Instituto Nacional de Investigación y Tecnología Agraria y Alimentaria (INIA)
Available online at www.inia.es/sjar
Spanish Journal of Agricultural Research 2010 8(3), 538-546
ISSN: 1695-971-X
eISSN: 2171-9292

# Valuation method for land pricing based on two cumulative distribution functions

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## Abstract

This paper introduces the well-known valuation method based on two cumulative distribution functions (VMTCDF) that presents advantages with respect to other comparative techniques such as classical synthetic, which need a hypothesis of proportionality that is extremely simplifying and quite implausible, and econometric methods, which need databases contrary to VMTCDF that requires very little information. The VMTCDF use a single explicative variable, which summarizes in an index the different external signs that influence the market value of the asset to be assessed. The main aim of this paper is to extend the VMTCDF to find, under uncertainty, the market value of an asset from a two-dimensional vector of the characteristics of this asset. For this reason a new two-dimensional distribution is presented, Pyramidal distribution, which serves as a probabilistic model in the extended VMTCDF, and some of its statistical properties are studied. Finally, a practical application on land pricing illustrates the use of the extended VMTCDF as a tool for asset valuation. The main conclusion to be drawn from this paper is that it is the first step to extend the VMTCDF to the multidimensional case.

**Additional key words**: appraisal, PERT method, two-dimensional distribution.

## Resumen

**El método de valoración de las dos funciones de distribución para fijar el precio de la tierra**

Este trabajo presenta el conocido método de valoración de las dos funciones de distribución (VMTCDF) que presenta ventajas respecto a otras técnicas comparativas, como el método sintético clásico, que necesita una hipótesis de proporcionalidad, que requiere una gran simplificación y es extremadamente improbable, y los métodos econométricos, que necesitan el uso de bases de datos, al contrario que el VMTCDF que requiere muy poca información. El VMTCDF emplea una sola variable explicativa, la cual resume en un único índice los diferentes signos externos que influyen en el valor de mercado del bien a valorar. El objetivo principal de este trabajo es extender el VMTCDF para encontrar, en ambiente de incertidumbre, el valor de mercado de un bien a partir de un vector bidimensional de las características de ese bien. Por este motivo se presenta una nueva distribución bidimensional, la distribución piramidal, que sirve de modelo probabilístico en la extensión del VMTCDF y se estudian algunas de sus propiedades estadísticas. Por último, se ilustra con una aplicación práctica sobre los precios de la tierra, el uso de VMTCDF extendido como instrumento para la valoración de activos. La principal conclusión que puede extraerse de este trabajo es que constituye el primer paso para extender el VMTCDF al caso multidimensional.

**Palabras clave adicionales**: distribución bidimensional, método PERT, valoración.

# Introduction

Asset pricing, under uncertainty, has been the subject of many analyses of econometric modelling and hedonic price indexes [see, *e.g.,* Banerjee *et al.* (2004), Deltas and Zacharias (2004), Caballer and Guadalajara (2005) and Benkard and Bajari (2005) and the references therein] as improvements on the classical synthetic method, but the weakness of these techniques arises in the absence of data. The asset valuation method based on two cumulative distribution functions (VMTCDF) was introduced by Ballestero (1971, 1973) as a new valuation method, based on two beta distributions, doubtless inspired by the PERT Method, which is currently very fashionable and which uses the beta distribution as a probabilistic model. These approaches constitute the trunk of a broad-ranging tree of knowledge,

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Received: 09-10-09; Accepted: 05-07-10.

with many solid branches made up of different studies that have made use of diverse probabilistic distributions for the two-beta method. Among many others, let us highlight the work of Romero (1977) with rectangular and triangular distributions, Ballestero and Caballer (1982) with a three-parameter beta distribution, tabulated by Caballer (1975), Alonso and Lozano (1985) with the normal distribution, Lozano (1996), García *et al.* (1999) and Herrerías *et al.* (2001) with the trapezoidal distribution. These all provide good examples of what has been termed by Caballer (2009) the Spanish school of appraisal.

The two-beta method, expanded with other probabilistic models, and therefore known as the VMTCDF, constitutes an original and very fruitful new path for comparative valuation methods. It straddles the classical synthetic models of the Italian school (Caballer, 2009), based on criteria of proportionality between the endogenous, or explained variable and the explicative variable, and the econometric methods of the Anglo-Saxon school (Caballer, 2009), which are inspired by linear regression analysis (Goldberg and Mark, 1988; Kincheloe, 1993) and by non-linear analysis (Brotman, 1990) of the explained variable over explicative variables.

