# Learning by Copying* 

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#### Abstract

We analyze the behavior of a multiproduct monopolist, a duopolist and consumers who are able to learn by copying. We show that when the effect of learning by copying is strong and the cost of copying is low enough, consumers decide to copy all goods, independently of their prices. This suggests that the DRM systems implemented by the digital industry have adverse consequences, because they hinder the use of original information goods and provide consumers with an incentive for copying. Moreover, we obtain two more kinds of equilibrium: one where each firm sells to the consumer who values its good more highly and another where each firm sells to all consumers. These results are robust when we consider that consumers' preferences are "opposed." Finally, by analyzing social welfare we show that, from a static perspective, the multiproduct monopoly provides a welfare at least as great as the duopoly and, from a dynamic perspective, a duopolist has at least the same incentive to create a new product as a monopolist.


Keywords: Consumers, Learning by Copying, Opposed Preferences, DRM, Copy, Piracy

JEL classification: K42; L11; L86; O34

[^0]
## 1 Introduction

In recent years major distribution companies of information goods (in particular record companies) have developed technological tools known as digital rights management (DRM) to prevent the copying of their goods. However, these tools have a negative impact on utility for consumers of original information goods, because they hinder the use of those goods. For instance, DRM for music can limit the uses of music files downloaded from online retailers, the number of computers to which the user can transfer his or her files (typically between 3 and 5) and the number of times a playlist can be burned on a CD-R (typically between 7 and 9) (Duchêne and Waelbroeck (2006)). Moreover, DRM can also authorize playing the content on a specific piece of hardware, and ensure that movies released for viewing in one region of the world cannot be viewed in another (Park and Scotchmer (2006)). However, the latest technological developments and the Internet have enabled consumers to overcome these restrictions so that consumers are able to learn by copying, to the point where it is possible that some consumers may prefer a copy to an original information good because they can use the copy more easily and in more devices and improve its quality.

This negative effect of DRM may explain why half the executives in the music industry support the distribution of files compatible with all computers and the elimination of DRM, according to a poll by Jupiter Research (ELPAIS.com (15-02-2007)). Moreover, at the time of writing this paper a story appeared in the news reporting agreements by Apple and Amazon with EMI for the sale of music without technical protection (ELPAIS.com (02-04-2007) and ELPAIS.com (24-05-2007)).

The fact that consumers copy information goods has been analyzed before from a theoretical viewpoint. ${ }^{1}$ Johnson (1985) shows that government intervention (taxes on copying and subsidies for the producers of creative works) may enhance the social surplus, which depends on the degree to which copying reduces the demand for originals as opposed to increasing total consumption, the elasticity of supply of creative works, and the value that consumers place on product variety. On the other hand, Bae and Choi (2006), who consider that the cost of copying depends on the reproduction cost and the degradation cost, show that the effects of copying on social welfare depend on the nature of the cost of copying.

There are also papers that analyze the role of DRM. Duchêne, Peitz and Waelbroeck (2005) explain how artists and producers could use DRM-based technologies to raise their profits. Moreover, Duchêne and Waelbroeck (2006) find that increasing copyright protection by legal authorities gives companies using information-push technology (in which the firms pay to provide information about the products they sell) incentives to raise the level of technological protection, which increases their profits. However it is detrimental to firms using information-pull technologies (where consumers spend resources to acquire information on products in which they have a special interest), who would prefer weak DRM protection. Park and Scotchmer (2006) show that DRM has a collusive impact through the sharing of its implementation costs.

[^1]Shy and Thisse (1999) analyze a duopoly model in which the utility of consumers depends on the number of consumers who use the original and copied software (network effects). They show that the software industry should choose not to protect the software when network effects are strong, because these effects make software more attractive to consumers, thereby enabling firms to raise prices.

The effect of sampling has been analyzed by Peitz and Waelbroeck (2006a). They show that, under sufficient heterogeneity of taste and diversity of products, the effect of sampling leads to higher profit, because consumers are willing to pay more when the match between product characteristics and buyers' tastes is closer.

Finally, the fact that copying technology by consumers exhibits increasing returns to scale has been analyzed by Belleflamme and Picard (2007). They analyze the pricing behavior of a monopolist and a duopolist in a model of vertical product differentiation, where there are two information goods which are perfectly (horizontally) differentiated and equally valued by users. Belleflamme and Picard (2007) show that in a monopoly equilibrium prices are neither unique nor symmetric, and that in a duopoly, when the cost of copying is high enough there is a symmetric equilibrium in pure strategies, and when the cost of copying is low enough there is no equilibrium in pure strategies although there is a symmetric equilibrium in mixed strategies. This paper also shows that the multiproduct monopolist has an incentive to set lower prices than the duopolist because it realizes that decreasing the price for one good increases demand for the other good by making copying less attractive. They call this result the Cournot effect. Finally, Belleflamme and Picard (2007) show that a multiproduct monopoly makes for greater welfare than a duopoly in the short run but provides lower incentives to create in the long run.

The present paper provides a new approach that gives more pessimistic results about DRM and copying by consumers than those obtained in the rest of the relevant literature. Using a different method from Belleflamme and Picard (2007), we analyze the process of learning by copying by consumers where they are able to make better copies as they make more copies, to the point that they may come to value copies more highly than originals. Under some conditions this is equivalent to assuming that the technology of copying exhibits increasing returns to scale as in Belleflamme and Picard (2007). We consider that information goods have independent content, and consumers value information goods differently, although each consumer values each good in the same way, as in Belleflamme and Picard (2007). However, the fact of learning by copying causes information goods to become interdependent, i.e. the demand for information goods depends on the price of other information goods.

We find a complete characterization of equilibrium, unlike Belleflamme and Picard (2007). An important conclusion is that when the effect of learning by copying is strong and the cost of copying is low enough, consumers decide to copy all goods, independently of their prices. This suggests that DRM systems implemented by the digital industry have adverse consequences because they hinder the use of original information goods and encourage consumers to copy.

We obtain two more kinds of equilibria: one where only the consumer who most highly values both goods buys them, and another where both consumers buy both goods. The existence of both kinds of equilibrium depends on the quantity that the consumer who values the goods least is willing to pay for
each one. Thus, the first kind of equilibrium exists when this quantity is low enough, and the second when it is high enough. This means that when the consumer who values the goods least is willing to pay enough, the duopolists want to sell to him.

We also show that whether or not there is a Cournot effect depends on the size of the gain on the second copy and the quantity that the consumer who values the goods least is willing to pay for each one, unlike Belleflamme and Picard (2007), who obtain that the Cournot effect always exists.

Finally, we extend the model to analyze the case where consumers have so-called "opposed preferences". This means that consumers value each good differently, and do not agree on the most and least valued information good. We consider these preferences because there are more and more information goods that can be copied by consumers, those goods are completely different and consumers value them differently. For instance, we can think of films, videogames, music and software programs.

In this case we also find two more kinds of equilibrium with regard to the case where the effect of learning by copying is strong and the cost of copying is low enough: one where each consumer only buys the good that he/she most values; and another where both consumers buy both goods. Moreover, we show that there is no Cournot effect. Therefore, whether or not there is a Cournot effect also depends on the preferences of consumers.

By analyzing social welfare we obtain that, from a static perspective, the multiproduct monopoly provides a social welfare at least as great as a duopoly, and from a dynamic perspective a duopolist has at least the same incentives to create a new good as a multiproduct monopolist, independently of the kind of consumer preferences that we consider. Thus, the results obtained in Belleflamme and Picard (2007) are robust when we consider a more general cost function and opposed preferences.

The rest of the paper is organized as follows. Section 2 describes and analyzes the model. Section 3 considers that consumers have "opposed preferences". Section 4 analyzes social welfare. The effect of copying on the sales of firms is analyzed in Section 5. Finally, Section 6 concludes.

