. In Proposition 5, I showed that for $a < \frac{1}{6}$, the centrist candidate wins for sure under a one round election system. So, for $a < \frac{1}{6}$ he would compete for sure in the second round and would win for sure under run-off rule as he would be located closer to the median voter compared to his opponent. Therefore, I focus on the case where $a \geq \frac{1}{6}$. The results are as follows:

Proposition 11: In equilibrium:

i. If $\frac{1}{6} \leq a < \frac{1}{4}$ the centrist candidate wins for sure choosing $x_C \in (a, 1-3a)$ or $x_C \in (3a, 1-a)$.

ii. If $a = \frac{1}{4}$, the centrist candidate can at best win with probability $\frac{1}{4}$ by choosing $x_C = \frac{1}{4}$ or $x_C = \frac{3}{4}$.

iii. If $a > \frac{1}{4}$, for any $x_C \in [a, 1-a]$, the leftist and rightist candidates win with equal probability.

In all type of equilibria, $x_L = a$ and $x_R = 1-a$.

Proof: Notice that $x_L = a$ and $x_R = 1-a$ are strictly dominant strategies for L and R respectively. Therefore, L can get at most half of the votes, which happens when $x_C = 1-a$. Similarly, R can get at most half of the votes, which happens when $x_C = a$. $s_C = \frac{1-2a}{2} < \frac{1}{2}$ since $a \geq \frac{1}{6}$. So, none of the candidates can get more than half of the votes in the first round for any a.

i. Suppose that, C locates himself to the left of $\frac{1}{2}$. Then, in order to get to the second round he has to get more votes than L i.e. $\frac{1-2a}{2} > a + \frac{x_C - a}{2}$ should be satisfied which holds for $x_C < 1-3a$. If C locates himself to the right of $\frac{1}{2}$, then, in order to get to the second round he has at least to get more votes than R i.e. $\frac{1-2a}{2} > a + \frac{1-a-x_C}{2}$ should be satisfied which holds for $x_C > 3a$. So, C should choose $x_C \in [a, 1-3a)$ or $x_C \in (3a, 1-a]$ to get to the second round. Clearly, it can only be satisfied if $a < \frac{1}{4}$ and these two intervals do not intersect for $\frac{1}{6} \leq a$. Choosing $x_C = a$ or $x_C = 1-a$, C would get in the second round the same amount of votes as his opponent (R and L respectively). Therefore, choosing $x_C \in (a, 1-3a)$ or $x_C \in (3a, 1-a)$, C wins the second round for sure.

ii. If $a = \frac{1}{4}$, then $a = 1-3a$, so to have the chance to get to the second round after the tie-braking, C has to choose $x_C = \frac{1}{4}$ or $x_C = \frac{3}{4}$. Doing so, he gets the same share of votes as his opponent in the second round meaning that C could win after two ties.

[10] In this case, for an arbitrary distribution, it should be expected that the centrist candidate has more chances of winning compared to the chances he has under plurality rule.

iii. If $a > \frac{1}{4}$, then for $x_L = a$ and $x_R = 1 - a$ which are strictly dominant strategies, $s_j > s_C$ $j \in \{L, R\}$ $\forall x_C \in [a, 1-a]$. So, L and R compete in the second round and win with equal probability getting the same share of votes. #

If we compare the results of the run-off rule with the results of Section 5.1, it can be said that the centrist candidate can win for a higher range of a as under plurality rule he wins for sure for $a < \frac{1}{6}$ and under run-off rule for $a < \frac{1}{4}$. Moreover, in the cases where the centrist candidate can never win, different from before, under run-off rule his policy point choice does not affect the outcome. For any choice of him, the other two candidates win with equal probability.

If $b > a$, that is when intervals do not intersect, and under a uniform distribution of ideal points of voters, the centrist candidate wins under a one-round electoral system only if $a < \frac{1}{6}$. So, for $a < \frac{1}{6}$ he would compete for sure in the second round and would win for sure under run-off rule as he would be located closer to the median voter compared to his opponent. Therefore, as before, I consider only the case where $a \geq \frac{1}{6}$. The result is as follows:

Proposition 12: For $b > a$, in equilibrium:

i. If $\frac{1}{6} \leq a < \frac{1}{4}$ and $1 - 3a > b$ the centrist candidate wins for sure choosing $x_C \in [b, 1-3a)$ or $x_C \in (3a, 1-b]$.

ii. If $\frac{1}{6} \leq a < \frac{1}{4}$ and $1 - 3a = b$, the centrist candidate can win by tieing once choosing $x_C = b$ or $x_C = 1 - b$.

iii. If $\frac{1}{6} \leq a < \frac{1}{4}$ and $1 - 3a < b$, for any $x_C \in [b, 1-b]$, the leftist and rightist candidates win with equal probability.

iv. If $a \geq \frac{1}{4}$, for any $x_C \in [b, 1-b]$, the leftist and rightist candidates win with equal probability

In all type of equilibria, $x_L = a$ and $x_R = 1 - a$.