In recent years, some authors have paid more attention to the study and generalization of probability models required in PERT methodology and valuation theory; see, *e.g.,* Johnson and Kotz (1999), Herrerías *et al.* (1999, 2003), van Dorp and Kotz (2002a,b, 2003) and in the analysis and development of the VMTCDF, see, *e.g.* Caballer (2008, 2009).

In this setting, the VMTCDF allows to appraise an asset under uncertainty, when the appraiser only possesses the minimum, maximum and most likely values, which may be supplied by expert judgement. This approach is a simplified version of the classical synthetic method, when both market value and asset characteristic follow the same distributions, and it has been used for various applications, such as the valuation of land, irrigation installations, forestry or businesses.

It is logical to consider the possibility that two variables could be affected by uncertainty (for example, production and location). In this case, and following the PERT methodology, the expert can be asked about the values *ai*, *mi*, *bi* $i = 1{,}2$, for each of the variables. The question is: what kind of distribution can be fitted to these values? The aim of this paper is to extend the VMTCDF to the case in which more than one asset characteristic is considered.

# Methodology

## Two-dimensional extension of the VMTCDF

In economic modelling, certain logical market rules are usually assumed. In particular, when we wish to obtain the market value of an asset from its characteristics, the following basic valuation principle is assumed: the asset with the highest characteristic value has the highest market value, which may be stated as follows:

Let $j$ and $k$ be two assets, with $i_j$ and $i_k$ being their values of the asset characteristics and $v_j$ and $v_k$ their market values, respectively. Thus, if $i_j < i_k$ then $v_j < v_k$.

Under this assumption, the VMTCDF is based on the equality between the cdf, $F$, of the market value, $V$, of the asset and the cdf, $G$, of the asset characteristic, $I$. Thus, the market value of an asset with characteristic $I = i_d$ by the VMTCDF is

$$F(v_d) = G(i_d) \text{ then } v_d = \phi(i_d) \qquad [1]$$

where $\phi = F^{-1} \cdot G$

This section provides an extension of the VMTCDF when the asset characteristic is a two-dimensional vector, since it is often necessary to determine the value of an asset through a particular set of characteristics which affect this asset, *i.e.,* using a two-dimensional vector, whose components are each of the one-dimensional characteristics of the asset. For this purpose, we assume the same basic valuation principle, *i.e.* that the asset with the highest characteristic vector has the highest market value, where the ordering between two vectors is determined by the orderings between the corresponding components of both vectors. In this context, the basic valuation principle can be established as follows:

Let $j$ and $k$ be two assets, with $(i_{1j}, i_{2j})$ and $(i_{1k}, i_{2k})$ being their characteristic vector values and $v_j$ and $v_k$ their market values, respectively. Thus, if

$$\sqrt{i_{1j}^{\ 2} + i_{2j}^{\ 2}} < \sqrt{i_{1k}^{\ 2} + i_{2k}^{\ 2}}$$

then: $v_j < v_k$.

Analogously to the one-dimensional case, the VMTCDF is based on the equality between the two cumulative distribution functions, $F$ of the market value and $G$ of the two characteristics of the asset, and so the appraisal of an asset with a vector of characteristic $I = (i_1, i_2)$ by the VMTCDF is:

$$F(v_d) = G(i_{1d}, i_{2d}) \text{ then } v_d = \phi(i_{1d}, i_{2d}) \qquad [2]$$

where $\phi = F^{-1} \cdot G$.

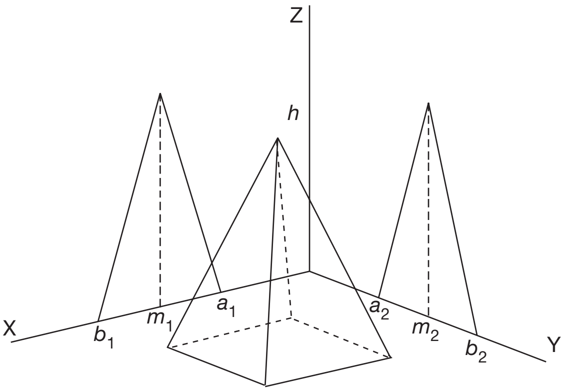


**Figure 1.** Graphical representation of the pyramidal distribution.

## The pyramidal distribution

A pyramidal distribution arises, in a natural way, as an extension to the two-dimensional domain of a univariate triangular distribution (specified by means of the minimum, $a$, maximum, $b$, and most likely, $m$, values of the variable's range). As shown in Figure 1, the graphical representation of the probability surface $z(x,y)$ is a pyramid.