## 2 The model with learning by copying

There are two firms, A and B, each of which produces a single information good, $a$ and $b$, respectively. These goods are independent of each other. We assume that there are two consumers (1 and 2) who value the information goods differently, although each consumer values each good in the same way, as in Belleflamme and Picard (2007). Let $V_{o}\left(V_{c}\right)$ and $v_{o}\left(v_{c}\right)$ be the valuation of consumers 1 and 2 of any original information good (copy), respectively. With no loss of generality, we assume that consumer 1 values any information good more highly independently if it is an original or a copy, i.e. $v_{o}<V_{o}$ and $v_{c}<V_{c}$.

We represent the process of learning by copying as an increase in the valuation of the second copy made by consumers, which we call the gain of the second copy and represent by $\Delta$. Thus, $V_{c}+\Delta$ and $v_{c}+\Delta$ are the valuations of the second copy made by consumers 1 and 2 , respectively.

Copying by consumers is assumed to be legal, because they make copies for private use and with no
intention of making a profit or using them collectively (Bravo-Bueno (2005)), as in Bae and Choi (2006), Johnson (1985) and Shy and Thisse (1999). ${ }^{2}$ This happens in those countries where intellectual property law allows private copying. We also assume that consumers bear a constant marginal cost of copying $c$. Thus, the utility of a consumer who buys or copies an information good, which he/she values at $v$, at cost $p$, is $v-p$, and otherwise it is zero.

We do not represent the process of learning by copying through the cost of copying, although our analysis is equivalent to this if we assume that the cost of the second copy is $c^{\prime}=c-\Delta$. Moreover, unlike Belleflamme and Picard (2007), we do not assume that the cost of copying is fixed, although our analysis is equivalent to this if $c=\Delta$. Thus, our analysis is equivalent to considering a more general cost function that can be decreasing, constant or increasing in the number of copies made by consumers, according to the gain of the second copy, $\Delta$.

The timing of the game is as follows. First, firms price the information goods. Next, consumers decide to buy or copy the information goods or do nothing after they have observed firms' prices.

We restrict our attention to subgame perfect equilibria (SPE) of the game, which we find by backward induction. Thus, we first look for the pattern of demand of consumers, and then for the optimal decisions of firms (monopolist and duopolist).

### 2.1 Consumers' behaviour

We begin by analyzing the pattern of demand of consumer 1, who has the following strategies: buying both information goods $\left(B_{a}^{1}, B_{b}^{1}\right)$, buying $a$ and copying $b\left(B_{a}^{1}, C_{b}^{1}\right)$, copying $a$ and buying $b\left(C_{a}^{1}, B_{b}^{1}\right)$, copying $a$ and $b\left(C_{a}^{1}, C_{b}^{1}\right)$, and doing nothing $(\varnothing)$. Given that the gain of the second copy does not depend on which good consumers copy first, consumer 1 is indifferent between copying $a$ and $b$ first. The utility of consumer 1 associated with these strategies is:

$$
U_{1}\left(p_{a}, p_{b}\right)=\left\{\begin{array}{cl}
2 V_{o}-p_{a}-p_{b} & \text { if consumer } 1 \text { buys } a \text { and } b  \tag{1}\\
V_{o}+V_{c}-p_{a}-c & \text { if consumer } 1 \text { buys } a \text { and copies } b \\
V_{c}+V_{o}-c-p_{b} & \text { if consumer } 1 \text { copies } a \text { and buys } b \\
2 V_{c}+\Delta-2 c & \text { if consumer } 1 \text { copies } a \text { and } b \\
0 & \text { otherwise, }
\end{array}\right.
$$

where $p_{j}$ is the price of information good $j=a, b$. We obtain the utility of consumer 2 in the same way, but taking into account that he/she values the original good at $v_{o}$ and the copy at $v_{c}$. To prevent problems with the existence of equilibrium, we assume that when a consumer is indifferent between buying and copying an information good, he/she decides to buy it.

Let $\delta_{V}=V_{o}-V_{c}>0$ and $\delta_{v}=v_{o}-v_{c}>0$ be the valuation gap for consumers 1 and 2 between any original information good and their copy, respectively. The levels of willingness to pay of consumers 1 and 2 are $c+\delta_{V}$ and $c+\delta_{v}$, respectively. We assume that $\delta_{V}>\delta_{v}$. This implies that the maximum

[^2]

Figure 1: The pattern of demand of consumers
price that consumer 1 is willing to pay for any original good is higher than that of consumer 2 , which is in keeping with the fact that consumer 1 values information goods more highly. By comparing the levels of utility obtained by each consumer, we get their patterns of demand, which are represented in Figure 1. Taking this into account, we find the demand for goods, which is illustrated in Figure 2. Notice that the demand differs according to the gain on the second copy, and that Figure $2(\mathrm{~b})$ is only possible if $\delta_{V}-\delta_{v}<c+\delta_{v}$.

An interesting result is that when $\Delta>2\left(c+\delta_{V}\right)$, consumers decide to copy all goods, independently of their prices. This suggests that the DRM systems implemented by the digital industry to prevent consumers from copying information goods have the adverse effect of destroying demand for original information goods. Thus, we assume that $0 \leq \Delta<c+\delta_{v}$ to guarantee that any consumer's strategy could be optimal under some prices, and since we are interested in copying becoming a profitable strategy by consumers, we assume that the cost of copying is low enough, i.e. $c<v_{c}$.

As can be seen in Figure 2, the process of learning by copying makes goods $a$ and $b$ complementary for some prices, although they are independent in content. Moreover, for the goods to become complementary it is not necessary for their prices to be similar enough, unlike Belleflamme and Picard (2007).

Before we characterize the case of a duopoly, in the next subsection we solve the model when both goods are sold by a monopolist on the market.

### 2.2 Multiproduct monopoly

From the demand for goods represented in Figure 2, we obtain the candidate strategies for equilibrium. These are shown in the following table, with the demand and profit associated with each one. ${ }^{3}$

[^3]
(a) $\Delta \leq \delta_{V}-\delta_{v}$

(b) $\delta_{V}-\delta_{v} \leq \Delta<c+\delta_{v}$

Figure 2: The demand for goods

| STRATEGY | DEMAND | PROFIT |
| :---: | :---: | :---: |
| (i)$p_{a}+p_{b}=2\left(c+\delta_{v}\right)-\Delta$ <br> $p_{a}, p_{b} \in\left[c+\delta_{v}-\Delta, c+\delta_{v}\right]$ | $B_{a}^{1} B_{b}^{1} ; B_{a}^{2} B_{b}^{2}$ | $\pi_{m}^{(\mathrm{i})}=4\left(c+\delta_{v}\right)-2 \Delta$ |
| (ii) $p_{a}=c+\delta_{v}-\Delta ; p_{b}=c+\delta_{V}$ | $B_{a}^{1} B_{b}^{1} ; B_{a}^{2} C_{b}^{2}$ | $\pi_{m}^{(\mathrm{iii})}=3 c+\delta_{V}+2\left(\delta_{v}-\Delta\right)$ |
| (iii)$p_{a}+p_{b}=2\left(c+\delta_{V}\right)-\Delta$ <br> $p_{a}, p_{b} \in\left[c+\delta_{V}-\Delta, c+\delta_{V}\right]$ | $B_{a}^{1} B_{b}^{1} ; C_{a}^{2} C_{b}^{2}$ | $\pi_{m}^{(\mathrm{iii)}}=2\left(c+\delta_{V}\right)-\Delta$ |

## Table 1

From Table 1, we observe that a higher cost of copying and a lower gain on the second copy imply higher prices and profits for every strategy.