Proof: Notice that $x_L = a$ and $x_R = 1 - a$ are strictly dominant strategies for L and R respectively. So, none of the candidates can get more than half of the votes in the first round for any a for a similar reasoning as in Proposition 12.

i. Suppose that, C locates himself to the left of $\frac{1}{2}$. Then, in order to get to the second round he has to get more votes than L i.e. $\frac{1-2a}{2} > a + \frac{x_C - x}{2}$ should be satisfied which holds for $x_C < 1 - 3a$. If C locates himself to the right of $\frac{1}{2}$, then, in order to get to the second round he has to get more votes than R i.e. $\frac{1-2a}{2} > a + \frac{1-a-x_C}{2}$ should be satisfied which holds for $x_C > 3a$. So, C should choose $x_C \in [b, 1-3a)$ or $x_C \in (3a, 1-b]$ to get to the second round. These are valid intervals only if $1 - 3a > b$, which could only hold if $a < \frac{1}{4}$ since $b > a$ should be satisfied. These two intervals do not intersect for $\frac{1}{6} \leq a$. So, if $\frac{1}{6} \leq a < \frac{1}{4}$ and $1 - 3a > b$, C gets to the second round and wins the election.

ii. If $\frac{1}{6} \leq a < \frac{1}{4}$ and $1 - 3a = b$, then C can only choose $x_C = b$ or $x_C = 1 - b$. So, he would get the same share of votes as L or R respectively and could get to the second round by tieing. He wins the second round for sure.

iii. If $\frac{1}{6} \leq a < \frac{1}{4}$ and $1 - 3a < b$, then C can not pass the first round since both L and R get more votes than C. He gets the same share of votes for any $x_C \in [b, 1 - b]$. L and R pass to the second round and tie in the second round getting the same share of votes.

iv. The same argument as in part (iii) holds. #

It can easily be concluded that the centrist candidate can win for a wider set of parameter values than in a one-round election as under plurality rule he could only win for $a < \frac{1}{6}$ while under run-off rule he can also win for $\frac{1}{6} < a < \frac{1}{4}$ if $1 - 3a > b$ holds.

To compare the result with the case where the intervals intersect, consider intervals of the same length for leftist and rightist candidates for both the cases where $b = a$ and where $b > a$ with $\frac{1}{4} > a > \frac{1}{6}$. In order to get to the second round, the policy choice of the centrist candidate has to be located close enough to at least one of his opponents. However, for $b > a$, he might not locate himself close enough to one of his opponents if b is too large (his interval too small), while for $b = a$ this would not occur since the intervals of the leftist and centrist candidates and the intervals of the rightist and centrist candidates intersect. Therefore, if $b > a$, the centrist candidate does not win for sure under a run-off system if $\frac{1}{4} > a$.

For $\frac{1}{2} > a > b$, the centrist candidate wins under a one-round electoral system only if $a < \frac{1}{6}$. So, for $a < \frac{1}{6}$ he would compete for sure in the second round and would win for sure under run-off rule as he would be located closer to the median voter compared to his opponent. Therefore, as before, I consider only the case where $a \geq \frac{1}{6}$. The result is as follows:

Proposition 13: For $\frac{1}{2} > a > b$, in equilibrium:

i. If $\frac{1}{6} \leq a < \frac{1}{4}$ the centrist candidate wins for sure choosing $x_C \in (a, 1 - 3a)$ or $x_C \in (3a, 1 - a)$.

ii. If $a = \frac{1}{4}$, the centrist candidate can win with probability $\frac{1}{4}$ by choosing $x_C = \frac{1}{4}$ or $x_C = \frac{3}{4}$.

iii. If $a > \frac{1}{4}$, for any $x_C \in [a, 1 - a]$, the leftist and rightist candidates win with equal probability.

In all equilibria, $x_L = a$ and $x_R = 1 - a$.