The faces of the pyramid can easily be determined, as these planes are defined by three points, two on the base and one corresponding to its vertex, with the coordinates ($m_1$, $m_2$, $h$). The altitude of this point, or the height of the pyramid, functions as a normalising constant for the bivariate distribution.

The pyramid is projected onto the plane $Z=0$ to obtain the ranges of the $X$ and $Y$ variables for each of the pyramid faces, together with the equations of the pyramid edges on the same plane. The different triangles that make up the ranges of $(x,y)$ are denoted by $Ti$ ($i=1,2,3,4$), see Figure 2a.

The probability density function (pdf) of the pyramidal distribution is as follows:

$$z(x,y)=\begin{cases} \dfrac{y-a_2}{m_2-a_2}h & \text{if } (x,y)\in \mathrm{T}_1 \\ \dfrac{b_1-x}{b_1-m_1}h & \text{if } (x,y)\in \mathrm{T}_2 \\ \dfrac{b_2-y}{b_2-m_2}h & \text{if } (x,y)\in \mathrm{T}_3 \\ \dfrac{x-a_1}{m_1-a_1}h & \text{if } (x,y)\in \mathrm{T}_4 \\ 0 & \text{otherwise} \end{cases} \qquad [3]$$

where the normalized constant is $h=\dfrac{3}{(b_1-a_1)(b_2-a_2)}$

It can easily be confirmed that:

a) Thus defined, [3] is a true density function, such that:

i) $z(x,y)\geq 0 \quad \forall (x,y)\in T_i\ (i=1,2,3,4)$ if, and only if, $\begin{cases} a_1<x<b_1 \\ a_2<y<b_2 \end{cases}$

ii) $\int_{a_1}^{b_1}\int_{a_2}^{b_2} z(x,y)dxdy=1$

b) The variables $(x,y)$ in [3] are not independent because the regions $Ti$ ($i=1,2,3,4$) are triangles, and then the range of one variable depends on that of another, see *e.g.* Herrerías *et al.* (1997).

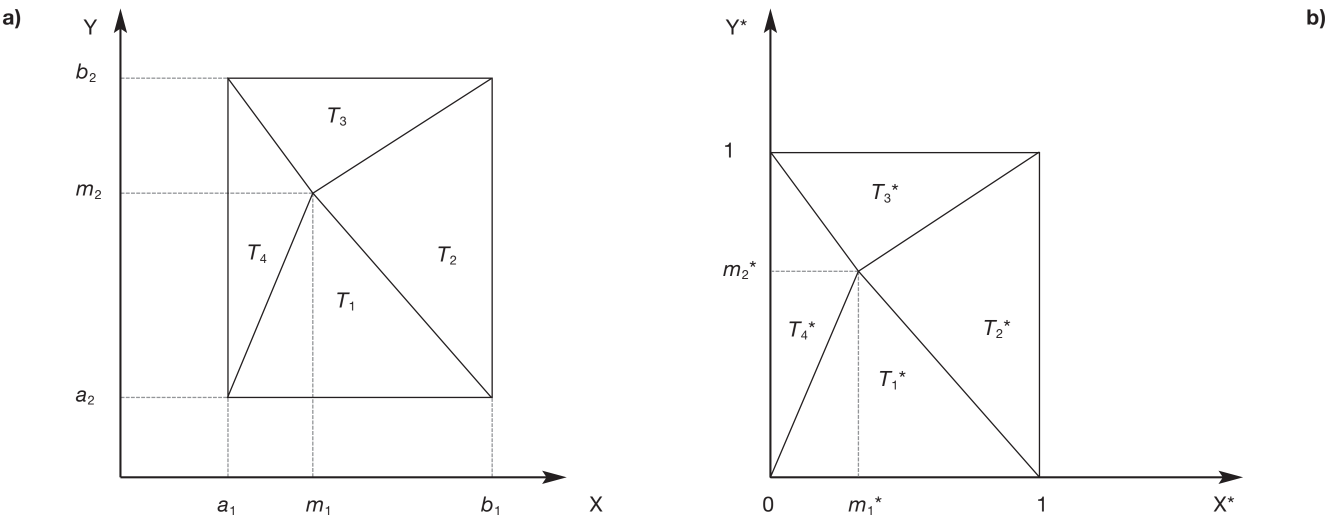


**Figure 2.** Ranges of the variables (X, Y) and (X*, Y*).

## Stochastic properties

From [3], the following marginal probability density functions are deduced:

$$f_1(x)=\begin{cases} h\dfrac{b_2-a_2}{m_1-a_1}(x-a_1)-\dfrac{h}{2}\dfrac{b_2-a_2}{(m_1-a_1)^2}(x-a_1)^2 & \text{if } a_1<x<m_1 \\ \\ h\dfrac{b_2-a_2}{b_1-m_1}(b_1-x)-\dfrac{h}{2}\dfrac{b_2-a_2}{(b_1-m_1)^2}(b_1-x)^2 & \text{if } m_1<x<b_1 \end{cases} \quad [4]$$

$$f_2(y)=\begin{cases} h\dfrac{b_1-a_1}{m_2-a_2}(y-a_2)-\dfrac{h}{2}\dfrac{b_1-a_1}{(m_2-a_2)^2}(y-a_2)^2 & \text{if } a_2<y<m_2 \\ \\ h\dfrac{b_1-a_1}{b_2-m_2}(b_2-y)-\dfrac{h}{2}\dfrac{b_1-a_1}{(b_2-m_2)^2}(b_2-y)^2 & \text{if } m_2<y<b_2 \end{cases} \quad [5]$$

where the normalized constant $h$ remains the same as seen above.

The mean vector is expressed as follows:

$$\vec{\mu}=\frac{1}{8}\begin{pmatrix} 3a_1+2m_1+3b_1 \\ 3a_2+2m_2+3b_2 \end{pmatrix} \quad [6]$$

Note that $E(x)$ and $E(y)$ are weighted means with a weighting factor of 3 for minimum and maximum values, and a weighting factor of 2 for the most probable value, unlike in the PERT method (MacCrimmon and Ryavec, 1964), in which a value of 1 was taken as a weighting factor for the minimum and maximum values, and a value of 4 for the most probable value, or the mean of the triangular distribution, which gives a unitary weight to each of the three values.

The variance-covariance matrix takes the following expression:

$$\Sigma=\frac{1}{320}\begin{pmatrix} 19(b_1-a_1)^2-12(b_1-m_1)(m_1-a_1) & 3\left(2m_1-a_1-b_1\right)\left(2m_2-a_2-b_2\right) \\ 3\left(2m_1-a_1-b_1\right)\left(2m_2-a_2-b_2\right) & 19(b_2-a_2)^2-12(b_2-m_2)(m_2-a_2) \end{pmatrix} \quad [7]$$

The linear correlation coefficient is expressed as follows:

$$\rho=\frac{3\left(2m_1-a_1-b_1\right)\left(2m_2-a_2-b_2\right)}{\sqrt{(19(b_1-a_1)^2-12(b_1-m_1)(m_1-a_1))\,(19(b_2-a_2)^2-12(b_2-m_2)(m_2-a_2))}} \quad [8]$$

Moreover, $\rho=0$ if, and only if $2m_1=a_1+b_1$ or $2m_2=a_2+b_2$, at least two of the faces of the pyramid are isosceles triangles.

## Cumulative distribution function

In calculating the cumulative distribution function (cdf), various cases must be distinguished:

1. If $(x_0, y_0) \in T_i$, see Figure 3a, then:

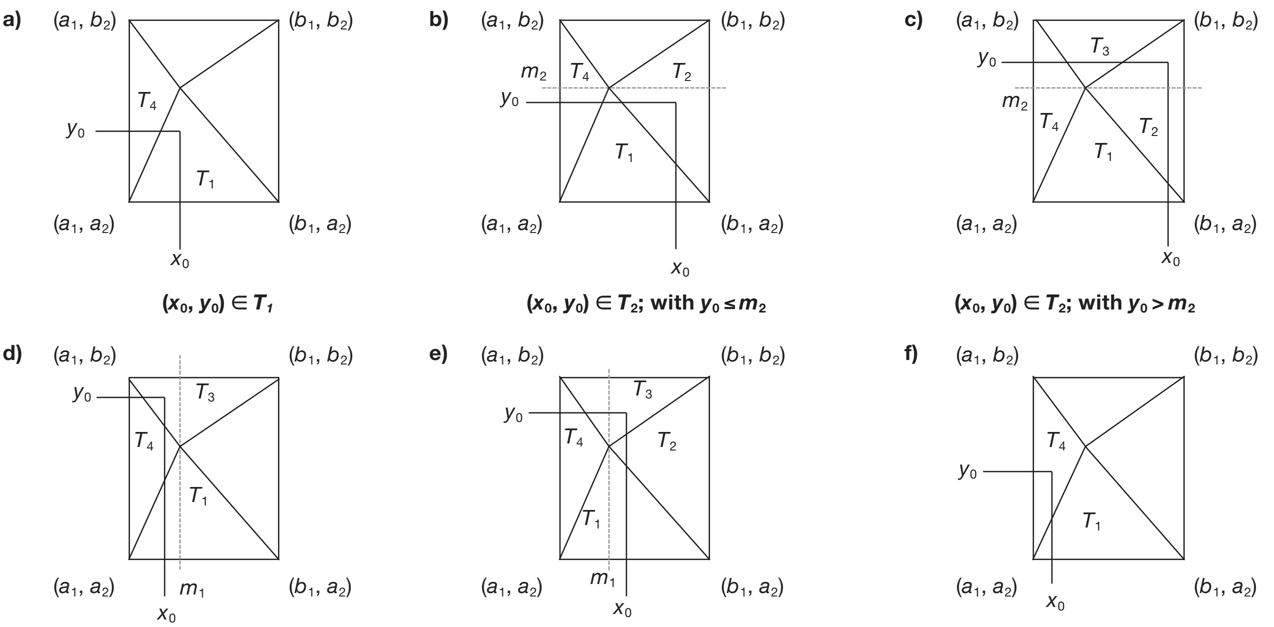


**Figure 3.** Cases to obtain the cumulative distribution function.

$$F(x_0,y_0)=\frac{h}{2}\frac{(\mathrm{x}_0-\mathrm{a}_1)(y_0-a_2)^2}{m_2-a_2}-\frac{h}{6}\frac{m_1-a_1}{(m_2-a_2)^2}(y_0-a_2)^3 \quad [9]$$

2. If $(x_0, y_0) \in T_2$, then the following two cases must be distinguished:

a) If $(x_0, y_0) \in T_2$; with $y_0 \le m_2$, see Figure 3b, then:

$$\begin{aligned}F(x_0,y_0)=&\frac{h}{6}\frac{b_1-a_1}{(m_2-a_2)^2}(a_2-y_0)^3-\frac{h}{6}\frac{a_2-m_2}{(b_1-m_1)^2}(b_1-x_0)^3+\\&+\frac{h}{2}\frac{b_1-a_1}{m_2-a_2}(y_0-a_2)^2-\frac{h}{2}\frac{(y_0-a_2)(b_1-x_0)^2}{b_1-m_1}\end{aligned} \quad [10]$$

b) If $(x_0, y_0) \in T_2$; with $y_0 > m_2$, see Figure 3c, then:

$$F(x_0,y_0)=1-\left[\begin{aligned}&\frac{h}{6}\frac{a_1-b_1}{(b_2-m_2)^2}(b_2-y_0)^3+\frac{h}{2}\frac{b_1-a_1}{b_2-m_2}(b_2-y_0)^2+\\&+\frac{h}{6}\frac{a_2-m_2}{(b_1-m_1)^2}(b_1-x_0)^3+\frac{h}{2}\frac{(y_0-a_2)(b_1-x_0)^2}{b_1-m_1}\end{aligned}\right] \quad [11]$$

3. If $(x_0, y_0) \in T_3$, then the following two cases must be distinguished:

a) If $(x_0, y_0) \in T_3$; with $x_0$ $m_1$, see Figure 3d, then:

$$\begin{aligned}F(x_0,y_0)=&\frac{h}{6}\frac{a_2-b_2}{(m_1-a_1)^2}(x_0-a_1)^3+\frac{h}{2}\frac{b_2-a_2}{m_1-a_1}(x_0-a_1)^2+\\&+\frac{h}{6}\frac{m_1-a_1}{(b_2-m_2)^2}(b_2-y_0)^3-\frac{h}{2}\frac{(x_0-a_1)(b_2-y_0)^2}{b_2-m_2}\end{aligned} \quad [12]$$

b) If $(x_0, y_0) \in T_3$; with $x_0 > m_1$, see Figure 3e, then:

$$F(x_0,y_0)=1-\left[\begin{aligned}&\frac{h}{6}\frac{a_1-m_1}{(b_2-m_2)^2}(b_2-y_0)^3-\frac{h}{2}\frac{(x_0-a_1)(b_2-y_0)^2}{m_2-b_2}+\\&+\frac{h}{6}\frac{a_2-b_2}{(b_1-m_1)^2}(b_1-x_0)^3+\frac{h}{2}\frac{b_2-a_2}{b_1-m_1}(b_1-x_0)^2\end{aligned}\right] \quad [13]$$