As we can see in the following proposition, the optimal decision of the monopolist depends on how much consumer 2 is willing to pay for each good. Thus, when consumer 2 is willing to pay enough for each good, the monopolist sells both goods to both consumers. Otherwise, the monopolist sells only to consumer 1.

Proposition 1 In any SPE, we have that:
(a) if $c+\delta_{v} \leq \delta_{V}-\delta_{v}+\Delta / 2$, the optimal strategy is (iii); and
(b) if $c+\delta_{v} \geq \delta_{V}-\delta_{v}+\Delta / 2$, the optimal strategy is (i).

Proof: see Appendix.

### 2.3 Duopoly

In this subsection, we analyze the model when each good is provided by a single firm. We begin by analyzing the case where the gain on the second copy is low enough, i.e. $\Delta \leq \delta_{V}-\delta_{v}$. Thus, the demand for goods is represented by Figure 2(a).

As we can see from Proposition 2 and Figure 3, there are two kinds of equilibrium: one where consumer 1 buys both goods and consumer 2 copies them, and the other where both consumers buy both goods. The first exists when the relative value of $c+\delta_{v}$ is low enough, and the second when it is high enough. This means that when consumer 2 is willing to pay enough, the duopolists want to sell to him/her. Moreover, for intermediate values of $c+\delta_{v}$, the two kinds of equilibrium coexist. Notice that both the monopolist and duopolists set the same average price for each kind of equilibrium.

Proposition 2 The SPE, when the gain on the second copy is low, are:
(a) $\left[p_{a}^{*}, p_{b}^{*},\left(B_{a}^{1}, B_{b}^{1}, C_{a}^{2}, C_{b}^{2}\right)\right]$, with $p_{a}^{*}+p_{b}^{*}=2\left(c+\delta_{V}\right)-\Delta$ and $p_{a}^{*}, p_{b}^{*} \in\left[c+\delta_{V}-\Delta, c+\delta_{V}\right]$, if $c+\delta_{v} \in$ $\left[\Delta, \delta_{V}-\delta_{v}\right] ;$
(b) $\left[p_{a}^{*}, p_{b}^{*},\left(B_{a}^{1}, B_{b}^{1}, C_{a}^{2}, C_{b}^{2}\right)\right]$, with $p_{a}^{*}+p_{b}^{*}=2\left(c+\delta_{V}\right)-\Delta$ and $p_{a}^{*}, p_{b}^{*} \in\left[c+\delta_{V}-\Delta, c+\delta_{V}\right]$, if $c+\delta_{v} \in$ $\left[\delta_{V}-\delta_{v}, \delta_{V}-\delta_{v}+\Delta\right] ;$
(c) $\left[p_{a}^{*}, p_{b}^{*},\left(B_{a}^{1}, B_{b}^{1}, C_{a}^{2}, C_{b}^{2}\right)\right]$, with $p_{a}^{*}+p_{b}^{*}=2\left(c+\delta_{V}\right)-\Delta$ and $p_{a}^{*}, p_{b}^{*} \in\left[2\left(c+\delta_{v}-\Delta\right), 2\left(\delta_{V}-\delta_{v}\right)+\Delta\right]$; and $\left[p_{a}^{*}, p_{b}^{*},\left(B_{a}^{1}, B_{b}^{1}, B_{a}^{2}, B_{b}^{2}\right)\right]$, with $p_{a}^{*}+p_{b}^{*}=2\left(c+\delta_{v}\right)-\Delta$ and $p_{a}^{*}, p_{b}^{*} \in\left[\left(c+\delta_{V}\right) / 2,\left(3 c-\delta_{V}+4 \delta_{v}\right.\right.$ $-2 \Delta) / 2]$, if $c+\delta_{v} \in\left[\delta_{V}-\delta_{v}+\Delta, \delta_{V}-\delta_{v}+3 \Delta / 2\right]$;
(d) $\left[p_{a}^{*}, p_{b}^{*},\left(B_{a}^{1}, B_{b}^{1}, B_{a}^{2}, B_{b}^{2}\right)\right]$, with $p_{a}^{*}+p_{b}^{*}=2\left(c+\delta_{v}\right)-\Delta$ and $p_{a}^{*}, p_{b}^{*} \in\left[\left(c+\delta_{V}\right) / 2,\left(3 c-\delta_{V}+4 \delta_{v}\right.\right.$ $-2 \Delta) / 2]$, if $c+\delta_{v} \in\left[\delta_{V}-\delta_{v}+3 \Delta / 2, \delta_{V}-\delta_{v}+2 \Delta\right]$; and
(e) $\left[p_{a}^{*}, p_{b}^{*},\left(B_{a}^{1}, B_{b}^{1}, B_{a}^{2}, B_{b}^{2}\right)\right]$, with $p_{a}^{*}+p_{b}^{*}=2\left(c+\delta_{v}\right)-\Delta$ and $p_{a}^{*}, p_{b}^{*} \in\left[c+\delta_{v}-\Delta, c+\delta_{v}\right]$, if $c+\delta_{v} \in$ $\left[\delta_{V}-\delta_{v}+2 \Delta,+\infty\right]$.

Proof: see Appendix.

We now analyze the model when the gain on the second copy is high enough, i.e. $\delta_{V}-\delta_{v}<\Delta<$ $\min \left(c+\delta_{v}, 2\left(\delta_{V}-\delta_{v}\right)\right)$, so the demand of goods is represented by Figure 2(b). As in the previous case, we obtain that when consumer 2 is willing to pay enough, the duopolist wants to sell to him/her, otherwise it only sells to consumer 1. This result is summarized in Proposition 3 and Figure 4.

Proposition 3 The SPE, when the gain on the second copy is high, are:
(a) $\left[p_{a}^{*}, p_{b}^{*},\left(B_{a}^{1} B_{b}^{1}, C_{a}^{2} C_{b}^{2}\right)\right]$, with $p_{a}^{*}+p_{b}^{*}=2\left(c+\delta_{V}\right)-\Delta$ and $p_{a}^{*}, p_{b}^{*} \in\left[c+\delta_{V}-\Delta, c+\delta_{V}\right]$, if $c+\delta_{v} \in$ $\left[\delta_{V}-\delta_{v}, \quad \delta_{V}-\delta_{v}+\Delta\right] ;$
(b) $\left[p_{a}^{*}, p_{b}^{*},\left(B_{a}^{1} B_{b}^{1}, C_{a}^{2} C_{b}^{2}\right)\right]$, with $p_{a}^{*}+p_{b}^{*}=2\left(c+\delta_{V}\right)-\Delta$ and $p_{a}^{*}, p_{b}^{*} \in\left[2\left(c+\delta_{v}-\Delta\right), 2\left(\delta_{V}-\delta_{v}\right)+\Delta\right]$; and $\left[p_{a}^{*}, p_{b}^{*},\left(B_{a}^{1} B_{b}^{1}, B_{a}^{2} B_{b}^{2}\right)\right]$, with $p_{a}^{*}+p_{b}^{*}=2\left(c+\delta_{v}\right)-\Delta$ and $p_{a}^{*}, p_{b}^{*} \in\left[\left(c+\delta_{V}\right) / 2,\left(3 c-\delta_{V}+4 \delta_{v}\right.\right.$ $-2 \Delta) / 2]$, if $c+\delta_{v} \in\left[\delta_{V}-\delta_{v}+\Delta, \min \left(3\left(\delta_{V}-\delta_{v}\right), \delta_{V}-\delta_{v}+3 \Delta / 2\right)\right]$;
(c) $\left[p_{a}^{*}, p_{b}^{*},\left(B_{a}^{1} B_{b}^{1}, B_{a}^{2} B_{b}^{2}\right)\right]$, with $p_{a}^{*}+p_{b}^{*}=2\left(c+\delta_{v}\right)-\Delta$ and $p_{a}^{*}, p_{b}^{*} \in\left[\left(c+\delta_{V}\right) / 2,\left(3 c-\delta_{V}+4 \delta_{v}\right.\right.$ $-2 \Delta) / 2]$, if $c+\delta_{v} \in\left[\delta_{V}-\delta_{v}+3 \Delta / 2,3\left(\delta_{V}-\delta_{v}\right)\right]$;