Proof: Notice that $x_L = a$ and $x_R = 1 - a$ are strictly dominant strategies for L and R respectively and C never chooses $x_C < a$ or $x_C > 1 - a$ since he would get no votes for such a choice. So, none of the candidates can get more than half of the votes in the first round for any a. So, the analysis becomes the same as in Proposition 11. #

Once again, under a run-off rule the centrist candidate can win under a higher range of parameters compared to plurality rule. (for $a < \frac{1}{6}$ under plurality and for $a < \frac{1}{4}$ under run-off) As in the one-round case the parameter b plays no role. The only important parameter is a, the one defining the length of the intervals of the leftist and rightist candidates.

Finally, for $a \geq \frac{1}{2}$, if Assumption 2 is maintained as it was done for the analysis of the one-round election, then the centrist candidate would continue to get no votes even under the run-off system and the leftist and rightist candidates would tie twice.

5.1 Discussion of the Results

As it was done for electoral competition under plurality rule, in the analysis above the equilibrium outcomes are described for different credibility sets of candidates, which cover all the possibilities. In all cases, for leftist and rightist candidates, it is a strictly dominant strategy to choose the policy point that is the most centrist one of their credibility sets. Therefore, the choice of the centrist candidate is the result of the maximization of his utility taking these strictly dominant strategies into account.

The winner depends on the distribution of the voters' ideal points. If the median voter is located in their credibility sets, the leftist and rightist candidates win without the need for a second round as they obtain absolute majority in the first round. If the ideal point of the median voter is located in the credibility set of the centrist candidate then the winner depends on the characteristics of the distribution and the size of the credibility sets of each candidate.

In that case, given that neither the leftist nor the rightist candidate gets more than half of the votes in the first round, that is, given that they do not win the election already in the first round, for a centrist candidate it is sufficient to come second in the first round to win the election. The fact that the leftist or the rightist candidate does not get more than half of the votes in the first round implies that the ideal point of the median voter is located in the interval of the centrist candidate. So, it can easily be argued that if the centrist candidate qualifies for the second round he will for sure be closer to the ideal point of the median voter than his opponent in this round. This is true because if it weren't the case, that is, if his opponent in the second round is closer to the ideal point of the median voter, this opponent would already have acquired more than half of the votes in the first round.

A centrist candidate would also win for sure under run-off rule for all parameter values for which he would win under plurality rule. Why is this true? If a centrist candidate wins under plurality rule for a given policy choice then he would also win the first round under the run-off rule for the same policy choice as the equilibrium policy choice of the leftist and rightist candidates is the same under both rules. We know that if a centrist candidate wins under plurality rule he is necessarily closer to the median voter compared to his two opponents. So, for the same policy choice he would win for sure the second round. Then the question that arises is whether the centrist candidate would win for a higher parameter range. For the plurality rule there exists a critical value of a beyond which the centrist candidate can never win. Suppose we take an a slightly larger ($a' = a + \varepsilon$). We know that the centrist candidate would win for sure if he gets to the second round. So, the only possibility that he does not win is that he becomes third for any policy choice in the first round. It can be shown that if $a \geq b$, that the centrist candidate can still win under run-off rule for ε sufficiently small. On the other hand, if $b > a$, then if b is sufficiently large, it can be found examples for which the centrist candidate can never win if a is only slightly higher than the critical value So, we can conclude that the centrist candidate would (weakly) win for a larger set of parameter values under run-off rule compared to plurality rule.

As before, I only analyzed symmetric intervals. It should not be difficult to argue that, the conclusions would be almost identical under asymmetric intervals.

6 Conclusion

The model I have analyzed uses the credibility that voters assign to candidates to introduce restrictions on the candidates' strategies. I assume that exogenously given restrictions about the policy space specify the policy points that candidates can credibly propose. This limitation can prevent full convergence in terms of candidates' policy choices.

For two candidate competition, we obtain that the policy choices in equilibrium are different from the proposed by the median voter theorem, when the credibility of candidates prevents total convergence to the median voter.

For three candidate competition, the results imply that in many cases, although a majority of voters could be considered as ideologically centrist, centrist candidates have less chances of winning. This conclusion verifies Duverger's statement mentioned before up to some point. It is not uniquely the plurality rule that leads to outcomes where only two candidates get a considerable amount of votes; credibility of candidates plays also an important role. Since candidates are assumed to be office-motivated, leftist and rightist candidates have incentives to converge to each other as much as their intervals allow.

The centrist candidate prefers to be as close as possible to one of his opponents for more asymmetric distributions of ideal points of voters, whereas for symmetric distributions he prefers to be as far as possible from all his opponents. The centrist candidate is better-off the larger is his interval and the smaller the interval of his opponents.

As it was discussed before, it can be said that the larger the interval of a candidate, the higher is his credibility. Therefore, whenever the intervals of the three candidates do not intersect, it can be concluded that as the credibility of a candidate increases, his probability of winning increases. A candidate with a larger interval would obtain higher share of votes; therefore as the credibility of a candidate increases, his utility also increases.