4. If $(x_0, y_0) \in T_4$, see Figure 3f, in which case:

$$F(x_0,y_0)=\frac{h}{2}\frac{(\mathrm{y}_0-\mathrm{a}_2)(x_0-a_1)^2}{m_1-a_1}-\frac{h}{6}\frac{m_2-a_2}{(m_1-a_1)^2}(x_0-a_1)^3 \quad [14]$$

## The standard pyramidal distribution

As usual, the finite range of a random variable can be transformed into a range over (0, 1). This standardised range is obtained by subtracting the smallest value from the original value of the variable and dividing the result by the range of the original variable, that is, by the difference between its largest and smallest values. Thus, if the range of variable $X$ is $(a,b)$, then the values of the standardised variable are obviously in (0,1). In the case of a pyramidal distribution, the ranges of the variables $X$ and $Y$, $R_X(a_1, b_1)$ and $R_Y(a_2, b_2)$ can be transformed into standardised ranges $R_{X*}(0,1)$ and $R_{Y*}(0,1)$, see Figure 2b, and so the formulas obtained above for the case of a general pyramidal distribution can be converted into much simpler and more straightforward ones for empirical application.

As an example, we now describe the general formulas obtained after performing the standardisation of the ranges of the variables.

By application of: $x^*=\dfrac{x-a_1}{b_1-a_1}$ and $y^*=\dfrac{y-a_2}{b_2-a_2}$

It is obtained: $a_1^*=0=a_2^*$, $b_1^*=1=b_2^*$, $m_1^*=\dfrac{m_1-a_1}{b_1-a_1}$ and $m_2^*=\dfrac{m_2-a_2}{b_2-a_2}$

The expression of the normalising constant $h$ is reduced to: $h^*=3$

And the pdf becomes:

$$f(x^*,y^*)=\begin{cases}\dfrac{3}{m_2^*}\mathrm{y}^* & \text{if } (x^*,y^*)\in \mathrm{T}_1\\ \dfrac{3}{1-m_1^*}(1-x^*) & \text{if } (x^*,y^*)\in \mathrm{T}_2\\ \dfrac{3}{1-m_2^*}(1-y^*) & \text{if } (x^*,y^*)\in \mathrm{T}_3\\ \dfrac{3}{m_1^*}\mathrm{x}^* & \text{if } (x^*,y^*)\in \mathrm{T}_4\\ 0 & \text{otherwise}\end{cases} \quad [15]$$

The mean vector is expressed as follows:

$$\vec{\mu}^*=\frac{1}{8}\begin{pmatrix}2m_1^*+3\\2m_2^*+3\end{pmatrix} \quad [16]$$

The variance-covariance matrix takes the following expression:

$$\Sigma^*=\frac{1}{320}\begin{pmatrix}12m_1^{*2}-12m_1^*+19 & 3\,(1-2m_1^*)(1-2m_2^*)\\ 3\,(1-2m_1^*)(1-2m_2^*) & 12m_2^{*2}-12m_2^*+19\end{pmatrix} \quad [17]$$

The linear correlation coefficient is expressed as follows:

$$\rho=\frac{3(1-2m_1^*)\,(1-2m_2^*)}{\sqrt{(12m_1^{*2}-12m_1^*+19)\;(12m_2^{*2}-12m_2^*+19)}} \quad [18]$$

Note that the highest value of the coefficient of the linear correlation is 0.1578947, which indicates that the pyramidal distribution would be a suitable probabilistic model when there is a moderately low linear correlation between the variables *X* and *Y*.

The cdf takes the following expression:

$$F(x^*,y^*)=\begin{cases} 0 & \text{if } x^*\le 0 \text{ and } y^*\le 0 \\[2mm] \frac{3}{2}\frac{x^* y^{*2}}{m_2^*}-\frac{1}{2}\frac{m_1^*}{m_2^{*2}}y^{*3} & \text{if } (x^*,y^*)\in T_1 \\[2mm] \frac{1}{2}\frac{m_2^*(1-x^*)^3}{(1-m_1^*)^2}-\frac{1}{2}\frac{y^{*3}}{m_2^{*2}}+\frac{3}{2}\frac{y^{*2}}{m_2^*}-\frac{3}{2}\frac{y^*(1-x^*)^2}{1-m_1^*} & \begin{array}{l}\text{if } (x^*,y^*)\in T_2\\ \text{and } y^*\le m_2^*\end{array} \\[2mm] \begin{array}{l}1+\frac{1}{2}\frac{(1-y^*)^3}{(1-m_2^*)^2}+\frac{1}{2}\frac{m_2^*(1-x^*)^3}{(1-m_1^*)^2}-\\ -\frac{3}{2}\frac{(1-y^*)^2}{1-m_2^*}-\frac{3}{2}\frac{y^*(1-x^*)^2}{1-m_1^*}\end{array} & \begin{array}{l}\text{if } (x^*,y^*)\in T_2\\ \text{and } y^*> m_2^*\end{array} \\[2mm] \frac{1}{2}\frac{m_1^*(1-y^*)^3}{(1-m_2^*)^2}-\frac{1}{2}\frac{x^{*3}}{m_1^{*2}}+\frac{3}{2}\frac{x^{*2}}{m_1^*}-\frac{3}{2}\frac{x^*(1-y^*)^2}{1-m_2^*} & \begin{array}{l}\text{if } (x^*,y^*)\in T_3\\ \text{and } x^*\le m_1^*\end{array} \\[2mm] \begin{array}{l}1+\frac{1}{2}\frac{(1-x^*)^3}{(1-m_1^*)^2}+\frac{1}{2}\frac{m_1^*(1-y^*)^3}{(1-m_2^*)^2}-\\ -\frac{3}{2}\frac{(1-x^*)^2}{1-m_1^*}-\frac{3}{2}\frac{x_0^*(1-y^*)^2}{1-m_2^*}\end{array} & \begin{array}{l}\text{if } (x^*,y^*)\in T_3\\ \text{and } x^*> m_1^*\end{array} \\[2mm] \frac{3}{2}\frac{x^{*2}y^*}{m_1^*}-\frac{1}{2}\frac{m_2^*}{m_1^{*2}}x^{*3} & \text{if } (x^*,y^*)\in T_4 \\[2mm] 1 & \text{if } x^*\ge 1 \text{ and } y^*\ge 1 \end{cases} \quad [19]$$

# Results

## Practical application

Note that many of the quality characteristics of a land plot (urban or rural) and other assets can be reduced to two major components: the status or location of the plot or business assets and the intrinsic quality of the plot or business. This intrinsic quality can sometimes be gauged by the profitability of the property, the production at the plot, the rent derived from the urban estates, the dividends paid on the shares, etc.

These two major components —production and location— usually bear little or no relationship, and so it is necessary to make use of two-dimensional distributions in which the variables do not present a high correlation.

Therefore, when considering the second practical case described by Guadalajara (1996), with respect to valuation of an agricultural plot, used for growing grapes, in Vinalopó Medio (province of Alicante, Spain). The vector of the characteristics considered to describe the market value (€ $ha^{-1}$) are the gross production of grapes (kg $ha^{-1}$), together with the percentage of sand in the soil of the plot.

Table 1 displays data of the minimum, maximum and most likely values for each variable; our aim is to evaluate a plot of agricultural land with an area of 1.2 ha, a gross production of 20,413 kg $ha^{-1}$ and a sand/soil content of 32%.

Until now, the VMTCDF has been applied considering a one-dimensional distribution; for example, triangular or trapezoidal models have been considered for both the characteristic of the asset and the market value.

This paper provides a two-dimensional extension of the VMTCDF. It seems logical to consider that the gross production of grapes will be related to the sand content of the soil, although this correlation may not be very strong.

With respect to the strength of the correlation between the two explicative variables, we know that the variables «production» and «sand index» are not stochastically independent; they are correlated, but this correlation is inverse and weak; in this respect, see the recent study by Martínez-Casasnovas *et al.* (2009), where for 35 test plots at the Costers del Segre (Lleida) vineyard they obtained a determination coefficient of $R^2 = 0.12$ and a correlation coefficient of $r = -0.34$, between the grape yield and the sand content.

Thus, the pyramidal model is considered as a probability model of a two-dimensional characteristics vector ($I_1$ , $I_2$) and the market value is believed to fit a triangular model.

It is necessary to obtain the cdf of ($I_1$ , $I_2$) = (20,413, 32) and so we must determine which region it belongs

**Table 1.** Agricultural plots used for growing grapes

| | $V$ = Market value (€ $ha^{-1}$) | $I_1$ = Gross production (kg $ha^{-1}$) | $I_2$ = Sand/soil content (%) |
|---|---|---|---|
| Minimum | 8,138.70 | 15,625 | 15 |
| Maximum | 15,025.30 | 26,042 | 50 |
| Most likely | 10,642.92 | 18,750 | 25 |
| Agricultural plot (1.2 ha) | ? | 20,413 | 32 |

*Source:* Own elaboration, from Guadalajara (1996).

to; it is easily established that (20,413, 32) belongs to the region $T_3$.