Figure 3: Equilibrium when the gain on the second copy is low enough
(d) $\left[p_{a}^{*}, p_{b}^{*},\left(B_{a}^{1} B_{b}^{1}, B_{a}^{2} B_{b}^{2}\right)\right]$, with $p_{a}^{*}+p_{b}^{*}=2\left(c+\delta_{v}\right)-\Delta$ and $p_{a}^{*}, p_{b}^{*} \in\left[2\left(\delta_{V}-\delta_{v}\right), 2 c-2\left(\delta_{V}-2 \delta_{v}\right)-\Delta\right]$; and $\left[p_{a}^{*}, p_{b}^{*},\left(B_{a}^{1} B_{b}^{1}, C_{a}^{2} C_{b}^{2}\right)\right]$, with $p_{a}^{*}+p_{b}^{*}=2\left(c+\delta_{V}\right)-\Delta$ and $p_{a}^{*}, p_{b}^{*} \in\left[2\left(c+\delta_{v}-\Delta\right), 2\left(\delta_{V}-\delta_{v}\right)\right.$ $+\Delta]$, if $c+\delta_{v} \in\left[3\left(\delta_{V}-\delta_{v}\right), \delta_{V}-\delta_{v}+3 \Delta / 2\right]$;
(e) $\left[p_{a}^{*}, p_{b}^{*},\left(B_{a}^{1} B_{b}^{1}, B_{a}^{2} B_{b}^{2}\right)\right]$, with $p_{a}^{*}+p_{b}^{*}=2\left(c+\delta_{v}\right)-\Delta$ and $p_{a}^{*}, p_{b}^{*} \in\left[2\left(\delta_{V}-\delta_{v}\right), 2 c-2\left(\delta_{V}-2 \delta_{v}\right)-\Delta\right]$, if $c+\delta_{v} \in\left[\max \left(3\left(\delta_{V}-\delta_{v}\right), \delta_{V}-\delta_{v}+3 \Delta / 2\right), 2\left(\delta_{V}-\delta_{v}\right)+\Delta\right]$; and,
(f) $\left[p_{a}^{*}, p_{b}^{*},\left(B_{a}^{1} B_{b}^{1}, B_{a}^{2} B_{b}^{2}\right)\right]$, with $p_{a}^{*}+p_{b}^{*}=2\left(c+\delta_{v}\right)-\Delta$ and $p_{a}^{*}, p_{b}^{*} \in\left[c+\delta_{v}-\Delta, c+\delta_{v}\right]$, if $c+\delta_{v} \in$ $\left[2\left(\delta_{V}-\delta_{v}\right)+\Delta,+\infty\right)$.

Proof: see Appendix.
Notice that these results are qualitatively equivalent to that obtained in a monopoly. In order to clarify the results obtained in this section, we show Figure 5, where the x-axis represents consumer 2's willingness to pay $\left(c+\delta_{v}\right)$ and the y-axis represents the gain on the second copy $(\Delta)$.

### 2.4 Static Comparative Analysis

To compare the results above and obtain relevant conclusions, let us assume that firms always choose an undominated SPE (USPE) in our game. Since we consider that firms face the same demand and do not bear any cost, we focus on symmetrical USPE (hereafter SUSPE). Thus, SUSPE are those equilibria that provide a maximal joint profit. As can be seen in the following proposition, the equilibrium where consumer 1 buys both goods and consumer 2 copies them is SUSPE if and only if $c+\delta_{v} \leq \delta_{V}-\delta_{v}+\Delta$; otherwise, the equilibrium where both consumers buy both goods is SUSPE.


Figure 4: Equilibrium when the gain on the second copy is high


Figure 5: The model with learning by copying

Proposition 4 The SUSPE, independently of the size of the gain on the second copy, is:
(a) $\left[p_{a}^{*}, p_{b}^{*},\left(B_{a}^{1}, B_{b}^{1}, C_{a}^{2}, C_{b}^{2}\right)\right]$, with $p_{a}^{*}=p_{b}^{*}=c+\delta_{V}-\Delta / 2$, if $c+\delta_{v} \in\left[\Delta, \delta_{V}-\delta_{v}+\Delta\right]$; and
(b) $\left[p_{a}^{*}, p_{b}^{*},\left(B_{a}^{1}, B_{b}^{1}, B_{a}^{2}, B_{b}^{2}\right)\right]$, with $p_{a}^{*}=p_{b}^{*}=c+\delta_{v}-\Delta / 2$, if $c+\delta_{v} \in\left[\delta_{V}-\delta_{v}+\Delta,+\infty\right)$.

By comparing SUSPE under a monopoly and a duopoly, we find that the "Cournot effect" only exists for some intermediate values of $c+\delta_{v}$. In particular, when $c+\delta_{v} \in\left[\delta_{V}-\delta_{v}+\Delta / 2, \delta_{V}-\delta_{v}+\Delta\right]$, independently of the size of the gain on the second copy. This effect means that the monopolist sets a lower average price than the duopolist because it knows that the two goods are complementary, i.e. decreasing the price for one good increases demand for the other good. This result differs from that obtained by Belleflamme and Picard (2007), who find that the Cournot effect always exists. Thus, unlike Belleflamme and Picard (2007), we find that the existence of the Cournot effect depends on the values of $\Delta$ and $c+\delta_{v}$.

## 3 The model with learning by copying: opposed preferences

Up to now we have considered that consumers value information goods (original or copy) differently, although each consumer values each good in the same way, as in Belleflamme and Picard (2007). In this section, we consider that consumers have so-called "opposed preferences". This means that they value each good differently and do not agree on the most and least highly valued information good. We consider these preferences because there are more and more information goods that can be copied by consumers, these goods are completely different and consumers value them differently.

With no loss of generality, we consider that consumer 1 values information good $a$ more highly than $b$, and consumer 2 values $b$ more, independently of whether it is an original or a copy. For the sake of simplicity and in keeping with the previous sections, the valuation of the goods that consumers value most is $V_{o}$ for the original and $V_{c}$ for the copy, while the valuation of the goods that they value least is $v_{o}$ for the original and $v_{c}$ for the copy.

The timing of the game is the same as for the previous model. First, firms price the information goods. Next, consumers decide to buy or copy the information goods or do nothing. We look for the SPE of the game.

### 3.1 Consumer behavior

Consumer 1 has the same strategies as in the previous section. Thus, consumer 1's utility function is:

$$
U_{1}\left(p_{a}, p_{b}\right)=\left\{\begin{array}{cl}
V_{o}+v_{o}-p_{a}-p_{b} & \text { if consumer } 1 \text { buys } a \text { and } b  \tag{2}\\
V_{o}+v_{c}-p_{a}-c & \text { if consumer } 1 \text { buys } a \text { and copies } b \\
V_{c}+v_{o}-c-p_{b} & \text { if consumer } 1 \text { copies } a \text { and buys } b \\
V_{c}+v_{c}+\Delta-2 c & \text { if consumer } 1 \text { copies } a \text { and } b \\
0 & \text { otherwise }
\end{array}\right.
$$



Figure 6: The demand for goods

In the same way, we get the utility of consumer 2 . Now $\delta_{V}\left(\delta_{v}\right)$ represents the gap in the valuations of consumers between the original information goods that they value most (least) and copies of them. Since consumers can copy instead of buying, their willingness to pay for the good that they value most (least) is $c+\delta_{V}\left(c+\delta_{v}\right)$. As before, we assume that $\delta_{V}>\delta_{v}$. This implies that the maximal price that consumers are willing to pay for the original information good that they value most is higher than that for the good which they value least.