It can easily be argued that not only the distribution of voters but also the election rule determines the winner in the model analyzed in this paper. The model shows that leftist and rightist candidates would prefer a one-round electoral competition (plurality rule) whereas a centrist candidate would be better off under a run-off rule. That is, the election rule might help to obtain more centrist outcomes. Broadly speaking, the electoral competition in UK can be considered as a three party competition where plurality rule applies. As Taagepera and Shugart (1989) state British Liberals favored changes in the electoral rule which continues up today, as the plurality rule favors the two parties (Labor and Conservatives) on the left and right wings, whereas these two parties are quite happy with the system.

If the intervals of credible policies of the leftist and rightist candidates intersect, a centrist candidate cannot get any vote. Therefore, one should expect that in countries where leftist and rightist parties have high credibility, the electoral competition would take place between two parties. Centrist parties would arise when there is a policy interval that cannot be reached by any leftist or rightist party. The chance of winning of a centrist candidate would increase with the size of his interval and with a decrease in the size of the intervals of his opponents.

One important assumption maintained during the analysis is that there is no abstention. One further step would be discard this assumption and think that a voter would not vote for any candidate if they are too far away from his ideal point. In this case we could reach equilibria in which leftist and rightist candidates might diverge more than they do without abstention as they would have two conflicting incentives, namely, being close to the centrist candidate to "steal" some of his votes, and secondly not to loose rather radical voters.

With regard to the voters, I assume, moreover, that voters vote truthfully. The next step would be to allow voters to vote strategically. Under strategic voting, pre-election polls would play an important role to provide voters information about the distribution of ideal points and to coordinate themselves. Under strategic voting with the aid of pre-election polls, I expect to verify Duverger's Law, that is I would obtain results where only the leftist and rightist candidates get a positive share of votes because voters whose most preferred candidate is the centrist one would prefer to vote for the less evil of the other two candidates if they expect that the centrist candidate has no chance of winning.

7 Appendix

Proposition 14: For $\frac{1}{2} > a > b$, if $a > \frac{1}{4}$, then there exists no equilibrium in pure strategies under Assumption 1.1 and a uniform distribution of ideal points of voters.

Proof: First notice that as in the previous cases any $x_L \in [0, b)$ is strictly dominated by $x_L = b$ and any $x_R \in (1-b, 1]$ is strictly dominated by $x_R = 1-b$. Therefore in any equilibrium $x_L \in [b, a]$ and $x_R, x_C \in [1-a, 1-b]$. Now, consider all possible equilibria. First, consider the possible equilibria with $x_C \in [a, 1-a]$. In that case, in equilibrium $x_L = a$ and $x_R = 1-a$, as any other strategy for them would be strictly dominated. So, $s_C = \frac{1-2a}{2}$. C would deviate to $a-\epsilon$ or $1-a+\epsilon$ if $a > \frac{1-2a}{2}$ i.e. if $a > \frac{1}{4}$. So there exists no equilibrium with $x_C \in [a, 1-a]$ if $a > \frac{1}{4}$. Now consider possible equilibria with x_L, x_C $\in [b, a]$. Then in any equilibrium $x_R = 1-a$ as it is a strictly dominant strategy. (Notice that the case with $x_R, x_C \in [1-a, 1-b]$ would just be the symmetric of the case at hand). Now consider the case where $x_L < x_C$. This cannot be part of an equilibrium as L would deviate to $x_L + \epsilon$ to increase his share of votes. So, we should have $x_L \geq x_C$. Now suppose that $x_L = x_C = k \in [b, a]$. Then, $s_C = \frac{1-a-k}{2}$ and $s_L = k$. C would deviate to $k-\epsilon$ if $s_L > s_C$ and L would deviate to $k+\epsilon$ if $s_L > s_C$. So, there can exist an equilibrium if $s_L = s_C$ i.e. if $s_L = s_C = k = \frac{1-a}{3}$. This would be possible if $\frac{1-a}{3} \leq a$ i.e. if $a \geq \frac{1}{4}$. But if $a > \frac{1}{4}$ then C would deviate to $1-a+\epsilon$ as it would increase its share of votes. So, there would exist an equilibrium iff $a = \frac{1}{4}$. Finally, consider the case $x_L > x_C$ where $x_C = k$. Then, C would deviate to $k+\epsilon$ to increase his share of votes. So, combining all possible cases it can be seen that there exists no equilibrium if $a > \frac{1}{4}$. #

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