

## NEW PARAMETRIZATIONS FOR THE PHOTON STRUCTURE FUNCTION\*

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In the last year four new parametrizations of the Hadronic Photon Structure Function at Next to Leading Order have appeared. In this talk, I briefly review the main features of the three of them: the FFNS<sub>CJK</sub>, CJK and AFG.

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### 1. Introduction

The photon structure function has been recognized as an interesting quantity for QCD since long ago [1, 2] because it was expected that the asymptotic point-like  $Q^2$  evolution could be calculated without additional assumptions. Unfortunately, further studies showed the need for a hadronic component that required extra assumptions at an input scale. However, a good knowledge of the parton content of the photon is still needed and useful for many phenomenological applications. A review of the situation in the early days can be found in these proceedings [3].

The main problem found in the study the Photon Structure Function some years ago was the lack of experimental data [4, 5]. Indeed, there were very few data and they covered a very limited region in the plane  $(x, Q^2)$ . The situation has improved very much in the last years with the measurements performed by the four LEP experiments. These measurements reduced the experimental errors in regions of  $(x, Q^2)$  that were already studied at previous experiments and also covered regions in this plane where

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there were no previous measurements. These has prompted the appearance in the last year of four new parametrizations for the Photon Structure Function at LO and NLO. They are, in chronological order: FFNS<sub>CJK</sub>, CJK [6], AFG05 [7] and SAL [8]. In this talk I will just cover the first three parametrizations for the fourth one will be covered in the next talk [9].

One important difference, certainly not the only one, among the new parametrizations is the way they deal with the heavy quark thresholds. There are three schemes to introduce these thresholds.

- The Fixed Flavor Number Scheme (FFNS), where one considers only the three light quarks and gluons as partons of the photon for all energy scales. The heavy quarks,  $c$  and  $b$ , contribute only as external particles in the final state produced in hard processes either in a direct production or through the partonic content of the photon. In the calculation of the heavy quark contributions one keeps their mass fixed to their physical value. This scheme is expected to give a poor description of the Photon Structure Function for energy scales much larger than the heavy quark masses, where one would expect the contributions of the heavy quarks to be similar to the ones of the light quarks.
- The Zero-mass Variable Flavor Number Scheme (ZVFNS). In this scheme the number of active flavors as partons of the photon increases in one whenever the energy goes through a heavy quark threshold. From then on, the heavy quark is treated as massless in the evolution of the parton densities, just in the same way as the light quarks are treated. This scheme is expected to solve the problem of the FFNS scheme at large energies but, obviously, should have problems at energies near the thresholds, where one cannot neglect the heavy quark masses.
- The Variable Flavor Number Scheme (VFNS) attempts to solve the problems of the previous schemes. Here, one considers both contributions: the heavy quarks are produced in the final state taking into account their masses but also they are included as massless partons of the photon. In this way both energy regions are treated properly. Unfortunately, this scheme is not free of problems either. It is clear that there is a double counting that should be avoided introducing some subtraction terms.

## 2. The FFNS<sub>CJK</sub> NLO and CJK NLO parametrizations

It is clear from the name that the FFNS<sub>CJK</sub> uses the Fixed Flavor Number Scheme, while the CJK parametrization is using the VFNS. Detailed expressions for the Photon Structure Function,  $F_2^\gamma(x, Q^2)$ , for both

parametrizations, involving a description of the way the subtraction terms are chosen in the CJK parametrization can be found in Ref. [6]. In addition, the CJK parametrization uses the ACOT( $\chi$ ) scheme. The idea is to enforce that the heavy quark distribution functions vanish for  $W = 2m_h$  (and below), where  $W$  is the invariant mass of the hadronic final state. This is achieved substituting the  $x$  variable by  $\chi_h = x(1 + 4m_h^2/Q^2)$  in the heavy quark densities. In this way  $\chi \rightarrow 1$  and  $q_h(\chi, Q^2) \rightarrow 0$  for  $W^2 = (1 - x)Q^2/x \rightarrow 4m_h^2$ .