As in the previous section, we obtain that when $\Delta>2 c+\delta_{V}+\delta_{v}$, consumers decide to copy all goods, independently of their prices. Thus, to ensure that any consumer's strategy could be optimal under some prices, we assume that $c<v_{c}$ and $\Delta \leq c+\delta_{v}$.

By comparing the levels of utility obtained by each consumer, we obtain their patterns of demand. Taking this into account we find the demand for goods, which is illustrated in Figure 6. Notice that for low prices consumers decide to buy, while for high prices they copy the information goods.

As shown in Figure 6, the demand for goods differs according to the gain on the second copy. In particular, when the gain on the second copy is low enough, there is an additional outcome, which represents the outcome where each consumer buys the good that he/she most values and copies the other. This is because it is easier to deter consumers from copying when the gain on the second copy is low enough, so firms set higher prices and only sell to the consumer that values their goods most highly. Notice that Figure 6(b) is only possible when $\delta_{V}-\delta_{v}<c+\delta_{v}$.

As in Section 2, the process of learning by copying makes the goods complementary for some prices, although they have independent content as in Belleflamme and Picard (2007).

### 3.2 Equilibrium

We begin by solving the model when both goods are sold by a monopolist on the market. We first analyze the case where the gain on the second copy is low enough, i.e. $\Delta \leq \delta_{V}-\delta_{v}$. Thus, the demand for goods
is represented in Figure 6(a). The candidate strategies for equilibrium are in the following table. ${ }^{4}$

| STRATEGY | DEMAND | PROFIT |
| :--- | :--- | :--- |
| (i) $p_{a}=c+\delta_{v} ; p_{b}=c+\delta_{V}-\Delta$ | $B_{a}^{1} C_{b}^{1} ; B_{a}^{2} B_{b}^{2}$ | $\pi_{m}^{(\mathrm{i})}=3 c+\delta_{V}+2 \delta_{v}-\Delta$ |
| (ii) $p_{a}=p_{b}=c+\delta_{V}-\Delta$ | $B_{a}^{1} C_{b}^{1} ; C_{a}^{2} B_{b}^{2}$ | $\pi_{m}^{(\mathrm{ii})}=2\left(c+\delta_{V}-\Delta\right)$ |
| (iii) $p_{a}=p_{b}=c+\delta_{v}$ | $B_{a}^{1} B_{b}^{1} ; B_{a}^{2} B_{b}^{2}$ | $\pi_{m}^{(\mathrm{iii})}=4\left(c+\delta_{v}\right)$ |

Table 2
As can be seen in Proposition 5, the optimal decision of the monopolist depends on the cost of copying, the gain on the second copy and the relative valuation of consumers of both goods.

Proposition 5 In any SPE, when $\Delta \leq \delta_{V}-\delta_{v}$, we have that:
(a) if $c+\delta_{v} \leq \delta_{V}-\delta_{v}-\Delta$, the optimal strategy is (ii); and
(b) if $c+\delta_{v} \geq \delta_{V}-\delta_{v}-\Delta$, the optimal strategy is (iii).

Proof: see Appendix.
We find that the monopolist sells to all consumers when they are willing to pay enough for the good that they least value. Otherwise, it only sells to the consumers that most value each good.

Finally, we analyze the case where the gain on the second copy is high enough, i.e. where $\Delta \in$ $\left[\delta_{V}-\delta_{v}, \min \left(c+\delta_{v}, 2\left(\delta_{V}-\delta_{v}\right)\right)\right]$. Thus, the demand for goods is represented in Figure 6(b). As can be easily shown, the optimal strategy in this case is (iv), where all consumers buy both goods. This result is shown in Proposition 6.
$\left.\begin{array}{||c|c|c||}\hline \hline \text { STRATEGY } & \text { DEMAND } & \text { PROFIT } \\ \hline \text { (iv) } p_{a}+p_{b}=2 c+\delta_{V}+\delta_{v}-\Delta \\ p_{a}, p_{b} \in\left[c+\delta_{V}-\Delta, c+\delta_{v}\right]\end{array} B_{a}^{1} B_{b}^{1} ; B_{a}^{2} B_{b}^{2}\right) \pi_{m}^{(\mathrm{iv})}=2\left(2 c+\delta_{V}+\delta_{v}-\Delta\right)$.

Table 3
Proposition 6 The equilibrium, when the gain on the second copy is high is: $\left[p_{a}^{*}, p_{b}^{*},\left(B_{a}^{1} B_{b}^{1}, B_{a}^{2} B_{b}^{2}\right)\right]$, with $p_{a}^{*}+p_{b}^{*}=2 c+\delta_{V}+\delta_{v}-\Delta$ and $p_{a}^{*}, p_{b}^{*} \in\left[c+\delta_{V}-\Delta, c+\delta_{v}\right]$.

We now analyze the case in which each good is sold by a single firm. As we can see in Proposition 7 , we obtain that the equilibrium is the same as that obtained in a monopoly.

Proposition 7 When consumers' preferences are opposed, the duopoly equilibrium coincides with that obtained under a monopoly.

Proof: see Appendix.
In contrast with previous literature on copying by end-users, we show that there is no Cournot effect when consumers have opposed preferences. Therefore, the existence of the Cournot effect also depends on the kind of preferences of consumers.

[^4]
## 4 Welfare analysis

In this section, we analyze social welfare and compare the results with those obtained by Belleflamme and Picard (2007). These authors show that from a static perspective the multiproduct monopoly provides greater welfare than the duopoly, but from a dynamic perspective the duopoly provides greater incentives to create a new good than the monopoly. In this section, as in Subsection 2.4, we consider SUSPE in order to provide clearer, more relevant conclusions.

Ex post efficiency We now conduct a welfare analysis when consumers' preferences are not opposed. In both a monopoly and a duopoly there are two possible outcomes: one where consumer 1 buys both goods and consumer 2 copies them (we represent this outcome by subscript $c$ ); and another where both consumers buy both goods (we represent this outcome by subscript $b$ ). We define social welfare as the sum of firms' profits and the consumer surplus. Thus, the welfare associated with each outcome is:

$$
\begin{align*}
& W_{c}=2 V_{o}+2\left(v_{c}-c\right)+\Delta  \tag{3}\\
& W_{b}=2\left(V_{o}+v_{o}\right)
\end{align*}
$$

Taking into account the results obtained in Proposition 1 and in Proposition 4 we deduce that social welfare in a monopoly and a duopoly, independently of the gain on the second copy, is

$$
\begin{gather*}
W^{m}= \begin{cases}W_{c} & \text { if } c+\delta_{v} \leq \delta_{V}-\delta_{v}+\Delta / 2 ; \text { and } \\
W_{b} & \text { if } c+\delta_{v} \geq \delta_{V}-\delta_{v}+\Delta / 2,\end{cases}  \tag{4}\\
W^{d}= \begin{cases}W_{c} & \text { if } \Delta \leq c+\delta_{v} \leq \delta_{V}-\delta_{v}+\Delta ; \text { and } \\
W_{b} & \text { if } c+\delta_{v} \geq \delta_{V}-\delta_{v}+\Delta .\end{cases} \tag{5}
\end{gather*}
$$

We have that $W_{b}>W_{c}$ if and only if $\Delta / 2<c+\delta_{v}$, which is always satisfied because we assume that $\Delta \leq c+\delta_{v}$. We find that in outcome $c$ increasing the cost of copying decreases the social welfare. Nevertheless this is a local conclusion. Taking into account the changes in firms' decisions, we obtain that a non local increase in the copying cost increases social welfare.