Note that $x_0 = 20{,}413$ is greater than $m_1 = 18{,}750$ then, from [13], we calculate the cdf of (20,413, 32) at region $T_3$ and obtain that $G$ (20,413, 32) = 0.319999

According to the VMTCDF, it is necessary to invert the cdf of the variable market value, which is assumed to fit a triangular model.

It is known that the cdf of a triangular model is:

$$F(x) = \begin{cases} 0 & \text{if } x \le a \\ \dfrac{(x-a)^2}{(b-a)(m-a)} & \text{if } a < x \le m \\ 1 - \dfrac{(b-x)^2}{(b-a)(b-m)} & \text{if } m \le x < b \\ 1 & \text{if } x \ge b \end{cases} \quad [20]$$

Replacing the value of the mode, $m = 10{,}642.92$ in the cdf of the market value produces a value of 0,363636443 (greater than the value returned by the pyramidal distribution: 0.319999).

Then, from the second branch of [20], we obtain:

$$\frac{(x - 8{,}138.70)^2}{(15{,}025.30 - 8{,}138.70)(10{,}642.92 - 8{,}138.70)} = \quad [21]$$

$$= 0.319999 \Rightarrow x = 10{,}487.8645$$

Multiplying this by the 1.2 ha of the land plot in question, we obtain an assessment of € 12,585.44 for this agricultural plot, which is practically the same as that achieved by Guadalajara (1996), € 12,583.02, as a result of the average of fourteen assessments obtained by four valuation methods – the Synthetic method (4 variants), the Beta method (4 variants), the Regression method (4 variants) and the analytical method (2 variants).

## Discussion

Among the advantages and disadvantages of the VMTCDF, with respect to other comparative techniques such as classical synthetic and econometric methods, the following are noteworthy:

a) Econometric and two distribution function methods avoid the hypothesis of proportionality employed by classical synthetic methods, a hypothesis that is extremely simplifying and quite implausible.

b) The VMTCDF requires very little information with respect to what is needed to apply econometric methods, which need databases that the valuer, in any given case, may not possess.

c) One of the advantages presented, to date, by econometric methods over other approaches is that the regression analysis combines various explicative variables in the model, while the two distribution function models, until now, have only used a single explicative variable, which summarizes in a weighted index the different external signs that influence the market value

**Table 2.** Valuation methods (€) used in Guadalajara (1996)

| Explanatory variable | Synthetic | | Two distribution function | |
|---|---|---|---|---|
| | **Criteria ends** | **Criterion source** | **Beta distribution** | **Triangular distribution** |
| Gross production | 13,567.84 | 13,620.43 | 13,319.93 | 13,775.52 |
| Sand/soil content | 13,780.21 | 13,684.80 | 13,319.93 | 14,018.86 |
| Average | 13,663.32 | | 13,608.56 | |
| | **Regression** | | **Analytical** | |
| Modality | Model 1 11,515.09 | Model 2 13,086.74 | Canon as lessee | Calculated as income |
| | Model 3 9,854.19 | Model 4 11,013.55 | 11,268.98 | 12,116.60 |
| Average | 11,367.39 | | 11,692.79 | |
| Average of four mean values | 12,583.02 | | | |

*Source:* Own elaboration, form Guadalajara (1996).

of the asset to be assessed (Ballestero and Rodríguez, 1999). For this reason, the present study represents an improvement, as it enables us to account for the market value, through the use of various explicative variables, using for this purpose the multivariate distribution functions of these explicative variables.

d) A disadvantage of the VMTCDF with respect to econometric models is that the latter can be used to obtain statistics on the reliability of the valuations produced, by means of prediction and structural permanence analyses of the regression model employed.

e) An advantage of the VMTCDF in its pyramidal variant is the economy and ease of calculation when using a single method, as shown in this paper, instead of applying the four different methods used by Guadalajara (1996) to reach a final assessment.

Table 2 shows that the highest market values are obtained with the VMTCDF, in its triangular variant. There is a sharp discrepancy between the values obtained by the synthetic method and VMTCDF, with respect to the values determined by the regression and analytical methods in their different modalities.

Ordering, from highest to lowest, the average values of the appraisals performed by the four different valuation methods used by Guadalajara (1996), and including the value obtained in this paper, it is found that:

Synthetic > VMTCDF > VMTCDF Pyramidal >
> Analytical> Regression.

## Acknowledgements

We are indebted to both referees and the editor whose comments improved the presentation and the content of an earlier version. This research has been partially supported by the Government of Andalusia, research group FQM-150.

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