Both parametrizations are written as a function of the quark and gluon distribution functions that obey an inhomogeneous DGLAP set of equations. In order to solve these equations we introduce the same input for both parametrizations at  $Q_0^2 = 0.765 \text{ GeV}^2$ , based on Vector Meson Dominance (VMD):

$$f^\gamma(x, Q_0^2) = \sum_V \frac{4\pi\alpha}{\hat{f}_V^2} f^V(x, Q_0^2), \tag{1}$$

with the sum running over all light vector mesons ( $V$ ) into which the photon can fluctuate. The parameters  $\hat{f}_V^2$  can be extracted from the experimental data on  $\Gamma(V \rightarrow e^+e^-)$  width. In practice we take into account the  $\rho^0$  meson while the contributions from the other mesons are accounted for via a parameter  $\kappa$

$$f^\gamma(x, Q_0^2) = \kappa \frac{4\pi\alpha}{\hat{f}_\rho^2} f^\rho(x, Q_0^2), \tag{2}$$

which is left as a free parameter in the fits. The scale  $Q_0$  has been fixed to this value because it is the one that allows a better fit to the experimental data.

For the  $\rho$  meson we assume the following form for the valence quark and gluon distributions

$$\begin{aligned} xv^\rho(x, Q_0^2) &= N_v x^\alpha (1 - x)^\beta, \\ xG^\rho(x, Q_0^2) &= \tilde{N}_G xv^\rho(x, Q_0^2) = N_G x^\alpha (1 - x)^\beta, \end{aligned} \tag{3}$$

where  $N_v$ ,  $N_G$ ,  $\alpha$  and  $\beta$  are free parameters. The sea quark distribution is assumed to vanish at this scale. This is similar to what was done in the GRV parametrization [11], but there the authors fixed the values of the parameters  $\alpha$  and  $\beta$  to the ones they had previously obtained for the pion distribution functions. Since there are more data available now, we prefer to leave these parameters as free parameters in the fit.

We have included in the fit all the available data in year 2004 except the DELPHI LEP2 data because they present three sets of mutually inconsistent

data. The total number of points used in the fit is 192 covering a kinematical range of  $0.001 \leq x \leq 0.65$  and  $1.3 < \text{GeV}^2 \leq Q^2 \leq 780 \text{ GeV}^2$ . The results of the fit are shown in Table I. Introducing the DELPHI LEP2 data in the fit the  $\chi^2/\text{DOF}$  increases to 1.50 (TWOAM), 1.54 (PHOJET) or 1.66 (PYTHIA), depending on the Monte Carlo used to analyze the data.

TABLE I

The  $\chi^2$  and parameters of the final fits for 192 data points for FFNS<sub>CJK</sub> NLO and CJK NLO models with assumed  $Q_0^2 = 0.765 \text{ GeV}^2$ . The  $\alpha$ ,  $\beta$  and  $\kappa$  errors are obtained from MINOS requiring  $\Delta\chi^2 = 1$ .

NLO models	$\chi^2$	$\chi^2/\text{DOF}$	$\kappa$	$\alpha$	$\beta$
FFNS <sub>CJK</sub>	243.3	1.29	$2.288^{+0.108}_{-0.096}$	$0.502^{+0.071}_{-0.066}$	$0.690^{+0.282}_{-0.252}$
CJK	256.8	1.37	$2.662^{+0.108}_{-0.099}$	$0.496^{+0.063}_{-0.057}$	$1.013^{+0.284}_{-0.255}$

A comparison of the CJK predictions with the recent L3 data for  $Q^2 = 12.4 \text{ GeV}^2$  and  $Q^2 = 16.7 \text{ GeV}^2$ , not included in the fit because they have been published after the fit was performed [12, 13] is shown in Fig. 1. Data for similar  $Q^2$  from CELLO [14], DELPHI [15], OPAL [16] and TOPAZ [17] are also included. We see that the CJK parametrization provides a good description of the data, even though not all the data sets are fully compatible with each other.

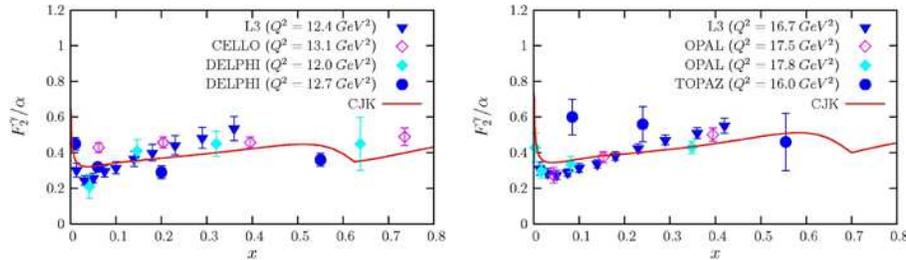


Fig. 1. Comparison of the CJK NLO prediction with various sets of data, including the new L3 data not included in the fit. The kink observed in the curve at large  $x$  is the charm quark threshold.