We compare social welfare in a monopoly and in a duopoly, and obtain that the former is at least as great as the latter, as in Belleflamme and Picard (2007). This result is shown in the following proposition.

Proposition 8 In any SUSPE, we have that $W^{m} \geq W^{d}$.

Next, we analyze social welfare when preferences are opposed. Notice that the social welfare associated with the outcome where both consumers buy both goods is the same as before, but the social welfare when each consumer buys the good that he/she values most and copies the other is $W_{b c}=2 V_{o}+2\left(v_{c}-c\right)$, which is lower than $W_{b}$ (i.e. $W_{b c}<W_{b}$ ). Therefore, as before, in outcome $c$ increasing the cost of copying decreases the social welfare. Nevertheless this is a local conclusion. Taking into account the changes in firms' decisions, we obtain that a non local increase in the copying cost increases social welfare.

As can be seen in Proposition 9, we find that social welfare in a monopoly is the same as in a duopoly. This is because the equilibrium in a monopoly is equal to the equilibrium in a duopoly (see Proposition 7).

Proposition 9 In any SUSPE, we have $W^{m}=W^{d}$.

Therefore, from a static perspective, the multiproduct monopoly provides social welfare at least as great as under as duopoly. To a certain extent, this result contradicts a statement made by Belleflamme and Picard (2007), who say that "social surplus is undoubtedly higher under a multiproduct monopoly than under a duopoly."

Ex ante efficiency Let $\pi_{d}$ be the profit of an entrant (duopolist), and let $\pi_{m}-\pi_{m}^{1}$ be the incumbent's incentive to create a new product, where $\pi_{m}$ is the profit of the incumbent and $\pi_{m}^{1}$ is the profit of a monopolist that sells only one product.

In the following proposition, we show that an entrant has at least the same incentives to create a new good as an incumbent, independently of the kind of consumer preferences that we consider. Thus, the results obtained in Belleflamme and Picard (2007) are robust when we consider a more general cost function and opposed preferences.

Proposition 10 An entrant has at least the same incentives to create a new product as an incumbent. In particular, $\pi_{d}=\pi_{m}-\pi_{m}^{1}$ if consumer preferences are opposed and $\Delta \leq \delta_{V}-\delta_{v} \leq c+\delta_{v}$; otherwise, we have $\pi_{d}>\pi_{m}-\pi_{m}^{1}$.

Proof: see Appendix.

## 5 Conclusions

We develop a model that lets us analyze the behavior of a multiproduct monopolist, a duopolist and consumers, where consumers are able to learn by copying. An important conclusion is that when the effect of learning by copying is strong and the cost of copying is low enough, consumers decide to copy all goods, independently of their prices. This suggests that the DRM systems implemented by the digital industry have an adverse effect, because they cause the gain on the second copy to become higher, with the consequent danger of deterring consumers from buying.

Unlike Belleflamme and Picard (2007), we find a complete characterization of equilibrium. Overall, we obtain two more kinds of equilibrium: one where each firm sells to a consumer, and another where both firms sell to all consumers. The existence of both kinds of equilibrium depends on how much the consumer who values the goods least is willing to pay for each one. Thus, if he/she is willing to pay a lot, firms will sell the good to him/her, otherwise they will not.

In this paper, unlike Belleflamme and Picard (2007), we show that the existence of the Cournot effect depends on the size of the gain on the second copy, the quantity that the consumer who values the goods least is willing to pay for each one and the kind of consumers' preferences.

By analyzing social welfare we obtain that from a static perspective the multiproduct monopoly provides at least as much social welfare as a duopoly, and from a dynamic perspective a duopolist has at least the same incentives to create a new good as a multiproduct monopolist, independently of the kind of consumers' preferences that we consider. Thus, the results obtained in Belleflamme and Picard (2007) are robust when we consider a more general cost function and opposed preferences.

Therefore, for future research, we suggest developing models that consider the existence of many goods that can be copied, different functions of consumers' preferences and consumers able to learn by copying.

## 6 Appendix

Proof of Proposition 1. By comparing the profits of the different strategies, we obtain that:
(a) $\pi_{m}^{(\mathrm{i})}>\pi_{m}^{(\mathrm{ii)}} ;$ iff $c+\delta_{v}>\delta_{V}-\delta_{v}$,
(b) $\pi_{m}^{(\mathrm{i})}>\pi_{m}^{(\mathrm{iii})}$, iff $c+\delta_{v}>\delta_{V}-\delta_{v}+\Delta / 2$; and
(c) $\pi_{m}^{(\mathrm{iii})}>\pi_{m}^{(\mathrm{iii})}$, iff $c+\delta_{v}>\delta_{V}-\delta_{v}+\Delta$.

Proof of Proposition 2. Taking into account the demand for goods in Figure 2(a), we look for the reaction function of each firm. The reaction function of firm $i=A, B$, when $c+\delta_{v} \leq \delta_{V}-\delta_{v}$, is:

$$
p_{i}\left(p_{j}\right)=\left\{\begin{array}{cl}
c+\delta_{V} & \text { if } p_{j} \leq c+\delta_{V}-\Delta  \tag{6}\\
2\left(c+\delta_{V}\right)-\Delta-p_{j} & \text { if } c+\delta_{V}-\Delta \leq p_{j} \leq c+\delta_{V} \\
c+\delta_{V}-\Delta & \text { if } c+\delta_{V} \leq p_{j}
\end{array}\right.
$$

when $\delta_{V}-\delta_{v} \leq c+\delta_{v} \leq \delta_{V}-\delta_{v}+\Delta$, is:

$$
p_{i}\left(p_{j}\right)=\left\{\begin{array}{cl}
c+\delta_{v} & \text { if } p_{j} \leq c+\delta_{v}-\Delta  \tag{7}\\
2\left(c+\delta_{v}\right)-\Delta-p_{j} & \text { if } c+\delta_{v}-\Delta \leq p_{j} \leq\left(3 c-\delta_{V}+4 \delta_{v}-2 \Delta\right) / 2 \\
c+\delta_{V} & \text { if }\left(3 c-\delta_{V}+4 \delta_{v}-2 \Delta\right) / 2 \leq p_{j} \leq c+\delta_{V}-\Delta \\
2\left(c+\delta_{V}\right)-\Delta-p_{j} & \text { if } c+\delta_{V}-\Delta \leq p_{j} \leq c+\delta_{V} \\
c+\delta_{V}-\Delta & \text { if } c+\delta_{V} \leq p_{j}
\end{array}\right.
$$

when $\delta_{V}-\delta_{v}+\Delta \leq c+\delta_{v} \leq \delta_{V}-\delta_{v}+2 \Delta$, is:

$$
p_{i}\left(p_{j}\right)=\left\{\begin{array}{cl}
c+\delta_{v} & \text { if } p_{j} \leq c+\delta_{v}-\Delta  \tag{8}\\
2\left(c+\delta_{v}\right)-\Delta-p_{j} & \text { if } c+\delta_{v}-\Delta \leq p_{j} \leq\left(3 c-\delta_{V}+4 \delta_{v}-2 \Delta\right) / 2 \\
c+\delta_{V} & \text { if }\left(3 c-\delta_{V}+4 \delta_{v}-2 \Delta\right) / 2 \leq p_{j} \leq c+\delta_{V}-\Delta \\
2\left(c+\delta_{V}\right)-\Delta-p_{j} & \text { if } c+\delta_{V}-\Delta \leq p_{j} \leq 2\left(\delta_{V}-\delta_{v}\right)+\Delta \\
c+\delta_{v}-\Delta & \text { if } 2\left(\delta_{V}-\delta_{v}\right)+\Delta \leq p_{j}
\end{array}\right.
$$

and when $\delta_{V}-\delta_{v}+2 \Delta \leq c+\delta_{v}$, is:

$$
p_{i}\left(p_{j}\right)=\left\{\begin{array}{cl}
c+\delta_{v} & \text { if } p_{j} \leq c+\delta_{v}-\Delta  \tag{9}\\
2\left(c+\delta_{v}\right)-\Delta-p_{j} & \text { if } c+\delta_{v}-\Delta \leq p_{j} \leq c+\delta_{v} \\
c+\delta_{v}-\Delta & \text { if } c+\delta_{v} \leq p_{j}
\end{array}\right.
$$

where $p_{i}$ and $p_{j}$ are the prices of the goods $i, j=a, b$. From the intersection of the two reaction functions we obtain the equilibrium, which is shown in Proposition 2 and in Figure 3.

Proof of Proposition 3. Taking into account the following relationship: ${ }^{5}$

$$
\begin{equation*}
\delta_{V}-\delta_{v}<\delta_{V}-\delta_{v}+\Delta<3\left(\delta_{V}-\delta_{v}\right)<2\left(\delta_{V}-\delta_{v}\right)+\Delta \tag{10}
\end{equation*}
$$

we calculate the reaction function of firm $i=A, B$ according to consumer 2's willingness to pay, $c+\delta_{v}$. Thus, the reaction function when $\delta_{V}-\delta_{v} \leq c+\delta_{v} \leq \delta_{V}-\delta_{v}+\Delta$, is:

$$
p_{i}\left(p_{j}\right)=\left\{\begin{array}{cl}
c+\delta_{v} & \text { if } p_{j} \leq c+\delta_{v}-\Delta  \tag{11}\\
2\left(c+\delta_{v}\right)-\Delta-p_{j} & \text { if } c+\delta_{v}-\Delta \leq p_{j} \leq\left(3 c-\delta_{V}+4 \delta_{v}-2 \Delta\right) / 2 \\
c+\delta_{V} & \text { if }\left(3 c-\delta_{V}+4 \delta_{v}-2 \Delta\right) / 2 \leq p_{j} \leq c+\delta_{V}-\Delta \\
2\left(c+\delta_{V}\right)-\Delta-p_{j} & \text { if } c+\delta_{V}-\Delta \leq p_{j} \leq c+\delta_{V} \\
c+\delta_{V}-\Delta & \text { if } c+\delta_{V} \leq p_{j}
\end{array}\right.
$$

when $\delta_{V}-\delta_{v}+\Delta \leq c+\delta_{v} \leq 3\left(\delta_{V}-\delta_{v}\right)$, is:

$$
p_{i}\left(p_{j}\right)=\left\{\begin{array}{cl}
c+\delta_{v} & \text { if } p_{j} \leq c+\delta_{v}-\Delta  \tag{12}\\
2\left(c+\delta_{v}\right)-\Delta-p_{j} & \text { if } c+\delta_{v}-\Delta \leq p_{j} \leq\left(3 c-\delta_{V}+4 \delta_{v}-2 \Delta\right) / 2 \\
c+\delta_{V} & \text { if }\left(3 c-\delta_{V}+4 \delta_{v}-2 \Delta\right) / 2 \leq p_{j} \leq c+\delta_{V}-\Delta \\
2\left(c+\delta_{V}\right)-\Delta-p_{j} & \text { if } c+\delta_{V}-\Delta \leq p_{j} \leq 2\left(\delta_{V}-\delta_{v}\right)+\Delta \\
c+\delta_{v}-\Delta & \text { if } 2\left(\delta_{V}-\delta_{v}\right)+\Delta \leq p_{j}
\end{array}\right.
$$

when $3\left(\delta_{V}-\delta_{v}\right) \leq c+\delta_{v} \leq 2\left(\delta_{V}-\delta_{v}\right)+\Delta$, is:

$$
p_{i}\left(p_{j}\right)=\left\{\begin{array}{cl}
c+\delta_{v} & \text { if } p_{j} \leq c+\delta_{v}-\Delta  \tag{13}\\
2\left(c+\delta_{v}\right)-\Delta-p_{j} & \text { if } c+\delta_{v}-\Delta \leq p_{j} \leq 2 c-2\left(\delta_{V}-2 \delta_{v}\right)-\Delta \\
2\left(c+\delta_{V}\right)-\Delta-p_{j} & \text { if } 2 c-2\left(\delta_{V}-2 \delta_{v}\right)-\Delta \leq p_{j} \leq 2\left(\delta_{V}-\delta_{v}\right)+\Delta \\
c+\delta_{v}-\Delta & \text { if } 2\left(\delta_{V}-\delta_{v}\right)+\Delta \leq p_{j}
\end{array}\right.
$$

and when $2\left(\delta_{V}-\delta_{v}\right)+\Delta \leq c+\delta_{v}$, is:

$$
p_{i}\left(p_{j}\right)=\left\{\begin{array}{cl}
c+\delta_{v} & \text { if } p_{j} \leq c+\delta_{v}-\Delta  \tag{14}\\
2\left(c+\delta_{v}\right)-\Delta-p_{j} & \text { if } c+\delta_{v}-\Delta \leq p_{j} \leq c+\delta_{v} \\
c+\delta_{v}-\Delta & \text { if } c+\delta_{v} \leq p_{j}
\end{array}\right.
$$

From the intersection of the two reaction functions we obtain the equilibrium, which is shown in Proposition 3 and in Figure 4.

Proof of Proposition 5. By comparing the profits of the different strategies we obtain that:
(a) $\pi_{m}^{(\mathrm{ii)}}>\pi_{m}^{(\mathrm{i})}$, if $c+\delta_{v}<\delta_{V}-\delta_{v}-\Delta$;
(b) $\pi_{m}^{(\mathrm{i})}>\pi_{m}^{(\mathrm{iii})}$, if $c+\delta_{v}<\delta_{V}-\delta_{v}-\Delta$; and
(c) $\pi_{m}^{(\text {ii) })}>\pi_{m}^{(\text {iii })}$, if $c+\delta_{v}<\delta_{V}-\delta_{v}-\Delta$.

[^5]Proof of Proposition 7. We begin by analyzing the case where the gain on the second copy is low enough, i.e. $\Delta \leq \delta_{V}-\delta_{v}$. Thus, the demand for goods is represented by Figure 6(a). Taking this into account, we look for the reaction function of each firm. The reaction function of firm $i=A, B$, when $c+\delta_{v} \leq \delta_{V}-\delta_{v}-\Delta$, is:

$$
p_{i}\left(p_{j}\right)=\left\{\begin{array}{cl}
c+\delta_{V} & \text { if } p_{j} \leq c+\delta_{v}-\Delta  \tag{15}\\
2 c+\delta_{V}+\delta_{v}-\Delta-p_{j} & \text { if } c+\delta_{v}-\Delta \leq p_{j} \leq c+\delta_{v} \\
c+\delta_{V}-\Delta & \text { if } c+\delta_{v} \leq p_{j}
\end{array}\right.
$$

when $\delta_{V}-\delta_{v}-\Delta \leq c+\delta_{v} \leq \delta_{V}-\delta_{v}$, is:

$$
p_{i}\left(p_{j}\right)=\left\{\begin{array}{cl}
c+\delta_{V} & \text { if } p_{j} \leq c+\delta_{v}-\Delta  \tag{16}\\
2 c+\delta_{V}+\delta_{v}-\Delta-p_{j} & \text { if } c+\delta_{v}-\Delta \leq p_{j} \leq \delta_{V}-\delta_{v}-\Delta \\
c+\delta_{v} & \text { if } \delta_{V}-\delta_{v}-\Delta \leq p_{j} \leq c+\delta_{V}-\Delta \\
2 c+\delta_{V}+\delta_{v}-\Delta-p_{j} & \text { if } c+\delta_{V}-\Delta \leq p_{j} \leq\left(3 c+\delta_{V}+2 \delta_{v}-\Delta\right) / 2 \\
c+\delta_{V}-\Delta & \text { if }\left(3 c+\delta_{V}+2 \delta_{v}-\Delta\right) / 2 \leq p_{j}
\end{array}\right.
$$

and when $c+\delta_{v} \geq \delta_{V}-\delta_{v}$, is:

$$
p_{i}\left(p_{j}\right)=\left\{\begin{array}{cl}
c+\delta_{v} & \text { if } p_{j} \leq c+\delta_{V}-\Delta  \tag{17}\\
2 c+\delta_{V}+\delta_{v}-\Delta-p_{j} & \text { if } c+\delta_{V}-\Delta \leq p_{j} \leq \min \{x, z\} \\
c+\delta_{V}-\Delta & \text { if } \min \{x, z\}=x \leq p_{j} \\
c+\delta_{v}-\Delta & \text { if } \min \{x, z\}=z \leq p_{j}
\end{array}\right.
$$

where $x=\left(3 c+\delta_{V}+2 \delta_{v}-\Delta\right) / 2$ and $z=c+\delta_{V}$. As can be seen in Figure 7 , from the intersection of the two reaction functions we obtain the equilibrium, which is same as that obtained in a monopoly when the gain on the second copy is low enough (see Proposition 5).

We now consider that the gain on the second copy is high enough, i.e. $\delta_{V}-\delta_{v} \leq \Delta \leq \min \left(c+\delta_{v}, 2\left(\delta_{V}\right.\right.$ $\left.-\delta_{v}\right)$ ), so the demand of the goods is represented by Figure $6(\mathrm{~b})$ and the reaction function of firm $i=A, B$ by the function (17). From the intersection of reaction functions we obtain the equilibrium, which is shown in Figure 8 and Proposition 6.

Proof of Proposition 10. To check that a duopolist has more incentives to create a new information good, we must check that $\pi_{d}>\pi_{m}-\pi_{m}^{1}$. The profit of a monopolist that sells only one product is:

$$
\pi_{m}^{1}=\left\{\begin{array}{cl}
c+\delta_{V} & \text { if } c+\delta_{v} \leq \delta_{V}-\delta_{v}  \tag{18}\\
2\left(c+\delta_{v}\right) & \text { if } c+\delta_{v} \geq \delta_{V}-\delta_{v}
\end{array}\right.
$$

We start by considering the preferences of Belleflamme and Picard (2007). We have that:

1. If $c+\delta_{v} \in\left[\Delta, \delta_{V}-\delta_{v}\right]$, we have $\pi_{d}=c+\delta_{V}-\Delta / 2, \pi_{m}=2\left(c+\delta_{V}\right)-\Delta$ and $\pi_{m}^{1}=c+\delta_{V}$. Thus, condition $\pi_{d}>\pi_{m}-\pi_{m}^{1}$ is satisfied.


Figure 7: Equilibrium when the gain on the second copy is low enough


Figure 8: Equilibrium when the gain on the second copy is high enough
2. If $c+\delta_{v} \in\left[\delta_{V}-\delta_{v}, \delta_{V}-\delta_{v}+\Delta / 2\right]$, we have $\pi_{d}=c+\delta_{V}-\Delta / 2, \pi_{m}=2\left(c+\delta_{V}\right)-\Delta$ and $\pi_{m}^{1}=2\left(c+\delta_{v}\right)$. Thus, condition $\pi_{d}>\pi_{m}-\pi_{m}^{1}$ is satisfied.
3. If $c+\delta_{v} \in\left[\delta_{V}-\delta_{v}+\Delta / 2, \delta_{V}-\delta_{v}+\Delta\right]$, we have $\pi_{d}=c+\delta_{V}-\Delta / 2, \pi_{m}=4\left(c+\delta_{v}\right)-2 \Delta$ and $\pi_{m}^{1}=2\left(c+\delta_{v}\right)$. Thus, condition $\pi_{d}>\pi_{m}-\pi_{m}^{1}$ is satisfied. And,
4. if $c+\delta_{v} \in\left[\delta_{V}-\delta_{v}+\Delta,+\infty\right)$, we have $\pi_{d}=2\left(c+\delta_{v}\right)-\Delta, \pi_{m}=4\left(c+\delta_{v}\right)-2 \Delta$ and $\pi_{m}^{1}=$ $2\left(c+\delta_{v}\right)$. Thus, condition $\pi_{d}>\pi_{m}-\pi_{m}^{1}$ is satisfied.

We now consider that consumers' preferences are opposed. When the gain on the second copy is low, we have:

1. If $c+\delta_{v} \in\left[\Delta, \delta_{V}-\delta_{v}-\Delta\right]$, we have $\pi_{d}=c+\delta_{V}-\Delta, \pi_{m}=2\left(c+\delta_{V}-\Delta\right)$ and $\pi_{m}^{1}=c+\delta_{V}$. Thus, condition $\pi_{d}>\pi_{m}-\pi_{m}^{1}$ is satisfied.
2. If $c+\delta_{v} \in\left[\delta_{V}-\delta_{v}-\Delta, \delta_{V}-\delta_{v}\right]$, we have $\pi_{d}=2\left(c+\delta_{v}\right), \pi_{m}=4\left(c+\delta_{v}\right)$ and $\pi_{m}^{1}=c+\delta_{V}$. Thus, condition $\pi_{d}>\pi_{m}-\pi_{m}^{1}$ is satisfied.
3. if $c+\delta_{v} \in\left[\delta_{V}-\delta_{v},+\infty\right)$, we have $\pi_{d}=2\left(c+\delta_{v}\right), \pi_{m}=4\left(c+\delta_{v}\right)$ and $\pi_{m}^{1}=2\left(c+\delta_{v}\right)$. Thus, we obtain $\pi_{d}=\pi_{m}-\pi_{m}^{1}$. So the monopolist has the same incentives to create a new good as a duopolist.

Finally, when the gain on the second copy is high, we have: $\pi_{d}=2 c+\delta_{V}+\delta_{v}-\Delta, \pi_{m}=$ $2\left(2 c+\delta_{V}+\delta_{v}-\Delta\right)$ and $\pi_{m}^{1}=2\left(c+\delta_{v}\right)$. Thus, condition $\pi_{d}>\pi_{m}-\pi_{m}^{1}$ is satisfied.

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[^1]:    ${ }^{1}$ See Peitz and Waelbroeck (2006b) for a survey of piracy in which copies are made exclusively by end consumers. However, there is another literature that analyzes the case of a single firm that illegally makes copies and sells them on the market (Poddar (2003) and Martínez-Sánchez (2007a)).

[^2]:    ${ }^{2}$ This is the case according to the penal codes of most countries (for instance, see articles 270 to 272 of the Spanish Penal Code).

[^3]:    ${ }^{3}$ Notice that the candidate strategy $\left(p_{a}=c+\delta_{V} ; p_{b}=c+\delta_{v}-\Delta\right)$ is not included in Table 1 because it is equivalent to strategy (ii).

[^4]:    ${ }^{4}$ Notice that strategy $\left(p_{a}=c+\delta_{V}-\Delta, p_{b}=c+\delta_{v}\right)$ is not included in Table 2 because it is equivalent to strategy (i).

[^5]:    ${ }^{5}$ For the sake of simplicity, we assume $\Delta \leq \min \left(c+\delta_{v}, 2\left(\delta_{V}-\delta_{v}\right)\right)$ to assure that relationship (10) holds.