### 3. The AFG NLO parametrization

The third parametrization I will briefly review here has been performed by Aurenche, Fontannaz and Guillet [7]. It is an update of a previous parametrization obtained by the same authors [18]. This parametrization

uses the ZVFNS with  $N_f = 5$ , however they keep terms  $O(m_h^2/Q^2)$  in the direct contribution in order to have a smooth threshold behavior. At  $Q^2 = Q_0^2$  the structure function is given by:

$$\frac{F_2^\gamma(x, Q_0^2)}{x} = C_\gamma(x) + \sum_{f=1}^{N_f} \left[ e_f^2(q_f^{\text{NP}}(Q_0^2) + \bar{q}_f^{\text{NP}}(Q_0^2) - C_{\gamma,c}^f \right], \quad (4)$$

where  $C_\gamma(x)$  is the direct contribution and  $C_{\gamma,c}^f$  is given by the ‘‘hand-bag’’ diagram. The non-perturbative input is also based on VMD, identifying the form of the parton distributions for the  $\rho$  meson with the ones for the pion obtained in Ref. [19], but leaving a normalization factor,  $C_{np}$ , as a free constant:

$$\begin{aligned} x u_{\text{valence}}^\gamma &= C_{np} \alpha \frac{4}{9} x u_{\text{valence}}^\pi = C_{np} \alpha \frac{4}{9} \frac{1}{B(p_2, 1 + p_3)} x^{p_2} (1 - x)^{p_3}, \\ x u_{\text{sea}}^\gamma &= C_{np} \alpha \frac{2}{3} x u_{\text{sea}}^\pi = C_{np} \alpha \frac{2}{3} C_s (1 - x)^{p_8}, \\ x G^\gamma &= C_{np} \frac{2}{3} x g^\pi = C_{np} \frac{2}{3} C_g (1 - x)^{p_{10}}, \end{aligned} \quad (5)$$

where  $p_2 = 0.48$ ,  $p_3 = 0.85$ ,  $p_8 = 7.5$ ,  $p_{10} = 1.9$ ,  $C_s = 1.2$ ,  $C_g = 0.447(1 + p_{10})$  and  $B(x, y)$  is the beta function. In summary, there are only two free parameters: the input scale  $Q_0$  and the normalization factor  $C_{np}$ .

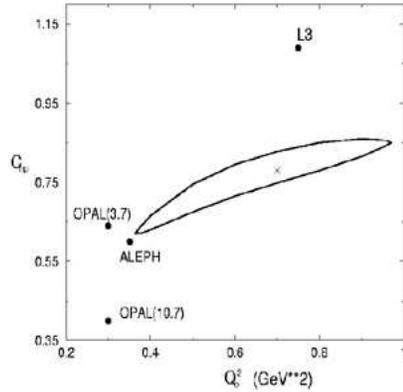


Fig. 2. Figure from Ref. [7] where it is presented the  $\Delta\chi^2 = 1$  contour in the  $(Q_0^2, C_{np})$  plane as well as the individual best fits for each LEP experiment. The point for DELPHI(12.7) is outside the figure, while for DEPHI (3.7) no minimum is found for  $Q_0^2 < 1.9 \text{ GeV}^2$ .

The values of the two free parameters are obtained performing a fit to all the LEP experimental data. The best fit, with a  $\chi^2/\text{DOF} = 1.03$ , gives

$Q_0 = 0.7 \text{ GeV}^2$  (very similar to the one used in the FFNS<sub>CJK</sub> and CJK parametrizations) and  $C_{np} = 0.78$ . Aurenche, Fontannaz and Guillet have also performed independent fits for each one of the four LEP experiments. The result is summarized in Fig. 2, where one can see that the best fits from the DELPHI experiment give very different values of the parameters compared with the ones obtained from the global fit as well as from each one of the other three experiments.

The authors have also explored two other parametrizations allowing for a harder gluon component modifying the value of  $p_{10}$  to  $p_{10} = 1.0$  or a softer gluon component with  $p_{10} = 4.0$ .

#### 4. Web pages

Instead of a summary I will finish just referring the interested reader to the web pages where he can find FORTRAN routines with these parametrizations:

- FFNS<sub>CJK</sub> and CJK: <http://www.fuw.edu.pl/~pjank/param.html>
- AFG05: [http://www.lapp.in2p3.fr/lapth/PHOX\\_FAMILY/main.html](http://www.lapp.in2p3.fr/lapth/PHOX_FAMILY/main.html)

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